Computer algebra independent integration tests

Summer 2022 edition

3-Logarithms/63-3.4-u-a+b-log-c-d+e-x^m-^n-^p

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [641]. This is test number [63].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (641)	0.00 (0)
Mathematica	96.88 (621)	3.12 (20)
Fricas	61.31 (393)	38.69 (248)
Maxima	61.00 (391)	39.00 (250)
Maple	54.91 (352)	45.09 (289)
Giac	54.76 (351)	45.24 (290)
Mupad	50.86 (326)	49.14 (315)
Sympy	31.36 (201)	68.64 (440)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

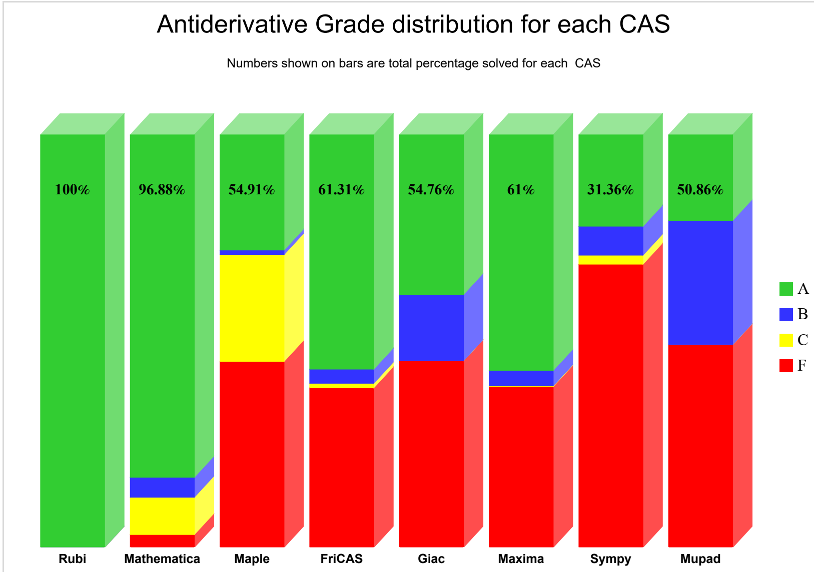
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

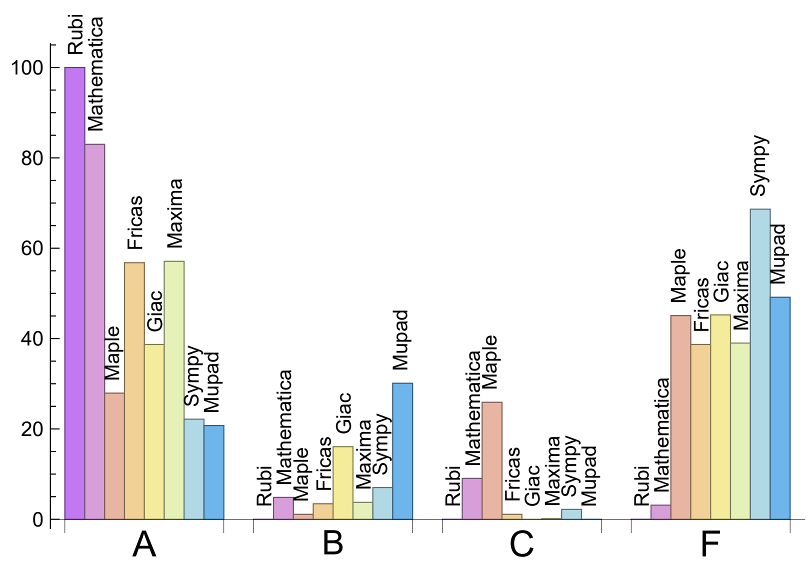
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	83.00	4.84	9.05	3.12
Maxima	57.10	3.74	0.16	39.00
Fricas	56.79	3.43	1.09	38.69
Giac	38.69	16.07	0.00	45.24
Maple	27.93	1.09	25.90	45.09
Sympy	22.15	7.02	2.18	68.64
Mupad	N/A	30.11	0.00	49.14

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	20	100.00 %	0.00 %	0.00 %
Maple	289	100.00 %	0.00 %	0.00 %
Fricas	248	100.00 %	0.00 %	0.00 %
Giac	290	99.31 %	0.34 %	0.34 %
Maxima	250	92.80 %	0.80 %	6.40 %
Sympy	440	42.05 %	42.50 %	15.45 %
Mupad	315	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

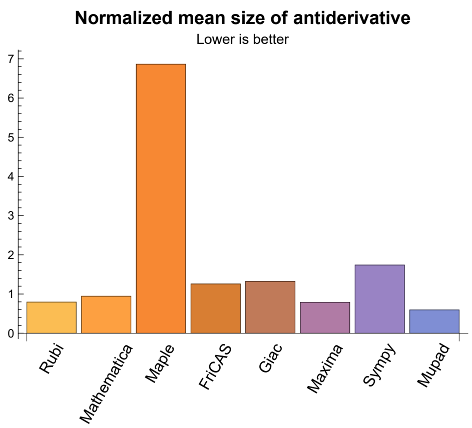
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

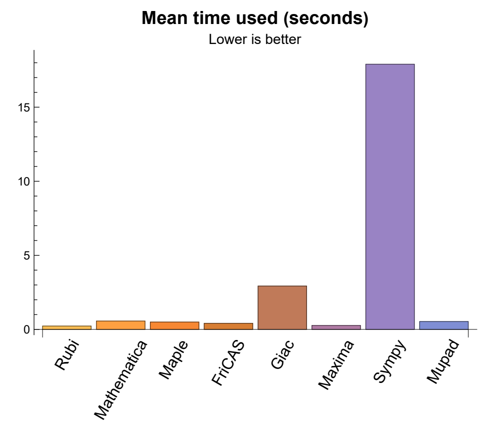
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.23	222.15	0.79	121.00	1.00
Mathematica	0.57	238.63	0.94	124.00	0.91
Maple	0.50	1194.09	6.86	120.00	1.13
Maxima	0.27	128.28	0.79	82.00	0.82
Fricas	0.41	282.16	1.26	80.00	0.97
Sympy	17.90	178.31	1.74	88.00	1.08
Giac	2.93	264.27	1.32	92.00	0.91
Mupad	0.53	121.17	0.59	48.00	0.79

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
1
2
3
4
5
6
7
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
5
10
15
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{98, 99, 100, 101, 104, 105, 106, 107, 108, 111, 112, 113, 114, 115, 118, 119, 120, 121, 122, 141, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 211, 216, 217, 218, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 381, 382, 383, 384, 385, 386, 387, 388, 398, 485, 486, 487, 488, 528, 529, 530, 531, 536, 537, 542, 543, 544, 545, 546, 550, 551, 552, 560, 561, 566, 567, 570, 571, 572, 573, 574, 577, 578, 579, 580, 581, 582, 583, 584, 588, 589, 590, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 625, 626, 627, 635, 640, 641\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {98, 99, 100, 101, 158, 159, 277, 298, 299, 485, 486, 487, 488, 528, 529, 530, 531}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{\}$

Mathematica $\{168, 292, 365, 366, 368, 369, 376\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

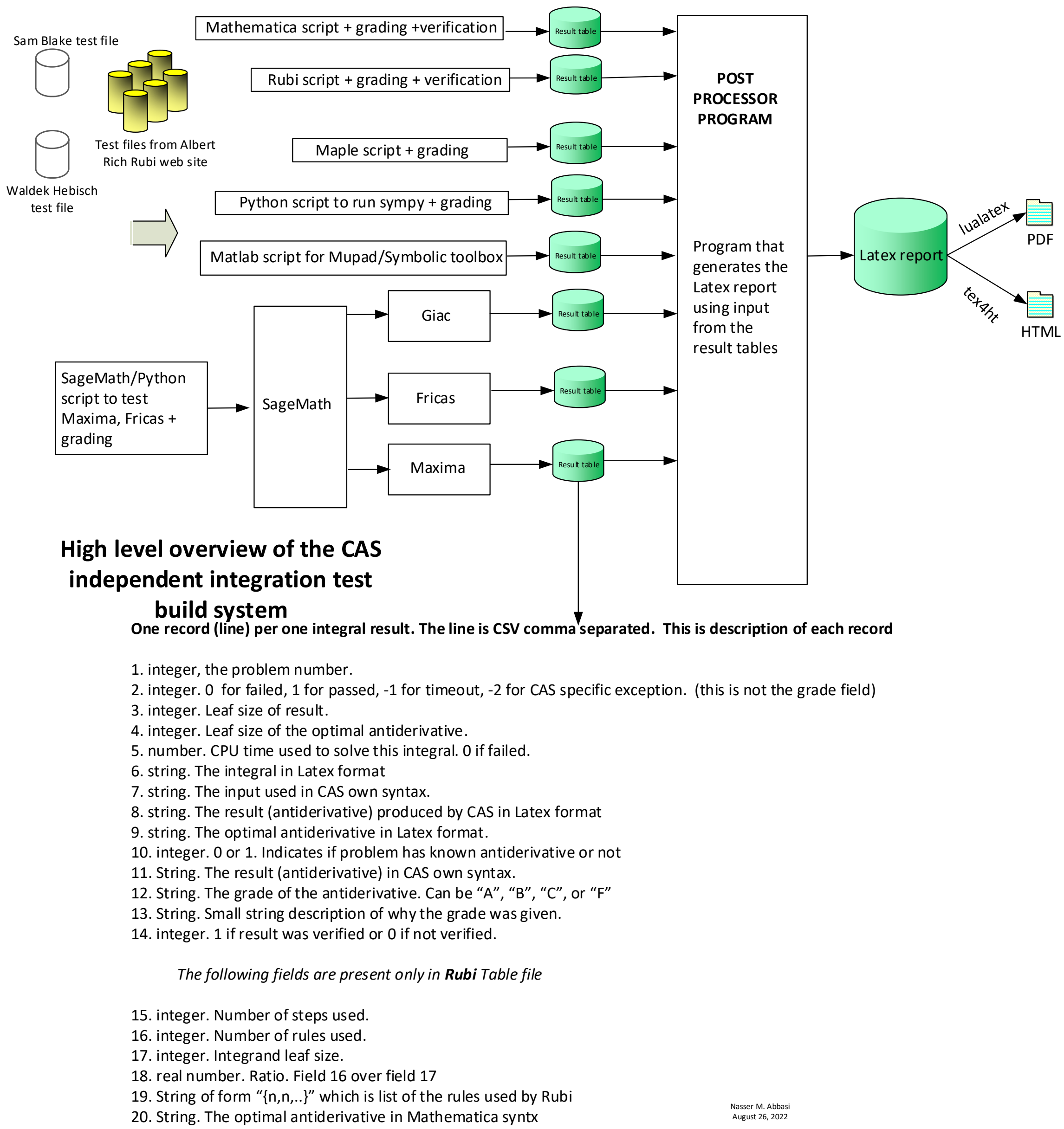
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 12, 13, 15, 16, 18, 19, 21, 22, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 37, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 91, 92, 93, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 132, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 194, 195, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 235, 236, 237, 240, 241, 242, 243, 244, 245, 246, 248, 249, 250, 251, 252, 253, 257, 258, 261, 262, 263, 264, 265, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 338, 339, 340, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 371, 372, 375, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 412, 413, 414, 415, 416, 417, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 436, 437, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 454, 455, 456, 457, 458, 459, 461, 462, 463, 464, 465, 466, 467, 469, 471, 472, 474, 475, 476, 477, 478, 481, 482, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 503, 504, 506, 507, 508, 510, 512, 513, 514, 515, 523, 524, 525, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 556, 557, 558, 559, 560, 561, 566, 567, 568, 569, 570, 571, 572, 573, 574, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 611, 612, 613, 616, 617, 618, 619, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 635, 638, 639, 640, 641 }

B grade: { 45, 80, 94, 95, 96, 131, 174, 175, 376, 390, 411, 418, 419, 432, 438, 453, 460, 473, 483, 500, 505, 516, 517, 518, 526, 620, 621, 622, 623, 636, 637 }

C grade: { 9, 11, 14, 17, 20, 23, 24, 36, 38, 89, 90, 133, 137, 191, 192, 193, 196, 197, 210, 233, 234, 238, 239, 247, 254, 255, 256, 259, 260, 266, 267, 321, 322, 335, 336, 337, 347, 359, 433, 434, 435, 468, 470, 479, 480, 501, 502, 509, 511, 519, 520, 521, 522, 609, 610, 614, 615, 634 }

F grade: { 370, 373, 374, 377, 538, 539, 540, 541, 553, 554, 555, 562, 563, 564, 565, 575, 576, 591, 592, 593 }

2.1.3 Maple

A grade: { 4, 5, 16, 18, 30, 32, 40, 43, 45, 49, 54, 98, 99, 100, 101, 104, 105, 106, 107, 108, 111, 112, 113, 114, 115, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 141, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 169, 172, 178, 179, 186, 187, 194, 205, 211, 216, 217, 218, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 358, 359, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 392, 393, 395, 396, 397, 398, 403, 424, 426, 445, 466, 485, 486, 487, 488, 492, 528, 529, 530, 531, 536, 537, 542, 543, 544, 545, 546, 550, 551, 552, 560, 561, 566, 567, 570, 571, 572, 573, 574, 577, 578, 579, 580, 581, 582, 583, 584, 588, 589, 590, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 625, 626, 627, 631, 632, 635, 639, 640, 641 }

B grade: { 170, 171, 391, 394, 511, 630, 633 }

C grade: { 1, 2, 3, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 19, 20, 21, 22, 23, 24, 25, 67, 73, 77, 78, 79, 81, 82, 83, 91, 92, 93, 102, 103, 109, 110, 116, 117, 129, 130, 132, 138, 139, 140, 148, 149, 150, 166, 173, 174, 175, 176, 177, 180, 181, 182, 183, 184, 185, 188, 189, 190, 191, 192, 193, 195, 196, 197, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 261, 262, 263, 268, 269, 270, 271, 288, 289, 290, 291, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 619, 620, 624, 629, 634 }

F grade: { 26, 27, 28, 29, 31, 33, 34, 35, 36, 37, 38, 39, 41, 42, 44, 46, 47, 48, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 74, 75, 76, 80, 84, 85, 86, 87, 88, 89, 90, 94, 95, 96, 97, 131, 133, 134, 135, 136, 137, 160, 163, 164, 165, 167, 168, 198, 199, 200, 201, 202, 203, 204, 206, 207, 208, 209, 210, 212, 213, 214, 215, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 264, 265, 266, 267, 272, 273, 274, 292, 293, 294, 295, 353, 354, 355, 356, 357, 399, 400, 401, 402, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 489, 490, 491, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 532, 533, 534, 535, 538, 539, 540, 541, 547, 548, 549, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 568, 569, 575, 576, 585, 586, 587, 591, 592, 593, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 621, 622, 623, 628, 636, 637, 638 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 46, 47, 48, 49, 51, 52, 53, 54, 64, 65, 66, 68, 69, 77, 78, 79, 80, 81, 82, 83, 91, 92, 93, 95, 96, 97, 98, 99, 101, 104, 105, 106, 107, 108, 111, 112, 113, 114, 115, 118, 119, 120, 121, 122, 129, 130, 132, 141, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 211, 216, 217, 218, 223, 224, 225, 243, 244, 245, 246, 264, 268, 269, 270, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 296, 298, 299, 300, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 341, 342, 348, 349, 350, 351, 352, 358, 371, 372, 375, 376, 393, 396, 398, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 415, 416, 417, 421, 422, 423, 424, 426, 427, 428, 433, 434, 435, 440, 441, 442, 443, 444, 445, 447, 448, 449, 450, 451, 452, 456, 457, 458, 459, 463, 464, 465, 466, 468, 469, 470, 471, 472, 473, 481, 482, 484, 485, 486, 487, 488, 489, 490, 491, 492, 494, 495, 496, 501, 502, 506, 507, 508, 509, 510, 511, 513, 514, 515, 519, 520, 527, 528, 529, 530, 531, 536, 537, 542, 543, 544, 545, 546, 550, 551, 552, 560, 561, 566, 567, 570, 571, 572, 573, 574, 577, 578, 579, 580, 581, 582, 583, 584, 588, 589, 590, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 625, 626, 627, 630, 631, 632, 635, 639, 640, 641 }

B grade: { 6, 19, 31, 41, 45, 50, 94, 180, 222, 340, 378, 379, 390, 391, 394, 404, 425, 439, 446, 467, 483, 493, 512, 633 }

C grade: { 263 }

F grade: { 55, 56, 57, 58, 59, 60, 61, 62, 63, 67, 70, 71, 72, 73, 74, 75, 76, 84, 85, 86, 87, 88, 89, 90, 100, 102, 103, 109, 110, 116, 117, 123, 124, 125, 126, 127, 128, 131, 133, 134, 135, 136, 137, 138, 139, 140, 148, 149, 150, 160, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 188, 195, 206, 207, 208, 209, 210, 212, 213, 214, 215, 219, 220, 221, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 265, 266, 267, 271, 272, 273, 274, 291, 292, 293, 294, 295, 297, 301, 343, 344, 345, 346, 347, 353, 354, 355, 356, 357, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 373, 374, 377, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 392, 395, 397, 399, 411, 412, 413, 414, 418, 419, 420, 429, 430, 431, 432, 436, 437, 438, 453, 454, 455, 460, 461, 462, 474, 475, 476, 477, 478, 479, 480, 497, 498, 499, 500, 503, 504, 505, 516, 517, 518, 521, 522, 523, 524, 525, 526, 532, 533, 534, 535, 538, 539, 540, 541, 547, 548, 549, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 568, 569, 575, 576, 585, 586, 587, 591, 592, 593, 616, 617, 618, 619, 620, 621, 622, 623, 624, 628, 629, 634, 636, 637, 638 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 64, 65, 66, 67, 68, 69, 73, 77, 78, 79, 91, 92, 93, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 167, 168, 169, 172, 173, 177, 178, 179, 181, 184, 185, 186, 187, 189, 194, 198, 199, 200, 202, 211, 216, 217, 218, 268, 269, 270, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 329, 330, 331, 332, 333, 334, 335, 336, 337, 350, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 395, 398, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 415, 416, 417, 421, 422, 423, 424, 426, 427, 428, 433, 434, 435, 440, 441, 442, 443, 444, 445, 447, 448, 449, 450, 451, 452, 456, 457, 458, 459, 463, 464, 465, 466, 468, 469, 470, 471, 472, 481, 482, 485, 486, 487, 488, 489, 490, 491, 492, 494, 495, 496, 501, 502, 506, 507, 508, 509, 510, 511, 513, 514, 515, 519, 520, 527, 528, 529, 530, 531, 536, 537, 542, 543, 544, 545, 546, 550, 551, 552, 560, 561, 566, 567, 570, 571, 572, 573, 574, 577, 578, 579, 580, 581, 582, 583, 584, 588, 589, 590, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 619, 620, 623, 624, 625, 626, 627, 630, 631, 632, 635, 639, 640, 641 }

B grade: { 170, 171, 176, 182, 183, 190, 203, 204, 439, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 621, 622, 633 }

C grade: { 191, 192, 193, 196, 197, 629, 634 }

F grade: { 6, 19, 31, 41, 50, 55, 56, 57, 58, 59, 60, 61, 62, 63, 70, 71, 72, 74, 75, 76, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 94, 95, 96, 97, 131, 132, 133, 134, 135, 136, 137, 160, 166, 174, 175, 180, 188, 195, 201, 205, 206, 207, 208, 209, 210, 212, 213, 214, 215, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 271, 272, 273, 274, 291, 292, 293, 294, 295, 313, 314, 326, 327, 328, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 370, 371, 372, 373, 374, 375, 376, 377, 393, 394, 396, 397, 399, 404, 411, 412, 413, 414, 418, 419, 420, 425, 429, 430, 431, 432, 436, 437, 438, 446, 453, 454, 455, 460, 461, 462, 467, 473, 474, 475, 476, 477, 478, 479, 480, 483, 484, 493, 497, 498, 499, 500, 503, 504, 505, 512, 516, 517, 518, 521, 522, 523, 524, 525, 526, 532, 533, 534, 535, 538, 539, 540, 541, 547, 548, 549, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 568, 569, 575, 576, 585, 586, 587, 591, 592, 593, 616, 617, 618, 628, 636, 637, 638 }

2.1.6 Sympy

A grade: { 1, 2, 4, 8, 10, 12, 13, 16, 17, 18, 20, 22, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 37, 39, 43, 46, 47, 48, 49, 56, 58, 59, 60, 77, 78, 79, 91, 92, 93, 98, 99, 100, 101, 104, 105, 106, 107, 108, 111, 112, 113, 114, 115, 118, 119, 120, 121, 122, 124, 126, 128, 129, 130, 141, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 178, 179, 191, 192, 193, 194, 200, 216, 217, 218, 268, 275, 277, 278, 280, 281, 282, 283, 284, 285, 286, 289, 290, 298, 299, 302, 303, 304, 306, 307, 308, 311, 312, 332, 386, 398, 400, 401, 402, 403, 421, 422, 423, 424, 442, 443, 444, 445, 465, 466, 486, 490, 491, 492, 511, 625, 635, 639, 640, 641 }

B grade: { 3, 5, 7, 9, 11, 36, 38, 40, 42, 44, 51, 52, 54, 176, 177, 183, 184, 185, 186, 187, 198, 199, 202, 203, 269, 270, 315, 318, 319, 320, 321, 322, 325, 333, 334, 335, 336, 405, 406, 426, 447, 494, 630, 631, 632 }

C grade: { 45, 70, 71, 72, 74, 75, 76, 169, 170, 171, 212, 213, 214, 215 }

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2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 11, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 36, 37, 38, 39, 40, 42, 43, 44, 54, 77, 78, 79, 92, 93, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 110, 111, 112, 113, 114, 115, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 178, 179, 181, 184, 185, 186, 187, 189, 191, 192, 193, 194, 196, 216, 217, 218, 268, 269, 270, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 312, 318, 319, 320, 321, 322, 332, 333, 334, 335, 336, 337, 381, 382, 383, 384, 385, 386, 387, 388, 398, 423, 424, 464, 465, 466, 468, 469, 470, 472, 482, 485, 486, 487, 488, 489, 490, 491, 492, 494, 495, 496, 508, 509, 510, 511, 513, 514, 515, 528, 529, 530, 531, 536, 537, 542, 543, 544, 545, 546, 550, 551, 552, 560, 561, 566, 567, 570, 571, 572, 573, 574, 577, 578, 579, 580, 581, 582, 583, 584, 588, 589, 590, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 625, 626, 627, 630, 631, 635, 640, 641 }

B grade: { 10, 12, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 45, 46, 47, 48, 49, 51, 52, 53, 91, 109, 116, 117, 148, 149, 176, 177, 182, 183, 190, 197, 198, 199, 200, 202, 203, 204, 310, 311, 315, 316, 317, 323, 324, 325, 329, 330, 331, 350, 390, 393, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 415, 416, 417, 421, 422, 426, 427, 428, 433, 434, 435, 439, 440, 441, 442, 443, 444, 445, 447, 448, 449, 450, 451, 452, 456, 457, 458, 459, 463, 471, 481, 501, 502, 506, 507, 629, 632, 633, 634, 639 }

C grade: { }

F grade: { 6, 19, 41, 50, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 94, 95, 96, 97, 131, 132, 133, 134, 135, 136, 137, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 180, 188, 195, 201, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 271, 272, 273, 274, 291, 292, 293, 294, 295, 313, 314, 326, 327, 328, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 389, 391, 392, 394, 395, 396, 397, 399, 404, 411, 412, 413, 414, 418, 419, 420, 425, 429, 430, 431, 432, 436, 437, 438, 446, 453, 454, 455, 460, 461, 462, 467, 473, 474, 475, 476, 477, 478, 479, 480, 483, 484, 493, 497, 498, 499, 500, 503, 504, 505, 512, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 532, 533, 534, 535, 538, 539, 540, 541, 547, 548, 549, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 568, 569, 575, 576, 585, 586, 587, 591, 592, 593, 616, 617, 618, 619, 620, 621, 622, 623, 624, 628, 636, 637, 638 }

2.1.8 Mupad

A grade: { 98, 99, 100, 101, 104, 105, 106, 107, 108, 111, 112, 113, 114, 115, 118, 119, 120, 121, 122, 141, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 211, 216, 217, 218, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 381, 382, 383, 384, 385, 386, 387, 388, 398, 485, 486, 487, 488, 528, 529, 530, 531, 536, 537, 542, 543, 544, 545, 546, 550, 551, 552, 560, 561, 566, 567, 570, 571, 572, 573, 574, 577, 578, 579, 580, 581, 582, 583, 584, 588, 589, 590, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 625, 626, 627, 635, 640, 641 }

B grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 77, 78, 79, 91, 92, 93, 124, 126, 128, 129, 130, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 202, 203, 204, 268, 269, 270, 288, 289, 290, 310, 311, 312, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 329, 330, 331, 332, 333, 334, 335, 336, 337, 350, 379, 390, 391, 393, 394, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 415, 416, 417, 421, 422, 423, 424, 426, 427, 428, 433, 434, 435, 439, 440, 441, 442, 443, 444, 445, 447, 448, 449, 450, 451, 452, 456, 457, 458, 459, 463, 465, 466, 469, 471, 472, 481, 482, 489, 490, 491, 492, 494, 495, 496, 501, 502, 506, 507, 508, 510, 511, 514, 519, 520, 527, 629, 630, 631, 632, 633, 634, 639 }

C grade: { }

F grade: { 6, 19, 31, 41, 50, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 94, 95, 96, 97, 102, 103, 109, 110, 116, 117, 123, 125, 127, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 148, 149, 150, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 180, 188, 195, 201, 205, 206, 207, 208, 209, 210, 212, 213, 214, 215, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 271, 272, 273, 274, 291, 292, 293, 294, 295, 313, 314, 326, 327, 328, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 380, 389, 392, 395, 396, 397, 399, 404, 411, 412, 413, 414, 418, 419, 420, 425, 429, 430, 431, 432, 436, 437, 438, 446, 453, 454, 455, 460, 461, 462, 464, 467, 468, 470, 473, 474, 475, 476, 477, 478, 479, 480, 483, 484, 493, 497, 498, 499, 500, 503, 504, 505, 509, 512, 513, 515, 516, 517, 518, 521, 522, 523, 524, 525, 526, 532, 533, 534, 535, 538, 539, 540, 541, 547, 548, 549, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 568, 569, 575, 576, 585, 586, 587, 591, 592, 593, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 628, 636, 637, 638 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	74	229	72	188	156	71	62
N.S.	1	1.00	0.92	2.86	0.90	2.35	1.95	0.89	0.78
time (sec)	N/A	0.029	0.035	0.373	0.487	0.427	34.420	3.512	0.221

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	1190	55	57	65	97	51
N.S.	1	1.00	1.00	20.17	0.93	0.97	1.10	1.64	0.86
time (sec)	N/A	0.034	0.011	0.334	0.273	0.372	0.893	4.791	0.224

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	62	217	59	152	141	59	50
N.S.	1	1.00	0.94	3.29	0.89	2.30	2.14	0.89	0.76
time (sec)	N/A	0.025	0.018	0.305	0.495	0.410	8.501	3.392	0.228

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	34	37	44	40	51	43	39
N.S.	1	1.00	0.97	1.06	1.26	1.14	1.46	1.23	1.11
time (sec)	N/A	0.018	0.008	0.478	0.266	0.426	0.314	4.784	0.226

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	46	45	107	100	41	37
N.S.	1	1.00	1.00	1.02	1.00	2.38	2.22	0.91	0.82
time (sec)	N/A	0.013	0.009	0.269	0.497	0.401	2.173	4.195	0.224

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	232	80	0	0	0	-1
N.S.	1	1.00	0.98	5.27	1.82	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.006	0.267	0.280	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	195	36	105	245	40	36
N.S.	1	1.00	1.00	4.43	0.82	2.39	5.57	0.91	0.82
time (sec)	N/A	0.014	0.008	0.217	0.507	0.419	7.954	4.067	0.221

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	45	45	173	44	43	65	58	41
N.S.	1	1.18	1.18	4.55	1.16	1.13	1.71	1.53	1.08
time (sec)	N/A	0.026	0.003	0.217	0.277	0.382	1.090	5.851	0.245

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	49	211	49	135	479	58	46
N.S.	1	1.00	0.82	3.52	0.82	2.25	7.98	0.97	0.77
time (sec)	N/A	0.018	0.003	0.315	0.487	0.411	40.551	4.615	0.252

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	56	198	54	58	83	132	56
N.S.	1	1.00	0.88	3.09	0.84	0.91	1.30	2.06	0.88
time (sec)	N/A	0.036	0.029	0.208	0.275	0.381	3.077	4.399	0.259

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	49	236	62	170	566	71	61
N.S.	1	1.00	0.66	3.19	0.84	2.30	7.65	0.96	0.82
time (sec)	N/A	0.024	0.003	0.358	0.477	0.394	169.059	3.531	0.261

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	68	206	69	71	97	191	68
N.S.	1	1.00	0.87	2.64	0.88	0.91	1.24	2.45	0.87
time (sec)	N/A	0.043	0.047	0.261	0.270	0.385	8.345	3.650	0.257

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	1190	55	57	65	97	51
N.S.	1	1.00	1.00	20.17	0.93	0.97	1.10	1.64	0.86
time (sec)	N/A	0.034	0.011	0.390	0.281	0.378	2.951	5.621	0.239

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	69	196	147	161	0	162	157
N.S.	1	1.00	0.43	1.23	0.92	1.01	0.00	1.02	0.99
time (sec)	N/A	0.082	0.004	0.413	0.499	0.400	0.000	2.857	2.503

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	147	194	144	144	0	160	129
N.S.	1	1.00	0.94	1.24	0.92	0.92	0.00	1.02	0.82
time (sec)	N/A	0.076	0.035	0.362	0.502	0.395	0.000	6.140	2.555

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	34	37	44	40	51	43	39
N.S.	1	1.00	0.97	1.06	1.26	1.14	1.46	1.23	1.11
time (sec)	N/A	0.021	0.007	0.471	0.286	0.383	0.828	2.810	0.232

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	53	184	131	150	178	150	121
N.S.	1	1.00	0.36	1.25	0.89	1.02	1.21	1.02	0.82
time (sec)	N/A	0.059	0.003	0.401	0.494	0.363	63.477	4.443	2.378

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	129	122	125	110	165	143	134
N.S.	1	1.00	0.97	0.92	0.94	0.83	1.24	1.08	1.01
time (sec)	N/A	0.058	0.026	0.359	0.501	0.394	27.612	3.493	0.458

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	180	80	0	0	0	-1
N.S.	1	1.00	0.98	4.09	1.82	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.006	0.302	0.270	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	47	184	119	126	165	137	149
N.S.	1	1.00	0.35	1.38	0.89	0.95	1.24	1.03	1.12
time (sec)	N/A	0.051	0.003	0.251	0.507	0.378	117.591	5.587	0.810

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	134	197	120	150	0	138	115
N.S.	1	1.00	0.96	1.42	0.86	1.08	0.00	0.99	0.83
time (sec)	N/A	0.056	0.024	0.324	0.511	0.399	0.000	3.898	2.606

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	173	44	43	65	58	41
N.S.	1	1.00	1.00	3.84	0.98	0.96	1.44	1.29	0.91
time (sec)	N/A	0.026	0.003	0.225	0.280	0.387	2.356	4.697	0.259

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	49	215	127	138	0	153	125
N.S.	1	1.00	0.32	1.42	0.84	0.91	0.00	1.01	0.83
time (sec)	N/A	0.063	0.003	0.301	0.499	0.363	0.000	3.736	2.361

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	49	216	128	172	0	149	156
N.S.	1	1.00	0.32	1.43	0.85	1.14	0.00	0.99	1.03
time (sec)	N/A	0.064	0.003	0.314	0.510	0.374	0.000	4.632	2.446

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	56	198	54	58	83	132	56
N.S.	1	1.00	0.88	3.09	0.84	0.91	1.30	2.06	0.88
time (sec)	N/A	0.035	0.028	0.260	0.274	0.391	9.269	5.685	0.284

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	85	0	74	89	100	308	77
N.S.	1	1.00	0.96	0.00	0.83	1.00	1.12	3.46	0.87
time (sec)	N/A	0.036	0.037	0.053	0.287	0.389	2.385	5.220	0.241

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	74	0	64	77	87	257	65
N.S.	1	1.00	0.99	0.00	0.85	1.03	1.16	3.43	0.87
time (sec)	N/A	0.028	0.024	0.043	0.282	0.369	1.348	3.927	0.214

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	62	0	51	64	73	210	53
N.S.	1	1.00	1.02	0.00	0.84	1.05	1.20	3.44	0.87
time (sec)	N/A	0.022	0.019	0.045	0.281	0.360	0.792	5.177	0.228

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	40	0	40	50	60	152	41
N.S.	1	1.00	0.85	0.00	0.85	1.06	1.28	3.23	0.87
time (sec)	N/A	0.015	0.017	0.046	0.278	0.372	0.477	4.140	0.246

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	37	30	27	33	36	96	27
N.S.	1	1.00	1.37	1.11	1.00	1.22	1.33	3.56	1.00
time (sec)	N/A	0.006	0.003	0.069	0.274	0.364	0.280	4.378	0.205

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	41	0	83	0	0	152	-1
N.S.	1	1.00	1.02	0.00	2.08	0.00	0.00	3.80	-0.02
time (sec)	N/A	0.026	0.003	0.053	0.264	0.000	0.000	4.627	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	37	50	36	37	63	40
N.S.	1	1.00	1.00	1.23	1.67	1.20	1.23	2.10	1.33
time (sec)	N/A	0.016	0.005	0.285	0.287	0.385	0.522	5.197	0.681

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	0	63	55	61	150	53
N.S.	1	1.00	1.00	0.00	1.07	0.93	1.03	2.54	0.90
time (sec)	N/A	0.026	0.011	0.040	0.268	0.365	0.787	4.256	0.340

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	74	66	75	234	65
N.S.	1	1.00	1.00	0.00	1.01	0.90	1.03	3.21	0.89
time (sec)	N/A	0.034	0.014	0.053	0.276	0.381	1.399	5.721	0.321

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	0	85	79	88	317	78
N.S.	1	1.00	1.00	0.00	0.98	0.91	1.01	3.64	0.90
time (sec)	N/A	0.040	0.017	0.050	0.287	0.355	2.009	2.166	0.388

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	49	0	59	178	148	75	56
N.S.	1	1.00	0.68	0.00	0.82	2.47	2.06	1.04	0.78
time (sec)	N/A	0.027	0.006	0.074	0.513	0.373	33.557	4.453	0.263

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	56	0	44	56	66	59	45
N.S.	1	1.00	1.10	0.00	0.86	1.10	1.29	1.16	0.88
time (sec)	N/A	0.025	0.016	0.055	0.261	0.379	1.650	6.330	0.245

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	47	0	48	141	133	63	44
N.S.	1	1.00	0.81	0.00	0.83	2.43	2.29	1.09	0.76
time (sec)	N/A	0.019	0.003	0.044	0.495	0.379	10.997	6.091	0.241

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	45	0	33	42	53	47	33
N.S.	1	1.00	1.22	0.00	0.89	1.14	1.43	1.27	0.89
time (sec)	N/A	0.011	0.003	0.049	0.276	0.373	0.682	4.503	0.209

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	43	38	33	107	95	42	33
N.S.	1	1.00	1.05	0.93	0.80	2.61	2.32	1.02	0.80
time (sec)	N/A	0.010	0.006	0.058	0.563	0.401	3.787	4.151	0.106

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	45	0	89	0	0	0	-1
N.S.	1	1.00	1.02	0.00	2.02	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.003	0.053	0.272	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	52	0	49	119	97	54	42
N.S.	1	1.00	1.04	0.00	0.98	2.38	1.94	1.08	0.84
time (sec)	N/A	0.019	0.011	0.045	0.551	0.357	8.695	4.933	0.241

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	34	37	54	41	53	57	47
N.S.	1	1.00	0.97	1.06	1.54	1.17	1.51	1.63	1.34
time (sec)	N/A	0.018	0.007	0.306	0.281	0.368	1.048	3.276	0.236

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	70	0	62	154	138	73	55
N.S.	1	1.00	1.03	0.00	0.91	2.26	2.03	1.07	0.81
time (sec)	N/A	0.025	0.017	0.038	0.515	0.416	26.330	4.350	0.254

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	34	9	35	11	8	110	8
N.S.	1	1.00	4.25	1.12	4.38	1.38	1.00	13.75	1.00
time (sec)	N/A	0.005	0.004	0.330	0.277	0.387	1.583	2.993	0.257

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	134	0	120	129	146	339	121
N.S.	1	1.00	0.88	0.00	0.78	0.84	0.95	2.22	0.79
time (sec)	N/A	0.078	0.094	0.057	0.289	0.489	13.632	4.643	0.334

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	112	0	98	105	119	255	97
N.S.	1	1.00	0.91	0.00	0.80	0.85	0.97	2.07	0.79
time (sec)	N/A	0.060	0.039	0.037	0.299	0.425	4.370	3.727	0.277

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	88	0	76	80	92	171	73
N.S.	1	1.00	0.95	0.00	0.82	0.86	0.99	1.84	0.78
time (sec)	N/A	0.042	0.029	0.060	0.281	0.429	1.456	3.393	0.267

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	52	50	51	61	97	47
N.S.	1	1.00	1.00	0.98	0.94	0.96	1.15	1.83	0.89
time (sec)	N/A	0.020	0.018	0.277	0.270	0.372	0.755	4.464	0.124

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	47	0	79	0	0	0	-1
N.S.	1	1.00	1.02	0.00	1.72	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.003	0.042	0.283	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	55	0	53	55	326	132	49
N.S.	1	1.00	0.87	0.00	0.84	0.87	5.17	2.10	0.78
time (sec)	N/A	0.030	0.029	0.032	0.277	0.416	10.807	5.653	0.660

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	90	0	76	84	403	232	72
N.S.	1	1.00	0.90	0.00	0.76	0.84	4.03	2.32	0.72
time (sec)	N/A	0.044	0.032	0.029	0.353	0.401	64.493	3.993	0.460

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	114	0	98	109	0	324	97
N.S.	1	1.00	0.88	0.00	0.75	0.84	0.00	2.49	0.75
time (sec)	N/A	0.060	0.048	0.039	0.319	0.405	0.000	4.524	0.563

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	33	32	31	28	133	31	33
N.S.	1	1.00	1.03	1.00	0.97	0.88	4.16	0.97	1.03
time (sec)	N/A	0.012	0.008	0.285	0.317	0.382	0.239	2.464	0.276

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	70	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.022	0.038	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	70	0	0	0	381	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	4.70	0.00	-0.01
time (sec)	N/A	0.030	0.021	0.035	0.000	0.000	31.778	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	56	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.021	0.020	0.040	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	56	0	0	0	218	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	3.25	0.00	-0.01
time (sec)	N/A	0.030	0.013	0.077	0.000	0.000	10.208	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	76	0	0	0	369	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	4.50	0.00	-0.01
time (sec)	N/A	0.040	0.022	0.069	0.000	0.000	32.791	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	76	0	0	0	382	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	4.49	0.00	-0.01
time (sec)	N/A	0.042	0.024	0.072	0.000	0.000	232.750	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	76	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.027	0.072	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	77	0	0	0	0	0	-1
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.029	0.069	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	77	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.029	0.055	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	92	0	115	109	0	0	-1
N.S.	1	1.00	0.65	0.00	0.82	0.77	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.057	0.066	0.284	0.374	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	74	0	95	90	0	0	-1
N.S.	1	1.00	0.66	0.00	0.85	0.80	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.032	0.051	0.285	0.393	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	48	0	70	60	0	0	-1
N.S.	1	1.00	0.70	0.00	1.01	0.87	0.00	0.00	-0.01
time (sec)	N/A	0.028	0.026	0.061	0.288	0.478	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	46	201	0	66	0	0	-1
N.S.	1	1.00	0.92	4.02	0.00	1.32	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.007	1.453	0.000	0.374	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	57	0	71	78	0	0	-1
N.S.	1	1.00	0.71	0.00	0.89	0.98	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.014	0.064	0.284	0.388	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	76	0	99	103	0	0	-1
N.S.	1	1.00	0.63	0.00	0.82	0.86	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.036	0.055	0.280	0.415	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	61	0	0	0	104	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	1.60	0.00	-0.02
time (sec)	N/A	0.021	0.025	0.046	0.000	0.000	7.902	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	61	0	0	0	104	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	1.60	0.00	-0.02
time (sec)	N/A	0.018	0.025	0.047	0.000	0.000	3.683	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	52	0	0	0	48	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.89	0.00	-0.02
time (sec)	N/A	0.014	0.023	0.048	0.000	0.000	1.769	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	177	0	63	0	0	-1
N.S.	1	1.00	0.98	4.02	0.00	1.43	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.004	1.233	0.000	0.376	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	59	0	0	0	46	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.70	0.00	-0.02
time (sec)	N/A	0.023	0.025	0.048	0.000	0.000	4.297	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	62	0	0	0	51	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.71	0.00	-0.01
time (sec)	N/A	0.022	0.021	0.049	0.000	0.000	8.872	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	62	0	0	0	51	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.73	0.00	-0.01
time (sec)	N/A	0.022	0.022	0.036	0.000	0.000	19.772	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	200	1436	145	189	182	370	126
N.S.	1	1.00	0.93	6.68	0.67	0.88	0.85	1.72	0.59
time (sec)	N/A	0.196	0.049	0.444	0.280	0.364	4.194	2.941	0.308

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	105	24297	120	148	139	216	100
N.S.	1	1.00	0.72	167.57	0.83	1.02	0.96	1.49	0.69
time (sec)	N/A	0.110	0.041	1.581	0.312	0.387	1.539	6.025	0.259

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	63	1034	97	96	90	96	70
N.S.	1	1.00	1.03	16.95	1.59	1.57	1.48	1.57	1.15
time (sec)	N/A	0.033	0.008	0.691	0.281	0.364	0.588	3.775	0.229

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	163	0	118	0	0	0	-1
N.S.	1	1.00	2.26	0.00	1.64	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.079	0.046	0.297	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	93	841	118	0	0	0	-1
N.S.	1	1.00	1.16	10.51	1.48	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.019	0.323	0.279	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	137	1080	142	0	0	0	-1
N.S.	1	1.00	1.06	8.37	1.10	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.154	0.057	0.300	0.288	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	205	1289	173	0	0	0	-1
N.S.	1	1.00	1.06	6.68	0.90	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.245	0.041	0.351	0.289	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	248	0	0	0	0	0	-1
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.259	0.138	0.064	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	223	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.215	0.093	0.042	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	193	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.175	0.058	0.052	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	173	0	0	0	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.040	0.049	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	207	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.190	0.066	0.046	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	277	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.217	0.192	0.032	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	334	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.250	0.161	0.036	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	309	5905	239	359	289	662	187
N.S.	1	1.00	0.93	17.68	0.72	1.07	0.87	1.98	0.56
time (sec)	N/A	0.242	0.139	1.308	0.288	0.398	6.588	4.590	0.356

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	237	241142	203	275	223	385	144
N.S.	1	1.00	1.12	1142.85	0.96	1.30	1.06	1.82	0.68
time (sec)	N/A	0.142	0.062	14.816	0.308	0.392	2.628	5.233	0.305

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	87	3925	164	176	143	169	103
N.S.	1	1.00	0.94	42.20	1.76	1.89	1.54	1.82	1.11
time (sec)	N/A	0.046	0.010	1.452	0.294	0.451	0.954	4.672	0.240

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	279	0	217	0	0	0	-1
N.S.	1	1.00	2.63	0.00	2.05	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.110	0.046	0.314	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	302	0	202	0	0	0	-1
N.S.	1	1.00	2.54	0.00	1.70	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.233	0.050	0.350	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	478	0	270	0	0	0	-1
N.S.	1	1.00	2.18	0.00	1.23	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.259	0.273	0.048	0.346	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	571	0	338	0	0	0	-1
N.S.	1	1.00	1.62	0.00	0.96	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.435	0.304	0.046	0.353	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	0	909	0	0	0	0	0	-1
N.S.	1	0.00	2.39	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.519	2.511	0.060	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	0	789	0	0	0	0	0	-1
N.S.	1	0.00	2.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.274	2.262	0.043	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	0	505	0	0	0	0	0	-1
N.S.	1	0.00	9.90	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.585	0.044	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	0	851	0	0	0	0	0	-1
N.S.	1	0.00	3.35	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.224	1.832	0.046	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	96	547	0	68	0	69	-1
N.S.	1	1.00	0.90	5.11	0.00	0.64	0.00	0.64	-0.01
time (sec)	N/A	0.100	0.095	0.649	0.000	0.359	0.000	3.085	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	272	0	29	0	31	-1
N.S.	1	1.00	1.00	5.33	0.00	0.57	0.00	0.61	-0.02
time (sec)	N/A	0.037	0.035	2.790	0.000	0.411	0.000	4.086	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.012	0.137	0.042	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.012	0.248	0.041	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.012	0.215	0.027	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.003	0.009	0.041	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.011	0.291	0.035	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	157	1474	0	141	0	313	-1
N.S.	1	1.00	1.14	10.68	0.00	1.02	0.00	2.27	-0.01
time (sec)	N/A	0.136	0.107	0.645	0.000	0.345	0.000	3.207	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	97	421	0	78	0	141	-1
N.S.	1	1.00	1.17	5.07	0.00	0.94	0.00	1.70	-0.01
time (sec)	N/A	0.051	0.037	2.141	0.000	0.367	0.000	5.893	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.011	0.206	0.004	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.011	1.008	0.035	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.011	0.237	0.004	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.003	0.267	0.005	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.011	0.798	0.005	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	185	1969	0	270	0	874	-1
N.S.	1	1.00	0.91	9.65	0.00	1.32	0.00	4.28	-0.00
time (sec)	N/A	0.188	0.140	0.756	0.000	0.362	0.000	2.628	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	113	716	0	157	0	406	-1
N.S.	1	1.00	0.99	6.28	0.00	1.38	0.00	3.56	-0.01
time (sec)	N/A	0.061	0.045	2.252	0.000	0.362	0.000	4.930	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.010	0.360	0.048	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.010	1.980	0.052	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.010	0.379	0.004	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.003	0.333	0.003	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.010	1.410	0.004	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	41	43	0	54	0	44	-1
N.S.	1	1.00	0.91	0.96	0.00	1.20	0.00	0.98	-0.02
time (sec)	N/A	0.060	0.068	1.713	0.000	0.358	0.000	3.821	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	23	0	19	27	20	18
N.S.	1	1.00	1.10	1.15	0.00	0.95	1.35	1.00	0.90
time (sec)	N/A	0.018	0.020	0.406	0.000	0.364	0.902	2.286	0.348

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	66	95	0	99	0	100	-1
N.S.	1	1.00	0.93	1.34	0.00	1.39	0.00	1.41	-0.01
time (sec)	N/A	0.083	0.075	2.082	0.000	0.388	0.000	4.850	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	48	0	55	49	47	46
N.S.	1	1.00	0.94	1.02	0.00	1.17	1.04	1.00	0.98
time (sec)	N/A	0.029	0.021	0.411	0.000	0.361	0.949	7.124	0.359

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	87	143	0	142	0	151	-1
N.S.	1	1.00	0.69	1.13	0.00	1.12	0.00	1.19	-0.01
time (sec)	N/A	0.116	0.086	2.117	0.000	0.378	0.000	5.306	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	56	70	0	79	70	71	74
N.S.	1	1.00	0.77	0.96	0.00	1.08	0.96	0.97	1.01
time (sec)	N/A	0.040	0.019	0.467	0.000	0.343	0.947	5.048	0.451

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	105	24313	120	146	136	232	100
N.S.	1	1.00	0.70	162.09	0.80	0.97	0.91	1.55	0.67
time (sec)	N/A	0.109	0.047	5.139	0.282	0.361	4.714	3.845	0.275

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	63	1036	97	103	100	104	71
N.S.	1	1.00	0.95	15.70	1.47	1.56	1.52	1.58	1.08
time (sec)	N/A	0.037	0.009	0.760	0.281	0.352	1.285	4.733	0.232

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	163	0	0	0	0	0	-1
N.S.	1	1.00	2.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.105	0.046	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	84	771	118	0	0	0	-1
N.S.	1	1.00	0.98	8.97	1.37	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.030	0.482	0.285	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1294	1300	823	0	0	0	0	0	-1
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.296	0.804	0.047	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1304	1310	1090	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.173	0.463	0.046	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1137	1143	742	0	0	0	0	0	-1
N.S.	1	1.01	0.65	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.901	0.585	0.058	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1170	1176	745	0	0	0	0	0	-1
N.S.	1	1.01	0.64	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.882	0.612	0.052	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1328	1334	847	0	0	0	0	0	-1
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.146	1.099	0.059	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	146	823	0	115	0	108	-1
N.S.	1	1.00	0.89	5.02	0.00	0.70	0.00	0.66	-0.01
time (sec)	N/A	0.161	0.151	0.722	0.000	0.367	0.000	5.807	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	96	547	0	68	0	69	-1
N.S.	1	1.00	0.90	5.11	0.00	0.64	0.00	0.64	-0.01
time (sec)	N/A	0.102	0.080	0.657	0.000	0.360	0.000	5.653	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	272	0	29	0	31	-1
N.S.	1	1.00	1.00	5.33	0.00	0.57	0.00	0.61	-0.02
time (sec)	N/A	0.041	0.037	2.578	0.000	0.367	0.000	2.749	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.011	0.135	0.044	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.011	0.231	0.046	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.012	0.185	0.033	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.006	0.167	0.035	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.003	0.008	0.036	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.011	0.267	0.041	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.011	0.282	0.043	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	290	2564	0	211	0	490	-1
N.S.	1	1.00	1.49	13.15	0.00	1.08	0.00	2.51	-0.01
time (sec)	N/A	0.252	0.175	0.797	0.000	0.377	0.000	5.347	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	157	1487	0	143	0	326	-1
N.S.	1	1.00	1.11	10.55	0.00	1.01	0.00	2.31	-0.01
time (sec)	N/A	0.142	0.092	0.645	0.000	0.350	0.000	3.001	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	97	421	0	84	0	154	-1
N.S.	1	1.00	1.17	5.07	0.00	1.01	0.00	1.86	-0.01
time (sec)	N/A	0.055	0.037	2.523	0.000	0.367	0.000	5.331	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.011	0.204	0.004	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.011	0.989	0.031	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.011	0.227	0.004	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.006	0.334	0.023	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.003	0.263	0.006	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.011	0.770	0.004	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.012	0.753	0.019	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	0	994	0	0	0	0	0	-1
N.S.	1	0.00	12.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	1.427	0.073	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	0	466	0	0	0	0	0	-1
N.S.	1	0.00	6.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	1.058	0.051	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	70	0	0	0	381	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	4.70	0.00	-0.01
time (sec)	N/A	0.027	0.018	0.045	0.000	0.000	32.283	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.012	0.242	0.052	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.011	0.375	0.007	0.000	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	171	0	239	260	0	0	-1
N.S.	1	1.00	0.46	0.00	0.64	0.70	0.00	0.00	-0.00
time (sec)	N/A	0.218	0.117	0.088	0.291	0.400	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	140	0	200	202	0	0	-1
N.S.	1	1.00	0.55	0.00	0.78	0.79	0.00	0.00	-0.00
time (sec)	N/A	0.130	0.082	0.069	0.304	0.424	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	74	0	146	128	0	0	-1
N.S.	1	1.00	0.73	0.00	1.45	1.27	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.019	0.078	0.297	0.394	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	168	1473	0	0	0	0	-1
N.S.	1	1.00	1.91	16.74	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.076	2.794	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	148	0	0	207	0	0	-1
N.S.	1	1.00	1.19	0.00	0.00	1.67	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.068	0.074	0.000	0.368	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	288	0	0	278	0	0	-1
N.S.	1	1.00	1.44	0.00	0.00	1.39	0.00	0.00	-0.00
time (sec)	N/A	0.182	0.200	0.071	0.000	0.381	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	0	13	14	0	-1
N.S.	1	1.00	1.00	1.08	0.00	1.00	1.08	0.00	-0.08
time (sec)	N/A	0.006	0.003	1.157	0.000	0.354	1.477	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	45	0	43	100	0	-1
N.S.	1	1.00	1.00	2.14	0.00	2.05	4.76	0.00	-0.05
time (sec)	N/A	0.019	0.003	1.091	0.000	0.378	1.632	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	46	0	44	100	0	-1
N.S.	1	1.00	1.00	2.19	0.00	2.10	4.76	0.00	-0.05
time (sec)	N/A	0.019	0.004	1.176	0.000	0.371	1.630	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	39	38	0	57	0	0	-1
N.S.	1	1.00	0.95	0.93	0.00	1.39	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.009	1.383	0.000	0.387	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	177	0	63	0	0	-1
N.S.	1	1.00	0.98	4.02	0.00	1.43	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.005	0.325	0.000	0.379	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	164	1356	0	0	0	0	-1
N.S.	1	1.00	2.08	17.16	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.049	2.504	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	270	6131	0	0	0	0	-1
N.S.	1	1.00	2.39	54.26	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.109	3.256	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	185	766	207	266	335	558	208
N.S.	1	1.00	1.32	5.47	1.48	1.90	2.39	3.99	1.49
time (sec)	N/A	0.055	0.145	0.724	0.283	0.362	0.955	3.189	0.314

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	121	537	135	172	202	313	131
N.S.	1	1.00	1.08	4.79	1.21	1.54	1.80	2.79	1.17
time (sec)	N/A	0.038	0.078	0.555	0.287	0.399	0.547	6.920	0.267

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	82	83	78	94	105	142	68
N.S.	1	1.00	0.98	0.99	0.93	1.12	1.25	1.69	0.81
time (sec)	N/A	0.023	0.035	0.348	0.312	0.392	0.310	2.671	0.254

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	36	35	32	36	39	29
N.S.	1	1.00	1.00	1.50	1.46	1.33	1.50	1.62	1.21
time (sec)	N/A	0.007	0.006	0.284	0.463	0.363	0.129	5.561	0.070

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	242	123	0	0	0	-1
N.S.	1	1.00	0.98	4.17	2.12	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.035	0.004	0.453	0.387	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	52	329	67	80	0	91	70
N.S.	1	1.00	0.76	4.84	0.99	1.18	0.00	1.34	1.03
time (sec)	N/A	0.020	0.036	0.541	0.356	0.432	0.000	3.242	1.072

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	80	582	121	231	0	266	96
N.S.	1	1.00	0.76	5.54	1.15	2.20	0.00	2.53	0.91
time (sec)	N/A	0.046	0.075	0.653	0.344	0.414	0.000	5.379	0.638

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	105	873	228	431	4571	495	145
N.S.	1	1.00	0.79	6.56	1.71	3.24	34.37	3.72	1.09
time (sec)	N/A	0.055	0.111	0.716	0.362	0.397	20.009	4.174	0.757

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	249	1330	172	486	527	273	222
N.S.	1	1.00	1.40	7.47	0.97	2.73	2.96	1.53	1.25
time (sec)	N/A	0.112	0.544	0.796	0.579	0.398	19.443	3.863	0.456

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	211	965	130	316	374	173	263
N.S.	1	1.00	1.50	6.84	0.92	2.24	2.65	1.23	1.87
time (sec)	N/A	0.087	0.320	0.620	0.589	0.369	9.453	4.327	3.276

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	83	93	83	206	199	100	81
N.S.	1	1.00	0.84	0.94	0.84	2.08	2.01	1.01	0.82
time (sec)	N/A	0.053	0.026	0.447	0.562	0.415	4.502	3.888	1.133

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	46	45	107	100	41	37
N.S.	1	1.00	1.00	1.02	1.00	2.38	2.22	0.91	0.82
time (sec)	N/A	0.014	0.011	0.067	0.566	0.431	2.246	4.099	0.081

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	201	366	0	0	0	0	-1
N.S.	1	1.00	1.00	1.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.176	0.064	0.435	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	137	755	106	257	0	158	337
N.S.	1	1.00	1.15	6.34	0.89	2.16	0.00	1.33	2.83
time (sec)	N/A	0.065	0.058	0.906	0.567	0.419	0.000	4.724	1.255

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	217	2684	198	724	0	420	272
N.S.	1	1.00	1.25	15.43	1.14	4.16	0.00	2.41	1.56
time (sec)	N/A	0.104	0.410	0.896	0.555	0.421	0.000	4.223	0.981

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	264	738	326	8463	265	407	536
N.S.	1	1.00	0.82	2.31	1.02	26.45	0.83	1.27	1.68
time (sec)	N/A	0.510	0.346	0.786	0.598	13.827	27.192	3.841	0.948

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	218	537	251	5590	173	298	358
N.S.	1	1.00	0.87	2.15	1.00	22.36	0.69	1.19	1.43
time (sec)	N/A	0.325	0.224	0.912	0.590	2.812	18.539	4.995	0.322

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	204	335	192	2251	112	220	210
N.S.	1	1.00	0.89	1.46	0.84	9.83	0.49	0.96	0.92
time (sec)	N/A	0.218	0.052	0.555	0.590	1.213	12.897	3.116	0.310

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	129	122	125	110	165	143	134
N.S.	1	1.00	0.97	0.92	0.94	0.83	1.24	1.08	1.01
time (sec)	N/A	0.056	0.028	0.053	0.613	0.381	27.393	4.719	0.004

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	313	261	0	0	0	0	-1
N.S.	1	1.00	1.02	0.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.335	0.112	0.575	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	202	1068	307	6743	0	398	736
N.S.	1	1.00	0.69	3.66	1.05	23.09	0.00	1.36	2.52
time (sec)	N/A	0.360	0.444	0.827	0.555	1.453	0.000	4.624	0.487

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	303	4085	500	12591	0	790	2227
N.S.	1	1.00	0.77	10.45	1.28	32.20	0.00	2.02	5.70
time (sec)	N/A	0.477	0.447	0.883	0.559	4.721	0.000	3.250	0.884

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	114	0	160	236	355	1659	184
N.S.	1	1.00	0.82	0.00	1.15	1.70	2.55	11.94	1.32
time (sec)	N/A	0.088	0.104	0.182	0.356	0.386	1.712	5.018	0.343

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	86	0	101	153	216	918	111
N.S.	1	1.00	0.84	0.00	0.99	1.50	2.12	9.00	1.09
time (sec)	N/A	0.063	0.063	0.209	0.343	0.361	0.993	3.866	0.321

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	70	0	58	85	112	394	57
N.S.	1	1.00	0.90	0.00	0.74	1.09	1.44	5.05	0.73
time (sec)	N/A	0.040	0.028	0.075	0.337	0.381	0.536	3.230	0.304

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	114	0	167	0	0	0	-1
N.S.	1	1.00	1.01	0.00	1.48	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.019	0.253	0.395	0.000	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	81	0	88	152	452	192	85
N.S.	1	1.00	1.00	0.00	1.09	1.88	5.58	2.37	1.05
time (sec)	N/A	0.055	0.049	0.220	0.327	0.453	2.635	2.790	0.532

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	113	0	162	423	3512	805	217
N.S.	1	1.00	0.89	0.00	1.28	3.33	27.65	6.34	1.71
time (sec)	N/A	0.085	0.135	0.243	0.427	0.856	48.698	4.523	1.082

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	164	0	290	814	0	1841	662
N.S.	1	1.00	0.94	0.00	1.66	4.65	0.00	10.52	3.78
time (sec)	N/A	0.122	0.190	0.224	0.402	4.144	0.000	5.617	1.853

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	80	126	82	0	0	0	-1
N.S.	1	1.00	0.76	1.20	0.78	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.029	2.628	0.288	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	239	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.524	0.443	0.199	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	176	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.169	0.170	0.243	0.000	0.000	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	77	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.041	0.208	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	123	0	0	0	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.055	0.251	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	310	0	0	0	0	0	-1
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.356	0.705	0.250	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.007	0.361	0.262	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	245	0	0	0	415	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	1.77	0.00	-0.00
time (sec)	N/A	0.165	0.102	0.216	0.000	0.000	18.163	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	173	0	0	0	284	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	1.57	0.00	-0.01
time (sec)	N/A	0.124	0.066	0.211	0.000	0.000	10.689	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	130	0	0	0	162	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	1.23	0.00	-0.01
time (sec)	N/A	0.091	0.088	0.055	0.000	0.000	6.296	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	52	0	0	0	48	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.89	0.00	-0.02
time (sec)	N/A	0.012	0.022	0.049	0.000	0.000	1.928	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.009	1.238	0.235	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.008	0.181	0.236	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.008	0.190	0.221	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	183	919	0	0	0	0	-1
N.S.	1	1.00	0.73	3.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.170	0.135	0.484	0.000	0.000	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	127	666	0	0	0	0	-1
N.S.	1	1.00	0.80	4.19	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.066	0.431	0.000	0.000	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	79	427	0	0	0	0	-1
N.S.	1	1.00	0.87	4.69	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.025	0.492	0.000	0.000	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	242	123	0	0	0	-1
N.S.	1	1.00	0.98	4.17	2.12	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.004	0.413	0.283	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	98	420	129	0	0	0	-1
N.S.	1	1.00	1.01	4.33	1.33	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.017	0.447	0.323	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	139	615	166	0	0	0	-1
N.S.	1	1.00	0.95	4.21	1.14	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.036	0.477	0.361	0.000	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	188	850	220	0	0	0	-1
N.S.	1	1.00	0.83	3.74	0.97	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.153	0.118	0.476	0.345	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	338	1083	0	0	0	0	-1
N.S.	1	1.00	0.86	2.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.304	0.233	0.497	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	271	825	0	0	0	0	-1
N.S.	1	1.00	0.87	2.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.229	0.111	0.482	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	225	576	0	0	0	0	-1
N.S.	1	1.00	0.88	2.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.192	0.079	0.513	0.000	0.000	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	201	366	0	0	0	0	-1
N.S.	1	1.00	1.00	1.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.129	0.021	0.367	0.000	0.000	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	232	624	0	0	0	0	-1
N.S.	1	1.00	0.94	2.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.206	0.050	0.424	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	268	831	0	0	0	0	-1
N.S.	1	1.00	0.88	2.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.242	0.128	0.522	0.000	0.000	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	320	1071	0	0	0	0	-1
N.S.	1	1.00	0.86	2.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.276	0.144	0.405	0.000	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	692	692	497	912	0	0	0	0	-1
N.S.	1	1.00	0.72	1.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.580	0.437	0.632	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	643	643	504	704	0	0	0	0	-1
N.S.	1	1.00	0.78	1.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.468	0.276	0.611	0.000	0.000	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	457	457	430	500	0	0	0	0	-1
N.S.	1	1.00	0.94	1.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.369	0.149	0.717	0.000	0.000	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	313	261	0	0	0	0	-1
N.S.	1	1.00	1.02	0.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.219	0.042	0.338	0.000	0.000	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	358	461	0	0	0	0	-1
N.S.	1	1.00	1.02	1.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.327	0.047	0.626	0.000	0.000	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	510	510	424	732	0	0	0	0	-1
N.S.	1	1.00	0.83	1.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.404	0.051	0.622	0.000	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	674	674	542	1025	0	0	0	0	-1
N.S.	1	1.00	0.80	1.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.481	0.273	0.613	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	344	0	0	0	0	0	-1
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.223	0.154	0.248	0.000	0.000	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	265	0	0	0	0	0	-1
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.189	0.082	0.246	0.000	0.000	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	149	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.149	0.045	0.238	0.000	0.000	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	114	0	167	0	0	0	-1
N.S.	1	1.00	1.01	0.00	1.48	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.014	0.030	0.298	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	139	0	188	0	0	0	-1
N.S.	1	1.00	0.87	0.00	1.18	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.171	0.044	0.244	0.333	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	166	0	246	0	0	0	-1
N.S.	1	1.00	0.84	0.00	1.24	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.198	0.069	0.232	0.337	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	241	0	311	0	0	0	-1
N.S.	1	1.00	0.84	0.00	1.08	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.234	0.154	0.186	0.326	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	421	375	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.393	0.310	0.250	0.000	0.000	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	319	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.322	0.161	0.225	0.000	0.000	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	271	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.289	0.124	0.263	0.000	0.000	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	242	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.218	0.047	0.201	0.000	0.000	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	264	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.301	0.084	0.223	0.000	0.000	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	320	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.323	0.146	0.225	0.000	0.000	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	364	0	0	0	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.366	0.195	0.218	0.000	0.000	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	714	714	505	0	0	0	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.645	0.290	0.210	0.000	0.000	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	666	666	443	0	0	0	0	0	-1
N.S.	1	1.00	0.67	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.492	0.171	0.199	0.000	0.000	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	488	488	403	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.399	0.100	0.252	0.000	0.000	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	350	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.281	0.089	0.278	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	395	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.369	0.087	0.222	0.000	0.000	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	557	557	429	0	0	0	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.460	0.160	0.185	0.000	0.000	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	737	737	520	0	0	0	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.561	0.207	0.195	0.000	0.000	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	749	749	867	327	0	0	0	0	-1
N.S.	1	1.00	1.16	0.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.750	0.412	0.832	0.000	0.000	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	533	533	564	504	0	0	0	0	-1
N.S.	1	1.00	1.06	0.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.354	0.248	0.904	0.000	0.000	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	178	419	311	0	0	0	-1
N.S.	1	1.00	0.78	1.83	1.36	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.173	0.081	0.558	0.521	0.000	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	373	0	381	0	0	0	-1
N.S.	1	1.00	1.04	0.00	1.06	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.300	0.163	0.249	0.550	0.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	597	597	706	0	0	0	0	0	-1
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.574	0.267	0.240	0.000	0.000	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	541	541	422	0	0	0	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.572	0.197	0.268	0.000	0.000	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	561	561	912	0	0	0	0	0	-1
N.S.	1	1.00	1.63	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.841	0.387	0.249	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	215	995	217	553	697	309	298
N.S.	1	1.00	0.64	2.94	0.64	1.64	2.06	0.91	0.88
time (sec)	N/A	0.174	0.194	0.650	0.507	0.406	133.187	5.892	0.380

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	151	686	146	373	478	201	193
N.S.	1	1.00	0.68	3.10	0.66	1.69	2.16	0.91	0.87
time (sec)	N/A	0.111	0.087	0.710	0.567	0.397	35.028	3.932	0.338

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	416	82	217	260	109	97
N.S.	1	1.00	1.00	3.56	0.70	1.85	2.22	0.93	0.83
time (sec)	N/A	0.060	0.030	0.370	0.554	0.362	8.813	4.792	0.324

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	533	533	564	504	0	0	0	0	-1
N.S.	1	1.00	1.06	0.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.335	0.193	0.341	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	751	751	1236	0	0	0	0	0	-1
N.S.	1	1.00	1.65	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.729	2.603	0.249	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	945	945	435	0	0	0	0	0	-1
N.S.	1	1.00	0.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.795	0.354	0.240	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	548	548	281	0	0	0	0	0	-1
N.S.	1	1.00	0.51	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.465	0.190	0.061	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.016	1.969	0.258	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.016	5.450	1.600	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	683	0	1460	0	0	0	0	0	-1
N.S.	1	0.00	2.14	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.904	2.989	0.067	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.017	2.616	0.254	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.018	9.794	0.280	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.016	0.322	0.183	0.000	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.009	0.205	0.048	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.019	0.390	0.242	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.017	1.206	0.237	0.000	0.000	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.015	0.562	0.004	0.000	0.000	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.009	0.450	0.019	0.000	0.000	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.017	2.480	0.004	0.000	0.000	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.016	5.724	0.004	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	258	1311	266	635	0	354	316
N.S.	1	1.00	0.70	3.58	0.73	1.73	0.00	0.97	0.86
time (sec)	N/A	0.214	0.177	0.988	0.552	0.378	0.000	3.969	3.328

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	178	869	168	415	440	225	317
N.S.	1	1.00	0.77	3.76	0.73	1.80	1.90	0.97	1.37
time (sec)	N/A	0.121	0.152	0.714	0.507	0.465	133.118	5.580	2.769

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	110	402	89	227	214	117	94
N.S.	1	1.00	1.00	3.65	0.81	2.06	1.95	1.06	0.85
time (sec)	N/A	0.067	0.040	0.421	0.498	0.399	17.347	3.270	0.923

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1165	1165	990	1180	0	0	0	0	-1
N.S.	1	1.00	0.85	1.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.146	0.589	1.074	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1861	1863	2168	0	0	0	0	0	-1
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.198	6.841	0.272	0.000	0.000	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1221	1221	1020	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.022	0.706	0.229	0.000	0.000	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	835	835	475	0	0	0	0	0	-1
N.S.	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.678	0.420	0.163	0.000	0.000	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	415	0	0	0	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.334	0.118	0.059	0.000	0.000	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.017	10.554	0.817	0.000	0.000	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.015	29.673	0.380	0.000	0.000	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1126	0	2539	0	0	0	0	0	-1
N.S.	1	0.00	2.25	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.685	6.346	0.265	0.000	0.000	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	518	0	1146	0	0	0	0	0	-1
N.S.	1	0.00	2.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.495	3.023	0.048	0.000	0.000	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.018	12.025	0.343	0.000	0.000	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.016	34.546	0.666	0.000	0.000	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.015	0.238	0.213	0.000	0.000	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.009	0.211	0.003	0.000	0.000	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.019	1.464	0.210	0.000	0.000	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.017	5.330	0.268	0.000	0.000	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.015	0.425	0.014	0.000	0.000	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.009	0.356	0.006	0.000	0.000	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.019	5.256	0.004	0.000	0.000	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.017	7.584	0.013	0.000	0.000	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	170	413	132	147	0	413	127
N.S.	1	1.00	1.20	2.91	0.93	1.04	0.00	2.91	0.89
time (sec)	N/A	0.155	0.034	0.489	0.274	0.357	0.000	3.967	0.324

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	140	387	111	120	156	281	103
N.S.	1	1.00	1.18	3.25	0.93	1.01	1.31	2.36	0.87
time (sec)	N/A	0.121	0.023	0.249	0.303	0.341	66.921	5.464	0.308

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	98	3275	101	94	126	155	78
N.S.	1	1.00	1.04	34.84	1.07	1.00	1.34	1.65	0.83
time (sec)	N/A	0.060	0.036	0.441	0.270	0.378	17.017	4.256	0.313

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	80	419	95	0	0	0	-1
N.S.	1	1.00	0.98	5.11	1.16	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.018	0.159	0.588	0.000	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	92	421	99	0	0	0	-1
N.S.	1	1.00	0.99	4.53	1.06	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.024	0.116	0.537	0.000	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	105	392	83	106	167	322	85
N.S.	1	1.00	1.13	4.22	0.89	1.14	1.80	3.46	0.91
time (sec)	N/A	0.091	0.031	0.153	0.276	0.388	79.746	3.787	0.369

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	130	428	107	134	0	515	113
N.S.	1	1.00	1.04	3.42	0.86	1.07	0.00	4.12	0.90
time (sec)	N/A	0.109	0.048	0.123	0.320	0.390	0.000	4.838	0.371

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	158	448	132	160	0	674	134
N.S.	1	1.00	1.07	3.03	0.89	1.08	0.00	4.55	0.91
time (sec)	N/A	0.128	0.082	0.192	0.331	0.345	0.000	3.575	0.399

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	118	453	111	283	320	138	126
N.S.	1	1.00	0.77	2.94	0.72	1.84	2.08	0.90	0.82
time (sec)	N/A	0.088	0.044	0.342	0.523	0.389	34.479	5.432	0.318

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	416	82	217	260	109	97
N.S.	1	1.00	1.00	3.56	0.70	1.85	2.22	0.93	0.83
time (sec)	N/A	0.057	0.029	0.036	0.533	0.355	8.831	5.909	0.002

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	93	62	427	59	208	204	78	83
N.S.	1	1.29	0.86	5.93	0.82	2.89	2.83	1.08	1.15
time (sec)	N/A	0.053	0.039	0.166	0.554	0.389	16.958	3.126	0.327

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	96	442	66	207	879	92	65
N.S.	1	1.00	0.89	4.09	0.61	1.92	8.14	0.85	0.60
time (sec)	N/A	0.064	0.033	0.183	0.485	0.390	40.835	5.398	0.365

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	101	474	86	276	1108	122	88
N.S.	1	1.00	0.72	3.39	0.61	1.97	7.91	0.87	0.63
time (sec)	N/A	0.079	0.008	0.095	0.488	0.356	164.835	3.908	0.377

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	205	687	218	252	0	773	224
N.S.	1	1.00	0.82	2.74	0.87	1.00	0.00	3.08	0.89
time (sec)	N/A	0.321	0.121	0.541	0.277	0.361	0.000	6.933	0.364

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	173	643	182	214	0	561	184
N.S.	1	1.00	0.82	3.06	0.87	1.02	0.00	2.67	0.88
time (sec)	N/A	0.245	0.095	0.578	0.278	0.384	0.000	3.864	0.342

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	135	605	151	169	235	354	142
N.S.	1	1.00	1.09	4.88	1.22	1.36	1.90	2.85	1.15
time (sec)	N/A	0.092	0.077	0.591	0.288	0.354	66.762	6.599	0.335

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	121	652	157	0	0	0	-1
N.S.	1	1.00	0.79	4.26	1.03	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.064	0.529	0.554	0.000	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	126	642	160	0	0	0	-1
N.S.	1	1.00	0.93	4.76	1.19	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.059	0.408	0.331	0.000	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	148	663	172	0	0	0	-1
N.S.	1	1.00	0.86	3.85	1.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.150	0.085	0.450	1.240	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	141	656	142	199	0	791	151
N.S.	1	1.00	1.08	5.05	1.09	1.53	0.00	6.08	1.16
time (sec)	N/A	0.140	0.091	0.454	0.266	0.369	0.000	5.802	0.415

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	184	713	184	243	0	1089	190
N.S.	1	1.00	0.85	3.30	0.85	1.12	0.00	5.04	0.88
time (sec)	N/A	0.197	0.115	0.497	0.284	0.372	0.000	3.471	0.427

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	215	748	220	282	0	1340	225
N.S.	1	1.00	0.85	2.96	0.87	1.11	0.00	5.30	0.89
time (sec)	N/A	0.227	0.162	0.508	0.268	0.367	0.000	5.508	0.458

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	188	761	182	465	559	246	235
N.S.	1	1.00	0.68	2.74	0.65	1.67	2.01	0.88	0.85
time (sec)	N/A	0.153	0.121	0.681	0.488	0.404	132.532	4.704	0.358

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	151	686	146	373	478	201	193
N.S.	1	1.00	0.68	3.10	0.66	1.69	2.16	0.91	0.87
time (sec)	N/A	0.105	0.090	0.372	0.568	0.408	35.284	5.344	0.002

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	112	702	108	345	400	168	180
N.S.	1	1.00	0.63	3.94	0.61	1.94	2.25	0.94	1.01
time (sec)	N/A	0.100	0.100	0.721	0.505	0.361	65.890	5.248	0.371

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	113	700	101	365	381	154	108
N.S.	1	1.00	0.67	4.14	0.60	2.16	2.25	0.91	0.64
time (sec)	N/A	0.099	0.100	0.446	0.569	0.393	80.888	5.448	0.392

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	156	736	114	373	1567	181	115
N.S.	1	1.00	0.78	3.68	0.57	1.86	7.84	0.90	0.58
time (sec)	N/A	0.113	0.047	0.443	0.536	0.356	169.597	4.790	0.393

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	161	784	148	456	0	222	149
N.S.	1	1.00	0.64	3.11	0.59	1.81	0.00	0.88	0.59
time (sec)	N/A	0.137	0.023	0.426	0.561	0.369	0.000	6.526	0.426

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	143	902	181	0	0	0	-1
N.S.	1	1.00	0.76	4.80	0.96	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.190	0.090	0.797	0.578	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	91	672	128	0	0	0	-1
N.S.	1	1.00	0.81	6.00	1.14	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.032	0.423	0.560	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	64	472	146	0	0	0	-1
N.S.	1	1.00	0.91	6.74	2.09	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.006	0.367	0.317	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	92	732	148	0	0	0	-1
N.S.	1	1.00	0.77	6.15	1.24	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.031	0.495	0.555	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	147	942	187	0	0	0	-1
N.S.	1	1.00	0.84	5.35	1.06	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.174	0.051	0.448	0.580	0.000	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	667	667	691	1011	0	0	0	0	-1
N.S.	1	1.00	1.04	1.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.494	0.392	0.547	0.000	0.000	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	585	585	680	746	0	0	0	0	-1
N.S.	1	1.00	1.16	1.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.388	0.219	0.497	0.000	0.000	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	533	533	564	504	0	0	0	0	-1
N.S.	1	1.00	1.06	0.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.278	0.122	0.331	0.000	0.000	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	581	581	673	755	0	0	0	0	-1
N.S.	1	1.00	1.16	1.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.384	0.220	0.492	0.000	0.000	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	651	651	754	1005	0	0	0	0	-1
N.S.	1	1.00	1.16	1.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.431	0.191	0.464	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	166	985	347	0	0	0	-1
N.S.	1	1.00	0.83	4.95	1.74	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.202	0.163	0.393	0.347	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	131	732	193	0	0	0	-1
N.S.	1	1.00	0.85	4.72	1.25	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.156	0.070	0.455	0.543	0.000	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	63	371	79	96	0	182	80
N.S.	1	1.00	0.76	4.47	0.95	1.16	0.00	2.19	0.96
time (sec)	N/A	0.051	0.037	0.508	0.277	0.352	0.000	4.437	1.458

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	170	984	208	0	0	0	-1
N.S.	1	1.00	0.85	4.90	1.03	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.189	0.075	0.463	0.545	0.000	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	208	1216	310	0	0	0	-1
N.S.	1	1.00	0.83	4.84	1.24	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.228	0.118	0.453	0.636	0.000	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	802	802	1349	0	0	0	0	0	-1
N.S.	1	1.00	1.68	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.191	2.932	0.235	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	746	746	1231	0	0	0	0	0	-1
N.S.	1	1.00	1.65	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.031	2.396	0.227	0.000	0.000	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	751	751	1236	0	0	0	0	0	-1
N.S.	1	1.00	1.65	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.587	2.325	0.009	0.000	0.000	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	803	803	1438	0	0	0	0	0	-1
N.S.	1	1.00	1.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.104	3.207	0.249	0.000	0.000	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	128	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.032	0.267	0.000	0.000	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	248	194	208	0	0	0	-1
N.S.	1	1.00	1.04	0.81	0.87	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.191	0.104	0.525	0.514	0.000	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	468	289	0	0	0	0	-1
N.S.	1	1.00	2.16	1.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.174	0.092	0.462	0.000	0.000	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	188	428	0	143	0	0	-1
N.S.	1	1.00	1.31	2.97	0.00	0.99	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.143	2.404	0.000	0.373	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	173	410	0	129	0	0	-1
N.S.	1	1.00	1.40	3.31	0.00	1.04	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.138	0.100	0.000	0.361	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	68	376	0	109	0	0	-1
N.S.	1	1.00	0.82	4.53	0.00	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.042	0.082	0.000	0.363	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	100	423	0	119	0	0	-1
N.S.	1	1.00	1.03	4.36	0.00	1.23	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.212	0.095	0.000	0.365	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	148	448	0	154	0	0	-1
N.S.	1	1.00	1.17	3.56	0.00	1.22	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.299	0.087	0.000	0.372	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	341	795	0	273	0	0	-1
N.S.	1	1.00	1.04	2.43	0.00	0.83	0.00	0.00	-0.00
time (sec)	N/A	0.210	0.153	1.341	0.000	0.378	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	291	734	0	229	0	0	-1
N.S.	1	1.00	1.15	2.89	0.00	0.90	0.00	0.00	-0.00
time (sec)	N/A	0.175	0.152	0.770	0.000	0.379	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	124	665	0	182	0	0	-1
N.S.	1	1.00	0.70	3.78	0.00	1.03	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.144	0.431	0.000	0.382	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	152	693	0	211	0	0	-1
N.S.	1	1.00	0.79	3.59	0.00	1.09	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.446	0.460	0.000	0.352	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	204	755	0	265	0	0	-1
N.S.	1	1.00	0.79	2.94	0.00	1.03	0.00	0.00	-0.00
time (sec)	N/A	0.195	0.411	0.459	0.000	0.396	0.000	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	0	695	0	0	0	0	-1
N.S.	1	1.00	0.00	2.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.299	4.151	0.903	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	92	532	169	0	0	0	-1
N.S.	1	1.00	0.76	4.40	1.40	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.058	0.780	0.790	0.000	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	64	298	123	0	0	0	-1
N.S.	1	1.00	0.91	4.26	1.76	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.019	0.925	0.646	0.000	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	0	461	0	0	0	0	-1
N.S.	1	1.00	0.00	2.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.308	1.321	0.847	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	419	419	0	1036	0	0	0	0	-1
N.S.	1	1.00	0.00	2.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.396	5.915	0.751	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	171	805	251	0	0	0	-1
N.S.	1	1.00	0.84	3.95	1.23	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.184	0.119	0.836	0.350	0.000	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	433	589	222	0	0	0	-1
N.S.	1	1.00	2.78	3.78	1.42	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.188	1.130	1.090	0.411	0.000	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	0	810	0	0	0	0	-1
N.S.	1	1.00	0.00	2.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.452	0.874	0.937	0.000	0.000	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	23	114	25	0	0	-1
N.S.	1	1.00	1.00	0.92	4.56	1.00	0.00	0.00	-0.04
time (sec)	N/A	0.104	0.060	1.178	0.371	0.378	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	23	118	25	0	0	21
N.S.	1	1.00	1.00	0.92	4.72	1.00	0.00	0.00	0.84
time (sec)	N/A	0.065	0.024	0.506	0.292	0.388	0.000	0.000	0.647

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	34	24	0	30	0	0	-1
N.S.	1	1.00	1.31	0.92	0.00	1.15	0.00	0.00	-0.04
time (sec)	N/A	0.102	0.054	2.566	0.000	0.447	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	608	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.053	0.254	0.278	0.000	0.000	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	307	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.051	0.199	0.291	0.000	0.000	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.069	0.252	0.313	0.000	0.000	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.072	0.221	0.349	0.000	0.000	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.057	1.590	0.428	0.000	0.000	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.055	1.163	0.699	0.000	0.000	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.069	1.199	0.772	0.000	0.000	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.072	0.176	0.618	0.000	0.000	0.000	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	75	66	0	80	0	0	-1
N.S.	1	1.00	1.09	0.96	0.00	1.16	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.045	0.691	0.000	0.359	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	12	34	9	59	11	0	68	8
N.S.	1	1.50	4.25	1.12	7.38	1.38	0.00	8.50	1.00
time (sec)	N/A	0.007	0.004	0.439	0.372	0.365	0.000	4.213	0.314

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	86	69	15	0	0	10
N.S.	1	1.00	1.00	7.17	5.75	1.25	0.00	0.00	0.83
time (sec)	N/A	0.012	0.004	0.094	0.371	0.345	0.000	0.000	0.289

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	0	19	0	0	-1
N.S.	1	1.00	1.00	1.07	0.00	1.36	0.00	0.00	-0.07
time (sec)	N/A	0.012	0.005	1.834	0.000	0.415	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	36	34	67	0	0	204	37
N.S.	1	1.00	1.03	0.97	1.91	0.00	0.00	5.83	1.06
time (sec)	N/A	0.040	0.005	0.720	0.275	0.000	0.000	4.340	0.406

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	40	118	77	0	0	0	33
N.S.	1	1.00	1.03	3.03	1.97	0.00	0.00	0.00	0.85
time (sec)	N/A	0.033	0.005	0.033	0.278	0.000	0.000	0.000	0.440

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	44	0	67	0	0	-1
N.S.	1	1.00	0.94	0.94	0.00	1.43	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.013	1.960	0.000	0.351	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	80	126	124	0	0	0	-1
N.S.	1	1.00	0.76	1.20	1.18	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.022	2.334	0.336	0.000	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	228	330	0	0	0	0	-1
N.S.	1	1.00	1.00	1.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.268	0.078	0.880	0.000	0.000	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	0.309	0.403	0.000	0.000	0.000	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	82	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.040	0.084	0.000	0.000	0.000	0.000	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	159	0	124	137	155	357	137
N.S.	1	1.00	0.96	0.00	0.75	0.83	0.93	2.15	0.83
time (sec)	N/A	0.090	0.095	0.066	0.275	0.456	9.719	2.708	0.514

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	131	0	104	113	128	271	111
N.S.	1	1.00	0.98	0.00	0.78	0.84	0.96	2.02	0.83
time (sec)	N/A	0.067	0.065	0.013	0.284	0.392	3.825	5.634	0.421

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	107	0	84	88	100	185	85
N.S.	1	1.00	1.05	0.00	0.82	0.86	0.98	1.81	0.83
time (sec)	N/A	0.049	0.026	0.014	0.281	0.378	1.902	4.158	0.409

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	53	59	60	66	107	52
N.S.	1	1.00	1.00	0.88	0.98	1.00	1.10	1.78	0.87
time (sec)	N/A	0.029	0.023	0.042	0.326	0.398	0.685	4.827	0.431

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	53	0	122	0	0	0	-1
N.S.	1	1.00	1.04	0.00	2.39	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.035	0.005	0.025	0.408	0.000	0.000	0.000	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	67	0	66	64	410	187	58
N.S.	1	1.00	0.96	0.00	0.94	0.91	5.86	2.67	0.83
time (sec)	N/A	0.039	0.035	0.012	0.271	0.392	14.690	3.943	0.802

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	104	0	84	95	493	366	83
N.S.	1	1.00	0.95	0.00	0.77	0.87	4.52	3.36	0.76
time (sec)	N/A	0.052	0.029	0.014	0.290	0.366	107.565	4.836	0.634

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	132	0	104	120	0	542	110
N.S.	1	1.00	0.94	0.00	0.74	0.85	0.00	3.84	0.78
time (sec)	N/A	0.066	0.093	0.014	0.274	0.374	0.000	4.914	0.756

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	480	480	341	0	317	451	0	956	434
N.S.	1	1.00	0.71	0.00	0.66	0.94	0.00	1.99	0.90
time (sec)	N/A	0.334	0.244	0.025	0.286	0.378	0.000	4.053	1.782

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	269	0	256	333	0	642	420
N.S.	1	1.00	0.79	0.00	0.75	0.97	0.00	1.88	1.23
time (sec)	N/A	0.243	0.165	0.031	0.294	0.385	0.000	4.238	0.563

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	150	0	184	213	0	361	186
N.S.	1	1.00	0.77	0.00	0.94	1.09	0.00	1.85	0.95
time (sec)	N/A	0.126	0.059	0.019	0.282	0.398	0.000	5.500	0.474

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	195	0	0	0	0	0	-1
N.S.	1	1.00	2.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.076	0.025	0.000	0.000	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	188	0	0	0	0	0	-1
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.192	0.120	0.020	0.000	0.000	0.000	0.000	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	353	0	0	0	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.397	0.229	0.035	0.000	0.000	0.000	0.000	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	538	0	0	0	0	0	-1
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.618	0.201	0.027	0.000	0.000	0.000	0.000	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	907	907	661	0	655	1098	0	2223	976
N.S.	1	1.00	0.73	0.00	0.72	1.21	0.00	2.45	1.08
time (sec)	N/A	0.666	0.449	0.043	0.323	0.436	0.000	4.968	8.180

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	595	595	517	0	537	798	0	1483	840
N.S.	1	1.00	0.87	0.00	0.90	1.34	0.00	2.49	1.41
time (sec)	N/A	0.404	0.321	0.037	0.314	0.445	0.000	4.574	0.771

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	241	0	394	496	0	763	350
N.S.	1	1.00	0.85	0.00	1.39	1.75	0.00	2.69	1.23
time (sec)	N/A	0.168	0.144	0.011	0.302	0.419	0.000	2.593	0.602

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	333	0	0	0	0	0	-1
N.S.	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.093	0.038	0.000	0.000	0.000	0.000	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	536	0	0	0	0	0	-1
N.S.	1	1.00	2.04	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.326	0.542	0.040	0.000	0.000	0.000	0.000	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	573	573	841	0	0	0	0	0	-1
N.S.	1	1.00	1.47	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.853	0.772	0.048	0.000	0.000	0.000	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	158	0	115	176	162	272	140
N.S.	1	1.00	0.92	0.00	0.67	1.03	0.95	1.59	0.82
time (sec)	N/A	0.085	0.096	0.014	0.289	0.383	59.311	4.712	0.977

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	130	0	95	152	134	236	106
N.S.	1	1.00	0.94	0.00	0.68	1.09	0.96	1.70	0.76
time (sec)	N/A	0.068	0.063	0.022	0.300	0.381	21.147	3.093	0.694

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	102	0	75	127	88	81	86
N.S.	1	1.00	0.95	0.00	0.70	1.19	0.82	0.76	0.80
time (sec)	N/A	0.048	0.023	0.016	0.312	0.391	7.493	4.255	0.831

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	62	94	52	93	76	56	44
N.S.	1	1.00	1.17	1.77	0.98	1.75	1.43	1.06	0.83
time (sec)	N/A	0.026	0.025	0.054	0.287	0.388	3.458	4.148	0.380

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	53	0	127	0	0	0	-1
N.S.	1	1.00	1.04	0.00	2.49	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.004	0.026	0.690	0.000	0.000	0.000	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	68	63	76	67	360	162	60
N.S.	1	1.00	1.05	0.97	1.17	1.03	5.54	2.49	0.92
time (sec)	N/A	0.035	0.025	0.028	0.285	0.360	51.831	3.446	0.425

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	109	0	94	91	0	349	87
N.S.	1	1.00	1.05	0.00	0.90	0.88	0.00	3.36	0.84
time (sec)	N/A	0.050	0.050	0.017	0.276	0.379	0.000	3.320	0.417

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	133	0	114	116	0	535	113
N.S.	1	1.00	0.98	0.00	0.84	0.85	0.00	3.93	0.83
time (sec)	N/A	0.063	0.062	0.014	0.294	0.428	0.000	4.291	0.442

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	404	379	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.621	0.724	0.025	0.000	0.000	0.000	0.000	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	307	0	0	0	0	0	-1
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.369	0.370	0.030	0.000	0.000	0.000	0.000	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	170	0	0	0	0	0	-1
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.195	0.093	0.016	0.000	0.000	0.000	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	386	0	0	0	0	0	-1
N.S.	1	1.00	4.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.225	0.026	0.000	0.000	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	298	0	252	227	0	503	193
N.S.	1	1.00	1.53	0.00	1.29	1.16	0.00	2.58	0.99
time (sec)	N/A	0.128	0.213	0.028	0.280	0.373	0.000	5.204	0.474

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	473	0	319	342	0	1071	424
N.S.	1	1.00	1.39	0.00	0.94	1.00	0.00	3.14	1.24
time (sec)	N/A	0.246	0.246	0.030	0.307	0.373	0.000	4.361	0.559

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	480	480	692	0	379	460	0	1639	440
N.S.	1	1.00	1.44	0.00	0.79	0.96	0.00	3.41	0.92
time (sec)	N/A	0.324	0.232	0.030	0.327	0.384	0.000	3.755	1.772

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	569	569	777	0	0	0	0	0	-1
N.S.	1	1.00	1.37	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.868	0.608	0.058	0.000	0.000	0.000	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	476	0	0	0	0	0	-1
N.S.	1	1.00	1.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.341	0.431	0.040	0.000	0.000	0.000	0.000	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	532	0	0	0	0	0	-1
N.S.	1	1.00	3.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.207	0.059	0.000	0.000	0.000	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	558	0	579	519	0	1127	357
N.S.	1	1.00	1.96	0.00	2.03	1.82	0.00	3.95	1.25
time (sec)	N/A	0.185	0.372	0.060	0.335	0.401	0.000	4.677	0.625

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	595	595	766	0	731	815	0	2389	846
N.S.	1	1.00	1.29	0.00	1.23	1.37	0.00	4.02	1.42
time (sec)	N/A	0.413	0.635	0.065	0.320	0.434	0.000	4.605	0.800

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	907	907	950	0	851	1115	0	3651	989
N.S.	1	1.00	1.05	0.00	0.94	1.23	0.00	4.03	1.09
time (sec)	N/A	0.637	1.030	0.359	0.335	0.425	0.000	6.086	8.179

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	219	0	164	186	216	529	189
N.S.	1	1.00	0.94	0.00	0.70	0.79	0.92	2.26	0.81
time (sec)	N/A	0.121	0.163	0.012	0.278	0.454	44.141	3.652	0.652

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	176	0	134	149	173	400	150
N.S.	1	1.00	0.95	0.00	0.72	0.81	0.94	2.16	0.81
time (sec)	N/A	0.090	0.100	0.015	0.272	0.380	9.844	5.900	0.506

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	133	0	104	114	131	271	111
N.S.	1	1.00	0.98	0.00	0.76	0.84	0.96	1.99	0.82
time (sec)	N/A	0.063	0.071	0.012	0.285	0.399	2.698	4.374	0.416

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	66	70	71	82	135	65
N.S.	1	1.00	1.00	0.86	0.91	0.92	1.06	1.75	0.84
time (sec)	N/A	0.039	0.033	0.025	0.269	0.376	0.746	4.377	0.342

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	53	0	183	0	0	0	-1
N.S.	1	1.00	1.04	0.00	3.59	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.004	0.011	0.435	0.000	0.000	0.000	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	84	0	77	79	450	280	74
N.S.	1	1.00	0.97	0.00	0.89	0.91	5.17	3.22	0.85
time (sec)	N/A	0.046	0.028	0.022	0.275	0.496	106.562	4.200	0.592

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	134	0	104	120	0	542	109
N.S.	1	1.00	0.94	0.00	0.73	0.84	0.00	3.79	0.76
time (sec)	N/A	0.062	0.100	0.011	0.275	0.373	0.000	3.933	0.657

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	177	0	135	156	0	808	154
N.S.	1	1.00	0.92	0.00	0.70	0.81	0.00	4.21	0.80
time (sec)	N/A	0.085	0.137	0.024	0.278	0.393	0.000	6.250	0.612

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	680	680	458	0	407	623	0	1427	608
N.S.	1	1.00	0.67	0.00	0.60	0.92	0.00	2.10	0.89
time (sec)	N/A	0.465	0.378	0.018	0.293	0.426	0.000	3.773	4.703

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	480	480	347	0	316	452	0	956	431
N.S.	1	1.00	0.72	0.00	0.66	0.94	0.00	1.99	0.90
time (sec)	N/A	0.311	0.255	0.023	0.307	0.402	0.000	3.286	1.720

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	249	0	217	272	0	479	290
N.S.	1	1.00	0.93	0.00	0.81	1.02	0.00	1.79	1.09
time (sec)	N/A	0.199	0.102	0.011	0.282	0.371	0.000	4.153	0.511

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	195	0	0	0	0	0	-1
N.S.	1	1.00	2.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.066	0.021	0.000	0.000	0.000	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	274	0	0	0	0	0	-1
N.S.	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.288	0.163	0.024	0.000	0.000	0.000	0.000	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	405	533	0	0	0	0	0	-1
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.627	0.195	0.017	0.000	0.000	0.000	0.000	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1835	1835	1117	0	1017	1985	0	4443	1802
N.S.	1	1.00	0.61	0.00	0.55	1.08	0.00	2.42	0.98
time (sec)	N/A	1.434	1.049	0.039	0.318	0.490	0.000	4.077	8.472

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1357	1357	895	0	835	1544	0	3333	1386
N.S.	1	1.00	0.66	0.00	0.62	1.14	0.00	2.46	1.02
time (sec)	N/A	1.019	0.708	0.028	0.311	0.455	0.000	3.780	8.254

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	907	907	673	0	657	1100	0	2223	979
N.S.	1	1.00	0.74	0.00	0.72	1.21	0.00	2.45	1.08
time (sec)	N/A	0.633	0.497	0.048	0.296	0.412	0.000	3.033	8.059

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	438	463	0	459	646	0	1105	558
N.S.	1	1.00	1.06	0.00	1.05	1.47	0.00	2.52	1.27
time (sec)	N/A	0.288	0.255	0.014	0.307	0.404	0.000	3.350	0.688

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	333	0	0	0	0	0	-1
N.S.	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.104	0.046	0.000	0.000	0.000	0.000	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	439	439	733	0	0	0	0	0	-1
N.S.	1	1.00	1.67	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.551	0.562	0.048	0.000	0.000	0.000	0.000	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	765	765	1074	0	0	0	0	0	-1
N.S.	1	1.00	1.40	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.732	1.151	0.050	0.000	0.000	0.000	0.000	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	135	0	106	121	0	261	113
N.S.	1	1.00	0.98	0.00	0.77	0.88	0.00	1.89	0.82
time (sec)	N/A	0.071	0.085	0.015	0.285	0.378	0.000	4.407	0.446

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	135	0	95	307	0	104	-1
N.S.	1	1.00	1.04	0.00	0.73	2.36	0.00	0.80	-0.01
time (sec)	N/A	0.057	0.072	0.013	0.512	0.420	0.000	3.088	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	94	0	76	77	95	82	74
N.S.	1	1.00	1.06	0.00	0.85	0.87	1.07	0.92	0.83
time (sec)	N/A	0.043	0.023	0.022	0.269	0.404	54.921	4.971	0.390

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	62	61	218	124	68	56
N.S.	1	1.00	1.00	0.86	0.85	3.03	1.72	0.94	0.78
time (sec)	N/A	0.037	0.022	0.028	0.506	0.383	2.403	4.112	0.394

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	0	128	0	0	0	-1
N.S.	1	1.00	1.00	0.00	2.33	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.035	0.011	0.017	0.409	0.000	0.000	0.000	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	59	0	56	210	0	61	-1
N.S.	1	1.00	0.87	0.00	0.82	3.09	0.00	0.90	-0.01
time (sec)	N/A	0.027	0.015	0.015	0.501	0.384	0.000	4.897	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	91	0	79	84	0	95	74
N.S.	1	1.00	0.97	0.00	0.84	0.89	0.00	1.01	0.79
time (sec)	N/A	0.047	0.027	0.024	0.270	0.366	0.000	5.206	0.610

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	65	0	88	303	0	94	-1
N.S.	1	1.00	0.53	0.00	0.72	2.46	0.00	0.76	-0.01
time (sec)	N/A	0.052	0.012	0.015	0.505	0.395	0.000	4.921	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	482	482	328	0	323	473	0	933	440
N.S.	1	1.00	0.68	0.00	0.67	0.98	0.00	1.94	0.91
time (sec)	N/A	0.329	0.261	0.022	0.318	0.437	0.000	4.442	1.754

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	239	0	232	286	0	316	299
N.S.	1	1.00	0.87	0.00	0.84	1.04	0.00	1.15	1.09
time (sec)	N/A	0.209	0.110	0.030	0.297	0.405	0.000	4.878	0.529

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	199	0	156	0	0	0	-1
N.S.	1	1.00	2.09	0.00	1.64	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.071	0.027	0.341	0.000	0.000	0.000	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	264	0	0	0	0	0	-1
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.281	0.207	0.029	0.000	0.000	0.000	0.000	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	539	0	0	0	0	0	-1
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.609	0.196	0.024	0.000	0.000	0.000	0.000	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	547	547	438	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.503	0.331	0.024	0.000	0.000	0.000	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	319	0	0	0	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.305	0.153	0.020	0.000	0.000	0.000	0.000	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	247	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.267	0.132	0.026	0.000	0.000	0.000	0.000	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	473	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.406	0.372	0.018	0.000	0.000	0.000	0.000	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	640	640	678	0	0	0	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.627	0.747	0.026	0.000	0.000	0.000	0.000	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	913	913	598	0	669	1142	0	2174	992
N.S.	1	1.00	0.65	0.00	0.73	1.25	0.00	2.38	1.09
time (sec)	N/A	0.672	0.721	0.030	0.310	0.454	0.000	4.297	8.104

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	428	0	489	671	0	778	575
N.S.	1	1.00	0.95	0.00	1.09	1.49	0.00	1.73	1.28
time (sec)	N/A	0.297	0.293	0.045	0.309	0.459	0.000	4.914	0.723

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	339	0	297	0	0	0	-1
N.S.	1	1.00	2.44	0.00	2.14	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.134	0.119	0.052	0.383	0.000	0.000	0.000	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	451	451	764	0	690	0	0	0	-1
N.S.	1	1.00	1.69	0.00	1.53	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.569	0.552	0.036	0.409	0.000	0.000	0.000	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	794	0	3146	0	0	0	0	0	-1
N.S.	1	0.00	3.96	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.940	8.185	0.046	0.000	0.000	0.000	0.000	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	486	0	598	0	0	0	0	0	-1
N.S.	1	0.00	1.23	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.708	0.897	0.013	0.000	0.000	0.000	0.000	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	0	1028	0	0	0	0	0	-1
N.S.	1	0.00	3.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.326	5.015	0.053	0.000	0.000	0.000	0.000	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	632	0	803	0	0	0	0	0	-1
N.S.	1	0.00	1.27	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.198	2.528	0.051	0.000	0.000	0.000	0.000	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	218	0	155	225	0	161	191
N.S.	1	1.00	0.91	0.00	0.65	0.94	0.00	0.67	0.80
time (sec)	N/A	0.118	0.162	0.013	0.285	0.379	0.000	4.401	0.796

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	175	0	125	188	180	131	153
N.S.	1	1.00	0.92	0.00	0.66	0.99	0.95	0.69	0.81
time (sec)	N/A	0.087	0.093	0.015	0.290	0.376	62.179	4.729	0.677

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	132	0	95	152	138	101	112
N.S.	1	1.00	0.94	0.00	0.67	1.08	0.98	0.72	0.79
time (sec)	N/A	0.063	0.068	0.013	0.289	0.382	13.524	4.464	0.792

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	79	115	62	109	92	66	59
N.S.	1	1.00	1.13	1.64	0.89	1.56	1.31	0.94	0.84
time (sec)	N/A	0.033	0.035	0.049	0.273	0.386	3.670	4.642	0.470

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	53	0	183	0	0	0	-1
N.S.	1	1.00	1.04	0.00	3.59	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.004	0.034	0.730	0.000	0.000	0.000	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	85	0	86	94	398	95	73
N.S.	1	1.00	1.04	0.00	1.05	1.15	4.85	1.16	0.89
time (sec)	N/A	0.047	0.028	0.015	0.280	0.350	237.547	3.987	0.430

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	135	0	114	149	0	123	113
N.S.	1	1.00	0.98	0.00	0.83	1.08	0.00	0.89	0.82
time (sec)	N/A	0.066	0.066	0.024	0.322	0.391	0.000	4.582	0.464

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	178	0	144	196	0	153	152
N.S.	1	1.00	0.95	0.00	0.77	1.05	0.00	0.82	0.81
time (sec)	N/A	0.088	0.111	0.014	0.291	0.394	0.000	3.428	0.526

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	572	572	495	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.078	1.064	0.029	0.000	0.000	0.000	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	400	383	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.623	0.642	0.040	0.000	0.000	0.000	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	336	0	0	0	0	0	-1
N.S.	1	1.00	1.48	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.304	0.271	0.017	0.000	0.000	0.000	0.000	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	389	0	0	0	0	0	-1
N.S.	1	1.00	4.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.137	0.030	0.000	0.000	0.000	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	374	0	285	329	0	787	299
N.S.	1	1.00	1.39	0.00	1.06	1.22	0.00	2.93	1.11
time (sec)	N/A	0.212	0.267	0.029	0.293	0.410	0.000	3.831	0.564

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	479	479	698	0	379	550	0	1639	439
N.S.	1	1.00	1.46	0.00	0.79	1.15	0.00	3.42	0.92
time (sec)	N/A	0.324	0.240	0.030	0.298	0.498	0.000	5.460	1.762

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	759	759	1006	0	0	0	0	0	-1
N.S.	1	1.00	1.33	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.753	1.028	0.069	0.000	0.000	0.000	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	436	675	0	0	0	0	0	-1
N.S.	1	1.00	1.55	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.591	0.461	0.029	0.000	0.000	0.000	0.000	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	527	0	0	0	0	0	-1
N.S.	1	1.00	3.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.197	0.056	0.000	0.000	0.000	0.000	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	438	666	0	645	750	0	1758	570
N.S.	1	1.00	1.52	0.00	1.47	1.71	0.00	4.01	1.30
time (sec)	N/A	0.301	0.508	0.060	0.323	0.416	0.000	3.970	0.740

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	907	907	962	0	851	1290	0	3651	992
N.S.	1	1.00	1.06	0.00	0.94	1.42	0.00	4.03	1.09
time (sec)	N/A	0.642	1.085	0.066	0.338	0.436	0.000	3.532	8.196

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	134	0	97	159	0	103	112
N.S.	1	1.00	0.94	0.00	0.68	1.11	0.00	0.72	0.78
time (sec)	N/A	0.069	0.082	0.015	0.283	0.387	0.000	4.266	0.702

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	65	0	86	389	0	97	-1
N.S.	1	1.00	0.54	0.00	0.71	3.21	0.00	0.80	-0.01
time (sec)	N/A	0.049	0.013	0.015	0.519	0.407	0.000	3.561	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	91	0	66	115	0	72	73
N.S.	1	1.00	0.97	0.00	0.70	1.22	0.00	0.77	0.78
time (sec)	N/A	0.044	0.019	0.013	0.290	0.412	0.000	4.616	0.592

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	53	168	54	274	112	57	51
N.S.	1	1.00	0.82	2.58	0.83	4.22	1.72	0.88	0.78
time (sec)	N/A	0.029	0.012	0.220	0.546	0.403	18.159	3.115	0.429

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	0	130	0	0	0	-1
N.S.	1	1.00	1.00	0.00	2.36	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.035	0.010	0.030	0.733	0.000	0.000	0.000	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	80	0	67	232	0	73	-1
N.S.	1	1.00	1.04	0.00	0.87	3.01	0.00	0.95	-0.01
time (sec)	N/A	0.033	0.036	0.017	0.507	0.484	0.000	4.580	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	94	0	88	80	0	104	74
N.S.	1	1.00	1.06	0.00	0.99	0.90	0.00	1.17	0.83
time (sec)	N/A	0.048	0.025	0.018	0.297	0.407	0.000	4.328	0.439

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	137	0	97	317	0	103	-1
N.S.	1	1.00	1.04	0.00	0.73	2.40	0.00	0.78	-0.01
time (sec)	N/A	0.058	0.048	0.013	0.539	0.366	0.000	3.717	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	968	0	0	0	0	0	-1
N.S.	1	1.00	2.35	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.601	0.320	0.040	0.000	0.000	0.000	0.000	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	542	0	0	0	0	0	-1
N.S.	1	1.00	2.27	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.283	0.315	0.030	0.000	0.000	0.000	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	199	0	0	0	0	0	-1
N.S.	1	1.00	2.09	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.126	0.025	0.000	0.000	0.000	0.000	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	691	0	299	294	0	0	302
N.S.	1	1.00	2.50	0.00	1.08	1.07	0.00	0.00	1.09
time (sec)	N/A	0.206	0.362	0.031	0.301	0.374	0.000	0.000	0.573

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	482	482	1021	0	389	483	0	0	440
N.S.	1	1.00	2.12	0.00	0.81	1.00	0.00	0.00	0.91
time (sec)	N/A	0.315	0.579	0.026	0.314	0.370	0.000	0.000	1.809

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	490	490	735	0	0	0	0	0	-1
N.S.	1	1.00	1.50	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.529	1.456	0.027	0.000	0.000	0.000	0.000	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	296	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.291	0.699	0.014	0.000	0.000	0.000	0.000	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	598	0	0	0	0	0	-1
N.S.	1	1.00	1.66	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.381	0.890	0.031	0.000	0.000	0.000	0.000	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	773	773	1014	0	0	0	0	0	-1
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.782	1.439	0.063	0.000	0.000	0.000	0.000	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	451	451	683	0	0	0	0	0	-1
N.S.	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.573	0.695	0.039	0.000	0.000	0.000	0.000	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	341	0	0	0	0	0	-1
N.S.	1	1.00	2.45	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.238	0.061	0.000	0.000	0.000	0.000	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	692	0	689	686	0	0	578
N.S.	1	1.00	1.54	0.00	1.53	1.53	0.00	0.00	1.29
time (sec)	N/A	0.305	0.745	0.045	0.316	0.400	0.000	0.000	0.745

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1278	0	764	0	0	0	0	0	-1
N.S.	1	0.00	0.60	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.992	2.941	0.058	0.000	0.000	0.000	0.000	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	738	0	824	0	0	0	0	0	-1
N.S.	1	0.00	1.12	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.813	3.517	0.026	0.000	0.000	0.000	0.000	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	483	0	1097	0	0	0	0	0	-1
N.S.	1	0.00	2.27	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.866	1.299	0.057	0.000	0.000	0.000	0.000	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	784	0	2726	0	0	0	0	0	-1
N.S.	1	0.00	3.48	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.267	5.979	0.065	0.000	0.000	0.000	0.000	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	730	730	435	0	0	0	0	0	-1
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.865	1.235	0.057	0.000	0.000	0.000	0.000	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	551	551	325	0	0	0	0	0	-1
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.569	1.133	0.012	0.000	0.000	0.000	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	229	0	0	0	0	0	-1
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.363	0.638	0.003	0.000	0.000	0.000	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	130	0	0	0	0	0	-1
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.151	0.330	0.003	0.000	0.000	0.000	0.000	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.033	0.257	0.003	0.000	0.000	0.000	0.000	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.032	0.430	0.002	0.000	0.000	0.000	0.000	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	907	907	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.944	0.346	0.034	0.000	0.000	0.000	0.000	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	677	677	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.643	0.234	0.003	0.000	0.000	0.000	0.000	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	445	445	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.417	0.170	0.004	0.000	0.000	0.000	0.000	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.173	0.089	0.002	0.000	0.000	0.000	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	0.088	0.003	0.000	0.000	0.000	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.033	0.089	0.003	0.000	0.000	0.000	0.000	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.028	0.892	0.016	0.000	0.000	0.000	0.000	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.013	0.222	0.004	0.000	0.000	0.000	0.000	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.033	0.270	0.006	0.000	0.000	0.000	0.000	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	131	0	0	0	0	0	-1
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.161	0.279	0.005	0.000	0.000	0.000	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	552	552	325	0	0	0	0	0	-1
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.564	0.927	0.013	0.000	0.000	0.000	0.000	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	926	926	525	0	0	0	0	0	-1
N.S.	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.037	3.738	0.005	0.000	0.000	0.000	0.000	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.031	0.168	0.017	0.000	0.000	0.000	0.000	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.014	0.078	0.004	0.000	0.000	0.000	0.000	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	0.127	0.004	0.000	0.000	0.000	0.000	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	216	213	0	0	0	0	0	0	-1
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.197	0.102	0.004	0.000	0.000	0.000	0.000	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	676	676	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.664	0.103	0.004	0.000	0.000	0.000	0.000	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1141	1141	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.187	0.101	0.005	0.000	0.000	0.000	0.000	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1121	1121	670	0	0	0	0	0	-1
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.259	3.347	0.005	0.000	0.000	0.000	0.000	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	831	831	501	0	0	0	0	0	-1
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.907	1.264	0.003	0.000	0.000	0.000	0.000	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	553	553	325	0	0	0	0	0	-1
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.560	1.112	0.002	0.000	0.000	0.000	0.000	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	174	0	0	0	0	0	-1
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.253	0.385	0.011	0.000	0.000	0.000	0.000	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.033	0.219	0.001	0.000	0.000	0.000	0.000	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.034	0.507	0.003	0.000	0.000	0.000	0.000	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1363	1363	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.454	0.470	0.006	0.000	0.000	0.000	0.000	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1035	1035	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.020	0.325	0.003	0.000	0.000	0.000	0.000	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	673	673	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.648	0.229	0.002	0.000	0.000	0.000	0.000	0.000

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.296	0.095	0.004	0.000	0.000	0.000	0.000	0.000

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.034	0.085	0.003	0.000	0.000	0.000	0.000	0.000

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.033	0.087	0.004	0.000	0.000	0.000	0.000	0.000

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	557	557	325	0	0	0	0	0	-1
N.S.	1	1.00	0.58	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.582	1.175	0.006	0.000	0.000	0.000	0.000	0.000

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	181	0	0	0	0	0	-1
N.S.	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.261	0.409	0.002	0.000	0.000	0.000	0.000	0.000

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.034	0.246	0.003	0.000	0.000	0.000	0.000	0.000

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.035	0.471	0.003	0.000	0.000	0.000	0.000	0.000

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	0.604	0.002	0.000	0.000	0.000	0.000	0.000

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.021	0.171	0.003	0.000	0.000	0.000	0.000	0.000

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.034	0.435	0.003	0.000	0.000	0.000	0.000	0.000

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	678	675	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.670	0.363	0.015	0.000	0.000	0.000	0.000	0.000

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	350	347	0	0	0	0	0	0	-1
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.314	0.210	0.002	0.000	0.000	0.000	0.000	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	0.118	0.004	0.000	0.000	0.000	0.000	0.000

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.035	0.117	0.003	0.000	0.000	0.000	0.000	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.037	0.128	0.002	0.000	0.000	0.000	0.000	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.022	0.071	0.003	0.000	0.000	0.000	0.000	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	0.093	0.003	0.000	0.000	0.000	0.000	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.028	1.290	0.009	0.000	0.000	0.000	0.000	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.021	0.241	0.013	0.000	0.000	0.000	0.000	0.000

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.034	0.270	0.004	0.000	0.000	0.000	0.000	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	175	0	0	0	0	0	-1
N.S.	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.266	0.366	0.005	0.000	0.000	0.000	0.000	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	554	554	325	0	0	0	0	0	-1
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.595	0.945	0.003	0.000	0.000	0.000	0.000	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	832	832	502	0	0	0	0	0	-1
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.894	0.988	0.005	0.000	0.000	0.000	0.000	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.030	0.173	0.016	0.000	0.000	0.000	0.000	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.023	0.083	0.004	0.000	0.000	0.000	0.000	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	0.128	0.004	0.000	0.000	0.000	0.000	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	342	339	0	0	0	0	0	0	-1
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.333	0.101	0.004	0.000	0.000	0.000	0.000	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	673	673	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.678	0.104	0.004	0.000	0.000	0.000	0.000	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1036	1036	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.046	0.101	0.004	0.000	0.000	0.000	0.000	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	1.074	0.009	0.000	0.000	0.000	0.000	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	1.182	0.005	0.000	0.000	0.000	0.000	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.028	0.603	0.005	0.000	0.000	0.000	0.000	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.022	0.226	0.010	0.000	0.000	0.000	0.000	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.034	0.367	0.005	0.000	0.000	0.000	0.000	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	0.361	0.005	0.000	0.000	0.000	0.000	0.000

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.037	0.214	0.018	0.000	0.000	0.000	0.000	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.037	0.156	0.005	0.000	0.000	0.000	0.000	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.029	0.187	0.005	0.000	0.000	0.000	0.000	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.023	0.086	0.004	0.000	0.000	0.000	0.000	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	0.126	0.004	0.000	0.000	0.000	0.000	0.000

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.037	0.105	0.005	0.000	0.000	0.000	0.000	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	631	631	344	0	538	1263	0	514	-1
N.S.	1	1.00	0.55	0.00	0.85	2.00	0.00	0.81	-0.00
time (sec)	N/A	0.588	0.322	0.064	0.512	0.439	0.000	5.656	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	603	603	316	0	517	1254	0	431	-1
N.S.	1	1.00	0.52	0.00	0.86	2.08	0.00	0.71	-0.00
time (sec)	N/A	0.535	0.307	0.018	0.516	0.400	0.000	3.140	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	588	588	271	0	495	1339	0	443	-1
N.S.	1	1.00	0.46	0.00	0.84	2.28	0.00	0.75	-0.00
time (sec)	N/A	0.479	0.284	0.021	0.522	0.391	0.000	4.399	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	620	620	309	0	512	1393	0	478	-1
N.S.	1	1.00	0.50	0.00	0.83	2.25	0.00	0.77	-0.00
time (sec)	N/A	0.520	0.118	0.017	0.525	0.445	0.000	3.095	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	641	641	100	0	526	1408	0	488	-1
N.S.	1	1.00	0.16	0.00	0.82	2.20	0.00	0.76	-0.00
time (sec)	N/A	0.562	0.050	0.029	0.512	0.422	0.000	5.446	0.000

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1002	1002	588	0	847	2326	0	820	-1
N.S.	1	1.00	0.59	0.00	0.85	2.32	0.00	0.82	-0.00
time (sec)	N/A	0.877	0.896	0.262	0.523	21.958	0.000	4.019	0.000

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	949	949	436	0	793	2334	0	649	-1
N.S.	1	1.00	0.46	0.00	0.84	2.46	0.00	0.68	-0.00
time (sec)	N/A	0.811	0.576	0.236	0.534	5.424	0.000	6.280	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	932	932	503	0	786	2347	0	638	-1
N.S.	1	1.00	0.54	0.00	0.84	2.52	0.00	0.68	-0.00
time (sec)	N/A	0.780	0.648	0.194	0.527	0.476	0.000	3.321	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	935	935	340	0	767	2420	0	660	-1
N.S.	1	1.00	0.36	0.00	0.82	2.59	0.00	0.71	-0.00
time (sec)	N/A	0.777	0.633	0.243	0.527	6.228	0.000	4.904	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	968	968	294	0	795	2436	0	674	-1
N.S.	1	1.00	0.30	0.00	0.82	2.52	0.00	0.70	-0.00
time (sec)	N/A	0.804	0.170	0.217	0.549	0.483	0.000	5.683	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1680	1680	1471	0	0	0	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.031	0.913	0.307	0.000	0.000	0.000	0.000	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1361	1361	1297	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.171	0.280	0.305	0.000	0.000	0.000	0.000	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1659	1659	1336	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.656	1.032	0.304	0.000	0.000	0.000	0.000	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	191	0	37	0	0	-1
N.S.	1	1.00	1.00	5.79	0.00	1.12	0.00	0.00	-0.03
time (sec)	N/A	0.017	0.011	0.449	0.000	0.367	0.000	0.000	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	220	1373	0	108	0	0	-1
N.S.	1	1.00	2.93	18.31	0.00	1.44	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.175	0.919	0.000	0.358	0.000	0.000	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	659	0	0	423	0	0	-1
N.S.	1	1.00	4.09	0.00	0.00	2.63	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.182	0.082	0.000	0.364	0.000	0.000	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	456	0	0	286	0	0	-1
N.S.	1	1.00	3.45	0.00	0.00	2.17	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.160	0.037	0.000	0.391	0.000	0.000	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	265	0	0	165	0	0	-1
N.S.	1	1.00	2.60	0.00	0.00	1.62	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.125	0.028	0.000	0.431	0.000	0.000	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	189	0	72	0	0	-1
N.S.	1	1.00	1.00	3.86	0.00	1.47	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.010	1.457	0.000	0.353	0.000	0.000	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.196	0.387	0.025	0.000	0.000	0.000	0.000	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.076	1.768	0.027	0.000	0.000	0.000	0.000	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	7.952	0.029	0.000	0.000	0.000	0.000	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	65	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.020	0.058	0.000	0.000	0.000	0.000	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	147	154	0	1346	0	265	192
N.S.	1	1.00	0.87	0.91	0.00	7.96	0.00	1.57	1.14
time (sec)	N/A	0.141	0.073	0.723	0.000	1.130	0.000	6.349	0.280

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	115	101	190	235	96	82
N.S.	1	1.00	1.00	1.83	1.60	3.02	3.73	1.52	1.30
time (sec)	N/A	0.035	0.025	0.310	0.518	0.344	104.352	7.278	0.134

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	44	57	57	50	80	69	46
N.S.	1	1.00	1.26	1.63	1.63	1.43	2.29	1.97	1.31
time (sec)	N/A	0.011	0.022	0.223	0.303	0.349	0.370	5.047	0.307

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	56	71	67	70	102	172	67
N.S.	1	1.00	1.24	1.58	1.49	1.56	2.27	3.82	1.49
time (sec)	N/A	0.019	0.035	0.053	0.290	0.343	0.655	8.945	0.177

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	61	129	101	288	0	137	163
N.S.	1	1.00	1.03	2.19	1.71	4.88	0.00	2.32	2.76
time (sec)	N/A	0.026	0.037	0.128	0.546	0.403	0.000	4.181	0.522

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	66	168	0	1424	0	319	499
N.S.	1	1.00	0.40	1.02	0.00	8.63	0.00	1.93	3.02
time (sec)	N/A	0.114	0.079	0.059	0.000	1.110	0.000	52.543	0.633

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.004	0.260	0.080	0.000	0.000	0.000	0.000	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	739	0	0	0	0	0	-1
N.S.	1	1.00	3.34	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.181	0.709	0.046	0.000	0.000	0.000	0.000	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	415	0	0	0	0	0	-1
N.S.	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.321	0.005	0.000	0.000	0.000	0.000	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	219	0	0	0	0	0	-1
N.S.	1	1.00	1.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.181	0.044	0.000	0.000	0.000	0.000	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	70	81	72	80	107	177	61
N.S.	1	1.00	1.40	1.62	1.44	1.60	2.14	3.54	1.22
time (sec)	N/A	0.026	0.027	0.030	0.387	0.401	0.569	5.627	0.419

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.004	0.368	0.039	0.000	0.000	0.000	0.000	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.004	0.580	0.044	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [378] had the largest ratio of [33]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	16	0.188
2	A	4	3	1.00	16	0.188
3	A	4	3	1.00	16	0.188
4	A	3	3	1.00	14	0.214
5	A	3	3	1.00	12	0.250
6	A	3	3	1.00	16	0.188
7	A	2	2	1.00	16	0.125
8	A	5	5	1.18	16	0.312
9	A	3	3	1.00	16	0.188
10	A	4	3	1.00	16	0.188
11	A	4	3	1.00	16	0.188
12	A	4	3	1.00	16	0.188
13	A	4	3	1.00	16	0.188
14	A	9	8	1.00	16	0.500
15	A	9	8	1.00	16	0.500
16	A	3	3	1.00	16	0.188
17	A	8	8	1.00	14	0.571
18	A	8	8	1.00	12	0.667
19	A	3	3	1.00	16	0.188
20	A	7	7	1.00	16	0.438
21	A	7	7	1.00	16	0.438
22	A	5	5	1.00	16	0.312
23	A	8	8	1.00	16	0.500
24	A	8	8	1.00	16	0.500
25	A	4	3	1.00	16	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	4	3	1.00	16	0.188
27	A	4	3	1.00	16	0.188
28	A	4	3	1.00	16	0.188
29	A	4	3	1.00	14	0.214
30	A	3	3	1.00	12	0.250
31	A	3	3	1.00	16	0.188
32	A	3	3	1.00	16	0.188
33	A	4	3	1.00	16	0.188
34	A	4	3	1.00	16	0.188
35	A	4	3	1.00	16	0.188
36	A	5	4	1.00	16	0.250
37	A	5	4	1.00	16	0.250
38	A	4	4	1.00	16	0.250
39	A	3	3	1.00	14	0.214
40	A	3	3	1.00	12	0.250
41	A	3	3	1.00	16	0.188
42	A	4	4	1.00	16	0.250
43	A	3	3	1.00	16	0.188
44	A	5	4	1.00	16	0.250
45	A	1	1	1.00	12	0.083
46	A	4	3	1.00	18	0.167
47	A	4	3	1.00	18	0.167
48	A	4	3	1.00	16	0.188
49	A	4	3	1.00	14	0.214
50	A	3	3	1.00	18	0.167
51	A	4	3	1.00	18	0.167
52	A	4	3	1.00	18	0.167
53	A	4	3	1.00	18	0.167
54	A	3	3	1.00	16	0.188
55	A	3	3	1.00	18	0.167
56	A	3	3	1.00	18	0.167
57	A	2	2	1.00	16	0.125
58	A	4	4	1.00	18	0.222
59	A	4	4	1.00	18	0.222
60	A	4	4	1.00	18	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	4	4	1.00	20	0.200
62	A	5	5	1.00	20	0.250
63	A	3	3	1.00	18	0.167
64	A	5	4	1.00	22	0.182
65	A	5	4	1.00	22	0.182
66	A	5	4	1.00	20	0.200
67	A	4	4	1.00	19	0.210
68	A	6	6	1.00	22	0.273
69	A	5	4	1.00	22	0.182
70	A	2	2	1.00	16	0.125
71	A	2	2	1.00	14	0.143
72	A	2	2	1.00	12	0.167
73	A	3	3	1.00	16	0.188
74	A	2	2	1.00	16	0.125
75	A	2	2	1.00	16	0.125
76	A	2	2	1.00	16	0.125
77	A	8	8	1.00	18	0.444
78	A	9	8	1.00	18	0.444
79	A	4	4	1.00	16	0.250
80	A	5	5	1.00	18	0.278
81	A	4	4	1.00	18	0.222
82	A	8	8	1.00	18	0.444
83	A	12	10	1.00	18	0.556
84	A	20	13	1.00	18	0.722
85	A	16	13	1.00	18	0.722
86	A	12	11	1.00	14	0.786
87	A	7	8	1.00	18	0.444
88	A	11	10	1.00	18	0.556
89	A	14	11	1.00	18	0.611
90	A	18	11	1.00	18	0.611
91	A	15	8	1.00	18	0.444
92	A	11	8	1.00	18	0.444
93	A	5	4	1.00	16	0.250
94	A	6	6	1.00	18	0.333
95	A	6	6	1.00	18	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	10	10	1.00	18	0.556
97	A	17	13	1.00	18	0.722
98	A	0	0	0.00	0	0.000
99	A	0	0	0.00	0	0.000
100	A	0	0	0.00	0	0.000
101	A	0	0	0.00	0	0.000
102	A	9	7	1.00	18	0.389
103	A	4	4	1.00	16	0.250
104	A	0	0	0.00	0	0.000
105	A	0	0	0.00	0	0.000
106	A	0	0	0.00	0	0.000
107	A	0	0	0.00	0	0.000
108	A	0	0	0.00	0	0.000
109	A	13	8	1.00	18	0.444
110	A	5	5	1.00	16	0.312
111	A	0	0	0.00	0	0.000
112	A	0	0	0.00	0	0.000
113	A	0	0	0.00	0	0.000
114	A	0	0	0.00	0	0.000
115	A	0	0	0.00	0	0.000
116	A	18	9	1.00	18	0.500
117	A	6	5	1.00	16	0.312
118	A	0	0	0.00	0	0.000
119	A	0	0	0.00	0	0.000
120	A	0	0	0.00	0	0.000
121	A	0	0	0.00	0	0.000
122	A	0	0	0.00	0	0.000
123	A	8	7	1.00	16	0.438
124	A	3	3	1.00	14	0.214
125	A	11	8	1.00	16	0.500
126	A	4	4	1.00	14	0.286
127	A	15	9	1.00	16	0.562
128	A	5	4	1.00	14	0.286
129	A	9	8	1.00	18	0.444
130	A	4	4	1.00	18	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	5	5	1.00	18	0.278
132	A	4	4	1.00	18	0.222
133	A	49	19	1.00	16	1.187
134	A	49	20	1.00	14	1.429
135	A	39	11	1.01	18	0.611
136	A	39	11	1.01	18	0.611
137	A	48	18	1.00	18	1.000
138	A	12	7	1.00	18	0.389
139	A	9	7	1.00	18	0.389
140	A	4	4	1.00	18	0.222
141	A	0	0	0.00	0	0.000
142	A	0	0	0.00	0	0.000
143	A	0	0	0.00	0	0.000
144	A	0	0	0.00	0	0.000
145	A	0	0	0.00	0	0.000
146	A	0	0	0.00	0	0.000
147	A	0	0	0.00	0	0.000
148	A	21	8	1.00	18	0.444
149	A	13	8	1.00	18	0.444
150	A	5	5	1.00	18	0.278
151	A	0	0	0.00	0	0.000
152	A	0	0	0.00	0	0.000
153	A	0	0	0.00	0	0.000
154	A	0	0	0.00	0	0.000
155	A	0	0	0.00	0	0.000
156	A	0	0	0.00	0	0.000
157	A	0	0	0.00	0	0.000
158	A	0	0	0.00	0	0.000
159	A	0	0	0.00	0	0.000
160	A	3	3	1.00	18	0.167
161	A	0	0	0.00	0	0.000
162	A	0	0	0.00	0	0.000
163	A	9	9	1.00	24	0.375
164	A	10	9	1.00	24	0.375
165	A	5	5	1.00	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	6	6	1.00	21	0.286
167	A	5	5	1.00	24	0.208
168	A	9	9	1.00	24	0.375
169	A	1	1	1.00	12	0.083
170	A	3	3	1.00	12	0.250
171	A	3	3	1.00	14	0.214
172	A	3	3	1.00	14	0.214
173	A	3	3	1.00	16	0.188
174	A	5	5	1.00	18	0.278
175	A	6	6	1.00	18	0.333
176	A	3	2	1.00	18	0.111
177	A	3	2	1.00	18	0.111
178	A	3	2	1.00	16	0.125
179	A	2	2	1.00	10	0.200
180	A	3	3	1.00	18	0.167
181	A	4	3	1.00	18	0.167
182	A	3	2	1.00	18	0.111
183	A	3	2	1.00	18	0.111
184	A	6	5	1.00	20	0.250
185	A	6	5	1.00	20	0.250
186	A	6	5	1.00	18	0.278
187	A	3	3	1.00	12	0.250
188	A	9	6	1.00	20	0.300
189	A	6	5	1.00	20	0.250
190	A	6	5	1.00	20	0.250
191	A	13	11	1.00	20	0.550
192	A	12	11	1.00	20	0.550
193	A	11	10	1.00	18	0.556
194	A	8	8	1.00	12	0.667
195	A	12	6	1.00	20	0.300
196	A	11	10	1.00	20	0.500
197	A	11	10	1.00	20	0.500
198	A	4	3	1.00	20	0.150
199	A	4	3	1.00	20	0.150
200	A	4	3	1.00	18	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	8	7	1.00	20	0.350
202	A	4	3	1.00	20	0.150
203	A	4	3	1.00	20	0.150
204	A	4	3	1.00	20	0.150
205	A	8	7	1.00	16	0.438
206	A	6	3	1.00	20	0.150
207	A	5	3	1.00	20	0.150
208	A	2	2	1.00	18	0.111
209	A	5	5	1.00	20	0.250
210	A	9	6	1.00	20	0.300
211	A	0	0	0.00	0	0.000
212	A	8	4	1.00	20	0.200
213	A	7	4	1.00	20	0.200
214	A	6	4	1.00	18	0.222
215	A	2	2	1.00	12	0.167
216	A	0	0	0.00	0	0.000
217	A	0	0	0.00	0	0.000
218	A	0	0	0.00	0	0.000
219	A	13	8	1.00	21	0.381
220	A	10	8	1.00	21	0.381
221	A	7	7	1.00	19	0.368
222	A	3	3	1.00	18	0.167
223	A	7	8	1.00	21	0.381
224	A	11	10	1.00	21	0.476
225	A	14	10	1.00	21	0.476
226	A	21	15	1.00	23	0.652
227	A	17	13	1.00	23	0.565
228	A	14	10	1.00	21	0.476
229	A	9	6	1.00	20	0.300
230	A	14	9	1.00	23	0.391
231	A	16	11	1.00	23	0.478
232	A	21	15	1.00	23	0.652
233	A	33	20	1.00	23	0.870
234	A	30	17	1.00	23	0.739
235	A	22	15	1.00	21	0.714

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	12	6	1.00	20	0.300
237	A	17	9	1.00	23	0.391
238	A	24	16	1.00	23	0.696
239	A	31	17	1.00	23	0.739
240	A	21	14	1.00	23	0.609
241	A	17	14	1.00	23	0.609
242	A	13	11	1.00	21	0.524
243	A	8	7	1.00	20	0.350
244	A	13	9	1.00	23	0.391
245	A	16	11	1.00	23	0.478
246	A	20	13	1.00	23	0.565
247	A	25	14	1.00	23	0.609
248	A	21	12	1.00	23	0.522
249	A	18	11	1.00	21	0.524
250	A	13	7	1.00	20	0.350
251	A	18	9	1.00	23	0.391
252	A	22	13	1.00	23	0.565
253	A	25	15	1.00	23	0.652
254	A	37	18	1.00	23	0.783
255	A	34	18	1.00	23	0.783
256	A	26	16	1.00	21	0.762
257	A	16	7	1.00	20	0.350
258	A	21	9	1.00	23	0.391
259	A	30	18	1.00	23	0.783
260	A	39	19	1.00	23	0.826
261	A	16	9	1.00	22	0.409
262	A	12	8	1.00	22	0.364
263	A	8	4	1.00	20	0.200
264	A	12	12	1.00	22	0.546
265	A	18	12	1.00	22	0.546
266	A	19	8	1.00	24	0.333
267	A	20	10	1.00	24	0.417
268	A	17	6	1.00	22	0.273
269	A	13	6	1.00	22	0.273
270	A	9	6	1.00	20	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	12	8	1.00	22	0.364
272	A	26	13	1.00	22	0.591
273	A	50	15	1.00	24	0.625
274	A	30	15	1.00	22	0.682
275	A	0	0	0.00	0	0.000
276	A	0	0	0.00	0	0.000
277	A	0	0	0.00	0	0.000
278	A	0	0	0.00	0	0.000
279	A	0	0	0.00	0	0.000
280	A	0	0	0.00	0	0.000
281	A	0	0	0.00	0	0.000
282	A	0	0	0.00	0	0.000
283	A	0	0	0.00	0	0.000
284	A	0	0	0.00	0	0.000
285	A	0	0	0.00	0	0.000
286	A	0	0	0.00	0	0.000
287	A	0	0	0.00	0	0.000
288	A	17	9	1.00	22	0.409
289	A	13	9	1.00	22	0.409
290	A	9	7	1.00	20	0.350
291	A	29	7	1.00	22	0.318
292	A	47	11	1.00	22	0.500
293	A	55	29	1.00	24	1.208
294	A	47	23	1.00	24	0.958
295	A	23	20	1.00	22	0.909
296	A	0	0	0.00	0	0.000
297	A	0	0	0.00	0	0.000
298	A	0	0	0.00	0	0.000
299	A	0	0	0.00	0	0.000
300	A	0	0	0.00	0	0.000
301	A	0	0	0.00	0	0.000
302	A	0	0	0.00	0	0.000
303	A	0	0	0.00	0	0.000
304	A	0	0	0.00	0	0.000
305	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	0	0	0.00	0	0.000
307	A	0	0	0.00	0	0.000
308	A	0	0	0.00	0	0.000
309	A	0	0	0.00	0	0.000
310	A	5	5	1.00	23	0.217
311	A	5	5	1.00	23	0.217
312	A	4	3	1.00	21	0.143
313	A	7	7	1.00	23	0.304
314	A	9	9	1.00	23	0.391
315	A	5	5	1.00	23	0.217
316	A	5	5	1.00	23	0.217
317	A	5	5	1.00	23	0.217
318	A	10	4	1.00	23	0.174
319	A	9	6	1.00	20	0.300
320	A	7	5	1.29	23	0.217
321	A	7	4	1.00	23	0.174
322	A	9	4	1.00	23	0.174
323	A	5	5	1.00	25	0.200
324	A	5	5	1.00	25	0.200
325	A	4	3	1.00	23	0.130
326	A	10	8	1.00	25	0.320
327	A	11	11	1.00	25	0.440
328	A	12	10	1.00	25	0.400
329	A	5	5	1.00	25	0.200
330	A	5	5	1.00	25	0.200
331	A	5	5	1.00	25	0.200
332	A	14	4	1.00	25	0.160
333	A	13	6	1.00	22	0.273
334	A	11	6	1.00	25	0.240
335	A	10	6	1.00	25	0.240
336	A	11	4	1.00	25	0.160
337	A	14	4	1.00	25	0.160
338	A	11	9	1.00	25	0.360
339	A	8	8	1.00	25	0.320
340	A	4	4	1.00	23	0.174

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	8	9	1.00	25	0.360
342	A	12	11	1.00	25	0.440
343	A	21	13	1.00	25	0.520
344	A	17	11	1.00	25	0.440
345	A	12	8	1.00	22	0.364
346	A	16	10	1.00	25	0.400
347	A	19	11	1.00	25	0.440
348	A	12	11	1.00	25	0.440
349	A	10	9	1.00	25	0.360
350	A	5	4	1.00	23	0.174
351	A	12	10	1.00	25	0.400
352	A	16	11	1.00	25	0.440
353	A	43	16	1.00	25	0.640
354	A	40	14	1.00	25	0.560
355	A	26	13	1.00	22	0.591
356	A	42	15	1.00	25	0.600
357	A	6	7	1.00	22	0.318
358	A	12	8	1.00	18	0.444
359	A	11	7	1.00	18	0.389
360	A	8	7	1.00	25	0.280
361	A	8	7	1.00	25	0.280
362	A	7	7	1.00	23	0.304
363	A	9	9	1.00	25	0.360
364	A	8	7	1.00	25	0.280
365	A	11	7	1.00	27	0.259
366	A	11	7	1.00	27	0.259
367	A	10	8	1.00	25	0.320
368	A	12	11	1.00	27	0.407
369	A	11	9	1.00	27	0.333
370	A	13	11	1.00	27	0.407
371	A	8	9	1.00	25	0.360
372	A	5	5	1.00	27	0.185
373	A	9	7	1.00	27	0.259
374	A	19	14	1.00	27	0.518
375	A	12	10	1.00	25	0.400

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
376	A	10	10	1.00	27	0.370
377	A	17	14	1.00	27	0.518
378	A	4	4	1.00	33	0.121
379	A	3	3	1.00	29	0.103
380	A	4	4	1.00	33	0.121
381	A	0	0	0.00	0	0.000
382	A	0	0	0.00	0	0.000
383	A	0	0	0.00	0	0.000
384	A	0	0	0.00	0	0.000
385	A	0	0	0.00	0	0.000
386	A	0	0	0.00	0	0.000
387	A	0	0	0.00	0	0.000
388	A	0	0	0.00	0	0.000
389	A	4	4	1.00	14	0.286
390	A	1	1	1.50	12	0.083
391	A	2	2	1.00	14	0.143
392	A	2	2	1.00	16	0.125
393	A	4	4	1.00	14	0.286
394	A	4	4	1.00	16	0.250
395	A	4	4	1.00	18	0.222
396	A	9	8	1.00	18	0.444
397	A	14	8	1.00	20	0.400
398	A	0	0	0.00	0	0.000
399	A	3	3	1.00	22	0.136
400	A	4	3	1.00	22	0.136
401	A	4	3	1.00	22	0.136
402	A	4	3	1.00	20	0.150
403	A	5	3	1.00	18	0.167
404	A	3	3	1.00	22	0.136
405	A	4	3	1.00	22	0.136
406	A	4	3	1.00	22	0.136
407	A	4	3	1.00	22	0.136
408	A	8	8	1.00	24	0.333
409	A	8	8	1.00	22	0.364
410	A	10	8	1.00	20	0.400

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	5	5	1.00	24	0.208
412	A	8	8	1.00	24	0.333
413	A	16	10	1.00	24	0.417
414	A	24	10	1.00	24	0.417
415	A	28	8	1.00	24	0.333
416	A	20	8	1.00	22	0.364
417	A	12	8	1.00	20	0.400
418	A	6	6	1.00	24	0.250
419	A	10	10	1.00	24	0.417
420	A	28	14	1.00	24	0.583
421	A	4	3	1.00	22	0.136
422	A	4	3	1.00	22	0.136
423	A	4	3	1.00	20	0.150
424	A	6	4	1.00	18	0.222
425	A	3	3	1.00	22	0.136
426	A	4	3	1.00	22	0.136
427	A	4	3	1.00	22	0.136
428	A	4	3	1.00	22	0.136
429	A	24	10	1.00	24	0.417
430	A	16	10	1.00	22	0.454
431	A	9	9	1.00	20	0.450
432	A	5	5	1.00	24	0.208
433	A	10	8	1.00	24	0.333
434	A	8	8	1.00	24	0.333
435	A	8	8	1.00	24	0.333
436	A	28	14	1.00	22	0.636
437	A	11	11	1.00	20	0.550
438	A	6	6	1.00	24	0.250
439	A	12	8	1.00	24	0.333
440	A	20	8	1.00	24	0.333
441	A	28	8	1.00	24	0.333
442	A	4	3	1.00	22	0.136
443	A	4	3	1.00	22	0.136
444	A	4	3	1.00	20	0.150
445	A	5	3	1.00	18	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
446	A	3	3	1.00	22	0.136
447	A	4	3	1.00	22	0.136
448	A	4	3	1.00	22	0.136
449	A	4	3	1.00	22	0.136
450	A	8	8	1.00	24	0.333
451	A	8	8	1.00	22	0.364
452	A	8	8	1.00	20	0.400
453	A	5	5	1.00	24	0.208
454	A	12	10	1.00	24	0.417
455	A	24	10	1.00	24	0.417
456	A	52	8	1.00	24	0.333
457	A	40	8	1.00	24	0.333
458	A	28	8	1.00	22	0.364
459	A	16	8	1.00	20	0.400
460	A	6	6	1.00	24	0.250
461	A	17	13	1.00	24	0.542
462	A	62	14	1.00	24	0.583
463	A	4	3	1.00	22	0.136
464	A	5	4	1.00	22	0.182
465	A	4	3	1.00	20	0.150
466	A	6	4	1.00	18	0.222
467	A	3	3	1.00	22	0.136
468	A	4	4	1.00	22	0.182
469	A	4	3	1.00	22	0.136
470	A	7	4	1.00	22	0.182
471	A	8	8	1.00	24	0.333
472	A	8	8	1.00	22	0.364
473	A	5	5	1.00	24	0.208
474	A	12	10	1.00	24	0.417
475	A	24	10	1.00	24	0.417
476	A	30	14	1.00	24	0.583
477	A	18	14	1.00	20	0.700
478	A	12	11	1.00	24	0.458
479	A	24	12	1.00	24	0.500
480	A	45	12	1.00	24	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
481	A	28	8	1.00	24	0.333
482	A	16	8	1.00	22	0.364
483	A	6	6	1.00	24	0.250
484	A	17	13	1.00	24	0.542
485	A	0	0	0.00	0	0.000
486	A	0	0	0.00	0	0.000
487	A	0	0	0.00	0	0.000
488	A	0	0	0.00	0	0.000
489	A	4	3	1.00	22	0.136
490	A	4	3	1.00	22	0.136
491	A	4	3	1.00	20	0.150
492	A	6	4	1.00	18	0.222
493	A	3	3	1.00	22	0.136
494	A	4	3	1.00	22	0.136
495	A	4	3	1.00	22	0.136
496	A	4	3	1.00	22	0.136
497	A	36	10	1.00	24	0.417
498	A	24	10	1.00	22	0.454
499	A	13	11	1.00	20	0.550
500	A	5	5	1.00	24	0.208
501	A	8	8	1.00	24	0.333
502	A	8	8	1.00	24	0.333
503	A	62	14	1.00	22	0.636
504	A	18	14	1.00	20	0.700
505	A	6	6	1.00	24	0.250
506	A	16	8	1.00	24	0.333
507	A	28	8	1.00	24	0.333
508	A	4	3	1.00	22	0.136
509	A	6	5	1.00	22	0.227
510	A	4	3	1.00	20	0.150
511	A	6	5	1.00	18	0.278
512	A	3	3	1.00	22	0.136
513	A	6	5	1.00	22	0.227
514	A	4	3	1.00	22	0.136
515	A	9	5	1.00	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
516	A	24	10	1.00	24	0.417
517	A	12	10	1.00	22	0.454
518	A	5	5	1.00	24	0.208
519	A	8	8	1.00	24	0.333
520	A	8	8	1.00	24	0.333
521	A	28	17	1.00	24	0.708
522	A	14	13	1.00	20	0.650
523	A	19	14	1.00	24	0.583
524	A	62	14	1.00	24	0.583
525	A	17	13	1.00	22	0.591
526	A	6	6	1.00	24	0.250
527	A	16	8	1.00	24	0.333
528	A	0	0	0.00	0	0.000
529	A	0	0	0.00	0	0.000
530	A	0	0	0.00	0	0.000
531	A	0	0	0.00	0	0.000
532	A	27	7	1.00	22	0.318
533	A	21	7	1.00	22	0.318
534	A	15	7	1.00	20	0.350
535	A	9	7	1.00	18	0.389
536	A	0	0	0.00	0	0.000
537	A	0	0	0.00	0	0.000
538	A	27	7	1.00	24	0.292
539	A	21	7	1.00	24	0.292
540	A	15	7	1.00	22	0.318
541	A	9	7	1.00	20	0.350
542	A	0	0	0.00	0	0.000
543	A	0	0	0.00	0	0.000
544	A	0	0	0.00	0	0.000
545	A	0	0	0.00	0	0.000
546	A	0	0	0.00	0	0.000
547	A	9	7	1.00	22	0.318
548	A	21	7	1.00	22	0.318
549	A	33	7	1.00	22	0.318
550	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
551	A	0	0	0.00	0	0.000
552	A	0	0	0.00	0	0.000
553	A	9	7	0.99	24	0.292
554	A	21	7	1.00	24	0.292
555	A	33	7	1.00	24	0.292
556	A	39	7	1.00	22	0.318
557	A	30	7	1.00	22	0.318
558	A	21	7	1.00	20	0.350
559	A	12	7	1.00	18	0.389
560	A	0	0	0.00	0	0.000
561	A	0	0	0.00	0	0.000
562	A	39	7	1.00	24	0.292
563	A	30	7	1.00	24	0.292
564	A	21	7	1.00	22	0.318
565	A	12	7	1.00	20	0.350
566	A	0	0	0.00	0	0.000
567	A	0	0	0.00	0	0.000
568	A	21	7	1.00	22	0.318
569	A	12	7	1.00	20	0.350
570	A	0	0	0.00	0	0.000
571	A	0	0	0.00	0	0.000
572	A	0	0	0.00	0	0.000
573	A	0	0	0.00	0	0.000
574	A	0	0	0.00	0	0.000
575	A	21	7	1.00	24	0.292
576	A	12	7	0.99	22	0.318
577	A	0	0	0.00	0	0.000
578	A	0	0	0.00	0	0.000
579	A	0	0	0.00	0	0.000
580	A	0	0	0.00	0	0.000
581	A	0	0	0.00	0	0.000
582	A	0	0	0.00	0	0.000
583	A	0	0	0.00	0	0.000
584	A	0	0	0.00	0	0.000
585	A	12	7	1.00	22	0.318

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
586	A	21	7	1.00	22	0.318
587	A	30	7	1.00	22	0.318
588	A	0	0	0.00	0	0.000
589	A	0	0	0.00	0	0.000
590	A	0	0	0.00	0	0.000
591	A	12	7	0.99	24	0.292
592	A	21	7	1.00	24	0.292
593	A	30	7	1.00	24	0.292
594	A	0	0	0.00	0	0.000
595	A	0	0	0.00	0	0.000
596	A	0	0	0.00	0	0.000
597	A	0	0	0.00	0	0.000
598	A	0	0	0.00	0	0.000
599	A	0	0	0.00	0	0.000
600	A	0	0	0.00	0	0.000
601	A	0	0	0.00	0	0.000
602	A	0	0	0.00	0	0.000
603	A	0	0	0.00	0	0.000
604	A	0	0	0.00	0	0.000
605	A	0	0	0.00	0	0.000
606	A	26	12	1.00	29	0.414
607	A	25	12	1.00	29	0.414
608	A	23	10	1.00	29	0.345
609	A	24	11	1.00	29	0.379
610	A	25	11	1.00	29	0.379
611	A	38	13	1.00	31	0.419
612	A	36	12	1.00	31	0.387
613	A	35	12	1.00	31	0.387
614	A	34	11	1.00	31	0.355
615	A	35	11	1.00	31	0.355
616	A	39	20	1.00	31	0.645
617	A	25	11	1.00	31	0.355
618	A	37	19	1.00	31	0.613
619	A	2	2	1.00	18	0.111
620	A	3	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
621	A	6	5	1.00	28	0.179
622	A	5	5	1.00	28	0.179
623	A	4	4	1.00	26	0.154
624	A	3	3	1.00	20	0.150
625	A	0	0	0.00	0	0.000
626	A	0	0	0.00	0	0.000
627	A	0	0	0.00	0	0.000
628	A	3	3	1.00	16	0.188
629	A	9	9	1.00	16	0.562
630	A	4	4	1.00	16	0.250
631	A	3	3	1.00	14	0.214
632	A	4	4	1.00	16	0.250
633	A	4	4	1.00	16	0.250
634	A	9	9	1.00	16	0.562
635	A	0	0	0.00	0	0.000
636	A	8	8	1.00	22	0.364
637	A	7	7	1.00	22	0.318
638	A	5	5	1.00	22	0.227
639	A	5	4	1.00	20	0.200
640	A	0	0	0.00	0	0.000
641	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^4 \log\left(c(a+bx^2)^p\right)\,dx$

Optimal. Leaf size=80

$$-\frac{2a^2px}{5b^2}+\frac{2apx^3}{15b}-\frac{2px^5}{25}+\frac{2a^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{5b^{5/2}}+\frac{1}{5}x^5\log\left(c\left(a+bx^2\right)^p\right)$$

```
[Out] -2/5*a^2*p*x/b^2+2/15*a*p*x^3/b-2/25*p*x^5+2/5*a^(5/2)*p*arctan(x*b^(1/2)/a
^(1/2))/b^(5/2)+1/5*x^5*ln(c*(b*x^2+a)^p)
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2505, 308, 211}

$$\frac{2a^{5/2}p\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{5b^{5/2}}-\frac{2a^2px}{5b^2}+\frac{1}{5}x^5\log\left(c\left(a+bx^2\right)^p\right)+\frac{2apx^3}{15b}-\frac{2px^5}{25}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (-2*a^2*p*x)/(5*b^2) + (2*a*p*x^3)/(15*b) - (2*p*x^5)/25 + (2*a^(5/2)*p*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(5*b^(5/2)) + (x^5*Log[c*(a + b*x^2)^p])/5
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^4 \log\left(c\left(a+bx^2\right)^p\right)\,dx &= \frac{1}{5}x^5\log\left(c\left(a+bx^2\right)^p\right)-\frac{1}{5}(2bp)\int \frac{x^6}{a+bx^2}\,dx\\ &= \frac{1}{5}x^5\log\left(c\left(a+bx^2\right)^p\right)-\frac{1}{5}(2bp)\int \left(\frac{a^2}{b^3}-\frac{ax^2}{b^2}+\frac{x^4}{b}-\frac{a^3}{b^3\left(a+bx^2\right)}\right)dx\\ &= -\frac{2a^2px}{5b^2}+\frac{2apx^3}{15b}-\frac{2px^5}{25}+\frac{1}{5}x^5\log\left(c\left(a+bx^2\right)^p\right)+\frac{\left(2a^3p\right)\int\frac{1}{a+bx^2}\,dx}{5b^2}\\ &= -\frac{2a^2px}{5b^2}+\frac{2apx^3}{15b}-\frac{2px^5}{25}+\frac{2a^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{5b^{5/2}}+\frac{1}{5}x^5\log\left(c\left(a+bx^2\right)^p\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 74, normalized size = 0.92

$$\frac{1}{75}\left(-\frac{30a^2px}{b^2}+\frac{10apx^3}{b}-6px^5+\frac{30a^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}+15x^5\log\left(c\left(a+bx^2\right)^p\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Log[c*(a + b*x^2)^p],x]
```

```
[Out] ((-30*a^2*p*x)/b^2 + (10*a*p*x^3)/b - 6*p*x^5 + (30*a^(5/2)*p*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/b^(5/2) + 15*x^5*Log[c*(a + b*x^2)^p])/75
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.37, size = 229, normalized size = 2.86

method	result
risch	$\frac{x^5\ln\left(\left(bx^2+a\right)^p\right)}{5}+\frac{i\pi x^5\mathrm{csgn}\left(i\left(bx^2+a\right)^p\right)\mathrm{csgn}\left(ic\left(bx^2+a\right)^p\right)^2}{10}-\frac{i\pi x^5\mathrm{csgn}\left(i\left(bx^2+a\right)^p\right)\mathrm{csgn}\left(ic\left(bx^2+a\right)^p\right)\mathrm{csgn}(ic)}{10}-\frac{i\pi x^5\mathrm{csgn}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*ln(c*(b*x^2+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^5*ln((b*x^2+a)^p)+1/10*I*Pi*x^5*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a
)^p)^2-1/10*I*Pi*x^5*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-1/
10*I*Pi*x^5*csgn(I*c*(b*x^2+a)^p)^3+1/10*I*Pi*x^5*csgn(I*c*(b*x^2+a)^p)^2*c
sgn(I*c)+1/5*ln(c)*x^5-2/25*p*x^5+2/15*a*p*x^3/b+1/5/b^3*(-b*a)^(1/2)*a^2*p
*ln(-(-b*a)^(1/2)*x+a)-1/5/b^3*(-b*a)^(1/2)*a^2*p*ln((-b*a)^(1/2)*x+a)-2/5*
a^2*p*x/b^2
```

Maxima [A]
time = 0.49, size = 72, normalized size = 0.90

$$\frac{1}{5}\,x^5\log\left(\left(bx^2+a\right)^p c\right)+\frac{2}{75}\,bp\left(\frac{15\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}-\frac{3\,b^2x^5-5\,abx^3+15\,a^2x}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*log((b*x^2 + a)^p*c) + 2/75*b*p*(15*a^3*arctan(b*x/sqrt(a*b))/(sqrt
(a*b)*b^3) - (3*b^2*x^5 - 5*a*b*x^3 + 15*a^2*x)/b^3)
```

Fricas [A]
time = 0.43, size = 188, normalized size = 2.35

$$\left[\frac{15\,b^2px^5\log\left(bx^2+a\right)-6\,b^2px^5+15\,b^2x^5\log\left(c\right)+10\,abpx^3+15\,a^2p\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-30\,a^2px}{75\,b^2},\ \frac{15\,b^2px^5\log\left(bx^2+a\right)-6\,b^2px^5+15\,b^2x^5\log\left(c\right)+10\,abpx^3+30\,a^2p\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-30\,a^2px}{75\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] [1/75*(15*b^2*p*x^5*log(b*x^2 + a) - 6*b^2*p*x^5 + 15*b^2*x^5*log(c) + 10*a
*b*p*x^3 + 15*a^2*p*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 +
a)) - 30*a^2*p*x)/b^2, 1/75*(15*b^2*p*x^5*log(b*x^2 + a) - 6*b^2*p*x^5 + 15
*b^2*x^5*log(c) + 10*a*b*p*x^3 + 30*a^2*p*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a)
 - 30*a^2*p*x)/b^2]
```

Sympy [A]
time = 34.42, size = 156, normalized size = 1.95

$$\begin{cases} \frac{x^5\log\left(0^p c\right)}{5} & \text{for}\ a=0\wedge b=0 \\ \frac{x^5\log\left(a^p c\right)}{5} & \text{for}\ b=0 \\ -\frac{2px^5}{25}+\frac{x^5\log\left(c\left(bx^2\right)^p\right)}{5} & \text{for}\ a=0 \\ \frac{2a^3p\log\left(x-\sqrt{-\frac{a}{b}}\right)}{5b^3\sqrt{-\frac{a}{b}}}-\frac{a^3\log\left(c\left(a+bx^2\right)^p\right)}{5b^3\sqrt{-\frac{a}{b}}}-\frac{2a^2px}{5b^2}+\frac{2apx^3}{15b}-\frac{2px^5}{25}+\frac{x^5\log\left(c\left(a+bx^2\right)^p\right)}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*ln(c*(b*x**2+a)**p),x)
```

```
[Out] Piecewise((x**5*log(0**p*c)/5, Eq(a, 0) & Eq(b, 0)), (x**5*log(a**p*c)/5, E
q(b, 0)), (-2*p*x**5/25 + x**5*log(c*(b*x**2)**p)/5, Eq(a, 0)), (2*a**3*p*l
```

```
og(x - sqrt(-a/b))/(5*b**3*sqrt(-a/b)) - a**3*log(c*(a + b*x**2)**p)/(5*b**
3*sqrt(-a/b)) - 2*a**2*p*x/(5*b**2) + 2*a*p*x**3/(15*b) - 2*p*x**5/25 + x**
5*log(c*(a + b*x**2)**p)/5, True))
```

Giac [A]
time = 3.51, size = 71, normalized size = 0.89

$$\frac{1}{5}\,px^5\log\left(bx^2+a\right)-\frac{1}{25}\,(2\,p-5\,\log\left(c\right))x^5+\frac{2\,apx^3}{15\,b}+\frac{2\,a^3p\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{5\,\sqrt{ab}\,b^2}-\frac{2\,a^2px}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] 1/5*p*x^5*log(b*x^2 + a) - 1/25*(2*p - 5*log(c))*x^5 + 2/15*a*p*x^3/b + 2/5
*a^3*p*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) - 2/5*a^2*p*x/b^2
```

Mupad [B]
time = 0.22, size = 62, normalized size = 0.78

$$\frac{x^5\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{5}-\frac{2\,p\,x^5}{25}+\frac{2\,a^{5/2}\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{5\,b^{5/2}}+\frac{2\,a\,p\,x^3}{15\,b}-\frac{2\,a^2\,p\,x}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*log(c*(a + b*x^2)^p),x)
```

```
[Out] (x^5*log(c*(a + b*x^2)^p))/5 - (2*p*x^5)/25 + (2*a^(5/2)*p*atan((b^(1/2)*x)
/a^(1/2)))/(5*b^(5/2)) + (2*a*p*x^3)/(15*b) - (2*a^2*p*x)/(5*b^2)
```

3.2 $\int x^3 \log\left(c\left(a+bx^2\right)^p\right)\,dx$

Optimal. Leaf size=59

$$\frac{apx^2}{4b}-\frac{px^4}{8}-\frac{a^2p\log\left(a+bx^2\right)}{4b^2}+\frac{1}{4}x^4\log\left(c\left(a+bx^2\right)^p\right)$$

```
[Out] 1/4*a*p*x^2/b-1/8*p*x^4-1/4*a^2*p*ln(b*x^2+a)/b^2+1/4*x^4*ln(c*(b*x^2+a)^p)
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2504, 2442, 45}

$$-\frac{a^2p\log\left(a+bx^2\right)}{4b^2}+\frac{1}{4}x^4\log\left(c\left(a+bx^2\right)^p\right)+\frac{apx^2}{4b}-\frac{px^4}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (a*p*x^2)/(4*b) - (p*x^4)/8 - (a^2*p*Log[a + b*x^2])/(4*b^2) + (x^4*Log[c*(
a + b*x^2)^p])/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3 \log\left(c(a+bx^2)^p\right) dx = \frac{1}{2}\text{Subst}\left(\int x\log\left(c(a+bx)^p\right) dx, x, x^2\right)$$
$$= \frac{1}{4}x^4\log\left(c(a+bx^2)^p\right) - \frac{1}{4}(bp)\text{Subst}\left(\int \frac{x^2}{a+bx} dx, x, x^2\right)$$
$$= \frac{1}{4}x^4\log\left(c(a+bx^2)^p\right) - \frac{1}{4}(bp)\text{Subst}\left(\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right) dx, x, x^2\right)$$
$$= \frac{apx^2}{4b} - \frac{px^4}{8} - \frac{a^2p\log(a+bx^2)}{4b^2} + \frac{1}{4}x^4\log\left(c(a+bx^2)^p\right)$$

Mathematica [A]
time = 0.01, size = 59, normalized size = 1.00

$$\frac{apx^2}{4b} - \frac{px^4}{8} - \frac{a^2p\log(a+bx^2)}{4b^2} + \frac{1}{4}x^4\log\left(c(a+bx^2)^p\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (a*p*x^2)/(4*b) - (p*x^4)/8 - (a^2*p*Log[a + b*x^2])/(4*b^2) + (x^4*Log[c*(
a + b*x^2)^p])/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.33, size = 1190, normalized size = 20.17

method	result	size
risch	Expression too large to display	1190

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(b*x^2+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*ln((b*x^2+a)^p)+1/8*(-2*a*b*p^2*x^2+b^2*p^2*x^4-Pi^2*b^2*x^4*csgn(I
*c*(b*x^2+a)^p)^6+4*ln(c)^2*b^2*x^4+4*ln(c)*a*b*p*x^2+2*I*Pi*ln(b*x^2+a)*a^
2*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+2*I*Pi*a*b*p*x^2*cs
gn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*Pi^2*b^2*x^4*csgn(I*(b*x^2+a)^p)^2*csgn(I
*c*(b*x^2+a)^p)^3*csgn(I*c)-2*I*Pi*ln(b*x^2+a)*a^2*p*csgn(I*c*(b*x^2+a)^p)^
2*csgn(I*c)+4*I*Pi*ln(c)*b^2*x^4*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-4*I*Pi*l
n(c)*b^2*x^4*csgn(I*c*(b*x^2+a)^p)^3+2*I*Pi*b^2*p*x^4*csgn(I*c*(b*x^2+a)^p)
^3-4*ln(c)*b^2*p*x^4-4*ln(c)*ln(b*x^2+a)*a^2*p+2*ln(b*x^2+a)*a^2*p^2-Pi^2*b
^2*x^4*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)^2+2*I*Pi*a*b*p*x^2*csgn(I*(b*x^2+a
)^p)*csgn(I*c*(b*x^2+a)^p)^2+2*I*Pi*b^2*p*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*
(b*x^2+a)^p)*csgn(I*c)-4*I*Pi*ln(c)*b^2*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b
```

```
*x^2+a)^p)*csgn(I*c)-2*I*Pi*ln(b*x^2+a)*a^2*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*
(b*x^2+a)^p)^2+a^2*p^2+2*Pi^2*b^2*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a
)^p)^5-2*I*Pi*b^2*p*x^4*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*I*Pi*ln(b*x^2+a
)*a^2*p*csgn(I*c*(b*x^2+a)^p)^3-2*I*Pi*a*b*p*x^2*csgn(I*(b*x^2+a)^p)*csgn(I
*c*(b*x^2+a)^p)*csgn(I*c)-Pi^2*b^2*x^4*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^
2+a)^p)^4+4*I*Pi*ln(c)*b^2*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+
2*Pi^2*b^2*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)^2+2*Pi
^2*b^2*x^4*csgn(I*c*(b*x^2+a)^p)^5*csgn(I*c)-2*I*Pi*b^2*p*x^4*csgn(I*(b*x^2
+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-Pi^2*b^2*x^4*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*
(b*x^2+a)^p)^2*csgn(I*c)^2-4*Pi^2*b^2*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x
^2+a)^p)^4*csgn(I*c)-2*I*Pi*a*b*p*x^2*csgn(I*c*(b*x^2+a)^p)^3)/b^2/(I*Pi*cs
gn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c
*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a
)^p)^2*csgn(I*c)+2*ln(c)-p)
```

Maxima [A]
time = 0.27, size = 55, normalized size = 0.93

$$\frac{1}{4}\,x^4\log\left(\left(bx^2+a\right)^p c\right)-\frac{1}{8}\,bp\left(\frac{2\,a^2\log\left(bx^2+a\right)}{b^3}+\frac{bx^4-2\,ax^2}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p),x, algorithm="maxima")
[Out] 1/4*x^4*log((b*x^2 + a)^p*c) - 1/8*b*p*(2*a^2*log(b*x^2 + a)/b^3 + (b*x^4 -
 2*a*x^2)/b^2)
```

Fricas [A]
time = 0.37, size = 57, normalized size = 0.97

$$-\frac{b^2px^4-2\,b^2x^4\log\left(c\right)-2\,abpx^2-2\,\left(b^2px^4-a^2p\right)\log\left(bx^2+a\right)}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p),x, algorithm="fricas")
[Out] -1/8*(b^2*p*x^4 - 2*b^2*x^4*log(c) - 2*a*b*p*x^2 - 2*(b^2*p*x^4 - a^2*p)*lo
g(b*x^2 + a))/b^2
```

Sympy [A]
time = 0.89, size = 65, normalized size = 1.10

$$\begin{cases} -\frac{a^2\log\left(c\left(a+bx^2\right)^p\right)}{4b^2}+\frac{apx^2}{4b}-\frac{px^4}{8}+\frac{x^4\log\left(c\left(a+bx^2\right)^p\right)}{4} & \text{for}\ b\neq 0\\ \frac{x^4\log\left(a^pc\right)}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(b*x**2+a)**p),x)
```

```
[Out] Piecewise((-a**2*log(c*(a + b*x**2)**p)/(4*b**2) + a*p*x**2/(4*b) - p*x**4/
8 + x**4*log(c*(a + b*x**2)**p)/4, Ne(b, 0)), (x**4*log(a**p*c)/4, True))
```

Giac [A]
time = 4.79, size = 97, normalized size = 1.64

$$\frac{2\left(bx^2+a\right)^2 p\log\left(bx^2+a\right)-\left(bx^2+a\right)^2 p+2\left(bx^2+a\right)^2\log\left(c\right)}{8\,b^2}+\frac{\left(bx^2-\left(bx^2+a\right)\log\left(bx^2+a\right)+a\right)ap-\left(bx^2+a\right)a\log\left(c\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] 1/8*(2*(b*x^2 + a)^2*p*log(b*x^2 + a) - (b*x^2 + a)^2*p + 2*(b*x^2 + a)^2*l
og(c))/b^2 + 1/2*((b*x^2 - (b*x^2 + a)*log(b*x^2 + a) + a)*a*p - (b*x^2 + a
)*a*log(c))/b^2
```

Mupad [B]
time = 0.22, size = 51, normalized size = 0.86

$$\frac{x^4\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{4}-\frac{p\,x^4}{8}-\frac{a^2\,p\,\ln\left(b\,x^2+a\right)}{4\,b^2}+\frac{a\,p\,x^2}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(c*(a + b*x^2)^p),x)
```

```
[Out] (x^4*log(c*(a + b*x^2)^p))/4 - (p*x^4)/8 - (a^2*p*log(a + b*x^2))/(4*b^2) +
 (a*p*x^2)/(4*b)
```

3.3 $\int x^2 \log\left(c\left(a+bx^2\right)^p\right)\,dx$

Optimal. Leaf size=66

$$\frac{2apx}{3b}-\frac{2px^3}{9}-\frac{2a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3b^{3/2}}+\frac{1}{3}x^3\log\left(c\left(a+bx^2\right)^p\right)$$

```
[Out] 2/3*a*p*x/b-2/9*p*x^3-2/3*a^(3/2)*p*arctan(x*b^(1/2)/a^(1/2))/b^(3/2)+1/3*x
^3*ln(c*(b*x^2+a)^p)
```

Rubi [A]
time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.188$, Rules used = {2505, 308, 211}

$$-\frac{2a^{3/2}p\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3b^{3/2}}+\frac{1}{3}x^3\log\left(c\left(a+bx^2\right)^p\right)+\frac{2apx}{3b}-\frac{2px^3}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (2*a*p*x)/(3*b) - (2*p*x^3)/9 - (2*a^(3/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
3*b^(3/2)) + (x^3*Log[c*(a + b*x^2)^p])/3
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2 \log \left(c\left(a+bx^2\right)^p\right) \, dx &= \frac{1}{3}x^3 \log \left(c\left(a+bx^2\right)^p\right) - \frac{1}{3}(2bp) \int \frac{x^4}{a+bx^2} \, dx \\ &= \frac{1}{3}x^3 \log \left(c\left(a+bx^2\right)^p\right) - \frac{1}{3}(2bp) \int \left(-\frac{a}{b^2} + \frac{x^2}{b} + \frac{a^2}{b^2\left(a+bx^2\right)}\right) dx \\ &= \frac{2apx}{3b} - \frac{2px^3}{9} + \frac{1}{3}x^3 \log \left(c\left(a+bx^2\right)^p\right) - \frac{\left(2a^2p\right) \int \frac{1}{a+bx^2} \, dx}{3b} \\ &= \frac{2apx}{3b} - \frac{2px^3}{9} - \frac{2a^{3/2}p \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3b^{3/2}} + \frac{1}{3}x^3 \log \left(c\left(a+bx^2\right)^p\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 62, normalized size = 0.94

$$\frac{1}{9}\left(\frac{6apx}{b} - 2px^3 - \frac{6a^{3/2}p \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}} + 3x^3 \log \left(c\left(a+bx^2\right)^p\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[c*(a + b*x^2)^p],x]
```

```
[Out] ((6*a*p*x)/b - 2*p*x^3 - (6*a^(3/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
+ 3*x^3*Log[c*(a + b*x^2)^p])/9
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.30, size = 217, normalized size = 3.29

method	result
risch	$\frac{x^3 \ln\left(\left(b\,x^2+a\right)^p\right)}{3} + \frac{i\pi\,x^3 \mathrm{csgn}\left(i\left(b\,x^2+a\right)^p\right)\mathrm{csgn}\left(ic\left(b\,x^2+a\right)^p\right)^2}{6} - \frac{i\pi\,x^3 \mathrm{csgn}\left(i\left(b\,x^2+a\right)^p\right)\mathrm{csgn}\left(ic\left(b\,x^2+a\right)^p\right)\mathrm{csgn}(ic)}{6} - \frac{i\pi\,x^3 \mathrm{csgn}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(b*x^2+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*ln((b*x^2+a)^p)+1/6*I*Pi*x^3*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)
^p)^2-1/6*I*Pi*x^3*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-1/6*
I*Pi*x^3*csgn(I*c*(b*x^2+a)^p)^3+1/6*I*Pi*x^3*csgn(I*c*(b*x^2+a)^p)^2*csgn(
I*c)+1/3*ln(c)*x^3-2/9*p*x^3+1/3/b^2*(-b*a)^(1/2)*a*p*ln(-(-b*a)^(1/2)*x-a)
-1/3/b^2*(-b*a)^(1/2)*a*p*ln((-b*a)^(1/2)*x-a)+2/3*a*p*x/b
```

Maxima [A]
time = 0.49, size = 59, normalized size = 0.89

$$\frac{1}{3}\,x^3\log\left(\left(bx^2+a\right)^p c\right)-\frac{2}{9}\,bp\left(\frac{3\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{bx^3-3\,ax}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log((b*x^2 + a)^p*c) - 2/9*b*p*(3*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a
*b)*b^2) + (b*x^3 - 3*a*x)/b^2)
```

Fricas [A]
time = 0.41, size = 152, normalized size = 2.30

$$\left[\frac{3\,bpx^3\log\left(bx^2+a\right)-2\,bpx^3+3\,bx^3\log\left(c\right)+3\,ap\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+6\,apx}{9\,b},\ \frac{3\,bpx^3\log\left(bx^2+a\right)-2\,bpx^3+3\,bx^3\log\left(c\right)-6\,ap\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+6\,apx}{9\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] [1/9*(3*b*p*x^3*log(b*x^2 + a) - 2*b*p*x^3 + 3*b*x^3*log(c) + 3*a*p*sqrt(-a
/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 6*a*p*x)/b, 1/9*(3*b*
p*x^3*log(b*x^2 + a) - 2*b*p*x^3 + 3*b*x^3*log(c) - 6*a*p*sqrt(a/b)*arctan(
b*x*sqrt(a/b)/a) + 6*a*p*x)/b]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(63) = 126$.
time = 8.50, size = 141, normalized size = 2.14

$$\begin{cases} \frac{x^3\log\left(0^p c\right)}{3} & \text{for}\ a=0\wedge b=0 \\ \frac{x^3\log\left(a^p c\right)}{3} & \text{for}\ b=0 \\ -\frac{2px^3}{9}+\frac{x^3\log\left(c\left(bx^2\right)^p\right)}{3} & \text{for}\ a=0 \\ -\frac{2a^2p\log\left(x-\sqrt{-\frac{a}{b}}\right)}{3b^2\sqrt{-\frac{a}{b}}}+\frac{a^2\log\left(c\left(a+bx^2\right)^p\right)}{3b^2\sqrt{-\frac{a}{b}}}+\frac{2apx}{3b}-\frac{2px^3}{9}+\frac{x^3\log\left(c\left(a+bx^2\right)^p\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(b*x**2+a)**p),x)
```

```
[Out] Piecewise((x**3*log(0**p*c)/3, Eq(a, 0) & Eq(b, 0)), (x**3*log(a**p*c)/3, E
q(b, 0)), (-2*p*x**3/9 + x**3*log(c*(b*x**2)**p)/3, Eq(a, 0)), (-2*a**2*p*l
og(x - sqrt(-a/b))/(3*b**2*sqrt(-a/b)) + a**2*log(c*(a + b*x**2)**p)/(3*b**
2*sqrt(-a/b)) + 2*a*p*x/(3*b) - 2*p*x**3/9 + x**3*log(c*(a + b*x**2)**p)/3,
 True))
```

Giac [A]
time = 3.39, size = 59, normalized size = 0.89

$$\frac{1}{3}px^3\log\left(bx^2+a\right)-\frac{1}{9}\left(2p-3\log\left(c\right)\right)x^3-\frac{2a^2p\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{3\sqrt{ab}\,b}+\frac{2apx}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] 1/3*p*x^3*log(b*x^2 + a) - 1/9*(2*p - 3*log(c))*x^3 - 2/3*a^2*p*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*b) + 2/3*a*p*x/b
```

Mupad [B]
time = 0.23, size = 50, normalized size = 0.76

$$\frac{x^3\ln\left(c\left(b\,x^2+a\right)^p\right)}{3}-\frac{2\,p\,x^3}{9}-\frac{2\,a^{3/2}\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3\,b^{3/2}}+\frac{2\,a\,p\,x}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(c*(a + b*x^2)^p),x)
```

```
[Out] (x^3*log(c*(a + b*x^2)^p))/3 - (2*p*x^3)/9 - (2*a^(3/2)*p*atan((b^(1/2)*x)/
a^(1/2)))/(3*b^(3/2)) + (2*a*p*x)/(3*b)
```

3.4 $\int x \log\left(c(a+bx^2)^p\right)\,dx$

Optimal. Leaf size=35

$$-\frac{px^2}{2}+\frac{(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2b}$$

```
[Out] -1/2*p*x^2+1/2*(b*x^2+a)*ln(c*(b*x^2+a)^p)/b
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2504, 2436, 2332}

$$\frac{(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2b}-\frac{px^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[c*(a + b*x^2)^p],x]
```

```
[Out] -1/2*(p*x^2) + ((a + b*x^2)*Log[c*(a + b*x^2)^p])/(2*b)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\log\left(c(a+bx^2)^p\right)\,dx=\frac{1}{2}\text{Subst}\left(\int\log\left(c(a+bx)^p\right)\,dx,x,x^2\right)$$
$$=\frac{\text{Subst}\left(\int\log\left(cx^p\right)\,dx,x,a+bx^2\right)}{2b}$$
$$=-\frac{px^2}{2}+\frac{(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2b}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 0.97

$$\frac{1}{2}\left(-px^2+\frac{(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[c*(a + b*x^2)^p],x]
[Out] (-(p*x^2) + ((a + b*x^2)*Log[c*(a + b*x^2)^p])/b)/2
```

Maple [A]
time = 0.48, size = 37, normalized size = 1.06

method	result
derivativedivides	$\frac{\ln(c(bx^2+a)^p)(bx^2+a)-(bx^2+a)p}{2b}$
default	$\frac{\ln(c(bx^2+a)^p)(bx^2+a)-(bx^2+a)p}{2b}$
norman	$-\frac{px^2}{2}+\frac{x^2\ln\left(c\,e^{p\ln(bx^2+a)}\right)}{2}+\frac{pa\ln(bx^2+a)}{2b}$
risch	$\frac{x^2\ln((bx^2+a)^p)}{2}+\frac{i\pi x^2\operatorname{csgn}(i(bx^2+a)^p)\operatorname{csgn}(ic(bx^2+a)^p)^2}{4}-\frac{i\pi x^2\operatorname{csgn}(i(bx^2+a)^p)\operatorname{csgn}(ic(bx^2+a)^p)\operatorname{csgn}(ic)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(b*x^2+a)^p),x,method=_RETURNVERBOSE)
[Out] 1/2/b*(ln(c*(b*x^2+a)^p)*(b*x^2+a)-(b*x^2+a)*p)
```

Maxima [A]
time = 0.27, size = 44, normalized size = 1.26

$$-\frac{1}{2}\,bp\left(\frac{x^2}{b}-\frac{a\log\left(bx^2+a\right)}{b^2}\right)+\frac{1}{2}\,x^2\log\left(\left(bx^2+a\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] -1/2*b*p*(x^2/b - a*log(b*x^2 + a)/b^2) + 1/2*x^2*log((b*x^2 + a)^p*c)
```

Fricas [A]
time = 0.43, size = 40, normalized size = 1.14

$$-\frac{bpx^2 - bx^2 \log(c) - (bpx^2 + ap) \log(bx^2 + a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] -1/2*(b*p*x^2 - b*x^2*log(c) - (b*p*x^2 + a*p)*log(b*x^2 + a))/b
```

Sympy [A]
time = 0.31, size = 51, normalized size = 1.46

$$\begin{cases} \frac{a \log\left(c(a+bx^2)^p\right)}{2b} - \frac{px^2}{2} + \frac{x^2 \log\left(c(a+bx^2)^p\right)}{2} & \text{for}\ b \neq 0 \\ \frac{x^2 \log(a^p c)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(b*x**2+a)**p),x)
```

```
[Out] Piecewise((a*log(c*(a + b*x**2)**p)/(2*b) - p*x**2/2 + x**2*log(c*(a + b*x*
*2)**p)/2, Ne(b, 0)), (x**2*log(a**p*c)/2, True))
```

Giac [A]
time = 4.78, size = 43, normalized size = 1.23

$$-\frac{(bx^2 - (bx^2 + a)\log(bx^2 + a) + a)p - (bx^2 + a)\log(c)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] -1/2*((b*x^2 - (b*x^2 + a)*log(b*x^2 + a) + a)*p - (b*x^2 + a)*log(c))/b
```

Mupad [B]
time = 0.23, size = 39, normalized size = 1.11

$$\frac{x^2\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{2} - \frac{p\,x^2}{2} + \frac{a\,p\,\ln\left(b\,x^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(c*(a + b*x^2)^p),x)
```

```
[Out] (x^2*log(c*(a + b*x^2)^p))/2 - (p*x^2)/2 + (a*p*log(a + b*x^2))/(2*b)
```

3.5 $\int \log\left(c(a+bx^2)^p\right)\,dx$

Optimal. Leaf size=45

$$-2px+\frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}+x\log\left(c\left(a+bx^2\right)^p\right)$$

```
[Out] -2*p*x+x*ln(c*(b*x^2+a)^p)+2*p*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {2498, 327, 211}

$$\frac{2\sqrt{a}\,p\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}+x\log\left(c\left(a+bx^2\right)^p\right)-2px$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p],x]
```

```
[Out] -2*p*x + (2*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + x*Log[c*(a + b
*x^2)^p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\int \log \left(c\left(a+bx^2\right)^p\right) \, dx = x \log \left(c\left(a+bx^2\right)^p\right) - (2bp) \int \frac{x^2}{a+bx^2} \, dx$$
$$= -2px + x \log \left(c\left(a+bx^2\right)^p\right) + (2ap) \int \frac{1}{a+bx^2} \, dx$$
$$= -2px + \frac{2\sqrt{a}\, p \tan^{-1}\left(\frac{\sqrt{b}\, x}{\sqrt{a}}\right)}{\sqrt{b}} + x \log \left(c\left(a+bx^2\right)^p\right)$$

Mathematica [A]
time = 0.01, size = 45, normalized size = 1.00

$$-2px + \frac{2\sqrt{a}\, p \tan^{-1}\left(\frac{\sqrt{b}\, x}{\sqrt{a}}\right)}{\sqrt{b}} + x \log \left(c\left(a+bx^2\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p],x]
[Out] -2*p*x + (2*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + x*Log[c*(a + b
*x^2)^p]
```

Maple [A]
time = 0.27, size = 46, normalized size = 1.02

method	result
default	$x \ln \left(c\left(b\,x^2+a\right)^p\right) - 2pb\left(\frac{x}{b} - \frac{a \arctan\left(\frac{bx}{\sqrt{ba}}\right)}{b\sqrt{ba}}\right)$
risch	$x \ln \left(\left(b\,x^2+a\right)^p\right) + \frac{i \operatorname{csgn}\left(ic\left(b\,x^2+a\right)^p\right)^2 \operatorname{csgn}\left(i\left(b\,x^2+a\right)^p\right) x\pi}{2} - \frac{i\pi x \operatorname{csgn}\left(i\left(b\,x^2+a\right)^p\right) \operatorname{csgn}\left(ic\left(b\,x^2+a\right)^p\right) \operatorname{csgn}(ic)}{2} - \frac{i\pi x \mathrm{cs}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p),x,method=_RETURNVERBOSE)
[Out] x*ln(c*(b*x^2+a)^p)-2*p*b*(x/b-a/b/(b*a)^(1/2)*arctan(b*x/(b*a)^(1/2)))
```

Maxima [A]
time = 0.50, size = 45, normalized size = 1.00

$$2\,bp\left(\frac{a \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\, b} - \frac{x}{b}\right) + x \log \left(\left(bx^2+a\right)^p c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] 2*b*p*(a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - x/b) + x*log((b*x^2 + a)^p*c
)
```

Fricas [A]
time = 0.40, size = 107, normalized size = 2.38

$$\left[px\log\left(bx^2+a\right)+p\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-2\,px+x\log\left(c\right),px\log\left(bx^2+a\right)+2\,p\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-2\,px+x\log\left(c\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] [p*x*log(b*x^2 + a) + p*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^
2 + a)) - 2*p*x + x*log(c), p*x*log(b*x^2 + a) + 2*p*sqrt(a/b)*arctan(b*x*s
qrt(a/b)/a) - 2*p*x + x*log(c)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(44) = 88$.
time = 2.17, size = 100, normalized size = 2.22

$$\begin{cases} x\log\left(0^p c\right) & \text{for}\ a=0\wedge b=0 \\ x\log\left(a^p c\right) & \text{for}\ b=0 \\ -2px+x\log\left(c(bx^2)^p\right) & \text{for}\ a=0 \\ \frac{2ap\log\left(x-\sqrt{-\frac{a}{b}}\right)}{b\sqrt{-\frac{a}{b}}}-\frac{a\log\left(c(a+bx^2)^p\right)}{b\sqrt{-\frac{a}{b}}}-2px+x\log\left(c(a+bx^2)^p\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p),x)
```

```
[Out] Piecewise((x*log(0**p*c), Eq(a, 0) & Eq(b, 0)), (x*log(a**p*c), Eq(b, 0)),
(-2*p*x + x*log(c*(b*x**2)**p), Eq(a, 0)), (2*a*p*log(x - sqrt(-a/b))/(b*sq
rt(-a/b)) - a*log(c*(a + b*x**2)**p)/(b*sqrt(-a/b)) - 2*p*x + x*log(c*(a +
b*x**2)**p), True))
```

Giac [A]
time = 4.19, size = 41, normalized size = 0.91

$$px\log\left(bx^2+a\right)+\frac{2\,ap\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}-(2\,p-\log\left(c\right))x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] p*x*log(b*x^2 + a) + 2*a*p*arctan(b*x/sqrt(a*b))/sqrt(a*b) - (2*p - log(c))
*x
```

Mupad [B]
time = 0.22, size = 37, normalized size = 0.82

$$x\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)-2\,p\,x+\frac{2\,\sqrt{a}\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p),x)
```

```
[Out] x*log(c*(a + b*x^2)^p) - 2*p*x + (2*a^(1/2)*p*atan((b^(1/2)*x)/a^(1/2)))/b^
(1/2)
```

3.6 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x}\,dx$

Optimal. Leaf size=44

$$\frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)+\frac{1}{2}p\text{Li}_2\left(1+\frac{bx^2}{a}\right)$$

```
[Out] 1/2*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)+1/2*p*polylog(2,1+b*x^2/a)
```

Rubi [A]
time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2504, 2441, 2352}

$$\frac{1}{2}p\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)+\frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/x,x]
```

```
[Out] (Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/2 + (p*PolyLog[2, 1 + (b*x^2)/a])/
2
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log \left(c \left(a+b x^2\right)^p\right)}{x} \, dx = \frac{1}{2}\text{Subst}\left(\int \frac{\log \left(c (a+b x)^p\right)}{x} \, dx,x,x^2\right)$$
$$= \frac{1}{2} \log \left(-\frac{b x^2}{a}\right) \log \left(c \left(a+b x^2\right)^p\right)-\frac{1}{2} (b p) \text{Subst}\left(\int \frac{\log \left(-\frac{b x}{a}\right)}{a+b x} \, dx,x,x^2\right)$$
$$= \frac{1}{2} \log \left(-\frac{b x^2}{a}\right) \log \left(c \left(a+b x^2\right)^p\right)+\frac{1}{2} p \text{Li}_2\left(1+\frac{b x^2}{a}\right)$$

Mathematica [A]
time = 0.01, size = 43, normalized size = 0.98

$$\frac{1}{2} \left(\log \left(-\frac{b x^2}{a}\right) \log \left(c \left(a+b x^2\right)^p\right)+p \text{Li}_2\left(\frac{a+b x^2}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/x,x]
```

```
[Out] (Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p] + p*PolyLog[2, (a + b*x^2)/a])/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.27, size = 232, normalized size = 5.27

method	result
risch	$\ln(x) \ln\left(\left(b\,x^2+a\right)^p\right) - p\ln(x)\ln\left(\frac{-bx+\sqrt{-ba}}{\sqrt{-ba}}\right) - p\ln(x)\ln\left(\frac{bx+\sqrt{-ba}}{\sqrt{-ba}}\right) - p\,\text{dilog}\left(\frac{-bx+\sqrt{-}}{\sqrt{-b}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)*ln((b*x^2+a)^p)-p*ln(x)*ln((-b*x+(-b*a)^(1/2))/(-b*a)^(1/2))-p*ln(x)*
ln((b*x+(-b*a)^(1/2))/(-b*a)^(1/2))-p*dilog((-b*x+(-b*a)^(1/2))/(-b*a)^(1/2
))-p*dilog((b*x+(-b*a)^(1/2))/(-b*a)^(1/2))+1/2*I*ln(x)*Pi*csgn(I*(b*x^2+a)
^p)*csgn(I*c*(b*x^2+a)^p)^2-1/2*I*ln(x)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*
x^2+a)^p)*csgn(I*c)-1/2*I*ln(x)*Pi*csgn(I*c*(b*x^2+a)^p)^3+1/2*I*ln(x)*Pi*c
sgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+ln(c)*ln(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(39) = 78.
time = 0.28, size = 80, normalized size = 1.82

$$\frac{1}{2} \, b p {\left(\frac{2 \, \log\left(b x^{2} + a\right) \log\left(x\right)}{b} - \frac{2 \, \log\left(\frac{b x^{2}}{a} + 1\right) \log\left(x\right) + {\rm Li}_2\left(-\frac{b x^{2}}{a}\right)}{b}\right)} - p \log\left(b x^{2} + a\right) \log\left(x\right) + \log\left({\left(b x^{2} + a\right)}^{p} c\right) \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x,x, algorithm="maxima")
```

```
[Out] 1/2*b*p*(2*log(b*x^2 + a)*log(x)/b - (2*log(b*x^2/a + 1)*log(x) + dilog(-b*
x^2/a))/b) - p*log(b*x^2 + a)*log(x) + log((b*x^2 + a)^p*c)*log(x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/x,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)/x,x)
```

```
[Out] int(log(c*(a + b*x^2)^p)/x, x)
```

3.7 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^2}\,dx$

Optimal. Leaf size=44

$$\frac{2\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\log\left(c(a+bx^2)^p\right)}{x}$$

```
[Out] -ln(c*(b*x^2+a)^p)/x+2*p*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2505, 211}

$$\frac{2\sqrt{b}\,p\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\log\left(c(a+bx^2)^p\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/x^2,x]
```

```
[Out] (2*Sqrt[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] - Log[c*(a + b*x^2)^p]/x
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int\frac{\log\left(c(a+bx^2)^p\right)}{x^2}\,dx&=-\frac{\log\left(c(a+bx^2)^p\right)}{x}+(2bp)\int\frac{1}{a+bx^2}\,dx\\&=\frac{2\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\log\left(c(a+bx^2)^p\right)}{x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 1.00

$$\frac{2\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/x^2,x]
[Out] (2*Sqrt[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] - Log[c*(a + b*x^2)^p]/x
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.22, size = 195, normalized size = 4.43

method	result
risch	$-\frac{\ln\left(\left(b\,x^2+a\right)^p\right)}{x}-\frac{i\pi a\,\mathrm{csgn}\left(i\left(b\,x^2+a\right)^p\right)\mathrm{csgn}\left(ic\left(b\,x^2+a\right)^p\right)^2-i\pi a\,\mathrm{csgn}\left(i\left(b\,x^2+a\right)^p\right)\mathrm{csgn}\left(ic\left(b\,x^2+a\right)^p\right)\mathrm{csgn}(ic)-i\pi a\mathrm{csgn}\left(ic\left(b\,x^2+a\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/x^2,x,method=_RETURNVERBOSE)
[Out] -1/x*ln((b*x^2+a)^p)-1/2*(I*Pi*a*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^
2-I*Pi*a*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*a*csgn(I*
c*(b*x^2+a)^p)^3+I*Pi*a*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-2*(-b*a)^(1/2)*p*
ln(-b*x-(-b*a)^(1/2))*x+2*(-b*a)^(1/2)*p*ln(-b*x+(-b*a)^(1/2))*x+2*ln(c)*a)
/a/x
```

Maxima [A]
time = 0.51, size = 36, normalized size = 0.82

$$\frac{2\,bp\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}-\frac{\log\left(\left(bx^2+a\right)^pc\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^2,x, algorithm="maxima")
[Out] 2*b*p*arctan(b*x/sqrt(a*b))/sqrt(a*b) - log((b*x^2 + a)^p*c)/x
```

Fricas [A]
time = 0.42, size = 105, normalized size = 2.39

$$\left[\frac{px\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-p\log\left(bx^2+a\right)-\log\left(c\right)}{x},\frac{2\,px\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-p\log\left(bx^2+a\right)-\log\left(c\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^2,x, algorithm="fricas")
```

```
[Out] [(p*x*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - p*log(b*
x^2 + a) - log(c))/x, (2*p*x*sqrt(b/a)*arctan(x*sqrt(b/a)) - p*log(b*x^2 +
a) - log(c))/x]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(39) = 78$.

time = 7.95, size = 245, normalized size = 5.57

$$\begin{cases} -\frac{\log(0^p c)}{x} & \text{for}\ a = 0 \wedge b = 0 \\ -\frac{\log(a^p c)}{x} & \text{for}\ b = 0 \\ -\frac{2p}{x} - \frac{\log(c(bx^2)^p)}{x} & \text{for}\ a = 0 \\ -\frac{a^2\log(c(a+bx^2)^p)}{a^2x+abx^3} - \frac{2apx\sqrt{-\frac{a}{b}}\log\left(x-\sqrt{-\frac{a}{b}}\right)}{\frac{a^2x}{b}+ax^3} - \frac{ax^2\log(c(a+bx^2)^p)}{\frac{a^2x}{b}+ax^3} + \frac{ax\sqrt{-\frac{a}{b}}\log(c(a+bx^2)^p)}{\frac{a^2x}{b}+ax^3} - \frac{2bpx^3\sqrt{-\frac{a}{b}}\log\left(x-\sqrt{-\frac{a}{b}}\right)}{\frac{a^2x}{b}+ax^3} + \frac{bx^3\sqrt{-\frac{a}{b}}\log(c(a+bx^2)^p)}{\frac{a^2x}{b}+ax^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/x**2,x)
```

```
[Out] Piecewise((-log(0**p*c)/x, Eq(a, 0) & Eq(b, 0)), (-log(a**p*c)/x, Eq(b, 0))
, (-2*p/x - log(c*(b*x**2)**p)/x, Eq(a, 0)), (-a**2*log(c*(a + b*x**2)**p)/
(a**2*x + a*b*x**3) - 2*a*p*x*sqrt(-a/b)*log(x - sqrt(-a/b))/(a**2*x/b + a*
x**3) - a*x**2*log(c*(a + b*x**2)**p)/(a**2*x/b + a*x**3) + a*x*sqrt(-a/b)*
log(c*(a + b*x**2)**p)/(a**2*x/b + a*x**3) - 2*b*p*x**3*sqrt(-a/b)*log(x -
sqrt(-a/b))/(a**2*x/b + a*x**3) + b*x**3*sqrt(-a/b)*log(c*(a + b*x**2)**p)/
(a**2*x/b + a*x**3), True))
```

Giac [A]

time = 4.07, size = 40, normalized size = 0.91

$$\frac{2\,bp\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} - \frac{p\log\left(bx^2+a\right)}{x} - \frac{\log\left(c\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^2,x, algorithm="giac")
```

```
[Out] 2*b*p*arctan(b*x/sqrt(a*b))/sqrt(a*b) - p*log(b*x^2 + a)/x - log(c)/x
```

Mupad [B]

time = 0.22, size = 36, normalized size = 0.82

$$\frac{2\,\sqrt{b}\,\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)/x^2,x)
```

```
[Out] (2*b^(1/2)*p*atan((b^(1/2)*x)/a^(1/2)))/a^(1/2) - log(c*(a + b*x^2)^p)/x
```

3.8 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^3}\,dx$

Optimal. Leaf size=38

$$\frac{bp\log(x)}{a}-\frac{(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2ax^2}$$

```
[Out] b*p*ln(x)/a-1/2*(b*x^2+a)*ln(c*(b*x^2+a)^p)/a/x^2
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.312$, Rules used = {2504, 2442, 36, 29, 31}

$$-\frac{\log\left(c(a+bx^2)^p\right)}{2x^2}-\frac{bp\log\left(a+bx^2\right)}{2a}+\frac{bp\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/x^3,x]
```

```
[Out] (b*p*Log[x])/a - (b*p*Log[a + b*x^2])/(2*a) - Log[c*(a + b*x^2)^p]/(2*x^2)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(c(a+bx^2)^p\right)}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x^2}\,dx,x,x^2\right)\\ &= -\frac{\log\left(c(a+bx^2)^p\right)}{2x^2}+\frac{1}{2}(bp)\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx,x,x^2\right)\\ &= -\frac{\log\left(c(a+bx^2)^p\right)}{2x^2}+\frac{(bp)\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2a}-\frac{(b^2p)\,\text{Subst}\left(\int \frac{1}{a+bx}\,dx,x,x^2\right)}{2a}\\ &= \frac{bp\log(x)}{a}-\frac{bp\log\left(a+bx^2\right)}{2a}-\frac{\log\left(c(a+bx^2)^p\right)}{2x^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 45, normalized size = 1.18

$$\frac{bp\log(x)}{a}-\frac{bp\log\left(a+bx^2\right)}{2a}-\frac{\log\left(c(a+bx^2)^p\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/x^3,x]
```

```
[Out] (b*p*Log[x])/a - (b*p*Log[a + b*x^2])/(2*a) - Log[c*(a + b*x^2)^p]/(2*x^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.22, size = 173, normalized size = 4.55

method	result
risch	$-\frac{\ln\left(\left(b\,x^2+a\right)^p\right)}{2x^2}-\frac{i\pi a\,\mathrm{csgn}\left(i\left(b\,x^2+a\right)^p\right)\mathrm{csgn}\left(ic\left(b\,x^2+a\right)^p\right)^2-i\pi a\,\mathrm{csgn}\left(i\left(b\,x^2+a\right)^p\right)\mathrm{csgn}\left(ic\left(b\,x^2+a\right)^p\right)\mathrm{csgn}(ic)-i\pi a\mathrm{csgn}\left(ic\left(b\,x^2\right.\right.}{4x^2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^2*ln((b*x^2+a)^p)-1/4*(I*Pi*a*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)
^p)^2-I*Pi*a*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*a*csg
n(I*c*(b*x^2+a)^p)^3+I*Pi*a*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-4*b*p*ln(x)*x
^2+2*b*p*ln(b*x^2+a)*x^2+2*ln(c)*a)/x^2/a
```

Maxima [A]
time = 0.28, size = 44, normalized size = 1.16

$$-\frac{1}{2}\,bp\left(\frac{\log\left(bx^2+a\right)}{a}-\frac{\log\left(x^2\right)}{a}\right)-\frac{\log\left(\left(bx^2+a\right)^p c\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b*p*(log(b*x^2 + a)/a - log(x^2)/a) - 1/2*log((b*x^2 + a)^p*c)/x^2
```

Fricas [A]
time = 0.38, size = 43, normalized size = 1.13

$$\frac{2\,bpx^2\log\left(x\right)-\left(bpx^2+ap\right)\log\left(bx^2+a\right)-a\log\left(c\right)}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*p*x^2*log(x) - (b*p*x^2 + a*p)*log(b*x^2 + a) - a*log(c))/(a*x^2)
```

Sympy [A]
time = 1.09, size = 65, normalized size = 1.71

$$\begin{cases} -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{2x^2}+\frac{bp\log\left(x\right)}{a}-\frac{b\log\left(c\left(a+bx^2\right)^p\right)}{2a} & \text{for } a\neq 0 \\ -\frac{p}{2x^2}-\frac{\log\left(c\left(bx^2\right)^p\right)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/x**3,x)
```

```
[Out] Piecewise((-log(c*(a + b*x**2)**p)/(2*x**2) + b*p*log(x)/a - b*log(c*(a + b
*x**2)**p)/(2*a), Ne(a, 0)), (-p/(2*x**2) - log(c*(b*x**2)**p)/(2*x**2), Tr
ue))
```

Giac [A]
time = 5.85, size = 58, normalized size = 1.53

$$-\frac{\frac{b^2p\log\left(bx^2+a\right)}{a}-\frac{b^2p\log\left(bx^2\right)}{a}+\frac{bp\log\left(bx^2+a\right)}{x^2}+\frac{b\log\left(c\right)}{x^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*p*log(b*x^2 + a)/a - b^2*p*log(b*x^2)/a + b*p*log(b*x^2 + a)/x^2
+ b*log(c)/x^2)/b
```

Mupad [B]
time = 0.24, size = 41, normalized size = 1.08

$$\frac{b\,p\,\ln\left(x\right)}{a}-\frac{b\,p\,\ln\left(b\,x^2+a\right)}{2\,a}-\frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)/x^3,x)
[Out] (b*p*log(x))/a - (b*p*log(a + b*x^2))/(2*a) - log(c*(a + b*x^2)^p)/(2*x^2)
```

3.9 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^4}\,dx$

Optimal. Leaf size=60

$$-\frac{2bp}{3ax}-\frac{2b^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{3x^3}$$

```
[Out] -2/3*b*p/a/x-2/3*b^(3/2)*p*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)-1/3*ln(c*(b*x^
2+a)^p)/x^3
```

Rubi [A]
time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2505, 331, 211}

$$-\frac{2b^{3/2}p\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{3x^3}-\frac{2bp}{3ax}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/x^4,x]
```

```
[Out] (-2*b*p)/(3*a*x) - (2*b^(3/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3*a^(3/2)) -
Log[c*(a + b*x^2)^p]/(3*x^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log \left(c(a+bx^2)^p\right)}{x^4}\,dx &= -\frac{\log \left(c(a+bx^2)^p\right)}{3x^3}+\frac{1}{3}(2bp)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx \\
&= -\frac{2bp}{3ax}-\frac{\log \left(c(a+bx^2)^p\right)}{3x^3}-\frac{(2b^2p)\int \frac{1}{a+bx^2}\,dx}{3a} \\
&= -\frac{2bp}{3ax}-\frac{2b^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{\log \left(c(a+bx^2)^p\right)}{3x^3}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.00, size = 49, normalized size = 0.82

$$
-\frac{2bp\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{bx^2}{a}\right)}{3ax}-\frac{\log \left(c(a+bx^2)^p\right)}{3x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/x^4,x]
```

```
[Out] (-2*b*p*Hypergeometric2F1[-1/2, 1, 1/2, -((b*x^2)/a)])/(3*a*x) - Log[c*(a +
 b*x^2)^p]/(3*x^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.32, size = 211, normalized size = 3.52

method	result
risch	$-\frac{\ln\left(\left(b\,x^2+a\right)^p\right)}{3x^3}-\frac{i\pi a\,\mathrm{csgn}\left(i\left(b\,x^2+a\right)^p\right)\mathrm{csgn}\left(ic\left(b\,x^2+a\right)^p\right)^2-i\pi a\,\mathrm{csgn}\left(i\left(b\,x^2+a\right)^p\right)\mathrm{csgn}\left(ic\left(b\,x^2+a\right)^p\right)\mathrm{csgn}(ic)-i\pi a\mathrm{csgn}\left(ic\left(b\,x^2\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/x^3*ln((b*x^2+a)^p)-1/6*(I*Pi*a*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)
^p)^2-I*Pi*a*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*a*csg
n(I*c*(b*x^2+a)^p)^3+I*Pi*a*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-2*sum(_R*ln((
3*_R^2*a^3+2*b^3*p^2)*x+a^2*b*p*_R),_R=RootOf(_Z^2*a^3+b^3*p^2))*x^3*a+4*x^
2*p*b+2*ln(c)*a)/x^3/a
```

Maxima [A]
time = 0.49, size = 49, normalized size = 0.82

$$-\frac{2}{3}\,bp\left(\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}+\frac{1}{ax}\right)-\frac{\log\left(\left(bx^2+a\right)^p c\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^4,x, algorithm="maxima")
```

```
[Out] -2/3*b*p*(b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/(a*x)) - 1/3*log((b*x^2
 + a)^p*c)/x^3
```

Fricas [A]
time = 0.41, size = 135, normalized size = 2.25

$$\left[\frac{bpx^3\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2\,bpx^2-ap\log\left(bx^2+a\right)-a\log\left(c\right)}{3\,ax^3},\,-\frac{2\,bpx^3\sqrt{\frac{b}{a}}\,\arctan\left(x\sqrt{\frac{b}{a}}\right)+2\,bpx^2+ap\log\left(bx^2+a\right)+a\log\left(c\right)}{3\,ax^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^4,x, algorithm="fricas")
```

```
[Out] [1/3*(b*p*x^3*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) -
2*b*p*x^2 - a*p*log(b*x^2 + a) - a*log(c))/(a*x^3), -1/3*(2*b*p*x^3*sqrt(b/
a)*arctan(x*sqrt(b/a)) + 2*b*p*x^2 + a*p*log(b*x^2 + a) + a*log(c))/(a*x^3)
]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 479 vs. 2(56) = 112.
time = 40.55, size = 479, normalized size = 7.98

$$\begin{cases} -\frac{\log\left(0^p c\right)}{3x^3} & \text{for}\ a=0\wedge b=0 \\ -\frac{\log\left(a^p c\right)}{3x^3} & \text{for}\ b=0 \\ -\frac{2p}{9x^3}-\frac{\log\left(c\left(bx^2\right)^p\right)}{3x^3} & \text{for}\ a=0 \\ -\frac{a^2\sqrt{-\frac{a}{b}}\log\left(c\left(a+bx^2\right)^p\right)}{3a^2x^3\sqrt{-\frac{a}{b}}+3abx^5\sqrt{-\frac{a}{b}}}-\frac{2apx^3\log\left(x-\sqrt{-\frac{a}{b}}\right)}{\frac{3a^2x^3\sqrt{-\frac{a}{b}}}{b}+3ax^5\sqrt{-\frac{a}{b}}}-\frac{2apx^2\sqrt{-\frac{a}{b}}}{\frac{3a^2x^3\sqrt{-\frac{a}{b}}}{b}+3ax^5\sqrt{-\frac{a}{b}}}+\frac{ax^3\log\left(c\left(a+bx^2\right)^p\right)}{\frac{3a^2x^3\sqrt{-\frac{a}{b}}}{b}+3ax^5\sqrt{-\frac{a}{b}}}-\frac{ax^2\sqrt{-\frac{a}{b}}\log\left(c\left(a+bx^2\right)^p\right)}{\frac{3a^2x^3\sqrt{-\frac{a}{b}}}{b}+3ax^5\sqrt{-\frac{a}{b}}}-\frac{2bpx^5\log\left(x-\sqrt{-\frac{a}{b}}\right)}{\frac{3a^2x^3\sqrt{-\frac{a}{b}}}{b}+3ax^5\sqrt{-\frac{a}{b}}}-\frac{2bpx^4\sqrt{-\frac{a}{b}}}{\frac{3a^2x^3\sqrt{-\frac{a}{b}}}{b}+3ax^5\sqrt{-\frac{a}{b}}}+\frac{bx^5\log\left(c\left(a+bx^2\right)^p\right)}{\frac{3a^2x^3\sqrt{-\frac{a}{b}}}{b}+3ax^5\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/x**4,x)
```

```
[Out] Piecewise((-log(0**p*c)/(3*x**3), Eq(a, 0) & Eq(b, 0)), (-log(a**p*c)/(3*x*
*3), Eq(b, 0)), (-2*p/(9*x**3) - log(c*(b*x**2)**p)/(3*x**3), Eq(a, 0)), (-
a**2*sqrt(-a/b)*log(c*(a + b*x**2)**p)/(3*a**2*x**3*sqrt(-a/b) + 3*a*b*x**5
```

```
*sqrt(-a/b)) - 2*a*p*x**3*log(x - sqrt(-a/b))/(3*a**2*x**3*sqrt(-a/b)/b + 3
*a*x**5*sqrt(-a/b)) - 2*a*p*x**2*sqrt(-a/b)/(3*a**2*x**3*sqrt(-a/b)/b + 3*a
*x**5*sqrt(-a/b)) + a*x**3*log(c*(a + b*x**2)**p)/(3*a**2*x**3*sqrt(-a/b)/b
 + 3*a*x**5*sqrt(-a/b)) - a*x**2*sqrt(-a/b)*log(c*(a + b*x**2)**p)/(3*a**2*
x**3*sqrt(-a/b)/b + 3*a*x**5*sqrt(-a/b)) - 2*b*p*x**5*log(x - sqrt(-a/b))/(
3*a**2*x**3*sqrt(-a/b)/b + 3*a*x**5*sqrt(-a/b)) - 2*b*p*x**4*sqrt(-a/b)/(3*
a**2*x**3*sqrt(-a/b)/b + 3*a*x**5*sqrt(-a/b)) + b*x**5*log(c*(a + b*x**2)**
p)/(3*a**2*x**3*sqrt(-a/b)/b + 3*a*x**5*sqrt(-a/b)), True))
```

Giac [A]
time = 4.61, size = 58, normalized size = 0.97

$$-\frac{2\,b^2p\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,a}-\frac{p\log\left(bx^2+a\right)}{3\,x^3}-\frac{2\,bpx^2+a\log\left(c\right)}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^4,x, algorithm="giac")
```

```
[Out] -2/3*b^2*p*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/3*p*log(b*x^2 + a)/x^3 -
 1/3*(2*b*p*x^2 + a*log(c))/(a*x^3)
```

Mupad [B]
time = 0.25, size = 46, normalized size = 0.77

$$-\frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{3\,x^3}-\frac{2\,b^{3/2}\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3\,a^{3/2}}-\frac{2\,b\,p}{3\,a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)/x^4,x)
```

```
[Out] - log(c*(a + b*x^2)^p)/(3*x^3) - (2*b^(3/2)*p*atan((b^(1/2)*x)/a^(1/2)))/(3
*a^(3/2)) - (2*b*p)/(3*a*x)
```

3.10 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^5} dx$

Optimal. Leaf size=64

$$-\frac{bp}{4ax^2}-\frac{b^2p\log(x)}{2a^2}+\frac{b^2p\log\left(a+bx^2\right)}{4a^2}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{4x^4}$$

```
[Out] -1/4*b*p/a/x^2-1/2*b^2*p*ln(x)/a^2+1/4*b^2*p*ln(b*x^2+a)/a^2-1/4*ln(c*(b*x^
2+a)^p)/x^4
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2504, 2442, 46}

$$\frac{b^2p\log\left(a+bx^2\right)}{4a^2}-\frac{b^2p\log(x)}{2a^2}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{4x^4}-\frac{bp}{4ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/x^5,x]
```

```
[Out] -1/4*(b*p)/(a*x^2) - (b^2*p*Log[x])/(2*a^2) + (b^2*p*Log[a + b*x^2])/(4*a^2
) - Log[c*(a + b*x^2)^p]/(4*x^4)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
```

```
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(c(a+bx^2)^p\right)}{x^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x^3}\,dx, x, x^2\right)\\
&= -\frac{\log\left(c(a+bx^2)^p\right)}{4x^4} + \frac{1}{4}(bp)\text{Subst}\left(\int \frac{1}{x^2(a+bx)}\,dx, x, x^2\right)\\
&= -\frac{\log\left(c(a+bx^2)^p\right)}{4x^4} + \frac{1}{4}(bp)\text{Subst}\left(\int \left(\frac{1}{ax^2} - \frac{b}{a^2x} + \frac{b^2}{a^2(a+bx)}\right)\,dx, x, x^2\right)\\
&= -\frac{bp}{4ax^2} - \frac{b^2p\log(x)}{2a^2} + \frac{b^2p\log\left(a+bx^2\right)}{4a^2} - \frac{\log\left(c(a+bx^2)^p\right)}{4x^4}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 56, normalized size = 0.88

$$\frac{1}{4}bp\left(-\frac{1}{ax^2} - \frac{2b\log(x)}{a^2} + \frac{b\log\left(a+bx^2\right)}{a^2}\right) - \frac{\log\left(c(a+bx^2)^p\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/x^5,x]
```

```
[Out] (b*p*(-(1/(a*x^2)) - (2*b*Log[x])/a^2 + (b*Log[a + b*x^2])/a^2))/4 - Log[c*
(a + b*x^2)^p]/(4*x^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.21, size = 198, normalized size = 3.09

method	result
risch	$-\frac{\ln\left(\left(b\,x^2+a\right)^p\right)}{4x^4} - \frac{4b^2p\ln(x)x^4-2b^2p\ln\left(-b\,x^2-a\right)x^4+i\pi\,a^2\mathrm{csgn}\left(i\left(b\,x^2+a\right)^p\right)\mathrm{csgn}\left(ic\left(b\,x^2+a\right)^p\right)^2-i\pi\,a^2\mathrm{csgn}\left(i\left(b\,x^2+a\right)^p\right)\mathrm{csgn}(i}{8a^2x^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/x^4*ln((b*x^2+a)^p)-1/8*(4*b^2*p*ln(x)*x^4-2*b^2*p*ln(-b*x^2-a)*x^4+I*
Pi*a^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*a^2*csgn(I*(b*x^2+a
)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*a^2*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*
a^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*a*b*p*x^2+2*ln(c)*a^2)/a^2/x^4
```

Maxima [A]
time = 0.27, size = 54, normalized size = 0.84

$$\frac{1}{4}\,bp\left(\frac{b\log\left(bx^2+a\right)}{a^2}-\frac{b\log\left(x^2\right)}{a^2}-\frac{1}{ax^2}\right)-\frac{\log\left(\left(bx^2+a\right)^p c\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*b*p*(b*log(b*x^2 + a)/a^2 - b*log(x^2)/a^2 - 1/(a*x^2)) - 1/4*log((b*x^
2 + a)^p*c)/x^4
```

Fricas [A]
time = 0.38, size = 58, normalized size = 0.91

$$-\frac{2\,b^2px^4\log\left(x\right)+abpx^2+a^2\log\left(c\right)-\left(b^2px^4-a^2p\right)\log\left(bx^2+a\right)}{4\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^5,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^2*p*x^4*log(x) + a*b*p*x^2 + a^2*log(c) - (b^2*p*x^4 - a^2*p)*log
(b*x^2 + a))/(a^2*x^4)
```

Sympy [A]
time = 3.08, size = 83, normalized size = 1.30

$$\begin{cases} -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{4x^4}-\frac{bp}{4ax^2}-\frac{b^2p\log\left(x\right)}{2a^2}+\frac{b^2\log\left(c\left(a+bx^2\right)^p\right)}{4a^2} & \text{for}\: a\neq 0 \\ -\frac{p}{8x^4}-\frac{\log\left(c\left(bx^2\right)^p\right)}{4x^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/x**5,x)
```

```
[Out] Piecewise((-log(c*(a + b*x**2)**p)/(4*x**4) - b*p/(4*a*x**2) - b**2*p*log(x
)/(2*a**2) + b**2*log(c*(a + b*x**2)**p)/(4*a**2), Ne(a, 0)), (-p/(8*x**4)
- log(c*(b*x**2)**p)/(4*x**4), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(56) = 112.
time = 4.40, size = 132, normalized size = 2.06

$$-\frac{\frac{b^3p\log\left(bx^2+a\right)}{\left(bx^2+a\right)^2-2\left(bx^2+a\right)a+a^2}-\frac{b^3p\log\left(bx^2+a\right)}{a^2}+\frac{b^3p\log\left(bx^2\right)}{a^2}+\frac{\left(bx^2+a\right)b^3p-ab^3p+ab^3\log\left(c\right)}{\left(bx^2+a\right)^2a-2\left(bx^2+a\right)a^2+a^3}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^5,x, algorithm="giac")
```

```
[Out] -1/4*(b^3*p*log(b*x^2 + a)/((b*x^2 + a)^2 - 2*(b*x^2 + a)*a + a^2) - b^3*p*
log(b*x^2 + a)/a^2 + b^3*p*log(b*x^2)/a^2 + ((b*x^2 + a)*b^3*p - a*b^3*p +
a*b^3*log(c))/((b*x^2 + a)^2*a - 2*(b*x^2 + a)*a^2 + a^3))/b
```

Mupad [B]
time = 0.26, size = 56, normalized size = 0.88

$$\frac{b^2\,p\,\ln\left(b\,x^2+a\right)}{4\,a^2}-\frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{4\,x^4}-\frac{b^2\,p\,\ln\left(x\right)}{2\,a^2}-\frac{b\,p}{4\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)/x^5,x)
```

```
[Out] (b^2*p*log(a + b*x^2))/(4*a^2) - log(c*(a + b*x^2)^p)/(4*x^4) - (b^2*p*log(
x))/(2*a^2) - (b*p)/(4*a*x^2)
```

3.11 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^6}\,dx$

Optimal. Leaf size=74

$$-\frac{2bp}{15ax^3}+\frac{2b^2p}{5a^2x}+\frac{2b^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{5a^{5/2}}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{5x^5}$$

```
[Out] -2/15*b*p/a/x^3+2/5*b^2*p/a^2/x+2/5*b^(5/2)*p*arctan(x*b^(1/2)/a^(1/2))/a^(
5/2)-1/5*ln(c*(b*x^2+a)^p)/x^5
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2505, 331, 211}

$$\frac{2b^{5/2}p\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{5a^{5/2}}+\frac{2b^2p}{5a^2x}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{5x^5}-\frac{2bp}{15ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/x^6,x]
```

```
[Out] (-2*b*p)/(15*a*x^3) + (2*b^2*p)/(5*a^2*x) + (2*b^(5/2)*p*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(5*a^(5/2)) - Log[c*(a + b*x^2)^p]/(5*x^5)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^6}\,dx &= -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{5x^5}+\frac{1}{5}(2bp)\int \frac{1}{x^4\left(a+bx^2\right)}\,dx\\ &= -\frac{2bp}{15ax^3}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{5x^5}-\frac{\left(2b^2p\right)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{5a}\\ &= -\frac{2bp}{15ax^3}+\frac{2b^2p}{5a^2x}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{5x^5}+\frac{\left(2b^3p\right)\int \frac{1}{a+bx^2}\,dx}{5a^2}\\ &= -\frac{2bp}{15ax^3}+\frac{2b^2p}{5a^2x}+\frac{2b^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{5a^{5/2}}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{5x^5}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.00, size = 49, normalized size = 0.66

$$-\frac{2bp\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\frac{bx^2}{a}\right)}{15ax^3}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/x^6,x]
```

```
[Out] (-2*b*p*Hypergeometric2F1[-3/2, 1, -1/2, -((b*x^2)/a)])/(15*a*x^3) - Log[c*
(a + b*x^2)^p]/(5*x^5)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.36, size = 236, normalized size = 3.19

method	result
risch	$-\frac{\ln\left(\left(b\,x^2+a\right)^p\right)}{5x^5}-\frac{-6\left(\sum_{\_R=\mathrm{RootOf}\left(a^5\_Z^2+b^5p^2\right)}\_R\ln\left(\left(3\_R^2a^5+2b^5p^2\right)x-a^3b^2p\_R\right)\right)a^2x^5+3i\pi\,a^2\mathrm{csgn}\left(i\left(b\,x^2+a\right)^p\right)\mathrm{cs}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/x^5*ln((b*x^2+a)^p)-1/30*(-6*sum(_R*ln((3*_R^2*a^5+2*b^5*p^2)*x-a^3*b^
2*p*_R),_R=RootOf(_Z^2*a^5+b^5*p^2))*a^2*x^5+3*I*Pi*a^2*csgn(I*(b*x^2+a)^p)
*csgn(I*c*(b*x^2+a)^p)^2-3*I*Pi*a^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^
p)*csgn(I*c)-3*I*Pi*a^2*csgn(I*c*(b*x^2+a)^p)^3+3*I*Pi*a^2*csgn(I*c*(b*x^2+
a)^p)^2*csgn(I*c)-12*b^2*p*x^4+4*a*b*p*x^2+6*ln(c)*a^2)/a^2/x^5
```

Maxima [A]
time = 0.48, size = 62, normalized size = 0.84

$$\frac{2}{15}\,bp\left(\frac{3\,b^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{2}}+\frac{3\,bx^{2}-a}{a^{2}x^{3}}\right)-\frac{\log\left(\left(bx^{2}+a\right)^{p}c\right)}{5\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^6,x, algorithm="maxima")
```

```
[Out] 2/15*b*p*(3*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + (3*b*x^2 - a)/(a^2*
x^3)) - 1/5*log((b*x^2 + a)^p*c)/x^5
```

Fricas [A]
time = 0.39, size = 170, normalized size = 2.30

$$\left[\frac{3\,b^{2}px^{5}\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)+6\,b^{2}px^{4}-2\,abpx^{2}-3\,a^{2}p\log\left(bx^{2}+a\right)-3\,a^{2}\log\left(c\right)}{15\,a^{2}x^{5}},\ \frac{6\,b^{2}px^{5}\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+6\,b^{2}px^{4}-2\,abpx^{2}-3\,a^{2}p\log\left(bx^{2}+a\right)-3\,a^{2}\log\left(c\right)}{15\,a^{2}x^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^6,x, algorithm="fricas")
```

```
[Out] [1/15*(3*b^2*p*x^5*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a
)) + 6*b^2*p*x^4 - 2*a*b*p*x^2 - 3*a^2*p*log(b*x^2 + a) - 3*a^2*log(c))/(a^
2*x^5), 1/15*(6*b^2*p*x^5*sqrt(b/a)*arctan(x*sqrt(b/a)) + 6*b^2*p*x^4 - 2*a
*b*p*x^2 - 3*a^2*p*log(b*x^2 + a) - 3*a^2*log(c))/(a^2*x^5)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 566 vs. 2(70) = 140.
time = 169.06, size = 566, normalized size = 7.65

$$\begin{cases} -\frac{\log\left(0^{p}c\right)}{5x^{5}} & \text{for}\ a=0\wedge b=0\\ -\frac{\log\left(a^{p}c\right)}{5x^{5}} & \text{for}\ b=0\\ -\frac{2p}{25x^{5}}-\frac{\log\left(c\left(bx^{2}\right)^{p}\right)}{5x^{5}} & \text{for}\ a=0\\ -\frac{3a^{3}\sqrt{-\frac{a}{b}}\log\left(c\left(a+bx^{2}\right)^{p}\right)}{15a^{3}x^{5}\sqrt{-\frac{a}{b}}+15a^{2}bx^{7}\sqrt{-\frac{a}{b}}}-\frac{2a^{2}px^{2}\sqrt{-\frac{a}{b}}}{\frac{15a^{3}x^{5}\sqrt{-\frac{a}{b}}}{b}+15a^{2}x^{7}\sqrt{-\frac{a}{b}}}-\frac{3a^{2}x^{2}\sqrt{-\frac{a}{b}}\log\left(c\left(a+bx^{2}\right)^{p}\right)}{\frac{15a^{3}x^{5}\sqrt{-\frac{a}{b}}}{b}+15a^{2}x^{7}\sqrt{-\frac{a}{b}}}+\frac{6abpx^{5}\log\left(x-\sqrt{-\frac{a}{b}}\right)}{\frac{15a^{3}x^{5}\sqrt{-\frac{a}{b}}}{b}+15a^{2}x^{7}\sqrt{-\frac{a}{b}}}+\frac{4abpx^{4}\sqrt{-\frac{a}{b}}}{\frac{15a^{3}x^{5}\sqrt{-\frac{a}{b}}}{b}+15a^{2}x^{7}\sqrt{-\frac{a}{b}}}-\frac{3abx^{5}\log\left(c\left(a+bx^{2}\right)^{p}\right)}{\frac{15a^{3}x^{5}\sqrt{-\frac{a}{b}}}{b}+15a^{2}x^{7}\sqrt{-\frac{a}{b}}}+\frac{6b^{2}px^{7}\log\left(x-\sqrt{-\frac{a}{b}}\right)}{\frac{15a^{3}x^{5}\sqrt{-\frac{a}{b}}}{b}+15a^{2}x^{7}\sqrt{-\frac{a}{b}}}+\frac{6b^{2}px^{6}\sqrt{-\frac{a}{b}}}{\frac{15a^{3}x^{5}\sqrt{-\frac{a}{b}}}{b}+15a^{2}x^{7}\sqrt{-\frac{a}{b}}}-\frac{3b^{2}x^{7}\log\left(c\left(a+bx^{2}\right)^{p}\right)}{\frac{15a^{3}x^{5}\sqrt{-\frac{a}{b}}}{b}+15a^{2}x^{7}\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/x**6,x)
```

```
[Out] Piecewise((-log(0**p*c)/(5*x**5), Eq(a, 0) & Eq(b, 0)), (-log(a**p*c)/(5*x*
*5), Eq(b, 0)), (-2*p/(25*x**5) - log(c*(b*x**2)**p)/(5*x**5), Eq(a, 0)), (
-3*a**3*sqrt(-a/b)*log(c*(a + b*x**2)**p)/(15*a**3*x**5*sqrt(-a/b) + 15*a**
2*b*x**7*sqrt(-a/b)) - 2*a**2*p*x**2*sqrt(-a/b)/(15*a**3*x**5*sqrt(-a/b)/b
+ 15*a**2*x**7*sqrt(-a/b)) - 3*a**2*x**2*sqrt(-a/b)*log(c*(a + b*x**2)**p)/
```

```
(15*a**3*x**5*sqrt(-a/b)/b + 15*a**2*x**7*sqrt(-a/b)) + 6*a*b*p*x**5*log(x
- sqrt(-a/b))/(15*a**3*x**5*sqrt(-a/b)/b + 15*a**2*x**7*sqrt(-a/b)) + 4*a*b
*p*x**4*sqrt(-a/b)/(15*a**3*x**5*sqrt(-a/b)/b + 15*a**2*x**7*sqrt(-a/b)) -
3*a*b*x**5*log(c*(a + b*x**2)**p)/(15*a**3*x**5*sqrt(-a/b)/b + 15*a**2*x**7
*sqrt(-a/b)) + 6*b**2*p*x**7*log(x - sqrt(-a/b))/(15*a**3*x**5*sqrt(-a/b)/b
 + 15*a**2*x**7*sqrt(-a/b)) + 6*b**2*p*x**6*sqrt(-a/b)/(15*a**3*x**5*sqrt(-
a/b)/b + 15*a**2*x**7*sqrt(-a/b)) - 3*b**2*x**7*log(c*(a + b*x**2)**p)/(15*
a**3*x**5*sqrt(-a/b)/b + 15*a**2*x**7*sqrt(-a/b)), True))
```

Giac [**A**]
time = 3.53, size = 71, normalized size = 0.96

$$\frac{2\,b^3p\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{5\,\sqrt{ab}\,a^2}-\frac{p\log\left(bx^2+a\right)}{5\,x^5}+\frac{6\,b^2px^4-2\,abpx^2-3\,a^2\log\left(c\right)}{15\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^6,x, algorithm="giac")
```

```
[Out] 2/5*b^3*p*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/5*p*log(b*x^2 + a)/x^5
+ 1/15*(6*b^2*p*x^4 - 2*a*b*p*x^2 - 3*a^2*log(c))/(a^2*x^5)
```

Mupad [**B**]
time = 0.26, size = 61, normalized size = 0.82

$$\frac{2\,b^{5/2}\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{5\,a^{5/2}}-\frac{\frac{2\,b\,p}{3\,a}-\frac{2\,b^2\,p\,x^2}{a^2}}{5\,x^3}-\frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)/x^6,x)
```

```
[Out] (2*b^(5/2)*p*atan((b^(1/2)*x)/a^(1/2)))/(5*a^(5/2)) - ((2*b*p)/(3*a) - (2*b
^2*p*x^2)/a^2)/(5*x^3) - log(c*(a + b*x^2)^p)/(5*x^5)
```

3.12 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^7}\,dx$

Optimal. Leaf size=78

$$-\frac{bp}{12ax^4}+\frac{b^2p}{6a^2x^2}+\frac{b^3p\log(x)}{3a^3}-\frac{b^3p\log\left(a+bx^2\right)}{6a^3}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{6x^6}$$

```
[Out] -1/12*b*p/a/x^4+1/6*b^2*p/a^2/x^2+1/3*b^3*p*ln(x)/a^3-1/6*b^3*p*ln(b*x^2+a)
/a^3-1/6*ln(c*(b*x^2+a)^p)/x^6
```

Rubi [A]
time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2504, 2442, 46}

$$-\frac{b^3p\log\left(a+bx^2\right)}{6a^3}+\frac{b^3p\log(x)}{3a^3}+\frac{b^2p}{6a^2x^2}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{6x^6}-\frac{bp}{12ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/x^7,x]
```

```
[Out] -1/12*(b*p)/(a*x^4) + (b^2*p)/(6*a^2*x^2) + (b^3*p*Log[x])/(3*a^3) - (b^3*p
*Log[a + b*x^2])/(6*a^3) - Log[c*(a + b*x^2)^p]/(6*x^6)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
```

```
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^7}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x^4}\,dx,x,x^2\right)\\ &= -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{6x^6}+\frac{1}{6}(bp)\text{Subst}\left(\int \frac{1}{x^3(a+bx)}\,dx,x,x^2\right)\\ &= -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{6x^6}+\frac{1}{6}(bp)\text{Subst}\left(\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)\,dx,x,\right.\\ &= -\frac{bp}{12ax^4}+\frac{b^2p}{6a^2x^2}+\frac{b^3p\log(x)}{3a^3}-\frac{b^3p\log\left(a+bx^2\right)}{6a^3}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{6x^6}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 68, normalized size = 0.87

$$-\frac{\frac{bpx^2\left(a\left(a-2bx^2\right)-4b^2x^4\log(x)+2b^2x^4\log\left(a+bx^2\right)\right)}{a^3}+2\log\left(c\left(a+bx^2\right)^p\right)}{12x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/x^7,x]
```

```
[Out] -1/12*((b*p*x^2*(a*(a - 2*b*x^2) - 4*b^2*x^4*Log[x] + 2*b^2*x^4*Log[a + b*x
^2]))/a^3 + 2*Log[c*(a + b*x^2)^p])/x^6
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.26, size = 206, normalized size = 2.64

method	result
risch	$-\frac{\ln\left(\left(b\,x^2+a\right)^p\right)}{6x^6}-\frac{-4b^3p\ln(x)x^6+2b^3p\ln\left(b\,x^2+a\right)x^6+i\pi\,a^3\mathrm{csgn}\left(i\left(b\,x^2+a\right)^p\right)\mathrm{csgn}\left(ic\left(b\,x^2+a\right)^p\right)^2-i\pi\,a^3\mathrm{csgn}\left(i\left(b\,x^2+a\right)^p\right)\mathrm{csgn}(i}{12}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/x^6*ln((b*x^2+a)^p)-1/12*(-4*b^3*p*ln(x)*x^6+2*b^3*p*ln(b*x^2+a)*x^6+I
*Pi*a^3*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*a^3*csgn(I*(b*x^2+
a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*a^3*csgn(I*c*(b*x^2+a)^p)^3+I*Pi
*a^3*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-2*a*b^2*p*x^4+a^2*b*p*x^2+2*ln(c)*a^
3)/a^3/x^6
```

Maxima [A]
time = 0.27, size = 69, normalized size = 0.88

$$-\frac{1}{12}\,bp\left(\frac{2\,b^2\log\left(bx^2+a\right)}{a^3}-\frac{2\,b^2\log\left(x^2\right)}{a^3}-\frac{2\,bx^2-a}{a^2x^4}\right)-\frac{\log\left(\left(bx^2+a\right)^p c\right)}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^7,x, algorithm="maxima")
```

```
[Out] -1/12*b*p*(2*b^2*log(b*x^2 + a)/a^3 - 2*b^2*log(x^2)/a^3 - (2*b*x^2 - a)/(a
^2*x^4)) - 1/6*log((b*x^2 + a)^p*c)/x^6
```

Fricas [A]
time = 0.38, size = 71, normalized size = 0.91

$$\frac{4\,b^3px^6\log\left(x\right)+2\,ab^2px^4-a^2bpx^2-2\,a^3\log\left(c\right)-2\,\left(b^3px^6+a^3p\right)\log\left(bx^2+a\right)}{12\,a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(4*b^3*p*x^6*log(x) + 2*a*b^2*p*x^4 - a^2*b*p*x^2 - 2*a^3*log(c) - 2*(
b^3*p*x^6 + a^3*p)*log(b*x^2 + a))/(a^3*x^6)
```

Sympy [A]
time = 8.34, size = 97, normalized size = 1.24

$$\begin{cases} -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{6x^6}-\frac{bp}{12ax^4}+\frac{b^2p}{6a^2x^2}+\frac{b^3p\log\left(x\right)}{3a^3}-\frac{b^3\log\left(c\left(a+bx^2\right)^p\right)}{6a^3} & \text{for}\ a\neq 0\\ -\frac{p}{18x^6}-\frac{\log\left(c\left(bx^2\right)^p\right)}{6x^6} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/x**7,x)
```

```
[Out] Piecewise((-log(c*(a + b*x**2)**p)/(6*x**6) - b*p/(12*a*x**4) + b**2*p/(6*a
**2*x**2) + b**3*p*log(x)/(3*a**3) - b**3*log(c*(a + b*x**2)**p)/(6*a**3),
Ne(a, 0)), (-p/(18*x**6) - log(c*(b*x**2)**p)/(6*x**6), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(68) = 136.
time = 3.65, size = 191, normalized size = 2.45

$$-\frac{\frac{2\,b^4p\log\left(bx^2+a\right)}{\left(bx^2+a\right)^3-3\,\left(bx^2+a\right)^2a+3\,\left(bx^2+a\right)a^2-a^3}+\frac{2\,b^4p\log\left(bx^2+a\right)}{a^3}-\frac{2\,b^4p\log\left(bx^2\right)}{a^3}-\frac{2\,\left(bx^2+a\right)^2b^4p-5\,\left(bx^2+a\right)ab^4p+3\,a^2b^4p-2\,a^2b^4\log\left(c\right)}{\left(bx^2+a\right)^3a^2-3\,\left(bx^2+a\right)^2a^3+3\,\left(bx^2+a\right)a^4-a^5}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^7,x, algorithm="giac")
```

```
[Out] -1/12*(2*b^4*p*log(b*x^2 + a)/((b*x^2 + a)^3 - 3*(b*x^2 + a)^2*a + 3*(b*x^2
 + a)*a^2 - a^3) + 2*b^4*p*log(b*x^2 + a)/a^3 - 2*b^4*p*log(b*x^2)/a^3 - (2
*(b*x^2 + a)^2*b^4*p - 5*(b*x^2 + a)*a*b^4*p + 3*a^2*b^4*p - 2*a^2*b^4*log(
c))/((b*x^2 + a)^3*a^2 - 3*(b*x^2 + a)^2*a^3 + 3*(b*x^2 + a)*a^4 - a^5))/b
```

Mupad [B]
time = 0.26, size = 68, normalized size = 0.87

$$\frac{b^2\,p}{6\,a^2\,x^2}-\frac{b^3\,p\,\ln\left(b\,x^2+a\right)}{6\,a^3}-\frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{6\,x^6}+\frac{b^3\,p\,\ln\left(x\right)}{3\,a^3}-\frac{b\,p}{12\,a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)/x^7,x)
```

```
[Out] (b^2*p)/(6*a^2*x^2) - (b^3*p*log(a + b*x^2))/(6*a^3) - log(c*(a + b*x^2)^p)
/(6*x^6) + (b^3*p*log(x))/(3*a^3) - (b*p)/(12*a*x^4)
```

3.13 $\int x^5 \log \left(c(a+bx^3)^p\right) \, dx$

Optimal. Leaf size=59

$$\frac{apx^3}{6b} - \frac{px^6}{12} - \frac{a^2p\log\left(a+bx^3\right)}{6b^2} + \frac{1}{6}x^6\log\left(c\left(a+bx^3\right)^p\right)$$

```
[Out] 1/6*a*p*x^3/b-1/12*p*x^6-1/6*a^2*p*ln(b*x^3+a)/b^2+1/6*x^6*ln(c*(b*x^3+a)^p
)
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2504, 2442, 45}

$$-\frac{a^2p\log\left(a+bx^3\right)}{6b^2} + \frac{1}{6}x^6\log\left(c\left(a+bx^3\right)^p\right) + \frac{apx^3}{6b} - \frac{px^6}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (a*p*x^3)/(6*b) - (p*x^6)/12 - (a^2*p*Log[a + b*x^3])/(6*b^2) + (x^6*Log[c*
(a + b*x^3)^p])/6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^5 \log \left(c\left(a+bx^3\right)^p\right)\,dx &= \frac{1}{3}\text{Subst}\left(\int x\log\left(c(a+bx)^p\right)\,dx,x,x^3\right)\\
&= \frac{1}{6}x^6\log\left(c\left(a+bx^3\right)^p\right)-\frac{1}{6}(bp)\text{Subst}\left(\int\frac{x^2}{a+bx}\,dx,x,x^3\right)\\
&= \frac{1}{6}x^6\log\left(c\left(a+bx^3\right)^p\right)-\frac{1}{6}(bp)\text{Subst}\left(\int\left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)\,dx,x,x^3\right)\\
&= \frac{apx^3}{6b}-\frac{px^6}{12}-\frac{a^2p\log\left(a+bx^3\right)}{6b^2}+\frac{1}{6}x^6\log\left(c\left(a+bx^3\right)^p\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 59, normalized size = 1.00

$$
\frac{apx^3}{6b}-\frac{px^6}{12}-\frac{a^2p\log\left(a+bx^3\right)}{6b^2}+\frac{1}{6}x^6\log\left(c\left(a+bx^3\right)^p\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (a*p*x^3)/(6*b) - (p*x^6)/12 - (a^2*p*Log[a + b*x^3])/(6*b^2) + (x^6*Log[c*
(a + b*x^3)^p])/6
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.39, size = 1190, normalized size = 20.17

method	result	size
risch	Expression too large to display	1190

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*ln(c*(b*x^3+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x^6*ln((b*x^3+a)^p)+1/12*(-Pi^2*b^2*x^6*csgn(I*c*(b*x^3+a)^p)^6+2*ln(b*
x^3+a)*a^2*p^2+4*ln(c)^2*b^2*x^6-2*I*Pi*b^2*p*x^6*csgn(I*(b*x^3+a)^p)*csgn(
I*c*(b*x^3+a)^p)^2+2*Pi^2*b^2*x^6*csgn(I*c*(b*x^3+a)^p)^5*csgn(I*c)+2*I*Pi*
b^2*p*x^6*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)-4*I*ln(c)*Pi*
b^2*x^6*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)-2*I*Pi*b^2*p*x^
6*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)-2*I*Pi*ln(b*x^3+a)*a^2*p*csgn(I*c*(b*x^
3+a)^p)^2*csgn(I*c)+2*Pi^2*b^2*x^6*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p
)^5-2*a*b*p^2*x^3-4*ln(c)*b^2*p*x^6-4*ln(c)*ln(b*x^3+a)*a^2*p+2*Pi^2*b^2*x^
6*csgn(I*(b*x^3+a)^p)^2*csgn(I*c*(b*x^3+a)^p)^3*csgn(I*c)+b^2*p^2*x^6+2*I*P
i*a*b*p*x^3*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)+2*I*Pi*a*b*p*x^3*csgn(I*(b*x^
```

```
3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2+2*I*Pi*ln(b*x^3+a)*a^2*p*csgn(I*(b*x^3+a)^p
)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)+4*ln(c)*a*b*p*x^3+a^2*p^2+2*Pi^2*b^2*x^6*
csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^3*csgn(I*c)^2-Pi^2*b^2*x^6*csgn(I
*(b*x^3+a)^p)^2*csgn(I*c*(b*x^3+a)^p)^4+2*I*Pi*b^2*p*x^6*csgn(I*c*(b*x^3+a)
^p)^3+4*I*ln(c)*Pi*b^2*x^6*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2-Pi^2
*b^2*x^6*csgn(I*(b*x^3+a)^p)^2*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)^2-2*I*Pi*l
n(b*x^3+a)*a^2*p*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2+2*I*Pi*ln(b*x^
3+a)*a^2*p*csgn(I*c*(b*x^3+a)^p)^3-4*I*ln(c)*Pi*b^2*x^6*csgn(I*c*(b*x^3+a)^
p)^3-2*I*Pi*a*b*p*x^3*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)+4
*I*ln(c)*Pi*b^2*x^6*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)-Pi^2*b^2*x^6*csgn(I*c
*(b*x^3+a)^p)^4*csgn(I*c)^2-4*Pi^2*b^2*x^6*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*
x^3+a)^p)^4*csgn(I*c)-2*I*Pi*a*b*p*x^3*csgn(I*c*(b*x^3+a)^p)^3)/b^2/(I*Pi*c
sgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2-I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*
c*(b*x^3+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^3+a)^p)^3+I*Pi*csgn(I*c*(b*x^3+
a)^p)^2*csgn(I*c)+2*ln(c)-p)
```

Maxima [A]
time = 0.28, size = 55, normalized size = 0.93

$$\frac{1}{6}\,x^6\log\left(\left(bx^3+a\right)^p c\right)-\frac{1}{12}\,bp\left(\frac{2\,a^2\log\left(bx^3+a\right)}{b^3}+\frac{bx^6-2\,ax^3}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] 1/6*x^6*log((b*x^3 + a)^p*c) - 1/12*b*p*(2*a^2*log(b*x^3 + a)/b^3 + (b*x^6
- 2*a*x^3)/b^2)
```

Fricas [A]
time = 0.38, size = 57, normalized size = 0.97

$$-\frac{b^2px^6-2\,b^2x^6\log\left(c\right)-2\,abpx^3-2\left(b^2px^6-a^2p\right)\log\left(bx^3+a\right)}{12\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] -1/12*(b^2*p*x^6 - 2*b^2*x^6*log(c) - 2*a*b*p*x^3 - 2*(b^2*p*x^6 - a^2*p)*l
og(b*x^3 + a))/b^2
```

Sympy [A]
time = 2.95, size = 65, normalized size = 1.10

$$\begin{cases} -\frac{a^2\log\left(c\left(a+bx^3\right)^p\right)}{6b^2}+\frac{apx^3}{6b}-\frac{px^6}{12}+\frac{x^6\log\left(c\left(a+bx^3\right)^p\right)}{6} & \text{for}\ b\neq 0\\ \frac{x^6\log\left(a^pc\right)}{6} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*ln(c*(b*x**3+a)**p),x)
```

```
[Out] Piecewise((-a**2*log(c*(a + b*x**3)**p)/(6*b**2) + a*p*x**3/(6*b) - p*x**6/
12 + x**6*log(c*(a + b*x**3)**p)/6, Ne(b, 0)), (x**6*log(a**p*c)/6, True))
```

Giac [A]
time = 5.62, size = 97, normalized size = 1.64

$$\frac{2\,(bx^3+a)^2p\log\left(bx^3+a\right)-(bx^3+a)^2p+2\,(bx^3+a)^2\log\left(c\right)}{12\,b^2}+\frac{(bx^3-(bx^3+a)\log\left(bx^3+a\right)+a)ap-(bx^3+a)a\log\left(c\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] 1/12*(2*(b*x^3 + a)^2*p*log(b*x^3 + a) - (b*x^3 + a)^2*p + 2*(b*x^3 + a)^2*
log(c))/b^2 + 1/3*((b*x^3 - (b*x^3 + a)*log(b*x^3 + a) + a)*a*p - (b*x^3 +
a)*a*log(c))/b^2
```

Mupad [B]
time = 0.24, size = 51, normalized size = 0.86

$$\frac{x^6\,\ln\left(c\,{\left(b\,x^3+a\right)}^p\right)}{6}-\frac{p\,x^6}{12}-\frac{a^2\,p\,\ln\left(b\,x^3+a\right)}{6\,b^2}+\frac{a\,p\,x^3}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*log(c*(a + b*x^3)^p),x)
```

```
[Out] (x^6*log(c*(a + b*x^3)^p))/6 - (p*x^6)/12 - (a^2*p*log(a + b*x^3))/(6*b^2)
+ (a*p*x^3)/(6*b)
```

3.14 $\int x^4 \log\left(c(a+bx^3)^p\right)\,dx$

Optimal. Leaf size=159

$$\frac{3apx^2}{10b}-\frac{3px^5}{25}+\frac{\sqrt{3}\,a^{5/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{5b^{5/3}}+\frac{a^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{5b^{5/3}}-\frac{a^{5/3}p\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{10b^{5/3}}$$

```
[Out] 3/10*a*p*x^2/b-3/25*p*x^5+1/5*a^(5/3)*p*ln(a^(1/3)+b^(1/3)*x)/b^(5/3)-1/10*
a^(5/3)*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(5/3)+1/5*x^5*ln(c*(b
*x^3+a)^p)+1/5*a^(5/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*
3^(1/2)/b^(5/3)
```

Rubi [A]
time = 0.08, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2505, 308, 298, 31, 648, 631, 210, 642}

$$\frac{\sqrt{3}\,a^{5/3}p\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{5b^{5/3}}-\frac{a^{5/3}p\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{10b^{5/3}}+\frac{a^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{5b^{5/3}}+\frac{1}{5}x^5\log\left(c\left(a+bx^3\right)^p\right)+\frac{3apx^2}{10b}-\frac{3px^5}{25}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (3*a*p*x^2)/(10*b) - (3*p*x^5)/25 + (Sqrt[3]*a^(5/3)*p*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(5*b^(5/3)) + (a^(5/3)*p*Log[a^(1/3) + b^(1/
3)*x])/(5*b^(5/3)) - (a^(5/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(10*b^(5/3)) + (x^5*Log[c*(a + b*x^3)^p])/5
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^4 \log \left(c\left(a+bx^3\right)^p\right)\, dx = \frac{1}{5}x^5 \log \left(c\left(a+bx^3\right)^p\right) - \frac{1}{5}(3bp)\int \frac{x^7}{a+bx^3}\, dx$$

$$= \frac{1}{5}x^5 \log \left(c\left(a+bx^3\right)^p\right) - \frac{1}{5}(3bp)\int \left(-\frac{ax}{b^2}+\frac{x^4}{b}+\frac{a^2x}{b^2\left(a+bx^3\right)}\right) dx$$

$$= \frac{3apx^2}{10b} - \frac{3px^5}{25} + \frac{1}{5}x^5 \log \left(c\left(a+bx^3\right)^p\right) - \frac{\left(3a^2p\right)\int \frac{x}{a+bx^3}\, dx}{5b}$$

$$= \frac{3apx^2}{10b} - \frac{3px^5}{25} + \frac{1}{5}x^5 \log \left(c\left(a+bx^3\right)^p\right) + \frac{\left(a^{5/3}p\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\, x}\, dx}{5b^{4/3}} - \frac{\left(a^{5/3}p\right)\int \cdots}{\cdots}$$

$$= \frac{3apx^2}{10b} - \frac{3px^5}{25} + \frac{a^{5/3}p \log \left(\sqrt[3]{a}+\sqrt[3]{b}\, x\right)}{5b^{5/3}} + \frac{1}{5}x^5 \log \left(c\left(a+bx^3\right)^p\right) - \frac{\left(a^{5/3}p\right)\int \cdots}{\cdots}$$

$$= \frac{3apx^2}{10b} - \frac{3px^5}{25} + \frac{a^{5/3}p \log \left(\sqrt[3]{a}+\sqrt[3]{b}\, x\right)}{5b^{5/3}} - \frac{a^{5/3}p \log \left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\, x+b^{2/3}x \cdots\right)}{10b^{5/3}}$$

$$= \frac{3apx^2}{10b} - \frac{3px^5}{25} + \frac{\sqrt{3}\, a^{5/3}p \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\, x}{\sqrt{3}\sqrt[3]{a}}\right)}{5b^{5/3}} + \frac{a^{5/3}p \log \left(\sqrt[3]{a}+\sqrt[3]{b}\, x\right)}{5b^{5/3}} - \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.00, size = 69, normalized size = 0.43

$$\frac{3apx^2}{10b} - \frac{3px^5}{25} - \frac{3apx^2\, {}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a}\right)}{10b} + \frac{1}{5}x^5 \log \left(c\left(a+bx^3\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (3*a*p*x^2)/(10*b) - (3*p*x^5)/25 - (3*a*p*x^2*Hypergeometric2F1[2/3, 1, 5/
3, -((b*x^3)/a)])/(10*b) + (x^5*Log[c*(a + b*x^3)^p])/5
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.41, size = 196, normalized size = 1.23

method	result
risch	$\frac{x^5 \ln\left(\left(x^3b+a\right)^p\right)}{5} - \frac{i\pi\, x^5 \mathrm{csgn}\left(ic\left(x^3b+a\right)^p\right)^3}{10} + \frac{i\pi\, x^5 \mathrm{csgn}\left(ic\left(x^3b+a\right)^p\right)^2 \mathrm{csgn}(ic)}{10} + \frac{i\pi\, x^5 \mathrm{csgn}\left(i\left(x^3b+a\right)^p\right) \mathrm{csgn}\left(ic\left(x^3b+a\right)^p\right)^2}{10} - \cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*ln(c*(b*x^3+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^5*ln((b*x^3+a)^p)-1/10*I*Pi*x^5*csgn(I*c*(b*x^3+a)^p)^3+1/10*I*Pi*x^5
*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)+1/10*I*Pi*x^5*csgn(I*(b*x^3+a)^p)*csgn(I
*c*(b*x^3+a)^p)^2-1/10*I*Pi*x^5*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*c
sgn(I*c)+1/5*ln(c)*x^5-3/25*p*x^5+3/10*a*p*x^2/b-1/5/b^2*a^2*p*sum(1/_R*ln(
x-_R),_R=RootOf(_Z^3*b+a))
```

Maxima [A]
time = 0.50, size = 147, normalized size = 0.92

$$\frac{1}{5}\,x^{5}\log\left(\left(bx^{3}+a\right)^{p}c\right)-\frac{1}{50}\,bp\left(\frac{10\,\sqrt{3}\,a^{2}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\,a^{2}\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{10\,a^{2}\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{3\left(2\,bx^{5}-5\,ax^{2}\right)}{b^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*log((b*x^3 + a)^p*c) - 1/50*b*p*(10*sqrt(3)*a^2*arctan(1/3*sqrt(3)*
(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(1/3)) + 5*a^2*log(x^2 - x*(a/b
)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(1/3)) - 10*a^2*log(x + (a/b)^(1/3))/(b^3
*(a/b)^(1/3)) + 3*(2*b*x^5 - 5*a*x^2)/b^2)
```

Fricas [A]
time = 0.40, size = 161, normalized size = 1.01

$$\frac{10\,bpx^{5}\log\left(bx^{3}+a\right)-6\,bpx^{5}+10\,bx^{5}\log\left(c\right)+15\,apx^{2}-10\,\sqrt{3}\,ap\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,bx\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}-\sqrt{3}\,a}{3\,a}\right)-5\,ap\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\log\left(ax^{2}-bx\left(\frac{a^{2}}{b^{2}}\right)^{\frac{2}{3}}+a\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\right)+10\,ap\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\log\left(ax+b\left(\frac{a^{2}}{b^{2}}\right)^{\frac{2}{3}}\right)}{50\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] 1/50*(10*b*p*x^5*log(b*x^3 + a) - 6*b*p*x^5 + 10*b*x^5*log(c) + 15*a*p*x^2
- 10*sqrt(3)*a*p*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3)
- sqrt(3)*a)/a) - 5*a*p*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a
*(a^2/b^2)^(1/3)) + 10*a*p*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/b
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*ln(c*(b*x**3+a)**p),x)
```

```
[Out] Timed out
```

Giac [A]
time = 2.86, size = 162, normalized size = 1.02

$$\frac{1}{10}\,a^{2}b^{4}p\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{ab^{5}}+\frac{2\sqrt{3}\left(-ab^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^{7}}-\frac{\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab^{7}}\right)+\frac{1}{5}\,px^{5}\log\left(bx^{3}+a\right)-\frac{1}{25}\,(3\,p-5\,\log\left(c\right))x^{5}+\frac{3\,apx^{2}}{10\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] 1/10*a^2*b^4*p*(2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5) + 2*sqrt(
3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*
b^7) - (-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^7)) + 1
/5*p*x^5*log(b*x^3 + a) - 1/25*(3*p - 5*log(c))*x^5 + 3/10*a*p*x^2/b
```

Mupad [B]
time = 2.50, size = 157, normalized size = 0.99

$$\frac{x^{5}\,\ln\left(c\,{\left(b\,x^{3}+a\right)}^{p}\right)}{5}-\frac{3\,p\,x^{5}}{25}+\frac{a^{5/3}\,p\,\ln\left(b^{1/3}\,x+a^{1/3}\right)}{5\,b^{5/3}}+\frac{3\,a\,p\,x^{2}}{10\,b}+\frac{a^{5/3}\,p\,\ln\left(\frac{9\,a^{4}\,p^{2}\,x}{25\,b}+\frac{9\,a^{13/3}\,p^{2}\,{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^{2}}{25\,b^{4/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{5\,b^{5/3}}-\frac{a^{5/3}\,p\,\ln\left(\frac{9\,a^{4}\,p^{2}\,x}{25\,b}+\frac{9\,a^{13/3}\,p^{2}\,{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^{2}}{25\,b^{4/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{5\,b^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*log(c*(a + b*x^3)^p),x)
```

```
[Out] (x^5*log(c*(a + b*x^3)^p))/5 - (3*p*x^5)/25 + (a^(5/3)*p*log(b^(1/3)*x + a^
(1/3)))/(5*b^(5/3)) + (3*a*p*x^2)/(10*b) + (a^(5/3)*p*log((9*a^4*p^2*x)/(25
*b) + (9*a^(13/3)*p^2*((3^(1/2)*1i)/2 - 1/2)^2)/(25*b^(4/3)))*((3^(1/2)*1i)
/2 - 1/2))/(5*b^(5/3)) - (a^(5/3)*p*log((9*a^4*p^2*x)/(25*b) + (9*a^(13/3)*
p^2*((3^(1/2)*1i)/2 + 1/2)^2)/(25*b^(4/3)))*((3^(1/2)*1i)/2 + 1/2))/(5*b^(5
/3))
```

3.15 $\int x^3 \log\left(c(a+bx^3)^p\right)\,dx$

Optimal. Leaf size=157

$$\frac{3apx}{4b}-\frac{3px^4}{16}+\frac{\sqrt{3}\,a^{4/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{4b^{4/3}}-\frac{a^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{4b^{4/3}}+\frac{a^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right.}{8b^{4/3}}$$

```
[Out] 3/4*a*p*x/b-3/16*p*x^4-1/4*a^(4/3)*p*ln(a^(1/3)+b^(1/3)*x)/b^(4/3)+1/8*a^(4
/3)*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(4/3)+1/4*x^4*ln(c*(b*x^3
+a)^p)+1/4*a^(4/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1
/2)/b^(4/3)
```

Rubi [A]
time = 0.08, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2505, 308, 206, 31, 648, 631, 210, 642}

$$\frac{\sqrt{3}\,a^{4/3}p\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{4b^{4/3}}+\frac{a^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{8b^{4/3}}-\frac{a^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{4b^{4/3}}+\frac{1}{4}x^4\log\left(c\left(a+bx^3\right)^p\right)+\frac{3apx}{4b}-\frac{3px^4}{16}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (3*a*p*x)/(4*b) - (3*p*x^4)/16 + (Sqrt[3]*a^(4/3)*p*ArcTan[(a^(1/3) - 2*b^(
1/3)*x)/(Sqrt[3]*a^(1/3))])/(4*b^(4/3)) - (a^(4/3)*p*Log[a^(1/3) + b^(1/3)*
x])/(4*b^(4/3)) + (a^(4/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(8*b^(4/3)) + (x^4*Log[c*(a + b*x^3)^p])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^3 \log \left(c(a+bx^3)^p\right) \, dx = \frac{1}{4}x^4 \log \left(c(a+bx^3)^p\right) - \frac{1}{4}(3bp) \int \frac{x^6}{a+bx^3} \, dx$$

$$= \frac{1}{4}x^4 \log \left(c(a+bx^3)^p\right) - \frac{1}{4}(3bp) \int \left(-\frac{a}{b^2} + \frac{x^3}{b} + \frac{a^2}{b^2(a+bx^3)}\right) dx$$

$$= \frac{3apx}{4b} - \frac{3px^4}{16} + \frac{1}{4}x^4 \log \left(c(a+bx^3)^p\right) - \frac{(3a^2p)\int \frac{1}{a+bx^3} \, dx}{4b}$$

$$= \frac{3apx}{4b} - \frac{3px^4}{16} + \frac{1}{4}x^4 \log \left(c(a+bx^3)^p\right) - \frac{(a^{4/3}p)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x} \, dx}{4b} - \frac{(a^{4/3}p)\int}{}$$

$$= \frac{3apx}{4b} - \frac{3px^4}{16} - \frac{a^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{4b^{4/3}} + \frac{1}{4}x^4 \log \left(c(a+bx^3)^p\right) + \frac{(a^{4/3}p)\int}{}$$

$$= \frac{3apx}{4b} - \frac{3px^4}{16} - \frac{a^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{4b^{4/3}} + \frac{a^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right.}{8b^{4/3}}$$

$$= \frac{3apx}{4b} - \frac{3px^4}{16} + \frac{\sqrt{3}\,a^{4/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{4b^{4/3}} - \frac{a^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{4b^{4/3}} +$$

Mathematica [A]
time = 0.03, size = 147, normalized size = 0.94

$$\frac{12a\sqrt[3]{b}\,px - 3b^{4/3}px^4 + 4\sqrt{3}\,a^{4/3}p\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 4a^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right) + 2a^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right) + 4b^{4/3}x^4\log\left(c(a+bx^3)^p\right)}{16b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (12*a*b^(1/3)*p*x - 3*b^(4/3)*p*x^4 + 4*Sqrt[3]*a^(4/3)*p*ArcTan[(1 - (2*b^
(1/3)*x)/a^(1/3))/Sqrt[3]] - 4*a^(4/3)*p*Log[a^(1/3) + b^(1/3)*x] + 2*a^(4/
3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 4*b^(4/3)*x^4*Log[c*(
a + b*x^3)^p])/(16*b^(4/3))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.36, size = 194, normalized size = 1.24

method	result

risch	$\frac{x^4\ln((x^3b+a)^p)}{4}+\frac{i\pi x^4\mathrm{csgn}(i(x^3b+a)^p)\mathrm{csgn}(ic(x^3b+a)^p)^2}{8}-\frac{i\pi x^4\mathrm{csgn}(i(x^3b+a)^p)\mathrm{csgn}(ic(x^3b+a)^p)\mathrm{csgn}(ic)}{8}-\frac{i\pi x^4\mathrm{csgn}(i\ldots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(b*x^3+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*ln((b*x^3+a)^p)+1/8*I*Pi*x^4*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)
^p)^2-1/8*I*Pi*x^4*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)-1/8*
I*Pi*x^4*csgn(I*c*(b*x^3+a)^p)^3+1/8*I*Pi*x^4*csgn(I*c*(b*x^3+a)^p)^2*csgn(
I*c)+1/4*ln(c)*x^4-3/16*p*x^4-1/4/b^2*a^2*p*sum(1/_R^2*ln(x-_R),_R=RootOf(_
Z^3*b+a))+3/4*a*p*x/b
```

Maxima [A]
time = 0.50, size = 144, normalized size = 0.92

$$\frac{1}{4}x^4\log\left((bx^3+a)^pc\right)-\frac{1}{16}bp\left(\frac{4\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{2a^2\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{4a^2\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{3(bx^4-4ax)}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log((b*x^3 + a)^p*c) - 1/16*b*p*(4*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(
2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(2/3)) - 2*a^2*log(x^2 - x*(a/b)
^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) + 4*a^2*log(x + (a/b)^(1/3))/(b^3*(
a/b)^(2/3)) + 3*(b*x^4 - 4*a*x)/b^2)
```

Fricas [A]
time = 0.40, size = 144, normalized size = 0.92

$$\frac{4bpx^4\log(bx^3+a)-3bpx^4+4bx^4\log(c)+4\sqrt{3}ap\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)-2ap\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)+4ap\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)+12apx}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] 1/16*(4*b*p*x^4*log(b*x^3 + a) - 3*b*p*x^4 + 4*b*x^4*log(c) + 4*sqrt(3)*a*p
*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) - 2*a*
p*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)) + 4*a*p*(-a/b)^(1/3
)*log(x - (-a/b)^(1/3)) + 12*a*p*x)/b
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(b*x**3+a)**p),x)
```

```
[Out] Timed out
```

Giac [A]
time = 6.14, size = 160, normalized size = 1.02

$$\frac{1}{8}a^2b^3p\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{ab^4}-\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^5}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab^5}\right)+\frac{1}{4}px^4\log\left(bx^3+a\right)-\frac{1}{16}\left(3p-4\log\left(c\right)\right)x^4+\frac{3apx}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] 1/8*a^2*b^3*p*(2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4) - 2*sqrt(3
)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b
^5) - (-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^5)) + 1/
4*p*x^4*log(b*x^3 + a) - 1/16*(3*p - 4*log(c))*x^4 + 3/4*a*p*x/b
```

Mupad [B]
time = 2.55, size = 129, normalized size = 0.82

$$\frac{x^4\,\ln\left(c\,{\left(b\,x^3+a\right)}^p\right)}{4}-\frac{3\,p\,x^4}{16}+\frac{3\,a\,p\,x}{4\,b}-\frac{a^{4/3}\,p\,\ln\left(b^{1/3}\,x+a^{1/3}\right)}{4\,b^{4/3}}+\frac{a^{4/3}\,p\,\ln\left(2\,b^{1/3}\,x-a^{1/3}-\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{4\,b^{4/3}}-\frac{a^{4/3}\,p\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{4\,b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(c*(a + b*x^3)^p),x)
```

```
[Out] (x^4*log(c*(a + b*x^3)^p))/4 - (3*p*x^4)/16 + (3*a*p*x)/(4*b) - (a^(4/3)*p*
log(b^(1/3)*x + a^(1/3)))/(4*b^(4/3)) + (a^(4/3)*p*log(2*b^(1/3)*x - 3^(1/2
)*a^(1/3)*1i - a^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(4*b^(4/3)) - (a^(4/3)*p*lo
g(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(4*b^
(4/3))
```

3.16 $\int x^2 \log\left(c(a+bx^3)^p\right)\,dx$

Optimal. Leaf size=35

$$-\frac{px^3}{3}+\frac{(a+bx^3)\log\left(c(a+bx^3)^p\right)}{3b}$$

```
[Out] -1/3*p*x^3+1/3*(b*x^3+a)*ln(c*(b*x^3+a)^p)/b
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.188$, Rules used = {2504, 2436, 2332}

$$\frac{(a+bx^3)\log\left(c(a+bx^3)^p\right)}{3b}-\frac{px^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[c*(a + b*x^3)^p],x]
```

```
[Out] -1/3*(p*x^3) + ((a + b*x^3)*Log[c*(a + b*x^3)^p])/(3*b)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^2 \log \left(c\left(a+bx^3\right)^p\right) \, dx &= \frac{1}{3}\text{Subst}\left(\int \log \left(c(a+bx)^p\right) \, dx, x, x^3\right) \\
&= \frac{\text{Subst}\left(\int \log \left(cx^p\right) \, dx, x, a+bx^3\right)}{3b} \\
&= -\frac{px^3}{3} + \frac{\left(a+bx^3\right) \log \left(c\left(a+bx^3\right)^p\right)}{3b}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 0.97

$$
\frac{1}{3}\left(-px^3 + \frac{\left(a+bx^3\right) \log \left(c\left(a+bx^3\right)^p\right)}{b}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (-(p*x^3) + ((a + b*x^3)*Log[c*(a + b*x^3)^p])/b)/3
```

Maple [A]
time = 0.47, size = 37, normalized size = 1.06

method	result
derivativedivides	$\frac{\ln\left(c\left(x^3b+a\right)^p\right)\left(x^3b+a\right)-\left(x^3b+a\right)p}{3b}$
default	$\frac{\ln\left(c\left(x^3b+a\right)^p\right)\left(x^3b+a\right)-\left(x^3b+a\right)p}{3b}$
risch	$\frac{x^3\ln\left(\left(x^3b+a\right)^p\right)}{3} - \frac{i\pi\, x^3 \text{csgn}\left(i\left(x^3b+a\right)^p\right)\text{csgn}\left(ic\left(x^3b+a\right)^p\right)\text{csgn}(ic)}{6} + \frac{i\pi\, x^3 \text{csgn}\left(ic\left(x^3b+a\right)^p\right)^2\text{csgn}(ic)}{6} + \frac{i\pi\, x^3\text{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(b*x^3+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/b*(ln(c*(b*x^3+a)^p)*(b*x^3+a)-(b*x^3+a)*p)
```

Maxima [A]
time = 0.29, size = 44, normalized size = 1.26

$$
\frac{1}{3}\,x^3 \log \left(\left(bx^3+a\right)^p c\right) - \frac{1}{3}\left(\frac{x^3}{b} - \frac{a \log \left(bx^3+a\right)}{b^2}\right)bp
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log((b*x^3 + a)^p*c) - 1/3*(x^3/b - a*log(b*x^3 + a)/b^2)*b*p
```

Fricas [A]
time = 0.38, size = 40, normalized size = 1.14

$$-\frac{bpx^3 - bx^3\log(c) - (bpx^3 + ap)\log(bx^3 + a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] -1/3*(b*p*x^3 - b*x^3*log(c) - (b*p*x^3 + a*p)*log(b*x^3 + a))/b
```

Sympy [A]
time = 0.83, size = 51, normalized size = 1.46

$$\begin{cases} \frac{a\log(c(a+bx^3)^p)}{3b} - \frac{px^3}{3} + \frac{x^3\log(c(a+bx^3)^p)}{3} & \text{for } b \neq 0 \\ \frac{x^3\log(a^pc)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(b*x**3+a)**p),x)
```

```
[Out] Piecewise((a*log(c*(a + b*x**3)**p)/(3*b) - p*x**3/3 + x**3*log(c*(a + b*x*
*3)**p)/3, Ne(b, 0)), (x**3*log(a**p*c)/3, True))
```

Giac [A]
time = 2.81, size = 43, normalized size = 1.23

$$-\frac{(bx^3 - (bx^3 + a)\log(bx^3 + a) + a)p - (bx^3 + a)\log(c)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] -1/3*((b*x^3 - (b*x^3 + a)*log(b*x^3 + a) + a)*p - (b*x^3 + a)*log(c))/b
```

Mupad [B]
time = 0.23, size = 39, normalized size = 1.11

$$\frac{x^3\ln\left(c\,(b\,x^3+a)^p\right)}{3} - \frac{p\,x^3}{3} + \frac{a\,p\,\ln(b\,x^3+a)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(c*(a + b*x^3)^p),x)
```

```
[Out] (x^3*log(c*(a + b*x^3)^p))/3 - (p*x^3)/3 + (a*p*log(a + b*x^3))/(3*b)
```

3.17 $\int x \log\left(c(a+bx^3)^p\right)\,dx$

Optimal. Leaf size=147

$$-\frac{3px^2}{4}-\frac{\sqrt{3}\,a^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2b^{2/3}}-\frac{a^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2b^{2/3}}+\frac{a^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{4b^{2/3}}+$$

```
[Out] -3/4*p*x^2-1/2*a^(2/3)*p*ln(a^(1/3)+b^(1/3)*x)/b^(2/3)+1/4*a^(2/3)*p*ln(a^(
2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(2/3)+1/2*x^2*ln(c*(b*x^3+a)^p)-1/2*a
^(2/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/b^(2/3)
```

Rubi [A]

time = 0.06, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {2505, 327, 298, 31, 648, 631, 210, 642}

$$-\frac{\sqrt{3}\,a^{2/3}p\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2b^{2/3}}+\frac{a^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{4b^{2/3}}-\frac{a^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2b^{2/3}}+\frac{1}{2}x^2\log\left(c\left(a+bx^3\right)^p\right)-\frac{3px^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (-3*p*x^2)/4 - (Sqrt[3]*a^(2/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a
^(1/3))])/(2*b^(2/3)) - (a^(2/3)*p*Log[a^(1/3) + b^(1/3)*x])/(2*b^(2/3)) +
(a^(2/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(4*b^(2/3)) + (x
^2*Log[c*(a + b*x^3)^p])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x \log\left(c\left(a+bx^3\right)^p\right)\,dx = \frac{1}{2}x^2\log\left(c\left(a+bx^3\right)^p\right) - \frac{1}{2}(3bp)\int \frac{x^4}{a+bx^3}\,dx$$
$$= -\frac{3px^2}{4} + \frac{1}{2}x^2\log\left(c\left(a+bx^3\right)^p\right) + \frac{1}{2}(3ap)\int \frac{x}{a+bx^3}\,dx$$
$$= -\frac{3px^2}{4} + \frac{1}{2}x^2\log\left(c\left(a+bx^3\right)^p\right) - \frac{\left(a^{2/3}p\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{2\sqrt[3]{b}} + \frac{\left(a^{2/3}p\right)\int \frac{\sqrt[3]{a}}{a^{2/3}-\sqrt[3]{a}}}{2\sqrt[3]{b}}$$
$$= -\frac{3px^2}{4} - \frac{a^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2b^{2/3}} + \frac{1}{2}x^2\log\left(c\left(a+bx^3\right)^p\right) + \frac{\left(a^{2/3}p\right)\int \frac{-\sqrt[3]{a}}{a^{2/3}-\sqrt[3]{}}}{4b^{2}}$$
$$= -\frac{3px^2}{4} - \frac{a^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2b^{2/3}} + \frac{a^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{4b^{2/3}} + \frac{1}{2}$$
$$= -\frac{3px^2}{4} - \frac{\sqrt{3}\,a^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{2b^{2/3}} - \frac{a^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2b^{2/3}} + \frac{a^{2/3}p\,\mathrm{lo}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.00, size = 53, normalized size = 0.36

$$-\frac{3px^2}{4} + \frac{3}{4}px^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a}\right) + \frac{1}{2}x^2\log\left(c\left(a+bx^3\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (-3*p*x^2)/4 + (3*p*x^2*Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)])/4 + (
x^2*Log[c*(a + b*x^3)^p])/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.40, size = 184, normalized size = 1.25

method	result
risch	$\frac{x^2\ln\left(\left(x^3b+a\right)^p\right)}{2} - \frac{i\pi x^2\operatorname{csgn}\left(i\left(x^3b+a\right)^p\right)\operatorname{csgn}\left(ic\left(x^3b+a\right)^p\right)\operatorname{csgn}(ic)}{4} + \frac{i\operatorname{csgn}(ic)\operatorname{csgn}\left(ic\left(x^3b+a\right)^p\right)^2x^2\pi}{4} + \frac{i\operatorname{csgn}\left(ic\left(x^3b+a\right)^p\right)^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(b*x^3+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*ln((b*x^3+a)^p)-1/4*I*Pi*x^2*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)
^p)*csgn(I*c)+1/4*I*csgn(I*c)*csgn(I*c*(b*x^3+a)^p)^2*x^2*Pi+1/4*I*csgn(I*c
*(b*x^3+a)^p)^2*csgn(I*(b*x^3+a)^p)*x^2*Pi-1/4*I*Pi*x^2*csgn(I*c*(b*x^3+a)^
p)^3+1/2*ln(c)*x^2-3/4*p*x^2+1/2*a*p/b*sum(1/_R*ln(x-_R),_R=RootOf(_Z^3*b+a
))
```

Maxima [A]
time = 0.49, size = 131, normalized size = 0.89

$$-\frac{1}{4}\,bp\left(\frac{3\,x^2}{b}-\frac{2\,\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{a\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2\,a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+\frac{1}{2}\,x^2\log\left(\left(bx^3+a\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] -1/4*b*p*(3*x^2/b - 2*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b
)^(1/3))/(b^2*(a/b)^(1/3)) - a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*
(a/b)^(1/3)) + 2*a*log(x + (a/b)^(1/3))/(b^2*(a/b)^(1/3))) + 1/2*x^2*log((b
*x^3 + a)^p*c)
```

Fricas [A]
time = 0.36, size = 150, normalized size = 1.02

$$\frac{1}{2}\,px^2\log\left(bx^3+a\right)-\frac{3}{4}\,px^2+\frac{1}{2}\,x^2\log\left(c\right)+\frac{1}{2}\,\sqrt{3}\,p\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,bx\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}+\sqrt{3}\,a}{3\,a}\right)-\frac{1}{4}\,p\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2-bx\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}}-a\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)+\frac{1}{2}\,p\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax+b\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] 1/2*p*x^2*log(b*x^3 + a) - 3/4*p*x^2 + 1/2*x^2*log(c) + 1/2*sqrt(3)*p*(-a^2
/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a^2/b^2)^(1/3) + sqrt(3)*a)/a) - 1/
4*p*(-a^2/b^2)^(1/3)*log(a*x^2 - b*x*(-a^2/b^2)^(2/3) - a*(-a^2/b^2)^(1/3))
 + 1/2*p*(-a^2/b^2)^(1/3)*log(a*x + b*(-a^2/b^2)^(2/3))
```

Sympy [A]
time = 63.48, size = 178, normalized size = 1.21

$$\begin{cases} \frac{x^2\log\left(0^pc\right)}{2} & \text{for}\: a=0\wedge b=0 \\ \frac{x^2\log\left(a^pc\right)}{2} & \text{for}\: b=0 \\ -\frac{3px^2}{4}+\frac{x^2\log\left(c\left(bx^3\right)^p\right)}{2} & \text{for}\: a=0 \\ -\frac{3px^2}{4}+\frac{3p\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(4x^2+4x\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{4}-\frac{\sqrt{3}\,p\left(-\frac{a}{b}\right)^{\frac{2}{3}}\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{2}+\frac{x^2\log\left(c\left(a+bx^3\right)^p\right)}{2}-\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(c\left(a+bx^3\right)^p\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(b*x**3+a)**p),x)
```

```
[Out] Piecewise((x**2*log(0**p*c)/2, Eq(a, 0) & Eq(b, 0)), (x**2*log(a**p*c)/2, E
q(b, 0)), (-3*p*x**2/4 + x**2*log(c*(b*x**3)**p)/2, Eq(a, 0)), (-3*p*x**2/4
 + 3*p*(-a/b)**(2/3)*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/4 -
sqrt(3)*p*(-a/b)**(2/3)*atan(2*sqrt(3)*x/(3*(-a/b)**(1/3)) + sqrt(3)/3)/2 +
 x**2*log(c*(a + b*x**3)**p)/2 - (-a/b)**(2/3)*log(c*(a + b*x**3)**p)/2, Tr
ue))
```

Giac [A]
time = 4.44, size = 150, normalized size = 1.02

$$-\frac{1}{4}\,ab^{2}p\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{ab^{2}}+\frac{2\sqrt{3}\left(-ab^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^{4}}-\frac{\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab^{4}}\right)+\frac{1}{2}\,px^{2}\log\left(bx^{3}+a\right)-\frac{1}{4}\,(3\,p-2\,\log\left(c\right))x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] -1/4*a*b^2*p*(2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2) + 2*sqrt(3)
*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^
4) - (-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^4)) + 1/2
*p*x^2*log(b*x^3 + a) - 1/4*(3*p - 2*log(c))*x^2
```

Mupad [B]
time = 2.38, size = 121, normalized size = 0.82

$$\frac{x^{2}\,\ln\left(c\,{\left(b\,x^{3}+a\right)}^{p}\right)}{2}-\frac{3\,p\,x^{2}}{4}-\frac{a^{2/3}\,p\,\ln\left(b^{1/3}\,x+a^{1/3}\right)}{2\,b^{2/3}}-\frac{a^{2/3}\,p\,\ln\left(4\,b^{1/3}\,x-2\,a^{1/3}-\sqrt{3}\,a^{1/3}\,2\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,b^{2/3}}+\frac{a^{2/3}\,p\,\ln\left(4\,b^{1/3}\,x-2\,a^{1/3}+\sqrt{3}\,a^{1/3}\,2\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(c*(a + b*x^3)^p),x)
```

```
[Out] (x^2*log(c*(a + b*x^3)^p))/2 - (3*p*x^2)/4 - (a^(2/3)*p*log(b^(1/3)*x + a^(
1/3)))/(2*b^(2/3)) - (a^(2/3)*p*log(4*b^(1/3)*x - 3^(1/2)*a^(1/3)*2i - 2*a^
(1/3))*((3^(1/2)*1i)/2 - 1/2))/(2*b^(2/3)) + (a^(2/3)*p*log(3^(1/2)*a^(1/3)
*2i + 4*b^(1/3)*x - 2*a^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(2*b^(2/3))
```

3.18 $\int \log\left(c(a+bx^3)^p\right)\,dx$

Optimal. Leaf size=133

$$-3px-\frac{\sqrt{3}\,\sqrt[3]{a}\,p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt[3]{b}}+\frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}-\frac{\sqrt[3]{a}\,p\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}+x$$

```
[Out] -3*p*x+a^(1/3)*p*ln(a^(1/3)+b^(1/3)*x)/b^(1/3)-1/2*a^(1/3)*p*ln(a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(1/3)+x*ln(c*(b*x^3+a)^p)-a^(1/3)*p*arctan(1/
3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/b^(1/3)
```

Rubi [A]
time = 0.06, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {2498, 327, 206, 31, 648, 631, 210, 642}

$$-\frac{\sqrt[3]{a}\,p\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\,\sqrt[3]{a}\,p\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt[3]{b}}+x\log\left(c\left(a+bx^3\right)^p\right)+\frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}-3px$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p],x]
```

```
[Out] -3*p*x - (Sqrt[3]*a^(1/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)])/b^(1/3) + (a^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (a^(1/3)*p*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)) + x*Log[c*(a + b*x
^3)^p]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\int \log\left(c\left(a+bx^3\right)^p\right)\,dx = x\log\left(c\left(a+bx^3\right)^p\right) - (3bp)\int \frac{x^3}{a+bx^3}\,dx$$

$$= -3px + x\log\left(c\left(a+bx^3\right)^p\right) + (3ap)\int \frac{1}{a+bx^3}\,dx$$

$$= -3px + x\log\left(c\left(a+bx^3\right)^p\right) + \left(\sqrt[3]{a}\,p\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx + \left(\sqrt[3]{a}\,p\right)\int \frac{2\sqrt[3]{a}}{a^{2/3}-\sqrt[3]{a}}$$

$$= -3px + \frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}} + x\log\left(c\left(a+bx^3\right)^p\right) + \frac{1}{2}\left(3a^{2/3}p\right)\int \frac{}{a^{2/3}-\sqrt[3]{a}}$$

$$= -3px + \frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}} - \frac{\sqrt[3]{a}\,p\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{2\sqrt[3]{b}} + x\log$$

$$= -3px - \frac{\sqrt{3}\,\sqrt[3]{a}\,p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt[3]{b}} + \frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}} - \frac{\sqrt[3]{a}\,p\log\left(\right.}{}$$

Mathematica [A]
time = 0.03, size = 129, normalized size = 0.97

$$-3px - \frac{\sqrt{3}\,\sqrt[3]{a}\,p\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} + \frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}} - \frac{\sqrt[3]{a}\,p\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{2\sqrt[3]{b}} + x\log\left(c\left(a+bx^3\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p],x]
```

```
[Out] -3*p*x - (Sqrt[3]*a^(1/3)*p*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^
(1/3) + (a^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (a^(1/3)*p*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)) + x*Log[c*(a + b*x^3)^p]
```

Maple [A]
time = 0.36, size = 122, normalized size = 0.92

method	result

default	$x\ln\left(c(x^3b+a)^p\right)-3pb\left(\frac{x}{b}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)a}{b}\right)$
risch	$x\ln\left((x^3b+a)^p\right)-\frac{i\pi x\,\mathrm{csgn}\left(i(x^3b+a)^p\right)\mathrm{csgn}\left(ic(x^3b+a)^p\right)\mathrm{csgn}(ic)}{2}+\frac{i\mathrm{csgn}(ic)\mathrm{csgn}\left(ic(x^3b+a)^p\right)^2x\pi}{2}+\frac{i\mathrm{csgn}\left(ic(x^3b+a)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(c*(b*x^3+a)^p)-3*p*b*(x/b-(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(
a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*a/b)
```

Maxima [A]
time = 0.50, size = 125, normalized size = 0.94

$$-\frac{1}{2}\,bp\left(\frac{6\,x}{b}-\frac{2\,\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{a\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{2\,a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+x\log\left((bx^3+a)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] -1/2*b*p*(6*x/b - 2*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^
(1/3))/(b^2*(a/b)^(2/3)) + a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a
/b)^(2/3)) - 2*a*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))) + x*log((b*x^3 + a
)^p*c)
```

Fricas [A]
time = 0.39, size = 110, normalized size = 0.83

$$px\log\left(bx^3+a\right)+\sqrt{3}\,p\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)-\frac{1}{2}p\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+p\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-3\,px+x\log\left(c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] p*x*log(b*x^3 + a) + sqrt(3)*p*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^
(2/3) - sqrt(3)*a)/a) - 1/2*p*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(
2/3)) + p*(a/b)^(1/3)*log(x + (a/b)^(1/3)) - 3*p*x + x*log(c)
```

Sympy [A]
time = 27.61, size = 165, normalized size = 1.24

$$\begin{cases} x\log\left(0^p c\right) & \text{for}\ a=0\wedge b=0 \\ -3px+x\log\left(c\left(bx^3\right)^p\right) & \text{for}\ a=0 \\ x\log\left(a^p c\right) & \text{for}\ b=0 \\ -3px+x\log\left(c\left(a+bx^3\right)^p\right)-\frac{3bp\left(-\frac{a}{b}\right)^{\frac{4}{3}}\log\left(4x^2+4x\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2a}-\frac{\sqrt{3}\,bp\left(-\frac{a}{b}\right)^{\frac{4}{3}}\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{a}+\frac{b\left(-\frac{a}{b}\right)^{\frac{4}{3}}\log\left(c\left(a+bx^3\right)^p\right)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p),x)
```

```
[Out] Piecewise((x*log(0**p*c), Eq(a, 0) & Eq(b, 0)), (-3*p*x + x*log(c*(b*x**3)*
*p), Eq(a, 0)), (x*log(a**p*c), Eq(b, 0)), (-3*p*x + x*log(c*(a + b*x**3)**
p) - 3*b*p*(-a/b)**(4/3)*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/
(2*a) - sqrt(3)*b*p*(-a/b)**(4/3)*atan(2*sqrt(3)*x/(3*(-a/b)**(1/3)) + sqrt
(3)/3)/a + b*(-a/b)**(4/3)*log(c*(a + b*x**3)**p)/a, True))
```

Giac [A]
time = 3.49, size = 143, normalized size = 1.08

$$-\frac{1}{2}\,abp\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{ab}-\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^2}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab^2}\right)+px\log\left(bx^3+a\right)-\left(3\,p-\log\left(c\right)\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] -1/2*a*b*p*(2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b) - 2*sqrt(3)*(-a
*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) -
 (-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2)) + p*x*log
(b*x^3 + a) - (3*p - log(c))*x
```

Mupad [B]
time = 0.46, size = 134, normalized size = 1.01

$$x\,\ln\left(c\,{\left(b\,x^3+a\right)}^p\right)-3\,p\,x-\frac{{\left(-a\right)}^{1/3}\,p\,\ln\left({\left(-a\right)}^{4/3}+a\,b^{1/3}\,x\right)}{b^{1/3}}+\frac{{\left(-a\right)}^{1/3}\,p\,\ln\left(2\,a\,b^{1/3}\,x-{\left(-a\right)}^{4/3}-\sqrt{3}\,{\left(-a\right)}^{4/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^{1/3}}-\frac{{\left(-a\right)}^{1/3}\,p\,\ln\left(2\,a\,b^{1/3}\,x-{\left(-a\right)}^{4/3}+\sqrt{3}\,{\left(-a\right)}^{4/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p),x)
```

```
[Out] x*log(c*(a + b*x^3)^p) - 3*p*x - ((-a)^(1/3)*p*log((-a)^(4/3) + a*b^(1/3)*x
))/b^(1/3) + ((-a)^(1/3)*p*log(2*a*b^(1/3)*x - 3^(1/2)*(-a)^(4/3)*1i - (-a)
^(4/3))*((3^(1/2)*1i)/2 + 1/2))/b^(1/3) - ((-a)^(1/3)*p*log(3^(1/2)*(-a)^(4
/3)*1i - (-a)^(4/3) + 2*a*b^(1/3)*x)*((3^(1/2)*1i)/2 - 1/2))/b^(1/3)
```

3.19 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x} dx$

Optimal. Leaf size=44

$$\frac{1}{3}\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right)+\frac{1}{3}p\text{Li}_2\left(1+\frac{bx^3}{a}\right)$$

```
[Out] 1/3*ln(-b*x^3/a)*ln(c*(b*x^3+a)^p)+1/3*p*polylog(2,1+b*x^3/a)
```

Rubi [A]
time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2504, 2441, 2352}

$$\frac{1}{3}p\text{PolyLog}\left(2,\frac{bx^3}{a}+1\right)+\frac{1}{3}\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/x,x]
```

```
[Out] (Log[-((b*x^3)/a)]*Log[c*(a + b*x^3)^p])/3 + (p*PolyLog[2, 1 + (b*x^3)/a])/
3
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log(c(a+bx^3)^p)}{x}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\log\left(-\frac{bx^3}{a}\right)\log\left(c(a+bx^3)^p\right) - \frac{1}{3}(bp)\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\log\left(-\frac{bx^3}{a}\right)\log\left(c(a+bx^3)^p\right) + \frac{1}{3}p\text{Li}_2\left(1+\frac{bx^3}{a}\right)$$

Mathematica [A]
time = 0.01, size = 43, normalized size = 0.98

$$\frac{1}{3}\left(\log\left(-\frac{bx^3}{a}\right)\log\left(c(a+bx^3)^p\right) + p\text{Li}_2\left(\frac{a+bx^3}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/x,x]
[Out] (Log[-((b*x^3)/a)]*Log[c*(a + b*x^3)^p] + p*PolyLog[2, (a + b*x^3)/a])/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.30, size = 180, normalized size = 4.09

method	result
risch	$\ln(x)\ln\left((x^3b+a)^p\right) - p\left(\sum_{\_R1=\text{RootOf}(b\_Z^3+a)}\left(\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\text{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)\right) + \frac{i\ln(x)\pi}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/x,x,method=_RETURNVERBOSE)
[Out] ln(x)*ln((b*x^3+a)^p)-p*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R1),_R1=Ro
otOf(_Z^3*b+a))+1/2*I*ln(x)*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2-
1/2*I*ln(x)*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)-1/2*I*ln
(x)*Pi*csgn(I*c*(b*x^3+a)^p)^3+1/2*I*ln(x)*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(
I*c)+ln(c)*ln(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(39) = 78.
time = 0.27, size = 80, normalized size = 1.82

$$\frac{1}{3}bp\left(\frac{3\log(bx^3+a)\log(x)}{b} - \frac{3\log\left(\frac{bx^3}{a}+1\right)\log(x)+\text{Li}_2\left(-\frac{bx^3}{a}\right)}{b}\right) - p\log(bx^3+a)\log(x) + \log\left((bx^3+a)^p c\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x,x, algorithm="maxima")
```

```
[Out] 1/3*b*p*(3*log(b*x^3 + a)*log(x)/b - (3*log(b*x^3/a + 1)*log(x) + dilog(-b*
x^3/a))/b) - p*log(b*x^3 + a)*log(x) + log((b*x^3 + a)^p*c)*log(x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^3 + a)^p*c)/x, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/x,x)
```

```
[Out] Integral(log(c*(a + b*x**3)**p)/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^3 + a)^p*c)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln \left(c\,{\left(b\,x^3+a\right)}^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)/x,x)
```

```
[Out] int(log(c*(a + b*x^3)^p)/x, x)
```

3.20 $\int \frac{\log\left(c(a+bx^3)^p\right)}{x^2}\,dx$

Optimal. Leaf size=133

$$-\frac{\sqrt{3}\,\sqrt[3]{b}\,p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt[3]{a}}-\frac{\sqrt[3]{b}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a}}+\frac{\sqrt[3]{b}\,p\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{2\sqrt[3]{a}}-\frac{\log\left(c\right.}{}$$

```
[Out] -b^(1/3)*p*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)+1/2*b^(1/3)*p*ln(a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/a^(1/3)-ln(c*(b*x^3+a)^p)/x-b^(1/3)*p*arctan(1/3*(a^(
1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/a^(1/3)
```

Rubi [A]
time = 0.05, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {2505, 298, 31, 648, 631, 210, 642}

$$\frac{\sqrt[3]{b}\,p\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{2\sqrt[3]{a}}-\frac{\sqrt{3}\,\sqrt[3]{b}\,p\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt[3]{a}}-\frac{\log\left(c(a+bx^3)^p\right)}{x}-\frac{\sqrt[3]{b}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/x^2,x]
```

```
[Out] -((Sqrt[3]*b^(1/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(
1/3)) - (b^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + (b^(1/3)*p*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*a^(1/3)) - Log[c*(a + b*x^3)^p]/x
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^2}\,dx = -\frac{\log\left(c(a+bx^3)^p\right)}{x} + (3bp)\int \frac{x}{a+bx^3}\,dx$$

$$= -\frac{\log\left(c(a+bx^3)^p\right)}{x} - \frac{\left(b^{2/3}p\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{\sqrt[3]{a}} + \frac{\left(b^{2/3}p\right)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{\sqrt[3]{a}}$$

$$= -\frac{\sqrt[3]{b}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a}} - \frac{\log\left(c(a+bx^3)^p\right)}{x} + \frac{\left(\sqrt[3]{b}\,p\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,d}{2\sqrt[3]{a}}$$

$$= -\frac{\sqrt[3]{b}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a}} + \frac{\sqrt[3]{b}\,p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{2\sqrt[3]{a}} - \frac{\log\left(c(a+b\right.}{x}$$

$$= -\frac{\sqrt{3}\sqrt[3]{b}\,p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}} - \frac{\sqrt[3]{b}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a}} + \frac{\sqrt[3]{b}\,p\log\left(a^{2/3}-\right.}{2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.00, size = 47, normalized size = 0.35

$$\frac{3bpx^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a}\right)}{2a}-\frac{\log\left(c(a+bx^3)^p\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/x^2,x]
```

```
[Out] (3*b*p*x^2*Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)])/(2*a) - Log[c*(a +
 b*x^3)^p]/x
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.25, size = 184, normalized size = 1.38

method	result
risch	$-\frac{\ln((x^3b+a)^p)}{x}+\frac{-i\pi\,\mathrm{csgn}(i(x^3b+a)^p)\mathrm{csgn}(ic(x^3b+a)^p)^2+i\pi\,\mathrm{csgn}(i(x^3b+a)^p)\mathrm{csgn}(ic(x^3b+a)^p)\mathrm{csgn}(ic)+i\pi\mathrm{csgn}(ic(x^3b+a)^p}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/x*ln((b*x^3+a)^p)+1/2*(-I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2
+I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)+I*Pi*csgn(I*c*(b*
x^3+a)^p)^3-I*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)+2*sum(_R*ln((-4*_R^3*a-3
*b*p^3)*x+a*p*_R^2),_R=RootOf(_Z^3*a+b*p^3))*x-2*ln(c))/x
```

Maxima [A]
time = 0.51, size = 119, normalized size = 0.89

$$\frac{1}{2}\,bp\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)-\frac{\log\left((bx^3+a)^pc\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*p*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(
a/b)^(1/3)) + log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(1/3)) - 2*lo
g(x + (a/b)^(1/3))/(b*(a/b)^(1/3))) - log((b*x^3 + a)^p*c)/x
```

Fricas [A]
time = 0.38, size = 126, normalized size = 0.95

$$\frac{2\sqrt{3}\,px\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-px\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(-\frac{b}{a}\right)^{\frac{2}{3}}-a\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)+2\,px\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)-2\,p\log\left(bx^3+a\right)-2\,\log\left(c\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*p*x*(-b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqr
t(3)) - p*x*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a*(-b/a)^(1/3)) + 2
*p*x*(-b/a)^(1/3)*log(b*x + a*(-b/a)^(2/3)) - 2*p*log(b*x^3 + a) - 2*log(c)
)/x
```

Sympy [A]
time = 117.59, size = 165, normalized size = 1.24

$$\begin{cases} -\frac{\log\left(0^{p}c\right)}{x} & \text{for}\ a=0\wedge b=0\\ -\frac{3p}{x}-\frac{\log\left(c\left(bx^3\right)^{p}\right)}{x} & \text{for}\ a=0\\ -\frac{\log\left(a^{p}c\right)}{x} & \text{for}\ b=0\\ -\frac{\log\left(c\left(a+bx^3\right)^{p}\right)}{x}+\frac{3bp\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(4x^2+4x\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2a}-\frac{\sqrt{3}\,bp\left(-\frac{a}{b}\right)^{\frac{2}{3}}\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{a}-\frac{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(c\left(a+bx^3\right)^{p}\right)}{a} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/x**2,x)
```

```
[Out] Piecewise((-log(0**p*c)/x, Eq(a, 0) & Eq(b, 0)), (-3*p/x - log(c*(b*x**3)**
p)/x, Eq(a, 0)), (-log(a**p*c)/x, Eq(b, 0)), (-log(c*(a + b*x**3)**p)/x + 3
*b*p*(-a/b)**(2/3)*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(2*a)
- sqrt(3)*b*p*(-a/b)**(2/3)*atan(2*sqrt(3)*x/(3*(-a/b)**(1/3)) + sqrt(3)/3)
/a - b*(-a/b)**(2/3)*log(c*(a + b*x**3)**p)/a, True))
```

Giac [A]
time = 5.59, size = 137, normalized size = 1.03

$$-\frac{bp\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a}-\frac{\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}p\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab}-\frac{p\log\left(bx^3+a\right)}{x}+\frac{\left(-ab^2\right)^{\frac{2}{3}}p\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,ab}-\frac{\log\left(c\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^2,x, algorithm="giac")
```

```
[Out] -b*p*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a - sqrt(3)*(-a*b^2)^(2/3)*p*a
rctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) - p*log(b*x^3 +
a)/x + 1/2*(-a*b^2)^(2/3)*p*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b)
- log(c)/x
```

Mupad [B]
time = 0.81, size = 149, normalized size = 1.12

$$\frac{(-b)^{1/3}\,p\,\ln\left(a^{1/3}\,(-b)^{8/3}+b^3\,x\right)}{a^{1/3}}-\frac{\ln\left(c\,(b\,x^3+a)^p\right)}{x}+\frac{(-b)^{1/3}\,p\,\ln\left(9\,b^3\,p^2\,x+9\,a^{1/3}\,(-b)^{8/3}\,p^2\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{1/3}}-\frac{(-b)^{1/3}\,p\,\ln\left(9\,b^3\,p^2\,x+9\,a^{1/3}\,(-b)^{8/3}\,p^2\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)/x^2,x)
```

```
[Out] ((-b)^(1/3)*p*log(a^(1/3)*(-b)^(8/3) + b^3*x))/a^(1/3) - log(c*(a + b*x^3)^
p)/x + ((-b)^(1/3)*p*log(9*b^3*p^2*x + 9*a^(1/3)*(-b)^(8/3)*p^2*((3^(1/2)*1
i)/2 - 1/2)^2)*((3^(1/2)*1i)/2 - 1/2))/a^(1/3) - ((-b)^(1/3)*p*log(9*b^3*p^
2*x + 9*a^(1/3)*(-b)^(8/3)*p^2*((3^(1/2)*1i)/2 + 1/2)^2)*((3^(1/2)*1i)/2 +
1/2))/a^(1/3)
```

3.21 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^3}\,dx$

Optimal. Leaf size=139

$$-\frac{\sqrt{3}\,b^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2a^{2/3}}+\frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2a^{2/3}}-\frac{b^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{4a^{2/3}}-\frac{\log\left(c(a$$

```
[Out] 1/2*b^(2/3)*p*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)-1/4*b^(2/3)*p*ln(a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)-1/2*ln(c*(b*x^3+a)^p)/x^2-1/2*b^(2/3)*p*ar
ctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/a^(2/3)
```

Rubi [A]
time = 0.06, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {2505, 206, 31, 648, 631, 210, 642}

$$-\frac{\sqrt{3}\,b^{2/3}p\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2a^{2/3}}-\frac{b^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{4a^{2/3}}+\frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2a^{2/3}}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/x^3,x]
```

```
[Out] -1/2*(Sqrt[3]*b^(2/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/
a^(2/3) + (b^(2/3)*p*Log[a^(1/3) + b^(1/3)*x])/(2*a^(2/3)) - (b^(2/3)*p*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(4*a^(2/3)) - Log[c*(a + b*x^3
)^p]/(2*x^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^3}\,dx = -\frac{\log\left(c(a+bx^3)^p\right)}{2x^2}+\frac{1}{2}(3bp)\int\frac{1}{a+bx^3}\,dx$$

$$= -\frac{\log\left(c(a+bx^3)^p\right)}{2x^2}+\frac{(bp)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{2a^{2/3}}+\frac{(bp)\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{2a^{2/3}}$$

$$= \frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2a^{2/3}}-\frac{\log\left(c(a+bx^3)^p\right)}{2x^2}-\frac{\left(b^{2/3}p\right)\int\frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{4a^{2/3}}$$

$$= \frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2a^{2/3}}-\frac{b^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{4a^{2/3}}-\frac{\log\left(c(a+bx^3\right.}{2x^2}$$

$$= -\frac{\sqrt{3}\,b^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2a^{2/3}}+\frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2a^{2/3}}-\frac{b^{2/3}p\log\left(a^{2/3}-\right.}{4a}$$

Mathematica [A]
time = 0.02, size = 134, normalized size = 0.96

$$-\frac{2\sqrt{3}\,b^{2/3}px^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2b^{2/3}px^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+b^{2/3}px^2\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)+2a^{2/3}\log\left(c(a+bx^3)^p\right)}{4a^{2/3}x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/x^3,x]
```

```
[Out] -1/4*(2*Sqrt[3]*b^(2/3)*p*x^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] -
 2*b^(2/3)*p*x^2*Log[a^(1/3) + b^(1/3)*x] + b^(2/3)*p*x^2*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*a^(2/3)*Log[c*(a + b*x^3)^p])/(a^(2/3)*x^
2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.32, size = 197, normalized size = 1.42

method	result
risch	$-\frac{\ln\left((x^3b+a)^p\right)}{2x^2}-\frac{i\pi\,\mathrm{csgn}\left(i(x^3b+a)^p\right)\mathrm{csgn}\left(ic(x^3b+a)^p\right)^2-i\pi\,\mathrm{csgn}\left(i(x^3b+a)^p\right)\mathrm{csgn}\left(ic(x^3b+a)^p\right)\mathrm{csgn}(ic)-i\pi\mathrm{csgn}\left(ic(x^3b+a)^p\right)^3+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^2*ln((b*x^3+a)^p)-1/4*(I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p
)^2-I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*
(b*x^3+a)^p)^3+I*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)-2*sum(_R*ln((-4*_R^3*
a^2+3*b^2*p^3)*x-p^2*a*_R*b),_R=RootOf(_Z^3*a^2-b^2*p^3))*x^2+2*ln(c))/x^2
```

Maxima [A]
time = 0.51, size = 120, normalized size = 0.86

$$\frac{1}{4}\,bp\left(\frac{2\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)-\frac{\log\left((bx^3+a)^pc\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*b*p*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(
a/b)^(2/3)) - log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) + 2*lo
g(x + (a/b)^(1/3))/(b*(a/b)^(2/3))) - 1/2*log((b*x^3 + a)^p*c)/x^2
```

Fricas [A]
time = 0.40, size = 150, normalized size = 1.08

$$\frac{2\sqrt{3}\,px^{2}\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,ax\left(\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)-px^{2}\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(b^{2}x^{2}-abx\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}+a^{2}\left(\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}\right)+2\,px^{2}\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\right)-2\,p\log\left(bx^{3}+a\right)-2\,\log\left(c\right)}{4\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*p*x^2*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)^(2
/3) - sqrt(3)*b)/b) - p*x^2*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*b*x*(b^2/a^2)^(
1/3) + a^2*(b^2/a^2)^(2/3)) + 2*p*x^2*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)
^(1/3)) - 2*p*log(b*x^3 + a) - 2*log(c))/x^2
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/x**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.90, size = 138, normalized size = 0.99

$$-\frac{1}{4}\,bp\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a}-\frac{2\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab}-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab}\right)-\frac{p\log\left(bx^{3}+a\right)}{2\,x^{2}}-\frac{\log\left(c\right)}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^3,x, algorithm="giac")
```

```
[Out] -1/4*b*p*(2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a - 2*sqrt(3)*(-a*b^2)^
(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) - (-a*b^2
)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b)) - 1/2*p*log(b*x^3 +
 a)/x^2 - 1/2*log(c)/x^2
```

Mupad [B]
time = 2.61, size = 115, normalized size = 0.83

$$\frac{b^{2/3}\,p\,\ln\left(b^{1/3}\,x+a^{1/3}\right)}{2\,a^{2/3}}-\frac{\ln\left(c\,{\left(b\,x^3+a\right)}^p\right)}{2\,x^2}-\frac{b^{2/3}\,p\,\ln\left(2\,b^{1/3}\,x-a^{1/3}-\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2\,a^{2/3}}+\frac{b^{2/3}\,p\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2\,a^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)/x^3,x)
```

```
[Out] (b^(2/3)*p*log(b^(1/3)*x + a^(1/3)))/(2*a^(2/3)) - log(c*(a + b*x^3)^p)/(2*
x^2) - (b^(2/3)*p*log(2*b^(1/3)*x - 3^(1/2)*a^(1/3)*1i - a^(1/3))*((3^(1/2)
*1i)/2 + 1/2))/(2*a^(2/3)) + (b^(2/3)*p*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*
x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(2*a^(2/3))
```

3.22 $\int \frac{\log\left(c(a+bx^3)^p\right)}{x^4}\,dx$

Optimal. Leaf size=45

$$\frac{bp\log(x)}{a}-\frac{bp\log\left(a+bx^3\right)}{3a}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{3x^3}$$

[Out] b*p*ln(x)/a-1/3*b*p*ln(b*x^3+a)/a-1/3*ln(c*(b*x^3+a)^p)/x^3

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2504, 2442, 36, 29, 31}

$$-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{3x^3}-\frac{bp\log\left(a+bx^3\right)}{3a}+\frac{bp\log(x)}{a}$$

Antiderivative was successfully verified.

[In] Int[Log[c*(a + b*x^3)^p]/x^4,x]

[Out] (b*p*Log[x])/a - (b*p*Log[a + b*x^3])/(3*a) - Log[c*(a + b*x^3)^p]/(3*x^3)

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\log \left(c\left(a+bx^3\right)^p\right)}{x^4}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\log \left(c(a+bx)^p\right)}{x^2}\,dx,x,x^3\right)\\
&= -\frac{\log \left(c\left(a+bx^3\right)^p\right)}{3x^3}+\frac{1}{3}(bp)\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx,x,x^3\right)\\
&= -\frac{\log \left(c\left(a+bx^3\right)^p\right)}{3x^3}+\frac{(bp)\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^3\right)}{3a}-\frac{\left(b^2p\right)\text{Subst}\left(\int \frac{1}{a+bx}\,dx,x,x^3\right)}{3a}\\
&= \frac{bp\log(x)}{a}-\frac{bp\log \left(a+bx^3\right)}{3a}-\frac{\log \left(c\left(a+bx^3\right)^p\right)}{3x^3}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 45, normalized size = 1.00

$$\frac{bp\log(x)}{a}-\frac{bp\log \left(a+bx^3\right)}{3a}-\frac{\log \left(c\left(a+bx^3\right)^p\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/x^4,x]
```

```
[Out] (b*p*Log[x])/a - (b*p*Log[a + b*x^3])/(3*a) - Log[c*(a + b*x^3)^p]/(3*x^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.22, size = 173, normalized size = 3.84

method	result
risch	$-\frac{\ln\left(\left(x^3b+a\right)^p\right)}{3x^3}-\frac{-6bp\ln(x)x^3+i\pi a\,\mathrm{csgn}\left(i\left(x^3b+a\right)^p\right)\mathrm{csgn}\left(ic\left(x^3b+a\right)^p\right)^2-i\pi a\,\mathrm{csgn}\left(i\left(x^3b+a\right)^p\right)\mathrm{csgn}\left(ic\left(x^3b+a\right)^p\right)\mathrm{csgn}(ic)-i\pi a\,\mathrm{cs}}{6x^3a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/x^3*ln((b*x^3+a)^p)-1/6*(-6*b*p*ln(x)*x^3+I*Pi*a*csgn(I*(b*x^3+a)^p)*c
sgn(I*c*(b*x^3+a)^p)^2-I*Pi*a*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csg
n(I*c)-I*Pi*a*csgn(I*c*(b*x^3+a)^p)^3+I*Pi*a*csgn(I*c*(b*x^3+a)^p)^2*csgn(I
*c)+2*b*p*ln(b*x^3+a)*x^3+2*ln(c)*a)/x^3/a
```

Maxima [A]
time = 0.28, size = 44, normalized size = 0.98

$$-\frac{1}{3}\,bp\left(\frac{\log\left(bx^3+a\right)}{a}-\frac{\log\left(x^3\right)}{a}\right)-\frac{\log\left(\left(bx^3+a\right)^p c\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^4,x, algorithm="maxima")
[Out] -1/3*b*p*(log(b*x^3 + a)/a - log(x^3)/a) - 1/3*log((b*x^3 + a)^p*c)/x^3
```

Fricas [A]
time = 0.39, size = 43, normalized size = 0.96

$$\frac{3\,bpx^3\log\left(x\right)-\left(bpx^3+ap\right)\log\left(bx^3+a\right)-a\log\left(c\right)}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^4,x, algorithm="fricas")
[Out] 1/3*(3*b*p*x^3*log(x) - (b*p*x^3 + a*p)*log(b*x^3 + a) - a*log(c))/(a*x^3)
```

Sympy [A]
time = 2.36, size = 65, normalized size = 1.44

$$\begin{cases} -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{3x^3}+\frac{bp\log\left(x\right)}{a}-\frac{b\log\left(c\left(a+bx^3\right)^p\right)}{3a} & \text{for}\ a\neq 0 \\ -\frac{p}{3x^3}-\frac{\log\left(c\left(bx^3\right)^p\right)}{3x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/x**4,x)
[Out] Piecewise((-log(c*(a + b*x**3)**p)/(3*x**3) + b*p*log(x)/a - b*log(c*(a + b
*x**3)**p)/(3*a), Ne(a, 0)), (-p/(3*x**3) - log(c*(b*x**3)**p)/(3*x**3), Tr
ue))
```

Giac [A]
time = 4.70, size = 58, normalized size = 1.29

$$-\frac{\frac{b^2p\log\left(bx^3+a\right)}{a}-\frac{b^2p\log\left(bx^3\right)}{a}+\frac{bp\log\left(bx^3+a\right)}{x^3}+\frac{b\log\left(c\right)}{x^3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^4,x, algorithm="giac")
[Out] -1/3*(b^2*p*log(b*x^3 + a)/a - b^2*p*log(b*x^3)/a + b*p*log(b*x^3 + a)/x^3
+ b*log(c)/x^3)/b
```

Mupad [B]
time = 0.26, size = 41, normalized size = 0.91

$$\frac{b\,p\,\ln\left(x\right)}{a}-\frac{b\,p\,\ln\left(b\,x^3+a\right)}{3\,a}-\frac{\ln\left(c\,{\left(b\,x^3+a\right)}^p\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)/x^4,x)
[Out] (b*p*log(x))/a - (b*p*log(a + b*x^3))/(3*a) - log(c*(a + b*x^3)^p)/(3*x^3)
```

3.23 $\int \frac{\log\left(c(a+bx^3)^p\right)}{x^5}\,dx$

Optimal. Leaf size=151

$$-\frac{3bp}{4ax}+\frac{\sqrt{3}\,b^{4/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{4a^{4/3}}+\frac{b^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{4a^{4/3}}-\frac{b^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{8a^{4/3}}-\text{l}$$

```
[Out] -3/4*b*p/a/x+1/4*b^(4/3)*p*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)-1/8*b^(4/3)*p*ln(a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)-1/4*ln(c*(b*x^3+a)^p)/x^4+1/4
*b^(4/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/a^(4/3
)
```

Rubi [A]
time = 0.06, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2505, 331, 298, 31, 648, 631, 210, 642}

$$\frac{\sqrt{3}\,b^{4/3}p\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{4a^{4/3}}-\frac{b^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{8a^{4/3}}+\frac{b^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{4a^{4/3}}-\frac{\log\left(c(a+bx^3)^p\right)}{4x^4}-\frac{3bp}{4ax}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/x^5,x]
```

```
[Out] (-3*b*p)/(4*a*x) + (Sqrt[3]*b^(4/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(4*a^(4/3)) + (b^(4/3)*p*Log[a^(1/3) + b^(1/3)*x])/(4*a^(4/3)
) - (b^(4/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(8*a^(4/3))
- Log[c*(a + b*x^3)^p]/(4*x^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
```

```
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x^5}\,dx = -\frac{\log \left(c(a+bx^3)^p\right)}{4x^4} + \frac{1}{4}(3bp)\int \frac{1}{x^2\left(a+bx^3\right)}\,dx$$
$$= -\frac{3bp}{4ax} - \frac{\log \left(c(a+bx^3)^p\right)}{4x^4} - \frac{(3b^2p)\int \frac{x}{a+bx^3}\,dx}{4a}$$
$$= -\frac{3bp}{4ax} - \frac{\log \left(c(a+bx^3)^p\right)}{4x^4} + \frac{(b^{5/3}p)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{4a^{4/3}} - \frac{(b^{5/3}p)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b}}{4a^{4/3}}$$
$$= -\frac{3bp}{4ax} + \frac{b^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{4a^{4/3}} - \frac{\log \left(c(a+bx^3)^p\right)}{4x^4} - \frac{(b^{4/3}p)\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+}}{8a^{4/3}}$$
$$= -\frac{3bp}{4ax} + \frac{b^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{4a^{4/3}} - \frac{b^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{8a^{4/3}} - \frac{\log(}{}$$
$$= -\frac{3bp}{4ax} + \frac{\sqrt{3}\,b^{4/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{4a^{4/3}} + \frac{b^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{4a^{4/3}} - \frac{b^{4/3}p\log(}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.00, size = 49, normalized size = 0.32

$$-\frac{3bp\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};-\frac{bx^3}{a}\right)}{4ax} - \frac{\log \left(c(a+bx^3)^p\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/x^5,x]
```

```
[Out] (-3*b*p*Hypergeometric2F1[-1/3, 1, 2/3, -((b*x^3)/a)])/(4*a*x) - Log[c*(a +
 b*x^3)^p]/(4*x^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.30, size = 215, normalized size = 1.42

method	result
risch	$-\frac{\ln((x^3b+a)^p)}{4x^4} - \frac{i\pi a\,\mathrm{csgn}(i(x^3b+a)^p)\mathrm{csgn}(ic(x^3b+a)^p)^2 - i\pi a\,\mathrm{csgn}(i(x^3b+a)^p)\mathrm{csgn}(ic(x^3b+a)^p)\mathrm{csgn}(ic) - i\pi a\mathrm{csgn}(ic(x^3b+a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/x^4*ln((b*x^3+a)^p)-1/8*(I*Pi*a*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)
^p)^2-I*Pi*a*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)-I*Pi*a*csg
n(I*c*(b*x^3+a)^p)^3+I*Pi*a*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)-2*sum(_R*ln((
-4*_R^3*a^4+3*b^4*p^3)*x-a^3*b*p*_R^2),_R=RootOf(_Z^3*a^4-b^4*p^3))*a*x^4+6
*b*p*x^3+2*ln(c)*a)/a/x^4
```

Maxima [A]
time = 0.50, size = 127, normalized size = 0.84

$$-\frac{1}{8}\,bp\left(\frac{2\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{a\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{6}{ax}\right)-\frac{\log\left(\left(bx^{3}+a\right)^{p}c\right)}{4\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^5,x, algorithm="maxima")
```

```
[Out] -1/8*b*p*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*
(a/b)^(1/3)) + log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(1/3)) - 2*l
og(x + (a/b)^(1/3))/(a*(a/b)^(1/3)) + 6/(a*x)) - 1/4*log((b*x^3 + a)^p*c)/x
^4
```

Fricas [A]
time = 0.36, size = 138, normalized size = 0.91

$$-\frac{2\,\sqrt{3}\,bpx^{4}\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}\,x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\,\sqrt{3}\right)+bpx^{4}\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^{2}-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-2\,bpx^{4}\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)+6\,bpx^{3}+2\,ap\log\left(bx^{3}+a\right)+2\,a\log\left(c\right)}{8\,ax^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^5,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(3)*b*p*x^4*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*
sqrt(3)) + b*p*x^4*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3))
 - 2*b*p*x^4*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3)) + 6*b*p*x^3 + 2*a*p*log(b
*x^3 + a) + 2*a*log(c))/(a*x^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/x**5,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.74, size = 153, normalized size = 1.01

$$\frac{1}{8}\,b^{2}p\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^{2}}+\frac{2\,\sqrt{3}\left(-ab^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^{2}b^{2}}-\frac{\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^{2}b^{2}}\right)-\frac{p\log\left(bx^{3}+a\right)}{4\,x^{4}}-\frac{3\,bpx^{3}+a\log\left(c\right)}{4\,ax^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*b^2*p*(2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 2*sqrt(3)*(-a*b^
2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^2) -
(-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^2)) - 1/4*p*
log(b*x^3 + a)/x^4 - 1/4*(3*b*p*x^3 + a*log(c))/(a*x^4)
```

Mupad [B]
time = 2.36, size = 125, normalized size = 0.83

$$\frac{b^{4/3}\,p\,\ln\left(b^{1/3}\,x+a^{1/3}\right)}{4\,a^{4/3}}-\frac{\ln\left(c\,{\left(b\,x^3+a\right)}^p\right)}{4\,x^4}-\frac{3\,b\,p}{4\,a\,x}+\frac{b^{4/3}\,p\,\ln\left(4\,b^{1/3}\,x-2\,a^{1/3}-\sqrt{3}\,a^{1/3}\,2{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{4\,a^{4/3}}-\frac{b^{4/3}\,p\,\ln\left(4\,b^{1/3}\,x-2\,a^{1/3}+\sqrt{3}\,a^{1/3}\,2{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{4\,a^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)/x^5,x)
```

```
[Out] (b^(4/3)*p*log(b^(1/3)*x + a^(1/3)))/(4*a^(4/3)) - log(c*(a + b*x^3)^p)/(4*
x^4) - (3*b*p)/(4*a*x) + (b^(4/3)*p*log(4*b^(1/3)*x - 3^(1/2)*a^(1/3)*2i -
2*a^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(4*a^(4/3)) - (b^(4/3)*p*log(3^(1/2)*a^(
1/3)*2i + 4*b^(1/3)*x - 2*a^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(4*a^(4/3))
```

3.24 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^6}\,dx$

Optimal. Leaf size=151

$$-\frac{3bp}{10ax^2}+\frac{\sqrt{3}\,b^{5/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{5a^{5/3}}-\frac{b^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{5a^{5/3}}+\frac{b^{5/3}p\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{10a^{5/3}}-$$

```
[Out] -3/10*b*p/a/x^2-1/5*b^(5/3)*p*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)+1/10*b^(5/3)*p*
ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)-1/5*ln(c*(b*x^3+a)^p)/x^5
+1/5*b^(5/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/a^
(5/3)
```

Rubi [A]
time = 0.06, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2505, 331, 206, 31, 648, 631, 210, 642}

$$\frac{\sqrt{3}\,b^{5/3}p\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{5a^{5/3}}+\frac{b^{5/3}p\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{10a^{5/3}}-\frac{b^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{5a^{5/3}}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{5x^5}-\frac{3bp}{10ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/x^6,x]
```

```
[Out] (-3*b*p)/(10*a*x^2) + (Sqrt[3]*b^(5/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sq
rt[3]*a^(1/3))])/(5*a^(5/3)) - (b^(5/3)*p*Log[a^(1/3) + b^(1/3)*x])/(5*a^(5
/3)) + (b^(5/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(10*a^(5/
3)) - Log[c*(a + b*x^3)^p]/(5*x^5)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^6}\,dx = -\frac{\log\left(c(a+bx^3)^p\right)}{5x^5} + \frac{1}{5}(3bp)\int \frac{1}{x^3\left(a+bx^3\right)}\,dx$$

$$= -\frac{3bp}{10ax^2} - \frac{\log\left(c(a+bx^3)^p\right)}{5x^5} - \frac{(3b^2p)\int \frac{1}{a+bx^3}\,dx}{5a}$$

$$= -\frac{3bp}{10ax^2} - \frac{\log\left(c(a+bx^3)^p\right)}{5x^5} - \frac{(b^2p)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{5a^{5/3}} - \frac{(b^2p)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x}}{5a^{5/3}}$$

$$= -\frac{3bp}{10ax^2} - \frac{b^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{5a^{5/3}} - \frac{\log\left(c(a+bx^3)^p\right)}{5x^5} + \frac{\left(b^{5/3}p\right)\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x}}{10a^{5/3}}$$

$$= -\frac{3bp}{10ax^2} - \frac{b^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{5a^{5/3}} + \frac{b^{5/3}p\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{10a^{5/3}} - \log$$

$$= -\frac{3bp}{10ax^2} + \frac{\sqrt{3}\,b^{5/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{5a^{5/3}} - \frac{b^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{5a^{5/3}} + \frac{b^{5/3}p\log\left(\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.00, size = 49, normalized size = 0.32

$$-\frac{3bp\,{}_2F_1\left(-\frac{2}{3},1;\frac{1}{3};-\frac{bx^3}{a}\right)}{10ax^2} - \frac{\log\left(c(a+bx^3)^p\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/x^6,x]
```

```
[Out] (-3*b*p*Hypergeometric2F1[-2/3, 1, 1/3, -((b*x^3)/a)])/(10*a*x^2) - Log[c*(
a + b*x^3)^p]/(5*x^5)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.31, size = 216, normalized size = 1.43

method	result
risch	$-\frac{\ln\left(\left(x^3b+a\right)^p\right)}{5x^5} - \frac{-2\left(\sum_{\_R=\mathrm{RootOf}\left(a^5\_Z^3+b^5p^3\right)} \_R\ln\left(\left(-4\_R^3a^5-3b^5p^3\right)x-a^2b^3p^2\_R\right)\right)a\,x^5+i\pi a\,\mathrm{csgn}\left(i\left(x^3b+a\right)^p\right)\mathrm{csgn}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/x^5*ln((b*x^3+a)^p)-1/10*(-2*sum(_R*ln((-4*_R^3*a^5-3*b^5*p^3)*x-a^2*b
^3*p^2*_R),_R=RootOf(_Z^3*a^5+b^5*p^3))*a*x^5+I*Pi*a*csgn(I*(b*x^3+a)^p)*cs
gn(I*c*(b*x^3+a)^p)^2-I*Pi*a*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn
(I*c)-I*Pi*a*csgn(I*c*(b*x^3+a)^p)^3+I*Pi*a*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*
c)+3*b*p*x^3+2*ln(c)*a)/a/x^5
```

Maxima [A]
time = 0.51, size = 128, normalized size = 0.85

$$-\frac{1}{10}\,bp\left(\frac{2\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{a\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{3}{ax^2}\right)-\frac{\log\left(\left(bx^3+a\right)^p c\right)}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^6,x, algorithm="maxima")
```

```
[Out] -1/10*b*p*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a
*(a/b)^(2/3)) - log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(2/3)) + 2*
log(x + (a/b)^(1/3))/(a*(a/b)^(2/3)) + 3/(a*x^2)) - 1/5*log((b*x^3 + a)^p*c
)/x^5
```

Fricas [A]
time = 0.37, size = 172, normalized size = 1.14

$$\frac{2\,\sqrt{3}\,bpx^5\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)-bpx^5\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+2\,bpx^5\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx-a\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)-3\,bpx^3-2\,ap\log\left(bx^3+a\right)-2\,a\log\left(c\right)}{10\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^6,x, algorithm="fricas")
```

```
[Out] 1/10*(2*sqrt(3)*b*p*x^5*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^
2)^(2/3) - sqrt(3)*b)/b) - b*p*x^5*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b
^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) + 2*b*p*x^5*(-b^2/a^2)^(1/3)*log(b*x
- a*(-b^2/a^2)^(1/3)) - 3*b*p*x^3 - 2*a*p*log(b*x^3 + a) - 2*a*log(c))/(a*x
^5)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/x**6,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.63, size = 149, normalized size = 0.99

$$\frac{b^{2}p\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{5\,a^{2}}-\frac{\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}bp\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{5\,a^{2}}-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}bp\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{10\,a^{2}}-\frac{p\log\left(bx^{3}+a\right)}{5\,x^{5}}-\frac{3\,bpx^{3}+2\,a\log\left(c\right)}{10\,ax^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^6,x, algorithm="giac")
```

```
[Out] 1/5*b^2*p*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 - 1/5*sqrt(3)*(-a*b^2
)^(1/3)*b*p*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^2 - 1/1
0*(-a*b^2)^(1/3)*b*p*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^2 - 1/5*p*l
og(b*x^3 + a)/x^5 - 1/10*(3*b*p*x^3 + 2*a*log(c))/(a*x^5)
```

Mupad [B]
time = 2.45, size = 156, normalized size = 1.03

$$\frac{(-b)^{5/3}\,p\,\ln\left(a^{1/3}\,(-b)^{11/3}-b^4\,x\right)}{5\,a^{5/3}}-\frac{\ln\left(c\,\left(b\,x^3+a\right)^p\right)}{5\,x^5}-\frac{3\,b\,p}{10\,a\,x^2}+\frac{(-b)^{5/3}\,p\,\ln\left(225\,a^2\,b^4\,p\,x-225\,a^{7/3}\,(-b)^{11/3}\,p\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{5\,a^{5/3}}-\frac{(-b)^{5/3}\,p\,\ln\left(225\,a^2\,b^4\,p\,x+225\,a^{7/3}\,(-b)^{11/3}\,p\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{5\,a^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)/x^6,x)
```

```
[Out] ((-b)^(5/3)*p*log(a^(1/3)*(-b)^(11/3) - b^4*x))/(5*a^(5/3)) - log(c*(a + b*
x^3)^p)/(5*x^5) - (3*b*p)/(10*a*x^2) + ((-b)^(5/3)*p*log(225*a^2*b^4*p*x -
225*a^(7/3)*(-b)^(11/3)*p*((3^(1/2)*1i)/2 - 1/2))*((3^(1/2)*1i)/2 - 1/2))/(
5*a^(5/3)) - ((-b)^(5/3)*p*log(225*a^2*b^4*p*x + 225*a^(7/3)*(-b)^(11/3)*p*
((3^(1/2)*1i)/2 + 1/2))*((3^(1/2)*1i)/2 + 1/2))/(5*a^(5/3))
```

3.25 $\int \frac{\log\left(c(a+bx^3)^p\right)}{x^7}\,dx$

Optimal. Leaf size=64

$$-\frac{bp}{6ax^3}-\frac{b^2p\log(x)}{2a^2}+\frac{b^2p\log\left(a+bx^3\right)}{6a^2}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{6x^6}$$

```
[Out] -1/6*b*p/a/x^3-1/2*b^2*p*ln(x)/a^2+1/6*b^2*p*ln(b*x^3+a)/a^2-1/6*ln(c*(b*x^
3+a)^p)/x^6
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.188$, Rules used = {2504, 2442, 46}

$$\frac{b^2p\log\left(a+bx^3\right)}{6a^2}-\frac{b^2p\log(x)}{2a^2}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{6x^6}-\frac{bp}{6ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/x^7,x]
```

```
[Out] -1/6*(b*p)/(a*x^3) - (b^2*p*Log[x])/(2*a^2) + (b^2*p*Log[a + b*x^3])/(6*a^2
) - Log[c*(a + b*x^3)^p]/(6*x^6)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
```

```
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log \left(c\left(a+b x^3\right)^p\right)}{x^7} \, dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\log \left(c(a+b x)^p\right)}{x^3} \, dx, x, x^3\right) \\
&= -\frac{\log \left(c\left(a+b x^3\right)^p\right)}{6 x^6}+\frac{1}{6}(b p)\text{Subst}\left(\int \frac{1}{x^2(a+b x)} \, dx, x, x^3\right) \\
&= -\frac{\log \left(c\left(a+b x^3\right)^p\right)}{6 x^6}+\frac{1}{6}(b p)\text{Subst}\left(\int \left(\frac{1}{a x^2}-\frac{b}{a^2 x}+\frac{b^2}{a^2(a+b x)}\right) \, dx, x, x^3\right) \\
&= -\frac{b p}{6 a x^3}-\frac{b^2 p \log (x)}{2 a^2}+\frac{b^2 p \log \left(a+b x^3\right)}{6 a^2}-\frac{\log \left(c\left(a+b x^3\right)^p\right)}{6 x^6}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 56, normalized size = 0.88

$$\frac{1}{6} b p \left(-\frac{1}{a x^3}-\frac{3 b \log (x)}{a^2}+\frac{b \log \left(a+b x^3\right)}{a^2}\right)-\frac{\log \left(c\left(a+b x^3\right)^p\right)}{6 x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/x^7,x]
```

```
[Out] (b*p*(-(1/(a*x^3)) - (3*b*Log[x])/a^2 + (b*Log[a + b*x^3])/a^2))/6 - Log[c*
(a + b*x^3)^p]/(6*x^6)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.26, size = 198, normalized size = 3.09

method	result
risch	$-\frac{\ln\left(\left(x^3 b+a\right)^p\right)}{6x^6}-\frac{6b^2 p\ln(x)x^6-2b^2 p\ln\left(-x^3 b-a\right)x^6+i\pi a^2\operatorname{csgn}\left(i\left(x^3 b+a\right)^p\right)\operatorname{csgn}\left(ic\left(x^3 b+a\right)^p\right)^2-i\pi a^2\operatorname{csgn}\left(i\left(x^3 b+a\right)^p\right)\operatorname{csgn}\left(ic(x^3\right.}{12a^2x^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/x^6*ln((b*x^3+a)^p)-1/12*(6*b^2*p*ln(x)*x^6-2*b^2*p*ln(-b*x^3-a)*x^6+I
*Pi*a^2*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2-I*Pi*a^2*csgn(I*(b*x^3+
a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)-I*Pi*a^2*csgn(I*c*(b*x^3+a)^p)^3+I*Pi
*a^2*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)+2*a*b*p*x^3+2*ln(c)*a^2)/a^2/x^6
```

Maxima [A]
time = 0.27, size = 54, normalized size = 0.84

$$\frac{1}{6}\,bp\left(\frac{b\log\left(bx^3+a\right)}{a^2}-\frac{b\log\left(x^3\right)}{a^2}-\frac{1}{ax^3}\right)-\frac{\log\left(\left(bx^3+a\right)^pc\right)}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^7,x, algorithm="maxima")
```

```
[Out] 1/6*b*p*(b*log(b*x^3 + a)/a^2 - b*log(x^3)/a^2 - 1/(a*x^3)) - 1/6*log((b*x^
3 + a)^p*c)/x^6
```

Fricas [A]
time = 0.39, size = 58, normalized size = 0.91

$$-\frac{3\,b^2px^6\log\left(x\right)+abpx^3+a^2\log\left(c\right)-\left(b^2px^6-a^2p\right)\log\left(bx^3+a\right)}{6\,a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^7,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b^2*p*x^6*log(x) + a*b*p*x^3 + a^2*log(c) - (b^2*p*x^6 - a^2*p)*log
(b*x^3 + a))/(a^2*x^6)
```

Sympy [A]
time = 9.27, size = 83, normalized size = 1.30

$$\begin{cases} -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{6x^6}-\frac{bp}{6ax^3}-\frac{b^2p\log\left(x\right)}{2a^2}+\frac{b^2\log\left(c\left(a+bx^3\right)^p\right)}{6a^2} & \text{for}\ a\neq 0\\ -\frac{p}{12x^6}-\frac{\log\left(c\left(bx^3\right)^p\right)}{6x^6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/x**7,x)
```

```
[Out] Piecewise((-log(c*(a + b*x**3)**p)/(6*x**6) - b*p/(6*a*x**3) - b**2*p*log(x
)/(2*a**2) + b**2*log(c*(a + b*x**3)**p)/(6*a**2), Ne(a, 0)), (-p/(12*x**6)
 - log(c*(b*x**3)**p)/(6*x**6), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(56) = 112.
time = 5.69, size = 132, normalized size = 2.06

$$-\frac{\frac{b^3p\log\left(bx^3+a\right)}{\left(bx^3+a\right)^2-2\left(bx^3+a\right)a+a^2}-\frac{b^3p\log\left(bx^3+a\right)}{a^2}+\frac{b^3p\log\left(bx^3\right)}{a^2}+\frac{\left(bx^3+a\right)b^3p-ab^3p+ab^3\log\left(c\right)}{\left(bx^3+a\right)^2a-2\left(bx^3+a\right)a^2+a^3}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^7,x, algorithm="giac")
```

```
[Out] -1/6*(b^3*p*log(b*x^3 + a)/((b*x^3 + a)^2 - 2*(b*x^3 + a)*a + a^2) - b^3*p*
log(b*x^3 + a)/a^2 + b^3*p*log(b*x^3)/a^2 + ((b*x^3 + a)*b^3*p - a*b^3*p +
a*b^3*log(c))/((b*x^3 + a)^2*a - 2*(b*x^3 + a)*a^2 + a^3))/b
```

Mupad [B]
time = 0.28, size = 56, normalized size = 0.88

$$\frac{b^2\,p\,\ln\left(b\,x^3+a\right)}{6\,a^2}-\frac{\ln\left(c\,{\left(b\,x^3+a\right)}^p\right)}{6\,x^6}-\frac{b^2\,p\,\ln\left(x\right)}{2\,a^2}-\frac{b\,p}{6\,a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)/x^7,x)
```

```
[Out] (b^2*p*log(a + b*x^3))/(6*a^2) - log(c*(a + b*x^3)^p)/(6*x^6) - (b^2*p*log(
x))/(2*a^2) - (b*p)/(6*a*x^3)
```

3.26 $\int x^4 \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx$

Optimal. Leaf size=89

$$-\frac{b^4px}{5a^4}+\frac{b^3px^2}{10a^3}-\frac{b^2px^3}{15a^2}+\frac{bpx^4}{20a}+\frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{b^5p\log(b+ax)}{5a^5}$$

```
[Out] -1/5*b^4*p*x/a^4+1/10*b^3*p*x^2/a^3-1/15*b^2*p*x^3/a^2+1/20*b*p*x^4/a+1/5*x
^5*ln(c*(a+b/x)^p)+1/5*b^5*p*ln(a*x+b)/a^5
```

Rubi [A]
time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2505, 269, 45}

$$\frac{b^5p\log(ax+b)}{5a^5}-\frac{b^4px}{5a^4}+\frac{b^3px^2}{10a^3}-\frac{b^2px^3}{15a^2}+\frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{bpx^4}{20a}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Log[c*(a + b/x)^p],x]
```

```
[Out] -1/5*(b^4*p*x)/a^4 + (b^3*p*x^2)/(10*a^3) - (b^2*p*x^3)/(15*a^2) + (b*p*x^4
)/(20*a) + (x^5*Log[c*(a + b/x)^p])/5 + (b^5*p*Log[b + a*x])/(5*a^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^4 \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx = \frac{1}{5}x^5 \log\left(c\left(a+\frac{b}{x}\right)^p\right) + \frac{1}{5}(bp)\int \frac{x^3}{a+\frac{b}{x}} dx$$
$$= \frac{1}{5}x^5 \log\left(c\left(a+\frac{b}{x}\right)^p\right) + \frac{1}{5}(bp)\int \frac{x^4}{b+ax} dx$$
$$= \frac{1}{5}x^5 \log\left(c\left(a+\frac{b}{x}\right)^p\right) + \frac{1}{5}(bp)\int \left(-\frac{b^3}{a^4}+\frac{b^2x}{a^3}-\frac{bx^2}{a^2}+\frac{x^3}{a}+\frac{b^4}{a^4(b+ax)}\right) dx$$
$$= -\frac{b^4px}{5a^4}+\frac{b^3px^2}{10a^3}-\frac{b^2px^3}{15a^2}+\frac{bpx^4}{20a}+\frac{1}{5}x^5 \log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{b^5p\log(b+ax)}{5a^5}$$

Mathematica [A]
time = 0.04, size = 85, normalized size = 0.96

$$\frac{abpx(-12b^3+6ab^2x-4a^2bx^2+3a^3x^3)+12b^5p\log\left(a+\frac{b}{x}\right)+12a^5x^5\log\left(c\left(a+\frac{b}{x}\right)^p\right)+12b^5p\log(x)}{60a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Log[c*(a + b/x)^p],x]
```

```
[Out] (a*b*p*x*(-12*b^3 + 6*a*b^2*x - 4*a^2*b*x^2 + 3*a^3*x^3) + 12*b^5*p*Log[a +
 b/x] + 12*a^5*x^5*Log[c*(a + b/x)^p] + 12*b^5*p*Log[x])/(60*a^5)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int x^4 \ln\left(c\left(a+\frac{b}{x}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*ln(c*(a+b/x)^p),x)
```

```
[Out] int(x^4*ln(c*(a+b/x)^p),x)
```

Maxima [A]
time = 0.29, size = 74, normalized size = 0.83

$$\frac{1}{5}x^5\log\left(\left(a+\frac{b}{x}\right)^p c\right)+\frac{1}{60}bp\left(\frac{12\,b^4\log(ax+b)}{a^5}+\frac{3\,a^3x^4-4\,a^2bx^3+6\,ab^2x^2-12\,b^3x}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(a+b/x)^p),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*log((a + b/x)^p*c) + 1/60*b*p*(12*b^4*log(a*x + b)/a^5 + (3*a^3*x^4
 - 4*a^2*b*x^3 + 6*a*b^2*x^2 - 12*b^3*x)/a^4)
```

Fricas [A]
time = 0.39, size = 89, normalized size = 1.00

$$\frac{12\,a^{5}px^{5}\log\left(\frac{ax+b}{x}\right)+12\,a^{5}x^{5}\log\left(c\right)+3\,a^{4}bpx^{4}-4\,a^{3}b^{2}px^{3}+6\,a^{2}b^{3}px^{2}-12\,ab^{4}px+12\,b^{5}p\log\left(ax+b\right)}{60\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(a+b/x)^p),x, algorithm="fricas")
```

```
[Out] 1/60*(12*a^5*p*x^5*log((a*x + b)/x) + 12*a^5*x^5*log(c) + 3*a^4*b*p*x^4 - 4
*a^3*b^2*p*x^3 + 6*a^2*b^3*p*x^2 - 12*a*b^4*p*x + 12*b^5*p*log(a*x + b))/a^
5
```

Sympy [A]
time = 2.39, size = 100, normalized size = 1.12

$$\begin{cases}\frac{x^{5}\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{5}+\frac{bpx^{4}}{20a}-\frac{b^{2}px^{3}}{15a^{2}}+\frac{b^{3}px^{2}}{10a^{3}}-\frac{b^{4}px}{5a^{4}}+\frac{b^{5}p\log\left(ax+b\right)}{5a^{5}} & \text{for}\ a\neq 0\\ \frac{px^{5}}{25}+\frac{x^{5}\log\left(c\left(\frac{b}{x}\right)^{p}\right)}{5} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*ln(c*(a+b/x)**p),x)
```

```
[Out] Piecewise((x**5*log(c*(a + b/x)**p)/5 + b*p*x**4/(20*a) - b**2*p*x**3/(15*a
**2) + b**3*p*x**2/(10*a**3) - b**4*p*x/(5*a**4) + b**5*p*log(a*x + b)/(5*a
**5), Ne(a, 0)), (p*x**5/25 + x**5*log(c*(b/x)**p)/5, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(77) = 154.
time = 5.22, size = 308, normalized size = 3.46

$$-\frac{\frac{12\,b^{6}p\log\left(\frac{ax+b}{x}\right)}{a^{5}-\frac{5\,(ax+b)a^{4}}{x}+\frac{10\,(ax+b)^{2}a^{3}}{x^{2}}-\frac{10\,(ax+b)^{3}a^{2}}{x^{3}}+\frac{5\,(ax+b)^{4}a}{x^{4}}-\frac{(ax+b)^{5}}{x^{5}}}+\frac{12\,b^{6}p\log\left(-a+\frac{ax+b}{x}\right)}{a^{5}}-\frac{12\,b^{6}p\log\left(\frac{ax+b}{x}\right)}{a^{5}}-\frac{25\,a^{4}b^{6}p-12\,a^{4}b^{6}\log\left(c\right)-\frac{77\,(ax+b)a^{3}b^{6}p}{x}+\frac{94\,(ax+b)^{2}a^{2}b^{6}p}{x^{2}}-\frac{54\,(ax+b)^{3}ab^{6}p}{x^{3}}+\frac{12\,(ax+b)^{4}b^{6}p}{x^{4}}}{a^{9}-\frac{5\,(ax+b)a^{8}}{x}+\frac{10\,(ax+b)^{2}a^{7}}{x^{2}}-\frac{10\,(ax+b)^{3}a^{6}}{x^{3}}+\frac{5\,(ax+b)^{4}a^{5}}{x^{4}}-\frac{(ax+b)^{5}a^{4}}{x^{5}}}}{60\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(a+b/x)^p),x, algorithm="giac")
```

```
[Out] -1/60*(12*b^6*p*log((a*x + b)/x)/(a^5 - 5*(a*x + b)*a^4/x + 10*(a*x + b)^2*
a^3/x^2 - 10*(a*x + b)^3*a^2/x^3 + 5*(a*x + b)^4*a/x^4 - (a*x + b)^5/x^5) +
 12*b^6*p*log(-a + (a*x + b)/x)/a^5 - 12*b^6*p*log((a*x + b)/x)/a^5 - (25*a
^4*b^6*p - 12*a^4*b^6*log(c) - 77*(a*x + b)*a^3*b^6*p/x + 94*(a*x + b)^2*a^
2*b^6*p/x^2 - 54*(a*x + b)^3*a*b^6*p/x^3 + 12*(a*x + b)^4*b^6*p/x^4)/(a^9 -
 5*(a*x + b)*a^8/x + 10*(a*x + b)^2*a^7/x^2 - 10*(a*x + b)^3*a^6/x^3 + 5*(a
*x + b)^4*a^5/x^4 - (a*x + b)^5*a^4/x^5))/b
```

Mupad [B]
time = 0.24, size = 77, normalized size = 0.87

$$\frac{x^5 \ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{5}-\frac{b^2 p x^3}{15 a^2}+\frac{b^3 p x^2}{10 a^3}+\frac{b^5 p \ln(b+a x)}{5 a^5}+\frac{b p x^4}{20 a}-\frac{b^4 p x}{5 a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*log(c*(a + b/x)^p),x)
```

```
[Out] (x^5*log(c*(a + b/x)^p))/5 - (b^2*p*x^3)/(15*a^2) + (b^3*p*x^2)/(10*a^3) +
(b^5*p*log(b + a*x))/(5*a^5) + (b*p*x^4)/(20*a) - (b^4*p*x)/(5*a^4)
```

3.27 $\int x^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx$

Optimal. Leaf size=75

$$\frac{b^3px}{4a^3}-\frac{b^2px^2}{8a^2}+\frac{bpx^3}{12a}+\frac{1}{4}x^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)-\frac{b^4p\log(b+ax)}{4a^4}$$

[Out] 1/4*b^3*p*x/a^3-1/8*b^2*p*x^2/a^2+1/12*b*p*x^3/a+1/4*x^4*ln(c*(a+b/x)^p)-1/4*b^4*p*ln(a*x+b)/a^4

Rubi [A]
time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2505, 269, 45}

$$-\frac{b^4p\log(ax+b)}{4a^4}+\frac{b^3px}{4a^3}-\frac{b^2px^2}{8a^2}+\frac{1}{4}x^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{bpx^3}{12a}$$

Antiderivative was successfully verified.

[In] Int[x^3*Log[c*(a + b/x)^p],x]

[Out] (b^3*p*x)/(4*a^3) - (b^2*p*x^2)/(8*a^2) + (b*p*x^3)/(12*a) + (x^4*Log[c*(a + b/x)^p])/4 - (b^4*p*Log[b + a*x])/(4*a^4)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)dx &= \frac{1}{4}x^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{4}(bp)\int\frac{x^2}{a+\frac{b}{x}}dx\\ &= \frac{1}{4}x^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{4}(bp)\int\frac{x^3}{b+ax}dx\\ &= \frac{1}{4}x^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{4}(bp)\int\left(\frac{b^2}{a^3}-\frac{bx}{a^2}+\frac{x^2}{a}-\frac{b^3}{a^3(b+ax)}\right)dx\\ &= \frac{b^3px}{4a^3}-\frac{b^2px^2}{8a^2}+\frac{bpx^3}{12a}+\frac{1}{4}x^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)-\frac{b^4p\log(b+ax)}{4a^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 74, normalized size = 0.99

$$\frac{abpx(6b^2-3abx+2a^2x^2)-6b^4p\log\left(a+\frac{b}{x}\right)+6a^4x^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)-6b^4p\log(x)}{24a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[c*(a + b/x)^p],x]
[Out] (a*b*p*x*(6*b^2 - 3*a*b*x + 2*a^2*x^2) - 6*b^4*p*Log[a + b/x] + 6*a^4*x^4*L
og[c*(a + b/x)^p] - 6*b^4*p*Log[x])/(24*a^4)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int x^3\ln\left(c\left(a+\frac{b}{x}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(a+b/x)^p),x)
[Out] int(x^3*ln(c*(a+b/x)^p),x)
```

Maxima [A]
time = 0.28, size = 64, normalized size = 0.85

$$\frac{1}{4}x^4\log\left(\left(a+\frac{b}{x}\right)^pc\right)-\frac{1}{24}bp\left(\frac{6\,b^3\log\left(ax+b\right)}{a^4}-\frac{2\,a^2x^3-3\,abx^2+6\,b^2x}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x)^p),x, algorithm="maxima")
[Out] 1/4*x^4*log((a + b/x)^p*c) - 1/24*b*p*(6*b^3*log(a*x + b)/a^4 - (2*a^2*x^3
- 3*a*b*x^2 + 6*b^2*x)/a^3)
```

Fricas [A]
time = 0.37, size = 77, normalized size = 1.03

$$\frac{6\,a^4px^4\log\left(\frac{ax+b}{x}\right)+6\,a^4x^4\log\left(c\right)+2\,a^3bpx^3-3\,a^2b^2px^2+6\,ab^3px-6\,b^4p\log\left(ax+b\right)}{24\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x)^p),x, algorithm="fricas")
```

```
[Out] 1/24*(6*a^4*p*x^4*log((a*x + b)/x) + 6*a^4*x^4*log(c) + 2*a^3*b*p*x^3 - 3*a
^2*b^2*p*x^2 + 6*a*b^3*p*x - 6*b^4*p*log(a*x + b))/a^4
```

Sympy [A]
time = 1.35, size = 87, normalized size = 1.16

$$\begin{cases} \frac{x^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4}+\frac{bpx^3}{12a}-\frac{b^2px^2}{8a^2}+\frac{b^3px}{4a^3}-\frac{b^4p\log\left(ax+b\right)}{4a^4} & \text{for } a\neq 0 \\ \frac{px^4}{16}+\frac{x^4\log\left(c\left(\frac{b}{x}\right)^p\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(a+b/x)**p),x)
```

```
[Out] Piecewise((x**4*log(c*(a + b/x)**p)/4 + b*p*x**3/(12*a) - b**2*p*x**2/(8*a*
*2) + b**3*p*x/(4*a**3) - b**4*p*log(a*x + b)/(4*a**4), Ne(a, 0)), (p*x**4/
16 + x**4*log(c*(b/x)**p)/4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(65) = 130.
time = 3.93, size = 257, normalized size = 3.43

$$\frac{\frac{6\,b^5p\log\left(\frac{ax+b}{x}\right)}{a^4-\frac{4\,(ax+b)a^3}{x}+\frac{6\,(ax+b)^2a^2}{x^2}-\frac{4\,(ax+b)^3a}{x^3}+\frac{(ax+b)^4}{x^4}}+\frac{6\,b^5p\log\left(-a+\frac{ax+b}{x}\right)}{a^4}-\frac{6\,b^5p\log\left(\frac{ax+b}{x}\right)}{a^4}-\frac{11\,a^3b^5p-6\,a^3b^5\log(c)-\frac{26\,(ax+b)a^2b^5p}{x}+\frac{21\,(ax+b)^2ab^5p}{x^2}-\frac{6\,(ax+b)^3b^5p}{x^3}}{a^7-\frac{4\,(ax+b)a^6}{x}+\frac{6\,(ax+b)^2a^5}{x^2}-\frac{4\,(ax+b)^3a^4}{x^3}+\frac{(ax+b)^4a^3}{x^4}}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x)^p),x, algorithm="giac")
```

```
[Out] 1/24*(6*b^5*p*log((a*x + b)/x)/(a^4 - 4*(a*x + b)*a^3/x + 6*(a*x + b)^2*a^2
/x^2 - 4*(a*x + b)^3*a/x^3 + (a*x + b)^4/x^4) + 6*b^5*p*log(-a + (a*x + b)/
x)/a^4 - 6*b^5*p*log((a*x + b)/x)/a^4 - (11*a^3*b^5*p - 6*a^3*b^5*log(c) -
26*(a*x + b)*a^2*b^5*p/x + 21*(a*x + b)^2*a*b^5*p/x^2 - 6*(a*x + b)^3*b^5*p
/x^3)/(a^7 - 4*(a*x + b)*a^6/x + 6*(a*x + b)^2*a^5/x^2 - 4*(a*x + b)^3*a^4/
x^3 + (a*x + b)^4*a^3/x^4))/b
```

Mupad [B]
time = 0.21, size = 65, normalized size = 0.87

$$\frac{x^4\,\ln\left(c\,{\left(a+\frac{b}{x}\right)}^p\right)}{4}-\frac{b^2\,p\,x^2}{8\,a^2}-\frac{b^4\,p\,\ln\left(b+a\,x\right)}{4\,a^4}+\frac{b\,p\,x^3}{12\,a}+\frac{b^3\,p\,x}{4\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(c*(a + b/x)^p),x)
```

```
[Out] (x^4*log(c*(a + b/x)^p))/4 - (b^2*p*x^2)/(8*a^2) - (b^4*p*log(b + a*x))/(4*
a^4) + (b*p*x^3)/(12*a) + (b^3*p*x)/(4*a^3)
```

3.28 $\int x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right) \, dx$

Optimal. Leaf size=61

$$-\frac{b^2 px}{3a^2}+\frac{bpx^2}{6a}+\frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{b^3 p\log(b+ax)}{3a^3}$$

```
[Out] -1/3*b^2*p*x/a^2+1/6*b*p*x^2/a+1/3*x^3*ln(c*(a+b/x)^p)+1/3*b^3*p*ln(a*x+b)/
a^3
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2505, 269, 45}

$$\frac{b^3 p\log(ax+b)}{3a^3}-\frac{b^2 px}{3a^2}+\frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{bpx^2}{6a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[c*(a + b/x)^p],x]
```

```
[Out] -1/3*(b^2*p*x)/a^2 + (b*p*x^2)/(6*a) + (x^3*Log[c*(a + b/x)^p])/3 + (b^3*p*
Log[b + a*x])/(3*a^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx &= \frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{3}(bp)\int\frac{x}{a+\frac{b}{x}}\,dx\\ &= \frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{3}(bp)\int\frac{x^2}{b+ax}\,dx\\ &= \frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{3}(bp)\int\left(-\frac{b}{a^2}+\frac{x}{a}+\frac{b^2}{a^2(b+ax)}\right)\,dx\\ &= -\frac{b^2px}{3a^2}+\frac{bpx^2}{6a}+\frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{b^3p\log(b+ax)}{3a^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 62, normalized size = 1.02

$$\frac{abpx(-2b+ax)+2b^3p\log\left(a+\frac{b}{x}\right)+2a^3x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)+2b^3p\log(x)}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[c*(a + b/x)^p],x]
```

```
[Out] (a*b*p*x*(-2*b + a*x) + 2*b^3*p*Log[a + b/x] + 2*a^3*x^3*Log[c*(a + b/x)^p]
 + 2*b^3*p*Log[x])/(6*a^3)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int x^2\ln\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(a+b/x)^p),x)
```

```
[Out] int(x^2*ln(c*(a+b/x)^p),x)
```

Maxima [A]
time = 0.28, size = 51, normalized size = 0.84

$$\frac{1}{3}\,x^3\log\left(\left(a+\frac{b}{x}\right)^p c\right)+\frac{1}{6}\,bp\left(\frac{2\,b^2\log\left(ax+b\right)}{a^3}+\frac{ax^2-2\,bx}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x)^p),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log((a + b/x)^p*c) + 1/6*b*p*(2*b^2*log(a*x + b)/a^3 + (a*x^2 - 2*b
*x)/a^2)
```

Fricas [A]
time = 0.36, size = 64, normalized size = 1.05

$$\frac{2\,a^3px^3\log\left(\frac{ax+b}{x}\right)+2\,a^3x^3\log\left(c\right)+a^2bpx^2-2\,ab^2px+2\,b^3p\log\left(ax+b\right)}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x)^p),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^3*p*x^3*log((a*x + b)/x) + 2*a^3*x^3*log(c) + a^2*b*p*x^2 - 2*a*b^
2*p*x + 2*b^3*p*log(a*x + b))/a^3
```

Sympy [A]
time = 0.79, size = 73, normalized size = 1.20

$$\begin{cases}\frac{x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3}+\frac{bpx^2}{6a}-\frac{b^2px}{3a^2}+\frac{b^3p\log\left(ax+b\right)}{3a^3} & \text{for } a\neq 0\\ \frac{px^3}{9}+\frac{x^3\log\left(c\left(\frac{b}{x}\right)^p\right)}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(a+b/x)**p),x)
```

```
[Out] Piecewise((x**3*log(c*(a + b/x)**p)/3 + b*p*x**2/(6*a) - b**2*p*x/(3*a**2)
+ b**3*p*log(a*x + b)/(3*a**3), Ne(a, 0)), (p*x**3/9 + x**3*log(c*(b/x)**p)
/3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(53) = 106.
time = 5.18, size = 210, normalized size = 3.44

$$-\frac{\frac{2\,b^4p\log\left(\frac{ax+b}{x}\right)}{a^3-\frac{3\,(ax+b)a^2}{x}+\frac{3\,(ax+b)^2a}{x^2}-\frac{(ax+b)^3}{x^3}}+\frac{2\,b^4p\log\left(-a+\frac{ax+b}{x}\right)}{a^3}-\frac{2\,b^4p\log\left(\frac{ax+b}{x}\right)}{a^3}-\frac{3\,a^2b^4p-2\,a^2b^4\log\left(c\right)-\frac{5\,(ax+b)ab^4p}{x}+\frac{2\,(ax+b)^2b^4p}{x^2}}{a^5-\frac{3\,(ax+b)a^4}{x}+\frac{3\,(ax+b)^2a^3}{x^2}-\frac{(ax+b)^3a^2}{x^3}}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x)^p),x, algorithm="giac")
```

```
[Out] -1/6*(2*b^4*p*log((a*x + b)/x)/(a^3 - 3*(a*x + b)*a^2/x + 3*(a*x + b)^2*a/x
^2 - (a*x + b)^3/x^3) + 2*b^4*p*log(-a + (a*x + b)/x)/a^3 - 2*b^4*p*log((a*
x + b)/x)/a^3 - (3*a^2*b^4*p - 2*a^2*b^4*log(c) - 5*(a*x + b)*a*b^4*p/x + 2
*(a*x + b)^2*b^4*p/x^2)/(a^5 - 3*(a*x + b)*a^4/x + 3*(a*x + b)^2*a^3/x^2 -
(a*x + b)^3*a^2/x^3))/b
```

Mupad [B]
time = 0.23, size = 53, normalized size = 0.87

$$\frac{x^3\,\ln\left(c\,{\left(a+\frac{b}{x}\right)}^p\right)}{3}+\frac{b^3\,p\,\ln\left(b+a\,x\right)}{3\,a^3}+\frac{b\,p\,x^2}{6\,a}-\frac{b^2\,p\,x}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(c*(a + b/x)^p),x)
[Out] (x^3*log(c*(a + b/x)^p))/3 + (b^3*p*log(b + a*x))/(3*a^3) + (b*p*x^2)/(6*a)
 - (b^2*p*x)/(3*a^2)
```

3.29 $\int x \log \left(c\left(a+\frac{b}{x}\right)^p\right) \, dx$

Optimal. Leaf size=47

$$\frac{bpx}{2a}+\frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)-\frac{b^2p\log(b+ax)}{2a^2}$$

```
[Out] 1/2*b*p*x/a+1/2*x^2*ln(c*(a+b/x)^p)-1/2*b^2*p*ln(a*x+b)/a^2
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2505, 199, 45}

$$-\frac{b^2p\log(ax+b)}{2a^2}+\frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{bpx}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[c*(a + b/x)^p],x]
```

```
[Out] (b*p*x)/(2*a) + (x^2*Log[c*(a + b/x)^p])/2 - (b^2*p*Log[b + a*x])/(2*a^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx &= \frac{1}{2}x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right) + \frac{1}{2}(bp) \int \frac{1}{a+\frac{b}{x}} dx \\
&= \frac{1}{2}x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right) + \frac{1}{2}(bp) \int \frac{x}{b+ax} dx \\
&= \frac{1}{2}x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right) + \frac{1}{2}(bp) \int \left(\frac{1}{a} - \frac{b}{a(b+ax)}\right) dx \\
&= \frac{bpx}{2a} + \frac{1}{2}x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right) - \frac{b^2 p \log(b+ax)}{2a^2}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 40, normalized size = 0.85

$$\frac{1}{2}\left(x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right) + \frac{bp(ax-b\log(b+ax))}{a^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[c*(a + b/x)^p],x]
[Out] (x^2*Log[c*(a + b/x)^p] + (b*p*(a*x - b*Log[b + a*x]))/a^2)/2
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int x \ln \left(c\left(a+\frac{b}{x}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(a+b/x)^p),x)
[Out] int(x*ln(c*(a+b/x)^p),x)
```

Maxima [A]
time = 0.28, size = 40, normalized size = 0.85

$$\frac{1}{2}\,bp\left(\frac{x}{a} - \frac{b\log\left(ax+b\right)}{a^2}\right) + \frac{1}{2}\,x^2\log\left(\left(a+\frac{b}{x}\right)^p c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x)^p),x, algorithm="maxima")
[Out] 1/2*b*p*(x/a - b*log(a*x + b)/a^2) + 1/2*x^2*log((a + b/x)^p*c)
```

Fricas [A]
time = 0.37, size = 50, normalized size = 1.06

$$\frac{a^2px^2\log\left(\frac{ax+b}{x}\right)+a^2x^2\log\left(c\right)+abpx-b^2p\log\left(ax+b\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x)^p),x, algorithm="fricas")
[Out] 1/2*(a^2*p*x^2*log((a*x + b)/x) + a^2*x^2*log(c) + a*b*p*x - b^2*p*log(a*x
+ b))/a^2
```

Sympy [A]
time = 0.48, size = 60, normalized size = 1.28

$$\begin{cases} \frac{x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2}+\frac{bpx}{2a}-\frac{b^2p\log\left(ax+b\right)}{2a^2} & \text{for } a\neq 0 \\ \frac{px^2}{4}+\frac{x^2\log\left(c\left(\frac{b}{x}\right)^p\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(a+b/x)**p),x)
[Out] Piecewise((x**2*log(c*(a + b/x)**p)/2 + b*p*x/(2*a) - b**2*p*log(a*x + b)/(
2*a**2), Ne(a, 0)), (p*x**2/4 + x**2*log(c*(b/x)**p)/2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(41) = 82.
time = 4.14, size = 152, normalized size = 3.23

$$\frac{\frac{b^3p\log\left(\frac{ax+b}{x}\right)}{a^2-\frac{2\,(ax+b)a}{x}+\frac{(ax+b)^2}{x^2}}+\frac{b^3p\log\left(-a+\frac{ax+b}{x}\right)}{a^2}-\frac{b^3p\log\left(\frac{ax+b}{x}\right)}{a^2}-\frac{ab^3p-ab^3\log(c)-\frac{(ax+b)b^3p}{x}}{a^3-\frac{2\,(ax+b)a^2}{x}+\frac{(ax+b)^2a}{x^2}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x)^p),x, algorithm="giac")
[Out] 1/2*(b^3*p*log((a*x + b)/x)/(a^2 - 2*(a*x + b)*a/x + (a*x + b)^2/x^2) + b^3
*p*log(-a + (a*x + b)/x)/a^2 - b^3*p*log((a*x + b)/x)/a^2 - (a*b^3*p - a*b^
3*log(c) - (a*x + b)*b^3*p/x)/(a^3 - 2*(a*x + b)*a^2/x + (a*x + b)^2*a/x^2)
)/b
```

Mupad [B]
time = 0.25, size = 41, normalized size = 0.87

$$\frac{x^2\,\ln\left(c\,{\left(a+\frac{b}{x}\right)}^p\right)}{2}+\frac{b\,p\,x}{2\,a}-\frac{b^2\,p\,\ln\left(b+a\,x\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(c*(a + b/x)^p),x)
[Out] (x^2*log(c*(a + b/x)^p))/2 + (b*p*x)/(2*a) - (b^2*p*log(b + a*x))/(2*a^2)
```

3.30 $\int \log \left(c\left(a+\frac{b}{x}\right)^p\right)\,dx$

Optimal. Leaf size=27

$$x \log \left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{b p \log (b+a x)}{a}$$

[Out] x*ln(c*(a+b/x)^p)+b*p*ln(a*x+b)/a

Rubi [A]

time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2498, 269, 31}

$$x \log \left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{b p \log (a x+b)}{a}$$

Antiderivative was successfully verified.

[In] Int[Log[c*(a + b/x)^p],x]

[Out] x*Log[c*(a + b/x)^p] + (b*p*Log[b + a*x])/a

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \log \left(c\left(a+\frac{b}{x}\right)^p\right)\,dx &= x \log \left(c\left(a+\frac{b}{x}\right)^p\right)+(b p) \int \frac{1}{\left(a+\frac{b}{x}\right) x}\,dx\\
&= x \log \left(c\left(a+\frac{b}{x}\right)^p\right)+(b p) \int \frac{1}{b+a x}\,dx\\
&= x \log \left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{b p \log (b+a x)}{a}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 37, normalized size = 1.37

$$\frac{bp\log\left(a+\frac{b}{x}\right)}{a}+x\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)+\frac{bp\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p],x]
[Out] (b*p*Log[a + b/x])/a + x*Log[c*(a + b/x)^p] + (b*p*Log[x])/a
```

Maple [A]
time = 0.07, size = 30, normalized size = 1.11

method	result	size
default	$x\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)+\frac{bp\ln(ax+b)}{a}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p),x,method=_RETURNVERBOSE)
[Out] x*ln(c*((a*x+b)/x)^p)+b*p*ln(a*x+b)/a
```

Maxima [A]
time = 0.27, size = 27, normalized size = 1.00

$$x\log\left(\left(a+\frac{b}{x}\right)^{p}c\right)+\frac{bp\log\left(ax+b\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p),x, algorithm="maxima")
[Out] x*log((a + b/x)^p*c) + b*p*log(a*x + b)/a
```

Fricas [A]
time = 0.36, size = 33, normalized size = 1.22

$$\frac{apx\log\left(\frac{ax+b}{x}\right)+bp\log\left(ax+b\right)+ax\log\left(c\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p),x, algorithm="fricas")
[Out] (a*p*x*log((a*x + b)/x) + b*p*log(a*x + b) + a*x*log(c))/a
```

Sympy [A]
time = 0.28, size = 36, normalized size = 1.33

$$\begin{cases} x\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{bp\log\left(ax+b\right)}{a} & \text{for } a\neq 0 \\ px+x\log\left(c\left(\frac{b}{x}\right)^p\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p),x)
```

```
[Out] Piecewise((x*log(c*(a + b/x)**p) + b*p*log(a*x + b)/a, Ne(a, 0)), (p*x + x*
log(c*(b/x)**p), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(27) = 54.
time = 4.38, size = 96, normalized size = 3.56

$$-\frac{\frac{b^2p\log\left(-a+\frac{ax+b}{x}\right)}{a}+\frac{b^2p\log\left(\frac{ax+b}{x}\right)}{a-\frac{ax+b}{x}}-\frac{b^2p\log\left(\frac{ax+b}{x}\right)}{a}+\frac{b^2\log(c)}{a-\frac{ax+b}{x}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p),x, algorithm="giac")
```

```
[Out] -(b^2*p*log(-a + (a*x + b)/x)/a + b^2*p*log((a*x + b)/x)/(a - (a*x + b)/x)
- b^2*p*log((a*x + b)/x)/a + b^2*log(c)/(a - (a*x + b)/x))/b
```

Mupad [B]
time = 0.20, size = 27, normalized size = 1.00

$$x\,\ln\left(c\,{\left(a+\frac{b}{x}\right)}^p\right)+\frac{b\,p\,\ln\left(b+a\,x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p),x)
```

```
[Out] x*log(c*(a + b/x)^p) + (b*p*log(b + a*x))/a
```

3.31 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx$

Optimal. Leaf size=40

$$-\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)-p\text{Li}_2\left(1+\frac{b}{ax}\right)$$

```
[Out] -ln(c*(a+b/x)^p)*ln(-b/a/x)-p*polylog(2,1+b/a/x)
```

Rubi [A]
time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2504, 2441, 2352}

$$\log\left(-\frac{b}{ax}\right)\left(-\log\left(c\left(a+\frac{b}{x}\right)^p\right)\right)-p\text{PolyLog}\left(2,\frac{b}{ax}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/x,x]
```

```
[Out] -(Log[c*(a + b/x)^p]*Log[-(b/(a*x))]) - p*PolyLog[2, 1 + b/(a*x)]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx = -\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x}\,dx, x, \frac{1}{x}\right)$$
$$= -\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right) + (bp)\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx, x, \frac{1}{x}\right)$$
$$= -\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right) - p\text{Li}_2\left(1+\frac{b}{ax}\right)$$

Mathematica [A]
time = 0.00, size = 41, normalized size = 1.02

$$-\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right) - p\text{Li}_2\left(\frac{a+\frac{b}{x}}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/x,x]
[Out] -(Log[c*(a + b/x)^p]*Log[-(b/(a*x))]) - p*PolyLog[2, (a + b/x)/a]
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/x,x)
[Out] int(ln(c*(a+b/x)^p)/x,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(39) = 78.
time = 0.26, size = 83, normalized size = 2.08

$$\frac{1}{2}bp\left(\frac{2\log\left(a+\frac{b}{x}\right)\log(x)}{b} + \frac{\log(x)^2}{b} - \frac{2\left(\log\left(\frac{ax}{b}+1\right)\log(x) + \text{Li}_2\left(-\frac{ax}{b}\right)\right)}{b}\right) - p\log\left(a+\frac{b}{x}\right)\log(x) + \log\left(\left(a+\frac{b}{x}\right)^p c\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x,x, algorithm="maxima")
[Out] 1/2*b*p*(2*log(a + b/x)*log(x)/b + log(x)^2/b - 2*(log(a*x/b + 1)*log(x) +
dilog(-a*x/b))/b) - p*log(a + b/x)*log(x) + log((a + b/x)^p*c)*log(x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x,x, algorithm="fricas")
[Out] integral(log(c*((a*x + b)/x)^p)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/x,x)
[Out] Integral(log(c*(a + b/x)**p)/x, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(39) = 78.
time = 4.63, size = 152, normalized size = 3.80

$$-\frac{\frac{b^{3}p\log\left(\frac{ax+b}{x}\right)}{a^{2}-\frac{2\,(ax+b)a}{x}+\frac{(ax+b)^{2}}{x^{2}}}+\frac{b^{3}p\log\left(-a+\frac{ax+b}{x}\right)}{a^{2}}-\frac{b^{3}p\log\left(\frac{ax+b}{x}\right)}{a^{2}}-\frac{ab^{3}p-ab^{3}\log(c)-\frac{(ax+b)b^{3}p}{x}}{a^{3}-\frac{2\,(ax+b)a^{2}}{x}+\frac{(ax+b)^{2}a}{x^{2}}}}{2\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x,x, algorithm="giac")
[Out] -1/2*(b^3*p*log((a*x + b)/x)/(a^2 - 2*(a*x + b)*a/x + (a*x + b)^2/x^2) + b^
3*p*log(-a + (a*x + b)/x)/a^2 - b^3*p*log((a*x + b)/x)/a^2 - (a*b^3*p - a*b
^3*log(c) - (a*x + b)*b^3*p/x)/(a^3 - 2*(a*x + b)*a^2/x + (a*x + b)^2*a/x^2
))/b^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(c\,{\left(a+\frac{b}{x}\right)}^{p}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)/x,x)
[Out] int(log(c*(a + b/x)^p)/x, x)
```

3.32 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2}\,dx$

Optimal. Leaf size=30

$$\frac{p}{x}-\frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{b}$$

```
[Out] p/x-(a+b/x)*ln(c*(a+b/x)^p)/b
```

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2504, 2436, 2332}

$$\frac{p}{x}-\frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/x^2,x]
```

```
[Out] p/x - ((a + b/x)*Log[c*(a + b/x)^p])/b
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2}\,dx = -\text{Subst}\left(\int \log\left(c(a+bx)^p\right)\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\text{Subst}\left(\int \log\left(cx^p\right)\,dx, x, a+\frac{b}{x}\right)}{b}$$
$$= \frac{p}{x} - \frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{b}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$\frac{p}{x} - \frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/x^2,x]
[Out] p/x - ((a + b/x)*Log[c*(a + b/x)^p])/b
```

Maple [A]
time = 0.28, size = 37, normalized size = 1.23

method	result	size
derivativedivides	$-\frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)\left(a+\frac{b}{x}\right)-\left(a+\frac{b}{x}\right)p}{b}$	37
default	$-\frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)\left(a+\frac{b}{x}\right)-\left(a+\frac{b}{x}\right)p}{b}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/x^2,x,method=_RETURNVERBOSE)
[Out] -1/b*(ln(c*(a+b/x)^p)*(a+b/x)-(a+b/x)*p)
```

Maxima [A]
time = 0.29, size = 50, normalized size = 1.67

$$-bp\left(\frac{a\log\left(ax+b\right)}{b^2} - \frac{a\log\left(x\right)}{b^2} - \frac{1}{bx}\right) - \frac{\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^2,x, algorithm="maxima")
[Out] -b*p*(a*log(a*x + b)/b^2 - a*log(x)/b^2 - 1/(b*x)) - log((a + b/x)^p*c)/x
```

Fricas [A]
time = 0.39, size = 36, normalized size = 1.20

$$\frac{bp - b\log(c) - (apx + bp)\log\left(\frac{ax+b}{x}\right)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^2,x, algorithm="fricas")
[Out] (b*p - b*log(c) - (a*p*x + b*p)*log((a*x + b)/x))/(b*x)
```

Sympy [A]
time = 0.52, size = 37, normalized size = 1.23

$$\begin{cases} -\frac{a\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{b} + \frac{p}{x} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x} & \text{for}\ b \neq 0 \\ -\frac{\log(a^{p}c)}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/x**2,x)
[Out] Piecewise((-a*log(c*(a + b/x)**p)/b + p/x - log(c*(a + b/x)**p)/x, Ne(b, 0)
), (-log(a**p*c)/x, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(30) = 60$.
time = 5.20, size = 63, normalized size = 2.10

$$-\frac{\frac{(ax+b)p\log\left(-b\left(\frac{a}{b}-\frac{ax+b}{bx}\right)+a\right)}{x} - \frac{(ax+b)p}{x} + \frac{(ax+b)\log(c)}{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^2,x, algorithm="giac")
[Out] -((a*x + b)*p*log(-b*(a/b - (a*x + b)/(b*x)) + a)/x - (a*x + b)*p/x + (a*x
+ b)*log(c)/x)/b
```

Mupad [B]
time = 0.68, size = 40, normalized size = 1.33

$$\frac{p}{x} - \frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x} - \frac{2\,a\,p\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)/x^2,x)
[Out] p/x - log(c*(a + b/x)^p)/x - (2*a*p*atanh((2*a*x)/b + 1))/b
```

3.33 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3}\,dx$

Optimal. Leaf size=59

$$\frac{p}{4x^2}-\frac{ap}{2bx}+\frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2x^2}$$

```
[Out] 1/4*p/x^2-1/2*a*p/b/x+1/2*a^2*p*ln(a+b/x)/b^2-1/2*ln(c*(a+b/x)^p)/x^2
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.188$, Rules used = {2504, 2442, 45}

$$\frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2x^2}-\frac{ap}{2bx}+\frac{p}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/x^3,x]
```

```
[Out] p/(4*x^2) - (a*p)/(2*b*x) + (a^2*p*Log[a + b/x])/(2*b^2) - Log[c*(a + b/x)^
p]/(2*x^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3}\,dx = -\text{Subst}\left(\int x\log\left(c(a+bx)^p\right)\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2x^2} + \frac{1}{2}(bp)\text{Subst}\left(\int \frac{x^2}{a+bx}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2x^2} + \frac{1}{2}(bp)\text{Subst}\left(\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)\,dx, x, \frac{1}{x}\right)$$
$$= \frac{p}{4x^2} - \frac{ap}{2bx} + \frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2x^2}$$

Mathematica [A]
time = 0.01, size = 59, normalized size = 1.00

$$\frac{p}{4x^2} - \frac{ap}{2bx} + \frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/x^3,x]
[Out] p/(4*x^2) - (a*p)/(2*b*x) + (a^2*p*Log[a + b/x])/(2*b^2) - Log[c*(a + b/x)^
p]/(2*x^2)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/x^3,x)
[Out] int(ln(c*(a+b/x)^p)/x^3,x)
```

Maxima [A]
time = 0.27, size = 63, normalized size = 1.07

$$\frac{1}{4}\,bp\left(\frac{2\,a^2\log\left(ax+b\right)}{b^3} - \frac{2\,a^2\log\left(x\right)}{b^3} - \frac{2\,ax-b}{b^2x^2}\right) - \frac{\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*b*p*(2*a^2*log(a*x + b)/b^3 - 2*a^2*log(x)/b^3 - (2*a*x - b)/(b^2*x^2))
 - 1/2*log((a + b/x)^p*c)/x^2
```

Fricas [A]
time = 0.37, size = 55, normalized size = 0.93

$$-\frac{2\,abpx-b^2p+2\,b^2\log\left(c\right)-2\,\left(a^2px^2-b^2p\right)\log\left(\frac{ax+b}{x}\right)}{4\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*b*p*x - b^2*p + 2*b^2*log(c) - 2*(a^2*p*x^2 - b^2*p)*log((a*x + b
)/x))/(b^2*x^2)
```

Sympy [A]
time = 0.79, size = 61, normalized size = 1.03

$$\begin{cases}\frac{a^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2b^2}-\frac{ap}{2bx}+\frac{p}{4x^2}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2x^2} & \text{for}\ b\neq 0\\ -\frac{\log\left(a^pc\right)}{2x^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/x**3,x)
```

```
[Out] Piecewise((a**2*log(c*(a + b/x)**p)/(2*b**2) - a*p/(2*b*x) + p/(4*x**2) - l
og(c*(a + b/x)**p)/(2*x**2), Ne(b, 0)), (-log(a**p*c)/(2*x**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(51) = 102.
time = 4.26, size = 150, normalized size = 2.54

$$\frac{\frac{4\,(ax+b)ap\log\left(-b\left(\frac{a}{b}-\frac{ax+b}{bx}\right)+a\right)}{bx}-\frac{4\,(ax+b)ap}{bx}-\frac{2\,(ax+b)^2p\log\left(-b\left(\frac{a}{b}-\frac{ax+b}{bx}\right)+a\right)}{bx^2}+\frac{4\,(ax+b)a\log(c)}{bx}+\frac{(ax+b)^2p}{bx^2}-\frac{2\,(ax+b)^2\log(c)}{bx^2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(4*(a*x + b)*a*p*log(-b*(a/b - (a*x + b)/(b*x)) + a)/(b*x) - 4*(a*x + b
)*a*p/(b*x) - 2*(a*x + b)^2*p*log(-b*(a/b - (a*x + b)/(b*x)) + a)/(b*x^2) +
 4*(a*x + b)*a*log(c)/(b*x) + (a*x + b)^2*p/(b*x^2) - 2*(a*x + b)^2*log(c)/
(b*x^2))/b
```

Mupad [B]
time = 0.34, size = 53, normalized size = 0.90

$$\frac{\frac{p}{2}-\frac{a\,p\,x}{b}}{2\,x^2}-\frac{\ln\left(c\,{\left(a+\frac{b}{x}\right)}^p\right)}{2\,x^2}+\frac{a^2\,p\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)/x^3,x)
```

```
[Out] (p/2 - (a*p*x)/b)/(2*x^2) - log(c*(a + b/x)^p)/(2*x^2) + (a^2*p*atanh((2*a*
x)/b + 1))/b^2
```

3.34 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^4}\,dx$

Optimal. Leaf size=73

$$\frac{p}{9x^3}-\frac{ap}{6bx^2}+\frac{a^2p}{3b^2x}-\frac{a^3p\log\left(a+\frac{b}{x}\right)}{3b^3}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3x^3}$$

```
[Out] 1/9*p/x^3-1/6*a*p/b/x^2+1/3*a^2*p/b^2/x-1/3*a^3*p*ln(a+b/x)/b^3-1/3*ln(c*(a
+b/x)^p)/x^3
```

Rubi [A]
time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2504, 2442, 45}

$$-\frac{a^3p\log\left(a+\frac{b}{x}\right)}{3b^3}+\frac{a^2p}{3b^2x}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3x^3}-\frac{ap}{6bx^2}+\frac{p}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/x^4,x]
```

```
[Out] p/(9*x^3) - (a*p)/(6*b*x^2) + (a^2*p)/(3*b^2*x) - (a^3*p*Log[a + b/x])/(3*b
^3) - Log[c*(a + b/x)^p]/(3*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
```

```
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^4}\,dx &= -\text{Subst}\left(\int x^2\log\left(c(a+bx)^p\right)\,dx,x,\frac{1}{x}\right)\\
&= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3x^3}+\frac{1}{3}(bp)\text{Subst}\left(\int \frac{x^3}{a+bx}\,dx,x,\frac{1}{x}\right)\\
&= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3x^3}+\frac{1}{3}(bp)\text{Subst}\left(\int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)\,dx,x,\frac{1}{x}\right)\\
&= \frac{p}{9x^3}-\frac{ap}{6bx^2}+\frac{a^2p}{3b^2x}-\frac{a^3p\log\left(a+\frac{b}{x}\right)}{3b^3}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3x^3}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 73, normalized size = 1.00

$$\frac{p}{9x^3}-\frac{ap}{6bx^2}+\frac{a^2p}{3b^2x}-\frac{a^3p\log\left(a+\frac{b}{x}\right)}{3b^3}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/x^4,x]
[Out] p/(9*x^3) - (a*p)/(6*b*x^2) + (a^2*p)/(3*b^2*x) - (a^3*p*Log[a + b/x])/(3*b
^3) - Log[c*(a + b/x)^p]/(3*x^3)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/x^4,x)
[Out] int(ln(c*(a+b/x)^p)/x^4,x)
```

Maxima [A]
time = 0.28, size = 74, normalized size = 1.01

$$-\frac{1}{18}\,bp\left(\frac{6\,a^3\log\left(ax+b\right)}{b^4}-\frac{6\,a^3\log\left(x\right)}{b^4}-\frac{6\,a^2x^2-3\,abx+2\,b^2}{b^3x^3}\right)-\frac{\log\left(\left(a+\frac{b}{x}\right)^pc\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^4,x, algorithm="maxima")
```

```
[Out] -1/18*b*p*(6*a^3*log(a*x + b)/b^4 - 6*a^3*log(x)/b^4 - (6*a^2*x^2 - 3*a*b*x
 + 2*b^2)/(b^3*x^3)) - 1/3*log((a + b/x)^p*c)/x^3
```

Fricas [A]
time = 0.38, size = 66, normalized size = 0.90

$$\frac{6\,a^2bpx^2 - 3\,ab^2px + 2\,b^3p - 6\,b^3\log\left(c\right) - 6\,(a^3px^3 + b^3p)\log\left(\frac{ax+b}{x}\right)}{18\,b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^4,x, algorithm="fricas")
```

```
[Out] 1/18*(6*a^2*b*p*x^2 - 3*a*b^2*p*x + 2*b^3*p - 6*b^3*log(c) - 6*(a^3*p*x^3 +
 b^3*p)*log((a*x + b)/x))/(b^3*x^3)
```

Sympy [A]
time = 1.40, size = 75, normalized size = 1.03

$$\begin{cases} -\frac{a^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3b^3} + \frac{a^2p}{3b^2x} - \frac{ap}{6bx^2} + \frac{p}{9x^3} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3x^3} & \text{for}\ b \neq 0 \\ -\frac{\log\left(a^pc\right)}{3x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/x**4,x)
```

```
[Out] Piecewise((-a**3*log(c*(a + b/x)**p)/(3*b**3) + a**2*p/(3*b**2*x) - a*p/(6*
b*x**2) + p/(9*x**3) - log(c*(a + b/x)**p)/(3*x**3), Ne(b, 0)), (-log(a**p*
c)/(3*x**3), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(63) = 126.
time = 5.72, size = 234, normalized size = 3.21

$$-\frac{\frac{18\,(ax+b)a^2p\log\left(-b\left(\frac{a}{b}-\frac{ax+b}{bx}\right)+a\right)}{b^2x} - \frac{18\,(ax+b)a^2p}{b^2x} - \frac{18\,(ax+b)^2ap\log\left(-b\left(\frac{a}{b}-\frac{ax+b}{bx}\right)+a\right)}{b^2x^2} + \frac{18\,(ax+b)a^2\log(c)}{b^2x} + \frac{9\,(ax+b)^2ap}{b^2x^2} + \frac{6\,(ax+b)^3p\log\left(-b\left(\frac{a}{b}-\frac{ax+b}{bx}\right)+a\right)}{b^2x^3} - \frac{18\,(ax+b)^2a\log(c)}{b^2x^2} - \frac{2\,(ax+b)^3p}{b^2x^3} + \frac{6\,(ax+b)^3\log(c)}{b^2x^3}}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^4,x, algorithm="giac")
```

```
[Out] -1/18*(18*(a*x + b)*a^2*p*log(-b*(a/b - (a*x + b)/(b*x)) + a)/(b^2*x) - 18*
(a*x + b)*a^2*p/(b^2*x) - 18*(a*x + b)^2*a*p*log(-b*(a/b - (a*x + b)/(b*x))
 + a)/(b^2*x^2) + 18*(a*x + b)*a^2*log(c)/(b^2*x) + 9*(a*x + b)^2*a*p/(b^2*
x^2) + 6*(a*x + b)^3*p*log(-b*(a/b - (a*x + b)/(b*x)) + a)/(b^2*x^3) - 18*(
```

```
a*x + b)^2*a*log(c)/(b^2*x^2) - 2*(a*x + b)^3*p/(b^2*x^3) + 6*(a*x + b)^3*l
og(c)/(b^2*x^3))/b
```

Mupad [B]
time = 0.32, size = 65, normalized size = 0.89

$$\frac{\frac{p}{3}+\frac{a^2\,p\,x^2}{b^2}-\frac{a\,p\,x}{2\,b}}{3\,x^3}-\frac{\ln\left(c\,{\left(a+\frac{b}{x}\right)}^p\right)}{3\,x^3}-\frac{2\,a^3\,p\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)/x^4,x)
```

```
[Out] (p/3 + (a^2*p*x^2)/b^2 - (a*p*x)/(2*b))/(3*x^3) - log(c*(a + b/x)^p)/(3*x^3
) - (2*a^3*p*atanh((2*a*x)/b + 1))/(3*b^3)
```

3.35 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^5}\,dx$

Optimal. Leaf size=87

$$\frac{p}{16x^4}-\frac{ap}{12bx^3}+\frac{a^2p}{8b^2x^2}-\frac{a^3p}{4b^3x}+\frac{a^4p\log\left(a+\frac{b}{x}\right)}{4b^4}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4x^4}$$

```
[Out] 1/16*p/x^4-1/12*a*p/b/x^3+1/8*a^2*p/b^2/x^2-1/4*a^3*p/b^3/x+1/4*a^4*p*ln(a+
b/x)/b^4-1/4*ln(c*(a+b/x)^p)/x^4
```

Rubi [A]
time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2504, 2442, 45}

$$\frac{a^4p\log\left(a+\frac{b}{x}\right)}{4b^4}-\frac{a^3p}{4b^3x}+\frac{a^2p}{8b^2x^2}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4x^4}-\frac{ap}{12bx^3}+\frac{p}{16x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/x^5,x]
```

```
[Out] p/(16*x^4) - (a*p)/(12*b*x^3) + (a^2*p)/(8*b^2*x^2) - (a^3*p)/(4*b^3*x) + (
a^4*p*Log[a + b/x])/(4*b^4) - Log[c*(a + b/x)^p]/(4*x^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
```

```
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^5}\,dx &= -\text{Subst}\left(\int x^3\log\left(c(a+bx)^p\right)\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4x^4}+\frac{1}{4}(bp)\text{Subst}\left(\int \frac{x^4}{a+bx}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4x^4}+\frac{1}{4}(bp)\text{Subst}\left(\int \left(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+bx)}\right)\,dx, x\right.\\
&= \frac{p}{16x^4}-\frac{ap}{12bx^3}+\frac{a^2p}{8b^2x^2}-\frac{a^3p}{4b^3x}+\frac{a^4p\log\left(a+\frac{b}{x}\right)}{4b^4}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4x^4}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 87, normalized size = 1.00

$$\frac{p}{16x^4}-\frac{ap}{12bx^3}+\frac{a^2p}{8b^2x^2}-\frac{a^3p}{4b^3x}+\frac{a^4p\log\left(a+\frac{b}{x}\right)}{4b^4}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/x^5,x]
[Out] p/(16*x^4) - (a*p)/(12*b*x^3) + (a^2*p)/(8*b^2*x^2) - (a^3*p)/(4*b^3*x) + (
a^4*p*Log[a + b/x])/(4*b^4) - Log[c*(a + b/x)^p]/(4*x^4)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/x^5,x)
[Out] int(ln(c*(a+b/x)^p)/x^5,x)
```

Maxima [A]
time = 0.29, size = 85, normalized size = 0.98

$$\frac{1}{48}\,bp\left(\frac{12\,a^4\log\left(ax+b\right)}{b^5}-\frac{12\,a^4\log\left(x\right)}{b^5}-\frac{12\,a^3x^3-6\,a^2bx^2+4\,ab^2x-3\,b^3}{b^4x^4}\right)-\frac{\log\left(\left(a+\frac{b}{x}\right)^pc\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^5,x, algorithm="maxima")
```

```
[Out] 1/48*b*p*(12*a^4*log(a*x + b)/b^5 - 12*a^4*log(x)/b^5 - (12*a^3*x^3 - 6*a^2
*b*x^2 + 4*a*b^2*x - 3*b^3)/(b^4*x^4)) - 1/4*log((a + b/x)^p*c)/x^4
```

Fricas [A]
time = 0.35, size = 79, normalized size = 0.91

$$-\frac{12\,a^3bpx^3-6\,a^2b^2px^2+4\,ab^3px-3\,b^4p+12\,b^4\log\left(c\right)-12\left(a^4px^4-b^4p\right)\log\left(\frac{ax+b}{x}\right)}{48\,b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^5,x, algorithm="fricas")
```

```
[Out] -1/48*(12*a^3*b*p*x^3 - 6*a^2*b^2*p*x^2 + 4*a*b^3*p*x - 3*b^4*p + 12*b^4*lo
g(c) - 12*(a^4*p*x^4 - b^4*p)*log((a*x + b)/x))/(b^4*x^4)
```

Sympy [A]
time = 2.01, size = 88, normalized size = 1.01

$$\begin{cases} \frac{a^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4b^4}-\frac{a^3p}{4b^3x}+\frac{a^2p}{8b^2x^2}-\frac{ap}{12bx^3}+\frac{p}{16x^4}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4x^4} & \text{for}\: b\neq 0 \\ -\frac{\log\left(a^pc\right)}{4x^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/x**5,x)
```

```
[Out] Piecewise((a**4*log(c*(a + b/x)**p)/(4*b**4) - a**3*p/(4*b**3*x) + a**2*p/(
8*b**2*x**2) - a*p/(12*b*x**3) + p/(16*x**4) - log(c*(a + b/x)**p)/(4*x**4)
, Ne(b, 0)), (-log(a**p*c)/(4*x**4), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(75) = 150.
time = 2.17, size = 317, normalized size = 3.64

$$\frac{\frac{48\,(ax+b)a^3p\log\left(-b\left(\frac{a}{b}-\frac{ax+b}{bx}\right)+a\right)}{b^3x}-\frac{48\,(ax+b)a^3p}{b^3x}-\frac{72\,(ax+b)^2a^2p\log\left(-b\left(\frac{a}{b}-\frac{ax+b}{bx}\right)+a\right)}{b^3x^2}+\frac{48\,(ax+b)a^3\log(c)}{b^3x}+\frac{36\,(ax+b)^2a^2p}{b^3x^2}+\frac{48\,(ax+b)^3ap\log\left(-b\left(\frac{a}{b}-\frac{ax+b}{bx}\right)+a\right)}{b^3x^3}-\frac{72\,(ax+b)^2a^2\log(c)}{b^3x^2}-\frac{16\,(ax+b)^3ap}{b^3x^3}-\frac{12\,(ax+b)^4p\log\left(-b\left(\frac{a}{b}-\frac{ax+b}{bx}\right)+a\right)}{b^3x^4}+\frac{48\,(ax+b)^3a\log(c)}{b^3x^3}+\frac{3\,(ax+b)^4p}{b^3x^4}-\frac{12\,(ax+b)^4\log(c)}{b^3x^4}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^5,x, algorithm="giac")
```

```
[Out] 1/48*(48*(a*x + b)*a^3*p*log(-b*(a/b - (a*x + b)/(b*x)) + a)/(b^3*x) - 48*(
a*x + b)*a^3*p/(b^3*x) - 72*(a*x + b)^2*a^2*p*log(-b*(a/b - (a*x + b)/(b*x)
) + a)/(b^3*x^2) + 48*(a*x + b)*a^3*log(c)/(b^3*x) + 36*(a*x + b)^2*a^2*p/(
b^3*x^2) + 48*(a*x + b)^3*a*p*log(-b*(a/b - (a*x + b)/(b*x)) + a)/(b^3*x^3)
 - 72*(a*x + b)^2*a^2*log(c)/(b^3*x^2) - 16*(a*x + b)^3*a*p/(b^3*x^3) - 12*
(a*x + b)^4*p*log(-b*(a/b - (a*x + b)/(b*x)) + a)/(b^3*x^4) + 48*(a*x + b)^
```

```
3*a*log(c)/(b^3*x^3) + 3*(a*x + b)^4*p/(b^3*x^4) - 12*(a*x + b)^4*log(c)/(b
^3*x^4))/b
```

Mupad [B]
time = 0.39, size = 78, normalized size = 0.90

$$\frac{\frac{p}{4}+\frac{a^2\,p\,x^2}{2\,b^2}-\frac{a^3\,p\,x^3}{b^3}-\frac{a\,p\,x}{3\,b}}{4\,x^4}-\frac{\ln\left(c\,{\left(a+\frac{b}{x}\right)}^p\right)}{4\,x^4}+\frac{a^4\,p\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)/x^5,x)
```

```
[Out] (p/4 + (a^2*p*x^2)/(2*b^2) - (a^3*p*x^3)/b^3 - (a*p*x)/(3*b))/(4*x^4) - log
(c*(a + b/x)^p)/(4*x^4) + (a^4*p*atanh((2*a*x)/b + 1))/(2*b^4)
```

3.36 $\int x^4 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx$

Optimal. Leaf size=72

$$-\frac{2b^2px}{5a^2}+\frac{2bpx^3}{15a}+\frac{2b^{5/2}p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{5a^{5/2}}+\frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

```
[Out] -2/5*b^2*p*x/a^2+2/15*b*p*x^3/a+2/5*b^(5/2)*p*arctan(x*a^(1/2)/b^(1/2))/a^(
5/2)+1/5*x^5*ln(c*(a+b/x^2)^p)
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2505, 269, 308, 211}

$$\frac{2b^{5/2}p\text{ArcTan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{5a^{5/2}}-\frac{2b^2px}{5a^2}+\frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{2bpx^3}{15a}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Log[c*(a + b/x^2)^p],x]
```

```
[Out] (-2*b^2*p*x)/(5*a^2) + (2*b*p*x^3)/(15*a) + (2*b^(5/2)*p*ArcTan[(Sqrt[a]*x)
/Sqrt[b]])/(5*a^(5/2)) + (x^5*Log[c*(a + b/x^2)^p])/5
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
```

```
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^4 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx &= \frac{1}{5}x^5 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{1}{5}(2bp)\int \frac{x^2}{a+\frac{b}{x^2}}\,dx\\
&= \frac{1}{5}x^5 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{1}{5}(2bp)\int \frac{x^4}{b+ax^2}\,dx\\
&= \frac{1}{5}x^5 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{1}{5}(2bp)\int \left(-\frac{b}{a^2}+\frac{x^2}{a}+\frac{b^2}{a^2(b+ax^2)}\right) dx\\
&= -\frac{2b^2px}{5a^2}+\frac{2bpx^3}{15a}+\frac{1}{5}x^5 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{(2b^3p)\int \frac{1}{b+ax^2}\,dx}{5a^2}\\
&= -\frac{2b^2px}{5a^2}+\frac{2bpx^3}{15a}+\frac{2b^{5/2}p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{5a^{5/2}}+\frac{1}{5}x^5 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.01, size = 49, normalized size = 0.68

$$\frac{2bpx^3\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\frac{b}{ax^2}\right)}{15a}+\frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Log[c*(a + b/x^2)^p],x]
[Out] (2*b*p*x^3*Hypergeometric2F1[-3/2, 1, -1/2, -(b/(a*x^2))])/(15*a) + (x^5*Lo
g[c*(a + b/x^2)^p])/5
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int x^4 \ln\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*ln(c*(a+b/x^2)^p),x)
[Out] int(x^4*ln(c*(a+b/x^2)^p),x)
```

Maxima [A]
time = 0.51, size = 59, normalized size = 0.82

$$\frac{1}{5}\,x^{5}\log\left(\left(a+\frac{b}{x^{2}}\right)^{p}c\right)+\frac{2}{15}\,bp\left(\frac{3\,b^{2}\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{2}}+\frac{ax^{3}-3\,bx}{a^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(a+b/x^2)^p),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*log((a + b/x^2)^p*c) + 2/15*b*p*(3*b^2*arctan(a*x/sqrt(a*b))/(sqrt(
a*b)*a^2) + (a*x^3 - 3*b*x)/a^2)
```

Fricas [A]
time = 0.37, size = 178, normalized size = 2.47

$$\left[\frac{3\,a^{2}px^{5}\log\left(\frac{ax^{2}+b}{x^{2}}\right)+3\,a^{2}x^{5}\log\left(c\right)+2\,abpx^{3}+3\,b^{2}p\sqrt{-\frac{b}{a}}\log\left(\frac{ax^{2}+2\,ax\sqrt{-\frac{b}{a}}-b}{ax^{2}+b}\right)-6\,b^{2}px}{15\,a^{2}},\ \frac{3\,a^{2}px^{5}\log\left(\frac{ax^{2}+b}{x^{2}}\right)+3\,a^{2}x^{5}\log\left(c\right)+2\,abpx^{3}+6\,b^{2}p\sqrt{\frac{b}{a}}\arctan\left(\frac{ax\sqrt{\frac{b}{a}}}{b}\right)-6\,b^{2}px}{15\,a^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(a+b/x^2)^p),x, algorithm="fricas")
```

```
[Out] [1/15*(3*a^2*p*x^5*log((a*x^2 + b)/x^2) + 3*a^2*x^5*log(c) + 2*a*b*p*x^3 +
3*b^2*p*sqrt(-b/a)*log((a*x^2 + 2*a*x*sqrt(-b/a) - b)/(a*x^2 + b)) - 6*b^2*
p*x)/a^2, 1/15*(3*a^2*p*x^5*log((a*x^2 + b)/x^2) + 3*a^2*x^5*log(c) + 2*a*b
*p*x^3 + 6*b^2*p*sqrt(b/a)*arctan(a*x*sqrt(b/a)/b) - 6*b^2*p*x)/a^2]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(70) = 140.
time = 33.56, size = 148, normalized size = 2.06

$$\begin{cases} \frac{x^{5}\log\left(0^{p}c\right)}{5} & \text{for}\: a=0 \wedge b=0 \\ \frac{2px^{5}}{25}+\frac{x^{5}\log\left(c\left(\frac{b}{x^{2}}\right)^{p}\right)}{5} & \text{for}\: a=0 \\ \frac{x^{5}\log\left(a^{p}c\right)}{5} & \text{for}\: b=0 \\ \frac{x^{5}\log\left(c\left(a+\frac{b}{x^{2}}\right)^{p}\right)}{5}+\frac{2bpx^{3}}{15a}-\frac{2b^{2}px}{5a^{2}}+\frac{b^{3}p\log\left(x-\sqrt{-\frac{b}{a}}\right)}{5a^{3}\sqrt{-\frac{b}{a}}}-\frac{b^{3}p\log\left(x+\sqrt{-\frac{b}{a}}\right)}{5a^{3}\sqrt{-\frac{b}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*ln(c*(a+b/x**2)**p),x)
```

```
[Out] Piecewise((x**5*log(0**p*c)/5, Eq(a, 0) & Eq(b, 0)), (2*p*x**5/25 + x**5*lo
g(c*(b/x**2)**p)/5, Eq(a, 0)), (x**5*log(a**p*c)/5, Eq(b, 0)), (x**5*log(c*
(a + b/x**2)**p)/5 + 2*b*p*x**3/(15*a) - 2*b**2*p*x/(5*a**2) + b**3*p*log(x
 - sqrt(-b/a))/(5*a**3*sqrt(-b/a)) - b**3*p*log(x + sqrt(-b/a))/(5*a**3*sqr
t(-b/a)), True))
```

Giac [A]
time = 4.45, size = 75, normalized size = 1.04

$$\frac{1}{5}\,px^5\log\left(ax^2+b\right)-\frac{1}{5}\,px^5\log\left(x^2\right)+\frac{1}{5}\,x^5\log\left(c\right)+\frac{2\,bpx^3}{15\,a}+\frac{2\,b^3p\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{5\,\sqrt{ab}\,a^2}-\frac{2\,b^2px}{5\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(a+b/x^2)^p),x, algorithm="giac")
```

```
[Out] 1/5*p*x^5*log(a*x^2 + b) - 1/5*p*x^5*log(x^2) + 1/5*x^5*log(c) + 2/15*b*p*x
^3/a + 2/5*b^3*p*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 2/5*b^2*p*x/a^2
```

Mupad [B]
time = 0.26, size = 56, normalized size = 0.78

$$\frac{x^5\,\ln\left(c\,{\left(a+\frac{b}{x^2}\right)}^p\right)}{5}+\frac{2\,b^{5/2}\,p\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{5\,a^{5/2}}+\frac{2\,b\,p\,x^3}{15\,a}-\frac{2\,b^2\,p\,x}{5\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*log(c*(a + b/x^2)^p),x)
```

```
[Out] (x^5*log(c*(a + b/x^2)^p))/5 + (2*b^(5/2)*p*atan((a^(1/2)*x)/b^(1/2)))/(5*a
^(5/2)) + (2*b*p*x^3)/(15*a) - (2*b^2*p*x)/(5*a^2)
```

3.37 $\int x^3 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) \, dx$

Optimal. Leaf size=51

$$\frac{bpx^2}{4a}+\frac{1}{4}x^4\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)-\frac{b^2p\log\left(b+ax^2\right)}{4a^2}$$

```
[Out] 1/4*b*p*x^2/a+1/4*x^4*ln(c*(a+b/x^2)^p)-1/4*b^2*p*ln(a*x^2+b)/a^2
```

Rubi [A]
time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2505, 269, 272, 45}

$$-\frac{b^2p\log\left(ax^2+b\right)}{4a^2}+\frac{1}{4}x^4\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{bpx^2}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[c*(a + b/x^2)^p],x]
```

```
[Out] (b*p*x^2)/(4*a) + (x^4*Log[c*(a + b/x^2)^p])/4 - (b^2*p*Log[b + a*x^2])/(4*
a^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
```

```
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx &= \frac{1}{4}x^4\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{1}{2}(bp)\int\frac{x}{a+\frac{b}{x^2}}\,dx\\ &= \frac{1}{4}x^4\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{1}{2}(bp)\int\frac{x^3}{b+ax^2}\,dx\\ &= \frac{1}{4}x^4\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{1}{4}(bp)\text{Subst}\left(\int\frac{x}{b+ax}\,dx,x,x^2\right)\\ &= \frac{1}{4}x^4\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{1}{4}(bp)\text{Subst}\left(\int\left(\frac{1}{a}-\frac{b}{a(b+ax)}\right)dx,x,x^2\right)\\ &= \frac{bpx^2}{4a}+\frac{1}{4}x^4\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)-\frac{b^2p\log\left(b+ax^2\right)}{4a^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 56, normalized size = 1.10

$$\frac{1}{4}x^4\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{1}{4}bp\left(\frac{x^2}{a}-\frac{b\log\left(a+\frac{b}{x^2}\right)}{a^2}-\frac{2b\log(x)}{a^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[c*(a + b/x^2)^p],x]
[Out] (x^4*Log[c*(a + b/x^2)^p])/4 + (b*p*(x^2/a - (b*Log[a + b/x^2])/a^2 - (2*b*
Log[x])/a^2))/4
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int x^3\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(a+b/x^2)^p),x)
[Out] int(x^3*ln(c*(a+b/x^2)^p),x)
```

Maxima [A]
time = 0.26, size = 44, normalized size = 0.86

$$\frac{1}{4}x^4\log\left(\left(a+\frac{b}{x^2}\right)^pc\right)+\frac{1}{4}bp\left(\frac{x^2}{a}-\frac{b\log\left(ax^2+b\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x^2)^p),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log((a + b/x^2)^p*c) + 1/4*b*p*(x^2/a - b*log(a*x^2 + b)/a^2)
```

Fricas [A]
time = 0.38, size = 56, normalized size = 1.10

$$\frac{a^2px^4\log\left(\frac{ax^2+b}{x^2}\right)+a^2x^4\log\left(c\right)+abpx^2-b^2p\log\left(ax^2+b\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x^2)^p),x, algorithm="fricas")
```

```
[Out] 1/4*(a^2*p*x^4*log((a*x^2 + b)/x^2) + a^2*x^4*log(c) + a*b*p*x^2 - b^2*p*lo
g(a*x^2 + b))/a^2
```

Sympy [A]
time = 1.65, size = 66, normalized size = 1.29

$$\begin{cases} \frac{x^4\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{4}+\frac{bpx^2}{4a}-\frac{b^2p\log\left(ax^2+b\right)}{4a^2} & \text{for}\ a\neq 0 \\ \frac{px^4}{8}+\frac{x^4\log\left(c\left(\frac{b}{x^2}\right)^p\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(a+b/x**2)**p),x)
```

```
[Out] Piecewise((x**4*log(c*(a + b/x**2)**p)/4 + b*p*x**2/(4*a) - b**2*p*log(a*x*
*2 + b)/(4*a**2), Ne(a, 0)), (p*x**4/8 + x**4*log(c*(b/x**2)**p)/4, True))
```

Giac [A]
time = 6.33, size = 59, normalized size = 1.16

$$\frac{1}{4}\,px^4\log\left(ax^2+b\right)-\frac{1}{4}\,px^4\log\left(x^2\right)+\frac{1}{4}\,x^4\log\left(c\right)+\frac{bpx^2}{4\,a}-\frac{b^2p\log\left(ax^2+b\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x^2)^p),x, algorithm="giac")
```

```
[Out] 1/4*p*x^4*log(a*x^2 + b) - 1/4*p*x^4*log(x^2) + 1/4*x^4*log(c) + 1/4*b*p*x^
2/a - 1/4*b^2*p*log(a*x^2 + b)/a^2
```

Mupad [B]
time = 0.24, size = 45, normalized size = 0.88

$$\frac{x^4\,\ln\left(c\,{\left(a+\frac{b}{x^2}\right)}^p\right)}{4}-\frac{b^2\,p\,\ln\left(a\,x^2+b\right)}{4\,a^2}+\frac{b\,p\,x^2}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(c*(a + b/x^2)^p),x)
```

```
[Out] (x^4*log(c*(a + b/x^2)^p))/4 - (b^2*p*log(b + a*x^2))/(4*a^2) + (b*p*x^2)/(
4*a)
```

3.38 $\int x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) \, dx$

Optimal. Leaf size=58

$$\frac{2bpx}{3a}-\frac{2b^{3/2}p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{3a^{3/2}}+\frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

```
[Out] 2/3*b*p*x/a-2/3*b^(3/2)*p*arctan(x*a^(1/2)/b^(1/2))/a^(3/2)+1/3*x^3*ln(c*(a
+b/x^2)^p)
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2505, 199, 327, 211}

$$-\frac{2b^{3/2}p\text{ArcTan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{3a^{3/2}}+\frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{2bpx}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[c*(a + b/x^2)^p],x]
```

```
[Out] (2*b*p*x)/(3*a) - (2*b^(3/2)*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(3*a^(3/2)) + (
x^3*Log[c*(a + b/x^2)^p])/3
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx &= \frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{1}{3}(2bp)\int\frac{1}{a+\frac{b}{x^2}}\,dx\\
&= \frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{1}{3}(2bp)\int\frac{x^2}{b+ax^2}\,dx\\
&= \frac{2bpx}{3a}+\frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)-\frac{(2b^2p)\int\frac{1}{b+ax^2}\,dx}{3a}\\
&= \frac{2bpx}{3a}-\frac{2b^{3/2}p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{3a^{3/2}}+\frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.00, size = 47, normalized size = 0.81

$$\frac{2bpx\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{b}{ax^2}\right)}{3a}+\frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[c*(a + b/x^2)^p],x]
[Out] (2*b*p*x*Hypergeometric2F1[-1/2, 1, 1/2, -(b/(a*x^2))])/(3*a) + (x^3*Log[c*
(a + b/x^2)^p])/3
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int x^2\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(a+b/x^2)^p),x)
[Out] int(x^2*ln(c*(a+b/x^2)^p),x)
```

Maxima [A]
time = 0.49, size = 48, normalized size = 0.83

$$\frac{1}{3}\,x^3\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)-\frac{2}{3}\,bp\left(\frac{b\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x^2)^p),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log((a + b/x^2)^p*c) - 2/3*b*p*(b*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*
a) - x/a)
```

Fricas [A]
time = 0.38, size = 141, normalized size = 2.43

$$\left[\frac{apx^3\log\left(\frac{ax^2+b}{x^2}\right)+ax^3\log\left(c\right)+bp\sqrt{-\frac{b}{a}}\,\log\left(\frac{ax^2-2\,ax\sqrt{-\frac{b}{a}}-b}{ax^2+b}\right)+2\,bpx}{3\,a},\ \frac{apx^3\log\left(\frac{ax^2+b}{x^2}\right)+ax^3\log\left(c\right)-2\,bp\sqrt{\frac{b}{a}}\,\arctan\left(\frac{ax\sqrt{\frac{b}{a}}}{b}\right)+2\,bpx}{3\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x^2)^p),x, algorithm="fricas")
```

```
[Out] [1/3*(a*p*x^3*log((a*x^2 + b)/x^2) + a*x^3*log(c) + b*p*sqrt(-b/a)*log((a*x
^2 - 2*a*x*sqrt(-b/a) - b)/(a*x^2 + b)) + 2*b*p*x)/a, 1/3*(a*p*x^3*log((a*x
^2 + b)/x^2) + a*x^3*log(c) - 2*b*p*sqrt(b/a)*arctan(a*x*sqrt(b/a)/b) + 2*b
*p*x)/a]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(54) = 108$.
time = 11.00, size = 133, normalized size = 2.29

$$\begin{cases} \frac{x^3\log\left(0^p c\right)}{3} & \text{for}\ a=0\wedge b=0 \\ \frac{2px^3}{9}+\frac{x^3\log\left(c\left(\frac{b}{x^2}\right)^p\right)}{3} & \text{for}\ a=0 \\ \frac{x^3\log\left(a^p c\right)}{3} & \text{for}\ b=0 \\ \frac{x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3}+\frac{2bpx}{3a}-\frac{b^2p\log\left(x-\sqrt{-\frac{b}{a}}\right)}{3a^2\sqrt{-\frac{b}{a}}}+\frac{b^2p\log\left(x+\sqrt{-\frac{b}{a}}\right)}{3a^2\sqrt{-\frac{b}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(a+b/x**2)**p),x)
```

```
[Out] Piecewise((x**3*log(0**p*c)/3, Eq(a, 0) & Eq(b, 0)), (2*p*x**3/9 + x**3*log
(c*(b/x**2)**p)/3, Eq(a, 0)), (x**3*log(a**p*c)/3, Eq(b, 0)), (x**3*log(c*(
a + b/x**2)**p)/3 + 2*b*p*x/(3*a) - b**2*p*log(x - sqrt(-b/a))/(3*a**2*sqrt
(-b/a)) + b**2*p*log(x + sqrt(-b/a))/(3*a**2*sqrt(-b/a)), True))
```

Giac [A]
time = 6.09, size = 63, normalized size = 1.09

$$\frac{1}{3}\,px^{3}\log\left(ax^{2}+b\right)-\frac{1}{3}\,px^{3}\log\left(x^{2}\right)+\frac{1}{3}\,x^{3}\log\left(c\right)-\frac{2\,b^{2}p\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,a}+\frac{2\,bpx}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x^2)^p),x, algorithm="giac")
```

```
[Out] 1/3*p*x^3*log(a*x^2 + b) - 1/3*p*x^3*log(x^2) + 1/3*x^3*log(c) - 2/3*b^2*p*
arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a) + 2/3*b*p*x/a
```

Mupad [B]
time = 0.24, size = 44, normalized size = 0.76

$$\frac{x^{3}\,\ln\left(c\,{\left(a+\frac{b}{x^{2}}\right)}^{p}\right)}{3}-\frac{2\,b^{3/2}\,p\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{3\,a^{3/2}}+\frac{2\,b\,p\,x}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(c*(a + b/x^2)^p),x)
```

```
[Out] (x^3*log(c*(a + b/x^2)^p))/3 - (2*b^(3/2)*p*atan((a^(1/2)*x)/b^(1/2)))/(3*a
^(3/2)) + (2*b*p*x)/(3*a)
```

3.39 $\int x \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx$

Optimal. Leaf size=37

$$\frac{1}{2}x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{bp \log \left(b+ax^2\right)}{2a}$$

```
[Out] 1/2*x^2*ln(c*(a+b/x^2)^p)+1/2*b*p*ln(a*x^2+b)/a
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2505, 269, 266}

$$\frac{1}{2}x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{bp \log \left(ax^2+b\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[c*(a + b/x^2)^p],x]
```

```
[Out] (x^2*Log[c*(a + b/x^2)^p])/2 + (b*p*Log[b + a*x^2])/(2*a)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx = \frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+(bp)\int\frac{1}{\left(a+\frac{b}{x^2}\right)x}dx$$
$$=\frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+(bp)\int\frac{x}{b+ax^2}dx$$
$$=\frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{bp\log\left(b+ax^2\right)}{2a}$$

Mathematica [A]
time = 0.00, size = 45, normalized size = 1.22

$$\frac{bp\log\left(a+\frac{b}{x^2}\right)}{2a}+\frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{bp\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[c*(a + b/x^2)^p],x]
[Out] (b*p*Log[a + b/x^2])/(2*a) + (x^2*Log[c*(a + b/x^2)^p])/2 + (b*p*Log[x])/a
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int x\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(a+b/x^2)^p),x)
[Out] int(x*ln(c*(a+b/x^2)^p),x)
```

Maxima [A]
time = 0.28, size = 33, normalized size = 0.89

$$\frac{1}{2}x^2\log\left(\left(a+\frac{b}{x^2}\right)^pc\right)+\frac{bp\log\left(ax^2+b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x^2)^p),x, algorithm="maxima")
[Out] 1/2*x^2*log((a + b/x^2)^p*c) + 1/2*b*p*log(a*x^2 + b)/a
```

Fricas [A]
time = 0.37, size = 42, normalized size = 1.14

$$\frac{apx^2\log\left(\frac{ax^2+b}{x^2}\right)+ax^2\log\left(c\right)+bp\log\left(ax^2+b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x^2)^p),x, algorithm="fricas")
```

```
[Out] 1/2*(a*p*x^2*log((a*x^2 + b)/x^2) + a*x^2*log(c) + b*p*log(a*x^2 + b))/a
```

Sympy [A]
time = 0.68, size = 53, normalized size = 1.43

$$\begin{cases} \frac{x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2}+\frac{bp\log\left(ax^2+b\right)}{2a} & \text{for } a\neq 0 \\ \frac{px^2}{2}+\frac{x^2\log\left(c\left(\frac{b}{x^2}\right)^p\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(a+b/x**2)**p),x)
```

```
[Out] Piecewise((x**2*log(c*(a + b/x**2)**p)/2 + b*p*log(a*x**2 + b)/(2*a), Ne(a,
 0)), (p*x**2/2 + x**2*log(c*(b/x**2)**p)/2, True))
```

Giac [A]
time = 4.50, size = 47, normalized size = 1.27

$$\frac{1}{2}\,px^2\log\left(ax^2+b\right)-\frac{1}{2}\,px^2\log\left(x^2\right)+\frac{1}{2}\,x^2\log\left(c\right)+\frac{bp\log\left(ax^2+b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x^2)^p),x, algorithm="giac")
```

```
[Out] 1/2*p*x^2*log(a*x^2 + b) - 1/2*p*x^2*log(x^2) + 1/2*x^2*log(c) + 1/2*b*p*lo
g(a*x^2 + b)/a
```

Mupad [B]
time = 0.21, size = 33, normalized size = 0.89

$$\frac{x^2\,\ln\left(c\,{\left(a+\frac{b}{x^2}\right)}^p\right)}{2}+\frac{b\,p\,\ln\left(a\,x^2+b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(c*(a + b/x^2)^p),x)
```

```
[Out] (x^2*log(c*(a + b/x^2)^p))/2 + (b*p*log(b + a*x^2))/(2*a)
```

3.40 $\int \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) \, dx$

Optimal. Leaf size=41

$$\frac{2\sqrt{b}\, p \tan^{-1}\left(\frac{\sqrt{a}\, x}{\sqrt{b}}\right)}{\sqrt{a}}+x \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)$$

[Out] x*ln(c*(a+b/x^2)^p)+2*p*arctan(x*a^(1/2)/b^(1/2))*b^(1/2)/a^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2498, 269, 211}

$$\frac{2\sqrt{b}\, p \text{ArcTan}\left(\frac{\sqrt{a}\, x}{\sqrt{b}}\right)}{\sqrt{a}}+x \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^2)^p],x]
```

```
[Out] (2*Sqrt[b]*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/Sqrt[a] + x*Log[c*(a + b/x^2)^p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\int \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = x\log \left(c\left(a+\frac{b}{x^2}\right)^p\right) + (2bp)\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^2}dx$$
$$= x\log \left(c\left(a+\frac{b}{x^2}\right)^p\right) + (2bp)\int \frac{1}{b+ax^2}dx$$
$$= \frac{2\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}} + x\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

Mathematica [A]
time = 0.01, size = 43, normalized size = 1.05

$$-\frac{2\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}\,x}\right)}{\sqrt{a}} + x\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^2)^p],x]
[Out] (-2*Sqrt[b]*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)])/Sqrt[a] + x*Log[c*(a + b/x^2)^p]
```

Maple [A]
time = 0.06, size = 38, normalized size = 0.93

method	result	size
default	$x\ln\left(c\left(\frac{x^2a+b}{x^2}\right)^p\right)+\frac{2pb\arctan\left(\frac{ax}{\sqrt{ba}}\right)}{\sqrt{ba}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^2)^p),x,method=_RETURNVERBOSE)
[Out] x*ln(c*((a*x^2+b)/x^2)^p)+2*p*b/(b*a)^(1/2)*arctan(a*x/(b*a)^(1/2))
```

Maxima [A]
time = 0.56, size = 33, normalized size = 0.80

$$\frac{2\,bp\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}} + x\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p),x, algorithm="maxima")
```

```
[Out] 2*b*p*arctan(a*x/sqrt(a*b))/sqrt(a*b) + x*log((a + b/x^2)^p*c)
```

Fricas [A]
time = 0.40, size = 107, normalized size = 2.61

$$\left[px\log\left(\frac{ax^2+b}{x^2}\right)+p\sqrt{-\frac{b}{a}}\log\left(\frac{ax^2+2\,ax\sqrt{-\frac{b}{a}}-b}{ax^2+b}\right)+x\log\left(c\right),px\log\left(\frac{ax^2+b}{x^2}\right)+2\,p\sqrt{\frac{b}{a}}\arctan\left(\frac{ax\sqrt{\frac{b}{a}}}{b}\right)+x\log\left(c\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p),x, algorithm="fricas")
```

```
[Out] [p*x*log((a*x^2 + b)/x^2) + p*sqrt(-b/a)*log((a*x^2 + 2*a*x*sqrt(-b/a) - b)
/(a*x^2 + b)) + x*log(c), p*x*log((a*x^2 + b)/x^2) + 2*p*sqrt(b/a)*arctan(a
*x*sqrt(b/a)/b) + x*log(c)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(39) = 78$.
time = 3.79, size = 95, normalized size = 2.32

$$\begin{cases} x\log\left(0^pc\right) & \text{for } a=0\wedge b=0 \\ 2px+x\log\left(c\left(\frac{b}{x^2}\right)^p\right) & \text{for } a=0 \\ x\log\left(a^pc\right) & \text{for } b=0 \\ x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{bp\log\left(x-\sqrt{-\frac{b}{a}}\right)}{a\sqrt{-\frac{b}{a}}}-\frac{bp\log\left(x+\sqrt{-\frac{b}{a}}\right)}{a\sqrt{-\frac{b}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**2)**p),x)
```

```
[Out] Piecewise((x*log(0**p*c), Eq(a, 0) & Eq(b, 0)), (2*p*x + x*log(c*(b/x**2)**
p), Eq(a, 0)), (x*log(a**p*c), Eq(b, 0)), (x*log(c*(a + b/x**2)**p) + b*p*l
og(x - sqrt(-b/a))/(a*sqrt(-b/a)) - b*p*log(x + sqrt(-b/a))/(a*sqrt(-b/a)),
 True))
```

Giac [A]
time = 4.15, size = 42, normalized size = 1.02

$$px\log\left(ax^2+b\right)-px\log\left(x^2\right)+\frac{2\,bp\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}}+x\log\left(c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p),x, algorithm="giac")
```

```
[Out] p*x*log(a*x^2 + b) - p*x*log(x^2) + 2*b*p*arctan(a*x/sqrt(a*b))/sqrt(a*b) +
 x*log(c)
```

Mupad [B]
time = 0.11, size = 33, normalized size = 0.80

$$x\,\ln\left(c\,{\left(a+\frac{b}{x^2}\right)}^p\right)+\frac{2\,\sqrt{b}\,p\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x^2)^p),x)
```

```
[Out] x*log(c*(a + b/x^2)^p) + (2*b^(1/2)*p*atan((a^(1/2)*x)/b^(1/2)))/a^(1/2)
```

3.41 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx$

Optimal. Leaf size=44

$$-\frac{1}{2}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)-\frac{1}{2}p\text{Li}_2\left(1+\frac{b}{ax^2}\right)$$

```
[Out] -1/2*ln(c*(a+b/x^2)^p)*ln(-b/a/x^2)-1/2*p*polylog(2,1+b/a/x^2)
```

Rubi [A]
time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2504, 2441, 2352}

$$-\frac{1}{2}p\text{PolyLog}\left(2,\frac{b}{ax^2}+1\right)-\frac{1}{2}\log\left(-\frac{b}{ax^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^2)^p]/x,x]
```

```
[Out] -1/2*(Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))]) - (p*PolyLog[2, 1 + b/(a*x^2)
])/2
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{\log \left(c(a+bx)^p\right)}{x}\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\frac{1}{2}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)+\frac{1}{2}(bp)\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx, x, \frac{1}{x^2}\right)$$
$$= -\frac{1}{2}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)-\frac{1}{2}p\text{Li}_2\left(1+\frac{b}{ax^2}\right)$$

Mathematica [**A**]
time = 0.00, size = 45, normalized size = 1.02

$$-\frac{1}{2}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)-\frac{1}{2}p\text{Li}_2\left(\frac{a+\frac{b}{x^2}}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^2)^p]/x,x]
```

```
[Out] -1/2*(Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))]) - (p*PolyLog[2, (a + b/x^2)/a
])/2
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^2)^p)/x,x)
```

```
[Out] int(ln(c*(a+b/x^2)^p)/x,x)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(39) = 78.
time = 0.27, size = 89, normalized size = 2.02

$$\frac{1}{2}bp\left(\frac{2\log\left(a+\frac{b}{x^2}\right)\log(x)}{b}+\frac{2\log(x)^2}{b}-\frac{2\log\left(\frac{ax^2}{b}+1\right)\log(x)+\text{Li}_2\left(-\frac{ax^2}{b}\right)}{b}\right)-p\log\left(a+\frac{b}{x^2}\right)\log(x)+\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x,x, algorithm="maxima")
```

```
[Out] 1/2*b*p*(2*log(a + b/x^2)*log(x)/b + 2*log(x)^2/b - (2*log(a*x^2/b + 1)*log
(x) + dilog(-a*x^2/b))/b) - p*log(a + b/x^2)*log(x) + log((a + b/x^2)^p*c)*
log(x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x,x, algorithm="fricas")
[Out] integral(log(c*((a*x^2 + b)/x^2)^p)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(c\left(a + \frac{b}{x^2}\right)^p\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**2)**p)/x,x)
[Out] Integral(log(c*(a + b/x**2)**p)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x,x, algorithm="giac")
[Out] integrate(log((a + b/x^2)^p*c)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln \left(c\,{\left(a + \frac{b}{x^2}\right)}^p\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x^2)^p)/x,x)
[Out] int(log(c*(a + b/x^2)^p)/x, x)
```

3.42 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2}\,dx$

Optimal. Leaf size=50

$$\frac{2p}{x}+\frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{b}}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}$$

[Out] 2*p/x-ln(c*(a+b/x^2)^p)/x+2*p*arctan(x*a^(1/2)/b^(1/2))*a^(1/2)/b^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2505, 269, 331, 211}

$$\frac{2\sqrt{a}\,p\text{ArcTan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{b}}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}+\frac{2p}{x}$$

Antiderivative was successfully verified.

[In] Int[Log[c*(a + b/x^2)^p]/x^2,x]

[Out] (2*p)/x + (2*Sqrt[a]*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/Sqrt[b] - Log[c*(a + b/x^2)^p]/x

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2}\,dx &= -\frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}-(2bp)\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^4}\,dx\\
&= -\frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}-(2bp)\int \frac{1}{x^2\left(b+ax^2\right)}\,dx\\
&= \frac{2p}{x}-\frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}+(2ap)\int \frac{1}{b+ax^2}\,dx\\
&= \frac{2p}{x}+\frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{b}}-\frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}
\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 52, normalized size = 1.04

$$\frac{2p}{x}-\frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}\,x}\right)}{\sqrt{b}}-\frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^2)^p]/x^2,x]
```

```
[Out] (2*p)/x - (2*Sqrt[a]*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)])/Sqrt[b] - Log[c*(a + b/
x^2)^p]/x
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^2)^p)/x^2,x)
```

```
[Out] int(ln(c*(a+b/x^2)^p)/x^2,x)
```

Maxima [A]
time = 0.55, size = 49, normalized size = 0.98

$$2\,bp\left(\frac{a\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{1}{bx}\right)-\frac{\log\left(\left(a+\frac{b}{x^2}\right)^pc\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^2,x, algorithm="maxima")
```

```
[Out] 2*b*p*(a*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b) + 1/(b*x)) - log((a + b/x^2)^p
*c)/x
```

Fricas [A]
time = 0.36, size = 119, normalized size = 2.38

$$\left[\frac{px\sqrt{-\frac{a}{b}}\log\left(\frac{ax^2+2\,bx\sqrt{-\frac{a}{b}}-b}{ax^2+b}\right)-p\log\left(\frac{ax^2+b}{x^2}\right)+2\,p-\log\left(c\right)}{x},\frac{2\,px\sqrt{\frac{a}{b}}\arctan\left(x\sqrt{\frac{a}{b}}\right)-p\log\left(\frac{ax^2+b}{x^2}\right)+2\,p-\log\left(c\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^2,x, algorithm="fricas")
```

```
[Out] [(p*x*sqrt(-a/b)*log((a*x^2 + 2*b*x*sqrt(-a/b) - b)/(a*x^2 + b)) - p*log((a
*x^2 + b)/x^2) + 2*p - log(c))/x, (2*p*x*sqrt(a/b)*arctan(x*sqrt(a/b)) - p*
log((a*x^2 + b)/x^2) + 2*p - log(c))/x]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(44) = 88$.
time = 8.70, size = 97, normalized size = 1.94

$$\begin{cases} -\frac{\log\left(0^pc\right)}{x} & \text{for}\: a=0\wedge b=0 \\ \frac{2p}{x}-\frac{\log\left(c\left(\frac{b}{x^2}\right)^p\right)}{x} & \text{for}\: a=0 \\ -\frac{\log\left(a^pc\right)}{x} & \text{for}\: b=0 \\ \frac{p\log\left(x-\sqrt{-\frac{b}{a}}\right)}{\sqrt{-\frac{b}{a}}}-\frac{p\log\left(x+\sqrt{-\frac{b}{a}}\right)}{\sqrt{-\frac{b}{a}}}+\frac{2p}{x}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**2)**p)/x**2,x)
```

```
[Out] Piecewise((-log(0**p*c)/x, Eq(a, 0) & Eq(b, 0)), (2*p/x - log(c*(b/x**2)**p
)/x, Eq(a, 0)), (-log(a**p*c)/x, Eq(b, 0)), (p*log(x - sqrt(-b/a))/sqrt(-b/
a) - p*log(x + sqrt(-b/a))/sqrt(-b/a) + 2*p/x - log(c*(a + b/x**2)**p)/x, T
rue))
```

Giac [A]
time = 4.93, size = 54, normalized size = 1.08

$$\frac{2\,ap\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}} - \frac{p\log\left(ax^2+b\right)}{x} + \frac{p\log\left(x^2\right)}{x} + \frac{2\,p-\log\left(c\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^2,x, algorithm="giac")
```

```
[Out] 2*a*p*arctan(a*x/sqrt(a*b))/sqrt(a*b) - p*log(a*x^2 + b)/x + p*log(x^2)/x +
 (2*p - log(c))/x
```

Mupad [B]
time = 0.24, size = 42, normalized size = 0.84

$$\frac{2\,p}{x} - \frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x} + \frac{2\,\sqrt{a}\,\,p\,\mathrm{atan}\left(\frac{\sqrt{a}\,\,x}{\sqrt{b}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x^2)^p)/x^2,x)
```

```
[Out] (2*p)/x - log(c*(a + b/x^2)^p)/x + (2*a^(1/2)*p*atan((a^(1/2)*x)/b^(1/2)))/
b^(1/2)
```

3.43 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3}\,dx$

Optimal. Leaf size=35

$$\frac{p}{2x^2}-\frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2b}$$

```
[Out] 1/2*p/x^2-1/2*(a+b/x^2)*ln(c*(a+b/x^2)^p)/b
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.188$, Rules used = {2504, 2436, 2332}

$$\frac{p}{2x^2}-\frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^2)^p]/x^3,x]
```

```
[Out] p/(2*x^2) - ((a + b/x^2)*Log[c*(a + b/x^2)^p])/(2*b)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3} dx = -\left(\frac{1}{2}\text{Subst}\left(\int \log \left(c(a+bx)^p\right) dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\frac{\text{Subst}\left(\int \log \left(cx^p\right) dx, x, a+\frac{b}{x^2}\right)}{2b}$$
$$= \frac{p}{2x^2} - \frac{\left(a+\frac{b}{x^2}\right)\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2b}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 0.97

$$\frac{1}{2}\left(\frac{p}{x^2} - \frac{\left(a+\frac{b}{x^2}\right)\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^2)^p]/x^3,x]
```

```
[Out] (p/x^2 - ((a + b/x^2)*Log[c*(a + b/x^2)^p])/b)/2
```

Maple [A]
time = 0.31, size = 37, normalized size = 1.06

method	result	size
derivativedivides	$-\frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)\left(a+\frac{b}{x^2}\right)-\left(a+\frac{b}{x^2}\right)p}{2b}$	37
default	$-\frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)\left(a+\frac{b}{x^2}\right)-\left(a+\frac{b}{x^2}\right)p}{2b}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^2)^p)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b*(ln(c*(a+b/x^2)^p)*(a+b/x^2)-(a+b/x^2)*p)
```

Maxima [A]
time = 0.28, size = 54, normalized size = 1.54

$$-\frac{1}{2}\,bp\left(\frac{a\log \left(ax^2+b\right)}{b^2} - \frac{a\log \left(x^2\right)}{b^2} - \frac{1}{bx^2}\right) - \frac{\log \left(\left(a+\frac{b}{x^2}\right)^p c\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b*p*(a*log(a*x^2 + b)/b^2 - a*log(x^2)/b^2 - 1/(b*x^2)) - 1/2*log((a +
 b/x^2)^p*c)/x^2
```

Fricas [A]
time = 0.37, size = 41, normalized size = 1.17

$$\frac{bp - b\log(c) - (apx^2 + bp)\log\left(\frac{ax^2+b}{x^2}\right)}{2\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^3,x, algorithm="fricas")
[Out] 1/2*(b*p - b*log(c) - (a*p*x^2 + b*p)*log((a*x^2 + b)/x^2))/(b*x^2)
```

Sympy [A]
time = 1.05, size = 53, normalized size = 1.51

$$\begin{cases} -\frac{a\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2b} + \frac{p}{2x^2} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2x^2} & \text{for}\ b \neq 0 \\ -\frac{\log(a^p c)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**2)**p)/x**3,x)
[Out] Piecewise((-a*log(c*(a + b/x**2)**p)/(2*b) + p/(2*x**2) - log(c*(a + b/x**2
)**p)/(2*x**2), Ne(b, 0)), (-log(a**p*c)/(2*x**2), True))
```

Giac [A]
time = 3.28, size = 57, normalized size = 1.63

$$-\frac{p\left(\frac{(ax^2+b)\log\left(\frac{ax^2+b}{x^2}\right)}{x^2} - \frac{ax^2+b}{x^2}\right) + \frac{(ax^2+b)\log(c)}{x^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^3,x, algorithm="giac")
[Out] -1/2*(p*((a*x^2 + b)*log((a*x^2 + b)/x^2)/x^2 - (a*x^2 + b)/x^2) + (a*x^2 +
 b)*log(c)/x^2)/b
```

Mupad [B]
time = 0.24, size = 47, normalized size = 1.34

$$\frac{p}{2\,x^2} - \frac{\ln\left(c\,\left(a+\frac{b}{x^2}\right)^p\right)}{2\,x^2} - \frac{a\,p\,\ln\left(a\,x^2+b\right)}{2\,b} + \frac{a\,p\,\ln\left(x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x^2)^p)/x^3,x)
[Out] p/(2*x^2) - log(c*(a + b/x^2)^p)/(2*x^2) - (a*p*log(b + a*x^2))/(2*b) + (a*
p*log(x))/b
```

3.44 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^4}\,dx$

Optimal. Leaf size=68

$$\frac{2p}{9x^3}-\frac{2ap}{3bx}-\frac{2a^{3/2}p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{3b^{3/2}}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3}$$

```
[Out] 2/9*p/x^3-2/3*a*p/b/x-2/3*a^(3/2)*p*arctan(x*a^(1/2)/b^(1/2))/b^(3/2)-1/3*l
n(c*(a+b/x^2)^p)/x^3
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2505, 269, 331, 211}

$$-\frac{2a^{3/2}p\text{ArcTan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{3b^{3/2}}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3}-\frac{2ap}{3bx}+\frac{2p}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^2)^p]/x^4,x]
```

```
[Out] (2*p)/(9*x^3) - (2*a*p)/(3*b*x) - (2*a^(3/2)*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])
/(3*b^(3/2)) - Log[c*(a + b/x^2)^p]/(3*x^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^4}\,dx &= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3}-\frac{1}{3}(2bp)\int\frac{1}{\left(a+\frac{b}{x^2}\right)x^6}\,dx\\
&= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3}-\frac{1}{3}(2bp)\int\frac{1}{x^4\left(b+ax^2\right)}\,dx\\
&= \frac{2p}{9x^3}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3}+\frac{1}{3}(2ap)\int\frac{1}{x^2\left(b+ax^2\right)}\,dx\\
&= \frac{2p}{9x^3}-\frac{2ap}{3bx}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3}-\frac{\left(2a^2p\right)\int\frac{1}{b+ax^2}\,dx}{3b}\\
&= \frac{2p}{9x^3}-\frac{2ap}{3bx}-\frac{2a^{3/2}p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{3b^{3/2}}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 70, normalized size = 1.03

$$\frac{2p}{9x^3}-\frac{2ap}{3bx}+\frac{2a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}\,x}\right)}{3b^{3/2}}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^2)^p]/x^4,x]
```

```
[Out] (2*p)/(9*x^3) - (2*a*p)/(3*b*x) + (2*a^(3/2)*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)])
/(3*b^(3/2)) - Log[c*(a + b/x^2)^p]/(3*x^3)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^2)^p)/x^4,x)
```

```
[Out] int(ln(c*(a+b/x^2)^p)/x^4,x)
```

Maxima [A]
time = 0.52, size = 62, normalized size = 0.91

$$-\frac{2}{9}\,bp\left(\frac{3\,a^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{3\,ax^2-b}{b^2x^3}\right)-\frac{\log\left(\left(a+\frac{b}{x^2}\right)^pc\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^4,x, algorithm="maxima")
```

```
[Out] -2/9*b*p*(3*a^2*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b^2) + (3*a*x^2 - b)/(b^2*
x^3)) - 1/3*log((a + b/x^2)^p*c)/x^3
```

Fricas [A]
time = 0.42, size = 154, normalized size = 2.26

$$\left[\frac{3\,apx^3\sqrt{-\frac{a}{b}}\log\left(\frac{ax^2-2\,bx\sqrt{-\frac{a}{b}}-b}{ax^2+b}\right)-6\,apx^2-3\,bp\log\left(\frac{ax^2+b}{x^2}\right)+2\,bp-3\,b\log\left(c\right)}{9\,bx^3},\ -\frac{6\,apx^3\sqrt{\frac{a}{b}}\arctan\left(x\sqrt{\frac{a}{b}}\right)+6\,apx^2+3\,bp\log\left(\frac{ax^2+b}{x^2}\right)-2\,bp+3\,b\log\left(c\right)}{9\,bx^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^4,x, algorithm="fricas")
```

```
[Out] [1/9*(3*a*p*x^3*sqrt(-a/b)*log((a*x^2 - 2*b*x*sqrt(-a/b) - b)/(a*x^2 + b))
 - 6*a*p*x^2 - 3*b*p*log((a*x^2 + b)/x^2) + 2*b*p - 3*b*log(c))/(b*x^3), -1/
9*(6*a*p*x^3*sqrt(a/b)*arctan(x*sqrt(a/b)) + 6*a*p*x^2 + 3*b*p*log((a*x^2 +
 b)/x^2) - 2*b*p + 3*b*log(c))/(b*x^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(63) = 126$.
time = 26.33, size = 138, normalized size = 2.03

$$\begin{cases} -\frac{\log\left(0^pc\right)}{3x^3} & \text{for}\: a=0\wedge b=0 \\ \frac{2p}{9x^3}-\frac{\log\left(c\left(\frac{b}{x^2}\right)^p\right)}{3x^3} & \text{for}\: a=0 \\ -\frac{\log\left(a^pc\right)}{3x^3} & \text{for}\: b=0 \\ -\frac{ap\log\left(x-\sqrt{-\frac{b}{a}}\right)}{3b\sqrt{-\frac{b}{a}}}+\frac{ap\log\left(x+\sqrt{-\frac{b}{a}}\right)}{3b\sqrt{-\frac{b}{a}}}-\frac{2ap}{3bx}+\frac{2p}{9x^3}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**2)**p)/x**4,x)
```

```
[Out] Piecewise((-log(0**p*c)/(3*x**3), Eq(a, 0) & Eq(b, 0)), (2*p/(9*x**3) - log
(c*(b/x**2)**p)/(3*x**3), Eq(a, 0)), (-log(a**p*c)/(3*x**3), Eq(b, 0)), (-a
*p*log(x - sqrt(-b/a))/(3*b*sqrt(-b/a)) + a*p*log(x + sqrt(-b/a))/(3*b*sqrt
(-b/a)) - 2*a*p/(3*b*x) + 2*p/(9*x**3) - log(c*(a + b/x**2)**p)/(3*x**3), T
rue))
```

Giac [A]
time = 4.35, size = 73, normalized size = 1.07

$$-\frac{2\,a^2p\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,b}-\frac{p\log\left(ax^2+b\right)}{3\,x^3}+\frac{p\log\left(x^2\right)}{3\,x^3}-\frac{6\,apx^2-2\,bp+3\,b\log\left(c\right)}{9\,bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^4,x, algorithm="giac")
```

```
[Out] -2/3*a^2*p*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/3*p*log(a*x^2 + b)/x^3 +
 1/3*p*log(x^2)/x^3 - 1/9*(6*a*p*x^2 - 2*b*p + 3*b*log(c))/(b*x^3)
```

Mupad [B]
time = 0.25, size = 55, normalized size = 0.81

$$\frac{\frac{2\,p}{3}-\frac{2\,a\,p\,x^2}{b}}{3\,x^3}-\frac{\ln\left(c\,{\left(a+\frac{b}{x^2}\right)}^p\right)}{3\,x^3}-\frac{2\,a^{3/2}\,p\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{3\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x^2)^p)/x^4,x)
```

```
[Out] ((2*p)/3 - (2*a*p*x^2)/b)/(3*x^3) - log(c*(a + b/x^2)^p)/(3*x^3) - (2*a^(3/
2)*p*atan((a^(1/2)*x)/b^(1/2)))/(3*b^(3/2))
```

3.45 $\int \frac{\log\left(1+\frac{b}{x}\right)}{x}\,dx$

Optimal. Leaf size=8

$$\text{Li}_2\left(-\frac{b}{x}\right)$$

```
[Out] polylog(2,-b/x)
```

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2438}

$$\text{PolyLog}\left(2,-\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + b/x]/x,x]
[Out] PolyLog[2, -(b/x)]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(1+\frac{b}{x}\right)}{x}\,dx = \text{Li}_2\left(-\frac{b}{x}\right)$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 34 vs. 2(8) = 16.
time = 0.00, size = 34, normalized size = 4.25

$$-\log\left(-\frac{b}{x}\right)\log\left(\frac{b+x}{x}\right)-\text{Li}_2\left(-\frac{-b-x}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + b/x]/x,x]
[Out] -(Log[-(b/x)]*Log[(b + x)/x]) - PolyLog[2, -((-b - x)/x)]
```

Maple [A]
time = 0.33, size = 9, normalized size = 1.12

method	result	size
derivativedivides	$\operatorname{dilog}\left(1+\frac{b}{x}\right)$	9
default	$\operatorname{dilog}\left(1+\frac{b}{x}\right)$	9
risch	$\operatorname{dilog}\left(1+\frac{b}{x}\right)$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1+b/x)/x,x,method=_RETURNVERBOSE)
```

```
[Out] dilog(1+b/x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(7) = 14$.
time = 0.28, size = 35, normalized size = 4.38

$$\log(b+x)\log(x) - \frac{1}{2}\log(x)^2 - \log(x)\log\left(\frac{x}{b}+1\right) - \operatorname{Li}_2\left(-\frac{x}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+b/x)/x,x, algorithm="maxima")
```

```
[Out] log(b + x)*log(x) - 1/2*log(x)^2 - log(x)*log(x/b + 1) - dilog(-x/b)
```

Fricas [A]
time = 0.39, size = 11, normalized size = 1.38

$$\operatorname{Li}_2\left(-\frac{b+x}{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+b/x)/x,x, algorithm="fricas")
```

```
[Out] dilog(-(b + x)/x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.58, size = 8, normalized size = 1.00

$$\operatorname{Li}_2\left(\frac{be^{i\pi}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+b/x)/x,x)
```

```
[Out] polylog(2, b*exp_polar(I*pi)/x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(7) = 14$.
time = 2.99, size = 110, normalized size = 13.75

$$-\frac{b^3\left(\frac{1}{\frac{b+x}{x}-1}-\log\left(\frac{|b+x|}{|x|}\right)+\log\left(\left|\frac{b+x}{x}-1\right|\right)\right)+\frac{b^3\log\left(-b\left(\frac{\left(b-\frac{1}{\frac{1}{b}-\frac{b+x}{bx}}\right)\left(\frac{1}{b}-\frac{b+x}{bx}\right)}{b}+\frac{1}{b}\right)+1\right)}{\left(\frac{b+x}{x}-1\right)^2}}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+b/x)/x,x, algorithm="giac")
```

```
[Out] -1/2*(b^3*(1/((b + x)/x - 1) - log(abs(b + x)/abs(x)) + log(abs((b + x)/x -
 1))) + b^3*log(-b*((b - 1/(1/b - (b + x)/(b*x)))*(1/b - (b + x)/(b*x))/b +
 1/b) + 1)/((b + x)/x - 1)^2)/b^2
```

Mupad [B]
time = 0.26, size = 8, normalized size = 1.00

$$\mathrm{polylog}\left(2,-\frac{b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(b/x + 1)/x,x)
```

```
[Out] polylog(2, -b/x)
```

3.46 $\int x^3 \log \left(c\left(a+b\sqrt{x}\right)^p\right) \, dx$

Optimal. Leaf size=153

$$\frac{a^7p\sqrt{x}}{4b^7}-\frac{a^6px}{8b^6}+\frac{a^5px^{3/2}}{12b^5}-\frac{a^4px^2}{16b^4}+\frac{a^3px^{5/2}}{20b^3}-\frac{a^2px^3}{24b^2}+\frac{apx^{7/2}}{28b}-\frac{px^4}{32}-\frac{a^8p\log\left(a+b\sqrt{x}\right)}{4b^8}+\frac{1}{4}x^4\log\left(c\left(a+b\sqrt{x}\right.\right.$$

```
[Out] -1/8*a^6*p*x/b^6+1/12*a^5*p*x^(3/2)/b^5-1/16*a^4*p*x^2/b^4+1/20*a^3*p*x^(5/
2)/b^3-1/24*a^2*p*x^3/b^2+1/28*a*p*x^(7/2)/b-1/32*p*x^4-1/4*a^8*p*ln(a+b*x^
(1/2))/b^8+1/4*x^4*ln(c*(a+b*x^(1/2))^p)+1/4*a^7*p*x^(1/2)/b^7
```

Rubi [A]
time = 0.08, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2504, 2442, 45}

$$-\frac{a^8p\log\left(a+b\sqrt{x}\right)}{4b^8}+\frac{a^7p\sqrt{x}}{4b^7}-\frac{a^6px}{8b^6}+\frac{a^5px^{3/2}}{12b^5}-\frac{a^4px^2}{16b^4}+\frac{a^3px^{5/2}}{20b^3}-\frac{a^2px^3}{24b^2}+\frac{1}{4}x^4\log\left(c\left(a+b\sqrt{x}\right)^p\right)+\frac{apx^{7/2}}{28b}-\frac{px^4}{32}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[c*(a + b*Sqrt[x])^p],x]
```

```
[Out] (a^7*p*Sqrt[x])/(4*b^7) - (a^6*p*x)/(8*b^6) + (a^5*p*x^(3/2))/(12*b^5) - (a
^4*p*x^2)/(16*b^4) + (a^3*p*x^(5/2))/(20*b^3) - (a^2*p*x^3)/(24*b^2) + (a*p
*x^(7/2))/(28*b) - (p*x^4)/32 - (a^8*p*Log[a + b*Sqrt[x]])/(4*b^8) + (x^4*L
og[c*(a + b*Sqrt[x])^p])/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
```

```
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3 \log \left(c\left(a+b\sqrt{x}\right)^p\right) dx &= 2\text{Subst}\left(\int x^7 \log \left(c(a+bx)^p\right) dx, x, \sqrt{x}\right) \\
&= \frac{1}{4}x^4 \log \left(c\left(a+b\sqrt{x}\right)^p\right) - \frac{1}{4}(bp)\text{Subst}\left(\int \frac{x^8}{a+bx} dx, x, \sqrt{x}\right) \\
&= \frac{1}{4}x^4 \log \left(c\left(a+b\sqrt{x}\right)^p\right) - \frac{1}{4}(bp)\text{Subst}\left(\int \left(-\frac{a^7}{b^8} + \frac{a^6 x}{b^7} - \frac{a^5 x^2}{b^6} + \frac{a^4 x^3}{b^5} - \frac{a^3}{b} \right.\right. \\
&= \frac{a^7 p\sqrt{x}}{4b^7} - \frac{a^6 px}{8b^6} + \frac{a^5 px^{3/2}}{12b^5} - \frac{a^4 px^2}{16b^4} + \frac{a^3 px^{5/2}}{20b^3} - \frac{a^2 px^3}{24b^2} + \frac{apx^{7/2}}{28b} - \frac{px^4}{32} - \frac{a}{}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 134, normalized size = 0.88

$$\frac{1}{4}\left(-\frac{p\left(-840a^7b\sqrt{x}+420a^6b^2x-280a^5b^3x^{3/2}+210a^4b^4x^2-168a^3b^5x^{5/2}+140a^2b^6x^3-120ab^7x^{7/2}+105b^8x^4+840a^8\log\left(a+b\sqrt{x}\right)\right)}{840b^8}+x^4\log\left(c\left(a+b\sqrt{x}\right)^p\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[c*(a + b*Sqrt[x])^p],x]
```

```
[Out] (-1/840*(p*(-840*a^7*b*Sqrt[x] + 420*a^6*b^2*x - 280*a^5*b^3*x^(3/2) + 210*
a^4*b^4*x^2 - 168*a^3*b^5*x^(5/2) + 140*a^2*b^6*x^3 - 120*a*b^7*x^(7/2) + 1
05*b^8*x^4 + 840*a^8*Log[a + b*Sqrt[x]]))/b^8 + x^4*Log[c*(a + b*Sqrt[x])^p
])/4
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int x^3 \ln \left(c\left(a+b\sqrt{x}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(a+b*x^(1/2))^p),x)
```

```
[Out] int(x^3*ln(c*(a+b*x^(1/2))^p),x)
```

Maxima [A]
time = 0.29, size = 120, normalized size = 0.78

$$\frac{1}{4}x^4\log\left(\left(b\sqrt{x}+a\right)^p c\right)-\frac{1}{3360}bp\left(\frac{840a^8\log\left(b\sqrt{x}+a\right)}{b^9}+\frac{105b^7x^4-120ab^6x^{\frac{7}{2}}+140a^2b^5x^3-168a^3b^4x^{\frac{5}{2}}+210a^4b^3x^2-280a^5b^2x^{\frac{3}{2}}+420a^6bx-840a^7\sqrt{x}}{b^8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b*x^(1/2))^p),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log((b*sqrt(x) + a)^p*c) - 1/3360*b*p*(840*a^8*log(b*sqrt(x) + a)/b
^9 + (105*b^7*x^4 - 120*a*b^6*x^(7/2) + 140*a^2*b^5*x^3 - 168*a^3*b^4*x^(5/
2) + 210*a^4*b^3*x^2 - 280*a^5*b^2*x^(3/2) + 420*a^6*b*x - 840*a^7*sqrt(x))
/b^8)
```

Fricas [A]
time = 0.49, size = 129, normalized size = 0.84

$$-\frac{105\,b^8px^4 - 840\,b^8x^4\log(c) + 140\,a^2b^6px^3 + 210\,a^4b^4px^2 + 420\,a^6b^2px - 840\,(b^8px^4 - a^8p)\log(b\sqrt{x}+a) - 8\,(15\,ab^7px^3 + 21\,a^3b^5px^2 + 35\,a^5b^3px + 105\,a^7bp)\sqrt{x}}{3360\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b*x^(1/2))^p),x, algorithm="fricas")
```

```
[Out] -1/3360*(105*b^8*p*x^4 - 840*b^8*x^4*log(c) + 140*a^2*b^6*p*x^3 + 210*a^4*b
^4*p*x^2 + 420*a^6*b^2*p*x - 840*(b^8*p*x^4 - a^8*p)*log(b*sqrt(x) + a) - 8
*(15*a*b^7*p*x^3 + 21*a^3*b^5*p*x^2 + 35*a^5*b^3*p*x + 105*a^7*b*p)*sqrt(x)
)/b^8
```

Sympy [A]
time = 13.63, size = 146, normalized size = 0.95

$$-\frac{bp\left(\frac{2a^8\left(\begin{cases}\frac{\sqrt{x}}{a} & \text{for } b=0\\ \frac{\log\left(a+b\sqrt{x}\right)}{b} & \text{otherwise}\end{cases}\right)}{b^8} - \frac{2a^7\sqrt{x}}{b^8} + \frac{a^6x}{b^7} - \frac{2a^5x^{\frac{3}{2}}}{3b^6} + \frac{a^4x^2}{2b^5} - \frac{2a^3x^{\frac{5}{2}}}{5b^4} + \frac{a^2x^3}{3b^3} - \frac{2ax^{\frac{7}{2}}}{7b^2} + \frac{x^4}{4b}\right)}{8} + \frac{x^4\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(a+b*x**(1/2))**p),x)
```

```
[Out] -b*p*(2*a**8*Piecewise((sqrt(x)/a, Eq(b, 0)), (log(a + b*sqrt(x))/b, True))
/b**8 - 2*a**7*sqrt(x)/b**8 + a**6*x/b**7 - 2*a**5*x**(3/2)/(3*b**6) + a**4
*x**2/(2*b**5) - 2*a**3*x**(5/2)/(5*b**4) + a**2*x**3/(3*b**3) - 2*a*x**(7/
2)/(7*b**2) + x**4/(4*b))/8 + x**4*log(c*(a + b*sqrt(x))**p)/4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 339 vs. 2(121) = 242.
time = 4.64, size = 339, normalized size = 2.22

$$\frac{840\,bx^4\log(c) + \left(\frac{840\left(b\sqrt{x}+a\right)^8\log\left(b\sqrt{x}+a\right)}{b^7} - \frac{6720\left(b\sqrt{x}+a\right)^7a\log\left(b\sqrt{x}+a\right)}{b^7} + \frac{23520\left(b\sqrt{x}+a\right)^6a^2\log\left(b\sqrt{x}+a\right)}{b^7} - \frac{47040\left(b\sqrt{x}+a\right)^5a^3\log\left(b\sqrt{x}+a\right)}{b^7} + \frac{58800\left(b\sqrt{x}+a\right)^4a^4\log\left(b\sqrt{x}+a\right)}{b^7} - \frac{47040\left(b\sqrt{x}+a\right)^3a^5\log\left(b\sqrt{x}+a\right)}{b^7} + \frac{23520\left(b\sqrt{x}+a\right)^2a^6\log\left(b\sqrt{x}+a\right)}{b^7} - \frac{6720\left(b\sqrt{x}+a\right)a^7\log\left(b\sqrt{x}+a\right)}{b^7} - \frac{105\left(b\sqrt{x}+a\right)^8}{b^7} + \frac{960\left(b\sqrt{x}+a\right)^7a}{b^7} - \frac{3920\left(b\sqrt{x}+a\right)^6a^2}{b^7} + \frac{9408\left(b\sqrt{x}+a\right)^5a^3}{b^7} - \frac{14700\left(b\sqrt{x}+a\right)^4a^4}{b^7} + \frac{15680\left(b\sqrt{x}+a\right)^3a^5}{b^7} - \frac{11760\left(b\sqrt{x}+a\right)^2a^6}{b^7} + \frac{6720\left(b\sqrt{x}+a\right)a^7}{b^7}\right)p}{3360\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b*x^(1/2))^p),x, algorithm="giac")
```

```
[Out] 1/3360*(840*b*x^4*log(c) + (840*(b*sqrt(x) + a)^8*log(b*sqrt(x) + a)/b^7 -
6720*(b*sqrt(x) + a)^7*a*log(b*sqrt(x) + a)/b^7 + 23520*(b*sqrt(x) + a)^6*a
^2*log(b*sqrt(x) + a)/b^7 - 47040*(b*sqrt(x) + a)^5*a^3*log(b*sqrt(x) + a)/
b^7 + 58800*(b*sqrt(x) + a)^4*a^4*log(b*sqrt(x) + a)/b^7 - 47040*(b*sqrt(x)
 + a)^3*a^5*log(b*sqrt(x) + a)/b^7 + 23520*(b*sqrt(x) + a)^2*a^6*log(b*sqrt
(x) + a)/b^7 - 6720*(b*sqrt(x) + a)*a^7*log(b*sqrt(x) + a)/b^7 - 105*(b*sqr
t(x) + a)^8/b^7 + 960*(b*sqrt(x) + a)^7*a/b^7 - 3920*(b*sqrt(x) + a)^6*a^2/
b^7 + 9408*(b*sqrt(x) + a)^5*a^3/b^7 - 14700*(b*sqrt(x) + a)^4*a^4/b^7 + 15
680*(b*sqrt(x) + a)^3*a^5/b^7 - 11760*(b*sqrt(x) + a)^2*a^6/b^7 + 6720*(b*s
qrt(x) + a)*a^7/b^7)*p)/b
```

Mupad [B]
time = 0.33, size = 121, normalized size = 0.79

$$\frac{x^4\,\ln\left(c\,{\left(a+b\,\sqrt{x}\,\right)}^p\right)}{4}-\frac{p\,x^4}{32}-\frac{a^8\,p\,\ln\left(a+b\,\sqrt{x}\,\right)}{4\,b^8}-\frac{a^2\,p\,x^3}{24\,b^2}-\frac{a^4\,p\,x^2}{16\,b^4}+\frac{a^3\,p\,x^{5/2}}{20\,b^3}+\frac{a^5\,p\,x^{3/2}}{12\,b^5}+\frac{a^7\,p\,\sqrt{x}}{4\,b^7}+\frac{a\,p\,x^{7/2}}{28\,b}-\frac{a^6\,p\,x}{8\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(c*(a + b*x^(1/2))^p),x)
```

```
[Out] (x^4*log(c*(a + b*x^(1/2))^p))/4 - (p*x^4)/32 - (a^8*p*log(a + b*x^(1/2)))/
(4*b^8) - (a^2*p*x^3)/(24*b^2) - (a^4*p*x^2)/(16*b^4) + (a^3*p*x^(5/2))/(20
*b^3) + (a^5*p*x^(3/2))/(12*b^5) + (a^7*p*x^(1/2))/(4*b^7) + (a*p*x^(7/2))/
(28*b) - (a^6*p*x)/(8*b^6)
```

3.47 $\int x^2 \log \left(c\left(a+b\sqrt{x}\right)^p\right)\, dx$

Optimal. Leaf size=123

$$\frac{a^5p\sqrt{x}}{3b^5}-\frac{a^4px}{6b^4}+\frac{a^3px^{3/2}}{9b^3}-\frac{a^2px^2}{12b^2}+\frac{apx^{5/2}}{15b}-\frac{px^3}{18}-\frac{a^6p\log\left(a+b\sqrt{x}\right)}{3b^6}+\frac{1}{3}x^3\log\left(c\left(a+b\sqrt{x}\right)^p\right)$$

```
[Out] -1/6*a^4*p*x/b^4+1/9*a^3*p*x^(3/2)/b^3-1/12*a^2*p*x^2/b^2+1/15*a*p*x^(5/2)/
b-1/18*p*x^3-1/3*a^6*p*ln(a+b*x^(1/2))/b^6+1/3*x^3*ln(c*(a+b*x^(1/2))^p)+1/
3*a^5*p*x^(1/2)/b^5
```

Rubi [A]
time = 0.06, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2504, 2442, 45}

$$-\frac{a^6p\log\left(a+b\sqrt{x}\right)}{3b^6}+\frac{a^5p\sqrt{x}}{3b^5}-\frac{a^4px}{6b^4}+\frac{a^3px^{3/2}}{9b^3}-\frac{a^2px^2}{12b^2}+\frac{1}{3}x^3\log\left(c\left(a+b\sqrt{x}\right)^p\right)+\frac{apx^{5/2}}{15b}-\frac{px^3}{18}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[c*(a + b*Sqrt[x])^p],x]
```

```
[Out] (a^5*p*Sqrt[x])/(3*b^5) - (a^4*p*x)/(6*b^4) + (a^3*p*x^(3/2))/(9*b^3) - (a^
2*p*x^2)/(12*b^2) + (a*p*x^(5/2))/(15*b) - (p*x^3)/18 - (a^6*p*Log[a + b*Sq
rt[x]])/(3*b^6) + (x^3*Log[c*(a + b*Sqrt[x])^p])/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
```

```
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^2 \log \left(c\left(a+b\sqrt{x}\right)^p\right) dx = 2\text{Subst}\left(\int x^5 \log \left(c(a+bx)^p\right) dx, x, \sqrt{x}\right)$$
$$= \frac{1}{3}x^3 \log \left(c\left(a+b\sqrt{x}\right)^p\right) - \frac{1}{3}(bp)\text{Subst}\left(\int \frac{x^6}{a+bx} dx, x, \sqrt{x}\right)$$
$$= \frac{1}{3}x^3 \log \left(c\left(a+b\sqrt{x}\right)^p\right) - \frac{1}{3}(bp)\text{Subst}\left(\int \left(-\frac{a^5}{b^6}+\frac{a^4x}{b^5}-\frac{a^3x^2}{b^4}+\frac{a^2x^3}{b^3}-\frac{ax}{b^2}\right.\right.$$
$$= \frac{a^5p\sqrt{x}}{3b^5} - \frac{a^4px}{6b^4} + \frac{a^3px^{3/2}}{9b^3} - \frac{a^2px^2}{12b^2} + \frac{apx^{5/2}}{15b} - \frac{px^3}{18} - \frac{a^6p\log\left(a+b\sqrt{x}\right)}{3b^6} +$$

Mathematica [A]
time = 0.04, size = 112, normalized size = 0.91

$$\frac{bp\sqrt{x}\left(60a^5-30a^4b\sqrt{x}+20a^3b^2x-15a^2b^3x^{3/2}+12ab^4x^2-10b^5x^{5/2}\right)-60a^6p\log\left(a+b\sqrt{x}\right)+60b^6x^3\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{180b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[c*(a + b*Sqrt[x])^p],x]
[Out] (b*p*Sqrt[x]*(60*a^5 - 30*a^4*b*Sqrt[x] + 20*a^3*b^2*x - 15*a^2*b^3*x^(3/2)
 + 12*a*b^4*x^2 - 10*b^5*x^(5/2)) - 60*a^6*p*Log[a + b*Sqrt[x]] + 60*b^6*x^
3*Log[c*(a + b*Sqrt[x])^p])/(180*b^6)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int x^2 \ln \left(c\left(a+b\sqrt{x}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(a+b*x^(1/2))^p),x)
[Out] int(x^2*ln(c*(a+b*x^(1/2))^p),x)
```

Maxima [A]
time = 0.30, size = 98, normalized size = 0.80

$$\frac{1}{3}x^3\log\left(\left(b\sqrt{x}+a\right)^pc\right)-\frac{1}{180}bp\left(\frac{60\,a^6\log\left(b\sqrt{x}+a\right)}{b^7}+\frac{10\,b^5x^3-12\,ab^4x^{\frac{5}{2}}+15\,a^2b^3x^2-20\,a^3b^2x^{\frac{3}{2}}+30\,a^4bx-60\,a^5\sqrt{x}}{b^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b*x^(1/2))^p),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log((b*sqrt(x) + a)^p*c) - 1/180*b*p*(60*a^6*log(b*sqrt(x) + a)/b^7
 + (10*b^5*x^3 - 12*a*b^4*x^(5/2) + 15*a^2*b^3*x^2 - 20*a^3*b^2*x^(3/2) + 3
0*a^4*b*x - 60*a^5*sqrt(x))/b^6)
```

Fricas [**A**]
time = 0.42, size = 105, normalized size = 0.85

$$-\frac{10\,b^{6}px^{3}-60\,b^{6}x^{3}\log\left(c\right)+15\,a^{2}b^{4}px^{2}+30\,a^{4}b^{2}px-60\,\left(b^{6}px^{3}-a^{6}p\right)\log\left(b\sqrt{x}+a\right)-4\,\left(3\,ab^{5}px^{2}+5\,a^{3}b^{3}px+15\,a^{5}bp\right)\sqrt{x}}{180\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b*x^(1/2))^p),x, algorithm="fricas")
```

```
[Out] -1/180*(10*b^6*p*x^3 - 60*b^6*x^3*log(c) + 15*a^2*b^4*p*x^2 + 30*a^4*b^2*p*
x - 60*(b^6*p*x^3 - a^6*p)*log(b*sqrt(x) + a) - 4*(3*a*b^5*p*x^2 + 5*a^3*b^
3*p*x + 15*a^5*b*p)*sqrt(x))/b^6
```

Sympy [**A**]
time = 4.37, size = 119, normalized size = 0.97

$$-\frac{bp\left(\frac{2a^{6}\left(\begin{cases}\frac{\sqrt{x}}{a} & \text{for}\: b=0\\\frac{\log\left(a+b\sqrt{x}\right)}{b} & \text{otherwise}\end{cases}\right)}{b^{6}}-\frac{2a^{5}\sqrt{x}}{b^{6}}+\frac{a^{4}x}{b^{5}}-\frac{2a^{3}x^{\frac{3}{2}}}{3b^{4}}+\frac{a^{2}x^{2}}{2b^{3}}-\frac{2ax^{\frac{5}{2}}}{5b^{2}}+\frac{x^{3}}{3b}\right)}{6}+\frac{x^{3}\log\left(c\left(a+b\sqrt{x}\right)^{p}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(a+b*x**(1/2))**p),x)
```

```
[Out] -b*p*(2*a**6*Piecewise((sqrt(x)/a, Eq(b, 0)), (log(a + b*sqrt(x))/b, True))
/b**6 - 2*a**5*sqrt(x)/b**6 + a**4*x/b**5 - 2*a**3*x**(3/2)/(3*b**4) + a**2
*x**2/(2*b**3) - 2*a*x**(5/2)/(5*b**2) + x**3/(3*b))/6 + x**3*log(c*(a + b*
sqrt(x))**p)/3
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(97) = 194.
time = 3.73, size = 255, normalized size = 2.07

$$\frac{60\,bx^{3}\log\left(c\right)+\left(\frac{60\,\left(b\sqrt{x}+a\right)^{6}\log\left(b\sqrt{x}+a\right)}{b^{5}}-\frac{360\,\left(b\sqrt{x}+a\right)^{5}a\log\left(b\sqrt{x}+a\right)}{b^{5}}+\frac{900\,\left(b\sqrt{x}+a\right)^{4}a^{2}\log\left(b\sqrt{x}+a\right)}{b^{5}}-\frac{1200\,\left(b\sqrt{x}+a\right)^{3}a^{3}\log\left(b\sqrt{x}+a\right)}{b^{5}}+\frac{900\,\left(b\sqrt{x}+a\right)^{2}a^{4}\log\left(b\sqrt{x}+a\right)}{b^{5}}-\frac{360\,\left(b\sqrt{x}+a\right)a^{5}\log\left(b\sqrt{x}+a\right)}{b^{5}}-\frac{10\,\left(b\sqrt{x}+a\right)^{6}}{b^{5}}+\frac{72\,\left(b\sqrt{x}+a\right)^{5}a}{b^{5}}-\frac{225\,\left(b\sqrt{x}+a\right)^{4}a^{2}}{b^{5}}+\frac{400\,\left(b\sqrt{x}+a\right)^{3}a^{3}}{b^{5}}-\frac{450\,\left(b\sqrt{x}+a\right)^{2}a^{4}}{b^{5}}+\frac{360\,\left(b\sqrt{x}+a\right)a^{5}}{b^{5}}\right)p}{180\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b*x^(1/2))^p),x, algorithm="giac")
```

```
[Out] 1/180*(60*b*x^3*log(c) + (60*(b*sqrt(x) + a)^6*log(b*sqrt(x) + a)/b^5 - 360
*(b*sqrt(x) + a)^5*a*log(b*sqrt(x) + a)/b^5 + 900*(b*sqrt(x) + a)^4*a^2*log
(b*sqrt(x) + a)/b^5 - 1200*(b*sqrt(x) + a)^3*a^3*log(b*sqrt(x) + a)/b^5 + 9
00*(b*sqrt(x) + a)^2*a^4*log(b*sqrt(x) + a)/b^5 - 360*(b*sqrt(x) + a)*a^5*l
og(b*sqrt(x) + a)/b^5 - 10*(b*sqrt(x) + a)^6/b^5 + 72*(b*sqrt(x) + a)^5*a/b
^5 - 225*(b*sqrt(x) + a)^4*a^2/b^5 + 400*(b*sqrt(x) + a)^3*a^3/b^5 - 450*(b
*sqrt(x) + a)^2*a^4/b^5 + 360*(b*sqrt(x) + a)*a^5/b^5)*p)/b
```

Mupad [B]
time = 0.28, size = 97, normalized size = 0.79

$$\frac{x^3\,\ln\left(c\,{\left(a+b\,\sqrt{x}\right)}^p\right)}{3}-\frac{p\,x^3}{18}-\frac{a^6\,p\,\ln\left(a+b\,\sqrt{x}\right)}{3\,b^6}-\frac{a^2\,p\,x^2}{12\,b^2}+\frac{a^3\,p\,x^{3/2}}{9\,b^3}+\frac{a^5\,p\,\sqrt{x}}{3\,b^5}+\frac{a\,p\,x^{5/2}}{15\,b}-\frac{a^4\,p\,x}{6\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(c*(a + b*x^(1/2))^p),x)
```

```
[Out] (x^3*log(c*(a + b*x^(1/2))^p))/3 - (p*x^3)/18 - (a^6*p*log(a + b*x^(1/2)))/
(3*b^6) - (a^2*p*x^2)/(12*b^2) + (a^3*p*x^(3/2))/(9*b^3) + (a^5*p*x^(1/2))/
(3*b^5) + (a*p*x^(5/2))/(15*b) - (a^4*p*x)/(6*b^4)
```

3.48 $\int x \log \left(c\left(a+b\sqrt{x}\right)^p\right)\,dx$

Optimal. Leaf size=93

$$\frac{a^3 p\sqrt{x}}{2b^3}-\frac{a^2 p x}{4b^2}+\frac{a p x^{3/2}}{6b}-\frac{p x^2}{8}-\frac{a^4 p \log\left(a+b\sqrt{x}\right)}{2b^4}+\frac{1}{2}x^2\log\left(c\left(a+b\sqrt{x}\right)^p\right)$$

```
[Out] -1/4*a^2*p*x/b^2+1/6*a*p*x^(3/2)/b-1/8*p*x^2-1/2*a^4*p*ln(a+b*x^(1/2))/b^4+
1/2*x^2*ln(c*(a+b*x^(1/2))^p)+1/2*a^3*p*x^(1/2)/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2504, 2442, 45}

$$-\frac{a^4 p \log\left(a+b\sqrt{x}\right)}{2b^4}+\frac{a^3 p\sqrt{x}}{2b^3}-\frac{a^2 p x}{4b^2}+\frac{1}{2}x^2\log\left(c\left(a+b\sqrt{x}\right)^p\right)+\frac{a p x^{3/2}}{6b}-\frac{p x^2}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[c*(a + b*Sqrt[x])^p],x]
```

```
[Out] (a^3*p*Sqrt[x])/(2*b^3) - (a^2*p*x)/(4*b^2) + (a*p*x^(3/2))/(6*b) - (p*x^2)
/8 - (a^4*p*Log[a + b*Sqrt[x]])/(2*b^4) + (x^2*Log[c*(a + b*Sqrt[x])^p])/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int x\log\left(c\left(a+b\sqrt{x}\right)^p\right)\,dx &= 2\text{Subst}\left(\int x^3\log\left(c(a+bx)^p\right)\,dx,x,\sqrt{x}\right)\\ &= \frac{1}{2}x^2\log\left(c\left(a+b\sqrt{x}\right)^p\right)-\frac{1}{2}(bp)\text{Subst}\left(\int\frac{x^4}{a+bx}\,dx,x,\sqrt{x}\right)\\ &= \frac{1}{2}x^2\log\left(c\left(a+b\sqrt{x}\right)^p\right)-\frac{1}{2}(bp)\text{Subst}\left(\int\left(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+\right.\right.\\ &= \frac{a^3p\sqrt{x}}{2b^3}-\frac{a^2px}{4b^2}+\frac{apx^{3/2}}{6b}-\frac{px^2}{8}-\frac{a^4p\log\left(a+b\sqrt{x}\right)}{2b^4}+\frac{1}{2}x^2\log\left(c\left(a+b\sqrt{x}\right.\right.\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 88, normalized size = 0.95

$$\frac{bp\sqrt{x}\left(12a^3-6a^2b\sqrt{x}+4ab^2x-3b^3x^{3/2}\right)-12a^4p\log\left(a+b\sqrt{x}\right)+12b^4x^2\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{24b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[c*(a + b*Sqrt[x])^p],x]
```

```
[Out] (b*p*Sqrt[x]*(12*a^3 - 6*a^2*b*Sqrt[x] + 4*a*b^2*x - 3*b^3*x^(3/2)) - 12*a^
4*p*Log[a + b*Sqrt[x]] + 12*b^4*x^2*Log[c*(a + b*Sqrt[x])^p])/(24*b^4)
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int x\ln\left(c\left(a+b\sqrt{x}\right)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(a+b*x^(1/2))^p),x)
```

```
[Out] int(x*ln(c*(a+b*x^(1/2))^p),x)
```

Maxima [**A**]
time = 0.28, size = 76, normalized size = 0.82

$$-\frac{1}{24}\,bp\left(\frac{12\,a^4\log\left(b\sqrt{x}+a\right)}{b^5}+\frac{3\,b^3x^2-4\,ab^2x^{\frac{3}{2}}+6\,a^2bx-12\,a^3\sqrt{x}}{b^4}\right)+\frac{1}{2}\,x^2\log\left(\left(b\sqrt{x}+a\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b*x^(1/2))^p),x, algorithm="maxima")
```

```
[Out] -1/24*b*p*(12*a^4*log(b*sqrt(x) + a)/b^5 + (3*b^3*x^2 - 4*a*b^2*x^(3/2) + 6
*a^2*b*x - 12*a^3*sqrt(x))/b^4) + 1/2*x^2*log((b*sqrt(x) + a)^p*c)
```

Fricas [A]
time = 0.43, size = 80, normalized size = 0.86

$$-\frac{3\,b^4px^2-12\,b^4x^2\log\left(c\right)+6\,a^2b^2px-12\,\left(b^4px^2-a^4p\right)\log\left(b\sqrt{x}+a\right)-4\,\left(ab^3px+3\,a^3bp\right)\sqrt{x}}{24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b*x^(1/2))^p),x, algorithm="fricas")
```

```
[Out] -1/24*(3*b^4*p*x^2 - 12*b^4*x^2*log(c) + 6*a^2*b^2*p*x - 12*(b^4*p*x^2 - a^
4*p)*log(b*sqrt(x) + a) - 4*(a*b^3*p*x + 3*a^3*b*p)*sqrt(x))/b^4
```

Sympy [A]
time = 1.46, size = 92, normalized size = 0.99

$$-\frac{bp\left(\frac{2a^4\left(\begin{cases}\frac{\sqrt{x}}{a} & \text{for}\ b=0\\ \frac{\log\left(a+b\sqrt{x}\right)}{b} & \text{otherwise}\end{cases}\right)}{b^4}-\frac{2a^3\sqrt{x}}{b^4}+\frac{a^2x}{b^3}-\frac{2ax^{\frac{3}{2}}}{3b^2}+\frac{x^2}{2b}\right)}{4}+\frac{x^2\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(a+b*x**(1/2))**p),x)
```

```
[Out] -b*p*(2*a**4*Piecewise((sqrt(x)/a, Eq(b, 0)), (log(a + b*sqrt(x))/b, True))
/b**4 - 2*a**3*sqrt(x)/b**4 + a**2*x/b**3 - 2*a*x**(3/2)/(3*b**2) + x**2/(2
*b))/4 + x**2*log(c*(a + b*sqrt(x))**p)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(73) = 146.
time = 3.39, size = 171, normalized size = 1.84

$$\frac{12\,bx^2\log\left(c\right)+\left(\frac{12\left(b\sqrt{x}+a\right)^4\log\left(b\sqrt{x}+a\right)}{b^3}-\frac{48\left(b\sqrt{x}+a\right)^3a\log\left(b\sqrt{x}+a\right)}{b^3}+\frac{72\left(b\sqrt{x}+a\right)^2a^2\log\left(b\sqrt{x}+a\right)}{b^3}-\frac{48\left(b\sqrt{x}+a\right)a^3\log\left(b\sqrt{x}+a\right)}{b^3}-\frac{3\left(b\sqrt{x}+a\right)^4}{b^3}+\frac{16\left(b\sqrt{x}+a\right)^3a}{b^3}-\frac{36\left(b\sqrt{x}+a\right)^2a^2}{b^3}+\frac{48\left(b\sqrt{x}+a\right)a^3}{b^3}\right)p}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b*x^(1/2))^p),x, algorithm="giac")
```

```
[Out] 1/24*(12*b*x^2*log(c) + (12*(b*sqrt(x) + a)^4*log(b*sqrt(x) + a)/b^3 - 48*(
b*sqrt(x) + a)^3*a*log(b*sqrt(x) + a)/b^3 + 72*(b*sqrt(x) + a)^2*a^2*log(b*
```

```
sqrt(x) + a)/b^3 - 48*(b*sqrt(x) + a)*a^3*log(b*sqrt(x) + a)/b^3 - 3*(b*sqr
t(x) + a)^4/b^3 + 16*(b*sqrt(x) + a)^3*a/b^3 - 36*(b*sqrt(x) + a)^2*a^2/b^3
 + 48*(b*sqrt(x) + a)*a^3/b^3)*p)/b
```

Mupad [B]
time = 0.27, size = 73, normalized size = 0.78

$$\frac{x^2 \ln\left(c\left(a+b\sqrt{x}\right)^p\right)}{2}-\frac{p\,x^2}{8}-\frac{a^4\,p\,\ln\left(a+b\sqrt{x}\right)}{2\,b^4}+\frac{a^3\,p\,\sqrt{x}}{2\,b^3}-\frac{a^2\,p\,x}{4\,b^2}+\frac{a\,p\,x^{3/2}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(c*(a + b*x^(1/2))^p),x)
```

```
[Out] (x^2*log(c*(a + b*x^(1/2))^p))/2 - (p*x^2)/8 - (a^4*p*log(a + b*x^(1/2)))/(
2*b^4) + (a^3*p*x^(1/2))/(2*b^3) - (a^2*p*x)/(4*b^2) + (a*p*x^(3/2))/(6*b)
```

3.49 $\int \log \left(c\left(a+b\sqrt{x}\right)^p\right) \, dx$

Optimal. Leaf size=53

$$\frac{ap\sqrt{x}}{b}-\frac{px}{2}-\frac{a^2p\log \left(a+b\sqrt{x}\right)}{b^2}+x\log \left(c\left(a+b\sqrt{x}\right)^p\right)$$

```
[Out] -1/2*p*x-a^2*p*ln(a+b*x^(1/2))/b^2+x*ln(c*(a+b*x^(1/2))^p)+a*p*x^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2498, 272, 45}

$$-\frac{a^2p\log \left(a+b\sqrt{x}\right)}{b^2}+x\log \left(c\left(a+b\sqrt{x}\right)^p\right)+\frac{ap\sqrt{x}}{b}-\frac{px}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*Sqrt[x])^p],x]
```

```
[Out] (a*p*Sqrt[x])/b - (p*x)/2 - (a^2*p*Log[a + b*Sqrt[x]])/b^2 + x*Log[c*(a + b
*Sqrt[x])^p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\int \log \left(c\left(a+b\sqrt{x}\right)^p\right) dx = x\log \left(c\left(a+b\sqrt{x}\right)^p\right) - \frac{1}{2}(bp)\int \frac{\sqrt{x}}{a+b\sqrt{x}}\,dx$$
$$= x\log \left(c\left(a+b\sqrt{x}\right)^p\right) - (bp)\text{Subst}\left(\int \frac{x^2}{a+bx}\,dx, x, \sqrt{x}\right)$$
$$= x\log \left(c\left(a+b\sqrt{x}\right)^p\right) - (bp)\text{Subst}\left(\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right) dx, x, \sqrt{x}\right)$$
$$= \frac{ap\sqrt{x}}{b} - \frac{px}{2} - \frac{a^2p\log \left(a+b\sqrt{x}\right)}{b^2} + x\log \left(c\left(a+b\sqrt{x}\right)^p\right)$$

Mathematica [A]
time = 0.02, size = 53, normalized size = 1.00

$$\frac{ap\sqrt{x}}{b} - \frac{px}{2} - \frac{a^2p\log \left(a+b\sqrt{x}\right)}{b^2} + x\log \left(c\left(a+b\sqrt{x}\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*Sqrt[x])^p],x]
[Out] (a*p*Sqrt[x])/b - (p*x)/2 - (a^2*p*Log[a + b*Sqrt[x]])/b^2 + x*Log[c*(a + b
*Sqrt[x])^p]
```

Maple [A]
time = 0.28, size = 52, normalized size = 0.98

method	result	size
default	$x\ln \left(c\left(a+b\sqrt{x}\right)^p\right) - \frac{pb\left(-\frac{2\left(-\frac{bx}{2}+a\sqrt{x}\right)}{b^2}+\frac{2a^2\ln \left(a+b\sqrt{x}\right)}{b^3}\right)}{2}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b*x^(1/2))^p),x,method=_RETURNVERBOSE)
[Out] x*ln(c*(a+b*x^(1/2))^p)-1/2*p*b*(-2/b^2*(-1/2*b*x+a*x^(1/2))+2*a^2/b^3*ln(a
+b*x^(1/2)))
```

Maxima [A]
time = 0.27, size = 50, normalized size = 0.94

$$-\frac{1}{2}\,bp\left(\frac{2\,a^2\log \left(b\sqrt{x}+a\right)}{b^3}+\frac{bx-2\,a\sqrt{x}}{b^2}\right)+x\log \left(\left(b\sqrt{x}+a\right)^p c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p),x, algorithm="maxima")
```

```
[Out] -1/2*b*p*(2*a^2*log(b*sqrt(x) + a)/b^3 + (b*x - 2*a*sqrt(x))/b^2) + x*log((
b*sqrt(x) + a)^p*c)
```

Fricas [**A**]
time = 0.37, size = 51, normalized size = 0.96

$$-\frac{b^2px - 2\,b^2x\log\left(c\right) - 2\,abp\sqrt{x} - 2\,(b^2px - a^2p)\log\left(b\sqrt{x} + a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*p*x - 2*b^2*x*log(c) - 2*a*b*p*sqrt(x) - 2*(b^2*p*x - a^2*p)*log(
b*sqrt(x) + a))/b^2
```

Sympy [**A**]
time = 0.75, size = 61, normalized size = 1.15

$$-\frac{bp\left(\frac{2a^2\left(\begin{cases}\frac{\sqrt{x}}{a} & \text{for}\ b = 0\\ \frac{\log\left(a+b\sqrt{x}\right)}{b} & \text{otherwise}\end{cases}\right)}{b^2} - \frac{2a\sqrt{x}}{b^2} + \frac{x}{b}\right)}{2} + x\log\left(c\left(a + b\sqrt{x}\right)^p\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b*x**(1/2))**p),x)
```

```
[Out] -b*p*(2*a**2*Piecewise((sqrt(x)/a, Eq(b, 0)), (log(a + b*sqrt(x))/b, True))
/b**2 - 2*a*sqrt(x)/b**2 + x/b)/2 + x*log(c*(a + b*sqrt(x))**p)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(45) = 90.
time = 4.46, size = 97, normalized size = 1.83

$$\frac{\frac{\left(2\left(b\sqrt{x}+a\right)^2\log\left(b\sqrt{x}+a\right) - 4\left(b\sqrt{x}+a\right)a\log\left(b\sqrt{x}+a\right) - \left(b\sqrt{x}+a\right)^2 + 4\left(b\sqrt{x}+a\right)a\right)p}{b} + \frac{2\left(\left(b\sqrt{x}+a\right)^2 - 2\left(b\sqrt{x}+a\right)a\right)\log\left(c\right)}{b}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p),x, algorithm="giac")
```

```
[Out] 1/2*((2*(b*sqrt(x) + a)^2*log(b*sqrt(x) + a) - 4*(b*sqrt(x) + a)*a*log(b*sq
rt(x) + a) - (b*sqrt(x) + a)^2 + 4*(b*sqrt(x) + a)*a)*p/b + 2*((b*sqrt(x) +
 a)^2 - 2*(b*sqrt(x) + a)*a)*log(c)/b)/b
```

Mupad [B]
time = 0.12, size = 47, normalized size = 0.89

$$x\,\ln\left(c\,{\left(a+b\,\sqrt{x}\right)}^p\right)-\frac{p\,\left(b^2\,x+2\,a^2\,\ln\left(a+b\,\sqrt{x}\right)-2\,a\,b\,\sqrt{x}\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^(1/2))^p),x)
```

```
[Out] x*log(c*(a + b*x^(1/2))^p) - (p*(b^2*x + 2*a^2*log(a + b*x^(1/2)) - 2*a*b*x
^(1/2)))/(2*b^2)
```

3.50 $\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}\,dx$

Optimal. Leaf size=46

$$2\log\left(c\left(a+b\sqrt{x}\right)^p\right)\log\left(-\frac{b\sqrt{x}}{a}\right)+2p\mathrm{Li}_2\left(1+\frac{b\sqrt{x}}{a}\right)$$

```
[Out] 2*ln(-b*x^(1/2)/a)*ln(c*(a+b*x^(1/2))^p)+2*p*polylog(2,1+b*x^(1/2)/a)
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2504, 2441, 2352}

$$2p\text{PolyLog}\left(2,\frac{b\sqrt{x}}{a}+1\right)+2\log\left(-\frac{b\sqrt{x}}{a}\right)\log\left(c\left(a+b\sqrt{x}\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*Sqrt[x])^p]/x,x]
```

```
[Out] 2*Log[c*(a + b*Sqrt[x])^p]*Log[-((b*Sqrt[x])/a)] + 2*p*PolyLog[2, 1 + (b*Sq
rt[x])/a]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{x}\,dx = 2\text{Subst}\left(\int \frac{\log \left(c(a+bx)^p\right)}{x}\,dx, x, \sqrt{x}\right)$$
$$= 2\log \left(c\left(a+b\sqrt{x}\right)^p\right)\log \left(-\frac{b\sqrt{x}}{a}\right) - (2bp)\text{Subst}\left(\int \frac{\log \left(-\frac{bx}{a}\right)}{a+bx}\,dx, x, \sqrt{x}\right)$$
$$= 2\log \left(c\left(a+b\sqrt{x}\right)^p\right)\log \left(-\frac{b\sqrt{x}}{a}\right) + 2p\text{Li}_2\left(1+\frac{b\sqrt{x}}{a}\right)$$

Mathematica [A]
time = 0.00, size = 47, normalized size = 1.02

$$2\log \left(c\left(a+b\sqrt{x}\right)^p\right)\log \left(-\frac{b\sqrt{x}}{a}\right) + 2p\text{Li}_2\left(\frac{a+b\sqrt{x}}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*Sqrt[x])^p]/x,x]
```

```
[Out] 2*Log[c*(a + b*Sqrt[x])^p]*Log[-((b*Sqrt[x])/a)] + 2*p*PolyLog[2, (a + b*Sq
rt[x])/a]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(c\left(a+b\sqrt{x}\right)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b*x^(1/2))^p)/x,x)
```

```
[Out] int(ln(c*(a+b*x^(1/2))^p)/x,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(39) = 78.
time = 0.28, size = 79, normalized size = 1.72

$$bp\left(\frac{\log \left(b\sqrt{x}+a\right)\log (x)}{b} - \frac{\log (x)\log \left(\frac{b\sqrt{x}}{a}+1\right)+2\,\text{Li}_2\left(-\frac{b\sqrt{x}}{a}\right)}{b}\right) - p\log \left(b\sqrt{x}+a\right)\log (x) + \log \left(\left(b\sqrt{x}+a\right)^p c\right)\log (x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x,x, algorithm="maxima")
```

[Out] b*p*(log(b*sqrt(x) + a)*log(x)/b - (log(x)*log(b*sqrt(x)/a + 1) + 2*dilog(-b*sqrt(x)/a))/b) - p*log(b*sqrt(x) + a)*log(x) + log((b*sqrt(x) + a)^p*c)*log(x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c*(a+b*x^(1/2))^p)/x,x, algorithm="fricas")

[Out] integral(log((b*sqrt(x) + a)^p*c)/x, x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(c\left(a + b\sqrt{x}\right)^{p}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(c*(a+b*x**(1/2))**p)/x,x)

[Out] Integral(log(c*(a + b*sqrt(x))**p)/x, x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c*(a+b*x^(1/2))^p)/x,x, algorithm="giac")

[Out] integrate(log((b*sqrt(x) + a)^p*c)/x, x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln \left(c\,{\left(a+b\,\sqrt{x}\right)}^p\right)}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(c*(a + b*x^(1/2))^p)/x,x)

[Out] int(log(c*(a + b*x^(1/2))^p)/x, x)

3.51 $\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^2}\,dx$

Optimal. Leaf size=63

$$-\frac{bp}{a\sqrt{x}}+\frac{b^2p\log\left(a+b\sqrt{x}\right)}{a^2}-\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}-\frac{b^2p\log(x)}{2a^2}$$

```
[Out] -1/2*b^2*p*ln(x)/a^2+b^2*p*ln(a+b*x^(1/2))/a^2-ln(c*(a+b*x^(1/2))^p)/x-b*p/
a/x^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2504, 2442, 46}

$$\frac{b^2p\log\left(a+b\sqrt{x}\right)}{a^2}-\frac{b^2p\log(x)}{2a^2}-\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}-\frac{bp}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*Sqrt[x])^p]/x^2,x]
```

```
[Out] -((b*p)/(a*Sqrt[x])) + (b^2*p*Log[a + b*Sqrt[x]])/a^2 - Log[c*(a + b*Sqrt[x
])^p]/x - (b^2*p*Log[x])/(2*a^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
```

```
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{x^2}\,dx &= 2\text{Subst}\left(\int \frac{\log \left(c(a+bx)^p\right)}{x^3}\,dx,x,\sqrt{x}\right)\\
&= -\frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{x}+(bp)\text{Subst}\left(\int \frac{1}{x^2(a+bx)}\,dx,x,\sqrt{x}\right)\\
&= -\frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{x}+(bp)\text{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)\,dx,x,\sqrt{x}\right)\\
&= -\frac{bp}{a\sqrt{x}}+\frac{b^2p\log \left(a+b\sqrt{x}\right)}{a^2}-\frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{x}-\frac{b^2p\log(x)}{2a^2}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 55, normalized size = 0.87

$$-\frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{x}-\frac{bp\left(\frac{2a}{\sqrt{x}}-2b\log \left(a+b\sqrt{x}\right)+b\log(x)\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*Sqrt[x])^p]/x^2,x]
```

```
[Out] -(Log[c*(a + b*Sqrt[x])^p]/x) - (b*p*((2*a)/Sqrt[x] - 2*b*Log[a + b*Sqrt[x]
] + b*Log[x]))/(2*a^2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(c\left(a+b\sqrt{x}\right)^p\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b*x^(1/2))^p)/x^2,x)
```

```
[Out] int(ln(c*(a+b*x^(1/2))^p)/x^2,x)
```

Maxima [A]
time = 0.28, size = 53, normalized size = 0.84

$$\frac{1}{2}\,bp\left(\frac{2\,b\log \left(b\sqrt{x}+a\right)}{a^2}-\frac{b\log \left(x\right)}{a^2}-\frac{2}{a\sqrt{x}}\right)-\frac{\log \left(\left(b\sqrt{x}+a\right)^p c\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*p*(2*b*log(b*sqrt(x) + a)/a^2 - b*log(x)/a^2 - 2/(a*sqrt(x))) - log((
b*sqrt(x) + a)^p*c)/x
```

Fricas [A]
time = 0.42, size = 55, normalized size = 0.87

$$-\frac{b^2px\log\left(\sqrt{x}\right)+abp\sqrt{x}+a^2\log\left(c\right)-\left(b^2px-a^2p\right)\log\left(b\sqrt{x}+a\right)}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^2,x, algorithm="fricas")
```

```
[Out] -(b^2*p*x*log(sqrt(x)) + a*b*p*sqrt(x) + a^2*log(c) - (b^2*p*x - a^2*p)*log
(b*sqrt(x) + a))/(a^2*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(56) = 112$.
time = 10.81, size = 326, normalized size = 5.17

$$\begin{cases} -\frac{2a^3\sqrt{x}\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{2a^3x^{\frac{3}{2}}+2a^2bx^2}-\frac{2a^2bpx}{2a^3x^{\frac{3}{2}}+2a^2bx^2}-\frac{2a^2bx\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{2a^3x^{\frac{3}{2}}+2a^2bx^2}-\frac{ab^2px^{\frac{3}{2}}\log\left(x\right)}{2a^3x^{\frac{3}{2}}+2a^2bx^2}-\frac{2ab^2px^{\frac{3}{2}}}{2a^3x^{\frac{3}{2}}+2a^2bx^2}+\frac{2ab^2x^{\frac{3}{2}}\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{2a^3x^{\frac{3}{2}}+2a^2bx^2}-\frac{b^3px^2\log\left(x\right)}{2a^3x^{\frac{3}{2}}+2a^2bx^2}+\frac{2b^3x^2\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{2a^3x^{\frac{3}{2}}+2a^2bx^2} & \text{for}\: a \neq 0 \\ -\frac{p}{2x}-\frac{\log\left(c\left(b\sqrt{x}\right)^p\right)}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b*x**(1/2))**p)/x**2,x)
```

```
[Out] Piecewise((-2*a**3*sqrt(x)*log(c*(a + b*sqrt(x))**p)/(2*a**3*x**(3/2) + 2*a
**2*b*x**2) - 2*a**2*b*p*x/(2*a**3*x**(3/2) + 2*a**2*b*x**2) - 2*a**2*b*x*l
og(c*(a + b*sqrt(x))**p)/(2*a**3*x**(3/2) + 2*a**2*b*x**2) - a*b**2*p*x**(3
/2)*log(x)/(2*a**3*x**(3/2) + 2*a**2*b*x**2) - 2*a*b**2*p*x**(3/2)/(2*a**3*
x**(3/2) + 2*a**2*b*x**2) + 2*a*b**2*x**(3/2)*log(c*(a + b*sqrt(x))**p)/(2*
a**3*x**(3/2) + 2*a**2*b*x**2) - b**3*p*x**2*log(x)/(2*a**3*x**(3/2) + 2*a*
*2*b*x**2) + 2*b**3*x**2*log(c*(a + b*sqrt(x))**p)/(2*a**3*x**(3/2) + 2*a**
2*b*x**2), Ne(a, 0)), (-p/(2*x) - log(c*(b*sqrt(x))**p)/x, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(55) = 110$.
time = 5.65, size = 132, normalized size = 2.10

$$-\frac{\frac{b^3p\log\left(b\sqrt{x}+a\right)}{\left(b\sqrt{x}+a\right)^2-2\left(b\sqrt{x}+a\right)a+a^2}-\frac{b^3p\log\left(b\sqrt{x}+a\right)}{a^2}+\frac{b^3p\log\left(b\sqrt{x}\right)}{a^2}+\frac{\left(b\sqrt{x}+a\right)b^3p-ab^3p+ab^3\log\left(c\right)}{\left(b\sqrt{x}+a\right)^2a-2\left(b\sqrt{x}+a\right)a^2+a^3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^2,x, algorithm="giac")
```

```
[Out] -(b^3*p*log(b*sqrt(x) + a)/((b*sqrt(x) + a)^2 - 2*(b*sqrt(x) + a)*a + a^2)
- b^3*p*log(b*sqrt(x) + a)/a^2 + b^3*p*log(b*sqrt(x))/a^2 + ((b*sqrt(x) + a
)*b^3*p - a*b^3*p + a*b^3*log(c))/((b*sqrt(x) + a)^2*a - 2*(b*sqrt(x) + a)*
a^2 + a^3))/b
```

Mupad [B]
time = 0.66, size = 49, normalized size = 0.78

$$\frac{2\,b^2\,p\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^2}-\frac{\ln\left(c\,{\left(a+b\,\sqrt{x}\right)}^p\right)}{x}-\frac{b\,p}{a\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^(1/2))^p)/x^2,x)
```

```
[Out] (2*b^2*p*atanh((2*b*x^(1/2))/a + 1))/a^2 - log(c*(a + b*x^(1/2))^p)/x - (b*
p)/(a*x^(1/2))
```

3.52 $\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^3} \, dx$

Optimal. Leaf size=100

$$-\frac{bp}{6ax^{3/2}}+\frac{b^2p}{4a^2x}-\frac{b^3p}{2a^3\sqrt{x}}+\frac{b^4p\log\left(a+b\sqrt{x}\right)}{2a^4}-\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{2x^2}-\frac{b^4p\log(x)}{4a^4}$$

```
[Out] -1/6*b*p/a/x^(3/2)+1/4*b^2*p/a^2/x-1/4*b^4*p*ln(x)/a^4+1/2*b^4*p*ln(a+b*x^(
1/2))/a^4-1/2*ln(c*(a+b*x^(1/2))^p)/x^2-1/2*b^3*p/a^3/x^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2504, 2442, 46}

$$\frac{b^4p\log\left(a+b\sqrt{x}\right)}{2a^4}-\frac{b^4p\log(x)}{4a^4}-\frac{b^3p}{2a^3\sqrt{x}}+\frac{b^2p}{4a^2x}-\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{2x^2}-\frac{bp}{6ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*Sqrt[x])^p]/x^3,x]
```

```
[Out] -1/6*(b*p)/(a*x^(3/2)) + (b^2*p)/(4*a^2*x) - (b^3*p)/(2*a^3*Sqrt[x]) + (b^4
*p*Log[a + b*Sqrt[x]])/(2*a^4) - Log[c*(a + b*Sqrt[x])^p]/(2*x^2) - (b^4*p*
Log[x])/(4*a^4)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
```

```
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^3}\,dx = 2\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x^5}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{2x^2} + \frac{1}{2}(bp)\text{Subst}\left(\int \frac{1}{x^4(a+bx)}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{2x^2} + \frac{1}{2}(bp)\text{Subst}\left(\int \left(\frac{1}{ax^4} - \frac{b}{a^2x^3} + \frac{b^2}{a^3x^2} - \frac{b^3}{a^4x} + \frac{b^4}{a^4(a+$$
$$= -\frac{bp}{6ax^{3/2}} + \frac{b^2p}{4a^2x} - \frac{b^3p}{2a^3\sqrt{x}} + \frac{b^4p\log\left(a+b\sqrt{x}\right)}{2a^4} - \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{2x^2} - \frac{b^4p$$

Mathematica [A]
time = 0.03, size = 90, normalized size = 0.90

$$\frac{abp\sqrt{x}\left(-2a^2+3ab\sqrt{x}-6b^2x\right)+6b^4px^2\log\left(a+b\sqrt{x}\right)-6a^4\log\left(c\left(a+b\sqrt{x}\right)^p\right)-3b^4px^2\log(x)}{12a^4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*Sqrt[x])^p]/x^3,x]
```

```
[Out] (a*b*p*Sqrt[x]*(-2*a^2 + 3*a*b*Sqrt[x] - 6*b^2*x) + 6*b^4*p*x^2*Log[a + b*S
qrt[x]] - 6*a^4*Log[c*(a + b*Sqrt[x])^p] - 3*b^4*p*x^2*Log[x])/(12*a^4*x^2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b*x^(1/2))^p)/x^3,x)
```

```
[Out] int(ln(c*(a+b*x^(1/2))^p)/x^3,x)
```

Maxima [A]
time = 0.35, size = 76, normalized size = 0.76

$$\frac{1}{12}\,bp\left(\frac{6\,b^3\log\left(b\sqrt{x}+a\right)}{a^4} - \frac{3\,b^3\log(x)}{a^4} - \frac{6\,b^2x - 3\,ab\sqrt{x} + 2\,a^2}{a^3x^{\frac{3}{2}}}\right) - \frac{\log\left(\left(b\sqrt{x}+a\right)^p c\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^3,x, algorithm="maxima")
```

```
[Out] 1/12*b*p*(6*b^3*log(b*sqrt(x) + a)/a^4 - 3*b^3*log(x)/a^4 - (6*b^2*x - 3*a*
b*sqrt(x) + 2*a^2)/(a^3*x^(3/2))) - 1/2*log((b*sqrt(x) + a)^p*c)/x^2
```

Fricas [A]
time = 0.40, size = 84, normalized size = 0.84

$$-\frac{6\,b^4px^2\log\left(\sqrt{x}\right)-3\,a^2b^2px+6\,a^4\log\left(c\right)-6\,\left(b^4px^2-a^4p\right)\log\left(b\sqrt{x}+a\right)+2\,\left(3\,ab^3px+a^3bp\right)\sqrt{x}}{12\,a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^3,x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^4*p*x^2*log(sqrt(x)) - 3*a^2*b^2*p*x + 6*a^4*log(c) - 6*(b^4*p*x
^2 - a^4*p)*log(b*sqrt(x) + a) + 2*(3*a*b^3*p*x + a^3*b*p)*sqrt(x))/(a^4*x^
2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(90) = 180$.
time = 64.49, size = 403, normalized size = 4.03

$$\begin{cases} -\frac{6a^5\sqrt{x}\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{12a^5x^{\frac{5}{2}}+12a^4bx^3}-\frac{2a^4bpx}{12a^5x^{\frac{5}{2}}+12a^4bx^3}-\frac{6a^4bx\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{12a^5x^{\frac{5}{2}}+12a^4bx^3}+\frac{a^3b^2px^{\frac{3}{2}}}{12a^5x^{\frac{5}{2}}+12a^4bx^3}-\frac{3a^2b^3px^2}{12a^5x^{\frac{5}{2}}+12a^4bx^3}-\frac{3ab^4px^{\frac{5}{2}}\log\left(x\right)}{12a^5x^{\frac{5}{2}}+12a^4bx^3}-\frac{6ab^4px^{\frac{5}{2}}}{12a^5x^{\frac{5}{2}}+12a^4bx^3}+\frac{6ab^4x^{\frac{5}{2}}\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{12a^5x^{\frac{5}{2}}+12a^4bx^3}-\frac{3b^5px^3\log\left(x\right)}{12a^5x^{\frac{5}{2}}+12a^4bx^3}+\frac{6b^5x^3\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{12a^5x^{\frac{5}{2}}+12a^4bx^3} & \text{for}\: a \neq 0 \\ -\frac{p}{8x^2}-\frac{\log\left(c\left(b\sqrt{x}\right)^p\right)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b*x**(1/2))**p)/x**3,x)
```

```
[Out] Piecewise((-6*a**5*sqrt(x)*log(c*(a + b*sqrt(x))**p)/(12*a**5*x**(5/2) + 12
*a**4*b*x**3) - 2*a**4*b*p*x/(12*a**5*x**(5/2) + 12*a**4*b*x**3) - 6*a**4*b
*x*log(c*(a + b*sqrt(x))**p)/(12*a**5*x**(5/2) + 12*a**4*b*x**3) + a**3*b**
2*p*x**(3/2)/(12*a**5*x**(5/2) + 12*a**4*b*x**3) - 3*a**2*b**3*p*x**2/(12*a
**5*x**(5/2) + 12*a**4*b*x**3) - 3*a*b**4*p*x**(5/2)*log(x)/(12*a**5*x**(5/
2) + 12*a**4*b*x**3) - 6*a*b**4*p*x**(5/2)/(12*a**5*x**(5/2) + 12*a**4*b*x*
*3) + 6*a*b**4*x**(5/2)*log(c*(a + b*sqrt(x))**p)/(12*a**5*x**(5/2) + 12*a*
*4*b*x**3) - 3*b**5*p*x**3*log(x)/(12*a**5*x**(5/2) + 12*a**4*b*x**3) + 6*b
**5*x**3*log(c*(a + b*sqrt(x))**p)/(12*a**5*x**(5/2) + 12*a**4*b*x**3), Ne(
a, 0)), (-p/(8*x**2) - log(c*(b*sqrt(x))**p)/(2*x**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(80) = 160$.
time = 3.99, size = 232, normalized size = 2.32

$$-\frac{\frac{6\,b^5p\log\left(b\sqrt{x}+a\right)}{\left(b\sqrt{x}+a\right)^4-4\,\left(b\sqrt{x}+a\right)^3a+6\,\left(b\sqrt{x}+a\right)^2a^2-4\,\left(b\sqrt{x}+a\right)a^3+a^4}-\frac{6\,b^5p\log\left(b\sqrt{x}+a\right)}{a^4}+\frac{6\,b^5p\log\left(b\sqrt{x}\right)}{a^4}+\frac{6\,\left(b\sqrt{x}+a\right)^3b^5p-21\,\left(b\sqrt{x}+a\right)^2ab^5p+26\,\left(b\sqrt{x}+a\right)a^2b^5p-11\,a^3b^5p+6\,a^3b^5\log(c)}{\left(b\sqrt{x}+a\right)^4a^3-4\,\left(b\sqrt{x}+a\right)^3a^4+6\,\left(b\sqrt{x}+a\right)^2a^5-4\,\left(b\sqrt{x}+a\right)a^6+a^7}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^3,x, algorithm="giac")
```

```
[Out] -1/12*(6*b^5*p*log(b*sqrt(x) + a)/((b*sqrt(x) + a)^4 - 4*(b*sqrt(x) + a)^3*
a + 6*(b*sqrt(x) + a)^2*a^2 - 4*(b*sqrt(x) + a)*a^3 + a^4) - 6*b^5*p*log(b*
sqrt(x) + a)/a^4 + 6*b^5*p*log(b*sqrt(x))/a^4 + (6*(b*sqrt(x) + a)^3*b^5*p
- 21*(b*sqrt(x) + a)^2*a*b^5*p + 26*(b*sqrt(x) + a)*a^2*b^5*p - 11*a^3*b^5*
p + 6*a^3*b^5*log(c))/((b*sqrt(x) + a)^4*a^3 - 4*(b*sqrt(x) + a)^3*a^4 + 6*
(b*sqrt(x) + a)^2*a^5 - 4*(b*sqrt(x) + a)*a^6 + a^7))/b
```

Mupad [B]
time = 0.46, size = 72, normalized size = 0.72

$$\frac{b^4\,p\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^4}-\frac{\ln\left(c\,{\left(a+b\,\sqrt{x}\right)}^p\right)}{2\,x^2}-\frac{\frac{b\,p}{3\,a}-\frac{b^2\,p\,\sqrt{x}}{2\,a^2}+\frac{b^3\,p\,x}{a^3}}{2\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^(1/2))^p)/x^3,x)
```

```
[Out] (b^4*p*atanh((2*b*x^(1/2))/a + 1))/a^4 - log(c*(a + b*x^(1/2))^p)/(2*x^2) -
 ((b*p)/(3*a) - (b^2*p*x^(1/2))/(2*a^2) + (b^3*p*x)/a^3)/(2*x^(3/2))
```

3.53 $\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^4}\,dx$

Optimal. Leaf size=130

$$-\frac{bp}{15ax^{5/2}}+\frac{b^2p}{12a^2x^2}-\frac{b^3p}{9a^3x^{3/2}}+\frac{b^4p}{6a^4x}-\frac{b^5p}{3a^5\sqrt{x}}+\frac{b^6p\log\left(a+b\sqrt{x}\right)}{3a^6}-\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{3x^3}-\frac{b^6p\log(x)}{6a^6}$$

```
[Out] -1/15*b*p/a/x^(5/2)+1/12*b^2*p/a^2/x^2-1/9*b^3*p/a^3/x^(3/2)+1/6*b^4*p/a^4/
x-1/6*b^6*p*ln(x)/a^6+1/3*b^6*p*ln(a+b*x^(1/2))/a^6-1/3*ln(c*(a+b*x^(1/2))^
p)/x^3-1/3*b^5*p/a^5/x^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2504, 2442, 46}

$$\frac{b^6p\log\left(a+b\sqrt{x}\right)}{3a^6}-\frac{b^6p\log(x)}{6a^6}-\frac{b^5p}{3a^5\sqrt{x}}+\frac{b^4p}{6a^4x}-\frac{b^3p}{9a^3x^{3/2}}+\frac{b^2p}{12a^2x^2}-\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{3x^3}-\frac{bp}{15ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*Sqrt[x])^p]/x^4,x]
```

```
[Out] -1/15*(b*p)/(a*x^(5/2)) + (b^2*p)/(12*a^2*x^2) - (b^3*p)/(9*a^3*x^(3/2)) +
(b^4*p)/(6*a^4*x) - (b^5*p)/(3*a^5*Sqrt[x]) + (b^6*p*Log[a + b*Sqrt[x]])/(3
*a^6) - Log[c*(a + b*Sqrt[x])^p]/(3*x^3) - (b^6*p*Log[x])/(6*a^6)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
```

```
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^4}\,dx &= 2\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x^7}\,dx, x, \sqrt{x}\right)\\ &= -\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{3x^3}+\frac{1}{3}(bp)\text{Subst}\left(\int \frac{1}{x^6(a+bx)}\,dx, x, \sqrt{x}\right)\\ &= -\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{3x^3}+\frac{1}{3}(bp)\text{Subst}\left(\int \left(\frac{1}{ax^6}-\frac{b}{a^2x^5}+\frac{b^2}{a^3x^4}-\frac{b^3}{a^4x^3}+\frac{b^4}{a^5x^2}\right.\right.\\ &= -\frac{bp}{15ax^{5/2}}+\frac{b^2p}{12a^2x^2}-\frac{b^3p}{9a^3x^{3/2}}+\frac{b^4p}{6a^4x}-\frac{b^5p}{3a^5\sqrt{x}}+\frac{b^6p\log\left(a+b\sqrt{x}\right)}{3a^6}-\frac{\log}{}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 114, normalized size = 0.88

$$\frac{abp\sqrt{x}\left(-12a^4+15a^3b\sqrt{x}-20a^2b^2x+30ab^3x^{3/2}-60b^4x^2\right)+60b^6px^3\log\left(a+b\sqrt{x}\right)-60a^6\log\left(c\left(a+b\sqrt{x}\right)^p\right)-30b^6px^3\log(x)}{180a^6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*Sqrt[x])^p]/x^4,x]
```

```
[Out] (a*b*p*Sqrt[x]*(-12*a^4 + 15*a^3*b*Sqrt[x] - 20*a^2*b^2*x + 30*a*b^3*x^(3/2
) - 60*b^4*x^2) + 60*b^6*p*x^3*Log[a + b*Sqrt[x]] - 60*a^6*Log[c*(a + b*Sqr
t[x])^p] - 30*b^6*p*x^3*Log[x])/(180*a^6*x^3)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b*x^(1/2))^p)/x^4,x)
```

```
[Out] int(ln(c*(a+b*x^(1/2))^p)/x^4,x)
```

Maxima [A]
time = 0.32, size = 98, normalized size = 0.75

$$\frac{1}{180}\,bp\left(\frac{60\,b^5\log\left(b\sqrt{x}+a\right)}{a^6}-\frac{30\,b^5\log\left(x\right)}{a^6}-\frac{60\,b^4x^2-30\,ab^3x^{\frac{3}{2}}+20\,a^2b^2x-15\,a^3b\sqrt{x}+12\,a^4}{a^5x^{\frac{5}{2}}}\right)-\frac{\log\left(\left(b\sqrt{x}+a\right)^pc\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^4,x, algorithm="maxima")
```

```
[Out] 1/180*b*p*(60*b^5*log(b*sqrt(x) + a)/a^6 - 30*b^5*log(x)/a^6 - (60*b^4*x^2
- 30*a*b^3*x^(3/2) + 20*a^2*b^2*x - 15*a^3*b*sqrt(x) + 12*a^4)/(a^5*x^(5/2)
)) - 1/3*log((b*sqrt(x) + a)^p*c)/x^3
```

Fricas [A]
time = 0.41, size = 109, normalized size = 0.84

$$-\frac{60\,b^{6}px^{3}\log\left(\sqrt{x}\right)-30\,a^{2}b^{4}px^{2}-15\,a^{4}b^{2}px+60\,a^{6}\log\left(c\right)-60\,\left(b^{6}px^{3}-a^{6}p\right)\log\left(b\sqrt{x}+a\right)+4\,\left(15\,ab^{5}px^{2}+5\,a^{3}b^{3}px+3\,a^{5}bp\right)\sqrt{x}}{180\,a^{6}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^4,x, algorithm="fricas")
```

```
[Out] -1/180*(60*b^6*p*x^3*log(sqrt(x)) - 30*a^2*b^4*p*x^2 - 15*a^4*b^2*p*x + 60*
a^6*log(c) - 60*(b^6*p*x^3 - a^6*p)*log(b*sqrt(x) + a) + 4*(15*a*b^5*p*x^2
+ 5*a^3*b^3*p*x + 3*a^5*b*p)*sqrt(x))/(a^6*x^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b*x**(1/2))**p)/x**4,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 324 vs. 2(104) = 208.
time = 4.52, size = 324, normalized size = 2.49

$$-\frac{\frac{60\,b^{7}p\log\left(b\sqrt{x}+a\right)}{\left(b\sqrt{x}+a\right)^{6}-6\,\left(b\sqrt{x}+a\right)^{5}a+15\,\left(b\sqrt{x}+a\right)^{4}a^{2}-20\,\left(b\sqrt{x}+a\right)^{3}a^{3}+15\,\left(b\sqrt{x}+a\right)^{2}a^{4}-6\,\left(b\sqrt{x}+a\right)a^{5}+a^{6}}-\frac{60\,b^{7}p\log\left(b\sqrt{x}+a\right)}{a^{6}}+\frac{60\,b^{7}p\log\left(b\sqrt{x}\right)}{a^{6}}+\frac{60\,\left(b\sqrt{x}+a\right)^{5}b^{7}p-330\,\left(b\sqrt{x}+a\right)^{4}ab^{7}p+740\,\left(b\sqrt{x}+a\right)^{3}a^{2}b^{7}p-855\,\left(b\sqrt{x}+a\right)^{2}a^{3}b^{7}p+522\,\left(b\sqrt{x}+a\right)a^{4}b^{7}p-137\,a^{5}b^{7}p+60\,a^{5}b^{7}\log\left(c\right)}{\left(b\sqrt{x}+a\right)^{6}a^{5}-6\,\left(b\sqrt{x}+a\right)^{5}a^{6}+15\,\left(b\sqrt{x}+a\right)^{4}a^{7}-20\,\left(b\sqrt{x}+a\right)^{3}a^{8}+15\,\left(b\sqrt{x}+a\right)^{2}a^{9}-6\,\left(b\sqrt{x}+a\right)a^{10}+a^{11}}}{180\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^4,x, algorithm="giac")
```

```
[Out] -1/180*(60*b^7*p*log(b*sqrt(x) + a)/((b*sqrt(x) + a)^6 - 6*(b*sqrt(x) + a)^
5*a + 15*(b*sqrt(x) + a)^4*a^2 - 20*(b*sqrt(x) + a)^3*a^3 + 15*(b*sqrt(x) +
 a)^2*a^4 - 6*(b*sqrt(x) + a)*a^5 + a^6) - 60*b^7*p*log(b*sqrt(x) + a)/a^6
+ 60*b^7*p*log(b*sqrt(x))/a^6 + (60*(b*sqrt(x) + a)^5*b^7*p - 330*(b*sqrt(x
) + a)^4*a*b^7*p + 740*(b*sqrt(x) + a)^3*a^2*b^7*p - 855*(b*sqrt(x) + a)^2*
a^3*b^7*p + 522*(b*sqrt(x) + a)*a^4*b^7*p - 137*a^5*b^7*p + 60*a^5*b^7*log(
c))/((b*sqrt(x) + a)^6*a^5 - 6*(b*sqrt(x) + a)^5*a^6 + 15*(b*sqrt(x) + a)^4
```

```
*a^7 - 20*(b*sqrt(x) + a)^3*a^8 + 15*(b*sqrt(x) + a)^2*a^9 - 6*(b*sqrt(x) +
 a)*a^10 + a^11))/b
```

Mupad [B]
time = 0.56, size = 97, normalized size = 0.75

$$\frac{2\,b^6\,p\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{3\,a^6}-\frac{\frac{b\,p}{5\,a}-\frac{b^2\,p\,\sqrt{x}}{4\,a^2}+\frac{b^5\,p\,x^2}{a^5}-\frac{b^4\,p\,x^{3/2}}{2\,a^4}+\frac{b^3\,p\,x}{3\,a^3}}{3\,x^{5/2}}-\frac{\ln\left(c\,{\left(a+b\,\sqrt{x}\right)}^p\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^(1/2))^p)/x^4,x)
```

```
[Out] (2*b^6*p*atanh((2*b*x^(1/2))/a + 1))/(3*a^6) - ((b*p)/(5*a) - (b^2*p*x^(1/2
))/(4*a^2) + (b^5*p*x^2)/a^5 - (b^4*p*x^(3/2))/(2*a^4) + (b^3*p*x)/(3*a^3))
/(3*x^(5/2)) - log(c*(a + b*x^(1/2))^p)/(3*x^3)
```

3.54 $\int \frac{\log\left(a+b\sqrt{x}\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=32

$$-2\sqrt{x}+\frac{2\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b}$$

```
[Out] -2*x^(1/2)+2*ln(a+b*x^(1/2))*(a+b*x^(1/2))/b
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2504, 2436, 2332}

$$\frac{2\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b}-2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[Log[a + b*Sqrt[x]]/Sqrt[x],x]
```

```
[Out] -2*Sqrt[x] + (2*(a + b*Sqrt[x])*Log[a + b*Sqrt[x]])/b
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log\left(a+b\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\text{Subst}\left(\int \log(a+bx)\,dx, x, \sqrt{x}\right)$$
$$= \frac{2\text{Subst}\left(\int \log(x)\,dx, x, a+b\sqrt{x}\right)}{b}$$
$$= -2\sqrt{x} + \frac{2\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.03

$$2\left(-\sqrt{x}+\frac{\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a + b*Sqrt[x]]/Sqrt[x],x]
[Out] 2*(-Sqrt[x] + ((a + b*Sqrt[x])*Log[a + b*Sqrt[x]])/b)
```

Maple [A]
time = 0.28, size = 32, normalized size = 1.00

method	result	size
derivativedivides	$\frac{2\left(a+b\sqrt{x}\right)\ln\left(a+b\sqrt{x}\right)-2b\sqrt{x}-2a}{b}$	32
default	$\frac{2\left(a+b\sqrt{x}\right)\ln\left(a+b\sqrt{x}\right)-2b\sqrt{x}-2a}{b}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a+b*x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/b*((a+b*x^(1/2))*ln(a+b*x^(1/2))-b*x^(1/2)-a)
```

Maxima [A]
time = 0.32, size = 31, normalized size = 0.97

$$\frac{2\left(\left(b\sqrt{x}+a\right)\log\left(b\sqrt{x}+a\right)-b\sqrt{x}-a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b*x^(1/2))/x^(1/2),x, algorithm="maxima")
[Out] 2*((b*sqrt(x) + a)*log(b*sqrt(x) + a) - b*sqrt(x) - a)/b
```

Fricas [A]
time = 0.38, size = 28, normalized size = 0.88

$$\frac{2\left(\left(b\sqrt{x}+a\right)\log\left(b\sqrt{x}+a\right)-b\sqrt{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b*x^(1/2))/x^(1/2),x, algorithm="fricas")
[Out] 2*((b*sqrt(x) + a)*log(b*sqrt(x) + a) - b*sqrt(x))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(27) = 54$.
time = 0.24, size = 133, normalized size = 4.16

$$\begin{cases} \frac{2a^2\log\left(a+b\sqrt{x}\right)}{ab+b^2\sqrt{x}}+\frac{2a^2}{ab+b^2\sqrt{x}}+\frac{4ab\sqrt{x}\log\left(a+b\sqrt{x}\right)}{ab+b^2\sqrt{x}}+\frac{2b^2x\log\left(a+b\sqrt{x}\right)}{ab+b^2\sqrt{x}}-\frac{2b^2x}{ab+b^2\sqrt{x}} & \text{for } b\neq 0 \\ 2\sqrt{x}\log\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a+b*x**(1/2))/x**(1/2),x)
[Out] Piecewise((2*a**2*log(a + b*sqrt(x))/(a*b + b**2*sqrt(x)) + 2*a**2/(a*b + b
**2*sqrt(x)) + 4*a*b*sqrt(x)*log(a + b*sqrt(x))/(a*b + b**2*sqrt(x)) + 2*b*
*2*x*log(a + b*sqrt(x))/(a*b + b**2*sqrt(x)) - 2*b**2*x/(a*b + b**2*sqrt(x)
), Ne(b, 0)), (2*sqrt(x)*log(a), True))
```

Giac [A]
time = 2.46, size = 31, normalized size = 0.97

$$\frac{2\left(\left(b\sqrt{x}+a\right)\log\left(b\sqrt{x}+a\right)-b\sqrt{x}-a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b*x^(1/2))/x^(1/2),x, algorithm="giac")
[Out] 2*((b*sqrt(x) + a)*log(b*sqrt(x) + a) - b*sqrt(x) - a)/b
```

Mupad [B]
time = 0.28, size = 33, normalized size = 1.03

$$2\sqrt{x}\ \ln\left(a+b\sqrt{x}\right)-2\sqrt{x}+\frac{2\,a\,\ln\left(a+b\sqrt{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a + b*x^(1/2))/x^(1/2),x)
[Out] 2*x^(1/2)*log(a + b*x^(1/2)) - 2*x^(1/2) + (2*a*log(a + b*x^(1/2)))/b
```

3.55 $\int (fx)^m \log \left(c(d+ex^3)^p\right) \, dx$

Optimal. Leaf size=81

$$-\frac{3ep(fx)^{4+m}\,{}_2F_1\left(1,\frac{4+m}{3};\frac{7+m}{3};-\frac{ex^3}{d}\right)}{df^4(1+m)(4+m)}+\frac{(fx)^{1+m}\log\left(c(d+ex^3)^p\right)}{f(1+m)}$$

```
[Out] -3*e*p*(f*x)^(4+m)*hypergeom([1, 4/3+1/3*m],[7/3+1/3*m],-e*x^3/d)/d/f^4/(1+
m)/(4+m)+(f*x)^(1+m)*ln(c*(e*x^3+d)^p)/f/(1+m)
```

Rubi [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2505, 16, 371}

$$\frac{(fx)^{m+1}\log\left(c(d+ex^3)^p\right)}{f(m+1)}-\frac{3ep(fx)^{m+4}\,{}_2F_1\left(1,\frac{m+4}{3};\frac{m+7}{3};-\frac{ex^3}{d}\right)}{df^4(m+1)(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Log[c*(d + e*x^3)^p],x]
```

```
[Out] (-3*e*p*(f*x)^(4 + m)*Hypergeometric2F1[1, (4 + m)/3, (7 + m)/3, -((e*x^3)/
d)])/(d*f^4*(1 + m)*(4 + m)) + ((f*x)^(1 + m)*Log[c*(d + e*x^3)^p])/(f*(1 +
 m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (fx)^m \log\left(c\left(d+ex^3\right)^p\right)\,dx = \frac{(fx)^{1+m}\log\left(c\left(d+ex^3\right)^p\right)}{f(1+m)} - \frac{(3ep)\int \frac{x^2(fx)^{1+m}}{d+ex^3}\,dx}{f(1+m)}$$
$$= \frac{(fx)^{1+m}\log\left(c\left(d+ex^3\right)^p\right)}{f(1+m)} - \frac{(3ep)\int \frac{(fx)^{3+m}}{d+ex^3}\,dx}{f^3(1+m)}$$
$$= -\frac{3ep(fx)^{4+m}\,{}_2F_1\left(1,\frac{4+m}{3};\frac{7+m}{3};-\frac{ex^3}{d}\right)}{df^4(1+m)(4+m)} + \frac{(fx)^{1+m}\log\left(c\left(d+ex^3\right)^p\right)}{f(1+m)}$$

Mathematica [**A**]
time = 0.02, size = 70, normalized size = 0.86

$$\frac{x(fx)^m\left(-3epx^3\,{}_2F_1\left(1,\frac{4+m}{3};\frac{7+m}{3};-\frac{ex^3}{d}\right)+d(4+m)\log\left(c\left(d+ex^3\right)^p\right)\right)}{d(1+m)(4+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Log[c*(d + e*x^3)^p],x]
```

```
[Out] (x*(f*x)^m*(-3*e*p*x^3*Hypergeometric2F1[1, (4 + m)/3, (7 + m)/3, -((e*x^3)
/d)] + d*(4 + m)*Log[c*(d + e*x^3)^p]))/(d*(1 + m)*(4 + m))
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int (fx)^m \ln\left(c\left(e\,x^3+d\right)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(e*x^3+d)^p),x)
```

```
[Out] int((f*x)^m*ln(c*(e*x^3+d)^p),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

```
[Out] f^m*x*x^m*log((x^3*e + d)^p)/(m + 1) + integrate((((f^m*(m + 1)*log(c) - 3*f
^m*p)*x^3*e + d*f^m*(m + 1)*log(c))*x^m/((m + 1)*x^3*e + d*(m + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^3+d)^p),x, algorithm="fricas")
[Out] integral((f*x)^m*log((x^3*e + d)^p*c), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(e*x**3+d)**p),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^3+d)^p),x, algorithm="giac")
[Out] integrate((f*x)^m*log((x^3*e + d)^p*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\left(e\,x^3+d\right)^p\right)\,\left(f\,x\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^3)^p)*(f*x)^m,x)
[Out] int(log(c*(d + e*x^3)^p)*(f*x)^m, x)
```

3.56 $\int (fx)^m \log \left(c(d+ex^2)^p\right) dx$

Optimal. Leaf size=81

$$-\frac{2ep(fx)^{3+m}\,{}_2F_1\left(1,\frac{3+m}{2};\frac{5+m}{2};-\frac{ex^2}{d}\right)}{df^3(1+m)(3+m)}+\frac{(fx)^{1+m}\log\left(c(d+ex^2)^p\right)}{f(1+m)}$$

```
[Out] -2*e*p*(f*x)^(3+m)*hypergeom([1, 3/2+1/2*m],[5/2+1/2*m],-e*x^2/d)/d/f^3/(1+
m)/(3+m)+(f*x)^(1+m)*ln(c*(e*x^2+d)^p)/f/(1+m)
```

Rubi [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2505, 16, 371}

$$\frac{(fx)^{m+1}\log\left(c(d+ex^2)^p\right)}{f(m+1)}-\frac{2ep(fx)^{m+3}\,{}_2F_1\left(1,\frac{m+3}{2};\frac{m+5}{2};-\frac{ex^2}{d}\right)}{df^3(m+1)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (-2*e*p*(f*x)^(3 + m)*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2)/
d)])/(d*f^3*(1 + m)*(3 + m)) + ((f*x)^(1 + m)*Log[c*(d + e*x^2)^p])/(f*(1 +
 m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (fx)^m \log \left(c\left(d+ex^2\right)^p\right) dx = \frac{(fx)^{1+m}\log\left(c\left(d+ex^2\right)^p\right)}{f(1+m)} - \frac{(2ep)\int \frac{x(fx)^{1+m}}{d+ex^2}dx}{f(1+m)}$$
$$= \frac{(fx)^{1+m}\log\left(c\left(d+ex^2\right)^p\right)}{f(1+m)} - \frac{(2ep)\int \frac{(fx)^{2+m}}{d+ex^2}dx}{f^2(1+m)}$$
$$= -\frac{2ep(fx)^{3+m}\,{}_2F_1\left(1,\frac{3+m}{2};\frac{5+m}{2};-\frac{ex^2}{d}\right)}{df^3(1+m)(3+m)} + \frac{(fx)^{1+m}\log\left(c\left(d+ex^2\right)^p\right)}{f(1+m)}$$

Mathematica [A]
time = 0.02, size = 70, normalized size = 0.86

$$\frac{x(fx)^m\left(-2epx^2\,{}_2F_1\left(1,\frac{3+m}{2};\frac{5+m}{2};-\frac{ex^2}{d}\right)+d(3+m)\log\left(c\left(d+ex^2\right)^p\right)\right)}{d(1+m)(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (x*(f*x)^m*(-2*e*p*x^2*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2)
/d)] + d*(3 + m)*Log[c*(d + e*x^2)^p]))/(d*(1 + m)*(3 + m))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (fx)^m \ln\left(c\left(e\,x^2+d\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(e*x^2+d)^p),x)
```

```
[Out] int((f*x)^m*ln(c*(e*x^2+d)^p),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] f^m*p*x*x^m*log(x^2*e + d)/(m + 1) + integrate(((f^m*(m + 1)*log(c) - 2*f^m
*p)*x^2*e + d*f^m*(m + 1)*log(c))*x^m/((m + 1)*x^2*e + d*(m + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] integral((f*x)^m*log((x^2*e + d)^p*c), x)
```

Sympy [A]
time = 31.78, size = 381, normalized size = 4.70

$$-2ep\left(\begin{cases} \frac{0^{m}\sqrt{-\frac{d}{e^{3}}}\log\left(-e\sqrt{-\frac{d}{e^{3}}}+x\right)}{2}-\frac{0^{m}\sqrt{-\frac{d}{e^{3}}}\log\left(e\sqrt{-\frac{d}{e^{3}}}+x\right)}{2}+\frac{0^{m}x}{e} & \text{for}\: \left(f=0\wedge m\neq -1\right)\vee f=0 \\ \frac{ff^{m}mx^{3}x^{m}\Phi\left(\frac{ex^{2}e^{i\pi}}{d},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4dfm\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)+4df\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}+\frac{3ff^{m}x^{3}x^{m}\Phi\left(\frac{ex^{2}e^{i\pi}}{d},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4dfm\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)+4df\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} & \text{for}\: m>-\infty\wedge m<\infty\wedge m\neq -1 \\ -\frac{\begin{cases} -\frac{\operatorname{Li}_{2}\left(\frac{ex^{2}e^{i\pi}}{d}\right)}{2} & \text{for}\: \frac{1}{\left|x\right|}<1\wedge\left|x\right|<1 \\ \log\left(d\right)\log\left(x\right)-\frac{\operatorname{Li}_{2}\left(\frac{ex^{2}e^{i\pi}}{d}\right)}{2} & \text{for}\: \left|x\right|<1 \\ -\log\left(d\right)\log\left(\frac{1}{x}\right)-\frac{\operatorname{Li}_{2}\left(\frac{ex^{2}e^{i\pi}}{d}\right)}{2} & \text{for}\: \frac{1}{\left|x\right|}<1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right)\log\left(d\right)+G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| x\right)\log\left(d\right)-\frac{\operatorname{Li}_{2}\left(\frac{ex^{2}e^{i\pi}}{d}\right)}{2} & \text{otherwise} \end{cases}}{2ef}+\frac{\log\left(fx\right)\log\left(d+ex^{2}\right)}{2ef} & \text{otherwise} \end{cases}\right)+\left(\begin{cases} 0^{m}x & \text{for}\: f=0 \\ \frac{\begin{cases} \frac{\left(fx\right)^{m+1}}{m+1} & \text{for}\: m\neq -1 \\ \log\left(fx\right) & \text{otherwise} \end{cases}}{f} & \text{otherwise} \end{cases}\right)\log\left(c\left(d+ex^{2}\right)^{p}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(e*x**2+d)**p),x)
```

```
[Out] -2*e*p*Piecewise((0**m*sqrt(-d/e**3)*log(-e*sqrt(-d/e**3) + x)/2 - 0**m*sqr
t(-d/e**3)*log(e*sqrt(-d/e**3) + x)/2 + 0**m*x/e, Eq(f, 0) | (Eq(f, 0) & Ne
(m, -1))), (f*f**m*m*x**3*x**m*lerchphi(e*x**2*exp_polar(I*pi)/d, 1, m/2 +
3/2)*gamma(m/2 + 3/2)/(4*d*f*m*gamma(m/2 + 5/2) + 4*d*f*gamma(m/2 + 5/2)) +
 3*f*f**m*x**3*x**m*lerchphi(e*x**2*exp_polar(I*pi)/d, 1, m/2 + 3/2)*gamma(
m/2 + 3/2)/(4*d*f*m*gamma(m/2 + 5/2) + 4*d*f*gamma(m/2 + 5/2)), (m > -oo) &
 (m < oo) & Ne(m, -1)), (-Piecewise((-polylog(2, e*x**2*exp_polar(I*pi)/d)/
2, (Abs(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x) - polylog(2, e*x**2*exp_p
olar(I*pi)/d)/2, Abs(x) < 1), (-log(d)*log(1/x) - polylog(2, e*x**2*exp_pol
ar(I*pi)/d)/2, 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(
d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) - polylog(2, e*x**2*exp_
polar(I*pi)/d)/2, True))/(2*e*f) + log(f*x)*log(d + e*x**2)/(2*e*f), True))
 + Piecewise((0**m*x, Eq(f, 0)), (Piecewise(((f*x)**(m + 1)/(m + 1), Ne(m,
-1)), (log(f*x), True))/f, True))*log(c*(d + e*x**2)**p)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log((x^2*e + d)^p*c), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\left(e\,x^2+d\right)^p\right)\,\left(f\,x\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)*(f*x)^m,x)
[Out] int(log(c*(d + e*x^2)^p)*(f*x)^m, x)
```

3.57 $\int (fx)^m \log \left(c(d+ex)^p\right) \, dx$

Optimal. Leaf size=69

$$-\frac{ep(fx)^{2+m} {}_2F_1\left(1,2+m;3+m;-\frac{ex}{d}\right)}{df^2(1+m)(2+m)}+\frac{(fx)^{1+m}\log \left(c(d+ex)^p\right)}{f(1+m)}$$

```
[Out] -e*p*(f*x)^(2+m)*hypergeom([1, 2+m],[3+m],-e*x/d)/d/f^2/(1+m)/(2+m)+(f*x)^(
1+m)*ln(c*(e*x+d)^p)/f/(1+m)
```

Rubi [A]
time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2442, 66}

$$\frac{(fx)^{m+1}\log \left(c(d+ex)^p\right)}{f(m+1)}-\frac{ep(fx)^{m+2} {}_2F_1\left(1,m+2;m+3;-\frac{ex}{d}\right)}{df^2(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Log[c*(d + e*x)^p],x]
```

```
[Out] -((e*p*(f*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, -((e*x)/d)])/(d*f^2
*(1 + m)*(2 + m))) + ((f*x)^(1 + m)*Log[c*(d + e*x)^p])/(f*(1 + m))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int (fx)^m \log \left(c(d+ex)^p\right) \, dx &= \frac{(fx)^{1+m}\log \left(c(d+ex)^p\right)}{f(1+m)}-\frac{(ep)\int \frac{(fx)^{1+m}}{d+ex} \, dx}{f(1+m)}\\ &= -\frac{ep(fx)^{2+m} {}_2F_1\left(1,2+m;3+m;-\frac{ex}{d}\right)}{df^2(1+m)(2+m)}+\frac{(fx)^{1+m}\log \left(c(d+ex)^p\right)}{f(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 56, normalized size = 0.81

$$\frac{x(fx)^m\left(-epx\,{}_2F_1\left(1,2+m;3+m;-\frac{ex}{d}\right)+d(2+m)\log\left(c(d+ex)^p\right)\right)}{d(1+m)(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Log[c*(d + e*x)^p],x]
[Out] (x*(f*x)^m*(-(e*p*x*Hypergeometric2F1[1, 2 + m, 3 + m, -((e*x)/d)]) + d*(2
+ m)*Log[c*(d + e*x)^p]))/(d*(1 + m)*(2 + m))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (fx)^m \ln\left(c(ex+d)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(e*x+d)^p),x)
[Out] int((f*x)^m*ln(c*(e*x+d)^p),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x+d)^p),x, algorithm="maxima")
[Out] f^m*x*x^m*log((x*e + d)^p)/(m + 1) + integrate((d*f^m*(m + 1)*log(c) + (f^m
*(m + 1)*log(c) - f^m*p)*x*e)*x^m/((m + 1)*x*e + d*(m + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x+d)^p),x, algorithm="fricas")
[Out] integral((f*x)^m*log((x*e + d)^p*c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m \log\left(c(d+ex)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(e*x+d)**p),x)
```

```
[Out] Integral((f*x)**m*log(c*(d + e*x)**p), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x+d)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log((x*e + d)^p*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\,{\left(d+e\,x\right)}^p\right)\,{\left(f\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x)^p)*(f*x)^m,x)
```

```
[Out] int(log(c*(d + e*x)^p)*(f*x)^m, x)
```

3.58 $\int (fx)^m \log \left(c\left(d+\frac{e}{x}\right)^p\right) \, dx$

Optimal. Leaf size=67

$$\frac{ep(fx)^m \,{}_2F_1\left(1,-m;1-m;-\frac{e}{dx}\right)}{dm(1+m)}+\frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f(1+m)}$$

```
[Out] e*p*(f*x)^m*hypergeom([1, -m],[1-m],-e/d/x)/d/m/(1+m)+(f*x)^(1+m)*ln(c*(d+e
/x)^p)/f/(1+m)
```

Rubi [A]

time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2505, 16, 346, 66}

$$\frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f(m+1)}+\frac{ep(fx)^m \,{}_2F_1\left(1,-m;1-m;-\frac{e}{dx}\right)}{dm(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Log[c*(d + e/x)^p],x]
```

```
[Out] (e*p*(f*x)^m*Hypergeometric2F1[1, -m, 1 - m, -(e/(d*x))])/(d*m*(1 + m)) + (
(f*x)^(1 + m)*Log[c*(d + e/x)^p])/(f*(1 + m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 346

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1), Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x
, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
```

```
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (fx)^m \log\left(c\left(d+\frac{e}{x}\right)^p\right)\,dx &= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f(1+m)}+\frac{(ep)\int\frac{(fx)^{1+m}}{\left(d+\frac{e}{x}\right)x^2}\,dx}{f(1+m)}\\ &= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f(1+m)}+\frac{(efp)\int\frac{(fx)^{-1+m}}{d+\frac{e}{x}}\,dx}{1+m}\\ &= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f(1+m)}-\frac{\left(ep\left(\frac{1}{x}\right)^m(fx)^m\right)\text{Subst}\left(\int\frac{x^{-1-m}}{d+ex}\,dx,x,\frac{1}{x}\right)}{1+m}\\ &= \frac{ep(fx)^m\,{}_2F_1\left(1,-m;1-m;-\frac{e}{dx}\right)}{dm(1+m)}+\frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 56, normalized size = 0.84

$$\frac{(fx)^m\left(ep\,{}_2F_1\left(1,-m;1-m;-\frac{e}{dx}\right)+dmx\log\left(c\left(d+\frac{e}{x}\right)^p\right)\right)}{dm(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Log[c*(d + e/x)^p],x]
```

```
[Out] ((f*x)^m*(e*p*Hypergeometric2F1[1, -m, 1 - m, -(e/(d*x))] + d*m*x*Log[c*(d
+ e/x)^p]))/(d*m*(1 + m))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (fx)^m \ln\left(c\left(d+\frac{e}{x}\right)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(d+e/x)^p),x)
```

```
[Out] int((f*x)^m*ln(c*(d+e/x)^p),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x)^p),x, algorithm="maxima")
```

```
[Out] (f^m*x*x^m*log((d*x + e)^p) - f^m*x*x^m*log(x^p))/(m + 1) + integrate((d*f^
m*(m + 1)*x*log(c) + (f^m*(m + 1)*log(c) + f^m*p)*e)*x^m/(d*(m + 1)*x + (m
+ 1)*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x)^p),x, algorithm="fricas")
```

```
[Out] integral((f*x)^m*log(c*((d*x + e)/x)^p), x)
```

Sympy [A]
time = 10.21, size = 218, normalized size = 3.25

$$ep\left(\begin{cases}\frac{0^{m}\log\left(dx+e\right)}{d} & \text{for}\ \left(f=0\wedge m\neq -1\right)\vee f=0\\ \frac{f^{m}mx^{m}\Phi\left(\frac{ee^{i\pi}}{dx},1,me^{i\pi}\right)\Gamma\left(-m\right)}{dm\Gamma\left(1-m\right)+d\Gamma\left(1-m\right)} & \text{for}\ m>-\infty\wedge m<\infty\wedge m\neq -1\\ \frac{\begin{cases}-\frac{1}{dx} & \text{for}\ e=0\\ \frac{\begin{cases}\operatorname{Li}_{2}\left(\frac{ee^{i\pi}}{dx}\right) & \text{for}\ \frac{1}{|x|}<1\wedge |x|<1\\ \log\left(d\right)\log\left(x\right)+\operatorname{Li}_{2}\left(\frac{ee^{i\pi}}{dx}\right) & \text{for}\ |x|<1\\ -\log\left(d\right)\log\left(\frac{1}{x}\right)+\operatorname{Li}_{2}\left(\frac{ee^{i\pi}}{dx}\right) & \text{for}\ \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle|x\right)\log\left(d\right)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle|x\right)\log\left(d\right)+\operatorname{Li}_{2}\left(\frac{ee^{i\pi}}{dx}\right) & \text{otherwise}\end{cases}}{e} & \text{otherwise}\end{cases}}{f}-\frac{\left(\begin{cases}\frac{1}{dx} & \text{for}\ e=0\\ \frac{\log\left(d+\frac{e}{x}\right)}{e} & \text{otherwise}\end{cases}\right)\log\left(fx\right)}{f} & \text{otherwise}\end{cases}\right)+\left(\begin{cases}0^{m}x & \text{for}\ f=0\\ \frac{\begin{cases}\frac{(fx)^{m+1}}{m+1} & \text{for}\ m\neq -1\\ \log\left(fx\right) & \text{otherwise}\end{cases}}{f} & \text{otherwise}\end{cases}\right)\log\left(c\left(d+\frac{e}{x}\right)^{p}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(d+e/x)**p),x)
```

```
[Out] e*p*Piecewise((0**m*log(d*x + e)/d, Eq(f, 0) | (Eq(f, 0) & Ne(m, -1))), (f*
*m*m*x**m*lerchphi(e*exp_polar(I*pi)/(d*x), 1, m*exp_polar(I*pi))*gamma(-m)
/(d*m*gamma(1 - m) + d*gamma(1 - m)), (m > -oo) & (m < oo) & Ne(m, -1)), (P
iecewise((-1/(d*x), Eq(e, 0)), (Piecewise((polylog(2, e*exp_polar(I*pi)/(d*
x)), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x) + polylog(2, e*exp_pola
r(I*pi)/(d*x)), Abs(x) < 1), (-log(d)*log(1/x) + polylog(2, e*exp_polar(I*p
i)/(d*x)), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(d) +
 meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) + polylog(2, e*exp_polar(I*p
i)/(d*x)), True))/e, True))/f - Piecewise((1/(d*x), Eq(e, 0)), (log(d + e/x
)/e, True))*log(f*x)/f, True)) + Piecewise((0**m*x, Eq(f, 0)), (Piecewise((
(f*x)**(m + 1)/(m + 1), Ne(m, -1)), (log(f*x), True))/f, True))*log(c*(d +
e/x)**p)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x)^p),x, algorithm="giac")
[Out] integrate((f*x)^m*log(c*(d + e/x)^p), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\left(d+\frac{e}{x}\right)^p\right)\,(f\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e/x)^p)*(f*x)^m,x)
[Out] int(log(c*(d + e/x)^p)*(f*x)^m, x)
```

3.59 $\int (fx)^m \log \left(c\left(d+\frac{e}{x^2}\right)^p\right) \, dx$

Optimal. Leaf size=82

$$-\frac{2efp(fx)^{-1+m} {}_2F_1\left(1,\frac{1-m}{2};\frac{3-m}{2};-\frac{e}{dx^2}\right)}{d\left(1-m^2\right)}+\frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f(1+m)}$$

```
[Out] -2*e*f*p*(f*x)^(-1+m)*hypergeom([1, 1/2-1/2*m],[3/2-1/2*m],-e/d/x^2)/d/(-m^
2+1)+(f*x)^(1+m)*ln(c*(d+e/x^2)^p)/f/(1+m)
```

Rubi [A]
time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2505, 16, 346, 371}

$$\frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f(m+1)}-\frac{2efp(fx)^{m-1} {}_2F_1\left(1,\frac{1-m}{2};\frac{3-m}{2};-\frac{e}{dx^2}\right)}{d\left(1-m^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Log[c*(d + e/x^2)^p],x]
```

```
[Out] (-2*e*f*p*(f*x)^(-1 + m)*Hypergeometric2F1[1, (1 - m)/2, (3 - m)/2, -(e/(d*
x^2))])/(d*(1 - m^2)) + ((f*x)^(1 + m)*Log[c*(d + e/x^2)^p])/(f*(1 + m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 346

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1), Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x
, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
```

```
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (fx)^m \log\left(c\left(d+\frac{e}{x^2}\right)^p\right)\,dx &= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f(1+m)}+\frac{(2ep)\int \frac{(fx)^{1+m}}{\left(d+\frac{e}{x^2}\right)x^3}\,dx}{f(1+m)}\\ &= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f(1+m)}+\frac{(2ef^2p)\int \frac{(fx)^{-2+m}}{d+\frac{e}{x^2}}\,dx}{1+m}\\ &= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f(1+m)}-\frac{\left(2efp\left(\frac{1}{x}\right)^{-1+m}(fx)^{-1+m}\right)\mathrm{Subst}\left(\int \frac{x^{-m}}{d+ex^2}\,dx,x\right.}{1+m}\\ &= -\frac{2efp(fx)^{-1+m}\,{}_2F_1\left(1,\frac{1-m}{2};\frac{3-m}{2};-\frac{e}{dx^2}\right)}{d\,(1-m^2)}+\frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 76, normalized size = 0.93

$$\frac{(fx)^m\left(2ep\,{}_2F_1\left(1,\frac{1}{2}-\frac{m}{2};\frac{3}{2}-\frac{m}{2};-\frac{e}{dx^2}\right)+d(-1+m)x^2\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)\right)}{d(-1+m)(1+m)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Log[c*(d + e/x^2)^p],x]
```

```
[Out] ((f*x)^m*(2*e*p*Hypergeometric2F1[1, 1/2 - m/2, 3/2 - m/2, -(e/(d*x^2))] +
d*(-1 + m)*x^2*Log[c*(d + e/x^2)^p]))/(d*(-1 + m)*(1 + m)*x)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int (fx)^m \ln\left(c\left(d+\frac{e}{x^2}\right)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(d+e/x^2)^p),x)
```

```
[Out] int((f*x)^m*ln(c*(d+e/x^2)^p),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x^2)^p),x, algorithm="maxima")
```

```
[Out] (f^m*p*x*x^m*log(d*x^2 + e) - 2*f^m*x*x^m*log(x^p))/(m + 1) + integrate((d*
f^m*(m + 1)*x^2*log(c) + (f^m*(m + 1)*log(c) + 2*f^m*p)*e)*x^m/(d*(m + 1)*x
^2 + (m + 1)*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x^2)^p),x, algorithm="fricas")
```

```
[Out] integral((f*x)^m*log(c*((d*x^2 + e)/x^2)^p), x)
```

Sympy [A]
time = 32.79, size = 369, normalized size = 4.50

$$2ep\left(\begin{cases} -\frac{0^{m}\sqrt{-\frac{1}{de}}\log\left(-e\sqrt{-\frac{1}{de}}+x\right)}{2}+\frac{0^{m}\sqrt{-\frac{1}{de}}\log\left(e\sqrt{-\frac{1}{de}}+x\right)}{2} & \text{for}\ (f=0\wedge m\neq -1)\vee f=0 \\ \frac{ff^{m}mx^{m}\Phi\left(\frac{ee^{i\pi}}{dx^{2}},1,\frac{1}{2}-\frac{m}{2}\right)\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}{4dfmx\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)+4dfx\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)}-\frac{ff^{m}x^{m}\Phi\left(\frac{ee^{i\pi}}{dx^{2}},1,\frac{1}{2}-\frac{m}{2}\right)\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}{4dfmx\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)+4dfx\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)} & \text{for}\ m>-\infty\wedge m<\infty\wedge m\neq -1 \\ \frac{\begin{cases} \frac{\operatorname{Li}_{2}\left(\frac{ee^{i\pi}}{dx^{2}}\right)}{2} & \text{for}\ \frac{1}{|x|}<1\wedge |x|<1 \\ \log(d)\log(x)+\frac{\operatorname{Li}_{2}\left(\frac{ee^{i\pi}}{dx^{2}}\right)}{2} & \text{for}\ |x|<1 \\ -\log(d)\log\left(\frac{1}{x}\right)+\frac{\operatorname{Li}_{2}\left(\frac{ee^{i\pi}}{dx^{2}}\right)}{2} & \text{for}\ \frac{1}{|x|}<1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right)\log(d)+G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| x\right)\log(d)+\frac{\operatorname{Li}_{2}\left(\frac{ee^{i\pi}}{dx^{2}}\right)}{2} & \text{otherwise} \end{cases}}{2ef}-\frac{\log(fx)\log\left(d+\frac{e}{x^{2}}\right)}{2ef} & \text{otherwise} \end{cases}\right)+\left(\begin{cases} 0^{m}x & \text{for}\ f=0 \\ \frac{\begin{cases} \frac{(fx)^{m+1}}{m+1} & \text{for}\ m\neq -1 \\ \log(fx) & \text{otherwise} \end{cases}}{f} & \text{otherwise} \end{cases}\right)\log\left(c\left(d+\frac{e}{x^{2}}\right)^{p}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(d+e/x**2)**p),x)
```

```
[Out] 2*e*p*Piecewise((-0**m*sqrt(-1/(d*e))*log(-e*sqrt(-1/(d*e)) + x)/2 + 0**m*s
qrt(-1/(d*e))*log(e*sqrt(-1/(d*e)) + x)/2, Eq(f, 0) | (Eq(f, 0) & Ne(m, -1)
)), (f*f**m*m*x**m*lerchphi(e*exp_polar(I*pi)/(d*x**2), 1, 1/2 - m/2)*gamma
(1/2 - m/2)/(4*d*f*m*x*gamma(3/2 - m/2) + 4*d*f*x*gamma(3/2 - m/2)) - f*f**
m*x**m*lerchphi(e*exp_polar(I*pi)/(d*x**2), 1, 1/2 - m/2)*gamma(1/2 - m/2)/
(4*d*f*m*x*gamma(3/2 - m/2) + 4*d*f*x*gamma(3/2 - m/2)), (m > -oo) & (m < o
o) & Ne(m, -1)), (Piecewise((polylog(2, e*exp_polar(I*pi)/(d*x**2))/2, (Abs
(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x) + polylog(2, e*exp_polar(I*pi)/(
d*x**2))/2, Abs(x) < 1), (-log(d)*log(1/x) + polylog(2, e*exp_polar(I*pi)/(
d*x**2))/2, 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(d)
+ meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) + polylog(2, e*exp_polar(I*
pi)/(d*x**2))/2, True))/(2*e*f) - log(f*x)*log(d + e/x**2)/(2*e*f), True))
+ Piecewise((0**m*x, Eq(f, 0)), (Piecewise(((f*x)**(m + 1)/(m + 1), Ne(m, -
1)), (log(f*x), True))/f, True))*log(c*(d + e/x**2)**p)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x^2)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log(c*(d + e/x^2)^p), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\left(d+\frac{e}{x^2}\right)^p\right)\,(f\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e/x^2)^p)*(f*x)^m,x)
```

```
[Out] int(log(c*(d + e/x^2)^p)*(f*x)^m, x)
```

3.60 $\int (fx)^m \log\left(c\left(d+\frac{e}{x^3}\right)^p\right)\,dx$

Optimal. Leaf size=85

$$-\frac{3ef^2p(fx)^{-2+m}\,{}_2F_1\left(1,\frac{2-m}{3};\frac{5-m}{3};-\frac{e}{dx^3}\right)}{d\,(2+m-m^2)}+\frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^3}\right)^p\right)}{f(1+m)}$$

```
[Out] -3*e*f^2*p*(f*x)^(-2+m)*hypergeom([1, 2/3-1/3*m],[5/3-1/3*m],-e/d/x^3)/d/(-
m^2+m+2)+(f*x)^(1+m)*ln(c*(d+e/x^3)^p)/f/(1+m)
```

Rubi [A]
time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2505, 16, 346, 371}

$$\frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{x^3}\right)^p\right)}{f(m+1)}-\frac{3ef^2p(fx)^{m-2}\,{}_2F_1\left(1,\frac{2-m}{3};\frac{5-m}{3};-\frac{e}{dx^3}\right)}{d\,(-m^2+m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Log[c*(d + e/x^3)^p],x]
```

```
[Out] (-3*e*f^2*p*(f*x)^(-2 + m)*Hypergeometric2F1[1, (2 - m)/3, (5 - m)/3, -(e/(
d*x^3))])/(d*(2 + m - m^2)) + ((f*x)^(1 + m)*Log[c*(d + e/x^3)^p])/(f*(1 +
m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 346

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1), Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x
, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
```

```
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int (fx)^m \log\left(c\left(d+\frac{e}{x^3}\right)^p\right)\,dx &= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^3}\right)^p\right)}{f(1+m)}+\frac{(3ep)\int \frac{(fx)^{1+m}}{\left(d+\frac{e}{x^3}\right)x^4}\,dx}{f(1+m)}\\
&= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^3}\right)^p\right)}{f(1+m)}+\frac{(3ef^3p)\int \frac{(fx)^{-3+m}}{d+\frac{e}{x^3}}\,dx}{1+m}\\
&= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^3}\right)^p\right)}{f(1+m)}-\frac{\left(3ef^2p\left(\frac{1}{x}\right)^{-2+m}(fx)^{-2+m}\right)\operatorname{Subst}\left(\int \frac{x^{1-m}}{d+ex^3}\,dx,\right.}{1+m}\\
&= -\frac{3ef^2p(fx)^{-2+m}\,{}_2F_1\left(1,\frac{2-m}{3};\frac{5-m}{3};-\frac{e}{dx^3}\right)}{d\left(2+m-m^2\right)}+\frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^3}\right)^p\right)}{f(1+m)}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 76, normalized size = 0.89

$$\frac{(fx)^m\left(3ep\,{}_2F_1\left(1,\frac{2}{3}-\frac{m}{3};\frac{5}{3}-\frac{m}{3};-\frac{e}{dx^3}\right)+d(-2+m)x^3\log\left(c\left(d+\frac{e}{x^3}\right)^p\right)\right)}{d(-2+m)(1+m)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Log[c*(d + e/x^3)^p],x]
[Out] ((f*x)^m*(3*e*p*Hypergeometric2F1[1, 2/3 - m/3, 5/3 - m/3, -(e/(d*x^3))] +
d*(-2 + m)*x^3*Log[c*(d + e/x^3)^p]))/(d*(-2 + m)*(1 + m)*x^2)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int (fx)^m \ln\left(c\left(d+\frac{e}{x^3}\right)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(d+e/x^3)^p),x)
[Out] int((f*x)^m*ln(c*(d+e/x^3)^p),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x^3)^p),x, algorithm="maxima")
```

```
[Out] (f^m*x*x^m*log((d*x^3 + e)^p) - 3*f^m*x*x^m*log(x^p))/(m + 1) + integrate((
d*f^m*(m + 1)*x^3*log(c) + (f^m*(m + 1)*log(c) + 3*f^m*p)*e)*x^m/(d*(m + 1)
*x^3 + (m + 1)*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x^3)^p),x, algorithm="fricas")
```

```
[Out] integral((f*x)^m*log(c*((d*x^3 + e)/x^3)^p), x)
```

Sympy [A]
time = 232.75, size = 382, normalized size = 4.49

$$3ep\left(\begin{cases} 0^{m}\operatorname{RootSum}{\left(27t^{3}de^{2} - 1, \left(t \mapsto t\log{\left(3te + x \right)}\right)\right)} & \text{for}\: f = 0 \wedge m \neq -1 \\ 0^{m}\operatorname{RootSum}{\left(27t^{3}de^{2} - 1, \left(t \mapsto t\log{\left(3te + x \right)}\right)\right)} & \text{for}\: f = 0 \\ \frac{ff^{m}mx^{m}\Phi\left(\frac{ee^{i\pi}}{dx^{3}}, 1, \frac{2}{3} - \frac{m}{3}\right)\Gamma\left(\frac{2}{3} - \frac{m}{3}\right)}{9dfmx^{2}\Gamma\left(\frac{5}{3} - \frac{m}{3}\right) + 9dfx^{2}\Gamma\left(\frac{5}{3} - \frac{m}{3}\right)} - \frac{2ff^{m}x^{m}\Phi\left(\frac{ee^{i\pi}}{dx^{3}}, 1, \frac{2}{3} - \frac{m}{3}\right)\Gamma\left(\frac{2}{3} - \frac{m}{3}\right)}{9dfmx^{2}\Gamma\left(\frac{5}{3} - \frac{m}{3}\right) + 9dfx^{2}\Gamma\left(\frac{5}{3} - \frac{m}{3}\right)} & \text{for}\: m > -\infty \wedge m < \infty \wedge m \neq -1 \\ \frac{\begin{cases} - \frac{1}{9dx^{3}} & \text{for}\: e = 0 \\ \frac{\begin{cases} \frac{\operatorname{Li}_{2}\left(\frac{ee^{i\pi}}{dx^{3}}\right)}{3} & \text{for}\: \frac{1}{\left|x\right|} < 1 \wedge \left|x\right| < 1 \\ \log{\left(d \right)}\log{\left(x \right)} + \frac{\operatorname{Li}_{2}\left(\frac{ee^{i\pi}}{dx^{3}}\right)}{3} & \text{for}\: \left|x\right| < 1 \\ - \log{\left(d \right)}\log{\left(\frac{1}{x} \right)} + \frac{\operatorname{Li}_{2}\left(\frac{ee^{i\pi}}{dx^{3}}\right)}{3} & \text{for}\: \frac{1}{\left|x\right|} < 1 \\ - {G_{2, 2}^{2, 0}\left(\begin{matrix} & 1, 1 \\0, 0 & \end{matrix} \middle| {x} \right)}\log{\left(d \right)} + {G_{2, 2}^{0, 2}\left(\begin{matrix} 1, 1 & \\ & 0, 0 \end{matrix} \middle| {x} \right)}\log{\left(d \right)} + \frac{\operatorname{Li}_{2}\left(\frac{ee^{i\pi}}{dx^{3}}\right)}{3} & \text{otherwise} \end{cases}}{3e} & \text{otherwise} \end{cases}}{f} - \frac{\left(\begin{cases} \frac{1}{3dx^{3}} & \text{for}\: e = 0 \\ \frac{\log{\left(d + \frac{e}{x^{3}} \right)}}{3e} & \text{otherwise} \end{cases}\right)\log{\left(fx \right)}}{f} & \text{otherwise} \end{cases}\right) + \left(\begin{cases} 0^{m}x & \text{for}\: f = 0 \\ \frac{\begin{cases} \frac{\left(fx\right)^{m + 1}}{m + 1} & \text{for}\: m \neq -1 \\ \log{\left(fx \right)} & \text{otherwise} \end{cases}}{f} & \text{otherwise} \end{cases}\right)\log{\left(c\left(d + \frac{e}{x^{3}}\right)^{p} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(d+e/x**3)**p),x)
```

```
[Out] 3*e*p*Piecewise((0**m*RootSum(27*_t**3*d*e**2 - 1, Lambda(_t, _t*log(3*_t*e
 + x))), Eq(f, 0) & Ne(m, -1)), (0**m*RootSum(27*_t**3*d*e**2 - 1, Lambda(_
t, _t*log(3*_t*e + x))), Eq(f, 0)), (f*f**m*m*x**m*lerchphi(e*exp_polar(I*p
i)/(d*x**3), 1, 2/3 - m/3)*gamma(2/3 - m/3)/(9*d*f*m*x**2*gamma(5/3 - m/3)
+ 9*d*f*x**2*gamma(5/3 - m/3)) - 2*f*f**m*x**m*lerchphi(e*exp_polar(I*pi)/(
d*x**3), 1, 2/3 - m/3)*gamma(2/3 - m/3)/(9*d*f*m*x**2*gamma(5/3 - m/3) + 9*
d*f*x**2*gamma(5/3 - m/3)), (m > -oo) & (m < oo) & Ne(m, -1)), (Piecewise((
-1/(9*d*x**3), Eq(e, 0)), (Piecewise((polylog(2, e*exp_polar(I*pi)/(d*x**3)
)/3, (Abs(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x) + polylog(2, e*exp_pola
r(I*pi)/(d*x**3))/3, Abs(x) < 1), (-log(d)*log(1/x) + polylog(2, e*exp_pola
r(I*pi)/(d*x**3))/3, 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x
)*log(d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) + polylog(2, e*exp
_polar(I*pi)/(d*x**3))/3, True))/(3*e), True))/f - Piecewise((1/(3*d*x**3),
 Eq(e, 0)), (log(d + e/x**3)/(3*e), True))*log(f*x)/f, True)) + Piecewise((
0**m*x, Eq(f, 0)), (Piecewise(((f*x)**(m + 1)/(m + 1), Ne(m, -1)), (log(f*x
), True))/f, True))*log(c*(d + e/x**3)**p)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x^3)^p),x, algorithm="giac")
[Out] integrate((f*x)^m*log(c*(d + e/x^3)^p), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\,{\left(d+\frac{e}{x^3}\right)}^p\right)\,{\left(f\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e/x^3)^p)*(f*x)^m,x)
[Out] int(log(c*(d + e/x^3)^p)*(f*x)^m, x)
```

3.61 $\int (fx)^m \log \left(c\left(d+e\sqrt{x}\right)^p\right) \, dx$

Optimal. Leaf size=83

$$-\frac{epx^{3/2}(fx)^m \, {}_2F_1\left(1,3+2m;2(2+m);-\frac{e\sqrt{x}}{d}\right)}{d\left(3+5m+2m^2\right)}+\frac{(fx)^{1+m}\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f(1+m)}$$

```
[Out] -e*p*x^(3/2)*(f*x)^m*hypergeom([1, 3+2*m],[4+2*m],-e*x^(1/2)/d)/d/(1+m)/(3+
2*m)+(f*x)^(1+m)*ln(c*(d+e*x^(1/2))^p)/f/(1+m)
```

Rubi [A]
time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2505, 20, 348, 66}

$$\frac{(fx)^{m+1}\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f(m+1)}-\frac{epx^{3/2}(fx)^m \, {}_2F_1\left(1,2m+3;2(m+2);-\frac{e\sqrt{x}}{d}\right)}{d\left(2m^2+5m+3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Log[c*(d + e*Sqrt[x])^p],x]
```

```
[Out] -((e*p*x^(3/2)*(f*x)^m*Hypergeometric2F1[1, 3 + 2*m, 2*(2 + m), -((e*Sqrt[x
])/d)])/(d*(3 + 5*m + 2*m^2))) + ((f*x)^(1 + m)*Log[c*(d + e*Sqrt[x])^p])/(
f*(1 + m))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (fx)^m \log\left(c\left(d+e\sqrt{x}\right)^p\right)\,dx = \frac{(fx)^{1+m}\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f(1+m)} - \frac{(ep)\int \frac{(fx)^{1+m}}{\left(d+e\sqrt{x}\right)\sqrt{x}}\,dx}{2f(1+m)}$$

$$= \frac{(fx)^{1+m}\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f(1+m)} - \frac{\left(epx^{-m}(fx)^m\right)\int \frac{x^{\frac{1}{2}+m}}{d+e\sqrt{x}}\,dx}{2(1+m)}$$

$$= \frac{(fx)^{1+m}\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f(1+m)} - \frac{\left(epx^{-m}(fx)^m\right)\text{Subst}\left(\int \frac{x^{-1+2\left(\frac{3}{2}+m\right)}}{d+ex}\,dx, x,\right.}{1+m}$$

$$= -\frac{epx^{3/2}(fx)^m\,{}_2F_1\left(1,3+2m;2(2+m);-\frac{e\sqrt{x}}{d}\right)}{d\left(3+5m+2m^2\right)} + \frac{(fx)^{1+m}\log\left(c\left(d+e\right.\right.}{f(1+m)}$$

Mathematica [A]
time = 0.03, size = 76, normalized size = 0.92

$$\frac{x(fx)^m\left(-ep\sqrt{x}\,{}_2F_1\left(1,3+2m;4+2m;-\frac{e\sqrt{x}}{d}\right)+d(3+2m)\log\left(c\left(d+e\sqrt{x}\right)^p\right)\right)}{d(1+m)(3+2m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Log[c*(d + e*Sqrt[x])^p],x]
```

```
[Out] (x*(f*x)^m*(-(e*p*Sqrt[x]*Hypergeometric2F1[1, 3 + 2*m, 4 + 2*m, -((e*Sqrt[
x])/d)]) + d*(3 + 2*m)*Log[c*(d + e*Sqrt[x])^p]))/(d*(1 + m)*(3 + 2*m))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int (fx)^m \ln\left(c\left(d+e\sqrt{x}\right)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(d+e*x^(1/2))^p),x)
```

```
[Out] int((f*x)^m*ln(c*(d+e*x^(1/2))^p),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e*x^(1/2))^p),x, algorithm="maxima")
```

```
[Out] f^m*p*integrate(1/2*x*e^(m*log(x) + 2)/(d^2*(m + 1) + d*(m + 1)*e^(1/2*log(
x) + 1)), x) + (d*f^m*(2*m + 3)*p*x*x^m*log(sqrt(x)*e + d) + d*f^m*(2*m + 3
)*x*x^m*log(c) - f^m*p*x^(3/2)*e^(m*log(x) + 1))/((2*m^2 + 5*m + 3)*d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e*x^(1/2))^p),x, algorithm="fricas")
```

```
[Out] integral((f*x)^m*log((sqrt(x)*e + d)^p*c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m \log \left(c\left(d+e\sqrt{x}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(d+e*x**(1/2))**p),x)
```

```
[Out] Integral((f*x)**m*log(c*(d + e*sqrt(x))**p), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e*x^(1/2))^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log((sqrt(x)*e + d)^p*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln \left(c \left(d+e \sqrt{x}\right)^p\right) (f x)^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^(1/2))^p)*(f*x)^m,x)
```

```
[Out] int(log(c*(d + e*x^(1/2))^p)*(f*x)^m, x)
```

3.62 $\int (fx)^m \log \left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right) dx$

Optimal. Leaf size=70

$$\frac{px(fx)^m\,{}_2F_1\left(1,2(1+m);3+2m;-\frac{d\sqrt{x}}{e}\right)}{2(1+m)^2}+\frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f(1+m)}$$

```
[Out] 1/2*p*x*(f*x)^m*hypergeom([1, 2+2*m],[3+2*m],-d*x^(1/2)/e)/(1+m)^2+(f*x)^(1
+m)*ln(c*(d+e/x^(1/2))^p)/f/(1+m)
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2505, 20, 269, 348, 66}

$$\frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f(m+1)}+\frac{px(fx)^m\,{}_2F_1\left(1,2(m+1);2m+3;-\frac{d\sqrt{x}}{e}\right)}{2(m+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Log[c*(d + e/Sqrt[x])^p],x]
```

```
[Out] (p*x*(f*x)^m*Hypergeometric2F1[1, 2*(1 + m), 3 + 2*m, -((d*Sqrt[x])/e)])/(2
*(1 + m)^2) + ((f*x)^(1 + m)*Log[c*(d + e/Sqrt[x])^p])/(f*(1 + m))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (fx)^m \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right) dx = \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f(1+m)} + \frac{(ep)\int \frac{(fx)^{1+m}}{\left(d+\frac{e}{\sqrt{x}}\right)x^{3/2}}\,dx}{2f(1+m)}$$

$$= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f(1+m)} + \frac{(epx^{-m}(fx)^m)\int \frac{x^{-\frac{1}{2}+m}}{d+\frac{e}{\sqrt{x}}}\,dx}{2(1+m)}$$

$$= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f(1+m)} + \frac{(epx^{-m}(fx)^m)\int \frac{x^m}{e+d\sqrt{x}}\,dx}{2(1+m)}$$

$$= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f(1+m)} + \frac{(epx^{-m}(fx)^m)\,\mathrm{Subst}\left(\int \frac{x^{-1+2(1+m)}}{e+dx}\,dx,\right.}{1+m}$$

$$= \frac{px(fx)^m\,{}_2F_1\left(1,2(1+m);3+2m;-\frac{d\sqrt{x}}{e}\right)}{2(1+m)^2} + \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{\sqrt{}}\right.\right.}{f(1+m)}$$

Mathematica [A]
time = 0.03, size = 77, normalized size = 1.10

$$\frac{\sqrt{x}\,(fx)^m\left(ep\,{}_2F_1\left(1,-1-2m;-2m;-\frac{e}{d\sqrt{x}}\right)+d(1+2m)\sqrt{x}\,\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\right)}{d(1+m)(1+2m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Log[c*(d + e/Sqrt[x])^p],x]
```

```
[Out] (Sqrt[x]*(f*x)^m*(e*p*Hypergeometric2F1[1, -1 - 2*m, -2*m, -(e/(d*Sqrt[x]))
] + d*(1 + 2*m)*Sqrt[x]*Log[c*(d + e/Sqrt[x])^p]))/(d*(1 + m)*(1 + 2*m))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int (fx)^m \ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(d+e/x^(1/2))^p),x)
[Out] int((f*x)^m*ln(c*(d+e/x^(1/2))^p),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x^(1/2))^p),x, algorithm="maxima")
[Out] d^2*f^m*p*integrate(1/2*x*x^m/(d*(m + 1)*e^(1/2*log(x) + 1) + (m + 1)*e^2),
 x) + 1/2*(2*(2*m^2 + 5*m + 3)*f^m*p*x*e^(m*log(x) + 1)*log(d*sqrt(x) + e)
- 2*(m*p + p)*d*f^m*x^(3/2)*x^m - 2*(2*m^2 + 5*m + 3)*f^m*x*e^(m*log(x) + 1
)*log(x^(1/2*p)) + (2*(2*m^2 + 5*m + 3)*f^m*log(c) + (2*m*p + 3*p)*f^m)*x*e
^(m*log(x) + 1))*e^(-1)/(2*m^3 + 7*m^2 + 8*m + 3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x^(1/2))^p),x, algorithm="fricas")
[Out] integral((f*x)^m*log(c*((d*x + sqrt(x)*e)/x)^p), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(d+e/x**(1/2))**p),x)
```

```
[Out] Integral((f*x)**m*log(c*(d + e/sqrt(x))**p), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e/x^(1/2))^p),x, algorithm="giac")
[Out] integrate((f*x)^m*log(c*(d + e/sqrt(x))^p), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)(f\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e/x^(1/2))^p)*(f*x)^m,x)
[Out] int(log(c*(d + e/x^(1/2))^p)*(f*x)^m, x)
```

3.63 $\int (fx)^m \log\left(c(d+ex^n)^p\right)\,dx$

Optimal. Leaf size=87

$$-\frac{enpx^{1+n}(fx)^m\,{}_2F_1\left(1,\frac{1+m+n}{n};\frac{1+m+2n}{n};-\frac{ex^n}{d}\right)}{d(1+m)(1+m+n)}+\frac{(fx)^{1+m}\log\left(c(d+ex^n)^p\right)}{f(1+m)}$$

```
[Out] -e*n*p*x^(1+n)*(f*x)^m*hypergeom([1, (1+m+n)/n],[(1+m+2*n)/n],-e*x^n/d)/d/(
1+m)/(1+m+n)+(f*x)^(1+m)*ln(c*(d+e*x^n)^p)/f/(1+m)
```

Rubi [A]
time = 0.03, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2505, 20, 371}

$$\frac{(fx)^{m+1}\log\left(c(d+ex^n)^p\right)}{f(m+1)}-\frac{enpx^{n+1}(fx)^m\,{}_2F_1\left(1,\frac{m+n+1}{n};\frac{m+2n+1}{n};-\frac{ex^n}{d}\right)}{d(m+1)(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -((e*n*p*x^(1 + n)*(f*x)^m*Hypergeometric2F1[1, (1 + m + n)/n, (1 + m + 2*n
)/n, -((e*x^n)/d)])/(d*(1 + m)*(1 + m + n))) + ((f*x)^(1 + m)*Log[c*(d + e*
x^n)^p])/(f*(1 + m))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (fx)^m \log\left(c(d+ex^n)^p\right)\,dx = \frac{(fx)^{1+m}\log\left(c(d+ex^n)^p\right)}{f(1+m)} - \frac{(enp)\int \frac{x^{-1+n}(fx)^{1+m}}{d+ex^n}\,dx}{f(1+m)}$$
$$= \frac{(fx)^{1+m}\log\left(c(d+ex^n)^p\right)}{f(1+m)} - \frac{\left(enpx^{-m}(fx)^m\right)\int \frac{x^{m+n}}{d+ex^n}\,dx}{1+m}$$
$$= -\frac{enpx^{1+n}(fx)^m\,{}_2F_1\left(1,\frac{1+m+n}{n};\frac{1+m+2n}{n};-\frac{ex^n}{d}\right)}{d(1+m)(1+m+n)} + \frac{(fx)^{1+m}\log\left(c(d+ex^n)^p\right)}{f(1+m)}$$

Mathematica [A]
time = 0.03, size = 77, normalized size = 0.89

$$\frac{x(fx)^m\left(-enpx^n\,{}_2F_1\left(1,\frac{1+m+n}{n};\frac{1+m+2n}{n};-\frac{ex^n}{d}\right)+d(1+m+n)\log\left(c(d+ex^n)^p\right)\right)}{d(1+m)(1+m+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Log[c*(d + e*x^n)^p],x]
```

```
[Out] (x*(f*x)^m*(-(e*n*p*x^n*Hypergeometric2F1[1, (1 + m + n)/n, (1 + m + 2*n)/n
, -((e*x^n)/d)]) + d*(1 + m + n)*Log[c*(d + e*x^n)^p]))/(d*(1 + m)*(1 + m +
 n))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (fx)^m \ln\left(c(d+e\,x^n)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(d+e*x^n)^p),x)
```

```
[Out] int((f*x)^m*ln(c*(d+e*x^n)^p),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] d*f^m*n*p*integrate(x^m/(d*(m + 1) + (m + 1)*e^(n*log(x) + 1)), x) + (f^m*(
m + 1)*x*x^m*log((d + e^(n*log(x) + 1))^p) - (f^m*n*p - f^m*(m + 1)*log(c))
*x*x^m)/(m^2 + 2*m + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e*x^n)^p),x, algorithm="fricas")
[Out] integral((f*x)^m*log((x^n*e + d)^p*c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m \log \left(c(d+ex^n)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(d+e*x**n)**p),x)
[Out] Integral((f*x)**m*log(c*(d + e*x**n)**p), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(d+e*x^n)^p),x, algorithm="giac")
[Out] integrate((f*x)^m*log((x^n*e + d)^p*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln \left(c\left(d+e\,x^n\right)^p\right)\,{\left(f\,x\right)}^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)*(f*x)^m,x)
[Out] int(log(c*(d + e*x^n)^p)*(f*x)^m, x)
```

3.64 $\int (fx)^{-1+3n} \log \left(c(d+ex^n)^p\right) \, dx$

Optimal. Leaf size=141

$$-\frac{p(fx)^{3n}}{9fn}-\frac{d^2px^{-2n}(fx)^{3n}}{3e^2fn}+\frac{dpx^{-n}(fx)^{3n}}{6efn}+\frac{d^3px^{-3n}(fx)^{3n}\log\left(d+ex^n\right)}{3e^3fn}+\frac{(fx)^{3n}\log\left(c(d+ex^n)^p\right)}{3fn}$$

```
[Out] -1/9*p*(f*x)^(3*n)/f/n-1/3*d^2*p*(f*x)^(3*n)/e^2/f/n/(x^(2*n))+1/6*d*p*(f*x
)^(3*n)/e/f/n/(x^n)+1/3*d^3*p*(f*x)^(3*n)*ln(d+e*x^n)/e^3/f/n/(x^(3*n))+1/3
*(f*x)^(3*n)*ln(c*(d+e*x^n)^p)/f/n
```

Rubi [A]
time = 0.05, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2505, 20, 272, 45}

$$\frac{(fx)^{3n}\log\left(c(d+ex^n)^p\right)}{3fn}+\frac{d^3px^{-3n}(fx)^{3n}\log\left(d+ex^n\right)}{3e^3fn}-\frac{d^2px^{-2n}(fx)^{3n}}{3e^2fn}+\frac{dpx^{-n}(fx)^{3n}}{6efn}-\frac{p(fx)^{3n}}{9fn}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(-1 + 3*n)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -1/9*(p*(f*x)^(3*n))/(f*n) - (d^2*p*(f*x)^(3*n))/(3*e^2*f*n*x^(2*n)) + (d*p
*(f*x)^(3*n))/(6*e*f*n*x^n) + (d^3*p*(f*x)^(3*n)*Log[d + e*x^n])/(3*e^3*f*n
*x^(3*n)) + ((f*x)^(3*n)*Log[c*(d + e*x^n)^p])/(3*f*n)
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int (fx)^{-1+3n}\log\left(c(d+ex^n)^p\right)\,dx &= \frac{(fx)^{3n}\log\left(c(d+ex^n)^p\right)}{3fn} - \frac{(ep)\int \frac{x^{-1+n}(fx)^{3n}}{d+ex^n}\,dx}{3f} \\
&= \frac{(fx)^{3n}\log\left(c(d+ex^n)^p\right)}{3fn} - \frac{\left(epx^{-3n}(fx)^{3n}\right)\int \frac{x^{-1+4n}}{d+ex^n}\,dx}{3f} \\
&= \frac{(fx)^{3n}\log\left(c(d+ex^n)^p\right)}{3fn} - \frac{\left(epx^{-3n}(fx)^{3n}\right)\mathrm{Subst}\left(\int \frac{x^3}{d+ex}\,dx, x, x^n\right)}{3fn} \\
&= \frac{(fx)^{3n}\log\left(c(d+ex^n)^p\right)}{3fn} - \frac{\left(epx^{-3n}(fx)^{3n}\right)\mathrm{Subst}\left(\int \left(\frac{d^2}{e^3} - \frac{dx}{e^2} + \frac{x^2}{e} - \frac{}{e^3}\right.\right.}{3fn} \\
&= -\frac{p(fx)^{3n}}{9fn} - \frac{d^2px^{-2n}(fx)^{3n}}{3e^2fn} + \frac{dpx^{-n}(fx)^{3n}}{6efn} + \frac{d^3px^{-3n}(fx)^{3n}\log(d+}{3e^3fn}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 92, normalized size = 0.65

$$
\frac{x^{-3n}(fx)^{3n}\left(-epx^n(6d^2-3dex^n+2e^2x^{2n})+6d^3p\log(d+ex^n)+6e^3x^{3n}\log\left(c(d+ex^n)^p\right)\right)}{18e^3fn}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^(-1 + 3*n)*Log[c*(d + e*x^n)^p],x]
[Out] ((f*x)^(3*n)*(-(e*p*x^n*(6*d^2 - 3*d*e*x^n + 2*e^2*x^(2*n))) + 6*d^3*p*Log[
d + e*x^n] + 6*e^3*x^(3*n)*Log[c*(d + e*x^n)^p]))/(18*e^3*f*n*x^(3*n))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$
\int (fx)^{-1+3n}\ln\left(c(d+e\,x^n)^p\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(-1+3*n)*ln(c*(d+e*x^n)^p),x)
[Out] int((f*x)^(-1+3*n)*ln(c*(d+e*x^n)^p),x)
```

Maxima [A]
time = 0.28, size = 115, normalized size = 0.82

$$\frac{ep\left(\frac{6\,d^3f^{3\,n}\log\left(\frac{ex^n+d}{e}\right)}{e^4n}-\frac{2\,e^2f^{3\,n}x^{3\,n}-3\,def^{3\,n}x^{2\,n}+6\,d^2f^{3\,n}x^n}{e^3n}\right)}{18\,f}+\frac{(fx)^{3\,n}\log\left((ex^n+d)^pc\right)}{3\,fn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+3*n)*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] 1/18*e*p*(6*d^3*f^(3*n)*log((e*x^n + d)/e)/(e^4*n) - (2*e^2*f^(3*n)*x^(3*n)
 - 3*d*e*f^(3*n)*x^(2*n) + 6*d^2*f^(3*n)*x^n)/(e^3*n))/f + 1/3*(f*x)^(3*n)*
log((e*x^n + d)^p*c)/(f*n)
```

Fricas [A]
time = 0.37, size = 109, normalized size = 0.77

$$-\frac{\left(6\,d^2f^{3\,n-1}px^ne-3\,df^{3\,n-1}px^{2\,n}e^2+2\,(pe^3-3\,e^3\log\left(c\right))f^{3\,n-1}x^{3\,n}-6\,(d^3f^{3\,n-1}p+f^{3\,n-1}px^{3\,n}e^3)\log\left(x^ne+d\right)\right)e^{(-3)}}{18\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+3*n)*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] -1/18*(6*d^2*f^(3*n - 1)*p*x^n*e - 3*d*f^(3*n - 1)*p*x^(2*n)*e^2 + 2*(p*e^3
 - 3*e^3*log(c))*f^(3*n - 1)*x^(3*n) - 6*(d^3*f^(3*n - 1)*p + f^(3*n - 1)*p
*x^(3*n)*e^3)*log(x^n*e + d))*e^(-3)/n
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^{3n-1}\log\left(c(d+ex^n)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(-1+3*n)*ln(c*(d+e*x**n)**p),x)
```

```
[Out] Integral((f*x)**(3*n - 1)*log(c*(d + e*x**n)**p), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+3*n)*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^(3*n - 1)*log((x^n*e + d)^p*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln \left(c\,{\left(d+e\,x^n\right)}^p\right)\,{\left(f\,x\right)}^{3\,n-1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)*(f*x)^(3*n - 1),x)
[Out] int(log(c*(d + e*x^n)^p)*(f*x)^(3*n - 1), x)
```

3.65 $\int (fx)^{-1+2n} \log \left(c(d+ex^n)^p\right) dx$

Optimal. Leaf size=112

$$-\frac{p(fx)^{2n}}{4fn}+\frac{dpx^{-n}(fx)^{2n}}{2efn}-\frac{d^2px^{-2n}(fx)^{2n}\log\left(d+ex^n\right)}{2e^2fn}+\frac{(fx)^{2n}\log\left(c(d+ex^n)^p\right)}{2fn}$$

```
[Out] -1/4*p*(f*x)^(2*n)/f/n+1/2*d*p*(f*x)^(2*n)/e/f/n/(x^n)-1/2*d^2*p*(f*x)^(2*n
)*ln(d+e*x^n)/e^2/f/n/(x^(2*n))+1/2*(f*x)^(2*n)*ln(c*(d+e*x^n)^p)/f/n
```

Rubi [A]
time = 0.04, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2505, 20, 272, 45}

$$\frac{(fx)^{2n}\log\left(c(d+ex^n)^p\right)}{2fn}-\frac{d^2px^{-2n}(fx)^{2n}\log\left(d+ex^n\right)}{2e^2fn}+\frac{dpx^{-n}(fx)^{2n}}{2efn}-\frac{p(fx)^{2n}}{4fn}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(-1 + 2*n)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -1/4*(p*(f*x)^(2*n))/(f*n) + (d*p*(f*x)^(2*n))/(2*e*f*n*x^n) - (d^2*p*(f*x)
^(2*n)*Log[d + e*x^n])/(2*e^2*f*n*x^(2*n)) + ((f*x)^(2*n)*Log[c*(d + e*x^n)
^p])/(2*f*n)
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (fx)^{-1+2n}\log\left(c(d+ex^n)^p\right)\,dx=\frac{(fx)^{2n}\log\left(c(d+ex^n)^p\right)}{2fn}-\frac{(ep)\int\frac{x^{-1+n}(fx)^{2n}}{d+ex^n}\,dx}{2f}$$
$$=\frac{(fx)^{2n}\log\left(c(d+ex^n)^p\right)}{2fn}-\frac{\left(epx^{-2n}(fx)^{2n}\right)\int\frac{x^{-1+3n}}{d+ex^n}\,dx}{2f}$$
$$=\frac{(fx)^{2n}\log\left(c(d+ex^n)^p\right)}{2fn}-\frac{\left(epx^{-2n}(fx)^{2n}\right)\operatorname{Subst}\left(\int\frac{x^2}{d+ex}\,dx,x,x^n\right)}{2fn}$$
$$=\frac{(fx)^{2n}\log\left(c(d+ex^n)^p\right)}{2fn}-\frac{\left(epx^{-2n}(fx)^{2n}\right)\operatorname{Subst}\left(\int\left(-\frac{d}{e^2}+\frac{x}{e}+\frac{d^2}{e^2(d+ex}\right.\right.}{2fn}$$
$$=-\frac{p(fx)^{2n}}{4fn}+\frac{dpx^{-n}(fx)^{2n}}{2efn}-\frac{d^2px^{-2n}(fx)^{2n}\log(d+ex^n)}{2e^2fn}+\frac{(fx)^{2n}\log}{2}$$

Mathematica [A]
time = 0.03, size = 74, normalized size = 0.66

$$-\frac{x^{-2n}(fx)^{2n}\left(2d^2p\log(d+ex^n)+ex^n\left(-2dp+epx^n-2ex^n\log\left(c(d+ex^n)^p\right)\right)\right)}{4e^2fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^(-1 + 2*n)*Log[c*(d + e*x^n)^p],x]
[Out] -1/4*((f*x)^(2*n)*(2*d^2*p*Log[d + e*x^n] + e*x^n*(-2*d*p + e*p*x^n - 2*e*x
^n*Log[c*(d + e*x^n)^p])))/(e^2*f*n*x^(2*n))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (fx)^{-1+2n}\ln\left(c(d+e\,x^n)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(-1+2*n)*ln(c*(d+e*x^n)^p),x)
[Out] int((f*x)^(-1+2*n)*ln(c*(d+e*x^n)^p),x)
```

Maxima [A]
time = 0.29, size = 95, normalized size = 0.85

$$-\frac{ep\left(\frac{2\,d^2f^{2\,n}\log\left(\frac{ex^n+d}{e}\right)}{e^3n}+\frac{ef^{2\,n}x^{2\,n}-2\,df^{2\,n}x^n}{e^2n}\right)}{4\,f}+\frac{(fx)^{2\,n}\log\left((ex^n+d)^pc\right)}{2\,fn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+2*n)*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] -1/4*e*p*(2*d^2*f^(2*n)*log((e*x^n + d)/e)/(e^3*n) + (e*f^(2*n)*x^(2*n) - 2
*d*f^(2*n)*x^n)/(e^2*n))/f + 1/2*(f*x)^(2*n)*log((e*x^n + d)^p*c)/(f*n)
```

Fricas [A]
time = 0.39, size = 90, normalized size = 0.80

$$\frac{(2\,df^{2\,n-1}px^ne-(pe^2-2\,e^2\log\left(c\right))f^{2\,n-1}x^{2\,n}-2\,(d^2f^{2\,n-1}p-f^{2\,n-1}px^{2\,n}e^2)\log\left(x^ne+d\right))e^{(-2)}}{4\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+2*n)*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] 1/4*(2*d*f^(2*n - 1)*p*x^n*e - (p*e^2 - 2*e^2*log(c))*f^(2*n - 1)*x^(2*n) -
 2*(d^2*f^(2*n - 1)*p - f^(2*n - 1)*p*x^(2*n)*e^2)*log(x^n*e + d))*e^(-2)/n
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(-1+2*n)*ln(c*(d+e*x**n)**p),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+2*n)*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^(2*n - 1)*log((x^n*e + d)^p*c), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \ln \left(c \left(d+e x^n\right)^p\right) (f x)^{2 n-1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)*(f*x)^(2*n - 1),x)
[Out] int(log(c*(d + e*x^n)^p)*(f*x)^(2*n - 1), x)
```

3.66 $\int (fx)^{-1+n} \log (c(d+ex^n)^p) \, dx$

Optimal. Leaf size=69

$$-\frac{p(fx)^n}{fn}+\frac{dpx^{-n}(fx)^n \log (d+ex^n)}{efn}+\frac{(fx)^n \log (c(d+ex^n)^p)}{fn}$$

```
[Out] -p*(f*x)^n/f/n+d*p*(f*x)^n*ln(d+e*x^n)/e/f/n/(x^n)+(f*x)^n*ln(c*(d+e*x^n)^p
)/f/n
```

Rubi [A]
time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2505, 20, 272, 45}

$$\frac{(fx)^n \log (c(d+ex^n)^p)}{fn}+\frac{dpx^{-n}(fx)^n \log (d+ex^n)}{efn}-\frac{p(fx)^n}{fn}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(-1 + n)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -((p*(f*x)^n)/(f*n)) + (d*p*(f*x)^n*Log[d + e*x^n])/(e*f*n*x^n) + ((f*x)^n*
Log[c*(d + e*x^n)^p])/(f*n)
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
```

```
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int (fx)^{-1+n}\log\left(c(d+ex^n)^p\right)\,dx &= \frac{(fx)^n\log\left(c(d+ex^n)^p\right)}{fn}-\frac{(ep)\int\frac{x^{-1+n}(fx)^n}{d+ex^n}\,dx}{f}\\
&= \frac{(fx)^n\log\left(c(d+ex^n)^p\right)}{fn}-\frac{\left(epx^{-n}(fx)^n\right)\int\frac{x^{-1+2n}}{d+ex^n}\,dx}{f}\\
&= \frac{(fx)^n\log\left(c(d+ex^n)^p\right)}{fn}-\frac{\left(epx^{-n}(fx)^n\right)\mathrm{Subst}\left(\int\frac{x}{d+ex}\,dx,x,x^n\right)}{fn}\\
&= \frac{(fx)^n\log\left(c(d+ex^n)^p\right)}{fn}-\frac{\left(epx^{-n}(fx)^n\right)\mathrm{Subst}\left(\int\left(\frac{1}{e}-\frac{d}{e(d+ex)}\right)\,dx,x,x^n\right)}{fn}\\
&= -\frac{p(fx)^n}{fn}+\frac{dpx^{-n}(fx)^n\log\left(d+ex^n\right)}{efn}+\frac{(fx)^n\log\left(c(d+ex^n)^p\right)}{fn}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 48, normalized size = 0.70

$$
\frac{x^{1-n}(fx)^{-1+n}\left(-px^n+\frac{(d+ex^n)\log\left(c(d+ex^n)^p\right)}{e}\right)}{n}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^(-1 + n)*Log[c*(d + e*x^n)^p],x]
[Out] (x^(1 - n)*(f*x)^(-1 + n)*(-(p*x^n) + ((d + e*x^n)*Log[c*(d + e*x^n)^p])/e)
)/n
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$
\int (fx)^{-1+n}\ln\left(c\left(d+e\,x^{n}\right)^{p}\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(-1+n)*ln(c*(d+e*x^n)^p),x)
[Out] int((f*x)^(-1+n)*ln(c*(d+e*x^n)^p),x)
```

Maxima [A]
time = 0.29, size = 70, normalized size = 1.01

$$-\frac{ep\left(\frac{f^n x^n}{en}-\frac{df^n\log\left(\frac{ex^n+d}{e}\right)}{e^2n}\right)}{f}+\frac{(fx)^n\log\left((ex^n+d)^pc\right)}{fn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+n)*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] -e*p*(f^n*x^n/(e*n) - d*f^n*log((e*x^n + d)/e)/(e^2*n))/f + (f*x)^n*log((e*
x^n + d)^p*c)/(f*n)
```

Fricas [A]
time = 0.48, size = 60, normalized size = 0.87

$$-\frac{((pe-e\log(c))f^{n-1}x^n-(f^{n-1}px^ne+df^{n-1}p)\log(x^ne+d))e^{(-1)}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+n)*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] -((p*e - e*log(c))*f^(n - 1)*x^n - (f^(n - 1)*p*x^n*e + d*f^(n - 1)*p)*log(
x^n*e + d))*e^(-1)/n
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(fx)^{n-1}\log\left(c(d+ex^n)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(-1+n)*ln(c*(d+e*x**n)**p),x)
```

```
[Out] Integral((f*x)**(n - 1)*log(c*(d + e*x**n)**p), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+n)*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^(n - 1)*log((x^n*e + d)^p*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln \left(c\left(d+e\,x^n\right)^p\right)\,\left(f\,x\right)^{n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)*(f*x)^(n - 1),x)
[Out] int(log(c*(d + e*x^n)^p)*(f*x)^(n - 1), x)
```

3.67 $\int \frac{\log(c(d+ex^n)^p)}{fx} \, dx$

Optimal. Leaf size=50

$$\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{fn}+\frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{fn}$$

```
[Out] ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/f/n+p*polylog(2,1+e*x^n/d)/f/n
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {12, 2504, 2441, 2352}

$$\frac{p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{fn}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{fn}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/(f*x),x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/(f*n) + (p*PolyLog[2, 1 + (e*x^n)/
d])/(f*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
```

```
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\log(c(d+ex^n)^p)}{fx}\,dx &= \frac{\int \frac{\log(c(d+ex^n)^p)}{x}\,dx}{f}\\ &= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{fn}\\ &= \frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{fn}-\frac{(ep)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx,x,x^n\right)}{fn}\\ &= \frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{fn}+\frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{fn}\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 46, normalized size = 0.92

$$\frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)+p\text{Li}_2\left(\frac{d+ex^n}{d}\right)}{fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(f*x),x]
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + p*PolyLog[2, (d + e*x^n)/d])/(f*n
)
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.45, size = 201, normalized size = 4.02

method	result
risch	$\frac{\ln(x)\ln((d+e\,x^n)^p)}{f}+\frac{i\ln(x)\pi\,\text{csgn}(i(d+e\,x^n)^p)\text{csgn}(ic(d+e\,x^n)^p)^2}{2f}-\frac{i\ln(x)\pi\,\text{csgn}(i(d+e\,x^n)^p)\text{csgn}(ic(d+e\,x^n)^p)\text{csgn}(ic)}{2f}-$ i

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)/f/x,x,method=_RETURNVERBOSE)
[Out] 1/f*ln(x)*ln((d+e*x^n)^p)+1/2*I/f*ln(x)*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+
e*x^n)^p)^2-1/2*I/f*ln(x)*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn
(I*c)-1/2*I/f*ln(x)*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I/f*ln(x)*Pi*csgn(I*c*(d
+e*x^n)^p)^2*csgn(I*c)+1/f*ln(c)*ln(x)-1/f*p/n*dilog((d+e*x^n)/d)-1/f*p*ln(
x)*ln((d+e*x^n)/d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/f/x,x, algorithm="maxima")
```

```
[Out] 1/2*(2*d*n*p*integrate(log(x)/(e*x*x^n + d*x), x) - n*p*log(x)^2 + 2*log((e
*x^n + d)^p)*log(x) + 2*log(c)*log(x))/f
```

Fricas [A]
time = 0.37, size = 66, normalized size = 1.32

$$\frac{np\log\left(x^{n}e+d\right)\log\left(x\right)-np\log\left(x\right)\log\left(\frac{x^{n}e+d}{d}\right)+n\log\left(c\right)\log\left(x\right)-p\mathrm{Li}_{2}\left(-\frac{x^{n}e+d}{d}+1\right)}{fn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/f/x,x, algorithm="fricas")
```

```
[Out] (n*p*log(x^n*e + d)*log(x) - n*p*log(x)*log((x^n*e + d)/d) + n*log(c)*log(x
) - p*dilog(-(x^n*e + d)/d + 1))/(f*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\log\left(c\left(d+ex^{n}\right)^{p}\right)}{x}\,dx}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/f/x,x)
```

```
[Out] Integral(log(c*(d + e*x**n)**p)/x, x)/f
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/f/x,x, algorithm="giac")
```

```
[Out] integrate(log((x^n*e + d)^p*c)/(f*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\ln\left(c\,{\left(d+e\,x^{n}\right)}^{p}\right)}{f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/(f*x),x)
[Out] int(log(c*(d + e*x^n)^p)/(f*x), x)
```

3.68 $\int (fx)^{-1-n} \log \left(c(d+ex^n)^p\right) dx$

Optimal. Leaf size=80

$$\frac{epx^n(fx)^{-n}\log(x)}{df}-\frac{epx^n(fx)^{-n}\log\left(d+ex^n\right)}{dfn}-\frac{(fx)^{-n}\log\left(c(d+ex^n)^p\right)}{fn}$$

```
[Out] e*p*x^n*ln(x)/d/f/((f*x)^n)-e*p*x^n*ln(d+e*x^n)/d/f/n/((f*x)^n)-ln(c*(d+e*x
^n)^p)/f/n/((f*x)^n)
```

Rubi [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2505, 19, 272, 36, 29, 31}

$$-\frac{(fx)^{-n}\log\left(c(d+ex^n)^p\right)}{fn}+\frac{epx^n\log(x)(fx)^{-n}}{df}-\frac{epx^n(fx)^{-n}\log\left(d+ex^n\right)}{dfn}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(-1 - n)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] (e*p*x^n*Log[x])/(d*f*(f*x)^n) - (e*p*x^n*Log[d + e*x^n])/(d*f*n*(f*x)^n) -
 Log[c*(d + e*x^n)^p]/(f*n*(f*x)^n)
```

Rule 19

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + n)*
((b*v)^n/(a*v)^n), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !I
ntegerQ[m] && !IntegerQ[n] && IntegerQ[m + n]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (fx)^{-1-n}\log\left(c(d+ex^n)^p\right)\,dx &= -\frac{(fx)^{-n}\log\left(c(d+ex^n)^p\right)}{fn}+\frac{(ep)\int\frac{x^{-1+n}(fx)^{-n}}{d+ex^n}\,dx}{f}\\ &= -\frac{(fx)^{-n}\log\left(c(d+ex^n)^p\right)}{fn}+\frac{\left(epx^n(fx)^{-n}\right)\int\frac{1}{x(d+ex^n)}\,dx}{f}\\ &= -\frac{(fx)^{-n}\log\left(c(d+ex^n)^p\right)}{fn}+\frac{\left(epx^n(fx)^{-n}\right)\text{Subst}\left(\int\frac{1}{x(d+ex)}\,dx,x,x^n\right)}{fn}\\ &= -\frac{(fx)^{-n}\log\left(c(d+ex^n)^p\right)}{fn}+\frac{\left(epx^n(fx)^{-n}\right)\text{Subst}\left(\int\frac{1}{x}\,dx,x,x^n\right)}{dfn}-\frac{(e^2p\\ &= \frac{epx^n(fx)^{-n}\log(x)}{df}-\frac{epx^n(fx)^{-n}\log\left(d+ex^n\right)}{dfn}-\frac{(fx)^{-n}\log\left(c(d+ex^n\right.}{fn}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 57, normalized size = 0.71

$$-\frac{(fx)^{-n}\left(-enpx^n\log(x)+epx^n\log\left(d+ex^n\right)+d\log\left(c(d+ex^n)^p\right)\right)}{dfn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^(-1 - n)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -((-(e*n*p*x^n*Log[x]) + e*p*x^n*Log[d + e*x^n] + d*Log[c*(d + e*x^n)^p])/(
d*f*n*(f*x)^n))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (fx)^{-1-n}\ln\left(c(d+e\,x^n)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(-1-n)*ln(c*(d+e*x^n)^p),x)
```

```
[Out] int((f*x)^(-1-n)*ln(c*(d+e*x^n)^p),x)
```

Maxima [A]
time = 0.28, size = 71, normalized size = 0.89

$$\frac{ep\left(\frac{\log(x)}{df^n} - \frac{\log\left(\frac{ex^n+d}{e}\right)}{df^n n}\right)}{f} - \frac{\log\left((ex^n+d)^p c\right)}{(fx)^n fn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-n)*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] e*p*(log(x)/(d*f^n) - log((e*x^n + d)/e)/(d*f^n*n))/f - log((e*x^n + d)^p*c
)/((f*x)^n*f*n)
```

Fricas [A]
time = 0.39, size = 78, normalized size = 0.98

$$\frac{f^{-n-1}npx^n e\log(x) - df^{-n-1}\log(c) - (f^{-n-1}px^n e + df^{-n-1}p)\log(x^n e + d)}{dnx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-n)*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] (f^(-n - 1)*n*p*x^n*e*log(x) - d*f^(-n - 1)*log(c) - (f^(-n - 1)*p*x^n*e +
d*f^(-n - 1)*p)*log(x^n*e + d))/(d*n*x^n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^{-n-1}\log\left(c(d+ex^n)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(-1-n)*ln(c*(d+e*x**n)**p),x)
```

```
[Out] Integral((f*x)**(-n - 1)*log(c*(d + e*x**n)**p), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-n)*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^(-n - 1)*log((x^n*e + d)^p*c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)}{{\left(f\,x\right)}^{n+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/(f*x)^(n + 1),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)/(f*x)^(n + 1), x)
```

3.69 $\int (fx)^{-1-2n}\log\left(c(d+ex^n)^p\right)\,dx$

Optimal. Leaf size=120

$$-\frac{epx^n(fx)^{-2n}}{2dfn}-\frac{e^2px^{2n}(fx)^{-2n}\log(x)}{2d^2f}+\frac{e^2px^{2n}(fx)^{-2n}\log\left(d+ex^n\right)}{2d^2fn}-\frac{(fx)^{-2n}\log\left(c(d+ex^n)^p\right)}{2fn}$$

```
[Out] -1/2*e*p*x^n/d/f/n/((f*x)^(2*n))-1/2*e^2*p*x^(2*n)*ln(x)/d^2/f/((f*x)^(2*n)
)+1/2*e^2*p*x^(2*n)*ln(d+e*x^n)/d^2/f/n/((f*x)^(2*n))-1/2*ln(c*(d+e*x^n)^p)
/f/n/((f*x)^(2*n))
```

Rubi [A]
time = 0.04, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2505, 20, 272, 46}

$$-\frac{(fx)^{-2n}\log\left(c(d+ex^n)^p\right)}{2fn}-\frac{e^2px^{2n}\log(x)(fx)^{-2n}}{2d^2f}+\frac{e^2px^{2n}(fx)^{-2n}\log\left(d+ex^n\right)}{2d^2fn}-\frac{epx^n(fx)^{-2n}}{2dfn}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(-1 - 2*n)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -1/2*(e*p*x^n)/(d*f*n*(f*x)^(2*n)) - (e^2*p*x^(2*n)*Log[x])/(2*d^2*f*(f*x)^
(2*n)) + (e^2*p*x^(2*n)*Log[d + e*x^n])/(2*d^2*f*n*(f*x)^(2*n)) - Log[c*(d
+ e*x^n)^p]/(2*f*n*(f*x)^(2*n))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (fx)^{-1-2n}\log\left(c(d+ex^n)^p\right)\,dx &= -\frac{(fx)^{-2n}\log\left(c(d+ex^n)^p\right)}{2fn}+\frac{(ep)\int\frac{x^{-1+n}(fx)^{-2n}}{d+ex^n}\,dx}{2f}\\ &= -\frac{(fx)^{-2n}\log\left(c(d+ex^n)^p\right)}{2fn}+\frac{\left(epx^{2n}(fx)^{-2n}\right)\int\frac{x^{-1-n}}{d+ex^n}\,dx}{2f}\\ &= -\frac{(fx)^{-2n}\log\left(c(d+ex^n)^p\right)}{2fn}+\frac{\left(epx^{2n}(fx)^{-2n}\right)\operatorname{Subst}\left(\int\frac{1}{x^2(d+ex)}\,dx,x,x^\right.}{2fn}\\ &= -\frac{(fx)^{-2n}\log\left(c(d+ex^n)^p\right)}{2fn}+\frac{\left(epx^{2n}(fx)^{-2n}\right)\operatorname{Subst}\left(\int\left(\frac{1}{dx^2}-\frac{e}{d^2x}+\frac{}{d^2}\right.\right.}{2fn}\\ &= -\frac{epx^n(fx)^{-2n}}{2dfn}-\frac{e^2px^{2n}(fx)^{-2n}\log(x)}{2d^2f}+\frac{e^2px^{2n}(fx)^{-2n}\log\left(d+ex^n\right)}{2d^2fn}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 76, normalized size = 0.63

$$-\frac{(fx)^{-2n}\left(e^2npx^{2n}\log(x)-e^2px^{2n}\log\left(d+ex^n\right)+d\left(epx^n+d\log\left(c(d+ex^n)^p\right)\right)\right)}{2d^2fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^(-1 - 2*n)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -1/2*(e^2*n*p*x^(2*n)*Log[x] - e^2*p*x^(2*n)*Log[d + e*x^n] + d*(e*p*x^n +
d*Log[c*(d + e*x^n)^p]))/(d^2*f*n*(f*x)^(2*n))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (fx)^{-1-2n}\ln\left(c(d+e\,x^n)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(-1-2*n)*ln(c*(d+e*x^n)^p),x)
```

```
[Out] int((f*x)^(-1-2*n)*ln(c*(d+e*x^n)^p),x)
```

Maxima [A]
time = 0.28, size = 99, normalized size = 0.82

$$-\frac{ep\left(\frac{e\log(x)}{d^2f^{2n}}-\frac{e\log\left(\frac{ex^n+d}{e}\right)}{d^2f^{2n}n}+\frac{1}{df^{2n}nx^n}\right)}{2f}-\frac{\log\left((ex^n+d)^pc\right)}{2\,(fx)^{2n}\,fn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-2*n)*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] -1/2*e*p*(e*log(x)/(d^2*f^(2*n)) - e*log((e*x^n + d)/e)/(d^2*f^(2*n)*n) + 1
/(d*f^(2*n)*n*x^n))/f - 1/2*log((e*x^n + d)^p*c)/((f*x)^(2*n)*f*n)
```

Fricas [A]
time = 0.41, size = 103, normalized size = 0.86

$$-\frac{f^{-2n-1}npx^{2n}e^2\log(x)+df^{-2n-1}px^ne+d^2f^{-2n-1}\log(c)+(d^2f^{-2n-1}p-f^{-2n-1}px^{2n}e^2)\log(x^ne+d)}{2\,d^2nx^{2n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-2*n)*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] -1/2*(f^(-2*n - 1)*n*p*x^(2*n)*e^2*log(x) + d*f^(-2*n - 1)*p*x^n*e + d^2*f^
(-2*n - 1)*log(c) + (d^2*f^(-2*n - 1)*p - f^(-2*n - 1)*p*x^(2*n)*e^2)*log(x
^n*e + d))/(d^2*n*x^(2*n))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(-1-2*n)*ln(c*(d+e*x**n)**p),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-2*n)*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^(-2*n - 1)*log((x^n*e + d)^p*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\left(d+e\,x^n\right)^p\right)}{{\left(f\,x\right)}^{2\,n+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/(f*x)^(2*n + 1),x)
[Out] int(log(c*(d + e*x^n)^p)/(f*x)^(2*n + 1), x)
```

3.70 $\int x^2 \log\left(c(d+ex^n)^p\right)\,dx$

Optimal. Leaf size=65

$$-\frac{enpx^{3+n}\,{}_2F_1\left(1,\frac{3+n}{n};2+\frac{3}{n};-\frac{ex^n}{d}\right)}{3d(3+n)}+\frac{1}{3}x^3\log\left(c(d+ex^n)^p\right)$$

```
[Out] -1/3*e*n*p*x^(3+n)*hypergeom([1, (3+n)/n],[2+3/n],-e*x^n/d)/d/(3+n)+1/3*x^3
*ln(c*(d+e*x^n)^p)
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2505, 371}

$$\frac{1}{3}x^3\log\left(c(d+ex^n)^p\right)-\frac{enpx^{n+3}\,{}_2F_1\left(1,\frac{n+3}{n};2+\frac{3}{n};-\frac{ex^n}{d}\right)}{3d(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -1/3*(e*n*p*x^(3 + n)*Hypergeometric2F1[1, (3 + n)/n, 2 + 3/n, -((e*x^n)/d)
])/(d*(3 + n)) + (x^3*Log[c*(d + e*x^n)^p])/3
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\log\left(c(d+ex^n)^p\right)\,dx&=\frac{1}{3}x^3\log\left(c(d+ex^n)^p\right)-\frac{1}{3}(enp)\int\frac{x^{2+n}}{d+ex^n}\,dx\\&=-\frac{enpx^{3+n}\,{}_2F_1\left(1,\frac{3+n}{n};2+\frac{3}{n};-\frac{ex^n}{d}\right)}{3d(3+n)}+\frac{1}{3}x^3\log\left(c(d+ex^n)^p\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 61, normalized size = 0.94

$$\frac{1}{3}x^3\left(-\frac{enpx^n\,{}_2F_1\left(1,\frac{3+n}{n};2+\frac{3}{n};-\frac{ex^n}{d}\right)}{d(3+n)}+\log\left(c(d+ex^n)^p\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[c*(d + e*x^n)^p],x]
[Out] (x^3*(-((e*n*p*x^n*Hypergeometric2F1[1, (3 + n)/n, 2 + 3/n, -((e*x^n)/d)])/
(d*(3 + n))) + Log[c*(d + e*x^n)^p]))/3
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int x^2 \ln\left(c(d+e\,x^n)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(d+e*x^n)^p),x)
[Out] int(x^2*ln(c*(d+e*x^n)^p),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(d+e*x^n)^p),x, algorithm="maxima")
[Out] -1/9*(n*p - 3*log(c))*x^3 + d*n*p*integrate(1/3*x^2/(e*x^n + d), x) + 1/3*x
^3*log((e*x^n + d)^p)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(d+e*x^n)^p),x, algorithm="fricas")
[Out] integral(x^2*log((x^n*e + d)^p*c), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 7.90, size = 104, normalized size = 1.60

$$\frac{x^3\log\left(c(d+ex^n)^p\right)}{3}-\frac{epx^3x^n\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{3}{n}\right)\Gamma\left(1+\frac{3}{n}\right)}{3d\Gamma\left(2+\frac{3}{n}\right)}-\frac{epx^3x^n\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{3}{n}\right)\Gamma\left(1+\frac{3}{n}\right)}{dn\Gamma\left(2+\frac{3}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(d+e*x**n)**p),x)
```

```
[Out] x**3*log(c*(d + e*x**n)**p)/3 - e*p*x**3*x**n*lerchphi(e*x**n*exp_polar(I*p
i)/d, 1, 1 + 3/n)*gamma(1 + 3/n)/(3*d*gamma(2 + 3/n)) - e*p*x**3*x**n*lerch
phi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 3/n)*gamma(1 + 3/n)/(d*n*gamma(2 + 3/n
))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate(x^2*log((x^n*e + d)^p*c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\,\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(c*(d + e*x^n)^p),x)
```

```
[Out] int(x^2*log(c*(d + e*x^n)^p), x)
```

3.71 $\int x \log(c(d+ex^n)^p)\,dx$

Optimal. Leaf size=65

$$-\frac{enpx^{2+n}\,{}_2F_1\left(1,\frac{2+n}{n};2\left(1+\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{2d(2+n)}+\frac{1}{2}x^2\log\left(c(d+ex^n)^p\right)$$

```
[Out] -1/2*e*n*p*x^(2+n)*hypergeom([1, (2+n)/n],[2+2/n],-e*x^n/d)/d/(2+n)+1/2*x^2
*ln(c*(d+e*x^n)^p)
```

Rubi [A]

time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2505, 371}

$$\frac{1}{2}x^2\log\left(c(d+ex^n)^p\right)-\frac{enpx^{n+2}\,{}_2F_1\left(1,\frac{n+2}{n};2\left(1+\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{2d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -1/2*(e*n*p*x^(2 + n)*Hypergeometric2F1[1, (2 + n)/n, 2*(1 + n^(-1)), -((e*
x^n)/d)])/(d*(2 + n)) + (x^2*Log[c*(d + e*x^n)^p])/2
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\log\left(c(d+ex^n)^p\right)\,dx&=\frac{1}{2}x^2\log\left(c(d+ex^n)^p\right)-\frac{1}{2}(enp)\int\frac{x^{1+n}}{d+ex^n}\,dx\\&=-\frac{enpx^{2+n}\,{}_2F_1\left(1,\frac{2+n}{n};2\left(1+\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{2d(2+n)}+\frac{1}{2}x^2\log\left(c(d+ex^n)^p\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 61, normalized size = 0.94

$$\frac{1}{2}x^2\left(-\frac{enpx^n\,{}_2F_1\left(1,\frac{2+n}{n};2+\frac{2}{n};-\frac{ex^n}{d}\right)}{d(2+n)}+\log\left(c(d+ex^n)^p\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[c*(d + e*x^n)^p],x]
[Out] (x^2*(-((e*n*p*x^n*Hypergeometric2F1[1, (2 + n)/n, 2 + 2/n, -((e*x^n)/d)])/
(d*(2 + n))) + Log[c*(d + e*x^n)^p]))/2
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int x\ln\left(c(d+e\,x^n)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(d+e*x^n)^p),x)
[Out] int(x*ln(c*(d+e*x^n)^p),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(d+e*x^n)^p),x, algorithm="maxima")
[Out] d*n*p*integrate(1/2*x/(e*x^n + d), x) - 1/4*(n*p - 2*log(c))*x^2 + 1/2*x^2*
log((e*x^n + d)^p)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(d+e*x^n)^p),x, algorithm="fricas")
[Out] integral(x*log((x^n*e + d)^p*c), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.68, size = 104, normalized size = 1.60

$$\frac{x^2\log\left(c(d+ex^n)^p\right)}{2}-\frac{epx^2x^n\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{2}{n}\right)\Gamma\left(1+\frac{2}{n}\right)}{2d\Gamma\left(2+\frac{2}{n}\right)}-\frac{epx^2x^n\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{2}{n}\right)\Gamma\left(1+\frac{2}{n}\right)}{dn\Gamma\left(2+\frac{2}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(d+e*x**n)**p),x)
```

```
[Out] x**2*log(c*(d + e*x**n)**p)/2 - e*p*x**2*x**n*lerchphi(e*x**n*exp_polar(I*p
i)/d, 1, 1 + 2/n)*gamma(1 + 2/n)/(2*d*gamma(2 + 2/n)) - e*p*x**2*x**n*lerch
phi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 2/n)*gamma(1 + 2/n)/(d*n*gamma(2 + 2/n
))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate(x*log((x^n*e + d)^p*c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(c*(d + e*x^n)^p),x)
```

```
[Out] int(x*log(c*(d + e*x^n)^p), x)
```

3.72 $\int \log(c(d+ex^n)^p)\,dx$

Optimal. Leaf size=54

$$-\frac{enpx^{1+n}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1+n)}+x\log\left(c(d+ex^n)^p\right)$$

```
[Out] -e*n*p*x^(1+n)*hypergeom([1, 1+1/n],[2+1/n],-e*x^n/d)/d/(1+n)+x*ln(c*(d+e*x
^n)^p)
```

Rubi [A]
time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2498, 371}

$$x\log\left(c(d+ex^n)^p\right)-\frac{enpx^{n+1}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p],x]
```

```
[Out] -((e*n*p*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^n)/d
)])/(d*(1 + n))) + x*Log[c*(d + e*x^n)^p]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \log\left(c(d+ex^n)^p\right)\,dx &= x\log\left(c(d+ex^n)^p\right)-(enp)\int\frac{x^n}{d+ex^n}\,dx\\ &= -\frac{enpx^{1+n}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1+n)}+x\log\left(c(d+ex^n)^p\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 52, normalized size = 0.96

$$x\left(-\frac{enpx^n\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1+n)}+\log\left(c(d+ex^n)^p\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p],x]
```

```
[Out] x*(-((e*n*p*x^n*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^n)/d)])
/(d*(1 + n))) + Log[c*(d + e*x^n)^p])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \ln\left(c(d+e\,x^n)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p),x)
```

```
[Out] int(ln(c*(d+e*x^n)^p),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] d*n*p*integrate(1/(e*x^n + d), x) - (n*p - log(c))*x + x*log((e*x^n + d)^p)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] integral(log((x^n*e + d)^p*c), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.77, size = 48, normalized size = 0.89

$$x\log\left(c(d+ex^n)^p\right)+\frac{px\Phi\left(\frac{dx^{-n}e^{i\pi}}{e},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n\Gamma\left(1+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p),x)
```

```
[Out] x*log(c*(d + e*x**n)**p) + p*x*lerchphi(d*exp_polar(I*pi)/(e*x**n), 1, exp_
polar(I*pi)/n)*gamma(1/n)/(n*gamma(1 + 1/n))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate(log((x^n*e + d)^p*c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(c\,{\left(d+e\,x^n\right)}^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p),x)
```

```
[Out] int(log(c*(d + e*x^n)^p), x)
```

3.73 $\int \frac{\log(c(d+ex^n)^p)}{x} \, dx$

Optimal. Leaf size=44

$$\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n}+\frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}$$

```
[Out] ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/n+p*polylog(2,1+e*x^n/d)/n
```

Rubi [A]
time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2504, 2441, 2352}

$$\frac{p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/x,x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n + (p*PolyLog[2, 1 + (e*x^n)/d])/
n
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}$$
$$= \frac{\log \left(-\frac{ex^n}{d}\right)\log \left(c(d+ex^n)^p\right)}{n} - \frac{(ep)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx, x, x^n\right)}{n}$$
$$= \frac{\log \left(-\frac{ex^n}{d}\right)\log \left(c(d+ex^n)^p\right)}{n} + \frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}$$

Mathematica [A]
time = 0.00, size = 43, normalized size = 0.98

$$\frac{\log \left(-\frac{ex^n}{d}\right)\log \left(c(d+ex^n)^p\right) + p\text{Li}_2\left(\frac{d+ex^n}{d}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/x,x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + p*PolyLog[2, (d + e*x^n)/d])/n
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.23, size = 177, normalized size = 4.02

method	result
risch	$\ln(x)\ln\left((d+e\,x^n)^p\right) + \frac{i\ln(x)\pi\,\mathrm{csgn}\left(i(d+e\,x^n)^p\right)\mathrm{csgn}\left(ic(d+e\,x^n)^p\right)^2}{2} - \frac{i\ln(x)\pi\,\mathrm{csgn}\left(i(d+e\,x^n)^p\right)\mathrm{csgn}\left(ic(d+e\,x^n)^p\right)\mathrm{csgn}(ic}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)*ln((d+e*x^n)^p)+1/2*I*ln(x)*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)
^p)^2-1/2*I*ln(x)*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/
2*I*ln(x)*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I*ln(x)*Pi*csgn(I*c*(d+e*x^n)^p)^2
*csgn(I*c)+ln(c)*ln(x)-p/n*dilog((d+e*x^n)/d)-p*ln(x)*ln((d+e*x^n)/d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

[Out] d*n*p*integrate(log(x)/(e*x*x^n + d*x), x) - 1/2*n*p*log(x)^2 + log((e*x^n + d)^p)*log(x) + log(c)*log(x)

Fricas [A]
time = 0.38, size = 63, normalized size = 1.43

$$\frac{np\log\left(x^{n}e+d\right)\log\left(x\right)-np\log\left(x\right)\log\left(\frac{x^{n}e+d}{d}\right)+n\log\left(c\right)\log\left(x\right)-p\mathrm{Li}_{2}\left(-\frac{x^{n}e+d}{d}+1\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")

[Out] (n*p*log(x^n*e + d)*log(x) - n*p*log(x)*log((x^n*e + d)/d) + n*log(c)*log(x) - p*dilog(-(x^n*e + d)/d + 1))/n

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left(c(d+ex^{n})^{p}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(c*(d+e*x**n)**p)/x,x)

[Out] Integral(log(c*(d + e*x**n)**p)/x, x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c*(d+e*x^n)^p)/x,x, algorithm="giac")

[Out] integrate(log((x^n*e + d)^p*c)/x, x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\ln\left(c\,{\left(d+e\,x^{n}\right)}^{p}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(c*(d + e*x^n)^p)/x,x)

[Out] int(log(c*(d + e*x^n)^p)/x, x)

3.74 $\int \frac{\log(c(d+ex^n)^p)}{x^2}\,dx$

Optimal. Leaf size=66

$$-\frac{enpx^{-1+n}\,{}_2F_1\left(1,-\frac{1-n}{n};2-\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1-n)}-\frac{\log\left(c(d+ex^n)^p\right)}{x}$$

```
[Out] -e*n*p*x^(-1+n)*hypergeom([1, (-1+n)/n],[2-1/n],-e*x^n/d)/d/(1-n)-ln(c*(d+e
*x^n)^p)/x
```

Rubi [A]
time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2505, 371}

$$-\frac{\log\left(c(d+ex^n)^p\right)}{x}-\frac{enpx^{n-1}\,{}_2F_1\left(1,-\frac{1-n}{n};2-\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/x^2,x]
```

```
[Out] -((e*n*p*x^(-1 + n)*Hypergeometric2F1[1, -((1 - n)/n), 2 - n^(-1), -((e*x^n
)/d)])/(d*(1 - n))) - Log[c*(d + e*x^n)^p]/x
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(c(d+ex^n)^p\right)}{x^2}\,dx &= -\frac{\log\left(c(d+ex^n)^p\right)}{x}+(enp)\int \frac{x^{-2+n}}{d+ex^n}\,dx\\ &= -\frac{enpx^{-1+n}\,{}_2F_1\left(1,-\frac{1-n}{n};2-\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1-n)}-\frac{\log\left(c(d+ex^n)^p\right)}{x}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 59, normalized size = 0.89

$$\frac{\frac{enpx^n\,{}_2F_1\left(1,\frac{-1+n}{n};2-\frac{1}{n};-\frac{ex^n}{d}\right)}{d(-1+n)}-\log\left(c(d+ex^n)^p\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/x^2,x]
```

```
[Out] ((e*n*p*x^n*Hypergeometric2F1[1, (-1 + n)/n, 2 - n^(-1), -((e*x^n)/d)])/(d*
(-1 + n)) - Log[c*(d + e*x^n)^p])/x
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(d+e\,x^{n}\right)^{p}\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)/x^2,x)
```

```
[Out] int(ln(c*(d+e*x^n)^p)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x^2,x, algorithm="maxima")
```

```
[Out] -d*n*p*integrate(1/(e*x^2*x^n + d*x^2), x) - (n*p + log((e*x^n + d)^p) + lo
g(c))/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x^2,x, algorithm="fricas")
```

```
[Out] integral(log((x^n*e + d)^p*c)/x^2, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.30, size = 46, normalized size = 0.70

$$-\frac{\log\left(c(d+ex^n)^p\right)}{x}+\frac{p\Phi\left(\frac{dx^{-n}e^{i\pi}}{e},1,\frac{1}{n}\right)\Gamma\left(-\frac{1}{n}\right)}{nx\Gamma\left(1-\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x**2,x)
```

```
[Out] -log(c*(d + e*x**n)**p)/x + p*lerchphi(d*exp_polar(I*pi)/(e*x**n), 1, 1/n)*
gamma(-1/n)/(n*x*gamma(1 - 1/n))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x^2,x, algorithm="giac")
```

```
[Out] integrate(log((x^n*e + d)^p*c)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(c\left(d+e\,x^n\right)^p\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/x^2,x)
```

```
[Out] int(log(c*(d + e*x^n)^p)/x^2, x)
```

3.75 $\int \frac{\log(c(d+ex^n)^p)}{x^3}\,dx$

Optimal. Leaf size=72

$$-\frac{enpx^{-2+n}\,{}_2F_1\left(1,-\frac{2-n}{n};2\left(1-\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{2d(2-n)}-\frac{\log\left(c(d+ex^n)^p\right)}{2x^2}$$

```
[Out] -1/2*e*n*p*x^(-2+n)*hypergeom([1, (-2+n)/n],[2-2/n],-e*x^n/d)/d/(2-n)-1/2*l
n(c*(d+e*x^n)^p)/x^2
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2505, 371}

$$-\frac{\log\left(c(d+ex^n)^p\right)}{2x^2}-\frac{enpx^{n-2}\,{}_2F_1\left(1,-\frac{2-n}{n};2\left(1-\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{2d(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/x^3,x]
```

```
[Out] -1/2*(e*n*p*x^(-2 + n)*Hypergeometric2F1[1, -((2 - n)/n), 2*(1 - n^(-1)), -
((e*x^n)/d)])/(d*(2 - n)) - Log[c*(d + e*x^n)^p]/(2*x^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(c(d+ex^n)^p\right)}{x^3}\,dx &= -\frac{\log\left(c(d+ex^n)^p\right)}{2x^2}+\frac{1}{2}(enp)\int \frac{x^{-3+n}}{d+ex^n}\,dx\\ &= -\frac{enpx^{-2+n}\,{}_2F_1\left(1,-\frac{2-n}{n};2\left(1-\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{2d(2-n)}-\frac{\log\left(c(d+ex^n)^p\right)}{2x^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 62, normalized size = 0.86

$$\frac{\frac{enpx^n\,{}_2F_1\left(1,\frac{-2+n}{n};2-\frac{2}{n};-\frac{ex^n}{d}\right)}{d(-2+n)}-\log\left(c(d+ex^n)^p\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/x^3,x]
```

```
[Out] ((e*n*p*x^n*Hypergeometric2F1[1, (-2 + n)/n, 2 - 2/n, -((e*x^n)/d)])/(d*(-2
 + n)) - Log[c*(d + e*x^n)^p])/(2*x^2)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c(d+e\,x^n)^p\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)/x^3,x)
```

```
[Out] int(ln(c*(d+e*x^n)^p)/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x^3,x, algorithm="maxima")
```

```
[Out] -d*n*p*integrate(1/2/(e*x^3*x^n + d*x^3), x) - 1/4*(n*p + 2*log((e*x^n + d)
^p) + 2*log(c))/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x^3,x, algorithm="fricas")
```

```
[Out] integral(log((x^n*e + d)^p*c)/x^3, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 8.87, size = 51, normalized size = 0.71

$$-\frac{\log\left(c(d+ex^n)^p\right)}{2x^2}+\frac{p\Phi\left(\frac{dx^{-n}e^{i\pi}}{e},1,\frac{2}{n}\right)\Gamma\left(-\frac{2}{n}\right)}{nx^2\Gamma\left(1-\frac{2}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x**3,x)
```

```
[Out] -log(c*(d + e*x**n)**p)/(2*x**2) + p*lerchphi(d*exp_polar(I*pi)/(e*x**n), 1
, 2/n)*gamma(-2/n)/(n*x**2*gamma(1 - 2/n))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x^3,x, algorithm="giac")
```

```
[Out] integrate(log((x^n*e + d)^p*c)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\left(d+e\,x^n\right)^p\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/x^3,x)
```

```
[Out] int(log(c*(d + e*x^n)^p)/x^3, x)
```

3.76 $\int \frac{\log(c(d+ex^n)^p)}{x^4}\,dx$

Optimal. Leaf size=70

$$-\frac{enpx^{-3+n}\,{}_2F_1\left(1,-\frac{3-n}{n};2-\frac{3}{n};-\frac{ex^n}{d}\right)}{3d(3-n)}-\frac{\log\left(c(d+ex^n)^p\right)}{3x^3}$$

```
[Out] -1/3*e*n*p*x^(-3+n)*hypergeom([1, (-3+n)/n],[2-3/n],-e*x^n/d)/d/(3-n)-1/3*l
n(c*(d+e*x^n)^p)/x^3
```

Rubi [A]
time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {2505, 371}

$$-\frac{\log\left(c(d+ex^n)^p\right)}{3x^3}-\frac{enpx^{n-3}\,{}_2F_1\left(1,-\frac{3-n}{n};2-\frac{3}{n};-\frac{ex^n}{d}\right)}{3d(3-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/x^4,x]
```

```
[Out] -1/3*(e*n*p*x^(-3 + n)*Hypergeometric2F1[1, -((3 - n)/n), 2 - 3/n, -((e*x^n
)/d)])/(d*(3 - n)) - Log[c*(d + e*x^n)^p]/(3*x^3)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(c(d+ex^n)^p\right)}{x^4}\,dx &= -\frac{\log\left(c(d+ex^n)^p\right)}{3x^3}+\frac{1}{3}(enp)\int \frac{x^{-4+n}}{d+ex^n}\,dx\\ &= -\frac{enpx^{-3+n}\,{}_2F_1\left(1,-\frac{3-n}{n};2-\frac{3}{n};-\frac{ex^n}{d}\right)}{3d(3-n)}-\frac{\log\left(c(d+ex^n)^p\right)}{3x^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 62, normalized size = 0.89

$$\frac{\frac{enpx^n\,{}_2F_1\left(1,\frac{-3+n}{n};2-\frac{3}{n};-\frac{ex^n}{d}\right)}{d(-3+n)}-\log\left(c(d+ex^n)^p\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/x^4,x]
```

```
[Out] ((e*n*p*x^n*Hypergeometric2F1[1, (-3 + n)/n, 2 - 3/n, -((e*x^n)/d)])/(d*(-3
 + n)) - Log[c*(d + e*x^n)^p])/(3*x^3)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c(d+e\,x^n)^p\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)/x^4,x)
```

```
[Out] int(ln(c*(d+e*x^n)^p)/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x^4,x, algorithm="maxima")
```

```
[Out] -d*n*p*integrate(1/3/(e*x^4*x^n + d*x^4), x) - 1/9*(n*p + 3*log((e*x^n + d)
^p) + 3*log(c))/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x^4,x, algorithm="fricas")
```

```
[Out] integral(log((x^n*e + d)^p*c)/x^4, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 19.77, size = 51, normalized size = 0.73

$$-\frac{\log\left(c(d+ex^n)^p\right)}{3x^3}+\frac{p\Phi\left(\frac{dx^{-n}e^{i\pi}}{e},1,\frac{3}{n}\right)\Gamma\left(-\frac{3}{n}\right)}{nx^3\Gamma\left(1-\frac{3}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x**4,x)
```

```
[Out] -log(c*(d + e*x**n)**p)/(3*x**3) + p*lerchphi(d*exp_polar(I*pi)/(e*x**n), 1
, 3/n)*gamma(-3/n)/(n*x**3*gamma(1 - 3/n))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x^4,x, algorithm="giac")
```

```
[Out] integrate(log((x^n*e + d)^p*c)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/x^4,x)
```

```
[Out] int(log(c*(d + e*x^n)^p)/x^4, x)
```

3.77 $\int x^5 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx$

Optimal. Leaf size=215

$$\frac{a^2p^2x^2}{b^2}-\frac{ap^2\left(a+bx^2\right)^2}{4b^3}+\frac{p^2\left(a+bx^2\right)^3}{27b^3}-\frac{a^3p^2\log^2\left(a+bx^2\right)}{6b^3}-\frac{a^2p\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b^3}+\frac{ap\left(a+bx^2\right)^2\log\ldots}{2\ldots}$$

```
[Out] a^2*p^2*x^2/b^2-1/4*a*p^2*(b*x^2+a)^2/b^3+1/27*p^2*(b*x^2+a)^3/b^3-1/6*a^3*
p^2*ln(b*x^2+a)^2/b^3-a^2*p*(b*x^2+a)*ln(c*(b*x^2+a)^p)/b^3+1/2*a*p*(b*x^2+
a)^2*ln(c*(b*x^2+a)^p)/b^3-1/9*p*(b*x^2+a)^3*ln(c*(b*x^2+a)^p)/b^3+1/3*a^3*
p*ln(b*x^2+a)*ln(c*(b*x^2+a)^p)/b^3+1/6*x^6*ln(c*(b*x^2+a)^p)^2
```

Rubi [A]
time = 0.20, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {2504, 2445, 2458, 45, 2372, 12, 14, 2338}

$$\frac{a^3p\log\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{3b^3}-\frac{a^3p^2\log^2\left(a+bx^2\right)}{6b^3}-\frac{a^2p\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b^3}+\frac{a^2p^2x^2}{b^2}-\frac{p\left(a+bx^2\right)^3\log\left(c\left(a+bx^2\right)^p\right)}{9b^3}+\frac{ap\left(a+bx^2\right)^2\log\left(c\left(a+bx^2\right)^p\right)}{2b^3}+\frac{p^2\left(a+bx^2\right)^3}{27b^3}-\frac{ap^2\left(a+bx^2\right)^2}{4b^3}+\frac{1}{6}x^6\log^2\left(c\left(a+bx^2\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] (a^2*p^2*x^2)/b^2 - (a*p^2*(a + b*x^2)^2)/(4*b^3) + (p^2*(a + b*x^2)^3)/(27
*b^3) - (a^3*p^2*Log[a + b*x^2]^2)/(6*b^3) - (a^2*p*(a + b*x^2)*Log[c*(a +
b*x^2)^p])/b^3 + (a*p*(a + b*x^2)^2*Log[c*(a + b*x^2)^p])/(2*b^3) - (p*(a +
 b*x^2)^3*Log[c*(a + b*x^2)^p])/(9*b^3) + (a^3*p*Log[a + b*x^2]*Log[c*(a +
b*x^2)^p])/(3*b^3) + (x^6*Log[c*(a + b*x^2)^p]^2)/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^5 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx = \frac{1}{2}\text{Subst}\left(\int x^2\log^2\left(c(a+bx)^p\right)\,dx, x, x^2\right)$$

$$= \frac{1}{6}x^6\log^2\left(c\left(a+bx^2\right)^p\right) - \frac{1}{3}(bp)\text{Subst}\left(\int \frac{x^3\log\left(c(a+bx)^p\right)}{a+bx}\,dx, x, x^2\right)$$

$$= \frac{1}{6}x^6\log^2\left(c\left(a+bx^2\right)^p\right) - \frac{1}{3}p\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^3\log\left(cx^p\right)}{x}\,dx, x, a+bx^2\right)$$

$$= -\frac{1}{18}p\left(\frac{18a^2(a+bx^2)}{b^3} - \frac{9a(a+bx^2)^2}{b^3} + \frac{2(a+bx^2)^3}{b^3} - \frac{6a^3\log(a+bx^2)}{b^3}\right)\text{lo}$$

$$= -\frac{1}{18}p\left(\frac{18a^2(a+bx^2)}{b^3} - \frac{9a(a+bx^2)^2}{b^3} + \frac{2(a+bx^2)^3}{b^3} - \frac{6a^3\log(a+bx^2)}{b^3}\right)\text{lo}$$

$$= -\frac{1}{18}p\left(\frac{18a^2(a+bx^2)}{b^3} - \frac{9a(a+bx^2)^2}{b^3} + \frac{2(a+bx^2)^3}{b^3} - \frac{6a^3\log(a+bx^2)}{b^3}\right)\text{lo}$$

$$= \frac{a^2p^2x^2}{b^2} - \frac{ap^2(a+bx^2)^2}{4b^3} + \frac{p^2(a+bx^2)^3}{27b^3} - \frac{1}{18}p\left(\frac{18a^2(a+bx^2)}{b^3} - \frac{9a(a+bx^2}{b^3}\right.$$

$$= \frac{a^2p^2x^2}{b^2} - \frac{ap^2(a+bx^2)^2}{4b^3} + \frac{p^2(a+bx^2)^3}{27b^3} - \frac{a^3p^2\log^2(a+bx^2)}{6b^3} - \frac{1}{18}p\left(\frac{18a^2(a}{b}\right.$$

Mathematica [A]
time = 0.05, size = 200, normalized size = 0.93

$$\frac{11a^2p^2x^2}{18b^2} - \frac{5ap^2x^4}{36b} + \frac{p^2x^6}{27} - \frac{5a^3p^2\log(a+bx^2)}{18b^3} - \frac{a^3p\log\left(c\left(a+bx^2\right)^p\right)}{3b^3} - \frac{a^2px^2\log\left(c\left(a+bx^2\right)^p\right)}{3b^2} + \frac{apx^4\log\left(c\left(a+bx^2\right)^p\right)}{6b} - \frac{1}{9}px^6\log\left(c\left(a+bx^2\right)^p\right) + \frac{a^3\log^2\left(c\left(a+bx^2\right)^p\right)}{6b^3} + \frac{1}{6}x^6\log^2\left(c\left(a+bx^2\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] (11*a^2*p^2*x^2)/(18*b^2) - (5*a*p^2*x^4)/(36*b) + (p^2*x^6)/27 - (5*a^3*p^
2*Log[a + b*x^2])/(18*b^3) - (a^3*p*Log[c*(a + b*x^2)^p])/(3*b^3) - (a^2*p*
x^2*Log[c*(a + b*x^2)^p])/(3*b^2) + (a*p*x^4*Log[c*(a + b*x^2)^p])/(6*b) -
(p*x^6*Log[c*(a + b*x^2)^p])/9 + (a^3*Log[c*(a + b*x^2)^p]^2)/(6*b^3) + (x^
6*Log[c*(a + b*x^2)^p]^2)/6
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.44, size = 1436, normalized size = 6.68

method	result	size

risch	Expression too large to display	1436

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*ln(c*(b*x^2+a)^p)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*ln(c)^2*x^6-1/6*a^3*p^2*ln(b*x^2+a)^2/b^3-11/18*a^3*p^2/b^3*ln(b*x^2+a)
+11/18*a^2*p^2*x^2/b^2-1/9*ln(c)*p*x^6-1/24*Pi^2*x^6*csgn(I*c*(b*x^2+a)^p)^
6+1/27*p^2*x^6-1/6*I*ln(c)*Pi*x^6*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)
*csgn(I*c)+1/18*I*Pi*p*x^6*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I
*c)-1/6*I/b^3*Pi*ln(b*x^2+a)*a^3*p*csgn(I*c*(b*x^2+a)^p)^3-1/12*I/b*Pi*a*p*
x^4*csgn(I*c*(b*x^2+a)^p)^3+1/6*I/b^2*Pi*a^2*p*x^2*csgn(I*c*(b*x^2+a)^p)^3+
1/6*x^6*ln((b*x^2+a)^p)^2-1/12*I/b*Pi*a*p*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*
(b*x^2+a)^p)*csgn(I*c)+1/6*I/b^2*Pi*a^2*p*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*
(b*x^2+a)^p)*csgn(I*c)-1/6*I/b^3*Pi*ln(b*x^2+a)*a^3*p*csgn(I*(b*x^2+a)^p)*c
sgn(I*c*(b*x^2+a)^p)*csgn(I*c)+1/6/b*ln(c)*a*p*x^4-1/3/b^2*ln(c)*a^2*p*x^2+
1/3/b^3*ln(c)*ln(b*x^2+a)*a^3*p-1/6*Pi^2*x^6*csgn(I*(b*x^2+a)^p)*csgn(I*c*(
b*x^2+a)^p)^4*csgn(I*c)-5/36/b*a*p^2*x^4-1/24*Pi^2*x^6*csgn(I*c*(b*x^2+a)^p
)^4*csgn(I*c)^2+1/12*Pi^2*x^6*csgn(I*c*(b*x^2+a)^p)^5*csgn(I*c)-1/24*Pi^2*x
^6*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^4+1/12*Pi^2*x^6*csgn(I*(b*x^
2+a)^p)*csgn(I*c*(b*x^2+a)^p)^5+1/6*I*ln(c)*Pi*x^6*csgn(I*c*(b*x^2+a)^p)^2*
csgn(I*c)+1/6*I*ln(c)*Pi*x^6*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-1/
18*I*Pi*p*x^6*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-1/18*I*Pi*p*x^6*csgn(I*(b*x
^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+1/18*(3*I*Pi*b^3*x^6*csgn(I*(b*x^2+a)^p)*c
sgn(I*c*(b*x^2+a)^p)^2-3*I*Pi*b^3*x^6*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a
)^p)*csgn(I*c)-3*I*Pi*b^3*x^6*csgn(I*c*(b*x^2+a)^p)^3+3*I*Pi*b^3*x^6*csgn(I
*c*(b*x^2+a)^p)^2*csgn(I*c)+6*ln(c)*b^3*x^6-2*b^3*p*x^6+3*a*b^2*p*x^4-6*a^2
*b*p*x^2+6*a^3*p*ln(b*x^2+a))/b^3*ln((b*x^2+a)^p)-1/24*Pi^2*x^6*csgn(I*(b*x
^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)^2+1/12*Pi^2*x^6*csgn(I*(b*x^2+
a)^p)*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)^2+1/12*Pi^2*x^6*csgn(I*(b*x^2+a)^p)
^2*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)-1/6*I*ln(c)*Pi*x^6*csgn(I*c*(b*x^2+a)^
p)^3+1/18*I*Pi*p*x^6*csgn(I*c*(b*x^2+a)^p)^3+1/6*I/b^3*Pi*ln(b*x^2+a)*a^3*p
*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-1/6*I/b^2*Pi*a^2*p*x^2*csgn(I*
(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-1/6*I/b^2*Pi*a^2*p*x^2*csgn(I*c*(b*x^2
+a)^p)^2*csgn(I*c)+1/12*I/b*Pi*a*p*x^4*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+1/
6*I/b^3*Pi*ln(b*x^2+a)*a^3*p*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+1/12*I/b*Pi*
a*p*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2
```

<u>Maxima</u> [**A**]
time = 0.28, size = 145, normalized size = 0.67

$$\frac{1}{6}\,x^{6}\log\left(\left(bx^{2}+a\right)^{p}c\right)^{2}+\frac{1}{18}\,bp\left(\frac{6\,a^{3}\log\left(bx^{2}+a\right)}{b^{4}}-\frac{2\,b^{2}x^{6}-3\,abx^{4}+6\,a^{2}x^{2}}{b^{3}}\right)\log\left(\left(bx^{2}+a\right)^{p}c\right)+\frac{\left(4\,b^{3}x^{6}-15\,ab^{2}x^{4}+66\,a^{2}bx^{2}-18\,a^{3}\log\left(bx^{2}+a\right)^{2}-66\,a^{3}\log\left(bx^{2}+a\right)\right)p^{2}}{108\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] 1/6*x^6*log((b*x^2 + a)^p*c)^2 + 1/18*b*p*(6*a^3*log(b*x^2 + a)/b^4 - (2*b^
2*x^6 - 3*a*b*x^4 + 6*a^2*x^2)/b^3)*log((b*x^2 + a)^p*c) + 1/108*(4*b^3*x^6
 - 15*a*b^2*x^4 + 66*a^2*b*x^2 - 18*a^3*log(b*x^2 + a)^2 - 66*a^3*log(b*x^2
 + a))*p^2/b^3
```

Fricas [A]
time = 0.36, size = 189, normalized size = 0.88

$$\frac{4\,b^{3}p^{2}x^{6}+18\,b^{3}x^{6}\log\left(c\right)^{2}-15\,ab^{2}p^{2}x^{4}+66\,a^{2}bp^{2}x^{2}+18\,\left(b^{3}p^{2}x^{6}+a^{3}p^{2}\right)\log\left(bx^{2}+a\right)^{2}-6\,\left(2\,b^{3}p^{2}x^{6}-3\,ab^{2}p^{2}x^{4}+6\,a^{2}bp^{2}x^{2}+11\,a^{3}p^{2}-6\,\left(b^{3}px^{6}+a^{3}p\right)\log\left(c\right)\right)\log\left(bx^{2}+a\right)-6\,\left(2\,b^{3}px^{6}-3\,ab^{2}px^{4}+6\,a^{2}bpx^{2}\right)\log\left(c\right)}{108\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/108*(4*b^3*p^2*x^6 + 18*b^3*x^6*log(c)^2 - 15*a*b^2*p^2*x^4 + 66*a^2*b*p^
2*x^2 + 18*(b^3*p^2*x^6 + a^3*p^2)*log(b*x^2 + a)^2 - 6*(2*b^3*p^2*x^6 - 3*
a*b^2*p^2*x^4 + 6*a^2*b*p^2*x^2 + 11*a^3*p^2 - 6*(b^3*p*x^6 + a^3*p)*log(c)
)*log(b*x^2 + a) - 6*(2*b^3*p*x^6 - 3*a*b^2*p*x^4 + 6*a^2*b*p*x^2)*log(c))/
b^3
```

Sympy [A]
time = 4.19, size = 182, normalized size = 0.85

$$\begin{cases} - \frac{11 a^{3} p \log{\left(c \left(a + b x^{2}\right)^{p} \right)}}{18 b^{3}} + \frac{a^{3} \log{\left(c \left(a + b x^{2}\right)^{p} \right)}^{2}}{6 b^{3}} + \frac{11 a^{2} p^{2} x^{2}}{18 b^{2}} - \frac{a^{2} p x^{2} \log{\left(c \left(a + b x^{2}\right)^{p} \right)}}{3 b^{2}} - \frac{5 a p^{2} x^{4}}{36 b} + \frac{a p x^{4} \log{\left(c \left(a + b x^{2}\right)^{p} \right)}}{6 b} + \frac{p^{2} x^{6}}{27} - \frac{p x^{6} \log{\left(c \left(a + b x^{2}\right)^{p} \right)}}{9} + \frac{x^{6} \log{\left(c \left(a + b x^{2}\right)^{p} \right)}^{2}}{6} & \text{for}\: b \neq 0 \\\frac{x^{6} \log{\left(a^{p} c \right)}^{2}}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Piecewise((-11*a**3*p*log(c*(a + b*x**2)**p)/(18*b**3) + a**3*log(c*(a + b*
x**2)**p)**2/(6*b**3) + 11*a**2*p**2*x**2/(18*b**2) - a**2*p*x**2*log(c*(a
+ b*x**2)**p)/(3*b**2) - 5*a*p**2*x**4/(36*b) + a*p*x**4*log(c*(a + b*x**2)
**p)/(6*b) + p**2*x**6/27 - p*x**6*log(c*(a + b*x**2)**p)/9 + x**6*log(c*(a
 + b*x**2)**p)**2/6, Ne(b, 0)), (x**6*log(a**p*c)**2/6, True))
```

Giac [A]
time = 2.94, size = 370, normalized size = 1.72

$$\frac{{\left(bx^{2}+a\right)}^{3}p^{2}\log\left(bx^{2}+a\right)^{2}}{6\,b^{3}}-\frac{{\left(bx^{2}+a\right)}^{2}ap^{2}\log\left(bx^{2}+a\right)^{2}}{2\,b^{3}}-\frac{{\left(bx^{2}+a\right)}^{3}p^{2}\log\left(bx^{2}+a\right)}{9\,b^{3}}+\frac{{\left(bx^{2}+a\right)}^{2}ap^{2}\log\left(bx^{2}+a\right)}{2\,b^{3}}+\frac{{\left(bx^{2}+a\right)}^{3}p\log\left(bx^{2}+a\right)\log\left(c\right)}{3\,b^{3}}-\frac{{\left(bx^{2}+a\right)}^{2}ap\log\left(bx^{2}+a\right)\log\left(c\right)}{b^{3}}+\frac{{\left(bx^{2}+a\right)}^{3}p^{2}}{27\,b^{3}}-\frac{{\left(bx^{2}+a\right)}^{2}ap^{2}}{4\,b^{3}}-\frac{{\left(bx^{2}+a\right)}^{3}p\log\left(c\right)}{9\,b^{3}}+\frac{{\left(bx^{2}+a\right)}^{2}ap\log\left(c\right)}{2\,b^{3}}+\frac{{\left(bx^{2}+a\right)}^{3}\log\left(c\right)^{2}}{6\,b^{3}}-\frac{{\left(bx^{2}+a\right)}^{2}a\log\left(c\right)^{2}}{2\,b^{3}}+\frac{{\left(2\,bx^{2}+\left(bx^{2}+a\right)\log\left(bx^{2}+a\right)^{2}-2\,\left(bx^{2}+a\right)\log\left(bx^{2}+a\right)+2\,a\right)}a^{2}p^{2}-2\,\left(bx^{2}-\left(bx^{2}+a\right)\log\left(bx^{2}+a\right)+a\right)a^{2}p\log\left(c\right)+\left(bx^{2}+a\right)a^{2}\log\left(c\right)^{2}}{2\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] 1/6*(b*x^2 + a)^3*p^2*log(b*x^2 + a)^2/b^3 - 1/2*(b*x^2 + a)^2*a*p^2*log(b*
x^2 + a)^2/b^3 - 1/9*(b*x^2 + a)^3*p^2*log(b*x^2 + a)/b^3 + 1/2*(b*x^2 + a)
^2*a*p^2*log(b*x^2 + a)/b^3 + 1/3*(b*x^2 + a)^3*p*log(b*x^2 + a)*log(c)/b^3
 - (b*x^2 + a)^2*a*p*log(b*x^2 + a)*log(c)/b^3 + 1/27*(b*x^2 + a)^3*p^2/b^3
```

```
 - 1/4*(b*x^2 + a)^2*a*p^2/b^3 - 1/9*(b*x^2 + a)^3*p*log(c)/b^3 + 1/2*(b*x^
2 + a)^2*a*p*log(c)/b^3 + 1/6*(b*x^2 + a)^3*log(c)^2/b^3 - 1/2*(b*x^2 + a)^
2*a*log(c)^2/b^3 + 1/2*((2*b*x^2 + (b*x^2 + a)*log(b*x^2 + a)^2 - 2*(b*x^2
+ a)*log(b*x^2 + a) + 2*a)*a^2*p^2 - 2*(b*x^2 - (b*x^2 + a)*log(b*x^2 + a)
+ a)*a^2*p*log(c) + (b*x^2 + a)*a^2*log(c)^2)/b^3
```

Mupad [B]
time = 0.31, size = 126, normalized size = 0.59

$$\frac{p^2\,x^6}{27}+\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^2\,\left(\frac{x^6}{6}+\frac{a^3}{6\,b^3}\right)-\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)\,\left(\frac{p\,x^6}{9}+\frac{a^2\,p\,x^2}{3\,b^2}-\frac{a\,p\,x^4}{6\,b}\right)-\frac{5\,a\,p^2\,x^4}{36\,b}-\frac{11\,a^3\,p^2\,\ln\left(b\,x^2+a\right)}{18\,b^3}+\frac{11\,a^2\,p^2\,x^2}{18\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*log(c*(a + b*x^2)^p)^2,x)
```

```
[Out] (p^2*x^6)/27 + log(c*(a + b*x^2)^p)^2*(x^6/6 + a^3/(6*b^3)) - log(c*(a + b*
x^2)^p)*((p*x^6)/9 + (a^2*p*x^2)/(3*b^2) - (a*p*x^4)/(6*b)) - (5*a*p^2*x^4)
/(36*b) - (11*a^3*p^2*log(a + b*x^2))/(18*b^3) + (11*a^2*p^2*x^2)/(18*b^2)
```

3.78 $\int x^3 \log^2\left(c(a+bx^2)^p\right)\,dx$

Optimal. Leaf size=145

$$-\frac{ap^2x^2}{b}+\frac{p^2(a+bx^2)^2}{8b^2}+\frac{ap(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b^2}-\frac{p(a+bx^2)^2\log\left(c(a+bx^2)^p\right)}{4b^2}-\frac{a(a+bx^2)\log^2\left(c(a\right.}{2b^2}$$

```
[Out] -a*p^2*x^2/b+1/8*p^2*(b*x^2+a)^2/b^2+a*p*(b*x^2+a)*ln(c*(b*x^2+a)^p)/b^2-1/
4*p*(b*x^2+a)^2*ln(c*(b*x^2+a)^p)/b^2-1/2*a*(b*x^2+a)*ln(c*(b*x^2+a)^p)^2/b
^2+1/4*(b*x^2+a)^2*ln(c*(b*x^2+a)^p)^2/b^2
```

Rubi [A]
time = 0.11, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$,
Rules used = {2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

$$\frac{(a+bx^2)^2\log^2\left(c(a+bx^2)^p\right)}{4b^2}-\frac{a(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2b^2}-\frac{p(a+bx^2)^2\log\left(c(a+bx^2)^p\right)}{4b^2}+\frac{ap(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b^2}+\frac{p^2(a+bx^2)^2}{8b^2}-\frac{ap^2x^2}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] -((a*p^2*x^2)/b) + (p^2*(a + b*x^2)^2)/(8*b^2) + (a*p*(a + b*x^2)*Log[c*(a
+ b*x^2)^p])/b^2 - (p*(a + b*x^2)^2*Log[c*(a + b*x^2)^p])/(4*b^2) - (a*(a +
 b*x^2)*Log[c*(a + b*x^2)^p]^2)/(2*b^2) + ((a + b*x^2)^2*Log[c*(a + b*x^2)^
p]^2)/(4*b^2)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
```

```
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3 \log^2\left(c(a+bx^2)^p\right) dx = \frac{1}{2}\text{Subst}\left(\int x \log^2\left(c(a+bx)^p\right) dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a \log^2\left(c(a+bx)^p\right)}{b} + \frac{(a+bx)\log^2\left(c(a+bx)^p\right)}{b}\right) dx, x, x^2\right)$$
$$= \frac{\text{Subst}\left(\int (a+bx)\log^2\left(c(a+bx)^p\right) dx, x, x^2\right)}{2b} - \frac{a\text{Subst}\left(\int \log^2\left(c(a+bx)^p\right) dx\right.}{2b}$$
$$= \frac{\text{Subst}\left(\int x \log^2\left(cx^p\right) dx, x, a+bx^2\right)}{2b^2} - \frac{a\text{Subst}\left(\int \log^2\left(cx^p\right) dx, x, a+bx^2\right)}{2b^2}$$
$$= -\frac{a(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2b^2} + \frac{(a+bx^2)^2\log^2\left(c(a+bx^2)^p\right)}{4b^2} - \frac{p\text{Subst}\left(\int x\right.}{}$$
$$= -\frac{ap^2x^2}{b} + \frac{p^2(a+bx^2)^2}{8b^2} + \frac{ap(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b^2} - \frac{p(a+bx^2)^2\log\left(c\right.}{4b^2}$$

Mathematica [A]
time = 0.04, size = 105, normalized size = 0.72

$$\frac{bp^2x^2(-6a+bx^2) + 2a^2p^2\log(a+bx^2) + 2p(2a^2+2abx^2-b^2x^4)\log\left(c(a+bx^2)^p\right) - 2(a^2-b^2x^4)\log^2\left(c(a+bx^2)^p\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] (b*p^2*x^2*(-6*a + b*x^2) + 2*a^2*p^2*Log[a + b*x^2] + 2*p*(2*a^2 + 2*a*b*x
^2 - b^2*x^4)*Log[c*(a + b*x^2)^p] - 2*(a^2 - b^2*x^4)*Log[c*(a + b*x^2)^p]
^2)/(8*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.58, size = 24297, normalized size = 167.57

method	result	size
risch	Expression too large to display	24297

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(b*x^2+a)^p)^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.31, size = 120, normalized size = 0.83

$$\frac{1}{4}x^4\log\left((bx^2+a)^pc\right)^2 - \frac{1}{4}bp\left(\frac{2a^2\log(bx^2+a)}{b^3} + \frac{bx^4-2ax^2}{b^2}\right)\log\left((bx^2+a)^pc\right) + \frac{\left(b^2x^4 - 6abx^2 + 2a^2\log(bx^2+a)^2 + 6a^2\log(bx^2+a)\right)p^2}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log((b*x^2 + a)^p*c)^2 - 1/4*b*p*(2*a^2*log(b*x^2 + a)/b^3 + (b*x^4
 - 2*a*x^2)/b^2)*log((b*x^2 + a)^p*c) + 1/8*(b^2*x^4 - 6*a*b*x^2 + 2*a^2*lo
g(b*x^2 + a)^2 + 6*a^2*log(b*x^2 + a))*p^2/b^2
```

Fricas [A]
time = 0.39, size = 148, normalized size = 1.02

$$\frac{b^2p^2x^4 + 2\,b^2x^4\log\left(c\right)^2 - 6\,abp^2x^2 + 2\,(b^2p^2x^4 - a^2p^2)\log\left(bx^2+a\right)^2 - 2\,(b^2p^2x^4 - 2\,abp^2x^2 - 3\,a^2p^2 - 2\,(b^2px^4 - a^2p)\log\left(c\right))\log\left(bx^2+a\right) - 2\,(b^2px^4 - 2\,abpx^2)\log\left(c\right)}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(b^2*p^2*x^4 + 2*b^2*x^4*log(c)^2 - 6*a*b*p^2*x^2 + 2*(b^2*p^2*x^4 - a^
2*p^2)*log(b*x^2 + a)^2 - 2*(b^2*p^2*x^4 - 2*a*b*p^2*x^2 - 3*a^2*p^2 - 2*(b
^2*p*x^4 - a^2*p)*log(c))*log(b*x^2 + a) - 2*(b^2*p*x^4 - 2*a*b*p*x^2)*log(
c))/b^2
```

Sympy [A]
time = 1.54, size = 139, normalized size = 0.96

$$\begin{cases} \frac{3a^2p\log\left(c(a+bx^2)^p\right)}{4b^2} - \frac{a^2\log\left(c(a+bx^2)^p\right)^2}{4b^2} - \frac{3ap^2x^2}{4b} + \frac{apx^2\log\left(c(a+bx^2)^p\right)}{2b} + \frac{p^2x^4}{8} - \frac{px^4\log\left(c(a+bx^2)^p\right)}{4} + \frac{x^4\log\left(c(a+bx^2)^p\right)^2}{4} & \text{for}\: b \neq 0 \\ \frac{x^4\log\left(a^pc\right)^2}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Piecewise((3*a**2*p*log(c*(a + b*x**2)**p)/(4*b**2) - a**2*log(c*(a + b*x**
2)**p)**2/(4*b**2) - 3*a*p**2*x**2/(4*b) + a*p*x**2*log(c*(a + b*x**2)**p)/
(2*b) + p**2*x**4/8 - p*x**4*log(c*(a + b*x**2)**p)/4 + x**4*log(c*(a + b*x
**2)**p)**2/4, Ne(b, 0)), (x**4*log(a**p*c)**2/4, True))
```

Giac [A]
time = 6.03, size = 216, normalized size = 1.49

$$\frac{2\,(bx^2+a)^2p^2\log\left(bx^2+a\right)^2 - 2\,(bx^2+a)^2p^2\log\left(bx^2+a\right) + 4\,(bx^2+a)^2p\log\left(bx^2+a\right)\log\left(c\right) + (bx^2+a)^2p^2 - 2\,(bx^2+a)^2p\log\left(c\right) + 2\,(bx^2+a)^2\log\left(c\right)^2}{8\,b^2} - \frac{\left(2\,bx^2 + (bx^2+a)\log\left(bx^2+a\right)^2 - 2\,(bx^2+a)\log\left(bx^2+a\right) + 2\,a\right)ap^2 - 2\,(bx^2 - (bx^2+a)\log\left(bx^2+a\right) + a)ap\log\left(c\right) + (bx^2+a)a\log\left(c\right)^2}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] 1/8*(2*(b*x^2 + a)^2*p^2*log(b*x^2 + a)^2 - 2*(b*x^2 + a)^2*p^2*log(b*x^2 +
 a) + 4*(b*x^2 + a)^2*p*log(b*x^2 + a)*log(c) + (b*x^2 + a)^2*p^2 - 2*(b*x^
2 + a)^2*p*log(c) + 2*(b*x^2 + a)^2*log(c)^2)/b^2 - 1/2*((2*b*x^2 + (b*x^2
```

```
+ a)*log(b*x^2 + a)^2 - 2*(b*x^2 + a)*log(b*x^2 + a) + 2*a)*a*p^2 - 2*(b*x^
2 - (b*x^2 + a)*log(b*x^2 + a) + a)*a*p*log(c) + (b*x^2 + a)*a*log(c)^2)/b^
2
```

Mupad [B]
time = 0.26, size = 100, normalized size = 0.69

$$\frac{p^2\,x^4}{8}-\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)\,\left(\frac{p\,x^4}{4}-\frac{a\,p\,x^2}{2\,b}\right)+{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}^2\,\left(\frac{x^4}{4}-\frac{a^2}{4\,b^2}\right)-\frac{3\,a\,p^2\,x^2}{4\,b}+\frac{3\,a^2\,p^2\,\ln\left(b\,x^2+a\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(c*(a + b*x^2)^p)^2,x)
```

```
[Out] (p^2*x^4)/8 - log(c*(a + b*x^2)^p)*((p*x^4)/4 - (a*p*x^2)/(2*b)) + log(c*(a
 + b*x^2)^p)^2*(x^4/4 - a^2/(4*b^2)) - (3*a*p^2*x^2)/(4*b) + (3*a^2*p^2*log
(a + b*x^2))/(4*b^2)
```

3.79 $\int x \log^2\left(c(a+bx^2)^p\right)\,dx$

Optimal. Leaf size=61

$$p^2x^2-\frac{p(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b}+\frac{(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2b}$$

```
[Out] p^2*x^2-p*(b*x^2+a)*ln(c*(b*x^2+a)^p)/b+1/2*(b*x^2+a)*ln(c*(b*x^2+a)^p)^2/b
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {2504, 2436, 2333, 2332}

$$\frac{(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2b}-\frac{p(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b}+p^2x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] p^2*x^2 - (p*(a + b*x^2)*Log[c*(a + b*x^2)^p])/b + ((a + b*x^2)*Log[c*(a +
b*x^2)^p]^2)/(2*b)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\log^2\left(c\left(a+bx^2\right)^p\right)\,dx=\frac{1}{2}\text{Subst}\left(\int\log^2\left(c(a+bx)^p\right)\,dx,x,x^2\right)$$
$$=\frac{\text{Subst}\left(\int\log^2\left(cx^p\right)\,dx,x,a+bx^2\right)}{2b}$$
$$=\frac{\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b}-\frac{p\text{Subst}\left(\int\log\left(cx^p\right)\,dx,x,a+bx^2\right)}{b}$$
$$=p^2x^2-\frac{p\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b}+\frac{\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b}$$

Mathematica [A]
time = 0.01, size = 63, normalized size = 1.03

$$\frac{1}{2}\left(\frac{\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{b}-2p\left(-px^2+\frac{\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] (((a + b*x^2)*Log[c*(a + b*x^2)^p]^2)/b - 2*p*(-(p*x^2) + ((a + b*x^2)*Log[
c*(a + b*x^2)^p])/b))/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.69, size = 1034, normalized size = 16.95

method	result	size
risch	Expression too large to display	1034

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(b*x^2+a)^p)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a*p^2/b*ln(b*x^2+a)+1/2*x^2*ln((b*x^2+a)^p)^2+1/b*ln(c)*ln(b*x^2+a)*a*p+1/
4*Pi^2*x^2*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)-1/8*Pi^2
*x^2*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)^2-1/2*Pi^2*x^2
*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)+1/4*Pi^2*x^2*csgn(I*
(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)^2-1/2*I/b*Pi*ln(b*x^2+a)*a*p
*csgn(I*c*(b*x^2+a)^p)^3-1/2*I*ln(c)*Pi*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b
*x^2+a)^p)*csgn(I*c)+1/2*I*Pi*p*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^
p)*csgn(I*c)-ln(c)*p*x^2-1/8*Pi^2*x^2*csgn(I*c*(b*x^2+a)^p)^6+1/2*ln(c)^2*x
^2+1/2*I/b*Pi*ln(b*x^2+a)*a*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+1
```

```
/2*I/b*Pi*ln(b*x^2+a)*a*p*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-1/2*I*ln(c)*Pi*
x^2*csgn(I*c*(b*x^2+a)^p)^3+1/2*I*Pi*p*x^2*csgn(I*c*(b*x^2+a)^p)^3+p^2*x^2+
1/2*I*ln(c)*Pi*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+1/2*I*ln(c)*
Pi*x^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-1/2*I*Pi*p*x^2*csgn(I*(b*x^2+a)^p)
*csgn(I*c*(b*x^2+a)^p)^2-1/2*I*Pi*p*x^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+1
/2*(I*Pi*b*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*b*x^2*csgn(
I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*b*x^2*csgn(I*c*(b*x^2+a
)^p)^3+I*Pi*b*x^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)*b*x^2-2*x^2*p*b
+2*p*a*ln(b*x^2+a))/b*ln((b*x^2+a)^p)-1/2*I/b*Pi*ln(b*x^2+a)*a*p*csgn(I*(b*
x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+1/4*Pi^2*x^2*csgn(I*c*(b*x^2+a)^p
)^5*csgn(I*c)-1/8*Pi^2*x^2*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)^2-1/8*Pi^2*x^2
*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^4+1/4*Pi^2*x^2*csgn(I*(b*x^2+a
)^p)*csgn(I*c*(b*x^2+a)^p)^5-1/2/b*a*p^2*ln(b*x^2+a)^2
```

Maxima [A]
time = 0.28, size = 97, normalized size = 1.59

$$-bp\left(\frac{x^2}{b}-\frac{a\log(bx^2+a)}{b^2}\right)\log\left((bx^2+a)^pc\right)+\frac{1}{2}x^2\log\left((bx^2+a)^pc\right)^2+\frac{\left(2\,bx^2-a\log(bx^2+a)^2-2\,a\log(bx^2+a)\right)p^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] -b*p*(x^2/b - a*log(b*x^2 + a)/b^2)*log((b*x^2 + a)^p*c) + 1/2*x^2*log((b*x
^2 + a)^p*c)^2 + 1/2*(2*b*x^2 - a*log(b*x^2 + a)^2 - 2*a*log(b*x^2 + a))*p^
2/b
```

Fricas [A]
time = 0.36, size = 96, normalized size = 1.57

$$\frac{2\,bp^2x^2-2\,bpx^2\log(c)+bx^2\log(c)^2+(bp^2x^2+ap^2)\log(bx^2+a)^2-2\,(bp^2x^2+ap^2-(bpx^2+ap)\log(c))\log(bx^2+a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*p^2*x^2 - 2*b*p*x^2*log(c) + b*x^2*log(c)^2 + (b*p^2*x^2 + a*p^2)*
log(b*x^2 + a)^2 - 2*(b*p^2*x^2 + a*p^2 - (b*p*x^2 + a*p)*log(c))*log(b*x^2
 + a))/b
```

Sympy [A]
time = 0.59, size = 90, normalized size = 1.48

$$\begin{cases}-\frac{ap\log\left(c(a+bx^2)^p\right)}{b}+\frac{a\log\left(c(a+bx^2)^p\right)^2}{2b}+p^2x^2-px^2\log\left(c(a+bx^2)^p\right)+\frac{x^2\log\left(c(a+bx^2)^p\right)^2}{2} & \text{for}\ b\neq 0\\ \frac{x^2\log(a^pc)^2}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Piecewise((-a*p*log(c*(a + b*x**2)**p)/b + a*log(c*(a + b*x**2)**p)**2/(2*b
) + p**2*x**2 - p*x**2*log(c*(a + b*x**2)**p) + x**2*log(c*(a + b*x**2)**p)
**2/2, Ne(b, 0)), (x**2*log(a**p*c)**2/2, True))
```

Giac [A]
time = 3.77, size = 96, normalized size = 1.57

$$\frac{\left(2\,bx^2+(bx^2+a)\log\left(bx^2+a\right)^2-2\,(bx^2+a)\log\left(bx^2+a\right)+2\,a\right)p^2-2\,(bx^2-(bx^2+a)\log\left(bx^2+a\right)+a)p\log\left(c\right)+(bx^2+a)\log\left(c\right)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] 1/2*((2*b*x^2 + (b*x^2 + a)*log(b*x^2 + a)^2 - 2*(b*x^2 + a)*log(b*x^2 + a)
 + 2*a)*p^2 - 2*(b*x^2 - (b*x^2 + a)*log(b*x^2 + a) + a)*p*log(c) + (b*x^2
+ a)*log(c)^2)/b
```

Mupad [B]
time = 0.23, size = 70, normalized size = 1.15

$$p^2\,x^2+\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^2\,\left(\frac{a}{2\,b}+\frac{x^2}{2}\right)-p\,x^2\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)-\frac{a\,p^2\,\ln\left(b\,x^2+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(c*(a + b*x^2)^p)^2,x)
```

```
[Out] p^2*x^2 + log(c*(a + b*x^2)^p)^2*(a/(2*b) + x^2/2) - p*x^2*log(c*(a + b*x^2
)^p) - (a*p^2*log(a + b*x^2))/b
```

3.80 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x}\,dx$

Optimal. Leaf size=72

$$\frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)+p\log\left(c\left(a+bx^2\right)^p\right)\text{Li}_2\left(1+\frac{bx^2}{a}\right)-p^2\text{Li}_3\left(1+\frac{bx^2}{a}\right)$$

```
[Out] 1/2*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)^2+p*ln(c*(b*x^2+a)^p)*polylog(2,1+b*x^2/
a)-p^2*polylog(3,1+b*x^2/a)
```

Rubi [A]
time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2504, 2443, 2481, 2421, 6724}

$$p\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)\log\left(c\left(a+bx^2\right)^p\right)+p^2\left(-\text{PolyLog}\left(3,\frac{bx^2}{a}+1\right)\right)+\frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^2/x,x]
```

```
[Out] (Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p]^2)/2 + p*Log[c*(a + b*x^2)^p]*PolyL
og[2, 1 + (b*x^2)/a] - p^2*PolyLog[3, 1 + (b*x^2)/a]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x}\,dx &= \frac{1}{2}\mathrm{Subst}\left(\int \frac{\log^2\left(c(a+bx)^p\right)}{x}\,dx, x, x^2\right)\\
&= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)-(bp)\mathrm{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{a+bx}\,dx\right.\\
&= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)-p\mathrm{Subst}\left(\int \frac{\log\left(cx^p\right)\log\left(-\frac{b\left(-\frac{a}{b}+\frac{x}{b}\right)}{a}\right)}{x}\,dx, x,\right.\\
&= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)+p\log\left(c\left(a+bx^2\right)^p\right)\mathrm{Li}_2\left(1+\frac{bx^2}{a}\right)-p^2\mathrm{Subs}\\
&= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)+p\log\left(c\left(a+bx^2\right)^p\right)\mathrm{Li}_2\left(1+\frac{bx^2}{a}\right)-p^2\mathrm{Li}_3\left(\right.
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 163 vs. 2(72) = 144.
time = 0.08, size = 163, normalized size = 2.26

$$\log(x)\left(-p\log\left(a+bx^2\right)+\log\left(c\left(a+bx^2\right)^p\right)\right)^2+2p\left(-p\log\left(a+bx^2\right)+\log\left(c\left(a+bx^2\right)^p\right)\right)\left(\log(x)\left(\log\left(a+bx^2\right)-\log\left(1+\frac{bx^2}{a}\right)\right)-\frac{1}{2}\mathrm{Li}_2\left(-\frac{bx^2}{a}\right)\right)+\frac{1}{2}p^2\left(\log\left(-\frac{bx^2}{a}\right)\log^2\left(a+bx^2\right)+2\log\left(a+bx^2\right)\mathrm{Li}_2\left(1+\frac{bx^2}{a}\right)-2\mathrm{Li}_3\left(1+\frac{bx^2}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2/x,x]
```

```
[Out] Log[x]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2 + 2*p*(-(p*Log[a + b*
x^2]) + Log[c*(a + b*x^2)^p])*(Log[x]*(Log[a + b*x^2] - Log[1 + (b*x^2)/a])
```

```
 - PolyLog[2, -((b*x^2)/a)]/2) + (p^2*(Log[-((b*x^2)/a)]*Log[a + b*x^2]^2 +
 2*Log[a + b*x^2]*PolyLog[2, 1 + (b*x^2)/a] - 2*PolyLog[3, 1 + (b*x^2)/a]))
/2
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c(b\,x^2+a)^p\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^2/x,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^2/x,x)
```

Maxima [A]
time = 0.30, size = 118, normalized size = 1.64

$$\frac{1}{2}\left(\log\left(bx^2+a\right)^2\log\left(-\frac{bx^2+a}{a}+1\right)+2\,\mathrm{Li}_2\left(\frac{bx^2+a}{a}\right)\log\left(bx^2+a\right)-2\,\mathrm{Li}_3(\frac{bx^2+a}{a})\right)p^2+\left(\log\left(bx^2+a\right)\log\left(-\frac{bx^2+a}{a}+1\right)+\mathrm{Li}_2\left(\frac{bx^2+a}{a}\right)\right)p\log\left(c\right)+\log\left(c\right)^2\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x,x, algorithm="maxima")
```

```
[Out] 1/2*(log(b*x^2 + a)^2*log(-(b*x^2 + a)/a + 1) + 2*dilog((b*x^2 + a)/a)*log(
b*x^2 + a) - 2*polylog(3, (b*x^2 + a)/a))*p^2 + (log(b*x^2 + a)*log(-(b*x^2
 + a)/a + 1) + dilog((b*x^2 + a)/a))*p*log(c) + log(c)^2*log(x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^2/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c(a+bx^2)^p\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**2/x,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**2/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x,x, algorithm="giac")
[Out] integrate(log((b*x^2 + a)^p*c)^2/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)^2/x,x)
[Out] int(log(c*(a + b*x^2)^p)^2/x, x)
```

3.81 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^3}\,dx$

Optimal. Leaf size=80

$$\frac{bp\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{a}-\frac{\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2ax^2}+\frac{bp^2\text{Li}_2\left(1+\frac{bx^2}{a}\right)}{a}$$

```
[Out] b*p*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)/a-1/2*(b*x^2+a)*ln(c*(b*x^2+a)^p)^2/a/x^
2+b*p^2*polylog(2,1+b*x^2/a)/a
```

Rubi [A]
time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2504, 2444, 2441, 2352}

$$\frac{bp^2\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{a}-\frac{\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2ax^2}+\frac{bp\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^2/x^3,x]
```

```
[Out] (b*p*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/a - ((a + b*x^2)*Log[c*(a + b*
x^2)^p]^2)/(2*a*x^2) + (b*p^2*PolyLog[2, 1 + (b*x^2)/a])/a
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Dist[b*e*n*(p/(e*f - d*g)), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\log^2\left(c(a+bx)^p\right)}{x^2}\,dx, x, x^2\right) \\
&= -\frac{(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2ax^2} + \frac{(bp)\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}\,dx, x, x^2\right)}{a} \\
&= \frac{bp\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{a} - \frac{(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2ax^2} - \frac{(b^2p^2)\,\text{Subst}\left(\right.}{} \\
&= \frac{bp\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{a} - \frac{(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2ax^2} + \frac{bp^2\text{Li}_2\left(1+\frac{bx^2}{a}\right.}{a}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 93, normalized size = 1.16

$$
\frac{bp\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{a} - \frac{b\log^2\left(c(a+bx^2)^p\right)}{2a} - \frac{\log^2\left(c(a+bx^2)^p\right)}{2x^2} + \frac{bp^2\text{Li}_2\left(\frac{a+bx^2}{a}\right)}{a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2/x^3,x]
```

```
[Out] (b*p*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/a - (b*Log[c*(a + b*x^2)^p]^2)
/(2*a) - Log[c*(a + b*x^2)^p]^2/(2*x^2) + (b*p^2*PolyLog[2, (a + b*x^2)/a])
/a
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.32, size = 841, normalized size = 10.51

method	result
risch	$-\frac{\ln\left(\left(b\,x^2+a\right)^p\right)^2}{2x^2}+\frac{2pb\ln\left(\left(b\,x^2+a\right)^p\right)\ln(x)}{a}-\frac{pb\ln\left(\left(b\,x^2+a\right)^p\right)\ln\left(b\,x^2+a\right)}{a}-\frac{2p^2b\ln(x)\ln\left(\frac{-bx+\sqrt{-ba}}{\sqrt{-ba}}\right)}{a}-\frac{2p^2b\ln(x)\,\mathrm{l}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^2*ln((b*x^2+a)^p)^2+2*p*b*ln((b*x^2+a)^p)/a*ln(x)-p*b*ln((b*x^2+a)^p
)/a*ln(b*x^2+a)-2*p^2*b/a*ln(x)*ln((-b*x+(-b*a)^(1/2))/(-b*a)^(1/2))-2*p^2*
b/a*ln(x)*ln((b*x+(-b*a)^(1/2))/(-b*a)^(1/2))-2*p^2*b/a*dilog((-b*x+(-b*a)^
(1/2))/(-b*a)^(1/2))-2*p^2*b/a*dilog((b*x+(-b*a)^(1/2))/(-b*a)^(1/2))+1/2*p
^2*b/a*ln(b*x^2+a)^2-1/2*I/x^2*ln((b*x^2+a)^p)*Pi*csgn(I*(b*x^2+a)^p)*csgn(
I*c*(b*x^2+a)^p)^2-1/2*I*p*b/a*ln(b*x^2+a)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*
(b*x^2+a)^p)^2-I*p*b/a*ln(x)*Pi*csgn(I*c*(b*x^2+a)^p)^3+1/2*I*p*b/a*ln(b*x^
2+a)*Pi*csgn(I*c*(b*x^2+a)^p)^3-1/x^2*ln((b*x^2+a)^p)*ln(c)+1/2*I/x^2*ln((b
*x^2+a)^p)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+I*p*b/a*l
n(x)*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+1/2*I*p*b/a*ln(b*x^2+a)*Pi*csgn(I
*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-1/2*I/x^2*ln((b*x^2+a)^p)*Pi*
csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*p*b/a*ln(x)*ln(c)-I*p*b/a*ln(x)*Pi*csgn
(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-1/2*I*p*b/a*ln(b*x^2+a)*Pi*
csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+1/2*I/x^2*ln((b*x^2+a)^p)*Pi*csgn(I*c*(b*
x^2+a)^p)^3+I*p*b/a*ln(x)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-p*
b/a*ln(b*x^2+a)*ln(c)-1/8*(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2
-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*
x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c))^2/x^2
```

Maxima [A]
time = 0.28, size = 118, normalized size = 1.48

$$\frac{1}{2}\,b^2p^2\left(\frac{\log\left(bx^2+a\right)^2}{ab}-\frac{2\left(2\,\log\left(\frac{bx^2}{a}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{bx^2}{a}\right)\right)}{ab}\right)-bp\left(\frac{\log\left(bx^2+a\right)}{a}-\frac{\log\left(x^2\right)}{a}\right)\log\left(\left(bx^2+a\right)^pc\right)-\frac{\log\left(\left(bx^2+a\right)^pc\right)^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*p^2*(log(b*x^2 + a)^2/(a*b) - 2*(2*log(b*x^2/a + 1)*log(x) + dilog(
-b*x^2/a))/(a*b)) - b*p*(log(b*x^2 + a)/a - log(x^2)/a)*log((b*x^2 + a)^p*c
) - 1/2*log((b*x^2 + a)^p*c)^2/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^2/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**2/x**3,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**2/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^2/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)^2/x^3,x)
```

```
[Out] int(log(c*(a + b*x^2)^p)^2/x^3, x)
```

3.82 $\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^5} dx$

Optimal. Leaf size=129

$$\frac{b^2p^2\log(x)}{a^2} - \frac{bp(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2a^2x^2} - \frac{\log^2\left(c(a+bx^2)^p\right)}{4x^4} - \frac{b^2p\log\left(c(a+bx^2)^p\right)\log\left(1-\frac{a}{a+bx^2}\right)}{2a^2} + \frac{b^2p^2\mathrm{Li}\ldots}{2\ldots}$$

```
[Out] b^2*p^2*ln(x)/a^2-1/2*b*p*(b*x^2+a)*ln(c*(b*x^2+a)^p)/a^2/x^2-1/4*ln(c*(b*x
^2+a)^p)^2/x^4-1/2*b^2*p*ln(c*(b*x^2+a)^p)*ln(1-a/(b*x^2+a))/a^2+1/2*b^2*p^
2*polylog(2,a/(b*x^2+a))/a^2
```

Rubi [A]
time = 0.15, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$,
Rules used = {2504, 2445, 2458, 2389, 2379, 2438, 2351, 31}

$$\frac{b^2p^2\text{PolyLog}\left(2,\frac{a}{a+bx^2}\right)}{2a^2} - \frac{b^2p\log\left(1-\frac{a}{a+bx^2}\right)\log\left(c(a+bx^2)^p\right)}{2a^2} + \frac{b^2p^2\log(x)}{a^2} - \frac{bp(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2a^2x^2} - \frac{\log^2\left(c(a+bx^2)^p\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^2/x^5,x]
```

```
[Out] (b^2*p^2*Log[x])/a^2 - (b*p*(a + b*x^2)*Log[c*(a + b*x^2)^p])/(2*a^2*x^2) -
 Log[c*(a + b*x^2)^p]^2/(4*x^4) - (b^2*p*Log[c*(a + b*x^2)^p]*Log[1 - a/(a
+ b*x^2)])/(2*a^2) + (b^2*p^2*PolyLog[2, a/(a + b*x^2)])/(2*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^5}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\log^2\left(c(a+bx)^p\right)}{x^3}\,dx,x,x^2\right)$$
$$= -\frac{\log^2\left(c(a+bx^2)^p\right)}{4x^4} + \frac{1}{2}(bp)\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x^2(a+bx)}\,dx,x,x^2\right)$$
$$= -\frac{\log^2\left(c(a+bx^2)^p\right)}{4x^4} + \frac{1}{2}p\text{Subst}\left(\int \frac{\log\left(cx^p\right)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx,x,a+bx^2\right)$$
$$= -\frac{\log^2\left(c(a+bx^2)^p\right)}{4x^4} + \frac{p\text{Subst}\left(\int \frac{\log(cx^p)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx,x,a+bx^2\right)}{2a} - \frac{(bp)\text{Subst}\left(\int \frac{\log(cx^p)}{x\left(-\frac{a}{b}+\frac{x}{b}\right.}\right.}{2a}$$
$$= -\frac{bp(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2a^2x^2} - \frac{\log^2\left(c(a+bx^2)^p\right)}{4x^4} - \frac{(bp)\text{Subst}\left(\int \frac{\log(cx^p)}{-\frac{a}{b}+\frac{x}{b}}\,dx,x,\right.}{2a^2}$$
$$= \frac{b^2p^2\log(x)}{a^2} - \frac{bp(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2a^2x^2} - \frac{b^2p\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{2a^2} +$$
$$= \frac{b^2p^2\log(x)}{a^2} - \frac{bp(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2a^2x^2} - \frac{b^2p\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{2a^2} +$$

Mathematica [A]
time = 0.06, size = 137, normalized size = 1.06

$$\frac{-\log^2\left(c(a+bx^2)^p\right) + \frac{bx^2\left(2bp^2x^2(2\log(x)-\log(a+bx^2))-2ap\log\left(c(a+bx^2)^p\right)+bx^2\log^2\left(c(a+bx^2)^p\right)-2bpx^2\left(\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)+p\text{Li}_2\left(1+\frac{bx^2}{a}\right)\right)\right)}{a^2}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2/x^5,x]
```

```
[Out] (-Log[c*(a + b*x^2)^p]^2 + (b*x^2*(2*b*p^2*x^2*(2*Log[x] - Log[a + b*x^2])
- 2*a*p*Log[c*(a + b*x^2)^p] + b*x^2*Log[c*(a + b*x^2)^p]^2 - 2*b*p*x^2*(Lo
g[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p] + p*PolyLog[2, 1 + (b*x^2)/a])))/a^2)/
(4*x^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.30, size = 1080, normalized size = 8.37

method	result	size
risch	Expression too large to display	1080

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*p^2*ln(x)/a^2-1/4*I*p*b/a/x^2*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+1/4*
I*p*b^2/a^2*ln(b*x^2+a)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-1/4*
I*p*b/a/x^2*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+1/4*I*p*b^2/a^2*
ln(b*x^2+a)*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-1/2*I*p*b^2/a^2*ln(x)*Pi*c
sgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-1/2*I*p*b^2/a^2*ln(x)*Pi*csgn(I*(b*x^2+a)^
p)*csgn(I*c*(b*x^2+a)^p)^2-1/4*I/x^4*ln((b*x^2+a)^p)*Pi*csgn(I*c*(b*x^2+a)^
p)^2*csgn(I*c)-1/4*p^2*b^2/a^2*ln(b*x^2+a)^2-1/2*p^2*b^2/a^2*ln(b*x^2+a)+p^
2*b^2/a^2*dilog((-b*x+(-b*a)^(1/2))/(-b*a)^(1/2))+p^2*b^2/a^2*dilog((b*x+(-
b*a)^(1/2))/(-b*a)^(1/2))+1/2*p*b^2*ln((b*x^2+a)^p)/a^2*ln(b*x^2+a)+1/2*I*p
*b^2/a^2*ln(x)*Pi*csgn(I*c*(b*x^2+a)^p)^3+1/4*I/x^4*ln((b*x^2+a)^p)*Pi*csgn
(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+1/4*I*p*b/a/x^2*Pi*csgn(I*c
*(b*x^2+a)^p)^3+p^2*b^2/a^2*ln(x)*ln((-b*x+(-b*a)^(1/2))/(-b*a)^(1/2))+p^2*
b^2/a^2*ln(x)*ln((b*x+(-b*a)^(1/2))/(-b*a)^(1/2))-1/4/x^4*ln((b*x^2+a)^p)^2
+1/4*I/x^4*ln((b*x^2+a)^p)*Pi*csgn(I*c*(b*x^2+a)^p)^3-1/2*p*b*ln((b*x^2+a)^
p)/a/x^2-p*b^2*ln((b*x^2+a)^p)/a^2*ln(x)-1/2*p*b/a/x^2*ln(c)-p*b^2/a^2*ln(x
)*ln(c)+1/2*p*b^2/a^2*ln(b*x^2+a)*ln(c)-1/4*I*p*b^2/a^2*ln(b*x^2+a)*Pi*csgn
(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-1/4*I*p*b^2/a^2*ln(b*x^2+a)
*Pi*csgn(I*c*(b*x^2+a)^p)^3-1/2/x^4*ln((b*x^2+a)^p)*ln(c)-1/4*I/x^4*ln((b*x
^2+a)^p)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+1/2*I*p*b^2/a^2*ln(
x)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+1/4*I*p*b/a/x^2*P
i*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-1/16*(I*Pi*csgn(I*(b*
x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+
a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*c
sgn(I*c)+2*ln(c))^2/x^4
```

Maxima [A]
time = 0.29, size = 142, normalized size = 1.10

$$-\frac{1}{4}b^2p^2\left(\frac{\log(bx^2+a)^2}{a^2}-\frac{2\left(2\log\left(\frac{bx^2}{a}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{bx^2}{a}\right)\right)}{a^2}+\frac{2\log(bx^2+a)}{a^2}-\frac{4\log(x)}{a^2}\right)+\frac{1}{2}bp\left(\frac{b\log(bx^2+a)}{a^2}-\frac{b\log(x^2)}{a^2}-\frac{1}{ax^2}\right)\log\left((bx^2+a)^pc\right)-\frac{\log\left((bx^2+a)^pc\right)^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*p^2*(log(b*x^2 + a)^2/a^2 - 2*(2*log(b*x^2/a + 1)*log(x) + dilog(-
b*x^2/a))/a^2 + 2*log(b*x^2 + a)/a^2 - 4*log(x)/a^2) + 1/2*b*p*(b*log(b*x^2
 + a)/a^2 - b*log(x^2)/a^2 - 1/(a*x^2))*log((b*x^2 + a)^p*c) - 1/4*log((b*x
^2 + a)^p*c)^2/x^4
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^5,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^2/x^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(c(a+bx^2)^p\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**2/x**5,x)
[Out] Integral(log(c*(a + b*x**2)**p)**2/x**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^5,x, algorithm="giac")
[Out] integrate(log((b*x^2 + a)^p*c)^2/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)^2/x^5,x)
[Out] int(log(c*(a + b*x^2)^p)^2/x^5, x)
```

3.83 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^7}\,dx$

Optimal. Leaf size=193

$$-\frac{b^2p^2}{6a^2x^2}-\frac{b^3p^2\log(x)}{a^3}+\frac{b^3p^2\log\left(a+bx^2\right)}{6a^3}-\frac{bp\log\left(c\left(a+bx^2\right)^p\right)}{6ax^4}+\frac{b^2p\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{3a^3x^2}-\frac{\log^2(c(a+}{6x^6}$$

```
[Out] -1/6*b^2*p^2/a^2/x^2-b^3*p^2*ln(x)/a^3+1/6*b^3*p^2*ln(b*x^2+a)/a^3-1/6*b*p*
ln(c*(b*x^2+a)^p)/a/x^4+1/3*b^2*p*(b*x^2+a)*ln(c*(b*x^2+a)^p)/a^3/x^2-1/6*l
n(c*(b*x^2+a)^p)^2/x^6+1/3*b^3*p*ln(c*(b*x^2+a)^p)*ln(1-a/(b*x^2+a))/a^3-1/
3*b^3*p^2*polylog(2,a/(b*x^2+a))/a^3
```

Rubi [A]
time = 0.24, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {2504, 2445, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$-\frac{b^3p^2\text{PolyLog}\left(2,\frac{a}{a+bx^2}\right)}{3a^3}+\frac{b^3p\log\left(1-\frac{a}{a+bx^2}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3a^3}+\frac{b^3p^2\log\left(a+bx^2\right)}{6a^3}-\frac{b^3p^2\log(x)}{a^3}+\frac{b^2p\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{3a^3x^2}-\frac{b^2p^2}{6a^2x^2}-\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{6x^6}-\frac{bp\log\left(c\left(a+bx^2\right)^p\right)}{6ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^2/x^7,x]
```

```
[Out] -1/6*(b^2*p^2)/(a^2*x^2) - (b^3*p^2*Log[x])/a^3 + (b^3*p^2*Log[a + b*x^2])/
(6*a^3) - (b*p*Log[c*(a + b*x^2)^p])/(6*a*x^4) + (b^2*p*(a + b*x^2)*Log[c*(
a + b*x^2)^p])/(3*a^3*x^2) - Log[c*(a + b*x^2)^p]^2/(6*x^6) + (b^3*p*Log[c*
(a + b*x^2)^p]*Log[1 - a/(a + b*x^2)])/(3*a^3) - (b^3*p^2*PolyLog[2, a/(a +
 b*x^2)])/(3*a^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
```

```
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^7}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\log^2\left(c(a+bx)^p\right)}{x^4}\,dx, x, x^2\right) \\
&= -\frac{\log^2\left(c(a+bx^2)^p\right)}{6x^6} + \frac{1}{3}(bp)\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x^3(a+bx)}\,dx, x, x^2\right) \\
&= -\frac{\log^2\left(c(a+bx^2)^p\right)}{6x^6} + \frac{1}{3}p\text{Subst}\left(\int \frac{\log\left(cx^p\right)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx, x, a+bx^2\right) \\
&= -\frac{\log^2\left(c(a+bx^2)^p\right)}{6x^6} + \frac{p\text{Subst}\left(\int \frac{\log(cx^p)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx, x, a+bx^2\right)}{3a} - \frac{(bp)\text{Subst}\left(\int \frac{\log(c}{x(-\frac{a}{b}-}\right.}{3} \\
&= -\frac{bp\log\left(c(a+bx^2)^p\right)}{6ax^4} - \frac{\log^2\left(c(a+bx^2)^p\right)}{6x^6} - \frac{(bp)\text{Subst}\left(\int \frac{\log(cx^p)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+b\right.}{3a^2} \\
&= -\frac{bp\log\left(c(a+bx^2)^p\right)}{6ax^4} + \frac{b^2p(a+bx^2)\log\left(c(a+bx^2)^p\right)}{3a^3x^2} - \frac{\log^2\left(c(a+bx^2)^p\right)}{6x^6} + \\
&= -\frac{b^2p^2}{6a^2x^2} - \frac{b^3p^2\log(x)}{a^3} + \frac{b^3p^2\log\left(a+bx^2\right)}{6a^3} - \frac{bp\log\left(c(a+bx^2)^p\right)}{6ax^4} + \frac{b^2p(a+bx^2}{} \\
&= -\frac{b^2p^2}{6a^2x^2} - \frac{b^3p^2\log(x)}{a^3} + \frac{b^3p^2\log\left(a+bx^2\right)}{6a^3} - \frac{bp\log\left(c(a+bx^2)^p\right)}{6ax^4} + \frac{b^2p(a+bx^2}{}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 205, normalized size = 1.06

$$-\frac{b^2p^2}{6a^2x^2} - \frac{b^3p^2\log(x)}{a^3} + \frac{b^3p^2\log\left(a+bx^2\right)}{2a^3} - \frac{bp\log\left(c(a+bx^2)^p\right)}{6ax^4} + \frac{b^2p\log\left(c(a+bx^2)^p\right)}{3a^2x^2} + \frac{b^3p\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{3a^3} - \frac{b^3\log^2\left(c(a+bx^2)^p\right)}{6a^3} - \frac{\log^2\left(c(a+bx^2)^p\right)}{6x^6} + \frac{b^3p^2\text{Li}_2\left(\frac{a+bx^2}{a}\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2/x^7,x]
```

```
[Out] -1/6*(b^2*p^2)/(a^2*x^2) - (b^3*p^2*Log[x])/a^3 + (b^3*p^2*Log[a + b*x^2])/
(2*a^3) - (b*p*Log[c*(a + b*x^2)^p])/(6*a*x^4) + (b^2*p*Log[c*(a + b*x^2)^p
```

```
])/(3*a^2*x^2) + (b^3*p*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/(3*a^3) - (
b^3*Log[c*(a + b*x^2)^p]^2)/(6*a^3) - Log[c*(a + b*x^2)^p]^2/(6*x^6) + (b^3
*p^2*PolyLog[2, (a + b*x^2)/a])/(3*a^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.35, size = 1289, normalized size = 6.68

method	result	size
risch	Expression too large to display	1289

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^2/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*I*p*b^2/a^2/x^2*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)
+1/6*I*p*b^2/a^2/x^2*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-1/12*I*
p*b/a/x^4*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-1/12*I*p*b/a/x^4*Pi*csgn(I*(
b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-1/6*I*p*b^3/a^3*ln(b*x^2+a)*Pi*csgn(I*(
b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-b^3*p^2*ln(x)/a^3+1/2*b^3*p^2*ln(b*x^2+
a)/a^3+1/3*I*p*b^3/a^3*ln(x)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2
+1/6*I*p*b^2/a^2/x^2*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+1/6*I/x^6*ln((b*x
^2+a)^p)*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-1/6*I*p*b^3
/a^3*ln(b*x^2+a)*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+1/3*I*p*b^3/a^3*ln(x)
*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+1/12*I*p*b/a/x^4*Pi*csgn(I*c*(b*x^2+a
)^p)^3+1/12*I*p*b/a/x^4*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I
*c)-1/24*(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x
^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*
csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c))^2/x^6-1/6*I/x^6*ln((b*x^2+a)^p)*
Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-1/3*I*p*b^3/a^3*ln(x)*Pi*csgn(I*c*(b*x
^2+a)^p)^3-2/3*p^2*b^3/a^3*dilog((-b*x+(-b*a)^(1/2))/(-b*a)^(1/2))-2/3*p^2*
b^3/a^3*dilog((b*x+(-b*a)^(1/2))/(-b*a)^(1/2))+1/6*p^2*b^3/a^3*ln(b*x^2+a)^
2-1/6*p*b/a/x^4*ln(c)+1/3*p*b^2/a^2/x^2*ln(c)-1/6*I/x^6*ln((b*x^2+a)^p)*Pi*
csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+1/6*I*p*b^3/a^3*ln(b*x^2+a)*Pi*
csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-1/3/x^6*ln((b*x^2+a)^p)
*ln(c)-1/6*b^2*p^2/a^2/x^2-1/6*I*p*b^2/a^2/x^2*Pi*csgn(I*c*(b*x^2+a)^p)^3-2
/3*p^2*b^3/a^3*ln(x)*ln((-b*x+(-b*a)^(1/2))/(-b*a)^(1/2))-2/3*p^2*b^3/a^3*l
n(x)*ln((b*x+(-b*a)^(1/2))/(-b*a)^(1/2))-1/6/x^6*ln((b*x^2+a)^p)^2+2/3*p*b^
3/a^3*ln(x)*ln(c)-1/3*p*b^3/a^3*ln(b*x^2+a)*ln(c)-1/3*I*p*b^3/a^3*ln(x)*Pi*
csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+1/6*I*p*b^3/a^3*ln(b*x^
2+a)*Pi*csgn(I*c*(b*x^2+a)^p)^3-1/6*p*b*ln((b*x^2+a)^p)/a/x^4+2/3*p*b^3*ln(
(b*x^2+a)^p)/a^3*ln(x)+1/6*I/x^6*ln((b*x^2+a)^p)*Pi*csgn(I*c*(b*x^2+a)^p)^3
-1/3*p*b^3*ln((b*x^2+a)^p)/a^3*ln(b*x^2+a)+1/3*p*b^2*ln((b*x^2+a)^p)/a^2/x^
2
```

Maxima [A]

time = 0.29, size = 173, normalized size = 0.90

$$-\frac{1}{6}\,b^2p^2\left(\frac{2\left(2\,\log\left(\frac{bx^2}{a}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{bx^2}{a}\right)\right)b}{a^3}-\frac{3\,b\log\left(bx^2+a\right)}{a^3}-\frac{bx^2\log\left(bx^2+a\right)^2-6\,bx^2\log\left(x\right)-a}{a^3x^2}\right)-\frac{1}{6}\,bp\left(\frac{2\,b^2\log\left(bx^2+a\right)}{a^3}-\frac{2\,b^2\log\left(x^2\right)}{a^3}-\frac{2\,bx^2-a}{a^2x^4}\right)\log\left(\left(bx^2+a\right)^pc\right)-\frac{\log\left(\left(bx^2+a\right)^pc\right)^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^7,x, algorithm="maxima")
```

```
[Out] -1/6*b^2*p^2*(2*(2*log(b*x^2/a + 1)*log(x) + dilog(-b*x^2/a))*b/a^3 - 3*b*l
og(b*x^2 + a)/a^3 - (b*x^2*log(b*x^2 + a)^2 - 6*b*x^2*log(x) - a)/(a^3*x^2)
) - 1/6*b*p*(2*b^2*log(b*x^2 + a)/a^3 - 2*b^2*log(x^2)/a^3 - (2*b*x^2 - a)/
(a^2*x^4))*log((b*x^2 + a)^p*c) - 1/6*log((b*x^2 + a)^p*c)^2/x^6
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^7,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^2/x^7, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c(a+bx^2)^p\right)^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**2/x**7,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**2/x**7, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^7,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^2/x^7, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)^2/x^7,x)
[Out] int(log(c*(a + b*x^2)^p)^2/x^7, x)
```

3.84 $\int x^4 \log^2\left(c\left(a+bx^2\right)^p\right) dx$

__Optimal__. Leaf size=336

$$\frac{184a^2p^2x}{75b^2}-\frac{64ap^2x^3}{225b}+\frac{8p^2x^5}{125}-\frac{184a^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{75b^{5/2}}+\frac{4ia^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{5b^{5/2}}+\frac{8a^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\mathrm{l}}{5b^{5/2}}$$

```
[Out] 184/75*a^2*p^2*x/b^2-64/225*a*p^2*x^3/b+8/125*p^2*x^5-184/75*a^(5/2)*p^2*ar
ctan(x*b^(1/2)/a^(1/2))/b^(5/2)+4/5*I*a^(5/2)*p^2*arctan(x*b^(1/2)/a^(1/2))
^2/b^(5/2)-4/5*a^2*p*x*ln(c*(b*x^2+a)^p)/b^2+4/15*a*p*x^3*ln(c*(b*x^2+a)^p)
/b-4/25*p*x^5*ln(c*(b*x^2+a)^p)+4/5*a^(5/2)*p*arctan(x*b^(1/2)/a^(1/2))*ln(
c*(b*x^2+a)^p)/b^(5/2)+1/5*x^5*ln(c*(b*x^2+a)^p)^2+8/5*a^(5/2)*p^2*arctan(x
*b^(1/2)/a^(1/2))*ln(2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))/b^(5/2)+4/5*I*a^(5/2)
*p^2*polylog(2,1-2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))/b^(5/2)
```

__Rubi__ [A]

time = 0.26, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {2507, 2526, 2498, 327, 211, 2505, 308, 2520, 12, 5040, 4964, 2449, 2352}

$$\frac{4ia^{5/2}p^2\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{5b^{5/2}}+\frac{4a^{5/2}p\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{5b^{5/2}}+\frac{4ia^{5/2}p^2\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{5b^{5/2}}-\frac{184a^{5/2}p^2\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{75b^{5/2}}+\frac{8a^{5/2}p^2\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{5b^{5/2}}-\frac{4a^2px\log\left(c\left(a+bx^2\right)^p\right)}{5b^2}+\frac{184a^2p^2x}{75b^2}+\frac{1}{5}x^5\log^2\left(c\left(a+bx^2\right)^p\right)-\frac{4}{25}px^5\log\left(c\left(a+bx^2\right)^p\right)+\frac{4apx^3\log\left(c\left(a+bx^2\right)^p\right)}{15b}-\frac{64ap^2x^3}{225b}+\frac{8p^2x^5}{125}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] (184*a^2*p^2*x)/(75*b^2) - (64*a*p^2*x^3)/(225*b) + (8*p^2*x^5)/125 - (184*
a^(5/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(75*b^(5/2)) + (((4*I)/5)*a^(5/2)*
p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/b^(5/2) + (8*a^(5/2)*p^2*ArcTan[(Sqrt[b]
*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/(5*b^(5/2)) - (4*a^2
*p*x*Log[c*(a + b*x^2)^p])/(5*b^2) + (4*a*p*x^3*Log[c*(a + b*x^2)^p])/(15*b
) - (4*p*x^5*Log[c*(a + b*x^2)^p])/25 + (4*a^(5/2)*p*ArcTan[(Sqrt[b]*x)/Sqr
t[a]]*Log[c*(a + b*x^2)^p])/(5*b^(5/2)) + (x^5*Log[c*(a + b*x^2)^p]^2)/5 +
(((4*I)/5)*a^(5/2)*p^2*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])
/b^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int x^4 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx = \frac{1}{5}x^5\log^2\left(c\left(a+bx^2\right)^p\right) - \frac{1}{5}(4bp)\int \frac{x^6\log\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx$$

$$= \frac{1}{5}x^5\log^2\left(c\left(a+bx^2\right)^p\right) - \frac{1}{5}(4bp)\int \left(\frac{a^2\log\left(c\left(a+bx^2\right)^p\right)}{b^3} - \frac{ax^2\log\left(c\left(a+bx^2\right.\right.}{b^2}\right.$$

$$= \frac{1}{5}x^5\log^2\left(c\left(a+bx^2\right)^p\right) - \frac{1}{5}(4p)\int x^4\log\left(c\left(a+bx^2\right)^p\right)\,dx - \frac{(4a^2p)\int\log\left(c(a\right.}{5b^2}$$

$$= -\frac{4a^2px\log\left(c\left(a+bx^2\right)^p\right)}{5b^2} + \frac{4apx^3\log\left(c\left(a+bx^2\right)^p\right)}{15b} - \frac{4}{25}px^5\log\left(c\left(a+bx^2\right)^p\right)$$

$$= \frac{8a^2p^2x}{5b^2} - \frac{4a^2px\log\left(c\left(a+bx^2\right)^p\right)}{5b^2} + \frac{4apx^3\log\left(c\left(a+bx^2\right)^p\right)}{15b} - \frac{4}{25}px^5\log\left(c(a\right.$$

$$= \frac{184a^2p^2x}{75b^2} - \frac{64ap^2x^3}{225b} + \frac{8p^2x^5}{125} - \frac{8a^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{5b^{5/2}} + \frac{4ia^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}}\right.}{5b^{5/2}}$$

$$= \frac{184a^2p^2x}{75b^2} - \frac{64ap^2x^3}{225b} + \frac{8p^2x^5}{125} - \frac{184a^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{75b^{5/2}} + \frac{4ia^{5/2}p^2\tan^{-1}\left(\right.}{5b^{5/2}}$$

$$= \frac{184a^2p^2x}{75b^2} - \frac{64ap^2x^3}{225b} + \frac{8p^2x^5}{125} - \frac{184a^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{75b^{5/2}} + \frac{4ia^{5/2}p^2\tan^{-1}\left(\right.}{5b^{5/2}}$$

$$= \frac{184a^2p^2x}{75b^2} - \frac{64ap^2x^3}{225b} + \frac{8p^2x^5}{125} - \frac{184a^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{75b^{5/2}} + \frac{4ia^{5/2}p^2\tan^{-1}\left(\right.}{5b^{5/2}}$$

Mathematica [A]
time = 0.14, size = 248, normalized size = 0.74

$$\frac{900ia^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2 + 60a^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-46p+30p\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)+15\log\left(c\left(a+bx^2\right)^p\right)\right) + \sqrt{b}\,x\left(8p^2(345a^2-40abx^2+9b^2x^4)-60p(15a^2-5abx^2+3b^2x^4)\log\left(c\left(a+bx^2\right)^p\right)+225b^2x^4\log^2\left(c\left(a+bx^2\right)^p\right)\right) + 900ia^{5/2}p^2\mathrm{Li}_2\left(\frac{i\sqrt{a}+\sqrt{b}\,x}{-i\sqrt{a}+\sqrt{b}\,x}\right)}{1125b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] ((900*I)*a^(5/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 + 60*a^(5/2)*p*ArcTan[(S
qrt[b]*x)/Sqrt[a]]*(-46*p + 30*p*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)] +
 15*Log[c*(a + b*x^2)^p]) + Sqrt[b]*x*(8*p^2*(345*a^2 - 40*a*b*x^2 + 9*b^2*
x^4) - 60*p*(15*a^2 - 5*a*b*x^2 + 3*b^2*x^4)*Log[c*(a + b*x^2)^p] + 225*b^2
*x^4*Log[c*(a + b*x^2)^p]^2) + (900*I)*a^(5/2)*p^2*PolyLog[2, (I*Sqrt[a] +
Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)])/(1125*b^(5/2))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int x^{4} \ln \left(c\left(b\,x^{2}+a\right)^{p}\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*ln(c*(b*x^2+a)^p)^2,x)
```

```
[Out] int(x^4*ln(c*(b*x^2+a)^p)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] 1/5*p^2*x^5*log(b*x^2 + a)^2 + integrate(1/5*(5*b*x^6*log(c)^2 + 5*a*x^4*lo
g(c)^2 - 2*((2*p^2 - 5*p*log(c))*b*x^6 - 5*a*p*x^4*log(c))*log(b*x^2 + a))/
(b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(x^4*log((b*x^2 + a)^p*c)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{4} \log \left(c\left(a+bx^{2}\right)^{p}\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Integral(x**4*log(c*(a + b*x**2)**p)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4*log((b*x^2 + a)^p*c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \ln\left(c\left(b\,x^2+a\right)^p\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*log(c*(a + b*x^2)^p)^2,x)
```

```
[Out] int(x^4*log(c*(a + b*x^2)^p)^2, x)
```

3.85 $\int x^2 \log^2\left(c(a+bx^2)^p\right) dx$

Optimal. Leaf size=294

$$-\frac{32ap^2x}{9b}+\frac{8p^2x^3}{27}+\frac{32a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{9b^{3/2}}-\frac{4ia^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{3b^{3/2}}-\frac{8a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\ldots}\right.}{3b^{3/2}}$$

```
[Out] -32/9*a*p^2*x/b+8/27*p^2*x^3+32/9*a^(3/2)*p^2*arctan(x*b^(1/2)/a^(1/2))/b^(
3/2)-4/3*I*a^(3/2)*p^2*arctan(x*b^(1/2)/a^(1/2))^2/b^(3/2)+4/3*a*p*x*ln(c*(
b*x^2+a)^p)/b-4/9*p*x^3*ln(c*(b*x^2+a)^p)-4/3*a^(3/2)*p*arctan(x*b^(1/2)/a^
(1/2))*ln(c*(b*x^2+a)^p)/b^(3/2)+1/3*x^3*ln(c*(b*x^2+a)^p)^2-8/3*a^(3/2)*p^
2*arctan(x*b^(1/2)/a^(1/2))*ln(2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))/b^(3/2)-4/3
*I*a^(3/2)*p^2*polylog(2,1-2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))/b^(3/2)
```

Rubi [A]
time = 0.22, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {2507, 2526, 2498, 327, 211, 2505, 308, 2520, 12, 5040, 4964, 2449, 2352}

$$-\frac{4ia^{3/2}p^2\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{3b^{3/2}}-\frac{4a^{3/2}p\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{3b^{3/2}}-\frac{4ia^{3/2}p^2\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{3b^{3/2}}+\frac{32a^{3/2}p^2\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{9b^{3/2}}-\frac{8a^{3/2}p^2\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{3b^{3/2}}+\frac{4apx\log\left(c(a+bx^2)^p\right)}{3b}+\frac{1}{3}x^3\log^2\left(c(a+bx^2)^p\right)-\frac{4}{9}px^3\log\left(c(a+bx^2)^p\right)-\frac{32ap^2x}{9b}+\frac{8p^2x^3}{27}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] (-32*a*p^2*x)/(9*b) + (8*p^2*x^3)/27 + (32*a^(3/2)*p^2*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(9*b^(3/2)) - (((4*I)/3)*a^(3/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2
)/b^(3/2) - (8*a^(3/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqr
t[a] + I*Sqrt[b]*x)])/(3*b^(3/2)) + (4*a*p*x*Log[c*(a + b*x^2)^p])/(3*b) -
(4*p*x^3*Log[c*(a + b*x^2)^p])/9 - (4*a^(3/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
*Log[c*(a + b*x^2)^p])/(3*b^(3/2)) + (x^3*Log[c*(a + b*x^2)^p]^2)/3 - (((4*
I)/3)*a^(3/2)*p^2*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/b^(3
/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx &= \frac{1}{3}x^3\log^2\left(c\left(a+bx^2\right)^p\right) - \frac{1}{3}(4bp)\int \frac{x^4\log\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx \\
&= \frac{1}{3}x^3\log^2\left(c\left(a+bx^2\right)^p\right) - \frac{1}{3}(4bp)\int \left(-\frac{a\log\left(c\left(a+bx^2\right)^p\right)}{b^2} + \frac{x^2\log\left(c(a+bx^2\right.}{b}\right. \\
&= \frac{1}{3}x^3\log^2\left(c\left(a+bx^2\right)^p\right) - \frac{1}{3}(4p)\int x^2\log\left(c\left(a+bx^2\right)^p\right)\,dx + \frac{(4ap)\int\log\left(c(a+\right.}{3b} \\
&= \frac{4apx\log\left(c\left(a+bx^2\right)^p\right)}{3b} - \frac{4}{9}px^3\log\left(c\left(a+bx^2\right)^p\right) - \frac{4a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\right.}{3b^{3/2}} \\
&= -\frac{8ap^2x}{3b} + \frac{4apx\log\left(c\left(a+bx^2\right)^p\right)}{3b} - \frac{4}{9}px^3\log\left(c\left(a+bx^2\right)^p\right) - \frac{4a^{3/2}p\tan^{-1}\left(\right.}{} \\
&= -\frac{32ap^2x}{9b} + \frac{8p^2x^3}{27} + \frac{8a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3b^{3/2}} - \frac{4ia^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{3b^{3/2}} + \frac{4ap}{} \\
&= -\frac{32ap^2x}{9b} + \frac{8p^2x^3}{27} + \frac{32a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{9b^{3/2}} - \frac{4ia^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{3b^{3/2}} - \frac{8a}{} \\
&= -\frac{32ap^2x}{9b} + \frac{8p^2x^3}{27} + \frac{32a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{9b^{3/2}} - \frac{4ia^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{3b^{3/2}} - \frac{8a}{} \\
&= -\frac{32ap^2x}{9b} + \frac{8p^2x^3}{27} + \frac{32a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{9b^{3/2}} - \frac{4ia^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{3b^{3/2}} - \frac{8a}{}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 223, normalized size = 0.76

$$\frac{-36ia^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2 - 12a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-8p+6p\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)+3\log\left(c\left(a+bx^2\right)^p\right)\right)+\sqrt{b}\,x\left(8p^2\left(-12a+bx^2\right)+12p\left(3a-bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)+9bx^2\log^2\left(c\left(a+bx^2\right)^p\right)\right)-36ia^{3/2}p^2\text{Li}_2\left(\frac{i\sqrt{a}+\sqrt{b}\,x}{-i\sqrt{a}+\sqrt{b}\,x}\right)}{27b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] ((-36*I)*a^(3/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 - 12*a^(3/2)*p*ArcTan[(S
qrt[b]*x)/Sqrt[a]]*(-8*p + 6*p*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)] + 3
*Log[c*(a + b*x^2)^p]) + Sqrt[b]*x*(8*p^2*(-12*a + b*x^2) + 12*p*(3*a - b*x
^2)*Log[c*(a + b*x^2)^p] + 9*b*x^2*Log[c*(a + b*x^2)^p]^2) - (36*I)*a^(3/2)
*p^2*PolyLog[2, (I*Sqrt[a] + Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)])/(27*b^
(3/2))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int x^{2} \ln \left(c \left(b \,x^{2}+a \right)^{p}\right)^{2}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(b*x^2+a)^p)^2,x)
```

```
[Out] int(x^2*ln(c*(b*x^2+a)^p)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] 1/3*p^2*x^3*log(b*x^2 + a)^2 + integrate(1/3*(3*b*x^4*log(c)^2 + 3*a*x^2*lo
g(c)^2 - 2*((2*p^2 - 3*p*log(c))*b*x^4 - 3*a*p*x^2*log(c))*log(b*x^2 + a))/
(b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*log((b*x^2 + a)^p*c)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2} \log{\left(c \left(a + b x^{2}\right)^{p} \right)}^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Integral(x**2*log(c*(a + b*x**2)**p)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*log((b*x^2 + a)^p*c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \ln\left(c\left(b\,x^2+a\right)^p\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(c*(a + b*x^2)^p)^2,x)
```

```
[Out] int(x^2*log(c*(a + b*x^2)^p)^2, x)
```

3.86 $\int \log^2\left(c\left(a+bx^2\right)^p\right)\,dx$

__Optimal__. Leaf size=237

$$8p^2x-\frac{8\sqrt{a}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{4i\sqrt{a}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{b}}+\frac{8\sqrt{a}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{\sqrt{b}}-4px$$

```
[Out] 8*p^2*x-4*p*x*ln(c*(b*x^2+a)^p)+x*ln(c*(b*x^2+a)^p)^2-8*p^2*arctan(x*b^(1/2
)/a^(1/2))*a^(1/2)/b^(1/2)+4*I*p^2*arctan(x*b^(1/2)/a^(1/2))^2*a^(1/2)/b^(1
/2)+4*p*arctan(x*b^(1/2)/a^(1/2))*ln(c*(b*x^2+a)^p)*a^(1/2)/b^(1/2)+8*p^2*a
rctan(x*b^(1/2)/a^(1/2))*ln(2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))*a^(1/2)/b^(1/2
)+4*I*p^2*polylog(2,1-2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))*a^(1/2)/b^(1/2)
```

__Rubi__ [A]
time = 0.17, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {2500, 2526, 2498, 327, 211, 2520, 12, 5040, 4964, 2449, 2352}

$$\frac{4i\sqrt{a}\,p^2\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{\sqrt{b}}+\frac{4\sqrt{a}\,p\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{b}}+\frac{4i\sqrt{a}\,p^2\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{b}}-\frac{8\sqrt{a}\,p^2\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{8\sqrt{a}\,p^2\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{\sqrt{b}}+x\log^2\left(c\left(a+bx^2\right)^p\right)-4px\log\left(c\left(a+bx^2\right)^p\right)+8p^2x$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] 8*p^2*x - (8*Sqrt[a]*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + ((4*I)*Sqrt
[a]*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/Sqrt[b] + (8*Sqrt[a]*p^2*ArcTan[(Sqr
t[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/Sqrt[b] - 4*p*x*
Log[c*(a + b*x^2)^p] + (4*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[c*(a +
b*x^2)^p])/Sqrt[b] + x*Log[c*(a + b*x^2)^p]^2 + ((4*I)*Sqrt[a]*p^2*PolyLog[
2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/Sqrt[b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2500

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[x^n*(
(a + b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
```

```
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \log^2\left(c\left(a+bx^2\right)^p\right)\,dx = x\log^2\left(c\left(a+bx^2\right)^p\right) - (4bp)\int \frac{x^2\log\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx$$

$$= x\log^2\left(c\left(a+bx^2\right)^p\right) - (4bp)\int \left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{b} - \frac{a\log\left(c\left(a+bx^2\right)^p\right)}{b\left(a+bx^2\right)}\right)\,dx$$

$$= x\log^2\left(c\left(a+bx^2\right)^p\right) - (4p)\int \log\left(c\left(a+bx^2\right)^p\right)\,dx + (4ap)\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,d$$

$$= -4px\log\left(c\left(a+bx^2\right)^p\right) + \frac{4\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{b}} + x\log^2\left(c(a+\right.$$

$$= 8p^2x - 4px\log\left(c\left(a+bx^2\right)^p\right) + \frac{4\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{b}} + x\log^2\left(\right.$$

$$= 8p^2x - \frac{8\sqrt{a}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{4i\sqrt{a}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{b}} - 4px\log\left(c\left(a+bx^2\right)\right.$$

$$= 8p^2x - \frac{8\sqrt{a}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{4i\sqrt{a}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{b}} + \frac{8\sqrt{a}\,p^2\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}}\right.}{}$$

$$= 8p^2x - \frac{8\sqrt{a}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{4i\sqrt{a}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{b}} + \frac{8\sqrt{a}\,p^2\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}}\right.}{}$$

$$= 8p^2x - \frac{8\sqrt{a}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{4i\sqrt{a}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{b}} + \frac{8\sqrt{a}\,p^2\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}}\right.}{}$$

Mathematica [A]
time = 0.06, size = 193, normalized size = 0.81

$$\frac{4i\sqrt{a}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2 + 4\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-2p + 2p\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right) + \log\left(c\left(a+bx^2\right)^p\right)\right) + \sqrt{b}\,x\left(8p^2 - 4p\log\left(c\left(a+bx^2\right)^p\right) + \log^2\left(c\left(a+bx^2\right)^p\right)\right) + 4i\sqrt{a}\,p^2\text{Li}_2\left(\frac{i\sqrt{a}+\sqrt{b}\,x}{-i\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] ((4*I)*Sqrt[a]*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 + 4*Sqrt[a]*p*ArcTan[(Sqrt
[b]*x)/Sqrt[a]]*(-2*p + 2*p*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)] + Log[
c*(a + b*x^2)^p]) + Sqrt[b]*x*(8*p^2 - 4*p*Log[c*(a + b*x^2)^p] + Log[c*(a
+ b*x^2)^p]^2) + (4*I)*Sqrt[a]*p^2*PolyLog[2, (I*Sqrt[a] + Sqrt[b]*x)/((-I)
*Sqrt[a] + Sqrt[b]*x)])/Sqrt[b]
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \ln \left(c \left(b\,x^{2}+a \right)^{p}\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^2,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] p^2*x*log(b*x^2 + a)^2 + integrate((b*x^2*log(c)^2 + a*log(c)^2 - 2*((2*p^2
 - p*log(c))*b*x^2 - a*p*log(c))*log(b*x^2 + a))/(b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log \left(c\left(a+bx^{2}\right)^{p}\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\left(b\,x^2+a\right)^p\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)^2,x)
```

```
[Out] int(log(c*(a + b*x^2)^p)^2, x)
```

3.87 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^2} \, dx$

Optimal. Leaf size=190

$$\frac{4i\sqrt{b}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}}+\frac{8\sqrt{b}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{\sqrt{a}}+\frac{4\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c(a+bx^2\right.}{\sqrt{a}}$$

```
[Out] -ln(c*(b*x^2+a)^p)^2/x+4*I*p^2*arctan(x*b^(1/2)/a^(1/2))^2*b^(1/2)/a^(1/2)+
4*p*arctan(x*b^(1/2)/a^(1/2))*ln(c*(b*x^2+a)^p)*b^(1/2)/a^(1/2)+8*p^2*arcta
n(x*b^(1/2)/a^(1/2))*ln(2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))*b^(1/2)/a^(1/2)+4*
I*p^2*polylog(2,1-2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))*b^(1/2)/a^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {2507, 211, 2520, 12, 5040, 4964, 2449, 2352}

$$\frac{4i\sqrt{b}\,p^2\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{\sqrt{a}}+\frac{4\sqrt{b}\,p\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{a}}+\frac{4i\sqrt{b}\,p^2\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}}+\frac{8\sqrt{b}\,p^2\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{\sqrt{a}}-\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^2/x^2,x]
```

```
[Out] ((4*I)*Sqrt[b]*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/Sqrt[a] + (8*Sqrt[b]*p^2*
ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/Sqrt[
a] + (4*Sqrt[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[c*(a + b*x^2)^p])/Sqrt[a]
 - Log[c*(a + b*x^2)^p]^2/x + ((4*I)*Sqrt[b]*p^2*PolyLog[2, 1 - (2*Sqrt[a])
/(Sqrt[a] + I*Sqrt[b]*x)])/Sqrt[a]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^2}\,dx = -\frac{\log^2\left(c(a+bx^2)^p\right)}{x} + (4bp)\int \frac{\log\left(c(a+bx^2)^p\right)}{a+bx^2}\,dx$$

$$= \frac{4\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{\sqrt{a}} - \frac{\log^2\left(c(a+bx^2)^p\right)}{x} - (8b^2p^2)\int \frac{x\tan}{\sqrt{a}\,\sqrt{}}$$

$$= \frac{4\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{\sqrt{a}} - \frac{\log^2\left(c(a+bx^2)^p\right)}{x} - \frac{(8b^{3/2}p^2)\int \frac{x\tan^{-}}{}}{\sqrt{a}}$$

$$= \frac{4i\sqrt{b}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}} + \frac{4\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{\sqrt{a}} - \frac{\log^2\left(c(a\right.}{x}$$

$$= \frac{4i\sqrt{b}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}} + \frac{8\sqrt{b}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{\sqrt{a}} + \frac{4\sqrt{b}\,p}{}$$

$$= \frac{4i\sqrt{b}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}} + \frac{8\sqrt{b}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{\sqrt{a}} + \frac{4\sqrt{b}\,p}{}$$

$$= \frac{4i\sqrt{b}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}} + \frac{8\sqrt{b}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{\sqrt{a}} + \frac{4\sqrt{b}\,p}{}$$

Mathematica [A]
time = 0.04, size = 173, normalized size = 0.91

$$\frac{4i\sqrt{b}\,p^2x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2 - \sqrt{a}\,\log^2\left(c(a+bx^2)^p\right) + 4\sqrt{b}\,px\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(2p\log\left(\frac{2i}{i-\frac{\sqrt{b}\,x}{\sqrt{a}}}\right) + \log\left(c(a+bx^2)^p\right)\right) + 4i\sqrt{b}\,p^2x\,\mathrm{Li}_2\left(\frac{i\sqrt{a}+\sqrt{b}\,x}{-i\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{a}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2/x^2,x]
```

```
[Out] ((4*I)*Sqrt[b]*p^2*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 - Sqrt[a]*Log[c*(a + b*x
^2)^p]^2 + 4*Sqrt[b]*p*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(2*p*Log[(2*I)/(I - (S
```

```
qrt[b]*x)/Sqrt[a])] + Log[c*(a + b*x^2)^p]) + (4*I)*Sqrt[b]*p^2*x*PolyLog[2
, (I*Sqrt[a] + Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)])/(Sqrt[a]*x)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c(b\,x^2+a)^p\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^2/x^2,x)
[Out] int(ln(c*(b*x^2+a)^p)^2/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^2,x, algorithm="maxima")
[Out] -p^2*log(b*x^2 + a)^2/x + integrate((b*x^2*log(c)^2 + a*log(c)^2 + 2*((2*p^
2 + p*log(c))*b*x^2 + a*p*log(c))*log(b*x^2 + a))/(b*x^4 + a*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^2,x, algorithm="fricas")
[Out] integral(log((b*x^2 + a)^p*c)^2/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c(a+bx^2)^p\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**2/x**2,x)
[Out] Integral(log(c*(a + b*x**2)**p)**2/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^2/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\left(b\,x^2+a\right)^p\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)^2/x^2,x)
```

```
[Out] int(log(c*(a + b*x^2)^p)^2/x^2, x)
```

3.88 $\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^4}\,dx$

Optimal. Leaf size=254

$$\frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{4ib^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{3a^{3/2}}-\frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{3a^{3/2}}-\frac{4bp\log\left(c(a+\right.}{3ax}$$

```
[Out] 8/3*b^(3/2)*p^2*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)-4/3*I*b^(3/2)*p^2*arctan(
x*b^(1/2)/a^(1/2))^2/a^(3/2)-4/3*b*p*ln(c*(b*x^2+a)^p)/a/x-4/3*b^(3/2)*p*ar
ctan(x*b^(1/2)/a^(1/2))*ln(c*(b*x^2+a)^p)/a^(3/2)-1/3*ln(c*(b*x^2+a)^p)^2/x
^3-8/3*b^(3/2)*p^2*arctan(x*b^(1/2)/a^(1/2))*ln(2*a^(1/2)/(a^(1/2)+I*x*b^(1
/2)))/a^(3/2)-4/3*I*b^(3/2)*p^2*polylog(2,1-2*a^(1/2)/(a^(1/2)+I*x*b^(1/2))
)/a^(3/2)
```

Rubi [A]
time = 0.19, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {2507, 2526, 2505, 211, 2520, 12, 5040, 4964, 2449, 2352}

$$-\frac{4ib^{3/2}p^2\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{3a^{3/2}}-\frac{4b^{3/2}p\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{3a^{3/2}}-\frac{4ib^{3/2}p^2\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{3a^{3/2}}+\frac{8b^{3/2}p^2\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{8b^{3/2}p^2\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{3a^{3/2}}-\frac{4bp\log\left(c(a+bx^2)^p\right)}{3ax}-\frac{\log^2\left(c(a+bx^2)^p\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^2/x^4,x]
```

```
[Out] (8*b^(3/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3*a^(3/2)) - (((4*I)/3)*b^(3/2
)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/a^(3/2) - (8*b^(3/2)*p^2*ArcTan[(Sqrt[
b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/(3*a^(3/2)) - (4*b
*p*Log[c*(a + b*x^2)^p])/(3*a*x) - (4*b^(3/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
*Log[c*(a + b*x^2)^p])/(3*a^(3/2)) - Log[c*(a + b*x^2)^p]^2/(3*x^3) - (((4*
I)/3)*b^(3/2)*p^2*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/a^(3
/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^4}\,dx = -\frac{\log^2\left(c(a+bx^2)^p\right)}{3x^3}+\frac{1}{3}(4bp)\int \frac{\log\left(c(a+bx^2)^p\right)}{x^2\left(a+bx^2\right)}\,dx$$

$$= -\frac{\log^2\left(c(a+bx^2)^p\right)}{3x^3}+\frac{1}{3}(4bp)\int \left(\frac{\log\left(c(a+bx^2)^p\right)}{ax^2}-\frac{b\log\left(c(a+bx^2)^p\right)}{a\left(a+bx^2\right)}\right)dx$$

$$= -\frac{\log^2\left(c(a+bx^2)^p\right)}{3x^3}+\frac{(4bp)\int \frac{\log\left(c(a+bx^2)^p\right)}{x^2}\,dx}{3a}-\frac{(4b^2p)\int \frac{\log\left(c(a+bx^2)^p\right)}{a+bx^2}\,dx}{3a}$$

$$= -\frac{4bp\log\left(c(a+bx^2)^p\right)}{3ax}-\frac{4b^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{3a^{3/2}}-\frac{\log^2\left(c(a+bx\right.}{3x^3}$$

$$= \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{4bp\log\left(c(a+bx^2)^p\right)}{3ax}-\frac{4b^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c(a+\right.}{3a^{3/2}}$$

$$= \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{4ib^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{3a^{3/2}}-\frac{4bp\log\left(c(a+bx^2)^p\right)}{3ax}-\frac{4b^{3/}}{}$$

$$= \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{4ib^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{3a^{3/2}}-\frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\right.}{3a^{3/2}}$$

$$= \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{4ib^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{3a^{3/2}}-\frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\right.}{3a^{3/2}}$$

$$= \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{4ib^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{3a^{3/2}}-\frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\right.}{3a^{3/2}}$$

Mathematica [**A**]
time = 0.07, size = 207, normalized size = 0.81

$$\frac{-4ib^{3/2}p^2x^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2-4b^{3/2}px^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-2p+2p\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)+\log\left(c(a+bx^2)^p\right)\right)-\sqrt{a}\,\log\left(c(a+bx^2)^p\right)\left(4bpx^2+a\log\left(c(a+bx^2)^p\right)\right)-4ib^{3/2}p^2x^3\mathrm{Li}_2\left(\frac{i\sqrt{a}+\sqrt{b}\,x}{-i\sqrt{a}+\sqrt{b}\,x}\right)}{3a^{3/2}x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2/x^4,x]
```

```
[Out] ((-4*I)*b^(3/2)*p^2*x^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 - 4*b^(3/2)*p*x^3*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]]*(-2*p + 2*p*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x
)] + Log[c*(a + b*x^2)^p]) - Sqrt[a]*Log[c*(a + b*x^2)^p]*(4*b*p*x^2 + a*Lo
g[c*(a + b*x^2)^p]) - (4*I)*b^(3/2)*p^2*x^3*PolyLog[2, (I*Sqrt[a] + Sqrt[b]
*x)/((-I)*Sqrt[a] + Sqrt[b]*x)])/(3*a^(3/2)*x^3)
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c(b\,x^2+a)^p\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^2/x^4,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^2/x^4,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*p^2*log(b*x^2 + a)^2/x^3 + integrate(1/3*(3*b*x^2*log(c)^2 + 3*a*log(c
)^2 + 2*((2*p^2 + 3*p*log(c))*b*x^2 + 3*a*p*log(c))*log(b*x^2 + a))/(b*x^6
+ a*x^4), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^2/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(c(a+bx^2)^p\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**2/x**4,x)
[Out] Integral(log(c*(a + b*x**2)**p)**2/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^4,x, algorithm="giac")
[Out] integrate(log((b*x^2 + a)^p*c)^2/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln \left(c\,{\left(b\,x^2+a\right)}^p\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)^2/x^4,x)
[Out] int(log(c*(a + b*x^2)^p)^2/x^4, x)
```

3.89 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^6}\,dx$

Optimal. Leaf size=296

$$-\frac{8b^2p^2}{15a^2x}-\frac{32b^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{15a^{5/2}}+\frac{4ib^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{5a^{5/2}}+\frac{8b^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{5a^{5/2}}-\frac{4}{}$$

```
[Out] -8/15*b^2*p^2/a^2/x-32/15*b^(5/2)*p^2*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)+4/5
*I*b^(5/2)*p^2*arctan(x*b^(1/2)/a^(1/2))^2/a^(5/2)-4/15*b*p*ln(c*(b*x^2+a)^
p)/a/x^3+4/5*b^2*p*ln(c*(b*x^2+a)^p)/a^2/x+4/5*b^(5/2)*p*arctan(x*b^(1/2)/a
^(1/2))*ln(c*(b*x^2+a)^p)/a^(5/2)-1/5*ln(c*(b*x^2+a)^p)^2/x^5+8/5*b^(5/2)*p
^2*arctan(x*b^(1/2)/a^(1/2))*ln(2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))/a^(5/2)+4/
5*I*b^(5/2)*p^2*polylog(2,1-2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))/a^(5/2)
```

Rubi [A]
time = 0.22, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {2507, 2526, 2505, 331, 211, 2520, 12, 5040, 4964, 2449, 2352}

$$\frac{4ib^{5/2}p^2\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{5a^{5/2}}+\frac{4b^{5/2}p\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{5a^{5/2}}+\frac{4ib^{5/2}p^2\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{5a^{5/2}}-\frac{32b^{5/2}p^2\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{15a^{5/2}}+\frac{8b^{5/2}p^2\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{5a^{5/2}}+\frac{4b^2p\log\left(c\left(a+bx^2\right)^p\right)}{5a^2x}-\frac{8b^2p^2}{15a^2x}-\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{5x^5}-\frac{4bp\log\left(c\left(a+bx^2\right)^p\right)}{15ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^2/x^6,x]
```

```
[Out] (-8*b^2*p^2)/(15*a^2*x) - (32*b^(5/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(15*
a^(5/2)) + (((4*I)/5)*b^(5/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/a^(5/2) +
(8*b^(5/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqr
t[b]*x)])/(5*a^(5/2)) - (4*b*p*Log[c*(a + b*x^2)^p])/(15*a*x^3) + (4*b^2*p*
Log[c*(a + b*x^2)^p])/(5*a^2*x) + (4*b^(5/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*
Log[c*(a + b*x^2)^p])/(5*a^(5/2)) - Log[c*(a + b*x^2)^p]^2/(5*x^5) + (((4*I
)/5)*b^(5/2)*p^2*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/a^(5/
2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^6}\,dx = -\frac{\log^2\left(c(a+bx^2)^p\right)}{5x^5} + \frac{1}{5}(4bp)\int \frac{\log\left(c(a+bx^2)^p\right)}{x^4\left(a+bx^2\right)}\,dx$$

$$= -\frac{\log^2\left(c(a+bx^2)^p\right)}{5x^5} + \frac{1}{5}(4bp)\int\left(\frac{\log\left(c(a+bx^2)^p\right)}{ax^4} - \frac{b\log\left(c(a+bx^2)^p\right)}{a^2x^2} + \frac{b^2\,\text{lo}}{a}\right.$$

$$= -\frac{\log^2\left(c(a+bx^2)^p\right)}{5x^5} + \frac{(4bp)\int\frac{\log\left(c(a+bx^2)^p\right)}{x^4}\,dx}{5a} - \frac{(4b^2p)\int\frac{\log\left(c(a+bx^2)^p\right)}{x^2}\,dx}{5a^2} + \frac{(4b^3p}{}$$

$$= -\frac{4bp\log\left(c(a+bx^2)^p\right)}{15ax^3} + \frac{4b^2p\log\left(c(a+bx^2)^p\right)}{5a^2x} + \frac{4b^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c(a\right.}{5a^{5/2}}$$

$$= -\frac{8b^2p^2}{15a^2x} - \frac{8b^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{5a^{5/2}} - \frac{4bp\log\left(c(a+bx^2)^p\right)}{15ax^3} + \frac{4b^2p\log\left(c(a+bx^2)^p\right.}{5a^2x}$$

$$= -\frac{8b^2p^2}{15a^2x} - \frac{32b^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{15a^{5/2}} + \frac{4ib^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{5a^{5/2}} - \frac{4bp\log\left(c(a+b\right.}{15ax^3}$$

$$= -\frac{8b^2p^2}{15a^2x} - \frac{32b^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{15a^{5/2}} + \frac{4ib^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{5a^{5/2}} + \frac{8b^{5/2}p^2\tan^{-1}\left(\right.}{}$$

$$= -\frac{8b^2p^2}{15a^2x} - \frac{32b^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{15a^{5/2}} + \frac{4ib^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{5a^{5/2}} + \frac{8b^{5/2}p^2\tan^{-1}\left(\right.}{}$$

$$= -\frac{8b^2p^2}{15a^2x} - \frac{32b^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{15a^{5/2}} + \frac{4ib^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{5a^{5/2}} + \frac{8b^{5/2}p^2\tan^{-1}\left(\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.19, size = 277, normalized size = 0.94

$$-\frac{3\log^2\left(c(a+bx^2)^p\right) + \frac{4bpx^2\left(6b^{3/2}px^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)+2\sqrt{a}\,bpx^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{bx^2}{a}\right)+a^{3/2}\log(c(a+bx^2)^p)-3\sqrt{a}\,bx^2\log(c(a+bx^2)^p)-3b^{3/2}x^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log(c(a+bx^2)^p)-3ib^{3/2}px^3\left(\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)-2i\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)\right)+\text{Li}_2\left(\frac{i\sqrt{a}+\sqrt{b}\,x}{-i\sqrt{a}+\sqrt{b}\,x}\right)\right)\right)}{a^{5/2}}}{15x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2/x^6,x]
```

```
[Out] -1/15*(3*Log[c*(a + b*x^2)^p]^2 + (4*b*p*x^2*(6*b^(3/2)*p*x^3*ArcTan[(Sqrt[
b]*x)/Sqrt[a]] + 2*Sqrt[a]*b*p*x^2*Hypergeometric2F1[-1/2, 1, 1/2, -((b*x^2
)/a)] + a^(3/2)*Log[c*(a + b*x^2)^p] - 3*Sqrt[a]*b*x^2*Log[c*(a + b*x^2)^p]
 - 3*b^(3/2)*x^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[c*(a + b*x^2)^p] - (3*I)*b
^(3/2)*p*x^3*(ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(ArcTan[(Sqrt[b]*x)/Sqrt[a]] - (2
*I)*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)]) + PolyLog[2, (I*Sqrt[a] + Sqr
t[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)])))/a^(5/2))/x^5
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(c(b\,x^2+a)^p\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^2/x^6,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^2/x^6,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^6,x, algorithm="maxima")
```

```
[Out] -1/5*p^2*log(b*x^2 + a)^2/x^5 + integrate(1/5*(5*b*x^2*log(c)^2 + 5*a*log(c
)^2 + 2*((2*p^2 + 5*p*log(c))*b*x^2 + 5*a*p*log(c))*log(b*x^2 + a))/(b*x^8
+ a*x^6), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^6,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^2/x^6, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(c(a+bx^2)^p\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**2/x**6,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**2/x**6, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^6,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^2/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)^2/x^6,x)
```

```
[Out] int(log(c*(a + b*x^2)^p)^2/x^6, x)
```

3.90 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^8}\,dx$

Optimal. Leaf size=338

$$-\frac{8b^2p^2}{105a^2x^3}+\frac{64b^3p^2}{105a^3x}+\frac{184b^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{105a^{7/2}}-\frac{4ib^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{7a^{7/2}}-\frac{8b^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2}{\sqrt{a}}\right.}{7a^{7/2}}$$

```
[Out] -8/105*b^2*p^2/a^2/x^3+64/105*b^3*p^2/a^3/x+184/105*b^(7/2)*p^2*arctan(x*b^
(1/2)/a^(1/2))/a^(7/2)-4/7*I*b^(7/2)*p^2*arctan(x*b^(1/2)/a^(1/2))^2/a^(7/2
)-4/35*b*p*ln(c*(b*x^2+a)^p)/a/x^5+4/21*b^2*p*ln(c*(b*x^2+a)^p)/a^2/x^3-4/7
*b^3*p*ln(c*(b*x^2+a)^p)/a^3/x-4/7*b^(7/2)*p*arctan(x*b^(1/2)/a^(1/2))*ln(c
*(b*x^2+a)^p)/a^(7/2)-1/7*ln(c*(b*x^2+a)^p)^2/x^7-8/7*b^(7/2)*p^2*arctan(x*
b^(1/2)/a^(1/2))*ln(2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))/a^(7/2)-4/7*I*b^(7/2)*
p^2*polylog(2,1-2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))/a^(7/2)
```

Rubi [A]
time = 0.25, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {2507, 2526, 2505, 331, 211, 2520, 12, 5040, 4964, 2449, 2352}

$$-\frac{4ib^{7/2}p^2\,\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{7a^{7/2}}-\frac{4b^{7/2}p\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{7a^{7/2}}-\frac{4ib^{7/2}p^2\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{7a^{7/2}}+\frac{184b^{7/2}p^2\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{105a^{7/2}}-\frac{8b^{7/2}p^2\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{7a^{7/2}}-\frac{4b^3p\log\left(c\left(a+bx^2\right)^p\right)}{7a^3x}+\frac{64b^3p^2}{105a^3x}+\frac{4b^2p\log\left(c\left(a+bx^2\right)^p\right)}{21a^2x^3}-\frac{8b^2p^2}{105a^2x^3}-\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{7x^7}-\frac{4bp\log\left(c\left(a+bx^2\right)^p\right)}{35ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^2/x^8,x]
```

```
[Out] (-8*b^2*p^2)/(105*a^2*x^3) + (64*b^3*p^2)/(105*a^3*x) + (184*b^(7/2)*p^2*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]])/(105*a^(7/2)) - (((4*I)/7)*b^(7/2)*p^2*ArcTan[(S
qrt[b]*x)/Sqrt[a]]^2)/a^(7/2) - (8*b^(7/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*
Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/(7*a^(7/2)) - (4*b*p*Log[c*(a + b
*x^2)^p])/(35*a*x^5) + (4*b^2*p*Log[c*(a + b*x^2)^p])/(21*a^2*x^3) - (4*b^3
*p*Log[c*(a + b*x^2)^p])/(7*a^3*x) - (4*b^(7/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a
]]*Log[c*(a + b*x^2)^p])/(7*a^(7/2)) - Log[c*(a + b*x^2)^p]^2/(7*x^7) - (((
4*I)/7)*b^(7/2)*p^2*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/a^
(7/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
```

```
& IntegerQ[s]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^8}\,dx = -\frac{\log^2\left(c(a+bx^2)^p\right)}{7x^7} + \frac{1}{7}(4bp)\int \frac{\log\left(c(a+bx^2)^p\right)}{x^6\left(a+bx^2\right)}\,dx$$

$$= -\frac{\log^2\left(c(a+bx^2)^p\right)}{7x^7} + \frac{1}{7}(4bp)\int \left(\frac{\log\left(c(a+bx^2)^p\right)}{ax^6} - \frac{b\log\left(c(a+bx^2)^p\right)}{a^2x^4} + \frac{b^2 \mathrm{lo}\cdots}{}\right.$$

$$= -\frac{\log^2\left(c(a+bx^2)^p\right)}{7x^7} + \frac{(4bp)\int \frac{\log(c(a+bx^2)^p)}{x^6}\,dx}{7a} - \frac{(4b^2p)\int \frac{\log(c(a+bx^2)^p)}{x^4}\,dx}{7a^2} + \frac{(4b^3 p\cdots}{}$$

$$= -\frac{4bp\log\left(c(a+bx^2)^p\right)}{35ax^5} + \frac{4b^2p\log\left(c(a+bx^2)^p\right)}{21a^2x^3} - \frac{4b^3p\log\left(c(a+bx^2)^p\right)}{7a^3x} - \frac{4b^{7/2}\cdots}{}$$

$$= -\frac{8b^2p^2}{105a^2x^3} + \frac{8b^3p^2}{21a^3x} + \frac{8b^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{7a^{7/2}} - \frac{4bp\log\left(c(a+bx^2)^p\right)}{35ax^5} + \frac{4b^2p\log\left(\cdots\right.}{21\cdots}$$

$$= -\frac{8b^2p^2}{105a^2x^3} + \frac{64b^3p^2}{105a^3x} + \frac{32b^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{21a^{7/2}} - \frac{4ib^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{7a^{7/2}} - \frac{4bp\cdots}{}$$

$$= -\frac{8b^2p^2}{105a^2x^3} + \frac{64b^3p^2}{105a^3x} + \frac{184b^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{105a^{7/2}} - \frac{4ib^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{7a^{7/2}} - \frac{8b\cdots}{}$$

$$= -\frac{8b^2p^2}{105a^2x^3} + \frac{64b^3p^2}{105a^3x} + \frac{184b^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{105a^{7/2}} - \frac{4ib^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{7a^{7/2}} - \frac{8b\cdots}{}$$

$$= -\frac{8b^2p^2}{105a^2x^3} + \frac{64b^3p^2}{105a^3x} + \frac{184b^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{105a^{7/2}} - \frac{4ib^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{7a^{7/2}} - \frac{8b\cdots}{}$$

<u>**Mathematica**</u> [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.16, size = 334, normalized size = 0.99

$$-\frac{\log^2\left(c(a+bx^2)^p\right)}{7x^7} + \frac{4bp\left(30b^{5/2}px^5\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right) - 2a^{3/2}bpx^2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\frac{bx^2}{a}\right) + 10\sqrt{a}\,b^2px^4\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{bx^2}{a}\right) - 3a^{5/2}\log\left(c(a+bx^2)^p\right) + 5a^{3/2}bx^2\log\left(c(a+bx^2)^p\right) - 15\sqrt{a}\,b^2x^4\log\left(c(a+bx^2)^p\right) - 15b^{5/2}x^5\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right) - 15ib^{5/2}px^5\left(\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right) - 2i\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)\right) + \mathrm{Li}_2\left(\frac{i\sqrt{a}+\sqrt{b}\,x}{-i\sqrt{a}+\sqrt{b}\,x}\right)\right)\right)}{105a^{7/2}x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2/x^8,x]
```

```
[Out] -1/7*Log[c*(a + b*x^2)^p]^2/x^7 + (4*b*p*(30*b^(5/2)*p*x^5*ArcTan[(Sqrt[b]*
x)/Sqrt[a]] - 2*a^(3/2)*b*p*x^2*Hypergeometric2F1[-3/2, 1, -1/2, -((b*x^2)/
a)] + 10*Sqrt[a]*b^2*p*x^4*Hypergeometric2F1[-1/2, 1, 1/2, -((b*x^2)/a)] -
3*a^(5/2)*Log[c*(a + b*x^2)^p] + 5*a^(3/2)*b*x^2*Log[c*(a + b*x^2)^p] - 15*
Sqrt[a]*b^2*x^4*Log[c*(a + b*x^2)^p] - 15*b^(5/2)*x^5*ArcTan[(Sqrt[b]*x)/Sq
rt[a]]*Log[c*(a + b*x^2)^p] - (15*I)*b^(5/2)*p*x^5*(ArcTan[(Sqrt[b]*x)/Sqrt
[a]]*(ArcTan[(Sqrt[b]*x)/Sqrt[a]] - (2*I)*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt
[b]*x)]) + PolyLog[2, (I*Sqrt[a] + Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)]))
)/(105*a^(7/2)*x^5)
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c(b\,x^2+a)^p\right)^2}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^2/x^8,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^2/x^8,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^8,x, algorithm="maxima")
```

```
[Out] -1/7*p^2*log(b*x^2 + a)^2/x^7 + integrate(1/7*(7*b*x^2*log(c)^2 + 7*a*log(c
)^2 + 2*((2*p^2 + 7*p*log(c))*b*x^2 + 7*a*p*log(c))*log(b*x^2 + a))/(b*x^10
 + a*x^8), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^8,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^2/x^8, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c(a+bx^2)^p\right)^2}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**2/x**8,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**2/x**8, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^8,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^2/x^8, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^2}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)^2/x^8,x)
```

```
[Out] int(log(c*(a + b*x^2)^p)^2/x^8, x)
```

3.91 $\int x^5 \log^3\left(c(a+bx^2)^p\right)\, dx$

Optimal. Leaf size=334

$$-\frac{3a^2p^3x^2}{b^2}+\frac{3ap^3(a+bx^2)^2}{8b^3}-\frac{p^3(a+bx^2)^3}{27b^3}+\frac{3a^2p^2(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b^3}-\frac{3ap^2(a+bx^2)^2\log\left(c(a+bx^2\right.}{4b^3}$$

```
[Out] -3*a^2*p^3*x^2/b^2+3/8*a*p^3*(b*x^2+a)^2/b^3-1/27*p^3*(b*x^2+a)^3/b^3+3*a^2
*p^2*(b*x^2+a)*ln(c*(b*x^2+a)^p)/b^3-3/4*a*p^2*(b*x^2+a)^2*ln(c*(b*x^2+a)^p
)/b^3+1/9*p^2*(b*x^2+a)^3*ln(c*(b*x^2+a)^p)/b^3-3/2*a^2*p*(b*x^2+a)*ln(c*(b
*x^2+a)^p)^2/b^3+3/4*a*p*(b*x^2+a)^2*ln(c*(b*x^2+a)^p)^2/b^3-1/6*p*(b*x^2+a
)^3*ln(c*(b*x^2+a)^p)^2/b^3+1/2*a^2*(b*x^2+a)*ln(c*(b*x^2+a)^p)^3/b^3-1/2*a
*(b*x^2+a)^2*ln(c*(b*x^2+a)^p)^3/b^3+1/6*(b*x^2+a)^3*ln(c*(b*x^2+a)^p)^3/b^
3
```

Rubi [A]
time = 0.24, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

$$\frac{3a^2p^2(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b^3}+\frac{a^2(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2b^3}-\frac{3a^2p(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2b^3}-\frac{3a^2p^3x^2}{b^2}+\frac{p^2(a+bx^2)^3\log\left(c(a+bx^2)^p\right)}{9b^3}-\frac{3ap^2(a+bx^2)^2\log\left(c(a+bx^2)^p\right)}{4b^3}+\frac{(a+bx^2)^3\log^3\left(c(a+bx^2)^p\right)}{6b^3}-\frac{a(a+bx^2)^2\log^3\left(c(a+bx^2)^p\right)}{2b^3}-\frac{p(a+bx^2)^3\log^2\left(c(a+bx^2)^p\right)}{6b^3}+\frac{3ap(a+bx^2)^2\log^2\left(c(a+bx^2)^p\right)}{4b^3}-\frac{p^3(a+bx^2)^3}{27b^3}+\frac{3ap^3(a+bx^2)^2}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] (-3*a^2*p^3*x^2)/b^2 + (3*a*p^3*(a + b*x^2)^2)/(8*b^3) - (p^3*(a + b*x^2)^3
)/(27*b^3) + (3*a^2*p^2*(a + b*x^2)*Log[c*(a + b*x^2)^p])/b^3 - (3*a*p^2*(a
 + b*x^2)^2*Log[c*(a + b*x^2)^p])/(4*b^3) + (p^2*(a + b*x^2)^3*Log[c*(a + b
*x^2)^p])/(9*b^3) - (3*a^2*p*(a + b*x^2)*Log[c*(a + b*x^2)^p]^2)/(2*b^3) +
(3*a*p*(a + b*x^2)^2*Log[c*(a + b*x^2)^p]^2)/(4*b^3) - (p*(a + b*x^2)^3*Log
[c*(a + b*x^2)^p]^2)/(6*b^3) + (a^2*(a + b*x^2)*Log[c*(a + b*x^2)^p]^3)/(2*
b^3) - (a*(a + b*x^2)^2*Log[c*(a + b*x^2)^p]^3)/(2*b^3) + ((a + b*x^2)^3*Lo
g[c*(a + b*x^2)^p]^3)/(6*b^3)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^5 \log^3\left(c\left(a+bx^2\right)^p\right)\,dx = \frac{1}{2}\text{Subst}\left(\int x^2\log^3\left(c(a+bx)^p\right)\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2\log^3\left(c(a+bx)^p\right)}{b^2} - \frac{2a(a+bx)\log^3\left(c(a+bx)^p\right)}{b^2} + \frac{(a+bx)^2}{\ }\right.\right.$$
$$= \frac{\text{Subst}\left(\int (a+bx)^2\log^3\left(c(a+bx)^p\right)\,dx, x, x^2\right)}{2b^2} - \frac{a\text{Subst}\left(\int (a+bx)\log^3\left(c(a+\right.\right.}{b^2}$$
$$= \frac{\text{Subst}\left(\int x^2\log^3\left(cx^p\right)\,dx, x, a+bx^2\right)}{2b^3} - \frac{a\text{Subst}\left(\int x\log^3\left(cx^p\right)\,dx, x, a+bx^2\right)}{b^3}$$
$$= \frac{a^2\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2b^3} - \frac{a\left(a+bx^2\right)^2\log^3\left(c\left(a+bx^2\right)^p\right)}{2b^3} + \frac{\left(a+bx^2\right)^3}{\ }$$
$$= -\frac{3a^2p\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b^3} + \frac{3ap\left(a+bx^2\right)^2\log^2\left(c\left(a+bx^2\right)^p\right)}{4b^3} - \frac{p(a}{\ }$$
$$= -\frac{3a^2p^3x^2}{b^2} + \frac{3ap^3\left(a+bx^2\right)^2}{8b^3} - \frac{p^3\left(a+bx^2\right)^3}{27b^3} + \frac{3a^2p^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)\right.}{b^3}$$

Mathematica [A]
time = 0.14, size = 309, normalized size = 0.93

$$-\frac{85a^2p^3x^2}{36b^2} + \frac{19ap^3x^4}{72b} - \frac{p^3x^6}{27} + \frac{19a^3p^3\log\left(a+bx^2\right)}{36b^3} + \frac{11a^3p^2\log\left(c\left(a+bx^2\right)^p\right)}{6b^3} + \frac{11a^2p^2x^2\log\left(c\left(a+bx^2\right)^p\right)}{6b^2} - \frac{5ap^2x^4\log\left(c\left(a+bx^2\right)^p\right)}{12b} + \frac{1}{9}p^2x^6\log\left(c\left(a+bx^2\right)^p\right) - \frac{11a^3p\log^2\left(c\left(a+bx^2\right)^p\right)}{12b^3} - \frac{a^2px^2\log^2\left(c\left(a+bx^2\right)^p\right)}{2b^2} + \frac{apx^4\log^2\left(c\left(a+bx^2\right)^p\right)}{4b} - \frac{1}{6}px^6\log^2\left(c\left(a+bx^2\right)^p\right) + \frac{a^3\log^3\left(c\left(a+bx^2\right)^p\right)}{6b^3} + \frac{1}{6}x^6\log^3\left(c\left(a+bx^2\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] (-85*a^2*p^3*x^2)/(36*b^2) + (19*a*p^3*x^4)/(72*b) - (p^3*x^6)/27 + (19*a^3
*p^3*Log[a + b*x^2])/(36*b^3) + (11*a^3*p^2*Log[c*(a + b*x^2)^p])/(6*b^3) +
 (11*a^2*p^2*x^2*Log[c*(a + b*x^2)^p])/(6*b^2) - (5*a*p^2*x^4*Log[c*(a + b*
x^2)^p])/(12*b) + (p^2*x^6*Log[c*(a + b*x^2)^p])/9 - (11*a^3*p*Log[c*(a + b
*x^2)^p]^2)/(12*b^3) - (a^2*p*x^2*Log[c*(a + b*x^2)^p]^2)/(2*b^2) + (a*p*x^
4*Log[c*(a + b*x^2)^p]^2)/(4*b) - (p*x^6*Log[c*(a + b*x^2)^p]^2)/6 + (a^3*L
og[c*(a + b*x^2)^p]^3)/(6*b^3) + (x^6*Log[c*(a + b*x^2)^p]^3)/6
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.31, size = 5905, normalized size = 17.68

method	result	size
risch	Expression too large to display	5905

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*ln(c*(b*x^2+a)^p)^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.29, size = 239, normalized size = 0.72

$$\frac{1}{6}x^{6}\log\left(\left(bx^{2}+a\right)^{p}c\right)^{3}+\frac{1}{12}bp\left(\frac{6a^{3}\log\left(bx^{2}+a\right)}{b^{4}}-\frac{2b^{2}x^{6}-3abx^{4}+6a^{2}x^{2}}{b^{3}}\right)\log\left(\left(bx^{2}+a\right)^{p}c\right)^{2}-\frac{1}{216}bp\left(\frac{\left(8b^{3}x^{6}-57ab^{2}x^{4}-36a^{3}\log\left(bx^{2}+a\right)^{3}+510a^{2}bx^{2}-198a^{3}\log\left(bx^{2}+a\right)^{2}-510a^{3}\log\left(bx^{2}+a\right)\right)p^{2}}{b^{4}}-\frac{6\left(4b^{3}x^{6}-15ab^{2}x^{4}+66a^{2}bx^{2}-18a^{3}\log\left(bx^{2}+a\right)^{2}-66a^{3}\log\left(bx^{2}+a\right)\right)p\log\left(\left(bx^{2}+a\right)^{p}c\right)}{b^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] 1/6*x^6*log((b*x^2 + a)^p*c)^3 + 1/12*b*p*(6*a^3*log(b*x^2 + a)/b^4 - (2*b^
2*x^6 - 3*a*b*x^4 + 6*a^2*x^2)/b^3)*log((b*x^2 + a)^p*c)^2 - 1/216*b*p*((8*
b^3*x^6 - 57*a*b^2*x^4 - 36*a^3*log(b*x^2 + a)^3 + 510*a^2*b*x^2 - 198*a^3*
log(b*x^2 + a)^2 - 510*a^3*log(b*x^2 + a))*p^2/b^4 - 6*(4*b^3*x^6 - 15*a*b^
2*x^4 + 66*a^2*b*x^2 - 18*a^3*log(b*x^2 + a)^2 - 66*a^3*log(b*x^2 + a))*p*l
og((b*x^2 + a)^p*c)/b^4)
```

Fricas [A]
time = 0.40, size = 359, normalized size = 1.07

$$-\frac{8b^{3}p^{3}x^{6}-36b^{3}x^{6}\log\left(c\right)^{3}-57ab^{2}p^{3}x^{4}+510a^{2}bp^{3}x^{2}-36\left(b^{3}p^{3}x^{6}+a^{3}p^{3}\right)\log\left(bx^{2}+a\right)^{3}+18\left(2b^{3}p^{3}x^{6}-3ab^{2}p^{3}x^{4}+6a^{2}bp^{3}x^{2}+11a^{3}p^{3}-6\left(b^{3}p^{2}x^{6}+a^{3}p^{2}\right)\log\left(c\right)\right)\log\left(bx^{2}+a\right)^{2}+18\left(2b^{3}px^{6}-3ab^{2}px^{4}+6a^{2}bpx^{2}\right)\log\left(c\right)^{2}-6\left(4b^{3}p^{3}x^{6}-15ab^{2}p^{3}x^{4}+66a^{2}bp^{3}x^{2}+85a^{3}p^{3}+18\left(b^{3}px^{6}+a^{3}p\right)\log\left(c\right)^{2}-6\left(2b^{3}p^{2}x^{6}-3ab^{2}p^{2}x^{4}+6a^{2}bp^{2}x^{2}+11a^{3}p^{2}\right)\log\left(c\right)\right)\log\left(bx^{2}+a\right)-6\left(4b^{3}p^{2}x^{6}-15ab^{2}p^{2}x^{4}+66a^{2}bp^{2}x^{2}\right)\log\left(c\right)}{216b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] -1/216*(8*b^3*p^3*x^6 - 36*b^3*x^6*log(c)^3 - 57*a*b^2*p^3*x^4 + 510*a^2*b*
p^3*x^2 - 36*(b^3*p^3*x^6 + a^3*p^3)*log(b*x^2 + a)^3 + 18*(2*b^3*p^3*x^6 -
 3*a*b^2*p^3*x^4 + 6*a^2*b*p^3*x^2 + 11*a^3*p^3 - 6*(b^3*p^2*x^6 + a^3*p^2)
*log(c))*log(b*x^2 + a)^2 + 18*(2*b^3*p*x^6 - 3*a*b^2*p*x^4 + 6*a^2*b*p*x^2
)*log(c)^2 - 6*(4*b^3*p^3*x^6 - 15*a*b^2*p^3*x^4 + 66*a^2*b*p^3*x^2 + 85*a^
3*p^3 + 18*(b^3*p*x^6 + a^3*p)*log(c)^2 - 6*(2*b^3*p^2*x^6 - 3*a*b^2*p^2*x^
4 + 6*a^2*b*p^2*x^2 + 11*a^3*p^2)*log(c))*log(b*x^2 + a) - 6*(4*b^3*p^2*x^6
 - 15*a*b^2*p^2*x^4 + 66*a^2*b*p^2*x^2)*log(c))/b^3
```

Sympy [A]
time = 6.59, size = 289, normalized size = 0.87

$$\begin{cases}\frac{85a^{3}p^{2}\log\left(c\left(a+bx^{2}\right)^{p}\right)}{36b^{3}}-\frac{11a^{3}p\log\left(c\left(a+bx^{2}\right)^{p}\right)^{2}}{12b^{3}}+\frac{a^{3}\log\left(c\left(a+bx^{2}\right)^{p}\right)^{3}}{6b^{3}}-\frac{85a^{2}p^{3}x^{2}}{36b^{2}}+\frac{11a^{2}p^{2}x^{2}\log\left(c\left(a+bx^{2}\right)^{p}\right)}{6b^{2}}-\frac{a^{2}px^{2}\log\left(c\left(a+bx^{2}\right)^{p}\right)^{2}}{2b^{2}}+\frac{19ap^{3}x^{4}}{72b}-\frac{5ap^{2}x^{4}\log\left(c\left(a+bx^{2}\right)^{p}\right)}{12b}+\frac{apx^{4}\log\left(c\left(a+bx^{2}\right)^{p}\right)^{2}}{4b}-\frac{p^{3}x^{6}}{27}+\frac{p^{2}x^{6}\log\left(c\left(a+bx^{2}\right)^{p}\right)}{9}-\frac{px^{6}\log\left(c\left(a+bx^{2}\right)^{p}\right)^{2}}{6}+\frac{x^{6}\log\left(c\left(a+bx^{2}\right)^{p}\right)^{3}}{6} & \text{for}\: b \neq 0 \\ \frac{x^{6}\log\left(a^{p}c\right)^{3}}{6} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Piecewise((85*a**3*p**2*log(c*(a + b*x**2)**p)/(36*b**3) - 11*a**3*p*log(c*
(a + b*x**2)**p)**2/(12*b**3) + a**3*log(c*(a + b*x**2)**p)**3/(6*b**3) - 8
5*a**2*p**3*x**2/(36*b**2) + 11*a**2*p**2*x**2*log(c*(a + b*x**2)**p)/(6*b*
*2) - a**2*p*x**2*log(c*(a + b*x**2)**p)**2/(2*b**2) + 19*a*p**3*x**4/(72*b
```

```
) - 5*a*p**2*x**4*log(c*(a + b*x**2)**p)/(12*b) + a*p*x**4*log(c*(a + b*x**
2)**p)**2/(4*b) - p**3*x**6/27 + p**2*x**6*log(c*(a + b*x**2)**p)/9 - p*x**
6*log(c*(a + b*x**2)**p)**2/6 + x**6*log(c*(a + b*x**2)**p)**3/6, Ne(b, 0))
, (x**6*log(a**p*c)**3/6, True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 662 vs. 2(314) = 628.
time = 4.59, size = 662, normalized size = 1.98

$$\frac{(bx^2+a)^3p^3\log(bx^2+a)^3}{6b^3} - \frac{(bx^2+a)^2ap^3\log(bx^2+a)^3}{2b^3} - \frac{(bx^2+a)^3p^3\log(bx^2+a)^2}{6b^3} + \frac{3(bx^2+a)^2ap^3\log(bx^2+a)^2}{4b^3} + \frac{(bx^2+a)^3p^2\log(bx^2+a)^2\log(c)}{2b^3} - \frac{3(bx^2+a)^2ap^2\log(bx^2+a)^2\log(c)}{2b^3} + \frac{(bx^2+a)^3p^3\log(bx^2+a)}{9b^3} - \frac{3(bx^2+a)^2ap^3\log(bx^2+a)}{4b^3} - \frac{(bx^2+a)^3p^2\log(bx^2+a)\log(c)}{3b^3} + \frac{3(bx^2+a)^2ap^2\log(bx^2+a)\log(c)}{2b^3} + \frac{(bx^2+a)^3p\log(bx^2+a)\log(c)^2}{2b^3} - \frac{3(bx^2+a)^2ap\log(bx^2+a)\log(c)^2}{2b^3} - \frac{(bx^2+a)^3p^3}{27b^3} + \frac{3(bx^2+a)^2ap^3}{8b^3} + \frac{(bx^2+a)^3p^2\log(c)}{9b^3} - \frac{3(bx^2+a)^2ap^2\log(c)}{4b^3} - \frac{(bx^2+a)^3p\log(c)^2}{6b^3} + \frac{3(bx^2+a)^2ap\log(c)^2}{4b^3} + \frac{(bx^2+a)^3\log(c)^3}{6b^3} - \frac{(bx^2+a)^2a\log(c)^3}{2b^3} + \frac{\left((bx^2+a)\log(bx^2+a)^3 - 6bx^2 - 3(bx^2+a)\log(bx^2+a)^2 + 6(bx^2+a)\log(bx^2+a) - 6a\right)a^2p^3 + 3\left(2bx^2 + (bx^2+a)\log(bx^2+a)^2 - 2(bx^2+a)\log(bx^2+a) + 2a\right)a^2p^2\log(c) - 3\left(bx^2 - (bx^2+a)\log(bx^2+a) + a\right)a^2p\log(c)^2 + (bx^2+a)a^2\log(c)^3}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] 1/6*(b*x^2 + a)^3*p^3*log(b*x^2 + a)^3/b^3 - 1/2*(b*x^2 + a)^2*a*p^3*log(b*
x^2 + a)^3/b^3 - 1/6*(b*x^2 + a)^3*p^3*log(b*x^2 + a)^2/b^3 + 3/4*(b*x^2 +
a)^2*a*p^3*log(b*x^2 + a)^2/b^3 + 1/2*(b*x^2 + a)^3*p^2*log(b*x^2 + a)^2*lo
g(c)/b^3 - 3/2*(b*x^2 + a)^2*a*p^2*log(b*x^2 + a)^2*log(c)/b^3 + 1/9*(b*x^2
 + a)^3*p^3*log(b*x^2 + a)/b^3 - 3/4*(b*x^2 + a)^2*a*p^3*log(b*x^2 + a)/b^3
 - 1/3*(b*x^2 + a)^3*p^2*log(b*x^2 + a)*log(c)/b^3 + 3/2*(b*x^2 + a)^2*a*p^
2*log(b*x^2 + a)*log(c)/b^3 + 1/2*(b*x^2 + a)^3*p*log(b*x^2 + a)*log(c)^2/b
^3 - 3/2*(b*x^2 + a)^2*a*p*log(b*x^2 + a)*log(c)^2/b^3 - 1/27*(b*x^2 + a)^3
*p^3/b^3 + 3/8*(b*x^2 + a)^2*a*p^3/b^3 + 1/9*(b*x^2 + a)^3*p^2*log(c)/b^3 -
 3/4*(b*x^2 + a)^2*a*p^2*log(c)/b^3 - 1/6*(b*x^2 + a)^3*p*log(c)^2/b^3 + 3/
4*(b*x^2 + a)^2*a*p*log(c)^2/b^3 + 1/6*(b*x^2 + a)^3*log(c)^3/b^3 - 1/2*(b*
x^2 + a)^2*a*log(c)^3/b^3 + 1/2*(((b*x^2 + a)*log(b*x^2 + a)^3 - 6*b*x^2 -
3*(b*x^2 + a)*log(b*x^2 + a)^2 + 6*(b*x^2 + a)*log(b*x^2 + a) - 6*a)*a^2*p^
3 + 3*(2*b*x^2 + (b*x^2 + a)*log(b*x^2 + a)^2 - 2*(b*x^2 + a)*log(b*x^2 + a
) + 2*a)*a^2*p^2*log(c) - 3*(b*x^2 - (b*x^2 + a)*log(b*x^2 + a) + a)*a^2*p*
log(c)^2 + (b*x^2 + a)*a^2*log(c)^3)/b^3
```

Mupad [**B**]
time = 0.36, size = 187, normalized size = 0.56

$$\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^3\,\left(\frac{x^6}{6}+\frac{a^3}{6\,b^3}\right)-\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^2\,\left(\frac{p\,x^6}{6}+\frac{11\,a^3\,p}{12\,b^3}+\frac{a^2\,p\,x^2}{2\,b^2}-\frac{a\,p\,x^4}{4\,b}\right)-\frac{p^3\,x^6}{27}+\frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)\,\left(\frac{b\,p^2\,x^6}{3}-\frac{5\,a\,p^2\,x^4}{4}+\frac{11\,a^2\,p^2\,x^2}{2\,b}\right)}{3\,b}+\frac{19\,a\,p^3\,x^4}{72\,b}+\frac{85\,a^3\,p^3\,\ln\left(b\,x^2+a\right)}{36\,b^3}-\frac{85\,a^2\,p^3\,x^2}{36\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*log(c*(a + b*x^2)^p)^3,x)
```

```
[Out] log(c*(a + b*x^2)^p)^3*(x^6/6 + a^3/(6*b^3)) - log(c*(a + b*x^2)^p)^2*((p*x
^6)/6 + (11*a^3*p)/(12*b^3) + (a^2*p*x^2)/(2*b^2) - (a*p*x^4)/(4*b)) - (p^3
*x^6)/27 + (log(c*(a + b*x^2)^p)*((b*p^2*x^6)/3 - (5*a*p^2*x^4)/4 + (11*a^2
*p^2*x^2)/(2*b)))/(3*b) + (19*a*p^3*x^4)/(72*b) + (85*a^3*p^3*log(a + b*x^2
))/(36*b^3) - (85*a^2*p^3*x^2)/(36*b^2)
```

3.92 $\int x^3 \log^3\left(c(a+bx^2)^p\right)\, dx$

Optimal. Leaf size=211

$$\frac{3ap^3x^2}{b}-\frac{3p^3(a+bx^2)^2}{16b^2}-\frac{3ap^2(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b^2}+\frac{3p^2(a+bx^2)^2\log\left(c(a+bx^2)^p\right)}{8b^2}+\frac{3ap(a+bx^2)\log^2}{2b^2}$$

```
[Out] 3*a*p^3*x^2/b-3/16*p^3*(b*x^2+a)^2/b^2-3*a*p^2*(b*x^2+a)*ln(c*(b*x^2+a)^p)/
b^2+3/8*p^2*(b*x^2+a)^2*ln(c*(b*x^2+a)^p)/b^2+3/2*a*p*(b*x^2+a)*ln(c*(b*x^2
+a)^p)^2/b^2-3/8*p*(b*x^2+a)^2*ln(c*(b*x^2+a)^p)^2/b^2-1/2*a*(b*x^2+a)*ln(c
*(b*x^2+a)^p)^3/b^2+1/4*(b*x^2+a)^2*ln(c*(b*x^2+a)^p)^3/b^2
```

Rubi [A]
time = 0.14, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

$$\frac{3p^2(a+bx^2)^2\log\left(c(a+bx^2)^p\right)}{8b^2}-\frac{3ap^2(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b^2}+\frac{(a+bx^2)^2\log^3\left(c(a+bx^2)^p\right)}{4b^2}-\frac{a(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2b^2}-\frac{3p(a+bx^2)^2\log^2\left(c(a+bx^2)^p\right)}{8b^2}+\frac{3ap(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2b^2}-\frac{3p^3(a+bx^2)^2}{16b^2}+\frac{3ap^3x^2}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] (3*a*p^3*x^2)/b - (3*p^3*(a + b*x^2)^2)/(16*b^2) - (3*a*p^2*(a + b*x^2)*Log
[c*(a + b*x^2)^p])/b^2 + (3*p^2*(a + b*x^2)^2*Log[c*(a + b*x^2)^p])/(8*b^2)
 + (3*a*p*(a + b*x^2)*Log[c*(a + b*x^2)^p]^2)/(2*b^2) - (3*p*(a + b*x^2)^2*
Log[c*(a + b*x^2)^p]^2)/(8*b^2) - (a*(a + b*x^2)*Log[c*(a + b*x^2)^p]^3)/(2
*b^2) + ((a + b*x^2)^2*Log[c*(a + b*x^2)^p]^3)/(4*b^2)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3 \log^3\left(c\left(a+bx^2\right)^p\right)\,dx = \frac{1}{2}\text{Subst}\left(\int x\log^3\left(c(a+bx)^p\right)\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a\log^3\left(c(a+bx)^p\right)}{b} + \frac{(a+bx)\log^3\left(c(a+bx)^p\right)}{b}\right)\,dx, x, x^2\right)$$
$$= \frac{\text{Subst}\left(\int (a+bx)\log^3\left(c(a+bx)^p\right)\,dx, x, x^2\right)}{2b} - \frac{a\text{Subst}\left(\int \log^3\left(c(a+bx)^p\right)\,dx,\right.}{2b}$$
$$= \frac{\text{Subst}\left(\int x\log^3\left(cx^p\right)\,dx, x, a+bx^2\right)}{2b^2} - \frac{a\text{Subst}\left(\int \log^3\left(cx^p\right)\,dx, x, a+bx^2\right)}{2b^2}$$
$$= -\frac{a(a+bx^2)\log^3\left(c\left(a+bx^2\right)^p\right)}{2b^2} + \frac{\left(a+bx^2\right)^2\log^3\left(c\left(a+bx^2\right)^p\right)}{4b^2} - \frac{(3p)\text{Subst}\left(\int\right.}{}$$
$$= \frac{3ap(a+bx^2)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b^2} - \frac{3p\left(a+bx^2\right)^2\log^2\left(c\left(a+bx^2\right)^p\right)}{8b^2} - \frac{a(a+bx^2}{}$$
$$= \frac{3ap^3x^2}{b} - \frac{3p^3\left(a+bx^2\right)^2}{16b^2} - \frac{3ap^2(a+bx^2)\log\left(c\left(a+bx^2\right)^p\right)}{b^2} + \frac{3p^2\left(a+bx^2\right)^2\log}{8b^2}$$

Mathematica [**A**]
time = 0.06, size = 237, normalized size = 1.12

$$\frac{21ap^3x^2}{8b} - \frac{3p^3x^4}{16} - \frac{3a^2p^3\log\left(a+bx^2\right)}{8b^2} - \frac{9a^2p^2\log\left(c\left(a+bx^2\right)^p\right)}{4b^2} - \frac{9ap^2x^2\log\left(c\left(a+bx^2\right)^p\right)}{4b} + \frac{3}{8}p^2x^4\log\left(c\left(a+bx^2\right)^p\right) + \frac{9a^2p\log^2\left(c\left(a+bx^2\right)^p\right)}{8b^2} + \frac{3apx^2\log^2\left(c\left(a+bx^2\right)^p\right)}{4b} - \frac{3}{8}px^4\log^2\left(c\left(a+bx^2\right)^p\right) - \frac{a^2\log^3\left(c\left(a+bx^2\right)^p\right)}{4b^2} + \frac{1}{4}x^4\log^3\left(c\left(a+bx^2\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] (21*a*p^3*x^2)/(8*b) - (3*p^3*x^4)/16 - (3*a^2*p^3*Log[a + b*x^2])/(8*b^2)
- (9*a^2*p^2*Log[c*(a + b*x^2)^p])/(4*b^2) - (9*a*p^2*x^2*Log[c*(a + b*x^2)
^p])/(4*b) + (3*p^2*x^4*Log[c*(a + b*x^2)^p])/8 + (9*a^2*p*Log[c*(a + b*x^2
)^p]^2)/(8*b^2) + (3*a*p*x^2*Log[c*(a + b*x^2)^p]^2)/(4*b) - (3*p*x^4*Log[c
*(a + b*x^2)^p]^2)/8 - (a^2*Log[c*(a + b*x^2)^p]^3)/(4*b^2) + (x^4*Log[c*(a
 + b*x^2)^p]^3)/4
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 14.82, size = 241142, normalized size = 1142.85

method	result	size
risch	Expression too large to display	241142

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(b*x^2+a)^p)^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.31, size = 203, normalized size = 0.96

$$\frac{1}{4}x^4\log\left(\left(bx^2+a\right)^pc\right)^3-\frac{3}{8}bp\left(\frac{2a^2\log\left(bx^2+a\right)}{b^3}+\frac{bx^4-2ax^2}{b^2}\right)\log\left(\left(bx^2+a\right)^pc\right)^2-\frac{1}{16}bp\left(\frac{\left(3b^2x^4+4a^2\log\left(bx^2+a\right)^3-42abx^2+18a^2\log\left(bx^2+a\right)^2+42a^2\log\left(bx^2+a\right)\right)p^2}{b^3}-\frac{6\left(b^2x^4-6abx^2+2a^2\log\left(bx^2+a\right)^2+6a^2\log\left(bx^2+a\right)\right)p\log\left(\left(bx^2+a\right)^pc\right)}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log((b*x^2 + a)^p*c)^3 - 3/8*b*p*(2*a^2*log(b*x^2 + a)/b^3 + (b*x^4
 - 2*a*x^2)/b^2)*log((b*x^2 + a)^p*c)^2 - 1/16*b*p*((3*b^2*x^4 + 4*a^2*log(
b*x^2 + a)^3 - 42*a*b*x^2 + 18*a^2*log(b*x^2 + a)^2 + 42*a^2*log(b*x^2 + a)
)*p^2/b^3 - 6*(b^2*x^4 - 6*a*b*x^2 + 2*a^2*log(b*x^2 + a)^2 + 6*a^2*log(b*x
^2 + a))*p*log((b*x^2 + a)^p*c)/b^3)
```

Fricas [A]
time = 0.39, size = 275, normalized size = 1.30

$$-\frac{3b^2p^3x^4-4b^2x^4\log\left(c\right)^3-42abp^3x^2-4\left(b^2p^3x^4-a^2p^3\right)\log\left(bx^2+a\right)^3+6\left(b^2p^3x^4-2abp^3x^2-3a^2p^3-2\left(b^2p^2x^4-a^2p^2\right)\log\left(c\right)\right)\log\left(bx^2+a\right)^2+6\left(b^2px^4-2abpx^2\right)\log\left(c\right)^2-6\left(b^2p^3x^4-6abp^3x^2-7a^2p^3+2\left(b^2px^4-a^2p\right)\log\left(c\right)^2-2\left(b^2p^2x^4-2abp^2x^2-3a^2p^2\right)\log\left(c\right)\right)\log\left(bx^2+a\right)-6\left(b^2p^2x^4-6abp^2x^2\right)\log\left(c\right)}{16b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(3*b^2*p^3*x^4 - 4*b^2*x^4*log(c)^3 - 42*a*b*p^3*x^2 - 4*(b^2*p^3*x^4
 - a^2*p^3)*log(b*x^2 + a)^3 + 6*(b^2*p^3*x^4 - 2*a*b*p^3*x^2 - 3*a^2*p^3 -
 2*(b^2*p^2*x^4 - a^2*p^2)*log(c))*log(b*x^2 + a)^2 + 6*(b^2*p*x^4 - 2*a*b*
p*x^2)*log(c)^2 - 6*(b^2*p^3*x^4 - 6*a*b*p^3*x^2 - 7*a^2*p^3 + 2*(b^2*p*x^4
 - a^2*p)*log(c)^2 - 2*(b^2*p^2*x^4 - 2*a*b*p^2*x^2 - 3*a^2*p^2)*log(c))*lo
g(b*x^2 + a) - 6*(b^2*p^2*x^4 - 6*a*b*p^2*x^2)*log(c))/b^2
```

Sympy [A]
time = 2.63, size = 223, normalized size = 1.06

$$\begin{cases} -\frac{21a^2p^2\log\left(c\left(a+bx^2\right)^p\right)}{8b^2}+\frac{9a^2p\log\left(c\left(a+bx^2\right)^p\right)^2}{8b^2}-\frac{a^2\log\left(c\left(a+bx^2\right)^p\right)^3}{4b^2}+\frac{21ap^3x^2}{8b}-\frac{9ap^2x^2\log\left(c\left(a+bx^2\right)^p\right)}{4b}+\frac{3apx^2\log\left(c\left(a+bx^2\right)^p\right)^2}{4b}-\frac{3p^3x^4}{16}+\frac{3p^2x^4\log\left(c\left(a+bx^2\right)^p\right)}{8}-\frac{3px^4\log\left(c\left(a+bx^2\right)^p\right)^2}{8}+\frac{x^4\log\left(c\left(a+bx^2\right)^p\right)^3}{4} & \text{for } b\neq 0 \\ \frac{x^4\log\left(a^pc\right)^3}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Piecewise((-21*a**2*p**2*log(c*(a + b*x**2)**p)/(8*b**2) + 9*a**2*p*log(c*(
a + b*x**2)**p)**2/(8*b**2) - a**2*log(c*(a + b*x**2)**p)**3/(4*b**2) + 21*
a*p**3*x**2/(8*b) - 9*a*p**2*x**2*log(c*(a + b*x**2)**p)/(4*b) + 3*a*p*x**2
*log(c*(a + b*x**2)**p)**2/(4*b) - 3*p**3*x**4/16 + 3*p**2*x**4*log(c*(a +
b*x**2)**p)/8 - 3*p*x**4*log(c*(a + b*x**2)**p)**2/8 + x**4*log(c*(a + b*x*
*2)**p)**3/4, Ne(b, 0)), (x**4*log(a**p*c)**3/4, True))
```

Giac [A]
time = 5.23, size = 385, normalized size = 1.82

$$\frac{4\,(bx^2+a)^2p^3\log(bx^2+a)^3-6\,(bx^2+a)^2p^3\log(bx^2+a)^2+12\,(bx^2+a)^2p^2\log(bx^2+a)^2\log(c)+6\,(bx^2+a)^2p^3\log(bx^2+a)-12\,(bx^2+a)^2p^2\log(bx^2+a)\log(c)+12\,(bx^2+a)^2p\log(bx^2+a)\log(c)^2-3\,(bx^2+a)^2p^3+6\,(bx^2+a)^2p^2\log(c)-6\,(bx^2+a)^2p\log(c)^2+4\,(bx^2+a)^2\log(c)^3}{16\,b^2}-\frac{\left((bx^2+a)\log(bx^2+a)^3-6\,bx^2-3\,(bx^2+a)\log(bx^2+a)^2+6\,(bx^2+a)\log(bx^2+a)-6\,a\right)ap^3+3\left(2\,bx^2+(bx^2+a)\log(bx^2+a)^2-2\,(bx^2+a)\log(bx^2+a)+2\,a\right)ap^2\log(c)-3\,(bx^2-(bx^2+a)\log(bx^2+a)+a)ap\log(c)^2+(bx^2+a)a\log(c)^3}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] 1/16*(4*(b*x^2 + a)^2*p^3*log(b*x^2 + a)^3 - 6*(b*x^2 + a)^2*p^3*log(b*x^2
+ a)^2 + 12*(b*x^2 + a)^2*p^2*log(b*x^2 + a)^2*log(c) + 6*(b*x^2 + a)^2*p^3
*log(b*x^2 + a) - 12*(b*x^2 + a)^2*p^2*log(b*x^2 + a)*log(c) + 12*(b*x^2 +
a)^2*p*log(b*x^2 + a)*log(c)^2 - 3*(b*x^2 + a)^2*p^3 + 6*(b*x^2 + a)^2*p^2*
log(c) - 6*(b*x^2 + a)^2*p*log(c)^2 + 4*(b*x^2 + a)^2*log(c)^3)/b^2 - 1/2*(
((b*x^2 + a)*log(b*x^2 + a)^3 - 6*b*x^2 - 3*(b*x^2 + a)*log(b*x^2 + a)^2 +
6*(b*x^2 + a)*log(b*x^2 + a) - 6*a)*a*p^3 + 3*(2*b*x^2 + (b*x^2 + a)*log(b*
x^2 + a)^2 - 2*(b*x^2 + a)*log(b*x^2 + a) + 2*a)*a*p^2*log(c) - 3*(b*x^2 -
(b*x^2 + a)*log(b*x^2 + a) + a)*a*p*log(c)^2 + (b*x^2 + a)*a*log(c)^3)/b^2
```

Mupad [B]
time = 0.30, size = 144, normalized size = 0.68

$$\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^2\,\left(\frac{9\,a^2\,p}{8\,b^2}-\frac{3\,p\,x^4}{8}+\frac{3\,a\,p\,x^2}{4\,b}\right)-\frac{3\,p^3\,x^4}{16}+\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)\,\left(\frac{3\,p^2\,x^4}{8}-\frac{9\,a\,p^2\,x^2}{4\,b}\right)+{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}^3\,\left(\frac{x^4}{4}-\frac{a^2}{4\,b^2}\right)+\frac{21\,a\,p^3\,x^2}{8\,b}-\frac{21\,a^2\,p^3\,\ln\left(b\,x^2+a\right)}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(c*(a + b*x^2)^p)^3,x)
```

```
[Out] log(c*(a + b*x^2)^p)^2*((9*a^2*p)/(8*b^2) - (3*p*x^4)/8 + (3*a*p*x^2)/(4*b)
) - (3*p^3*x^4)/16 + log(c*(a + b*x^2)^p)*((3*p^2*x^4)/8 - (9*a*p^2*x^2)/(4
*b)) + log(c*(a + b*x^2)^p)^3*(x^4/4 - a^2/(4*b^2)) + (21*a*p^3*x^2)/(8*b)
- (21*a^2*p^3*log(a + b*x^2))/(8*b^2)
```

3.93 $\int x \log^3\left(c(a+bx^2)^p\right)\,dx$

Optimal. Leaf size=93

$$-3p^3x^2+\frac{3p^2(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b}-\frac{3p(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2b}+\frac{(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2b}$$

```
[Out] -3*p^3*x^2+3*p^2*(b*x^2+a)*ln(c*(b*x^2+a)^p)/b-3/2*p*(b*x^2+a)*ln(c*(b*x^2+
a)^p)^2/b+1/2*(b*x^2+a)*ln(c*(b*x^2+a)^p)^3/b
```

Rubi [A]
time = 0.05, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2504, 2436, 2333, 2332}

$$\frac{3p^2(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b}+\frac{(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2b}-\frac{3p(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2b}-3p^3x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] -3*p^3*x^2 + (3*p^2*(a + b*x^2)*Log[c*(a + b*x^2)^p])/b - (3*p*(a + b*x^2)*
Log[c*(a + b*x^2)^p]^2)/(2*b) + ((a + b*x^2)*Log[c*(a + b*x^2)^p]^3)/(2*b)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
```

```
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int x\log^3\left(c\left(a+bx^2\right)^p\right)\,dx &= \frac{1}{2}\text{Subst}\left(\int \log^3\left(c(a+bx)^p\right)\,dx, x, x^2\right)\\ &= \frac{\text{Subst}\left(\int \log^3\left(cx^p\right)\,dx, x, a+bx^2\right)}{2b}\\ &= \frac{\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2b}-\frac{(3p)\text{Subst}\left(\int \log^2\left(cx^p\right)\,dx, x, a+bx^2\right)}{2b}\\ &= -\frac{3p\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b}+\frac{\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2b}+\frac{\left(3p^2\right)\text{Subst}\left(\int\right.}{}\\ &= -3p^3x^2+\frac{3p^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b}-\frac{3p\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b}+\frac{(a}{}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 87, normalized size = 0.94

$$\frac{-6bp^3x^2+6p^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)-3p\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)+\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] (-6*b*p^3*x^2 + 6*p^2*(a + b*x^2)*Log[c*(a + b*x^2)^p] - 3*p*(a + b*x^2)*Lo
g[c*(a + b*x^2)^p]^2 + (a + b*x^2)*Log[c*(a + b*x^2)^p]^3)/(2*b)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.45, size = 3925, normalized size = 42.20

method	result	size
risch	Expression too large to display	3925

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(b*x^2+a)^p)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*I*Pi^3*x^2*csgn(I*c*(b*x^2+a)^p)^9-3/2/b*ln(c)*a*p^2*ln(b*x^2+a)^2+3/2
/b*ln(c)^2*ln(b*x^2+a)*a*p-3/b*ln(c)*ln(b*x^2+a)*a*p^2-3/8*ln(c)*Pi^2*x^2*c
sgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^4+3/4*ln(c)*Pi^2*x^2*csgn(I*(b*x
^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^5+3/4*ln(c)*Pi^2*x^2*csgn(I*c*(b*x^2+a)^p)^5
*csgn(I*c)-3/8*ln(c)*Pi^2*x^2*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)^2+3/8*Pi^2*
p*x^2*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^4-3/4*Pi^2*p*x^2*csgn(I*(
```

b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^5-3/4*Pi^2*p*x^2*csgn(I*c*(b*x^2+a)^p)^5*
csgn(I*c)+3/8*Pi^2*p*x^2*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)^2-1/16*I*Pi^3*x^
2*csgn(I*(b*x^2+a)^p)^3*csgn(I*c*(b*x^2+a)^p)^6+3/16*I*Pi^3*x^2*csgn(I*(b*x
^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^7-3/2*I*ln(c)*Pi*p*x^2*csgn(I*c*(b*x^2+a)^
p)^2*csgn(I*c)-3/2*I*Pi*p^2*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*c
sgn(I*c)-3/16*I*Pi^3*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^8-3/16*I
*Pi^3*x^2*csgn(I*c*(b*x^2+a)^p)^8*csgn(I*c)+3/16*I*Pi^3*x^2*csgn(I*c*(b*x^2
+a)^p)^7*csgn(I*c)^2-1/16*I*Pi^3*x^2*csgn(I*c*(b*x^2+a)^p)^6*csgn(I*c)^3-3/
4*I*ln(c)^2*Pi*x^2*csgn(I*c*(b*x^2+a)^p)^3-3/2*I*Pi*p^2*x^2*csgn(I*c*(b*x^2
+a)^p)^3+3/4/b*Pi^2*ln(b*x^2+a)*a*p*csgn(I*c*(b*x^2+a)^p)^5*csgn(I*c)-3/8/b
*Pi^2*ln(b*x^2+a)*a*p*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)^2-3/8/b*Pi^2*ln(b*x
^2+a)*a*p*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^4+3/4/b*Pi^2*ln(b*x^2
+a)*a*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^5+3/4*I/b*Pi*a*p^2*csgn(I
c(b*x^2+a)^p)^3*ln(b*x^2+a)^2+3/2*I/b*Pi*ln(b*x^2+a)*a*p^2*csgn(I*c*(b*x^
2+a)^p)^3-3/4*I*ln(c)^2*Pi*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*cs
gn(I*c)-3/2*I*ln(c)*Pi*p*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-3/
2*I/b*ln(c)*Pi*ln(b*x^2+a)*a*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*cs
gn(I*c)+3*a*p^3/b*ln(b*x^2+a)-3/2*ln(c)^2*p*x^2+3*ln(c)*p^2*x^2+1/2*x^2*ln(
(b*x^2+a)^p)^3-3*p^3*x^2+3/4/b*Pi^2*ln(b*x^2+a)*a*p*csgn(I*(b*x^2+a)^p)^2*c
sgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)-3/8/b*Pi^2*ln(b*x^2+a)*a*p*csgn(I*(b*x^2+a
)^p)^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)^2-3/2/b*Pi^2*ln(b*x^2+a)*a*p*csgn(
I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)+3/4/b*Pi^2*ln(b*x^2+a)*a*p
csgn(I(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)^2-3/4*I/b*Pi*a*p^2*c
sgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2*ln(b*x^2+a)^2-3/4*I/b*Pi*a*p^2*c
sgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)*ln(b*x^2+a)^2-3/2*I/b*ln(c)*Pi*ln(b*x^2+a)
*a*p*csgn(I*c*(b*x^2+a)^p)^3-3/2*I/b*Pi*ln(b*x^2+a)*a*p^2*csgn(I*(b*x^2+a)^
p)*csgn(I*c*(b*x^2+a)^p)^2-3/2*I/b*Pi*ln(b*x^2+a)*a*p^2*csgn(I*c*(b*x^2+a)^
p)^2*csgn(I*c)+3/2*I*ln(c)*Pi*p*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^
p)*csgn(I*c)-3/8*ln(c)*Pi^2*x^2*csgn(I*c*(b*x^2+a)^p)^6+3/8*Pi^2*p*x^2*csgn
(I*c*(b*x^2+a)^p)^6+1/2/b*a*p^3*ln(b*x^2+a)^3+3/2/b*a*p^3*ln(b*x^2+a)^2+3/4
*I/b*Pi*a*p^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)*ln(b*x^2+
a)^2+3/2*I/b*ln(c)*Pi*ln(b*x^2+a)*a*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a
)^p)^2+3/2*I/b*ln(c)*Pi*ln(b*x^2+a)*a*p*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+3
/2*I/b*Pi*ln(b*x^2+a)*a*p^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(
I*c)+3/4*(I*Pi*b*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*b*x^2
csgn(I(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*b*x^2*csgn(I*c*(b
*x^2+a)^p)^3+I*Pi*b*x^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)*b*x^2-2*x
^2*p*b+2*p*a*ln(b*x^2+a))/b*ln((b*x^2+a)^p)^2+3/8*(8*x^2*b*p^2-8*ln(c)*b*p*
x^2+8*ln(c)*ln(b*x^2+a)*a*p-Pi^2*b*x^2*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)^2+
2*Pi^2*b*x^2*csgn(I*c*(b*x^2+a)^p)^5*csgn(I*c)+4*ln(c)^2*b*x^2-4*a*p^2*ln(b
*x^2+a)^2-Pi^2*b*x^2*csgn(I*c*(b*x^2+a)^p)^6+4*I*Pi*b*p*x^2*csgn(I*(b*x^2+a
)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-4*I*Pi*b*p*x^2*csgn(I*(b*x^2+a)^p)*csg
n(I*c*(b*x^2+a)^p)^2-4*I*Pi*b*p*x^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+4*I*P
i*ln(b*x^2+a)*a*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-8*ln(b*x^2+a)
*a*p^2-4*I*Pi*ln(b*x^2+a)*a*p*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csg

```
n(I*c)-Pi^2*b*x^2*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^4+2*Pi^2*b*x^
2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^5-4*I*ln(c)*Pi*b*x^2*csgn(I*(b*
x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+4*I*Pi*ln(b*x^2+a)*a*p*csgn(I*c*(
b*x^2+a)^p)^2*csgn(I*c)+4*I*ln(c)*Pi*b*x^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c
)+4*I*ln(c)*Pi*b*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+2*Pi^2*b*x
^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)^2+2*Pi^2*b*x^2*csg
n(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)-4*I*Pi*ln(b*x^2+a)*a*p
*csgn(I*c*(b*x^2+a)^p)^3-4*I*ln(c)*Pi*b*x^2*csgn(I*c*(b*x^2+a)^p)^3+4*I*Pi*
b*p*x^2*csgn(I*c*(b*x^2+a)^p)^3-Pi^2*b*x^2*csgn(I*(b*x^2+a)^p)^2*csgn(I*c*(
b*x^2+a)^p)^2*csgn(I*c)^2-4*Pi^2*b*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+
a)^p)^4*csgn(I*c))/b*ln((b*x^2+a)^p)+1/2*ln(c)^3*x^2+3/4*ln(c)*Pi^2*x^2*csg
n(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)-3/8*ln(c)*Pi^2*x^2*csg
n(I*(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)^2-3/2*ln(c)*Pi^2*x^2*c
sgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^4*csgn(I*c)+3/4*ln(c)*Pi^2*x^2*csg
n(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c)^2-3/4*Pi^2*p*x^2*csgn(I*
(b*x^2+a)^p)^2*csgn(I*c*(b*x^2+a)^p)^3*csgn(I*c...
```

Maxima [A]
time = 0.29, size = 164, normalized size = 1.76

$$-\frac{3}{2}\,bp\left(\frac{x^2}{b}-\frac{a\log\left(bx^2+a\right)}{b^2}\right)\log\left(\left(bx^2+a\right)^p c\right)^2+\frac{1}{2}\,x^2\log\left(\left(bx^2+a\right)^p c\right)^3+\frac{1}{2}\,bp\left(\frac{\left(a\log\left(bx^2+a\right)^3-6\,bx^2+3\,a\log\left(bx^2+a\right)^2+6\,a\log\left(bx^2+a\right)\right)p^2}{b^2}+\frac{3\left(2\,bx^2-a\log\left(bx^2+a\right)^2-2\,a\log\left(bx^2+a\right)\right)p\log\left(\left(bx^2+a\right)^p c\right)}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
[Out] -3/2*b*p*(x^2/b - a*log(b*x^2 + a)/b^2)*log((b*x^2 + a)^p*c)^2 + 1/2*x^2*lo
g((b*x^2 + a)^p*c)^3 + 1/2*b*p*((a*log(b*x^2 + a)^3 - 6*b*x^2 + 3*a*log(b*x
^2 + a)^2 + 6*a*log(b*x^2 + a))*p^2/b^2 + 3*(2*b*x^2 - a*log(b*x^2 + a)^2 -
 2*a*log(b*x^2 + a))*p*log((b*x^2 + a)^p*c)/b^2)
```

Fricas [A]
time = 0.45, size = 176, normalized size = 1.89

$$-\frac{6\,bp^3x^2-6\,bp^2x^2\log\left(c\right)+3\,bpx^2\log\left(c\right)^2-bx^2\log\left(c\right)^3-\left(bp^3x^2+ap^3\right)\log\left(bx^2+a\right)^3+3\left(bp^3x^2+ap^3-\left(bp^2x^2+ap^2\right)\log\left(c\right)\right)\log\left(bx^2+a\right)^2-3\left(2\,bp^3x^2+2\,ap^3+\left(bpx^2+ap\right)\log\left(c\right)^2-2\left(bp^2x^2+ap^2\right)\log\left(c\right)\right)\log\left(bx^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
[Out] -1/2*(6*b*p^3*x^2 - 6*b*p^2*x^2*log(c) + 3*b*p*x^2*log(c)^2 - b*x^2*log(c)^
3 - (b*p^3*x^2 + a*p^3)*log(b*x^2 + a)^3 + 3*(b*p^3*x^2 + a*p^3 - (b*p^2*x^
2 + a*p^2)*log(c))*log(b*x^2 + a)^2 - 3*(2*b*p^3*x^2 + 2*a*p^3 + (b*p*x^2 +
 a*p)*log(c)^2 - 2*(b*p^2*x^2 + a*p^2)*log(c))*log(b*x^2 + a))/b
```

Sympy [A]
time = 0.95, size = 143, normalized size = 1.54

$$\begin{cases} \frac{3ap^2\log\left(c\left(a+bx^2\right)^p\right)}{b}-\frac{3ap\log\left(c\left(a+bx^2\right)^p\right)^2}{2b}+\frac{a\log\left(c\left(a+bx^2\right)^p\right)^3}{2b}-3p^3x^2+3p^2x^2\log\left(c\left(a+bx^2\right)^p\right)-\frac{3px^2\log\left(c\left(a+bx^2\right)^p\right)^2}{2}+\frac{x^2\log\left(c\left(a+bx^2\right)^p\right)^3}{2} & \text{for}\: b\neq 0 \\ \frac{x^2\log\left(a^pc\right)^3}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Piecewise((3*a*p**2*log(c*(a + b*x**2)**p)/b - 3*a*p*log(c*(a + b*x**2)**p)
**2/(2*b) + a*log(c*(a + b*x**2)**p)**3/(2*b) - 3*p**3*x**2 + 3*p**2*x**2*l
og(c*(a + b*x**2)**p) - 3*p*x**2*log(c*(a + b*x**2)**p)**2/2 + x**2*log(c*(
a + b*x**2)**p)**3/2, Ne(b, 0)), (x**2*log(a**p*c)**3/2, True))
```

Giac [A]
time = 4.67, size = 169, normalized size = 1.82

$$\frac{\left((bx^2+a)\log\left(bx^2+a\right)^3-6\,bx^2-3\,(bx^2+a)\log\left(bx^2+a\right)^2+6\,(bx^2+a)\log\left(bx^2+a\right)-6\,a\right)p^3+3\left(2\,bx^2+(bx^2+a)\log\left(bx^2+a\right)^2-2\,(bx^2+a)\log\left(bx^2+a\right)+2\,a\right)p^2\log\left(c\right)-3\,(bx^2-(bx^2+a)\log\left(bx^2+a\right)+a)p\log\left(c\right)^2+(bx^2+a)\log\left(c\right)^3}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] 1/2*(((b*x^2 + a)*log(b*x^2 + a)^3 - 6*b*x^2 - 3*(b*x^2 + a)*log(b*x^2 + a)
^2 + 6*(b*x^2 + a)*log(b*x^2 + a) - 6*a)*p^3 + 3*(2*b*x^2 + (b*x^2 + a)*log
(b*x^2 + a)^2 - 2*(b*x^2 + a)*log(b*x^2 + a) + 2*a)*p^2*log(c) - 3*(b*x^2 -
 (b*x^2 + a)*log(b*x^2 + a) + a)*p*log(c)^2 + (b*x^2 + a)*log(c)^3)/b
```

Mupad [B]
time = 0.24, size = 103, normalized size = 1.11

$$\ln\left(c\left(b\,x^2+a\right)^p\right)^3\left(\frac{a}{2\,b}+\frac{x^2}{2}\right)-\ln\left(c\left(b\,x^2+a\right)^p\right)^2\left(\frac{3\,p\,x^2}{2}+\frac{3\,a\,p}{2\,b}\right)-3\,p^3\,x^2+3\,p^2\,x^2\,\ln\left(c\left(b\,x^2+a\right)^p\right)+\frac{3\,a\,p^3\,\ln\left(b\,x^2+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(c*(a + b*x^2)^p)^3,x)
```

```
[Out] log(c*(a + b*x^2)^p)^3*(a/(2*b) + x^2/2) - log(c*(a + b*x^2)^p)^2*((3*p*x^2
)/2 + (3*a*p)/(2*b)) - 3*p^3*x^2 + 3*p^2*x^2*log(c*(a + b*x^2)^p) + (3*a*p^
3*log(a + b*x^2))/b
```

3.94 $\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x}\,dx$

Optimal. Leaf size=106

$$\frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^3\left(c\left(a+bx^2\right)^p\right)+\frac{3}{2}p\log^2\left(c\left(a+bx^2\right)^p\right)\text{Li}_2\left(1+\frac{bx^2}{a}\right)-3p^2\log\left(c\left(a+bx^2\right)^p\right)\text{Li}_3\left(1+\frac{bx^2}{a}\right)$$

```
[Out] 1/2*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)^3+3/2*p*ln(c*(b*x^2+a)^p)^2*polylog(2,1+
b*x^2/a)-3*p^2*ln(c*(b*x^2+a)^p)*polylog(3,1+b*x^2/a)+3*p^3*polylog(4,1+b*x
^2/a)
```

Rubi [A]
time = 0.11, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2504, 2443, 2481, 2421, 2430, 6724}

$$-3p^2\text{PolyLog}\left(3,\frac{bx^2}{a}+1\right)\log\left(c\left(a+bx^2\right)^p\right)+\frac{3}{2}p\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)\log^2\left(c\left(a+bx^2\right)^p\right)+3p^3\text{PolyLog}\left(4,\frac{bx^2}{a}+1\right)+\frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^3\left(c\left(a+bx^2\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^3/x,x]
```

```
[Out] (Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p]^3)/2 + (3*p*Log[c*(a + b*x^2)^p]^2*
PolyLog[2, 1 + (b*x^2)/a])/2 - 3*p^2*Log[c*(a + b*x^2)^p]*PolyLog[3, 1 + (b
*x^2)/a] + 3*p^3*PolyLog[4, 1 + (b*x^2)/a]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
```

```
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\log^3\left(c(a+bx)^p\right)}{x}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^3\left(c\left(a+bx^2\right)^p\right) - \frac{1}{2}(3bp)\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)\log^2\left(c(a+bx)^p\right)}{a+bx}\right.$$

$$= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^3\left(c\left(a+bx^2\right)^p\right) - \frac{1}{2}(3p)\text{Subst}\left(\int \frac{\log^2\left(cx^p\right)\log\left(-\frac{b\left(-\frac{a}{b}+\frac{x}{b}\right)}{a}\right)}{x}\right.$$

$$= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^3\left(c\left(a+bx^2\right)^p\right) + \frac{3}{2}p\log^2\left(c\left(a+bx^2\right)^p\right)\text{Li}_2\left(1+\frac{bx^2}{a}\right) - \left(3p^2\right.$$

$$= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^3\left(c\left(a+bx^2\right)^p\right) + \frac{3}{2}p\log^2\left(c\left(a+bx^2\right)^p\right)\text{Li}_2\left(1+\frac{bx^2}{a}\right) - 3p^2\,\text{l}$$

$$= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^3\left(c\left(a+bx^2\right)^p\right) + \frac{3}{2}p\log^2\left(c\left(a+bx^2\right)^p\right)\text{Li}_2\left(1+\frac{bx^2}{a}\right) - 3p^2\,\text{l}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 279 vs. $2(106) = 212$.
time = 0.11, size = 279, normalized size = 2.63

$$\log(x)\left(-p\log\left(a+bx^2\right)+\log\left(c\left(a+bx^2\right)^p\right)\right)^3+3p\left(-p\log\left(a+bx^2\right)+\log\left(c\left(a+bx^2\right)^p\right)\right)^2\left(\log(x)\left(\log\left(a+bx^2\right)-\log\left(1+\frac{bx^2}{a}\right)\right)-\frac{1}{2}\mathrm{Li}_2\left(-\frac{bx^2}{a}\right)\right)-\frac{3}{2}p^2\left(p\log\left(a+bx^2\right)-\log\left(c\left(a+bx^2\right)^p\right)\right)\left(\log\left(-\frac{bx^2}{a}\right)\log^2\left(a+bx^2\right)+2\log\left(a+bx^2\right)\mathrm{Li}_2\left(1+\frac{bx^2}{a}\right)-2\mathrm{Li}_3\left(1+\frac{bx^2}{a}\right)\right)+\frac{1}{2}p^3\left(\log\left(-\frac{bx^2}{a}\right)\log^3\left(a+bx^2\right)+3\log^2\left(a+bx^2\right)\mathrm{Li}_2\left(1+\frac{bx^2}{a}\right)-6\log\left(a+bx^2\right)\mathrm{Li}_3\left(1+\frac{bx^2}{a}\right)+6\mathrm{Li}_4\left(1+\frac{bx^2}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^3/x,x]
```

```
[Out] Log[x]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^3 + 3*p*(-(p*Log[a + b*
x^2]) + Log[c*(a + b*x^2)^p])^2*(Log[x]*(Log[a + b*x^2] - Log[1 + (b*x^2)/a
]) - PolyLog[2, -((b*x^2)/a)]/2) - (3*p^2*(p*Log[a + b*x^2] - Log[c*(a + b*
x^2)^p])*(Log[-((b*x^2)/a)]*Log[a + b*x^2]^2 + 2*Log[a + b*x^2]*PolyLog[2,
1 + (b*x^2)/a] - 2*PolyLog[3, 1 + (b*x^2)/a]))/2 + (p^3*(Log[-((b*x^2)/a)]*
Log[a + b*x^2]^3 + 3*Log[a + b*x^2]^2*PolyLog[2, 1 + (b*x^2)/a] - 6*Log[a +
 b*x^2]*PolyLog[3, 1 + (b*x^2)/a] + 6*PolyLog[4, 1 + (b*x^2)/a]))/2
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c(b\,x^2+a)^p\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^3/x,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^3/x,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(101) = 202$.
time = 0.31, size = 217, normalized size = 2.05

$$\frac{1}{2}\left(\log\left(bx^2+a\right)^3\log\left(-\frac{bx^2+a}{a}+1\right)+3\,\mathrm{Li}_2\left(\frac{bx^2+a}{a}\right)\log\left(bx^2+a\right)^2-6\,\log\left(bx^2+a\right)\mathrm{Li}_3\left(\frac{bx^2+a}{a}\right)+6\,\mathrm{Li}_4\left(\frac{bx^2+a}{a}\right)\right)p^3+\frac{3}{2}\left(\log\left(bx^2+a\right)^2\log\left(-\frac{bx^2+a}{a}+1\right)+2\,\mathrm{Li}_2\left(\frac{bx^2+a}{a}\right)\log\left(bx^2+a\right)-2\,\mathrm{Li}_3\left(\frac{bx^2+a}{a}\right)\right)p^2\log\left(c\right)+\frac{3}{2}\left(\log\left(bx^2+a\right)\log\left(-\frac{bx^2+a}{a}+1\right)+\mathrm{Li}_2\left(\frac{bx^2+a}{a}\right)\right)p\log\left(c\right)^2+\log\left(c\right)^3\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x,x, algorithm="maxima")
```

```
[Out] 1/2*(log(b*x^2 + a)^3*log(-(b*x^2 + a)/a + 1) + 3*dilog((b*x^2 + a)/a)*log(
b*x^2 + a)^2 - 6*log(b*x^2 + a)*polylog(3, (b*x^2 + a)/a) + 6*polylog(4, (b
*x^2 + a)/a))*p^3 + 3/2*(log(b*x^2 + a)^2*log(-(b*x^2 + a)/a + 1) + 2*dilog
((b*x^2 + a)/a)*log(b*x^2 + a) - 2*polylog(3, (b*x^2 + a)/a))*p^2*log(c) +
3/2*(log(b*x^2 + a)*log(-(b*x^2 + a)/a + 1) + dilog((b*x^2 + a)/a))*p*log(c
)^2 + log(c)^3*log(x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^3/x, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(c(a + bx^2)^p\right)^3}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**3/x,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**3/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^3/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln \left(c\,(b\,x^2 + a)^p\right)^3}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)^3/x,x)
```

```
[Out] int(log(c*(a + b*x^2)^p)^3/x, x)
```

3.95 $\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^3}\,dx$

Optimal. Leaf size=119

$$\frac{3bp\log\left(-\frac{bx^2}{a}\right)\log^2\left(c(a+bx^2)^p\right)}{2a}-\frac{(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2ax^2}+\frac{3bp^2\log\left(c(a+bx^2)^p\right)\text{Li}_2\left(1+\frac{bx^2}{a}\right)}{a}-\frac{3bp^3\text{L}}{}$$

```
[Out] 3/2*b*p*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)^2/a-1/2*(b*x^2+a)*ln(c*(b*x^2+a)^p)^
3/a/x^2+3*b*p^2*ln(c*(b*x^2+a)^p)*polylog(2,1+b*x^2/a)/a-3*b*p^3*polylog(3,
1+b*x^2/a)/a
```

Rubi [A]
time = 0.10, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2504, 2444, 2443, 2481, 2421, 6724}

$$\frac{3bp^2\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)\log\left(c(a+bx^2)^p\right)}{a}-\frac{3bp^3\text{PolyLog}\left(3,\frac{bx^2}{a}+1\right)}{a}-\frac{(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2ax^2}+\frac{3bp\log\left(-\frac{bx^2}{a}\right)\log^2\left(c(a+bx^2)^p\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^3/x^3,x]
```

```
[Out] (3*b*p*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p]^2)/(2*a) - ((a + b*x^2)*Log[c
*(a + b*x^2)^p]^3)/(2*a*x^2) + (3*b*p^2*Log[c*(a + b*x^2)^p]*PolyLog[2, 1 +
 (b*x^2)/a])/a - (3*b*p^3*PolyLog[3, 1 + (b*x^2)/a])/a
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
```

```
- d*g)*(f + g*x))), x] - Dist[b*e*n*(p/(e*f - d*g)), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\log^3\left(c(a+bx)^p\right)}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2ax^2} + \frac{(3bp)\text{Subst}\left(\int \frac{\log^2(c(a+bx)^p)}{x}\,dx, x, x^2\right)}{2a}$$

$$= \frac{3bp\log\left(-\frac{bx^2}{a}\right)\log^2\left(c(a+bx^2)^p\right)}{2a} - \frac{(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2ax^2} - \frac{(3b^2p^2)\,\text{Subst}}{}$$

$$= \frac{3bp\log\left(-\frac{bx^2}{a}\right)\log^2\left(c(a+bx^2)^p\right)}{2a} - \frac{(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2ax^2} - \frac{(3bp^2)\,\text{Subst}\Big(}{}$$

$$= \frac{3bp\log\left(-\frac{bx^2}{a}\right)\log^2\left(c(a+bx^2)^p\right)}{2a} - \frac{(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2ax^2} + \frac{3bp^2\log\left(c(a\right.}{}$$

$$= \frac{3bp\log\left(-\frac{bx^2}{a}\right)\log^2\left(c(a+bx^2)^p\right)}{2a} - \frac{(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2ax^2} + \frac{3bp^2\log\left(c(a\right.}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 302 vs. 2(119) = 238.
time = 0.23, size = 302, normalized size = 2.54

$$-\frac{-6bp^3x^2\log(x)\log^2(a+bx^2) + 3bp^3x^2\log\left(-\frac{bx^2}{a}\right)\log^2(a+bx^2) + bp^3x^2\log^3(a+bx^2) + 12bp^2x^2\log(x)\log(a+bx^2)\log\left(c(a+bx^2)^p\right) - 6bp^2x^2\log\left(-\frac{bx^2}{a}\right)\log(a+bx^2)\log\left(c(a+bx^2)^p\right) - 3bp^2x^2\log^2(a+bx^2)\log\left(c(a+bx^2)^p\right) - 6bpx^2\log(x)\log^2\left(c(a+bx^2)^p\right) + 3bpx^2\log(a+bx^2)\log^2\left(c(a+bx^2)^p\right) + a\log^3\left(c(a+bx^2)^p\right) - 6bp^2x^2\log\left(c(a+bx^2)^p\right)\text{Li}_2\left(1+\frac{bx^2}{a}\right) + 6bp^3x^2\text{Li}_3\left(1+\frac{bx^2}{a}\right)}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^3/x^3,x]
```

```
[Out] -1/2*(-6*b*p^3*x^2*Log[x]*Log[a + b*x^2]^2 + 3*b*p^3*x^2*Log[-((b*x^2)/a)]*
Log[a + b*x^2]^2 + b*p^3*x^2*Log[a + b*x^2]^3 + 12*b*p^2*x^2*Log[x]*Log[a +
 b*x^2]*Log[c*(a + b*x^2)^p] - 6*b*p^2*x^2*Log[-((b*x^2)/a)]*Log[a + b*x^2]
*Log[c*(a + b*x^2)^p] - 3*b*p^2*x^2*Log[a + b*x^2]^2*Log[c*(a + b*x^2)^p] -
 6*b*p*x^2*Log[x]*Log[c*(a + b*x^2)^p]^2 + 3*b*p*x^2*Log[a + b*x^2]*Log[c*(
a + b*x^2)^p]^2 + a*Log[c*(a + b*x^2)^p]^3 - 6*b*p^2*x^2*Log[c*(a + b*x^2)^
p]*PolyLog[2, 1 + (b*x^2)/a] + 6*b*p^3*x^2*PolyLog[3, 1 + (b*x^2)/a])/(a*x^
2)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c(b\,x^2+a)^p\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^3/x^3,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^3/x^3,x)
```

Maxima [A]
time = 0.35, size = 202, normalized size = 1.70

$$\frac{1}{2}\left(\frac{3\left(\log\left(bx^2+a\right)^2\log\left(-\frac{bx^2+a}{a}+1\right)+2\,\mathrm{Li}_2\left(\frac{bx^2+a}{a}\right)\log\left(bx^2+a\right)-2\,\mathrm{Li}_3\left(\frac{bx^2+a}{a}\right)\right)p^2}{a}+\frac{6\left(\log\left(bx^2+a\right)\log\left(-\frac{bx^2+a}{a}+1\right)+\mathrm{Li}_2\left(\frac{bx^2+a}{a}\right)\right)p\log\left(c\right)}{a}+\frac{6\,\log\left(c\right)^2\log\left(x\right)}{a}-\frac{p^2\log\left(bx^2+a\right)^3+3\,p\log\left(bx^2+a\right)^2\log\left(c\right)+3\,\log\left(bx^2+a\right)\log\left(c\right)^2}{a}\right)bp-\frac{\log\left(\left(bx^2+a\right)^pc\right)^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(3*(log(b*x^2 + a)^2*log(-(b*x^2 + a)/a + 1) + 2*dilog((b*x^2 + a)/a)*l
og(b*x^2 + a) - 2*polylog(3, (b*x^2 + a)/a))*p^2/a + 6*(log(b*x^2 + a)*log(
-(b*x^2 + a)/a + 1) + dilog((b*x^2 + a)/a))*p*log(c)/a + 6*log(c)^2*log(x)/
a - (p^2*log(b*x^2 + a)^3 + 3*p*log(b*x^2 + a)^2*log(c) + 3*log(b*x^2 + a)*
log(c)^2)/a)*b*p - 1/2*log((b*x^2 + a)^p*c)^3/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^3,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^3/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left(c\left(a+bx^2\right)^p\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**3/x**3,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**3/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^3,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^3/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln \left(c\,{\left(b\,x^2+a\right)}^p\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)^3/x^3,x)
```

```
[Out] int(log(c*(a + b*x^2)^p)^3/x^3, x)
```

3.96 $\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^5} \, dx$

Optimal. Leaf size=219

$$\frac{3b^2p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{2a^2}-\frac{3bp(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{4a^2x^2}-\frac{\log^3\left(c(a+bx^2)^p\right)}{4x^4}-\frac{3b^2p\log^2\left(c(a+b\right.}{}$$

```
[Out] 3/2*b^2*p^2*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)/a^2-3/4*b*p*(b*x^2+a)*ln(c*(b*x^
2+a)^p)^2/a^2/x^2-1/4*ln(c*(b*x^2+a)^p)^3/x^4-3/4*b^2*p*ln(c*(b*x^2+a)^p)^2
*ln(1-a/(b*x^2+a))/a^2+3/2*b^2*p^2*ln(c*(b*x^2+a)^p)*polylog(2,a/(b*x^2+a))
/a^2+3/2*b^2*p^3*polylog(2,1+b*x^2/a)/a^2+3/2*b^2*p^3*polylog(3,a/(b*x^2+a)
)/a^2
```

Rubi [A]
time = 0.26, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {2504, 2445, 2458, 2389, 2379, 2421, 6724, 2355, 2354, 2438}

$$\frac{3b^2p^2\text{PolyLog}\left(2,\frac{a}{a+bx^2}\right)\log\left(c(a+bx^2)^p\right)}{2a^2}+\frac{3b^2p^3\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{2a^2}+\frac{3b^2p^3\text{PolyLog}\left(3,\frac{a}{a+bx^2}\right)}{2a^2}+\frac{3b^2p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{2a^2}-\frac{3b^2p\log\left(1-\frac{a}{a+bx^2}\right)\log^2\left(c(a+bx^2)^p\right)}{4a^2}-\frac{3bp(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{4a^2x^2}-\frac{\log^3\left(c(a+bx^2)^p\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^3/x^5,x]
```

```
[Out] (3*b^2*p^2*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/(2*a^2) - (3*b*p*(a + b*
x^2)*Log[c*(a + b*x^2)^p]^2)/(4*a^2*x^2) - Log[c*(a + b*x^2)^p]^3/(4*x^4) -
 (3*b^2*p*Log[c*(a + b*x^2)^p]^2*Log[1 - a/(a + b*x^2)])/(4*a^2) + (3*b^2*p
^2*Log[c*(a + b*x^2)^p]*PolyLog[2, a/(a + b*x^2)])/(2*a^2) + (3*b^2*p^3*Pol
yLog[2, 1 + (b*x^2)/a])/(2*a^2) + (3*b^2*p^3*PolyLog[3, a/(a + b*x^2)])/(2*
a^2)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\log^3\left(c(a+bx)^p\right)}{x^3}\,dx, x, x^2\right)\\
&= -\frac{\log^3\left(c(a+bx^2)^p\right)}{4x^4}+\frac{1}{4}(3bp)\text{Subst}\left(\int \frac{\log^2\left(c(a+bx)^p\right)}{x^2(a+bx)}\,dx, x, x^2\right)\\
&= -\frac{\log^3\left(c(a+bx^2)^p\right)}{4x^4}+\frac{1}{4}(3p)\text{Subst}\left(\int \frac{\log^2\left(cx^p\right)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx^2\right)\\
&= -\frac{\log^3\left(c(a+bx^2)^p\right)}{4x^4}+\frac{(3p)\text{Subst}\left(\int \frac{\log^2(cx^p)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx^2\right)}{4a}-\frac{(3bp)\text{Subst}\left(\int \frac{l\ldots}{x\ldots}\right.}{\ldots}\\
&= -\frac{3bp(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{4a^2x^2}-\frac{\log^3\left(c(a+bx^2)^p\right)}{4x^4}-\frac{(3bp)\text{Subst}\left(\int \frac{\log^2(cx^p)}{-\frac{a}{b}+\frac{x}{b}}\ldots\right.}{4a^2}\\
&= \frac{3b^2p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{2a^2}-\frac{3bp(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{4a^2x^2}-\frac{3b^2p\log\ldots}{\ldots}\\
&= \frac{3b^2p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{2a^2}-\frac{3bp(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{4a^2x^2}-\frac{3b^2p\log\ldots}{\ldots}\\
&= \frac{3b^2p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{2a^2}-\frac{3bp(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{4a^2x^2}-\frac{3b^2p\log\ldots}{\ldots}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 478 vs. 2(219) = 438.
time = 0.27, size = 478, normalized size = 2.18

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^3/x^5,x]
```

```
[Out] (-12*b^2*p^3*x^4*Log[x]*Log[a + b*x^2] + 6*b^2*p^3*x^4*Log[-((b*x^2)/a)]*Lo
g[a + b*x^2] + 3*b^2*p^3*x^4*Log[a + b*x^2]^2 - 6*b^2*p^3*x^4*Log[x]*Log[a
+ b*x^2]^2 + 3*b^2*p^3*x^4*Log[-((b*x^2)/a)]*Log[a + b*x^2]^2 + b^2*p^3*x^4
*Log[a + b*x^2]^3 + 12*b^2*p^2*x^4*Log[x]*Log[c*(a + b*x^2)^p] - 6*b^2*p^2*
x^4*Log[a + b*x^2]*Log[c*(a + b*x^2)^p] + 12*b^2*p^2*x^4*Log[x]*Log[a + b*x
^2]*Log[c*(a + b*x^2)^p] - 6*b^2*p^2*x^4*Log[-((b*x^2)/a)]*Log[a + b*x^2]*L
og[c*(a + b*x^2)^p] - 3*b^2*p^2*x^4*Log[a + b*x^2]^2*Log[c*(a + b*x^2)^p] -
 3*a*b*p*x^2*Log[c*(a + b*x^2)^p]^2 - 6*b^2*p*x^4*Log[x]*Log[c*(a + b*x^2)^
p]^2 + 3*b^2*p*x^4*Log[a + b*x^2]*Log[c*(a + b*x^2)^p]^2 - a^2*Log[c*(a + b
*x^2)^p]^3 + 6*b^2*p^2*x^4*(p - Log[c*(a + b*x^2)^p])*PolyLog[2, 1 + (b*x^2
)/a] + 6*b^2*p^3*x^4*PolyLog[3, 1 + (b*x^2)/a])/(4*a^2*x^4)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(c(b\,x^2+a)^p\right)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^3/x^5,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^3/x^5,x)
```

Maxima [A]
time = 0.35, size = 270, normalized size = 1.23

$$-\frac{1}{4}\left(\frac{3\left(\log\left(bx^2+a\right)^2\log\left(-\frac{bx^2+a}{a}+1\right)+2\,\mathrm{Li}_2\left(\frac{bx^2+a}{a}\right)\log\left(bx^2+a\right)-2\,\mathrm{Li}_3\left(\frac{bx^2+a}{a}\right)\right)bp^2}{a^2}-\frac{6\left(p^2-p\log\left(c\right)\right)\left(\log\left(bx^2+a\right)\log\left(-\frac{bx^2+a}{a}+1\right)+\mathrm{Li}_2\left(\frac{bx^2+a}{a}\right)\right)b}{a^2}-\frac{6\left(2\,p\log\left(c\right)-\log\left(c\right)^2\right)b\log\left(x\right)}{a^2}-\frac{bp^2x^2\log\left(bx^2+a\right)^3-3\left(\left(p^2-p\log\left(c\right)\right)bx^2+ap^2\right)\log\left(bx^2+a\right)^2-3\,a\log\left(c\right)^2-3\left(\left(2\,p\log\left(c\right)-\log\left(c\right)^2\right)bx^2+2\,ap\log\left(c\right)\right)\log\left(bx^2+a\right)}{a^2x^2}\right)bp-\frac{\log\left(\left(bx^2+a\right)^pc\right)^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*(3*(log(b*x^2 + a)^2*log(-(b*x^2 + a)/a + 1) + 2*dilog((b*x^2 + a)/a)*
log(b*x^2 + a) - 2*polylog(3, (b*x^2 + a)/a))*b*p^2/a^2 - 6*(p^2 - p*log(c)
)*(log(b*x^2 + a)*log(-(b*x^2 + a)/a + 1) + dilog((b*x^2 + a)/a))*b/a^2 - 6
*(2*p*log(c) - log(c)^2)*b*log(x)/a^2 - (b*p^2*x^2*log(b*x^2 + a)^3 - 3*((p
^2 - p*log(c))*b*x^2 + a*p^2)*log(b*x^2 + a)^2 - 3*a*log(c)^2 - 3*((2*p*log
(c) - log(c)^2)*b*x^2 + 2*a*p*log(c))*log(b*x^2 + a))/(a^2*x^2))*b*p - 1/4*
log((b*x^2 + a)^p*c)^3/x^4
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^5,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^3/x^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(c(a+bx^2)^p\right)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**3/x**5,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**3/x**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^5,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^3/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)^3/x^5,x)
```

```
[Out] int(log(c*(a + b*x^2)^p)^3/x^5, x)
```

3.97 $\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^7}\,dx$

Optimal. Leaf size=352

$$\frac{b^3p^3\log(x)}{a^3}-\frac{b^2p^2(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2a^3x^2}-\frac{b^3p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{a^3}-\frac{bp\log^2\left(c(a+bx^2)^p\right)}{4ax^4}+\frac{b^2p(\cdots}{}$$

```
[Out] b^3*p^3*ln(x)/a^3-1/2*b^2*p^2*(b*x^2+a)*ln(c*(b*x^2+a)^p)/a^3/x^2-b^3*p^2*l
n(-b*x^2/a)*ln(c*(b*x^2+a)^p)/a^3-1/4*b*p*ln(c*(b*x^2+a)^p)^2/a/x^4+1/2*b^2
*p*(b*x^2+a)*ln(c*(b*x^2+a)^p)^2/a^3/x^2-1/6*ln(c*(b*x^2+a)^p)^3/x^6-1/2*b^
3*p^2*ln(c*(b*x^2+a)^p)*ln(1-a/(b*x^2+a))/a^3+1/2*b^3*p*ln(c*(b*x^2+a)^p)^2
*ln(1-a/(b*x^2+a))/a^3+1/2*b^3*p^3*polylog(2,a/(b*x^2+a))/a^3-b^3*p^2*ln(c*
(b*x^2+a)^p)*polylog(2,a/(b*x^2+a))/a^3-b^3*p^3*polylog(2,1+b*x^2/a)/a^3-b^
3*p^3*polylog(3,a/(b*x^2+a))/a^3
```

Rubi [A]
time = 0.43, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {2504, 2445, 2458, 2389, 2379, 2421, 6724, 2355, 2354, 2438, 2356, 2351, 31}

$$-\frac{b^3p^2\text{PolyLog}\left(2,\frac{a}{a+bx^2}\right)\log\left(c(a+bx^2)^p\right)}{a^3}+\frac{b^3p^3\text{PolyLog}\left(2,\frac{a}{a+bx^2}\right)}{2a^3}-\frac{b^3p^3\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{a^3}-\frac{b^3p^3\text{PolyLog}\left(3,\frac{a}{a+bx^2}\right)}{a^3}-\frac{b^3p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{a^3}-\frac{b^3p^2\log\left(1-\frac{a}{a+bx^2}\right)\log\left(c(a+bx^2)^p\right)}{2a^3}+\frac{b^3p\log\left(1-\frac{a}{a+bx^2}\right)\log^2\left(c(a+bx^2)^p\right)}{2a^3}+\frac{b^3p^3\log(x)}{a^3}-\frac{b^2p^2(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2a^3x^2}+\frac{b^2p(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2a^3x^2}-\frac{\log^3\left(c(a+bx^2)^p\right)}{6x^6}-\frac{bp\log^2\left(c(a+bx^2)^p\right)}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]^3/x^7,x]
```

```
[Out] (b^3*p^3*Log[x])/a^3 - (b^2*p^2*(a + b*x^2)*Log[c*(a + b*x^2)^p])/(2*a^3*x^
2) - (b^3*p^2*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/a^3 - (b*p*Log[c*(a +
 b*x^2)^p]^2)/(4*a*x^4) + (b^2*p*(a + b*x^2)*Log[c*(a + b*x^2)^p]^2)/(2*a^3
*x^2) - Log[c*(a + b*x^2)^p]^3/(6*x^6) - (b^3*p^2*Log[c*(a + b*x^2)^p]*Log[
1 - a/(a + b*x^2)])/(2*a^3) + (b^3*p*Log[c*(a + b*x^2)^p]^2*Log[1 - a/(a +
b*x^2)])/(2*a^3) + (b^3*p^3*PolyLog[2, a/(a + b*x^2)])/(2*a^3) - (b^3*p^2*L
og[c*(a + b*x^2)^p]*PolyLog[2, a/(a + b*x^2)])/a^3 - (b^3*p^3*PolyLog[2, 1
+ (b*x^2)/a])/a^3 - (b^3*p^3*PolyLog[3, a/(a + b*x^2)])/a^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^7}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\log^3\left(c(a+bx)^p\right)}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{\log^3\left(c(a+bx^2)^p\right)}{6x^6} + \frac{1}{2}(bp)\text{Subst}\left(\int \frac{\log^2\left(c(a+bx)^p\right)}{x^3(a+bx)}\,dx, x, x^2\right)$$

$$= -\frac{\log^3\left(c(a+bx^2)^p\right)}{6x^6} + \frac{1}{2}p\text{Subst}\left(\int \frac{\log^2\left(cx^p\right)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx, x, a+bx^2\right)$$

$$= -\frac{\log^3\left(c(a+bx^2)^p\right)}{6x^6} + \frac{p\text{Subst}\left(\int \frac{\log^2(cx^p)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx, x, a+bx^2\right)}{2a} - \frac{(bp)\text{Subst}\left(\int \frac{\log^2(c}{x(-\frac{a}{b}-}\right.}{2}$$

$$= -\frac{bp\log^2\left(c(a+bx^2)^p\right)}{4ax^4} - \frac{\log^3\left(c(a+bx^2)^p\right)}{6x^6} - \frac{(bp)\text{Subst}\left(\int \frac{\log^2(cx^p)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+b\right.}{2a^2}$$

$$= -\frac{bp\log^2\left(c(a+bx^2)^p\right)}{4ax^4} + \frac{b^2p(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2a^3x^2} - \frac{\log^3\left(c(a+bx^2)^p\right)}{6x^6} +$$

$$= -\frac{b^2p^2(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2a^3x^2} - \frac{b^3p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{a^3} - \frac{bp\log^2\left(\right.}{}$$

$$= \frac{b^3p^3\log(x)}{a^3} - \frac{b^2p^2(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2a^3x^2} - \frac{3b^3p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2\right.}{2a^3}$$

$$= \frac{b^3p^3\log(x)}{a^3} - \frac{b^2p^2(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2a^3x^2} - \frac{3b^3p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2\right.}{2a^3}$$

Mathematica [A]
time = 0.30, size = 571, normalized size = 1.62

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^3/x^7,x]
```

```
[Out] -1/12*(-6*b^3*p^3*x^6*Log[-((b*x^2)/a)] + 6*b^3*p^3*x^6*Log[a + b*x^2] - 36
*b^3*p^3*x^6*Log[x]*Log[a + b*x^2] + 18*b^3*p^3*x^6*Log[-((b*x^2)/a)]*Log[a
 + b*x^2] + 9*b^3*p^3*x^6*Log[a + b*x^2]^2 - 12*b^3*p^3*x^6*Log[x]*Log[a +
b*x^2]^2 + 6*b^3*p^3*x^6*Log[-((b*x^2)/a)]*Log[a + b*x^2]^2 + 2*b^3*p^3*x^6
*Log[a + b*x^2]^3 + 6*a*b^2*p^2*x^4*Log[c*(a + b*x^2)^p] + 36*b^3*p^2*x^6*L
og[x]*Log[c*(a + b*x^2)^p] - 18*b^3*p^2*x^6*Log[a + b*x^2]*Log[c*(a + b*x^2
)^p] + 24*b^3*p^2*x^6*Log[x]*Log[a + b*x^2]*Log[c*(a + b*x^2)^p] - 12*b^3*p
^2*x^6*Log[-((b*x^2)/a)]*Log[a + b*x^2]*Log[c*(a + b*x^2)^p] - 6*b^3*p^2*x^
```

```
6*Log[a + b*x^2]^2*Log[c*(a + b*x^2)^p] + 3*a^2*b*p*x^2*Log[c*(a + b*x^2)^p
]^2 - 6*a*b^2*p*x^4*Log[c*(a + b*x^2)^p]^2 - 12*b^3*p*x^6*Log[x]*Log[c*(a +
 b*x^2)^p]^2 + 6*b^3*p*x^6*Log[a + b*x^2]*Log[c*(a + b*x^2)^p]^2 + 2*a^3*Lo
g[c*(a + b*x^2)^p]^3 + 6*b^3*p^2*x^6*(3*p - 2*Log[c*(a + b*x^2)^p])*PolyLog
[2, 1 + (b*x^2)/a] + 12*b^3*p^3*x^6*PolyLog[3, 1 + (b*x^2)/a])/(a^3*x^6)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(c(b\,x^2+a)^p\right)^3}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^3/x^7,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^3/x^7,x)
```

Maxima [A]
time = 0.35, size = 338, normalized size = 0.96

$$\frac{1}{12}\left(\frac{6\left(\log\left(bx^2+a\right)^2\log\left(-\frac{bx^2+a}{a}+1\right)+2\,\mathrm{Li}_2\left(\frac{bx^2+a}{a}\right)\log\left(bx^2+a\right)-2\,\mathrm{Li}_3\left(\frac{bx^2+a}{a}\right)\right)b^2p^2}{a^3}-\frac{6\left(3\,p^2-2\,p\log\left(c\right)\right)\left(\log\left(bx^2+a\right)\log\left(-\frac{bx^2+a}{a}+1\right)+\mathrm{Li}_2\left(\frac{bx^2+a}{a}\right)\right)b^2}{a^3}+\frac{12\left(p^2-3\,p\log\left(c\right)+\log\left(c\right)^2\right)b^2\log\left(x\right)}{a^3}-\frac{2\,b^2p^2x^4\log\left(bx^2+a\right)^3+6\left(p\log\left(c\right)-\log\left(c\right)^2\right)abx^2+3\,a^2\log\left(c\right)^2-3\left(\left(3\,p^2-2\,p\log\left(c\right)\right)b^2x^4+2\,abp^2x^2-a^2p^2\right)\log\left(bx^2+a\right)^2+6\left(\left(p^2-3\,p\log\left(c\right)+\log\left(c\right)^2\right)b^2x^4+\left(p^2-2\,p\log\left(c\right)\right)abx^2+a^2p\log\left(c\right)\right)\log\left(bx^2+a\right)}{a^3x^4}\right)bp-\frac{\log\left(\left(bx^2+a\right)^pc\right)^3}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^7,x, algorithm="maxima")
```

```
[Out] 1/12*(6*(log(b*x^2 + a)^2*log(-(b*x^2 + a)/a + 1) + 2*dilog((b*x^2 + a)/a)*
log(b*x^2 + a) - 2*polylog(3, (b*x^2 + a)/a))*b^2*p^2/a^3 - 6*(3*p^2 - 2*p*
log(c))*(log(b*x^2 + a)*log(-(b*x^2 + a)/a + 1) + dilog((b*x^2 + a)/a))*b^2
/a^3 + 12*(p^2 - 3*p*log(c) + log(c)^2)*b^2*log(x)/a^3 - (2*b^2*p^2*x^4*log
(b*x^2 + a)^3 + 6*(p*log(c) - log(c)^2)*a*b*x^2 + 3*a^2*log(c)^2 - 3*((3*p^
2 - 2*p*log(c))*b^2*x^4 + 2*a*b*p^2*x^2 - a^2*p^2)*log(b*x^2 + a)^2 + 6*((p
^2 - 3*p*log(c) + log(c)^2)*b^2*x^4 + (p^2 - 2*p*log(c))*a*b*x^2 + a^2*p*lo
g(c))*log(b*x^2 + a))/(a^3*x^4))*b*p - 1/6*log((b*x^2 + a)^p*c)^3/x^6
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^7,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^3/x^7, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(c(a+bx^2)^p\right)^3}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**3/x**7,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**3/x**7, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^7,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^3/x^7, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^3}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)^3/x^7,x)
```

```
[Out] int(log(c*(a + b*x^2)^p)^3/x^7, x)
```

3.98 $\int x^2 \log^3\left(c(a+bx^2)^p\right)\,dx$

Optimal. Leaf size=380

$$\frac{208ap^3x}{9b}-\frac{16p^3x^3}{27}-\frac{208a^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{9b^{3/2}}+\frac{32ia^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{3b^{3/2}}+\frac{64a^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2}{\sqrt{a}\right.}}{3b^{3/2}}$$

```
[Out] 208/9*a*p^3*x/b-16/27*p^3*x^3-208/9*a^(3/2)*p^3*arctan(x*b^(1/2)/a^(1/2))/b
^(3/2)+32/3*I*a^(3/2)*p^3*arctan(x*b^(1/2)/a^(1/2))^2/b^(3/2)-32/3*a*p^2*x*
ln(c*(b*x^2+a)^p)/b+8/9*p^2*x^3*ln(c*(b*x^2+a)^p)+32/3*a^(3/2)*p^2*arctan(x
*b^(1/2)/a^(1/2))*ln(c*(b*x^2+a)^p)/b^(3/2)+2*a*p*x*ln(c*(b*x^2+a)^p)^2/b-2
/3*p*x^3*ln(c*(b*x^2+a)^p)^2+1/3*x^3*ln(c*(b*x^2+a)^p)^3+64/3*a^(3/2)*p^3*a
rctan(x*b^(1/2)/a^(1/2))*ln(2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))/b^(3/2)+32/3*I
*a^(3/2)*p^3*polylog(2,1-2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))/b^(3/2)-2*a^2*p*U
nintegrable(ln(c*(b*x^2+a)^p)^2/(b*x^2+a),x)/b
```

Rubi [A]

time = 0.52, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^2 \log^3\left(c(a+bx^2)^p\right)\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] (208*a*p^3*x)/(9*b) - (16*p^3*x^3)/27 - (208*a^(3/2)*p^3*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(9*b^(3/2)) + (((32*I)/3)*a^(3/2)*p^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]
]^2)/b^(3/2) + (64*a^(3/2)*p^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/
(Sqrt[a] + I*Sqrt[b]*x)])/(3*b^(3/2)) - (32*a*p^2*x*Log[c*(a + b*x^2)^p])/(
3*b) + (8*p^2*x^3*Log[c*(a + b*x^2)^p])/9 + (32*a^(3/2)*p^2*ArcTan[(Sqrt[b]
*x)/Sqrt[a]]*Log[c*(a + b*x^2)^p])/(3*b^(3/2)) + (2*a*p*x*Log[c*(a + b*x^2)
^p]^2)/b - (2*p*x^3*Log[c*(a + b*x^2)^p]^2)/3 + (x^3*Log[c*(a + b*x^2)^p]^3
)/3 + (((32*I)/3)*a^(3/2)*p^3*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[
b]*x)])/b^(3/2) - (2*a^2*p*Defer[Int][Log[c*(a + b*x^2)^p]^2/(a + b*x^2), x
])/b
```

Rubi steps

$$\int x^2 \log^3\left(c(a+bx^2)^p\right) dx = \frac{1}{3}x^3 \log^3\left(c(a+bx^2)^p\right) - (2bp)\int \frac{x^4 \log^2\left(c(a+bx^2)^p\right)}{a+bx^2} dx$$

$$= \frac{1}{3}x^3 \log^3\left(c(a+bx^2)^p\right) - (2bp)\int \left(-\frac{a\log^2\left(c(a+bx^2)^p\right)}{b^2} + \frac{x^2\log^2\left(c(a+bx\right.}{b}\right.$$

$$= \frac{1}{3}x^3 \log^3\left(c(a+bx^2)^p\right) - (2p)\int x^2 \log^2\left(c(a+bx^2)^p\right) dx + \frac{(2ap)\int \log^2\left(c(a\right.}{b}$$

$$= \frac{2apx\log^2\left(c(a+bx^2)^p\right)}{b} - \frac{2}{3}px^3\log^2\left(c(a+bx^2)^p\right) + \frac{1}{3}x^3\log^3\left(c(a+bx^2)^p\right) -$$

$$= \frac{2apx\log^2\left(c(a+bx^2)^p\right)}{b} - \frac{2}{3}px^3\log^2\left(c(a+bx^2)^p\right) + \frac{1}{3}x^3\log^3\left(c(a+bx^2)^p\right) -$$

$$= \frac{2apx\log^2\left(c(a+bx^2)^p\right)}{b} - \frac{2}{3}px^3\log^2\left(c(a+bx^2)^p\right) + \frac{1}{3}x^3\log^3\left(c(a+bx^2)^p\right) -$$

$$= -\frac{32ap^2x\log\left(c(a+bx^2)^p\right)}{3b} + \frac{8}{9}p^2x^3\log\left(c(a+bx^2)^p\right) + \frac{32a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}}\right.}{3b}$$

$$= \frac{64ap^3x}{3b} - \frac{32ap^2x\log\left(c(a+bx^2)^p\right)}{3b} + \frac{8}{9}p^2x^3\log\left(c(a+bx^2)^p\right) + \frac{32a^{3/2}p^2\tan}{}$$

$$= \frac{208ap^3x}{9b} - \frac{16p^3x^3}{27} - \frac{64a^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3b^{3/2}} + \frac{32ia^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{3b^{3/2}} -$$

$$= \frac{208ap^3x}{9b} - \frac{16p^3x^3}{27} - \frac{208a^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{9b^{3/2}} + \frac{32ia^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{3b^{3/2}}$$

$$= \frac{208ap^3x}{9b} - \frac{16p^3x^3}{27} - \frac{208a^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{9b^{3/2}} + \frac{32ia^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{3b^{3/2}}$$

$$= \frac{208ap^3x}{9b} - \frac{16p^3x^3}{27} - \frac{208a^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{9b^{3/2}} + \frac{32ia^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{3b^{3/2}}$$

Mathematica [**A**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 909 vs. 2(380) = 760.
time = 2.51, size = 909, normalized size = 2.39

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] (2*a*p*x*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2)/b - (2*a^(3/2)*p*A
rcTan[(Sqrt[b]*x)/Sqrt[a]]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2)/
b^(3/2) + p*x^3*Log[a + b*x^2]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])
^2 + (x^3*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2*(-2*p - p*Log[a +
b*x^2] + Log[c*(a + b*x^2)^p]))/3 + 3*p^2*(-(p*Log[a + b*x^2]) + Log[c*(a +
 b*x^2)^p])*((x^3*Log[a + b*x^2]^2)/3 - (4*((9*I)*a^(3/2)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]]^2 + 3*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(-8 + 6*Log[(2*Sqrt[a]
)/(Sqrt[a] + I*Sqrt[b]*x)] + 3*Log[a + b*x^2]) + Sqrt[b]*x*(24*a - 2*b*x^2
+ (-9*a + 3*b*x^2)*Log[a + b*x^2]) + (9*I)*a^(3/2)*PolyLog[2, (I*Sqrt[a] +
Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)]))/(27*b^(3/2))) + (p^3*(416*Sqrt[-a]
*a^(3/2)*Sqrt[(b*x^2)/(a + b*x^2)]*Sqrt[a + b*x^2]*ArcSin[Sqrt[a]/Sqrt[a +
b*x^2]] + (2*Sqrt[-a]*b*x^2*(624*a - 16*b*x^2 + (-288*a + 24*b*x^2)*Log[a +
 b*x^2] + 18*(3*a - b*x^2)*Log[a + b*x^2]^2 + 9*b*x^2*Log[a + b*x^2]^3))/3
+ 36*Sqrt[-a]*a^(3/2)*Sqrt[(b*x^2)/(a + b*x^2)]*(8*Sqrt[a]*HypergeometricPF
Q[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}, a/(a + b*x^2)] + Log[a + b*x^2]*(4
*Sqrt[a]*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, a/(a + b*x^2)] + Sq
rt[a + b*x^2]*ArcSin[Sqrt[a]/Sqrt[a + b*x^2]]*Log[a + b*x^2])) - 48*a^2*(4*
Sqrt[b*x^2]*ArcTanh[Sqrt[b*x^2]/Sqrt[-a]]*(Log[a + b*x^2] - Log[1 + (b*x^2)
/a]) - Sqrt[-a]*Sqrt[-((b*x^2)/a)]*(Log[1 + (b*x^2)/a]^2 - 4*Log[1 + (b*x^2
)/a]*Log[(1 + Sqrt[-((b*x^2)/a)])/2] + 2*Log[(1 + Sqrt[-((b*x^2)/a)])/2]^2
- 4*PolyLog[2, 1/2 - Sqrt[-((b*x^2)/a)]/2]))))/(18*Sqrt[-a]*b^2*x)
```

Maple [**A**]
time = 0.06, size = 0, normalized size = 0.00

$$\int x^2 \ln\left(c\left(b\,x^2+a\right)^p\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(b*x^2+a)^p)^3,x)
```

```
[Out] int(x^2*ln(c*(b*x^2+a)^p)^3,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] 1/3*p^3*x^3*log(b*x^2 + a)^3 + integrate((b*x^4*log(c)^3 + a*x^2*log(c)^3 -
 ((2*p^3 - 3*p^2*log(c))*b*x^4 - 3*a*p^2*x^2*log(c))*log(b*x^2 + a)^2 + 3*(
b*p*x^4*log(c)^2 + a*p*x^2*log(c)^2)*log(b*x^2 + a))/(b*x^2 + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] integral(x^2*log((b*x^2 + a)^p*c)^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \log\left(c(a+bx^2)^p\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Integral(x**2*log(c*(a + b*x**2)**p)**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*log((b*x^2 + a)^p*c)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \ln\left(c\left(b\,x^2+a\right)^p\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(c*(a + b*x^2)^p)^3,x)
```

```
[Out] int(x^2*log(c*(a + b*x^2)^p)^3, x)
```

3.99 $\int \log^3\left(c\left(a+bx^2\right)^p\right)\,dx$

Optimal. Leaf size=290

$$-48p^3x+\frac{48\sqrt{a}\,p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}-\frac{24i\sqrt{a}\,p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{b}}-\frac{48\sqrt{a}\,p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{\sqrt{b}}+$$

```
[Out] -48*p^3*x+24*p^2*x*ln(c*(b*x^2+a)^p)-6*p*x*ln(c*(b*x^2+a)^p)^2+x*ln(c*(b*x^
2+a)^p)^3+48*p^3*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(1/2)-24*I*p^3*arctan(
x*b^(1/2)/a^(1/2))^2*a^(1/2)/b^(1/2)-24*p^2*arctan(x*b^(1/2)/a^(1/2))*ln(c*
(b*x^2+a)^p)*a^(1/2)/b^(1/2)-48*p^3*arctan(x*b^(1/2)/a^(1/2))*ln(2*a^(1/2)/
(a^(1/2)+I*x*b^(1/2)))*a^(1/2)/b^(1/2)-24*I*p^3*polylog(2,1-2*a^(1/2)/(a^(1
/2)+I*x*b^(1/2)))*a^(1/2)/b^(1/2)+6*a*p*Unintegrable(ln(c*(b*x^2+a)^p)^2/(b
*x^2+a),x)
```

Rubi [A]

time = 0.27, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \log^3\left(c\left(a+bx^2\right)^p\right)\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] -48*p^3*x + (48*Sqrt[a]*p^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] - ((24*I)*
Sqrt[a]*p^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/Sqrt[b] - (48*Sqrt[a]*p^3*ArcTan
[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/Sqrt[b] + 2
4*p^2*x*Log[c*(a + b*x^2)^p] - (24*Sqrt[a]*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*
Log[c*(a + b*x^2)^p])/Sqrt[b] - 6*p*x*Log[c*(a + b*x^2)^p]^2 + x*Log[c*(a +
 b*x^2)^p]^3 - ((24*I)*Sqrt[a]*p^3*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*
Sqrt[b]*x)])/Sqrt[b] + 6*a*p*Defer[Int][Log[c*(a + b*x^2)^p]^2/(a + b*x^2),
 x]
```

Rubi steps

$$\int \log^3\left(c\left(a+bx^2\right)^p\right)\,dx = x\log^3\left(c\left(a+bx^2\right)^p\right)-(6bp)\int \frac{x^2\log^2\left(c(a+bx^2)^p\right)}{a+bx^2}\,dx$$

$$= x\log^3\left(c\left(a+bx^2\right)^p\right)-(6bp)\int \left(\frac{\log^2\left(c(a+bx^2)^p\right)}{b}-\frac{a\log^2\left(c(a+bx^2)^p\right)}{b\left(a+bx^2\right)}\right)\,dx$$

$$= x\log^3\left(c\left(a+bx^2\right)^p\right)-(6p)\int \log^2\left(c\left(a+bx^2\right)^p\right)\,dx+(6ap)\int \frac{\log^2\left(c(a+bx^2)^p\right)}{a+bx^2}$$

$$= -6px\log^2\left(c\left(a+bx^2\right)^p\right)+x\log^3\left(c\left(a+bx^2\right)^p\right)+(6ap)\int \frac{\log^2\left(c(a+bx^2)^p\right)}{a+bx^2}\,dx$$

$$= -6px\log^2\left(c\left(a+bx^2\right)^p\right)+x\log^3\left(c\left(a+bx^2\right)^p\right)+(6ap)\int \frac{\log^2\left(c(a+bx^2)^p\right)}{a+bx^2}\,dx$$

$$= -6px\log^2\left(c\left(a+bx^2\right)^p\right)+x\log^3\left(c\left(a+bx^2\right)^p\right)+(6ap)\int \frac{\log^2\left(c(a+bx^2)^p\right)}{a+bx^2}\,dx$$

$$= 24p^2x\log\left(c\left(a+bx^2\right)^p\right)-\frac{24\sqrt{a}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{\sqrt{b}}-6px\log^2$$

$$= -48p^3x+24p^2x\log\left(c\left(a+bx^2\right)^p\right)-\frac{24\sqrt{a}\,p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{\sqrt{b}}$$

$$= -48p^3x+\frac{48\sqrt{a}\,p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}-\frac{24i\sqrt{a}\,p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{b}}+24p^2x\log\left(c\right.$$

$$= -48p^3x+\frac{48\sqrt{a}\,p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}-\frac{24i\sqrt{a}\,p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{b}}-\frac{48\sqrt{a}\,p^3\tan}{}$$

$$= -48p^3x+\frac{48\sqrt{a}\,p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}-\frac{24i\sqrt{a}\,p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{b}}-\frac{48\sqrt{a}\,p^3\tan}{}$$

$$= -48p^3x+\frac{48\sqrt{a}\,p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}-\frac{24i\sqrt{a}\,p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{b}}-\frac{48\sqrt{a}\,p^3\tan}{}$$

<u>**Mathematica**</u> [A] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 789 vs. $2(290) = 580$.

time = 2.26, size = 789, normalized size = 2.72

$$\frac{6\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-p\log\left(a+bx^2\right)+\log\left(c\left(a+bx^2\right)^p\right)\right)^2}{\sqrt{b}}+3px\log\left(a+bx^2\right)\left(-p\log\left(a+bx^2\right)+\log\left(c\left(a+bx^2\right)^p\right)\right)^2+x\left(-p\log\left(a+bx^2\right)+\log\left(c\left(a+bx^2\right)^p\right)\right)^2\left(-6p-p\log\left(a+bx^2\right)+\log\left(c\left(a+bx^2\right)^p\right)\right)-\frac{3p^2\left(p\log\left(a+bx^2\right)-\log\left(c\left(a+bx^2\right)^p\right)\right)\left(4i\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2+4\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-2+2\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)+\log\left(a+bx^2\right)\right)+\sqrt{b}\,x\left(8-4\log\left(a+bx^2\right)+\log^2\left(a+bx^2\right)\right)+4i\sqrt{a}\,\mathrm{Li}_2\left(\frac{i\sqrt{a}+\sqrt{b}\,x}{-i\sqrt{a}+\sqrt{b}\,x}\right)\right)}{\sqrt{b}}+\frac{p^3\left(-48\sqrt{-a^2}\sqrt{\frac{bx^2}{a+bx^2}}\sqrt{a+bx^2}\sin^{-1}\left(\frac{\sqrt{a}}{\sqrt{a+bx^2}}\right)+\sqrt{-a}\,bx^2\left(-48+24\log\left(a+bx^2\right)-6\log^2\left(a+bx^2\right)+\log^3\left(a+bx^2\right)\right)-6\sqrt{-a^2}\sqrt{\frac{bx^2}{a+bx^2}}\left(8\sqrt{a}\,{}_4F_3\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2},\frac{3}{2};\frac{a}{a+bx^2}\right)+\log\left(a+bx^2\right)\left(4\sqrt{a}\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{a}{a+bx^2}\right)+\sqrt{a+bx^2}\sin^{-1}\left(\frac{\sqrt{a}}{\sqrt{a+bx^2}}\right)\log\left(a+bx^2\right)\right)\right)+24a\sqrt{bx^2}\tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{-a}}\right)\left(\log\left(a+bx^2\right)-\log\left(1+\frac{bx^2}{a}\right)\right)+6(-a)^{3/2}\sqrt{-\frac{bx^2}{a}}\left(\log^2\left(1+\frac{bx^2}{a}\right)-4\log\left(1+\frac{bx^2}{a}\right)\log\left(\frac{1}{2}\left(1+\sqrt{-\frac{bx^2}{a}}\right)\right)+2\log^2\left(\frac{1}{2}\left(1+\sqrt{-\frac{bx^2}{a}}\right)\right)-4\,\mathrm{Li}_2\left(\frac{1}{2}-\frac{1}{2}\sqrt{-\frac{bx^2}{a}}\right)\right)\right)}{\sqrt{-a}\,bx}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] (6*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(-(p*Log[a + b*x^2]) + Log[c*(a +
b*x^2)^p])^2)/Sqrt[b] + 3*p*x*Log[a + b*x^2]*(-(p*Log[a + b*x^2]) + Log[c*(
a + b*x^2)^p])^2 + x*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2*(-6*p -
 p*Log[a + b*x^2] + Log[c*(a + b*x^2)^p]) - (3*p^2*(p*Log[a + b*x^2] - Log[
c*(a + b*x^2)^p])*((4*I)*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 + 4*Sqrt[a]*
ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(-2 + 2*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)
] + Log[a + b*x^2]) + Sqrt[b]*x*(8 - 4*Log[a + b*x^2] + Log[a + b*x^2]^2) +
 (4*I)*Sqrt[a]*PolyLog[2, (I*Sqrt[a] + Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x
)]))/Sqrt[b] + (p^3*(-48*Sqrt[-a^2]*Sqrt[(b*x^2)/(a + b*x^2)]*Sqrt[a + b*x^
2]*ArcSin[Sqrt[a]/Sqrt[a + b*x^2]] + Sqrt[-a]*b*x^2*(-48 + 24*Log[a + b*x^2
] - 6*Log[a + b*x^2]^2 + Log[a + b*x^2]^3) - 6*Sqrt[-a^2]*Sqrt[(b*x^2)/(a +
 b*x^2)]*(8*Sqrt[a]*HypergeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}
, a/(a + b*x^2)] + Log[a + b*x^2]*(4*Sqrt[a]*HypergeometricPFQ[{1/2, 1/2, 1
/2}, {3/2, 3/2}, a/(a + b*x^2)] + Sqrt[a + b*x^2]*ArcSin[Sqrt[a]/Sqrt[a + b
*x^2]]*Log[a + b*x^2])) + 24*a*Sqrt[b*x^2]*ArcTanh[Sqrt[b*x^2]/Sqrt[-a]]*(L
og[a + b*x^2] - Log[1 + (b*x^2)/a]) + 6*(-a)^(3/2)*Sqrt[-((b*x^2)/a)]*(Log[
1 + (b*x^2)/a]^2 - 4*Log[1 + (b*x^2)/a]*Log[(1 + Sqrt[-((b*x^2)/a)])/2] + 2
*Log[(1 + Sqrt[-((b*x^2)/a)])/2]^2 - 4*PolyLog[2, 1/2 - Sqrt[-((b*x^2)/a)]/
2])))/(Sqrt[-a]*b*x)
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \ln\left(c\left(b\,x^2+a\right)^p\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^3,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] p^3*x*log(b*x^2 + a)^3 + integrate((b*x^2*log(c)^3 + a*log(c)^3 - 3*((2*p^3
 - p^2*log(c))*b*x^2 - a*p^2*log(c))*log(b*x^2 + a)^2 + 3*(b*p*x^2*log(c)^2
 + a*p*log(c)^2)*log(b*x^2 + a))/(b*x^2 + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log \left(c\left(a+bx^2\right)^p\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\left(b\,x^2+a\right)^p\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)^3,x)
```

```
[Out] int(log(c*(a + b*x^2)^p)^3, x)
```

3.100 $\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^2}\,dx$

Optimal. Leaf size=51

$$-\frac{\log^3\left(c(a+bx^2)^p\right)}{x}+6bp\text{Int}\left(\frac{\log^2\left(c(a+bx^2)^p\right)}{a+bx^2},x\right)$$

```
[Out] -ln(c*(b*x^2+a)^p)^3/x+6*b*p*Unintegrable(ln(c*(b*x^2+a)^p)^2/(b*x^2+a),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[c*(a + b*x^2)^p]^3/x^2,x]
```

```
[Out] -(Log[c*(a + b*x^2)^p]^3/x) + 6*b*p*Defer[Int][Log[c*(a + b*x^2)^p]^2/(a + b*x^2), x]
```

Rubi steps

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^2}\,dx=-\frac{\log^3\left(c(a+bx^2)^p\right)}{x}+(6bp)\int \frac{\log^2\left(c(a+bx^2)^p\right)}{a+bx^2}\,dx$$

Mathematica [A] Result contains complex when optimal does not.
time = 0.58, size = 505, normalized size = 9.90

$$\frac{p^3\left(-96\sqrt{a}\sqrt{1-\frac{a}{a+bx^2}}\,{}_4F_3\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2},\frac{3}{2};\frac{a}{a+bx^2}\right)-48\sqrt{a}\sqrt{1-\frac{a}{a+bx^2}}\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{a}{a+bx^2}\right)\log\left(a+bx^2\right)-2\log^2\left(a+bx^2\right)\left(6\sqrt{a+bx^2}\sqrt{1-\frac{a}{a+bx^2}}\sin^{-1}\left(\frac{\sqrt{a}}{\sqrt{a+bx^2}}\right)+\sqrt{a}\log\left(a+bx^2\right)\right)\right)}{2\sqrt{a}\,x}+\frac{6\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-p\log\left(a+bx^2\right)+\log\left(c(a+bx^2)^p\right)\right)^2}{\sqrt{a}}-\frac{3p\log\left(a+bx^2\right)\left(-p\log\left(a+bx^2\right)+\log\left(c(a+bx^2)^p\right)\right)^2}{x}-\frac{\left(-p\log\left(a+bx^2\right)+\log\left(c(a+bx^2)^p\right)\right)^3}{x}+3p^2\left(-p\log\left(a+bx^2\right)+\log\left(c(a+bx^2)^p\right)\right)\left(-\frac{\log^2\left(a+bx^2\right)}{x}+\frac{4\sqrt{b}\left(\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(i\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)+2\log\left(\frac{2i}{i-\frac{\sqrt{b}\,x}{\sqrt{a}}}\right)+\log\left(a+bx^2\right)\right)+i\,\text{Li}_2\left(\frac{i\sqrt{a}+\sqrt{b}\,x}{-i\sqrt{a}+\sqrt{b}\,x}\right)\right)}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^3/x^2,x]
```

```
[Out] (p^3*(-96*Sqrt[a]*Sqrt[1 - a/(a + b*x^2)]*HypergeometricPFQ[{1/2, 1/2, 1/2,
 1/2}, {3/2, 3/2, 3/2}, a/(a + b*x^2)] - 48*Sqrt[a]*Sqrt[1 - a/(a + b*x^2)]
*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, a/(a + b*x^2)]*Log[a + b*x^
2] - 2*Log[a + b*x^2]^2*(6*Sqrt[a + b*x^2]*Sqrt[1 - a/(a + b*x^2)]*ArcSin[S
qrt[a]/Sqrt[a + b*x^2]] + Sqrt[a]*Log[a + b*x^2])))/(2*Sqrt[a]*x) + (6*Sqrt
```

```
[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^
p])^2)/Sqrt[a] - (3*p*Log[a + b*x^2]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^
2)^p])^2)/x - (-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^3/x + 3*p^2*(-(p
*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])*(-(Log[a + b*x^2]^2/x) + (4*Sqrt[b
]*(ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(I*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 2*Log[(2*I)
/(I - (Sqrt[b]*x)/Sqrt[a])] + Log[a + b*x^2]) + I*PolyLog[2, (I*Sqrt[a] + S
qrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)]))/Sqrt[a])
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(c(b\,x^2+a)^p\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^3/x^2,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^3/x^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^3/x^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(c(a+bx^2)^p\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**3/x**2,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**3/x**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^3/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)^3/x^2,x)
```

```
[Out] int(log(c*(a + b*x^2)^p)^3/x^2, x)
```

3.101 $\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^4}\,dx$

Optimal. Leaf size=254

$$\frac{8ib^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{a^{3/2}}+\frac{16b^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{a^{3/2}}+\frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c(a+bx^2\right.}{a^{3/2}}$$

```
[Out] 8*I*b^(3/2)*p^3*arctan(x*b^(1/2)/a^(1/2))^2/a^(3/2)+8*b^(3/2)*p^2*arctan(x*
b^(1/2)/a^(1/2))*ln(c*(b*x^2+a)^p)/a^(3/2)-2*b*p*ln(c*(b*x^2+a)^p)^2/a/x-1/
3*ln(c*(b*x^2+a)^p)^3/x^3+16*b^(3/2)*p^3*arctan(x*b^(1/2)/a^(1/2))*ln(2*a^(
1/2)/(a^(1/2)+I*x*b^(1/2)))/a^(3/2)+8*I*b^(3/2)*p^3*polylog(2,1-2*a^(1/2)/(
a^(1/2)+I*x*b^(1/2)))/a^(3/2)-2*b^2*p*Unintegrable(ln(c*(b*x^2+a)^p)^2/(b*x
^2+a),x)/a
```

Rubi [A]
time = 0.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[c*(a + b*x^2)^p]^3/x^4,x]
```

```
[Out] ((8*I)*b^(3/2)*p^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/a^(3/2) + (16*b^(3/2)*p^3
*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/a^(3
/2) + (8*b^(3/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[c*(a + b*x^2)^p])/a^(3
/2) - (2*b*p*Log[c*(a + b*x^2)^p]^2)/(a*x) - Log[c*(a + b*x^2)^p]^3/(3*x^3)
 + ((8*I)*b^(3/2)*p^3*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/
a^(3/2) - (2*b^2*p*Defer[Int][Log[c*(a + b*x^2)^p]^2/(a + b*x^2), x])/a
```

Rubi steps

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^4}\,dx = -\frac{\log^3\left(c(a+bx^2)^p\right)}{3x^3} + (2bp)\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^2\left(a+bx^2\right)}\,dx$$

$$= -\frac{\log^3\left(c(a+bx^2)^p\right)}{3x^3} + (2bp)\int \left(\frac{\log^2\left(c(a+bx^2)^p\right)}{ax^2} - \frac{b\log^2\left(c(a+bx^2)^p\right)}{a\left(a+bx^2\right)}\right)dx$$

$$= -\frac{\log^3\left(c(a+bx^2)^p\right)}{3x^3} + \frac{(2bp)\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^2}\,dx}{a} - \frac{(2b^2p)\int \frac{\log^2\left(c(a+bx^2)^p\right)}{a+bx^2}\,dx}{a}$$

$$= -\frac{2bp\log^2\left(c(a+bx^2)^p\right)}{ax} - \frac{\log^3\left(c(a+bx^2)^p\right)}{3x^3} - \frac{(2b^2p)\int \frac{\log^2\left(c(a+bx^2)^p\right)}{a+bx^2}\,dx}{a} + \frac{(8b^2p$$

$$= \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{a^{3/2}} - \frac{2bp\log^2\left(c(a+bx^2)^p\right)}{ax} - \frac{\log^3\left(c(a+b$$

$$= \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{a^{3/2}} - \frac{2bp\log^2\left(c(a+bx^2)^p\right)}{ax} - \frac{\log^3\left(c(a+b$$

$$= \frac{8ib^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{a^{3/2}} + \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{a^{3/2}} - \frac{2bp\log^2\left(c($$

$$= \frac{8ib^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{a^{3/2}} + \frac{16b^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{a^{3/2}} + \frac{8b^{3/2}p^2\,t$$

$$= \frac{8ib^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{a^{3/2}} + \frac{16b^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{a^{3/2}} + \frac{8b^{3/2}p^2\,t$$

$$= \frac{8ib^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{a^{3/2}} + \frac{16b^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{a^{3/2}} + \frac{8b^{3/2}p^2\,t$$

Mathematica [A] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 851 vs. 2(254) = 508.
time = 1.83, size = 851, normalized size = 3.35

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^3/x^4,x]
```

```
[Out] (a^2*(p*Log[a + b*x^2] - Log[c*(a + b*x^2)^p])^3 - 6*a*b*p*x^2*(-(p*Log[a +
 b*x^2]) + Log[c*(a + b*x^2)^p])^2 - 6*Sqrt[a]*b^(3/2)*p*x^3*ArcTan[(Sqrt[b
]*x)/Sqrt[a]]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2 - 3*a^2*p*Log[
a + b*x^2]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2 + 3*Sqrt[a]*p^2*(
p*Log[a + b*x^2] - Log[c*(a + b*x^2)^p])*(a^(3/2)*Log[a + b*x^2]^2 + 4*b*x^
2*(I*Sqrt[b]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 + Sqrt[a]*Log[a + b*x^2] + Sqr
t[b]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(-2 + 2*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqr
t[b]*x)] + Log[a + b*x^2]) + I*Sqrt[b]*x*PolyLog[2, (I*Sqrt[a] + Sqrt[b]*x)
/((-I)*Sqrt[a] + Sqrt[b]*x)])) + p^3*(48*a*b*x^2*Sqrt[(b*x^2)/(a + b*x^2)]*
HypergeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}, a/(a + b*x^2)] + 2
4*Sqrt[-a]*(b*x^2)^(3/2)*ArcTanh[Sqrt[b*x^2]/Sqrt[-a]]*Log[a + b*x^2] + 24*
a*b*x^2*Sqrt[(b*x^2)/(a + b*x^2)]*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2,
3/2}, a/(a + b*x^2)]*Log[a + b*x^2] - 6*a*b*x^2*Log[a + b*x^2]^2 + 6*Sqrt[a
]*((b*x^2)/(a + b*x^2))^(3/2)*(a + b*x^2)^(3/2)*ArcSin[Sqrt[a]/Sqrt[a + b*x
^2]]*Log[a + b*x^2]^2 - a^2*Log[a + b*x^2]^3 - 24*Sqrt[-a]*(b*x^2)^(3/2)*Ar
cTanh[Sqrt[b*x^2]/Sqrt[-a]]*Log[1 + (b*x^2)/a] - 6*a^2*(-((b*x^2)/a))^(3/2)
*Log[1 + (b*x^2)/a]^2 + 24*a^2*(-((b*x^2)/a))^(3/2)*Log[1 + (b*x^2)/a]*Log[
(1 + Sqrt[-((b*x^2)/a)])/2] - 12*a^2*(-((b*x^2)/a))^(3/2)*Log[(1 + Sqrt[-((
b*x^2)/a)])/2]^2 + 24*a^2*(-((b*x^2)/a))^(3/2)*PolyLog[2, 1/2 - Sqrt[-((b*x
^2)/a)]/2]))/(3*a^2*x^3)
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(c(b\,x^{2}+a)^{p}\right)^{3}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)^3/x^4,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^3/x^4,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*p^3*log(b*x^2 + a)^3/x^3 + integrate((b*x^2*log(c)^3 + a*log(c)^3 + ((
2*p^3 + 3*p^2*log(c))*b*x^2 + 3*a*p^2*log(c))*log(b*x^2 + a)^2 + 3*(b*p*x^2
*log(c)^2 + a*p*log(c)^2)*log(b*x^2 + a))/(b*x^6 + a*x^4), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^4,x, algorithm="fricas")
[Out] integral(log((b*x^2 + a)^p*c)^3/x^4, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c(a+bx^2)^p\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)**3/x**4,x)
[Out] Integral(log(c*(a + b*x**2)**p)**3/x**4, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^4,x, algorithm="giac")
[Out] integrate(log((b*x^2 + a)^p*c)^3/x^4, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)^3/x^4,x)
[Out] int(log(c*(a + b*x^2)^p)^3/x^4, x)
```

3.102 $\int \frac{x^3}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$

Optimal. Leaf size=107

$$-\frac{a\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)}{2b^2p}+\frac{\left(a+bx^2\right)^2\left(c\left(a+bx^2\right)^p\right)^{-2/p}\operatorname{Ei}\left(\frac{2\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)}{2b^2p}$$

```
[Out] -1/2*a*(b*x^2+a)*Ei(ln(c*(b*x^2+a)^p)/p)/b^2/p/((c*(b*x^2+a)^p)^(1/p))+1/2*
(b*x^2+a)^2*Ei(2*ln(c*(b*x^2+a)^p)/p)/b^2/p/((c*(b*x^2+a)^p)^(2/p))
```

Rubi [A]
time = 0.10, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {2504, 2446, 2436, 2337, 2209, 2437, 2347}

$$\frac{\left(a+bx^2\right)^2\left(c\left(a+bx^2\right)^p\right)^{-2/p}\operatorname{Ei}\left(\frac{2\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{2b^2p}-\frac{a\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{2b^2p}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Log[c*(a + b*x^2)^p],x]
```

```
[Out] -1/2*(a*(a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(b^2*p*(c*(a + b
*x^2)^p)^p^(-1)) + ((a + b*x^2)^2*ExpIntegralEi[(2*Log[c*(a + b*x^2)^p])/p]
)/(2*b^2*p*(c*(a + b*x^2)^p)^(2/p))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\log\left(c(a+bx)^p\right)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a}{b\log\left(c(a+bx)^p\right)}+\frac{a+bx}{b\log\left(c(a+bx)^p\right)}\right)\,dx,x,x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{a+bx}{\log(c(a+bx)^p)}\,dx,x,x^2\right)}{2b}-\frac{a\text{Subst}\left(\int \frac{1}{\log(c(a+bx)^p)}\,dx,x,x^2\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \frac{x}{\log(cx^p)}\,dx,x,a+bx^2\right)}{2b^2}-\frac{a\text{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx,x,a+bx^2\right)}{2b^2}\\
&= \frac{\left(\left(a+bx^2\right)^2\left(c(a+bx^2)^p\right)^{-2/p}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{p}}}{x}\,dx,x,\log\left(c(a+bx^2)^p\right)\right)}{2b^2p}-\frac{\left(a(a+bx\right.}{}\\
&= -\frac{a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\text{Ei}\left(\frac{\log(c(a+bx^2)^p)}{p}\right)}{2b^2p}+\frac{(a+bx^2)^2\left(c(a+bx^2)^p\right)^{-2/p}\text{Ei}}{2b^2p}
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 96, normalized size = 0.90

$$-\frac{(a+bx^2)\left(c(a+bx^2)^p\right)^{-2/p}\left(a\left(c(a+bx^2)^p\right)^{\frac{1}{p}}\operatorname{Ei}\left(\frac{\log(c(a+bx^2)^p)}{p}\right)-(a+bx^2)\operatorname{Ei}\left(\frac{2\log(c(a+bx^2)^p)}{p}\right)\right)}{2b^2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Log[c*(a + b*x^2)^p],x]
```

```
[Out] -1/2*((a + b*x^2)*(a*(c*(a + b*x^2)^p)^p^(-1)*ExpIntegralEi[Log[c*(a + b*x^
2)^p]/p] - (a + b*x^2)*ExpIntegralEi[(2*Log[c*(a + b*x^2)^p])/p]))/(b^2*p*(
c*(a + b*x^2)^p)^(2/p))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.65, size = 547, normalized size = 5.11

method	result
risch	$-\frac{(b x^2+a)^2 c^{-\frac{2}{p}} ((b x^2+a)^p)^{-\frac{2}{p}} \mathrm{e}^{\frac{i\pi \operatorname{csgn}(ic(b x^2+a)^p)(\operatorname{csgn}(i(b x^2+a)^p)-\operatorname{csgn}(ic(b x^2+a)^p))(\operatorname{csgn}(ic)-\operatorname{csgn}(ic(b x^2+a)^p))}{p}} \text{expIntegr}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/ln(c*(b*x^2+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b^2/p*(b*x^2+a)^2*c^(-2/p)*((b*x^2+a)^p)^(-2/p)*exp(I*Pi*csgn(I*c*(b*x
^2+a)^p)*(csgn(I*(b*x^2+a)^p)-csgn(I*c*(b*x^2+a)^p))*(csgn(I*c)-csgn(I*c*(b
*x^2+a)^p))/p)*Ei(1,-2*ln(b*x^2+a)-(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^
2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csg
n(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+2*ln((b
*x^2+a)^p)-2*p*ln(b*x^2+a))/p)+1/2/b^2*a/p*(b*x^2+a)*c^(-1/p)*((b*x^2+a)^p)
^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)*(csgn(I*(b*x^2+a)^p)-csgn(I*c*(b
*x^2+a)^p))*(csgn(I*c)-csgn(I*c*(b*x^2+a)^p))/p)*Ei(1,-ln(b*x^2+a)-1/2*(I*P
i*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn
(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x
^2+a)^p)^2*csgn(I*c)+2*ln(c)+2*ln((b*x^2+a)^p)-2*p*ln(b*x^2+a))/p)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] integrate(x^3/log((b*x^2 + a)^p*c), x)
```

Fricas [A]
time = 0.36, size = 68, normalized size = 0.64

$$-\frac{ac^{\left(\frac{1}{p}\right)}\operatorname{log\_integral}\left((bx^2+a)c^{\left(\frac{1}{p}\right)}\right)-\operatorname{log\_integral}\left((b^2x^4+2\,abx^2+a^2)c^{\frac{2}{p}}\right)}{2\,b^2c^{\frac{2}{p}}p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] -1/2*(a*c^(1/p)*log_integral((b*x^2 + a)*c^(1/p)) - log_integral((b^2*x^4 +
 2*a*b*x^2 + a^2)*c^(2/p)))/(b^2*c^(2/p)*p)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/ln(c*(b*x**2+a)**p),x)
```

```
[Out] Integral(x**3/log(c*(a + b*x**2)**p), x)
```

Giac [A]
time = 3.09, size = 69, normalized size = 0.64

$$-\frac{a\operatorname{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2+a\right)\right)}{2\,b^2c^{\left(\frac{1}{p}\right)}p}+\frac{\operatorname{Ei}\left(\frac{2\,\log(c)}{p}+2\,\log\left(bx^2+a\right)\right)}{2\,b^2c^{\frac{2}{p}}p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] -1/2*a*Ei(log(c)/p + log(b*x^2 + a))/(b^2*c^(1/p)*p) + 1/2*Ei(2*log(c)/p +
2*log(b*x^2 + a))/(b^2*c^(2/p)*p)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^3}{\ln\left(c\left(b\,x^2+a\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/log(c*(a + b*x^2)^p),x)
```

```
[Out] int(x^3/log(c*(a + b*x^2)^p), x)
```

3.103 $\int \frac{x}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$

Optimal. Leaf size=51

$$\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)}{2bp}$$

```
[Out] 1/2*(b*x^2+a)*Ei(ln(c*(b*x^2+a)^p)/p)/b/p/((c*(b*x^2+a)^p)^(1/p))
```

Rubi [A]
time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2504, 2436, 2337, 2209}

$$\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{2bp}$$

Antiderivative was successfully verified.

```
[In] Int[x/Log[c*(a + b*x^2)^p],x]
```

```
[Out] ((a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(2*b*p*(c*(a + b*x^2)^p
)^p^(-1))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\log\left(c(a+bx)^p\right)}\,dx, x, x^2\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx, x, a+bx^2\right)}{2b}\\ &= \frac{\left(\left(a+bx^2\right)\left(c(a+bx^2)^p\right)^{-1/p}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{p}}}{x}\,dx, x, \log\left(c(a+bx^2)^p\right)\right)}{2bp}\\ &= \frac{\left(a+bx^2\right)\left(c(a+bx^2)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c(a+bx^2)^p\right)}{p}\right)}{2bp}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 51, normalized size = 1.00

$$\frac{\left(a+bx^2\right)\left(c(a+bx^2)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c(a+bx^2)^p\right)}{p}\right)}{2bp}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Log[c*(a + b*x^2)^p],x]
```

```
[Out] ((a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(2*b*p*(c*(a + b*x^2)^p
)^p^(-1))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.79, size = 272, normalized size = 5.33

method	result
risch	$-\frac{(bx^2+a)\left((bx^2+a)^p\right)^{-\frac{1}{p}} c^{-\frac{1}{p}} e^{\frac{i\pi\,\text{csgn}\left(ic\left(bx^2+a\right)^p\right)\left(\text{csgn}\left(i\left(bx^2+a\right)^p\right)-\text{csgn}\left(ic\left(bx^2+a\right)^p\right)\right)\left(\text{csgn}(ic)-\text{csgn}\left(ic\left(bx^2+a\right)^p\right)\right)}{2p}}\,\text{expIntegral}\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/ln(c*(b*x^2+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b/p*(b*x^2+a)*((b*x^2+a)^p)^(-1/p)*c^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(b*x
^2+a)^p)*(csgn(I*(b*x^2+a)^p)-csgn(I*c*(b*x^2+a)^p))*(csgn(I*c)-csgn(I*c*(b
```

```
*x^2+a)^p))/p)*Ei(1,-ln(b*x^2+a)-1/2*(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*
x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*c
sgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+2*ln(
(b*x^2+a)^p)-2*p*ln(b*x^2+a))/p)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)^p),x, algorithm="maxima")
[Out] integrate(x/log((b*x^2 + a)^p*c), x)
```

Fricas [A]
time = 0.41, size = 29, normalized size = 0.57

$$\frac{\text{log\_integral}\left((bx^2+a)c^{\left(\frac{1}{p}\right)}\right)}{2\,bc^{\left(\frac{1}{p}\right)}p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)^p),x, algorithm="fricas")
[Out] 1/2*log_integral((b*x^2 + a)*c^(1/p))/(b*c^(1/p)*p)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/ln(c*(b*x**2+a)**p),x)
[Out] Integral(x/log(c*(a + b*x**2)**p), x)
```

Giac [A]
time = 4.09, size = 31, normalized size = 0.61

$$\frac{\text{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2+a\right)\right)}{2\,bc^{\left(\frac{1}{p}\right)}p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)^p),x, algorithm="giac")
[Out] 1/2*Ei(log(c)/p + log(b*x^2 + a))/(b*c^(1/p)*p)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/log(c*(a + b*x^2)^p),x)
[Out] int(x/log(c*(a + b*x^2)^p), x)
```

3.104 $\int \frac{1}{x\log\left(c\left(a+bx^2\right)^p\right)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x\log\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x/ln(c*(b*x^2+a)^p),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*Log[c*(a + b*x^2)^p]),x]
```

```
[Out] Defer[Int][1/(x*Log[c*(a + b*x^2)^p]), x]
```

Rubi steps

$$\int \frac{1}{x\log\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{x\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Mathematica [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{x\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*Log[c*(a + b*x^2)^p]),x]
```

```
[Out] Integrate[1/(x*Log[c*(a + b*x^2)^p]), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{x\ln\left(c\left(b\,x^2+a\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(c*(b*x^2+a)^p),x)
[Out] int(1/x/ln(c*(b*x^2+a)^p),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(b*x^2+a)^p),x, algorithm="maxima")
[Out] integrate(1/(x*log((b*x^2 + a)^p*c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(b*x^2+a)^p),x, algorithm="fricas")
[Out] integral(1/(x*log((b*x^2 + a)^p*c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \log \left(c \left(a + b x^{2}\right)^{p}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(c*(b*x**2+a)**p),x)
[Out] Integral(1/(x*log(c*(a + b*x**2)**p)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(b*x^2+a)^p),x, algorithm="giac")
[Out] integrate(1/(x*log((b*x^2 + a)^p*c)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*log(c*(a + b*x^2)^p)),x)
[Out] int(1/(x*log(c*(a + b*x^2)^p)), x)
```

3.105 $\int \frac{1}{x^3 \log\left(c\left(a+bx^2\right)^p\right)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x^3 \log \left(c \left(a+b x^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x^3/ln(c*(b*x^2+a)^p),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^3 \log \left(c \left(a+b x^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^3*Log[c*(a + b*x^2)^p]),x]
[Out] Defer[Int][1/(x^3*Log[c*(a + b*x^2)^p]), x]
```

Rubi steps

$$\int \frac{1}{x^3 \log \left(c \left(a+b x^2\right)^p\right)}\,dx=\int \frac{1}{x^3 \log \left(c \left(a+b x^2\right)^p\right)}\,dx$$

Mathematica [A]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \log \left(c \left(a+b x^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^3*Log[c*(a + b*x^2)^p]),x]
[Out] Integrate[1/(x^3*Log[c*(a + b*x^2)^p]), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \ln \left(c \left(b\,x^2+a\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/ln(c*(b*x^2+a)^p),x)
[Out] int(1/x^3/ln(c*(b*x^2+a)^p),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p),x, algorithm="maxima")
[Out] integrate(1/(x^3*log((b*x^2 + a)^p*c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p),x, algorithm="fricas")
[Out] integral(1/(x^3*log((b*x^2 + a)^p*c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \log \left(c \left(a+b x^2\right)^p\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/ln(c*(b*x**2+a)**p),x)
[Out] Integral(1/(x**3*log(c*(a + b*x**2)**p)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p),x, algorithm="giac")
[Out] integrate(1/(x^3*log((b*x^2 + a)^p*c)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^3\,\ln \left(c\,{\left(b\,x^2+a\right)}^p\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*log(c*(a + b*x^2)^p)),x)
[Out] int(1/(x^3*log(c*(a + b*x^2)^p)), x)
```

3.106 $\int \frac{x^2}{\log(c(a+bx^2)^p)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(x^2/ln(c*(b*x^2+a)^p),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2/Log[c*(a + b*x^2)^p],x]
[Out] Defer[Int][x^2/Log[c*(a + b*x^2)^p], x]
```

Rubi steps

$$\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Mathematica [A]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2/Log[c*(a + b*x^2)^p],x]
[Out] Integrate[x^2/Log[c*(a + b*x^2)^p], x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\ln\left(c\left(b\,x^2+a\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/ln(c*(b*x^2+a)^p),x)
[Out] int(x^2/ln(c*(b*x^2+a)^p),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(b*x^2+a)^p),x, algorithm="maxima")
[Out] integrate(x^2/log((b*x^2 + a)^p*c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(b*x^2+a)^p),x, algorithm="fricas")
[Out] integral(x^2/log((b*x^2 + a)^p*c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/ln(c*(b*x**2+a)**p),x)
[Out] Integral(x**2/log(c*(a + b*x**2)**p), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(b*x^2+a)^p),x, algorithm="giac")
[Out] integrate(x^2/log((b*x^2 + a)^p*c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^2}{\ln\left(c\left(b\,x^2+a\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/log(c*(a + b*x^2)^p),x)
[Out] int(x^2/log(c*(a + b*x^2)^p), x)
```

3.107 $\int \frac{1}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{\log\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/ln(c*(b*x^2+a)^p),x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[c*(a + b*x^2)^p]^(-1),x]
[Out] Defer[Int][Log[c*(a + b*x^2)^p]^(-1), x]
```

Rubi steps

$$\int \frac{1}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Mathematica [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^(-1),x]
[Out] Integrate[Log[c*(a + b*x^2)^p]^(-1), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\ln\left(c\left(b\,x^2+a\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(c*(b*x^2+a)^p),x)
[Out] int(1/ln(c*(b*x^2+a)^p),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(b*x^2+a)^p),x, algorithm="maxima")
[Out] integrate(1/log((b*x^2 + a)^p*c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(b*x^2+a)^p),x, algorithm="fricas")
[Out] integral(1/log((b*x^2 + a)^p*c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\log \left(c \left(a + bx^{2}\right)^{p}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(c*(b*x**2+a)**p),x)
[Out] Integral(1/log(c*(a + b*x**2)**p), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(b*x^2+a)^p),x, algorithm="giac")
[Out] integrate(1/log((b*x^2 + a)^p*c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\ln \left(c \left(b\, x^2 + a\right)^p\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/log(c*(a + b*x^2)^p),x)
[Out] int(1/log(c*(a + b*x^2)^p), x)
```

3.108 $\int \frac{1}{x^2 \log\left(c\left(a+bx^2\right)^p\right)} \, dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x^2 \log \left(c \left(a+b x^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x^2/ln(c*(b*x^2+a)^p),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2 \log \left(c \left(a+b x^2\right)^p\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*Log[c*(a + b*x^2)^p]),x]
```

```
[Out] Defer[Int][1/(x^2*Log[c*(a + b*x^2)^p]), x]
```

Rubi steps

$$\int \frac{1}{x^2 \log \left(c \left(a+b x^2\right)^p\right)} \, dx=\int \frac{1}{x^2 \log \left(c \left(a+b x^2\right)^p\right)} \, dx$$

Mathematica [A]
time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \log \left(c \left(a+b x^2\right)^p\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*Log[c*(a + b*x^2)^p]),x]
```

```
[Out] Integrate[1/(x^2*Log[c*(a + b*x^2)^p]), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \ln \left(c \left(b\,x^2+a\right)^p\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/ln(c*(b*x^2+a)^p),x)
[Out] int(1/x^2/ln(c*(b*x^2+a)^p),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p),x, algorithm="maxima")
[Out] integrate(1/(x^2*log((b*x^2 + a)^p*c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p),x, algorithm="fricas")
[Out] integral(1/(x^2*log((b*x^2 + a)^p*c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \log \left(c \left(a + bx^2\right)^p\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/ln(c*(b*x**2+a)**p),x)
[Out] Integral(1/(x**2*log(c*(a + b*x**2)**p)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p),x, algorithm="giac")
[Out] integrate(1/(x^2*log((b*x^2 + a)^p*c)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^2\, \ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*log(c*(a + b*x^2)^p)),x)
[Out] int(1/(x^2*log(c*(a + b*x^2)^p)), x)
```

3.109 $\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$

Optimal. Leaf size=138

$$-\frac{a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c(a+bx^2)^p\right)}{p}\right)}{2b^2p^2}+\frac{(a+bx^2)^2\left(c(a+bx^2)^p\right)^{-2/p}\operatorname{Ei}\left(\frac{2\log\left(c(a+bx^2)^p\right)}{p}\right)}{b^2p^2}-\frac{x^2(a}{2bp\log(c}$$

```
[Out] -1/2*a*(b*x^2+a)*Ei(ln(c*(b*x^2+a)^p)/p)/b^2/p^2/((c*(b*x^2+a)^p)^(1/p))+(b
*x^2+a)^2*Ei(2*ln(c*(b*x^2+a)^p)/p)/b^2/p^2/((c*(b*x^2+a)^p)^(2/p))-1/2*x^2
*(b*x^2+a)/b/p/ln(c*(b*x^2+a)^p)
```

Rubi [A]

time = 0.14, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {2504, 2447, 2446, 2436, 2337, 2209, 2437, 2347}

$$\frac{(a+bx^2)^2\left(c(a+bx^2)^p\right)^{-2/p}\operatorname{Ei}\left(\frac{2\log\left(c(bx^2+a)^p\right)}{p}\right)}{b^2p^2}-\frac{a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c(bx^2+a)^p\right)}{p}\right)}{2b^2p^2}-\frac{x^2(a+bx^2)}{2bp\log\left(c\left(a+bx^2\right)^p\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] -1/2*(a*(a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(b^2*p^2*(c*(a +
 b*x^2)^p)^p^(-1)) + ((a + b*x^2)^2*ExpIntegralEi[(2*Log[c*(a + b*x^2)^p])/
p])/(b^2*p^2*(c*(a + b*x^2)^p)^(2/p)) - (x^2*(a + b*x^2))/(2*b*p*Log[c*(a +
 b*x^2)^p])
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2447

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[q*((e*f - d*g)/
(b*e*n*(p + 1))), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{\log^2\left(c(a+bx)^p\right)}\,dx, x, x^2\right)$$

$$= -\frac{x^2(a+bx^2)}{2bp\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{x}{\log(c(a+bx)^p)}\,dx, x, x^2\right)}{p} + \frac{a\text{Subst}\left(\int \frac{1}{\log(c(a+bx)^p)}\right.}{2bp}$$

$$= -\frac{x^2(a+bx^2)}{2bp\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \left(-\frac{a}{b\log(c(a+bx)^p)} + \frac{a+bx}{b\log(c(a+bx)^p)}\right)dx, x, x^2\right)}{p} + \cdots$$

$$= -\frac{x^2(a+bx^2)}{2bp\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{a+bx}{\log(c(a+bx)^p)}\,dx, x, x^2\right)}{bp} - \frac{a\text{Subst}\left(\int \frac{1}{\log(c(a+bx)^p)}\right.}{bp}$$

$$= \frac{a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\text{Ei}\left(\frac{\log(c(a+bx^2)^p)}{p}\right)}{2b^2p^2} - \frac{x^2(a+bx^2)}{2bp\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\cdots\right.}{}$$

$$= \frac{a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\text{Ei}\left(\frac{\log(c(a+bx^2)^p)}{p}\right)}{2b^2p^2} - \frac{x^2(a+bx^2)}{2bp\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\left((a+b\cdots\right.}{}$$

$$= -\frac{a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\text{Ei}\left(\frac{\log(c(a+bx^2)^p)}{p}\right)}{2b^2p^2} + \frac{(a+bx^2)^2\left(c(a+bx^2)^p\right)^{-2/p}\text{E}\cdots}{b^2p^2}$$

Mathematica [A]
time = 0.11, size = 157, normalized size = 1.14

$$-\frac{(a+bx^2)\left(c(a+bx^2)^p\right)^{-2/p}\left(bpx^2\left(c(a+bx^2)^p\right)^{2/p} + a\left(c(a+bx^2)^p\right)^{\frac{1}{p}}\text{Ei}\left(\frac{\log(c(a+bx^2)^p)}{p}\right)\log\left(c(a+bx^2)^p\right) - 2(a+bx^2)\text{Ei}\left(\frac{2\log(c(a+bx^2)^p)}{p}\right)\log\left(c(a+bx^2)^p\right)\right)}{2b^2p^2\log\left(c\left(a+bx^2\right)^p\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] -1/2*((a + b*x^2)*(b*p*x^2*(c*(a + b*x^2)^p)^(2/p) + a*(c*(a + b*x^2)^p)^p^
(-1)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p]*Log[c*(a + b*x^2)^p] - 2*(a + b*
x^2)*ExpIntegralEi[(2*Log[c*(a + b*x^2)^p])/p]*Log[c*(a + b*x^2)^p]))/(b^2*
p^2*(c*(a + b*x^2)^p)^(2/p)*Log[c*(a + b*x^2)^p])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.64, size = 1474, normalized size = 10.68

method	result	size
risch	Expression too large to display	1474

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/ln(c*(b*x^2+a)^p)^2,x,method=_RETURNVERBOSE)
```

```
[Out] I*x^2*(b*x^2+a)/p/b/(Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-Pi*csgn
(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-Pi*csgn(I*c*(b*x^2+a)^p)^3+
Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-2*I*ln(c)-2*I*ln((b*x^2+a)^p))-1/p^2*c
^(-2/p)*((b*x^2+a)^p)^(-2/p)*exp(I*Pi*csgn(I*c*(b*x^2+a)^p)*(csgn(I*(b*x^2+
a)^p)-csgn(I*c*(b*x^2+a)^p))*(csgn(I*c)-csgn(I*c*(b*x^2+a)^p))/p)*Ei(1,-2*l
n(b*x^2+a)+I*(-Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+Pi*csgn(I*(b*
x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+Pi*csgn(I*c*(b*x^2+a)^p)^3-Pi*csg
n(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*I*ln(c)+2*I*(ln((b*x^2+a)^p)-p*ln(b*x^2+a)
))/p)*x^4-2/p^2/b*c^(-2/p)*((b*x^2+a)^p)^(-2/p)*exp(I*Pi*csgn(I*c*(b*x^2+a)
^p)*(csgn(I*(b*x^2+a)^p)-csgn(I*c*(b*x^2+a)^p))*(csgn(I*c)-csgn(I*c*(b*x^2+
a)^p))/p)*Ei(1,-2*ln(b*x^2+a)+I*(-Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)
^p)^2+Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+Pi*csgn(I*c*(b
*x^2+a)^p)^3-Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*I*ln(c)+2*I*(ln((b*x^2+
a)^p)-p*ln(b*x^2+a)))/p)*a*x^2-1/p^2/b^2*c^(-2/p)*((b*x^2+a)^p)^(-2/p)*exp(
I*Pi*csgn(I*c*(b*x^2+a)^p)*(csgn(I*(b*x^2+a)^p)-csgn(I*c*(b*x^2+a)^p))*(csg
n(I*c)-csgn(I*c*(b*x^2+a)^p))/p)*Ei(1,-2*ln(b*x^2+a)+I*(-Pi*csgn(I*(b*x^2+a
)^p)*csgn(I*c*(b*x^2+a)^p)^2+Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*c
sgn(I*c)+Pi*csgn(I*c*(b*x^2+a)^p)^3-Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*
I*ln(c)+2*I*(ln((b*x^2+a)^p)-p*ln(b*x^2+a)))/p)*a^2+1/2/p^2/b*a*c^(-1/p)*((
b*x^2+a)^p)^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)*(csgn(I*(b*x^2+a)^p)-
csgn(I*c*(b*x^2+a)^p))*(csgn(I*c)-csgn(I*c*(b*x^2+a)^p))/p)*Ei(1,-ln(b*x^2+
a)+1/2*I*(-Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+Pi*csgn(I*(b*x^2+
a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+Pi*csgn(I*c*(b*x^2+a)^p)^3-Pi*csgn(I*
c*(b*x^2+a)^p)^2*csgn(I*c)+2*I*ln(c)+2*I*(ln((b*x^2+a)^p)-p*ln(b*x^2+a)))/p
)*x^2+1/2/p^2/b^2*a^2*c^(-1/p)*((b*x^2+a)^p)^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(
b*x^2+a)^p)*(csgn(I*(b*x^2+a)^p)-csgn(I*c*(b*x^2+a)^p))*(csgn(I*c)-csgn(I*c
*(b*x^2+a)^p))/p)*Ei(1,-ln(b*x^2+a)+1/2*I*(-Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c
*(b*x^2+a)^p)^2+Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+Pi*c
sgn(I*c*(b*x^2+a)^p)^3-Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*I*ln(c)+2*I*(
ln((b*x^2+a)^p)-p*ln(b*x^2+a)))/p)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^4 + a*x^2)/(b*p^2*log(b*x^2 + a) + b*p*log(c)) + integrate((2*b*x
^3 + a*x)/(b*p^2*log(b*x^2 + a) + b*p*log(c)), x)
```

Fricas [A]

time = 0.34, size = 141, normalized size = 1.02

$$-\frac{\left(ap\log\left(bx^2+a\right)+a\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}\,\mathrm{log\_integral}\left(\left(bx^2+a\right)c^{\left(\frac{1}{p}\right)}\right)+\left(b^2px^4+abpx^2\right)c^{\frac{2}{p}}-2\left(p\log\left(bx^2+a\right)+\log\left(c\right)\right)\mathrm{log\_integral}\left(\left(b^2x^4+2\,abx^2+a^2\right)c^{\frac{2}{p}}\right)}{2\left(b^2p^3\log\left(bx^2+a\right)+b^2p^2\log\left(c\right)\right)c^{\frac{2}{p}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((a*p*log(b*x^2 + a) + a*log(c))*c^(1/p)*log_integral((b*x^2 + a)*c^(1
/p)) + (b^2*p*x^4 + a*b*p*x^2)*c^(2/p) - 2*(p*log(b*x^2 + a) + log(c))*log_
integral((b^2*x^4 + 2*a*b*x^2 + a^2)*c^(2/p)))/((b^2*p^3*log(b*x^2 + a) + b
^2*p^2*log(c))*c^(2/p))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\log\left(c\left(a+bx^2\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Integral(x**3/log(c*(a + b*x**2)**p)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(136) = 272.
time = 3.21, size = 313, normalized size = 2.27

$$\frac{1}{2}\,a\left(\frac{\left(bx^2+a\right)p}{b^2p^3\log\left(bx^2+a\right)+b^2p^2\log\left(c\right)}-\frac{p\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2+a\right)\right)\log\left(bx^2+a\right)}{\left(b^2p^3\log\left(bx^2+a\right)+b^2p^2\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}}-\frac{\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2+a\right)\right)\log\left(c\right)}{\left(b^2p^3\log\left(bx^2+a\right)+b^2p^2\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}}\right)-\frac{\frac{\left(bx^2+a\right)^2p}{bp^3\log(bx^2+a)+bp^2\log(c)}-\frac{2\,p\mathrm{Ei}\left(\frac{2\log(c)}{p}+2\log\left(bx^2+a\right)\right)\log\left(bx^2+a\right)}{\left(bp^3\log(bx^2+a)+bp^2\log(c)\right)c^{\frac{2}{p}}}-\frac{2\,\mathrm{Ei}\left(\frac{2\log(c)}{p}+2\log\left(bx^2+a\right)\right)\log(c)}{\left(bp^3\log(bx^2+a)+bp^2\log(c)\right)c^{\frac{2}{p}}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] 1/2*a*((b*x^2 + a)*p/(b^2*p^3*log(b*x^2 + a) + b^2*p^2*log(c)) - p*Ei(log(c
)/p + log(b*x^2 + a))*log(b*x^2 + a)/((b^2*p^3*log(b*x^2 + a) + b^2*p^2*log
(c))*c^(1/p)) - Ei(log(c)/p + log(b*x^2 + a))*log(c)/((b^2*p^3*log(b*x^2 +
a) + b^2*p^2*log(c))*c^(1/p))) - 1/2*((b*x^2 + a)^2*p/(b*p^3*log(b*x^2 + a)
 + b*p^2*log(c)) - 2*p*Ei(2*log(c)/p + 2*log(b*x^2 + a))*log(b*x^2 + a)/((b
*p^3*log(b*x^2 + a) + b*p^2*log(c))*c^(2/p)) - 2*Ei(2*log(c)/p + 2*log(b*x^
2 + a))*log(c)/((b*p^3*log(b*x^2 + a) + b*p^2*log(c))*c^(2/p)))/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/log(c*(a + b*x^2)^p)^2,x)
[Out] int(x^3/log(c*(a + b*x^2)^p)^2, x)
```

3.110 $\int \frac{x}{\log^2\left(c\left(a+bx^2\right)^p\right)} \, dx$

Optimal. Leaf size=83

$$\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)}{2bp^2}-\frac{a+bx^2}{2bp\log\left(c\left(a+bx^2\right)^p\right)}$$

```
[Out] 1/2*(b*x^2+a)*Ei(ln(c*(b*x^2+a)^p)/p)/b/p^2/((c*(b*x^2+a)^p)^(1/p))+1/2*(-b
*x^2-a)/b/p/ln(c*(b*x^2+a)^p)
```

Rubi [A]
time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2504, 2436, 2334, 2337, 2209}

$$\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{2bp^2}-\frac{a+bx^2}{2bp\log\left(c\left(a+bx^2\right)^p\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] ((a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(2*b*p^2*(c*(a + b*x^2)
^p)^p^(-1)) - (a + b*x^2)/(2*b*p*Log[c*(a + b*x^2)^p])
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\log^2\left(c(a+bx)^p\right)}\,dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{\log^2(cx^p)}\,dx, x, a+bx^2\right)}{2b}\\
&= -\frac{a+bx^2}{2bp\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx, x, a+bx^2\right)}{2bp}\\
&= -\frac{a+bx^2}{2bp\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\left(\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{p}}}{x}\,dx, x, \log\left(c\right.\right.}{2bp^2}\\
&= \frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)}{2bp^2} - \frac{a+bx^2}{2bp\log\left(c\left(a+bx^2\right)^p\right)}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 97, normalized size = 1.17

$$
-\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\left(p\left(c\left(a+bx^2\right)^p\right)^{\frac{1}{p}}-\text{Ei}\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)\log\left(c\left(a+bx^2\right)^p\right)\right)}{2bp^2\log\left(c\left(a+bx^2\right)^p\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] -1/2*((a + b*x^2)*(p*(c*(a + b*x^2)^p)^p^(-1) - ExpIntegralEi[Log[c*(a + b*
x^2)^p]/p]*Log[c*(a + b*x^2)^p]))/(b*p^2*(c*(a + b*x^2)^p)^p^(-1)*Log[c*(a
+ b*x^2)^p])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.14, size = 421, normalized size = 5.07

method	result
risch	$-\frac{b\,x^2+a}{\left(2\ln(c)+2\ln((b\,x^2+a)^p)+i\pi\,\mathrm{csgn}(i(b\,x^2+a)^p)\mathrm{csgn}(ic(b\,x^2+a)^p)^2-i\pi\,\mathrm{csgn}(i(b\,x^2+a)^p)\mathrm{csgn}(ic(b\,x^2+a)^p)\mathrm{csgn}(ic)-i\pi\mathrm{csgn}(ic(b\,x^2+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/ln(c*(b*x^2+a)^p)^2,x,method=_RETURNVERBOSE)

[Out] -1/(2*ln(c)+2*ln((b*x^2+a)^p)+I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c))/p/b*(b*x^2+a)-1/2/p^2/b*(b*x^2+a)*c^(-1/p)*((b*x^2+a)^p)^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)*(csgn(I*(b*x^2+a)^p)-csgn(I*c*(b*x^2+a)^p))*(csgn(I*c)-csgn(I*c*(b*x^2+a)^p))/p)*Ei(1,-ln(b*x^2+a)-1/2*(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+2*ln((b*x^2+a)^p)-2*p*ln(b*x^2+a))/p)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")

[Out] -1/2*(b*x^2 + a)/(b*p^2*log(b*x^2 + a) + b*p*log(c)) + integrate(x/(p^2*log(b*x^2 + a) + p*log(c)), x)

Fricas [A]
time = 0.37, size = 78, normalized size = 0.94

$$-\frac{(bpx^2+ap)c^{\left(\frac{1}{p}\right)}-(p\log(bx^2+a)+\log(c))\,\mathrm{log\_integral}\left((bx^2+a)c^{\left(\frac{1}{p}\right)}\right)}{2\,(bp^3\log(bx^2+a)+bp^2\log(c))c^{\left(\frac{1}{p}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")

[Out] -1/2*((b*p*x^2 + a*p)*c^(1/p) - (p*log(b*x^2 + a) + log(c))*log_integral((b*x^2 + a)*c^(1/p)))/((b*p^3*log(b*x^2 + a) + b*p^2*log(c))*c^(1/p))

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\log(c(a+bx^2)^p)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Integral(x/log(c*(a + b*x**2)**p)**2, x)
```

Giac [A]
time = 5.89, size = 141, normalized size = 1.70

$$-\frac{(bx^2+a)p}{2\left(bp^3\log\left(bx^2+a\right)+bp^2\log\left(c\right)\right)}+\frac{p\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2+a\right)\right)\log\left(bx^2+a\right)}{2\left(bp^3\log\left(bx^2+a\right)+bp^2\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}}+\frac{\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2+a\right)\right)\log\left(c\right)}{2\left(bp^3\log\left(bx^2+a\right)+bp^2\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b*x^2 + a)*p/(b*p^3*log(b*x^2 + a) + b*p^2*log(c)) + 1/2*p*Ei(log(c)/
p + log(b*x^2 + a))*log(b*x^2 + a)/((b*p^3*log(b*x^2 + a) + b*p^2*log(c))*c
^(1/p)) + 1/2*Ei(log(c)/p + log(b*x^2 + a))*log(c)/((b*p^3*log(b*x^2 + a) +
 b*p^2*log(c))*c^(1/p))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/log(c*(a + b*x^2)^p)^2,x)
```

```
[Out] int(x/log(c*(a + b*x^2)^p)^2, x)
```

3.111 $\int \frac{1}{x\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x\log^2\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x/ln(c*(b*x^2+a)^p)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*Log[c*(a + b*x^2)^p]^2),x]
```

```
[Out] Defer[Int][1/(x*Log[c*(a + b*x^2)^p]^2), x]
```

Rubi steps

$$\int \frac{1}{x\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Mathematica [A]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{1}{x\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*Log[c*(a + b*x^2)^p]^2),x]
```

```
[Out] Integrate[1/(x*Log[c*(a + b*x^2)^p]^2), x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\ln\left(c\left(b\,x^2+a\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(c*(b*x^2+a)^p)^2,x)
```

```
[Out] int(1/x/ln(c*(b*x^2+a)^p)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] -a*integrate(1/(b*p^2*x^3*log(b*x^2 + a) + b*p*x^3*log(c)), x) - 1/2*(b*x^2
 + a)/(b*p^2*x^2*log(b*x^2 + a) + b*p*x^2*log(c))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(x*log((b*x^2 + a)^p*c)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \log \left(c \left(a + b x^{2}\right)^{p}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Integral(1/(x*log(c*(a + b*x**2)**p)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x*log((b*x^2 + a)^p*c)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x \ln \left(c \left(b\, x^2+a\right)^p\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*log(c*(a + b*x^2)^p)^2),x)
[Out] int(1/(x*log(c*(a + b*x^2)^p)^2), x)
```

3.112 $\int \frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)} dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x^3/ln(c*(b*x^2+a)^p)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)} dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^3*Log[c*(a + b*x^2)^p]^2),x]
```

```
[Out] Defer[Int][1/(x^3*Log[c*(a + b*x^2)^p]^2), x]
```

Rubi steps

$$\int \frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)} dx = \int \frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)} dx$$

Mathematica [A]
time = 1.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^3*Log[c*(a + b*x^2)^p]^2),x]
```

```
[Out] Integrate[1/(x^3*Log[c*(a + b*x^2)^p]^2), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \ln\left(c\left(b\,x^2+a\right)^p\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/ln(c*(b*x^2+a)^p)^2,x)
```

```
[Out] int(1/x^3/ln(c*(b*x^2+a)^p)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^2 + a)/(b*p^2*x^4*log(b*x^2 + a) + b*p*x^4*log(c)) - integrate((b
*x^2 + 2*a)/(b*p^2*x^5*log(b*x^2 + a) + b*p*x^5*log(c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(x^3*log((b*x^2 + a)^p*c)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \log \left(c \left(a + bx^2\right)^p\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Integral(1/(x**3*log(c*(a + b*x**2)**p)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^3*log((b*x^2 + a)^p*c)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^3 \ln \left(c \left(b\, x^2+a\right)^p\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*log(c*(a + b*x^2)^p)^2),x)
```

```
[Out] int(1/(x^3*log(c*(a + b*x^2)^p)^2), x)
```

3.113 $\int \frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$

Optimal. Leaf size=21

$$\operatorname{Int}\left(\frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(x^2/ln(c*(b*x^2+a)^p)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2/Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] Defer[Int][x^2/Log[c*(a + b*x^2)^p]^2, x]
```

Rubi steps

$$\int \frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Mathematica [A]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2/Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] Integrate[x^2/Log[c*(a + b*x^2)^p]^2, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\ln\left(c\left(b\,x^2+a\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/ln(c*(b*x^2+a)^p)^2,x)
```

```
[Out] int(x^2/ln(c*(b*x^2+a)^p)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^3 + a*x)/(b*p^2*log(b*x^2 + a) + b*p*log(c)) + integrate(1/2*(3*b
*x^2 + a)/(b*p^2*log(b*x^2 + a) + b*p*log(c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2/log((b*x^2 + a)^p*c)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Integral(x**2/log(c*(a + b*x**2)**p)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/log((b*x^2 + a)^p*c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^2}{\ln\left(c\left(b\,x^2+a\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/log(c*(a + b*x^2)^p)^2,x)
[Out] int(x^2/log(c*(a + b*x^2)^p)^2, x)
```

3.114 $\int \frac{1}{\log^2(c(a+bx^2)^p)}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/ln(c*(b*x^2+a)^p)^2,x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[c*(a + b*x^2)^p]^(-2),x]
```

```
[Out] Defer[Int][Log[c*(a + b*x^2)^p]^(-2), x]
```

Rubi steps

$$\int \frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Mathematica [A]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^(-2),x]
```

```
[Out] Integrate[Log[c*(a + b*x^2)^p]^(-2), x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\ln\left(c\left(b\,x^2+a\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(c*(b*x^2+a)^p)^2,x)
```

```
[Out] int(1/ln(c*(b*x^2+a)^p)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^2 + a)/(b*p^2*x*log(b*x^2 + a) + b*p*x*log(c)) + integrate(1/2*(b
*x^2 - a)/(b*p^2*x^2*log(b*x^2 + a) + b*p*x^2*log(c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^(-2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\log\left(c\left(a+bx^2\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**(-2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^(-2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\ln\left(c\left(b\,x^2+a\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/log(c*(a + b*x^2)^p)^2,x)
[Out] int(1/log(c*(a + b*x^2)^p)^2, x)
```

3.115 $\int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x^2/ln(c*(b*x^2+a)^p)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*Log[c*(a + b*x^2)^p]^2),x]
```

```
[Out] Defer[Int][1/(x^2*Log[c*(a + b*x^2)^p]^2), x]
```

Rubi steps

$$\int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Mathematica [A]
time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*Log[c*(a + b*x^2)^p]^2),x]
```

```
[Out] Integrate[1/(x^2*Log[c*(a + b*x^2)^p]^2), x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \ln\left(c\left(b\,x^2+a\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/ln(c*(b*x^2+a)^p)^2,x)
```

```
[Out] int(1/x^2/ln(c*(b*x^2+a)^p)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^2 + a)/(b*p^2*x^3*log(b*x^2 + a) + b*p*x^3*log(c)) - integrate(1/
2*(b*x^2 + 3*a)/(b*p^2*x^4*log(b*x^2 + a) + b*p*x^4*log(c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(x^2*log((b*x^2 + a)^p*c)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \log{\left(c \left(a + b x^{2}\right)^{p} \right)}^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Integral(1/(x**2*log(c*(a + b*x**2)**p)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*log((b*x^2 + a)^p*c)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^2 \ln\left(c\left(b\,x^2+a\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*log(c*(a + b*x^2)^p)^2),x)
```

```
[Out] int(1/(x^2*log(c*(a + b*x^2)^p)^2), x)
```

3.116 $\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)^p\right)} \, dx$

Optimal. Leaf size=204

$$-\frac{a\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)}{4b^2p^3}+\frac{\left(a+bx^2\right)^2\left(c\left(a+bx^2\right)^p\right)^{-2/p}\text{Ei}\left(\frac{2\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)}{b^2p^3}-\frac{x^2}{4bp\log^2}$$

```
[Out] -1/4*a*(b*x^2+a)*Ei(ln(c*(b*x^2+a)^p)/p)/b^2/p^3/((c*(b*x^2+a)^p)^(1/p))+(b
*x^2+a)^2*Ei(2*ln(c*(b*x^2+a)^p)/p)/b^2/p^3/((c*(b*x^2+a)^p)^(2/p))-1/4*x^2
*(b*x^2+a)/b/p/ln(c*(b*x^2+a)^p)^2-1/4*a*(b*x^2+a)/b^2/p^2/ln(c*(b*x^2+a)^p
)-1/2*x^2*(b*x^2+a)/b/p^2/ln(c*(b*x^2+a)^p)
```

Rubi [A]
time = 0.19, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2504, 2447, 2446, 2436, 2337, 2209, 2437, 2347, 2334}

$$\frac{\left(a+bx^2\right)^2\left(c\left(a+bx^2\right)^p\right)^{-2/p}\text{Ei}\left(\frac{2\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{b^2p^3}-\frac{a\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{4b^2p^3}-\frac{a\left(a+bx^2\right)}{4b^2p^2\log\left(c\left(a+bx^2\right)^p\right)}-\frac{x^2\left(a+bx^2\right)}{2bp^2\log\left(c\left(a+bx^2\right)^p\right)}-\frac{x^2\left(a+bx^2\right)}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] -1/4*(a*(a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(b^2*p^3*(c*(a +
 b*x^2)^p)^p^(-1)) + ((a + b*x^2)^2*ExpIntegralEi[(2*Log[c*(a + b*x^2)^p])/
p])/(b^2*p^3*(c*(a + b*x^2)^p)^(2/p)) - (x^2*(a + b*x^2))/(4*b*p*Log[c*(a +
 b*x^2)^p]^2) - (a*(a + b*x^2))/(4*b^2*p^2*Log[c*(a + b*x^2)^p]) - (x^2*(a
+ b*x^2))/(2*b*p^2*Log[c*(a + b*x^2)^p])
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
```

```
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2447

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[q*((e*f - d*g)/
(b*e*n*(p + 1))), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{\log^3\left(c(a+bx)^p\right)}\,dx, x, x^2\right)$$

$$= -\frac{x^2(a+bx^2)}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{x}{\log^2(c(a+bx)^p)}\,dx, x, x^2\right)}{2p} + \frac{a\text{Subst}\left(\int \frac{1}{\log^2(c(a+b}\right.}{4bp}$$

$$= -\frac{x^2(a+bx^2)}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{x^2(a+bx^2)}{2bp^2\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{x}{\log(c(a+bx)^p)}\,dx, x, x\right.}{p^2}$$

$$= -\frac{x^2(a+bx^2)}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{a(a+bx^2)}{4b^2p^2\log\left(c\left(a+bx^2\right)^p\right)} - \frac{x^2(a+bx^2)}{2bp^2\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Su}}{}$$

$$= -\frac{x^2(a+bx^2)}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{a(a+bx^2)}{4b^2p^2\log\left(c\left(a+bx^2\right)^p\right)} - \frac{x^2(a+bx^2)}{2bp^2\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Su}}{}$$

$$= \frac{3a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)}{4b^2p^3} - \frac{x^2(a+bx^2)}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{}{4b^2p}$$

$$= \frac{3a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)}{4b^2p^3} - \frac{x^2(a+bx^2)}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{}{4b^2p}$$

$$= -\frac{a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)}{4b^2p^3} + \frac{(a+bx^2)^2\left(c(a+bx^2)^p\right)^{-2/p}}{b^2p^3}$$

Mathematica [A]
time = 0.14, size = 185, normalized size = 0.91

$$-\frac{(a+bx^2)\left(c(a+bx^2)^p\right)^{-2/p}\left(a\left(c(a+bx^2)^p\right)^{\frac{1}{p}}\text{Ei}\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)\log^2\left(c(a+bx^2)^p\right) - 4(a+bx^2)\text{Ei}\left(\frac{2\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)\log^2\left(c(a+bx^2)^p\right) + p\left(c(a+bx^2)^p\right)^{2/p}\left(bpx^2+(a+2bx^2)\log\left(c(a+bx^2)^p\right)\right)\right)}{4b^2p^3\log^2\left(c\left(a+bx^2\right)^p\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] -1/4*((a + b*x^2)*(a*(c*(a + b*x^2)^p)^p^(-1)*ExpIntegralEi[Log[c*(a + b*x^
2)^p]/p]*Log[c*(a + b*x^2)^p]^2 - 4*(a + b*x^2)*ExpIntegralEi[(2*Log[c*(a +
 b*x^2)^p])/p]*Log[c*(a + b*x^2)^p]^2 + p*(c*(a + b*x^2)^p)^(2/p)*(b*p*x^2
+ (a + 2*b*x^2)*Log[c*(a + b*x^2)^p])))/(b^2*p^3*(c*(a + b*x^2)^p)^(2/p)*Lo
g[c*(a + b*x^2)^p]^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.76, size = 1969, normalized size = 9.65

method	result	size
risch	Expression too large to display	1969

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/ln(c*(b*x^2+a)^p)^3,x,method=_RETURNVERBOSE)

[Out] -1/2*(2*b^2*p*x^4+2*a*b*p*x^2+3*I*Pi*a*b*x^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-3*I*Pi*a*b*x^2*csgn(I*c*(b*x^2+a)^p)^3-I*Pi*a^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*a^2*csgn(I*c*(b*x^2+a)^p)^3-2*I*Pi*b^2*x^4*csgn(I*c*(b*x^2+a)^p)^3+3*I*Pi*a*b*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+2*I*Pi*b^2*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+I*Pi*a^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+4*ln(c)*b^2*x^4-3*I*Pi*a*b*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+I*Pi*a^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*I*Pi*b^2*x^4*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-2*I*Pi*b^2*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+4*b^2*x^4*ln((b*x^2+a)^p)+6*ln(c)*a*b*x^2+6*a*b*x^2*ln((b*x^2+a)^p)+2*ln(c)*a^2+2*a^2*ln((b*x^2+a)^p))/(2*ln(c)+2*ln((b*x^2+a)^p)+I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c))^2/p^2/b^2-1/p^3*((b*x^2+a)^p)^(-2/p)*c^(-2/p)*exp(I*Pi*csgn(I*c*(b*x^2+a)^p)*(csgn(I*(b*x^2+a)^p)-csgn(I*c*(b*x^2+a)^p))*(csgn(I*c)-csgn(I*c*(b*x^2+a)^p))/p)*Ei(1,-2*ln(b*x^2+a)-(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+2*ln((b*x^2+a)^p)-2*p*ln(b*x^2+a))/p)*x^4-2/b/p^3*((b*x^2+a)^p)^(-2/p)*c^(-2/p)*exp(I*Pi*csgn(I*c*(b*x^2+a)^p)*(csgn(I*(b*x^2+a)^p)-csgn(I*c*(b*x^2+a)^p))*(csgn(I*c)-csgn(I*c*(b*x^2+a)^p))/p)*Ei(1,-2*ln(b*x^2+a)-(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+2*ln((b*x^2+a)^p)-2*p*ln(b*x^2+a))/p)*a*x^2-1/b^2/p^3*((b*x^2+a)^p)^(-2/p)*c^(-2/p)*exp(I*Pi*csgn(I*c*(b*x^2+a)^p)*(csgn(I*(b*x^2+a)^p)-csgn(I*c*(b*x^2+a)^p))*(csgn(I*c)-csgn(I*c*(b*x^2+a)^p))/p)*Ei(1,-2*ln(b*x^2+a)-(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+2*ln((b*x^2+a)^p)-2*p*ln(b*x^2+a))/p)*a^2+1/4/b/p^3*a*((b*x^2+a)^p)^(-1/p)*c^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)*(csgn(I*(b*x^2+a)^p)-csgn(I*c*(b*x^2+a)^p))*(csgn(I*c)-csgn(I*c*(b*x^2+a)^p))/p)*Ei(1,-ln(b*x^2+a)-1/2*(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+2*ln((b*x^2+a)^p)-2*p*ln(b*x^2+a))/p)*x^2+1/4/b^2/p^3*a^2*((b*x^2+a)^p)^(-1/p)*c^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)*(csgn(I*(b*x^2+a)^p)-csgn(I*c*(b*x^2+a)^p))*(csgn(I*c)-csgn(I*c*(b*x^2+a)^p))/p)*Ei(1,-ln(b*x^2+a)-1/2*(I*Pi*csgn(I*(b*x^2+

```
a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p
)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(
I*c)+2*ln(c)+2*ln((b*x^2+a)^p)-2*p*ln(b*x^2+a))/p)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*(p + 2*log(c))*x^4 + a*b*(p + 3*log(c))*x^2 + a^2*log(c) + (2*b^2
*p*x^4 + 3*a*b*p*x^2 + a^2*p)*log(b*x^2 + a))/(b^2*p^4*log(b*x^2 + a)^2 + 2
*b^2*p^3*log(b*x^2 + a)*log(c) + b^2*p^2*log(c)^2) + integrate(1/2*(4*b*x^3
 + 3*a*x)/(b*p^3*log(b*x^2 + a) + b*p^2*log(c)), x)
```

Fricas [A]
time = 0.36, size = 270, normalized size = 1.32

$$-\frac{\left(ap^2\log\left(bx^2+a\right)^2+2\,ap\log\left(bx^2+a\right)\log\left(c\right)+a\log\left(c\right)^2\right)c^{\left(\frac{1}{p}\right)}\log\_integral\left(\left(bx^2+a\right)c^{\left(\frac{1}{p}\right)}\right)+\left(b^2p^2x^4+abp^2x^2+\left(2\,b^2p^2x^4+3\,abp^2x^2+a^2p^2\right)\log\left(bx^2+a\right)+\left(2\,b^2px^4+3\,abpx^2+a^2p\right)\log\left(c\right)\right)c^{\frac{2}{p}}-4\left(p^2\log\left(bx^2+a\right)^2+2\,p\log\left(bx^2+a\right)\log\left(c\right)+\log\left(c\right)^2\right)\log\_integral\left(\left(b^2x^4+2\,abx^2+a^2\right)c^{\frac{2}{p}}\right)}{4\left(b^2p^5\log\left(bx^2+a\right)^2+2\,b^2p^4\log\left(bx^2+a\right)\log\left(c\right)+b^2p^3\log\left(c\right)^2\right)c^{\frac{2}{p}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] -1/4*((a*p^2*log(b*x^2 + a)^2 + 2*a*p*log(b*x^2 + a)*log(c) + a*log(c)^2)*c
^(1/p)*log_integral((b*x^2 + a)*c^(1/p)) + (b^2*p^2*x^4 + a*b*p^2*x^2 + (2*
b^2*p^2*x^4 + 3*a*b*p^2*x^2 + a^2*p^2)*log(b*x^2 + a) + (2*b^2*p*x^4 + 3*a*
b*p*x^2 + a^2*p)*log(c))*c^(2/p) - 4*(p^2*log(b*x^2 + a)^2 + 2*p*log(b*x^2
+ a)*log(c) + log(c)^2)*log_integral((b^2*x^4 + 2*a*b*x^2 + a^2)*c^(2/p)))/
((b^2*p^5*log(b*x^2 + a)^2 + 2*b^2*p^4*log(b*x^2 + a)*log(c) + b^2*p^3*log(
c)^2)*c^(2/p))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\log\left(c\left(a+bx^2\right)^p\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Integral(x**3/log(c*(a + b*x**2)**p)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 874 vs. 2(198) = 396.
time = 2.63, size = 874, normalized size = 4.28

$$\frac{1}{4}\left(\frac{\left(bx^2+a\right)p^2\log\left(bx^2+a\right)}{b^2p^5\log\left(bx^2+a\right)^2+2\,b^2p^4\log\left(bx^2+a\right)\log\left(c\right)+b^2p^3\log\left(c\right)^2}-\frac{p^2{\rm Ei}\left(\frac{\log\left(c\right)}{p}+\log\left(bx^2+a\right)\right)\log\left(bx^2+a\right)^2}{\left(b^2p^5\log\left(bx^2+a\right)^2+2\,b^2p^4\log\left(bx^2+a\right)\log\left(c\right)+b^2p^3\log\left(c\right)^2\right)c^{\left(\frac{1}{p}\right)}}+\frac{\left(bx^2+a\right)p^2}{b^2p^5\log\left(bx^2+a\right)^2+2\,b^2p^4\log\left(bx^2+a\right)\log\left(c\right)+b^2p^3\log\left(c\right)^2}+\frac{\left(bx^2+a\right)p\log\left(c\right)}{b^2p^5\log\left(bx^2+a\right)^2+2\,b^2p^4\log\left(bx^2+a\right)\log\left(c\right)+b^2p^3\log\left(c\right)^2}-\frac{2\,p{\rm Ei}\left(\frac{\log\left(c\right)}{p}+\log\left(bx^2+a\right)\right)\log\left(bx^2+a\right)\log\left(c\right)}{\left(b^2p^5\log\left(bx^2+a\right)^2+2\,b^2p^4\log\left(bx^2+a\right)\log\left(c\right)+b^2p^3\log\left(c\right)^2\right)c^{\left(\frac{1}{p}\right)}}-\frac{{\rm Ei}\left(\frac{\log\left(c\right)}{p}+\log\left(bx^2+a\right)\right)\log\left(c\right)^2}{\left(b^2p^5\log\left(bx^2+a\right)^2+2\,b^2p^4\log\left(bx^2+a\right)\log\left(c\right)+b^2p^3\log\left(c\right)^2\right)c^{\left(\frac{1}{p}\right)}}\right)a-\frac{\frac{2\left(bx^2+a\right)^2p^2\log\left(bx^2+a\right)}{bp^5\log\left(bx^2+a\right)^2+2\,bp^4\log\left(bx^2+a\right)\log\left(c\right)+bp^3\log\left(c\right)^2}+\frac{\left(bx^2+a\right)^2p^2}{bp^5\log\left(bx^2+a\right)^2+2\,bp^4\log\left(bx^2+a\right)\log\left(c\right)+bp^3\log\left(c\right)^2}-\frac{4\,p^2{\rm Ei}\left(\frac{2\log\left(c\right)}{p}+2\,\log\left(bx^2+a\right)\right)\log\left(bx^2+a\right)^2}{\left(bp^5\log\left(bx^2+a\right)^2+2\,bp^4\log\left(bx^2+a\right)\log\left(c\right)+bp^3\log\left(c\right)^2\right)c^{\frac{2}{p}}}+\frac{2\left(bx^2+a\right)^2p\log\left(c\right)}{bp^5\log\left(bx^2+a\right)^2+2\,bp^4\log\left(bx^2+a\right)\log\left(c\right)+bp^3\log\left(c\right)^2}-\frac{8\,p{\rm Ei}\left(\frac{2\log\left(c\right)}{p}+2\,\log\left(bx^2+a\right)\right)\log\left(bx^2+a\right)\log\left(c\right)}{\left(bp^5\log\left(bx^2+a\right)^2+2\,bp^4\log\left(bx^2+a\right)\log\left(c\right)+bp^3\log\left(c\right)^2\right)c^{\frac{2}{p}}}-\frac{4\,{\rm Ei}\left(\frac{2\log\left(c\right)}{p}+2\,\log\left(bx^2+a\right)\right)\log\left(c\right)^2}{\left(bp^5\log\left(bx^2+a\right)^2+2\,bp^4\log\left(bx^2+a\right)\log\left(c\right)+bp^3\log\left(c\right)^2\right)c^{\frac{2}{p}}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] 1/4*((b*x^2 + a)*p^2*log(b*x^2 + a)/(b^2*p^5*log(b*x^2 + a)^2 + 2*b^2*p^4*l
og(b*x^2 + a)*log(c) + b^2*p^3*log(c)^2) - p^2*Ei(log(c)/p + log(b*x^2 + a)
)*log(b*x^2 + a)^2/((b^2*p^5*log(b*x^2 + a)^2 + 2*b^2*p^4*log(b*x^2 + a)*lo
g(c) + b^2*p^3*log(c)^2)*c^(1/p)) + (b*x^2 + a)*p^2/(b^2*p^5*log(b*x^2 + a)
^2 + 2*b^2*p^4*log(b*x^2 + a)*log(c) + b^2*p^3*log(c)^2) + (b*x^2 + a)*p*lo
g(c)/(b^2*p^5*log(b*x^2 + a)^2 + 2*b^2*p^4*log(b*x^2 + a)*log(c) + b^2*p^3*
log(c)^2) - 2*p*Ei(log(c)/p + log(b*x^2 + a))*log(b*x^2 + a)*log(c)/((b^2*p
^5*log(b*x^2 + a)^2 + 2*b^2*p^4*log(b*x^2 + a)*log(c) + b^2*p^3*log(c)^2)*c
^(1/p)) - Ei(log(c)/p + log(b*x^2 + a))*log(c)^2/((b^2*p^5*log(b*x^2 + a)^2
 + 2*b^2*p^4*log(b*x^2 + a)*log(c) + b^2*p^3*log(c)^2)*c^(1/p)))*a - 1/4*(2
*(b*x^2 + a)^2*p^2*log(b*x^2 + a)/(b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(b*x
^2 + a)*log(c) + b*p^3*log(c)^2) + (b*x^2 + a)^2*p^2/(b*p^5*log(b*x^2 + a)^
2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2) - 4*p^2*Ei(2*log(c)/p +
 2*log(b*x^2 + a))*log(b*x^2 + a)^2/((b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(
b*x^2 + a)*log(c) + b*p^3*log(c)^2)*c^(2/p)) + 2*(b*x^2 + a)^2*p*log(c)/(b*
p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2) - 8*
p*Ei(2*log(c)/p + 2*log(b*x^2 + a))*log(b*x^2 + a)*log(c)/((b*p^5*log(b*x^2
 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2)*c^(2/p)) - 4*Ei(2
*log(c)/p + 2*log(b*x^2 + a))*log(c)^2/((b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*l
og(b*x^2 + a)*log(c) + b*p^3*log(c)^2)*c^(2/p)))/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\ln\left(c\left(b\,x^2+a\right)^p\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/log(c*(a + b*x^2)^p)^3,x)
```

```
[Out] int(x^3/log(c*(a + b*x^2)^p)^3, x)
```

3.117 $\int \frac{x}{\log^3\left(c\left(a+bx^2\right)^p\right)} \, dx$

Optimal. Leaf size=114

$$\frac{(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log(c(a+bx^2)^p)}{p}\right)}{4bp^3}-\frac{a+bx^2}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)}-\frac{a+bx^2}{4bp^2\log\left(c\left(a+bx^2\right)^p\right)}$$

```
[Out] 1/4*(b*x^2+a)*Ei(ln(c*(b*x^2+a)^p)/p)/b/p^3/((c*(b*x^2+a)^p)^(1/p))+1/4*(-b
*x^2-a)/b/p/ln(c*(b*x^2+a)^p)^2+1/4*(-b*x^2-a)/b/p^2/ln(c*(b*x^2+a)^p)
```

Rubi [A]
time = 0.06, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2504, 2436, 2334, 2337, 2209}

$$\frac{(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log(c(bx^2+a)^p)}{p}\right)}{4bp^3}-\frac{a+bx^2}{4bp^2\log\left(c\left(a+bx^2\right)^p\right)}-\frac{a+bx^2}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] ((a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(4*b*p^3*(c*(a + b*x^2)
^p)^p^(-1)) - (a + b*x^2)/(4*b*p*Log[c*(a + b*x^2)^p]^2) - (a + b*x^2)/(4*b
*p^2*Log[c*(a + b*x^2)^p])
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{x}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{\log^3\left(c(a+bx)^p\right)}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\log^3(cx^p)}\,dx, x, a+bx^2\right)}{2b}$$

$$= -\frac{a+bx^2}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{1}{\log^2(cx^p)}\,dx, x, a+bx^2\right)}{4bp}$$

$$= -\frac{a+bx^2}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{a+bx^2}{4bp^2\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx, x, a+bx^2\right.}{4bp^2}$$

$$= -\frac{a+bx^2}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{a+bx^2}{4bp^2\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\left(\left(a+bx^2\right)\left(c(a+bx^2)^p\right)^{-1/p}\right.}{}$$

$$= \frac{\left(a+bx^2\right)\left(c(a+bx^2)^p\right)^{-1/p}\text{Ei}\left(\frac{\log(c(a+bx^2)^p)}{p}\right)}{4bp^3} - \frac{a+bx^2}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{a}{4bp^2\log}$$

Mathematica [A]
time = 0.04, size = 113, normalized size = 0.99

$$-\frac{\left(a+bx^2\right)\left(c(a+bx^2)^p\right)^{-1/p}\left(-\text{Ei}\left(\frac{\log(c(a+bx^2)^p)}{p}\right)\log^2\left(c(a+bx^2)^p\right)+p\left(c(a+bx^2)^p\right)^{\frac{1}{p}}\left(p+\log\left(c(a+bx^2)^p\right)\right)\right)}{4bp^3\log^2\left(c\left(a+bx^2\right)^p\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] -1/4*((a + b*x^2)*(-(ExpIntegralEi[Log[c*(a + b*x^2)^p]/p]*Log[c*(a + b*x^2
)^p]^2) + p*(c*(a + b*x^2)^p)^p^(-1)*(p + Log[c*(a + b*x^2)^p])))/(b*p^3*(c
*(a + b*x^2)^p)^p^(-1)*Log[c*(a + b*x^2)^p]^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.25, size = 716, normalized size = 6.28

method	result
risch	$-\frac{i\pi b\,x^{2}\operatorname{csgn}\left(i\left(b\,x^{2}+a\right)^{p}\right)\operatorname{csgn}\left(ic\left(b\,x^{2}+a\right)^{p}\right)^{2}-i\pi b\,x^{2}\operatorname{csgn}\left(i\left(b\,x^{2}+a\right)^{p}\right)\operatorname{csgn}\left(ic\left(b\,x^{2}+a\right)^{p}\right)\operatorname{csgn}\left(ic\right)-i\pi b\,x^{2}\operatorname{csgn}\left(ic\left(b\,x^{2}+a\right)^{p}\right)^{3}+i\pi b}{2p^{2}\left(2\ln\left(c\right)+2\ln\left(\left(b\,x^{2}\right.\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/ln(c*(b*x^2+a)^p)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(I*Pi*b*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*b*x^2*csg
n(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*b*x^2*csgn(I*c*(b*x^2
+a)^p)^3+I*Pi*b*x^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+I*Pi*a*csgn(I*(b*x^2+
a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*a*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)
^p)*csgn(I*c)-I*Pi*a*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*a*csgn(I*c*(b*x^2+a)^p)^2
*csgn(I*c)+2*ln(c)*b*x^2+2*b*x^2*ln((b*x^2+a)^p)+2*ln(c)*a+2*a*ln((b*x^2+a)
^p)+2*x^2*p*b+2*a*p)/p^2/(2*ln(c)+2*ln((b*x^2+a)^p)+I*Pi*csgn(I*(b*x^2+a)^p
)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*cs
gn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)
)^2/b-1/4/p^3/b*(b*x^2+a)*c^(-1/p)*((b*x^2+a)^p)^(-1/p)*exp(1/2*I*Pi*csgn(I
*c*(b*x^2+a)^p)*(csgn(I*(b*x^2+a)^p)-csgn(I*c*(b*x^2+a)^p))*(csgn(I*c)-csgn
(I*c*(b*x^2+a)^p))/p)*Ei(1,-ln(b*x^2+a)-1/2*(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(
I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)
-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c
)+2*ln((b*x^2+a)^p)-2*p*ln(b*x^2+a))/p)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b*(p + log(c))*x^2 + a*(p + log(c)) + (b*p*x^2 + a*p)*log(b*x^2 + a))
/(b*p^4*log(b*x^2 + a)^2 + 2*b*p^3*log(b*x^2 + a)*log(c) + b*p^2*log(c)^2)
+ integrate(1/2*x/(p^3*log(b*x^2 + a) + p^2*log(c)), x)
```

Fricas [A]
time = 0.36, size = 157, normalized size = 1.38

$$-\frac{(bp^{2}x^{2}+ap^{2}+(bp^{2}x^{2}+ap^{2})\log\left(bx^{2}+a\right)+(bpx^{2}+ap)\log\left(c\right))c^{\left(\frac{1}{p}\right)}-\left(p^{2}\log\left(bx^{2}+a\right)^{2}+2\,p\log\left(bx^{2}+a\right)\log\left(c\right)+\log\left(c\right)^{2}\right)\operatorname{log\_integral}\left((bx^{2}+a)c^{\left(\frac{1}{p}\right)}\right)}{4\left(bp^{5}\log\left(bx^{2}+a\right)^{2}+2\,bp^{4}\log\left(bx^{2}+a\right)\log\left(c\right)+bp^{3}\log\left(c\right)^{2}\right)c^{\left(\frac{1}{p}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] -1/4*((b*p^2*x^2 + a*p^2 + (b*p^2*x^2 + a*p^2)*log(b*x^2 + a) + (b*p*x^2 +
a*p)*log(c))*c^(1/p) - (p^2*log(b*x^2 + a)^2 + 2*p*log(b*x^2 + a)*log(c) +
log(c)^2)*log_integral((b*x^2 + a)*c^(1/p)))/((b*p^5*log(b*x^2 + a)^2 + 2*b
*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2)*c^(1/p))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\log\left(c\left(a+bx^2\right)^p\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Integral(x/log(c*(a + b*x**2)**p)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(108) = 216.
time = 4.93, size = 406, normalized size = 3.56

$$-\frac{(bx^2+a)p^2\log\left(bx^2+a\right)}{4\left(bp^5\log\left(bx^2+a\right)^2+2\,bp^4\log\left(bx^2+a\right)\log\left(c\right)+bp^3\log\left(c\right)^2\right)}+\frac{p^2\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2+a\right)\right)\log\left(bx^2+a\right)^2}{4\left(bp^5\log\left(bx^2+a\right)^2+2\,bp^4\log\left(bx^2+a\right)\log\left(c\right)+bp^3\log\left(c\right)^2\right)c^{\left(\frac{1}{p}\right)}}-\frac{(bx^2+a)p^2}{4\left(bp^5\log\left(bx^2+a\right)^2+2\,bp^4\log\left(bx^2+a\right)\log\left(c\right)+bp^3\log\left(c\right)^2\right)}-\frac{(bx^2+a)p\log\left(c\right)}{4\left(bp^5\log\left(bx^2+a\right)^2+2\,bp^4\log\left(bx^2+a\right)\log\left(c\right)+bp^3\log\left(c\right)^2\right)}+\frac{p\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2+a\right)\right)\log\left(bx^2+a\right)\log\left(c\right)}{2\left(bp^5\log\left(bx^2+a\right)^2+2\,bp^4\log\left(bx^2+a\right)\log\left(c\right)+bp^3\log\left(c\right)^2\right)c^{\left(\frac{1}{p}\right)}}+\frac{\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2+a\right)\right)\log\left(c\right)^2}{4\left(bp^5\log\left(bx^2+a\right)^2+2\,bp^4\log\left(bx^2+a\right)\log\left(c\right)+bp^3\log\left(c\right)^2\right)c^{\left(\frac{1}{p}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] -1/4*(b*x^2 + a)*p^2*log(b*x^2 + a)/(b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(b
*x^2 + a)*log(c) + b*p^3*log(c)^2) + 1/4*p^2*Ei(log(c)/p + log(b*x^2 + a))*
log(b*x^2 + a)^2/((b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) +
 b*p^3*log(c)^2)*c^(1/p)) - 1/4*(b*x^2 + a)*p^2/(b*p^5*log(b*x^2 + a)^2 + 2
*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2) - 1/4*(b*x^2 + a)*p*log(c)/(
b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2) +
1/2*p*Ei(log(c)/p + log(b*x^2 + a))*log(b*x^2 + a)*log(c)/((b*p^5*log(b*x^2
 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2)*c^(1/p)) + 1/4*Ei
(log(c)/p + log(b*x^2 + a))*log(c)^2/((b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log
(b*x^2 + a)*log(c) + b*p^3*log(c)^2)*c^(1/p))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/log(c*(a + b*x^2)^p)^3,x)
```

```
[Out] int(x/log(c*(a + b*x^2)^p)^3, x)
```

3.118 $\int \frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x/ln(c*(b*x^2+a)^p)^3,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*Log[c*(a + b*x^2)^p]^3),x]
```

```
[Out] Defer[Int][1/(x*Log[c*(a + b*x^2)^p]^3), x]
```

Rubi steps

$$\int \frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Mathematica [A]
time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*Log[c*(a + b*x^2)^p]^3),x]
```

```
[Out] Integrate[1/(x*Log[c*(a + b*x^2)^p]^3), x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{x\ln\left(c\left(b\,x^2+a\right)^p\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(c*(b*x^2+a)^p)^3,x)
```

```
[Out] int(1/x/ln(c*(b*x^2+a)^p)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*p*x^4 + a*b*(p - log(c))*x^2 - a^2*log(c) - (a*b*p*x^2 + a^2*p)*l
og(b*x^2 + a))/(b^2*p^4*x^4*log(b*x^2 + a)^2 + 2*b^2*p^3*x^4*log(b*x^2 + a)
*log(c) + b^2*p^2*x^4*log(c)^2) + integrate(1/2*(a*b*x^2 + 2*a^2)/(b^2*p^3*
x^5*log(b*x^2 + a) + b^2*p^2*x^5*log(c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] integral(1/(x*log((b*x^2 + a)^p*c)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \log{\left(c \left(a + b x^{2}\right)^{p} \right)}^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Integral(1/(x*log(c*(a + b*x**2)**p)**3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x*log((b*x^2 + a)^p*c)^3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x \ln \left(c \left(b\, x^2+a\right)^p\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*log(c*(a + b*x^2)^p)^3),x)
[Out] int(1/(x*log(c*(a + b*x^2)^p)^3), x)
```

3.119 $\int \frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)} \, dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x^3/ln(c*(b*x^2+a)^p)^3,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^3*Log[c*(a + b*x^2)^p]^3),x]
[Out] Defer[Int][1/(x^3*Log[c*(a + b*x^2)^p]^3), x]
```

Rubi steps

$$\int \frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)} \, dx$$

Mathematica [A]
time = 1.98, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^3*Log[c*(a + b*x^2)^p]^3),x]
[Out] Integrate[1/(x^3*Log[c*(a + b*x^2)^p]^3), x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \ln\left(c\left(b\,x^2+a\right)^p\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/ln(c*(b*x^2+a)^p)^3,x)
```

```
[Out] int(1/x^3/ln(c*(b*x^2+a)^p)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*(p - log(c))*x^4 + a*b*(p - 3*log(c))*x^2 - 2*a^2*log(c) - (b^2*p
*x^4 + 3*a*b*p*x^2 + 2*a^2*p)*log(b*x^2 + a))/(b^2*p^4*x^6*log(b*x^2 + a)^2
 + 2*b^2*p^3*x^6*log(b*x^2 + a)*log(c) + b^2*p^2*x^6*log(c)^2) + integrate(
1/2*(b^2*x^4 + 6*a*b*x^2 + 6*a^2)/(b^2*p^3*x^7*log(b*x^2 + a) + b^2*p^2*x^7
*log(c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] integral(1/(x^3*log((b*x^2 + a)^p*c)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \log \left(c \left(a + bx^2\right)^p\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Integral(1/(x**3*log(c*(a + b*x**2)**p)**3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x^3*log((b*x^2 + a)^p*c)^3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^3 \ln\left(c\left(b\,x^2+a\right)^p\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*log(c*(a + b*x^2)^p)^3),x)
[Out] int(1/(x^3*log(c*(a + b*x^2)^p)^3), x)
```

3.120 $\int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)} dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)},x\right)$$

[Out] Unintegrable(x^2/ln(c*(b*x^2+a)^p)^3,x)

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)} dx$$

Verification is not applicable to the result.

[In] Int[x^2/Log[c*(a + b*x^2)^p]^3,x]

[Out] Defer[Int][x^2/Log[c*(a + b*x^2)^p]^3, x]

Rubi steps

$$\int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)} dx=\int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)} dx$$

Mathematica [A]
time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)} dx$$

Verification is not applicable to the result.

[In] Integrate[x^2/Log[c*(a + b*x^2)^p]^3,x]

[Out] Integrate[x^2/Log[c*(a + b*x^2)^p]^3, x]

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\ln\left(c\left(b\,x^2+a\right)^p\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/ln(c*(b*x^2+a)^p)^3,x)
```

```
[Out] int(x^2/ln(c*(b*x^2+a)^p)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(b^2*(2*p + 3*log(c))*x^4 + 2*a*b*(p + 2*log(c))*x^2 + a^2*log(c) + (3
*b^2*p*x^4 + 4*a*b*p*x^2 + a^2*p)*log(b*x^2 + a))/(b^2*p^4*x*log(b*x^2 + a)
^2 + 2*b^2*p^3*x*log(b*x^2 + a)*log(c) + b^2*p^2*x*log(c)^2) + integrate(1/
8*(9*b^2*x^4 + 4*a*b*x^2 - a^2)/(b^2*p^3*x^2*log(b*x^2 + a) + b^2*p^2*x^2*l
og(c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] integral(x^2/log((b*x^2 + a)^p*c)^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Integral(x**2/log(c*(a + b*x**2)**p)**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2/log((b*x^2 + a)^p*c)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^2}{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/log(c*(a + b*x^2)^p)^3,x)
[Out] int(x^2/log(c*(a + b*x^2)^p)^3, x)
```

3.121 $\int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)} \, dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/ln(c*(b*x^2+a)^p)^3,x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[Log[c*(a + b*x^2)^p]^(-3),x]
```

```
[Out] Defer[Int][Log[c*(a + b*x^2)^p]^(-3), x]
```

Rubi steps

$$\int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)} \, dx$$

Mathematica [A]
time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[c*(a + b*x^2)^p]^(-3),x]
```

```
[Out] Integrate[Log[c*(a + b*x^2)^p]^(-3), x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\ln\left(c\left(b\,x^2+a\right)^p\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(c*(b*x^2+a)^p)^3,x)
```

```
[Out] int(1/ln(c*(b*x^2+a)^p)^3,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(b^2*(2*p + log(c))*x^4 + 2*a*b*p*x^2 - a^2*log(c) + (b^2*p*x^4 - a^2*
p)*log(b*x^2 + a))/(b^2*p^4*x^3*log(b*x^2 + a)^2 + 2*b^2*p^3*x^3*log(b*x^2
+ a)*log(c) + b^2*p^2*x^3*log(c)^2) + integrate(1/8*(b^2*x^4 + 3*a^2)/(b^2*
p^3*x^4*log(b*x^2 + a) + b^2*p^2*x^4*log(c)), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^(-3), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\log \left(c \left(a + bx^{2}\right)^{p}\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**(-3), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^(-3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\ln \left(c \left(b\, x^2 + a\right)^p\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/log(c*(a + b*x^2)^p)^3,x)
```

```
[Out] int(1/log(c*(a + b*x^2)^p)^3, x)
```

3.122 $\int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)} dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)}, x\right)$$

```
[Out] Unintegrable(1/x^2/ln(c*(b*x^2+a)^p)^3,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)} dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*Log[c*(a + b*x^2)^p]^3),x]
```

```
[Out] Defer[Int][1/(x^2*Log[c*(a + b*x^2)^p]^3), x]
```

Rubi steps

$$\int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)} dx = \int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)} dx$$

Mathematica [A]
time = 1.41, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*Log[c*(a + b*x^2)^p]^3),x]
```

```
[Out] Integrate[1/(x^2*Log[c*(a + b*x^2)^p]^3), x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \ln\left(c\left(b\,x^2+a\right)^p\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/ln(c*(b*x^2+a)^p)^3,x)
```

```
[Out] int(1/x^2/ln(c*(b*x^2+a)^p)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(b^2*(2*p - log(c))*x^4 + 2*a*b*(p - 2*log(c))*x^2 - 3*a^2*log(c) - (b
^2*p*x^4 + 4*a*b*p*x^2 + 3*a^2*p)*log(b*x^2 + a))/(b^2*p^4*x^5*log(b*x^2 +
a)^2 + 2*b^2*p^3*x^5*log(b*x^2 + a)*log(c) + b^2*p^2*x^5*log(c)^2) + integr
ate(1/8*(b^2*x^4 + 12*a*b*x^2 + 15*a^2)/(b^2*p^3*x^6*log(b*x^2 + a) + b^2*p
^2*x^6*log(c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] integral(1/(x^2*log((b*x^2 + a)^p*c)^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \log\left(c\left(a+bx^{2}\right)^{p}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Integral(1/(x**2*log(c*(a + b*x**2)**p)**3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*log((b*x^2 + a)^p*c)^3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^2 \ln\left(c\left(b\,x^2+a\right)^p\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*log(c*(a + b*x^2)^p)^3),x)
[Out] int(1/(x^2*log(c*(a + b*x^2)^p)^3), x)
```

3.123 $\int \frac{x^3}{\log(c(a+bx^2))}\,dx$

Optimal. Leaf size=45

$$\frac{\text{Ei}(2\log(c(a+bx^2)))}{2b^2c^2}-\frac{a\text{li}(c(a+bx^2))}{2b^2c}$$

```
[Out] 1/2*Ei(2*ln(c*(b*x^2+a)))/b^2/c^2-1/2*a*Li(c*(b*x^2+a))/b^2/c
```

Rubi [A]

time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {2504, 2446, 2436, 2335, 2437, 2346, 2209}

$$\frac{\text{Ei}(2\log(c(bx^2+a)))}{2b^2c^2}-\frac{a\text{li}(c(bx^2+a))}{2b^2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Log[c*(a + b*x^2)],x]
```

```
[Out] ExpIntegralEi[2*Log[c*(a + b*x^2)]]/(2*b^2*c^2) - (a*LogIntegral[c*(a + b*x
^2)])/(2*b^2*c)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\log \left(c \left(a+b x^2\right)\right)} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\log(c(a+bx))} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a}{b\log(c(a+bx))}+\frac{a+bx}{b\log(c(a+bx))}\right) \, dx, x, x^2\right) \\
&= \frac{\text{Subst}\left(\int \frac{a+bx}{\log(c(a+bx))} \, dx, x, x^2\right)}{2b}-\frac{a\text{Subst}\left(\int \frac{1}{\log(c(a+bx))} \, dx, x, x^2\right)}{2b} \\
&= \frac{\text{Subst}\left(\int \frac{x}{\log(cx)} \, dx, x, a+bx^2\right)}{2b^2}-\frac{a\text{Subst}\left(\int \frac{1}{\log(cx)} \, dx, x, a+bx^2\right)}{2b^2} \\
&= -\frac{a\text{li}(c(a+bx^2))}{2b^2c}+\frac{\text{Subst}\left(\int \frac{e^{2x}}{x} \, dx, x, \log\left(c(a+bx^2)\right)\right)}{2b^2c^2} \\
&= \frac{\text{Ei}(2\log\left(c(a+bx^2)\right))}{2b^2c^2}-\frac{a\text{li}(c(a+bx^2))}{2b^2c}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 41, normalized size = 0.91

$$\frac{-ac\text{Ei}(\log\left(ac+bcx^2\right))+\text{Ei}(2\log\left(ac+bcx^2\right))}{2b^2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Log[c*(a + b*x^2)],x]
```

```
[Out] (-(a*c*ExpIntegralEi[Log[a*c + b*c*x^2]]) + ExpIntegralEi[2*Log[a*c + b*c*x
^2]])/(2*b^2*c^2)
```

Maple [A]
time = 1.71, size = 43, normalized size = 0.96

method	result	size
default	$\frac{-\operatorname{expIntegral}\left(1,-2\ln\left(c\left(b\,x^{2}+a\right)\right)\right)+ca\operatorname{expIntegral}\left(1,-\ln\left(c\left(b\,x^{2}+a\right)\right)\right)}{2c^{2}b^{2}}$	43
risch	$\frac{a\operatorname{expIntegral}\left(1,-\ln\left(c\left(b\,x^{2}+a\right)\right)\right)}{2c\,b^{2}}-\frac{\operatorname{expIntegral}\left(1,-2\ln\left(c\left(b\,x^{2}+a\right)\right)\right)}{2c^{2}b^{2}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/ln(c*(b*x^2+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c^2/b^2*(-Ei(1,-2*ln(c*(b*x^2+a)))+c*a*Ei(1,-ln(c*(b*x^2+a))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)),x, algorithm="maxima")
```

```
[Out] integrate(x^3/log((b*x^2 + a)*c), x)
```

Fricas [A]
time = 0.36, size = 54, normalized size = 1.20

$$-\frac{ac\operatorname{log\_integral}\left(bcx^{2}+ac\right)-\operatorname{log\_integral}\left(b^{2}c^{2}x^{4}+2\,abc^{2}x^{2}+a^{2}c^{2}\right)}{2\,b^{2}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*c*log_integral(b*c*x^2 + a*c) - log_integral(b^2*c^2*x^4 + 2*a*b*c^
2*x^2 + a^2*c^2))/(b^2*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{3}}{\log\left(ac+bcx^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/ln(c*(b*x**2+a)),x)
```

```
[Out] Integral(x**3/log(a*c + b*c*x**2), x)
```

Giac [A]
time = 3.82, size = 44, normalized size = 0.98

$$-\frac{a\mathrm{Ei}(\log{(bcx^2+ac)})}{2\,b^2c}+\frac{\mathrm{Ei}(2\,\log{(bcx^2+ac)})}{2\,b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a)),x, algorithm="giac")
```

```
[Out] -1/2*a*Ei(log(b*c*x^2 + a*c))/(b^2*c) + 1/2*Ei(2*log(b*c*x^2 + a*c))/(b^2*c
^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\ln\left(c\,\left(b\,x^2+a\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/log(c*(a + b*x^2)),x)
```

```
[Out] int(x^3/log(c*(a + b*x^2)), x)
```

3.124 $\int \frac{x}{\log(c(a+bx^2))}\,dx$

Optimal. Leaf size=20

$$\frac{\mathrm{li}(c(a+bx^2))}{2bc}$$

[Out] 1/2*Li(c*(b*x^2+a))/b/c

Rubi [A]
time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2504, 2436, 2335}

$$\frac{\mathrm{li}(c(bx^2+a))}{2bc}$$

Antiderivative was successfully verified.

[In] Int[x/Log[c*(a + b*x^2)],x]

[Out] LogIntegral[c*(a + b*x^2)]/(2*b*c)

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{x}{\log \left(c \left(a+b x^2\right)\right)} \, dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{\log(c(a+bx))} \, dx, x, x^2\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\log(cx)} \, dx, x, a+bx^2\right)}{2b}$$
$$= \frac{\text{li}(c(a+bx^2))}{2bc}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 1.10

$$\frac{\text{Ei}(\log \left(ac+bcx^2\right))}{2bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Log[c*(a + b*x^2)],x]
[Out] ExpIntegralEi[Log[a*c + b*c*x^2]]/(2*b*c)
```

Maple [A]
time = 0.41, size = 23, normalized size = 1.15

method	result	size
derivativedivides	$-\frac{\text{expIntegral}\left(1,-\ln\left(c\left(b\,x^2+a\right)\right)\right)}{2bc}$	23
default	$-\frac{\text{expIntegral}\left(1,-\ln\left(c\left(b\,x^2+a\right)\right)\right)}{2bc}$	23
risch	$-\frac{\text{expIntegral}\left(1,-\ln\left(c\left(b\,x^2+a\right)\right)\right)}{2bc}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/ln(c*(b*x^2+a)),x,method=_RETURNVERBOSE)
[Out] -1/2/b/c*Ei(1,-ln(c*(b*x^2+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)),x, algorithm="maxima")
[Out] integrate(x/log((b*x^2 + a)*c), x)
```

Fricas [A]
time = 0.36, size = 19, normalized size = 0.95

$$\frac{\text{log\_integral}\left(bcx^{2}+ac\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)),x, algorithm="fricas")
```

```
[Out] 1/2*log_integral(b*c*x^2 + a*c)/(b*c)
```

Sympy [A]
time = 0.90, size = 27, normalized size = 1.35

$$\begin{cases} \frac{x^{2}}{2\log{\left(ac \right)}} & \text{for}\: b = 0 \\ 0 & \text{for}\: c = 0 \\ \frac{\operatorname{Ei}{\left(\log{\left(ac + bcx^{2} \right)} \right)}}{2bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/ln(c*(b*x**2+a)),x)
```

```
[Out] Piecewise((x**2/(2*log(a*c)), Eq(b, 0)), (0, Eq(c, 0)), (Ei(log(a*c + b*c*x
**2))/(2*b*c), True))
```

Giac [A]
time = 2.29, size = 20, normalized size = 1.00

$$\frac{\text{Ei}(\log\left(bcx^{2}+ac\right))}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a)),x, algorithm="giac")
```

```
[Out] 1/2*Ei(log(b*c*x^2 + a*c))/(b*c)
```

Mupad [B]
time = 0.35, size = 18, normalized size = 0.90

$$\frac{\text{logint}(c\,(b\,x^{2}+a))}{2\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/log(c*(a + b*x^2)),x)
```

```
[Out] logint(c*(a + b*x^2))/(2*b*c)
```

3.125 $\int \frac{x^3}{\log^2(c(a+bx^2))}\,dx$

Optimal. Leaf size=71

$$\frac{\operatorname{Ei}(2\log(c(a+bx^2)))}{b^2c^2}-\frac{x^2(a+bx^2)}{2b\log(c(a+bx^2))}-\frac{a\operatorname{li}(c(a+bx^2))}{2b^2c}$$

```
[Out] Ei(2*ln(c*(b*x^2+a)))/b^2/c^2-1/2*a*Li(c*(b*x^2+a))/b^2/c-1/2*x^2*(b*x^2+a)
/b/ln(c*(b*x^2+a))
```

Rubi [A]
time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2504, 2447, 2446, 2436, 2335, 2437, 2346, 2209}

$$\frac{\operatorname{Ei}(2\log(c(bx^2+a)))}{b^2c^2}-\frac{a\operatorname{li}(c(bx^2+a))}{2b^2c}-\frac{x^2(a+bx^2)}{2b\log(c(a+bx^2))}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Log[c*(a + b*x^2)]^2,x]
```

```
[Out] ExpIntegralEi[2*Log[c*(a + b*x^2)]]/(b^2*c^2) - (x^2*(a + b*x^2))/(2*b*Log[
c*(a + b*x^2)]) - (a*LogIntegral[c*(a + b*x^2)])/(2*b^2*c)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2447

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[q*((e*f - d*g)/
(b*e*n*(p + 1))), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{\log^2(c(a+bx))}\,dx, x, x^2\right)$$

$$= -\frac{x^2(a+bx^2)}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{a\text{Subst}\left(\int \frac{1}{\log(c(a+bx))}\,dx, x, x^2\right)}{2b} + \text{Subst}\left(\int \frac{x}{\log(c(a+bx}\right.$$

$$= -\frac{x^2(a+bx^2)}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{a\text{Subst}\left(\int \frac{1}{\log(cx)}\,dx, x, a+bx^2\right)}{2b^2} + \text{Subst}\left(\int \left(-\frac{a}{b\log(c(a}\right.\right.$$

$$= -\frac{x^2(a+bx^2)}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{a\text{li}(c(a+bx^2))}{2b^2c} + \frac{\text{Subst}\left(\int \frac{a+bx}{\log(c(a+bx))}\,dx, x, x^2\right)}{b} - \frac{a\text{Subs}}{}$$

$$= -\frac{x^2(a+bx^2)}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{a\text{li}(c(a+bx^2))}{2b^2c} + \frac{\text{Subst}\left(\int \frac{x}{\log(cx)}\,dx, x, a+bx^2\right)}{b^2} - \frac{a\text{Sub}}{}$$

$$= -\frac{x^2(a+bx^2)}{2b\log\left(c\left(a+bx^2\right)\right)} - \frac{a\text{li}(c(a+bx^2))}{2b^2c} + \frac{\text{Subst}\left(\int \frac{e^{2x}}{x}\,dx, x, \log\left(c(a+bx^2)\right)\right)}{b^2c^2}$$

$$= \frac{\text{Ei}(2\log\left(c(a+bx^2)\right))}{b^2c^2} - \frac{x^2(a+bx^2)}{2b\log\left(c\left(a+bx^2\right)\right)} - \frac{a\text{li}(c(a+bx^2))}{2b^2c}$$

Mathematica [A]
time = 0.08, size = 66, normalized size = 0.93

$$-\frac{\frac{a\text{Ei}(\log(c(a+bx^2)))}{c} - \frac{2\text{Ei}(2\log(c(a+bx^2)))}{c^2} + \frac{bx^2(a+bx^2)}{\log(c(a+bx^2))}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Log[c*(a + b*x^2)]^2,x]
```

```
[Out] -1/2*((a*ExpIntegralEi[Log[c*(a + b*x^2)]])/c - (2*ExpIntegralEi[2*Log[c*(a
 + b*x^2)]])/c^2 + (b*x^2*(a + b*x^2))/Log[c*(a + b*x^2)])/b^2
```

Maple [A]
time = 2.08, size = 95, normalized size = 1.34

method	result	size
risch	$-\frac{x^2(b x^2+a)}{2b\ln(c(b x^2+a))}+\frac{a\,\text{expIntegral}(1,-\ln(c(b x^2+a)))}{2c\,b^2}-\frac{\text{expIntegral}(1,-2\ln(c(b x^2+a)))}{c^2b^2}$	74
default	$\frac{-\frac{c^2\left(b x^2+a\right)^2}{\ln\left(c\left(b x^2+a\right)\right)}-2\,\text{expIntegral}(1,-2\ln(c(b x^2+a)))-ca\left(-\frac{c\left(b x^2+a\right)}{\ln\left(c\left(b x^2+a\right)\right)}-\text{expIntegral}(1,-\ln(c(b x^2+a)))\right)}{2c^2b^2}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/ln(c*(b*x^2+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c^2/b^2*(-1/ln(c*(b*x^2+a))*c^2*(b*x^2+a)^2-2*Ei(1,-2*ln(c*(b*x^2+a)))-
c*a*(-1/ln(c*(b*x^2+a))*c*(b*x^2+a)-Ei(1,-ln(c*(b*x^2+a)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^4 + a*x^2)/(b*log(b*x^2 + a) + b*log(c)) + integrate((2*b*x^3 + a
*x)/(b*log(b*x^2 + a) + b*log(c)), x)
```

Fricas [A]
time = 0.39, size = 99, normalized size = 1.39

$$-\frac{b^2c^2x^4 + abc^2x^2 + (ac\,\mathrm{log\_integral}\,(bcx^2+ac) - 2\,\mathrm{log\_integral}\,(b^2c^2x^4 + 2\,abc^2x^2 + a^2c^2))\log\left(bcx^2+ac\right)}{2\,b^2c^2\log\left(bcx^2+ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*c^2*x^4 + a*b*c^2*x^2 + (a*c*log_integral(b*c*x^2 + a*c) - 2*log_
integral(b^2*c^2*x^4 + 2*a*b*c^2*x^2 + a^2*c^2))*log(b*c*x^2 + a*c))/(b^2*c
^2*log(b*c*x^2 + a*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{-ax^2 - bx^4}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{\int \frac{ax}{\log\left(ac+bcx^2\right)}\,dx + \int \frac{2bx^3}{\log\left(ac+bcx^2\right)}\,dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/ln(c*(b*x**2+a))**2,x)
```

```
[Out] (-a*x**2 - b*x**4)/(2*b*log(c*(a + b*x**2))) + (Integral(a*x/log(a*c + b*c*
x**2), x) + Integral(2*b*x**3/log(a*c + b*c*x**2), x))/b
```

Giac [A]
time = 4.85, size = 100, normalized size = 1.41

$$\frac{a\left(\frac{bcx^2+ac}{\log(bcx^2+ac)} - \mathrm{Ei}(\log\left(bcx^2+ac\right))\right)}{2\,b^2c} - \frac{\frac{(bcx^2+ac)^2}{\log(bcx^2+ac)} - 2\,\mathrm{Ei}(2\,\log\left(bcx^2+ac\right))}{2\,b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a))^2,x, algorithm="giac")
```

```
[Out] 1/2*a*((b*c*x^2 + a*c)/log(b*c*x^2 + a*c) - Ei(log(b*c*x^2 + a*c)))/(b^2*c)
 - 1/2*((b*c*x^2 + a*c)^2/log(b*c*x^2 + a*c) - 2*Ei(2*log(b*c*x^2 + a*c)))/
(b^2*c^2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\ln\left(c\,\left(b\,x^2+a\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/log(c*(a + b*x^2))^2,x)
```

```
[Out] int(x^3/log(c*(a + b*x^2))^2, x)
```

3.126 $\int \frac{x}{\log^2(c(a+bx^2))} \, dx$

Optimal. Leaf size=47

$$-\frac{a+bx^2}{2b\log\left(c\left(a+bx^2\right)\right)}+\frac{\text{li}(c(a+bx^2))}{2bc}$$

```
[Out] 1/2*Li(c*(b*x^2+a))/b/c+1/2*(-b*x^2-a)/b/ln(c*(b*x^2+a))
```

Rubi [A]
time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2504, 2436, 2334, 2335}

$$\frac{\text{li}(c(bx^2+a))}{2bc}-\frac{a+bx^2}{2b\log\left(c\left(a+bx^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Log[c*(a + b*x^2)]^2,x]
```

```
[Out] -1/2*(a + b*x^2)/(b*Log[c*(a + b*x^2)]) + LogIntegral[c*(a + b*x^2)]/(2*b*c
)
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
```

```
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\log^2\left(c\left(a+bx^2\right)\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\log^2(c(a+bx))}\,dx,x,x^2\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{\log^2(cx)}\,dx,x,a+bx^2\right)}{2b}\\ &= -\frac{a+bx^2}{2b\log\left(c\left(a+bx^2\right)\right)}+\frac{\text{Subst}\left(\int \frac{1}{\log(cx)}\,dx,x,a+bx^2\right)}{2b}\\ &= -\frac{a+bx^2}{2b\log\left(c\left(a+bx^2\right)\right)}+\frac{\text{li}(c(a+bx^2))}{2bc}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 0.94

$$\frac{\frac{\text{Ei}(\log(c(a+bx^2)))}{c}-\frac{a+bx^2}{\log(c(a+bx^2))}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Log[c*(a + b*x^2)]^2,x]
[Out] (ExpIntegralEi[Log[c*(a + b*x^2)]]/c - (a + b*x^2)/Log[c*(a + b*x^2)])/(2*b
)
```

Maple [A]
time = 0.41, size = 48, normalized size = 1.02

method	result	size
derivativedivides	$\frac{-\frac{c(b x^2+a)}{\ln(c(b x^2+a))}-\text{expIntegral}(1,-\ln(c(b x^2+a)))}{2bc}$	48
default	$\frac{-\frac{c(b x^2+a)}{\ln(c(b x^2+a))}-\text{expIntegral}(1,-\ln(c(b x^2+a)))}{2bc}$	48
risch	$-\frac{b x^2+a}{2\ln(c(b x^2+a))b}-\frac{\text{expIntegral}(1,-\ln(c(b x^2+a)))}{2bc}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/ln(c*(b*x^2+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b/c*(-1/ln(c*(b*x^2+a))*c*(b*x^2+a)-Ei(1,-ln(c*(b*x^2+a))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^2 + a)/(b*log(b*x^2 + a) + b*log(c)) + integrate(x/(log(b*x^2 + a
) + log(c)), x)
```

Fricas [A]
time = 0.36, size = 55, normalized size = 1.17

$$-\frac{bcx^2 + ac - \log\left(bcx^2 + ac\right) \text{log\_integral}\left(bcx^2 + ac\right)}{2\,bc \log\left(bcx^2 + ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b*c*x^2 + a*c - log(b*c*x^2 + a*c)*log_integral(b*c*x^2 + a*c))/(b*c*
log(b*c*x^2 + a*c))
```

Sympy [A]
time = 0.95, size = 49, normalized size = 1.04

$$\begin{cases} \frac{x^2}{2\log\left(ac\right)} & \text{for}\ b = 0 \\ 0 & \text{for}\ c = 0 \\ \frac{\text{Ei}\left(\log\left(ac + bcx^2\right)\right)}{2bc} & \text{otherwise} \end{cases} + \frac{-a - bx^2}{2b\log\left(c\left(a + bx^2\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/ln(c*(b*x**2+a))**2,x)
```

```
[Out] Piecewise((x**2/(2*log(a*c)), Eq(b, 0)), (0, Eq(c, 0)), (Ei(log(a*c + b*c*x
**2))/(2*b*c), True)) + (-a - b*x**2)/(2*b*log(c*(a + b*x**2)))
```

Giac [A]
time = 7.12, size = 47, normalized size = 1.00

$$-\frac{\frac{bcx^2 + ac}{\log(bcx^2 + ac)} - \text{Ei}(\log\left(bcx^2 + ac\right))}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a))^2,x, algorithm="giac")
```

```
[Out] -1/2*((b*c*x^2 + a*c)/log(b*c*x^2 + a*c) - Ei(log(b*c*x^2 + a*c)))/(b*c)
```

Mupad [B]
time = 0.36, size = 46, normalized size = 0.98

$$\frac{\mathrm{logint}(c\,(b\,x^2+a))}{2\,b\,c}-\frac{\frac{b\,x^2}{2}+\frac{a}{2}}{b\,\ln\left(c\,\left(b\,x^2+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/log(c*(a + b*x^2))^2,x)
```

```
[Out] logint(c*(a + b*x^2))/(2*b*c) - (a/2 + (b*x^2)/2)/(b*log(c*(a + b*x^2)))
```

3.127 $\int \frac{x^3}{\log^3(c(a+bx^2))}\,dx$

Optimal. Leaf size=127

$$\frac{\operatorname{Ei}(2\log(c(a+bx^2)))}{b^2c^2}-\frac{x^2(a+bx^2)}{4b\log^2(c(a+bx^2))}-\frac{a(a+bx^2)}{4b^2\log(c(a+bx^2))}-\frac{x^2(a+bx^2)}{2b\log(c(a+bx^2))}-\frac{a\operatorname{li}(c(a+bx^2))}{4b^2c}$$

```
[Out] Ei(2*ln(c*(b*x^2+a)))/b^2/c^2-1/4*a*Li(c*(b*x^2+a))/b^2/c-1/4*x^2*(b*x^2+a)
/b/ln(c*(b*x^2+a))^2-1/4*a*(b*x^2+a)/b^2/ln(c*(b*x^2+a))-1/2*x^2*(b*x^2+a)/
b/ln(c*(b*x^2+a))
```

Rubi [A]
time = 0.12, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {2504, 2447, 2446, 2436, 2335, 2437, 2346, 2209, 2334}

$$\frac{\operatorname{Ei}(2\log(c(bx^2+a)))}{b^2c^2}-\frac{a\operatorname{li}(c(bx^2+a))}{4b^2c}-\frac{a(a+bx^2)}{4b^2\log(c(a+bx^2))}-\frac{x^2(a+bx^2)}{4b\log^2(c(a+bx^2))}-\frac{x^2(a+bx^2)}{2b\log(c(a+bx^2))}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Log[c*(a + b*x^2)]^3,x]
```

```
[Out] ExpIntegralEi[2*Log[c*(a + b*x^2)]]/(b^2*c^2) - (x^2*(a + b*x^2))/(4*b*Log[
c*(a + b*x^2)]^2) - (a*(a + b*x^2))/(4*b^2*Log[c*(a + b*x^2)]) - (x^2*(a +
b*x^2))/(2*b*Log[c*(a + b*x^2)]) - (a*LogIntegral[c*(a + b*x^2)])/(4*b^2*c)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2447

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[q*((e*f - d*g)/
(b*e*n*(p + 1))), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{\log^3(c(a+bx))}\,dx, x, x^2\right)$$

$$= -\frac{x^2(a+bx^2)}{4b\log^2\left(c\left(a+bx^2\right)\right)} + \frac{1}{2}\text{Subst}\left(\int \frac{x}{\log^2(c(a+bx))}\,dx, x, x^2\right) + \frac{a\text{Subst}\left(\int \frac{1}{\log^2(c(a}\right.}{4b}$$

$$= -\frac{x^2(a+bx^2)}{4b\log^2\left(c\left(a+bx^2\right)\right)} - \frac{x^2(a+bx^2)}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{a\text{Subst}\left(\int \frac{1}{\log^2(cx)}\,dx, x, a+bx^2\right)}{4b^2} +$$

$$= -\frac{x^2(a+bx^2)}{4b\log^2\left(c\left(a+bx^2\right)\right)} - \frac{a(a+bx^2)}{4b^2\log\left(c\left(a+bx^2\right)\right)} - \frac{x^2(a+bx^2)}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{a\text{Subst}\left(\int \frac{}{\log}\right.}{}$$

$$= -\frac{x^2(a+bx^2)}{4b\log^2\left(c\left(a+bx^2\right)\right)} - \frac{a(a+bx^2)}{4b^2\log\left(c\left(a+bx^2\right)\right)} - \frac{x^2(a+bx^2)}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{3a\text{li}(c(a+bx}{4b^2c}$$

$$= -\frac{x^2(a+bx^2)}{4b\log^2\left(c\left(a+bx^2\right)\right)} - \frac{a(a+bx^2)}{4b^2\log\left(c\left(a+bx^2\right)\right)} - \frac{x^2(a+bx^2)}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{3a\text{li}(c(a+bx}{4b^2c}$$

$$= -\frac{x^2(a+bx^2)}{4b\log^2\left(c\left(a+bx^2\right)\right)} - \frac{a(a+bx^2)}{4b^2\log\left(c\left(a+bx^2\right)\right)} - \frac{x^2(a+bx^2)}{2b\log\left(c\left(a+bx^2\right)\right)} - \frac{a\text{li}(c(a+bx^2}{4b^2c}$$

$$= \frac{\text{Ei}(2\log\left(c(a+bx^2)\right))}{b^2c^2} - \frac{x^2(a+bx^2)}{4b\log^2\left(c\left(a+bx^2\right)\right)} - \frac{a(a+bx^2)}{4b^2\log\left(c\left(a+bx^2\right)\right)} - \frac{x^2(a+b}{2b\log\left(c\left(a-\right.\right.}$$

Mathematica [A]
time = 0.09, size = 87, normalized size = 0.69

$$-\frac{\frac{a\text{Ei}\left(\log\left(c(a+bx^2)\right)\right)}{c} - \frac{4\text{Ei}\left(2\log\left(c(a+bx^2)\right)\right)}{c^2} + \frac{(a+bx^2)\left(bx^2+(a+2bx^2)\log\left(c(a+bx^2)\right)\right)}{\log^2\left(c(a+bx^2)\right)}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Log[c*(a + b*x^2)]^3,x]
```

```
[Out] -1/4*((a*ExpIntegralEi[Log[c*(a + b*x^2)]])/c - (4*ExpIntegralEi[2*Log[c*(a
 + b*x^2)]])/c^2 + ((a + b*x^2)*(b*x^2 + (a + 2*b*x^2)*Log[c*(a + b*x^2)]))
/Log[c*(a + b*x^2)]^2)/b^2
```

Maple [A]
time = 2.12, size = 143, normalized size = 1.13

method	result
risch	$-\frac{(b\,x^2+a)\left(2\ln(c(b\,x^2+a))b\,x^2+b\,x^2+\ln(c(b\,x^2+a))a\right)}{4b^2\ln(c(b\,x^2+a))^2} + \frac{a\,\text{expIntegral}(1,-\ln(c(b\,x^2+a)))}{4c\,b^2} - \frac{\text{expIntegral}(1,-2\ln(c(b\,x^2+a)))}{c^2b^2}$

default	$\frac{-\frac{c^2(bx^2+a)^2}{2\ln(c(bx^2+a))^2}-\frac{c^2(bx^2+a)^2}{\ln(c(bx^2+a))}-2\,\mathrm{expIntegral}(1,-2\ln(c(bx^2+a)))-ca\left(-\frac{c(bx^2+a)}{2\ln(c(bx^2+a))^2}-\frac{c(bx^2+a)}{2\ln(c(bx^2+a))}-\frac{\mathrm{expIntegral}(1,-\ln}{2}\right.}{2c^2b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/ln(c*(b*x^2+a))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c^2/b^2*(-1/2/ln(c*(b*x^2+a))^2*c^2*(b*x^2+a)^2-1/ln(c*(b*x^2+a))*c^2*(
b*x^2+a)^2-2*Ei(1,-2*ln(c*(b*x^2+a)))-c*a*(-1/2/ln(c*(b*x^2+a))^2*c*(b*x^2+
a)-1/2/ln(c*(b*x^2+a))*c*(b*x^2+a)-1/2*Ei(1,-ln(c*(b*x^2+a)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*x^4*(2*log(c) + 1) + a*b*x^2*(3*log(c) + 1) + a^2*log(c) + (2*b^2
*x^4 + 3*a*b*x^2 + a^2)*log(b*x^2 + a))/(b^2*log(b*x^2 + a)^2 + 2*b^2*log(b
*x^2 + a)*log(c) + b^2*log(c)^2) + integrate(1/2*(4*b*x^3 + 3*a*x)/(b*log(b
*x^2 + a) + b*log(c)), x)
```

Fricas [A]
time = 0.38, size = 142, normalized size = 1.12

$$-\frac{b^2c^2x^4+abc^2x^2+(ac\,\mathrm{log\_integral}\,(bcx^2+ac)-4\,\mathrm{log\_integral}\,(b^2c^2x^4+2\,abc^2x^2+a^2c^2))\log(bcx^2+ac)^2+(2\,b^2c^2x^4+3\,abc^2x^2+a^2c^2)\log(bcx^2+ac)}{4\,b^2c^2\log(bcx^2+ac)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*c^2*x^4 + a*b*c^2*x^2 + (a*c*log_integral(b*c*x^2 + a*c) - 4*log_
integral(b^2*c^2*x^4 + 2*a*b*c^2*x^2 + a^2*c^2))*log(b*c*x^2 + a*c)^2 + (2*
b^2*c^2*x^4 + 3*a*b*c^2*x^2 + a^2*c^2)*log(b*c*x^2 + a*c))/(b^2*c^2*log(b*c
*x^2 + a*c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{3ax}{\log(ac+bcx^2)}\,dx+\int\frac{4bx^3}{\log(ac+bcx^2)}\,dx}{2b}+\frac{-abx^2-b^2x^4+(-a^2-3abx^2-2b^2x^4)\log(c(a+bx^2))}{4b^2\log(c(a+bx^2))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/ln(c*(b*x**2+a))**3,x)
```

```
[Out] (Integral(3*a*x/log(a*c + b*c*x**2), x) + Integral(4*b*x**3/log(a*c + b*c*x
**2), x))/(2*b) + (-a*b*x**2 - b**2*x**4 + (-a**2 - 3*a*b*x**2 - 2*b**2*x**
4)*log(c*(a + b*x**2)))/(4*b**2*log(c*(a + b*x**2))**2)
```

Giac [A]
time = 5.31, size = 151, normalized size = 1.19

$$\frac{a\left(\frac{bcx^2+ac}{\log(bcx^2+ac)}+\frac{bcx^2+ac}{\log(bcx^2+ac)^2}-\mathrm{Ei}(\log(bcx^2+ac))\right)}{4\,b^2c}-\frac{\frac{2\,(bcx^2+ac)^2}{\log(bcx^2+ac)}+\frac{(bcx^2+ac)^2}{\log(bcx^2+ac)^2}-4\,\mathrm{Ei}(2\,\log(bcx^2+ac))}{4\,b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(b*x^2+a))^3,x, algorithm="giac")
```

```
[Out] 1/4*a*((b*c*x^2 + a*c)/log(b*c*x^2 + a*c) + (b*c*x^2 + a*c)/log(b*c*x^2 + a
*c)^2 - Ei(log(b*c*x^2 + a*c)))/(b^2*c) - 1/4*(2*(b*c*x^2 + a*c)^2/log(b*c*
x^2 + a*c) + (b*c*x^2 + a*c)^2/log(b*c*x^2 + a*c)^2 - 4*Ei(2*log(b*c*x^2 +
a*c)))/(b^2*c^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\ln\left(c\,\left(b\,x^2+a\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/log(c*(a + b*x^2))^3,x)
```

```
[Out] int(x^3/log(c*(a + b*x^2))^3, x)
```

3.128 $\int \frac{x}{\log^3(c(a+bx^2))}\,dx$

Optimal. Leaf size=73

$$-\frac{a+bx^2}{4b\log^2\left(c\left(a+bx^2\right)\right)}-\frac{a+bx^2}{4b\log\left(c\left(a+bx^2\right)\right)}+\frac{\operatorname{li}(c(a+bx^2))}{4bc}$$

```
[Out] 1/4*Li(c*(b*x^2+a))/b/c+1/4*(-b*x^2-a)/b/ln(c*(b*x^2+a))^2+1/4*(-b*x^2-a)/b
/ln(c*(b*x^2+a))
```

Rubi [A]
time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2504, 2436, 2334, 2335}

$$\frac{\operatorname{li}(c(bx^2+a))}{4bc}-\frac{a+bx^2}{4b\log^2\left(c\left(a+bx^2\right)\right)}-\frac{a+bx^2}{4b\log\left(c\left(a+bx^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Log[c*(a + b*x^2)]^3,x]
```

```
[Out] -1/4*(a + b*x^2)/(b*Log[c*(a + b*x^2)]^2) - (a + b*x^2)/(4*b*Log[c*(a + b*x
^2)]) + LogIntegral[c*(a + b*x^2)]/(4*b*c)
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\log^3(c(a+bx))}\,dx, x, x^2\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{\log^3(cx)}\,dx, x, a+bx^2\right)}{2b} \\
&= -\frac{a+bx^2}{4b\log^2\left(c\left(a+bx^2\right)\right)} + \frac{\text{Subst}\left(\int \frac{1}{\log^2(cx)}\,dx, x, a+bx^2\right)}{4b} \\
&= -\frac{a+bx^2}{4b\log^2\left(c\left(a+bx^2\right)\right)} - \frac{a+bx^2}{4b\log\left(c\left(a+bx^2\right)\right)} + \frac{\text{Subst}\left(\int \frac{1}{\log(cx)}\,dx, x, a+bx^2\right)}{4b} \\
&= -\frac{a+bx^2}{4b\log^2\left(c\left(a+bx^2\right)\right)} - \frac{a+bx^2}{4b\log\left(c\left(a+bx^2\right)\right)} + \frac{\text{li}\left(c\left(a+bx^2\right)\right)}{4bc}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 56, normalized size = 0.77

$$\frac{\text{Ei}\left(\log\left(c\left(a+bx^2\right)\right)\right) - \frac{c\left(a+bx^2\right)\left(1+\log\left(c\left(a+bx^2\right)\right)\right)}{\log^2\left(c\left(a+bx^2\right)\right)}}{4bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Log[c*(a + b*x^2)]^3,x]
[Out] (ExpIntegralEi[Log[c*(a + b*x^2)]] - (c*(a + b*x^2)*(1 + Log[c*(a + b*x^2)]
))/Log[c*(a + b*x^2)]^2)/(4*b*c)
```

Maple [A]
time = 0.47, size = 70, normalized size = 0.96

method	result	size
derivativedivides	$\frac{-\frac{c\left(b\,x^2+a\right)}{2\ln\left(c\left(b\,x^2+a\right)\right)^2}-\frac{c\left(b\,x^2+a\right)}{2\ln\left(c\left(b\,x^2+a\right)\right)}-\frac{\text{expIntegral}\left(1,-\ln\left(c\left(b\,x^2+a\right)\right)\right)}{2}}{2bc}$	70
default	$\frac{-\frac{c\left(b\,x^2+a\right)}{2\ln\left(c\left(b\,x^2+a\right)\right)^2}-\frac{c\left(b\,x^2+a\right)}{2\ln\left(c\left(b\,x^2+a\right)\right)}-\frac{\text{expIntegral}\left(1,-\ln\left(c\left(b\,x^2+a\right)\right)\right)}{2}}{2bc}$	70

risch	$-\frac{\ln(c(b\,x^2+a))b\,x^2+b\,x^2+\ln(c(b\,x^2+a))a+a}{4b\ln(c(b\,x^2+a))^2}-\frac{\text{expIntegral}(1,-\ln(c(b\,x^2+a)))}{4bc}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/ln(c*(b*x^2+a))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b/c*(-1/2/ln(c*(b*x^2+a))^2*c*(b*x^2+a)-1/2/ln(c*(b*x^2+a))*c*(b*x^2+a)
-1/2*Ei(1,-ln(c*(b*x^2+a))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b*x^2*(log(c) + 1) + a*(log(c) + 1) + (b*x^2 + a)*log(b*x^2 + a))/(b*
log(b*x^2 + a)^2 + 2*b*log(b*x^2 + a)*log(c) + b*log(c)^2) + integrate(1/2*
x/(log(b*x^2 + a) + log(c)), x)
```

Fricas [A]
time = 0.34, size = 79, normalized size = 1.08

$$-\frac{bcx^2-\log\left(bcx^2+ac\right)^2\operatorname{log\_integral}\left(bcx^2+ac\right)+ac+\left(bcx^2+ac\right)\log\left(bcx^2+ac\right)}{4\,bc\log\left(bcx^2+ac\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b*c*x^2 - log(b*c*x^2 + a*c)^2*log_integral(b*c*x^2 + a*c) + a*c + (b
*c*x^2 + a*c)*log(b*c*x^2 + a*c))/(b*c*log(b*c*x^2 + a*c)^2)
```

Sympy [A]
time = 0.95, size = 70, normalized size = 0.96

$$\frac{\begin{cases}\frac{x^2}{2\log(ac)} & \text{for}\ b=0\\ 0 & \text{for}\ c=0\\ \frac{\operatorname{Ei}\left(\log\left(ac+bcx^2\right)\right)}{2bc} & \text{otherwise}\end{cases}}{2}+\frac{-a-bx^2+\left(-a-bx^2\right)\log\left(c(a+bx^2)\right)}{4b\log\left(c\left(a+bx^2\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/ln(c*(b*x**2+a))**3,x)
```

```
[Out] Piecewise((x**2/(2*log(a*c)), Eq(b, 0)), (0, Eq(c, 0)), (Ei(log(a*c + b*c*x
**2))/(2*b*c), True))/2 + (-a - b*x**2 + (-a - b*x**2)*log(c*(a + b*x**2)))
/(4*b*log(c*(a + b*x**2))**2)
```

Giac [A]
time = 5.05, size = 71, normalized size = 0.97

$$-\frac{\frac{bcx^2+ac}{\log(bcx^2+ac)}+\frac{bcx^2+ac}{\log(bcx^2+ac)^2}-\mathrm{Ei}(\log(bcx^2+ac))}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(b*x^2+a))^3,x, algorithm="giac")
```

```
[Out] -1/4*((b*c*x^2 + a*c)/log(b*c*x^2 + a*c) + (b*c*x^2 + a*c)/log(b*c*x^2 + a*
c)^2 - Ei(log(b*c*x^2 + a*c)))/(b*c)
```

Mupad [B]
time = 0.45, size = 74, normalized size = 1.01

$$\frac{\mathrm{logint}(c\,(b\,x^2+a))}{4\,b\,c}-\frac{\frac{a\,c}{4}+\ln\left(c\,(b\,x^2+a)\right)\,\left(\frac{b\,c\,x^2}{4}+\frac{a\,c}{4}\right)+\frac{b\,c\,x^2}{4}}{b\,c\ln\left(c\,\left(b\,x^2+a\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/log(c*(a + b*x^2))^3,x)
```

```
[Out] logint(c*(a + b*x^2))/(4*b*c) - ((a*c)/4 + log(c*(a + b*x^2))*((a*c)/4 + (b
*c*x^2)/4) + (b*c*x^2)/4)/(b*c*log(c*(a + b*x^2))^2)
```

3.129 $\int x^5 \log^2\left(c(d+ex^3)^p\right)\,dx$

Optimal. Leaf size=150

$$-\frac{2dp^2x^3}{3e}+\frac{p^2(d+ex^3)^2}{12e^2}+\frac{2dp(d+ex^3)\log\left(c(d+ex^3)^p\right)}{3e^2}-\frac{p(d+ex^3)^2\log\left(c(d+ex^3)^p\right)}{6e^2}-\frac{d(d+ex^3)\log^2\left(c\right.}{3e^2}$$

```
[Out] -2/3*d*p^2*x^3/e+1/12*p^2*(e*x^3+d)^2/e^2+2/3*d*p*(e*x^3+d)*ln(c*(e*x^3+d)^
p)/e^2-1/6*p*(e*x^3+d)^2*ln(c*(e*x^3+d)^p)/e^2-1/3*d*(e*x^3+d)*ln(c*(e*x^3+
d)^p)^2/e^2+1/6*(e*x^3+d)^2*ln(c*(e*x^3+d)^p)^2/e^2
```

Rubi [A]
time = 0.11, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

$$\frac{(d+ex^3)^2\log^2\left(c(d+ex^3)^p\right)}{6e^2}-\frac{d(d+ex^3)\log^2\left(c(d+ex^3)^p\right)}{3e^2}-\frac{p(d+ex^3)^2\log\left(c(d+ex^3)^p\right)}{6e^2}+\frac{2dp(d+ex^3)\log\left(c(d+ex^3)^p\right)}{3e^2}+\frac{p^2(d+ex^3)^2}{12e^2}-\frac{2dp^2x^3}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] (-2*d*p^2*x^3)/(3*e) + (p^2*(d + e*x^3)^2)/(12*e^2) + (2*d*p*(d + e*x^3)*Lo
g[c*(d + e*x^3)^p])/(3*e^2) - (p*(d + e*x^3)^2*Log[c*(d + e*x^3)^p])/(6*e^2
) - (d*(d + e*x^3)*Log[c*(d + e*x^3)^p]^2)/(3*e^2) + ((d + e*x^3)^2*Log[c*(
d + e*x^3)^p]^2)/(6*e^2)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
```

```
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^5 \log^2\left(c\left(d+ex^3\right)^p\right)\,dx = \frac{1}{3}\text{Subst}\left(\int x\log^2\left(c(d+ex)^p\right)\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{d\log^2\left(c(d+ex)^p\right)}{e} + \frac{(d+ex)\log^2\left(c(d+ex)^p\right)}{e}\right)\,dx, x, x^3\right)$$
$$= \frac{\text{Subst}\left(\int (d+ex)\log^2\left(c(d+ex)^p\right)\,dx, x, x^3\right)}{3e} - \frac{d\text{Subst}\left(\int \log^2\left(c(d+ex)^p\right)\,dx\right.}{3e}$$
$$= \frac{\text{Subst}\left(\int x\log^2\left(cx^p\right)\,dx, x, d+ex^3\right)}{3e^2} - \frac{d\text{Subst}\left(\int \log^2\left(cx^p\right)\,dx, x, d+ex^3\right)}{3e^2}$$
$$= -\frac{d(d+ex^3)\log^2\left(c\left(d+ex^3\right)^p\right)}{3e^2} + \frac{\left(d+ex^3\right)^2\log^2\left(c\left(d+ex^3\right)^p\right)}{6e^2} - \frac{p\text{Subst}\left(\int\right.}{}$$
$$= -\frac{2dp^2x^3}{3e} + \frac{p^2\left(d+ex^3\right)^2}{12e^2} + \frac{2dp(d+ex^3)\log\left(c\left(d+ex^3\right)^p\right)}{3e^2} - \frac{p\left(d+ex^3\right)^2\log}{6e}$$

Mathematica [A]
time = 0.05, size = 105, normalized size = 0.70

$$\frac{ep^2x^3(-6d+ex^3)+2d^2p^2\log\left(d+ex^3\right)+2p(2d^2+2dex^3-e^2x^6)\log\left(c\left(d+ex^3\right)^p\right)-2(d^2-e^2x^6)\log^2\left(c\left(d+ex^3\right)^p\right)}{12e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] (e*p^2*x^3*(-6*d + e*x^3) + 2*d^2*p^2*Log[d + e*x^3] + 2*p*(2*d^2 + 2*d*e*x
^3 - e^2*x^6)*Log[c*(d + e*x^3)^p] - 2*(d^2 - e^2*x^6)*Log[c*(d + e*x^3)^p]
^2)/(12*e^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 5.14, size = 24313, normalized size = 162.09

method	result	size
risch	Expression too large to display	24313

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*ln(c*(e*x^3+d)^p)^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.28, size = 120, normalized size = 0.80

$$\frac{1}{6}x^6\log\left(\left(ex^3+d\right)^pc\right)^2 - \frac{1}{6}ep\left(\frac{2d^2\log\left(ex^3+d\right)}{e^3} + \frac{ex^6-2dx^3}{e^2}\right)\log\left(\left(ex^3+d\right)^pc\right) + \frac{\left(e^2x^6-6dex^3+2d^2\log\left(ex^3+d\right)^2+6d^2\log\left(ex^3+d\right)\right)p^2}{12e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] 1/6*x^6*log((e*x^3 + d)^p*c)^2 - 1/6*e*p*(2*d^2*log(e*x^3 + d)/e^3 + (e*x^6
 - 2*d*x^3)/e^2)*log((e*x^3 + d)^p*c) + 1/12*(e^2*x^6 - 6*d*e*x^3 + 2*d^2*l
og(e*x^3 + d)^2 + 6*d^2*log(e*x^3 + d))*p^2/e^2
```

Fricas [A]
time = 0.36, size = 146, normalized size = 0.97

$$\frac{1}{12}\left(p^2x^6e^2+2\,x^6e^2\log\left(c\right)^2-6\,dp^2x^3e+2\left(p^2x^6e^2-d^2p^2\right)\log\left(x^3e+d\right)^2-2\left(p^2x^6e^2-2\,dp^2x^3e-3\,d^2p^2-2\left(px^6e^2-d^2p\right)\log\left(c\right)\right)\log\left(x^3e+d\right)-2\left(px^6e^2-2\,dpx^3e\right)\log\left(c\right)\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(p^2*x^6*e^2 + 2*x^6*e^2*log(c)^2 - 6*d*p^2*x^3*e + 2*(p^2*x^6*e^2 - d
^2*p^2)*log(x^3*e + d)^2 - 2*(p^2*x^6*e^2 - 2*d*p^2*x^3*e - 3*d^2*p^2 - 2*(
p*x^6*e^2 - d^2*p)*log(c))*log(x^3*e + d) - 2*(p*x^6*e^2 - 2*d*p*x^3*e)*log
(c))*e^(-2)
```

Sympy [A]
time = 4.71, size = 136, normalized size = 0.91

$$\begin{cases} \frac{d^2p\log\left(c\left(d+ex^3\right)^p\right)}{2e^2}-\frac{d^2\log\left(c\left(d+ex^3\right)^p\right)^2}{6e^2}-\frac{dp^2x^3}{2e}+\frac{dpx^3\log\left(c\left(d+ex^3\right)^p\right)}{3e}+\frac{p^2x^6}{12}-\frac{px^6\log\left(c\left(d+ex^3\right)^p\right)}{6}+\frac{x^6\log\left(c\left(d+ex^3\right)^p\right)^2}{6} & \text{for}\ e\neq 0 \\ \frac{x^6\log\left(cd^p\right)^2}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Piecewise((d**2*p*log(c*(d + e*x**3)**p)/(2*e**2) - d**2*log(c*(d + e*x**3)
**p)**2/(6*e**2) - d*p**2*x**3/(2*e) + d*p*x**3*log(c*(d + e*x**3)**p)/(3*e
) + p**2*x**6/12 - p*x**6*log(c*(d + e*x**3)**p)/6 + x**6*log(c*(d + e*x**3
)**p)**2/6, Ne(e, 0)), (x**6*log(c*d**p)**2/6, True))
```

Giac [A]
time = 3.84, size = 232, normalized size = 1.55

$$\frac{1}{12}\left(2\left(x^3e+d\right)^2p^2\log\left(x^3e+d\right)^2-2\left(x^3e+d\right)^2p^2\log\left(x^3e+d\right)+4\left(x^3e+d\right)^2p\log\left(x^3e+d\right)\log\left(c\right)+\left(x^3e+d\right)^2p^2-2\left(x^3e+d\right)^2p\log\left(c\right)+2\left(x^3e+d\right)^2\log\left(c\right)^2\right)e^{(-2)}-\frac{1}{3}\left(\left(2\,x^3e+\left(x^3e+d\right)\log\left(x^3e+d\right)^2-2\left(x^3e+d\right)\log\left(x^3e+d\right)+2\,d\right)dp^2-2\left(x^3e-\left(x^3e+d\right)\log\left(x^3e+d\right)+d\right)dp\log\left(c\right)+\left(x^3e+d\right)d\log\left(c\right)^2\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] 1/12*(2*(x^3*e + d)^2*p^2*log(x^3*e + d)^2 - 2*(x^3*e + d)^2*p^2*log(x^3*e
+ d) + 4*(x^3*e + d)^2*p*log(x^3*e + d)*log(c) + (x^3*e + d)^2*p^2 - 2*(x^3
*e + d)^2*p*log(c) + 2*(x^3*e + d)^2*log(c)^2)*e^(-2) - 1/3*((2*x^3*e + (x^
```

```
3*e + d)*log(x^3*e + d)^2 - 2*(x^3*e + d)*log(x^3*e + d) + 2*d)*d*p^2 - 2*(
x^3*e - (x^3*e + d)*log(x^3*e + d) + d)*d*p*log(c) + (x^3*e + d)*d*log(c)^2
)*e^(-2)
```

Mupad [B]
time = 0.28, size = 100, normalized size = 0.67

$$\frac{p^2\,x^6}{12}-\ln\left(c\,{\left(e\,x^3+d\right)}^p\right)\,\left(\frac{p\,x^6}{6}-\frac{d\,p\,x^3}{3\,e}\right)+{\ln\left(c\,{\left(e\,x^3+d\right)}^p\right)}^2\,\left(\frac{x^6}{6}-\frac{d^2}{6\,e^2}\right)-\frac{d\,p^2\,x^3}{2\,e}+\frac{d^2\,p^2\,\ln\left(e\,x^3+d\right)}{2\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*log(c*(d + e*x^3)^p)^2,x)
```

```
[Out] (p^2*x^6)/12 - log(c*(d + e*x^3)^p)*((p*x^6)/6 - (d*p*x^3)/(3*e)) + log(c*(
d + e*x^3)^p)^2*(x^6/6 - d^2/(6*e^2)) - (d*p^2*x^3)/(2*e) + (d^2*p^2*log(d
+ e*x^3))/(2*e^2)
```

3.130 $\int x^2 \log^2\left(c(d+ex^3)^p\right)\,dx$

Optimal. Leaf size=66

$$\frac{2p^2x^3}{3}-\frac{2p(d+ex^3)\log\left(c(d+ex^3)^p\right)}{3e}+\frac{(d+ex^3)\log^2\left(c(d+ex^3)^p\right)}{3e}$$

```
[Out] 2/3*p^2*x^3-2/3*p*(e*x^3+d)*ln(c*(e*x^3+d)^p)/e+1/3*(e*x^3+d)*ln(c*(e*x^3+d
)^p)^2/e
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.222$, Rules used = {2504, 2436, 2333, 2332}

$$\frac{(d+ex^3)\log^2\left(c(d+ex^3)^p\right)}{3e}-\frac{2p(d+ex^3)\log\left(c(d+ex^3)^p\right)}{3e}+\frac{2p^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] (2*p^2*x^3)/3 - (2*p*(d + e*x^3)*Log[c*(d + e*x^3)^p])/(3*e) + ((d + e*x^3)
*Log[c*(d + e*x^3)^p]^2)/(3*e)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
```

```
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^2 \log^2\left(c\left(d+ex^3\right)^p\right) dx &= \frac{1}{3}\text{Subst}\left(\int \log^2\left(c(d+ex)^p\right) dx, x, x^3\right)\\
&= \frac{\text{Subst}\left(\int \log^2\left(cx^p\right) dx, x, d+ex^3\right)}{3e}\\
&= \frac{\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{3e} - \frac{(2p)\text{Subst}\left(\int \log\left(cx^p\right) dx, x, d+ex^3\right)}{3e}\\
&= \frac{2p^2x^3}{3} - \frac{2p\left(d+ex^3\right)\log\left(c\left(d+ex^3\right)^p\right)}{3e} + \frac{\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{3e}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 63, normalized size = 0.95

$$
\frac{1}{3}\left(\frac{\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{e} - 2p\left(-px^3 + \frac{\left(d+ex^3\right)\log\left(c\left(d+ex^3\right)^p\right)}{e}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] (((d + e*x^3)*Log[c*(d + e*x^3)^p]^2)/e - 2*p*(-(p*x^3) + ((d + e*x^3)*Log[
c*(d + e*x^3)^p])/e))/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.76, size = 1036, normalized size = 15.70

method	result	size
risch	Expression too large to display	1036

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(e*x^3+d)^p)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*ln(c)*p*x^3-1/12*Pi^2*x^3*csgn(I*c*(e*x^3+d)^p)^6+1/3*ln(c)^2*x^3-2/3*
d*p^2/e*ln(e*x^3+d)+1/3*x^3*ln((e*x^3+d)^p)^2-1/3*I/e*Pi*ln(e*x^3+d)*d*p*cs
gn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)+1/3*I*ln(c)*Pi*x^3*csgn(I
*c*(e*x^3+d)^p)^2*csgn(I*c)-1/3*I*Pi*p*x^3*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*
x^3+d)^p)^2-1/3*I*Pi*p*x^3*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2/3*p^2*x^3+1/
3*I/e*Pi*ln(e*x^3+d)*d*p*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2+1/3*I/
e*Pi*ln(e*x^3+d)*d*p*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)-1/12*Pi^2*x^3*csgn(I
```

```
*(e*x^3+d)^p)^2*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)^2-1/3*Pi^2*x^3*csgn(I*(e*
x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^4*csgn(I*c)+1/6*Pi^2*x^3*csgn(I*(e*x^3+d)^p
)*csgn(I*c*(e*x^3+d)^p)^3*csgn(I*c)^2+2/3/e*ln(c)*ln(e*x^3+d)*d*p-1/3*I*ln(
c)*Pi*x^3*csgn(I*c*(e*x^3+d)^p)^3+1/3*I*Pi*p*x^3*csgn(I*c*(e*x^3+d)^p)^3-1/
3*I/e*Pi*ln(e*x^3+d)*d*p*csgn(I*c*(e*x^3+d)^p)^3-1/3*I*ln(c)*Pi*x^3*csgn(I*
(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)+1/3*I*Pi*p*x^3*csgn(I*(e*x^3+d
)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)+1/3*I*ln(c)*Pi*x^3*csgn(I*(e*x^3+d)^p)
*csgn(I*c*(e*x^3+d)^p)^2+1/6*Pi^2*x^3*csgn(I*(e*x^3+d)^p)^2*csgn(I*c*(e*x^3
+d)^p)^3*csgn(I*c)-1/3/e*d*p^2*ln(e*x^3+d)^2-1/12*Pi^2*x^3*csgn(I*(e*x^3+d)
^p)^2*csgn(I*c*(e*x^3+d)^p)^4+1/6*Pi^2*x^3*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*
x^3+d)^p)^5+1/6*Pi^2*x^3*csgn(I*c*(e*x^3+d)^p)^5*csgn(I*c)-1/12*Pi^2*x^3*cs
gn(I*c*(e*x^3+d)^p)^4*csgn(I*c)^2+1/3*(I*Pi*e*x^3*csgn(I*(e*x^3+d)^p)*csgn(
I*c*(e*x^3+d)^p)^2-I*Pi*e*x^3*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csg
n(I*c)-I*Pi*e*x^3*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*e*x^3*csgn(I*c*(e*x^3+d)^p)^
2*csgn(I*c)+2*ln(c)*e*x^3-2*x^3*p*e+2*d*p*ln(e*x^3+d))/e*ln((e*x^3+d)^p)
```

Maxima [A]
time = 0.28, size = 97, normalized size = 1.47

$$\frac{1}{3}\,x^3\log\left(\left(ex^3+d\right)^p c\right)^2-\frac{2}{3}\left(\frac{x^3}{e}-\frac{d\log\left(ex^3+d\right)}{e^2}\right)ep\log\left(\left(ex^3+d\right)^p c\right)+\frac{\left(2\,ex^3-d\log\left(ex^3+d\right)^2-2\,d\log\left(ex^3+d\right)\right)p^2}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log((e*x^3 + d)^p*c)^2 - 2/3*(x^3/e - d*log(e*x^3 + d)/e^2)*e*p*log
((e*x^3 + d)^p*c) + 1/3*(2*e*x^3 - d*log(e*x^3 + d)^2 - 2*d*log(e*x^3 + d))
*p^2/e
```

Fricas [A]
time = 0.35, size = 103, normalized size = 1.56

$$\frac{1}{3}\left(2\,p^2x^3e-2\,px^3e\log\left(c\right)+x^3e\log\left(c\right)^2+\left(p^2x^3e+dp^2\right)\log\left(x^3e+d\right)^2-2\left(p^2x^3e+dp^2-\left(px^3e+dp\right)\log\left(c\right)\right)\log\left(x^3e+d\right)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*p^2*x^3*e - 2*p*x^3*e*log(c) + x^3*e*log(c)^2 + (p^2*x^3*e + d*p^2)*
log(x^3*e + d)^2 - 2*(p^2*x^3*e + d*p^2 - (p*x^3*e + d*p)*log(c))*log(x^3*e
 + d))*e^(-1)
```

Sympy [A]
time = 1.28, size = 100, normalized size = 1.52

$$\begin{cases} -\frac{2dp\log\left(c\left(d+ex^3\right)^p\right)}{3e}+\frac{d\log\left(c\left(d+ex^3\right)^p\right)^2}{3e}+\frac{2p^2x^3}{3}-\frac{2px^3\log\left(c\left(d+ex^3\right)^p\right)}{3}+\frac{x^3\log\left(c\left(d+ex^3\right)^p\right)^2}{3} & \text{for}\ e\neq 0\\ \frac{x^3\log\left(cd^p\right)^2}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Piecewise((-2*d*p*log(c*(d + e*x**3)**p)/(3*e) + d*log(c*(d + e*x**3)**p)**
2/(3*e) + 2*p**2*x**3/3 - 2*p*x**3*log(c*(d + e*x**3)**p)/3 + x**3*log(c*(d
 + e*x**3)**p)**2/3, Ne(e, 0)), (x**3*log(c*d**p)**2/3, True))
```

Giac [A]
time = 4.73, size = 104, normalized size = 1.58

$$\frac{1}{3}\left(\left(2x^3e + (x^3e+d)\log(x^3e+d)^2 - 2(x^3e+d)\log(x^3e+d) + 2d\right)p^2 - 2\left(x^3e - (x^3e+d)\log(x^3e+d) + d\right)p\log(c) + (x^3e+d)\log(c)^2\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] 1/3*((2*x^3*e + (x^3*e + d)*log(x^3*e + d)^2 - 2*(x^3*e + d)*log(x^3*e + d)
 + 2*d)*p^2 - 2*(x^3*e - (x^3*e + d)*log(x^3*e + d) + d)*p*log(c) + (x^3*e
+ d)*log(c)^2)*e^(-1)
```

Mupad [B]
time = 0.23, size = 71, normalized size = 1.08

$$\frac{2\,p^2\,x^3}{3}+\ln\left(c\,{\left(e\,x^3+d\right)}^p\right)^2\,\left(\frac{d}{3\,e}+\frac{x^3}{3}\right)-\frac{2\,p\,x^3\,\ln\left(c\,{\left(e\,x^3+d\right)}^p\right)}{3}-\frac{2\,d\,p^2\,\ln\left(e\,x^3+d\right)}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(c*(d + e*x^3)^p)^2,x)
```

```
[Out] (2*p^2*x^3)/3 + log(c*(d + e*x^3)^p)^2*(d/(3*e) + x^3/3) - (2*p*x^3*log(c*(
d + e*x^3)^p))/3 - (2*d*p^2*log(d + e*x^3))/(3*e)
```

3.131 $\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x}\,dx$

Optimal. Leaf size=77

$$\frac{1}{3}\log\left(-\frac{ex^3}{d}\right)\log^2\left(c\left(d+ex^3\right)^p\right)+\frac{2}{3}p\log\left(c\left(d+ex^3\right)^p\right)\text{Li}_2\left(1+\frac{ex^3}{d}\right)-\frac{2}{3}p^2\text{Li}_3\left(1+\frac{ex^3}{d}\right)$$

```
[Out] 1/3*ln(-e*x^3/d)*ln(c*(e*x^3+d)^p)^2+2/3*p*ln(c*(e*x^3+d)^p)*polylog(2,1+e*
x^3/d)-2/3*p^2*polylog(3,1+e*x^3/d)
```

Rubi [A]
time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2504, 2443, 2481, 2421, 6724}

$$\frac{2}{3}p\text{PolyLog}\left(2,\frac{ex^3}{d}+1\right)\log\left(c\left(d+ex^3\right)^p\right)-\frac{2}{3}p^2\text{PolyLog}\left(3,\frac{ex^3}{d}+1\right)+\frac{1}{3}\log\left(-\frac{ex^3}{d}\right)\log^2\left(c\left(d+ex^3\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^3)^p]^2/x,x]
```

```
[Out] (Log[-((e*x^3)/d)]*Log[c*(d + e*x^3)^p]^2)/3 + (2*p*Log[c*(d + e*x^3)^p]*Po
lyLog[2, 1 + (e*x^3)/d])/3 - (2*p^2*PolyLog[3, 1 + (e*x^3)/d])/3
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\log^2\left(c(d+ex)^p\right)}{x}\,dx, x, x^3\right)\\
&= \frac{1}{3}\log\left(-\frac{ex^3}{d}\right)\log^2\left(c\left(d+ex^3\right)^p\right)-\frac{1}{3}(2ep)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)\log\left(c(d+ex)^p\right)}{d+ex}\right.\\
&= \frac{1}{3}\log\left(-\frac{ex^3}{d}\right)\log^2\left(c\left(d+ex^3\right)^p\right)-\frac{1}{3}(2p)\text{Subst}\left(\int \frac{\log\left(cx^p\right)\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}\right.\\
&= \frac{1}{3}\log\left(-\frac{ex^3}{d}\right)\log^2\left(c\left(d+ex^3\right)^p\right)+\frac{2}{3}p\log\left(c\left(d+ex^3\right)^p\right)\text{Li}_2\left(1+\frac{ex^3}{d}\right)-\frac{1}{3}(2p\\
&= \frac{1}{3}\log\left(-\frac{ex^3}{d}\right)\log^2\left(c\left(d+ex^3\right)^p\right)+\frac{2}{3}p\log\left(c\left(d+ex^3\right)^p\right)\text{Li}_2\left(1+\frac{ex^3}{d}\right)-\frac{2}{3}p^2\text{L}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 163 vs. 2(77) = 154.
time = 0.10, size = 163, normalized size = 2.12

$$
\log(x)\left(-p\log\left(d+ex^3\right)+\log\left(c\left(d+ex^3\right)^p\right)\right)^2+2p\left(-p\log\left(d+ex^3\right)+\log\left(c\left(d+ex^3\right)^p\right)\right)\left(\log(x)\left(\log\left(d+ex^3\right)-\log\left(1+\frac{ex^3}{d}\right)\right)-\frac{1}{3}\text{Li}_2\left(-\frac{ex^3}{d}\right)\right)+\frac{1}{3}p^2\left(\log\left(-\frac{ex^3}{d}\right)\log^2\left(d+ex^3\right)+2\log\left(d+ex^3\right)\text{Li}_2\left(1+\frac{ex^3}{d}\right)-2\text{Li}_3\left(1+\frac{ex^3}{d}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^3)^p]^2/x,x]
```

```
[Out] Log[x]*(-(p*Log[d + e*x^3]) + Log[c*(d + e*x^3)^p])^2 + 2*p*(-(p*Log[d + e*
x^3]) + Log[c*(d + e*x^3)^p])*(Log[x]*(Log[d + e*x^3] - Log[1 + (e*x^3)/d])
 - PolyLog[2, -((e*x^3)/d)]/3) + (p^2*(Log[-((e*x^3)/d)]*Log[d + e*x^3]^2 +
 2*Log[d + e*x^3]*PolyLog[2, 1 + (e*x^3)/d] - 2*PolyLog[3, 1 + (e*x^3)/d]))
/3
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(c(e\,x^3+d)^p\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^3+d)^p)^2/x,x)
```

```
[Out] int(ln(c*(e*x^3+d)^p)^2/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x,x, algorithm="maxima")
```

```
[Out] integrate(log((e*x^3 + d)^p*c)^2/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x,x, algorithm="fricas")
```

```
[Out] integral(log((x^3*e + d)^p*c)^2/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(c(d+ex^3)^p\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**3+d)**p)**2/x,x)
```

```
[Out] Integral(log(c*(d + e*x**3)**p)**2/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x,x, algorithm="giac")
[Out] integrate(log((x^3*e + d)^p*c)^2/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(e\,x^3+d\right)}^p\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^3)^p)^2/x,x)
[Out] int(log(c*(d + e*x^3)^p)^2/x, x)
```

3.132 $\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^4}\,dx$

Optimal. Leaf size=86

$$\frac{2ep\log\left(-\frac{ex^3}{d}\right)\log\left(c\left(d+ex^3\right)^p\right)}{3d}-\frac{\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{3dx^3}+\frac{2ep^2\text{Li}_2\left(1+\frac{ex^3}{d}\right)}{3d}$$

```
[Out] 2/3*e*p*ln(-e*x^3/d)*ln(c*(e*x^3+d)^p)/d-1/3*(e*x^3+d)*ln(c*(e*x^3+d)^p)^2/
d/x^3+2/3*e*p^2*polylog(2,1+e*x^3/d)/d
```

Rubi [A]
time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2504, 2444, 2441, 2352}

$$\frac{2ep^2\text{PolyLog}\left(2,\frac{ex^3}{d}+1\right)}{3d}-\frac{\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{3dx^3}+\frac{2ep\log\left(-\frac{ex^3}{d}\right)\log\left(c\left(d+ex^3\right)^p\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^3)^p]^2/x^4,x]
```

```
[Out] (2*e*p*Log[-((e*x^3)/d)]*Log[c*(d + e*x^3)^p])/(3*d) - ((d + e*x^3)*Log[c*(
d + e*x^3)^p]^2)/(3*d*x^3) + (2*e*p^2*PolyLog[2, 1 + (e*x^3)/d])/(3*d)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Dist[b*e*n*(p/(e*f - d*g)), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^4}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\log^2\left(c(d+ex)^p\right)}{x^2}\,dx, x, x^3\right)$$
$$= -\frac{(d+ex^3)\log^2\left(c(d+ex^3)^p\right)}{3dx^3} + \frac{(2ep)\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^3\right)}{3d}$$
$$= \frac{2ep\log\left(-\frac{ex^3}{d}\right)\log\left(c(d+ex^3)^p\right)}{3d} - \frac{(d+ex^3)\log^2\left(c(d+ex^3)^p\right)}{3dx^3} - \frac{(2e^2p^2)\,\text{Subs}}{}$$
$$= \frac{2ep\log\left(-\frac{ex^3}{d}\right)\log\left(c(d+ex^3)^p\right)}{3d} - \frac{(d+ex^3)\log^2\left(c(d+ex^3)^p\right)}{3dx^3} + \frac{2ep^2\text{Li}_2\left(1+\right.}{3d}$$

Mathematica [A]
time = 0.03, size = 84, normalized size = 0.98

$$\frac{2epx^3\log\left(-\frac{ex^3}{d}\right)\log\left(c(d+ex^3)^p\right) - (d+ex^3)\log^2\left(c(d+ex^3)^p\right) + 2ep^2x^3\text{Li}_2\left(1+\frac{ex^3}{d}\right)}{3dx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^3)^p]^2/x^4,x]
```

```
[Out] (2*e*p*x^3*Log[-((e*x^3)/d)]*Log[c*(d + e*x^3)^p] - (d + e*x^3)*Log[c*(d +
e*x^3)^p]^2 + 2*e*p^2*x^3*PolyLog[2, 1 + (e*x^3)/d])/(3*d*x^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.48, size = 771, normalized size = 8.97

method	result
risch	$-\frac{\ln\left((e\,x^3+d)^p\right)^2}{3x^3} + \frac{2pe\ln\left((e\,x^3+d)^p\right)\ln(x)}{d} - \frac{2pe\ln\left((e\,x^3+d)^p\right)\ln(e\,x^3+d)}{3d} - \frac{2p^2e\left(\sum_{\_R1=\text{RootOf}(\_Z^3e+d)}\left(\ln(x)\ln\left(\frac{\_R}{}\right.\right.\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^3+d)^p)^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/x^3*ln((e*x^3+d)^p)^2+2*p*e*ln((e*x^3+d)^p)/d*ln(x)-2/3*p*e*ln((e*x^3+
d)^p)/d*ln(e*x^3+d)-2*p^2*e/d*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R1),
_R1=RootOf(_Z^3*e+d))+1/3*p^2*e/d*ln(e*x^3+d)^2+1/3*I*p*e/d*ln(e*x^3+d)*Pi*
csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)+I*p*e/d*ln(x)*Pi*csgn(I
*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2+1/3*I/x^3*ln((e*x^3+d)^p)*Pi*csgn(I*c
*(e*x^3+d)^p)^3-1/3*I*p*e/d*ln(e*x^3+d)*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c
)-2/3/x^3*ln((e*x^3+d)^p)*ln(c)-1/3*I*p*e/d*ln(e*x^3+d)*Pi*csgn(I*(e*x^3+d)
^p)*csgn(I*c*(e*x^3+d)^p)^2+1/3*I/x^3*ln((e*x^3+d)^p)*Pi*csgn(I*(e*x^3+d)^p
)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*p*e/d*ln(x)*Pi*csgn(I*(e*x^3+d)^p)*csgn
(I*c*(e*x^3+d)^p)*csgn(I*c)+1/3*I*p*e/d*ln(e*x^3+d)*Pi*csgn(I*c*(e*x^3+d)^p
)^3+2*p*e/d*ln(x)*ln(c)-I*p*e/d*ln(x)*Pi*csgn(I*c*(e*x^3+d)^p)^3-1/3*I/x^3*
ln((e*x^3+d)^p)*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+I*p*e/d*ln(x)*Pi*csgn(
I*c*(e*x^3+d)^p)^2*csgn(I*c)-1/3*I/x^3*ln((e*x^3+d)^p)*Pi*csgn(I*(e*x^3+d)^
p)*csgn(I*c*(e*x^3+d)^p)^2-2/3*p*e/d*ln(e*x^3+d)*ln(c)-1/12*(I*Pi*csgn(I*(e
*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3
+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*
csgn(I*c)+2*ln(c))^2/x^3
```

Maxima [A]
time = 0.29, size = 118, normalized size = 1.37

$$\frac{1}{3}\,e^{2}p^{2}\left(\frac{\log\left(ex^{3}+d\right)^{2}}{de}-\frac{2\left(3\,\log\left(\frac{ex^{3}}{d}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{ex^{3}}{d}\right)\right)}{de}\right)-\frac{2}{3}\,ep\left(\frac{\log\left(ex^{3}+d\right)}{d}-\frac{\log\left(x^{3}\right)}{d}\right)\log\left(\left(ex^{3}+d\right)^{p}c\right)-\frac{\log\left(\left(ex^{3}+d\right)^{p}c\right)^{2}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*e^2*p^2*(log(e*x^3 + d)^2/(d*e) - 2*(3*log(e*x^3/d + 1)*log(x) + dilog(
-e*x^3/d))/(d*e)) - 2/3*e*p*(log(e*x^3 + d)/d - log(x^3)/d)*log((e*x^3 + d)
^p*c) - 1/3*log((e*x^3 + d)^p*c)^2/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral(log((x^3*e + d)^p*c)^2/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left(c\left(d+ex^{3}\right)^{p}\right)^{2}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**3+d)**p)**2/x**4,x)
```

```
[Out] Integral(log(c*(d + e*x**3)**p)**2/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate(log((x^3*e + d)^p*c)^2/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(e\,x^3+d\right)}^p\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^3)^p)^2/x^4,x)
```

```
[Out] int(log(c*(d + e*x^3)^p)^2/x^4, x)
```

3.133 $\int x \log^2\left(c(d+ex^3)^p\right)\,dx$

Optimal. Leaf size=1294

$$\frac{9p^2x^2}{4}+\frac{3\sqrt{3}\,d^{2/3}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,x}{\sqrt{3}\,\sqrt[3]{d}}\right)}{2e^{2/3}}+\frac{3d^{2/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{2e^{2/3}}+\frac{d^{2/3}p^2\log^2\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{2e^{2/3}}+\frac{d^{2/3}p^2\,\mathrm{lo}\cdots}{}$$

```
[Out] -(-1)^(1/3)*d^(2/3)*p^2*ln((-1)^(1/3)*(d^(1/3)+e^(1/3)*x)/(1+(-1)^(1/3))/d^
(1/3))*ln(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/e^(2/3)+(-1)^(2/3)*d^(2/3)*p^2*ln((
-1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))*ln(d^(1/3)
+(-1)^(2/3)*e^(1/3)*x)/e^(2/3)+(-1)^(2/3)*d^(2/3)*p^2*ln(-(-1)^(2/3)*(d^(1/
3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/e^(2
/3)-(-1)^(2/3)*d^(2/3)*p^2*ln((-1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+
(-1)^(1/3))/d^(1/3))*ln((d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/
3))/e^(2/3)-(-1)^(2/3)*d^(2/3)*p^2*ln(-(-1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-
1)^(2/3))/d^(1/3))*ln((d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3)
)/e^(2/3)+(-1)^(1/3)*d^(2/3)*p^2*ln(-(-1)^(1/3)*((-1)^(2/3)*d^(1/3)+e^(1/3)
*x)/(1-(-1)^(2/3))/d^(1/3))*ln(-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(
1-(-1)^(2/3))/d^(1/3))/e^(2/3)-(-1)^(1/3)*d^(2/3)*p^2*ln(d^(1/3)-(-1)^(1/3)
*e^(1/3)*x)*ln(-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^
(1/3))/e^(2/3)+(-1)^(1/3)*d^(2/3)*p*ln(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)*ln(c*(
e*x^3+d)^p)/e^(2/3)+d^(2/3)*p^2*polylog(2,(d^(1/3)+e^(1/3)*x)/(1+(-1)^(1/3)
)/d^(1/3))/e^(2/3)-3/2*p*x^2*ln(c*(e*x^3+d)^p)-(-1)^(2/3)*d^(2/3)*p*ln(d^(1
/3)+(-1)^(2/3)*e^(1/3)*x)*ln(c*(e*x^3+d)^p)/e^(2/3)+1/2*x^2*ln(c*(e*x^3+d)^
p)^2+d^(2/3)*p^2*ln(d^(1/3)+e^(1/3)*x)*ln((-(-1)^(2/3)*d^(1/3)-e^(1/3)*x)/(
1-(-1)^(2/3))/d^(1/3))/e^(2/3)+d^(2/3)*p^2*ln(d^(1/3)+e^(1/3)*x)*ln((-1)^(1
/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/e^(2/3)-d^(2/3)*
p*ln(d^(1/3)+e^(1/3)*x)*ln(c*(e*x^3+d)^p)/e^(2/3)-1/2*(-1)^(1/3)*d^(2/3)*p^
2*ln(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)^2/e^(2/3)+1/2*(-1)^(2/3)*d^(2/3)*p^2*ln(
d^(1/3)+(-1)^(2/3)*e^(1/3)*x)^2/e^(2/3)+3/2*d^(2/3)*p^2*arctan(1/3*(d^(1/3)
-2*e^(1/3)*x)/d^(1/3)*3^(1/2))*3^(1/2)/e^(2/3)-(-1)^(2/3)*d^(2/3)*p^2*polyl
og(2,-(-1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/e^(2/3)-(-1)^(
1/3)*d^(2/3)*p^2*polylog(2,(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^
(1/3))/e^(2/3)-(-1)^(2/3)*d^(2/3)*p^2*polylog(2,(-1)^(1/3)*(d^(1/3)-(-1)^(1
/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/e^(2/3)+(-1)^(1/3)*d^(2/3)*p^2*polyl
og(2,-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/e^(
2/3)+3/2*d^(2/3)*p^2*ln(d^(1/3)+e^(1/3)*x)/e^(2/3)+1/2*d^(2/3)*p^2*ln(d^(1/
3)+e^(1/3)*x)^2/e^(2/3)-3/4*d^(2/3)*p^2*ln(d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3
)*x^2)/e^(2/3)+9/4*p^2*x^2+d^(2/3)*p^2*polylog(2,2*(d^(1/3)+e^(1/3)*x)/d^(1
/3)/(3-I*3^(1/2)))/e^(2/3)
```

Rubi [A]

time = 1.30, antiderivative size = 1300, normalized size of antiderivative = 1.00, number of

steps used = 49, number of rules used = 19, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.187$,
Rules used = {2507, 2526, 2505, 327, 298, 31, 648, 631, 210, 642, 2512, 266, 2463, 2437, 2338, 2441, 2440, 2438, 12}

Antiderivative was successfully verified.

```
[In] Int[x*Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] (9*p^2*x^2)/4 + (3*Sqrt[3]*d^(2/3)*p^2*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt
[3]*d^(1/3))])/(2*e^(2/3)) + (3*d^(2/3)*p^2*Log[d^(1/3) + e^(1/3)*x])/(2*e^
(2/3)) + (d^(2/3)*p^2*Log[d^(1/3) + e^(1/3)*x]^2)/(2*e^(2/3)) + (d^(2/3)*p^
2*Log[d^(1/3) + e^(1/3)*x]*Log[-(((-1)^(2/3)*d^(1/3) + e^(1/3)*x)/((1 - (-1
)^(2/3))*d^(1/3)))])/e^(2/3) - ((-1)^(1/3)*d^(2/3)*p^2*Log[((-1)^(1/3)*(d^(
1/3) + e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^(1/3) - (-1)^(1/3)*e^(
1/3)*x])/e^(2/3) - ((-1)^(1/3)*d^(2/3)*p^2*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)
*x]^2)/(2*e^(2/3)) + ((-1)^(2/3)*d^(2/3)*p^2*Log[-(((-1)^(2/3)*(d^(1/3) + e
^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))]*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x]
)/e^(2/3) + ((-1)^(2/3)*d^(2/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e
^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x])
/e^(2/3) + ((-1)^(2/3)*d^(2/3)*p^2*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x]^2)/(
2*e^(2/3)) - ((-1)^(2/3)*d^(2/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*
e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[(d^(1/3) + (-1)^(2/3)*e^(1/3)*x
)/((1 + (-1)^(1/3))*d^(1/3))])/e^(2/3) + (d^(2/3)*p^2*Log[d^(1/3) + e^(1/3)
*x]*Log[((-1)^(1/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(
1/3))])/e^(2/3) - ((-1)^(1/3)*d^(2/3)*p^2*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*
x]*Log[-(((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 - (-1)^(2/3))*d^
(1/3)))])/e^(2/3) - (3*d^(2/3)*p^2*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3
)*x^2])/(4*e^(2/3)) - (3*p*x^2*Log[c*(d + e*x^3)^p])/2 - (d^(2/3)*p*Log[d^(
1/3) + e^(1/3)*x]*Log[c*(d + e*x^3)^p])/e^(2/3) + ((-1)^(1/3)*d^(2/3)*p*Log
[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/e^(2/3) - ((-1)^(2/3
)*d^(2/3)*p*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/e^(2/
3) + (x^2*Log[c*(d + e*x^3)^p]^2)/2 + (d^(2/3)*p^2*PolyLog[2, (d^(1/3) + e^
(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/e^(2/3) + (d^(2/3)*p^2*PolyLog[2, (2*
(d^(1/3) + e^(1/3)*x))/((3 - I*Sqrt[3])*d^(1/3))])/e^(2/3) - ((-1)^(1/3)*d^
(2/3)*p^2*PolyLog[2, -(((-1)^(1/3)*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/((1 -
(-1)^(2/3))*d^(1/3)))])/e^(2/3) - ((-1)^(1/3)*d^(2/3)*p^2*PolyLog[2, (d^(1/
3) - (-1)^(1/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/e^(2/3) - ((-1)^(2/
3)*d^(2/3)*p^2*PolyLog[2, ((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1
 + (-1)^(1/3))*d^(1/3))])/e^(2/3) + ((-1)^(2/3)*d^(2/3)*p^2*PolyLog[2, (d^(
1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))])/e^(2/3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
```

```
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int x\log^2\left(c\left(d+ex^3\right)^p\right)\,dx=\frac{1}{2}x^2\log^2\left(c\left(d+ex^3\right)^p\right)-(3ep)\int\frac{x^4\log\left(c\left(d+ex^3\right)^p\right)}{d+ex^3}\,dx$$

$$=\frac{1}{2}x^2\log^2\left(c\left(d+ex^3\right)^p\right)-(3ep)\int\left(\frac{x\log\left(c\left(d+ex^3\right)^p\right)}{e}-\frac{dx\log\left(c\left(d+ex^3\right)^p\right)}{e\left(d+ex^3\right)}\cdots\right.$$

$$=\frac{1}{2}x^2\log^2\left(c\left(d+ex^3\right)^p\right)-(3p)\int x\log\left(c\left(d+ex^3\right)^p\right)\,dx+(3dp)\int\frac{x\log\left(c\left(d\cdots\right.\right.}{d+e\cdots}$$

$$=-\frac{3}{2}px^2\log\left(c\left(d+ex^3\right)^p\right)+\frac{1}{2}x^2\log^2\left(c\left(d+ex^3\right)^p\right)+(3dp)\int\left(-\frac{\log\left(c\left(d\cdots\right.\right.}{3\sqrt[3]{d}\,\sqrt[3]{e}\,\left(\cdots\right.}\right.$$

$$=\frac{9p^2x^2}{4}-\frac{3}{2}px^2\log\left(c\left(d+ex^3\right)^p\right)+\frac{1}{2}x^2\log^2\left(c\left(d+ex^3\right)^p\right)-\frac{\left(d^{2/3}p\right)\int\frac{\log\left(c\left(d+e\cdots\right.\right.}{\sqrt[3]{d}+\sqrt[3]{\cdots}}}{\sqrt[3]{e}}\cdots$$

$$=\frac{9p^2x^2}{4}-\frac{3}{2}px^2\log\left(c\left(d+ex^3\right)^p\right)-\frac{d^{2/3}p\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)\log\left(c\left(d+ex^3\right)^p\right)}{e^{2/3}}\cdots$$

$$=\frac{9p^2x^2}{4}+\frac{3d^{2/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{2e^{2/3}}-\frac{3}{2}px^2\log\left(c\left(d+ex^3\right)^p\right)-\frac{d^{2/3}p\log\left(\sqrt[3]{d}\cdots\right.}{\cdots}$$

$$=\frac{9p^2x^2}{4}+\frac{3d^{2/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{2e^{2/3}}-\frac{3d^{2/3}p^2\log\left(d^{2/3}-\sqrt[3]{d}\,\sqrt[3]{e}\,x+e^{2/3}x^2\right)}{4e^{2/3}}\cdots$$

$$=\frac{9p^2x^2}{4}+\frac{3\sqrt{3}\,d^{2/3}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,x}{\sqrt{3}\,\sqrt[3]{d}}\right)}{2e^{2/3}}+\frac{3d^{2/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{2e^{2/3}}+\frac{d^{2/\cdots}}{\cdots}$$

$$=\frac{9p^2x^2}{4}+\frac{3\sqrt{3}\,d^{2/3}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,x}{\sqrt{3}\,\sqrt[3]{d}}\right)}{2e^{2/3}}+\frac{3d^{2/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{2e^{2/3}}+\frac{d^{2/\cdots}}{\cdots}$$

$$=\frac{9p^2x^2}{4}+\frac{3\sqrt{3}\,d^{2/3}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,x}{\sqrt{3}\,\sqrt[3]{d}}\right)}{2e^{2/3}}+\frac{3d^{2/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{2e^{2/3}}+\frac{d^{2/\cdots}}{\cdots}$$

$$=\frac{9p^2x^2}{4}+\frac{3\sqrt{3}\,d^{2/3}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,x}{\sqrt{3}\,\sqrt[3]{d}}\right)}{2e^{2/3}}+\frac{3d^{2/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{2e^{2/3}}+\frac{d^{2/\cdots}}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.80, size = 823, normalized size = 0.64

$$\frac{1}{4}\left(2x^2\log^2\left(c\left(d+ex^3\right)^p\right)+\frac{p\left(-9e^{2/3}px^2\left(-1+{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{ex^3}{d}\right)\right)-6e^{2/3}x^2\log\left(c\left(d+ex^3\right)^p\right)-4d^{2/3}\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)\log\left(c\left(d+ex^3\right)^p\right)+4\sqrt[3]{-1}d^{2/3}\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(c\left(d+ex^3\right)^p\right)-4(-1)^{2/3}d^{2/3}\log\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x\right)\log\left(c\left(d+ex^3\right)^p\right)-2\sqrt[3]{-1}d^{2/3}p\left(\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)\left(2\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)+\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)+2\log\left(\frac{(-1)^{2/3}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{\left(-1+(-1)^{2/3}\right)\sqrt[3]{d}}\right)\right)+2\operatorname{Li}_2\left(\frac{\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)+2\operatorname{Li}_2\left(\frac{-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x}{\left(-1+(-1)^{2/3}\right)\sqrt[3]{d}}\right)\right)+2(-1)^{2/3}d^{2/3}p\left(\log\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x\right)\left(2\log\left(\frac{(-1)^{2/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(-1+(-1)^{2/3}\right)\sqrt[3]{d}}\right)+2\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)+\log\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x\right)\right)+2\operatorname{Li}_2\left(\frac{\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)+2\operatorname{Li}_2\left(\frac{\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)\right)+2d^{2/3}p\left(\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)\left(\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)+2\left(\log\left(\frac{\sqrt[3]{-1}\sqrt[3]{d}-\sqrt[3]{e}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)+\log\left(\frac{i+\sqrt{3}-\frac{2i\sqrt[3]{e}x}{\sqrt[3]{d}}}{3i+\sqrt{3}}\right)\right)\right)+2\operatorname{Li}_2\left(\frac{\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)+2\operatorname{Li}_2\left(\frac{2i\left(1+\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{3i+\sqrt{3}}\right)\right)\right)}{e^{2/3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] (2*x^2*Log[c*(d + e*x^3)^p]^2 + (p*(-9*e^(2/3)*p*x^2*(-1 + Hypergeometric2F
1[2/3, 1, 5/3, -((e*x^3)/d)]) - 6*e^(2/3)*x^2*Log[c*(d + e*x^3)^p] - 4*d^(2
/3)*Log[-d^(1/3) - e^(1/3)*x]*Log[c*(d + e*x^3)^p] + 4*(-1)^(1/3)*d^(2/3)*L
og[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p] - 4*(-1)^(2/3)*d^(
2/3)*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p] - 2*(-1)^(1/
3)*d^(2/3)*p*(Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*(2*Log[((-1)^(1/3)*(d^(1
/3) + e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))] + Log[-d^(1/3) + (-1)^(1/3)*e
^(1/3)*x] + 2*Log[((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((-1 + (-1)
^(2/3))*d^(1/3))]) + 2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1/3)*x)/((1 + (-
1)^(1/3))*d^(1/3))] + 2*PolyLog[2, (-d^(1/3) + (-1)^(1/3)*e^(1/3)*x)/((-1 +
 (-1)^(2/3))*d^(1/3))]) + 2*(-1)^(2/3)*d^(2/3)*p*(Log[-d^(1/3) - (-1)^(2/3)
*e^(1/3)*x]*(2*Log[((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/((-1 + (-1)^(2/3))*d^
(1/3))] + 2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 + (-1)^(1
/3))*d^(1/3))] + Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]) + 2*PolyLog[2, (d^(1
/3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + 2*PolyLog[2, (d^(
1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))]) + 2*d^(2/3)*p*(Lo
g[-d^(1/3) - e^(1/3)*x]*(Log[-d^(1/3) - e^(1/3)*x] + 2*(Log[((-1)^(1/3)*d^(
1/3) - e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + Log[(I + Sqrt[3] - ((2*I)*e
^(1/3)*x)/d^(1/3))/(3*I + Sqrt[3])])) + 2*PolyLog[2, (d^(1/3) + e^(1/3)*x)/
((1 + (-1)^(1/3))*d^(1/3))] + 2*PolyLog[2, ((2*I)*(1 + (e^(1/3)*x)/d^(1/3))
)/(3*I + Sqrt[3])])))/e^(2/3))/4
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int x \ln \left(c \left(e\, x^{3}+d \right)^{p}\right)^{2} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(e*x^3+d)^p)^2,x)
```

```
[Out] int(x*ln(c*(e*x^3+d)^p)^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(x*log((x^3*e + d)^p*c)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \log \left(c(d + ex^3)^p\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Integral(x*log(c*(d + e*x**3)**p)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(x*log((x^3*e + d)^p*c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x \ln \left(c \left(e\,x^3 + d\right)^p\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(c*(d + e*x^3)^p)^2,x)
```

```
[Out] int(x*log(c*(d + e*x^3)^p)^2, x)
```

3.134 $\int \log^2\left(c(d+ex^3)^p\right)\,dx$

Optimal. Leaf size=1304

$$18p^2x+\frac{6\sqrt{3}\,\sqrt[3]{d}\,p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,x}{\sqrt{3}\,\sqrt[3]{d}}\right)}{\sqrt[3]{e}}-\frac{\sqrt[3]{d}\,p^2\log^2\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)}{\sqrt[3]{e}}-\frac{6\sqrt[3]{d}\,p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\sqrt[3]{e}}-\frac{2\sqrt[3]{d}\cdots}{\cdots}$$

```
[Out] x*ln(c*(e*x^3+d)^p)^2+18*p^2*x-6*p*x*ln(c*(e*x^3+d)^p)-2*(-1)^(2/3)*d^(1/3)
*p^2*ln((-1)^(1/3)*(d^(1/3)+e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))*ln(-d^(1/3)+
(-1)^(1/3)*e^(1/3)*x)/e^(1/3)+2*(-1)^(1/3)*d^(1/3)*p^2*ln(-(-1)^(2/3)*(d^(1
/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln(-d^(1/3)-(-1)^(2/3)*e^(1/3)*x)/e^
(1/3)+2*(-1)^(1/3)*d^(1/3)*p^2*ln((-1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)
/(1+(-1)^(1/3))/d^(1/3))*ln(-d^(1/3)-(-1)^(2/3)*e^(1/3)*x)/e^(1/3)-2*(-1)^(
1/3)*d^(1/3)*p^2*ln((-1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+(-1)^(1/3)
)/d^(1/3))*ln((d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/e^(1/3
)-2*(-1)^(1/3)*d^(1/3)*p^2*ln(-(-1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3)
)/d^(1/3))*ln((d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/e^(1/3
)+2*(-1)^(2/3)*d^(1/3)*p^2*ln(-(-1)^(1/3)*((-1)^(2/3)*d^(1/3)+e^(1/3)*x)/(1
-(-1)^(2/3))/d^(1/3))*ln(-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1)
^(2/3))/d^(1/3))/e^(1/3)-2*(-1)^(2/3)*d^(1/3)*p^2*ln(-d^(1/3)+(-1)^(1/3)*e^
(1/3)*x)*ln(-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/
3))/e^(1/3)+2*(-1)^(2/3)*d^(1/3)*p*ln(-d^(1/3)+(-1)^(1/3)*e^(1/3)*x)*ln(c*(
e*x^3+d)^p)/e^(1/3)-2*(-1)^(1/3)*d^(1/3)*p*ln(-d^(1/3)-(-1)^(2/3)*e^(1/3)*x
)*ln(c*(e*x^3+d)^p)/e^(1/3)+(-1)^(1/3)*d^(1/3)*p^2*ln(-d^(1/3)-(-1)^(2/3)*e
^(1/3)*x)^2/e^(1/3)-(-1)^(2/3)*d^(1/3)*p^2*ln(-d^(1/3)+(-1)^(1/3)*e^(1/3)*x
)^2/e^(1/3)-2*d^(1/3)*p^2*ln(-d^(1/3)-e^(1/3)*x)*ln((-(-1)^(2/3)*d^(1/3)-e^
(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/e^(1/3)-2*d^(1/3)*p^2*ln(-d^(1/3)-e^(1/3)*
x)*ln((-1)^(1/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/e^(
1/3)+2*d^(1/3)*p*ln(-d^(1/3)-e^(1/3)*x)*ln(c*(e*x^3+d)^p)/e^(1/3)+6*d^(1/3)
*p^2*arctan(1/3*(d^(1/3)-2*e^(1/3)*x)/d^(1/3)*3^(1/2))*3^(1/2)/e^(1/3)-2*d^
(1/3)*p^2*polylog(2,(d^(1/3)+e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/e^(1/3)-2*(
-1)^(1/3)*d^(1/3)*p^2*polylog(2,-(-1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/
3))/d^(1/3))/e^(1/3)-2*(-1)^(2/3)*d^(1/3)*p^2*polylog(2,(d^(1/3)-(-1)^(1/3)
*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/e^(1/3)-2*(-1)^(1/3)*d^(1/3)*p^2*polylo
g(2,(-1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/e^(1/
3)+2*(-1)^(2/3)*d^(1/3)*p^2*polylog(2,-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1/
3)*x)/(1-(-1)^(2/3))/d^(1/3))/e^(1/3)-d^(1/3)*p^2*ln(-d^(1/3)-e^(1/3)*x)^2/
e^(1/3)+3*d^(1/3)*p^2*ln(d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/e^(1/3)-2*d
^(1/3)*p^2*polylog(2,2*(d^(1/3)+e^(1/3)*x)/d^(1/3)/(3-I*3^(1/2)))/e^(1/3)-6
*d^(1/3)*p^2*ln(d^(1/3)+e^(1/3)*x)/e^(1/3)
```

Rubi [A]

time = 1.17, antiderivative size = 1310, normalized size of antiderivative = 1.00, number of steps used = 49, number of rules used = 20, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.429$, Rules used = {2500, 2526, 2498, 327, 206, 31, 648, 631, 210, 642, 2521, 2512, 266, 2463, 2437, 2338, 2441, 2440, 2438, 12}

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] 18*p^2*x + (6*Sqrt[3]*d^(1/3)*p^2*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d
^(1/3))])/e^(1/3) - (d^(1/3)*p^2*Log[-d^(1/3) - e^(1/3)*x]^2)/e^(1/3) - (6*
d^(1/3)*p^2*Log[d^(1/3) + e^(1/3)*x])/e^(1/3) - (2*d^(1/3)*p^2*Log[-d^(1/3)
 - e^(1/3)*x]*Log[-(((-1)^(2/3)*d^(1/3) + e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1
/3)))])/e^(1/3) - (2*(-1)^(2/3)*d^(1/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) + e^(1
/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x])/e
^(1/3) - ((-1)^(2/3)*d^(1/3)*p^2*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]^2)/e^
(1/3) + (2*(-1)^(1/3)*d^(1/3)*p^2*Log[-(((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/
((1 - (-1)^(2/3))*d^(1/3)))]*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x])/e^(1/3)
+ (2*(-1)^(1/3)*d^(1/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x
))/((1 + (-1)^(1/3))*d^(1/3))]*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x])/e^(1/3
) + ((-1)^(1/3)*d^(1/3)*p^2*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]^2)/e^(1/3)
 - (2*(-1)^(1/3)*d^(1/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*
x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[(d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 +
(-1)^(1/3))*d^(1/3))])/e^(1/3) - (2*d^(1/3)*p^2*Log[-d^(1/3) - e^(1/3)*x]*L
og[((-1)^(1/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))
])/e^(1/3) - (2*(-1)^(2/3)*d^(1/3)*p^2*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]
*Log[-(((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1
/3)))])/e^(1/3) + (3*d^(1/3)*p^2*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*
x^2])/e^(1/3) - 6*p*x*Log[c*(d + e*x^3)^p] + (2*d^(1/3)*p*Log[-d^(1/3) - e^
(1/3)*x]*Log[c*(d + e*x^3)^p])/e^(1/3) + (2*(-1)^(2/3)*d^(1/3)*p*Log[-d^(1/
3) + (-1)^(1/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/e^(1/3) - (2*(-1)^(1/3)*d^
(1/3)*p*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/e^(1/3)
+ x*Log[c*(d + e*x^3)^p]^2 - (2*d^(1/3)*p^2*PolyLog[2, (d^(1/3) + e^(1/3)*x
)/((1 + (-1)^(1/3))*d^(1/3))])/e^(1/3) - (2*d^(1/3)*p^2*PolyLog[2, (2*(d^(1
/3) + e^(1/3)*x))/((3 - I*Sqrt[3])*d^(1/3))])/e^(1/3) - (2*(-1)^(2/3)*d^(1/
3)*p^2*PolyLog[2, -(((-1)^(1/3)*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/((1 - (-1
)^(2/3))*d^(1/3)))])/e^(1/3) - (2*(-1)^(2/3)*d^(1/3)*p^2*PolyLog[2, (d^(1/3
) - (-1)^(1/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/e^(1/3) - (2*(-1)^(1
/3)*d^(1/3)*p^2*PolyLog[2, ((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((
1 + (-1)^(1/3))*d^(1/3))])/e^(1/3) + (2*(-1)^(1/3)*d^(1/3)*p^2*PolyLog[2, (
d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))])/e^(1/3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2500

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[x^n*(
(a + b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int \log^2\left(c\left(d+ex^3\right)^p\right)\,dx = x\log^2\left(c\left(d+ex^3\right)^p\right) - (6ep)\int \frac{x^3\log\left(c\left(d+ex^3\right)^p\right)}{d+ex^3}\,dx$$

$$= x\log^2\left(c\left(d+ex^3\right)^p\right) - (6ep)\int \left(\frac{\log\left(c\left(d+ex^3\right)^p\right)}{e} - \frac{d\log\left(c\left(d+ex^3\right)^p\right)}{e\left(d+ex^3\right)}\right)dx$$

$$= x\log^2\left(c\left(d+ex^3\right)^p\right) - (6p)\int \log\left(c\left(d+ex^3\right)^p\right)\,dx + (6dp)\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{d+ex^3}$$

$$= -6px\log\left(c\left(d+ex^3\right)^p\right) + x\log^2\left(c\left(d+ex^3\right)^p\right) + (6dp)\int \left(-\frac{\log\left(c\left(d+ex^3\right)\right.}{3d^{2/3}\left(-\sqrt[3]{d}-\sqrt[3]{\right.}}\right.$$

$$= 18p^2x - 6px\log\left(c\left(d+ex^3\right)^p\right) + x\log^2\left(c\left(d+ex^3\right)^p\right) - \left(2\sqrt[3]{d}\,p\right)\int \frac{\log\left(c(d+\right.}{-\sqrt[3]{d}-}$$

$$= 18p^2x - 6px\log\left(c\left(d+ex^3\right)^p\right) + \frac{2\sqrt[3]{d}\,p\log\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt[3]{e}}$$

$$= 18p^2x - \frac{6\sqrt[3]{d}\,p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\sqrt[3]{e}} - 6px\log\left(c\left(d+ex^3\right)^p\right) + \frac{2\sqrt[3]{d}\,p\log\left(-\sqrt[3]{\right.}}{}$$

$$= 18p^2x - \frac{6\sqrt[3]{d}\,p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\sqrt[3]{e}} + \frac{3\sqrt[3]{d}\,p^2\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}\,x+e^{2/3}x^2\right)}{\sqrt[3]{e}}$$

$$= 18p^2x + \frac{6\sqrt{3}\sqrt[3]{d}\,p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt[3]{e}} - \frac{6\sqrt[3]{d}\,p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\sqrt[3]{e}} - \frac{2\sqrt[3]{\,}}{}$$

$$= 18p^2x + \frac{6\sqrt{3}\sqrt[3]{d}\,p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt[3]{e}} - \frac{\sqrt[3]{d}\,p^2\log^2\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)}{\sqrt[3]{e}} - \frac{6\sqrt{\,}}{}$$

$$= 18p^2x + \frac{6\sqrt{3}\sqrt[3]{d}\,p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt[3]{e}} - \frac{\sqrt[3]{d}\,p^2\log^2\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)}{\sqrt[3]{e}} - \frac{6\sqrt{\,}}{}$$

$$= 18p^2x + \frac{6\sqrt{3}\sqrt[3]{d}\,p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt[3]{e}} - \frac{\sqrt[3]{d}\,p^2\log^2\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)}{\sqrt[3]{e}} - \frac{6\sqrt{\,}}{}$$

Mathematica [A]
time = 0.46, size = 1090, normalized size = 0.84

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] x*Log[c*(d + e*x^3)^p]^2 - 6*e*p*(-1/2*(p*((6*x)/e - ((2*d^(1/3)*Log[d^(1/3
) + e^(1/3)*x])/e^(1/3) - d^(1/3)*((2*Sqrt[3]*ArcTan[(d^(1/3) - 2*e^(1/3)*x
)/(Sqrt[3]*d^(1/3))])/e^(1/3) + Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x
^2]/e^(1/3)))/e)) + (x*Log[c*(d + e*x^3)^p])/e - (d^(1/3)*Log[-d^(1/3) - e^
(1/3)*x]*Log[c*(d + e*x^3)^p])/(3*e^(4/3)) - ((-1)^(2/3)*d^(1/3)*Log[-d^(1/
3) + (-1)^(1/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/(3*e^(4/3)) + ((-1)^(1/3)*
d^(1/3)*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/(3*e^(4/
3)) + (d^(1/3)*p*(Log[-d^(1/3) - e^(1/3)*x]^2/e^(1/3) + (2*Log[-d^(1/3) - e
^(1/3)*x]*Log[-(((-1)^(2/3)*d^(1/3) + e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))
)])/e^(1/3) + (2*Log[-d^(1/3) - e^(1/3)*x]*Log[((-1)^(1/3)*(d^(1/3) + (-1)^
(2/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))])/e^(1/3) + (2*PolyLog[2, (d^(
1/3) + e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/e^(1/3) + (2*PolyLog[2, (d^(
1/3) + e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))])/e^(1/3)))/(6*e) + ((-1)^(2/3
)*d^(1/3)*p*((2*Log[((-1)^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + (-1)^(1/3))*d^
(1/3))]*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x])/e^(1/3) + Log[-d^(1/3) + (-1)
^(1/3)*e^(1/3)*x]^2/e^(1/3) + (2*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*Log[-
(((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))]
)/e^(1/3) + (2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1/3)*x)/((1 + (-1)^(1/3)
)*d^(1/3))])/e^(1/3) + (2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1/3)*x)/((1 -
 (-1)^(2/3))*d^(1/3))])/e^(1/3)))/(6*e) - ((-1)^(1/3)*d^(1/3)*p*((2*Log[-((
(-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))]*Log[-d^(1/3)
 - (-1)^(2/3)*e^(1/3)*x])/e^(1/3) + (2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3
)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)
*x])/e^(1/3) + Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]^2/e^(1/3) + (2*PolyLog[
2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/e^(1/3) +
(2*PolyLog[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))])
/e^(1/3)))/(6*e))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \ln \left(c\left(e\,x^{3}+d\right)^{p}\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^3+d)^p)^2,x)
```

```
[Out] int(ln(c*(e*x^3+d)^p)^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(log((x^3*e + d)^p*c)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log \left(c(d + ex^3)^p\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Integral(log(c*(d + e*x**3)**p)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(log((x^3*e + d)^p*c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\left(e\,x^3+d\right)^p\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^3)^p)^2,x)
```

```
[Out] int(log(c*(d + e*x^3)^p)^2, x)
```

3.135 $\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^2}\,dx$

Optimal. Leaf size=1137

$$\frac{\sqrt[3]{e}\,p^2\log^2\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\sqrt[3]{d}}+\frac{2\sqrt[3]{e}\,p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)\log\left(-\frac{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}\,x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{\sqrt[3]{d}}-\frac{2\sqrt[3]{-1}\,\sqrt[3]{e}\,p^2\log\left(\frac{\sqrt[3]{-}\ldots}{(1\ldots}\right.}{}$$

```
[Out] e^(1/3)*p^2*ln(d^(1/3)+e^(1/3)*x)^2/d^(1/3)+2*e^(1/3)*p^2*ln(d^(1/3)+e^(1/3
)*x)*ln((-(-1)^(2/3)*d^(1/3)-e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(1/3)-2*(
-1)^(1/3)*e^(1/3)*p^2*ln((-1)^(1/3)*(d^(1/3)+e^(1/3)*x)/(1+(-1)^(1/3))/d^(1
/3))*ln(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/d^(1/3)-(-1)^(1/3)*e^(1/3)*p^2*ln(d^(
1/3)-(-1)^(1/3)*e^(1/3)*x)^2/d^(1/3)+2*(-1)^(2/3)*e^(1/3)*p^2*ln(-(-1)^(2/3
)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln(d^(1/3)+(-1)^(2/3)*e^(1/3)
*x)/d^(1/3)+2*(-1)^(2/3)*e^(1/3)*p^2*ln((-1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1
/3)*x)/(1+(-1)^(1/3))/d^(1/3))*ln(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/d^(1/3)+(-1
)^(2/3)*e^(1/3)*p^2*ln(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)^2/d^(1/3)-2*(-1)^(2/3)
*e^(1/3)*p^2*ln((-1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^
(1/3))*ln((d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(1/3)+2*
e^(1/3)*p^2*ln(d^(1/3)+e^(1/3)*x)*ln((-1)^(1/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)
*x)/(1+(-1)^(1/3))/d^(1/3))/d^(1/3)-2*(-1)^(2/3)*e^(1/3)*p^2*ln(-(-1)^(2/3)
*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln((d^(1/3)+(-1)^(2/3)*e^(1/3)
*x)/(1-(-1)^(2/3))/d^(1/3))/d^(1/3)+2*(-1)^(1/3)*e^(1/3)*p^2*ln(-(-1)^(1/3)
*((-1)^(2/3)*d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln(-(-1)^(2/3)*(d^(
1/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(1/3)-2*(-1)^(1/3)*e^(
1/3)*p^2*ln(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)*ln(-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3
)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(1/3)-2*e^(1/3)*p*ln(d^(1/3)+e^(1/3)
*x)*ln(c*(e*x^3+d)^p)/d^(1/3)+2*(-1)^(1/3)*e^(1/3)*p*ln(d^(1/3)-(-1)^(1/3)*
e^(1/3)*x)*ln(c*(e*x^3+d)^p)/d^(1/3)-2*(-1)^(2/3)*e^(1/3)*p*ln(d^(1/3)+(-1)
^(2/3)*e^(1/3)*x)*ln(c*(e*x^3+d)^p)/d^(1/3)-ln(c*(e*x^3+d)^p)^2/x+2*e^(1/3)
*p^2*polylog(2,(d^(1/3)+e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(1/3)-2*(-1)^(
2/3)*e^(1/3)*p^2*polylog(2,-(-1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d
^(1/3))/d^(1/3)-2*(-1)^(1/3)*e^(1/3)*p^2*polylog(2,(d^(1/3)-(-1)^(1/3)*e^(1
/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(1/3)-2*(-1)^(2/3)*e^(1/3)*p^2*polylog(2,(
-1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(1/3)+2*
(-1)^(1/3)*e^(1/3)*p^2*polylog(2,-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)
/(1-(-1)^(2/3))/d^(1/3))/d^(1/3)+2*e^(1/3)*p^2*polylog(2,2*(d^(1/3)+e^(1/3)
*x)/d^(1/3)/(3-I*3^(1/2)))/d^(1/3)
```

Rubi [A]
time = 0.90, antiderivative size = 1143, normalized size of antiderivative = 1.01, number of steps used = 39, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$,

Rules used = {2507, 2526, 2512, 266, 2463, 2437, 2338, 2441, 2440, 2438, 12}

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^3)^p]^2/x^2,x]
```

```
[Out] (e^(1/3)*p^2*Log[d^(1/3) + e^(1/3)*x]^2)/d^(1/3) + (2*e^(1/3)*p^2*Log[d^(1/
3) + e^(1/3)*x]*Log[-(((-1)^(2/3)*d^(1/3) + e^(1/3)*x)/((1 - (-1)^(2/3))*d^
(1/3)))])/d^(1/3) - (2*(-1)^(1/3)*e^(1/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) + e^
(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x])/
d^(1/3) - ((-1)^(1/3)*e^(1/3)*p^2*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]^2)/d^
(1/3) + (2*(-1)^(2/3)*e^(1/3)*p^2*Log[-(((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/
((1 - (-1)^(2/3))*d^(1/3)))]*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x])/d^(1/3) +
 (2*(-1)^(2/3)*e^(1/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x)
)/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x])/d^(1/3)
+ ((-1)^(2/3)*e^(1/3)*p^2*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x]^2)/d^(1/3) -
(2*(-1)^(2/3)*e^(1/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))
/((1 + (-1)^(1/3))*d^(1/3))]*Log[(d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1
)^(1/3))*d^(1/3))])/d^(1/3) + (2*e^(1/3)*p^2*Log[d^(1/3) + e^(1/3)*x]*Log[(
(-1)^(1/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))])/d
^(1/3) - (2*(-1)^(1/3)*e^(1/3)*p^2*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]*Log[
-(((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))
])/d^(1/3) - (2*e^(1/3)*p*Log[d^(1/3) + e^(1/3)*x]*Log[c*(d + e*x^3)^p])/d^
(1/3) + (2*(-1)^(1/3)*e^(1/3)*p*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]*Log[c*(
d + e*x^3)^p])/d^(1/3) - (2*(-1)^(2/3)*e^(1/3)*p*Log[d^(1/3) + (-1)^(2/3)*e
^(1/3)*x]*Log[c*(d + e*x^3)^p])/d^(1/3) - Log[c*(d + e*x^3)^p]^2/x + (2*e^(
1/3)*p^2*PolyLog[2, (d^(1/3) + e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/d^(1
/3) + (2*e^(1/3)*p^2*PolyLog[2, (2*(d^(1/3) + e^(1/3)*x))/((3 - I*Sqrt[3])*
d^(1/3))])/d^(1/3) - (2*(-1)^(1/3)*e^(1/3)*p^2*PolyLog[2, -(((-1)^(1/3)*((-
1)^(2/3)*d^(1/3) + e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))])/d^(1/3) - (2*(
-1)^(1/3)*e^(1/3)*p^2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1/3)*x)/((1 + (-1
)^(1/3))*d^(1/3))])/d^(1/3) - (2*(-1)^(2/3)*e^(1/3)*p^2*PolyLog[2, ((-1)^(1
/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))])/d^(1/3)
+ (2*(-1)^(2/3)*e^(1/3)*p^2*PolyLog[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1
 - (-1)^(2/3))*d^(1/3))])/d^(1/3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^2}\,dx = -\frac{\log^2\left(c(d+ex^3)^p\right)}{x} + (6ep)\int \frac{x\log\left(c(d+ex^3)^p\right)}{d+ex^3}\,dx$$

$$= -\frac{\log^2\left(c(d+ex^3)^p\right)}{x} + (6ep)\int \left(-\frac{\log\left(c(d+ex^3)^p\right)}{3\sqrt[3]{d}\,\sqrt[3]{e}\,\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)} - \frac{(-1)^{2/3}\log\left(\cdots\right.}{3\sqrt[3]{d}\,\sqrt[3]{e}\,\left(\sqrt[3]{d}\cdots\right.}\right.$$

$$= -\frac{\log^2\left(c(d+ex^3)^p\right)}{x} - \frac{\left(2e^{2/3}p\right)\int \frac{\log\left(c(d+ex^3)^p\right)}{\sqrt[3]{d}+\sqrt[3]{e}\,x}\,dx}{\sqrt[3]{d}} + \frac{\left(2\sqrt[3]{-1}\,e^{2/3}p\right)\int \frac{\log(c(d+ex\cdots}{\sqrt[3]{d}+(-1)^{2/\cdots}}}{\sqrt[3]{d}}$$

$$= -\frac{2\sqrt[3]{e}\,p\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)\log\left(c(d+ex^3)^p\right)}{\sqrt[3]{d}} + \frac{2\sqrt[3]{-1}\,\sqrt[3]{e}\,p\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\cdots\right.}{\sqrt[3]{d}}$$

$$= -\frac{2\sqrt[3]{e}\,p\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)\log\left(c(d+ex^3)^p\right)}{\sqrt[3]{d}} + \frac{2\sqrt[3]{-1}\,\sqrt[3]{e}\,p\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\cdots\right.}{\sqrt[3]{d}}$$

$$= -\frac{2\sqrt[3]{e}\,p\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)\log\left(c(d+ex^3)^p\right)}{\sqrt[3]{d}} + \frac{2\sqrt[3]{-1}\,\sqrt[3]{e}\,p\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\cdots\right.}{\sqrt[3]{d}}$$

$$= \frac{2\sqrt[3]{e}\,p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)\log\left(-\frac{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}\,x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{\sqrt[3]{d}} - \frac{2\sqrt[3]{-1}\,\sqrt[3]{e}\,p^2\log\left(\frac{\sqrt[3]{-\cdots}}{(1\cdots}\right.}{\cdots}$$

$$= \frac{\sqrt[3]{e}\,p^2\log^2\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\sqrt[3]{d}} + \frac{2\sqrt[3]{e}\,p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)\log\left(-\frac{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}\,x\cdots}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right.}{\sqrt[3]{d}}$$

$$= \frac{\sqrt[3]{e}\,p^2\log^2\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\sqrt[3]{d}} + \frac{2\sqrt[3]{e}\,p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)\log\left(-\frac{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}\,x\cdots}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right.}{\sqrt[3]{d}}$$

$$= \frac{\sqrt[3]{e}\,p^2\log^2\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\sqrt[3]{d}} + \frac{2\sqrt[3]{e}\,p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)\log\left(-\frac{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}\,x\cdots}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right.}{\sqrt[3]{d}}$$

Mathematica [A]

time = 0.58, size = 742, normalized size = 0.65

$$-\frac{\log^2\left(c(d+ex^3)^p\right)}{x}-\frac{\sqrt[3]{e}\,p\left(2\log\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)\log\left(c(d+ex^3)^p\right)-2\sqrt[3]{-1}\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}\,x\right)\log\left(c(d+ex^3)^p\right)+2(-1)^{2/3}\log\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}\,x\right)\log\left(c(d+ex^3)^p\right)+\sqrt[3]{-1}\,p\left(\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}\,x\right)\left(2\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)+\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}\,x\right)+2\log\left(\frac{(-1)^{2/3}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}\,x\right)}{\left(-1+(-1)^{2/3}\right)\sqrt[3]{d}}\right)\right)+2\mathrm{Li}_2\left(\frac{\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)+2\mathrm{Li}_2\left(\frac{-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}\,x}{\left(-1+(-1)^{2/3}\right)\sqrt[3]{d}}\right)\right)-(-1)^{2/3}p\left(\log\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}\,x\right)\left(2\log\left(\frac{(-1)^{2/3}\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\left(-1+(-1)^{2/3}\right)\sqrt[3]{d}}\right)+2\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}\,x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)+\log\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}\,x\right)\right)+2\mathrm{Li}_2\left(\frac{\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)+2\mathrm{Li}_2\left(\frac{\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}\,x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)\right)-p\left(\log\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)\left(\log\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)+2\left(\log\left(\frac{\sqrt[3]{-1}\sqrt[3]{d}-\sqrt[3]{e}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)+\log\left(\frac{i+\sqrt{3}-\frac{2i\sqrt[3]{e}\,x}{\sqrt[3]{d}}}{3i+\sqrt{3}}\right)\right)\right)+2\mathrm{Li}_2\left(\frac{\sqrt[3]{d}+\sqrt[3]{e}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)+2\mathrm{Li}_2\left(\frac{2i\left(1+\frac{\sqrt[3]{e}\,x}{\sqrt[3]{d}}\right)}{3i+\sqrt{3}}\right)\right)\right)}{\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^3)^p]^2/x^2,x]
```

```
[Out] -(Log[c*(d + e*x^3)^p]^2/x) - (e^(1/3)*p*(2*Log[-d^(1/3) - e^(1/3)*x]*Log[c
*(d + e*x^3)^p] - 2*(-1)^(1/3)*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*Log[c*(
d + e*x^3)^p] + 2*(-1)^(2/3)*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]*Log[c*(d
+ e*x^3)^p] + (-1)^(1/3)*p*(Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*(2*Log[((-
1)^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))] + Log[-d^(1/3)
+ (-1)^(1/3)*e^(1/3)*x] + 2*Log[((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x
))/((-1 + (-1)^(2/3))*d^(1/3))]) + 2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1/
3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + 2*PolyLog[2, (-d^(1/3) + (-1)^(1/3)*e^(
1/3)*x)/((-1 + (-1)^(2/3))*d^(1/3))]) - (-1)^(2/3)*p*(Log[-d^(1/3) - (-1)^(
2/3)*e^(1/3)*x]*(2*Log[((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/((-1 + (-1)^(2/3)
)*d^(1/3))] + 2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 + (-1
)^(1/3))*d^(1/3))] + Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]) + 2*PolyLog[2, (
d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + 2*PolyLog[2,
(d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))]) - p*(Log[-d^(
1/3) - e^(1/3)*x]*(Log[-d^(1/3) - e^(1/3)*x] + 2*(Log[((-1)^(1/3)*d^(1/3) -
 e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + Log[(I + Sqrt[3] - ((2*I)*e^(1/3)
*x)/d^(1/3))/(3*I + Sqrt[3])])) + 2*PolyLog[2, (d^(1/3) + e^(1/3)*x)/((1 +
(-1)^(1/3))*d^(1/3))] + 2*PolyLog[2, ((2*I)*(1 + (e^(1/3)*x)/d^(1/3)))/(3*I
 + Sqrt[3])])))/d^(1/3)
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c(e\,x^3+d)^p\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^3+d)^p)^2/x^2,x)
```

```
[Out] int(ln(c*(e*x^3+d)^p)^2/x^2,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^2,x, algorithm="fricas")
[Out] integral(log((x^3*e + d)^p*c)^2/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(c(d+ex^3)^p\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**3+d)**p)**2/x**2,x)
[Out] Integral(log(c*(d + e*x**3)**p)**2/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^2,x, algorithm="giac")
[Out] integrate(log((x^3*e + d)^p*c)^2/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln \left(c\,{\left(e\,x^3+d\right)}^p\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^3)^p)^2/x^2,x)
[Out] int(log(c*(d + e*x^3)^p)^2/x^2, x)
```

3.136 $\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^3}\,dx$

Optimal. Leaf size=1170

$$-\frac{e^{2/3}p^2\log^2\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)}{2d^{2/3}}-\frac{e^{2/3}p^2\log\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)\log\left(-\frac{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}\,x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{d^{2/3}}-\frac{(-1)^{2/3}e^{2/3}p^2\log\left(\cdots\right.}{\cdots}$$

```
[Out] -1/2*e^(2/3)*p^2*ln(-d^(1/3)-e^(1/3)*x)^2/d^(2/3)-e^(2/3)*p^2*ln(-d^(1/3)-e
^(1/3)*x)*ln((-(-1)^(2/3)*d^(1/3)-e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(2/3
)-(-1)^(2/3)*e^(2/3)*p^2*ln((-1)^(1/3)*(d^(1/3)+e^(1/3)*x)/(1+(-1)^(1/3))/d
^(1/3))*ln(-d^(1/3)+(-1)^(1/3)*e^(1/3)*x)/d^(2/3)-1/2*(-1)^(2/3)*e^(2/3)*p^
2*ln(-d^(1/3)+(-1)^(1/3)*e^(1/3)*x)^2/d^(2/3)+(-1)^(1/3)*e^(2/3)*p^2*ln(-(-
1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln(-d^(1/3)-(-1)^(2/3)
*e^(1/3)*x)/d^(2/3)+(-1)^(1/3)*e^(2/3)*p^2*ln((-1)^(1/3)*(d^(1/3)-(-1)^(1/3
)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))*ln(-d^(1/3)-(-1)^(2/3)*e^(1/3)*x)/d^(2
/3)+1/2*(-1)^(1/3)*e^(2/3)*p^2*ln(-d^(1/3)-(-1)^(2/3)*e^(1/3)*x)^2/d^(2/3)-
(-1)^(1/3)*e^(2/3)*p^2*ln((-1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+(-1)
^(1/3))/d^(1/3))*ln((d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/
d^(2/3)-e^(2/3)*p^2*ln(-d^(1/3)-e^(1/3)*x)*ln((-1)^(1/3)*(d^(1/3)+(-1)^(2/3
)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(2/3)-(-1)^(1/3)*e^(2/3)*p^2*ln(-(-1
)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln((d^(1/3)+(-1)^(2/3)*
e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(2/3)+(-1)^(2/3)*e^(2/3)*p^2*ln(-(-1)^
(1/3)*((-1)^(2/3)*d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln(-(-1)^(2/3)
*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(2/3)-(-1)^(2/3)*
e^(2/3)*p^2*ln(-d^(1/3)+(-1)^(1/3)*e^(1/3)*x)*ln(-(-1)^(2/3)*(d^(1/3)+(-1)^
(2/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(2/3)+e^(2/3)*p*ln(-d^(1/3)-e^(1
/3)*x)*ln(c*(e*x^3+d)^p)/d^(2/3)+(-1)^(2/3)*e^(2/3)*p*ln(-d^(1/3)+(-1)^(1/3
)*e^(1/3)*x)*ln(c*(e*x^3+d)^p)/d^(2/3)-(-1)^(1/3)*e^(2/3)*p*ln(-d^(1/3)-(-1
)^(2/3)*e^(1/3)*x)*ln(c*(e*x^3+d)^p)/d^(2/3)-1/2*ln(c*(e*x^3+d)^p)^2/x^2-e^
(2/3)*p^2*polylog(2,(d^(1/3)+e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(2/3)-(-1
)^(1/3)*e^(2/3)*p^2*polylog(2,-(-1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3)
)/d^(1/3))/d^(2/3)-(-1)^(2/3)*e^(2/3)*p^2*polylog(2,(d^(1/3)-(-1)^(1/3)*e^(
1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(2/3)-(-1)^(1/3)*e^(2/3)*p^2*polylog(2,(-
1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(2/3)+(-1
)^(2/3)*e^(2/3)*p^2*polylog(2,-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1
-(-1)^(2/3))/d^(1/3))/d^(2/3)-e^(2/3)*p^2*polylog(2,2*(d^(1/3)+e^(1/3)*x)/d
^(1/3)/(3-I*3^(1/2)))/d^(2/3)
```

Rubi [A]

time = 0.88, antiderivative size = 1176, normalized size of antiderivative = 1.01, number of steps used = 39, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {2507, 2521, 2512, 266, 2463, 2437, 2338, 2441, 2440, 2438, 12}

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^3)^p]^2/x^3,x]
```

```
[Out] -1/2*(e^(2/3)*p^2*Log[-d^(1/3) - e^(1/3)*x]^2)/d^(2/3) - (e^(2/3)*p^2*Log[-
d^(1/3) - e^(1/3)*x]*Log[-(((-1)^(2/3)*d^(1/3) + e^(1/3)*x)/((1 - (-1)^(2/3
))*d^(1/3)))])/d^(2/3) - ((-1)^(2/3)*e^(2/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) +
 e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*
x])/d^(2/3) - ((-1)^(2/3)*e^(2/3)*p^2*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]^
2)/(2*d^(2/3)) + ((-1)^(1/3)*e^(2/3)*p^2*Log[-(((-1)^(2/3)*(d^(1/3) + e^(1/
3)*x))/((1 - (-1)^(2/3))*d^(1/3)))]*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x])/d
^(2/3) + ((-1)^(1/3)*e^(2/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1
/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x])/d
^(2/3) + ((-1)^(1/3)*e^(2/3)*p^2*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]^2)/(2
*d^(2/3)) - ((-1)^(1/3)*e^(2/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e
^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[(d^(1/3) + (-1)^(2/3)*e^(1/3)*x)
/((1 + (-1)^(1/3))*d^(1/3))])/d^(2/3) - (e^(2/3)*p^2*Log[-d^(1/3) - e^(1/3)
*x]*Log[((-1)^(1/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(
1/3))])/d^(2/3) - ((-1)^(2/3)*e^(2/3)*p^2*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)
*x]*Log[-(((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 - (-1)^(2/3))*d
^(1/3)))])/d^(2/3) + (e^(2/3)*p*Log[-d^(1/3) - e^(1/3)*x]*Log[c*(d + e*x^3)
^p])/d^(2/3) + ((-1)^(2/3)*e^(2/3)*p*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*L
og[c*(d + e*x^3)^p])/d^(2/3) - ((-1)^(1/3)*e^(2/3)*p*Log[-d^(1/3) - (-1)^(2
/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/d^(2/3) - Log[c*(d + e*x^3)^p]^2/(2*x^
2) - (e^(2/3)*p^2*PolyLog[2, (d^(1/3) + e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3
))])/d^(2/3) - (e^(2/3)*p^2*PolyLog[2, (2*(d^(1/3) + e^(1/3)*x))/((3 - I*Sq
rt[3])*d^(1/3))])/d^(2/3) - ((-1)^(2/3)*e^(2/3)*p^2*PolyLog[2, -(((-1)^(1/3
)*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))])/d^(2/3) -
 ((-1)^(2/3)*e^(2/3)*p^2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1/3)*x)/((1 +
(-1)^(1/3))*d^(1/3))])/d^(2/3) - ((-1)^(1/3)*e^(2/3)*p^2*PolyLog[2, ((-1)^(
1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))])/d^(2/3)
 + ((-1)^(1/3)*e^(2/3)*p^2*PolyLog[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1
- (-1)^(2/3))*d^(1/3))])/d^(2/3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^3}\,dx = -\frac{\log^2\left(c(d+ex^3)^p\right)}{2x^2} + (3ep)\int \frac{\log\left(c(d+ex^3)^p\right)}{d+ex^3}\,dx$$

$$= -\frac{\log^2\left(c(d+ex^3)^p\right)}{2x^2} + (3ep)\int \left(-\frac{\log\left(c(d+ex^3)^p\right)}{3d^{2/3}\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)} - \frac{\log\left(c(d+ex^3\right.}{3d^{2/3}\left(-\sqrt[3]{d}+\sqrt[3]{-\right.}\right.$$

$$= -\frac{\log^2\left(c(d+ex^3)^p\right)}{2x^2} - \frac{(ep)\int \frac{\log(c(d+ex^3)^p)}{-\sqrt[3]{d}-\sqrt[3]{e}\,x}\,dx}{d^{2/3}} - \frac{(ep)\int \frac{\log(c(d+ex^3)^p)}{-\sqrt[3]{d}+\sqrt[3]{-1}\,\sqrt[3]{e}\,x}\,dx}{d^{2/3}} - \frac{(e}{}$$

$$= \frac{e^{2/3}p\log\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)\log\left(c(d+ex^3)^p\right)}{d^{2/3}} + \frac{(-1)^{2/3}e^{2/3}p\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\,\sqrt[3]{}\right.}{d^{2/3}}$$

$$= \frac{e^{2/3}p\log\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)\log\left(c(d+ex^3)^p\right)}{d^{2/3}} + \frac{(-1)^{2/3}e^{2/3}p\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\,\sqrt[3]{}\right.}{d^{2/3}}$$

$$= \frac{e^{2/3}p\log\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)\log\left(c(d+ex^3)^p\right)}{d^{2/3}} + \frac{(-1)^{2/3}e^{2/3}p\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\,\sqrt[3]{}\right.}{d^{2/3}}$$

$$= -\frac{e^{2/3}p^2\log\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)\log\left(-\frac{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}\,x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{d^{2/3}} - \frac{(-1)^{2/3}e^{2/3}p^2\log\left(\frac{\sqrt[3]{-}}{(}\right.}{}$$

$$= -\frac{e^{2/3}p^2\log^2\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)}{2d^{2/3}} - \frac{e^{2/3}p^2\log\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)\log\left(-\frac{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{}}{\left(1-(-1)^{2/3}\right)\sqrt[3]{}}\right.}{d^{2/3}}$$

$$= -\frac{e^{2/3}p^2\log^2\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)}{2d^{2/3}} - \frac{e^{2/3}p^2\log\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)\log\left(-\frac{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{}}{\left(1-(-1)^{2/3}\right)\sqrt[3]{}}\right.}{d^{2/3}}$$

$$= -\frac{e^{2/3}p^2\log^2\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)}{2d^{2/3}} - \frac{e^{2/3}p^2\log\left(-\sqrt[3]{d}-\sqrt[3]{e}\,x\right)\log\left(-\frac{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{}}{\left(1-(-1)^{2/3}\right)\sqrt[3]{}}\right.}{d^{2/3}}$$

Mathematica [A]

time = 0.61, size = 745, normalized size = 0.64

$$\frac{1}{2}\left(-\frac{\log^2\left(c(d+ex^3)^p\right)}{x^2}+\frac{e^{2/3}p\left(2\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)+2(-1)^{2/3}\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)-2\sqrt[3]{-1}\log\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)-(-1)^{2/3}p\left(\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)\left(2\log\left(\frac{\sqrt[3]{-1}(\sqrt[3]{d}+\sqrt[3]{e}x)}{(1+\sqrt[3]{-1})\sqrt[3]{d}}\right)+\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)+2\log\left(\frac{(-1)^{2/3}(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x)}{(-1+(-1)^{2/3})\sqrt[3]{d}}\right)\right)+2\operatorname{Li}_2\left(\frac{\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x}{(1+\sqrt[3]{-1})\sqrt[3]{d}}\right)+2\operatorname{Li}_2\left(\frac{-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x}{(-1+(-1)^{2/3})\sqrt[3]{d}}\right)\right)+\sqrt[3]{-1}p\left(\log\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x\right)\left(2\log\left(\frac{(-1)^{2/3}(\sqrt[3]{d}+\sqrt[3]{e}x)}{(-1+(-1)^{2/3})\sqrt[3]{d}}\right)+2\log\left(\frac{\sqrt[3]{-1}(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x)}{(1+\sqrt[3]{-1})\sqrt[3]{d}}\right)+\log\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x\right)\right)+2\operatorname{Li}_2\left(\frac{\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x}{(1+\sqrt[3]{-1})\sqrt[3]{d}}\right)+2\operatorname{Li}_2\left(\frac{\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x}{(1-(-1)^{2/3})\sqrt[3]{d}}\right)\right)-p\left(\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)\left(\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)+2\left(\log\left(\frac{\sqrt[3]{-1}\sqrt[3]{d}-\sqrt[3]{e}x}{(1+\sqrt[3]{-1})\sqrt[3]{d}}\right)+\log\left(\frac{i+\sqrt{3}-\frac{2i\sqrt[3]{e}x}{\sqrt[3]{d}}}{3i+\sqrt{3}}\right)\right)\right)+2\operatorname{Li}_2\left(\frac{\sqrt[3]{d}+\sqrt[3]{e}x}{(1+\sqrt[3]{-1})\sqrt[3]{d}}\right)+2\operatorname{Li}_2\left(\frac{2i\left(1+\frac{\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{3i+\sqrt{3}}\right)\right)\right)}{d^{2/3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^3)^p]^2/x^3,x]
```

```
[Out] (-(Log[c*(d + e*x^3)^p]^2/x^2) + (e^(2/3)*p*(2*Log[-d^(1/3) - e^(1/3)*x]*Lo
g[c*(d + e*x^3)^p] + 2*(-1)^(2/3)*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*Log[
c*(d + e*x^3)^p] - 2*(-1)^(1/3)*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]*Log[c*
(d + e*x^3)^p] - (-1)^(2/3)*p*(Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*(2*Log[
((-1)^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))] + Log[-d^(1/
3) + (-1)^(1/3)*e^(1/3)*x] + 2*Log[((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3
)*x))/((-1 + (-1)^(2/3))*d^(1/3))]) + 2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^
(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + 2*PolyLog[2, (-d^(1/3) + (-1)^(1/3)*
e^(1/3)*x)/((-1 + (-1)^(2/3))*d^(1/3))]) + (-1)^(1/3)*p*(Log[-d^(1/3) - (-1
)^(2/3)*e^(1/3)*x]*(2*Log[((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/((-1 + (-1)^(2
/3))*d^(1/3))] + 2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 +
(-1)^(1/3))*d^(1/3))] + Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]) + 2*PolyLog[2
, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + 2*PolyLog[
2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))]) - p*(Log[-
d^(1/3) - e^(1/3)*x]*(Log[-d^(1/3) - e^(1/3)*x] + 2*(Log[((-1)^(1/3)*d^(1/3
) - e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + Log[(I + Sqrt[3] - ((2*I)*e^(1
/3)*x)/d^(1/3))/(3*I + Sqrt[3])])) + 2*PolyLog[2, (d^(1/3) + e^(1/3)*x)/((1
 + (-1)^(1/3))*d^(1/3))] + 2*PolyLog[2, ((2*I)*(1 + (e^(1/3)*x)/d^(1/3)))/(
3*I + Sqrt[3])])))/d^(2/3))/2
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c(e\,x^3+d)^p\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^3+d)^p)^2/x^3,x)
```

```
[Out] int(ln(c*(e*x^3+d)^p)^2/x^3,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(log((x^3*e + d)^p*c)^2/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(c(d+ex^3)^p\right)^2}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**3+d)**p)**2/x**3,x)
```

```
[Out] Integral(log(c*(d + e*x**3)**p)**2/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(log((x^3*e + d)^p*c)^2/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln \left(c\,{\left(e\,x^3+d\right)}^p\right)^2}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^3)^p)^2/x^3,x)
```

```
[Out] int(log(c*(d + e*x^3)^p)^2/x^3, x)
```

3.137 $\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^5} dx$

Optimal. Leaf size=1328

$$-\frac{3\sqrt{3}\,e^{4/3}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,x}{\sqrt{3}\,\sqrt[3]{d}}\right)}{2d^{4/3}}-\frac{3e^{4/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{2d^{4/3}}-\frac{e^{4/3}p^2\log^2\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{4d^{4/3}}-\frac{e^{4/3}p^2\log\left(\sqrt[3]{\right.}}{}$$

```
[Out] -1/4*(-1)^(2/3)*e^(4/3)*p^2*ln(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)^2/d^(4/3)-1/2*
e^(4/3)*p^2*ln(d^(1/3)+e^(1/3)*x)*ln((-1)^(1/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)
*x)/(1+(-1)^(1/3))/d^(1/3))/d^(4/3)-3/2*e*p*ln(c*(e*x^3+d)^p)/d/x+1/2*e^(4/
3)*p*ln(d^(1/3)+e^(1/3)*x)*ln(c*(e*x^3+d)^p)/d^(4/3)-3/2*e^(4/3)*p^2*arctan
(1/3*(d^(1/3)-2*e^(1/3)*x)/d^(1/3)*3^(1/2))*3^(1/2)/d^(4/3)+1/2*(-1)^(1/3)*
e^(4/3)*p^2*ln((-1)^(1/3)*(d^(1/3)+e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))*ln(d^
(1/3)-(-1)^(1/3)*e^(1/3)*x)/d^(4/3)-1/2*(-1)^(2/3)*e^(4/3)*p^2*ln(-(-1)^(2/
3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln(d^(1/3)+(-1)^(2/3)*e^(1/3
)*x)/d^(4/3)-1/2*(-1)^(2/3)*e^(4/3)*p^2*ln((-1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e
^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))*ln(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/d^(4/3)+
1/2*(-1)^(2/3)*e^(4/3)*p^2*ln((-1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+
(-1)^(1/3))/d^(1/3))*ln((d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/
3))/d^(4/3)+1/2*(-1)^(2/3)*e^(4/3)*p^2*ln(-(-1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(
1-(-1)^(2/3))/d^(1/3))*ln((d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(
1/3))/d^(4/3)-1/2*(-1)^(1/3)*e^(4/3)*p^2*ln(-(-1)^(1/3)*((-1)^(2/3)*d^(1/3)
+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln(-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1
/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(4/3)+1/2*(-1)^(1/3)*e^(4/3)*p^2*ln(d^(1/3
)-(-1)^(1/3)*e^(1/3)*x)*ln(-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-
1)^(2/3))/d^(1/3))/d^(4/3)-1/2*(-1)^(1/3)*e^(4/3)*p*ln(d^(1/3)-(-1)^(1/3)*e
^(1/3)*x)*ln(c*(e*x^3+d)^p)/d^(4/3)+1/2*(-1)^(2/3)*e^(4/3)*p*ln(d^(1/3)+(-1
)^(2/3)*e^(1/3)*x)*ln(c*(e*x^3+d)^p)/d^(4/3)-1/4*ln(c*(e*x^3+d)^p)^2/x^4-1/
2*e^(4/3)*p^2*ln(d^(1/3)+e^(1/3)*x)*ln((-(-1)^(2/3)*d^(1/3)-e^(1/3)*x)/(1-(
-1)^(2/3))/d^(1/3))/d^(4/3)+1/4*(-1)^(1/3)*e^(4/3)*p^2*ln(d^(1/3)-(-1)^(1/3
)*e^(1/3)*x)^2/d^(4/3)-1/2*e^(4/3)*p^2*polylog(2,(d^(1/3)+e^(1/3)*x)/(1+(-1
)^(1/3))/d^(1/3))/d^(4/3)+1/2*(-1)^(2/3)*e^(4/3)*p^2*polylog(2,-(-1)^(2/3)*
(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(4/3)+1/2*(-1)^(1/3)*e^(4/3)*
p^2*polylog(2,(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(4/3
)+1/2*(-1)^(2/3)*e^(4/3)*p^2*polylog(2,(-1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/
3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(4/3)-1/2*(-1)^(1/3)*e^(4/3)*p^2*polylog(2,
-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(4/3)-
1/2*e^(4/3)*p^2*polylog(2,2*(d^(1/3)+e^(1/3)*x)/d^(1/3)/(3-I*3^(1/2)))/d^(4
/3)-3/2*e^(4/3)*p^2*ln(d^(1/3)+e^(1/3)*x)/d^(4/3)-1/4*e^(4/3)*p^2*ln(d^(1/3
)+e^(1/3)*x)^2/d^(4/3)+3/4*e^(4/3)*p^2*ln(d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)
*x^2)/d^(4/3)
```

Rubi [A]

time = 1.15, antiderivative size = 1334, normalized size of antiderivative = 1.00, number of steps used = 48, number of rules used = 18, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {2507, 2526, 2505, 298, 31, 648, 631, 210, 642, 2512, 266, 2463, 2437, 2338, 2441, 2440, 2438, 12}

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^3)^p]^2/x^5,x]
```

```
[Out] (-3*Sqrt[3]*e^(4/3)*p^2*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/
(2*d^(4/3)) - (3*e^(4/3)*p^2*Log[d^(1/3) + e^(1/3)*x])/(2*d^(4/3)) - (e^(4/
3)*p^2*Log[d^(1/3) + e^(1/3)*x]^2)/(4*d^(4/3)) - (e^(4/3)*p^2*Log[d^(1/3) +
 e^(1/3)*x]*Log[-(((-1)^(2/3)*d^(1/3) + e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3
)))])/(2*d^(4/3)) + ((-1)^(1/3)*e^(4/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) + e^(1
/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x])/(2
*d^(4/3)) + ((-1)^(1/3)*e^(4/3)*p^2*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]^2)/
(4*d^(4/3)) - ((-1)^(2/3)*e^(4/3)*p^2*Log[-(((-1)^(2/3)*(d^(1/3) + e^(1/3)*
x))/((1 - (-1)^(2/3))*d^(1/3)))]*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x])/(2*d^
(4/3)) - ((-1)^(2/3)*e^(4/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1
/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x])/(2
*d^(4/3)) - ((-1)^(2/3)*e^(4/3)*p^2*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x]^2)/
(4*d^(4/3)) + ((-1)^(2/3)*e^(4/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)
*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[(d^(1/3) + (-1)^(2/3)*e^(1/3)*
x)/((1 + (-1)^(1/3))*d^(1/3))])/(2*d^(4/3)) - (e^(4/3)*p^2*Log[d^(1/3) + e^
(1/3)*x]*Log[((-1)^(1/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 + (-1)^(1/3)
)*d^(1/3))])/(2*d^(4/3)) + ((-1)^(1/3)*e^(4/3)*p^2*Log[d^(1/3) - (-1)^(1/3)
*e^(1/3)*x]*Log[-(((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 - (-1)^
(2/3))*d^(1/3)))])/(2*d^(4/3)) + (3*e^(4/3)*p^2*Log[d^(2/3) - d^(1/3)*e^(1/
3)*x + e^(2/3)*x^2])/(4*d^(4/3)) - (3*e*p*Log[c*(d + e*x^3)^p])/(2*d*x) + (
e^(4/3)*p*Log[d^(1/3) + e^(1/3)*x]*Log[c*(d + e*x^3)^p])/(2*d^(4/3)) - ((-1
)^(1/3)*e^(4/3)*p*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])
/(2*d^(4/3)) + ((-1)^(2/3)*e^(4/3)*p*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x]*Lo
g[c*(d + e*x^3)^p])/(2*d^(4/3)) - Log[c*(d + e*x^3)^p]^2/(4*x^4) - (e^(4/3)
*p^2*PolyLog[2, (d^(1/3) + e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/(2*d^(4/
3)) - (e^(4/3)*p^2*PolyLog[2, (2*(d^(1/3) + e^(1/3)*x))/((3 - I*Sqrt[3])*d^
(1/3))])/(2*d^(4/3)) + ((-1)^(1/3)*e^(4/3)*p^2*PolyLog[2, -(((-1)^(1/3)*((-
1)^(2/3)*d^(1/3) + e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))])/(2*d^(4/3)) +
((-1)^(1/3)*e^(4/3)*p^2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1/3)*x)/((1 + (
-1)^(1/3))*d^(1/3))])/(2*d^(4/3)) + ((-1)^(2/3)*e^(4/3)*p^2*PolyLog[2, ((-1
)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))])/(2*d
^(4/3)) - ((-1)^(2/3)*e^(4/3)*p^2*PolyLog[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*
x)/((1 - (-1)^(2/3))*d^(1/3))])/(2*d^(4/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

<u>Rule 2338</u>

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

<u>Rule 2437</u>

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

<u>Rule 2438</u>

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

<u>Rule 2440</u>

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

<u>Rule 2441</u>

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

<u>Rule 2463</u>

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

<u>Rule 2505</u>

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

<u>Rule 2507</u>

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^5}\,dx = -\frac{\log^2\left(c(d+ex^3)^p\right)}{4x^4} + \frac{1}{2}(3ep)\int \frac{\log\left(c(d+ex^3)^p\right)}{x^2\left(d+ex^3\right)}\,dx$$

$$= -\frac{\log^2\left(c(d+ex^3)^p\right)}{4x^4} + \frac{1}{2}(3ep)\int \left(\frac{\log\left(c(d+ex^3)^p\right)}{dx^2} - \frac{ex\log\left(c(d+ex^3)^p\right)}{d\left(d+ex^3\right)}\right)\,dx$$

$$= -\frac{\log^2\left(c(d+ex^3)^p\right)}{4x^4} + \frac{(3ep)\int \frac{\log(c(d+ex^3)^p)}{x^2}\,dx}{2d} - \frac{(3e^2p)\int \frac{x\log(c(d+ex^3)^p)}{d+ex^3}\,dx}{2d}$$

$$= -\frac{3ep\log\left(c(d+ex^3)^p\right)}{2dx} - \frac{\log^2\left(c(d+ex^3)^p\right)}{4x^4} - \frac{(3e^2p)\int \left(-\frac{\log(c(d+ex^3)^p)}{3\sqrt[3]{d}\sqrt[3]{e}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}\right.}{}$$

$$= -\frac{3ep\log\left(c(d+ex^3)^p\right)}{2dx} - \frac{\log^2\left(c(d+ex^3)^p\right)}{4x^4} + \frac{\left(e^{5/3}p\right)\int \frac{\log(c(d+ex^3)^p)}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{2d^{4/3}} - \frac{\left(\sqrt[3]{-1}\right.}{}$$

$$= -\frac{3e^{4/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2d^{4/3}} - \frac{3ep\log\left(c(d+ex^3)^p\right)}{2dx} + \frac{e^{4/3}p\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\mathrm{l}}{2d^{4/3}}$$

$$= -\frac{3e^{4/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2d^{4/3}} + \frac{3e^{4/3}p^2\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{4d^{4/3}} - \frac{3ep\log}{}$$

$$= -\frac{3\sqrt{3}e^{4/3}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{2d^{4/3}} - \frac{3e^{4/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2d^{4/3}} + \frac{3e^{4/3}p^2\log\left(d\right.}{}$$

$$= -\frac{3\sqrt{3}e^{4/3}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{2d^{4/3}} - \frac{3e^{4/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2d^{4/3}} - \frac{e^{4/3}p^2\log\left(\sqrt[3]{}\right.}{}$$

$$= -\frac{3\sqrt{3}e^{4/3}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{2d^{4/3}} - \frac{3e^{4/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2d^{4/3}} - \frac{e^{4/3}p^2\log^2\left(\sqrt[3]{}\right.}{4d^{4}}$$

$$= -\frac{3\sqrt{3}e^{4/3}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{2d^{4/3}} - \frac{3e^{4/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2d^{4/3}} - \frac{e^{4/3}p^2\log^2\left(\sqrt{}\right.}{4d^{4}}$$

$$= -\frac{3\sqrt{3}e^{4/3}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{2d^{4/3}} - \frac{3e^{4/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2d^{4/3}} - \frac{e^{4/3}p^2\log^2\left(\sqrt[3]{}\right.}{4d^{4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.10, size = 847, normalized size = 0.64

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^3)^p]^2/x^5,x]
```

```
[Out] (-Log[c*(d + e*x^3)^p]^2 + (e*p*x^3*(9*e*p*x^3*Hypergeometric2F1[2/3, 1, 5/
3, -((e*x^3)/d)] - 6*d*Log[c*(d + e*x^3)^p] + 2*d^(2/3)*e^(1/3)*x*Log[-d^(1
/3) - e^(1/3)*x]*Log[c*(d + e*x^3)^p] - 2*(-1)^(1/3)*d^(2/3)*e^(1/3)*x*Log[
-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p] + 2*(-1)^(2/3)*d^(2/3
)*e^(1/3)*x*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p] + (-1
)^(1/3)*d^(2/3)*e^(1/3)*p*x*(Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*(2*Log[((
-1)^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))] + Log[-d^(1/3)
 + (-1)^(1/3)*e^(1/3)*x] + 2*Log[((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*
x))/((-1 + (-1)^(2/3))*d^(1/3))]) + 2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1
/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + 2*PolyLog[2, (-d^(1/3) + (-1)^(1/3)*e^
(1/3)*x)/((-1 + (-1)^(2/3))*d^(1/3))]) - (-1)^(2/3)*d^(2/3)*e^(1/3)*p*x*(Lo
g[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]*(2*Log[((-1)^(2/3)*(d^(1/3) + e^(1/3)*x)
)/((-1 + (-1)^(2/3))*d^(1/3))] + 2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^
(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))] + Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x
]) + 2*PolyLog[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3
))] + 2*PolyLog[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/
3))]) - d^(2/3)*e^(1/3)*p*x*(Log[-d^(1/3) - e^(1/3)*x]*(Log[-d^(1/3) - e^(1
/3)*x] + 2*(Log[((-1)^(1/3)*d^(1/3) - e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))
] + Log[(I + Sqrt[3] - ((2*I)*e^(1/3)*x)/d^(1/3))/(3*I + Sqrt[3])])) + 2*Po
lyLog[2, (d^(1/3) + e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + 2*PolyLog[2, (
(2*I)*(1 + (e^(1/3)*x)/d^(1/3)))/(3*I + Sqrt[3])])))/d^2)/(4*x^4)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(c(e\,x^3+d)^p\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^3+d)^p)^2/x^5,x)
```

```
[Out] int(ln(c*(e*x^3+d)^p)^2/x^5,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^5,x, algorithm="fricas")
```

```
[Out] integral(log((x^3*e + d)^p*c)^2/x^5, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(c(d+ex^3)^p\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**3+d)**p)**2/x**5,x)
```

```
[Out] Integral(log(c*(d + e*x**3)**p)**2/x**5, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^5,x, algorithm="giac")
```

```
[Out] integrate(log((x^3*e + d)^p*c)^2/x^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln \left(c\left(e\,x^3+d\right)^p\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^3)^p)^2/x^5,x)
```

```
[Out] int(log(c*(d + e*x^3)^p)^2/x^5, x)
```

3.138 $\int \frac{x^8}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal. Leaf size=164

$$\frac{d^2\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(d+ex^3\right)^p\right)}{p}\right)}{3e^3p}-\frac{2d\left(d+ex^3\right)^2\left(c\left(d+ex^3\right)^p\right)^{-2/p}\text{Ei}\left(\frac{2\log\left(c\left(d+ex^3\right)^p\right)}{p}\right)}{3e^3p}+(d+e$$

```
[Out] 1/3*d^2*(e*x^3+d)*Ei(ln(c*(e*x^3+d)^p)/p)/e^3/p/((c*(e*x^3+d)^p)^(1/p))-2/3
*d*(e*x^3+d)^2*Ei(2*ln(c*(e*x^3+d)^p)/p)/e^3/p/((c*(e*x^3+d)^p)^(2/p))+1/3*
(e*x^3+d)^3*Ei(3*ln(c*(e*x^3+d)^p)/p)/e^3/p/((c*(e*x^3+d)^p)^(3/p))
```

Rubi [A]
time = 0.16, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {2504, 2446, 2436, 2337, 2209, 2437, 2347}

$$\frac{d^2\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3e^3p}+\frac{\left(d+ex^3\right)^3\left(c\left(d+ex^3\right)^p\right)^{-3/p}\text{Ei}\left(\frac{3\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3e^3p}-\frac{2d\left(d+ex^3\right)^2\left(c\left(d+ex^3\right)^p\right)^{-2/p}\text{Ei}\left(\frac{2\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3e^3p}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/Log[c*(d + e*x^3)^p],x]
```

```
[Out] (d^2*(d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(3*e^3*p*(c*(d + e*
x^3)^p)^p^(-1)) - (2*d*(d + e*x^3)^2*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p])
/p])/(3*e^3*p*(c*(d + e*x^3)^p)^(2/p)) + ((d + e*x^3)^3*ExpIntegralEi[(3*Lo
g[c*(d + e*x^3)^p])/p])/(3*e^3*p*(c*(d + e*x^3)^p)^(3/p))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{\log\left(c(d+ex)^p\right)}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{d^2}{e^2\log\left(c(d+ex)^p\right)}-\frac{2d(d+ex)}{e^2\log\left(c(d+ex)^p\right)}+\frac{(d+ex)^2}{e^2\log\left(c(d+ex)^p\right)}\right)\,dx,x\right.\\
&= \frac{\text{Subst}\left(\int \frac{(d+ex)^2}{\log(c(d+ex)^p)}\,dx,x,x^3\right)}{3e^2}-\frac{(2d)\text{Subst}\left(\int \frac{d+ex}{\log(c(d+ex)^p)}\,dx,x,x^3\right)}{3e^2}+\frac{d^2\text{Subst}\left(\int\right.}{}\\
&= \frac{\text{Subst}\left(\int \frac{x^2}{\log(cx^p)}\,dx,x,d+ex^3\right)}{3e^3}-\frac{(2d)\text{Subst}\left(\int \frac{x}{\log(cx^p)}\,dx,x,d+ex^3\right)}{3e^3}+\frac{d^2\text{Subst}\left(\right.}{}\\
&= \frac{\left(\left(d+ex^3\right)^3\left(c\left(d+ex^3\right)^p\right)^{-3/p}\right)\text{Subst}\left(\int \frac{e^{\frac{3x}{p}}}{x}\,dx,x,\log\left(c\left(d+ex^3\right)^p\right)\right)}{3e^3p}-\frac{\left(2d(d+\right.}{}\\
&= \frac{d^2\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(d+ex^3\right)^p\right)}{p}\right)}{3e^3p}-\frac{2d\left(d+ex^3\right)^2\left(c\left(d+ex^3\right)^p\right)^{-2/p}\,\text{I}}{3e^3p}
\end{aligned}$$

Mathematica [A]
time = 0.15, size = 146, normalized size = 0.89

$$\frac{(d+ex^3)\left(c(d+ex^3)^p\right)^{-3/p}\left(d^2\left(c(d+ex^3)^p\right)^{2/p}\text{Ei}\left(\frac{\log(c(d+ex^3)^p)}{p}\right)-(d+ex^3)\left(2d\left(c(d+ex^3)^p\right)^{\frac{1}{p}}\text{Ei}\left(\frac{2\log(c(d+ex^3)^p)}{p}\right)-(d+ex^3)\text{Ei}\left(\frac{3\log(c(d+ex^3)^p)}{p}\right)\right)\right)}{3e^3p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/Log[c*(d + e*x^3)^p],x]
```

```
[Out] ((d + e*x^3)*(d^2*(c*(d + e*x^3)^p)^(2/p)*ExpIntegralEi[Log[c*(d + e*x^3)^p
]/p] - (d + e*x^3)*(2*d*(c*(d + e*x^3)^p)^p^(-1)*ExpIntegralEi[(2*Log[c*(d
+ e*x^3)^p])/p] - (d + e*x^3)*ExpIntegralEi[(3*Log[c*(d + e*x^3)^p])/p])))/
(3*e^3*p*(c*(d + e*x^3)^p)^(3/p))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.72, size = 823, normalized size = 5.02

method	result
risch	$-\frac{(e\,x^3+d)^3 c^{-\frac{3}{p}}\left((e\,x^3+d)^p\right)^{-\frac{3}{p}} \mathrm{e}^{\frac{3i\pi\,\mathrm{csgn}\left(ic\left(e\,x^3+d\right)^p\right)\left(-\mathrm{csgn}\left(ic\left(e\,x^3+d\right)^p\right)+\mathrm{csgn}(ic)\right)\left(-\mathrm{csgn}\left(ic\left(e\,x^3+d\right)^p\right)+\mathrm{csgn}\left(i\left(e\,x^3+d\right)^p\right)\right)}{2p}}\,\mathrm{expIn}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/ln(c*(e*x^3+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/e^3/p*(e*x^3+d)^3*c^(-3/p)*((e*x^3+d)^p)^(-3/p)*exp(3/2*I*Pi*csgn(I*c*
(e*x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csg
n(I*(e*x^3+d)^p))/p)*Ei(1,-3*ln(e*x^3+d)-3/2*(I*Pi*csgn(I*(e*x^3+d)^p)*csgn
(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c
)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(
c)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)-1/3/e^3*d^2/p*(e*x^3+d)*c^(-1/p)*(
(e*x^3+d)^p)^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(e*x^3+d)^p)*(-csgn(I*c*(e*x^3+d)
^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*(e*x^3+d)^p))/p)*Ei(1,-ln(e*x
^3+d)-1/2*(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*
x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi
*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d
))/p)+2/3/e^3*d/p*(e*x^3+d)^2*c^(-2/p)*((e*x^3+d)^p)^(-2/p)*exp(I*Pi*csgn(I
*c*(e*x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+
csgn(I*(e*x^3+d)^p))/p)*Ei(1,-2*ln(e*x^3+d)-(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(
I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)
-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c
)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

```
[Out] integrate(x^8/log((e*x^3 + d)^p*c), x)
```

Fricas [A]
time = 0.37, size = 115, normalized size = 0.70

$$\frac{\left(c^{\frac{2}{p}}d^2\,\mathrm{log\_integral}\left((x^3e+d)c^{\left(\frac{1}{p}\right)}\right)-2\,c^{\left(\frac{1}{p}\right)}d\,\mathrm{log\_integral}\left((x^6e^2+2\,dx^3e+d^2)c^{\frac{2}{p}}\right)+\mathrm{log\_integral}\left((x^9e^3+3\,dx^6e^2+3\,d^2x^3e+d^3)c^{\frac{3}{p}}\right)\right)e^{(-3)}}{3\,c^{\frac{3}{p}}p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/log(c*(e*x^3+d)^p),x, algorithm="fricas")
```

```
[Out] 1/3*(c^(2/p)*d^2*log_integral((x^3*e + d)*c^(1/p)) - 2*c^(1/p)*d*log_integr
al((x^6*e^2 + 2*d*x^3*e + d^2)*c^(2/p)) + log_integral((x^9*e^3 + 3*d*x^6*e
^2 + 3*d^2*x^3*e + d^3)*c^(3/p)))*e^(-3)/(c^(3/p)*p)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/ln(c*(e*x**3+d)**p),x)
```

```
[Out] Integral(x**8/log(c*(d + e*x**3)**p), x)
```

Giac [A]
time = 5.81, size = 108, normalized size = 0.66

$$\frac{d^2\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(x^3e+d\right)\right)e^{(-3)}}{3\,c^{\left(\frac{1}{p}\right)}p}-\frac{2\,d\mathrm{Ei}\left(\frac{2\,\log(c)}{p}+2\,\log\left(x^3e+d\right)\right)e^{(-3)}}{3\,c^{\frac{2}{p}}p}+\frac{\mathrm{Ei}\left(\frac{3\,\log(c)}{p}+3\,\log\left(x^3e+d\right)\right)e^{(-3)}}{3\,c^{\frac{3}{p}}p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/log(c*(e*x^3+d)^p),x, algorithm="giac")
```

```
[Out] 1/3*d^2*Ei(log(c)/p + log(x^3*e + d))*e^(-3)/(c^(1/p)*p) - 2/3*d*Ei(2*log(c
)/p + 2*log(x^3*e + d))*e^(-3)/(c^(2/p)*p) + 1/3*Ei(3*log(c)/p + 3*log(x^3*
e + d))*e^(-3)/(c^(3/p)*p)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8}{\ln\left(c\,\left(e\,x^3+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/log(c*(d + e*x^3)^p),x)
[Out] int(x^8/log(c*(d + e*x^3)^p), x)
```

3.139 $\int \frac{x^5}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal. Leaf size=107

$$-\frac{d\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(d+ex^3\right)^p\right)}{p}\right)}{3e^2p}+\frac{\left(d+ex^3\right)^2\left(c\left(d+ex^3\right)^p\right)^{-2/p}\text{Ei}\left(\frac{2\log\left(c\left(d+ex^3\right)^p\right)}{p}\right)}{3e^2p}$$

```
[Out] -1/3*d*(e*x^3+d)*Ei(ln(c*(e*x^3+d)^p)/p)/e^2/p/((c*(e*x^3+d)^p)^(1/p))+1/3*
(e*x^3+d)^2*Ei(2*ln(c*(e*x^3+d)^p)/p)/e^2/p/((c*(e*x^3+d)^p)^(2/p))
```

Rubi [A]
time = 0.10, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {2504, 2446, 2436, 2337, 2209, 2437, 2347}

$$\frac{\left(d+ex^3\right)^2\left(c\left(d+ex^3\right)^p\right)^{-2/p}\text{Ei}\left(\frac{2\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3e^2p}-\frac{d\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3e^2p}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Log[c*(d + e*x^3)^p],x]
```

```
[Out] -1/3*(d*(d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(e^2*p*(c*(d + e
*x^3)^p)^p^(-1)) + ((d + e*x^3)^2*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p])/p]
)/(3*e^2*p*(c*(d + e*x^3)^p)^(2/p))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{\log\left(c(d+ex)^p\right)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{d}{e\log\left(c(d+ex)^p\right)}+\frac{d+ex}{e\log\left(c(d+ex)^p\right)}\right)\,dx, x, x^3\right)\\
&= \frac{\text{Subst}\left(\int \frac{d+ex}{\log(c(d+ex)^p)}\,dx, x, x^3\right)}{3e}-\frac{d\text{Subst}\left(\int \frac{1}{\log(c(d+ex)^p)}\,dx, x, x^3\right)}{3e}\\
&= \frac{\text{Subst}\left(\int \frac{x}{\log(cx^p)}\,dx, x, d+ex^3\right)}{3e^2}-\frac{d\text{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx, x, d+ex^3\right)}{3e^2}\\
&= \frac{\left(\left(d+ex^3\right)^2\left(c(d+ex^3)^p\right)^{-2/p}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{p}}}{x}\,dx, x, \log\left(c(d+ex^3)^p\right)\right)}{3e^2p}-\frac{\left(d(d+\cdots\right.}{\cdots}\\
&= -\frac{d(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\text{Ei}\left(\frac{\log(c(d+ex^3)^p)}{p}\right)}{3e^2p}+\frac{\left(d+ex^3\right)^2\left(c(d+ex^3)^p\right)^{-2/p}\cdots}{3e^2p}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 96, normalized size = 0.90

$$-\frac{(d+ex^3)\left(c(d+ex^3)^p\right)^{-2/p}\left(d\left(c(d+ex^3)^p\right)^{\frac{1}{p}}\text{Ei}\left(\frac{\log(c(d+ex^3)^p)}{p}\right)-(d+ex^3)\,\text{Ei}\left(\frac{2\log(c(d+ex^3)^p)}{p}\right)\right)}{3e^2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Log[c*(d + e*x^3)^p],x]
```

```
[Out] -1/3*((d + e*x^3)*(d*(c*(d + e*x^3)^p)^p^(-1)*ExpIntegralEi[Log[c*(d + e*x^
3)^p]/p] - (d + e*x^3)*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p])/p]))/(e^2*p*(
c*(d + e*x^3)^p)^(2/p))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.66, size = 547, normalized size = 5.11

method	result
risch	$-\frac{(e\,x^3+d)^2 c^{-\frac{2}{p}}\left((e\,x^3+d)^p\right)^{-\frac{2}{p}} \mathrm{e}^{\frac{i\pi\,\mathrm{csgn}\left(ic\left(e\,x^3+d\right)^p\right)\left(-\mathrm{csgn}\left(ic\left(e\,x^3+d\right)^p\right)+\mathrm{csgn}(ic)\right)\left(-\mathrm{csgn}\left(ic\left(e\,x^3+d\right)^p\right)+\mathrm{csgn}\left(i\left(e\,x^3+d\right)^p\right)\right)}{p}}\,\mathrm{expInteg}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/ln(c*(e*x^3+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/e^2/p*(e*x^3+d)^2*c^(-2/p)*((e*x^3+d)^p)^(-2/p)*exp(I*Pi*csgn(I*c*(e*x
^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*
(e*x^3+d)^p))/p)*Ei(1,-2*ln(e*x^3+d)-(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*
x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*c
sgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln(
(e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)+1/3/e^2*d/p*(e*x^3+d)*c^(-1/p)*((e*x^3+d)^
p)^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(e*x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I
*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*(e*x^3+d)^p))/p)*Ei(1,-ln(e*x^3+d)-1/2*
(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*
csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*
(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

```
[Out] integrate(x^5/log((e*x^3 + d)^p*c), x)
```

Fricas [A]
time = 0.36, size = 68, normalized size = 0.64

$$-\frac{\left(c^{\left(\frac{1}{p}\right)}d\operatorname{log\_integral}\left((x^3e+d)c^{\left(\frac{1}{p}\right)}\right)-\operatorname{log\_integral}\left((x^6e^2+2\,dx^3e+d^2)c^{\frac{2}{p}}\right)\right)e^{(-2)}}{3\,c^{\frac{2}{p}}p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/log(c*(e*x^3+d)^p),x, algorithm="fricas")
```

```
[Out] -1/3*(c^(1/p)*d*log_integral((x^3*e + d)*c^(1/p)) - log_integral((x^6*e^2 +
 2*d*x^3*e + d^2)*c^(2/p)))*e^(-2)/(c^(2/p)*p)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^5}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/ln(c*(e*x**3+d)**p),x)
```

```
[Out] Integral(x**5/log(c*(d + e*x**3)**p), x)
```

Giac [A]
time = 5.65, size = 69, normalized size = 0.64

$$-\frac{d\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(x^3e+d\right)\right)e^{(-2)}}{3\,c^{\left(\frac{1}{p}\right)}p}+\frac{\mathrm{Ei}\left(\frac{2\,\log(c)}{p}+2\,\log\left(x^3e+d\right)\right)e^{(-2)}}{3\,c^{\frac{2}{p}}p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/log(c*(e*x^3+d)^p),x, algorithm="giac")
```

```
[Out] -1/3*d*Ei(log(c)/p + log(x^3*e + d))*e^(-2)/(c^(1/p)*p) + 1/3*Ei(2*log(c)/p
 + 2*log(x^3*e + d))*e^(-2)/(c^(2/p)*p)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^5}{\ln\left(c\left(e\,x^3+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/log(c*(d + e*x^3)^p),x)
```

```
[Out] int(x^5/log(c*(d + e*x^3)^p), x)
```

3.140 $\int \frac{x^2}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal. Leaf size=51

$$\frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(d+ex^3\right)^p\right)}{p}\right)}{3ep}$$

```
[Out] 1/3*(e*x^3+d)*Ei(ln(c*(e*x^3+d)^p)/p)/e/p/((c*(e*x^3+d)^p)^(1/p))
```

Rubi [A]
time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2504, 2436, 2337, 2209}

$$\frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3ep}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Log[c*(d + e*x^3)^p],x]
```

```
[Out] ((d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(3*e*p*(c*(d + e*x^3)^p
)^p^(-1))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\log\left(c(d+ex)^p\right)}\,dx, x, x^3\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx, x, d+ex^3\right)}{3e}\\
&= \frac{\left(\left(d+ex^3\right)\left(c(d+ex^3)^p\right)^{-1/p}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{p}}}{x}\,dx, x, \log\left(c(d+ex^3)^p\right)\right)}{3ep}\\
&= \frac{\left(d+ex^3\right)\left(c(d+ex^3)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c(d+ex^3)^p\right)}{p}\right)}{3ep}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 51, normalized size = 1.00

$$
\frac{\left(d+ex^3\right)\left(c(d+ex^3)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c(d+ex^3)^p\right)}{p}\right)}{3ep}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Log[c*(d + e*x^3)^p],x]
```

```
[Out] ((d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(3*e*p*(c*(d + e*x^3)^p
)^p^(-1))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.58, size = 272, normalized size = 5.33

method	result
risch	$-\frac{(e\,x^3+d)c^{-\frac{1}{p}}\left((e\,x^3+d)^p\right)^{-\frac{1}{p}}\mathrm{e}^{\frac{i\pi\,\text{csgn}\left(ic\left(e\,x^3+d\right)^p\right)\left(-\text{csgn}\left(ic\left(e\,x^3+d\right)^p\right)+\text{csgn}(ic)\right)\left(-\text{csgn}\left(ic\left(e\,x^3+d\right)^p\right)+\text{csgn}\left(i\left(e\,x^3+d\right)^p\right)\right)}{2p}}\text{expInte}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/ln(c*(e*x^3+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/e/p*(e*x^3+d)*c^(-1/p)*((e*x^3+d)^p)^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(e*x
^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*
```

```
(e*x^3+d)^p))/p)*Ei(1,-ln(e*x^3+d)-1/2*(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(
e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi
*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*l
n((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

```
[Out] integrate(x^2/log((e*x^3 + d)^p*c), x)
```

Fricas [A]
time = 0.37, size = 29, normalized size = 0.57

$$\frac{e^{(-1)}\,\mathrm{log\_integral}\left((x^3e+d)c^{\left(\frac{1}{p}\right)}\right)}{3\,c^{\left(\frac{1}{p}\right)}p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(e*x^3+d)^p),x, algorithm="fricas")
```

```
[Out] 1/3*e^(-1)*log_integral((x^3*e + d)*c^(1/p))/(c^(1/p)*p)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/ln(c*(e*x**3+d)**p),x)
```

```
[Out] Integral(x**2/log(c*(d + e*x**3)**p), x)
```

Giac [A]
time = 2.75, size = 31, normalized size = 0.61

$$\frac{\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(x^3e+d\right)\right)e^{(-1)}}{3\,c^{\left(\frac{1}{p}\right)}p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(e*x^3+d)^p),x, algorithm="giac")
[Out] 1/3*Ei(log(c)/p + log(x^3*e + d))*e^(-1)/(c^(1/p)*p)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\ln\left(c\left(e\,x^3+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/log(c*(d + e*x^3)^p),x)
[Out] int(x^2/log(c*(d + e*x^3)^p), x)
```

3.141 $\int \frac{1}{x \log\left(c\left(d+ex^3\right)^p\right)} \, dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x \log \left(c \left(d+e x^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x/ln(c*(e*x^3+d)^p),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x \log \left(c \left(d+e x^3\right)^p\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*Log[c*(d + e*x^3)^p]),x]
[Out] Defer[Int][1/(x*Log[c*(d + e*x^3)^p]), x]
```

Rubi steps

$$\int \frac{1}{x \log \left(c \left(d+e x^3\right)^p\right)} \, dx = \int \frac{1}{x \log \left(c \left(d+e x^3\right)^p\right)} \, dx$$

Mathematica [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{x \log \left(c \left(d+e x^3\right)^p\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*Log[c*(d + e*x^3)^p]),x]
[Out] Integrate[1/(x*Log[c*(d + e*x^3)^p]), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{x \ln \left(c \left(e \,x^{3}+d \right)^{p}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(c*(e*x^3+d)^p),x)
[Out] int(1/x/ln(c*(e*x^3+d)^p),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(e*x^3+d)^p),x, algorithm="maxima")
[Out] integrate(1/(x*log((e*x^3 + d)^p*c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(e*x^3+d)^p),x, algorithm="fricas")
[Out] integral(1/(x*log((x^3*e + d)^p*c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \log \left(c \left(d + e x^{3}\right)^{p}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(c*(e*x**3+d)**p),x)
[Out] Integral(1/(x*log(c*(d + e*x**3)**p)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(e*x^3+d)^p),x, algorithm="giac")
[Out] integrate(1/(x*log((x^3*e + d)^p*c)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x\,\ln \left(c\,{\left(e\,x^3+d\right)}^p\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*log(c*(d + e*x^3)^p)),x)
[Out] int(1/(x*log(c*(d + e*x^3)^p)), x)
```

3.142 $\int \frac{1}{x^4 \log\left(c\left(d+ex^3\right)^p\right)} dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x^4 \log \left(c \left(d+e x^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x^4/ln(c*(e*x^3+d)^p),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^4 \log \left(c \left(d+e x^3\right)^p\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^4*Log[c*(d + e*x^3)^p]),x]
```

```
[Out] Defer[Int][1/(x^4*Log[c*(d + e*x^3)^p]), x]
```

Rubi steps

$$\int \frac{1}{x^4 \log \left(c \left(d+e x^3\right)^p\right)} \, dx = \int \frac{1}{x^4 \log \left(c \left(d+e x^3\right)^p\right)} \, dx$$

Mathematica [A]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \log \left(c \left(d+e x^3\right)^p\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^4*Log[c*(d + e*x^3)^p]),x]
```

```
[Out] Integrate[1/(x^4*Log[c*(d + e*x^3)^p]), x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \ln \left(c \left(e\,x^3+d\right)^p\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/ln(c*(e*x^3+d)^p),x)
[Out] int(1/x^4/ln(c*(e*x^3+d)^p),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/log(c*(e*x^3+d)^p),x, algorithm="maxima")
[Out] integrate(1/(x^4*log((e*x^3 + d)^p*c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/log(c*(e*x^3+d)^p),x, algorithm="fricas")
[Out] integral(1/(x^4*log((x^3*e + d)^p*c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \log \left(c \left(d + ex^3\right)^p\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/ln(c*(e*x**3+d)**p),x)
[Out] Integral(1/(x**4*log(c*(d + e*x**3)**p)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/log(c*(e*x^3+d)^p),x, algorithm="giac")
[Out] integrate(1/(x^4*log((x^3*e + d)^p*c)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^4\, \ln \left(c\,{\left(e\,x^3+d\right)}^p\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*log(c*(d + e*x^3)^p)),x)
[Out] int(1/(x^4*log(c*(d + e*x^3)^p)), x)
```

3.143 $\int \frac{x^3}{\log(c(d+ex^3)^p)}\,dx$

Optimal. Leaf size=21

$$\mathrm{Int}\left(\frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)},x\right)$$

`[Out] Unintegrable(x^3/ln(c*(e*x^3+d)^p),x)`

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is not applicable to the result.

`[In] Int[x^3/Log[c*(d + e*x^3)^p],x]`

`[Out] Defer[Int][x^3/Log[c*(d + e*x^3)^p], x]`

Rubi steps

$$\int \frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Mathematica [A]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is not applicable to the result.

`[In] Integrate[x^3/Log[c*(d + e*x^3)^p],x]`

`[Out] Integrate[x^3/Log[c*(d + e*x^3)^p], x]`

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\ln\left(c\left(e\,x^3+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/ln(c*(e*x^3+d)^p),x)
[Out] int(x^3/ln(c*(e*x^3+d)^p),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(e*x^3+d)^p),x, algorithm="maxima")
[Out] integrate(x^3/log((e*x^3 + d)^p*c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(e*x^3+d)^p),x, algorithm="fricas")
[Out] integral(x^3/log((x^3*e + d)^p*c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/ln(c*(e*x**3+d)**p),x)
[Out] Integral(x**3/log(c*(d + e*x**3)**p), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(e*x^3+d)^p),x, algorithm="giac")
[Out] integrate(x^3/log((x^3*e + d)^p*c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^3}{\ln\left(c\left(e\,x^3+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/log(c*(d + e*x^3)^p),x)
[Out] int(x^3/log(c*(d + e*x^3)^p), x)
```

3.144 $\int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal. Leaf size=19

$$\operatorname{Int}\left(\frac{x}{\log\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable(x/ln(c*(e*x^3+d)^p),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/Log[c*(d + e*x^3)^p],x]
[Out] Defer[Int][x/Log[c*(d + e*x^3)^p], x]
```

Rubi steps

$$\int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Mathematica [A]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/Log[c*(d + e*x^3)^p],x]
[Out] Integrate[x/Log[c*(d + e*x^3)^p], x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x}{\ln\left(c\left(e\,x^3+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/ln(c*(e*x^3+d)^p),x)
[Out] int(x/ln(c*(e*x^3+d)^p),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(e*x^3+d)^p),x, algorithm="maxima")
[Out] integrate(x/log((e*x^3 + d)^p*c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(e*x^3+d)^p),x, algorithm="fricas")
[Out] integral(x/log((x^3*e + d)^p*c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\log\left(c\left(d+ex^{3}\right)^{p}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/ln(c*(e*x**3+d)**p),x)
[Out] Integral(x/log(c*(d + e*x**3)**p), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(e*x^3+d)^p),x, algorithm="giac")
[Out] integrate(x/log((x^3*e + d)^p*c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x}{\ln\left(c\left(e\,x^3+d\right)^p\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/log(c*(d + e*x^3)^p),x)
[Out] int(x/log(c*(d + e*x^3)^p), x)
```

3.145 $\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{\log\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/ln(c*(e*x^3+d)^p),x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[c*(d + e*x^3)^p]^(-1),x]
[Out] Defer[Int][Log[c*(d + e*x^3)^p]^(-1), x]
```

Rubi steps

$$\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Mathematica [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^3)^p]^(-1),x]
[Out] Integrate[Log[c*(d + e*x^3)^p]^(-1), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\ln\left(c\left(e\,x^3+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(c*(e*x^3+d)^p),x)
[Out] int(1/ln(c*(e*x^3+d)^p),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(e*x^3+d)^p),x, algorithm="maxima")
[Out] integrate(1/log((e*x^3 + d)^p*c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(e*x^3+d)^p),x, algorithm="fricas")
[Out] integral(1/log((x^3*e + d)^p*c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\log \left(c \left(d + e x^{3}\right)^{p}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(c*(e*x**3+d)**p),x)
[Out] Integral(1/log(c*(d + e*x**3)**p), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(e*x^3+d)^p),x, algorithm="giac")
[Out] integrate(1/log((x^3*e + d)^p*c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\ln \left(c\,{\left(e\,x^3+d\right)}^p\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/log(c*(d + e*x^3)^p),x)
[Out] int(1/log(c*(d + e*x^3)^p), x)
```

3.146 $\int \frac{1}{x^2 \log\left(c\left(d+ex^3\right)^p\right)} \, dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x^2 \log\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x^2/ln(c*(e*x^3+d)^p),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2 \log\left(c\left(d+ex^3\right)^p\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*Log[c*(d + e*x^3)^p]),x]
```

```
[Out] Defer[Int][1/(x^2*Log[c*(d + e*x^3)^p]), x]
```

Rubi steps

$$\int \frac{1}{x^2 \log\left(c\left(d+ex^3\right)^p\right)} \, dx = \int \frac{1}{x^2 \log\left(c\left(d+ex^3\right)^p\right)} \, dx$$

Mathematica [A]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \log\left(c\left(d+ex^3\right)^p\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*Log[c*(d + e*x^3)^p]),x]
```

```
[Out] Integrate[1/(x^2*Log[c*(d + e*x^3)^p]), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \ln\left(c\left(e\,x^3+d\right)^p\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/ln(c*(e*x^3+d)^p),x)
```

```
[Out] int(1/x^2/ln(c*(e*x^3+d)^p),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*log((e*x^3 + d)^p*c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(e*x^3+d)^p),x, algorithm="fricas")
```

```
[Out] integral(1/(x^2*log((x^3*e + d)^p*c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \log \left(c \left(d+e x^3\right)^p\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/ln(c*(e*x**3+d)**p),x)
```

```
[Out] Integral(1/(x**2*log(c*(d + e*x**3)**p)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(e*x^3+d)^p),x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*log((x^3*e + d)^p*c)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^2\, \ln \left(c\, {\left(e\, x^3+d\right)}^p\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*log(c*(d + e*x^3)^p)),x)
[Out] int(1/(x^2*log(c*(d + e*x^3)^p)), x)
```

3.147 $\int \frac{1}{x^3 \log\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x^3 \log \left(c \left(d+e x^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x^3/ln(c*(e*x^3+d)^p),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^3 \log \left(c \left(d+e x^3\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^3*Log[c*(d + e*x^3)^p]),x]
[Out] Defer[Int][1/(x^3*Log[c*(d + e*x^3)^p]), x]
```

Rubi steps

$$\int \frac{1}{x^3 \log \left(c \left(d+e x^3\right)^p\right)}\,dx=\int \frac{1}{x^3 \log \left(c \left(d+e x^3\right)^p\right)}\,dx$$

Mathematica [A]
time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \log \left(c \left(d+e x^3\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^3*Log[c*(d + e*x^3)^p]),x]
[Out] Integrate[1/(x^3*Log[c*(d + e*x^3)^p]), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \ln \left(c \left(e\,x^3+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/ln(c*(e*x^3+d)^p),x)
[Out] int(1/x^3/ln(c*(e*x^3+d)^p),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(e*x^3+d)^p),x, algorithm="maxima")
[Out] integrate(1/(x^3*log((e*x^3 + d)^p*c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(e*x^3+d)^p),x, algorithm="fricas")
[Out] integral(1/(x^3*log((x^3*e + d)^p*c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/ln(c*(e*x**3+d)**p),x)
[Out] Integral(1/(x**3*log(c*(d + e*x**3)**p)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(e*x^3+d)^p),x, algorithm="giac")
[Out] integrate(1/(x^3*log((x^3*e + d)^p*c)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^3\,\ln\left(c\left(e\,x^3+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*log(c*(d + e*x^3)^p)),x)
[Out] int(1/(x^3*log(c*(d + e*x^3)^p)), x)
```

3.148 $\int \frac{x^8}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal. Leaf size=195

$$\frac{d^2(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c(d+ex^3)^p\right)}{p}\right)}{3e^3p^2}-\frac{4d(d+ex^3)^2\left(c(d+ex^3)^p\right)^{-2/p}\operatorname{Ei}\left(\frac{2\log\left(c(d+ex^3)^p\right)}{p}\right)}{3e^3p^2}+\frac{(d+e}{}$$

```
[Out] 1/3*d^2*(e*x^3+d)*Ei(ln(c*(e*x^3+d)^p)/p)/e^3/p^2/((c*(e*x^3+d)^p)^(1/p))-4
/3*d*(e*x^3+d)^2*Ei(2*ln(c*(e*x^3+d)^p)/p)/e^3/p^2/((c*(e*x^3+d)^p)^(2/p))+
(e*x^3+d)^3*Ei(3*ln(c*(e*x^3+d)^p)/p)/e^3/p^2/((c*(e*x^3+d)^p)^(3/p))-1/3*x
^6*(e*x^3+d)/e/p/ln(c*(e*x^3+d)^p)
```

Rubi [A]

time = 0.25, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {2504, 2447, 2446, 2436, 2337, 2209, 2437, 2347}

$$\frac{d^2(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c(ex^3+d)^p\right)}{p}\right)}{3e^3p^2}+\frac{(d+ex^3)^3\left(c(d+ex^3)^p\right)^{-3/p}\operatorname{Ei}\left(\frac{3\log\left(c(ex^3+d)^p\right)}{p}\right)}{e^3p^2}-\frac{4d(d+ex^3)^2\left(c(d+ex^3)^p\right)^{-2/p}\operatorname{Ei}\left(\frac{2\log\left(c(ex^3+d)^p\right)}{p}\right)}{3e^3p^2}-\frac{x^6(d+ex^3)}{3ep\log\left(c\left(d+ex^3\right)^p\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] (d^2*(d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(3*e^3*p^2*(c*(d +
e*x^3)^p)^p^(-1)) - (4*d*(d + e*x^3)^2*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p
])/p])/(3*e^3*p^2*(c*(d + e*x^3)^p)^(2/p)) + ((d + e*x^3)^3*ExpIntegralEi[(
3*Log[c*(d + e*x^3)^p])/p])/(e^3*p^2*(c*(d + e*x^3)^p)^(3/p)) - (x^6*(d + e
*x^3))/(3*e*p*Log[c*(d + e*x^3)^p])
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2447

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[q*((e*f - d*g)/
(b*e*n*(p + 1))), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{x^8}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{\log^2\left(c(d+ex)^p\right)}\,dx, x, x^3\right)$$

$$= -\frac{x^6(d+ex^3)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{x^2}{\log(c(d+ex)^p)}\,dx, x, x^3\right)}{p} + \frac{(2d)\text{Subst}\left(\int \frac{x}{\log(c(d+}\right.}{3ep}$$

$$= -\frac{x^6(d+ex^3)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{\text{Subst}\left(\int \left(\frac{d^2}{e^2\log(c(d+ex)^p)} - \frac{2d(d+ex)}{e^2\log(c(d+ex)^p)} + \frac{(d+ex)^2}{e^2\log(c(d+ex)^p}\right.\right.}{p}$$

$$= -\frac{x^6(d+ex^3)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{(d+ex)^2}{\log(c(d+ex)^p)}\,dx, x, x^3\right)}{e^2p} + \frac{(2d)\text{Subst}\left(\int \frac{d+e}{\log(c(d+}\right.}{3e^2p}$$

$$= -\frac{x^6(d+ex^3)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{x^2}{\log(cx^p)}\,dx, x, d+ex^3\right)}{e^3p} + \frac{(2d)\text{Subst}\left(\int \frac{x}{\log(cx^p}\right.}{3e^3}$$

$$= -\frac{x^6(d+ex^3)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{\left((d+ex^3)^3\left(c(d+ex^3)^p\right)^{-3/p}\right)\text{Subst}\left(\int \frac{e^{\frac{3x}{p}}}{x}\,dx, x, \log\right.}{e^3p^2}$$

$$= \frac{d^2(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\text{Ei}\left(\frac{\log(c(d+ex^3)^p)}{p}\right)}{3e^3p^2} - \frac{4d(d+ex^3)^2\left(c(d+ex^3)^p\right)^{-2}}{3e^3p^2}$$

Mathematica [A]
time = 0.17, size = 290, normalized size = 1.49

$$\frac{(d+ex^3)\left(c(d+ex^3)^p\right)^{-3/p}\left(-e^2px^6\left(c(d+ex^3)^p\right)^{3/p} + d^2\left(c(d+ex^3)^p\right)^{2/p}\text{Ei}\left(\frac{\log(c(d+ex^3)^p)}{p}\right)\log\left(c(d+ex^3)^p\right) - 4d(d+ex^3)\left(c(d+ex^3)^p\right)^{\frac{1}{p}}\text{Ei}\left(\frac{2\log(c(d+ex^3)^p)}{p}\right)\log\left(c(d+ex^3)^p\right) + 3d^2\text{Ei}\left(\frac{3\log(c(d+ex^3)^p)}{p}\right)\log\left(c(d+ex^3)^p\right) + 6dex^3\text{Ei}\left(\frac{3\log(c(d+ex^3)^p)}{p}\right)\log\left(c(d+ex^3)^p\right) + 3e^2x^6\text{Ei}\left(\frac{3\log(c(d+ex^3)^p)}{p}\right)\log\left(c(d+ex^3)^p\right)\right)}{3e^3p^2\log\left(c\left(d+ex^3\right)^p\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] ((d + e*x^3)*(-(e^2*p*x^6*(c*(d + e*x^3)^p)^(3/p)) + d^2*(c*(d + e*x^3)^p)^
(2/p)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p]*Log[c*(d + e*x^3)^p] - 4*d*(d +
 e*x^3)*(c*(d + e*x^3)^p)^p^(-1)*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p])/p]*
Log[c*(d + e*x^3)^p] + 3*d^2*ExpIntegralEi[(3*Log[c*(d + e*x^3)^p])/p]*Log[
c*(d + e*x^3)^p] + 6*d*e*x^3*ExpIntegralEi[(3*Log[c*(d + e*x^3)^p])/p]*Log[
c*(d + e*x^3)^p] + 3*e^2*x^6*ExpIntegralEi[(3*Log[c*(d + e*x^3)^p])/p]*Log[
c*(d + e*x^3)^p]))/(3*e^3*p^2*(c*(d + e*x^3)^p)^(3/p)*Log[c*(d + e*x^3)^p])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.80, size = 2564, normalized size = 13.15

method	result	size
risch	Expression too large to display	2564

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/ln(c*(e*x^3+d)^p)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/p/e*x^6*(e*x^3+d)/(2*ln(c)+2*ln((e*x^3+d)^p)+I*Pi*csgn(I*(e*x^3+d)^p)*
csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn
(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c))-
1/3/p^2/e^2*d^2*c^(-1/p)*((e*x^3+d)^p)^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(e*x^3+
d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*(e*
x^3+d)^p))/p)*Ei(1,-ln(e*x^3+d)-1/2*(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x
^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*cs
gn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((
e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)*x^3-1/3/p^2/e^3*d^3*c^(-1/p)*((e*x^3+d)^p)^
(-1/p)*exp(1/2*I*Pi*csgn(I*c*(e*x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c)
)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*(e*x^3+d)^p))/p)*Ei(1,-ln(e*x^3+d)-1/2*(I*
Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csg
n(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*
x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)-1/p^2*c
^(-3/p)*((e*x^3+d)^p)^(-3/p)*exp(3/2*I*Pi*csgn(I*c*(e*x^3+d)^p)*(-csgn(I*c*
(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*(e*x^3+d)^p))/p)*Ei(
1,-3*ln(e*x^3+d)-3/2*(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi
*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d
)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p)-2*p
*ln(e*x^3+d))/p)*x^9-3/p^2/e*c^(-3/p)*((e*x^3+d)^p)^(-3/p)*exp(3/2*I*Pi*csg
n(I*c*(e*x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^
p)+csgn(I*(e*x^3+d)^p))/p)*Ei(1,-3*ln(e*x^3+d)-3/2*(I*Pi*csgn(I*(e*x^3+d)^p
)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*cs
gn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)
+2*ln(c)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)*d*x^6-3/p^2/e^2*c^(-3/p)*((e
*x^3+d)^p)^(-3/p)*exp(3/2*I*Pi*csgn(I*c*(e*x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p
)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*(e*x^3+d)^p))/p)*Ei(1,-3*ln(e*x
^3+d)-3/2*(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*
x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi
*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d
))/p)*d^2*x^3-1/p^2/e^3*c^(-3/p)*((e*x^3+d)^p)^(-3/p)*exp(3/2*I*Pi*csgn(I*c
*(e*x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+cs
gn(I*(e*x^3+d)^p))/p)*Ei(1,-3*ln(e*x^3+d)-3/2*(I*Pi*csgn(I*(e*x^3+d)^p)*csg
n(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*
c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln
(c)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)*d^3+4/3/p^2/e*d*c^(-2/p)*((e*x^3+
d)^p)^(-2/p)*exp(I*Pi*csgn(I*c*(e*x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*
c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*(e*x^3+d)^p))/p)*Ei(1,-2*ln(e*x^3+d)-(I*
Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csg
n(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*
```

```
x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)*x^6+8/3
/p^2/e^2*d^2*c^(-2/p)*((e*x^3+d)^p)^(-2/p)*exp(I*Pi*csgn(I*c*(e*x^3+d)^p)*(
-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*(e*x^3+d)^
p))/p)*Ei(1,-2*ln(e*x^3+d)-(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^
2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e
*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^
p)-2*p*ln(e*x^3+d))/p)*x^3+4/3/p^2/e^3*d^3*c^(-2/p)*((e*x^3+d)^p)^(-2/p)*ex
p(I*Pi*csgn(I*c*(e*x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*
(e*x^3+d)^p)+csgn(I*(e*x^3+d)^p))/p)*Ei(1,-2*ln(e*x^3+d)-(I*Pi*csgn(I*(e*x^
3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)
^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csg
n(I*c)+2*ln(c)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(e*x^9 + d*x^6)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)) + integrate((3*e
*x^8 + 2*d*x^5)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)), x)
```

Fricas [A]
time = 0.38, size = 211, normalized size = 1.08

$$-\frac{4\,(dp\log\left(x^3e+d\right)+d\log\left(c\right))c^{\left(\frac{1}{p}\right)}\operatorname{log\_integral}\left((x^6e^2+2\,dx^3e+d^2)c^{\frac{2}{p}}\right)-(d^2p\log\left(x^3e+d\right)+d^2\log\left(c\right))c^{\frac{2}{p}}\operatorname{log\_integral}\left((x^3e+d)c^{\left(\frac{1}{p}\right)}\right)+(px^9e^3+dpx^6e^2)c^{\frac{3}{p}}-3\,(p\log\left(x^3e+d\right)+\log\left(c\right))\operatorname{log\_integral}\left((x^9e^3+3\,dx^6e^2+3\,d^2x^3e+d^3)c^{\frac{3}{p}}\right)}{3\,(p^3e^3\log\left(x^3e+d\right)+p^2e^3\log\left(c\right))c^{\frac{3}{p}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(4*(d*p*log(x^3*e + d) + d*log(c))*c^(1/p)*log_integral((x^6*e^2 + 2*d
*x^3*e + d^2)*c^(2/p)) - (d^2*p*log(x^3*e + d) + d^2*log(c))*c^(2/p)*log_in
tegral((x^3*e + d)*c^(1/p)) + (p*x^9*e^3 + d*p*x^6*e^2)*c^(3/p) - 3*(p*log(
x^3*e + d) + log(c))*log_integral((x^9*e^3 + 3*d*x^6*e^2 + 3*d^2*x^3*e + d^
3)*c^(3/p)))/((p^3*e^3*log(x^3*e + d) + p^2*e^3*log(c))*c^(3/p))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\log\left(c\left(d+ex^3\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Integral(x**8/log(c*(d + e*x**3)**p)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(200) = 400$.
time = 5.35, size = 490, normalized size = 2.51

$$-\frac{1}{3}d^{2}\left(\frac{(x^{3}e+d)p}{p^{3}e^{3}\log(x^{3}e+d)+p^{2}e^{3}\log(c)}-\frac{p\mathrm{Ei}\left(\frac{\log(c)}{p}+\log(x^{3}e+d)\right)\log(x^{3}e+d)}{(p^{3}e^{3}\log(x^{3}e+d)+p^{2}e^{3}\log(c))c^{\left(\frac{1}{p}\right)}}-\frac{\mathrm{Ei}\left(\frac{\log(c)}{p}+\log(x^{3}e+d)\right)\log(c)}{(p^{3}e^{3}\log(x^{3}e+d)+p^{2}e^{3}\log(c))c^{\left(\frac{1}{p}\right)}}\right)-\frac{(x^{3}e+d)^{3}p}{3(p^{3}e^{3}\log(x^{3}e+d)+p^{2}e^{3}\log(c))}+\frac{2(x^{3}e+d)^{2}dp}{3(p^{3}e^{3}\log(x^{3}e+d)+p^{2}e^{3}\log(c))}-\frac{4dp\mathrm{Ei}\left(\frac{2\log(c)}{p}+2\log(x^{3}e+d)\right)\log(x^{3}e+d)}{3(p^{3}e^{3}\log(x^{3}e+d)+p^{2}e^{3}\log(c))c^{\frac{2}{p}}}+\frac{p\mathrm{Ei}\left(\frac{3\log(c)}{p}+3\log(x^{3}e+d)\right)\log(x^{3}e+d)}{(p^{3}e^{3}\log(x^{3}e+d)+p^{2}e^{3}\log(c))c^{\frac{3}{p}}}-\frac{4d\mathrm{Ei}\left(\frac{2\log(c)}{p}+2\log(x^{3}e+d)\right)\log(c)}{3(p^{3}e^{3}\log(x^{3}e+d)+p^{2}e^{3}\log(c))c^{\frac{2}{p}}}+\frac{\mathrm{Ei}\left(\frac{3\log(c)}{p}+3\log(x^{3}e+d)\right)\log(c)}{(p^{3}e^{3}\log(x^{3}e+d)+p^{2}e^{3}\log(c))c^{\frac{3}{p}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] -1/3*d^2*((x^3*e + d)*p/(p^3*e^3*log(x^3*e + d) + p^2*e^3*log(c)) - p*Ei(lo
g(c)/p + log(x^3*e + d))*log(x^3*e + d)/((p^3*e^3*log(x^3*e + d) + p^2*e^3*
log(c))*c^(1/p)) - Ei(log(c)/p + log(x^3*e + d))*log(c)/((p^3*e^3*log(x^3*e
 + d) + p^2*e^3*log(c))*c^(1/p))) - 1/3*(x^3*e + d)^3*p/(p^3*e^3*log(x^3*e
+ d) + p^2*e^3*log(c)) + 2/3*(x^3*e + d)^2*d*p/(p^3*e^3*log(x^3*e + d) + p^
2*e^3*log(c)) - 4/3*d*p*Ei(2*log(c)/p + 2*log(x^3*e + d))*log(x^3*e + d)/((
p^3*e^3*log(x^3*e + d) + p^2*e^3*log(c))*c^(2/p)) + p*Ei(3*log(c)/p + 3*log
(x^3*e + d))*log(x^3*e + d)/((p^3*e^3*log(x^3*e + d) + p^2*e^3*log(c))*c^(3
/p)) - 4/3*d*Ei(2*log(c)/p + 2*log(x^3*e + d))*log(c)/((p^3*e^3*log(x^3*e +
 d) + p^2*e^3*log(c))*c^(2/p)) + Ei(3*log(c)/p + 3*log(x^3*e + d))*log(c)/(
(p^3*e^3*log(x^3*e + d) + p^2*e^3*log(c))*c^(3/p))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8}{\ln\left(c\,{\left(e\,x^3+d\right)}^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/log(c*(d + e*x^3)^p)^2,x)
```

```
[Out] int(x^8/log(c*(d + e*x^3)^p)^2, x)
```

3.149 $\int \frac{x^5}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal. Leaf size=141

$$-\frac{d(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c(d+ex^3)^p\right)}{p}\right)}{3e^2p^2}+\frac{2(d+ex^3)^2\left(c(d+ex^3)^p\right)^{-2/p}\operatorname{Ei}\left(\frac{2\log\left(c(d+ex^3)^p\right)}{p}\right)}{3e^2p^2}-\frac{x}{3ep\log}$$

```
[Out] -1/3*d*(e*x^3+d)*Ei(ln(c*(e*x^3+d)^p)/p)/e^2/p^2/((c*(e*x^3+d)^p)^(1/p))+2/
3*(e*x^3+d)^2*Ei(2*ln(c*(e*x^3+d)^p)/p)/e^2/p^2/((c*(e*x^3+d)^p)^(2/p))-1/3
*x^3*(e*x^3+d)/e/p/ln(c*(e*x^3+d)^p)
```

Rubi [A]
time = 0.14, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {2504, 2447, 2446, 2436, 2337, 2209, 2437, 2347}

$$\frac{2(d+ex^3)^2\left(c(d+ex^3)^p\right)^{-2/p}\operatorname{Ei}\left(\frac{2\log\left(c(ex^3+d)^p\right)}{p}\right)}{3e^2p^2}-\frac{d(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c(ex^3+d)^p\right)}{p}\right)}{3e^2p^2}-\frac{x^3(d+ex^3)}{3ep\log\left(c\left(d+ex^3\right)^p\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] -1/3*(d*(d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(e^2*p^2*(c*(d +
 e*x^3)^p)^p^(-1)) + (2*(d + e*x^3)^2*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p]
)/p])/(3*e^2*p^2*(c*(d + e*x^3)^p)^(2/p)) - (x^3*(d + e*x^3))/(3*e*p*Log[c*
(d + e*x^3)^p])
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2447

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[q*((e*f - d*g)/
(b*e*n*(p + 1))), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{x^5}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x}{\log^2\left(c(d+ex)^p\right)}\,dx, x, x^3\right)$$

$$= -\frac{x^3(d+ex^3)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{2\text{Subst}\left(\int \frac{x}{\log(c(d+ex)^p)}\,dx, x, x^3\right)}{3p} + \frac{d\text{Subst}\left(\int \frac{1}{\log(c(d+ex}\right.}{3ep}$$

$$= -\frac{x^3(d+ex^3)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{2\text{Subst}\left(\int \left(-\frac{d}{e\log(c(d+ex)^p)} + \frac{d+ex}{e\log(c(d+ex)^p)}\right)\,dx, x, x^3\right)}{3p}$$

$$= -\frac{x^3(d+ex^3)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{2\text{Subst}\left(\int \frac{d+ex}{\log(c(d+ex)^p)}\,dx, x, x^3\right)}{3ep} - \frac{(2d)\text{Subst}\left(\int \frac{}{\log(c(d}\right.}{3ep}$$

$$= \frac{d(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\text{Ei}\left(\frac{\log(c(d+ex^3)^p)}{p}\right)}{3e^2p^2} - \frac{x^3(d+ex^3)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{2\text{Subs}}{}$$

$$= \frac{d(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\text{Ei}\left(\frac{\log(c(d+ex^3)^p)}{p}\right)}{3e^2p^2} - \frac{x^3(d+ex^3)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{\left(2(d\right.}{}$$

$$= -\frac{d(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\text{Ei}\left(\frac{\log(c(d+ex^3)^p)}{p}\right)}{3e^2p^2} + \frac{2(d+ex^3)^2\left(c(d+ex^3)^p\right)^{-2/}}{3e^2p^2}$$

Mathematica [A]
time = 0.09, size = 157, normalized size = 1.11

$$-\frac{(d+ex^3)\left(c(d+ex^3)^p\right)^{-2/p}\left(epx^3\left(c(d+ex^3)^p\right)^{2/p} + d\left(c(d+ex^3)^p\right)^{\frac{1}{p}}\text{Ei}\left(\frac{\log(c(d+ex^3)^p)}{p}\right)\log\left(c(d+ex^3)^p\right) - 2(d+ex^3)\text{Ei}\left(\frac{2\log(c(d+ex^3)^p)}{p}\right)\log\left(c(d+ex^3)^p\right)\right)}{3e^2p^2\log\left(c\left(d+ex^3\right)^p\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] -1/3*((d + e*x^3)*(e*p*x^3*(c*(d + e*x^3)^p)^(2/p) + d*(c*(d + e*x^3)^p)^p^
(-1)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p]*Log[c*(d + e*x^3)^p] - 2*(d + e*
x^3)*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p])/p]*Log[c*(d + e*x^3)^p]))/(e^2*
p^2*(c*(d + e*x^3)^p)^(2/p)*Log[c*(d + e*x^3)^p])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.64, size = 1487, normalized size = 10.55

method	result	size
risch	Expression too large to display	1487

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/ln(c*(e*x^3+d)^p)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/p/e*x^3*(e*x^3+d)/(2*ln(c)+2*ln((e*x^3+d)^p)+I*Pi*csgn(I*(e*x^3+d)^p)*
csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn
(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c))-
2/3/p^2*c^(-2/p)*((e*x^3+d)^p)^(-2/p)*exp(I*Pi*csgn(I*c*(e*x^3+d)^p)*(-csgn
(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*(e*x^3+d)^p))/p
)*Ei(1,-2*ln(e*x^3+d)-(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*P
i*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+
d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p)-2*
p*ln(e*x^3+d))/p)*x^6-4/3/p^2/e*c^(-2/p)*((e*x^3+d)^p)^(-2/p)*exp(I*Pi*csgn
(I*c*(e*x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p
)+csgn(I*(e*x^3+d)^p))/p)*Ei(1,-2*ln(e*x^3+d)-(I*Pi*csgn(I*(e*x^3+d)^p)*csg
n(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*
c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln
(c)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)*d*x^3-2/3/p^2/e^2*c^(-2/p)*((e*x^
3+d)^p)^(-2/p)*exp(I*Pi*csgn(I*c*(e*x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(
I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*(e*x^3+d)^p))/p)*Ei(1,-2*ln(e*x^3+d)-(
I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*c
sgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(
e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)*d^2+1
/3/p^2/e*d*c^(-1/p)*((e*x^3+d)^p)^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(e*x^3+d)^p)
*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*(e*x^3+d
)^p))/p)*Ei(1,-ln(e*x^3+d)-1/2*(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)
^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*
c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3
+d)^p)-2*p*ln(e*x^3+d))/p)*x^3+1/3/p^2/e^2*d^2*c^(-1/p)*((e*x^3+d)^p)^(-1/p
)*exp(1/2*I*Pi*csgn(I*c*(e*x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-c
sgn(I*c*(e*x^3+d)^p)+csgn(I*(e*x^3+d)^p))/p)*Ei(1,-ln(e*x^3+d)-1/2*(I*Pi*cs
gn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c
*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d
)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(e*x^6 + d*x^3)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)) + integrate((2*e
*x^5 + d*x^2)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)), x)
```

Fricas [A]

time = 0.35, size = 143, normalized size = 1.01

$$-\frac{(dp\log(x^3e+d)+d\log(c))c^{\left(\frac{1}{p}\right)}\,\mathrm{log\_integral}\left((x^3e+d)c^{\left(\frac{1}{p}\right)}\right)+(px^6e^2+dpx^3e)c^{\frac{2}{p}}-2\,(p\log(x^3e+d)+\log(c))\,\mathrm{log\_integral}\left((x^6e^2+2\,dx^3e+d^2)c^{\frac{2}{p}}\right)}{3\,(p^3e^2\log(x^3e+d)+p^2e^2\log(c))c^{\frac{2}{p}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] -1/3*((d*p*log(x^3*e + d) + d*log(c))*c^(1/p)*log_integral((x^3*e + d)*c^(1
/p)) + (p*x^6*e^2 + d*p*x^3*e)*c^(2/p) - 2*(p*log(x^3*e + d) + log(c))*log_
integral((x^6*e^2 + 2*d*x^3*e + d^2)*c^(2/p)))/((p^3*e^2*log(x^3*e + d) + p
^2*e^2*log(c))*c^(2/p))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\log\left(c\left(d+ex^3\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Integral(x**5/log(c*(d + e*x**3)**p)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(142) = 284.
time = 3.00, size = 326, normalized size = 2.31

$$\frac{1}{3}\,d\left(\frac{(x^3e+d)p}{p^3e^2\log(x^3e+d)+p^2e^2\log(c)}-\frac{p\mathrm{Ei}\left(\frac{\log(c)}{p}+\log(x^3e+d)\right)\log(x^3e+d)}{(p^3e^2\log(x^3e+d)+p^2e^2\log(c))c^{\left(\frac{1}{p}\right)}}-\frac{\mathrm{Ei}\left(\frac{\log(c)}{p}+\log(x^3e+d)\right)\log(c)}{(p^3e^2\log(x^3e+d)+p^2e^2\log(c))c^{\left(\frac{1}{p}\right)}}\right)-\frac{1}{3}\left(\frac{(x^3e+d)^2p}{p^3e\log(x^3e+d)+p^2e\log(c)}-\frac{2\,p\mathrm{Ei}\left(\frac{2\,\log(c)}{p}+2\,\log(x^3e+d)\right)\log(x^3e+d)}{(p^3e\log(x^3e+d)+p^2e\log(c))c^{\frac{2}{p}}}-\frac{2\,\mathrm{Ei}\left(\frac{2\,\log(c)}{p}+2\,\log(x^3e+d)\right)\log(c)}{(p^3e\log(x^3e+d)+p^2e\log(c))c^{\frac{2}{p}}}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] 1/3*d*((x^3*e + d)*p/(p^3*e^2*log(x^3*e + d) + p^2*e^2*log(c)) - p*Ei(log(c
)/p + log(x^3*e + d))*log(x^3*e + d)/((p^3*e^2*log(x^3*e + d) + p^2*e^2*log
(c))*c^(1/p)) - Ei(log(c)/p + log(x^3*e + d))*log(c)/((p^3*e^2*log(x^3*e +
d) + p^2*e^2*log(c))*c^(1/p))) - 1/3*((x^3*e + d)^2*p/(p^3*e*log(x^3*e + d)
 + p^2*e*log(c)) - 2*p*Ei(2*log(c)/p + 2*log(x^3*e + d))*log(x^3*e + d)/((p
^3*e*log(x^3*e + d) + p^2*e*log(c))*c^(2/p)) - 2*Ei(2*log(c)/p + 2*log(x^3*
e + d))*log(c)/((p^3*e*log(x^3*e + d) + p^2*e*log(c))*c^(2/p)))*e^(-1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/log(c*(d + e*x^3)^p)^2,x)
```

```
[Out] int(x^5/log(c*(d + e*x^3)^p)^2, x)
```

3.150 $\int \frac{x^2}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal. Leaf size=83

$$\frac{(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c(d+ex^3)^p\right)}{p}\right)}{3ep^2}-\frac{d+ex^3}{3ep\log\left(c\left(d+ex^3\right)^p\right)}$$

```
[Out] 1/3*(e*x^3+d)*Ei(ln(c*(e*x^3+d)^p)/p)/e/p^2/((c*(e*x^3+d)^p)^(1/p))+1/3*(-e
*x^3-d)/e/p/ln(c*(e*x^3+d)^p)
```

Rubi [A]
time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2504, 2436, 2334, 2337, 2209}

$$\frac{(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log\left(c(ex^3+d)^p\right)}{p}\right)}{3ep^2}-\frac{d+ex^3}{3ep\log\left(c\left(d+ex^3\right)^p\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] ((d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(3*e*p^2*(c*(d + e*x^3)
^p)^p^(-1)) - (d + e*x^3)/(3*e*p*Log[c*(d + e*x^3)^p])
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\log^2\left(c(d+ex)^p\right)}\,dx, x, x^3\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{\log^2(cx^p)}\,dx, x, d+ex^3\right)}{3e}\\
&= -\frac{d+ex^3}{3ep\log\left(c\left(d+ex^3\right)^p\right)}+\frac{\text{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx, x, d+ex^3\right)}{3ep}\\
&= -\frac{d+ex^3}{3ep\log\left(c\left(d+ex^3\right)^p\right)}+\frac{\left(\left(d+ex^3\right)\left(c(d+ex^3)^p\right)^{-1/p}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{p}}}{x}\,dx, x, \log (\right.}{3ep^2}\\
&= \frac{\left(d+ex^3\right)\left(c(d+ex^3)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c(d+ex^3)^p\right)}{p}\right)}{3ep^2}-\frac{d+ex^3}{3ep\log\left(c\left(d+ex^3\right)^p\right)}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 97, normalized size = 1.17

$$
-\frac{\left(d+ex^3\right)\left(c(d+ex^3)^p\right)^{-1/p}\left(p\left(c(d+ex^3)^p\right)^{\frac{1}{p}}-\text{Ei}\left(\frac{\log\left(c(d+ex^3)^p\right)}{p}\right)\log\left(c(d+ex^3)^p\right)\right)}{3ep^2\log\left(c\left(d+ex^3\right)^p\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] -1/3*((d + e*x^3)*(p*(c*(d + e*x^3)^p)^p^(-1) - ExpIntegralEi[Log[c*(d + e*
x^3)^p]/p]*Log[c*(d + e*x^3)^p]))/(e*p^2*(c*(d + e*x^3)^p)^p^(-1)*Log[c*(d
+ e*x^3)^p])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.52, size = 421, normalized size = 5.07

method	result
risch	$-\frac{2(e\,x^3+d)}{3\left(2\ln(c)+2\ln((e\,x^3+d)^p)+i\pi\,\mathrm{csgn}(i(e\,x^3+d)^p)\mathrm{csgn}(ic(e\,x^3+d)^p)^2-i\pi\,\mathrm{csgn}(i(e\,x^3+d)^p)\mathrm{csgn}(ic(e\,x^3+d)^p)\mathrm{csgn}(ic)-i\pi\mathrm{csgn}(ic(e\,x^3\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/ln(c*(e*x^3+d)^p)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(2*ln(c)+2*ln((e*x^3+d)^p)+I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)
^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*
c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c))/p/e*(e*x^3+d)-1/3/
p^2/e*(e*x^3+d)*c^(-1/p)*((e*x^3+d)^p)^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(e*x^3+
d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*(e*
x^3+d)^p))/p)*Ei(1,-ln(e*x^3+d)-1/2*(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x
^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*cs
gn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((
e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(e*x^3 + d)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)) + integrate(x^2/(p*l
og((e*x^3 + d)^p) + p*log(c)), x)
```

Fricas [A]
time = 0.37, size = 84, normalized size = 1.01

$$-\frac{(px^3e+dp)c^{\left(\frac{1}{p}\right)}-(p\log\left(x^3e+d\right)+\log\left(c\right))\,\mathrm{log\_integral}\left((x^3e+d)c^{\left(\frac{1}{p}\right)}\right)}{3\,(p^3e\log\left(x^3e+d\right)+p^2e\log\left(c\right))c^{\left(\frac{1}{p}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] -1/3*((p*x^3*e + d*p)*c^(1/p) - (p*log(x^3*e + d) + log(c))*log_integral((x
^3*e + d)*c^(1/p)))/((p^3*e*log(x^3*e + d) + p^2*e*log(c))*c^(1/p))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\log\left(c\left(d+ex^3\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Integral(x**2/log(c*(d + e*x**3)**p)**2, x)
```

Giac [A]
time = 5.33, size = 154, normalized size = 1.86

$$-\frac{(x^3e+d)p}{3\left(p^3e\log\left(x^3e+d\right)+p^2e\log\left(c\right)\right)}+\frac{p\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(x^3e+d\right)\right)\log\left(x^3e+d\right)}{3\left(p^3e\log\left(x^3e+d\right)+p^2e\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}}+\frac{\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(x^3e+d\right)\right)\log\left(c\right)}{3\left(p^3e\log\left(x^3e+d\right)+p^2e\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] -1/3*(x^3*e + d)*p/(p^3*e*log(x^3*e + d) + p^2*e*log(c)) + 1/3*p*Ei(log(c)/
p + log(x^3*e + d))*log(x^3*e + d)/((p^3*e*log(x^3*e + d) + p^2*e*log(c))*c
^(1/p)) + 1/3*Ei(log(c)/p + log(x^3*e + d))*log(c)/((p^3*e*log(x^3*e + d) +
 p^2*e*log(c))*c^(1/p))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\ln\left(c\,{\left(e\,x^3+d\right)}^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/log(c*(d + e*x^3)^p)^2,x)
```

```
[Out] int(x^2/log(c*(d + e*x^3)^p)^2, x)
```

3.151 $\int \frac{1}{x \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x \log^2\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x/ln(c*(e*x^3+d)^p)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*Log[c*(d + e*x^3)^p]^2),x]
```

```
[Out] Defer[Int][1/(x*Log[c*(d + e*x^3)^p]^2), x]
```

Rubi steps

$$\int \frac{1}{x \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx = \int \frac{1}{x \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$$

Mathematica [A]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1}{x \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*Log[c*(d + e*x^3)^p]^2),x]
```

```
[Out] Integrate[1/(x*Log[c*(d + e*x^3)^p]^2), x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \ln\left(c\left(e\,x^3+d\right)^p\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(c*(e*x^3+d)^p)^2,x)
```

```
[Out] int(1/x/ln(c*(e*x^3+d)^p)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -d*integrate(1/(e*p*x^4*log((e*x^3 + d)^p) + e*p*x^4*log(c)), x) - 1/3*(e*x
^3 + d)/(e*p*x^3*log((e*x^3 + d)^p) + e*p*x^3*log(c))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(x*log((x^3*e + d)^p*c)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \log{\left(c \left(d + e x^{3}\right)^{p} \right)}^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Integral(1/(x*log(c*(d + e*x**3)**p)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x*log((x^3*e + d)^p*c)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x\ln\left(c\left(e\,x^3+d\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*log(c*(d + e*x^3)^p)^2),x)
[Out] int(1/(x*log(c*(d + e*x^3)^p)^2), x)
```

3.152 $\int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x^4/ln(c*(e*x^3+d)^p)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^4*Log[c*(d + e*x^3)^p]^2),x]
```

```
[Out] Defer[Int][1/(x^4*Log[c*(d + e*x^3)^p]^2), x]
```

Rubi steps

$$\int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx = \int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$$

Mathematica [A]
time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^4*Log[c*(d + e*x^3)^p]^2),x]
```

```
[Out] Integrate[1/(x^4*Log[c*(d + e*x^3)^p]^2), x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \ln\left(c\left(e\,x^3+d\right)^p\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/ln(c*(e*x^3+d)^p)^2,x)
[Out] int(1/x^4/ln(c*(e*x^3+d)^p)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
[Out] -1/3*(e*x^3 + d)/(e*p*x^6*log((e*x^3 + d)^p) + e*p*x^6*log(c)) - integrate(
(e*x^3 + 2*d)/(e*p*x^7*log((e*x^3 + d)^p) + e*p*x^7*log(c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
[Out] integral(1/(x^4*log((x^3*e + d)^p*c)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \log \left(c \left(d + e x^3\right)^p\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/ln(c*(e*x**3+d)**p)**2,x)
[Out] Integral(1/(x**4*log(c*(d + e*x**3)**p)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
[Out] integrate(1/(x^4*log((x^3*e + d)^p*c)^2), x)
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^4 \ln \left(c \left(e\, x^3 + d\right)^p\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*log(c*(d + e*x^3)^p)^2),x)
```

```
[Out] int(1/(x^4*log(c*(d + e*x^3)^p)^2), x)
```

3.153 $\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable(x^3/ln(c*(e*x^3+d)^p)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^3/Log[c*(d + e*x^3)^p]^2,x]
[Out] Defer[Int][x^3/Log[c*(d + e*x^3)^p]^2, x]
```

Rubi steps

$$\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Mathematica [A]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^3/Log[c*(d + e*x^3)^p]^2,x]
[Out] Integrate[x^3/Log[c*(d + e*x^3)^p]^2, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/ln(c*(e*x^3+d)^p)^2,x)
```

```
[Out] int(x^3/ln(c*(e*x^3+d)^p)^2,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(e*x^4 + d*x)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)) + integrate(1/3*(4
*e*x^3 + d)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3/log((x^3*e + d)^p*c)^2, x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Integral(x**3/log(c*(d + e*x**3)**p)**2, x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3/log((x^3*e + d)^p*c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^3}{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/log(c*(d + e*x^3)^p)^2,x)
[Out] int(x^3/log(c*(d + e*x^3)^p)^2, x)
```

3.154 $\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)},x\right)$$

`[Out] Unintegrable(x/ln(c*(e*x^3+d)^p)^2,x)`

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$$

Verification is not applicable to the result.

`[In] Int[x/Log[c*(d + e*x^3)^p]^2,x]`

`[Out] Defer[Int][x/Log[c*(d + e*x^3)^p]^2, x]`

Rubi steps

$$\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)} \, dx=\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$$

Mathematica [A]
time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$$

Verification is not applicable to the result.

`[In] Integrate[x/Log[c*(d + e*x^3)^p]^2,x]`

`[Out] Integrate[x/Log[c*(d + e*x^3)^p]^2, x]`

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x}{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/ln(c*(e*x^3+d)^p)^2,x)
[Out] int(x/ln(c*(e*x^3+d)^p)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
[Out] -1/3*(e*x^3 + d)/(e*p*x*log((e*x^3 + d)^p) + e*p*x*log(c)) + integrate(1/3*
(2*e*x^3 - d)/(e*p*x^2*log((e*x^3 + d)^p) + e*p*x^2*log(c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
[Out] integral(x/log((x^3*e + d)^p*c)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/ln(c*(e*x**3+d)**p)**2,x)
[Out] Integral(x/log(c*(d + e*x**3)**p)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
[Out] integrate(x/log((x^3*e + d)^p*c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x}{\ln \left(c \left(e\,x^3+d\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/log(c*(d + e*x^3)^p)^2,x)
```

```
[Out] int(x/log(c*(d + e*x^3)^p)^2, x)
```

3.155 $\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)},x\right)$$

`[Out] Unintegrable(1/ln(c*(e*x^3+d)^p)^2,x)`

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is not applicable to the result.

`[In] Int[Log[c*(d + e*x^3)^p]^(-2),x]`

`[Out] Defer[Int][Log[c*(d + e*x^3)^p]^(-2), x]`

Rubi steps

$$\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Mathematica [A]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is not applicable to the result.

`[In] Integrate[Log[c*(d + e*x^3)^p]^(-2),x]`

`[Out] Integrate[Log[c*(d + e*x^3)^p]^(-2), x]`

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(c*(e*x^3+d)^p)^2,x)
```

```
[Out] int(1/ln(c*(e*x^3+d)^p)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(e*x^3 + d)/(e*p*x^2*log((e*x^3 + d)^p) + e*p*x^2*log(c)) + integrate(
1/3*(e*x^3 - 2*d)/(e*p*x^3*log((e*x^3 + d)^p) + e*p*x^3*log(c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(log((x^3*e + d)^p*c)^(-2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\log \left(c \left(d + e x^{3}\right)^{p}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Integral(log(c*(d + e*x**3)**p)**(-2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(log((x^3*e + d)^p*c)^(-2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/log(c*(d + e*x^3)^p)^2,x)
```

```
[Out] int(1/log(c*(d + e*x^3)^p)^2, x)
```

3.156 $\int \frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)} dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x^2/ln(c*(e*x^3+d)^p)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)} dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*Log[c*(d + e*x^3)^p]^2),x]
[Out] Defer[Int][1/(x^2*Log[c*(d + e*x^3)^p]^2), x]
```

Rubi steps

$$\int \frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)} dx = \int \frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)} dx$$

Mathematica [A]
time = 0.77, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*Log[c*(d + e*x^3)^p]^2),x]
[Out] Integrate[1/(x^2*Log[c*(d + e*x^3)^p]^2), x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \ln\left(c\left(e\,x^3+d\right)^p\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/ln(c*(e*x^3+d)^p)^2,x)
```

```
[Out] int(1/x^2/ln(c*(e*x^3+d)^p)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(e*x^3 + d)/(e*p*x^4*log((e*x^3 + d)^p) + e*p*x^4*log(c)) - integrate(
1/3*(e*x^3 + 4*d)/(e*p*x^5*log((e*x^3 + d)^p) + e*p*x^5*log(c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(x^2*log((x^3*e + d)^p*c)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \log \left(c \left(d + e x^{3}\right)^{p}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Integral(1/(x**2*log(c*(d + e*x**3)**p)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*log((x^3*e + d)^p*c)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^2 \ln\left(c\left(e\,x^3+d\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*log(c*(d + e*x^3)^p)^2),x)
[Out] int(1/(x^2*log(c*(d + e*x^3)^p)^2), x)
```

3.157 $\int \frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal. Leaf size=21

$$\operatorname{Int}\left(\frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x^3/ln(c*(e*x^3+d)^p)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^3*Log[c*(d + e*x^3)^p]^2),x]
[Out] Defer[Int][1/(x^3*Log[c*(d + e*x^3)^p]^2), x]
```

Rubi steps

$$\int \frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Mathematica [A]
time = 0.75, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^3*Log[c*(d + e*x^3)^p]^2),x]
[Out] Integrate[1/(x^3*Log[c*(d + e*x^3)^p]^2), x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \ln\left(c\left(e\,x^3+d\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/ln(c*(e*x^3+d)^p)^2,x)
[Out] int(1/x^3/ln(c*(e*x^3+d)^p)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
[Out] -1/3*(e*x^3 + d)/(e*p*x^5*log((e*x^3 + d)^p) + e*p*x^5*log(c)) - integrate(
1/3*(2*e*x^3 + 5*d)/(e*p*x^6*log((e*x^3 + d)^p) + e*p*x^6*log(c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
[Out] integral(1/(x^3*log((x^3*e + d)^p*c)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \log \left(c \left(d + ex^3\right)^p\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/ln(c*(e*x**3+d)**p)**2,x)
[Out] Integral(1/(x**3*log(c*(d + e*x**3)**p)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
[Out] integrate(1/(x^3*log((x^3*e + d)^p*c)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^3 \ln \left(c \left(e\, x^3 + d\right)^p\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*log(c*(d + e*x^3)^p)^2),x)
[Out] int(1/(x^3*log(c*(d + e*x^3)^p)^2), x)
```

3.158 $\int (fx)^m \log^3\left(c(d+ex^2)^p\right)\,dx$

Optimal. Leaf size=77

$$\frac{(fx)^{1+m}\log^3\left(c(d+ex^2)^p\right)}{f(1+m)}-\frac{6ep\text{Int}\left(\frac{(fx)^{2+m}\log^2\left(c(d+ex^2)^p\right)}{d+ex^2},x\right)}{f^2(1+m)}$$

```
[Out] (f*x)^(1+m)*ln(c*(e*x^2+d)^p)^3/f/(1+m)-6*e*p*Unintegrable((f*x)^(2+m)*ln(c
*(e*x^2+d)^p)^2/(e*x^2+d),x)/f^2/(1+m)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (fx)^m \log^3\left(c(d+ex^2)^p\right)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(f*x)^m*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] ((f*x)^(1 + m)*Log[c*(d + e*x^2)^p]^3)/(f*(1 + m)) - (6*e*p*Defer[Int][((f*
x)^(2 + m)*Log[c*(d + e*x^2)^p]^2)/(d + e*x^2), x])/(f^2*(1 + m))
```

Rubi steps

$$\int (fx)^m \log^3\left(c(d+ex^2)^p\right)\,dx=\frac{(fx)^{1+m}\log^3\left(c(d+ex^2)^p\right)}{f(1+m)}-\frac{(6ep)\int\frac{(fx)^{2+m}\log^2\left(c(d+ex^2)^p\right)}{d+ex^2}\,dx}{f^2(1+m)}$$

Mathematica [A] Leaf count is larger than twice the leaf count of optimal. 994 vs. 2(77) = 154.
time = 1.43, size = 994, normalized size = 12.91

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] ((f*x)^m*((1 + m)*p^3*x^2*Log[d + e*x^2]^3 + (6*p^3*(-((e*x^2)/d))^(1/2 - m
/2)*(-((1 + m)*(d + e*x^2)*HypergeometricPFQ[{1, 1, 1, 1/2 - m/2}, {2, 2, 2
}, 1 + (e*x^2)/d]) + (1 + m)*(d + e*x^2)*HypergeometricPFQ[{1, 1, 1/2 - m/2
}, {2, 2}, 1 + (e*x^2)/d]*Log[d + e*x^2] + d*(-1 + (-((e*x^2)/d))^((1 + m)/
```

```
2))*Log[d + e*x^2]^2))/e + (6*d*(1 + m)*p^3*((e*x^2)/(d + e*x^2))^(1/2 - m/
2)*(8*HypergeometricPFQ[{1/2 - m/2, 1/2 - m/2, 1/2 - m/2, 1/2 - m/2}, {3/2
- m/2, 3/2 - m/2, 3/2 - m/2}, d/(d + e*x^2)] + (-1 + m)*Log[d + e*x^2]*(-4*
HypergeometricPFQ[{1/2 - m/2, 1/2 - m/2, 1/2 - m/2}, {3/2 - m/2, 3/2 - m/2}
, d/(d + e*x^2)] + (-1 + m)*Hypergeometric2F1[1/2 - m/2, 1/2 - m/2, 3/2 - m
/2, d/(d + e*x^2)]*Log[d + e*x^2])))/(e*(-1 + m)^3) - (3*p^2*(-((e*x^2)/d))
^(1/2 - m/2)*(-((1 + m)*(d + e*x^2)*HypergeometricPFQ[{1, 1, 1, 1/2 - m/2},
 {2, 2, 2}, 1 + (e*x^2)/d]) + (1 + m)*(d + e*x^2)*HypergeometricPFQ[{1, 1,
1/2 - m/2}, {2, 2}, 1 + (e*x^2)/d]*Log[d + e*x^2] + d*(-1 + (-((e*x^2)/d))^
((1 + m)/2))*Log[d + e*x^2]^2)*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])
)/e - (3*m*p^2*(-((e*x^2)/d))^(1/2 - m/2)*(-((1 + m)*(d + e*x^2)*Hypergeome
tricPFQ[{1, 1, 1, 1/2 - m/2}, {2, 2, 2}, 1 + (e*x^2)/d]) + (1 + m)*(d + e*x
^2)*HypergeometricPFQ[{1, 1, 1/2 - m/2}, {2, 2}, 1 + (e*x^2)/d]*Log[d + e*x
^2] + d*(-1 + (-((e*x^2)/d))^((1 + m)/2))*Log[d + e*x^2]^2)*(-(p*Log[d + e*
x^2]) + Log[c*(d + e*x^2)^p]))/e + (3*p*x^2*(-2*e*x^2*Hypergeometric2F1[1,
(3 + m)/2, (5 + m)/2, -((e*x^2)/d)] + d*(3 + m)*Log[d + e*x^2])*(-(p*Log[d
+ e*x^2]) + Log[c*(d + e*x^2)^p])^2)/(d*(3 + m)) + (3*m*p*x^2*(-2*e*x^2*Hyp
ergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2)/d)] + d*(3 + m)*Log[d + e
*x^2])*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/(d*(3 + m)) + x^2*(-
(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^3 + m*x^2*(-(p*Log[d + e*x^2]) +
 Log[c*(d + e*x^2)^p])^3))/((1 + m)^2*x)
```

Maple [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int (fx)^{m} \ln \left(c\left(e\,x^{2}+d\right)^{p}\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(e*x^2+d)^p)^3,x)
```

```
[Out] int((f*x)^m*ln(c*(e*x^2+d)^p)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p)^3,x, algorithm="maxima")
```

```
[Out] f^m*x*x^m*log((e*x^2 + d)^p)^3/(m + 1) + integrate((3*(d*f^m*(m + 1)*log(c)
 + (e*f^m*(m + 1)*log(c) - 2*e*f^m*p)*x^2)*x^m*log((e*x^2 + d)^p)^2 + 3*(e*
f^m*(m + 1)*x^2*log(c)^2 + d*f^m*(m + 1)*log(c)^2)*x^m*log((e*x^2 + d)^p) +
 (e*f^m*(m + 1)*x^2*log(c)^3 + d*f^m*(m + 1)*log(c)^3)*x^m)/(e*(m + 1)*x^2
+ d*(m + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p)^3,x, algorithm="fricas")
[Out] integral((f*x)^m*log((x^2*e + d)^p*c)^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m \log \left(c\left(d+ex^2\right)^p\right)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(e*x**2+d)**p)**3,x)
[Out] Integral((f*x)**m*log(c*(d + e*x**2)**p)**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p)^3,x, algorithm="giac")
[Out] integrate((f*x)^m*log((x^2*e + d)^p*c)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\left(e\,x^2+d\right)^p\right)^3\,{\left(f\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^3*(f*x)^m,x)
[Out] int(log(c*(d + e*x^2)^p)^3*(f*x)^m, x)
```

3.159 $\int (fx)^m \log^2\left(c\left(d+ex^2\right)^p\right)\,dx$

Optimal. Leaf size=75

$$\frac{(fx)^{1+m}\log^2\left(c\left(d+ex^2\right)^p\right)}{f(1+m)}-\frac{4ep\text{Int}\left(\frac{(fx)^{2+m}\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2},x\right)}{f^2(1+m)}$$

```
[Out] (f*x)^(1+m)*ln(c*(e*x^2+d)^p)^2/f/(1+m)-4*e*p*Unintegrable((f*x)^(2+m)*ln(c
*(e*x^2+d)^p)/(e*x^2+d),x)/f^2/(1+m)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (fx)^m \log^2\left(c\left(d+ex^2\right)^p\right)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(f*x)^m*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] ((f*x)^(1 + m)*Log[c*(d + e*x^2)^p]^2)/(f*(1 + m)) - (4*e*p*Defer[Int][((f*
x)^(2 + m)*Log[c*(d + e*x^2)^p])/(d + e*x^2), x])/(f^2*(1 + m))
```

Rubi steps

$$\int (fx)^m \log^2\left(c\left(d+ex^2\right)^p\right)\,dx=\frac{(fx)^{1+m}\log^2\left(c\left(d+ex^2\right)^p\right)}{f(1+m)}-\frac{(4ep)\int\frac{(fx)^{2+m}\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx}{f^2(1+m)}$$

Mathematica [A] Leaf count is larger than twice the leaf count of optimal. 466 vs. 2(75) = 150.
time = 1.06, size = 466, normalized size = 6.21

$$\frac{(fx)^m\left(4p^2x\left(\frac{2ex^2\,{}_2F_1\left(1,\frac{3+m}{2};\frac{5+m}{2};-\frac{ex^2}{d}\right)}{d(3+m)}-\log\left(d+ex^2\right)\right)+(1+m)p^2x\log^2\left(d+ex^2\right)+\frac{4d(1+m)p^2\left(\frac{ex^2}{d+ex^2}\right)^{\frac{1}{2}-\frac{m}{2}}\left(-2\,{}_3F_2\left(\frac{1}{2}-\frac{m}{2},\frac{1}{2}-\frac{m}{2},\frac{1}{2}-\frac{m}{2};\frac{3}{2}-\frac{m}{2},\frac{3}{2}-\frac{m}{2};\frac{d}{d+ex^2}\right)+(-1+m)\,{}_2F_1\left(\frac{1}{2}-\frac{m}{2},\frac{1}{2}-\frac{m}{2};\frac{3}{2}-\frac{m}{2};\frac{d}{d+ex^2}\right)\log(d+ex^2)\right)}{e(-1+m)^2x}+\frac{2p\left(2ex^3\,{}_2F_1\left(1,\frac{3+m}{2};\frac{5+m}{2};-\frac{ex^2}{d}\right)-d(3+m)x\log(d+ex^2)\right)\left(p\log(d+ex^2)-\log\left(c\left(d+ex^2\right)^p\right)\right)}{d(3+m)}-\frac{2mp\left(-2ex^3\,{}_2F_1\left(1,\frac{3+m}{2};\frac{5+m}{2};-\frac{ex^2}{d}\right)+d(3+m)x\log(d+ex^2)\right)\left(p\log(d+ex^2)-\log\left(c\left(d+ex^2\right)^p\right)\right)}{d(3+m)}+x\left(-p\log\left(d+ex^2\right)+\log\left(c\left(d+ex^2\right)^p\right)\right)^2+mx\left(-p\log\left(d+ex^2\right)+\log\left(c\left(d+ex^2\right)^p\right)\right)^2\right)}{(1+m)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] ((f*x)^m*(4*p^2*x*((2*e*x^2*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((e
*x^2)/d)])/(d*(3 + m)) - Log[d + e*x^2]) + (1 + m)*p^2*x*Log[d + e*x^2]^2 +
 (4*d*(1 + m)*p^2*((e*x^2)/(d + e*x^2))^(1/2 - m/2)*(-2*HypergeometricPFQ[{
1/2 - m/2, 1/2 - m/2, 1/2 - m/2}, {3/2 - m/2, 3/2 - m/2}, d/(d + e*x^2)] +
```

```
(-1 + m)*Hypergeometric2F1[1/2 - m/2, 1/2 - m/2, 3/2 - m/2, d/(d + e*x^2)]*
Log[d + e*x^2]))/(e*(-1 + m)^2*x) + (2*p*(2*e*x^3*Hypergeometric2F1[1, (3 +
 m)/2, (5 + m)/2, -((e*x^2)/d)] - d*(3 + m)*x*Log[d + e*x^2])*(p*Log[d + e*
x^2] - Log[c*(d + e*x^2)^p]))/(d*(3 + m)) - (2*m*p*(-2*e*x^3*Hypergeometric
2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2)/d)] + d*(3 + m)*x*Log[d + e*x^2])*(p
*Log[d + e*x^2] - Log[c*(d + e*x^2)^p]))/(d*(3 + m)) + x*(-(p*Log[d + e*x^2
]) + Log[c*(d + e*x^2)^p])^2 + m*x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)
^p])^2))/(1 + m)^2
```

Maple [**A**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \left(f x\right)^{m} \ln \left(c \left(e\,x^{2}+d\right)^{p}\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(e*x^2+d)^p)^2,x)
[Out] int((f*x)^m*ln(c*(e*x^2+d)^p)^2,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
[Out] f^m*x*x^m*log((e*x^2 + d)^p)^2/(m + 1) + integrate((2*(d*f^m*(m + 1)*log(c)
 + (e*f^m*(m + 1)*log(c) - 2*e*f^m*p)*x^2)*x^m*log((e*x^2 + d)^p) + (e*f^m*
(m + 1)*x^2*log(c)^2 + d*f^m*(m + 1)*log(c)^2)*x^m)/(e*(m + 1)*x^2 + d*(m +
 1)), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
[Out] integral((f*x)^m*log((x^2*e + d)^p*c)^2, x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(f x\right)^{m} \log \left(c \left(d + e x^{2}\right)^{p}\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Integral((f*x)**m*log(c*(d + e*x**2)**p)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log((x^2*e + d)^p*c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\left(e\,x^2+d\right)^p\right)^2\,{\left(f\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^2*(f*x)^m,x)
```

```
[Out] int(log(c*(d + e*x^2)^p)^2*(f*x)^m, x)
```

3.160 $\int (fx)^m \log\left(c(d+ex^2)^p\right)\,dx$

Optimal. Leaf size=81

$$-\frac{2ep(fx)^{3+m}\,{}_2F_1\left(1,\frac{3+m}{2};\frac{5+m}{2};-\frac{ex^2}{d}\right)}{df^3(1+m)(3+m)}+\frac{(fx)^{1+m}\log\left(c(d+ex^2)^p\right)}{f(1+m)}$$

```
[Out] -2*e*p*(f*x)^(3+m)*hypergeom([1, 3/2+1/2*m],[5/2+1/2*m],-e*x^2/d)/d/f^3/(1+
m)/(3+m)+(f*x)^(1+m)*ln(c*(e*x^2+d)^p)/f/(1+m)
```

Rubi [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2505, 16, 371}

$$\frac{(fx)^{m+1}\log\left(c(d+ex^2)^p\right)}{f(m+1)}-\frac{2ep(fx)^{m+3}\,{}_2F_1\left(1,\frac{m+3}{2};\frac{m+5}{2};-\frac{ex^2}{d}\right)}{df^3(m+1)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (-2*e*p*(f*x)^(3 + m)*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2)/
d)])/(d*f^3*(1 + m)*(3 + m)) + ((f*x)^(1 + m)*Log[c*(d + e*x^2)^p])/(f*(1 +
 m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (fx)^m \log \left(c(d+ex^2)^p\right) dx = \frac{(fx)^{1+m}\log\left(c(d+ex^2)^p\right)}{f(1+m)} - \frac{(2ep)\int \frac{x(fx)^{1+m}}{d+ex^2}\,dx}{f(1+m)}$$
$$= \frac{(fx)^{1+m}\log\left(c(d+ex^2)^p\right)}{f(1+m)} - \frac{(2ep)\int \frac{(fx)^{2+m}}{d+ex^2}\,dx}{f^2(1+m)}$$
$$= -\frac{2ep(fx)^{3+m}\,{}_2F_1\left(1,\frac{3+m}{2};\frac{5+m}{2};-\frac{ex^2}{d}\right)}{df^3(1+m)(3+m)} + \frac{(fx)^{1+m}\log\left(c(d+ex^2)^p\right)}{f(1+m)}$$

Mathematica [**A**]
time = 0.02, size = 70, normalized size = 0.86

$$\frac{x(fx)^m\left(-2epx^2\,{}_2F_1\left(1,\frac{3+m}{2};\frac{5+m}{2};-\frac{ex^2}{d}\right)+d(3+m)\log\left(c(d+ex^2)^p\right)\right)}{d(1+m)(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Log[c*(d + e*x^2)^p],x]
[Out] (x*(f*x)^m*(-2*e*p*x^2*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2)
/d)] + d*(3 + m)*Log[c*(d + e*x^2)^p]))/(d*(1 + m)*(3 + m))
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int (fx)^m \ln\left(c\left(e\,x^2+d\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*ln(c*(e*x^2+d)^p),x)
[Out] int((f*x)^m*ln(c*(e*x^2+d)^p),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p),x, algorithm="maxima")
[Out] f^m*x*x^m*log((e*x^2 + d)^p)/(m + 1) + integrate((d*f^m*(m + 1)*log(c) + (e
*f^m*(m + 1)*log(c) - 2*e*f^m*p)*x^2)*x^m/(e*(m + 1)*x^2 + d*(m + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] integral((f*x)^m*log((x^2*e + d)^p*c), x)
```

Sympy [A]
time = 32.28, size = 381, normalized size = 4.70

$$-2ep\left(\begin{cases} \frac{0^m\sqrt{-\frac{d}{e^3}}\log\left(-e\sqrt{-\frac{d}{e^3}}+x\right)}{2} - \frac{0^m\sqrt{-\frac{d}{e^3}}\log\left(e\sqrt{-\frac{d}{e^3}}+x\right)}{2} + \frac{0^m x}{e} & \text{for } (f=0 \wedge m\neq -1) \vee f=0 \\ \frac{ff^m m x^3 x^m \Phi\left(\frac{ex^2e^{i\pi}}{d},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4dfm\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)+4df\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{3ff^m x^3 x^m \Phi\left(\frac{ex^2e^{i\pi}}{d},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4dfm\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)+4df\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} & \text{for } m>-\infty \wedge m<\infty \wedge m\neq -1 \\ -\frac{\begin{cases} -\frac{\operatorname{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{for } \frac{1}{|x|}<1 \wedge |x|<1 \\ \log(d)\log(x) - \frac{\operatorname{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{for } |x|<1 \\ -\log(d)\log\left(\frac{1}{x}\right) - \frac{\operatorname{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{for } \frac{1}{|x|}<1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right)\log(d) + G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| x\right)\log(d) - \frac{\operatorname{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{otherwise} \end{cases}}{2ef} + \frac{\log(fx)\log(d+ex^2)}{2ef} & \text{otherwise} \end{cases}\right) + \left(\begin{cases} 0^m x & \text{for } f=0 \\ \frac{\begin{cases} \frac{(fx)^{m+1}}{m+1} & \text{for } m\neq -1 \\ \log(fx) & \text{otherwise} \end{cases}}{f} & \text{otherwise} \end{cases}\right)\log\left(c\left(d+ex^2\right)^p\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*ln(c*(e*x**2+d)**p),x)
```

```
[Out] -2*e*p*Piecewise((0**m*sqrt(-d/e**3)*log(-e*sqrt(-d/e**3) + x)/2 - 0**m*sqr
t(-d/e**3)*log(e*sqrt(-d/e**3) + x)/2 + 0**m*x/e, Eq(f, 0) | (Eq(f, 0) & Ne
(m, -1))), (f*f**m*m*x**3*x**m*lerchphi(e*x**2*exp_polar(I*pi)/d, 1, m/2 +
3/2)*gamma(m/2 + 3/2)/(4*d*f*m*gamma(m/2 + 5/2) + 4*d*f*gamma(m/2 + 5/2)) +
 3*f*f**m*x**3*x**m*lerchphi(e*x**2*exp_polar(I*pi)/d, 1, m/2 + 3/2)*gamma(
m/2 + 3/2)/(4*d*f*m*gamma(m/2 + 5/2) + 4*d*f*gamma(m/2 + 5/2)), (m > -oo) &
 (m < oo) & Ne(m, -1)), (-Piecewise((-polylog(2, e*x**2*exp_polar(I*pi)/d)/
2, (Abs(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x) - polylog(2, e*x**2*exp_p
olar(I*pi)/d)/2, Abs(x) < 1), (-log(d)*log(1/x) - polylog(2, e*x**2*exp_pol
ar(I*pi)/d)/2, 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(
d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) - polylog(2, e*x**2*exp_
polar(I*pi)/d)/2, True))/(2*e*f) + log(f*x)*log(d + e*x**2)/(2*e*f), True))
 + Piecewise((0**m*x, Eq(f, 0)), (Piecewise(((f*x)**(m + 1)/(m + 1), Ne(m,
-1)), (log(f*x), True))/f, True))*log(c*(d + e*x**2)**p)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log((x^2*e + d)^p*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\left(e\,x^2+d\right)^p\right)\,\left(f\,x\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)*(f*x)^m,x)
[Out] int(log(c*(d + e*x^2)^p)*(f*x)^m, x)
```

3.161 $\int \frac{(fx)^m}{\log(c(d+ex^2)^p)}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable((f*x)^m/ln(c*(e*x^2+d)^p),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(f*x)^m/Log[c*(d + e*x^2)^p],x]
```

```
[Out] Defer[Int][(f*x)^m/Log[c*(d + e*x^2)^p], x]
```

Rubi steps

$$\int \frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [A]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f*x)^m/Log[c*(d + e*x^2)^p],x]
```

```
[Out] Integrate[(f*x)^m/Log[c*(d + e*x^2)^p], x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m}{\ln\left(c\left(e\,x^2+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m/ln(c*(e*x^2+d)^p),x)
[Out] int((f*x)^m/ln(c*(e*x^2+d)^p),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/log(c*(e*x^2+d)^p),x, algorithm="maxima")
[Out] integrate((f*x)^m/log((e*x^2 + d)^p*c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/log(c*(e*x^2+d)^p),x, algorithm="fricas")
[Out] integral((f*x)^m/log((x^2*e + d)^p*c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m/ln(c*(e*x**2+d)**p),x)
[Out] Integral((f*x)**m/log(c*(d + e*x**2)**p), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/log(c*(e*x^2+d)^p),x, algorithm="giac")
[Out] integrate((f*x)^m/log((x^2*e + d)^p*c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(f\,x)^m}{\ln\left(c\,\left(e\,x^2+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m/log(c*(d + e*x^2)^p),x)
[Out] int((f*x)^m/log(c*(d + e*x^2)^p), x)
```

3.162 $\int \frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$

Optimal. Leaf size=23

$$\mathrm{Int}\left(\frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable((f*x)^m/ln(c*(e*x^2+d)^p)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(f*x)^m/Log[c*(d + e*x^2)^p]^2,x]
[Out] Defer[Int][(f*x)^m/Log[c*(d + e*x^2)^p]^2, x]
```

Rubi steps

$$\int \frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [A]
time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f*x)^m/Log[c*(d + e*x^2)^p]^2,x]
[Out] Integrate[(f*x)^m/Log[c*(d + e*x^2)^p]^2, x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m}{\ln\left(c\left(e\,x^2+d\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m/ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int((f*x)^m/ln(c*(e*x^2+d)^p)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e*f^m*x^2 + d*f^m)*x^m/(e*p*x*log((e*x^2 + d)^p) + e*p*x*log(c)) + in
tegrate(1/2*(e*f^m*(m + 1)*x^2 + d*f^m*(m - 1))*x^m/(e*p*x^2*log((e*x^2 + d
)^p) + e*p*x^2*log(c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral((f*x)^m/log((x^2*e + d)^p*c)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m/ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Integral((f*x)**m/log(c*(d + e*x**2)**p)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((f*x)^m/log((x^2*e + d)^p*c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(f\,x)^m}{\ln\left(c\left(e\,x^2+d\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m/log(c*(d + e*x^2)^p)^2,x)
[Out] int((f*x)^m/log(c*(d + e*x^2)^p)^2, x)
```

3.163 $\int (fx)^{-1+3n}\log^2\left(c(d+ex^n)^p\right)\,dx$

Optimal. Leaf size=372

$$\frac{2d^2p^2x^{1-2n}(fx)^{-1+3n}}{e^2n}-\frac{dp^2x^{1-3n}(fx)^{-1+3n}\left(d+ex^n\right)^2}{2e^3n}+\frac{2p^2x^{1-3n}(fx)^{-1+3n}\left(d+ex^n\right)^3}{27e^3n}-\frac{d^3p^2x^{1-3n}(fx)^{-1+3n}}{3e^3n}$$

```
[Out] 2*d^2*p^2*x^(1-2*n)*(f*x)^(-1+3*n)/e^2/n-1/2*d*p^2*x^(1-3*n)*(f*x)^(-1+3*n)
*(d+e*x^n)^2/e^3/n+2/27*p^2*x^(1-3*n)*(f*x)^(-1+3*n)*(d+e*x^n)^3/e^3/n-1/3*
d^3*p^2*x^(1-3*n)*(f*x)^(-1+3*n)*ln(d+e*x^n)^2/e^3/n-2*d^2*p*x^(1-3*n)*(f*x
)^(-1+3*n)*(d+e*x^n)*ln(c*(d+e*x^n)^p)/e^3/n+d*p*x^(1-3*n)*(f*x)^(-1+3*n)*(
d+e*x^n)^2*ln(c*(d+e*x^n)^p)/e^3/n-2/9*p*x^(1-3*n)*(f*x)^(-1+3*n)*(d+e*x^n)
^3*ln(c*(d+e*x^n)^p)/e^3/n+2/3*d^3*p*x^(1-3*n)*(f*x)^(-1+3*n)*ln(d+e*x^n)*l
n(c*(d+e*x^n)^p)/e^3/n+1/3*x*(f*x)^(-1+3*n)*ln(c*(d+e*x^n)^p)^2/n
```

Rubi [A]

time = 0.22, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2506, 2504, 2445, 2458, 45, 2372, 12, 14, 2338}

$$\frac{2d^3px^{1-3n}(fx)^{3n-1}\log\left(d+ex^n\right)\log\left(c(d+ex^n)^p\right)}{3e^3n}-\frac{2d^2px^{1-3n}(fx)^{3n-1}\left(d+ex^n\right)\log\left(c(d+ex^n)^p\right)}{e^3n}-\frac{2px^{1-3n}(fx)^{3n-1}\left(d+ex^n\right)^3\log\left(c(d+ex^n)^p\right)}{9e^3n}+\frac{dpx^{1-3n}(fx)^{3n-1}\left(d+ex^n\right)^2\log\left(c(d+ex^n)^p\right)}{e^3n}+\frac{x(fx)^{3n-1}\log^2\left(c(d+ex^n)^p\right)}{3n}-\frac{d^3p^2x^{1-3n}(fx)^{3n-1}\log^2\left(d+ex^n\right)}{3e^3n}+\frac{2d^2p^2x^{1-2n}(fx)^{3n-1}}{e^2n}+\frac{2p^2x^{1-3n}(fx)^{3n-1}\left(d+ex^n\right)^3}{27e^3n}-\frac{dp^2x^{1-3n}(fx)^{3n-1}\left(d+ex^n\right)^2}{2e^3n}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(-1 + 3*n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] (2*d^2*p^2*x^(1 - 2*n)*(f*x)^(-1 + 3*n))/(e^2*n) - (d*p^2*x^(1 - 3*n)*(f*x)
^(-1 + 3*n)*(d + e*x^n)^2)/(2*e^3*n) + (2*p^2*x^(1 - 3*n)*(f*x)^(-1 + 3*n)*
(d + e*x^n)^3)/(27*e^3*n) - (d^3*p^2*x^(1 - 3*n)*(f*x)^(-1 + 3*n)*Log[d + e
*x^n]^2)/(3*e^3*n) - (2*d^2*p*x^(1 - 3*n)*(f*x)^(-1 + 3*n)*(d + e*x^n)*Log[
c*(d + e*x^n)^p])/(e^3*n) + (d*p*x^(1 - 3*n)*(f*x)^(-1 + 3*n)*(d + e*x^n)^2
*Log[c*(d + e*x^n)^p])/(e^3*n) - (2*p*x^(1 - 3*n)*(f*x)^(-1 + 3*n)*(d + e*x
^n)^3*Log[c*(d + e*x^n)^p])/(9*e^3*n) + (2*d^3*p*x^(1 - 3*n)*(f*x)^(-1 + 3*
n)*Log[d + e*x^n]*Log[c*(d + e*x^n)^p])/(3*e^3*n) + (x*(f*x)^(-1 + 3*n)*Log
[c*(d + e*x^n)^p]^2)/(3*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2506

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_)*(
x_))^(m_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[x^m*(a + b*Log[c*(d + e*x^n)^
p])^q, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x] && IntegerQ[Simpl
ify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\int (fx)^{-1+3n}\log^2\left(c(d+ex^n)^p\right)\,dx &= \left(x^{1-3n}(fx)^{-1+3n}\right)\int x^{-1+3n}\log^2\left(c(d+ex^n)^p\right)\,dx\\
&= \frac{\left(x^{1-3n}(fx)^{-1+3n}\right)\operatorname{Subst}\left(\int x^2\log^2\left(c(d+ex)^p\right)\,dx,x,x^n\right)}{n}\\
&= \frac{x(fx)^{-1+3n}\log^2\left(c(d+ex^n)^p\right)}{3n}-\frac{\left(2epx^{1-3n}(fx)^{-1+3n}\right)\operatorname{Subst}\left(\int \frac{x^3\log(c(}{d+}\right.}{3n}\\
&= \frac{x(fx)^{-1+3n}\log^2\left(c(d+ex^n)^p\right)}{3n}-\frac{\left(2px^{1-3n}(fx)^{-1+3n}\right)\operatorname{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)}{}\right.}{3n}\\
&= -\frac{px^{1-3n}(fx)^{-1+3n}\left(\frac{18d^2(d+ex^n)}{e^3}-\frac{9d(d+ex^n)^2}{e^3}+\frac{2(d+ex^n)^3}{e^3}-\frac{6d^3\log(d+ex^n)}{e^3}\right)}{9n}\\
&= -\frac{px^{1-3n}(fx)^{-1+3n}\left(\frac{18d^2(d+ex^n)}{e^3}-\frac{9d(d+ex^n)^2}{e^3}+\frac{2(d+ex^n)^3}{e^3}-\frac{6d^3\log(d+ex^n)}{e^3}\right)}{9n}\\
&= -\frac{px^{1-3n}(fx)^{-1+3n}\left(\frac{18d^2(d+ex^n)}{e^3}-\frac{9d(d+ex^n)^2}{e^3}+\frac{2(d+ex^n)^3}{e^3}-\frac{6d^3\log(d+ex^n)}{e^3}\right)}{9n}\\
&= \frac{2d^2p^2x^{1-2n}(fx)^{-1+3n}}{e^2n}-\frac{dp^2x^{1-3n}(fx)^{-1+3n}\left(d+ex^n\right)^2}{2e^3n}+\frac{2p^2x^{1-3n}(fx}{}\\
&= \frac{2d^2p^2x^{1-2n}(fx)^{-1+3n}}{e^2n}-\frac{dp^2x^{1-3n}(fx)^{-1+3n}\left(d+ex^n\right)^2}{2e^3n}+\frac{2p^2x^{1-3n}(fx}{}
\end{aligned}$$

Mathematica [A]

time = 0.12, size = 171, normalized size = 0.46

$$\frac{x^{-3n}(fx)^{3n}\left(-18d^3p^2\log^2(d+ex^n)+6d^3p\log(d+ex^n)\left(-11p+6\log\left(c(d+ex^n)^p\right)\right)+ex^n\left(p^2(66d^2-15dex^n+4e^2x^{2n})-6p(6d^2-3dex^n+2e^2x^{2n})\log\left(c(d+ex^n)^p\right)+18e^2x^{2n}\log^2\left(c(d+ex^n)^p\right)\right)\right)}{54e^3fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^(-1 + 3*n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] ((f*x)^(3*n)*(-18*d^3*p^2*Log[d + e*x^n]^2 + 6*d^3*p*Log[d + e*x^n]*(-11*p
+ 6*Log[c*(d + e*x^n)^p]) + e*x^n*(p^2*(66*d^2 - 15*d*e*x^n + 4*e^2*x^(2*n)
```

```
) - 6*p*(6*d^2 - 3*d*e*x^n + 2*e^2*x^(2*n))*Log[c*(d + e*x^n)^p] + 18*e^2*x
^(2*n)*Log[c*(d + e*x^n)^p]^2)))/(54*e^3*f*n*x^(3*n))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int (fx)^{-1+3n} \ln \left(c(d+e\,x^n)^p\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(-1+3*n)*ln(c*(d+e*x^n)^p)^2,x)
```

```
[Out] int((f*x)^(-1+3*n)*ln(c*(d+e*x^n)^p)^2,x)
```

Maxima [A]
time = 0.29, size = 239, normalized size = 0.64

$$\frac{ep\left(\frac{6\,d^3f^{3\,n}\log\left(\frac{ex^n+d}{e}\right)}{e^4n} - \frac{2\,e^2f^{3\,n}x^{3\,n}-3\,def^{3\,n}x^{2\,n}+6\,d^2f^{3\,n}x^n}{e^3n}\right)\log\left((ex^n+d)^pc\right)}{9\,f} + \frac{(fx)^{3\,n}\log\left((ex^n+d)^pc\right)^2}{3\,fn} - \frac{\left(18\,d^3f^{3\,n}\log\left(ex^n+d\right)^2 - 4\,e^3f^{3\,n}x^{3\,n} + 15\,de^2f^{3\,n}x^{2\,n} - 66\,d^2ef^{3\,n}x^n - 6\,(6\,f^{3\,n}\log\left(e\right) - 11\,f^{3\,n})d^3\log\left(ex^n+d\right)\right)p^2}{54\,e^3fn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+3*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="maxima")
```

```
[Out] 1/9*e*p*(6*d^3*f^(3*n)*log((e*x^n + d)/e)/(e^4*n) - (2*e^2*f^(3*n)*x^(3*n)
- 3*d*e*f^(3*n)*x^(2*n) + 6*d^2*f^(3*n)*x^n)/(e^3*n))*log((e*x^n + d)^p*c)/
f + 1/3*(f*x)^(3*n)*log((e*x^n + d)^p*c)^2/(f*n) - 1/54*(18*d^3*f^(3*n)*log
(e*x^n + d)^2 - 4*e^3*f^(3*n)*x^(3*n) + 15*d*e^2*f^(3*n)*x^(2*n) - 66*d^2*e
*f^(3*n)*x^n - 6*(6*f^(3*n)*log(e) - 11*f^(3*n))*d^3*log(e*x^n + d))*p^2/(e
^3*f*n)
```

Fricas [A]
time = 0.40, size = 260, normalized size = 0.70

$$\frac{\left(2\,\left(2\,p^2e^3 - 6\,pe^3\log\left(c\right) + 9\,e^3\log\left(c\right)^2\right)f^{3\,n-1}x^{3\,n} - 3\,\left(5\,dp^2e^2 - 6\,dpe^2\log\left(c\right)\right)f^{3\,n-1}x^{2\,n} + 6\,\left(11\,d^2p^2e - 6\,d^2pe\log\left(c\right)\right)f^{3\,n-1}x^n + 18\,\left(d^3f^{3\,n-1}p^2 + f^{3\,n-1}p^2x^{3\,n}e^3\right)\log\left(x^ne+d\right)^2 - 6\,\left(6\,d^2f^{3\,n-1}p^2x^ne - 3\,df^{3\,n-1}p^2x^{2\,n}e^2 + 2\,\left(p^2e^3 - 3\,pe^3\log\left(c\right)\right)f^{3\,n-1}x^{3\,n} + \left(11\,d^3p^2 - 6\,d^3p\log\left(c\right)\right)f^{3\,n-1}\right)\log\left(x^ne+d\right)\right)e^{(-3)}}{54\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+3*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/54*(2*(2*p^2*e^3 - 6*p*e^3*log(c) + 9*e^3*log(c)^2)*f^(3*n - 1)*x^(3*n) -
 3*(5*d*p^2*e^2 - 6*d*p*e^2*log(c))*f^(3*n - 1)*x^(2*n) + 6*(11*d^2*p^2*e -
 6*d^2*p*e*log(c))*f^(3*n - 1)*x^n + 18*(d^3*f^(3*n - 1)*p^2 + f^(3*n - 1)*
p^2*x^(3*n)*e^3)*log(x^n*e + d)^2 - 6*(6*d^2*f^(3*n - 1)*p^2*x^n*e - 3*d*f^
(3*n - 1)*p^2*x^(2*n)*e^2 + 2*(p^2*e^3 - 3*p*e^3*log(c))*f^(3*n - 1)*x^(3*n
) + (11*d^3*p^2 - 6*d^3*p*log(c))*f^(3*n - 1))*log(x^n*e + d))*e^(-3)/n
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(-1+3*n)*ln(c*(d+e*x**n)**p)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+3*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((f*x)^(3*n - 1)*log((x^n*e + d)^p*c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\,{\left(d+e\,x^n\right)}^p\right)^2\,{\left(f\,x\right)}^{3\,n-1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)^2*(f*x)^(3*n - 1),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)^2*(f*x)^(3*n - 1), x)
```

3.164 $\int (fx)^{-1+2n} \log^2 \left(c(d+ex^n)^p\right) \, dx$

Optimal. Leaf size=255

$$-\frac{2dp^2x^{1-n}(fx)^{-1+2n}}{en}+\frac{p^2x^{1-2n}(fx)^{-1+2n}\left(d+ex^n\right)^2}{4e^2n}+\frac{2dpx^{1-2n}(fx)^{-1+2n}\left(d+ex^n\right)\log\left(c(d+ex^n)^p\right)}{e^2n}-px^{1-2}$$

```
[Out] -2*d*p^2*x^(1-n)*(f*x)^(-1+2*n)/e/n+1/4*p^2*x^(1-2*n)*(f*x)^(-1+2*n)*(d+e*x
^n)^2/e^2/n+2*d*p*x^(1-2*n)*(f*x)^(-1+2*n)*(d+e*x^n)*ln(c*(d+e*x^n)^p)/e^2/
n-1/2*p*x^(1-2*n)*(f*x)^(-1+2*n)*(d+e*x^n)^2*ln(c*(d+e*x^n)^p)/e^2/n-d*x^(1
-2*n)*(f*x)^(-1+2*n)*(d+e*x^n)*ln(c*(d+e*x^n)^p)^2/e^2/n+1/2*x^(1-2*n)*(f*x
)^(-1+2*n)*(d+e*x^n)^2*ln(c*(d+e*x^n)^p)^2/e^2/n
```

Rubi [A]
time = 0.13, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2506, 2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

$$\frac{x^{1-2n}(fx)^{2n-1}\left(d+ex^n\right)^2\log^2\left(c(d+ex^n)^p\right)}{2e^2n}-\frac{dx^{1-2n}(fx)^{2n-1}\left(d+ex^n\right)\log^2\left(c(d+ex^n)^p\right)}{e^2n}-\frac{px^{1-2n}(fx)^{2n-1}\left(d+ex^n\right)^2\log\left(c(d+ex^n)^p\right)}{2e^2n}+\frac{2dpx^{1-2n}(fx)^{2n-1}\left(d+ex^n\right)\log\left(c(d+ex^n)^p\right)}{e^2n}+\frac{p^2x^{1-2n}(fx)^{2n-1}\left(d+ex^n\right)^2}{4e^2n}-\frac{2dp^2x^{1-n}(fx)^{2n-1}}{en}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(-1 + 2*n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] (-2*d*p^2*x^(1 - n)*(f*x)^(-1 + 2*n))/(e*n) + (p^2*x^(1 - 2*n)*(f*x)^(-1 +
2*n)*(d + e*x^n)^2)/(4*e^2*n) + (2*d*p*x^(1 - 2*n)*(f*x)^(-1 + 2*n)*(d + e*
x^n)*Log[c*(d + e*x^n)^p])/(e^2*n) - (p*x^(1 - 2*n)*(f*x)^(-1 + 2*n)*(d + e
*x^n)^2*Log[c*(d + e*x^n)^p])/(2*e^2*n) - (d*x^(1 - 2*n)*(f*x)^(-1 + 2*n)*(
d + e*x^n)*Log[c*(d + e*x^n)^p]^2)/(e^2*n) + (x^(1 - 2*n)*(f*x)^(-1 + 2*n)*
(d + e*x^n)^2*Log[c*(d + e*x^n)^p]^2)/(2*e^2*n)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2506

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_)*(
x_))^(m_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[x^m*(a + b*Log[c*(d + e*x^n)^
p])^q, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x] && IntegerQ[Simpl
ify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\int (fx)^{-1+2n} \log^2\left(c(d+ex^n)^p\right)\,dx &= \left(x^{1-2n}(fx)^{-1+2n}\right)\int x^{-1+2n}\log^2\left(c(d+ex^n)^p\right)\,dx \\
&= \frac{\left(x^{1-2n}(fx)^{-1+2n}\right)\text{Subst}\left(\int x\log^2\left(c(d+ex)^p\right)\,dx, x, x^n\right)}{n} \\
&= \frac{\left(x^{1-2n}(fx)^{-1+2n}\right)\text{Subst}\left(\int\left(-\frac{d\log^2(c(d+ex)^p)}{e}+\frac{(d+ex)\log^2(c(d+ex)^p)}{e}\right)\,dx, x,}{n} \\
&= \frac{\left(x^{1-2n}(fx)^{-1+2n}\right)\text{Subst}\left(\int(d+ex)\log^2\left(c(d+ex)^p\right)\,dx, x, x^n\right)}{en} - \frac{(dx^{1-}}{} \\
&= \frac{\left(x^{1-2n}(fx)^{-1+2n}\right)\text{Subst}\left(\int x\log^2\left(cx^p\right)\,dx, x, d+ex^n\right)}{e^2n} - \frac{(dx^{1-2n}(fx)^{-1}}{} \\
&= -\frac{dx^{1-2n}(fx)^{-1+2n}\left(d+ex^n\right)\log^2\left(c(d+ex^n)^p\right)}{e^2n} + \frac{x^{1-2n}(fx)^{-1+2n}\left(d+\right.}{2} \\
&= -\frac{2dp^2x^{1-n}(fx)^{-1+2n}}{en} + \frac{p^2x^{1-2n}(fx)^{-1+2n}\left(d+ex^n\right)^2}{4e^2n} + \frac{2dpx^{1-2n}(fx)^{-}}{}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 140, normalized size = 0.55

$$\frac{x^{-2n}(fx)^{2n}\left(2d^2p^2\log^2\left(d+ex^n\right)+2d^2p\log\left(d+ex^n\right)\left(3p-2\log\left(c(d+ex^n)^p\right)\right)+ex^n\left(p^2(-6d+ex^n)+2p(2d-ex^n)\log\left(c(d+ex^n)^p\right)+2ex^n\log^2\left(c(d+ex^n)^p\right)\right)\right)}{4e^2fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^(-1 + 2*n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] ((f*x)^(2*n)*(2*d^2*p^2*Log[d + e*x^n]^2 + 2*d^2*p*Log[d + e*x^n]*(3*p - 2*
Log[c*(d + e*x^n)^p]) + e*x^n*(p^2*(-6*d + e*x^n) + 2*p*(2*d - e*x^n)*Log[c
*(d + e*x^n)^p] + 2*e*x^n*Log[c*(d + e*x^n)^p]^2)))/(4*e^2*f*n*x^(2*n))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int (fx)^{-1+2n}\ln\left(c(d+e\,x^n)^p\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(-1+2*n)*ln(c*(d+e*x^n)^p)^2,x)
```

```
[Out] int((f*x)^(-1+2*n)*ln(c*(d+e*x^n)^p)^2,x)
```

Maxima [A]
time = 0.30, size = 200, normalized size = 0.78

$$-\frac{ep\left(\frac{2\,d^2f^{2\,n}\log\left(\frac{ex^n+d}{e}\right)}{e^3n}+\frac{ef^{2\,n}x^{2\,n}-2\,df^{2\,n}x^n}{e^2n}\right)\log\left((ex^n+d)^pc\right)}{2\,f}+\frac{(fx)^{2\,n}\log\left((ex^n+d)^pc\right)^2}{2\,fn}+\frac{\left(2\,d^2f^{2\,n}\log\left(ex^n+d\right)^2+e^2f^{2\,n}x^{2\,n}-6\,def^{2\,n}x^n-2\left(2\,f^{2\,n}\log\left(e\right)-3\,f^{2\,n}\right)d^2\log\left(ex^n+d\right)\right)p^2}{4\,e^2fn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+2*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*e*p*(2*d^2*f^(2*n)*log((e*x^n + d)/e)/(e^3*n) + (e*f^(2*n)*x^(2*n) - 2
*d*f^(2*n)*x^n)/(e^2*n))*log((e*x^n + d)^p*c)/f + 1/2*(f*x)^(2*n)*log((e*x^
n + d)^p*c)^2/(f*n) + 1/4*(2*d^2*f^(2*n)*log(e*x^n + d)^2 + e^2*f^(2*n)*x^(
2*n) - 6*d*e*f^(2*n)*x^n - 2*(2*f^(2*n)*log(e) - 3*f^(2*n))*d^2*log(e*x^n +
 d))*p^2/(e^2*f*n)
```

Fricas [A]
time = 0.42, size = 202, normalized size = 0.79

$$\frac{\left(\left(p^{2} e^{2} - 2\, p e^{2} \log\left(c\right) + 2\, e^{2} \log\left(c\right)^{2}\right) f^{2\, n - 1} x^{2\, n} - 2\, \left(3\, d p^{2} e - 2\, d p e \log\left(c\right)\right) f^{2\, n - 1} x^{n} - 2\, \left(d^{2} f^{2\, n - 1} p^{2} - f^{2\, n - 1} p^{2} x^{2\, n} e^{2}\right) \log\left(x^{n} e + d\right)^{2} + 2\, \left(2\, d f^{2\, n - 1} p^{2} x^{n} e - \left(p^{2} e^{2} - 2\, p e^{2} \log\left(c\right)\right) f^{2\, n - 1} x^{2\, n} + \left(3\, d^{2} p^{2} - 2\, d^{2} p \log\left(c\right)\right) f^{2\, n - 1}\right) \log\left(x^{n} e + d\right)\right) e^{\left(-2\right)}}{4\, n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+2*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((p^2*e^2 - 2*p*e^2*log(c) + 2*e^2*log(c)^2)*f^(2*n - 1)*x^(2*n) - 2*(3
*d*p^2*e - 2*d*p*e*log(c))*f^(2*n - 1)*x^n - 2*(d^2*f^(2*n - 1)*p^2 - f^(2*
n - 1)*p^2*x^(2*n)*e^2)*log(x^n*e + d)^2 + 2*(2*d*f^(2*n - 1)*p^2*x^n*e - (
p^2*e^2 - 2*p*e^2*log(c))*f^(2*n - 1)*x^(2*n) + (3*d^2*p^2 - 2*d^2*p*log(c)
)*f^(2*n - 1))*log(x^n*e + d))*e^(-2)/n
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(-1+2*n)*ln(c*(d+e*x**n)**p)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+2*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((f*x)^(2*n - 1)*log((x^n*e + d)^p*c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\,{\left(d+e\,x^n\right)}^p\right)^2\,{\left(f\,x\right)}^{2\,n-1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)^2*(f*x)^(2*n - 1),x)
[Out] int(log(c*(d + e*x^n)^p)^2*(f*x)^(2*n - 1), x)
```

3.165 $\int (fx)^{-1+n} \log^2 \left(c(d+ex^n)^p\right) \, dx$

Optimal. Leaf size=101

$$\frac{2p^2x(fx)^{-1+n}}{n}-\frac{2px^{1-n}(fx)^{-1+n}\left(d+ex^n\right)\log\left(c(d+ex^n)^p\right)}{en}+\frac{x^{1-n}(fx)^{-1+n}\left(d+ex^n\right)\log^2\left(c(d+ex^n)^p\right)}{en}$$

```
[Out] 2*p^2*x*(f*x)^(-1+n)/n-2*p*x^(1-n)*(f*x)^(-1+n)*(d+e*x^n)*ln(c*(d+e*x^n)^p)
/e/n+x^(1-n)*(f*x)^(-1+n)*(d+e*x^n)*ln(c*(d+e*x^n)^p)^2/e/n
```

Rubi [A]
time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2506, 2504, 2436, 2333, 2332}

$$\frac{x^{1-n}(fx)^{n-1}\left(d+ex^n\right)\log^2\left(c(d+ex^n)^p\right)}{en}-\frac{2px^{1-n}(fx)^{n-1}\left(d+ex^n\right)\log\left(c(d+ex^n)^p\right)}{en}+\frac{2p^2x(fx)^{n-1}}{n}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(-1 + n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] (2*p^2*x*(f*x)^(-1 + n))/n - (2*p*x^(1 - n)*(f*x)^(-1 + n)*(d + e*x^n)*Log[
c*(d + e*x^n)^p])/(e*n) + (x^(1 - n)*(f*x)^(-1 + n)*(d + e*x^n)*Log[c*(d +
e*x^n)^p]^2)/(e*n)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
```

```
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2506

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_)*(
x_))^(m_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[x^m*(a + b*Log[c*(d + e*x^n)^
p])^q, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x] && IntegerQ[Simpl
ify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}\int (fx)^{-1+n}\log^2\left(c(d+ex^n)^p\right)\,dx &= \left(x^{1-n}(fx)^{-1+n}\right)\int x^{-1+n}\log^2\left(c(d+ex^n)^p\right)\,dx\\ &= \frac{\left(x^{1-n}(fx)^{-1+n}\right)\text{Subst}\left(\int \log^2\left(c(d+ex)^p\right)\,dx,x,x^n\right)}{n}\\ &= \frac{\left(x^{1-n}(fx)^{-1+n}\right)\text{Subst}\left(\int \log^2\left(cx^p\right)\,dx,x,d+ex^n\right)}{en}\\ &= \frac{x^{1-n}(fx)^{-1+n}\,(d+ex^n)\log^2\left(c(d+ex^n)^p\right)}{en}-\frac{\left(2px^{1-n}(fx)^{-1+n}\right)\text{Subst}\left(\int\right.}{en}\\ &= \frac{2p^2x(fx)^{-1+n}}{n}-\frac{2px^{1-n}(fx)^{-1+n}\,(d+ex^n)\log\left(c(d+ex^n)^p\right)}{en}+\frac{x^{1-n}(fx}{}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 74, normalized size = 0.73

$$\frac{x^{-n}(fx)^n\left(2ep^2x^n-2p(d+ex^n)\log\left(c(d+ex^n)^p\right)+(d+ex^n)\log^2\left(c(d+ex^n)^p\right)\right)}{efn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^(-1 + n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] ((f*x)^n*(2*e*p^2*x^n - 2*p*(d + e*x^n)*Log[c*(d + e*x^n)^p] + (d + e*x^n)*
Log[c*(d + e*x^n)^p]^2))/(e*f*n*x^n)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (fx)^{-1+n}\ln\left(c(d+e\,x^n)^p\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(-1+n)*ln(c*(d+e*x^n)^p)^2,x)
[Out] int((f*x)^(-1+n)*ln(c*(d+e*x^n)^p)^2,x)
```

Maxima [A]
time = 0.30, size = 146, normalized size = 1.45

$$-\frac{2\,ep\left(\frac{f^n x^n}{en}-\frac{df^n \log\left(\frac{ex^n+d}{e}\right)}{e^2 n}\right)\log\left((ex^n+d)^p c\right)}{f}+\frac{(fx)^n \log\left((ex^n+d)^p c\right)^2}{fn}-\frac{\left(df^n \log\left(ex^n+d\right)^2-2\,ef^n x^n-2\left(f^n \log\left(e\right)-f^n\right)d\log\left(ex^n+d\right)\right)p^2}{efn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+n)*log(c*(d+e*x^n)^p)^2,x, algorithm="maxima")
[Out] -2*e*p*(f^n*x^n/(e*n) - d*f^n*log((e*x^n + d)/e)/(e^2*n))*log((e*x^n + d)^p
*c)/f + (f*x)^n*log((e*x^n + d)^p*c)^2/(f*n) - (d*f^n*log(e*x^n + d)^2 - 2*
e*f^n*x^n - 2*(f^n*log(e) - f^n)*d*log(e*x^n + d))*p^2/(e*f*n)
```

Fricas [A]
time = 0.39, size = 128, normalized size = 1.27

$$\frac{\left(\left(2\,p^2 e-2\,pe\log\left(c\right)+e\log\left(c\right)^2\right)f^{n-1}x^n+\left(f^{n-1}p^2 x^n e+df^{n-1}p^2\right)\log\left(x^n e+d\right)^2-2\left(\left(p^2 e-pe\log\left(c\right)\right)f^{n-1}x^n+\left(dp^2-dp\log\left(c\right)\right)f^{n-1}\right)\log\left(x^n e+d\right)\right)e^{(-1)}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+n)*log(c*(d+e*x^n)^p)^2,x, algorithm="fricas")
[Out] ((2*p^2*e - 2*p*e*log(c) + e*log(c)^2)*f^(n - 1)*x^n + (f^(n - 1)*p^2*x^n*e
 + d*f^(n - 1)*p^2)*log(x^n*e + d)^2 - 2*((p^2*e - p*e*log(c))*f^(n - 1)*x^
n + (d*p^2 - d*p*log(c))*f^(n - 1))*log(x^n*e + d))*e^(-1)/n
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(-1+n)*ln(c*(d+e*x**n)**p)**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1+n)*log(c*(d+e*x^n)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((f*x)^(n - 1)*log((x^n*e + d)^p*c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\,{\left(d+e\,x^n\right)}^p\right)^2\,{\left(f\,x\right)}^{n-1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)^2*(f*x)^(n - 1),x)
[Out] int(log(c*(d + e*x^n)^p)^2*(f*x)^(n - 1), x)
```

3.166 $\int \frac{\log^2(c(d+ex^n)^p)}{fx}\,dx$

Optimal. Leaf size=88

$$\frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c(d+ex^n)^p\right)}{fn}+\frac{2p\log\left(c(d+ex^n)^p\right)\mathrm{Li}_2\left(1+\frac{ex^n}{d}\right)}{fn}-\frac{2p^2\mathrm{Li}_3\left(1+\frac{ex^n}{d}\right)}{fn}$$

```
[Out] ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)^2/f/n+2*p*ln(c*(d+e*x^n)^p)*polylog(2,1+e*x^
n/d)/f/n-2*p^2*polylog(3,1+e*x^n/d)/f/n
```

Rubi [A]
time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {12, 2504, 2443, 2481, 2421, 6724}

$$\frac{2p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)\log\left(c(d+ex^n)^p\right)}{fn}-\frac{2p^2\text{PolyLog}\left(3,\frac{ex^n}{d}+1\right)}{fn}+\frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c(d+ex^n)^p\right)}{fn}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]^2/(f*x),x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p]^2)/(f*n) + (2*p*Log[c*(d + e*x^n)^p
]*PolyLog[2, 1 + (e*x^n)/d])/(f*n) - (2*p^2*PolyLog[3, 1 + (e*x^n)/d])/(f*n
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log^2\left(c(d+ex^n)^p\right)}{fx}\,dx &= \frac{\int \frac{\log^2(c(d+ex^n)^p)}{x}\,dx}{f} \\
&= \frac{\text{Subst}\left(\int \frac{\log^2(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{fn} \\
&= \frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c(d+ex^n)^p\right)}{fn} - \frac{(2ep)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^p)}{d+ex}\,dx, x, x^n\right)}{fn} \\
&= \frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c(d+ex^n)^p\right)}{fn} - \frac{(2p)\text{Subst}\left(\int \frac{\log(cx^p)\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}\,dx, x, d+ex^n\right.}{fn} \\
&= \frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c(d+ex^n)^p\right)}{fn} + \frac{2p\log\left(c(d+ex^n)^p\right)\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{fn} - \frac{(2p^2)\,\text{Subst}\left(\right.}{} \\
&= \frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c(d+ex^n)^p\right)}{fn} + \frac{2p\log\left(c(d+ex^n)^p\right)\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{fn} - \frac{2p^2\text{Li}_3\left(1+\frac{ex^n}{d}\right.}{fn}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 168, normalized size = 1.91

$$\frac{\log(x)\left(-p\log\left(d+ex^n\right)+\log\left(c\left(d+ex^n\right)^p\right)\right)^2+2p\left(-p\log\left(d+ex^n\right)+\log\left(c\left(d+ex^n\right)^p\right)\right)\left(\log(x)\left(\log\left(d+ex^n\right)-\log\left(1+\frac{ex^n}{d}\right)\right)-\frac{\text{Li}_2\left(-\frac{ex^n}{d}\right)}{n}\right)+\frac{p^2\left(\log\left(-\frac{ex^n}{d}\right)\log^2\left(d+ex^n\right)+2\log\left(d+ex^n\right)\text{Li}_2\left(1+\frac{ex^n}{d}\right)-2\text{Li}_3\left(1+\frac{ex^n}{d}\right)\right)}{n}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]^2/(f*x),x]
```

```
[Out] (Log[x]*(-(p*Log[d + e*x^n]) + Log[c*(d + e*x^n)^p])^2 + 2*p*(-(p*Log[d + e
*x^n]) + Log[c*(d + e*x^n)^p])*(Log[x]*(Log[d + e*x^n] - Log[1 + (e*x^n)/d]
) - PolyLog[2, -((e*x^n)/d)]/n) + (p^2*(Log[-((e*x^n)/d)]*Log[d + e*x^n]^2
+ 2*Log[d + e*x^n]*PolyLog[2, 1 + (e*x^n)/d] - 2*PolyLog[3, 1 + (e*x^n)/d])
)/n)/f
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.79, size = 1473, normalized size = 16.74

method	result	size
risch	Expression too large to display	1473

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)^2/f/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f*ln(x)*Pi^2*csgn(I*(d+e*x^n)^p)^2*csgn(I*c*(d+e*x^n)^p)^4+I/f/n*ln((d
+e*x^n)^p)*Pi*ln(x^n)*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-I/f*ln(x)
*ln(c)*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)+I/f/n*ln((d+e
*x^n)^p)*Pi*ln(x^n)*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)-I/f/n*Pi*dilog((d+e*x
^n)/d)*p*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)-2/f/n*ln(c)*ln(x^n)*ln((d+e*x^n)
/d)*p+I/f/n*Pi*ln(x^n)*ln((d+e*x^n)/d)*p*csgn(I*c*(d+e*x^n)^p)^3-I/f/n*ln((
d+e*x^n)^p)*Pi*ln(x^n)*csgn(I*c*(d+e*x^n)^p)^3+I/f*ln(x)*ln(c)*Pi*csgn(I*(d
+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-I/f/n*Pi*dilog((d+e*x^n)/d)*p*csgn(I*(d+
e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-1/4/f*ln(x)*Pi^2*csgn(I*c*(d+e*x^n)^p)^6+
1/f/n*ln((d+e*x^n)^p)^2*ln(e*x^n)-2/f/n*polylog(3,(d+e*x^n)/d)*p^2+I/f/n*Pi
*dilog((d+e*x^n)/d)*p*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-2
/f/n*ln(c)*dilog((d+e*x^n)/d)*p-I/f/n*Pi*ln(x^n)*ln((d+e*x^n)/d)*p*csgn(I*(
d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-I/f/n*ln((d+e*x^n)^p)*Pi*ln(x^n)*csgn(I
*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)+1/2/f*ln(x)*Pi^2*csgn(I*(d+e*
x^n)^p)^2*csgn(I*c*(d+e*x^n)^p)^3*csgn(I*c)+I/f*ln(x)*ln(c)*Pi*csgn(I*c*(d+
e*x^n)^p)^2*csgn(I*c)-1/4/f*ln(x)*Pi^2*csgn(I*(d+e*x^n)^p)^2*csgn(I*c*(d+e*
x^n)^p)^2*csgn(I*c)^2-1/f*ln(x)*Pi^2*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)
^p)^4*csgn(I*c)+1/2/f*ln(x)*Pi^2*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^
3*csgn(I*c)^2-I/f*ln(x)*ln(c)*Pi*csgn(I*c*(d+e*x^n)^p)^3-I/f/n*Pi*ln(x^n)*l
n((d+e*x^n)/d)*p*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+1/f/n*ln(d+e*x^n)^2*ln(e
*x^n)*p^2+1/f/n*ln(1-(d+e*x^n)/d)*ln(d+e*x^n)^2*p^2-1/4/f*ln(x)*Pi^2*csgn(I
```

```
*c*(d+e*x^n)^p)^4*csgn(I*c)^2+1/2/f*ln(x)*Pi^2*csgn(I*(d+e*x^n)^p)*csgn(I*c
*(d+e*x^n)^p)^5+1/2/f*ln(x)*Pi^2*csgn(I*c*(d+e*x^n)^p)^5*csgn(I*c)-2/f/n*ln
(d+e*x^n)^2*ln(-e*x^n/d)*p^2+2/f/n*polylog(2,(d+e*x^n)/d)*ln(d+e*x^n)*p^2-2
/f/n*ln(d+e*x^n)*dilog(-e*x^n/d)*p^2+2/f/n*ln((d+e*x^n)^p)*dilog(-e*x^n/d)*
p+2/f/n*ln(c)*ln((d+e*x^n)^p)*ln(x^n)+2/f/n*ln((d+e*x^n)^p)*ln(d+e*x^n)*ln(
-e*x^n/d)*p-2/f/n*ln((d+e*x^n)^p)*ln(d+e*x^n)*ln(e*x^n)*p+I/f/n*Pi*dilog((d
+e*x^n)/d)*p*csgn(I*c*(d+e*x^n)^p)^3+1/f*ln(x)*ln(c)^2+I/f/n*Pi*ln(x^n)*ln(
(d+e*x^n)/d)*p*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^2/f/x,x, algorithm="maxima")
```

```
[Out] (log((d + e^(n*log(x) + 1))^p)^2*log(x) - integrate(-(d*log(c)^2 + e^(n*log
(x) + 1)*log(c)^2 - 2*((n*p*e*log(x) - e*log(c))*x^n - d*log(c))*log((d + e
^(n*log(x) + 1))^p))/(d*x + x*e^(n*log(x) + 1)), x))/f
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^2/f/x,x, algorithm="fricas")
```

```
[Out] integral(log((x^n*e + d)^p*c)^2/(f*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\log{\left(c \left(d + e x^{n}\right)^{p} \right)}^{2}}{x}\, dx}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)**2/f/x,x)
```

```
[Out] Integral(log(c*(d + e*x**n)**p)**2/x, x)/f
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^2/f/x,x, algorithm="giac")
```

```
[Out] integrate(log((x^n*e + d)^p*c)^2/(f*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln \left(c \left(d+e\,x^n\right)^p\right)^2}{f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)^2/(f*x),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)^2/(f*x), x)
```

3.167 $\int (fx)^{-1-n} \log^2 (c(d+ex^n)^p) \, dx$

Optimal. Leaf size=124

$$\frac{2epx^{1+n}(fx)^{-1-n}\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{dn}-\frac{x(fx)^{-1-n}\left(d+ex^n\right)\log^2\left(c(d+ex^n)^p\right)}{dn}+\frac{2ep^2x^{1+n}(fx)^{-1-n}\mathrm{L}}{dn}$$

```
[Out] 2*e*p*x^(1+n)*(f*x)^(-1-n)*ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/d/n-x*(f*x)^(-1-n
)*(d+e*x^n)*ln(c*(d+e*x^n)^p)^2/d/n+2*e*p^2*x^(1+n)*(f*x)^(-1-n)*polylog(2,
1+e*x^n/d)/d/n
```

Rubi [A]
time = 0.07, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2506, 2504, 2444, 2441, 2352}

$$\frac{2ep^2x^{n+1}(fx)^{-n-1}\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{dn}-\frac{x(fx)^{-n-1}\left(d+ex^n\right)\log^2\left(c(d+ex^n)^p\right)}{dn}+\frac{2epx^{n+1}(fx)^{-n-1}\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{dn}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(-1 - n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] (2*e*p*x^(1 + n)*(f*x)^(-1 - n)*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/(d*
n) - (x*(f*x)^(-1 - n)*(d + e*x^n)*Log[c*(d + e*x^n)^p]^2)/(d*n) + (2*e*p^2
*x^(1 + n)*(f*x)^(-1 - n)*PolyLog[2, 1 + (e*x^n)/d])/(d*n)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Dist[b*e*n*(p/(e*f - d*g)), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2506

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_)*(
x_))^(m_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[x^m*(a + b*Log[c*(d + e*x^n)^
p])^q, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x] && IntegerQ[Simpl
ify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}\int (fx)^{-1-n}\log^2\left(c(d+ex^n)^p\right)\,dx &= \left(x^{1+n}(fx)^{-1-n}\right)\int x^{-1-n}\log^2\left(c(d+ex^n)^p\right)\,dx\\ &= \frac{\left(x^{1+n}(fx)^{-1-n}\right)\text{Subst}\left(\int \frac{\log^2(c(d+ex)^p)}{x^2}\,dx,x,x^n\right)}{n}\\ &= -\frac{x(fx)^{-1-n}\left(d+ex^n\right)\log^2\left(c(d+ex^n)^p\right)}{dn}+\frac{\left(2epx^{1+n}(fx)^{-1-n}\right)\text{Subst}\left(\right.}{dn}\\ &= \frac{2epx^{1+n}(fx)^{-1-n}\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{dn}-\frac{x(fx)^{-1-n}\left(d+ex^n\right)\text{l}}{dn}\\ &= \frac{2epx^{1+n}(fx)^{-1-n}\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{dn}-\frac{x(fx)^{-1-n}\left(d+ex^n\right)\text{l}}{dn}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 148, normalized size = 1.19

$$-\frac{(fx)^{-n}\left(2ep^2x^n\log\left(-\frac{dx^{-n}}{e}\right)\log\left(-e-dx^{-n}\right)-ep^2x^n\log^2\left(-e-dx^{-n}\right)+2epx^n\log\left(-e-dx^{-n}\right)\log\left(c(d+ex^n)^p\right)+d\log^2\left(c(d+ex^n)^p\right)+2ep^2x^n\text{Li}_2\left(1+\frac{dx^{-n}}{e}\right)\right)}{dfn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^(-1 - n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] -((2*e*p^2*x^n*Log[-(d/(e*x^n))]*Log[-e - d/x^n] - e*p^2*x^n*Log[-e - d/x^n
]^2 + 2*e*p*x^n*Log[-e - d/x^n]*Log[c*(d + e*x^n)^p] + d*Log[c*(d + e*x^n)^
p]^2 + 2*e*p^2*x^n*PolyLog[2, 1 + d/(e*x^n)])/(d*f*n*(f*x)^n))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int (fx)^{-1-n}\ln\left(c(d+e\,x^n)^p\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(-1-n)*ln(c*(d+e*x^n)^p)^2,x)
```

```
[Out] int((f*x)^(-1-n)*ln(c*(d+e*x^n)^p)^2,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-n)*log(c*(d+e*x^n)^p)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((-n)-1>0)', see `assume?` for more
details
```

Fricas [**A**]
time = 0.37, size = 207, normalized size = 1.67

$$-\frac{2 f^{-n-1} n p^2 x^n e \log(x) \log\left(\frac{x^n e+d}{d}\right) - 2 f^{-n-1} n p x^n e \log(c) \log(x) + 2 f^{-n-1} p^2 x^n \mathrm{Li}_2\left(-\frac{x^n e+d}{d}+1\right) e + d f^{-n-1} \log(c)^2 + \left(f^{-n-1} p^2 x^n e + d f^{-n-1} p^2\right) \log(x^n e+d)^2 + 2\left(d f^{-n-1} p \log(c) - \left(n p^2 e \log(x) - p e \log(c)\right) f^{-n-1} x^n\right) \log(x^n e+d)}{d n x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-n)*log(c*(d+e*x^n)^p)^2,x, algorithm="fricas")
```

```
[Out] -(2*f^(-n - 1)*n*p^2*x^n*e*log(x)*log((x^n*e + d)/d) - 2*f^(-n - 1)*n*p*x^n
*e*log(c)*log(x) + 2*f^(-n - 1)*p^2*x^n*dilog(-(x^n*e + d)/d + 1)*e + d*f^(
-n - 1)*log(c)^2 + (f^(-n - 1)*p^2*x^n*e + d*f^(-n - 1)*p^2)*log(x^n*e + d)
^2 + 2*(d*f^(-n - 1)*p*log(c) - (n*p^2*e*log(x) - p*e*log(c))*f^(-n - 1)*x^
n)*log(x^n*e + d))/(d*n*x^n)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(-1-n)*ln(c*(d+e*x**n)**p)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-n)*log(c*(d+e*x^n)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((f*x)^(-n - 1)*log((x^n*e + d)^p*c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)^2}{{\left(f\,x\right)}^{n+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)^2/(f*x)^(n + 1),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)^2/(f*x)^(n + 1), x)
```

3.168 $\int (fx)^{-1-2n} \log^2 (c(d+ex^n)^p) \, dx$

Optimal. Leaf size=200

$$\frac{e^2p^2x^{1+2n}(fx)^{-1-2n}\log(x)}{d^2} - \frac{epx^{1+n}(fx)^{-1-2n}\left(d+ex^n\right)\log\left(c(d+ex^n)^p\right)}{d^2n} - \frac{x(fx)^{-1-2n}\log^2\left(c(d+ex^n)^p\right)}{2n} -$$

```
[Out] e^2*p^2*x^(1+2*n)*(f*x)^(-1-2*n)*ln(x)/d^2-e*p*x^(1+n)*(f*x)^(-1-2*n)*(d+e*
x^n)*ln(c*(d+e*x^n)^p)/d^2/n-1/2*x*(f*x)^(-1-2*n)*ln(c*(d+e*x^n)^p)^2/n-e^2
*p*x^(1+2*n)*(f*x)^(-1-2*n)*ln(c*(d+e*x^n)^p)*ln(1-d/(d+e*x^n))/d^2/n+e^2*p
^2*x^(1+2*n)*(f*x)^(-1-2*n)*polylog(2,d/(d+e*x^n))/d^2/n
```

Rubi [A]

time = 0.18, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2506, 2504, 2445, 2458, 2389, 2379, 2438, 2351, 31}

$$\frac{e^2p^2x^{2n+1}(fx)^{-2n-1}\text{PolyLog}\left(2,\frac{d}{d+ex^n}\right)}{d^2n} - \frac{e^2px^{2n+1}(fx)^{-2n-1}\log\left(1-\frac{d}{d+ex^n}\right)\log\left(c(d+ex^n)^p\right)}{d^2n} - \frac{epx^{n+1}(fx)^{-2n-1}\left(d+ex^n\right)\log\left(c(d+ex^n)^p\right)}{d^2n} - \frac{x(fx)^{-2n-1}\log^2\left(c(d+ex^n)^p\right)}{2n} + \frac{e^2p^2x^{2n+1}\log(x)(fx)^{-2n-1}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(-1 - 2*n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] (e^2*p^2*x^(1 + 2*n)*(f*x)^(-1 - 2*n)*Log[x])/d^2 - (e*p*x^(1 + n)*(f*x)^(-
1 - 2*n)*(d + e*x^n)*Log[c*(d + e*x^n)^p])/(d^2*n) - (x*(f*x)^(-1 - 2*n)*Lo
g[c*(d + e*x^n)^p]^2)/(2*n) - (e^2*p*x^(1 + 2*n)*(f*x)^(-1 - 2*n)*Log[c*(d
+ e*x^n)^p]*Log[1 - d/(d + e*x^n)])/(d^2*n) + (e^2*p^2*x^(1 + 2*n)*(f*x)^(-
1 - 2*n)*PolyLog[2, d/(d + e*x^n)])/(d^2*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2506

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_)*(
x_))^(m_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[x^m*(a + b*Log[c*(d + e*x^n)^
p])^q, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x] && IntegerQ[Simpl
ify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])
```

Rubi steps

$$\int (fx)^{-1-2n}\log^2\left(c(d+ex^n)^p\right)\,dx = \left(x^{1+2n}(fx)^{-1-2n}\right)\int x^{-1-2n}\log^2\left(c(d+ex^n)^p\right)\,dx$$

$$= \frac{\left(x^{1+2n}(fx)^{-1-2n}\right)\mathrm{Subst}\left(\int \frac{\log^2(c(d+ex)^p)}{x^3}\,dx, x, x^n\right)}{n}$$

$$= -\frac{x(fx)^{-1-2n}\log^2\left(c(d+ex^n)^p\right)}{2n} + \frac{\left(epx^{1+2n}(fx)^{-1-2n}\right)\mathrm{Subst}\left(\int \frac{\log(c(d+e}{x^2(d+e}\right.}{n}$$

$$= -\frac{x(fx)^{-1-2n}\log^2\left(c(d+ex^n)^p\right)}{2n} + \frac{\left(px^{1+2n}(fx)^{-1-2n}\right)\mathrm{Subst}\left(\int \frac{\log(cx^p)}{x\left(-\frac{d}{e}+\frac{x}{e}\right.}\right.}{n}$$

$$= -\frac{x(fx)^{-1-2n}\log^2\left(c(d+ex^n)^p\right)}{2n} + \frac{\left(px^{1+2n}(fx)^{-1-2n}\right)\mathrm{Subst}\left(\int \frac{\log(cx^p)}{\left(-\frac{d}{e}+\frac{x}{e}\right)}\right.}{dn}$$

$$= -\frac{epx^{1+n}(fx)^{-1-2n}\left(d+ex^n\right)\log\left(c(d+ex^n)^p\right)}{d^2n} - \frac{x(fx)^{-1-2n}\log^2\left(c(d+\right.}{2n}$$

$$= \frac{e^2p^2x^{1+2n}(fx)^{-1-2n}\log(x)}{d^2} - \frac{epx^{1+n}(fx)^{-1-2n}\left(d+ex^n\right)\log\left(c(d+ex^n\right.}{d^2n}$$

$$= \frac{e^2p^2x^{1+2n}(fx)^{-1-2n}\log(x)}{d^2} - \frac{epx^{1+n}(fx)^{-1-2n}\left(d+ex^n\right)\log\left(c(d+ex^n\right.}{d^2n}$$

Mathematica [A]
time = 0.20, size = 288, normalized size = 1.44

$$\frac{(fx)^{-2n}\left(e^2n^2p^2x^{2n}\log^2(x)+e^2p^2x^{2n}\log^2\left(e+dx^{-n}\right)-2e^2p^2x^{2n}\log\left(e-ex^{-n}\right)-2e^2p^2x^{2n}\log\left(e+dx^{-n}\right)\log\left(e-ex^{-n}\right)-2depx^n\log\left(c(d+ex^n)^p\right)+2e^2px^{2n}\log\left(e-ex^{-n}\right)\log\left(c(d+ex^n)^p\right)-d^2\log^2\left(c(d+ex^n)^p\right)+2e^2npx^{2n}\log(x)\left(p+p\log\left(e+dx^{-n}\right)-p\log\left(e-ex^{-n}\right)-\log\left(c(d+ex^n)^p\right)+p\log\left(1+\frac{ex^n}{d}\right)\right)+2e^2p^2x^{2n}\mathrm{Li}_2\left(-\frac{ex^n}{d}\right)\right)}{2d^2fn}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f*x)^(-1 - 2*n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] (e^2*n^2*p^2*x^(2*n)*Log[x]^2 + e^2*p^2*x^(2*n)*Log[e + d/x^n]^2 - 2*e^2*p^
2*x^(2*n)*Log[e - e/x^n] - 2*e^2*p^2*x^(2*n)*Log[e + d/x^n]*Log[e - e/x^n]
- 2*d*e*p*x^n*Log[c*(d + e*x^n)^p] + 2*e^2*p*x^(2*n)*Log[e - e/x^n]*Log[c*(
d + e*x^n)^p] - d^2*Log[c*(d + e*x^n)^p]^2 + 2*e^2*n*p*x^(2*n)*Log[x]*(p +
p*Log[e + d/x^n] - p*Log[e - e/x^n] - Log[c*(d + e*x^n)^p] + p*Log[1 + (e*x
^n)/d]) + 2*e^2*p^2*x^(2*n)*PolyLog[2, -((e*x^n)/d)])/(2*d^2*f*n*(f*x)^(2*n
))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int (fx)^{-1-2n}\ln\left(c(d+e\,x^n)^p\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(-1-2*n)*ln(c*(d+e*x^n)^p)^2,x)
```

```
[Out] int((f*x)^(-1-2*n)*ln(c*(d+e*x^n)^p)^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-2*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((-2*n)-1>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 0.38, size = 278, normalized size = 1.39

$$\frac{2\,f^{-2\,n-1}np^2x^{2\,n}e^2\log\left(x\right)\log\left(\frac{x^ne+d}{d}\right)+2\,f^{-2\,n-1}p^2x^{2\,n}\mathrm{Li}_2\left(-\frac{x^ne+d}{d}+1\right)e^2-2\,df^{-2\,n-1}px^ne\log\left(c\right)-d^2f^{-2\,n-1}\log\left(c\right)^2+2\,(np^2e^2-npe^2\log\left(c\right))f^{-2\,n-1}x^{2\,n}\log\left(x\right)-(d^2f^{-2\,n-1}p^2-f^{-2\,n-1}p^2x^{2\,n}e^2)\log\left(x^ne+d\right)^2-2\,(df^{-2\,n-1}p^2x^ne+d^2f^{-2\,n-1}p\log\left(c\right)+(np^2e^2\log\left(x\right)+p^2e^2-pe^2\log\left(c\right))f^{-2\,n-1}x^{2\,n})\log\left(x^ne+d\right)}{2\,d^2nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-2*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*f^(-2*n - 1)*n*p^2*x^(2*n)*e^2*log(x)*log((x^n*e + d)/d) + 2*f^(-2*n
 - 1)*p^2*x^(2*n)*dilog(-(x^n*e + d)/d + 1)*e^2 - 2*d*f^(-2*n - 1)*p*x^n*e*
log(c) - d^2*f^(-2*n - 1)*log(c)^2 + 2*(n*p^2*e^2 - n*p*e^2*log(c))*f^(-2*n
 - 1)*x^(2*n)*log(x) - (d^2*f^(-2*n - 1)*p^2 - f^(-2*n - 1)*p^2*x^(2*n)*e^2
)*log(x^n*e + d)^2 - 2*(d*f^(-2*n - 1)*p^2*x^n*e + d^2*f^(-2*n - 1)*p*log(c
) + (n*p^2*e^2*log(x) + p^2*e^2 - p*e^2*log(c))*f^(-2*n - 1)*x^(2*n))*log(x
^n*e + d))/(d^2*n*x^(2*n))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(-1-2*n)*ln(c*(d+e*x**n)**p)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(-1-2*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="giac")
[Out] integrate((f*x)^(-2*n - 1)*log((x^n*e + d)^p*c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)^2}{{\left(f\,x\right)}^{2\,n+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)^2/(f*x)^(2*n + 1),x)
[Out] int(log(c*(d + e*x^n)^p)^2/(f*x)^(2*n + 1), x)
```

3.169 $\int \frac{\log(1+ex^n)}{x}\,dx$

Optimal. Leaf size=13

$$-\frac{\text{Li}_2(-ex^n)}{n}$$

```
[Out] -polylog(2,-e*x^n)/n
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2438}

$$-\frac{\text{PolyLog}(2,-ex^n)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + e*x^n]/x,x]
```

```
[Out] -(PolyLog[2, -(e*x^n)]/n)
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(1+ex^n\right)}{x}\,dx=-\frac{\text{Li}_2(-ex^n)}{n}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$-\frac{\text{Li}_2(-ex^n)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + e*x^n]/x,x]
```

```
[Out] -(PolyLog[2, -(e*x^n)]/n)
```

Maple [A]
time = 1.16, size = 14, normalized size = 1.08

method	result	size
derivativedivides	$-\frac{\mathrm{dilog}(1+e\,x^n)}{n}$	14
default	$-\frac{\mathrm{dilog}(1+e\,x^n)}{n}$	14
meijerg	$-\frac{\mathrm{polylog}(2,-e\,x^n)}{n}$	14
risch	$-\frac{\mathrm{dilog}(1+e\,x^n)}{n}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1+e*x^n)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/n*dilog(1+e*x^n)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+e*x^n)/x,x, algorithm="maxima")
```

```
[Out] -1/2*n*log(x)^2 + n*integrate(log(x)/(x*e^(n*log(x) + 1) + x), x) + log(x)*
log(e^(n*log(x) + 1) + 1)
```

Fricas [A]
time = 0.35, size = 13, normalized size = 1.00

$$-\frac{\mathrm{Li}_2(-x^n e)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+e*x^n)/x,x, algorithm="fricas")
```

```
[Out] -dilog(-x^n*e)/n
```

Sympy [C] Result contains complex when optimal does not.
time = 1.48, size = 14, normalized size = 1.08

$$-\frac{\mathrm{Li}_2\left(ex^n e^{i\pi}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+e*x**n)/x,x)
```

```
[Out] -polylog(2, e*x**n*exp_polar(I*pi))/n
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+e*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(log(x^n*e + 1)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\ln\left(e\,x^n+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*x^n + 1)/x,x)
```

```
[Out] int(log(e*x^n + 1)/x, x)
```

3.170 $\int \frac{\log(2+ex^n)}{x} \, dx$

Optimal. Leaf size=21

$$\log(2)\log(x) - \frac{\text{Li}_2\left(-\frac{ex^n}{2}\right)}{n}$$

[Out] ln(2)*ln(x)-polylog(2,-1/2*e*x^n)/n

Rubi [A]
time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2504, 2439, 2438}

$$\log(2)\log(x) - \frac{\text{PolyLog}\left(2, -\frac{ex^n}{2}\right)}{n}$$

Antiderivative was successfully verified.

[In] Int[Log[2 + e*x^n]/x,x]

[Out] Log[2]*Log[x] - PolyLog[2, -1/2*(e*x^n)]/n

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + e*(x/d)]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log(2+ex^n)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\log(2+ex)}{x}\,dx, x, x^n\right)}{n}$$
$$= \log(2)\log(x) + \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{ex}{2}\right)}{x}\,dx, x, x^n\right)}{n}$$
$$= \log(2)\log(x) - \frac{\text{Li}_2\left(-\frac{ex^n}{2}\right)}{n}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 1.00

$$\log(2)\log(x) - \frac{\text{Li}_2\left(-\frac{ex^n}{2}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[2 + e*x^n]/x,x]
```

```
[Out] Log[2]*Log[x] - PolyLog[2, -1/2*(e*x^n)]/n
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(19) = 38$.
time = 1.09, size = 45, normalized size = 2.14

method	result	size
risch	$\ln(x)\ln(2+e\,x^n) - \frac{\text{dilog}\left(\frac{e\,x^n}{2}+1\right)}{n} - \ln(x)\ln\left(\frac{e\,x^n}{2}+1\right)$	40
derivativedivides	$\frac{\left(\ln(2+e\,x^n)-\ln\left(\frac{e\,x^n}{2}+1\right)\right)\ln\left(-\frac{e\,x^n}{2}\right)-\text{dilog}\left(\frac{e\,x^n}{2}+1\right)}{n}$	45
default	$\frac{\left(\ln(2+e\,x^n)-\ln\left(\frac{e\,x^n}{2}+1\right)\right)\ln\left(-\frac{e\,x^n}{2}\right)-\text{dilog}\left(\frac{e\,x^n}{2}+1\right)}{n}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(2+e*x^n)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*((ln(2+e*x^n)-ln(1/2*e*x^n+1))*ln(-1/2*e*x^n)-dilog(1/2*e*x^n+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(2+e*x^n)/x,x, algorithm="maxima")

[Out] -1/2*n*log(x)^2 + 2*n*integrate(log(x)/(x*e^(n*log(x) + 1) + 2*x), x) + log(x)*log(e^(n*log(x) + 1) + 2)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(19) = 38$.
time = 0.38, size = 43, normalized size = 2.05

$$\frac{n\log\left(x^{n}e+2\right)\log\left(x\right)-n\log\left(\frac{1}{2}\,x^{n}e+1\right)\log\left(x\right)-\mathrm{Li}_2\left(-\frac{1}{2}\,x^{n}e\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(2+e*x^n)/x,x, algorithm="fricas")

[Out] (n*log(x^n*e + 2)*log(x) - n*log(1/2*x^n*e + 1)*log(x) - dilog(-1/2*x^n*e))/n

Sympy [C] Result contains complex when optimal does not.
time = 1.63, size = 100, normalized size = 4.76

$$\begin{cases} -\frac{\mathrm{Li}_2\left(\frac{ex^{n}e^{i\pi}}{2}\right)}{n} & \text{for}\ \frac{1}{|x|}<1\wedge|x|<1\\ \log\left(2\right)\log\left(x\right)-\frac{\mathrm{Li}_2\left(\frac{ex^{n}e^{i\pi}}{2}\right)}{n} & \text{for}\ |x|<1\\ -\log\left(2\right)\log\left(\frac{1}{x}\right)-\frac{\mathrm{Li}_2\left(\frac{ex^{n}e^{i\pi}}{2}\right)}{n} & \text{for}\ \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle|x\right)\log\left(2\right)+G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0\end{matrix}\middle|x\right)\log\left(2\right)-\frac{\mathrm{Li}_2\left(\frac{ex^{n}e^{i\pi}}{2}\right)}{n} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(2+e*x**n)/x,x)

[Out] Piecewise((-polylog(2, e*x**n*exp_polar(I*pi)/2)/n, (Abs(x) < 1) & (1/Abs(x) < 1)), (log(2)*log(x) - polylog(2, e*x**n*exp_polar(I*pi)/2)/n, Abs(x) < 1), (-log(2)*log(1/x) - polylog(2, e*x**n*exp_polar(I*pi)/2)/n, 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(2) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(2) - polylog(2, e*x**n*exp_polar(I*pi)/2)/n, True))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(2+e*x^n)/x,x, algorithm="giac")

```
[Out] integrate(log(x^n*e + 2)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\ln \left(e\, x^n+2\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*x^n + 2)/x,x)
```

```
[Out] int(log(e*x^n + 2)/x, x)
```

3.171 $\int \frac{\log(2(3+ex^n))}{x} \, dx$

Optimal. Leaf size=21

$$\log(6)\log(x) - \frac{\text{Li}_2\left(-\frac{ex^n}{3}\right)}{n}$$

[Out] ln(6)*ln(x)-polylog(2,-1/3*e*x^n)/n

Rubi [A]
time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2504, 2439, 2438}

$$\log(6)\log(x) - \frac{\text{PolyLog}\left(2, -\frac{ex^n}{3}\right)}{n}$$

Antiderivative was successfully verified.

[In] Int[Log[2*(3 + e*x^n)]/x,x]

[Out] Log[6]*Log[x] - PolyLog[2, -1/3*(e*x^n)]/n

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + e*(x/d)]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log(2(3+ex^n))}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\log(2(3+ex))}{x}\,dx, x, x^n\right)}{n}$$
$$= \log(6)\log(x) + \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{ex}{3}\right)}{x}\,dx, x, x^n\right)}{n}$$
$$= \log(6)\log(x) - \frac{\text{Li}_2\left(-\frac{ex^n}{3}\right)}{n}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 1.00

$$\log(6)\log(x) - \frac{\text{Li}_2\left(-\frac{ex^n}{3}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[2*(3 + e*x^n)]/x,x]
```

```
[Out] Log[6]*Log[x] - PolyLog[2, -1/3*(e*x^n)]/n
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(19) = 38$.
time = 1.18, size = 46, normalized size = 2.19

method	result	size
risch	$\ln(x)\ln(6+2e\,x^n) - \frac{\text{dilog}\left(\frac{e\,x^n}{3}+1\right)}{n} - \ln(x)\ln\left(\frac{e\,x^n}{3}+1\right)$	41
derivativedivides	$\frac{\left(\ln(6+2e\,x^n)-\ln\left(\frac{e\,x^n}{3}+1\right)\right)\ln\left(-\frac{e\,x^n}{3}\right)-\text{dilog}\left(\frac{e\,x^n}{3}+1\right)}{n}$	46
default	$\frac{\left(\ln(6+2e\,x^n)-\ln\left(\frac{e\,x^n}{3}+1\right)\right)\ln\left(-\frac{e\,x^n}{3}\right)-\text{dilog}\left(\frac{e\,x^n}{3}+1\right)}{n}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(6+2*e*x^n)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*((ln(6+2*e*x^n)-ln(1/3*e*x^n+1))*ln(-1/3*e*x^n)-dilog(1/3*e*x^n+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(6+2*e*x^n)/x,x, algorithm="maxima")
```

```
[Out] -1/2*n*log(x)^2 + 3*n*integrate(log(x)/(x*e^(n*log(x) + 1) + 3*x), x) + log
(2)*log(x) + log(x)*log(e^(n*log(x) + 1) + 3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(19) = 38$.
time = 0.37, size = 44, normalized size = 2.10

$$\frac{n\log\left(2\,x^n e+6\right)\log\left(x\right)-n\log\left(\frac{1}{3}\,x^n e+1\right)\log\left(x\right)-\mathrm{Li}_2\left(-\frac{1}{3}\,x^n e\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(6+2*e*x^n)/x,x, algorithm="fricas")
```

```
[Out] (n*log(2*x^n*e + 6)*log(x) - n*log(1/3*x^n*e + 1)*log(x) - dilog(-1/3*x^n*e
))/n
```

Sympy [C] Result contains complex when optimal does not.
time = 1.63, size = 100, normalized size = 4.76

$$\begin{cases} -\frac{\mathrm{Li}_2\left(\frac{ex^n e^{i\pi}}{3}\right)}{n} & \text{for}\ \frac{1}{|x|}<1\wedge|x|<1 \\ \log\left(6\right)\log\left(x\right)-\frac{\mathrm{Li}_2\left(\frac{ex^n e^{i\pi}}{3}\right)}{n} & \text{for}\ |x|<1 \\ -\log\left(6\right)\log\left(\frac{1}{x}\right)-\frac{\mathrm{Li}_2\left(\frac{ex^n e^{i\pi}}{3}\right)}{n} & \text{for}\ \frac{1}{|x|}<1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle|x\right)\log\left(6\right)+G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle|x\right)\log\left(6\right)-\frac{\mathrm{Li}_2\left(\frac{ex^n e^{i\pi}}{3}\right)}{n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(6+2*e*x**n)/x,x)
```

```
[Out] Piecewise((-polylog(2, e*x**n*exp_polar(I*pi)/3)/n, (Abs(x) < 1) & (1/Abs(x
) < 1)), (log(6)*log(x) - polylog(2, e*x**n*exp_polar(I*pi)/3)/n, Abs(x) <
1), (-log(6)*log(1/x) - polylog(2, e*x**n*exp_polar(I*pi)/3)/n, 1/Abs(x) <
1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(6) + meijerg(((1, 1), ()),
 ((), (0, 0)), x)*log(6) - polylog(2, e*x**n*exp_polar(I*pi)/3)/n, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(6+2*e*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(log(2*x^n*e + 6)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\ln\left(2\,e\,x^n+6\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(2*e*x^n + 6)/x,x)
[Out] int(log(2*e*x^n + 6)/x, x)
```

3.172 $\int \frac{\log(c(d+ex^n))}{x} dx$

Optimal. Leaf size=41

$$\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)\right)}{n}+\frac{\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}$$

```
[Out] ln(-e*x^n/d)*ln(c*(d+e*x^n))/n+polylog(2,1+e*x^n/d)/n
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2504, 2441, 2352}

$$\frac{\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)]/x,x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)])/n + PolyLog[2, 1 + (e*x^n)/d]/n
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log (c(d+ex^n))}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\log(c(d+ex))}{x}\,dx, x, x^n\right)}{n}$$
$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log (c(d+ex^n))}{n} - \frac{e\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx, x, x^n\right)}{n}$$
$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log (c(d+ex^n))}{n} + \frac{\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.95

$$\frac{\log\left(-\frac{ex^n}{d}\right)\log (c(d+ex^n)) + \text{Li}_2\left(\frac{d+ex^n}{d}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)]/x,x]
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)] + PolyLog[2, (d + e*x^n)/d])/n
```

Maple [A]
time = 1.38, size = 38, normalized size = 0.93

method	result
derivativedivides	$\frac{\text{dilog}\left(-\frac{e\,x^n}{d}\right)+\ln(ce\,x^n+cd)\ln\left(-\frac{e\,x^n}{d}\right)}{n}$
default	$\frac{\text{dilog}\left(-\frac{e\,x^n}{d}\right)+\ln(ce\,x^n+cd)\ln\left(-\frac{e\,x^n}{d}\right)}{n}$
risch	$\ln(x)\ln(d+e\,x^n) + \frac{i\ln(x)\pi\,\text{csgn}(i(d+e\,x^n))\text{csgn}(ic(d+e\,x^n))^2}{2} - \frac{i\ln(x)\pi\,\text{csgn}(i(d+e\,x^n))\text{csgn}(ic(d+e\,x^n))\text{c}}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n))/x,x,method=_RETURNVERBOSE)
[Out] 1/n*(dilog(-e*x^n/d)+ln(c*e*x^n+c*d)*ln(-e*x^n/d))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n))/x,x, algorithm="maxima")
```

```
[Out] d*n*integrate(log(x)/(d*x + x*e^(n*log(x) + 1)), x) - 1/2*n*log(x)^2 + log(
c)*log(x) + log(d + e^(n*log(x) + 1))*log(x)
```

Fricas [A]
time = 0.39, size = 57, normalized size = 1.39

$$\frac{n\log\left(cx^{n}e+cd\right)\log\left(x\right)-n\log\left(x\right)\log\left(\frac{x^{n}e+d}{d}\right)-\mathrm{Li}_{2}\left(-\frac{x^{n}e+d}{d}+1\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n))/x,x, algorithm="fricas")
```

```
[Out] (n*log(c*x^n*e + c*d)*log(x) - n*log(x)*log((x^n*e + d)/d) - dilog(-(x^n*e
+ d)/d + 1))/n
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left(cd+cex^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n))/x,x)
```

```
[Out] Integral(log(c*d + c*e*x**n)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n))/x,x, algorithm="giac")
```

```
[Out] integrate(log((x^n*e + d)*c)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\ln\left(c\left(d+e\,x^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n))/x,x)
```

```
[Out] int(log(c*(d + e*x^n))/x, x)
```

3.173 $\int \frac{\log(c(d+ex^n)^p)}{x} \, dx$

Optimal. Leaf size=44

$$\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n}+\frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}$$

```
[Out] ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/n+p*polylog(2,1+e*x^n/d)/n
```

Rubi [A]
time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2504, 2441, 2352}

$$\frac{p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/x,x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n + (p*PolyLog[2, 1 + (e*x^n)/d])/
n
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\log(c(d+ex^n)^p)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}$$
$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} - \frac{(ep)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx, x, x^n\right)}{n}$$
$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} + \frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}$$

Mathematica [A]
time = 0.01, size = 43, normalized size = 0.98

$$\frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p) + p\text{Li}_2\left(\frac{d+ex^n}{d}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/x,x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + p*PolyLog[2, (d + e*x^n)/d])/n
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.32, size = 177, normalized size = 4.02

method	result
risch	$\ln(x)\ln\left((d+e\,x^n)^p\right) + \frac{i\ln(x)\pi\,\text{csgn}(i(d+e\,x^n)^p)\text{csgn}(ic(d+e\,x^n)^p)^2}{2} - \frac{i\ln(x)\pi\,\text{csgn}(i(d+e\,x^n)^p)\text{csgn}(ic(d+e\,x^n)^p)\text{csgn}(ic}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)*ln((d+e*x^n)^p)+1/2*I*ln(x)*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)
^p)^2-1/2*I*ln(x)*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/
2*I*ln(x)*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I*ln(x)*Pi*csgn(I*c*(d+e*x^n)^p)^2
*csgn(I*c)+ln(c)*ln(x)-p/n*dilog((d+e*x^n)/d)-p*ln(x)*ln((d+e*x^n)/d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] d*n*p*integrate(log(x)/(d*x + x*e^(n*log(x) + 1)), x) - 1/2*n*p*log(x)^2 +
log(c)*log(x) + log((d + e^(n*log(x) + 1))^p)*log(x)
```

Fricas [A]
time = 0.38, size = 63, normalized size = 1.43

$$\frac{np\log\left(x^{n}e+d\right)\log\left(x\right)-np\log\left(x\right)\log\left(\frac{x^{n}e+d}{d}\right)+n\log\left(c\right)\log\left(x\right)-p\mathrm{Li}_2\left(-\frac{x^{n}e+d}{d}+1\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
[Out] (n*p*log(x^n*e + d)*log(x) - n*p*log(x)*log((x^n*e + d)/d) + n*log(c)*log(x
) - p*dilog(-(x^n*e + d)/d + 1))/n
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left(c(d+ex^{n})^{p}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x,x)
[Out] Integral(log(c*(d + e*x**n)**p)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
[Out] integrate(log((x^n*e + d)^p*c)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\ln\left(c\,{\left(d+e\,x^{n}\right)}^{p}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/x,x)
[Out] int(log(c*(d + e*x^n)^p)/x, x)
```

3.174 $\int \frac{\log^2(c(d+ex^n)^p)}{x} \, dx$

Optimal. Leaf size=79

$$\frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c(d+ex^n)^p\right)}{n}+\frac{2p\log\left(c(d+ex^n)^p\right)\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}-\frac{2p^2\text{Li}_3\left(1+\frac{ex^n}{d}\right)}{n}$$

```
[Out] ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)^2/n+2*p*ln(c*(d+e*x^n)^p)*polylog(2,1+e*x^n/
d)/n-2*p^2*polylog(3,1+e*x^n/d)/n
```

Rubi [A]
time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2504, 2443, 2481, 2421, 6724}

$$\frac{2p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)\log\left(c(d+ex^n)^p\right)}{n}-\frac{2p^2\text{PolyLog}\left(3,\frac{ex^n}{d}+1\right)}{n}+\frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c(d+ex^n)^p\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]^2/x,x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p]^2)/n + (2*p*Log[c*(d + e*x^n)^p]*Po
lyLog[2, 1 + (e*x^n)/d])/n - (2*p^2*PolyLog[3, 1 + (e*x^n)/d])/n
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log^2(c(d+ex^n)^p)}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{\log^2(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n} \\
&= \frac{\log\left(-\frac{ex^n}{d}\right)\log^2(c(d+ex^n)^p)}{n} - \frac{(2ep)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^p)}{d+ex}\,dx, x, x^n\right)}{n} \\
&= \frac{\log\left(-\frac{ex^n}{d}\right)\log^2(c(d+ex^n)^p)}{n} - \frac{(2p)\text{Subst}\left(\int \frac{\log(cx^p)\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}\,dx, x, d+ex
\end{aligned}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^2(c(d+ex^n)^p)}{n} + \frac{2p\log(c(d+ex^n)^p)\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n} - \frac{(2p^2)\text{Subst}}{}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^2(c(d+ex^n)^p)}{n} + \frac{2p\log(c(d+ex^n)^p)\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n} - \frac{2p^2\text{Li}_3(1+}{n}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 164 vs. 2(79) = 158.
time = 0.05, size = 164, normalized size = 2.08

$$\log(x)\left(-p\log(d+ex^n)+\log(c(d+ex^n)^p)\right)^2 + 2p(-p\log(d+ex^n)+\log(c(d+ex^n)^p))\left(\log(x)\left(\log(d+ex^n)-\log\left(1+\frac{ex^n}{d}\right)\right) - \frac{\text{Li}_2\left(-\frac{ex^n}{d}\right)}{n}\right) + \frac{p^2\left(\log\left(-\frac{ex^n}{d}\right)\log^2(d+ex^n)+2\log(d+ex^n)\text{Li}_2\left(1+\frac{ex^n}{d}\right)-2\text{Li}_3\left(1+\frac{ex^n}{d}\right)\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]^2/x,x]
```

```
[Out] Log[x]*(-(p*Log[d + e*x^n]) + Log[c*(d + e*x^n)^p])^2 + 2*p*(-(p*Log[d + e*
x^n]) + Log[c*(d + e*x^n)^p])*(Log[x]*(Log[d + e*x^n] - Log[1 + (e*x^n)/d])
 - PolyLog[2, -((e*x^n)/d)]/n) + (p^2*(Log[-((e*x^n)/d)]*Log[d + e*x^n]^2 +
 2*Log[d + e*x^n]*PolyLog[2, 1 + (e*x^n)/d] - 2*PolyLog[3, 1 + (e*x^n)/d]))
/n
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.50, size = 1356, normalized size = 17.16

method	result	size
risch	Expression too large to display	1356

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Pi^2*ln(x)*csgn(I*c*(d+e*x^n)^p)^6+2/n*ln((d+e*x^n)^p)*ln(d+e*x^n)*ln(
-e*x^n/d)*p-2/n*ln(c)*ln(x^n)*ln((d+e*x^n)/d)*p-1/4*Pi^2*ln(x)*csgn(I*c*(d+
e*x^n)^p)^4*csgn(I*c)^2-I*ln(c)*Pi*ln(x)*csgn(I*c*(d+e*x^n)^p)^3-I/n*ln((d+
e*x^n)^p)*Pi*ln(x^n)*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)+I/
n*Pi*dilog((d+e*x^n)/d)*p*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*
c)-2/n*ln((d+e*x^n)^p)*ln(d+e*x^n)*ln(e*x^n)*p-I/n*Pi*ln(x^n)*ln((d+e*x^n)/
d)*p*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+I/n*Pi*dilog((d+e*x^n)/d)*p*csgn(I*c
*(d+e*x^n)^p)^3+I*ln(c)*Pi*ln(x)*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^
2-I/n*Pi*ln(x^n)*ln((d+e*x^n)/d)*p*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p
)^2+I/n*ln((d+e*x^n)^p)*Pi*ln(x^n)*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)-I/n*Pi
*dilog((d+e*x^n)/d)*p*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2+1/2*Pi^2*
ln(x)*csgn(I*(d+e*x^n)^p)^2*csgn(I*c*(d+e*x^n)^p)^3*csgn(I*c)+1/2*Pi^2*ln(x
)*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^3*csgn(I*c)^2+I/n*Pi*ln(x^n)*ln
((d+e*x^n)/d)*p*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)+I*ln(c)
*Pi*ln(x)*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)-I/n*ln((d+e*x^n)^p)*Pi*ln(x^n)*
csgn(I*c*(d+e*x^n)^p)^3+ln(c)^2*ln(x)-1/4*Pi^2*ln(x)*csgn(I*(d+e*x^n)^p)^2*
csgn(I*c*(d+e*x^n)^p)^4+1/2*Pi^2*ln(x)*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^
n)^p)^5+1/2*Pi^2*ln(x)*csgn(I*c*(d+e*x^n)^p)^5*csgn(I*c)-1/4*Pi^2*ln(x)*csg
n(I*(d+e*x^n)^p)^2*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)^2+2/n*ln((d+e*x^n)^p)*
ln(c)*ln(x^n)-2/n*ln(c)*dilog((d+e*x^n)/d)*p+I/n*Pi*ln(x^n)*ln((d+e*x^n)/d)
*p*csgn(I*c*(d+e*x^n)^p)^3+1/n*ln(d+e*x^n)^2*ln(e*x^n)*p^2+1/n*ln(1-(d+e*x^
n)/d)*ln(d+e*x^n)^2*p^2-2/n*ln(d+e*x^n)^2*ln(-e*x^n/d)*p^2+2/n*polylog(2,(d
+e*x^n)/d)*ln(d+e*x^n)*p^2-2/n*ln(d+e*x^n)*dilog(-e*x^n/d)*p^2+2/n*ln((d+e*
x^n)^p)*dilog(-e*x^n/d)*p-I*ln(c)*Pi*ln(x)*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+
e*x^n)^p)*csgn(I*c)-Pi^2*ln(x)*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^4*
csgn(I*c)-I/n*Pi*dilog((d+e*x^n)/d)*p*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+I/n
*ln((d+e*x^n)^p)*Pi*ln(x^n)*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2+1/n
*ln((d+e*x^n)^p)^2*ln(e*x^n)-2/n*polylog(3,(d+e*x^n)/d)*p^2
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^2/x,x, algorithm="maxima")
[Out] log((d + e^(n*log(x) + 1))^p)^2*log(x) - integrate(-(d*log(c)^2 + e^(n*log(
x) + 1)*log(c)^2 - 2*((n*p*e*log(x) - e*log(c))*x^n - d*log(c))*log((d + e^
(n*log(x) + 1))^p))/(d*x + x*e^(n*log(x) + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^2/x,x, algorithm="fricas")
[Out] integral(log((x^n*e + d)^p*c)^2/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log{\left(c \left(d + e x^{n}\right)^{p} \right)}^{2}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)**2/x,x)
[Out] Integral(log(c*(d + e*x**n)**p)**2/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^2/x,x, algorithm="giac")
[Out] integrate(log((x^n*e + d)^p*c)^2/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)}^2}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)^2/x,x)
[Out] int(log(c*(d + e*x^n)^p)^2/x, x)
```

3.175 $\int \frac{\log^3(c(d+ex^n)^p)}{x}\,dx$

Optimal. Leaf size=113

$$\frac{\log\left(-\frac{ex^n}{d}\right)\log^3\left(c(d+ex^n)^p\right)}{n}+\frac{3p\log^2\left(c(d+ex^n)^p\right)\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}-\frac{6p^2\log\left(c(d+ex^n)^p\right)\text{Li}_3\left(1+\frac{ex^n}{d}\right)}{n}+\frac{6p^3\text{Li}$$

```
[Out] ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)^3/n+3*p*ln(c*(d+e*x^n)^p)^2*polylog(2,1+e*x^
n/d)/n-6*p^2*ln(c*(d+e*x^n)^p)*polylog(3,1+e*x^n/d)/n+6*p^3*polylog(4,1+e*x
^n/d)/n
```

Rubi [A]
time = 0.10, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2504, 2443, 2481, 2421, 2430, 6724}

$$-\frac{6p^2\text{PolyLog}\left(3,\frac{ex^n}{d}+1\right)\log\left(c(d+ex^n)^p\right)}{n}+\frac{3p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)\log^2\left(c(d+ex^n)^p\right)}{n}+\frac{6p^3\text{PolyLog}\left(4,\frac{ex^n}{d}+1\right)}{n}+\frac{\log\left(-\frac{ex^n}{d}\right)\log^3\left(c(d+ex^n)^p\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]^3/x,x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p]^3)/n + (3*p*Log[c*(d + e*x^n)^p]^2*
PolyLog[2, 1 + (e*x^n)/d])/n - (6*p^2*Log[c*(d + e*x^n)^p]*PolyLog[3, 1 + (
e*x^n)/d])/n + (6*p^3*PolyLog[4, 1 + (e*x^n)/d])/n
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log^3\left(c(d+ex^n)^p\right)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\log^3(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^3\left(c(d+ex^n)^p\right)}{n} - \frac{(3ep)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)\log^2(c(d+ex)^p)}{d+ex}\,dx, x, x^n\right)}{n}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^3\left(c(d+ex^n)^p\right)}{n} - \frac{(3p)\text{Subst}\left(\int \frac{\log^2(cx^p)\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}\,dx, x, d+e\ldots\right.}{n}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^3\left(c(d+ex^n)^p\right)}{n} + \frac{3p\log^2\left(c(d+ex^n)^p\right)\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n} - \frac{(6p^2)\,\text{Subs}\ldots}{}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^3\left(c(d+ex^n)^p\right)}{n} + \frac{3p\log^2\left(c(d+ex^n)^p\right)\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n} - \frac{6p^2\log\left(c(\ldots\right.}{}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^3\left(c(d+ex^n)^p\right)}{n} + \frac{3p\log^2\left(c(d+ex^n)^p\right)\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n} - \frac{6p^2\log\left(c(\ldots\right.}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 270 vs. 2(113) = 226.
time = 0.11, size = 270, normalized size = 2.39

$$\frac{-np^3\log(x)\log^3(d+ex^n)+p^3\log\left(-\frac{ex^n}{d}\right)\log^3(d+ex^n)+3np^2\log(x)\log^2(d+ex^n)\log\left(c(d+ex^n)^p\right)-3p^2\log\left(-\frac{ex^n}{d}\right)\log^2(d+ex^n)\log\left(c(d+ex^n)^p\right)-3np\log(x)\log(d+ex^n)\log^2\left(c(d+ex^n)^p\right)+3p\log\left(-\frac{ex^n}{d}\right)\log(d+ex^n)\log^2\left(c(d+ex^n)^p\right)+n\log(x)\log^3\left(c(d+ex^n)^p\right)+3p\log^2\left(c(d+ex^n)^p\right)\mathrm{Li}_2\left(1+\frac{ex^n}{d}\right)-6p^2\log\left(c(d+ex^n)^p\right)\mathrm{Li}_3\left(1+\frac{ex^n}{d}\right)+6p^3\mathrm{Li}_4\left(1+\frac{ex^n}{d}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]^3/x,x]
```

```
[Out] (-(n*p^3*Log[x]*Log[d + e*x^n]^3) + p^3*Log[-((e*x^n)/d)]*Log[d + e*x^n]^3
+ 3*n*p^2*Log[x]*Log[d + e*x^n]^2*Log[c*(d + e*x^n)^p] - 3*p^2*Log[-((e*x^n
)/d)]*Log[d + e*x^n]^2*Log[c*(d + e*x^n)^p] - 3*n*p*Log[x]*Log[d + e*x^n]*L
og[c*(d + e*x^n)^p]^2 + 3*p*Log[-((e*x^n)/d)]*Log[d + e*x^n]*Log[c*(d + e*x
^n)^p]^2 + n*Log[x]*Log[c*(d + e*x^n)^p]^3 + 3*p*Log[c*(d + e*x^n)^p]^2*Pol
yLog[2, 1 + (e*x^n)/d] - 6*p^2*Log[c*(d + e*x^n)^p]*PolyLog[3, 1 + (e*x^n)/
d] + 6*p^3*PolyLog[4, 1 + (e*x^n)/d])/n
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.26, size = 6131, normalized size = 54.26

method	result	size
risch	Expression too large to display	6131

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^3/x,x, algorithm="maxima")
```

```
[Out] log((d + e^(n*log(x) + 1))^p)^3*log(x) - integrate(-(d*log(c)^3 + e^(n*log(
x) + 1)*log(c)^3 - 3*((n*p*e*log(x) - e*log(c))*x^n - d*log(c))*log((d + e^
(n*log(x) + 1))^p)^2 + 3*(d*log(c)^2 + e^(n*log(x) + 1)*log(c)^2)*log((d +
e^(n*log(x) + 1))^p))/(d*x + x*e^(n*log(x) + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^3/x,x, algorithm="fricas")
```

```
[Out] integral(log((x^n*e + d)^p*c)^3/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log{\left(c(d+ex^n)^p \right)}^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)**3/x,x)
```

```
[Out] Integral(log(c*(d + e*x**n)**p)**3/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^3/x,x, algorithm="giac")
```

```
[Out] integrate(log((x^n*e + d)^p*c)^3/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)^3/x,x)
```

```
[Out] int(log(c*(d + e*x^n)^p)^3/x, x)
```

3.176 $\int (d+ex)^3 \log\left(c(a+bx)^p\right) dx$

Optimal. Leaf size=140

$$-\frac{(bd-ae)^3px}{4b^3}-\frac{(bd-ae)^2p(d+ex)^2}{8b^2e}-\frac{(bd-ae)p(d+ex)^3}{12be}-\frac{p(d+ex)^4}{16e}-\frac{(bd-ae)^4p\log(a+bx)}{4b^4e}+\frac{(d+ex}{}$$

```
[Out] -1/4*(-a*e+b*d)^3*p*x/b^3-1/8*(-a*e+b*d)^2*p*(e*x+d)^2/b^2/e-1/12*(-a*e+b*d
)*p*(e*x+d)^3/b/e-1/16*p*(e*x+d)^4/e-1/4*(-a*e+b*d)^4*p*ln(b*x+a)/b^4/e+1/4
*(e*x+d)^4*ln(c*(b*x+a)^p)/e
```

Rubi [A]

time = 0.05, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2442, 45}

$$-\frac{p(bd-ae)^4\log(a+bx)}{4b^4e}-\frac{px(bd-ae)^3}{4b^3}-\frac{p(d+ex)^2(bd-ae)^2}{8b^2e}+\frac{(d+ex)^4\log\left(c(a+bx)^p\right)}{4e}-\frac{p(d+ex)^3(bd-ae)}{12be}-\frac{p(d+ex)^4}{16e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*Log[c*(a + b*x)^p],x]
```

```
[Out] -1/4*((b*d - a*e)^3*p*x)/b^3 - ((b*d - a*e)^2*p*(d + e*x)^2)/(8*b^2*e) - ((
b*d - a*e)*p*(d + e*x)^3)/(12*b*e) - (p*(d + e*x)^4)/(16*e) - ((b*d - a*e)^
4*p*Log[a + b*x])/(4*b^4*e) + ((d + e*x)^4*Log[c*(a + b*x)^p])/(4*e)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int (d+ex)^3 \log \left(c(a+bx)^p\right) \, dx = \frac{(d+ex)^4 \log \left(c(a+bx)^p\right)}{4e} - \frac{(bp) \int \frac{(d+ex)^4}{a+bx} \, dx}{4e}$$

$$= \frac{(d+ex)^4 \log \left(c(a+bx)^p\right)}{4e} - \frac{(bp) \int \left(\frac{e(bd-ae)^3}{b^4} + \frac{(bd-ae)^4}{b^4(a+bx)} + \frac{e(bd-ae)^2(d+ex)}{b^3} + \right.}{4e}$$

$$= -\frac{(bd-ae)^3 px}{4b^3} - \frac{(bd-ae)^2 p(d+ex)^2}{8b^2 e} - \frac{(bd-ae)p(d+ex)^3}{12be} - \frac{p(d+e}{16e}$$

Mathematica [A]
time = 0.14, size = 185, normalized size = 1.32

$$-\frac{bpx(-12a^3e^3 + 6a^2be^2(8d+ex) - 4ab^2e(18d^2+6dex+e^2x^2) + b^3(48d^3+36d^2ex+16de^2x^2+3e^3x^3)) + 12a^2e(6b^2d^2-4abde+a^2e^2)\,p\log(a+bx) - 12b^3(4ad^3+bx(4d^3+6d^2ex+4de^2x^2+e^3x^3))\log\left(c(a+bx)^p\right)}{48b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*Log[c*(a + b*x)^p],x]
```

```
[Out] -1/48*(b*p*x*(-12*a^3*e^3 + 6*a^2*b*e^2*(8*d + e*x) - 4*a*b^2*e*(18*d^2 + 6
*d*e*x + e^2*x^2) + b^3*(48*d^3 + 36*d^2*e*x + 16*d*e^2*x^2 + 3*e^3*x^3)) +
 12*a^2*e*(6*b^2*d^2 - 4*a*b*d*e + a^2*e^2)*p*Log[a + b*x] - 12*b^3*(4*a*d^
3 + b*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3))*Log[c*(a + b*x)^p])/b^
4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.72, size = 766, normalized size = 5.47

method	result
risch	$\frac{e^3 ap\,x^3}{12b} - \frac{e^3 a^2 p\,x^2}{8b^2} - d^3 px + \frac{d^3 pa \ln(bx+a)}{b} + e^2 \ln(c)\, d\, x^3 + \frac{3e \ln(c) d^2 x^2}{2} - \frac{\ln(bx+a) d^4 p}{4e} - \frac{e^2 a^2 dpx}{b^2} + \frac{3ea\, d^2 pa}{2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*ln(c*(b*x+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/b*e^3*a*p*x^3-1/8/b^2*e^3*a^2*p*x^2-d^3*p*x+d^3*p/b*a*ln(b*x+a)+e^2*ln
(c)*d*x^3+3/2*e*ln(c)*d^2*x^2-1/4/e*ln(b*x+a)*d^4*p-1/b^2*e^2*a^2*d*p*x+3/2
/b*e*a*d^2*p*x+1/b^3*e^2*ln(b*x+a)*a^3*d*p+1/2*I*Pi*d^3*x*csgn(I*c*(b*x+a)^
p)^2*csgn(I*c)-3/4*I*e*Pi*d^2*x^2*csgn(I*c*(b*x+a)^p)^3+1/4*e^3*ln(c)*x^4+l
n(c)*d^3*x-1/16*e^3*p*x^4+1/4*(e*x+d)^4/e*ln((b*x+a)^p)-1/2*I*e^2*Pi*d*x^3*
csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)-3/4*I*e*Pi*d^2*x^2*csgn(I*(
b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)-1/2*I*Pi*d^3*x*csgn(I*(b*x+a)^p)*cs
gn(I*c*(b*x+a)^p)*csgn(I*c)-1/8*I*e^3*Pi*x^4*csgn(I*(b*x+a)^p)*csgn(I*c*(b*
x+a)^p)*csgn(I*c)+1/2*I*e^2*Pi*d*x^3*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^
2+1/2*I*e^2*Pi*d*x^3*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)+3/4*I*e*Pi*d^2*x^2*csg
```

```
n(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2+3/4*I*e*Pi*d^2*x^2*csgn(I*c*(b*x+a)^p)
^2*csgn(I*c)-1/2*I*Pi*d^3*x*csgn(I*c*(b*x+a)^p)^3-1/8*I*e^3*Pi*x^4*csgn(I*c
*(b*x+a)^p)^3+1/4/b^3*e^3*a^3*p*x-1/4/b^4*e^3*ln(b*x+a)*a^4*p-3/4*d^2*e*p*x
^2-1/3*d*e^2*p*x^3+1/2/b*e^2*a*d*p*x^2-3/2/b^2*e*ln(b*x+a)*a^2*d^2*p+1/8*I*
e^3*Pi*x^4*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2+1/8*I*e^3*Pi*x^4*csgn(I*
c*(b*x+a)^p)^2*csgn(I*c)-1/2*I*e^2*Pi*d*x^3*csgn(I*c*(b*x+a)^p)^3+1/2*I*Pi*
d^3*x*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2
```

Maxima [A]
time = 0.28, size = 207, normalized size = 1.48

$$-\frac{1}{48}\,bp\left(\frac{3\,b^3x^4e^3+4\,(4\,b^3de^2-ab^2e^3)x^3+6\,(6\,b^3d^2e-4\,ab^2de^2+a^2be^3)x^2+12\,(4\,b^3d^3-6\,ab^2d^2e+4\,a^2bde^2-a^3e^3)x}{b^4}-\frac{12\,(4\,ab^3d^3-6\,a^2b^2d^2e+4\,a^3bde^2-a^4e^3)\log\left(bx+a\right)}{b^5}\right)+\frac{1}{4}\left(x^4e^3+4\,dx^3e^2+6\,d^2x^2e+4\,d^3x\right)\log\left((bx+a)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(b*x+a)^p),x, algorithm="maxima")
```

```
[Out] -1/48*b*p*((3*b^3*x^4*e^3 + 4*(4*b^3*d*e^2 - a*b^2*e^3)*x^3 + 6*(6*b^3*d^2*
e - 4*a*b^2*d*e^2 + a^2*b*e^3)*x^2 + 12*(4*b^3*d^3 - 6*a*b^2*d^2*e + 4*a^2*
b*d*e^2 - a^3*e^3)*x)/b^4 - 12*(4*a*b^3*d^3 - 6*a^2*b^2*d^2*e + 4*a^3*b*d*e
^2 - a^4*e^3)*log(b*x + a)/b^5) + 1/4*(x^4*e^3 + 4*d*x^3*e^2 + 6*d^2*x^2*e
+ 4*d^3*x)*log((b*x + a)^p*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(131) = 262$.
time = 0.36, size = 266, normalized size = 1.90

$$-\frac{48\,b^4d^3px+(3\,b^4px^4-4\,ab^3px^3+6\,a^2b^2px^2-12\,a^3bpx)e^3+8\,(2\,b^4dpx^3-3\,ab^3dpx^2+6\,a^2b^2dpx)e^2+36\,(b^4d^2px^2-2\,ab^3d^2px)e-12\,(4\,b^4d^3px+4\,ab^3d^3p+(b^4px^4-a^4p)e^3+4\,(b^4dpx^3+a^3bdp)e^2+6\,(b^4d^2px^2-a^2b^2d^2p)e)\log\left(bx+a\right)-12\,(b^4x^4e^3+4\,b^4dx^3e^2+6\,b^4d^2x^2e+4\,b^4d^3x)\log\left(c\right)}{48\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(b*x+a)^p),x, algorithm="fricas")
```

```
[Out] -1/48*(48*b^4*d^3*p*x + (3*b^4*p*x^4 - 4*a*b^3*p*x^3 + 6*a^2*b^2*p*x^2 - 12
*a^3*b*p*x)*e^3 + 8*(2*b^4*d*p*x^3 - 3*a*b^3*d*p*x^2 + 6*a^2*b^2*d*p*x)*e^2
 + 36*(b^4*d^2*p*x^2 - 2*a*b^3*d^2*p*x)*e - 12*(4*b^4*d^3*p*x + 4*a*b^3*d^3
*p + (b^4*p*x^4 - a^4*p)*e^3 + 4*(b^4*d*p*x^3 + a^3*b*d*p)*e^2 + 6*(b^4*d^2
*p*x^2 - a^2*b^2*d^2*p)*e)*log(b*x + a) - 12*(b^4*x^4*e^3 + 4*b^4*d*x^3*e^2
 + 6*b^4*d^2*x^2*e + 4*b^4*d^3*x)*log(c))/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(117) = 234$.
time = 0.95, size = 335, normalized size = 2.39

$$\begin{cases} -\frac{a^4e^3\log\left(c(a+bx)^p\right)}{4b^4}+\frac{a^3de^2\log\left(c(a+bx)^p\right)}{b^3}+\frac{a^3e^3px}{4b^3}-\frac{3a^2d^2e\log\left(c(a+bx)^p\right)}{2b^2}-\frac{a^2de^2px}{b^2}-\frac{a^2e^3px^2}{8b^2}+\frac{ad^3\log\left(c(a+bx)^p\right)}{b}+\frac{3ad^2epx}{2b}+\frac{ade^2px^2}{2b}+\frac{ae^3px^3}{12b}-d^3px+d^3x\log\left(c(a+bx)^p\right)-\frac{3d^2epx^2}{4}+\frac{3d^2ex^2\log\left(c(a+bx)^p\right)}{2}-\frac{de^2px^3}{3}+de^2x^3\log\left(c(a+bx)^p\right)-\frac{e^3px^4}{16}+\frac{e^3x^4\log\left(c(a+bx)^p\right)}{4} & \text{for}\: b \neq 0 \\ \left(d^3x+\frac{3d^2ex^2}{2}+de^2x^3+\frac{e^3x^4}{4}\right)\log\left(a^pc\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*ln(c*(b*x+a)**p),x)
```

```
[Out] Piecewise((-a**4*e**3*log(c*(a + b*x)**p)/(4*b**4) + a**3*d*e**2*log(c*(a +
 b*x)**p)/b**3 + a**3*e**3*p*x/(4*b**3) - 3*a**2*d**2*e*log(c*(a + b*x)**p)
/(2*b**2) - a**2*d*e**2*p*x/b**2 - a**2*e**3*p*x**2/(8*b**2) + a*d**3*log(c
*(a + b*x)**p)/b + 3*a*d**2*e*p*x/(2*b) + a*d*e**2*p*x**2/(2*b) + a*e**3*p*
x**3/(12*b) - d**3*p*x + d**3*x*log(c*(a + b*x)**p) - 3*d**2*e*p*x**2/4 + 3
*d**2*e*x**2*log(c*(a + b*x)**p)/2 - d*e**2*p*x**3/3 + d*e**2*x**3*log(c*(a
 + b*x)**p) - e**3*p*x**4/16 + e**3*x**4*log(c*(a + b*x)**p)/4, Ne(b, 0)),
((d**3*x + 3*d**2*e*x**2/2 + d*e**2*x**3 + e**3*x**4/4)*log(a**p*c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 558 vs. 2(131) = 262.
time = 3.19, size = 558, normalized size = 3.99

$$\frac{(bx+a)d^3p\log(bx+a)}{b}+\frac{3(bx+a)^2d^2pe\log(bx+a)}{2b^2}-\frac{3(bx+a)ad^2pe\log(bx+a)}{b^2}-\frac{(bx+a)d^3p}{b}-\frac{3(bx+a)^2d^2pe}{4b^2}+\frac{3(bx+a)ad^2pe}{b^2}+\frac{(bx+a)^3dpe^2\log(bx+a)}{b^3}-\frac{3(bx+a)^2adpe^2\log(bx+a)}{b^3}+\frac{3(bx+a)a^2dpe^2\log(bx+a)}{b^3}+\frac{(bx+a)d^3\log(c)}{b}+\frac{3(bx+a)^2d^2e\log(c)}{2b^2}-\frac{3(bx+a)ad^2e\log(c)}{b^2}-\frac{(bx+a)^3dpe^2}{3b^3}+\frac{3(bx+a)^2adpe^2}{2b^3}-\frac{3(bx+a)a^2dpe^2}{b^3}+\frac{(bx+a)^4pe^3\log(bx+a)}{4b^4}-\frac{(bx+a)^3ape^3\log(bx+a)}{b^4}+\frac{3(bx+a)^2a^2pe^3\log(bx+a)}{2b^4}-\frac{(bx+a)a^3pe^3\log(bx+a)}{b^4}+\frac{(bx+a)^3de^2\log(c)}{b^3}-\frac{3(bx+a)^2ade^2\log(c)}{b^3}+\frac{3(bx+a)a^2de^2\log(c)}{b^3}-\frac{(bx+a)^4pe^3}{16b^4}+\frac{(bx+a)^3ape^3}{3b^4}-\frac{3(bx+a)^2a^2pe^3}{4b^4}+\frac{(bx+a)a^3pe^3}{b^4}+\frac{(bx+a)^4e^3\log(c)}{4b^4}-\frac{(bx+a)^3ae^3\log(c)}{b^4}+\frac{3(bx+a)^2a^2e^3\log(c)}{2b^4}-\frac{(bx+a)a^3e^3\log(c)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(b*x+a)^p),x, algorithm="giac")
```

```
[Out] (b*x + a)*d^3*p*log(b*x + a)/b + 3/2*(b*x + a)^2*d^2*p*e*log(b*x + a)/b^2 -
 3*(b*x + a)*a*d^2*p*e*log(b*x + a)/b^2 - (b*x + a)*d^3*p/b - 3/4*(b*x + a)
^2*d^2*p*e/b^2 + 3*(b*x + a)*a*d^2*p*e/b^2 + (b*x + a)^3*d*p*e^2*log(b*x +
a)/b^3 - 3*(b*x + a)^2*a*d*p*e^2*log(b*x + a)/b^3 + 3*(b*x + a)*a^2*d*p*e^2
*log(b*x + a)/b^3 + (b*x + a)*d^3*log(c)/b + 3/2*(b*x + a)^2*d^2*e*log(c)/b
^2 - 3*(b*x + a)*a*d^2*e*log(c)/b^2 - 1/3*(b*x + a)^3*d*p*e^2/b^3 + 3/2*(b*
x + a)^2*a*d*p*e^2/b^3 - 3*(b*x + a)*a^2*d*p*e^2/b^3 + 1/4*(b*x + a)^4*p*e^
3*log(b*x + a)/b^4 - (b*x + a)^3*a*p*e^3*log(b*x + a)/b^4 + 3/2*(b*x + a)^2
*a^2*p*e^3*log(b*x + a)/b^4 - (b*x + a)*a^3*p*e^3*log(b*x + a)/b^4 + (b*x +
 a)^3*d*e^2*log(c)/b^3 - 3*(b*x + a)^2*a*d*e^2*log(c)/b^3 + 3*(b*x + a)*a^2
*d*e^2*log(c)/b^3 - 1/16*(b*x + a)^4*p*e^3/b^4 + 1/3*(b*x + a)^3*a*p*e^3/b^
4 - 3/4*(b*x + a)^2*a^2*p*e^3/b^4 + (b*x + a)*a^3*p*e^3/b^4 + 1/4*(b*x + a)
^4*e^3*log(c)/b^4 - (b*x + a)^3*a*e^3*log(c)/b^4 + 3/2*(b*x + a)^2*a^2*e^3*
log(c)/b^4 - (b*x + a)*a^3*e^3*log(c)/b^4
```

Mupad [B]
time = 0.31, size = 208, normalized size = 1.49

$$\ln\left(c\,{\left(a+b\,x\right)}^p\right)\,\left(d^3\,x+\frac{3\,d^2\,e\,x^2}{2}+d\,e^2\,x^3+\frac{e^3\,x^4}{4}\right)+x^2\,\left(\frac{a\,\left(d\,e^2\,p-\frac{a\,e^3\,p}{4\,b}\right)}{2\,b}-\frac{3\,d^2\,e\,p}{4}\right)-x\,\left(d^3\,p+\frac{a\,\left(\frac{a\,\left(d\,e^2\,p-\frac{a\,e^3\,p}{4\,b}\right)}{b}-\frac{3\,d^2\,e\,p}{2}\right)}{b}\right)-x^3\,\left(\frac{d\,e^2\,p}{3}-\frac{a\,e^3\,p}{12\,b}\right)-\frac{e^3\,p\,x^4}{16}-\frac{\ln\left(a+b\,x\right)\,\left(p\,a^4\,e^3-4\,p\,a^3\,b\,d\,e^2+6\,p\,a^2\,b^2\,d^2\,e-4\,p\,a\,b^3\,d^3\right)}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x)^p)*(d + e*x)^3,x)
```

```
[Out] log(c*(a + b*x)^p)*(d^3*x + (e^3*x^4)/4 + (3*d^2*e*x^2)/2 + d*e^2*x^3) + x^
2*((a*(d*e^2*p - (a*e^3*p)/(4*b)))/(2*b) - (3*d^2*e*p)/4) - x*(d^3*p + (a*(
(a*(d*e^2*p - (a*e^3*p)/(4*b)))/b - (3*d^2*e*p)/2))/b) - x^3*((d*e^2*p)/3 -
 (a*e^3*p)/(12*b)) - (e^3*p*x^4)/16 - (log(a + b*x)*(a^4*e^3*p - 4*a*b^3*d^
3*p - 4*a^3*b*d*e^2*p + 6*a^2*b^2*d^2*e*p))/(4*b^4)
```

3.177 $\int (d+ex)^2 \log\left(c(a+bx)^p\right)\,dx$

Optimal. Leaf size=112

$$-\frac{(bd-ae)^2px}{3b^2}-\frac{(bd-ae)p(d+ex)^2}{6be}-\frac{p(d+ex)^3}{9e}-\frac{(bd-ae)^3p\log(a+bx)}{3b^3e}+\frac{(d+ex)^3\log\left(c(a+bx)^p\right)}{3e}$$

```
[Out] -1/3*(-a*e+b*d)^2*p*x/b^2-1/6*(-a*e+b*d)*p*(e*x+d)^2/b/e-1/9*p*(e*x+d)^3/e-
1/3*(-a*e+b*d)^3*p*ln(b*x+a)/b^3/e+1/3*(e*x+d)^3*ln(c*(b*x+a)^p)/e
```

Rubi [A]
time = 0.04, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2442, 45}

$$-\frac{p(bd-ae)^3\log(a+bx)}{3b^3e}-\frac{px(bd-ae)^2}{3b^2}+\frac{(d+ex)^3\log\left(c(a+bx)^p\right)}{3e}-\frac{p(d+ex)^2(bd-ae)}{6be}-\frac{p(d+ex)^3}{9e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*Log[c*(a + b*x)^p],x]
```

```
[Out] -1/3*((b*d - a*e)^2*p*x)/b^2 - ((b*d - a*e)*p*(d + e*x)^2)/(6*b*e) - (p*(d
+ e*x)^3)/(9*e) - ((b*d - a*e)^3*p*Log[a + b*x])/(3*b^3*e) + ((d + e*x)^3*L
og[c*(a + b*x)^p])/(3*e)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int (d+ex)^2 \log \left(c(a+bx)^p\right) \, dx = \frac{(d+ex)^3 \log \left(c(a+bx)^p\right)}{3e} - \frac{(bp) \int \frac{(d+ex)^3}{a+bx} \, dx}{3e}$$

$$= \frac{(d+ex)^3 \log \left(c(a+bx)^p\right)}{3e} - \frac{(bp) \int \left(\frac{e(bd-ae)^2}{b^3} + \frac{(bd-ae)^3}{b^3(a+bx)} + \frac{e(bd-ae)(d+ex)}{b^2} + \right.}{3e}$$

$$= -\frac{(bd-ae)^2 px}{3b^2} - \frac{(bd-ae)p(d+ex)^2}{6be} - \frac{p(d+ex)^3}{9e} - \frac{(bd-ae)^3 p \log(a}{3b^3 e}$$

Mathematica [A]
time = 0.08, size = 121, normalized size = 1.08

$$\frac{6a^2e(-3bd+ae)p\log(a+bx)+b(-px(6a^2e^2-3abe(6d+ex)+b^2(18d^2+9dex+2e^2x^2))+6b(3ad^2+bx(3d^2+3dex+e^2x^2))\log\left(c(a+bx)^p\right))}{18b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*Log[c*(a + b*x)^p],x]
```

```
[Out] (6*a^2*e*(-3*b*d + a*e)*p*Log[a + b*x] + b*(-(p*x*(6*a^2*e^2 - 3*a*b*e*(6*d
 + e*x) + b^2*(18*d^2 + 9*d*e*x + 2*e^2*x^2))) + 6*b*(3*a*d^2 + b*x*(3*d^2
+ 3*d*e*x + e^2*x^2))*Log[c*(a + b*x)^p]))/(18*b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.56, size = 537, normalized size = 4.79

method	result
risch	$-\frac{ie^2\pi x^3 \text{csgn}(i(bx+a)^p)\text{csgn}(ic(bx+a)^p)\text{csgn}(ic)}{6} + \ln(c)\, d^2x + \frac{i\pi\, d^2x\, \text{csgn}(i(bx+a)^p)\text{csgn}(ic(bx+a)^p)^2}{2} + \frac{i\pi\, d^2x\text{csgn}(ic(b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*ln(c*(b*x+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] ln(c)*d^2*x-1/2*d*e*p*x^2+d^2*p/b*a*ln(b*x+a)+1/b*e*a*d*p*x-1/b^2*e*ln(b*x+
a)*a^2*d*p+1/6*I*e^2*Pi*x^3*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2+1/6*I*e
^2*Pi*x^3*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)-1/2*I*e*Pi*d*x^2*csgn(I*c*(b*x+a)
^p)^3-1/6*I*e^2*Pi*x^3*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)+1/2*
I*e*Pi*d*x^2*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2+1/2*I*e*Pi*d*x^2*csgn(
I*c*(b*x+a)^p)^2*csgn(I*c)-1/2*I*Pi*d^2*x*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a
)^p)*csgn(I*c)+1/3*(e*x+d)^3/e*ln((b*x+a)^p)+e*ln(c)*d*x^2-1/3/e*ln(b*x+a)*
d^3*p+1/3*e^2*ln(c)*x^3-1/9*e^2*p*x^3-d^2*p*x+1/2*I*Pi*d^2*x*csgn(I*(b*x+a)
^p)*csgn(I*c*(b*x+a)^p)^2+1/2*I*Pi*d^2*x*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)-1/
2*I*e*Pi*d*x^2*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)-1/2*I*Pi*d^2
*x*csgn(I*c*(b*x+a)^p)^3+1/6/b*e^2*a*p*x^2-1/3/b^2*e^2*a^2*p*x+1/3/b^3*e^2*
ln(b*x+a)*a^3*p-1/6*I*e^2*Pi*x^3*csgn(I*c*(b*x+a)^p)^3
```

Maxima [A]
time = 0.29, size = 135, normalized size = 1.21

$$-\frac{1}{18}\,bp\left(\frac{2\,b^2x^3e^2+3\,(3\,b^2de-abe^2)x^2+6\,(3\,b^2d^2-3\,abde+a^2e^2)x}{b^3}-\frac{6\,(3\,ab^2d^2-3\,a^2bde+a^3e^2)\log(bx+a)}{b^4}\right)+\frac{1}{3}\,(x^3e^2+3\,dx^2e+3\,d^2x)\log\left((bx+a)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(b*x+a)^p),x, algorithm="maxima")
```

```
[Out] -1/18*b*p*((2*b^2*x^3*e^2 + 3*(3*b^2*d*e - a*b*e^2)*x^2 + 6*(3*b^2*d^2 - 3*
a*b*d*e + a^2*e^2)*x)/b^3 - 6*(3*a*b^2*d^2 - 3*a^2*b*d*e + a^3*e^2)*log(b*x
 + a)/b^4) + 1/3*(x^3*e^2 + 3*d*x^2*e + 3*d^2*x)*log((b*x + a)^p*c)
```

Fricas [A]
time = 0.40, size = 172, normalized size = 1.54

$$-\frac{18\,b^3d^2px+(2\,b^3px^3-3\,ab^2px^2+6\,a^2bpx)e^2+9\,(b^3dpx^2-2\,ab^2dpx)e-6\,(3\,b^3d^2px+3\,ab^2d^2p+(b^3px^3+a^3p)e^2+3\,(b^3dpx^2-a^2bdp)e)\log(bx+a)-6\,(b^3x^3e^2+3\,b^3dx^2e+3\,b^3d^2x)\log(c)}{18\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(b*x+a)^p),x, algorithm="fricas")
```

```
[Out] -1/18*(18*b^3*d^2*p*x + (2*b^3*p*x^3 - 3*a*b^2*p*x^2 + 6*a^2*b*p*x)*e^2 + 9
*(b^3*d*p*x^2 - 2*a*b^2*d*p*x)*e - 6*(3*b^3*d^2*p*x + 3*a*b^2*d^2*p + (b^3*
p*x^3 + a^3*p)*e^2 + 3*(b^3*d*p*x^2 - a^2*b*d*p)*e)*log(b*x + a) - 6*(b^3*x
^3*e^2 + 3*b^3*d*x^2*e + 3*b^3*d^2*x)*log(c))/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(94) = 188.
time = 0.55, size = 202, normalized size = 1.80

$$\begin{cases}\frac{a^3e^2\log(c(a+bx)^p)}{3b^3}-\frac{a^2de\log(c(a+bx)^p)}{b^2}-\frac{a^2e^2px}{3b^2}+\frac{ad^2\log(c(a+bx)^p)}{b}+\frac{adepx}{b}+\frac{ae^2px^2}{6b}-d^2px+d^2x\log(c(a+bx)^p)-\frac{depx^2}{2}+dex^2\log(c(a+bx)^p)-\frac{e^2px^3}{9}+\frac{e^2x^3\log(c(a+bx)^p)}{3} & \text{for}\ b\neq 0\\ \left(d^2x+dex^2+\frac{e^2x^3}{3}\right)\log(a^pc) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*ln(c*(b*x+a)**p),x)
```

```
[Out] Piecewise((a**3*e**2*log(c*(a + b*x)**p)/(3*b**3) - a**2*d*e*log(c*(a + b*x
)**p)/b**2 - a**2*e**2*p*x/(3*b**2) + a*d**2*log(c*(a + b*x)**p)/b + a*d*e*
p*x/b + a*e**2*p*x**2/(6*b) - d**2*p*x + d**2*x*log(c*(a + b*x)**p) - d*e*p
*x**2/2 + d*e*x**2*log(c*(a + b*x)**p) - e**2*p*x**3/9 + e**2*x**3*log(c*(a
 + b*x)**p)/3, Ne(b, 0)), ((d**2*x + d*e*x**2 + e**2*x**3/3)*log(a**p*c), T
rue))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(104) = 208.
time = 6.92, size = 313, normalized size = 2.79

$$\frac{(bx+a)d^2p\log(bx+a)}{b}+\frac{(bx+a)^2dpe\log(bx+a)}{b^2}-\frac{2\,(bx+a)adpe\log(bx+a)}{b^2}-\frac{(bx+a)d^2p}{b}-\frac{(bx+a)^2dpe}{2\,b^2}+\frac{2\,(bx+a)adpe}{b^2}+\frac{(bx+a)^3pe^2\log(bx+a)}{3\,b^3}-\frac{(bx+a)^2ape^2\log(bx+a)}{b^3}+\frac{(bx+a)a^2pe^2\log(bx+a)}{b^3}+\frac{(bx+a)d^2\log(c)}{b}+\frac{(bx+a)^2de\log(c)}{b^2}-\frac{2\,(bx+a)ade\log(c)}{b^2}-\frac{(bx+a)^3pe^2}{9\,b^3}+\frac{(bx+a)^2ape^2}{2\,b^3}-\frac{(bx+a)a^2pe^2}{b^3}+\frac{(bx+a)^3e^2\log(c)}{3\,b^3}-\frac{(bx+a)^2ae^2\log(c)}{b^3}+\frac{(bx+a)a^2e^2\log(c)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2*log(c*(b*x+a)^p),x, algorithm="giac")

[Out] (b*x + a)*d^2*p*log(b*x + a)/b + (b*x + a)^2*d*p*e*log(b*x + a)/b^2 - 2*(b*x + a)*a*d*p*e*log(b*x + a)/b^2 - (b*x + a)*d^2*p/b - 1/2*(b*x + a)^2*d*p*e/b^2 + 2*(b*x + a)*a*d*p*e/b^2 + 1/3*(b*x + a)^3*p*e^2*log(b*x + a)/b^3 - (b*x + a)^2*a*p*e^2*log(b*x + a)/b^3 + (b*x + a)*a^2*p*e^2*log(b*x + a)/b^3 + (b*x + a)*d^2*log(c)/b + (b*x + a)^2*d*e*log(c)/b^2 - 2*(b*x + a)*a*d*e*log(c)/b^2 - 1/9*(b*x + a)^3*p*e^2/b^3 + 1/2*(b*x + a)^2*a*p*e^2/b^3 - (b*x + a)*a^2*p*e^2/b^3 + 1/3*(b*x + a)^3*e^2*log(c)/b^3 - (b*x + a)^2*a*e^2*log(c)/b^3 + (b*x + a)*a^2*e^2*log(c)/b^3

Mupad [B]
time = 0.27, size = 131, normalized size = 1.17

$$\ln\left(c\,{\left(a+b\,x\right)}^p\right)\,\left(d^2\,x+d\,e\,x^2+\frac{e^2\,x^3}{3}\right)-x^2\,\left(\frac{d\,e\,p}{2}-\frac{a\,e^2\,p}{6\,b}\right)-x\,\left(d^2\,p-\frac{a\,\left(d\,e\,p-\frac{a\,e^2\,p}{3\,b}\right)}{b}\right)-\frac{e^2\,p\,x^3}{9}+\frac{\ln\left(a+b\,x\right)\,\left(p\,a^3\,e^2-3\,p\,a^2\,b\,d\,e+3\,p\,a\,b^2\,d^2\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(c*(a + b*x)^p)*(d + e*x)^2,x)

[Out] log(c*(a + b*x)^p)*(d^2*x + (e^2*x^3)/3 + d*e*x^2) - x^2*((d*e*p)/2 - (a*e^2*p)/(6*b)) - x*(d^2*p - (a*(d*e*p - (a*e^2*p)/(3*b)))/b) - (e^2*p*x^3)/9 + (log(a + b*x)*(a^3*e^2*p + 3*a*b^2*d^2*p - 3*a^2*b*d*e*p))/(3*b^3)

3.178 $\int (d+ex)\log\left(c(a+bx)^p\right)\,dx$

Optimal. Leaf size=84

$$-\frac{(bd-ae)px}{2b}-\frac{p(d+ex)^2}{4e}-\frac{(bd-ae)^2p\log(a+bx)}{2b^2e}+\frac{(d+ex)^2\log\left(c(a+bx)^p\right)}{2e}$$

```
[Out] -1/2*(-a*e+b*d)*p*x/b-1/4*p*(e*x+d)^2/e-1/2*(-a*e+b*d)^2*p*ln(b*x+a)/b^2/e+
1/2*(e*x+d)^2*ln(c*(b*x+a)^p)/e
```

Rubi [A]
time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2442, 45}

$$-\frac{p(bd-ae)^2\log(a+bx)}{2b^2e}+\frac{(d+ex)^2\log\left(c(a+bx)^p\right)}{2e}-\frac{px(bd-ae)}{2b}-\frac{p(d+ex)^2}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*Log[c*(a + b*x)^p],x]
```

```
[Out] -1/2*((b*d - a*e)*p*x)/b - (p*(d + e*x)^2)/(4*e) - ((b*d - a*e)^2*p*Log[a +
 b*x])/(2*b^2*e) + ((d + e*x)^2*Log[c*(a + b*x)^p])/(2*e)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int (d+ex)\log\left(c(a+bx)^p\right)\,dx &= \frac{(d+ex)^2\log\left(c(a+bx)^p\right)}{2e}-\frac{(bp)\int\frac{(d+ex)^2}{a+bx}\,dx}{2e}\\ &= \frac{(d+ex)^2\log\left(c(a+bx)^p\right)}{2e}-\frac{(bp)\int\left(\frac{e(bd-ae)}{b^2}+\frac{(bd-ae)^2}{b^2(a+bx)}+\frac{e(d+ex)}{b}\right)dx}{2e}\\ &= -\frac{(bd-ae)px}{2b}-\frac{p(d+ex)^2}{4e}-\frac{(bd-ae)^2p\log(a+bx)}{2b^2e}+\frac{(d+ex)^2\log\left(c(a\right.}{2e}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 82, normalized size = 0.98

$$-dpx+\frac{aepx}{2b}-\frac{1}{4}epx^2-\frac{a^2ep\log(a+bx)}{2b^2}+\frac{1}{2}ex^2\log\left(c(a+bx)^p\right)+\frac{d(a+bx)\log\left(c(a+bx)^p\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*Log[c*(a + b*x)^p],x]
```

```
[Out] -(d*p*x) + (a*e*p*x)/(2*b) - (e*p*x^2)/4 - (a^2*e*p*Log[a + b*x])/(2*b^2) +
 (e*x^2*Log[c*(a + b*x)^p])/2 + (d*(a + b*x)*Log[c*(a + b*x)^p])/b
```

Maple [A]
time = 0.35, size = 83, normalized size = 0.99

method	result
norman	$dx\ln\left(c\,\mathrm{e}^{p\ln(bx+a)}\right)-\frac{ep\,x^2}{4}+\frac{e\,x^2\ln\left(c\,\mathrm{e}^{p\ln(bx+a)}\right)}{2}+\frac{p(ae-2bd)x}{2b}-\frac{p\left(e\,a^2-2adb\right)\ln(bx+a)}{2b^2}$
default	$d\ln\left(c(bx+a)^p\right)x-dpx+\frac{dpa\ln(bx+a)}{b}+\frac{e\,x^2\ln\left(c\,\mathrm{e}^{p\ln(bx+a)}\right)}{2}-\frac{ep\,x^2}{4}-\frac{pe\,a^2\ln(bx+a)}{2b^2}+\frac{aepx}{2b}$
risch	$\left(\frac{1}{2}e\,x^2+dx\right)\ln\left((bx+a)^p\right)-\frac{i\pi e\,x^2\mathrm{csgn}(ic(bx+a)^p)^3}{4}-\frac{i\pi dx\mathrm{csgn}(ic(bx+a)^p)^3}{2}+\frac{i\pi dx\,\mathrm{csgn}(i(bx+a)^p)\mathrm{csgn}(ic(bx+a)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*ln(c*(b*x+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] d*ln(c*(b*x+a)^p)*x-d*p*x+d*p/b*a*ln(b*x+a)+1/2*e*x^2*ln(c*exp(p*ln(b*x+a))
)-1/4*e*p*x^2-1/2*p*e*a^2/b^2*ln(b*x+a)+1/2*a*e*p/b*x
```

Maxima [A]
time = 0.31, size = 78, normalized size = 0.93

$$-\frac{1}{4}\,bp\left(\frac{bx^2e+2\,(2\,bd-ae)x}{b^2}-\frac{2\,(2\,abd-a^2e)\log\left(bx+a\right)}{b^3}\right)+\frac{1}{2}\,\left(x^2e+2\,dx\right)\log\left((bx+a)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(b*x+a)^p),x, algorithm="maxima")
```

```
[Out] -1/4*b*p*((b*x^2*e + 2*(2*b*d - a*e)*x)/b^2 - 2*(2*a*b*d - a^2*e)*log(b*x +
 a)/b^3) + 1/2*(x^2*e + 2*d*x)*log((b*x + a)^p*c)
```

Fricas [A]
time = 0.39, size = 94, normalized size = 1.12

$$-\frac{4\,b^2dpx+(b^2px^2-2\,abpx)e-2\,(2\,b^2dpx+2\,abdp+(b^2px^2-a^2p)e)\log\left(bx+a\right)-2\,(b^2x^2e+2\,b^2dx)\log\left(c\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(b*x+a)^p),x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^2*d*p*x + (b^2*p*x^2 - 2*a*b*p*x)*e - 2*(2*b^2*d*p*x + 2*a*b*d*p
+ (b^2*p*x^2 - a^2*p)*e)*log(b*x + a) - 2*(b^2*x^2*e + 2*b^2*d*x)*log(c))/b
^2
```

Sympy [A]
time = 0.31, size = 105, normalized size = 1.25

$$\begin{cases} -\frac{a^2e\log\left(c(a+bx)^p\right)}{2b^2} + \frac{ad\log\left(c(a+bx)^p\right)}{b} + \frac{aepx}{2b} - dpx + dx\log\left(c(a+bx)^p\right) - \frac{epx^2}{4} + \frac{ex^2\log\left(c(a+bx)^p\right)}{2} & \text{for}\ b \neq 0 \\ \left(dx + \frac{ex^2}{2}\right)\log\left(a^pc\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*ln(c*(b*x+a)**p),x)
```

```
[Out] Piecewise((-a**2*e*log(c*(a + b*x)**p)/(2*b**2) + a*d*log(c*(a + b*x)**p)/b
 + a*e*p*x/(2*b) - d*p*x + d*x*log(c*(a + b*x)**p) - e*p*x**2/4 + e*x**2*lo
g(c*(a + b*x)**p)/2, Ne(b, 0)), ((d*x + e*x**2/2)*log(a**p*c), True))
```

Giac [A]
time = 2.67, size = 142, normalized size = 1.69

$$\frac{(bx+a)dp\log\left(bx+a\right)}{b} + \frac{(bx+a)^2pe\log\left(bx+a\right)}{2\,b^2} - \frac{(bx+a)ape\log\left(bx+a\right)}{b^2} - \frac{(bx+a)dp}{b} - \frac{(bx+a)^2pe}{4\,b^2} + \frac{(bx+a)ape}{b^2} + \frac{(bx+a)d\log\left(c\right)}{b} + \frac{(bx+a)^2e\log\left(c\right)}{2\,b^2} - \frac{(bx+a)ae\log\left(c\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(b*x+a)^p),x, algorithm="giac")
```

```
[Out] (b*x + a)*d*p*log(b*x + a)/b + 1/2*(b*x + a)^2*p*e*log(b*x + a)/b^2 - (b*x
+ a)*a*p*e*log(b*x + a)/b^2 - (b*x + a)*d*p/b - 1/4*(b*x + a)^2*p*e/b^2 + (
b*x + a)*a*p*e/b^2 + (b*x + a)*d*log(c)/b + 1/2*(b*x + a)^2*e*log(c)/b^2 -
(b*x + a)*a*e*log(c)/b^2
```

Mupad [B]
time = 0.25, size = 68, normalized size = 0.81

$$\ln\left(c\,{\left(a+b\,x\right)}^p\right)\,\left(\frac{e\,x^2}{2}+d\,x\right)-x\,\left(d\,p-\frac{a\,e\,p}{2\,b}\right)-\frac{e\,p\,x^2}{4}-\frac{\ln\left(a+b\,x\right)\,\left(a^2\,e\,p-2\,a\,b\,d\,p\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x)^p)*(d + e*x),x)
```

```
[Out] log(c*(a + b*x)^p)*(d*x + (e*x^2)/2) - x*(d*p - (a*e*p)/(2*b)) - (e*p*x^2)/
4 - (log(a + b*x)*(a^2*e*p - 2*a*b*d*p))/(2*b^2)
```

3.179 $\int \log\left(c(a+bx)^p\right) dx$

Optimal. Leaf size=24

$$-px+\frac{(a+bx)\log\left(c(a+bx)^p\right)}{b}$$

```
[Out] -p*x+(b*x+a)*ln(c*(b*x+a)^p)/b
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2436, 2332}

$$\frac{(a+bx)\log\left(c(a+bx)^p\right)}{b}-px$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x)^p],x]
```

```
[Out] -(p*x) + ((a + b*x)*Log[c*(a + b*x)^p])/b
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \log\left(c(a+bx)^p\right) dx&=\frac{\text{Subst}\left(\int \log\left(cx^p\right) dx,x,a+bx\right)}{b}\\&=-px+\frac{(a+bx)\log\left(c(a+bx)^p\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 1.00

$$-px+\frac{(a+bx)\log\left(c(a+bx)^p\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x)^p],x]
[Out] -(p*x) + ((a + b*x)*Log[c*(a + b*x)^p])/b
```

Maple [A]
time = 0.28, size = 36, normalized size = 1.50

method	result
norman	$x \ln\left(c\,{e}^{p\ln(bx+a)}\right)+\frac{pa\ln(bx+a)}{b}-px$
default	$\ln\left(c(bx+a)^{p}\right)x-pb\left(\frac{x}{b}-\frac{a\ln(bx+a)}{b^{2}}\right)$
risch	$x\ln\left((bx+a)^{p}\right)-\frac{i\pi x\,\mathrm{csgn}(i(bx+a)^{p})\mathrm{csgn}(ic(bx+a)^{p})\mathrm{csgn}(ic)}{2}+\frac{i\pi x\mathrm{csgn}(ic(bx+a)^{p})^{2}\mathrm{csgn}(ic)}{2}+\frac{i\pi x\,\mathrm{csgn}(i(bx+a)^{p})\mathrm{csgn}(i$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x+a)^p),x,method=_RETURNVERBOSE)
[Out] ln(c*(b*x+a)^p)*x-p*b*(x/b-a/b^2*ln(b*x+a))
```

Maxima [A]
time = 0.46, size = 35, normalized size = 1.46

$$-bp\left(\frac{x}{b}-\frac{a\log\left(bx+a\right)}{b^{2}}\right)+x\log\left(\left(bx+a\right)^{p}c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p),x, algorithm="maxima")
[Out] -b*p*(x/b - a*log(b*x + a)/b^2) + x*log((b*x + a)^p*c)
```

Fricas [A]
time = 0.36, size = 32, normalized size = 1.33

$$-\frac{bpx-bx\log\left(c\right)-\left(bpx+ap\right)\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p),x, algorithm="fricas")
[Out] -(b*p*x - b*x*log(c) - (b*p*x + a*p)*log(b*x + a))/b
```

Sympy [A]
time = 0.13, size = 36, normalized size = 1.50

$$\begin{cases}\frac{a\log\left(c(a+bx)^{p}\right)}{b}-px+x\log\left(c(a+bx)^{p}\right) & \text{for}\ b\neq 0\\ x\log\left(a^{p}c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x+a)**p),x)
```

```
[Out] Piecewise((a*log(c*(a + b*x)**p)/b - p*x + x*log(c*(a + b*x)**p), Ne(b, 0))
, (x*log(a**p*c), True))
```

Giac [A]
time = 5.56, size = 39, normalized size = 1.62

$$\frac{(bx+a)p\log(bx+a)}{b} - \frac{(bx+a)p}{b} + \frac{(bx+a)\log(c)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p),x, algorithm="giac")
```

```
[Out] (b*x + a)*p*log(b*x + a)/b - (b*x + a)*p/b + (b*x + a)*log(c)/b
```

Mupad [B]
time = 0.07, size = 29, normalized size = 1.21

$$x\,\ln\left(c\,{\left(a+b\,x\right)}^p\right)-p\,x+\frac{a\,p\,\ln\left(a+b\,x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x)^p),x)
```

```
[Out] x*log(c*(a + b*x)^p) - p*x + (a*p*log(a + b*x))/b
```

3.180 $\int \frac{\log(c(a+bx)^p)}{d+ex}\,dx$

Optimal. Leaf size=58

$$\frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e}+\frac{p\text{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{e}$$

```
[Out] ln(c*(b*x+a)^p)*ln(b*(e*x+d)/(-a*e+b*d))/e+p*polylog(2,-e*(b*x+a)/(-a*e+b*d
))/e
```

Rubi [A]
time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2441, 2440, 2438}

$$\frac{p\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e}+\frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e + (p*PolyLog[2, -((e*
(a + b*x))/(b*d - a*e))])/e
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rubi steps

$$\int \frac{\log\left(c(a+bx)^p\right)}{d+ex}\,dx = \frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e} - \frac{(bp)\int \frac{\log\left(\frac{b(d+ex)}{bd-ae}\right)}{a+bx}\,dx}{e}$$
$$= \frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e} - \frac{p\text{Subst}\left(\int \frac{\log\left(1+\frac{ex}{bd-ae}\right)}{x}\,dx, x, a+bx\right)}{e}$$
$$= \frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e} + \frac{p\text{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{e}$$

Mathematica [A]
time = 0.00, size = 57, normalized size = 0.98

$$\frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e} + \frac{p\text{Li}_2\left(\frac{e(a+bx)}{-bd+ae}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e + (p*PolyLog[2, (e*(a
 + b*x))/(-(b*d) + a*e)])/e
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.45, size = 242, normalized size = 4.17

method	result
risch	$\frac{\ln(ex+d)\ln((bx+a)^p)}{e} - \frac{p\,\text{dilog}\left(\frac{(ex+d)b+ae-bd}{ae-bd}\right)}{e} - \frac{p\ln(ex+d)\ln\left(\frac{(ex+d)b+ae-bd}{ae-bd}\right)}{e} + \frac{i\ln(ex+d)\pi\,\text{csgn}(i(bx+a)^p)\text{csgn}(ic(bx+}{2e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] ln(e*x+d)/e*ln((b*x+a)^p)-1/e*p*dilog(((e*x+d)*b+a*e-b*d)/(a*e-b*d))-1/e*p*
ln(e*x+d)*ln(((e*x+d)*b+a*e-b*d)/(a*e-b*d))+1/2*I*ln(e*x+d)/e*Pi*csgn(I*(b*
x+a)^p)*csgn(I*c*(b*x+a)^p)^2-1/2*I*ln(e*x+d)/e*Pi*csgn(I*(b*x+a)^p)*csgn(I
*c*(b*x+a)^p)*csgn(I*c)-1/2*I*ln(e*x+d)/e*Pi*csgn(I*c*(b*x+a)^p)^3+1/2*I*ln
(e*x+d)/e*Pi*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)+ln(e*x+d)/e*ln(c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(59) = 118.
time = 0.39, size = 123, normalized size = 2.12

$$bp\left(\frac{\log\left(bx+a\right)\log\left(xe+d\right)}{b} - \frac{\log\left(xe+d\right)\log\left(-\frac{bxe+bd}{bd-ae}+1\right) + \text{Li}_2\left(\frac{bxe+bd}{bd-ae}\right)}{b}\right)e^{(-1)} - pe^{(-1)}\log\left(bx+a\right)\log\left(xe+d\right) + e^{(-1)}\log\left(\left(bx+a\right)^p c\right)\log\left(xe+d\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] b*p*(log(b*x + a)*log(x*e + d)/b - (log(x*e + d)*log(-(b*x*e + b*d)/(b*d -
a*e) + 1) + dilog((b*x*e + b*d)/(b*d - a*e)))/b)*e^(-1) - p*e^(-1)*log(b*x
+ a)*log(x*e + d) + e^(-1)*log((b*x + a)^p*c)*log(x*e + d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x + a)^p*c)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c(a+bx)^p\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x+a)**p)/(e*x+d),x)
```

```
[Out] Integral(log(c*(a + b*x)**p)/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x + a)^p*c)/(x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(c\,{\left(a+b\,x\right)}^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x)^p)/(d + e*x),x)
```

```
[Out] int(log(c*(a + b*x)^p)/(d + e*x), x)
```

3.181 $\int \frac{\log(c(a+bx)^p)}{(d+ex)^2} dx$

Optimal. Leaf size=68

$$\frac{bp\log(a+bx)}{e(bd-ae)} - \frac{\log\left(c(a+bx)^p\right)}{e(d+ex)} - \frac{bp\log(d+ex)}{e(bd-ae)}$$

```
[Out] b*p*ln(b*x+a)/e/(-a*e+b*d)-ln(c*(b*x+a)^p)/e/(e*x+d)-b*p*ln(e*x+d)/e/(-a*e+
b*d)
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2442, 36, 31}

$$-\frac{\log\left(c(a+bx)^p\right)}{e(d+ex)} + \frac{bp\log(a+bx)}{e(bd-ae)} - \frac{bp\log(d+ex)}{e(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x)^p]/(d + e*x)^2,x]
```

```
[Out] (b*p*Log[a + b*x])/(e*(b*d - a*e)) - Log[c*(a + b*x)^p]/(e*(d + e*x)) - (b*
p*Log[d + e*x])/(e*(b*d - a*e))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int \frac{\log\left(c(a+bx)^p\right)}{(d+ex)^2}\,dx = -\frac{\log\left(c(a+bx)^p\right)}{e(d+ex)} + \frac{(bp)\int \frac{1}{(a+bx)(d+ex)}\,dx}{e}$$
$$= -\frac{\log\left(c(a+bx)^p\right)}{e(d+ex)} - \frac{(bp)\int \frac{1}{d+ex}\,dx}{bd-ae} + \frac{(b^2p)\int \frac{1}{a+bx}\,dx}{e(bd-ae)}$$
$$= \frac{bp\log(a+bx)}{e(bd-ae)} - \frac{\log\left(c(a+bx)^p\right)}{e(d+ex)} - \frac{bp\log(d+ex)}{e(bd-ae)}$$

Mathematica [A]
time = 0.04, size = 52, normalized size = 0.76

$$\frac{-\frac{\log(c(a+bx)^p)}{d+ex} + \frac{bp(\log(a+bx)-\log(d+ex))}{bd-ae}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x)^p]/(d + e*x)^2,x]
```

```
[Out] (-(Log[c*(a + b*x)^p]/(d + e*x)) + (b*p*(Log[a + b*x] - Log[d + e*x]))/(b*d
 - a*e))/e
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.54, size = 329, normalized size = 4.84

method	result
risch	$-\frac{\ln((bx+a)^p)}{e(ex+d)} - \frac{i\pi ae\,\mathrm{csgn}(i(bx+a)^p)\mathrm{csgn}(ic(bx+a)^p)^2 - i\pi ae\,\mathrm{csgn}(i(bx+a)^p)\mathrm{csgn}(ic(bx+a)^p)\mathrm{csgn}(ic) - i\pi ae\mathrm{csgn}(ic(bx+a)^p)^3 + i\pi a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x+a)^p)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/e/(e*x+d)*ln((b*x+a)^p)-1/2*(I*Pi*a*e*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)
^p)^2-I*Pi*a*e*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)-I*Pi*a*e*csg
n(I*c*(b*x+a)^p)^3+I*Pi*a*e*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)-I*Pi*b*d*csgn(I
*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2+I*Pi*b*d*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+
a)^p)*csgn(I*c)+I*Pi*b*d*csgn(I*c*(b*x+a)^p)^3-I*Pi*b*d*csgn(I*c*(b*x+a)^p)
^2*csgn(I*c)+2*ln(b*x+a)*b*e*p*x-2*ln(-e*x-d)*b*e*p*x+2*ln(b*x+a)*b*d*p-2*l
n(-e*x-d)*b*d*p+2*ln(c)*a*e-2*d*b*ln(c))/(e*x+d)/e/(a*e-b*d)
```

Maxima [A]
time = 0.36, size = 67, normalized size = 0.99

$$bp\left(\frac{\log(bx+a)}{bd-ae} - \frac{\log(xe+d)}{bd-ae}\right)e^{(-1)} - \frac{e^{(-1)}\log\left((bx+a)^p c\right)}{xe+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] b*p*(log(b*x + a)/(b*d - a*e) - log(x*e + d)/(b*d - a*e))*e^(-1) - e^(-1)*l
og((b*x + a)^p*c)/(x*e + d)
```

Fricas [A]
time = 0.43, size = 80, normalized size = 1.18

$$\frac{(bpx+ap)e\log(bx+a)-(bpxe+bdp)\log(xe+d)-(bd-ae)\log(c)}{bd^2e-axe^3+(bdx-ad)e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] ((b*p*x + a*p)*e*log(b*x + a) - (b*p*x*e + b*d*p)*log(x*e + d) - (b*d - a*e
)*log(c))/(b*d^2*e - a*x*e^3 + (b*d*x - a*d)*e^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x+a)**p)/(e*x+d)**2,x)
```

```
[Out] Exception raised: NotImplementedError >> no valid subset found
```

Giac [A]
time = 3.24, size = 91, normalized size = 1.34

$$\frac{bpxe\log(bx+a)-bpxe\log(xe+d)+ape\log(bx+a)-bdp\log(xe+d)-bd\log(c)+ae\log(c)}{bdxe^2+bd^2e-axe^3-ade^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] (b*p*x*e*log(b*x + a) - b*p*x*e*log(x*e + d) + a*p*e*log(b*x + a) - b*d*p*l
og(x*e + d) - b*d*log(c) + a*e*log(c))/(b*d*x*e^2 + b*d^2*e - a*x*e^3 - a*d
*e^2)
```

Mupad [B]
time = 1.07, size = 70, normalized size = 1.03

$$-\frac{\ln\left(c\,{\left(a+b\,x\right)}^p\right)}{e\,\left(d+e\,x\right)}+\frac{b\,p\,\mathrm{atan}\left(\frac{a\,e\,1{}\mathrm{i}+b\,d\,1{}\mathrm{i}+b\,e\,x\,2{}\mathrm{i}}{a\,e-b\,d}\right)\,2{}\mathrm{i}}{a\,e^2-b\,d\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x)^p)/(d + e*x)^2,x)
```

```
[Out] (b*p*atan((a*e*1i + b*d*1i + b*e*x*2i)/(a*e - b*d))*2i)/(a*e^2 - b*d*e) - l
og(c*(a + b*x)^p)/(e*(d + e*x))
```

3.182 $\int \frac{\log(c(a+bx)^p)}{(d+ex)^3} \, dx$

Optimal. Leaf size=105

$$\frac{bp}{2e(bd-ae)(d+ex)}+\frac{b^2p\log(a+bx)}{2e(bd-ae)^2}-\frac{\log\left(c(a+bx)^p\right)}{2e(d+ex)^2}-\frac{b^2p\log(d+ex)}{2e(bd-ae)^2}$$

```
[Out] 1/2*b*p/e/(-a*e+b*d)/(e*x+d)+1/2*b^2*p*ln(b*x+a)/e/(-a*e+b*d)^2-1/2*ln(c*(b
*x+a)^p)/e/(e*x+d)^2-1/2*b^2*p*ln(e*x+d)/e/(-a*e+b*d)^2
```

Rubi [A]
time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2442, 46}

$$\frac{b^2p\log(a+bx)}{2e(bd-ae)^2}-\frac{b^2p\log(d+ex)}{2e(bd-ae)^2}-\frac{\log\left(c(a+bx)^p\right)}{2e(d+ex)^2}+\frac{bp}{2e(d+ex)(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x)^p]/(d + e*x)^3,x]
```

```
[Out] (b*p)/(2*e*(b*d - a*e)*(d + e*x)) + (b^2*p*Log[a + b*x])/(2*e*(b*d - a*e)^2
) - Log[c*(a + b*x)^p]/(2*e*(d + e*x)^2) - (b^2*p*Log[d + e*x])/(2*e*(b*d -
 a*e)^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int \frac{\log (c(a+bx)^p)}{(d+ex)^3}\,dx = -\frac{\log (c(a+bx)^p)}{2e(d+ex)^2} + \frac{(bp)\int \frac{1}{(a+bx)(d+ex)^2}\,dx}{2e}$$
$$= -\frac{\log (c(a+bx)^p)}{2e(d+ex)^2} + \frac{(bp)\int \left(\frac{b^2}{(bd-ae)^2(a+bx)} - \frac{e}{(bd-ae)(d+ex)^2} - \frac{be}{(bd-ae)^2(d+ex)}\right)\,dx}{2e}$$
$$= \frac{bp}{2e(bd-ae)(d+ex)} + \frac{b^2p\log(a+bx)}{2e(bd-ae)^2} - \frac{\log (c(a+bx)^p)}{2e(d+ex)^2} - \frac{b^2p\log(d+ex)}{2e(bd-ae)^2}$$

Mathematica [A]
time = 0.08, size = 80, normalized size = 0.76

$$\frac{-\log (c(a+bx)^p) + \frac{bp(d+ex)(bd-ae+b(d+ex)\log(a+bx)-b(d+ex)\log(d+ex))}{(bd-ae)^2}}{2e(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x)^p]/(d + e*x)^3,x]
```

```
[Out] (-Log[c*(a + b*x)^p] + (b*p*(d + e*x)*(b*d - a*e + b*(d + e*x)*Log[a + b*x]
 - b*(d + e*x)*Log[d + e*x]))/(b*d - a*e)^2)/(2*e*(d + e*x)^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.65, size = 582, normalized size = 5.54

method	result
risch	$-\frac{\ln((bx+a)^p)}{2e(ex+d)^2} - \frac{2ab\,e^2px-2b^2depx+i\pi\,b^2d^2\mathrm{csgn}(i(bx+a)^p)\mathrm{csgn}(ic(bx+a)^p)^2-2i\pi abde\,\mathrm{csgn}(i(bx+a)^p)\mathrm{csgn}(ic(bx+a)^p)^2+i\pi\,b^2d}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x+a)^p)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/e/(e*x+d)^2*ln((b*x+a)^p)-1/4*(2*a*b*e^2*p*x-2*b^2*d*e*p*x+I*Pi*b^2*d^
2*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)-I*Pi*b^2*d^2*csgn(I*(b*x+a)^p)*csgn(I*c*(
b*x+a)^p)*csgn(I*c)-I*Pi*b^2*d^2*csgn(I*c*(b*x+a)^p)^3+2*b^2*d^2*ln(c)-2*b^
2*d^2*p-2*I*Pi*a*b*d*e*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)-2*I*Pi*a*b*d*e*csgn(
I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2+2*a*b*d*p*e+2*ln(e*x+d)*b^2*d^2*p-2*ln(-
b*x-a)*b^2*d^2*p+2*I*Pi*a*b*d*e*csgn(I*c*(b*x+a)^p)^3+2*ln(c)*a^2*e^2+I*Pi*
a^2*e^2*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2+I*Pi*a^2*e^2*csgn(I*c*(b*x+
a)^p)^2*csgn(I*c)-I*Pi*a^2*e^2*csgn(I*c*(b*x+a)^p)^3+2*ln(e*x+d)*b^2*e^2*p*
x^2-2*ln(-b*x-a)*b^2*e^2*p*x^2-4*ln(c)*a*b*d*e+I*Pi*b^2*d^2*csgn(I*(b*x+a)^
p)*csgn(I*c*(b*x+a)^p)^2+2*I*Pi*a*b*d*e*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^
p)*csgn(I*c)-I*Pi*a^2*e^2*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)+4
*ln(e*x+d)*b^2*d*e*p*x-4*ln(-b*x-a)*b^2*d*e*p*x)/(e*x+d)^2/(a*e-b*d)^2/e
```

Maxima [A]
time = 0.34, size = 121, normalized size = 1.15

$$\frac{1}{2}\,bp\left(\frac{b\log\left(bx+a\right)}{b^2d^2-2\,abde+a^2e^2}-\frac{b\log\left(xe+d\right)}{b^2d^2-2\,abde+a^2e^2}+\frac{1}{bd^2-ade+(bde-ae^2)x}\right)e^{(-1)}-\frac{e^{(-1)}\log\left((bx+a)^pc\right)}{2\,(xe+d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*b*p*(b*log(b*x + a)/(b^2*d^2 - 2*a*b*d*e + a^2*e^2) - b*log(x*e + d)/(b
^2*d^2 - 2*a*b*d*e + a^2*e^2) + 1/(b*d^2 - a*d*e + (b*d*e - a*e^2)*x))*e^(-
1) - 1/2*e^(-1)*log((b*x + a)^p*c)/(x*e + d)^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(99) = 198.
time = 0.41, size = 231, normalized size = 2.20

$$\frac{b^2d^2p-abpxe^2+(b^2dpx-abdp)e+((b^2px^2-a^2p)e^2+2\,(b^2dpx+abdp)e)\log\left(bx+a\right)-(b^2px^2e^2+2\,b^2dpxe+b^2d^2p)\log\left(xe+d\right)-(b^2d^2-2\,abde+a^2e^2)\log\left(c\right)}{2\,(b^2d^4e+a^2x^2e^5-2\,(abdx^2-a^2dx)e^4+(b^2d^2x^2-4\,abd^2x+a^2d^2)e^3+2\,(b^2d^3x-abd^3)e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*p - a*b*p*x*e^2 + (b^2*d*p*x - a*b*d*p)*e + ((b^2*p*x^2 - a^2*
p)*e^2 + 2*(b^2*d*p*x + a*b*d*p)*e)*log(b*x + a) - (b^2*p*x^2*e^2 + 2*b^2*d
*p*x*e + b^2*d^2*p)*log(x*e + d) - (b^2*d^2 - 2*a*b*d*e + a^2*e^2)*log(c))/
(b^2*d^4*e + a^2*x^2*e^5 - 2*(a*b*d*x^2 - a^2*d*x)*e^4 + (b^2*d^2*x^2 - 4*a
*b*d^2*x + a^2*d^2)*e^3 + 2*(b^2*d^3*x - a*b*d^3)*e^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x+a)**p)/(e*x+d)**3,x)
```

```
[Out] Exception raised: NotImplementedError >> no valid subset found
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(99) = 198.
time = 5.38, size = 266, normalized size = 2.53

$$\frac{b^2px^2e^2\log\left(bx+a\right)+2\,b^2dpxe\log\left(bx+a\right)-b^2px^2e^2\log\left(xe+d\right)-2\,b^2dpxe\log\left(xe+d\right)+b^2dpxe+2\,abdpe\log\left(bx+a\right)-b^2d^2p\log\left(xe+d\right)+b^2d^2p-abpxe^2-abdpe-a^2pe^2\log\left(bx+a\right)-b^2d^2\log\left(c\right)+2\,abde\log\left(c\right)-a^2e^2\log\left(c\right)}{2\,(b^2d^2x^2e^3+2\,b^2d^3xe^2+b^2d^4e-2\,abdx^2e^4-4\,abd^2xe^3-2\,abd^3e^2+a^2x^2e^5+2\,a^2dxe^4+a^2d^2e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*p*x^2*e^2*log(b*x + a) + 2*b^2*d*p*x*e*log(b*x + a) - b^2*p*x^2*e^
2*log(x*e + d) - 2*b^2*d*p*x*e*log(x*e + d) + b^2*d*p*x*e + 2*a*b*d*p*e*log
(b*x + a) - b^2*d^2*p*log(x*e + d) + b^2*d^2*p - a*b*p*x*e^2 - a*b*d*p*e -
a^2*p*e^2*log(b*x + a) - b^2*d^2*log(c) + 2*a*b*d*e*log(c) - a^2*e^2*log(c)
)/(b^2*d^2*x^2*e^3 + 2*b^2*d^3*x*e^2 + b^2*d^4*e - 2*a*b*d*x^2*e^4 - 4*a*b*
d^2*x*e^3 - 2*a*b*d^3*e^2 + a^2*x^2*e^5 + 2*a^2*d*x*e^4 + a^2*d^2*e^3)
```

Mupad [B]
time = 0.64, size = 96, normalized size = 0.91

$$-\frac{\ln\left(c\,{\left(a+b\,x\right)}^p\right)}{2\,e\,{\left(d+e\,x\right)}^2}-\frac{b\,p}{2\,e\,\left(a\,e-b\,d\right)\,\left(d+e\,x\right)}-\frac{b^2\,p\,\mathrm{atan}\left(\frac{a\,e\,1{}\mathrm{i}+b\,d\,1{}\mathrm{i}+b\,e\,x\,2{}\mathrm{i}}{a\,e-b\,d}\right)\,1{}\mathrm{i}}{e\,{\left(a\,e-b\,d\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x)^p)/(d + e*x)^3,x)
```

```
[Out] - log(c*(a + b*x)^p)/(2*e*(d + e*x)^2) - (b*p)/(2*e*(a*e - b*d)*(d + e*x))
- (b^2*p*atan((a*e*1i + b*d*1i + b*e*x*2i)/(a*e - b*d))*1i)/(e*(a*e - b*d)^
2)
```

3.183 $\int \frac{\log(c(a+bx)^p)}{(d+ex)^4} dx$

Optimal. Leaf size=133

$$\frac{bp}{6e(bd-ae)(d+ex)^2}+\frac{b^2p}{3e(bd-ae)^2(d+ex)}+\frac{b^3p\log(a+bx)}{3e(bd-ae)^3}-\frac{\log\left(c(a+bx)^p\right)}{3e(d+ex)^3}-\frac{b^3p\log(d+ex)}{3e(bd-ae)^3}$$

```
[Out] 1/6*b*p/e/(-a*e+b*d)/(e*x+d)^2+1/3*b^2*p/e/(-a*e+b*d)^2/(e*x+d)+1/3*b^3*p*l
n(b*x+a)/e/(-a*e+b*d)^3-1/3*ln(c*(b*x+a)^p)/e/(e*x+d)^3-1/3*b^3*p*ln(e*x+d)
/e/(-a*e+b*d)^3
```

Rubi [A]
time = 0.06, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2442, 46}

$$\frac{b^3p\log(a+bx)}{3e(bd-ae)^3}-\frac{b^3p\log(d+ex)}{3e(bd-ae)^3}+\frac{b^2p}{3e(d+ex)(bd-ae)^2}-\frac{\log\left(c(a+bx)^p\right)}{3e(d+ex)^3}+\frac{bp}{6e(d+ex)^2(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x)^p]/(d + e*x)^4,x]
```

```
[Out] (b*p)/(6*e*(b*d - a*e)*(d + e*x)^2) + (b^2*p)/(3*e*(b*d - a*e)^2*(d + e*x))
 + (b^3*p*Log[a + b*x])/(3*e*(b*d - a*e)^3) - Log[c*(a + b*x)^p]/(3*e*(d +
e*x)^3) - (b^3*p*Log[d + e*x])/(3*e*(b*d - a*e)^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int \frac{\log \left(c(a+bx)^p\right)}{(d+ex)^4} \, dx = -\frac{\log \left(c(a+bx)^p\right)}{3e(d+ex)^3} + \frac{(bp)\int \frac{1}{(a+bx)(d+ex)^3} \, dx}{3e}$$

$$= -\frac{\log \left(c(a+bx)^p\right)}{3e(d+ex)^3} + \frac{(bp)\int \left(\frac{b^3}{(bd-ae)^3(a+bx)} - \frac{e}{(bd-ae)(d+ex)^3} - \frac{be}{(bd-ae)^2(d+ex)^2} - \frac{b^2}{(bd-ae)\ldots}\right.}{3e}$$

$$= \frac{bp}{6e(bd-ae)(d+ex)^2} + \frac{b^2p}{3e(bd-ae)^2(d+ex)} + \frac{b^3p\log(a+bx)}{3e(bd-ae)^3} - \frac{\log \left(c(a+bx)^p\right)}{3e(d+ex)^3}$$

Mathematica [A]
time = 0.11, size = 105, normalized size = 0.79

$$\frac{-2\log \left(c(a+bx)^p\right) + \frac{bp(d+ex)\left((bd-ae)(3bd-ae+2bex)+2b^2(d+ex)^2\log(a+bx)-2b^2(d+ex)^2\log(d+ex)\right)}{(bd-ae)^3}}{6e(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x)^p]/(d + e*x)^4,x]
```

```
[Out] (-2*Log[c*(a + b*x)^p] + (b*p*(d + e*x)*((b*d - a*e)*(3*b*d - a*e + 2*b*e*x
) + 2*b^2*(d + e*x)^2*Log[a + b*x] - 2*b^2*(d + e*x)^2*Log[d + e*x]))/(b*d
- a*e)^3)/(6*e*(d + e*x)^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.72, size = 873, normalized size = 6.56

method	result
risch	$-\frac{\ln((bx+a)^p)}{3e(ex+d)^3} + \frac{-i\pi a^3e^3\text{csgn}(ic(bx+a)^p)^2\text{csgn}(ic)+i\pi b^3d^3\text{csgn}(ic(bx+a)^p)^2\text{csgn}(ic)+2ab^2e^3px^2-2b^3de^2px^2-a^2be^3px-5b^3d\ldots}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x+a)^p)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/e/(e*x+d)^3*ln((b*x+a)^p)+1/6*(-3*I*Pi*a^2*b*d*e^2*csgn(I*(b*x+a)^p)*c
sgn(I*c*(b*x+a)^p)*csgn(I*c)+3*I*Pi*a*b^2*d^2*e*csgn(I*(b*x+a)^p)*csgn(I*c*
(b*x+a)^p)*csgn(I*c)+3*I*Pi*a^2*b*d*e^2*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^
p)^2+2*a*b^2*e^3*p*x^2-2*b^3*d*e^2*p*x^2-a^2*b*e^3*p*x-5*b^3*d^2*e*p*x-a^2*
b*d*p*e^2+4*a*b^2*d^2*p*e-3*b^3*d^3*p-3*I*Pi*a*b^2*d^2*e*csgn(I*c*(b*x+a)^p
)^2*csgn(I*c)+3*I*Pi*a^2*b*d*e^2*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)-3*I*Pi*a*b
^2*d^2*e*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2-6*ln(b*x+a)*b^3*d*e^2*p*x^
2+6*ln(-e*x-d)*b^3*d*e^2*p*x^2-6*ln(b*x+a)*b^3*d^2*e*p*x+6*ln(-e*x-d)*b^3*d
^2*e*p*x-I*Pi*a^3*e^3*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)+2*ln(-e*x-d)*b^3*d^3*
p+I*Pi*b^3*d^3*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)-I*Pi*a^3*e^3*csgn(I*(b*x+a)^
```

```
p)*csgn(I*c*(b*x+a)^p)^2-2*ln(b*x+a)*b^3*d^3*p+3*I*Pi*a*b^2*d^2*e*csgn(I*c*
(b*x+a)^p)^3-I*Pi*b^3*d^3*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)+6
*a*b^2*d*e^2*p*x+2*ln(c)*b^3*d^3-2*ln(c)*a^3*e^3+I*Pi*a^3*e^3*csgn(I*(b*x+a
)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)-3*I*Pi*a^2*b*d*e^2*csgn(I*c*(b*x+a)^p)^3
+I*Pi*b^3*d^3*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2-6*ln(c)*a*b^2*d^2*e-2
*ln(b*x+a)*b^3*e^3*p*x^3+2*ln(-e*x-d)*b^3*e^3*p*x^3+6*ln(c)*a^2*b*d*e^2-I*P
i*b^3*d^3*csgn(I*c*(b*x+a)^p)^3+I*Pi*a^3*e^3*csgn(I*c*(b*x+a)^p)^3)/(e*x+d)
^3/(a^2*e^2-2*a*b*d*e+b^2*d^2)/(a*e-b*d)/e
```

Maxima [A]
time = 0.36, size = 228, normalized size = 1.71

$$\frac{1}{6}\left(\frac{2\,b^2\log\left(bx+a\right)}{b^3d^3-3\,ab^2d^2e+3\,a^2bde^2-a^3e^3}-\frac{2\,b^2\log\left(xe+d\right)}{b^3d^3-3\,ab^2d^2e+3\,a^2bde^2-a^3e^3}+\frac{2\,bxe+3\,bd-ae}{b^2d^4-2\,abd^3e+a^2d^2e^2+(b^2d^2e^2-2\,abde^3+a^2e^4)x^2+2\,(b^2d^3e-2\,abd^2e^2+a^2de^3)x}\right)bpe^{(-1)}-\frac{e^{(-1)}\log\left((bx+a)^pc\right)}{3\,(xe+d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^2*log(b*x + a)/(b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2 - a^3*e^3)
 - 2*b^2*log(x*e + d)/(b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2 - a^3*e^3) +
 (2*b*x*e + 3*b*d - a*e)/(b^2*d^4 - 2*a*b*d^3*e + a^2*d^2*e^2 + (b^2*d^2*e^
2 - 2*a*b*d*e^3 + a^2*e^4)*x^2 + 2*(b^2*d^3*e - 2*a*b*d^2*e^2 + a^2*d*e^3)*
x))*b*p*e^(-1) - 1/3*e^(-1)*log((b*x + a)^p*c)/(x*e + d)^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(126) = 252$.
time = 0.40, size = 431, normalized size = 3.24

$$\frac{3\,b^3d^3p-(2\,ab^2px^2-a^2bpx)e^3+(2\,b^3dpx^2-6\,ab^2dpx+a^2bdp)e^2+(5\,b^3d^2px-4\,ab^2d^2p)e+2\,((b^3px^3+a^3p)e^3+3\,(b^3dpx^2-a^2bdp)e^2+3\,(b^3d^2px+ab^2d^2p)e)\log\left(bx+a\right)-2\,(b^3px^3e^3+3\,b^3dpx^2e^2+3\,b^3d^2pxe+b^3d^3p)\log\left(xe+d\right)-2\,(b^3d^3-3\,ab^2d^2e+3\,a^2bde^2-a^3e^3)\log\left(c\right)}{6\,(b^3d^6e-a^3x^3e^7+3\,(a^2bdx^3-a^3dx^2)e^6-3\,(ab^2d^2x^3-3\,a^2bd^2x^2+a^3d^2x)e^5+(b^3d^3x^3-9\,ab^2d^3x^2+9\,a^2bd^3x-a^3d^3)e^4+3\,(b^3d^4x^2-3\,ab^2d^4x+a^2bd^4)e^3+3\,(b^3d^5x-ab^2d^5)e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^3*d^3*p - (2*a*b^2*p*x^2 - a^2*b*p*x)*e^3 + (2*b^3*d*p*x^2 - 6*a*b
^2*d*p*x + a^2*b*d*p)*e^2 + (5*b^3*d^2*p*x - 4*a*b^2*d^2*p)*e + 2*((b^3*p*x
^3 + a^3*p)*e^3 + 3*(b^3*d*p*x^2 - a^2*b*d*p)*e^2 + 3*(b^3*d^2*p*x + a*b^2*
d^2*p)*e)*log(b*x + a) - 2*(b^3*p*x^3*e^3 + 3*b^3*d*p*x^2*e^2 + 3*b^3*d^2*p
*x*e + b^3*d^3*p)*log(x*e + d) - 2*(b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2
 - a^3*e^3)*log(c))/(b^3*d^6*e - a^3*x^3*e^7 + 3*(a^2*b*d*x^3 - a^3*d*x^2)*
e^6 - 3*(a*b^2*d^2*x^3 - 3*a^2*b*d^2*x^2 + a^3*d^2*x)*e^5 + (b^3*d^3*x^3 -
9*a*b^2*d^3*x^2 + 9*a^2*b*d^3*x - a^3*d^3)*e^4 + 3*(b^3*d^4*x^2 - 3*a*b^2*d
^4*x + a^2*b*d^4)*e^3 + 3*(b^3*d^5*x - a*b^2*d^5)*e^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 4571 vs. $2(109) = 218$.
time = 20.01, size = 4571, normalized size = 34.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(c*(b*x+a)**p)/(e*x+d)**4,x)

```
[Out] Piecewise((-p/(9*d**3*e + 27*d**2*e**2*x + 27*d*e**3*x**2 + 9*e**4*x**3) -
3*log(c*(b*d/e + b*x)**p)/(9*d**3*e + 27*d**2*e**2*x + 27*d*e**3*x**2 + 9*e
**4*x**3), Eq(a, b*d/e)), ((a*log(c*(a + b*x)**p)/b - p*x + x*log(c*(a + b*
x)**p))/d**4, Eq(e, 0)), (-2*a**3*e**3*log(c*(a + b*x)**p)/(6*a**3*d**3*e**
4 + 18*a**3*d**2*e**5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*x**3 - 18*a**2*
b*d**4*e**3 - 54*a**2*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x**2 - 18*a**2*b*
d*e**6*x**3 + 18*a*b**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x + 54*a*b**2*d**3*
e**4*x**2 + 18*a*b**2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b**3*d**5*e**2*x
- 18*b**3*d**4*e**3*x**2 - 6*b**3*d**3*e**4*x**3) - a**2*b*d*e**2*p/(6*a**3
*d**3*e**4 + 18*a**3*d**2*e**5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*x**3 -
 18*a**2*b*d**4*e**3 - 54*a**2*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x**2 - 1
8*a**2*b*d*e**6*x**3 + 18*a*b**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x + 54*a*b
**2*d**3*e**4*x**2 + 18*a*b**2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b**3*d**
5*e**2*x - 18*b**3*d**4*e**3*x**2 - 6*b**3*d**3*e**4*x**3) + 6*a**2*b*d*e**
2*log(c*(a + b*x)**p)/(6*a**3*d**3*e**4 + 18*a**3*d**2*e**5*x + 18*a**3*d*e
**6*x**2 + 6*a**3*e**7*x**3 - 18*a**2*b*d**4*e**3 - 54*a**2*b*d**3*e**4*x -
 54*a**2*b*d**2*e**5*x**2 - 18*a**2*b*d*e**6*x**3 + 18*a*b**2*d**5*e**2 + 5
4*a*b**2*d**4*e**3*x + 54*a*b**2*d**3*e**4*x**2 + 18*a*b**2*d**2*e**5*x**3
- 6*b**3*d**6*e - 18*b**3*d**5*e**2*x - 18*b**3*d**4*e**3*x**2 - 6*b**3*d**
3*e**4*x**3) - a**2*b*e**3*p*x/(6*a**3*d**3*e**4 + 18*a**3*d**2*e**5*x + 18
*a**3*d*e**6*x**2 + 6*a**3*e**7*x**3 - 18*a**2*b*d**4*e**3 - 54*a**2*b*d**3
*e**4*x - 54*a**2*b*d**2*e**5*x**2 - 18*a**2*b*d*e**6*x**3 + 18*a*b**2*d**5
*e**2 + 54*a*b**2*d**4*e**3*x + 54*a*b**2*d**3*e**4*x**2 + 18*a*b**2*d**2*e
**5*x**3 - 6*b**3*d**6*e - 18*b**3*d**5*e**2*x - 18*b**3*d**4*e**3*x**2 - 6
*b**3*d**3*e**4*x**3) + 4*a*b**2*d**2*e*p/(6*a**3*d**3*e**4 + 18*a**3*d**2*
e**5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*x**3 - 18*a**2*b*d**4*e**3 - 54*
a**2*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x**2 - 18*a**2*b*d*e**6*x**3 + 18*
a*b**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x + 54*a*b**2*d**3*e**4*x**2 + 18*a*
b**2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b**3*d**5*e**2*x - 18*b**3*d**4*e*
*3*x**2 - 6*b**3*d**3*e**4*x**3) - 6*a*b**2*d**2*e*log(c*(a + b*x)**p)/(6*a
**3*d**3*e**4 + 18*a**3*d**2*e**5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*x**
3 - 18*a**2*b*d**4*e**3 - 54*a**2*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x**2
- 18*a**2*b*d*e**6*x**3 + 18*a*b**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x + 54*
a*b**2*d**3*e**4*x**2 + 18*a*b**2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b**3*
d**5*e**2*x - 18*b**3*d**4*e**3*x**2 - 6*b**3*d**3*e**4*x**3) + 6*a*b**2*d*
e**2*p*x/(6*a**3*d**3*e**4 + 18*a**3*d**2*e**5*x + 18*a**3*d*e**6*x**2 + 6*
a**3*e**7*x**3 - 18*a**2*b*d**4*e**3 - 54*a**2*b*d**3*e**4*x - 54*a**2*b*d*
*2*e**5*x**2 - 18*a**2*b*d*e**6*x**3 + 18*a*b**2*d**5*e**2 + 54*a*b**2*d**4
*e**3*x + 54*a*b**2*d**3*e**4*x**2 + 18*a*b**2*d**2*e**5*x**3 - 6*b**3*d**6
*e - 18*b**3*d**5*e**2*x - 18*b**3*d**4*e**3*x**2 - 6*b**3*d**3*e**4*x**3)
+ 2*a*b**2*e**3*p*x**2/(6*a**3*d**3*e**4 + 18*a**3*d**2*e**5*x + 18*a**3*d*
```

```
e**6*x**2 + 6*a**3*e**7*x**3 - 18*a**2*b*d**4*e**3 - 54*a**2*b*d**3*e**4*x
- 54*a**2*b*d**2*e**5*x**2 - 18*a**2*b*d*e**6*x**3 + 18*a*b**2*d**5*e**2 +
54*a*b**2*d**4*e**3*x + 54*a*b**2*d**3*e**4*x**2 + 18*a*b**2*d**2*e**5*x**3
 - 6*b**3*d**6*e - 18*b**3*d**5*e**2*x - 18*b**3*d**4*e**3*x**2 - 6*b**3*d*
*3*e**4*x**3) + 2*b**3*d**3*p*log(d/e + x)/(6*a**3*d**3*e**4 + 18*a**3*d**2
*e**5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*x**3 - 18*a**2*b*d**4*e**3 - 54
*a**2*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x**2 - 18*a**2*b*d*e**6*x**3 + 18
*a*b**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x + 54*a*b**2*d**3*e**4*x**2 + 18*a
*b**2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b**3*d**5*e**2*x - 18*b**3*d**4*e
**3*x**2 - 6*b**3*d**3*e**4*x**3) - 3*b**3*d**3*p/(6*a**3*d**3*e**4 + 18*a*
*3*d**2*e**5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*x**3 - 18*a**2*b*d**4*e*
*3 - 54*a**2*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x**2 - 18*a**2*b*d*e**6*x*
*3 + 18*a*b**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x + 54*a*b**2*d**3*e**4*x**2
 + 18*a*b**2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b**3*d**5*e**2*x - 18*b**3
*d**4*e**3*x**2 - 6*b**3*d**3*e**4*x**3) + 6*b**3*d**2*e*p*x*log(d/e + x)/(
6*a**3*d**3*e**4 + 18*a**3*d**2*e**5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*
x**3 - 18*a**2*b*d**4*e**3 - 54*a**2*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x*
*2 - 18*a**2*b*d*e**6*x**3 + 18*a*b**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x +
54*a*b**2*d**3*e**4*x**2 + 18*a*b**2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b*
*3*d**5*e**2*x - 18*b**3*d**4*e**3*x**2 - 6*b**3*d**3*e**4*x**3) - 5*b**3*d
**2*e*p*x/(6*a**3*d**3*e**4 + 18*a**3*d**2*e**5*x + 18*a**3*d*e**6*x**2 + 6
*a**3*e**7*x**3 - 18*a**2*b*d**4*e**3 - 54*a**2*b*d**3*e**4*x - 54*a**2*b*d
**2*e**5*x**2 - 18*a**2*b*d*e**6*x**3 + 18*a*b**2*d**5*e**2 + 54*a*b**2*d**
4*e**3*x + 54*a*b**2*d**3*e**4*x**2 + 18*a*b**2*d**2*e**5*x**3 - 6*b**3*d**
6*e - 18*b**3*d**5*e**2*x - 18*b**3*d**4*e**3*x**2 - 6*b**3*d**3*e**4*x**3)
 - 6*b**3*d**2*e*x*log(c*(a + b*x)**p)/(6*a**3*...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(126) = 252$.
time = 4.17, size = 495, normalized size = 3.72

$$\frac{2\,b^3px^3e^3\log\left(bx+a\right)+6\,b^3dpx^2e^2\log\left(bx+a\right)+6\,b^3d^2pxe\log\left(bx+a\right)-2\,b^3px^3e^3\log\left(xe+d\right)-6\,b^3dpx^2e^2\log\left(xe+d\right)-6\,b^3d^2pxe\log\left(xe+d\right)+2\,b^3dpx^2e^2+5\,b^3d^2pxe+6\,ab^2d^2pe\log\left(bx+a\right)-2\,b^3d^3p\log\left(xe+d\right)+3\,b^3d^3p-2\,ab^2px^2e^3-6\,ab^2dpxe^2-4\,ab^2d^2pe-6\,a^2bdpe^2\log\left(bx+a\right)-2\,b^3d^3\log\left(c\right)+6\,ab^2d^2e\log\left(c\right)+a^2bpxe^3+a^2bdpe^2+2\,a^3pe^3\log\left(bx+a\right)-6\,a^2bde^2\log\left(c\right)+2\,a^3e^3\log\left(c\right)}{6\,\left(b^3d^3x^3e^4+3\,b^3d^4x^2e^3+3\,b^3d^5xe^2+b^3d^6e-3\,ab^2d^2x^3e^5-9\,ab^2d^3x^2e^4-9\,ab^2d^4xe^3-3\,ab^2d^5e^2+3\,a^2bdx^3e^6+9\,a^2bd^2x^2e^5+9\,a^2bd^3xe^4+3\,a^2bd^4e^3-a^3x^3e^7-3\,a^3dx^2e^6-3\,a^3d^2xe^5-a^3d^3e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] 1/6*(2*b^3*p*x^3*e^3*log(b*x + a) + 6*b^3*d*p*x^2*e^2*log(b*x + a) + 6*b^3*
d^2*p*x*e*log(b*x + a) - 2*b^3*p*x^3*e^3*log(x*e + d) - 6*b^3*d*p*x^2*e^2*l
og(x*e + d) - 6*b^3*d^2*p*x*e*log(x*e + d) + 2*b^3*d*p*x^2*e^2 + 5*b^3*d^2*
p*x*e + 6*a*b^2*d^2*p*e*log(b*x + a) - 2*b^3*d^3*p*log(x*e + d) + 3*b^3*d^3
*p - 2*a*b^2*p*x^2*e^3 - 6*a*b^2*d*p*x*e^2 - 4*a*b^2*d^2*p*e - 6*a^2*b*d*p*
e^2*log(b*x + a) - 2*b^3*d^3*log(c) + 6*a*b^2*d^2*e*log(c) + a^2*b*p*x*e^3
+ a^2*b*d*p*e^2 + 2*a^3*p*e^3*log(b*x + a) - 6*a^2*b*d*e^2*log(c) + 2*a^3*e
^3*log(c))/(b^3*d^3*x^3*e^4 + 3*b^3*d^4*x^2*e^3 + 3*b^3*d^5*x*e^2 + b^3*d^6
*e - 3*a*b^2*d^2*x^3*e^5 - 9*a*b^2*d^3*x^2*e^4 - 9*a*b^2*d^4*x*e^3 - 3*a*b^
```

```
2*d^5*e^2 + 3*a^2*b*d*x^3*e^6 + 9*a^2*b*d^2*x^2*e^5 + 9*a^2*b*d^3*x*e^4 + 3
*a^2*b*d^4*e^3 - a^3*x^3*e^7 - 3*a^3*d*x^2*e^6 - 3*a^3*d^2*x*e^5 - a^3*d^3*
e^4)
```

Mupad [B]
time = 0.76, size = 145, normalized size = 1.09

$$\frac{b^2\,p\,x}{3\,{\left(a\,e-b\,d\right)}^2\,{\left(d+e\,x\right)}^2}-\frac{\ln\left(c\,{\left(a+b\,x\right)}^p\right)}{3\,e\,{\left(d+e\,x\right)}^3}-\frac{a\,b\,p}{6\,{\left(a\,e-b\,d\right)}^2\,{\left(d+e\,x\right)}^2}+\frac{b^2\,d\,p}{2\,e\,{\left(a\,e-b\,d\right)}^2\,{\left(d+e\,x\right)}^2}+\frac{b^3\,p\,\mathrm{atan}\left(\frac{a\,e\,1{}\mathrm{i}+b\,d\,1{}\mathrm{i}+b\,e\,x\,2{}\mathrm{i}}{a\,e-b\,d}\right)\,2{}\mathrm{i}}{3\,e\,{\left(a\,e-b\,d\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x)^p)/(d + e*x)^4,x)
```

```
[Out] (b^2*p*x)/(3*(a*e - b*d)^2*(d + e*x)^2) - log(c*(a + b*x)^p)/(3*e*(d + e*x)
^3) + (b^3*p*atan((a*e*1i + b*d*1i + b*e*x*2i)/(a*e - b*d))*2i)/(3*e*(a*e -
 b*d)^3) - (a*b*p)/(6*(a*e - b*d)^2*(d + e*x)^2) + (b^2*d*p)/(2*e*(a*e - b*
d)^2*(d + e*x)^2)
```

3.184 $\int (d+ex)^3 \log \left(c(a+bx^2)^p\right) \, dx$

Optimal. Leaf size=178

$$-\frac{2d(bd^2-ae^2)\,px}{b}-\frac{e(6bd^2-ae^2)\,px^2}{4b}-\frac{2}{3}de^2px^3-\frac{1}{8}e^3px^4+\frac{2\sqrt{a}\,d(bd^2-ae^2)\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}-\frac{(b^2d^4-6ab}{}$$

```
[Out] -2*d*(-a*e^2+b*d^2)*p*x/b-1/4*e*(-a*e^2+6*b*d^2)*p*x^2/b-2/3*d*e^2*p*x^3-1/
8*e^3*p*x^4-1/4*(a^2*e^4-6*a*b*d^2*e^2+b^2*d^4)*p*ln(b*x^2+a)/b^2/e+1/4*(e*
x+d)^4*ln(c*(b*x^2+a)^p)/e+2*d*(-a*e^2+b*d^2)*p*arctan(x*b^(1/2)/a^(1/2))*a
^(1/2)/b^(3/2)
```

Rubi [A]
time = 0.11, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2513, 815, 649, 211, 266}

$$-\frac{p(a^2e^4-6abd^2e^2+b^2d^4)\log(a+bx^2)}{4b^2e}+\frac{2\sqrt{a}\,dp\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bd^2-ae^2)}{b^{3/2}}+\frac{(d+ex)^4\log\left(c(a+bx^2)^p\right)}{4e}-\frac{epx^2(6bd^2-ae^2)}{4b}-\frac{2dpx(bd^2-ae^2)}{b}-\frac{2}{3}de^2px^3-\frac{1}{8}e^3px^4$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (-2*d*(b*d^2 - a*e^2)*p*x)/b - (e*(6*b*d^2 - a*e^2)*p*x^2)/(4*b) - (2*d*e^2
*p*x^3)/3 - (e^3*p*x^4)/8 + (2*Sqrt[a]*d*(b*d^2 - a*e^2)*p*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/b^(3/2) - ((b^2*d^4 - 6*a*b*d^2*e^2 + a^2*e^4)*p*Log[a + b*x^2
])/(4*b^2*e) + ((d + e*x)^4*Log[c*(a + b*x^2)^p])/(4*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\int (d+ex)^3 \log\left(c\left(a+bx^2\right)^p\right)\, dx = \frac{(d+ex)^4 \log\left(c\left(a+bx^2\right)^p\right)}{4e} - \frac{(bp)\int \frac{x(d+ex)^4}{a+bx^2}\, dx}{2e}$$

$$= \frac{(d+ex)^4 \log\left(c\left(a+bx^2\right)^p\right)}{4e} - \frac{(bp)\int \left(\frac{4de\left(bd^2-ae^2\right)}{b^2} + \frac{e^2\left(6bd^2-ae^2\right)x}{b^2} + \frac{4de^3x}{b}\right.}{}$$

$$= -\frac{2d\left(bd^2-ae^2\right)px}{b} - \frac{e\left(6bd^2-ae^2\right)px^2}{4b} - \frac{2}{3}de^2px^3 - \frac{1}{8}e^3px^4 + \frac{(d+ex}{}$$

$$= -\frac{2d\left(bd^2-ae^2\right)px}{b} - \frac{e\left(6bd^2-ae^2\right)px^2}{4b} - \frac{2}{3}de^2px^3 - \frac{1}{8}e^3px^4 + \frac{(d+ex}{}$$

$$= -\frac{2d\left(bd^2-ae^2\right)px}{b} - \frac{e\left(6bd^2-ae^2\right)px^2}{4b} - \frac{2}{3}de^2px^3 - \frac{1}{8}e^3px^4 + \frac{2\sqrt{a}\, d(}{}$$

Mathematica [A]
time = 0.54, size = 249, normalized size = 1.40

$$\frac{-6\left(b^2d^4+4\sqrt{-a}\, b^{3/2}d^3e-6abd^2e^2+4(-a)^{3/2}\sqrt{b}\, de^3+a^2e^4\right)p\log\left(\sqrt{-a}-\sqrt{b}\, x\right)-6\left(b^2d^4-4\sqrt{-a}\, b^{3/2}d^3e-6abd^2e^2+4\sqrt{-a}\, a\sqrt{b}\, de^3+a^2e^4\right)p\log\left(\sqrt{-a}+\sqrt{b}\, x\right)+b\left(6ae^3px(8d+ex)-bepx\left(48d^3+36d^2ex+16de^2x^2+3e^3x^3\right)+6b(d+ex)^4\log\left(c\left(a+bx^2\right)^p\right)\right)}{24b^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (-6*(b^2*d^4 + 4*Sqrt[-a]*b^(3/2)*d^3*e - 6*a*b*d^2*e^2 + 4*(-a)^(3/2)*Sqrt
[b]*d*e^3 + a^2*e^4)*p*Log[Sqrt[-a] - Sqrt[b]*x] - 6*(b^2*d^4 - 4*Sqrt[-a]*
b^(3/2)*d^3*e - 6*a*b*d^2*e^2 + 4*Sqrt[-a]*a*Sqrt[b]*d*e^3 + a^2*e^4)*p*Log
[Sqrt[-a] + Sqrt[b]*x] + b*(6*a*e^3*p*x*(8*d + e*x) - b*e*p*x*(48*d^3 + 36*
d^2*e*x + 16*d*e^2*x^2 + 3*e^3*x^3) + 6*b*(d + e*x)^4*Log[c*(a + b*x^2)^p])
)/(24*b^2*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.80, size = 1330, normalized size = 7.47

method	result	size
risch	Expression too large to display	1330

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*ln(c*(b*x^2+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b*a*e^3*p*x^2-2*d^3*p*x-1/4/e*p*ln(-a^2*d*e^3+a*b*d^3*e+(-a^3*b*d^2*e^6
+2*a^2*b^2*d^4*e^4-a*b^3*d^6*e^2)^(1/2)*x)*d^4-1/4/e*p*ln(-a^2*d*e^3+a*b*d^
3*e-(-a^3*b*d^2*e^6+2*a^2*b^2*d^4*e^4-a*b^3*d^6*e^2)^(1/2)*x)*d^4+e^2*ln(c)
*d*x^3+3/2*e*ln(c)*d^2*x^2+1/4*e^3*ln(c)*x^4-1/4*e^3/b^2*p*ln(-a^2*d*e^3+a*
b*d^3*e+(-a^3*b*d^2*e^6+2*a^2*b^2*d^4*e^4-a*b^3*d^6*e^2)^(1/2)*x)*a^2+1/e/b
^2*p*ln(-a^2*d*e^3+a*b*d^3*e-(-a^3*b*d^2*e^6+2*a^2*b^2*d^4*e^4-a*b^3*d^6*e^
2)^(1/2)*x)*(-a^3*b*d^2*e^6+2*a^2*b^2*d^4*e^4-a*b^3*d^6*e^2)^(1/2)-1/4*e^3/
b^2*p*ln(-a^2*d*e^3+a*b*d^3*e-(-a^3*b*d^2*e^6+2*a^2*b^2*d^4*e^4-a*b^3*d^6*e
^2)^(1/2)*x)*a^2-1/e/b^2*p*ln(-a^2*d*e^3+a*b*d^3*e+(-a^3*b*d^2*e^6+2*a^2*b^
2*d^4*e^4-a*b^3*d^6*e^2)^(1/2)*x)*(-a^3*b*d^2*e^6+2*a^2*b^2*d^4*e^4-a*b^3*d
^6*e^2)^(1/2)-1/2*I*Pi*d^3*csgn(I*c*(b*x^2+a)^p)^3*x-1/8*I*e^3*Pi*x^4*csgn(
I*c*(b*x^2+a)^p)^3-1/8*e^3*p*x^4+1/2*I*Pi*d^3*csgn(I*(b*x^2+a)^p)*csgn(I*c*
(b*x^2+a)^p)^2*x+1/2*I*Pi*d^3*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)*x+1/8*I*e^3
*Pi*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+1/8*I*e^3*Pi*x^4*csgn(I
*c*(b*x^2+a)^p)^2*csgn(I*c)-1/2*I*e^2*Pi*d*x^3*csgn(I*c*(b*x^2+a)^p)^3-3/4*
I*e*Pi*d^2*x^2*csgn(I*c*(b*x^2+a)^p)^3-1/2*I*e^2*Pi*d*x^3*csgn(I*(b*x^2+a)^
p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-3/4*I*e*Pi*d^2*x^2*csgn(I*(b*x^2+a)^p)*c
sgn(I*c*(b*x^2+a)^p)*csgn(I*c)-1/2*I*Pi*d^3*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b
*x^2+a)^p)*csgn(I*c)*x-1/8*I*e^3*Pi*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2
+a)^p)*csgn(I*c)+1/2*I*e^2*Pi*d*x^3*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^
p)^2+1/2*I*e^2*Pi*d*x^3*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+3/4*I*e*Pi*d^2*x^
2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+3/4*I*e*Pi*d^2*x^2*csgn(I*c*(
b*x^2+a)^p)^2*csgn(I*c)+ln(c)*d^3*x-3/2*d^2*e*p*x^2+2/b*a*d*p*e^2*x-2/3*d*e
^2*p*x^3+3/2*e/b*p*ln(-a^2*d*e^3+a*b*d^3*e+(-a^3*b*d^2*e^6+2*a^2*b^2*d^4*e^
4-a*b^3*d^6*e^2)^(1/2)*x)*a*d^2+3/2*e/b*p*ln(-a^2*d*e^3+a*b*d^3*e-(-a^3*b*d
^2*e^6+2*a^2*b^2*d^4*e^4-a*b^3*d^6*e^2)^(1/2)*x)*a*d^2+1/4*(e*x+d)^4/e*ln((
b*x^2+a)^p)
```

Maxima [A]
time = 0.58, size = 172, normalized size = 0.97

$$\frac{1}{24}\,bp\left(\frac{48\,(abd^3-a^2de^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}-\frac{3\,bx^4e^3+16\,bdx^3e^2+6\,(6\,bd^2e-ae^3)x^2+48\,(bd^3-ade^2)x}{b^2}+\frac{6\,(6\,abd^2e-a^2e^3)\log\left(bx^2+a\right)}{b^3}\right)+\frac{1}{4}\left(x^4e^3+4\,dx^3e^2+6\,d^2x^2e+4\,d^3x\right)\log\left(\left(bx^2+a\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] 1/24*b*p*(48*(a*b*d^3 - a^2*d*e^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) -
(3*b*x^4*e^3 + 16*b*d*x^3*e^2 + 6*(6*b*d^2*e - a*e^3)*x^2 + 48*(b*d^3 - a*d
*e^2)*x)/b^2 + 6*(6*a*b*d^2*e - a^2*e^3)*log(b*x^2 + a)/b^3) + 1/4*(x^4*e^3
 + 4*d*x^3*e^2 + 6*d^2*x^2*e + 4*d^3*x)*log((b*x^2 + a)^p*c)
```

Fricas [**A**]
time = 0.40, size = 486, normalized size = 2.73

$$\left[-\frac{36\,b^2d^2px^2e+48\,b^2d^3px-24\,(b^2d^3p-abdpe^2)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+3\,(b^2px^4-2\,abpx^2)e^3+16\,(b^2dpx^3-3\,abdpx)e^2-6\,(4\,b^2dpx^3e^2+4\,b^2d^3px+(b^2px^4-a^2p)e^3+6\,(b^2d^2px^2+abd^2p)e)\log\left(bx^2+a\right)-6\,(b^2x^4e^3+4\,b^2dx^3e^2+6\,b^2d^2x^2e+4\,b^2d^3x)\log\left(c\right)}{24\,b^2},\ -\frac{36\,b^2d^2px^2e+48\,b^2d^3px-48\,(b^2d^3p-abdpe^2)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+3\,(b^2px^4-2\,abpx^2)e^3+16\,(b^2dpx^3-3\,abdpx)e^2-6\,(4\,b^2dpx^3e^2+4\,b^2d^3px+(b^2px^4-a^2p)e^3+6\,(b^2d^2px^2+abd^2p)e)\log\left(bx^2+a\right)-6\,(b^2x^4e^3+4\,b^2dx^3e^2+6\,b^2d^2x^2e+4\,b^2d^3x)\log\left(c\right)}{24\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] [-1/24*(36*b^2*d^2*p*x^2*e + 48*b^2*d^3*p*x - 24*(b^2*d^3*p - a*b*d*p*e^2)*
sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 3*(b^2*p*x^4 -
 2*a*b*p*x^2)*e^3 + 16*(b^2*d*p*x^3 - 3*a*b*d*p*x)*e^2 - 6*(4*b^2*d*p*x^3*e
^2 + 4*b^2*d^3*p*x + (b^2*p*x^4 - a^2*p)*e^3 + 6*(b^2*d^2*p*x^2 + a*b*d^2*p
)*e)*log(b*x^2 + a) - 6*(b^2*x^4*e^3 + 4*b^2*d*x^3*e^2 + 6*b^2*d^2*x^2*e +
4*b^2*d^3*x)*log(c))/b^2, -1/24*(36*b^2*d^2*p*x^2*e + 48*b^2*d^3*p*x - 48*(
b^2*d^3*p - a*b*d*p*e^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 3*(b^2*p*x^4 -
 2*a*b*p*x^2)*e^3 + 16*(b^2*d*p*x^3 - 3*a*b*d*p*x)*e^2 - 6*(4*b^2*d*p*x^3*e
^2 + 4*b^2*d^3*p*x + (b^2*p*x^4 - a^2*p)*e^3 + 6*(b^2*d^2*p*x^2 + a*b*d^2*p
)*e)*log(b*x^2 + a) - 6*(b^2*x^4*e^3 + 4*b^2*d*x^3*e^2 + 6*b^2*d^2*x^2*e +
4*b^2*d^3*x)*log(c))/b^2]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(170) = 340$.
time = 19.44, size = 527, normalized size = 2.96

$$\begin{cases}\left(d^3x+\frac{3d^2ex^2}{2}+de^2x^3+\frac{e^3x^4}{4}\right)\log\left(0^pc\right) & \text{for}\ a=0\wedge b=0\\ \left(d^3x+\frac{3d^2ex^2}{2}+de^2x^3+\frac{e^3x^4}{4}\right)\log\left(a^pc\right) & \text{for}\ b=0\\ -2d^3px+d^3x\log\left(c(bx^2)^p\right)-\frac{3d^2epx^2}{2}+\frac{3d^2ex^2\log\left(c(bx^2)^p\right)}{2}-\frac{2de^2px^3}{3}+de^2x^3\log\left(c(bx^2)^p\right)-\frac{e^3px^4}{8}+\frac{e^3x^4\log\left(c(bx^2)^p\right)}{4} & \text{for}\ a=0\\ -\frac{2a^2de^2p\log\left(x-\sqrt{-\frac{a}{b}}\right)}{b^2\sqrt{-\frac{a}{b}}}+\frac{a^2de^2\log\left(c(a+bx^2)^p\right)}{b^2\sqrt{-\frac{a}{b}}}-\frac{a^2e^3\log\left(c(a+bx^2)^p\right)}{4b^2}+\frac{2ad^3p\log\left(x-\sqrt{-\frac{a}{b}}\right)}{b\sqrt{-\frac{a}{b}}}-\frac{ad^3\log\left(c(a+bx^2)^p\right)}{b\sqrt{-\frac{a}{b}}}+\frac{3ad^2e\log\left(c(a+bx^2)^p\right)}{2b}+\frac{2ade^2px}{b}+\frac{ae^3px^2}{4b}-2d^3px+d^3x\log\left(c(a+bx^2)^p\right)-\frac{3d^2epx^2}{2}+\frac{3d^2ex^2\log\left(c(a+bx^2)^p\right)}{2}-\frac{2de^2px^3}{3}+de^2x^3\log\left(c(a+bx^2)^p\right)-\frac{e^3px^4}{8}+\frac{e^3x^4\log\left(c(a+bx^2)^p\right)}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*ln(c*(b*x**2+a)**p),x)
```

```
[Out] Piecewise(((d**3*x + 3*d**2*e*x**2/2 + d*e**2*x**3 + e**3*x**4/4)*log(0**p*
c), Eq(a, 0) & Eq(b, 0)), ((d**3*x + 3*d**2*e*x**2/2 + d*e**2*x**3 + e**3*x
**4/4)*log(a**p*c), Eq(b, 0)), (-2*d**3*p*x + d**3*x*log(c*(b*x**2)**p) - 3
*d**2*e*p*x**2/2 + 3*d**2*e*x**2*log(c*(b*x**2)**p)/2 - 2*d*e**2*p*x**3/3 +
 d*e**2*x**3*log(c*(b*x**2)**p) - e**3*p*x**4/8 + e**3*x**4*log(c*(b*x**2)*
*p)/4, Eq(a, 0)), (-2*a**2*d*e**2*p*log(x - sqrt(-a/b))/(b**2*sqrt(-a/b)) +
 a**2*d*e**2*log(c*(a + b*x**2)**p)/(b**2*sqrt(-a/b)) - a**2*e**3*log(c*(a
+ b*x**2)**p)/(4*b**2) + 2*a*d**3*p*log(x - sqrt(-a/b))/(b*sqrt(-a/b)) - a*
d**3*log(c*(a + b*x**2)**p)/(b*sqrt(-a/b)) + 3*a*d**2*e*log(c*(a + b*x**2)*
*p)/(2*b) + 2*a*d*e**2*p*x/b + a*e**3*p*x**2/(4*b) - 2*d**3*p*x + d**3*x*lo
```

```
g(c*(a + b*x**2)**p) - 3*d**2*e*p*x**2/2 + 3*d**2*e*x**2*log(c*(a + b*x**2)
**p)/2 - 2*d*e**2*p*x**3/3 + d*e**2*x**3*log(c*(a + b*x**2)**p) - e**3*p*x*
*4/8 + e**3*x**4*log(c*(a + b*x**2)**p)/4, True))
```

Giac [A]
time = 3.86, size = 273, normalized size = 1.53

$$\frac{2\,(abd^3p-a^2dpe^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{6\,b^2px^4e^3\log\left(bx^2+a\right)+24\,b^2dpx^3e^2\log\left(bx^2+a\right)+36\,b^2d^2px^2e\log\left(bx^2+a\right)-3\,b^2px^4e^3-16\,b^2dpx^3e^2-36\,b^2d^2px^2e+24\,b^2d^3px\log\left(bx^2+a\right)+6\,b^2x^4e^3\log\left(c\right)+24\,b^2dx^3e^2\log\left(c\right)+36\,b^2d^2x^2e\log\left(c\right)-48\,b^2d^3px+36\,abd^2pe\log\left(bx^2+a\right)+24\,b^2d^3x\log\left(c\right)+6\,abpx^2e^3+48\,abdpxe^2-6\,a^2pe^3\log\left(bx^2+a\right)}{24\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] 2*(a*b*d^3*p - a^2*d*p*e^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + 1/24*(6*b
^2*p*x^4*e^3*log(b*x^2 + a) + 24*b^2*d*p*x^3*e^2*log(b*x^2 + a) + 36*b^2*d^
2*p*x^2*e*log(b*x^2 + a) - 3*b^2*p*x^4*e^3 - 16*b^2*d*p*x^3*e^2 - 36*b^2*d^
2*p*x^2*e + 24*b^2*d^3*p*x*log(b*x^2 + a) + 6*b^2*x^4*e^3*log(c) + 24*b^2*d
*x^3*e^2*log(c) + 36*b^2*d^2*x^2*e*log(c) - 48*b^2*d^3*p*x + 36*a*b*d^2*p*e
*log(b*x^2 + a) + 24*b^2*d^3*x*log(c) + 6*a*b*p*x^2*e^3 + 48*a*b*d*p*x*e^2
- 6*a^2*p*e^3*log(b*x^2 + a))/b^2
```

Mupad [B]
time = 0.46, size = 222, normalized size = 1.25

$$\frac{e^3\,x^4\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{4}-2\,d^3\,p\,x-\frac{e^3\,p\,x^4}{8}+d^3\,x\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)+\frac{3\,d^2\,e\,x^2\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{2}+d\,e^2\,x^3\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)-\frac{3\,d^2\,e\,p\,x^2}{2}-\frac{2\,d\,e^2\,p\,x^3}{3}+\frac{a\,e^3\,p\,x^2}{4\,b}+\frac{2\,\sqrt{a}\,d^3\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}-\frac{a^2\,e^3\,p\,\ln\left(b\,x^2+a\right)}{4\,b^2}-\frac{2\,a^{3/2}\,d\,e^2\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}+\frac{2\,a\,d\,e^2\,p\,x}{b}+\frac{3\,a\,d^2\,e\,p\,\ln\left(b\,x^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)*(d + e*x)^3,x)
```

```
[Out] (e^3*x^4*log(c*(a + b*x^2)^p))/4 - 2*d^3*p*x - (e^3*p*x^4)/8 + d^3*x*log(c*
(a + b*x^2)^p) + (3*d^2*e*x^2*log(c*(a + b*x^2)^p))/2 + d*e^2*x^3*log(c*(a
+ b*x^2)^p) - (3*d^2*e*p*x^2)/2 - (2*d*e^2*p*x^3)/3 + (a*e^3*p*x^2)/(4*b) +
 (2*a^(1/2)*d^3*p*atan((b^(1/2)*x)/a^(1/2)))/b^(1/2) - (a^2*e^3*p*log(a + b
*x^2))/(4*b^2) - (2*a^(3/2)*d*e^2*p*atan((b^(1/2)*x)/a^(1/2)))/b^(3/2) + (2
*a*d*e^2*p*x)/b + (3*a*d^2*e*p*log(a + b*x^2))/(2*b)
```

3.185 $\int (d+ex)^2 \log \left(c\left(a+bx^2\right)^p\right) \, dx$

Optimal. Leaf size=141

$$-\frac{2(3bd^2-ae^2)\,px}{3b}-depx^2-\frac{2}{9}e^2px^3+\frac{2\sqrt{a}\,(3bd^2-ae^2)\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3b^{3/2}}-\frac{d(bd^2-3ae^2)\,p\log\left(a+bx^2\right)}{3be}+\frac{(d}{}$$

```
[Out] -2/3*(-a*e^2+3*b*d^2)*p*x/b-d*e*p*x^2-2/9*e^2*p*x^3-1/3*d*(-3*a*e^2+b*d^2)*
p*ln(b*x^2+a)/b/e+1/3*(e*x+d)^3*ln(c*(b*x^2+a)^p)/e+2/3*(-a*e^2+3*b*d^2)*p*
arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(3/2)
```

Rubi [A]
time = 0.09, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2513, 815, 649, 211, 266}

$$\frac{2\sqrt{a}\,p\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(3bd^2-ae^2)}{3b^{3/2}}+\frac{(d+ex)^3\log\left(c\left(a+bx^2\right)^p\right)}{3e}-\frac{dp(bd^2-3ae^2)\log\left(a+bx^2\right)}{3be}-\frac{2px(3bd^2-ae^2)}{3b}-depx^2-\frac{2}{9}e^2px^3$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (-2*(3*b*d^2 - a*e^2)*p*x)/(3*b) - d*e*p*x^2 - (2*e^2*p*x^3)/9 + (2*Sqrt[a]
*(3*b*d^2 - a*e^2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3*b^(3/2)) - (d*(b*d^2 -
 3*a*e^2)*p*Log[a + b*x^2])/(3*b*e) + ((d + e*x)^3*Log[c*(a + b*x^2)^p])/(3
*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\int (d+ex)^2 \log \left(c\left(a+bx^2\right)^p\right)\,dx = \frac{(d+ex)^3 \log \left(c\left(a+bx^2\right)^p\right)}{3e} - \frac{(2bp)\int \frac{x(d+ex)^3}{a+bx^2}\,dx}{3e}$$

$$= \frac{(d+ex)^3 \log \left(c\left(a+bx^2\right)^p\right)}{3e} - \frac{(2bp)\int \left(\frac{e(3bd^2-ae^2)}{b^2} + \frac{3de^2x}{b} + \frac{e^3x^2}{b} - \frac{ae(3bd^2}{}\right.}{3e}$$

$$= -\frac{2(3bd^2-ae^2)\,px}{3b} - depx^2 - \frac{2}{9}e^2px^3 + \frac{(d+ex)^3 \log \left(c\left(a+bx^2\right)^p\right)}{3e} + \frac{(2}{}$$

$$= -\frac{2(3bd^2-ae^2)\,px}{3b} - depx^2 - \frac{2}{9}e^2px^3 + \frac{(d+ex)^3 \log \left(c\left(a+bx^2\right)^p\right)}{3e} - \frac{(2}{}$$

$$= -\frac{2(3bd^2-ae^2)\,px}{3b} - depx^2 - \frac{2}{9}e^2px^3 + \frac{2\sqrt{a}\,(3bd^2-ae^2)\,p\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}}\right.}{3b^{3/2}}$$

Mathematica [A]
time = 0.32, size = 211, normalized size = 1.50

$$\frac{3\left(-b^{3/2}d^3-3\sqrt{-a}\,bd^2e+3a\sqrt{b}\,de^2+\sqrt{-a}\,ae^3\right)p\log\left(\sqrt{-a}-\sqrt{b}\,x\right)-3\left(b^{3/2}d^3-3\sqrt{-a}\,bd^2e-3a\sqrt{b}\,de^2+\sqrt{-a}\,ae^3\right)p\log\left(\sqrt{-a}+\sqrt{b}\,x\right)+\sqrt{b}\left(6ae^3px-bepx(18d^2+9dex+2e^2x^2)+3b(d+ex)^3\log\left(c(a+bx^2)^p\right)\right)}{9b^{3/2}e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (3*(-(b^(3/2)*d^3) - 3*Sqrt[-a]*b*d^2*e + 3*a*Sqrt[b]*d*e^2 + Sqrt[-a]*a*e^
3)*p*Log[Sqrt[-a] - Sqrt[b]*x] - 3*(b^(3/2)*d^3 - 3*Sqrt[-a]*b*d^2*e - 3*a*
Sqrt[b]*d*e^2 + Sqrt[-a]*a*e^3)*p*Log[Sqrt[-a] + Sqrt[b]*x] + Sqrt[b]*(6*a*
e^3*p*x - b*e*p*x*(18*d^2 + 9*d*e*x + 2*e^2*x^2) + 3*b*(d + e*x)^3*Log[c*(a
 + b*x^2)^p]))/(9*b^(3/2)*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.62, size = 965, normalized size = 6.84

method	result
risch	$-\frac{p \ln\left(-a^2e^3+3ab\,d^2e+\sqrt{-a^3b\,e^6+6a^2b^2d^2e^4-9a\,b^3d^4e^2}\,x\right)\sqrt{-a^3b\,e^6+6a^2b^2d^2e^4-9a\,b^3d^4e^2}}{3e\,b^2}+\frac{p\,\mathrm{l}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*ln(c*(b*x^2+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/e/b^2*p*ln(-a^2*e^3+3*a*b*d^2*e+(-a^3*b*e^6+6*a^2*b^2*d^2*e^4-9*a*b^3*
d^4*e^2)^(1/2)*x)*(-a^3*b*e^6+6*a^2*b^2*d^2*e^4-9*a*b^3*d^4*e^2)^(1/2)+1/3/
e/b^2*p*ln(-a^2*e^3+3*a*b*d^2*e-(-a^3*b*e^6+6*a^2*b^2*d^2*e^4-9*a*b^3*d^4*e
^2)^(1/2)*x)*(-a^3*b*e^6+6*a^2*b^2*d^2*e^4-9*a*b^3*d^4*e^2)^(1/2)-1/2*I*Pi*
d^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)*x-1/6*I*e^2*Pi*x^3*
csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+1/2*I*e*Pi*d*x^2*csgn(I
*c*(b*x^2+a)^p)^2*csgn(I*c)+1/2*I*e*Pi*d*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(
b*x^2+a)^p)^2-d*e*p*x^2-1/3/e*p*ln(-a^2*e^3+3*a*b*d^2*e+(-a^3*b*e^6+6*a^2*b
^2*d^2*e^4-9*a*b^3*d^4*e^2)^(1/2)*x)*d^3-1/3/e*p*ln(-a^2*e^3+3*a*b*d^2*e-(-
a^3*b*e^6+6*a^2*b^2*d^2*e^4-9*a*b^3*d^4*e^2)^(1/2)*x)*d^3+ln(c)*d*e*x^2+2/3
/b*a*p*e^2*x+1/3*e^2*ln(c)*x^3+1/6*I*e^2*Pi*x^3*csgn(I*(b*x^2+a)^p)*csgn(I*
c*(b*x^2+a)^p)^2-1/2*I*e*Pi*d*x^2*csgn(I*c*(b*x^2+a)^p)^3+1/2*I*Pi*d^2*csgn
(I*c*(b*x^2+a)^p)^2*csgn(I*c)*x+1/2*I*Pi*d^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(
b*x^2+a)^p)^2*x-2/9*e^2*p*x^3-2*d^2*p*x+1/3*(e*x+d)^3/e*ln((b*x^2+a)^p)+e/b
*p*ln(-a^2*e^3+3*a*b*d^2*e+(-a^3*b*e^6+6*a^2*b^2*d^2*e^4-9*a*b^3*d^4*e^2)^(
1/2)*x)*a*d+e/b*p*ln(-a^2*e^3+3*a*b*d^2*e-(-a^3*b*e^6+6*a^2*b^2*d^2*e^4-9*a
*b^3*d^4*e^2)^(1/2)*x)*a*d+1/6*I*e^2*Pi*x^3*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*
c)-1/2*I*e*Pi*d*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+ln(
c)*d^2*x-1/2*I*Pi*d^2*csgn(I*c*(b*x^2+a)^p)^3*x-1/6*I*e^2*Pi*x^3*csgn(I*c*(
b*x^2+a)^p)^3
```

Maxima [A]

time = 0.59, size = 130, normalized size = 0.92

$$\frac{1}{9}\left(\frac{9\,ade\log\left(bx^2+a\right)}{b^2}+\frac{6\left(3\,abd^2-a^2e^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}-\frac{2\,bx^3e^2+9\,bdx^2e+6\left(3\,bd^2-ae^2\right)x}{b^2}\right)bp+\frac{1}{3}\left(x^3e^2+3\,dx^2e+3\,d^2x\right)\log\left(\left(bx^2+a\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] 1/9*(9*a*d*e*log(b*x^2 + a)/b^2 + 6*(3*a*b*d^2 - a^2*e^2)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*b^2) - (2*b*x^3*e^2 + 9*b*d*x^2*e + 6*(3*b*d^2 - a*e^2)*x)/
b^2)*b*p + 1/3*(x^3*e^2 + 3*d*x^2*e + 3*d^2*x)*log((b*x^2 + a)^p*c)
```

Fricas [A]

time = 0.37, size = 316, normalized size = 2.24

$$\left[-\frac{9\,bdpx^2e+18\,bd^2px-3\left(3\,bd^2p-ape^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+2\left(bpx^3-3\,apx\right)e^2-3\left(bpx^3e^2+3\,bd^2px+3\left(bdpx^2+adp\right)e\right)\log\left(bx^2+a\right)-3\left(bx^3e^2+3\,bdx^2e+3\,bd^2x\right)\log\left(c\right)}{9\,b},-\frac{9\,bdpx^2e+18\,bd^2px-6\left(3\,bd^2p-ape^2\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+2\left(bpx^3-3\,apx\right)e^2-3\left(bpx^3e^2+3\,bd^2px+3\left(bdpx^2+adp\right)e\right)\log\left(bx^2+a\right)-3\left(bx^3e^2+3\,bdx^2e+3\,bd^2x\right)\log\left(c\right)}{9\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] [-1/9*(9*b*d*p*x^2*e + 18*b*d^2*p*x - 3*(3*b*d^2*p - a*p*e^2)*sqrt(-a/b)*lo
g((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 2*(b*p*x^3 - 3*a*p*x)*e^2 -
 3*(b*p*x^3*e^2 + 3*b*d^2*p*x + 3*(b*d*p*x^2 + a*d*p)*e)*log(b*x^2 + a) - 3
*(b*x^3*e^2 + 3*b*d*x^2*e + 3*b*d^2*x)*log(c))/b, -1/9*(9*b*d*p*x^2*e + 18*
b*d^2*p*x - 6*(3*b*d^2*p - a*p*e^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 2*(
b*p*x^3 - 3*a*p*x)*e^2 - 3*(b*p*x^3*e^2 + 3*b*d^2*p*x + 3*(b*d*p*x^2 + a*d*
p)*e)*log(b*x^2 + a) - 3*(b*x^3*e^2 + 3*b*d*x^2*e + 3*b*d^2*x)*log(c))/b]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(131) = 262$.
time = 9.45, size = 374, normalized size = 2.65

$$\begin{cases} \left(d^{2}x + dex^{2} + \frac{e^{2}x^{3}}{3}\right)\log\left(0^{p}c\right) & \text{for}\: a = 0 \wedge b = 0 \\ \left(d^{2}x + dex^{2} + \frac{e^{2}x^{3}}{3}\right)\log\left(a^{p}c\right) & \text{for}\: b = 0 \\ -2d^{2}px + d^{2}x\log\left(c(bx^{2})^{p}\right) - depx^{2} + dex^{2}\log\left(c(bx^{2})^{p}\right) - \frac{2e^{2}px^{3}}{9} + \frac{e^{2}x^{3}\log\left(c(bx^{2})^{p}\right)}{3} & \text{for}\: a = 0 \\ -\frac{2a^{2}e^{2}p\log\left(x-\sqrt{-\frac{a}{b}}\right)}{3b^{2}\sqrt{-\frac{a}{b}}} + \frac{a^{2}e^{2}\log\left(c(a+bx^{2})^{p}\right)}{3b^{2}\sqrt{-\frac{a}{b}}} + \frac{2ad^{2}p\log\left(x-\sqrt{-\frac{a}{b}}\right)}{b\sqrt{-\frac{a}{b}}} - \frac{ad^{2}\log\left(c(a+bx^{2})^{p}\right)}{b\sqrt{-\frac{a}{b}}} + \frac{ade\log\left(c(a+bx^{2})^{p}\right)}{b} + \frac{2ae^{2}px}{3b} - 2d^{2}px + d^{2}x\log\left(c(a+bx^{2})^{p}\right) - depx^{2} + dex^{2}\log\left(c(a+bx^{2})^{p}\right) - \frac{2e^{2}px^{3}}{9} + \frac{e^{2}x^{3}\log\left(c(a+bx^{2})^{p}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*ln(c*(b*x**2+a)**p),x)
```

```
[Out] Piecewise((((d**2*x + d*e*x**2 + e**2*x**3/3)*log(0**p*c), Eq(a, 0) & Eq(b,
0)), ((d**2*x + d*e*x**2 + e**2*x**3/3)*log(a**p*c), Eq(b, 0)), (-2*d**2*p*
x + d**2*x*log(c*(b*x**2)**p) - d*e*p*x**2 + d*e*x**2*log(c*(b*x**2)**p) -
2*e**2*p*x**3/9 + e**2*x**3*log(c*(b*x**2)**p)/3, Eq(a, 0)), (-2*a**2*e**2*
p*log(x - sqrt(-a/b))/(3*b**2*sqrt(-a/b)) + a**2*e**2*log(c*(a + b*x**2)**p
)/(3*b**2*sqrt(-a/b)) + 2*a*d**2*p*log(x - sqrt(-a/b))/(b*sqrt(-a/b)) - a*d
**2*log(c*(a + b*x**2)**p)/(b*sqrt(-a/b)) + a*d*e*log(c*(a + b*x**2)**p)/b
+ 2*a*e**2*p*x/(3*b) - 2*d**2*p*x + d**2*x*log(c*(a + b*x**2)**p) - d*e*p*x
**2 + d*e*x**2*log(c*(a + b*x**2)**p) - 2*e**2*p*x**3/9 + e**2*x**3*log(c*(
a + b*x**2)**p)/3, True))
```

Giac [A]
time = 4.33, size = 173, normalized size = 1.23

$$\frac{2\left(3\,abd^{2}p - a^{2}pe^{2}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,b} + \frac{3\,bpx^{3}e^{2}\log\left(bx^{2}+a\right) + 9\,bdpx^{2}e\log\left(bx^{2}+a\right) - 2\,bpx^{3}e^{2} - 9\,bdpx^{2}e + 9\,bd^{2}px\log\left(bx^{2}+a\right) + 3\,bx^{3}e^{2}\log\left(c\right) + 9\,bdx^{2}e\log\left(c\right) - 18\,bd^{2}px + 9\,adpe\log\left(bx^{2}+a\right) + 9\,bd^{2}x\log\left(c\right) + 6\,apxe^{2}}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] 2/3*(3*a*b*d^2*p - a^2*p*e^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + 1/9*(3*
b*p*x^3*e^2*log(b*x^2 + a) + 9*b*d*p*x^2*e*log(b*x^2 + a) - 2*b*p*x^3*e^2 -
 9*b*d*p*x^2*e + 9*b*d^2*p*x*log(b*x^2 + a) + 3*b*x^3*e^2*log(c) + 9*b*d*x^
```

```
2*e*log(c) - 18*b*d^2*p*x + 9*a*d*p*e*log(b*x^2 + a) + 9*b*d^2*x*log(c) + 6
*a*p*x*e^2)/b
```

Mupad [B]
time = 3.28, size = 263, normalized size = 1.87

$$\frac{e^2\,x^3\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{3}-2\,d^2\,p\,x-\frac{2\,e^2\,p\,x^3}{9}+d^2\,x\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)+d\,e\,x^2\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)-d\,e\,p\,x^2+\frac{2\,a\,e^2\,p\,x}{3\,b}-\frac{2\,\sqrt{a}\,d^2\,p\,\mathrm{atan}\left(\frac{3\,\sqrt{a}\,b^{3/2}\,d^2\,p\,x}{a^2\,e^2\,p-3\,a\,b\,d^2\,p}-\frac{a^{3/2}\,\sqrt{b}\,e^2\,p\,x}{a^2\,e^2\,p-3\,a\,b\,d^2\,p}\right)}{\sqrt{b}}+\frac{2\,a^{3/2}\,e^2\,p\,\mathrm{atan}\left(\frac{3\,\sqrt{a}\,b^{3/2}\,d^2\,p\,x}{a^2\,e^2\,p-3\,a\,b\,d^2\,p}-\frac{a^{3/2}\,\sqrt{b}\,e^2\,p\,x}{a^2\,e^2\,p-3\,a\,b\,d^2\,p}\right)}{3\,b^{3/2}}+\frac{a\,d\,e\,p\,\ln\left(b\,x^2+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)*(d + e*x)^2,x)
```

```
[Out] (e^2*x^3*log(c*(a + b*x^2)^p))/3 - 2*d^2*p*x - (2*e^2*p*x^3)/9 + d^2*x*log(
c*(a + b*x^2)^p) + d*e*x^2*log(c*(a + b*x^2)^p) - d*e*p*x^2 + (2*a*e^2*p*x)
/(3*b) - (2*a^(1/2)*d^2*p*atan((3*a^(1/2)*b^(3/2)*d^2*p*x)/(a^2*e^2*p - 3*a
*b*d^2*p) - (a^(3/2)*b^(1/2)*e^2*p*x)/(a^2*e^2*p - 3*a*b*d^2*p)))/b^(1/2) +
 (2*a^(3/2)*e^2*p*atan((3*a^(1/2)*b^(3/2)*d^2*p*x)/(a^2*e^2*p - 3*a*b*d^2*p
) - (a^(3/2)*b^(1/2)*e^2*p*x)/(a^2*e^2*p - 3*a*b*d^2*p)))/(3*b^(3/2)) + (a*
d*e*p*log(a + b*x^2))/b
```

3.186 $\int (d+ex)\log\left(c\left(a+bx^2\right)^p\right)\,dx$

Optimal. Leaf size=99

$$-2dpx-\frac{1}{2}epx^2+\frac{2\sqrt{a}\,dp\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}-\frac{(bd^2-ae^2)\,p\log\left(a+bx^2\right)}{2be}+\frac{(d+ex)^2\log\left(c\left(a+bx^2\right)^p\right)}{2e}$$

```
[Out] -2*d*p*x-1/2*e*p*x^2-1/2*(-a*e^2+b*d^2)*p*ln(b*x^2+a)/b/e+1/2*(e*x+d)^2*ln(
c*(b*x^2+a)^p)/e+2*d*p*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2513, 815, 649, 211, 266}

$$\frac{2\sqrt{a}\,dp\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{(d+ex)^2\log\left(c\left(a+bx^2\right)^p\right)}{2e}-\frac{p(bd^2-ae^2)\log\left(a+bx^2\right)}{2be}-2dpx-\frac{1}{2}epx^2$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*Log[c*(a + b*x^2)^p],x]
```

```
[Out] -2*d*p*x - (e*p*x^2)/2 + (2*Sqrt[a]*d*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b
] - ((b*d^2 - a*e^2)*p*Log[a + b*x^2])/(2*b*e) + ((d + e*x)^2*Log[c*(a + b*
x^2)^p])/(2*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$
\begin{aligned}
\int (d+ex)\log\left(c\left(a+bx^2\right)^p\right)\,dx &= \frac{(d+ex)^2\log\left(c\left(a+bx^2\right)^p\right)}{2e}-\frac{(bp)\int\frac{x(d+ex)^2}{a+bx^2}\,dx}{e}\\
&= \frac{(d+ex)^2\log\left(c\left(a+bx^2\right)^p\right)}{2e}-\frac{(bp)\int\left(\frac{2de}{b}+\frac{e^2x}{b}-\frac{2ade-\left(bd^2-ae^2\right)x}{b\left(a+bx^2\right)}\right)dx}{e}\\
&= -2dpx-\frac{1}{2}epx^2+\frac{(d+ex)^2\log\left(c\left(a+bx^2\right)^p\right)}{2e}+\frac{p\int\frac{2ade-\left(bd^2-ae^2\right)x}{a+bx^2}\,dx}{e}\\
&= -2dpx-\frac{1}{2}epx^2+\frac{(d+ex)^2\log\left(c\left(a+bx^2\right)^p\right)}{2e}+(2adp)\int\frac{1}{a+bx^2}\,dx+\\
&= -2dpx-\frac{1}{2}epx^2+\frac{2\sqrt{a}\,dp\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}-\frac{\left(bd^2-ae^2\right)p\log\left(a+bx^2\right)}{2be}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 83, normalized size = 0.84

$$
-2dpx+\frac{2\sqrt{a}\,dp\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}+dx\log\left(c\left(a+bx^2\right)^p\right)+\frac{1}{2}e\left(-px^2+\frac{\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*Log[c*(a + b*x^2)^p],x]
```

```
[Out] -2*d*p*x + (2*Sqrt[a]*d*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + d*x*Log[c*
(a + b*x^2)^p] + (e*(-(p*x^2) + ((a + b*x^2)*Log[c*(a + b*x^2)^p])/b))/2
```

Maple [A]
time = 0.45, size = 93, normalized size = 0.94

method	result
default	$d\ln\left(c(b\,x^2+a)^p\right)x-2dpx+\frac{2dpa\arctan\left(\frac{bx}{\sqrt{ba}}\right)}{\sqrt{ba}}+\frac{e\ln(c(b\,x^2+a)^p)x^2}{2}-\frac{ep\,x^2}{2}+\frac{e\ln(c(b\,x^2+a)^p)a}{2b}-\frac{aep}{2b}$
risch	$\left(\frac{1}{2}e\,x^2+dx\right)\ln\left((b\,x^2+a)^p\right)+\frac{i\pi d\,\mathrm{csgn}(i(b\,x^2+a)^p)\mathrm{csgn}(ic(b\,x^2+a)^p)^2x}{2}+\frac{i\mathrm{csgn}(ic(b\,x^2+a)^p)^2\mathrm{csgn}(i(b\,x^2+a)^p)x^2e\pi}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*ln(c*(b*x^2+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] d*ln(c*(b*x^2+a)^p)*x-2*d*p*x+2*d*p*a/(b*a)^(1/2)*arctan(b*x/(b*a)^(1/2))+1
/2*e*ln(c*(b*x^2+a)^p)*x^2-1/2*e*p*x^2+1/2*e/b*ln(c*(b*x^2+a)^p)*a-1/2*a*e*
p/b
```

Maxima [A]
time = 0.56, size = 83, normalized size = 0.84

$$\frac{1}{2}\left(\frac{4\,ad\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{ae\log\left(bx^2+a\right)}{b^2}-\frac{x^2e+4\,dx}{b}\right)bp+\frac{1}{2}\left(x^2e+2\,dx\right)\log\left(\left(bx^2+a\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] 1/2*(4*a*d*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + a*e*log(b*x^2 + a)/b^2 - (
x^2*e + 4*d*x)/b)*b*p + 1/2*(x^2*e + 2*d*x)*log((b*x^2 + a)^p*c)
```

Fricas [A]
time = 0.42, size = 206, normalized size = 2.08

$$\left[-\frac{bpx^2e-2\,bdp\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+4\,bdpx-(2\,bdpx+(bpx^2+ap)e)\log\left(bx^2+a\right)-(bx^2e+2\,bdx)\log\left(c\right)}{2\,b},-\frac{bpx^2e-4\,bdp\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+4\,bdpx-(2\,bdpx+(bpx^2+ap)e)\log\left(bx^2+a\right)-(bx^2e+2\,bdx)\log\left(c\right)}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] [-1/2*(b*p*x^2*e - 2*b*d*p*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b
*x^2 + a)) + 4*b*d*p*x - (2*b*d*p*x + (b*p*x^2 + a*p)*e)*log(b*x^2 + a) - (
b*x^2*e + 2*b*d*x)*log(c))/b, -1/2*(b*p*x^2*e - 4*b*d*p*sqrt(a/b)*arctan(b*
x*sqrt(a/b)/a) + 4*b*d*p*x - (2*b*d*p*x + (b*p*x^2 + a*p)*e)*log(b*x^2 + a)
 - (b*x^2*e + 2*b*d*x)*log(c))/b]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(92) = 184$.

time = 4.50, size = 199, normalized size = 2.01

$$\begin{cases} \left(dx + \frac{ex^2}{2}\right)\log\left(0^p c\right) & \text{for } a = 0 \wedge b = 0 \\ \left(dx + \frac{ex^2}{2}\right)\log\left(a^p c\right) & \text{for } b = 0 \\ -2dpx + dx\log\left(c(bx^2)^p\right) - \frac{epx^2}{2} + \frac{ex^2\log\left(c(bx^2)^p\right)}{2} & \text{for } a = 0 \\ \frac{2adp\log\left(x - \sqrt{-\frac{a}{b}}\right)}{b\sqrt{-\frac{a}{b}}} - \frac{ad\log\left(c(a+bx^2)^p\right)}{b\sqrt{-\frac{a}{b}}} + \frac{ae\log\left(c(a+bx^2)^p\right)}{2b} - 2dpx + dx\log\left(c(a+bx^2)^p\right) - \frac{epx^2}{2} + \frac{ex^2\log\left(c(a+bx^2)^p\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*ln(c*(b*x**2+a)**p),x)
```

```
[Out] Piecewise(((d*x + e*x**2/2)*log(0**p*c), Eq(a, 0) & Eq(b, 0)), ((d*x + e*x*
*2/2)*log(a**p*c), Eq(b, 0)), (-2*d*p*x + d*x*log(c*(b*x**2)**p) - e*p*x**2
/2 + e*x**2*log(c*(b*x**2)**p)/2, Eq(a, 0)), (2*a*d*p*log(x - sqrt(-a/b))/(
b*sqrt(-a/b)) - a*d*log(c*(a + b*x**2)**p)/(b*sqrt(-a/b)) + a*e*log(c*(a +
b*x**2)**p)/(2*b) - 2*d*p*x + d*x*log(c*(a + b*x**2)**p) - e*p*x**2/2 + e*x
**2*log(c*(a + b*x**2)**p)/2, True))
```

Giac [A]

time = 3.89, size = 100, normalized size = 1.01

$$\frac{2\,adp\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} + \frac{bpx^2e\log\left(bx^2+a\right) - bpx^2e + 2\,bdpx\log\left(bx^2+a\right) + bx^2e\log\left(c\right) - 4\,bdpx + ape\log\left(bx^2+a\right) + 2\,bdx\log\left(c\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] 2*a*d*p*arctan(b*x/sqrt(a*b))/sqrt(a*b) + 1/2*(b*p*x^2*e*log(b*x^2 + a) - b
*p*x^2*e + 2*b*d*p*x*log(b*x^2 + a) + b*x^2*e*log(c) - 4*b*d*p*x + a*p*e*lo
g(b*x^2 + a) + 2*b*d*x*log(c))/b
```

Mupad [B]

time = 1.13, size = 81, normalized size = 0.82

$$d\,x\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right) - \frac{e\,p\,x^2}{2} - 2\,d\,p\,x + \frac{e\,x^2\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{2} + \frac{a\,e\,p\,\ln\left(b\,x^2+a\right)}{2\,b} + \frac{2\,\sqrt{a}\,\,d\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)*(d + e*x),x)
```

```
[Out] d*x*log(c*(a + b*x^2)^p) - (e*p*x^2)/2 - 2*d*p*x + (e*x^2*log(c*(a + b*x^2)
^p))/2 + (a*e*p*log(a + b*x^2))/(2*b) + (2*a^(1/2)*d*p*atan((b^(1/2)*x)/a^(
1/2)))/b^(1/2)
```

3.187 $\int \log\left(c(a+bx^2)^p\right)\, dx$

Optimal. Leaf size=45

$$-2px+\frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}+x\log\left(c\left(a+bx^2\right)^p\right)$$

```
[Out] -2*p*x+x*ln(c*(b*x^2+a)^p)+2*p*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2498, 327, 211}

$$\frac{2\sqrt{a}\,p\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}+x\log\left(c\left(a+bx^2\right)^p\right)-2px$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p],x]
```

```
[Out] -2*p*x + (2*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + x*Log[c*(a + b
*x^2)^p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\int \log\left(c\left(a+bx^2\right)^p\right)\,dx = x\log\left(c\left(a+bx^2\right)^p\right) - (2bp)\int \frac{x^2}{a+bx^2}\,dx$$
$$= -2px + x\log\left(c\left(a+bx^2\right)^p\right) + (2ap)\int \frac{1}{a+bx^2}\,dx$$
$$= -2px + \frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}} + x\log\left(c\left(a+bx^2\right)^p\right)$$

Mathematica [A]
time = 0.01, size = 45, normalized size = 1.00

$$-2px + \frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}} + x\log\left(c\left(a+bx^2\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p],x]
[Out] -2*p*x + (2*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + x*Log[c*(a + b
*x^2)^p]
```

Maple [A]
time = 0.07, size = 46, normalized size = 1.02

method	result
default	$x\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)-2pb\left(\frac{x}{b}-\frac{a\arctan\left(\frac{bx}{\sqrt{ba}}\right)}{b\sqrt{ba}}\right)$
risch	$x\ln\left(\left(b\,x^{2}+a\right)^{p}\right)+\frac{i\,\mathrm{csgn}\left(ic\left(b\,x^{2}+a\right)^{p}\right)^{2}\mathrm{csgn}\left(i\left(b\,x^{2}+a\right)^{p}\right)x\pi}{2}-\frac{i\pi x\,\mathrm{csgn}\left(i\left(b\,x^{2}+a\right)^{p}\right)\mathrm{csgn}\left(ic\left(b\,x^{2}+a\right)^{p}\right)\mathrm{csgn}\left(ic\right)}{2}-i\pi x\mathrm{cs}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p),x,method=_RETURNVERBOSE)
[Out] x*ln(c*(b*x^2+a)^p)-2*p*b*(x/b-a/b/(b*a)^(1/2)*arctan(b*x/(b*a)^(1/2)))
```

Maxima [A]
time = 0.57, size = 45, normalized size = 1.00

$$2\,bp\left(\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}-\frac{x}{b}\right)+x\log\left(\left(bx^2+a\right)^p c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] 2*b*p*(a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - x/b) + x*log((b*x^2 + a)^p*c
)
```

Fricas [A]
time = 0.43, size = 107, normalized size = 2.38

$$\left[px\log\left(bx^2+a\right)+p\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-2\,px+x\log\left(c\right),px\log\left(bx^2+a\right)+2\,p\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-2\,px+x\log\left(c\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] [p*x*log(b*x^2 + a) + p*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^
2 + a)) - 2*p*x + x*log(c), p*x*log(b*x^2 + a) + 2*p*sqrt(a/b)*arctan(b*x*s
qrt(a/b)/a) - 2*p*x + x*log(c)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(44) = 88$.
time = 2.25, size = 100, normalized size = 2.22

$$\begin{cases} x\log\left(0^pc\right) & \text{for}\ a=0\wedge b=0 \\ x\log\left(a^pc\right) & \text{for}\ b=0 \\ -2px+x\log\left(c(bx^2)^p\right) & \text{for}\ a=0 \\ \frac{2ap\log\left(x-\sqrt{-\frac{a}{b}}\right)}{b\sqrt{-\frac{a}{b}}}-\frac{a\log\left(c(a+bx^2)^p\right)}{b\sqrt{-\frac{a}{b}}}-2px+x\log\left(c(a+bx^2)^p\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p),x)
```

```
[Out] Piecewise((x*log(0**p*c), Eq(a, 0) & Eq(b, 0)), (x*log(a**p*c), Eq(b, 0)),
(-2*p*x + x*log(c*(b*x**2)**p), Eq(a, 0)), (2*a*p*log(x - sqrt(-a/b))/(b*sq
rt(-a/b)) - a*log(c*(a + b*x**2)**p)/(b*sqrt(-a/b)) - 2*p*x + x*log(c*(a +
b*x**2)**p), True))
```

Giac [A]
time = 4.10, size = 41, normalized size = 0.91

$$px\log\left(bx^2+a\right)+\frac{2\,ap\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}-\left(2\,p-\log\left(c\right)\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] p*x*log(b*x^2 + a) + 2*a*p*arctan(b*x/sqrt(a*b))/sqrt(a*b) - (2*p - log(c))
*x
```

Mupad [B]
time = 0.08, size = 37, normalized size = 0.82

$$x\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)-2\,p\,x+\frac{2\,\sqrt{a}\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p),x)
```

```
[Out] x*log(c*(a + b*x^2)^p) - 2*p*x + (2*a^(1/2)*p*atan((b^(1/2)*x)/a^(1/2)))/b^
(1/2)
```

3.188 $\int \frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=201

$$-\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{e}-\frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)\log(d+ex)}{e}+\frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e}$$

```
[Out] ln(e*x+d)*ln(c*(b*x^2+a)^p)/e-p*ln(e*x+d)*ln(e*((-a)^(1/2)-x*b^(1/2))/(e*(-
a)^(1/2)+d*b^(1/2)))/e-p*ln(e*x+d)*ln(-e*((-a)^(1/2)+x*b^(1/2))/(-e*(-a)^(1
/2)+d*b^(1/2)))/e-p*polylog(2,(e*x+d)*b^(1/2)/(-e*(-a)^(1/2)+d*b^(1/2)))/e-
p*polylog(2,(e*x+d)*b^(1/2)/(e*(-a)^(1/2)+d*b^(1/2)))/e
```

Rubi [A]
time = 0.18, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2512, 266, 2463, 2441, 2440, 2438}

$$-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{b}\,(d+ex)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)}{e}-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{b}\,(d+ex)}{\sqrt{-a}\,e+\sqrt{b}\,d}\right)}{e}+\frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e}-\frac{p\log(d+ex)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{-a}\,e+\sqrt{b}\,d}\right)}{e}-\frac{p\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/(d + e*x),x]
```

```
[Out] -((p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])
/e) - (p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d
+ e*x])/e + (Log[d + e*x]*Log[c*(a + b*x^2)^p])/e - (p*PolyLog[2, (Sqrt[b]*
(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/e - (p*PolyLog[2, (Sqrt[b]*(d + e*x))
/(Sqrt[b]*d + Sqrt[-a]*e)])/e
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx = \frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e} - \frac{(2bp)\int \frac{x\log(d+ex)}{a+bx^2}\,dx}{e}$$

$$= \frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e} - \frac{(2bp)\int \left(-\frac{\log(d+ex)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\log(d+ex)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{\right.}\right.}}{e}$$

$$= \frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e} + \frac{\left(\sqrt{b}\,p\right)\int \frac{\log(d+ex)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{e} - \frac{\left(\sqrt{b}\,p\right)\int \frac{\log(d+ex)}{\sqrt{-a}+\sqrt{\right.}}}{e}$$

$$= -\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)\log(d+ex)}{e}$$

$$= -\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)\log(d+ex)}{e}$$

$$= -\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)\log(d+ex)}{e}$$

Mathematica [A]
time = 0.06, size = 201, normalized size = 1.00

$$-\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e} - \frac{p\text{Li}_2\left(\frac{\sqrt{b}\,(d+ex)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)}{e} - \frac{p\text{Li}_2\left(\frac{\sqrt{b}\,(d+ex)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/(d + e*x),x]
```

```
[Out] -((p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])
/e) - (p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d
+ e*x])/e + (Log[d + e*x]*Log[c*(a + b*x^2)^p])/e - (p*PolyLog[2, (Sqrt[b]*
(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/e - (p*PolyLog[2, (Sqrt[b]*(d + e*x))
/(Sqrt[b]*d + Sqrt[-a]*e)])/e
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.44, size = 366, normalized size = 1.82

method	result
risch	$\frac{\ln(ex+d)\ln((b\,x^2+a)^p)}{e}-\frac{p\ln(ex+d)\ln\left(\frac{e\sqrt{-ba}-(ex+d)b+bd}{e\sqrt{-ba}+bd}\right)}{e}-\frac{p\ln(ex+d)\ln\left(\frac{e\sqrt{-ba}+(ex+d)b-bd}{e\sqrt{-ba}-bd}\right)}{e}-\frac{p\,\mathrm{dilog}\left(\frac{e\sqrt{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] ln(e*x+d)/e*ln((b*x^2+a)^p)-p/e*ln(e*x+d)*ln((e*(-b*a)^(1/2)-(e*x+d)*b+b*d)
/(e*(-b*a)^(1/2)+b*d))-p/e*ln(e*x+d)*ln((e*(-b*a)^(1/2)+(e*x+d)*b-b*d)/(e*(
-b*a)^(1/2)-b*d))-p/e*dilog((e*(-b*a)^(1/2)-(e*x+d)*b+b*d)/(e*(-b*a)^(1/2)+
b*d))-p/e*dilog((e*(-b*a)^(1/2)+(e*x+d)*b-b*d)/(e*(-b*a)^(1/2)-b*d))+1/2*I*
ln(e*x+d)/e*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-1/2*I*ln(e*x+d)/
e*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-1/2*I*ln(e*x+d)/e*
Pi*csgn(I*c*(b*x^2+a)^p)^3+1/2*I*ln(e*x+d)/e*Pi*csgn(I*c*(b*x^2+a)^p)^2*csg
n(I*c)+ln(e*x+d)/e*ln(c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/(e*x+d),x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)/(x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)/(d + e*x),x)
```

```
[Out] int(log(c*(a + b*x^2)^p)/(d + e*x), x)
```

3.189 $\int \frac{\log\left(c(a+bx^2)^p\right)}{(d+ex)^2}\,dx$

Optimal. Leaf size=119

$$\frac{2\sqrt{a}\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{bd^2+ae^2}-\frac{2bdp\log(d+ex)}{e\left(bd^2+ae^2\right)}+\frac{bdp\log\left(a+bx^2\right)}{e\left(bd^2+ae^2\right)}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{e(d+ex)}$$

```
[Out] -2*b*d*p*ln(e*x+d)/e/(a*e^2+b*d^2)+b*d*p*ln(b*x^2+a)/e/(a*e^2+b*d^2)-ln(c*(
b*x^2+a)^p)/e/(e*x+d)+2*p*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)*b^(1/2)/(a*e^2+
b*d^2)
```

Rubi [A]
time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2513, 815, 649, 211, 266}

$$\frac{2\sqrt{a}\sqrt{b}\,p\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{ae^2+bd^2}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{e(d+ex)}+\frac{bdp\log\left(a+bx^2\right)}{e\left(ae^2+bd^2\right)}-\frac{2bdp\log(d+ex)}{e\left(ae^2+bd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/(d + e*x)^2,x]
```

```
[Out] (2*Sqrt[a]*Sqrt[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b*d^2 + a*e^2) - (2*b*d*
p*Log[d + e*x])/(e*(b*d^2 + a*e^2)) + (b*d*p*Log[a + b*x^2])/(e*(b*d^2 + a*
e^2)) - Log[c*(a + b*x^2)^p]/(e*(d + e*x))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(c(a+bx^2)^p\right)}{(d+ex)^2}\,dx &= -\frac{\log\left(c(a+bx^2)^p\right)}{e(d+ex)}+\frac{(2bp)\int \frac{x}{(d+ex)(a+bx^2)}\,dx}{e}\\
&= -\frac{\log\left(c(a+bx^2)^p\right)}{e(d+ex)}+\frac{(2bp)\int \left(-\frac{de}{(bd^2+ae^2)(d+ex)}+\frac{ae+bdx}{(bd^2+ae^2)(a+bx^2)}\right)dx}{e}\\
&= -\frac{2bdp\log(d+ex)}{e\left(bd^2+ae^2\right)}-\frac{\log\left(c(a+bx^2)^p\right)}{e(d+ex)}+\frac{(2bp)\int \frac{ae+bdx}{a+bx^2}\,dx}{e\left(bd^2+ae^2\right)}\\
&= -\frac{2bdp\log(d+ex)}{e\left(bd^2+ae^2\right)}-\frac{\log\left(c(a+bx^2)^p\right)}{e(d+ex)}+\frac{(2abp)\int \frac{1}{a+bx^2}\,dx}{bd^2+ae^2}+\frac{(2b^2dp)\int \frac{x}{a+bx^2}\,dx}{e\left(bd^2+ae^2\right)}\\
&= \frac{2\sqrt{a}\,\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{bd^2+ae^2}-\frac{2bdp\log(d+ex)}{e\left(bd^2+ae^2\right)}+\frac{bdp\log\left(a+bx^2\right)}{e\left(bd^2+ae^2\right)}-\frac{\log\left(c(a+bx\right.}{e(d+ex)}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 137, normalized size = 1.15

$$\frac{2\sqrt{a}\,\sqrt{b}\,ep(d+ex)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)-2bdp(d+ex)\log(d+ex)+bd^2p\log\left(a+bx^2\right)+bdepx\log\left(a+bx^2\right)-bd^2\log\left(c(a+bx^2)^p\right)-ae^2\log\left(c(a+bx^2)^p\right)}{e\left(bd^2+ae^2\right)(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/(d + e*x)^2,x]
```

```
[Out] (2*Sqrt[a]*Sqrt[b]*e*p*(d + e*x)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] - 2*b*d*p*(d +
 e*x)*Log[d + e*x] + b*d^2*p*Log[a + b*x^2] + b*d*e*p*x*Log[a + b*x^2] - b*
d^2*Log[c*(a + b*x^2)^p] - a*e^2*Log[c*(a + b*x^2)^p])/(e*(b*d^2 + a*e^2)*(
d + e*x))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.91, size = 755, normalized size = 6.34

method	result
risch	$-\frac{\ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{e(ex+d)}+\frac{-i\pi b\,d^{2}\mathrm{csgn}\left(i\left(b\,x^{2}+a\right)^{p}\right)\mathrm{csgn}\left(ic\left(b\,x^{2}+a\right)^{p}\right)^{2}+i\pi a\mathrm{csgn}\left(ic\left(b\,x^{2}+a\right)^{p}\right)^{3}e^{2}-i\pi b\,d^{2}\mathrm{csgn}\left(ic\left(b\,x^{2}+a\right)^{p}\right)^{2}\mathrm{csgn}(ic)+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/e/(e*x+d)*ln((b*x^2+a)^p)+1/2*(-I*Pi*b*d^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*
(b*x^2+a)^p)^2+I*Pi*a*csgn(I*c*(b*x^2+a)^p)^3*e^2-I*Pi*b*d^2*csgn(I*c*(b*x^
2+a)^p)^2*csgn(I*c)+I*Pi*a*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I
*c)*e^2-I*Pi*a*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)*e^2-I*Pi*a*csgn(I*(b*x^2+a
)^p)*csgn(I*c*(b*x^2+a)^p)^2*e^2+I*Pi*b*d^2*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*b*
d^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+2*sum(_R*ln(((3*a*e
^4-b*d^2*e^2)*_R^2-b*d*e*p*_R+2*b*p^2)*x+4*a*d*e^3*_R^2-a*e^2*p*_R),_R=Root
Of((a*e^4+b*d^2*e^2)*_Z^2-2*b*d*e*p*_Z+b*p^2))*a*e^4*x+2*sum(_R*ln(((3*a*e^
4-b*d^2*e^2)*_R^2-b*d*e*p*_R+2*b*p^2)*x+4*a*d*e^3*_R^2-a*e^2*p*_R),_R=RootO
f((a*e^4+b*d^2*e^2)*_Z^2-2*b*d*e*p*_Z+b*p^2))*b*d^2*e^2*x-4*ln(e*x+d)*b*d*e
*p*x+2*sum(_R*ln(((3*a*e^4-b*d^2*e^2)*_R^2-b*d*e*p*_R+2*b*p^2)*x+4*a*d*e^3*
_R^2-a*e^2*p*_R),_R=RootOf((a*e^4+b*d^2*e^2)*_Z^2-2*b*d*e*p*_Z+b*p^2))*a*d*
e^3+2*sum(_R*ln(((3*a*e^4-b*d^2*e^2)*_R^2-b*d*e*p*_R+2*b*p^2)*x+4*a*d*e^3*_
R^2-a*e^2*p*_R),_R=RootOf((a*e^4+b*d^2*e^2)*_Z^2-2*b*d*e*p*_Z+b*p^2))*b*d^3
*e-4*ln(e*x+d)*b*d^2*p-2*ln(c)*a*e^2-2*d^2*b*ln(c))/(e*x+d)/e/(a*e^2+b*d^2)
```

Maxima [A]
time = 0.57, size = 106, normalized size = 0.89

$$\left(\frac{2\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)e}{(bd^2+ae^2)\sqrt{ab}}+\frac{d\log\left(bx^2+a\right)}{bd^2+ae^2}-\frac{2\,d\log\left(xe+d\right)}{bd^2+ae^2}\right)bpe^{(-1)}-\frac{e^{(-1)}\log\left(\left(bx^2+a\right)^pc\right)}{xe+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] (2*a*arctan(b*x/sqrt(a*b))*e/((b*d^2 + a*e^2)*sqrt(a*b)) + d*log(b*x^2 + a)
/(b*d^2 + a*e^2) - 2*d*log(x*e + d)/(b*d^2 + a*e^2))*b*p*e^(-1) - e^(-1)*lo
g((b*x^2 + a)^p*c)/(x*e + d)
```

Fricas [A]
time = 0.42, size = 257, normalized size = 2.16

$$\left[\frac{(pxe^2+dpe)\sqrt{-ab}\log\left(\frac{bx^2+2\sqrt{-ab}x-a}{bx^2+a}\right)+(bdpxe-ape^2)\log\left(bx^2+a\right)-2\left(bdpxe+bd^2p\right)\log\left(xe+d\right)-(bd^2+ae^2)\log\left(c\right)}{bd^2xe^2+bd^3e+axe^4+ade^3},\frac{2\left(pxe^2+dpe\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)+(bdpxe-ape^2)\log\left(bx^2+a\right)-2\left(bdpxe+bd^2p\right)\log\left(xe+d\right)-(bd^2+ae^2)\log\left(c\right)}{bd^2xe^2+bd^3e+axe^4+ade^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [((p*x*e^2 + d*p*e)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)
) + (b*d*p*x*e - a*p*e^2)*log(b*x^2 + a) - 2*(b*d*p*x*e + b*d^2*p)*log(x*e
+ d) - (b*d^2 + a*e^2)*log(c))/(b*d^2*x*e^2 + b*d^3*e + a*x*e^4 + a*d*e^3),
 (2*(p*x*e^2 + d*p*e)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (b*d*p*x*e - a*p*e^
2)*log(b*x^2 + a) - 2*(b*d*p*x*e + b*d^2*p)*log(x*e + d) - (b*d^2 + a*e^2)*
log(c))/(b*d^2*x*e^2 + b*d^3*e + a*x*e^4 + a*d*e^3)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/(e*x+d)**2,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [A]
time = 4.72, size = 158, normalized size = 1.33

$$\frac{bdp\log\left(bx^2+a\right)}{bd^2e+ae^3}+\frac{2\,abp\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(bd^2+ae^2\right)\sqrt{ab}}-\frac{2\,bdpxe\log\left(xe+d\right)+bd^2p\log\left(bx^2+a\right)+2\,bd^2p\log\left(xe+d\right)+ape^2\log\left(bx^2+a\right)+bd^2\log\left(c\right)+ae^2\log\left(c\right)}{bd^2xe^2+bd^3e+axe^4+ade^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] b*d*p*log(b*x^2 + a)/(b*d^2*e + a*e^3) + 2*a*b*p*arctan(b*x/sqrt(a*b))/((b*
d^2 + a*e^2)*sqrt(a*b)) - (2*b*d*p*x*e*log(x*e + d) + b*d^2*p*log(b*x^2 + a
) + 2*b*d^2*p*log(x*e + d) + a*p*e^2*log(b*x^2 + a) + b*d^2*log(c) + a*e^2*
log(c))/(b*d^2*x*e^2 + b*d^3*e + a*x*e^4 + a*d*e^3)
```

Mupad [B]
time = 1.26, size = 337, normalized size = 2.83

$$\frac{\ln\left(\frac{4\,b^3\,p^2\,x}{e}-\frac{p\,\left(b\,d+e\,\sqrt{-a\,b}\right)\,\left(2\,a\,b^2\,e\,p+2\,b^3\,d\,p\,x-\frac{2\,b^2\,e\,p\,\left(b\,d+e\,\sqrt{-a\,b}\right)\,\left(-b\,x\,d^2+4\,a\,d\,e+3\,a\,x\,e^2\right)}{b\,d^2\,e+a\,e^3}\right)}{b\,d^2\,e+a\,e^3}\right)\,\left(b\,d\,p+e\,p\,\sqrt{-a\,b}\right)}{b\,d^2\,e+a\,e^3}-\frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{e\,\left(d+e\,x\right)}+\frac{\ln\left(\frac{4\,b^3\,p^2\,x}{e}-\frac{p\,\left(b\,d-e\,\sqrt{-a\,b}\right)\,\left(2\,a\,b^2\,e\,p+2\,b^3\,d\,p\,x-\frac{2\,b^2\,e\,p\,\left(b\,d-e\,\sqrt{-a\,b}\right)\,\left(-b\,x\,d^2+4\,a\,d\,e+3\,a\,x\,e^2\right)}{b\,d^2\,e+a\,e^3}\right)}{b\,d^2\,e+a\,e^3}\right)\,\left(b\,d\,p-e\,p\,\sqrt{-a\,b}\right)}{b\,d^2\,e+a\,e^3}-\frac{2\,b\,d\,p\,\ln\left(d+e\,x\right)}{b\,d^2\,e+a\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)/(d + e*x)^2,x)
```

```
[Out] (log((4*b^3*p^2*x)/e - (p*(b*d + e*(-a*b)^(1/2))*(2*a*b^2*e*p + 2*b^3*d*p*x
 - (2*b^2*e*p*(b*d + e*(-a*b)^(1/2))*(4*a*d*e + 3*a*e^2*x - b*d^2*x))/(a*e^
```

```
3 + b*d^2*e)))/(a*e^3 + b*d^2*e))*(b*d*p + e*p*(-a*b)^(1/2)))/(a*e^3 + b*d^
2*e) - log(c*(a + b*x^2)^p)/(e*(d + e*x)) + (log((4*b^3*p^2*x)/e - (p*(b*d
- e*(-a*b)^(1/2))*(2*a*b^2*e*p + 2*b^3*d*p*x - (2*b^2*e*p*(b*d - e*(-a*b)^(
1/2))*(4*a*d*e + 3*a*e^2*x - b*d^2*x))/(a*e^3 + b*d^2*e)))/(a*e^3 + b*d^2*e
))*(b*d*p - e*p*(-a*b)^(1/2)))/(a*e^3 + b*d^2*e) - (2*b*d*p*log(d + e*x))/(
a*e^3 + b*d^2*e)
```

3.190 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{(d+ex)^3}\,dx$

Optimal. Leaf size=174

$$\frac{bdp}{e\left(bd^2+ae^2\right)(d+ex)}+\frac{2\sqrt{a}\,b^{3/2}dp\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\left(bd^2+ae^2\right)^2}-\frac{b(bd^2-ae^2)\,p\log(d+ex)}{e\left(bd^2+ae^2\right)^2}+\frac{b(bd^2-ae^2)\,p\log\left(a+bx^2\right)}{2e\left(bd^2+ae^2\right)^2}$$

```
[Out] b*d*p/e/(a*e^2+b*d^2)/(e*x+d)-b*(-a*e^2+b*d^2)*p*ln(e*x+d)/e/(a*e^2+b*d^2)^
2+1/2*b*(-a*e^2+b*d^2)*p*ln(b*x^2+a)/e/(a*e^2+b*d^2)^2-1/2*ln(c*(b*x^2+a)^p
)/e/(e*x+d)^2+2*b^(3/2)*d*p*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/(a*e^2+b*d^2)
^2
```

Rubi [A]
time = 0.10, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2513, 815, 649, 211, 266}

$$\frac{2\sqrt{a}\,b^{3/2}dp\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\left(ae^2+bd^2\right)^2}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{2e(d+ex)^2}+\frac{bp(bd^2-ae^2)\log\left(a+bx^2\right)}{2e\left(ae^2+bd^2\right)^2}+\frac{bdp}{e(d+ex)\left(ae^2+bd^2\right)}-\frac{bp(bd^2-ae^2)\log(d+ex)}{e\left(ae^2+bd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/(d + e*x)^3,x]
```

```
[Out] (b*d*p)/(e*(b*d^2 + a*e^2)*(d + e*x)) + (2*Sqrt[a]*b^(3/2)*d*p*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/(b*d^2 + a*e^2)^2 - (b*(b*d^2 - a*e^2)*p*Log[d + e*x])/(e*
(b*d^2 + a*e^2)^2) + (b*(b*d^2 - a*e^2)*p*Log[a + b*x^2])/(2*e*(b*d^2 + a*e
^2)^2) - Log[c*(a + b*x^2)^p]/(2*e*(d + e*x)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log\left(c(a+bx^2)^p\right)}{(d+ex)^3}\,dx &= -\frac{\log\left(c(a+bx^2)^p\right)}{2e(d+ex)^2}+\frac{(bp)\int \frac{x}{(d+ex)^2(a+bx^2)}\,dx}{e}\\
&= -\frac{\log\left(c(a+bx^2)^p\right)}{2e(d+ex)^2}+\frac{(bp)\int \left(-\frac{de}{(bd^2+ae^2)(d+ex)^2}+\frac{e(-bd^2+ae^2)}{(bd^2+ae^2)^2(d+ex)}+\frac{b(2ade+(bd^2-ae^2)\ldots}{(bd^2+ae^2)^2(a+bx^2\ldots}\right.}{e}\\
&= \frac{bdp}{e\left(bd^2+ae^2\right)(d+ex)}-\frac{b(bd^2-ae^2)\,p\log(d+ex)}{e\left(bd^2+ae^2\right)^2}-\frac{\log\left(c(a+bx^2)^p\right)}{2e(d+ex)^2}+\frac{(b^2p)\int\ldots}{e}\\
&= \frac{bdp}{e\left(bd^2+ae^2\right)(d+ex)}-\frac{b(bd^2-ae^2)\,p\log(d+ex)}{e\left(bd^2+ae^2\right)^2}-\frac{\log\left(c(a+bx^2)^p\right)}{2e(d+ex)^2}+\frac{(2ab^2d\ldots}{(bd\ldots}\\
&= \frac{bdp}{e\left(bd^2+ae^2\right)(d+ex)}+\frac{2\sqrt{a}\,b^{3/2}dp\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\left(bd^2+ae^2\right)^2}-\frac{b(bd^2-ae^2)\,p\log(d+ex)}{e\left(bd^2+ae^2\right)^2}\ldots
\end{aligned}
$$

Mathematica [A]
time = 0.41, size = 217, normalized size = 1.25

$$
\frac{\frac{bp(d+ex)\left(\left(\sqrt{-a}\,bd^2+2a\sqrt{b}\,de+(-a)^{3/2}e^2\right)(d+ex)\log\left(\sqrt{-a}-\sqrt{b}\,x\right)+\left(\sqrt{-a}\,bd^2-2a\sqrt{b}\,de+(-a)^{3/2}e^2\right)(d+ex)\log\left(\sqrt{-a}+\sqrt{b}\,x\right)+2\sqrt{-a}\,(bd^3+ade^2-(bd^2-ae^2)(d+ex)\log(d+ex))\right)}{\sqrt{-a}\,(bd^2+ae^2)^2}-\log\left(c(a+bx^2)^p\right)}{2e(d+ex)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/(d + e*x)^3,x]
```

```
[Out] ((b*p*(d + e*x)*((Sqrt[-a]*b*d^2 + 2*a*Sqrt[b]*d*e + (-a)^(3/2)*e^2)*(d + e
*x)*Log[Sqrt[-a] - Sqrt[b]*x] + (Sqrt[-a]*b*d^2 - 2*a*Sqrt[b]*d*e + (-a)^(3
/2)*e^2)*(d + e*x)*Log[Sqrt[-a] + Sqrt[b]*x] + 2*Sqrt[-a]*(b*d^3 + a*d*e^2
```

```
- (b*d^2 - a*e^2)*(d + e*x)*Log[d + e*x])))/(Sqrt[-a]*(b*d^2 + a*e^2)^2) -
Log[c*(a + b*x^2)^p])/(2*e*(d + e*x)^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.90, size = 2684, normalized size = 15.43

method	result	size
risch	Expression too large to display	2684

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/e/(e*x+d)^2*ln((b*x^2+a)^p)+1/4*(I*Pi*b^2*d^4*csgn(I*c*(b*x^2+a)^p)^3+
4*b^2*d^4*p+I*Pi*b^2*d^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c
)-2*I*Pi*a*b*d^2*e^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-2*I*Pi*a*b*d^2*e^2*c
sgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+4*a*d^2*b*e^2*p+2*sum(_R*ln(((3*
a^3*e^8+5*a^2*b*d^2*e^6+a*b^2*d^4*e^4-b^3*d^6*e^2)*_R^2+(3*a^2*b*e^5*p+2*a*
b^2*d^2*e^3*p-b^3*d^4*e*p)*_R+2*b^3*d^2*p^2)*x+(4*a^3*d*e^7+8*a^2*b*d^3*e^5
+4*a*b^2*d^5*e^3)*_R^2+2*a*b^2*d*e*p^2),_R=RootOf((a^2*e^6+2*a*b*d^2*e^4+b^
2*d^4*e^2)*_Z^2+(2*a*b*e^3*p-2*b^2*d^2*e*p)*_Z+b^2*p^2))*a^2*e^7*x^2+2*sum(
_R*ln(((3*a^3*e^8+5*a^2*b*d^2*e^6+a*b^2*d^4*e^4-b^3*d^6*e^2)*_R^2+(3*a^2*b*
e^5*p+2*a*b^2*d^2*e^3*p-b^3*d^4*e*p)*_R+2*b^3*d^2*p^2)*x+(4*a^3*d*e^7+8*a^2
*b*d^3*e^5+4*a*b^2*d^5*e^3)*_R^2+2*a*b^2*d*e*p^2),_R=RootOf((a^2*e^6+2*a*b*
d^2*e^4+b^2*d^4*e^2)*_Z^2+(2*a*b*e^3*p-2*b^2*d^2*e*p)*_Z+b^2*p^2))*a^2*d^2*
e^5+2*sum(_R*ln(((3*a^3*e^8+5*a^2*b*d^2*e^6+a*b^2*d^4*e^4-b^3*d^6*e^2)*_R^2
+(3*a^2*b*e^5*p+2*a*b^2*d^2*e^3*p-b^3*d^4*e*p)*_R+2*b^3*d^2*p^2)*x+(4*a^3*d
*e^7+8*a^2*b*d^3*e^5+4*a*b^2*d^5*e^3)*_R^2+2*a*b^2*d*e*p^2),_R=RootOf((a^2*
e^6+2*a*b*d^2*e^4+b^2*d^4*e^2)*_Z^2+(2*a*b*e^3*p-2*b^2*d^2*e*p)*_Z+b^2*p^2)
)*b^2*d^6*e-4*ln(e*x+d)*b^2*d^4*p-I*Pi*b^2*d^4*csgn(I*(b*x^2+a)^p)*csgn(I*c
*(b*x^2+a)^p)^2+2*sum(_R*ln(((3*a^3*e^8+5*a^2*b*d^2*e^6+a*b^2*d^4*e^4-b^3*d
^6*e^2)*_R^2+(3*a^2*b*e^5*p+2*a*b^2*d^2*e^3*p-b^3*d^4*e*p)*_R+2*b^3*d^2*p^2
)*x+(4*a^3*d*e^7+8*a^2*b*d^3*e^5+4*a*b^2*d^5*e^3)*_R^2+2*a*b^2*d*e*p^2),_R=
RootOf((a^2*e^6+2*a*b*d^2*e^4+b^2*d^4*e^2)*_Z^2+(2*a*b*e^3*p-2*b^2*d^2*e*p)
*_Z+b^2*p^2))*b^2*d^4*e^3*x^2+4*sum(_R*ln(((3*a^3*e^8+5*a^2*b*d^2*e^6+a*b^2
*d^4*e^4-b^3*d^6*e^2)*_R^2+(3*a^2*b*e^5*p+2*a*b^2*d^2*e^3*p-b^3*d^4*e*p)*_R
+2*b^3*d^2*p^2)*x+(4*a^3*d*e^7+8*a^2*b*d^3*e^5+4*a*b^2*d^5*e^3)*_R^2+2*a*b^
2*d*e*p^2),_R=RootOf((a^2*e^6+2*a*b*d^2*e^4+b^2*d^4*e^2)*_Z^2+(2*a*b*e^3*p-
2*b^2*d^2*e*p)*_Z+b^2*p^2))*a^2*d*e^6*x+4*sum(_R*ln(((3*a^3*e^8+5*a^2*b*d^2
*e^6+a*b^2*d^4*e^4-b^3*d^6*e^2)*_R^2+(3*a^2*b*e^5*p+2*a*b^2*d^2*e^3*p-b^3*d
^4*e*p)*_R+2*b^3*d^2*p^2)*x+(4*a^3*d*e^7+8*a^2*b*d^3*e^5+4*a*b^2*d^5*e^3)*_
R^2+2*a*b^2*d*e*p^2),_R=RootOf((a^2*e^6+2*a*b*d^2*e^4+b^2*d^4*e^2)*_Z^2+(2*
a*b*e^3*p-2*b^2*d^2*e*p)*_Z+b^2*p^2))*b^2*d^5*e^2*x+4*sum(_R*ln(((3*a^3*e^8
+5*a^2*b*d^2*e^6+a*b^2*d^4*e^4-b^3*d^6*e^2)*_R^2+(3*a^2*b*e^5*p+2*a*b^2*d^2
*e^3*p-b^3*d^4*e*p)*_R+2*b^3*d^2*p^2)*x+(4*a^3*d*e^7+8*a^2*b*d^3*e^5+4*a*b^
2*d^5*e^3)*_R^2+2*a*b^2*d*e*p^2),_R=RootOf((a^2*e^6+2*a*b*d^2*e^4+b^2*d^4*e
```

```
^2)*_Z^2+(2*a*b*e^3*p-2*b^2*d^2*e*p)*_Z+b^2*p^2))*a*b*d^4*e^3+8*ln(e*x+d)*a
*b*d*e^3*p*x+4*a*d*p*e^3*x*b+4*ln(e*x+d)*a*b*e^4*p*x^2-4*ln(e*x+d)*b^2*d^2*
e^2*p*x^2-8*ln(e*x+d)*b^2*d^3*e*p*x+4*ln(e*x+d)*a*b*d^2*e^2*p+4*sum(_R*ln((
(3*a^3*e^8+5*a^2*b*d^2*e^6+a*b^2*d^4*e^4-b^3*d^6*e^2)*_R^2+(3*a^2*b*e^5*p+2
*a*b^2*d^2*e^3*p-b^3*d^4*e*p)*_R+2*b^3*d^2*p^2)*x+(4*a^3*d*e^7+8*a^2*b*d^3*
e^5+4*a*b^2*d^5*e^3)*_R^2+2*a*b^2*d*e*p^2),_R=RootOf((a^2*e^6+2*a*b*d^2*e^4
+b^2*d^4*e^2)*_Z^2+(2*a*b*e^3*p-2*b^2*d^2*e*p)*_Z+b^2*p^2))*a*b*d^2*e^5*x^2
+8*sum(_R*ln(((3*a^3*e^8+5*a^2*b*d^2*e^6+a*b^2*d^4*e^4-b^3*d^6*e^2)*_R^2+(3
*a^2*b*e^5*p+2*a*b^2*d^2*e^3*p-b^3*d^4*e*p)*_R+2*b^3*d^2*p^2)*x+(4*a^3*d*e^
7+8*a^2*b*d^3*e^5+4*a*b^2*d^5*e^3)*_R^2+2*a*b^2*d*e*p^2),_R=RootOf((a^2*e^6
+2*a*b*d^2*e^4+b^2*d^4*e^2)*_Z^2+(2*a*b*e^3*p-2*b^2*d^2*e*p)*_Z+b^2*p^2))*a
*b*d^3*e^4*x-I*Pi*b^2*d^4*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-I*Pi*a^2*e^4*cs
gn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*I*Pi*a*b*d^2*e^2*csgn(I*(b*x^2+a)^p)*csgn
(I*c*(b*x^2+a)^p)*csgn(I*c)-2*ln(c)*b^2*d^4-2*ln(c)*a^2*e^4-4*ln(c)*a*b*d^2
*e^2+4*d^3*p*x*e*b^2-I*Pi*a^2*e^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)
^2+I*Pi*a^2*e^4*csgn(I*c*(b*x^2+a)^p)^3+2*I*Pi*a*b*d^2*e^2*csgn(I*c*(b*x^2+
a)^p)^3+I*Pi*a^2*e^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c))/(
e*x+d)^2/e/(a*e^2+b*d^2)^2
```

Maxima [**A**]
time = 0.55, size = 198, normalized size = 1.14

$$\frac{1}{2}\left(\frac{4\,abd\arctan\left(\frac{bx}{\sqrt{ab}}\right)e}{(b^2d^4+2\,abd^2e^2+a^2e^4)\sqrt{ab}}+\frac{(bd^2-ae^2)\log\left(bx^2+a\right)}{b^2d^4+2\,abd^2e^2+a^2e^4}-\frac{2\,(bd^2-ae^2)\log\left(xe+d\right)}{b^2d^4+2\,abd^2e^2+a^2e^4}+\frac{2\,d}{bd^3+ade^2+(bd^2e+ae^3)x}\right)bpe^{(-1)}-\frac{e^{(-1)}\log\left((bx^2+a)^pc\right)}{2\,(xe+d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(4*a*b*d*arctan(b*x/sqrt(a*b))*e/((b^2*d^4 + 2*a*b*d^2*e^2 + a^2*e^4)*s
qrt(a*b)) + (b*d^2 - a*e^2)*log(b*x^2 + a)/(b^2*d^4 + 2*a*b*d^2*e^2 + a^2*e
^4) - 2*(b*d^2 - a*e^2)*log(x*e + d)/(b^2*d^4 + 2*a*b*d^2*e^2 + a^2*e^4) +
2*d/(b*d^3 + a*d*e^2 + (b*d^2*e + a*e^3)*x))*b*p*e^(-1) - 1/2*e^(-1)*log((b
*x^2 + a)^p*c)/(x*e + d)^2
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(155) = 310.
time = 0.42, size = 724, normalized size = 4.16

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] [1/2*(2*b^2*d^3*p*x*e + 2*b^2*d^4*p + 2*a*b*d*p*x*e^3 + 2*a*b*d^2*p*e^2 + 2
*(b*d*p*x^2*e^3 + 2*b*d^2*p*x*e^2 + b*d^3*p*e)*sqrt(-a*b)*log((b*x^2 + 2*sq
```

```
rt(-a*b)*x - a)/(b*x^2 + a)) + (2*b^2*d^3*p*x*e - 2*a*b*d*p*x*e^3 - (a*b*p*
x^2 + a^2*p)*e^4 + (b^2*d^2*p*x^2 - 3*a*b*d^2*p)*e^2)*log(b*x^2 + a) - 2*(2
*b^2*d^3*p*x*e + b^2*d^4*p - a*b*p*x^2*e^4 - 2*a*b*d*p*x*e^3 + (b^2*d^2*p*x
^2 - a*b*d^2*p)*e^2)*log(x*e + d) - (b^2*d^4 + 2*a*b*d^2*e^2 + a^2*e^4)*log
(c))/(2*b^2*d^5*x*e^2 + b^2*d^6*e + 4*a*b*d^3*x*e^4 + a^2*x^2*e^7 + 2*a^2*d
*x*e^6 + (2*a*b*d^2*x^2 + a^2*d^2)*e^5 + (b^2*d^4*x^2 + 2*a*b*d^4)*e^3), 1/
2*(2*b^2*d^3*p*x*e + 2*b^2*d^4*p + 2*a*b*d*p*x*e^3 + 2*a*b*d^2*p*e^2 + 4*(b
*d*p*x^2*e^3 + 2*b*d^2*p*x*e^2 + b*d^3*p*e)*sqrt(a*b)*arctan(sqrt(a*b)*x/a)
 + (2*b^2*d^3*p*x*e - 2*a*b*d*p*x*e^3 - (a*b*p*x^2 + a^2*p)*e^4 + (b^2*d^2*
p*x^2 - 3*a*b*d^2*p)*e^2)*log(b*x^2 + a) - 2*(2*b^2*d^3*p*x*e + b^2*d^4*p -
 a*b*p*x^2*e^4 - 2*a*b*d*p*x*e^3 + (b^2*d^2*p*x^2 - a*b*d^2*p)*e^2)*log(x*e
 + d) - (b^2*d^4 + 2*a*b*d^2*e^2 + a^2*e^4)*log(c))/(2*b^2*d^5*x*e^2 + b^2*
d^6*e + 4*a*b*d^3*x*e^4 + a^2*x^2*e^7 + 2*a^2*d*x*e^6 + (2*a*b*d^2*x^2 + a^
2*d^2)*e^5 + (b^2*d^4*x^2 + 2*a*b*d^4)*e^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. 2(155) = 310.
time = 4.22, size = 420, normalized size = 2.41

$$\frac{2\,ab^2dp\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2d^4+2\,abd^2e^2+a^2e^4)\sqrt{ab}}+\frac{(b^2d^2p-abpe^2)\log\left(bx^2+a\right)}{2\,(b^2d^4e+2\,abd^2e^3+a^2e^5)}-\frac{2\,b^2d^2px^2e^2\log\left(xe+d\right)+4\,b^2d^3pxe\log\left(xe+d\right)-2\,b^2d^3pxe+b^2d^4p\log\left(bx^2+a\right)+2\,b^2d^4p\log\left(xe+d\right)-2\,b^2d^4p+2\,abd^2pe^2\log\left(bx^2+a\right)-2\,abpx^2e^4\log\left(xe+d\right)-4\,abdpxe^3\log\left(xe+d\right)-2\,abd^2pe^2\log\left(xe+d\right)+b^2d^4\log\left(c\right)-2\,abdpxe^3-2\,abd^2pe^2+2\,abd^2e^2\log\left(c\right)+a^2pe^4\log\left(bx^2+a\right)+a^2e^4\log\left(c\right)}{2\,(b^2d^4x^2e^3+2\,b^2d^5xe^2+b^2d^6e+2\,abd^2x^2e^5+4\,abd^3xe^4+2\,abd^4e^3+a^2x^2e^7+2\,a^2dxe^6+a^2d^2e^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 2*a*b^2*d*p*arctan(b*x/sqrt(a*b))/((b^2*d^4 + 2*a*b*d^2*e^2 + a^2*e^4)*sqrt
(a*b)) + 1/2*(b^2*d^2*p - a*b*p*e^2)*log(b*x^2 + a)/(b^2*d^4*e + 2*a*b*d^2*
e^3 + a^2*e^5) - 1/2*(2*b^2*d^2*p*x^2*e^2*log(x*e + d) + 4*b^2*d^3*p*x*e*lo
g(x*e + d) - 2*b^2*d^3*p*x*e + b^2*d^4*p*log(b*x^2 + a) + 2*b^2*d^4*p*log(x
*e + d) - 2*b^2*d^4*p + 2*a*b*d^2*p*e^2*log(b*x^2 + a) - 2*a*b*p*x^2*e^4*lo
g(x*e + d) - 4*a*b*d*p*x*e^3*log(x*e + d) - 2*a*b*d^2*p*e^2*log(x*e + d) +
b^2*d^4*log(c) - 2*a*b*d*p*x*e^3 - 2*a*b*d^2*p*e^2 + 2*a*b*d^2*e^2*log(c) +
 a^2*p*e^4*log(b*x^2 + a) + a^2*e^4*log(c))/(b^2*d^4*x^2*e^3 + 2*b^2*d^5*x*
e^2 + b^2*d^6*e + 2*a*b*d^2*x^2*e^5 + 4*a*b*d^3*x*e^4 + 2*a*b*d^4*e^3 + a^2
*x^2*e^7 + 2*a^2*d*x*e^6 + a^2*d^2*e^5)
```

Mupad [B]
time = 0.98, size = 272, normalized size = 1.56

$$\frac{\ln\left(b^2\,x+\sqrt{-a\,b^3}\right)\,\left(b^2\,d^2\,p-a\,b\,e^2\,p+2\,d\,e\,p\,\sqrt{-a\,b^3}\right)}{2\,\left(a^2\,e^5+2\,a\,b\,d^2\,e^3+b^2\,d^4\,e\right)}-\frac{\ln\left(d+e\,x\right)\,\left(b^2\,d^2\,p-a\,b\,e^2\,p\right)}{a^2\,e^5+2\,a\,b\,d^2\,e^3+b^2\,d^4\,e}-\frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{2\,e\,\left(d^2+2\,d\,e\,x+e^2\,x^2\right)}-\frac{\ln\left(b^2\,x-\sqrt{-a\,b^3}\right)\,\left(a\,b\,e^2\,p-b^2\,d^2\,p+2\,d\,e\,p\,\sqrt{-a\,b^3}\right)}{2\,\left(a^2\,e^5+2\,a\,b\,d^2\,e^3+b^2\,d^4\,e\right)}+\frac{b\,d\,p}{\left(x\,e^2+d\,e\right)\,\left(b\,d^2+a\,e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)/(d + e*x)^3,x)
```

```
[Out] (log(b^2*x + (-a*b^3)^(1/2))*(b^2*d^2*p - a*b*e^2*p + 2*d*e*p*(-a*b^3)^(1/2
)))/(2*(a^2*e^5 + b^2*d^4*e + 2*a*b*d^2*e^3)) - (log(d + e*x)*(b^2*d^2*p -
a*b*e^2*p))/(a^2*e^5 + b^2*d^4*e + 2*a*b*d^2*e^3) - log(c*(a + b*x^2)^p)/(2
*e*(d^2 + e^2*x^2 + 2*d*e*x)) - (log(b^2*x - (-a*b^3)^(1/2))*(a*b*e^2*p - b
^2*d^2*p + 2*d*e*p*(-a*b^3)^(1/2)))/(2*(a^2*e^5 + b^2*d^4*e + 2*a*b*d^2*e^3
)) + (b*d*p)/((d*e + e^2*x)*(a*e^2 + b*d^2))
```

3.191 $\int (d+ex)^3 \log \left(c(a+bx^3)^p\right) \, dx$

Optimal. Leaf size=320

$$-\frac{3(4bd^3-ae^3)\,px}{4b}-\frac{9}{4}d^2epx^2-de^2px^3-\frac{3}{16}e^3px^4-\frac{\sqrt{3}\,\sqrt[3]{a}\,\left(4bd^3+6\sqrt[3]{a}\,b^{2/3}d^2e-ae^3\right)p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{4b^{4/3}}$$

```
[Out] -3/4*(-a*e^3+4*b*d^3)*p*x/b-9/4*d^2*e*p*x^2-d*e^2*p*x^3-3/16*e^3*p*x^4+1/4*
a^(1/3)*(4*b*d^3-6*a^(1/3)*b^(2/3)*d^2*e-a*e^3)*p*ln(a^(1/3)+b^(1/3)*x)/b^(
4/3)-1/8*a^(1/3)*(4*b*d^3-6*a^(1/3)*b^(2/3)*d^2*e-a*e^3)*p*ln(a^(2/3)-a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/b^(4/3)-1/4*d*(-4*a*e^3+b*d^3)*p*ln(b*x^3+a)/b/e+
1/4*(e*x+d)^4*ln(c*(b*x^3+a)^p)/e-1/4*a^(1/3)*(4*b*d^3+6*a^(1/3)*b^(2/3)*d^
2*e-a*e^3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/b^(4
/3)
```

Rubi [A]

time = 0.51, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {2513, 1850, 1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$-\frac{\sqrt[3]{a}\,p\left(-6\sqrt[3]{a}\,b^{2/3}d^2e-ae^3+4bd^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{8b^{4/3}}-\frac{\sqrt{3}\,\sqrt[3]{a}\,p\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(6\sqrt[3]{a}\,b^{2/3}d^2e-ae^3+4bd^3\right)}{4b^{4/3}}+\frac{\sqrt[3]{a}\,p\left(-6\sqrt[3]{a}\,b^{2/3}d^2e-ae^3+4bd^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{4b^{4/3}}+\frac{(d+ex)^4\log\left(c(a+bx^3)^p\right)}{4e}-\frac{dp(bd^3-4ae^3)\log\left(a+bx^3\right)}{4be}-\frac{3px(4bd^3-ae^3)}{4b}-\frac{9}{4}d^2epx^2-de^2px^3-\frac{3}{16}e^3px^4$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (-3*(4*b*d^3 - a*e^3)*p*x)/(4*b) - (9*d^2*e*p*x^2)/4 - d*e^2*p*x^3 - (3*e^3
*p*x^4)/16 - (Sqrt[3]*a^(1/3)*(4*b*d^3 + 6*a^(1/3)*b^(2/3)*d^2*e - a*e^3)*p
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(4*b^(4/3)) + (a^(1/3)*
(4*b*d^3 - 6*a^(1/3)*b^(2/3)*d^2*e - a*e^3)*p*Log[a^(1/3) + b^(1/3)*x])/(4*
b^(4/3)) - (a^(1/3)*(4*b*d^3 - 6*a^(1/3)*b^(2/3)*d^2*e - a*e^3)*p*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(8*b^(4/3)) - (d*(b*d^3 - 4*a*e^3)*p
*Log[a + b*x^3])/(4*b*e) + ((d + e*x)^4*Log[c*(a + b*x^3)^p])/(4*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
```

```
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\int (d+ex)^3 \log \left(c\left(a+bx^3\right)^p\right)\, dx = \frac{(d+ex)^4 \log \left(c\left(a+bx^3\right)^p\right)}{4e} - \frac{(3bp)\int \frac{x^2(d+ex)^4}{a+bx^3}\, dx}{4e}$$

$$= -\frac{3}{16}e^3px^4 + \frac{(d+ex)^4 \log \left(c\left(a+bx^3\right)^p\right)}{4e} - \frac{(3p)\int \frac{x^2\left(4bd^4+4e\left(4bd^3-ae^3\right)x+24}{a+bx^3}}{16e}$$

$$= -de^2px^3 - \frac{3}{16}e^3px^4 + \frac{(d+ex)^4 \log \left(c\left(a+bx^3\right)^p\right)}{4e} - \frac{p\int \frac{x^2\left(12bd\left(bd^3-4ae^3\right)}{}}{}$$

$$= -de^2px^3 - \frac{3}{16}e^3px^4 + \frac{(d+ex)^4 \log \left(c\left(a+bx^3\right)^p\right)}{4e} - \frac{p\int \left(12e\left(4bd^3-a\right.\right.}{}$$

$$= -\frac{3\left(4bd^3-ae^3\right)px}{4b} - \frac{9}{4}d^2epx^2 - de^2px^3 - \frac{3}{16}e^3px^4 + \frac{(d+ex)^4 \log \left(c(a\right.}{4e}$$

$$= -\frac{3\left(4bd^3-ae^3\right)px}{4b} - \frac{9}{4}d^2epx^2 - de^2px^3 - \frac{3}{16}e^3px^4 + \frac{(d+ex)^4 \log \left(c(a\right.}{4e}$$

$$= -\frac{3\left(4bd^3-ae^3\right)px}{4b} - \frac{9}{4}d^2epx^2 - de^2px^3 - \frac{3}{16}e^3px^4 - \frac{d\left(bd^3-4ae^3\right)p\,\mathrm{lo}}{4be}$$

$$= -\frac{3\left(4bd^3-ae^3\right)px}{4b} - \frac{9}{4}d^2epx^2 - de^2px^3 - \frac{3}{16}e^3px^4 + \frac{\sqrt[3]{a}\left(4bd^3-6\sqrt[3]{a}\right.}{}$$

$$= -\frac{3\left(4bd^3-ae^3\right)px}{4b} - \frac{9}{4}d^2epx^2 - de^2px^3 - \frac{3}{16}e^3px^4 + \frac{\sqrt[3]{a}\left(4bd^3-6\sqrt[3]{a}\right.}{}$$

$$= -\frac{3\left(4bd^3-ae^3\right)px}{4b} - \frac{9}{4}d^2epx^2 - de^2px^3 - \frac{3}{16}e^3px^4 - \frac{\sqrt{3}\sqrt[3]{a}\left(4bd^3+\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.35, size = 264, normalized size = 0.82

$$\frac{\frac{3e\left(-4bd^3+ae^3\right)px}{b} - 9d^2e^2px^2 - 4de^3px^3 - \frac{3}{4}e^4px^4 + 9d^2e^2px^2\, {}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a}\right) + \frac{\sqrt[3]{a}\,e\left(4bd^3-ae^3\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{4/3}} + \frac{\sqrt[3]{a}\,e\left(-4bd^3+ae^3\right)p\left(2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\right)}{2b^{4/3}} - \frac{d\left(bd^3-4ae^3\right)p\log\left(a+bx^3\right)}{b} + (d+ex)^4\log\left(c\left(a+bx^3\right)^p\right)}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*Log[c*(a + b*x^3)^p],x]
```

```
[Out] ((3*e*(-4*b*d^3 + a*e^3)*p*x)/b - 9*d^2*e^2*p*x^2 - 4*d*e^3*p*x^3 - (3*e^4*
p*x^4)/4 + 9*d^2*e^2*p*x^2*Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)] + (
a^(1/3)*e*(4*b*d^3 - a*e^3)*p*Log[a^(1/3) + b^(1/3)*x])/b^(4/3) + (a^(1/3)*
```

```
e*(-4*b*d^3 + a*e^3)*p*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(2*b^(4/3)) - (d*(b*d
^3 - 4*a*e^3)*p*Log[a + b*x^3])/b + (d + e*x)^4*Log[c*(a + b*x^3)^p])/(4*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.79, size = 738, normalized size = 2.31

method	result
risch	$\frac{3ie\pi\, d^2x^2 \operatorname{csgn}\left(ic\left(x^3b+a\right)^p\right)^2 \operatorname{csgn}(ic)}{4} - 3d^3px + e^2\ln(c)\, d\, x^3 + \frac{3e\ln(c)d^2x^2}{2} - \frac{ie^2\pi d\, x^3 \operatorname{csgn}\left(i\left(x^3b+a\right)^p\right)\operatorname{csgn}\left(ic\left(x^3b+a\right)\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*ln(c*(b*x^3+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] -3*d^3*p*x+1/4/b^2*p/e*sum((b*d*(4*a*e^3-b*d^3)*_R^2+6*a*b*d^2*e^2*_R-a^2*e
^4+4*a*b*d^3*e)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))+e^2*ln(c)*d*x^3+3/2*e*ln
(c)*d^2*x^2-1/2*I*e^2*Pi*d*x^3*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*cs
gn(I*c)-3/4*I*e*Pi*d^2*x^2*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I
*c)+1/4*e^3*ln(c)*x^4-3/16*e^3*p*x^4-1/2*I*e^2*Pi*d*x^3*csgn(I*c*(b*x^3+a)^
p)^3-3/4*I*e*Pi*d^2*x^2*csgn(I*c*(b*x^3+a)^p)^3+3/4*e^3/b*a*p*x+1/4*(e*x+d)
^4/e*ln((b*x^3+a)^p)-1/8*I*e^3*Pi*x^4*csgn(I*c*(b*x^3+a)^p)^3-1/2*I*Pi*d^3*
x*csgn(I*c*(b*x^3+a)^p)^3-1/2*I*Pi*d^3*x*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^
3+a)^p)*csgn(I*c)-1/8*I*e^3*Pi*x^4*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p
)*csgn(I*c)+1/2*I*e^2*Pi*d*x^3*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2+
1/2*I*e^2*Pi*d*x^3*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)+3/4*I*e*Pi*d^2*x^2*csg
n(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2+3/4*I*e*Pi*d^2*x^2*csgn(I*c*(b*x^3
+a)^p)^2*csgn(I*c)+ln(c)*d^3*x-9/4*d^2*e*p*x^2+1/2*I*Pi*d^3*x*csgn(I*(b*x^3
+a)^p)*csgn(I*c*(b*x^3+a)^p)^2+1/2*I*Pi*d^3*x*csgn(I*c*(b*x^3+a)^p)^2*csgn(
I*c)+1/8*I*e^3*Pi*x^4*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2+1/8*I*e^3
*Pi*x^4*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)-d*e^2*p*x^3
```

Maxima [A]
time = 0.60, size = 326, normalized size = 1.02

$$\frac{1}{16}bp\left(\frac{4\sqrt{3}\left(6abd^2\left(\frac{a}{b}\right)^{\frac{2}{3}}e+4abd^3\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}e^3\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^2}-\frac{3bx^4e^3+16bdx^3e^2+36bd^2x^2e+12\left(4bd^3-ae^3\right)x}{b^2}+\frac{2\left(6abd^2\left(\frac{a}{b}\right)^{\frac{1}{3}}e-4abd^3+8abd\left(\frac{a}{b}\right)^{\frac{2}{3}}e^2+a^2e^3\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{4\left(6abd^2\left(\frac{a}{b}\right)^{\frac{1}{3}}e-4abd^3-4abd\left(\frac{a}{b}\right)^{\frac{2}{3}}e^2+a^2e^3\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+\frac{1}{4}\left(x^4e^3+4dx^3e^2+6d^2x^2e+4d^3x\right)\log\left(\left(bx^3+a\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] 1/16*b*p*(4*sqrt(3)*(6*a*b*d^2*(a/b)^(2/3)*e + 4*a*b*d^3*(a/b)^(1/3) - a^2*
(a/b)^(1/3)*e^3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2
) - (3*b*x^4*e^3 + 16*b*d*x^3*e^2 + 36*b*d^2*x^2*e + 12*(4*b*d^3 - a*e^3)*x
)/b^2 + 2*(6*a*b*d^2*(a/b)^(1/3)*e - 4*a*b*d^3 + 8*a*b*d*(a/b)^(2/3)*e^2 +
a^2*e^3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) - 4*(6*a*
```

```
b*d^2*(a/b)^(1/3)*e - 4*a*b*d^3 - 4*a*b*d*(a/b)^(2/3)*e^2 + a^2*e^3)*log(x
+ (a/b)^(1/3))/(b^3*(a/b)^(2/3))) + 1/4*(x^4*e^3 + 4*d*x^3*e^2 + 6*d^2*x^2*
e + 4*d^3*x)*log((b*x^3 + a)^p*c)
```

Fricas [C] Result contains complex when optimal does not.
time = 13.83, size = 8463, normalized size = 26.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] -1/16*(16*b*d*p*x^3*e^2 + 36*b*d^2*p*x^2*e + 48*b*d^3*p*x - 2*(8*a*d*p*e^2/
b - 4*(1/2)^(2/3)*(8*a^2*d^2*p^2*e^4/b^2 - (12*a*b*d^5*p^2*e + 5*a^2*d^2*p^
2*e^4)/b^2)*(-I*sqrt(3) + 1)/(128*a^3*d^3*p^3*e^6/b^3 - 24*(12*a*b*d^5*p^2*
e + 5*a^2*d^2*p^2*e^4)*a*d*p*e^2/b^3 + (64*b^3*d^9 + 168*a*b^2*d^6*e^3 + 12
*a^2*b*d^3*e^6 - a^3*e^9)*a*p^3/b^4 + (64*a*b^3*d^9*p^3 + 24*a^2*b^2*d^6*p^
3*e^3 + 4*a^3*b*d^3*p^3*e^6 - a^4*p^3*e^9)/b^4)^(1/3) - (1/2)^(1/3)*(128*a^
3*d^3*p^3*e^6/b^3 - 24*(12*a*b*d^5*p^2*e + 5*a^2*d^2*p^2*e^4)*a*d*p*e^2/b^3
 + (64*b^3*d^9 + 168*a*b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e^9)*a*p^3/b^4
+ (64*a*b^3*d^9*p^3 + 24*a^2*b^2*d^6*p^3*e^3 + 4*a^3*b*d^3*p^3*e^6 - a^4*p^
3*e^9)/b^4)^(1/3)*(I*sqrt(3) + 1))*b*log(-64*b^3*d^9*p^2*x - 168*a*b^2*d^6*
p^2*x*e^3 - 224*a*b^2*d^7*p^2*e^2 - 12*a^2*b*d^3*p^2*x*e^6 - 56*a^2*b*d^4*p
^2*e^5 - 3/2*(8*a*d*p*e^2/b - 4*(1/2)^(2/3)*(8*a^2*d^2*p^2*e^4/b^2 - (12*a*
b*d^5*p^2*e + 5*a^2*d^2*p^2*e^4)/b^2)*(-I*sqrt(3) + 1)/(128*a^3*d^3*p^3*e^6
/b^3 - 24*(12*a*b*d^5*p^2*e + 5*a^2*d^2*p^2*e^4)*a*d*p*e^2/b^3 + (64*b^3*d^
9 + 168*a*b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e^9)*a*p^3/b^4 + (64*a*b^3*d
^9*p^3 + 24*a^2*b^2*d^6*p^3*e^3 + 4*a^3*b*d^3*p^3*e^6 - a^4*p^3*e^9)/b^4)^(
1/3) - (1/2)^(1/3)*(128*a^3*d^3*p^3*e^6/b^3 - 24*(12*a*b*d^5*p^2*e + 5*a^2*
d^2*p^2*e^4)*a*d*p*e^2/b^3 + (64*b^3*d^9 + 168*a*b^2*d^6*e^3 + 12*a^2*b*d^3
*e^6 - a^3*e^9)*a*p^3/b^4 + (64*a*b^3*d^9*p^3 + 24*a^2*b^2*d^6*p^3*e^3 + 4*
a^3*b*d^3*p^3*e^6 - a^4*p^3*e^9)/b^4)^(1/3)*(I*sqrt(3) + 1))^2*b^3*d^2*e +
a^3*p^2*x*e^9 + 4*a^3*d*p^2*e^8 - 1/2*(16*b^3*d^6*p - 56*a*b^2*d^3*p*e^3 +
a^2*b*p*e^6)*(8*a*d*p*e^2/b - 4*(1/2)^(2/3)*(8*a^2*d^2*p^2*e^4/b^2 - (12*a*
b*d^5*p^2*e + 5*a^2*d^2*p^2*e^4)/b^2)*(-I*sqrt(3) + 1)/(128*a^3*d^3*p^3*e^6
/b^3 - 24*(12*a*b*d^5*p^2*e + 5*a^2*d^2*p^2*e^4)*a*d*p*e^2/b^3 + (64*b^3*d^
9 + 168*a*b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e^9)*a*p^3/b^4 + (64*a*b^3*d
^9*p^3 + 24*a^2*b^2*d^6*p^3*e^3 + 4*a^3*b*d^3*p^3*e^6 - a^4*p^3*e^9)/b^4)^(
1/3) - (1/2)^(1/3)*(128*a^3*d^3*p^3*e^6/b^3 - 24*(12*a*b*d^5*p^2*e + 5*a^2*
d^2*p^2*e^4)*a*d*p*e^2/b^3 + (64*b^3*d^9 + 168*a*b^2*d^6*e^3 + 12*a^2*b*d^3
*e^6 - a^3*e^9)*a*p^3/b^4 + (64*a*b^3*d^9*p^3 + 24*a^2*b^2*d^6*p^3*e^3 + 4*
a^3*b*d^3*p^3*e^6 - a^4*p^3*e^9)/b^4)^(1/3)*(I*sqrt(3) + 1))) + 3*(b*p*x^4
- 4*a*p*x)*e^3 - (24*a*d*p*e^2 - 2*sqrt(3/2)*sqrt(1/2)*b*sqrt(-(384*a*b*d^5
*p^2*e - 32*a^2*d^2*p^2*e^4 - 16*(8*a*d*p*e^2/b - 4*(1/2)^(2/3)*(8*a^2*d^2*
p^2*e^4/b^2 - (12*a*b*d^5*p^2*e + 5*a^2*d^2*p^2*e^4)/b^2)*(-I*sqrt(3) + 1)/
```

```
(128*a^3*d^3*p^3*e^6/b^3 - 24*(12*a*b*d^5*p^2*e + 5*a^2*d^2*p^2*e^4)*a*d*p*
e^2/b^3 + (64*b^3*d^9 + 168*a*b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e^9)*a*p
^3/b^4 + (64*a*b^3*d^9*p^3 + 24*a^2*b^2*d^6*p^3*e^3 + 4*a^3*b*d^3*p^3*e^6 -
 a^4*p^3*e^9)/b^4)^(1/3) - (1/2)^(1/3)*(128*a^3*d^3*p^3*e^6/b^3 - 24*(12*a*
b*d^5*p^2*e + 5*a^2*d^2*p^2*e^4)*a*d*p*e^2/b^3 + (64*b^3*d^9 + 168*a*b^2*d^
6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e^9)*a*p^3/b^4 + (64*a*b^3*d^9*p^3 + 24*a^2*
b^2*d^6*p^3*e^3 + 4*a^3*b*d^3*p^3*e^6 - a^4*p^3*e^9)/b^4)^(1/3)*(I*sqrt(3)
+ 1))*a*b*d*p*e^2 + (8*a*d*p*e^2/b - 4*(1/2)^(2/3)*(8*a^2*d^2*p^2*e^4/b^2 -
 (12*a*b*d^5*p^2*e + 5*a^2*d^2*p^2*e^4)/b^2)*(-I*sqrt(3) + 1)/(128*a^3*d^3*
p^3*e^6/b^3 - 24*(12*a*b*d^5*p^2*e + 5*a^2*d^2*p^2*e^4)*a*d*p*e^2/b^3 + (64
*b^3*d^9 + 168*a*b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e^9)*a*p^3/b^4 + (64*
a*b^3*d^9*p^3 + 24*a^2*b^2*d^6*p^3*e^3 + 4*a^3*b*d^3*p^3*e^6 - a^4*p^3*e^9)
/b^4)^(1/3) - (1/2)^(1/3)*(128*a^3*d^3*p^3*e^6/b^3 - 24*(12*a*b*d^5*p^2*e +
 5*a^2*d^2*p^2*e^4)*a*d*p*e^2/b^3 + (64*b^3*d^9 + 168*a*b^2*d^6*e^3 + 12*a^
2*b*d^3*e^6 - a^3*e^9)*a*p^3/b^4 + (64*a*b^3*d^9*p^3 + 24*a^2*b^2*d^6*p^3*e
^3 + 4*a^3*b*d^3*p^3*e^6 - a^4*p^3*e^9)/b^4)^(1/3)*(I*sqrt(3) + 1))^2*b^2)/
b^2) - (8*a*d*p*e^2/b - 4*(1/2)^(2/3)*(8*a^2*d^2*p^2*e^4/b^2 - (12*a*b*d^5*
p^2*e + 5*a^2*d^2*p^2*e^4)/b^2)*(-I*sqrt(3) + 1)/(128*a^3*d^3*p^3*e^6/b^3 -
 24*(12*a*b*d^5*p^2*e + 5*a^2*d^2*p^2*e^4)*a*d*p*e^2/b^3 + (64*b^3*d^9 + 16
8*a*b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e^9)*a*p^3/b^4 + (64*a*b^3*d^9*p^3
 + 24*a^2*b^2*d^6*p^3*e^3 + 4*a^3*b*d^3*p^3*e^6 - a^4*p^3*e^9)/b^4)^(1/3) -
 (1/2)^(1/3)*(128*a^3*d^3*p^3*e^6/b^3 - 24*(12*a*b*d^5*p^2*e + 5*a^2*d^2*p^
2*e^4)*a*d*p*e^2/b^3 + (64*b^3*d^9 + 168*a*b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 -
 a^3*e^9)*a*p^3/b^4 + (64*a*b^3*d^9*p^3 + 24*a^2*b^2*d^6*p^3*e^3 + 4*a^3*b*
d^3*p^3*e^6 - a^4*p^3*e^9)/b^4)^(1/3)*(I*sqrt(3) + 1))*b)*log(-64*b^3*d^9*p
^2*x - 168*a*b^2*d^6*p^2*x*e^3 + 112*a*b^2*d^7*p^2*e^2 - 12*a^2*b*d^3*p^2*x
*e^6 + 28*a^2*b*d^4*p^2*e^5 + 3/4*(8*a*d*p*e^2/b - 4*(1/2)^(2/3)*(8*a^2*d^2
*p^2*e^4/b^2 - (12*a*b*d^5*p^2*e + 5*a^2*d^2*p^2*e^4)/b^2)*(-I*sqrt(3) + 1)
/(128*a^3*d^3*p^3*e^6/b^3 - 24*(12*a*b*d^5*p^2*e + 5*a^2*d^2*p^2*e^4)*a*d*p
*e^2/b^3 + (64*b^3*d^9 + 168*a*b^2*d^6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e^9)*a*
p^3/b^4 + (64*a*b^3*d^9*p^3 + 24*a^2*b^2*d^6*p^3*e^3 + 4*a^3*b*d^3*p^3*e^6
- a^4*p^3*e^9)/b^4)^(1/3) - (1/2)^(1/3)*(128*a^3*d^3*p^3*e^6/b^3 - 24*(12*a
*b*d^5*p^2*e + 5*a^2*d^2*p^2*e^4)*a*d*p*e^2/b^3 + (64*b^3*d^9 + 168*a*b^2*d
^6*e^3 + 12*a^2*b*d^3*e^6 - a^3*e^9)*a*p^3/b^4 ...
```

Sympy [A]
time = 27.19, size = 265, normalized size = 0.83

$$-\frac{3a^2e^3p\operatorname{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(3ta+x\right)\right)\right)}{4b}+3ad^3p\operatorname{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(3ta+x\right)\right)\right)+\frac{9ad^2ep\operatorname{RootSum}\left(27t^3ab^2+1,\left(t\mapsto t\log\left(9t^2ab+x\right)\right)\right)}{2}+ade^2p\left(\begin{cases}\frac{x^3}{a} & \text{for } b=0\\ \frac{\log\left(a+bx^3\right)}{b} & \text{otherwise}\end{cases}\right)+\frac{3ae^3px}{4b}-3d^3px+d^3x\log\left(c\left(a+bx^3\right)^p\right)-\frac{9d^2epx^2}{4}+\frac{3d^2ex^2\log\left(c\left(a+bx^3\right)^p\right)}{2}-de^2px^3+de^2x^3\log\left(c\left(a+bx^3\right)^p\right)-\frac{3e^3px^4}{16}+\frac{e^3x^4\log\left(c\left(a+bx^3\right)^p\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*ln(c*(b*x**3+a)**p),x)
```

```
[Out] -3*a**2*e**3*p*RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(3*_t*a + x)))
/(4*b) + 3*a*d**3*p*RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(3*_t*a +
```

```
 x))) + 9*a*d**2*e*p*RootSum(27*_t**3*a*b**2 + 1, Lambda(_t, _t*log(9*_t**2
*a*b + x)))/2 + a*d*e**2*p*Piecewise((x**3/a, Eq(b, 0)), (log(a + b*x**3)/b
, True)) + 3*a*e**3*p*x/(4*b) - 3*d**3*p*x + d**3*x*log(c*(a + b*x**3)**p)
- 9*d**2*e*p*x**2/4 + 3*d**2*e*x**2*log(c*(a + b*x**3)**p)/2 - d*e**2*p*x**
3 + d*e**2*x**3*log(c*(a + b*x**3)**p) - 3*e**3*p*x**4/16 + e**3*x**4*log(c
*(a + b*x**3)**p)/4
```

Giac [A]

time = 3.84, size = 407, normalized size = 1.27

$$-\frac{\left(6\,abd^2p\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+4\,abd^3p-a^2pe^3\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{4\,ab}+\frac{4\,bpx^4e^3\log\left(bx^3+a\right)+16\,bdpx^3e^2\log\left(bx^3+a\right)+24\,bd^2px^2e\log\left(bx^3+a\right)-3\,bpx^4e^3-16\,bdpx^3e^2-36\,bd^2px^2e+16\,bd^3px\log\left(bx^3+a\right)+4\,bx^4e^3\log\left(c\right)+16\,bdx^3e^2\log\left(c\right)+24\,bd^2x^2e\log\left(c\right)-48\,bd^3px+16\,bd^3x\log\left(c\right)+16\,adpe^2\log\left(bx^3+a\right)+12\,apxe^3}{16\,b}+\frac{\left(4\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}bd^3p-6\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}d^2pe-\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}ape^3\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{4\,b^2}+\frac{\left(4\left(-ab^2\right)^{\frac{1}{3}}bd^3p+6\left(-ab^2\right)^{\frac{2}{3}}d^2pe-\left(-ab^2\right)^{\frac{1}{3}}ape^3\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] -1/4*(6*a*b*d^2*p*(-a/b)^(1/3)*e + 4*a*b*d^3*p - a^2*p*e^3)*(-a/b)^(1/3)*lo
g(abs(x - (-a/b)^(1/3)))/(a*b) + 1/16*(4*b*p*x^4*e^3*log(b*x^3 + a) + 16*b*
d*p*x^3*e^2*log(b*x^3 + a) + 24*b*d^2*p*x^2*e*log(b*x^3 + a) - 3*b*p*x^4*e^
3 - 16*b*d*p*x^3*e^2 - 36*b*d^2*p*x^2*e + 16*b*d^3*p*x*log(b*x^3 + a) + 4*b
*x^4*e^3*log(c) + 16*b*d*x^3*e^2*log(c) + 24*b*d^2*x^2*e*log(c) - 48*b*d^3*
p*x + 16*b*d^3*x*log(c) + 16*a*d*p*e^2*log(b*x^3 + a) + 12*a*p*x*e^3)/b + 1
/4*(4*sqrt(3)*(-a*b^2)^(1/3)*b*d^3*p - 6*sqrt(3)*(-a*b^2)^(2/3)*d^2*p*e - s
qrt(3)*(-a*b^2)^(1/3)*a*p*e^3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/
b)^(1/3))/b^2 + 1/8*(4*(-a*b^2)^(1/3)*b*d^3*p + 6*(-a*b^2)^(2/3)*d^2*p*e -
(-a*b^2)^(1/3)*a*p*e^3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^2
```

Mupad [B]

time = 0.95, size = 536, normalized size = 1.68

$$\ln\left(c\,{\left(b\,x^3+a\right)}^p\right)\,\left(d^3\,x+\frac{3\,d^2\,e\,x^2}{2}+d\,e^2\,x^3+\frac{e^3\,x^4}{4}\right)-x\,\left(3\,d^3\,p-\frac{3\,a\,e^3\,p}{4\,b}\right)+\left(\sum_{k=1}^{3}\ln\left(x\,\left(\frac{9\,a^3\,d\,e^5\,p^2}{4}+\frac{45\,b\,a^2\,d^4\,e^2\,p^2}{4}\right)+\mathrm{root}\left(64\,b^4\,c^3-192\,a\,b^3\,c^2\,d\,e^2\,p+288\,a\,b^3\,c\,d^5\,e\,p^2+120\,a^2\,b^2\,c\,d^2\,e^4\,p^2-4\,a^3\,b\,d^3\,e^6\,p^3-24\,a^2\,b^2\,d^6\,e^3\,p^3-64\,a\,b^3\,d^9\,p^3+a^4\,e^9\,p^3,c,k\right)\,\left(x\,\left(9\,a\,b^2\,d^3\,p-\frac{9\,a^2\,b\,e^3\,p}{4}\right)+\mathrm{root}\left(64\,b^4\,c^3-192\,a\,b^3\,c^2\,d\,e^2\,p+288\,a\,b^3\,c\,d^5\,e\,p^2+120\,a^2\,b^2\,c\,d^2\,e^4\,p^2-4\,a^3\,b\,d^3\,e^6\,p^3-24\,a^2\,b^2\,d^6\,e^3\,p^3-64\,a\,b^3\,d^9\,p^3+a^4\,e^9\,p^3,c,k\right)\,a\,b^2\,9-18\,a^2\,b\,d\,e^2\,p\right)+\frac{45\,a^3\,d^2\,e^4\,p^2}{8}+\frac{27\,a^2\,b\,d^5\,e\,p^2}{2}\right)\,\mathrm{root}\left(64\,b^4\,c^3-192\,a\,b^3\,c^2\,d\,e^2\,p+288\,a\,b^3\,c\,d^5\,e\,p^2+120\,a^2\,b^2\,c\,d^2\,e^4\,p^2-4\,a^3\,b\,d^3\,e^6\,p^3-24\,a^2\,b^2\,d^6\,e^3\,p^3-64\,a\,b^3\,d^9\,p^3+a^4\,e^9\,p^3,c,k\right)\right)-\frac{3\,e^3\,p\,x^4}{16}-\frac{9\,d^2\,e\,p\,x^2}{4}-d\,e^2\,p\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)*(d + e*x)^3,x)
```

```
[Out] log(c*(a + b*x^3)^p)*(d^3*x + (e^3*x^4)/4 + (3*d^2*e*x^2)/2 + d*e^2*x^3) -
x*(3*d^3*p - (3*a*e^3*p)/(4*b)) + symsum(log(x*((9*a^3*d*e^5*p^2)/4 + (45*a
^2*b*d^4*e^2*p^2)/4) + root(64*b^4*c^3 - 192*a*b^3*c^2*d*e^2*p + 288*a*b^3*
c*d^5*e*p^2 + 120*a^2*b^2*c*d^2*e^4*p^2 - 4*a^3*b*d^3*e^6*p^3 - 24*a^2*b^2*
d^6*e^3*p^3 - 64*a*b^3*d^9*p^3 + a^4*e^9*p^3, c, k)*(x*(9*a*b^2*d^3*p - (9*
a^2*b*e^3*p)/4) + 9*root(64*b^4*c^3 - 192*a*b^3*c^2*d*e^2*p + 288*a*b^3*c*d
^5*e*p^2 + 120*a^2*b^2*c*d^2*e^4*p^2 - 4*a^3*b*d^3*e^6*p^3 - 24*a^2*b^2*d^6
*e^3*p^3 - 64*a*b^3*d^9*p^3 + a^4*e^9*p^3, c, k)*a*b^2 - 18*a^2*b*d*e^2*p)
+ (45*a^3*d^2*e^4*p^2)/8 + (27*a^2*b*d^5*e*p^2)/2)*root(64*b^4*c^3 - 192*a*
b^3*c^2*d*e^2*p + 288*a*b^3*c*d^5*e*p^2 + 120*a^2*b^2*c*d^2*e^4*p^2 - 4*a^3
*b*d^3*e^6*p^3 - 24*a^2*b^2*d^6*e^3*p^3 - 64*a*b^3*d^9*p^3 + a^4*e^9*p^3, c
, k), k, 1, 3) - (3*e^3*p*x^4)/16 - (9*d^2*e*p*x^2)/4 - d*e^2*p*x^3
```

3.192 $\int (d+ex)^2 \log \left(c(a+bx^3)^p\right) \, dx$

Optimal. Leaf size=250

$$-3d^2px-\frac{3}{2}depx^2-\frac{1}{3}e^2px^3-\frac{\sqrt{3}\,\sqrt[3]{a}\,d\left(\sqrt[3]{b}\,d+\sqrt[3]{a}\,e\right)p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{b^{2/3}}+\frac{\sqrt[3]{a}\,d\left(\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)p\log\left(\right.}{b^{2/3}}$$

```
[Out] -3*d^2*p*x-3/2*d*e*p*x^2-1/3*e^2*p*x^3+a^(1/3)*d*(b^(1/3)*d-a^(1/3)*e)*p*ln
(a^(1/3)+b^(1/3)*x)/b^(2/3)-1/2*a^(1/3)*d*(b^(1/3)*d-a^(1/3)*e)*p*ln(a^(2/3
)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(2/3)-1/3*(-a*e^3+b*d^3)*p*ln(b*x^3+a)/b
/e+1/3*(e*x+d)^3*ln(c*(b*x^3+a)^p)/e-a^(1/3)*d*(b^(1/3)*d+a^(1/3)*e)*p*arct
an(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/b^(2/3)
```

Rubi [A]

time = 0.32, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {2513, 1850, 1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$-\frac{\sqrt[3]{a}\,dp\left(\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{2b^{2/3}}-\frac{\sqrt{3}\,\sqrt[3]{a}\,dp\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(\sqrt[3]{a}\,e+\sqrt[3]{b}\,d\right)}{b^{2/3}}+\frac{\sqrt[3]{a}\,dp\left(\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{2/3}}+\frac{(d+ex)^3\log\left(c(a+bx^3)^p\right)}{3e}-\frac{p(bd^3-ae^3)\log\left(a+bx^3\right)}{3be}-3d^2px-\frac{3}{2}depx^2-\frac{1}{3}e^2px^3$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*Log[c*(a + b*x^3)^p],x]
```

```
[Out] -3*d^2*p*x - (3*d*e*p*x^2)/2 - (e^2*p*x^3)/3 - (Sqrt[3]*a^(1/3)*d*(b^(1/3)*
d + a^(1/3)*e)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/b^(2/3)
 + (a^(1/3)*d*(b^(1/3)*d - a^(1/3)*e)*p*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) -
 (a^(1/3)*d*(b^(1/3)*d - a^(1/3)*e)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/(2*b^(2/3)) - ((b*d^3 - a*e^3)*p*Log[a + b*x^3])/(3*b*e) + ((d +
 e*x)^3*Log[c*(a + b*x^3)^p])/(3*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\int (d+ex)^2 \log\left(c\left(a+bx^3\right)^p\right)\,dx = \frac{(d+ex)^3\log\left(c\left(a+bx^3\right)^p\right)}{3e} - \frac{(bp)\int \frac{x^2(d+ex)^3}{a+bx^3}\,dx}{e}$$

$$= -\frac{1}{3}e^2px^3 + \frac{(d+ex)^3\log\left(c\left(a+bx^3\right)^p\right)}{3e} - \frac{p\int \frac{x^2\left(3\left(bd^3-ae^3\right)+9bd^2ex+9bde^2x^2\right)}{a+bx^3}\,d}{3e}$$

$$= -\frac{1}{3}e^2px^3 + \frac{(d+ex)^3\log\left(c\left(a+bx^3\right)^p\right)}{3e} - \frac{p\int \left(9d^2e+9de^2x-\frac{3\left(3ad^2e+3a\right.}{}\right.}{3e}$$

$$= -3d^2px - \frac{3}{2}depx^2 - \frac{1}{3}e^2px^3 + \frac{(d+ex)^3\log\left(c\left(a+bx^3\right)^p\right)}{3e} + \frac{p\int \frac{3ad^2e+3a}{}}{}$$

$$= -3d^2px - \frac{3}{2}depx^2 - \frac{1}{3}e^2px^3 + \frac{(d+ex)^3\log\left(c\left(a+bx^3\right)^p\right)}{3e} + \frac{p\int \frac{3ad^2e+3a}{a+bx^3}}{e}$$

$$= -3d^2px - \frac{3}{2}depx^2 - \frac{1}{3}e^2px^3 - \frac{\left(bd^3-ae^3\right)p\log\left(a+bx^3\right)}{3be} + \frac{(d+ex)^3\log}{}$$

$$= -3d^2px - \frac{3}{2}depx^2 - \frac{1}{3}e^2px^3 + \frac{\sqrt[3]{a}\,d\left(\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{2/3}}$$

$$= -3d^2px - \frac{3}{2}depx^2 - \frac{1}{3}e^2px^3 + \frac{\sqrt[3]{a}\,d\left(\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{2/3}}$$

$$= -3d^2px - \frac{3}{2}depx^2 - \frac{1}{3}e^2px^3 - \frac{\sqrt{3}\,\sqrt[3]{a}\,d\left(\sqrt[3]{b}\,d+\sqrt[3]{a}\,e\right)p\tan^{-1}\left(\frac{\sqrt[3]{a}-}{\sqrt{3}}\right.}{b^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.22, size = 218, normalized size = 0.87

$$-\frac{\frac{p\left(18bd^2ex+9bde^2x^2+2be^3x^3-9bde^2x^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a}\right)-6\sqrt[3]{a}\,b^{2/3}d^2e\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+3\sqrt[3]{a}\,b^{2/3}d^2e\left(2\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)\right)+2(bd^3-ae^3)\log(a+bx^3)\right)}{2b}+(d+ex)^3\log\left(c(a+bx^3)^p\right)}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (-1/2*(p*(18*b*d^2*e*x + 9*b*d*e^2*x^2 + 2*b*e^3*x^3 - 9*b*d*e^2*x^2*Hyperg
eometric2F1[2/3, 1, 5/3, -((b*x^3)/a)] - 6*a^(1/3)*b^(2/3)*d^2*e*Log[a^(1/3
) + b^(1/3)*x] + 3*a^(1/3)*b^(2/3)*d^2*e*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*
x)/a^(1/3))/Sqrt[3]] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]) + 2*
(b*d^3 - a*e^3)*Log[a + b*x^3]))/b + (d + e*x)^3*Log[c*(a + b*x^3)^p])/(3*e
)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.91, size = 537, normalized size = 2.15

method	result
risch	$\frac{(ex+d)^3\ln\left(\left(x^3b+a\right)^p\right)}{3e}-\frac{ie^2\pi\,x^3\mathrm{csgn}\left(i\left(x^3b+a\right)^p\right)\mathrm{csgn}\left(ic\left(x^3b+a\right)^p\right)\mathrm{csgn}(ic)}{6}+\frac{i\pi\,d^2\mathrm{csgn}\left(i\left(x^3b+a\right)^p\right)\mathrm{csgn}\left(ic\left(x^3b+a\right)^p\right)^2x}{2}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*ln(c*(b*x^3+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(e*x+d)^3/e*ln((b*x^3+a)^p)-1/6*I*e^2*Pi*x^3*csgn(I*(b*x^3+a)^p)*csgn(I
*c*(b*x^3+a)^p)*csgn(I*c)+1/2*I*Pi*d^2*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+
a)^p)^2*x-1/6*I*e^2*Pi*x^3*csgn(I*c*(b*x^3+a)^p)^3+1/2*I*e*Pi*d*x^2*csgn(I*
(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2-1/2*I*e*Pi*d*x^2*csgn(I*(b*x^3+a)^p)*c
sgn(I*c*(b*x^3+a)^p)*csgn(I*c)-1/2*I*Pi*d^2*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b
*x^3+a)^p)*csgn(I*c)*x-1/2*I*Pi*d^2*csgn(I*c*(b*x^3+a)^p)^3*x+1/6*I*e^2*Pi*
x^3*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2+1/2*I*e*Pi*d*x^2*csgn(I*c*(
b*x^3+a)^p)^2*csgn(I*c)+1/2*I*Pi*d^2*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)*x+1/
6*I*e^2*Pi*x^3*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)-1/2*I*e*Pi*d*x^2*csgn(I*c*
(b*x^3+a)^p)^3+1/3*e^2*ln(c)*x^3-1/3*e^2*p*x^3+ln(c)*d*e*x^2-3/2*d*e*p*x^2+
ln(c)*d^2*x-3*d^2*p*x+1/3*p/b/e*sum(((a*e^3-b*d^3)*_R^2+3*e^2*d*a*_R+3*e*d^
2*a)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Maxima [A]
time = 0.59, size = 251, normalized size = 1.00

$$-\frac{1}{6}\,bp\left(\frac{2\,x^3e^2+9\,dx^2e+18\,d^2x}{b}-\frac{6\,\sqrt{3}\,\left(abd\left(\frac{a}{b}\right)^{\frac{2}{3}}e+abd^2\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\,\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^2}-\frac{\left(3\,ad\left(\frac{a}{b}\right)^{\frac{1}{3}}e-3\,ad^2+2\,a\left(\frac{a}{b}\right)^{\frac{2}{3}}e^2\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\,\left(3\,ad\left(\frac{a}{b}\right)^{\frac{1}{3}}e-3\,ad^2-a\left(\frac{a}{b}\right)^{\frac{2}{3}}e^2\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+\frac{1}{3}\,\left(x^3e^2+3\,dx^2e+3\,d^2x\right)\log\left(\left(bx^3+a\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] -1/6*b*p*((2*x^3*e^2 + 9*d*x^2*e + 18*d^2*x)/b - 6*sqrt(3)*(a*b*d*(a/b)^(2/
3)*e + a*b*d^2*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1
/3))/(a*b^2) - (3*a*d*(a/b)^(1/3)*e - 3*a*d^2 + 2*a*(a/b)^(2/3)*e^2)*log(x^
2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a/b)^(2/3)) + 2*(3*a*d*(a/b)^(1/3)*e
 - 3*a*d^2 - a*(a/b)^(2/3)*e^2)*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))) + 1
/3*(x^3*e^2 + 3*d*x^2*e + 3*d^2*x)*log((b*x^3 + a)^p*c)
```

Fricas [C] Result contains complex when optimal does not.
time = 2.81, size = 5590, normalized size = 22.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] -1/12*(4*b*p*x^3*e^2 + 18*b*d*p*x^2*e + 36*b*d^2*p*x + 2*(2*(1/2)^(2/3)*(a^
2*p^2*e^4/b^2 - (9*a*b*d^3*p^2*e + a^2*p^2*e^4)/b^2)*(-I*sqrt(3) + 1)/(27*(
b*d^3 + a*e^3)*a*d^3*p^3/b^2 + 2*a^3*p^3*e^6/b^3 - 3*(9*a*b*d^3*p^2*e + a^2
*p^2*e^4)*a*p*e^2/b^3 + (27*a*b^2*d^6*p^3 + a^3*p^3*e^6)/b^3)^(1/3) - 2*a*p
*e^2/b + (1/2)^(1/3)*(27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 + 2*a^3*p^3*e^6/b^3
- 3*(9*a*b*d^3*p^2*e + a^2*p^2*e^4)*a*p*e^2/b^3 + (27*a*b^2*d^6*p^3 + a^3*p
^3*e^6)/b^3)^(1/3)*(I*sqrt(3) + 1))*b*log(9*b^2*d^5*p^2*x + 9*a*b*d^2*p^2*x
*e^3 + 15*a*b*d^3*p^2*e^2 + a^2*p^2*e^5 + 1/4*(2*(1/2)^(2/3)*(a^2*p^2*e^4/b
^2 - (9*a*b*d^3*p^2*e + a^2*p^2*e^4)/b^2)*(-I*sqrt(3) + 1)/(27*(b*d^3 + a*e
^3)*a*d^3*p^3/b^2 + 2*a^3*p^3*e^6/b^3 - 3*(9*a*b*d^3*p^2*e + a^2*p^2*e^4)*a
*p*e^2/b^3 + (27*a*b^2*d^6*p^3 + a^3*p^3*e^6)/b^3)^(1/3) - 2*a*p*e^2/b + (1
/2)^(1/3)*(27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 + 2*a^3*p^3*e^6/b^3 - 3*(9*a*b*
d^3*p^2*e + a^2*p^2*e^4)*a*p*e^2/b^3 + (27*a*b^2*d^6*p^3 + a^3*p^3*e^6)/b^3
)^(1/3)*(I*sqrt(3) + 1))^2*b^2*e - 1/2*(3*b^2*d^3*p - 2*a*b*p*e^3)*(2*(1/2)
^(2/3)*(a^2*p^2*e^4/b^2 - (9*a*b*d^3*p^2*e + a^2*p^2*e^4)/b^2)*(-I*sqrt(3)
+ 1)/(27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 + 2*a^3*p^3*e^6/b^3 - 3*(9*a*b*d^3*p
^2*e + a^2*p^2*e^4)*a*p*e^2/b^3 + (27*a*b^2*d^6*p^3 + a^3*p^3*e^6)/b^3)^(1/
3) - 2*a*p*e^2/b + (1/2)^(1/3)*(27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 + 2*a^3*p^
3*e^6/b^3 - 3*(9*a*b*d^3*p^2*e + a^2*p^2*e^4)*a*p*e^2/b^3 + (27*a*b^2*d^6*p
^3 + a^3*p^3*e^6)/b^3)^(1/3)*(I*sqrt(3) + 1))) - (6*a*p*e^2 + (2*(1/2)^(2/3
)*(a^2*p^2*e^4/b^2 - (9*a*b*d^3*p^2*e + a^2*p^2*e^4)/b^2)*(-I*sqrt(3) + 1)/
(27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 + 2*a^3*p^3*e^6/b^3 - 3*(9*a*b*d^3*p^2*e
+ a^2*p^2*e^4)*a*p*e^2/b^3 + (27*a*b^2*d^6*p^3 + a^3*p^3*e^6)/b^3)^(1/3) -
2*a*p*e^2/b + (1/2)^(1/3)*(27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 + 2*a^3*p^3*e^6
/b^3 - 3*(9*a*b*d^3*p^2*e + a^2*p^2*e^4)*a*p*e^2/b^3 + (27*a*b^2*d^6*p^3 +
a^3*p^3*e^6)/b^3)^(1/3)*(I*sqrt(3) + 1))*b + 3*sqrt(1/3)*b*sqrt(-(144*a*b*d
^3*p^2*e + 4*a^2*p^2*e^4 + 4*(2*(1/2)^(2/3)*(a^2*p^2*e^4/b^2 - (9*a*b*d^3*p
```

```
^2*e + a^2*p^2*e^4)/b^2)*(-I*sqrt(3) + 1)/(27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2
 + 2*a^3*p^3*e^6/b^3 - 3*(9*a*b*d^3*p^2*e + a^2*p^2*e^4)*a*p*e^2/b^3 + (27*
a*b^2*d^6*p^3 + a^3*p^3*e^6)/b^3)^(1/3) - 2*a*p*e^2/b + (1/2)^(1/3)*(27*(b*
d^3 + a*e^3)*a*d^3*p^3/b^2 + 2*a^3*p^3*e^6/b^3 - 3*(9*a*b*d^3*p^2*e + a^2*p
^2*e^4)*a*p*e^2/b^3 + (27*a*b^2*d^6*p^3 + a^3*p^3*e^6)/b^3)^(1/3)*(I*sqrt(3
) + 1))*a*b*p*e^2 + (2*(1/2)^(2/3)*(a^2*p^2*e^4/b^2 - (9*a*b*d^3*p^2*e + a^
2*p^2*e^4)/b^2)*(-I*sqrt(3) + 1)/(27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 + 2*a^3*
p^3*e^6/b^3 - 3*(9*a*b*d^3*p^2*e + a^2*p^2*e^4)*a*p*e^2/b^3 + (27*a*b^2*d^6
*p^3 + a^3*p^3*e^6)/b^3)^(1/3) - 2*a*p*e^2/b + (1/2)^(1/3)*(27*(b*d^3 + a*e
^3)*a*d^3*p^3/b^2 + 2*a^3*p^3*e^6/b^3 - 3*(9*a*b*d^3*p^2*e + a^2*p^2*e^4)*a
*p*e^2/b^3 + (27*a*b^2*d^6*p^3 + a^3*p^3*e^6)/b^3)^(1/3)*(I*sqrt(3) + 1))^2
*b^2)/b^2))*log(18*b^2*d^5*p^2*x + 18*a*b*d^2*p^2*x*e^3 - 15*a*b*d^3*p^2*e^
2 - a^2*p^2*e^5 - 1/4*(2*(1/2)^(2/3)*(a^2*p^2*e^4/b^2 - (9*a*b*d^3*p^2*e +
a^2*p^2*e^4)/b^2)*(-I*sqrt(3) + 1)/(27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 + 2*a^
3*p^3*e^6/b^3 - 3*(9*a*b*d^3*p^2*e + a^2*p^2*e^4)*a*p*e^2/b^3 + (27*a*b^2*d
^6*p^3 + a^3*p^3*e^6)/b^3)^(1/3) - 2*a*p*e^2/b + (1/2)^(1/3)*(27*(b*d^3 + a
*e^3)*a*d^3*p^3/b^2 + 2*a^3*p^3*e^6/b^3 - 3*(9*a*b*d^3*p^2*e + a^2*p^2*e^4)
*a*p*e^2/b^3 + (27*a*b^2*d^6*p^3 + a^3*p^3*e^6)/b^3)^(1/3)*(I*sqrt(3) + 1))
^2*b^2*e + 1/2*(3*b^2*d^3*p - 2*a*b*p*e^3)*(2*(1/2)^(2/3)*(a^2*p^2*e^4/b^2
- (9*a*b*d^3*p^2*e + a^2*p^2*e^4)/b^2)*(-I*sqrt(3) + 1)/(27*(b*d^3 + a*e^3)
*a*d^3*p^3/b^2 + 2*a^3*p^3*e^6/b^3 - 3*(9*a*b*d^3*p^2*e + a^2*p^2*e^4)*a*p*
e^2/b^3 + (27*a*b^2*d^6*p^3 + a^3*p^3*e^6)/b^3)^(1/3) - 2*a*p*e^2/b + (1/2)
^(1/3)*(27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 + 2*a^3*p^3*e^6/b^3 - 3*(9*a*b*d^3
*p^2*e + a^2*p^2*e^4)*a*p*e^2/b^3 + (27*a*b^2*d^6*p^3 + a^3*p^3*e^6)/b^3)^(
1/3)*(I*sqrt(3) + 1)) + 3/4*sqrt(1/3)*(6*b^2*d^3*p + 2*a*b*p*e^3 + (2*(1/2)
^(2/3)*(a^2*p^2*e^4/b^2 - (9*a*b*d^3*p^2*e + a^2*p^2*e^4)/b^2)*(-I*sqrt(3)
+ 1)/(27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 + 2*a^3*p^3*e^6/b^3 - 3*(9*a*b*d^3*p
^2*e + a^2*p^2*e^4)*a*p*e^2/b^3 + (27*a*b^2*d^6*p^3 + a^3*p^3*e^6)/b^3)^(1/
3) - 2*a*p*e^2/b + (1/2)^(1/3)*(27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 + 2*a^3*p^
3*e^6/b^3 - 3*(9*a*b*d^3*p^2*e + a^2*p^2*e^4)*a*p*e^2/b^3 + (27*a*b^2*d^6*p
^3 + a^3*p^3*e^6)/b^3)^(1/3)*(I*sqrt(3) + 1))*b^2*e)*sqrt(-(144*a*b*d^3*p^2
*e + 4*a^2*p^2*e^4 + 4*(2*(1/2)^(2/3)*(a^2*p^2*e^4/b^2 - (9*a*b*d^3*p^2*e +
 a^2*p^2*e^4)/b^2)*(-I*sqrt(3) + 1)/(27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 + 2*a
^3*p^3*e^6/b^3 - 3*(9*a*b*d^3*p^2*e + a^2*p^2*e^4)*a*p*e^2/b^3 + (27*a*b^2*
d^6*p^3 + a^3*p^3*e^6)/b^3)^(1/3) - 2*a*p*e^2/b + (1/2)^(1/3)*(27*(b*d^3 +
a*e^3)*a*d^3*p^3/b^2 + 2*a^3*p^3*e^6/b^3 - 3*(9*a*b*d^3*p^2*e + a^2*p^2*e^4
)*a*p*e^2/b^3 + (27*a*b^2*d^6*p^3 + a^3*p^3*e^6)/b^3)^(1/3)*(I*sqrt(3) + 1)
)*a*b*p*e^2 + (2*(1/2)^(2/3)*(a^2*p^2*e^4/b^2 - (9*a*b*d^3*p^2*e + a^2*p^2*
e^4)/b^2)*(-I*sqrt(3) + 1)/(27*(b*d^3 + a*e^3)*a*d^3*p^3/b^2 + 2*a^3*p^3*e^
6/b^3 - 3*(9*a*b*d^3*p^2*e + a^2*p^2*e^4)*a*p*e...
```

Sympy [A]
time = 18.54, size = 173, normalized size = 0.69

$$3ad^2p\operatorname{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(3ta+x\right)\right)\right)+3adep\operatorname{RootSum}\left(27t^3ab^2+1,\left(t\mapsto t\log\left(9t^2ab+x\right)\right)\right)+\frac{ae^2p\left(\begin{cases}\frac{x^3}{a} & \text{for}\ b=0\\ \frac{\log\left(a+bx^3\right)}{b} & \text{otherwise}\end{cases}\right)}{3}-3d^2px+d^2x\log\left(c\left(a+bx^3\right)^p\right)-\frac{3depx^2}{2}+dex^2\log\left(c\left(a+bx^3\right)^p\right)-\frac{e^2px^3}{3}+\frac{e^2x^3\log\left(c\left(a+bx^3\right)^p\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*ln(c*(b*x**3+a)**p),x)
```

```
[Out] 3*a*d**2*p*RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(3*_t*a + x))) + 3
*a*d*e*p*RootSum(27*_t**3*a*b**2 + 1, Lambda(_t, _t*log(9*_t**2*a*b + x)))
+ a*e**2*p*Piecewise((x**3/a, Eq(b, 0)), (log(a + b*x**3)/b, True))/3 - 3*d
**2*p*x + d**2*x*log(c*(a + b*x**3)**p) - 3*d*e*p*x**2/2 + d*e*x**2*log(c*(
a + b*x**3)**p) - e**2*p*x**3/3 + e**2*x**3*log(c*(a + b*x**3)**p)/3
```

Giac [A]
time = 5.00, size = 298, normalized size = 1.19

$$-\frac{\left(adp\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+ad^{2}p\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a}+\frac{2\,bpx^{3}e^{2}\log\left(bx^{3}+a\right)+6\,bdpx^{2}e\log\left(bx^{3}+a\right)-2\,bpx^{3}e^{2}-9\,bdpx^{2}e+6\,bd^{2}px\log\left(bx^{3}+a\right)+2\,bx^{3}e^{2}\log\left(c\right)+6\,bdx^{2}e\log\left(c\right)-18\,bd^{2}px+6\,bd^{2}x\log\left(c\right)+2\,ape^{2}\log\left(bx^{3}+a\right)}{6\,b}+\frac{\left(\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}bd^{2}p-\sqrt{3}\left(-ab^{2}\right)^{\frac{2}{3}}dpe\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{2}}+\frac{\left(\left(-ab^{2}\right)^{\frac{1}{3}}bd^{2}p+\left(-ab^{2}\right)^{\frac{2}{3}}dpe\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] -(a*d*p*(-a/b)^(1/3)*e + a*d^2*p)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a
 + 1/6*(2*b*p*x^3*e^2*log(b*x^3 + a) + 6*b*d*p*x^2*e*log(b*x^3 + a) - 2*b*p
*x^3*e^2 - 9*b*d*p*x^2*e + 6*b*d^2*p*x*log(b*x^3 + a) + 2*b*x^3*e^2*log(c)
+ 6*b*d*x^2*e*log(c) - 18*b*d^2*p*x + 6*b*d^2*x*log(c) + 2*a*p*e^2*log(b*x^
3 + a))/b + (sqrt(3)*(-a*b^2)^(1/3)*b*d^2*p - sqrt(3)*(-a*b^2)^(2/3)*d*p*e)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^2 + 1/2*((-a*b^2)^
(1/3)*b*d^2*p + (-a*b^2)^(2/3)*d*p*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/
3))/b^2
```

Mupad [B]
time = 0.32, size = 358, normalized size = 1.43

$$\left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(27\,b^{3}\,c^{3}-27\,a\,b^{2}\,c^{2}\,e^{2}\,p+81\,a\,b^{2}\,c\,d^{3}\,e\,p^{2}+9\,a^{2}\,b\,c\,e^{4}\,p^{2}-27\,a\,b^{2}\,d^{6}\,p^{3}-a^{3}\,e^{6}\,p^{3},c,k\right)\,\left(\mathrm{root}\left(27\,b^{3}\,c^{3}-27\,a\,b^{2}\,c^{2}\,e^{2}\,p+81\,a\,b^{2}\,c\,d^{3}\,e\,p^{2}+9\,a^{2}\,b\,c\,e^{4}\,p^{2}-27\,a\,b^{2}\,d^{6}\,p^{3}-a^{3}\,e^{6}\,p^{3},c,k\right)\,a\,b^{2}\,9-6\,a^{2}\,b\,e^{2}\,p+9\,a\,b^{2}\,d^{2}\,p\,x\right)+a^{3}\,e^{4}\,p^{2}+9\,a^{2}\,b\,d^{3}\,e\,p^{2}+6\,a^{2}\,b\,d^{2}\,e^{2}\,p^{2}\,x\right)\,\mathrm{root}\left(27\,b^{3}\,c^{3}-27\,a\,b^{2}\,c^{2}\,e^{2}\,p+81\,a\,b^{2}\,c\,d^{3}\,e\,p^{2}+9\,a^{2}\,b\,c\,e^{4}\,p^{2}-27\,a\,b^{2}\,d^{6}\,p^{3}-a^{3}\,e^{6}\,p^{3},c,k\right)\right)+\ln\left(c\,{\left(b\,x^{3}+a\right)}^{p}\right)\,\left(d^{2}\,x+d\,e\,x^{2}+\frac{e^{2}\,x^{3}}{3}\right)-3\,d^{2}\,p\,x-\frac{e^{2}\,p\,x^{3}}{3}-\frac{3\,d\,e\,p\,x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)*(d + e*x)^2,x)
```

```
[Out] symsum(log(root(27*b^3*c^3 - 27*a*b^2*c^2*e^2*p + 81*a*b^2*c*d^3*e*p^2 + 9*
a^2*b*c*e^4*p^2 - 27*a*b^2*d^6*p^3 - a^3*e^6*p^3, c, k)*(9*root(27*b^3*c^3
- 27*a*b^2*c^2*e^2*p + 81*a*b^2*c*d^3*e*p^2 + 9*a^2*b*c*e^4*p^2 - 27*a*b^2*
d^6*p^3 - a^3*e^6*p^3, c, k)*a*b^2 - 6*a^2*b*e^2*p + 9*a*b^2*d^2*p*x) + a^3
*e^4*p^2 + 9*a^2*b*d^3*e*p^2 + 6*a^2*b*d^2*e^2*p^2*x)*root(27*b^3*c^3 - 27*
a*b^2*c^2*e^2*p + 81*a*b^2*c*d^3*e*p^2 + 9*a^2*b*c*e^4*p^2 - 27*a*b^2*d^6*p
^3 - a^3*e^6*p^3, c, k), k, 1, 3) + log(c*(a + b*x^3)^p)*(d^2*x + (e^2*x^3)
/3 + d*e*x^2) - 3*d^2*p*x - (e^2*p*x^3)/3 - (3*d*e*p*x^2)/2
```

3.193 $\int (d+ex)\log\left(c(a+bx^3)^p\right)\,dx$

Optimal. Leaf size=229

$$-3dpx-\frac{3}{4}epx^2-\frac{\sqrt{3}\,\sqrt[3]{a}\left(2\sqrt[3]{b}\,d+\sqrt[3]{a}\,e\right)p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2b^{2/3}}+\frac{\sqrt[3]{a}\left(2\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{2b^{2/3}}$$

```
[Out] -3*d*p*x-3/4*e*p*x^2+1/2*a^(1/3)*(2*b^(1/3)*d-a^(1/3)*e)*p*ln(a^(1/3)+b^(1/
3)*x)/b^(2/3)-1/4*a^(1/3)*(2*b^(1/3)*d-a^(1/3)*e)*p*ln(a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/b^(2/3)-1/2*d^2*p*ln(b*x^3+a)/e+1/2*(e*x+d)^2*ln(c*(b*x^
3+a)^p)/e-1/2*a^(1/3)*(2*b^(1/3)*d+a^(1/3)*e)*p*arctan(1/3*(a^(1/3)-2*b^(1/
3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/b^(2/3)
```

Rubi [A]

time = 0.22, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {2513, 1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$-\frac{\sqrt[3]{a}\,p\left(2\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{4b^{2/3}}-\frac{\sqrt{3}\,\sqrt[3]{a}\,p\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(\sqrt[3]{a}\,e+2\sqrt[3]{b}\,d\right)}{2b^{2/3}}+\frac{\sqrt[3]{a}\,p\left(2\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2b^{2/3}}+\frac{(d+ex)^2\log\left(c(a+bx^3)^p\right)}{2e}-\frac{d^2p\log\left(a+bx^3\right)}{2e}-3dpx-\frac{3}{4}epx^2$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*Log[c*(a + b*x^3)^p],x]
```

```
[Out] -3*d*p*x - (3*e*p*x^2)/4 - (Sqrt[3]*a^(1/3)*(2*b^(1/3)*d + a^(1/3)*e)*p*Arc
Tan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2*b^(2/3)) + (a^(1/3)*(2*b
^(1/3)*d - a^(1/3)*e)*p*Log[a^(1/3) + b^(1/3)*x])/(2*b^(2/3)) - (a^(1/3)*(2
*b^(1/3)*d - a^(1/3)*e)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(
4*b^(2/3)) - (d^2*p*Log[a + b*x^3])/(2*e) + ((d + e*x)^2*Log[c*(a + b*x^3)^
p])/(2*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
```

```
&& (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$
\begin{aligned}
\int (d+ex)\log\left(c\left(a+bx^3\right)^p\right)\,dx &= \frac{(d+ex)^2\log\left(c\left(a+bx^3\right)^p\right)}{2e} - \frac{(3bp)\int \frac{x^2(d+ex)^2}{a+bx^3}\,dx}{2e} \\
&= \frac{(d+ex)^2\log\left(c\left(a+bx^3\right)^p\right)}{2e} - \frac{(3bp)\int \left(\frac{2de}{b}+\frac{e^2x}{b}-\frac{2ade+ae^2x-bd^2x^2}{b(a+bx^3)}\right)dx}{2e} \\
&= -3dpx-\frac{3}{4}epx^2+\frac{(d+ex)^2\log\left(c\left(a+bx^3\right)^p\right)}{2e}+\frac{(3p)\int \frac{2ade+ae^2x-bd^2x^2}{a+bx^3}\,dx}{2e} \\
&= -3dpx-\frac{3}{4}epx^2+\frac{(d+ex)^2\log\left(c\left(a+bx^3\right)^p\right)}{2e}+\frac{(3p)\int \frac{2ade+ae^2x}{a+bx^3}\,dx}{2e}-\frac{(3 \cdots}{\cdots} \\
&= -3dpx-\frac{3}{4}epx^2-\frac{d^2p\log\left(a+bx^3\right)}{2e}+\frac{(d+ex)^2\log\left(c\left(a+bx^3\right)^p\right)}{2e}+\frac{p\int \cdots}{\cdots} \\
&= -3dpx-\frac{3}{4}epx^2+\frac{\sqrt[3]{a}\left(2\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2b^{2/3}}-\frac{d^2p\log \cdots}{2 \cdots} \\
&= -3dpx-\frac{3}{4}epx^2+\frac{\sqrt[3]{a}\left(2\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2b^{2/3}}-\frac{\sqrt[3]{a}\left(2 \cdots\right.}{\cdots} \\
&= -3dpx-\frac{3}{4}epx^2-\frac{\sqrt{3}\,\sqrt[3]{a}\left(2\sqrt[3]{b}\,d+\sqrt[3]{a}\,e\right)p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2b^{2/3}}+\cdots
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 204, normalized size = 0.89

$$
-3dpx-\frac{3}{4}epx^2+\frac{\sqrt{3}\,\sqrt[3]{a}\,dp\tan^{-1}\left(\frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{\sqrt{3}\,\sqrt[3]{a}\,\sqrt[3]{b}}\right)}{\sqrt[3]{b}}+\frac{3}{4}epx^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a}\right)+\frac{\sqrt[3]{a}\,dp\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}-\frac{\sqrt[3]{a}\,dp\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}+dx\log\left(c\left(a+bx^3\right)^p\right)+\frac{1}{2}ex^2\log\left(c\left(a+bx^3\right)^p\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*Log[c*(a + b*x^3)^p],x]
```

```
[Out] -3*d*p*x - (3*e*p*x^2)/4 + (Sqrt[3]*a^(1/3)*d*p*ArcTan[(-(a^(1/3)*b^(1/3))
+ 2*b^(2/3)*x)/(Sqrt[3]*a^(1/3)*b^(1/3))]])/b^(1/3) + (3*e*p*x^2*Hypergeomet
ric2F1[2/3, 1, 5/3, -((b*x^3)/a)])/4 + (a^(1/3)*d*p*Log[a^(1/3) + b^(1/3)*x
])/b^(1/3) - (a^(1/3)*d*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(
2*b^(1/3)) + d*x*Log[c*(a + b*x^3)^p] + (e*x^2*Log[c*(a + b*x^3)^p])/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.56, size = 335, normalized size = 1.46

method	result
risch	$\left(\frac{1}{2}e\,x^{2}+dx\right)\ln\left(\left(x^{3}b+a\right)^{p}\right)+\frac{i\operatorname{csgn}\left(ic\left(x^{3}b+a\right)^{p}\right)^{2}\operatorname{csgn}\left(i\left(x^{3}b+a\right)^{p}\right)x^{2}e\pi}{4}-\frac{i\pi e\,x^{2}\operatorname{csgn}\left(i\left(x^{3}b+a\right)^{p}\right)\operatorname{csgn}\left(ic\left(x^{3}b+a\right)^{p}\right)\operatorname{cs}}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*ln(c*(b*x^3+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*e*x^2+d*x)*ln((b*x^3+a)^p)+1/4*I*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*(b*x^3
+a)^p)*x^2*e*Pi-1/4*I*Pi*e*x^2*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*cs
gn(I*c)-1/4*I*Pi*e*x^2*csgn(I*c*(b*x^3+a)^p)^3+1/4*I*csgn(I*c)*csgn(I*c*(b*
x^3+a)^p)^2*x^2*e*Pi+1/2*I*Pi*d*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2
*x-1/2*I*Pi*d*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)*x-1/2*I*P
i*d*csgn(I*c*(b*x^3+a)^p)^3*x+1/2*I*Pi*d*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)*
x+1/2*ln(c)*e*x^2-3/4*e*p*x^2+ln(c)*d*x-3*d*p*x+1/2*a*p/b*sum((_R*e+2*d)/_R
^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Maxima [A]
time = 0.59, size = 192, normalized size = 0.84

$$-\frac{1}{4}\,bp\left(\frac{3\,(x^{2}e+4\,dx)}{b}-\frac{2\,\sqrt{3}\,\left(a\left(\frac{a}{b}\right)^{\frac{1}{3}}e+2\,ad\right)\arctan\left(\frac{\sqrt{3}\,\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(a\left(\frac{a}{b}\right)^{\frac{1}{3}}e-2\,ad\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\left(a\left(\frac{a}{b}\right)^{\frac{1}{3}}e-2\,ad\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+\frac{1}{2}\,(x^{2}e+2\,dx)\log\left(\left(bx^{3}+a\right)^{p}c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] -1/4*b*p*(3*(x^2*e + 4*d*x)/b - 2*sqrt(3)*(a*(a/b)^(1/3)*e + 2*a*d)*arctan(
1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^2*(a/b)^(2/3)) - (a*(a/b)^(
1/3)*e - 2*a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a/b)^(2/3)) +
2*(a*(a/b)^(1/3)*e - 2*a*d)*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))) + 1/2*(
x^2*e + 2*d*x)*log((b*x^3 + a)^p*c)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.21, size = 2251, normalized size = 9.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] -3/4*p*x^2*e - 3*d*p*x + 1/4*(4*(1/2)^(2/3)*a*d*p^2*(-I*sqrt(3) + 1)*e/(((8
*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*p^3*e^3)/b^2)^(1/3)*b) - (
```

```
1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*p^3*e^3)/b^2
)^(1/3)*(I*sqrt(3) + 1))*log(8*b*d^3*p^2*x + 2*(4*(1/2)^(2/3)*a*d*p^2*(-I*s
qrt(3) + 1)*e/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*p^3*e^3)
/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3
 - a^2*p^3*e^3)/b^2)^(1/3)*(I*sqrt(3) + 1))*b*d^2*p + a*p^2*x*e^3 + 4*a*d*p^
2*e^2 + 1/4*(4*(1/2)^(2/3)*a*d*p^2*(-I*sqrt(3) + 1)*e/(((8*b*d^3 + a*e^3)*a
*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*p^3*e^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*
d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*p^3*e^3)/b^2)^(1/3)*(I*sqrt(3
) + 1))^2*b*e) - 1/8*(4*(1/2)^(2/3)*a*d*p^2*(-I*sqrt(3) + 1)*e/(((8*b*d^3 +
 a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*p^3*e^3)/b^2)^(1/3)*b) - (1/2)^(1/
3)*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*p^3*e^3)/b^2)^(1/3)*
(I*sqrt(3) + 1) - sqrt(3)*sqrt(-(32*a*d*p^2*e + (4*(1/2)^(2/3)*a*d*p^2*(-I*
sqrt(3) + 1)*e/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*p^3*e^3
)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3
 - a^2*p^3*e^3)/b^2)^(1/3)*(I*sqrt(3) + 1))^2*b)/b))*log(8*b*d^3*p^2*x - (4
*(1/2)^(2/3)*a*d*p^2*(-I*sqrt(3) + 1)*e/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*
a*b*d^3*p^3 - a^2*p^3*e^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a
*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*p^3*e^3)/b^2)^(1/3)*(I*sqrt(3) + 1))*b*d^2*
p + a*p^2*x*e^3 - 2*a*d*p^2*e^2 - 1/8*(4*(1/2)^(2/3)*a*d*p^2*(-I*sqrt(3) +
1)*e/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*p^3*e^3)/b^2)^(1/
3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*p^3
*e^3)/b^2)^(1/3)*(I*sqrt(3) + 1))^2*b*e + 1/8*sqrt(3)*(8*b*d^2*p - (4*(1/2)
^(2/3)*a*d*p^2*(-I*sqrt(3) + 1)*e/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^
3*p^3 - a^2*p^3*e^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b
^2 + (8*a*b*d^3*p^3 - a^2*p^3*e^3)/b^2)^(1/3)*(I*sqrt(3) + 1))*b*e)*sqrt(-(
32*a*d*p^2*e + (4*(1/2)^(2/3)*a*d*p^2*(-I*sqrt(3) + 1)*e/(((8*b*d^3 + a*e^3
)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*p^3*e^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8
*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*p^3*e^3)/b^2)^(1/3)*(I*sqr
t(3) + 1))^2*b)/b)) - 1/8*(4*(1/2)^(2/3)*a*d*p^2*(-I*sqrt(3) + 1)*e/(((8*b*
d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*p^3*e^3)/b^2)^(1/3)*b) - (1/2
)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*p^3*e^3)/b^2)^(
1/3)*(I*sqrt(3) + 1) + sqrt(3)*sqrt(-(32*a*d*p^2*e + (4*(1/2)^(2/3)*a*d*p^2
*(-I*sqrt(3) + 1)*e/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*p^
3*e^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^
3*p^3 - a^2*p^3*e^3)/b^2)^(1/3)*(I*sqrt(3) + 1))^2*b)/b))*log(8*b*d^3*p^2*x
 - (4*(1/2)^(2/3)*a*d*p^2*(-I*sqrt(3) + 1)*e/(((8*b*d^3 + a*e^3)*a*p^3/b^2
+ (8*a*b*d^3*p^3 - a^2*p^3*e^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e
^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*p^3*e^3)/b^2)^(1/3)*(I*sqrt(3) + 1))*b
*d^2*p + a*p^2*x*e^3 - 2*a*d*p^2*e^2 - 1/8*(4*(1/2)^(2/3)*a*d*p^2*(-I*sqrt(
3) + 1)*e/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*p^3*e^3)/b^2
)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^
2*p^3*e^3)/b^2)^(1/3)*(I*sqrt(3) + 1))^2*b*e - 1/8*sqrt(3)*(8*b*d^2*p - (4*
(1/2)^(2/3)*a*d*p^2*(-I*sqrt(3) + 1)*e/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a
*b*d^3*p^3 - a^2*p^3*e^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*
p^3/b^2 + (8*a*b*d^3*p^3 - a^2*p^3*e^3)/b^2)^(1/3)*(I*sqrt(3) + 1))*b*e)*sq
```

```
rt(-(32*a*d*p^2*e + (4*(1/2)^(2/3)*a*d*p^2*(-I*sqrt(3) + 1)*e/(((8*b*d^3 +
a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*p^3*e^3)/b^2)^(1/3)*b) - (1/2)^(1/3
)*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*p^3*e^3)/b^2)^(1/3)*(
I*sqrt(3) + 1))^2*b)/b)) + 1/2*(p*x^2*e + 2*d*p*x)*log(b*x^3 + a) + 1/2*(x^
2*e + 2*d*x)*log(c)
```

Sympy [A]
time = 12.90, size = 112, normalized size = 0.49

$$3adp\operatorname{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(3ta+x\right)\right)\right)+\frac{3aep\operatorname{RootSum}\left(27t^3ab^2+1,\left(t\mapsto t\log\left(9t^2ab+x\right)\right)\right)}{2}-3dpx+dx\log\left(c\left(a+bx^3\right)^p\right)-\frac{3epx^2}{4}+\frac{ex^2\log\left(c\left(a+bx^3\right)^p\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*ln(c*(b*x**3+a)**p),x)
```

```
[Out] 3*a*d*p*RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(3*_t*a + x))) + 3*a*
e*p*RootSum(27*_t**3*a*b**2 + 1, Lambda(_t, _t*log(9*_t**2*a*b + x)))/2 - 3
*d*p*x + d*x*log(c*(a + b*x**3)**p) - 3*e*p*x**2/4 + e*x**2*log(c*(a + b*x*
*3)**p)/2
```

Giac [A]
time = 3.12, size = 220, normalized size = 0.96

$$\frac{1}{2}px^2e\log\left(bx^3+a\right)-\frac{3}{4}px^2e+dpx\log\left(bx^3+a\right)+\frac{1}{2}x^2e\log\left(c\right)-3dpx+dx\log\left(c\right)-\frac{\left(ap\left(-\frac{a}{b}\right)^{\frac{1}{3}}e+2adp\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{2a}+\frac{\left(2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}bdp-\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}pe\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{2b^2}+\frac{\left(2\left(-ab^2\right)^{\frac{1}{3}}bdp+\left(-ab^2\right)^{\frac{2}{3}}pe\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] 1/2*p*x^2*e*log(b*x^3 + a) - 3/4*p*x^2*e + d*p*x*log(b*x^3 + a) + 1/2*x^2*e
*log(c) - 3*d*p*x + d*x*log(c) - 1/2*(a*p*(-a/b)^(1/3)*e + 2*a*d*p)*(-a/b)^
(1/3)*log(abs(x - (-a/b)^(1/3)))/a + 1/2*(2*sqrt(3)*(-a*b^2)^(1/3)*b*d*p -
sqrt(3)*(-a*b^2)^(2/3)*p*e)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^
(1/3))/b^2 + 1/4*(2*(-a*b^2)^(1/3)*b*d*p + (-a*b^2)^(2/3)*p*e)*log(x^2 + x*
(-a/b)^(1/3) + (-a/b)^(2/3))/b^2
```

Mupad [B]
time = 0.31, size = 210, normalized size = 0.92

$$\left(\sum_{k=1}^{3}\ln\left(\operatorname{root}\left(8b^2c^3+12abcdep^2-8abd^3p^3+a^2e^3p^3,c,k\right)\,\left(\operatorname{root}\left(8b^2c^3+12abcdep^2-8abd^3p^3+a^2e^3p^3,c,k\right)\,ab^2\,9+9ab^2dpx\right)+\frac{9a^2bdep^2}{2}+\frac{9a^2be^2p^2x}{4}\right)\operatorname{root}\left(8b^2c^3+12abcdep^2-8abd^3p^3+a^2e^3p^3,c,k\right)\right)+\ln\left(c\left(bx^3+a\right)^p\right)\left(\frac{ex^2}{2}+dx\right)-\frac{3epx^2}{4}-3dpx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)*(d + e*x),x)
```

```
[Out] symsum(log(root(8*b^2*c^3 + 12*a*b*c*d*e*p^2 - 8*a*b*d^3*p^3 + a^2*e^3*p^3,
 c, k)*(9*root(8*b^2*c^3 + 12*a*b*c*d*e*p^2 - 8*a*b*d^3*p^3 + a^2*e^3*p^3,
c, k)*a*b^2 + 9*a*b^2*d*p*x) + (9*a^2*b*d*e*p^2)/2 + (9*a^2*b*e^2*p^2*x)/4)
*root(8*b^2*c^3 + 12*a*b*c*d*e*p^2 - 8*a*b*d^3*p^3 + a^2*e^3*p^3, c, k), k,
 1, 3) + log(c*(a + b*x^3)^p)*(d*x + (e*x^2)/2) - (3*e*p*x^2)/4 - 3*d*p*x
```

3.194 $\int \log \left(c(a+bx^3)^p\right) \, dx$

Optimal. Leaf size=133

$$-3px-\frac{\sqrt{3}\sqrt[3]{a}\,p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}}+\frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}-\frac{\sqrt[3]{a}\,p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}+$$

```
[Out] -3*p*x+a^(1/3)*p*ln(a^(1/3)+b^(1/3)*x)/b^(1/3)-1/2*a^(1/3)*p*ln(a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(1/3)+x*ln(c*(b*x^3+a)^p)-a^(1/3)*p*arctan(1/
3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/b^(1/3)
```

Rubi [A]
time = 0.06, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$,
Rules used = {2498, 327, 206, 31, 648, 631, 210, 642}

$$-\frac{\sqrt[3]{a}\,p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\sqrt[3]{a}\,p\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}}+x\log\left(c\left(a+bx^3\right)^p\right)+\frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}-3px$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p],x]
```

```
[Out] -3*p*x - (Sqrt[3]*a^(1/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)])/b^(1/3) + (a^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (a^(1/3)*p*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)) + x*Log[c*(a + b*x
^3)^p]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\int \log\left(c\left(a+bx^3\right)^p\right)\,dx = x\log\left(c\left(a+bx^3\right)^p\right) - (3bp)\int \frac{x^3}{a+bx^3}\,dx$$
$$= -3px + x\log\left(c\left(a+bx^3\right)^p\right) + (3ap)\int \frac{1}{a+bx^3}\,dx$$
$$= -3px + x\log\left(c\left(a+bx^3\right)^p\right) + \left(\sqrt[3]{a}\,p\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx + \left(\sqrt[3]{a}\,p\right)\int \frac{2\sqrt[3]{a}}{a^{2/3}-\sqrt[3]{a}}$$
$$= -3px + \frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}} + x\log\left(c\left(a+bx^3\right)^p\right) + \frac{1}{2}\left(3a^{2/3}p\right)\int \frac{}{a^{2/3}-\sqrt[3]{}}$$
$$= -3px + \frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}} - \frac{\sqrt[3]{a}\,p\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{2\sqrt[3]{b}} + x\log$$
$$= -3px - \frac{\sqrt{3}\,\sqrt[3]{a}\,p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt[3]{b}} + \frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}} - \frac{\sqrt[3]{a}\,p\log}{}$$

Mathematica [A]
time = 0.03, size = 129, normalized size = 0.97

$$-3px - \frac{\sqrt{3}\,\sqrt[3]{a}\,p\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} + \frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}} - \frac{\sqrt[3]{a}\,p\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{2\sqrt[3]{b}} + x\log\left(c\left(a+bx^3\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p],x]
```

```
[Out] -3*p*x - (Sqrt[3]*a^(1/3)*p*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^
(1/3) + (a^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (a^(1/3)*p*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)) + x*Log[c*(a + b*x^3)^p]
```

Maple [A]
time = 0.05, size = 122, normalized size = 0.92

method	result

default	$x \ln \left(c(x^3 b+a)^p\right)-3 p b\left(\frac{x}{b}-\frac{\left(\frac{\ln \left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln \left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3} \arctan \left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right) a}{b}\right)$
risch	$x \ln \left((x^3 b+a)^p\right)-\frac{i \pi x \operatorname{csgn}\left(i(x^3 b+a)^p\right) \operatorname{csgn}\left(i c(x^3 b+a)^p\right) \operatorname{csgn}(i c)}{2}+\frac{i \operatorname{csgn}(i c) \operatorname{csgn}\left(i c(x^3 b+a)^p\right)^2 x \pi}{2}+\frac{i \operatorname{csgn}\left(i c(x^3 b+a)^p\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(c*(b*x^3+a)^p)-3*p*b*(x/b-(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(
a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*a/b)
```

Maxima [A]
time = 0.61, size = 125, normalized size = 0.94

$$-\frac{1}{2} b p\left(\frac{6 x}{b}-\frac{2 \sqrt{3} a \arctan \left(\frac{\sqrt{3}\left(2 x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{a \log \left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{2 a \log \left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+x \log \left((b x^3+a)^p c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] -1/2*b*p*(6*x/b - 2*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^
(1/3))/(b^2*(a/b)^(2/3)) + a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a
/b)^(2/3)) - 2*a*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))) + x*log((b*x^3 + a
)^p*c)
```

Fricas [A]
time = 0.38, size = 110, normalized size = 0.83

$$px\log\left(bx^3+a\right)+\sqrt{3}\,p\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)-\frac{1}{2}\,p\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+p\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-3\,px+x\log\left(c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] p*x*log(b*x^3 + a) + sqrt(3)*p*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^
(2/3) - sqrt(3)*a)/a) - 1/2*p*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(
2/3)) + p*(a/b)^(1/3)*log(x + (a/b)^(1/3)) - 3*p*x + x*log(c)
```

Sympy [A]
time = 27.39, size = 165, normalized size = 1.24

$$\begin{cases} x\log\left(0^p c\right) & \text{for}\: a = 0 \wedge b = 0 \\ -3px + x\log\left(c\left(bx^3\right)^p\right) & \text{for}\: a = 0 \\ x\log\left(a^p c\right) & \text{for}\: b = 0 \\ -3px + x\log\left(c\left(a+bx^3\right)^p\right) - \frac{3bp\left(-\frac{a}{b}\right)^{\frac{4}{3}}\log\left(4x^2+4x\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2a} - \frac{\sqrt{3}\,bp\left(-\frac{a}{b}\right)^{\frac{4}{3}}\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{a} + \frac{b\left(-\frac{a}{b}\right)^{\frac{4}{3}}\log\left(c\left(a+bx^3\right)^p\right)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p),x)
```

```
[Out] Piecewise((x*log(0**p*c), Eq(a, 0) & Eq(b, 0)), (-3*p*x + x*log(c*(b*x**3)*
*p), Eq(a, 0)), (x*log(a**p*c), Eq(b, 0)), (-3*p*x + x*log(c*(a + b*x**3)**
p) - 3*b*p*(-a/b)**(4/3)*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/
(2*a) - sqrt(3)*b*p*(-a/b)**(4/3)*atan(2*sqrt(3)*x/(3*(-a/b)**(1/3)) + sqrt
(3)/3)/a + b*(-a/b)**(4/3)*log(c*(a + b*x**3)**p)/a, True))
```

Giac [A]
time = 4.72, size = 143, normalized size = 1.08

$$-\frac{1}{2}\,abp\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{ab}-\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^2}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab^2}\right)+px\log\left(bx^3+a\right)-\left(3\,p-\log\left(c\right)\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] -1/2*a*b*p*(2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b) - 2*sqrt(3)*(-a
*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) -
 (-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2)) + p*x*log
(b*x^3 + a) - (3*p - log(c))*x
```

Mupad [B]
time = 0.00, size = 134, normalized size = 1.01

$$x\,\ln\left(c\,{\left(b\,x^3+a\right)}^p\right)-3\,p\,x-\frac{{\left(-a\right)}^{1/3}\,p\,\ln\left({\left(-a\right)}^{4/3}+a\,b^{1/3}\,x\right)}{b^{1/3}}+\frac{{\left(-a\right)}^{1/3}\,p\,\ln\left(2\,a\,b^{1/3}\,x-{\left(-a\right)}^{4/3}-\sqrt{3}\,{\left(-a\right)}^{4/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^{1/3}}-\frac{{\left(-a\right)}^{1/3}\,p\,\ln\left(2\,a\,b^{1/3}\,x-{\left(-a\right)}^{4/3}+\sqrt{3}\,{\left(-a\right)}^{4/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p),x)
```

```
[Out] x*log(c*(a + b*x^3)^p) - 3*p*x - ((-a)^(1/3)*p*log((-a)^(4/3) + a*b^(1/3)*x
))/b^(1/3) + ((-a)^(1/3)*p*log(2*a*b^(1/3)*x - 3^(1/2)*(-a)^(4/3)*1i - (-a)
^(4/3))*((3^(1/2)*1i)/2 + 1/2))/b^(1/3) - ((-a)^(1/3)*p*log(3^(1/2)*(-a)^(4
/3)*1i - (-a)^(4/3) + 2*a*b^(1/3)*x)*((3^(1/2)*1i)/2 - 1/2))/b^(1/3)
```

3.195 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=308

$$-\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)\log(d+ex)}{e}-\frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-(-1)^{2/3}\sqrt[3]{a}\,e}\right)\log(d+ex)}{e}-\frac{p\log\left(\frac{\sqrt[3]{-1}\,e\left(\sqrt[3]{a}+\right.}{\sqrt[3]{b}\,d+\sqrt[3]{-}}\right.}{}$$

```
[Out] -p*ln(-e*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-a^(1/3)*e))*ln(e*x+d)/e-p*ln(-e*((-
1)^(2/3)*a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))*ln(e*x+d)/e-p
*ln((-1)^(1/3)*e*(a^(1/3)+(-1)^(2/3)*b^(1/3)*x)/(b^(1/3)*d+(-1)^(1/3)*a^(1/
3)*e))*ln(e*x+d)/e+ln(e*x+d)*ln(c*(b*x^3+a)^p)/e-p*polylog(2,b^(1/3)*(e*x+d
)/(b^(1/3)*d-a^(1/3)*e))/e-p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d+(-1)^(1/3
)*a^(1/3)*e))/e-p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e
))/e
```

Rubi [A]
time = 0.34, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2512, 266, 2463, 2441, 2440, 2438}

$$-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}\,(d+ex)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)}{e}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}\,(d+ex)}{\sqrt[3]{-1}\,\sqrt[3]{a}\,e+\sqrt[3]{b}\,d}\right)}{e}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[3]{b}\,(d+ex)}{\sqrt[3]{b}\,d-(-1)^{2/3}\sqrt[3]{a}\,e}\right)}{e}+\frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e}-\frac{p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)}{e}-\frac{p\log(d+ex)\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-(-1)^{2/3}\sqrt[3]{a}\,e}\right)}{e}-\frac{p\log(d+ex)\log\left(\frac{\sqrt[3]{-1}\,e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x\right)}{\sqrt[3]{-1}\,\sqrt[3]{a}\,e+\sqrt[3]{b}\,d}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/(d + e*x),x]
```

```
[Out] -((p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x]
)/e) - (p*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3
)*a^(1/3)*e))]*Log[d + e*x])/e - (p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)
*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/e + (Log[d +
 e*x]*Log[c*(a + b*x^3)^p])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*
d - a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1
/3)*a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2
/3)*a^(1/3)*e)])/e
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx = \frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e} - \frac{(3bp)\int \frac{x^2\log(d+ex)}{a+bx^3}\,dx}{e}$$

$$= \frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e} - \frac{(3bp)\int\left(\frac{\log(d+ex)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{\log(d+ex)}{3b^{2/3}\left(-\sqrt[3]{-1}\,\sqrt[3]{a}+\sqrt[3]{\ldots}\right.}\right.}{e}$$

$$= \frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e} - \frac{\left(\sqrt[3]{b}\,p\right)\int \frac{\log(d+ex)}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{e} - \frac{\left(\sqrt[3]{b}\,p\right)\int \frac{\log(d+e\ldots}{-\sqrt[3]{-1}\,\sqrt[3]{a}\ldots}}{e}$$

$$= -\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-(-1)^{2/3}\sqrt[3]{a}\,e}\right)\log(d+\ldots}{e}$$

$$= -\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-(-1)^{2/3}\sqrt[3]{a}\,e}\right)\log(d+\ldots}{e}$$

$$= -\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-(-1)^{2/3}\sqrt[3]{a}\,e}\right)\log(d+\ldots}{e}$$

Mathematica [A]
time = 0.11, size = 313, normalized size = 1.02

$$-\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{(-1)^{2/3}e\left(\sqrt[3]{a}-\sqrt[3]{-1}\,\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-(-1)^{2/3}\sqrt[3]{a}\,e}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{\sqrt[3]{-1}\,e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d+\sqrt[3]{-1}\,\sqrt[3]{a}\,e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e} - \frac{p\,\text{Li}_2\left(\frac{\sqrt[3]{b}\,(d+ex)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)}{e} - \frac{p\,\text{Li}_2\left(\frac{\sqrt[3]{b}\,(d+ex)}{\sqrt[3]{b}\,d+\sqrt[3]{-1}\,\sqrt[3]{a}\,e}\right)}{e} - \frac{p\,\text{Li}_2\left(\frac{\sqrt[3]{b}\,(d+ex)}{\sqrt[3]{b}\,d-(-1)^{2/3}\sqrt[3]{a}\,e}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/(d + e*x),x]
```

```
[Out] -((p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x]
)/e) - (p*Log[-(((-1)^(2/3)*e*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/(b^(1/3)*d
- (-1)^(2/3)*a^(1/3)*e))]*Log[d + e*x])/e - (p*Log[((-1)^(1/3)*e*(a^(1/3) +
 (-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/e
 + (Log[d + e*x]*Log[c*(a + b*x^3)^p])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x)
)/(b^(1/3)*d - a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*
d + (-1)^(1/3)*a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*
d - (-1)^(2/3)*a^(1/3)*e)])/e
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.58, size = 261, normalized size = 0.85

method	result
risch	$\frac{\ln(ex+d)\ln\left((x^3b+a)^p\right)}{e} - \frac{p\left(\sum_{\_R1=\mathrm{RootOf}\left(\_Z^3b-3bd\_Z^2+3b\,d^2\_Z+e^3a-b\,d^3\right)}\left(\ln(ex+d)\ln\left(\frac{-ex+\_R1-d}{\_R1}\right)+\mathrm{dilog}\left(\frac{-ex+\_R1-}{\_R1}\right.\right.\right.}{e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] ln(e*x+d)/e*ln((b*x^3+a)^p)-p/e*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((-
e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3))+1/2*I
*ln(e*x+d)/e*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2-1/2*I*ln(e*x+d)
/e*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)-1/2*I*ln(e*x+d)/e
*Pi*csgn(I*c*(b*x^3+a)^p)^3+1/2*I*ln(e*x+d)/e*Pi*csgn(I*c*(b*x^3+a)^p)^2*cs
gn(I*c)+ln(e*x+d)/e*ln(c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((b*x^3 + a)^p*c)/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x^3 + a)^p*c)/(x*e + d), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(log((b*x^3 + a)^p*c)/(x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(b\,x^3+a\right)}^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)/(d + e*x),x)
[Out] int(log(c*(a + b*x^3)^p)/(d + e*x), x)
```

3.196 $\int \frac{\log\left(c(a+bx^3)^p\right)}{(d+ex)^2}\,dx$

Optimal. Leaf size=292

$$-\frac{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\,p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{2/3}d^2+\sqrt[3]{a}\sqrt[3]{b}\,de+a^{2/3}e^2}+\frac{\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{b}\,d+\sqrt[3]{a}\,e\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{bd^3-ae^3}-\frac{3bd^2p\log(d+ex)}{e\left(bd^3-ae^3\right)}-\frac{\sqrt[3]{}}{}$$

```
[Out] a^(1/3)*b^(1/3)*(b^(1/3)*d+a^(1/3)*e)*p*ln(a^(1/3)+b^(1/3)*x)/(-a*e^3+b*d^3
)-3*b*d^2*p*ln(e*x+d)/e/(-a*e^3+b*d^3)-1/2*a^(1/3)*b^(1/3)*(b^(1/3)*d+a^(1/
3)*e)*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-a*e^3+b*d^3)+b*d^2*p*ln
(b*x^3+a)/e/(-a*e^3+b*d^3)-ln(c*(b*x^3+a)^p)/e/(e*x+d)-a^(1/3)*b^(1/3)*p*ar
ctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/(b^(2/3)*d^2+a^(1/3
)*b^(1/3)*d*e+a^(2/3)*e^2)
```

Rubi [A]
time = 0.36, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2513, 6857, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$-\frac{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\,p\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}e^2+\sqrt[3]{a}\sqrt[3]{b}\,de+b^{2/3}d^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}\,p\left(\sqrt[3]{a}\,e+\sqrt[3]{b}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{2\left(bd^3-ae^3\right)}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{e(d+ex)}+\frac{\sqrt[3]{a}\sqrt[3]{b}\,p\left(\sqrt[3]{a}\,e+\sqrt[3]{b}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{bd^3-ae^3}+\frac{bd^2p\log\left(a+bx^3\right)}{e\left(bd^3-ae^3\right)}-\frac{3bd^2p\log(d+ex)}{e\left(bd^3-ae^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/(d + e*x)^2,x]
```

```
[Out] -((Sqrt[3]*a^(1/3)*b^(1/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))])/(b^(2/3)*d^2 + a^(1/3)*b^(1/3)*d*e + a^(2/3)*e^2)) + (a^(1/3)*b^(1/3)*
(b^(1/3)*d + a^(1/3)*e)*p*Log[a^(1/3) + b^(1/3)*x])/(b*d^3 - a*e^3) - (3*b*
d^2*p*Log[d + e*x])/(e*(b*d^3 - a*e^3)) - (a^(1/3)*b^(1/3)*(b^(1/3)*d + a^(
1/3)*e)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*(b*d^3 - a*e^3
)) + (b*d^2*p*Log[a + b*x^3])/(e*(b*d^3 - a*e^3)) - Log[c*(a + b*x^3)^p]/(e
*(d + e*x))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
```

```
&& (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{(d+ex)^2}\,dx = -\frac{\log\left(c(a+bx^3)^p\right)}{e(d+ex)} + \frac{(3bp)\int \frac{x^2}{(d+ex)(a+bx^3)}\,dx}{e}$$

$$= -\frac{\log\left(c(a+bx^3)^p\right)}{e(d+ex)} + \frac{(3bp)\int \left(-\frac{d^2e}{(bd^3-ae^3)(d+ex)} + \frac{ade-ae^2x+bd^2x^2}{(bd^3-ae^3)(a+bx^3)}\right)dx}{e}$$

$$= -\frac{3bd^2p\log(d+ex)}{e\left(bd^3-ae^3\right)} - \frac{\log\left(c(a+bx^3)^p\right)}{e(d+ex)} + \frac{(3bp)\int \frac{ade-ae^2x+bd^2x^2}{a+bx^3}\,dx}{e\left(bd^3-ae^3\right)}$$

$$= -\frac{3bd^2p\log(d+ex)}{e\left(bd^3-ae^3\right)} - \frac{\log\left(c(a+bx^3)^p\right)}{e(d+ex)} + \frac{(3bp)\int \frac{ade-ae^2x}{a+bx^3}\,dx}{e\left(bd^3-ae^3\right)} + \frac{(3b^2d^2p)\int \frac{x^2}{a+bx^3}\,d\cdots}{e\left(bd^3-ae^3\right)}$$

$$= -\frac{3bd^2p\log(d+ex)}{e\left(bd^3-ae^3\right)} + \frac{bd^2p\log\left(a+bx^3\right)}{e\left(bd^3-ae^3\right)} - \frac{\log\left(c(a+bx^3)^p\right)}{e(d+ex)} + \frac{\left(b^{2/3}p\right)\int \frac{\sqrt[3]{a}\left(2a\sqrt[3]{\cdots}\right.}{\cdots}}{\cdots}$$

$$= \frac{\sqrt[3]{a}\,b^{2/3}\left(d+\frac{\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{bd^3-ae^3} - \frac{3bd^2p\log(d+ex)}{e\left(bd^3-ae^3\right)} + \frac{bd^2p\log\left(a+bx^3\cdots\right.}{e\left(bd^3-ae^3\right)}$$

$$= \frac{\sqrt[3]{a}\,b^{2/3}\left(d+\frac{\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{bd^3-ae^3} - \frac{3bd^2p\log(d+ex)}{e\left(bd^3-ae^3\right)} - \frac{\sqrt[3]{a}\,b^{2/3}\left(d+\frac{\sqrt[3]{\cdots}}{\sqrt[3]{\cdots}}\right.}{\cdots}$$

$$= -\frac{\sqrt{3}\,\sqrt[3]{a}\,\sqrt[3]{b}\left(\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{bd^3-ae^3} + \frac{\sqrt[3]{a}\,b^{2/3}\left(d+\frac{\sqrt[3]{a}\,e}{\sqrt[3]{b}}\right)p\log\cdots}{bd^3-ae^3\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.44, size = 202, normalized size = 0.69

$$-\frac{\frac{3be^2px^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a}\right)+b^{2/3}dp\left(2\sqrt{3}\,\sqrt[3]{a}\,e\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\sqrt[3]{a}\,e\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+6\sqrt[3]{b}\,d\log(d+ex)+\sqrt[3]{a}\,e\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)-2\sqrt[3]{b}\,d\log\left(a+bx^3\right)\right)}{2bd^3-2ae^3}+\frac{\log\left(c(a+bx^3)^p\right)}{d+ex}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/(d + e*x)^2,x]
```

```
[Out] -(((3*b*e^2*p*x^2*Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)] + b^(2/3)*d*
p*(2*Sqrt[3]*a^(1/3)*e*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*a^(1
/3)*e*Log[a^(1/3) + b^(1/3)*x] + 6*b^(1/3)*d*Log[d + e*x] + a^(1/3)*e*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*b^(1/3)*d*Log[a + b*x^3]))/(2
*b*d^3 - 2*a*e^3) + Log[c*(a + b*x^3)^p]/(d + e*x))/e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.83, size = 1068, normalized size = 3.66

method	result	size
risch	Expression too large to display	1068

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/e/(e*x+d)*ln((b*x^3+a)^p)+1/2*(-I*Pi*b*d^3*csgn(I*(b*x^3+a)^p)*csgn(I*c*
(b*x^3+a)^p)*csgn(I*c)-I*Pi*a*e^3*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)+I*Pi*a*
e^3*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)+I*Pi*a*e^3*csgn(I*c
*(b*x^3+a)^p)^3+I*Pi*b*d^3*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)-I*Pi*b*d^3*csg
n(I*c*(b*x^3+a)^p)^3+I*Pi*b*d^3*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2
-I*Pi*a*e^3*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2+2*sum(_R*ln(((-4*a*
e^7-2*b*d^3*e^4)*_R^3-3*_R^2*b*d^2*e^3*p+8*_R*b*d*e^2*p^2-3*b*e*p^3)*x+(-5*
a*d*e^6-b*d^4*e^3)*_R^3+(a*e^5*p-b*d^3*e^2*p)*_R^2+5*b*d^2*e*p^2*_R-3*b*d*p
^3),_R=RootOf((a*e^6-b*d^3*e^3)*_Z^3+3*b*d^2*e^2*p*_Z^2-3*b*d*e*p^2*_Z+b*p^
3))*a*e^5*x-2*sum(_R*ln(((-4*a*e^7-2*b*d^3*e^4)*_R^3-3*_R^2*b*d^2*e^3*p+8*_
R*b*d*e^2*p^2-3*b*e*p^3)*x+(-5*a*d*e^6-b*d^4*e^3)*_R^3+(a*e^5*p-b*d^3*e^2*p
)*_R^2+5*b*d^2*e*p^2*_R-3*b*d*p^3),_R=RootOf((a*e^6-b*d^3*e^3)*_Z^3+3*b*d^2
*e^2*p*_Z^2-3*b*d*e*p^2*_Z+b*p^3))*b*d^3*e^2*x+2*sum(_R*ln(((-4*a*e^7-2*b*d
^3*e^4)*_R^3-3*_R^2*b*d^2*e^3*p+8*_R*b*d*e^2*p^2-3*b*e*p^3)*x+(-5*a*d*e^6-b
*d^4*e^3)*_R^3+(a*e^5*p-b*d^3*e^2*p)*_R^2+5*b*d^2*e*p^2*_R-3*b*d*p^3),_R=Ro
otOf((a*e^6-b*d^3*e^3)*_Z^3+3*b*d^2*e^2*p*_Z^2-3*b*d*e*p^2*_Z+b*p^3))*a*d*e
^4-2*sum(_R*ln(((-4*a*e^7-2*b*d^3*e^4)*_R^3-3*_R^2*b*d^2*e^3*p+8*_R*b*d*e^2
*p^2-3*b*e*p^3)*x+(-5*a*d*e^6-b*d^4*e^3)*_R^3+(a*e^5*p-b*d^3*e^2*p)*_R^2+5*
b*d^2*e*p^2*_R-3*b*d*p^3),_R=RootOf((a*e^6-b*d^3*e^3)*_Z^3+3*b*d^2*e^2*p*_Z
^2-3*b*d*e*p^2*_Z+b*p^3))*b*d^4*e+6*ln(-e*x-d)*b*d^2*e*p*x+6*ln(-e*x-d)*b*d
^3*p-2*ln(c)*a*e^3+2*d^3*b*ln(c))/(e*x+d)/e/(a*e^3-b*d^3)
```

Maxima [A]
time = 0.56, size = 307, normalized size = 1.05

$$-\frac{1}{2}\left(\frac{6\,d^2\log\left(xe+d\right)}{bd^3-ae^3}-\frac{2\,\sqrt{3}\left(ad\left(\frac{a}{b}\right)^{\frac{1}{3}}e-a\left(\frac{a}{b}\right)^{\frac{2}{3}}e^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\left(b^2d^3\left(\frac{a}{b}\right)^{\frac{2}{3}}-ab\left(\frac{a}{b}\right)^{\frac{2}{3}}e^3\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\left(2\,bd^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-ade-a\left(\frac{a}{b}\right)^{\frac{1}{3}}e^2\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b^2d^3\left(\frac{a}{b}\right)^{\frac{2}{3}}-ab\left(\frac{a}{b}\right)^{\frac{2}{3}}e^3}-\frac{2\left(bd^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+ade+a\left(\frac{a}{b}\right)^{\frac{1}{3}}e^2\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^2d^3\left(\frac{a}{b}\right)^{\frac{2}{3}}-ab\left(\frac{a}{b}\right)^{\frac{2}{3}}e^3}\right)bpe^{(-1)}-\frac{e^{(-1)}\log\left(\left(bx^3+a\right)^pc\right)}{xe+d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d)^2,x, algorithm="maxima")

[Out] -1/2*(6*d^2*log(x*e + d)/(b*d^3 - a*e^3) - 2*sqrt(3)*(a*d*(a/b)^(1/3)*e - a*(a/b)^(2/3)*e^2)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^2*d^3*(a/b)^(2/3) - a*b*(a/b)^(2/3)*e^3)*(a/b)^(1/3)) - (2*b*d^2*(a/b)^(2/3) - a*d*e - a*(a/b)^(1/3)*e^2)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*d^3*(a/b)^(2/3) - a*b*(a/b)^(2/3)*e^3) - 2*(b*d^2*(a/b)^(2/3) + a*d*e + a*(a/b)^(1/3)*e^2)*log(x + (a/b)^(1/3))/(b^2*d^3*(a/b)^(2/3) - a*b*(a/b)^(2/3)*e^3))*b*p*e^(-1) - e^(-1)*log((b*x^3 + a)^p*c)/(x*e + d)

Fricas [C] Result contains complex when optimal does not.
time = 1.45, size = 6743, normalized size = 23.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d)^2,x, algorithm="fricas")

[Out] 1/4*(2*(b*d^3*x*e^2 + b*d^4*e - a*x*e^5 - a*d*e^4)*(2*b*d^2*p/(b*d^3*e - a*e^4) - (b^2*d^4*p^2/(b*d^3*e - a*e^4)^2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(-I*sqrt(3) + 1)/(b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3) - (b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3)*(I*sqrt(3) + 1))*log(3/2*(2*b*d^2*p/(b*d^3*e - a*e^4) - (b^2*d^4*p^2/(b*d^3*e - a*e^4)^2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(-I*sqrt(3) + 1)/(b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3) - (b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3)*(I*sqrt(3) + 1))*b*d^2*p*e + b*p^2*x*e - 2*b*d*p^2 - 1/4*(b*d^3*e^2 - a*e^5)*(2*b*d^2*p/(b*d^3*e - a*e^4) - (b^2*d^4*p^2/(b*d^3*e - a*e^4)^2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(-I*sqrt(3) + 1)/(b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3) - (b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3)*(I*sqrt(3) + 1))^2) + (6*b*d^2*p*x*e + 6*b*d^3*p - (b*d^3*x*e^2 + b*d^4*e - a*x*e^5 - a*d*e^4)*(2*b*d^2*p/(b*d^3*e - a*e^4) - (b^2*d^4*p^2/(b*d^3*e - a*e^4)^2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(-I*sqrt(3) + 1)/(b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3) - (b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^

```
3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3)*(I*sqrt(3) + 1)) + sqrt(3
)*(b*d^3*x*e^2 + b*d^4*e - a*x*e^5 - a*d*e^4)*sqrt(-(4*b^2*d^4*p^2 - 16*a*b
*d*p^2*e^3 + (b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(2*b*d^2*p/(b*d^3*e -
a*e^4) - (b^2*d^4*p^2/(b*d^3*e - a*e^4)^2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(-
I*sqrt(3) + 1)/(b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e
^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^
3/(b*d^3 - a*e^3)^2)^(1/3) - (b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3
*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6
) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3)*(I*sqrt(3) + 1))^2 - 4*(b^2*d^5*p*
e - a*b*d^2*p*e^4)*(2*b*d^2*p/(b*d^3*e - a*e^4) - (b^2*d^4*p^2/(b*d^3*e - a
*e^4)^2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(-I*sqrt(3) + 1)/(b^3*d^6*p^3/(b*d^3
*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1
/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3) - (b^3*
d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e
 - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)
^(1/3)*(I*sqrt(3) + 1)))/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)))*log(-3/2
*(2*b*d^2*p/(b*d^3*e - a*e^4) - (b^2*d^4*p^2/(b*d^3*e - a*e^4)^2 - b*d*p^2/
(b*d^3*e^2 - a*e^5))*(-I*sqrt(3) + 1)/(b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/
2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^
3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3) - (b^3*d^6*p^3/(b*d^3*e -
 a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b
*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3)*(I*sqrt(3)
+ 1))*b*d^2*p*e + 2*b*p^2*x*e + 2*b*d*p^2 + 1/4*(b*d^3*e^2 - a*e^5)*(2*b*d^
2*p/(b*d^3*e - a*e^4) - (b^2*d^4*p^2/(b*d^3*e - a*e^4)^2 - b*d*p^2/(b*d^3*e
^2 - a*e^5))*(-I*sqrt(3) + 1)/(b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^
3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^
6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3) - (b^3*d^6*p^3/(b*d^3*e - a*e^4)^
3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*
d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2)^(1/3)*(I*sqrt(3) + 1))^2
+ 1/4*sqrt(3)*(b*d^3*e^2 - a*e^5)*(2*b*d^2*p/(b*d^3*e - a*e^4) - (b^2*d^4*p
^2/(b*d^3*e - a*e^4)^2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(-I*sqrt(3) + 1)/(b^3
*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*e^5)*(b*d^3*
e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^3 - a*e^3)^2
)^(1/3) - (b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 -
a*e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*
d^3 - a*e^3)^2)^(1/3)*(I*sqrt(3) + 1))*sqrt(-(4*b^2*d^4*p^2 - 16*a*b*d*p^2*
e^3 + (b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(2*b*d^2*p/(b*d^3*e - a*e^4)
- (b^2*d^4*p^2/(b*d^3*e - a*e^4)^2 - b*d*p^2/(b*d^3*e^2 - a*e^5))*(-I*sqrt(
3) + 1)/(b^3*d^6*p^3/(b*d^3*e - a*e^4)^3 - 3/2*b^2*d^3*p^3/((b*d^3*e^2 - a*
e^5)*(b*d^3*e - a*e^4)) + 1/2*b*p^3/(b*d^3*e^3 - a*e^6) + 1/2*a*b*p^3/(b*d^
3 - a*e^3)^2)^(1/3) - (b^3*d^6*p^3/(b*d^3*e - a...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.62, size = 398, normalized size = 1.36

$$\frac{bd^2p\log\left(|bx^3+a|\right)}{bd^3e-ae^4}+\frac{\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}bp\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^2d^2-\left(-ab^2\right)^{\frac{1}{3}}bde+\left(-ab^2\right)^{\frac{2}{3}}e^2}-\frac{\left(ab^3d^4pe^2-ab^3d^3p\left(-\frac{a}{b}\right)^{\frac{1}{3}}e^3-a^2b^2dpe^5+a^2b^2p\left(-\frac{a}{b}\right)^{\frac{1}{3}}e^6\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{ab^3d^6e^2-2a^2b^2d^3e^5+a^3be^8}+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}bdp-\left(-ab^2\right)^{\frac{2}{3}}pe\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\left(b^2d^3-abe^3\right)}-\frac{3bd^2pxe\log\left(xe+d\right)+bd^3p\log\left(bx^3+a\right)+3bd^3p\log\left(xe+d\right)+bd^3\log\left(c\right)-ape^3\log\left(bx^3+a\right)-ae^3\log\left(c\right)}{bd^3xe^2+bd^4e-axe^5-ade^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] b*d^2*p*log(abs(b*x^3 + a))/(b*d^3*e - a*e^4) + sqrt(3)*(-a*b^2)^(1/3)*b*p*
arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(b^2*d^2 - (-a*b^2)^(
1/3)*b*d*e + (-a*b^2)^(2/3)*e^2) - (a*b^3*d^4*p*e^2 - a*b^3*d^3*p*(-a/b)^(1
/3)*e^3 - a^2*b^2*d*p*e^5 + a^2*b^2*p*(-a/b)^(1/3)*e^6)*(-a/b)^(1/3)*log(ab
s(x - (-a/b)^(1/3)))/(a*b^3*d^6*e^2 - 2*a^2*b^2*d^3*e^5 + a^3*b*e^8) + 1/2*
((-a*b^2)^(1/3)*b*d*p - (-a*b^2)^(2/3)*p*e)*log(x^2 + x*(-a/b)^(1/3) + (-a/
b)^(2/3))/(b^2*d^3 - a*b*e^3) - (3*b*d^2*p*x*e*log(x*e + d) + b*d^3*p*log(b
*x^3 + a) + 3*b*d^3*p*log(x*e + d) + b*d^3*log(c) - a*p*e^3*log(b*x^3 + a)
- a*e^3*log(c))/(b*d^3*x*e^2 + b*d^4*e - a*x*e^5 - a*d*e^4)
```

Mupad [B]
time = 0.49, size = 736, normalized size = 2.52

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)/(d + e*x)^2,x)
```

```
[Out] symsum(log(-(27*a*b^4*d*p^3 + 27*a*b^4*e*p^3*x + 9*root(b*d^3*e^3*z^3 - a*e
^6*z^3 - 3*b*d^2*e^2*p*z^2 + 3*b*d*e*p^2*z - b*p^3, z, k)^3*a*b^4*d^4*e^3 +
 45*root(b*d^3*e^3*z^3 - a*e^6*z^3 - 3*b*d^2*e^2*p*z^2 + 3*b*d*e*p^2*z - b*
p^3, z, k)^3*a^2*b^3*d*e^6 - 9*root(b*d^3*e^3*z^3 - a*e^6*z^3 - 3*b*d^2*e^2
*p*z^2 + 3*b*d*e*p^2*z - b*p^3, z, k)^2*a^2*b^3*e^5*p + 36*root(b*d^3*e^3*z
^3 - a*e^6*z^3 - 3*b*d^2*e^2*p*z^2 + 3*b*d*e*p^2*z - b*p^3, z, k)^3*a^2*b^3
*e^7*x + 9*root(b*d^3*e^3*z^3 - a*e^6*z^3 - 3*b*d^2*e^2*p*z^2 + 3*b*d*e*p^2
*z - b*p^3, z, k)^2*a*b^4*d^3*e^2*p + 18*root(b*d^3*e^3*z^3 - a*e^6*z^3 - 3
*b*d^2*e^2*p*z^2 + 3*b*d*e*p^2*z - b*p^3, z, k)^3*a*b^4*d^3*e^4*x - 45*root
(b*d^3*e^3*z^3 - a*e^6*z^3 - 3*b*d^2*e^2*p*z^2 + 3*b*d*e*p^2*z - b*p^3, z,
k)*a*b^4*d^2*e*p^2 - 72*root(b*d^3*e^3*z^3 - a*e^6*z^3 - 3*b*d^2*e^2*p*z^2
+ 3*b*d*e*p^2*z - b*p^3, z, k)*a*b^4*d*e^2*p^2*x + 27*root(b*d^3*e^3*z^3 -
a*e^6*z^3 - 3*b*d^2*e^2*p*z^2 + 3*b*d*e*p^2*z - b*p^3, z, k)^2*a*b^4*d^2*e^
3*p*x)/e^2)*root(b*d^3*e^3*z^3 - a*e^6*z^3 - 3*b*d^2*e^2*p*z^2 + 3*b*d*e*p^
2*z - b*p^3, z, k), k, 1, 3) - log(c*(a + b*x^3)^p)/(d*e + e^2*x) + (3*b*d^
2*p*log(d + e*x))/(a*e^4 - b*d^3*e)
```

3.197 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{(d+ex)^3}\,dx$

Optimal. Leaf size=391

$$\frac{3bd^2p}{2e\left(bd^3-ae^3\right)(d+ex)}-\frac{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}\left(2bd^3-3\sqrt[3]{a}\,b^{2/3}d^2e+ae^3\right)p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2\left(bd^3-ae^3\right)^2}+\frac{\sqrt[3]{a}\,b^{2/3}\left(2bd^3+3\right.}{}$$

```
[Out] 3/2*b*d^2*p/e/(-a*e^3+b*d^3)/(e*x+d)+1/2*a^(1/3)*b^(2/3)*(2*b*d^3+3*a^(1/3)
*b^(2/3)*d^2*e+a*e^3)*p*ln(a^(1/3)+b^(1/3)*x)/(-a*e^3+b*d^3)^2-3/2*b*d*(2*a
*e^3+b*d^3)*p*ln(e*x+d)/e/(-a*e^3+b*d^3)^2-1/4*a^(1/3)*b^(2/3)*(2*b*d^3+3*a
^(1/3)*b^(2/3)*d^2*e+a*e^3)*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-a
*e^3+b*d^3)^2+1/2*b*d*(2*a*e^3+b*d^3)*p*ln(b*x^3+a)/e/(-a*e^3+b*d^3)^2-1/2*
ln(c*(b*x^3+a)^p)/e/(e*x+d)^2-1/2*a^(1/3)*b^(2/3)*(2*b*d^3-3*a^(1/3)*b^(2/3
)*d^2*e+a*e^3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/
(-a*e^3+b*d^3)^2
```

Rubi [A]
time = 0.48, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2513, 6857, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$-\frac{\sqrt[3]{a}\,b^{2/3}p\left(3\sqrt[3]{a}\,b^{2/3}d^2e+ae^3+2bd^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{4\left(bd^3-ae^3\right)^2}-\frac{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}p\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(-3\sqrt[3]{a}\,b^{2/3}d^2e+ae^3+2bd^3\right)}{2\left(bd^3-ae^3\right)^2}+\frac{\sqrt[3]{a}\,b^{2/3}p\left(3\sqrt[3]{a}\,b^{2/3}d^2e+ae^3+2bd^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2\left(bd^3-ae^3\right)^2}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2e(d+ex)^2}+\frac{bdp\left(2ae^3+bd^3\right)\log\left(a+bx^3\right)}{2e\left(bd^3-ae^3\right)^2}-\frac{3bdp\left(2ae^3+bd^3\right)\log(d+ex)}{2e\left(bd^3-ae^3\right)^2}+\frac{3bd^2p}{2e(d+ex)\left(bd^3-ae^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/(d + e*x)^3,x]
```

```
[Out] (3*b*d^2*p)/(2*e*(b*d^3 - a*e^3)*(d + e*x)) - (Sqrt[3]*a^(1/3)*b^(2/3)*(2*b
*d^3 - 3*a^(1/3)*b^(2/3)*d^2*e + a*e^3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(S
qrt[3]*a^(1/3))])/(2*(b*d^3 - a*e^3)^2) + (a^(1/3)*b^(2/3)*(2*b*d^3 + 3*a^(
1/3)*b^(2/3)*d^2*e + a*e^3)*p*Log[a^(1/3) + b^(1/3)*x])/(2*(b*d^3 - a*e^3)^
2) - (3*b*d*(b*d^3 + 2*a*e^3)*p*Log[d + e*x])/(2*e*(b*d^3 - a*e^3)^2) - (a^
(1/3)*b^(2/3)*(2*b*d^3 + 3*a^(1/3)*b^(2/3)*d^2*e + a*e^3)*p*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(4*(b*d^3 - a*e^3)^2) + (b*d*(b*d^3 + 2*a*
e^3)*p*Log[a + b*x^3])/(2*e*(b*d^3 - a*e^3)^2) - Log[c*(a + b*x^3)^p]/(2*e*
(d + e*x)^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
```

```
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{(d+ex)^3}\,dx &= -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2e(d+ex)^2}+\frac{(3bp)\int\frac{x^2}{(d+ex)^2(a+bx^3)}\,dx}{2e}\\
&= -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2e(d+ex)^2}+\frac{(3bp)\int\left(-\frac{d^2e}{(bd^3-ae^3)(d+ex)^2}-\frac{de(bd^3+2ae^3)}{(bd^3-ae^3)^2(d+ex)}+\frac{ae(2bd^3+ae^3)-3a}{(bd^3-}\right.}{2e}\\
&= \frac{3bd^2p}{2e\left(bd^3-ae^3\right)(d+ex)}-\frac{3bd(bd^3+2ae^3)\,p\log(d+ex)}{2e\left(bd^3-ae^3\right)^2}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2e(d+ex)^2}+\frac{(3}{}\\
&= \frac{3bd^2p}{2e\left(bd^3-ae^3\right)(d+ex)}-\frac{3bd(bd^3+2ae^3)\,p\log(d+ex)}{2e\left(bd^3-ae^3\right)^2}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2e(d+ex)^2}+\frac{(3}{}\\
&= \frac{3bd^2p}{2e\left(bd^3-ae^3\right)(d+ex)}-\frac{3bd(bd^3+2ae^3)\,p\log(d+ex)}{2e\left(bd^3-ae^3\right)^2}+\frac{bd(bd^3+2ae^3)\,p\log(a}{2e\left(bd^3-ae^3\right)^2}\\
&= \frac{3bd^2p}{2e\left(bd^3-ae^3\right)(d+ex)}+\frac{\sqrt[3]{a}\,b^{2/3}\left(2bd^3+3\sqrt[3]{a}\,b^{2/3}d^2e+ae^3\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right.}{2\left(bd^3-ae^3\right)^2}\\
&= \frac{3bd^2p}{2e\left(bd^3-ae^3\right)(d+ex)}+\frac{\sqrt[3]{a}\,b^{2/3}\left(2bd^3+3\sqrt[3]{a}\,b^{2/3}d^2e+ae^3\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right.}{2\left(bd^3-ae^3\right)^2}\\
&= \frac{3bd^2p}{2e\left(bd^3-ae^3\right)(d+ex)}-\frac{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}\left(2bd^3-3\sqrt[3]{a}\,b^{2/3}d^2e+ae^3\right)p\tan^{-1}\left(\frac{\sqrt[3]{a}-}{\sqrt{3}}\right.}{2\left(bd^3-ae^3\right)^2}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.45, size = 303, normalized size = 0.77

$$\frac{\frac{b^{2/3}p(d+ex)\left(6\sqrt[3]{b}\,d^2\left(bd^3-ae^3\right)-9b^{4/3}d^2e^2x^2(d+ex)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a}\right)+2\sqrt[3]{a}\,e\left(2bd^3+ae^3\right)(d+ex)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-6\sqrt[3]{b}\,d\left(bd^3+2ae^3\right)(d+ex)\log(d+ex)-\sqrt[3]{a}\,e\left(2bd^3+ae^3\right)(d+ex)\left(2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\right)+2\sqrt[3]{b}\,d\left(bd^3+2ae^3\right)(d+ex)\log\left(a+bx^3\right)\right)}{\left(bd^3-ae^3\right)^2}-2\log\left(c\left(a+bx^3\right)^p\right)}{4e(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/(d + e*x)^3,x]
```

```
[Out] ((b^(2/3)*p*(d + e*x)*(6*b^(1/3)*d^2*(b*d^3 - a*e^3) - 9*b^(4/3)*d^2*e^2*x^
2*(d + e*x)*Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)] + 2*a^(1/3)*e*(2*b
*d^3 + a*e^3)*(d + e*x)*Log[a^(1/3) + b^(1/3)*x] - 6*b^(1/3)*d*(b*d^3 + 2*a
*e^3)*(d + e*x)*Log[d + e*x] - a^(1/3)*e*(2*b*d^3 + a*e^3)*(d + e*x)*(2*Sqr
t[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2]) + 2*b^(1/3)*d*(b*d^3 + 2*a*e^3)*(d + e*x)*Log[a + b
*x^3]))/(b*d^3 - a*e^3)^2 - 2*Log[c*(a + b*x^3)^p])/(4*e*(d + e*x)^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.88, size = 4085, normalized size = 10.45

method	result	size
risch	Expression too large to display	4085

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/e/(e*x+d)^2*ln((b*x^3+a)^p)+1/4*(2*sum(_R*ln(((-4*a^3*e^13+6*a^2*b*d^3
*e^10-2*b^3*d^9*e^4)*_R^3+(14*a^2*b*d*e^9*p-10*a*b^2*d^4*e^6*p-4*b^3*d^7*e^
3*p)*_R^2+(3*a*b^2*d^2*e^5*p^2+6*b^3*d^5*e^2*p^2)*_R+3*a*b^2*e^4*p^3)*x+(-5
*a^3*d*e^12+9*a^2*b*d^4*e^9-3*a*b^2*d^7*e^6-b^3*d^10*e^3)*_R^3+(8*a^2*b*d^2
*e^8*p-7*a*b^2*d^5*e^5*p-b^3*d^8*e^2*p)*_R^2+(-a^2*b*e^7*p^2+5*a*b^2*d^3*e^
4*p^2+5*b^3*d^6*e*p^2)*_R-3*a*b^2*d*e^3*p^3-3*b^3*d^4*p^3),_R=RootOf((a^2*e
^9-2*a*b*d^3*e^6+b^2*d^6*e^3)*_Z^3+(-6*a*b*d*e^5*p-3*b^2*d^4*e^2*p)*_Z^2+3*
b^2*d^2*e*p^2*_Z-b^2*p^3))*b^2*d^6*e^3*x^2+4*sum(_R*ln(((-4*a^3*e^13+6*a^2*
b*d^3*e^10-2*b^3*d^9*e^4)*_R^3+(14*a^2*b*d*e^9*p-10*a*b^2*d^4*e^6*p-4*b^3*d
^7*e^3*p)*_R^2+(3*a*b^2*d^2*e^5*p^2+6*b^3*d^5*e^2*p^2)*_R+3*a*b^2*e^4*p^3)*
x+(-5*a^3*d*e^12+9*a^2*b*d^4*e^9-3*a*b^2*d^7*e^6-b^3*d^10*e^3)*_R^3+(8*a^2*
b*d^2*e^8*p-7*a*b^2*d^5*e^5*p-b^3*d^8*e^2*p)*_R^2+(-a^2*b*e^7*p^2+5*a*b^2*d
^3*e^4*p^2+5*b^3*d^6*e*p^2)*_R-3*a*b^2*d*e^3*p^3-3*b^3*d^4*p^3),_R=RootOf((
a^2*e^9-2*a*b*d^3*e^6+b^2*d^6*e^3)*_Z^3+(-6*a*b*d*e^5*p-3*b^2*d^4*e^2*p)*_Z
^2+3*b^2*d^2*e*p^2*_Z-b^2*p^3))*a^2*d*e^8*x+4*sum(_R*ln(((-4*a^3*e^13+6*a^2
*b*d^3*e^10-2*b^3*d^9*e^4)*_R^3+(14*a^2*b*d*e^9*p-10*a*b^2*d^4*e^6*p-4*b^3*
d^7*e^3*p)*_R^2+(3*a*b^2*d^2*e^5*p^2+6*b^3*d^5*e^2*p^2)*_R+3*a*b^2*e^4*p^3)
*x+(-5*a^3*d*e^12+9*a^2*b*d^4*e^9-3*a*b^2*d^7*e^6-b^3*d^10*e^3)*_R^3+(8*a^2
*b*d^2*e^8*p-7*a*b^2*d^5*e^5*p-b^3*d^8*e^2*p)*_R^2+(-a^2*b*e^7*p^2+5*a*b^2*
d^3*e^4*p^2+5*b^3*d^6*e*p^2)*_R-3*a*b^2*d*e^3*p^3-3*b^3*d^4*p^3),_R=RootOf(
(a^2*e^9-2*a*b*d^3*e^6+b^2*d^6*e^3)*_Z^3+(-6*a*b*d*e^5*p-3*b^2*d^4*e^2*p)*_
Z^2+3*b^2*d^2*e*p^2*_Z-b^2*p^3))*b^2*d^7*e^2*x-4*sum(_R*ln(((-4*a^3*e^13+6*
a^2*b*d^3*e^10-2*b^3*d^9*e^4)*_R^3+(14*a^2*b*d*e^9*p-10*a*b^2*d^4*e^6*p-4*b
^3*d^7*e^3*p)*_R^2+(3*a*b^2*d^2*e^5*p^2+6*b^3*d^5*e^2*p^2)*_R+3*a*b^2*e^4*p
^3)*x+(-5*a^3*d*e^12+9*a^2*b*d^4*e^9-3*a*b^2*d^7*e^6-b^3*d^10*e^3)*_R^3+(8*
```

```
a^2*b*d^2*e^8*p-7*a*b^2*d^5*e^5*p-b^3*d^8*e^2*p)*_R^2+(-a^2*b*e^7*p^2+5*a*b
^2*d^3*e^4*p^2+5*b^3*d^6*e*p^2)*_R-3*a*b^2*d*e^3*p^3-3*b^3*d^4*p^3),_R=Root
Of((a^2*e^9-2*a*b*d^3*e^6+b^2*d^6*e^3)*_Z^3+(-6*a*b*d*e^5*p-3*b^2*d^4*e^2*p
)*_Z^2+3*b^2*d^2*e*p^2*_Z-b^2*p^3))*a*b*d^5*e^4+4*ln(c)*a*b*d^3*e^3-I*Pi*a^
2*e^6*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)-I*Pi*b^2*d^6*csgn(I*(b*x^3+a)^p)*cs
gn(I*c*(b*x^3+a)^p)^2+2*I*Pi*a*b*d^3*e^3*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)+
6*b^2*d^6*p-12*ln(e*x+d)*a*b*d*e^5*p*x^2-24*ln(e*x+d)*a*b*d^2*e^4*p*x+2*I*P
i*a*b*d^3*e^3*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2+2*sum(_R*ln(((-4*
a^3*e^13+6*a^2*b*d^3*e^10-2*b^3*d^9*e^4)*_R^3+(14*a^2*b*d*e^9*p-10*a*b^2*d^
4*e^6*p-4*b^3*d^7*e^3*p)*_R^2+(3*a*b^2*d^2*e^5*p^2+6*b^3*d^5*e^2*p^2)*_R+3*
a*b^2*e^4*p^3)*x+(-5*a^3*d*e^12+9*a^2*b*d^4*e^9-3*a*b^2*d^7*e^6-b^3*d^10*e^
3)*_R^3+(8*a^2*b*d^2*e^8*p-7*a*b^2*d^5*e^5*p-b^3*d^8*e^2*p)*_R^2+(-a^2*b*e^
7*p^2+5*a*b^2*d^3*e^4*p^2+5*b^3*d^6*e*p^2)*_R-3*a*b^2*d*e^3*p^3-3*b^3*d^4*p
^3),_R=RootOf((a^2*e^9-2*a*b*d^3*e^6+b^2*d^6*e^3)*_Z^3+(-6*a*b*d*e^5*p-3*b^
2*d^4*e^2*p)*_Z^2+3*b^2*d^2*e*p^2*_Z-b^2*p^3))*a^2*e^9*x^2+2*sum(_R*ln(((-4
*a^3*e^13+6*a^2*b*d^3*e^10-2*b^3*d^9*e^4)*_R^3+(14*a^2*b*d*e^9*p-10*a*b^2*d
^4*e^6*p-4*b^3*d^7*e^3*p)*_R^2+(3*a*b^2*d^2*e^5*p^2+6*b^3*d^5*e^2*p^2)*_R+3
*a*b^2*e^4*p^3)*x+(-5*a^3*d*e^12+9*a^2*b*d^4*e^9-3*a*b^2*d^7*e^6-b^3*d^10*e
^3)*_R^3+(8*a^2*b*d^2*e^8*p-7*a*b^2*d^5*e^5*p-b^3*d^8*e^2*p)*_R^2+(-a^2*b*e
^7*p^2+5*a*b^2*d^3*e^4*p^2+5*b^3*d^6*e*p^2)*_R-3*a*b^2*d*e^3*p^3-3*b^3*d^4*
p^3),_R=RootOf((a^2*e^9-2*a*b*d^3*e^6+b^2*d^6*e^3)*_Z^3+(-6*a*b*d*e^5*p-3*b
^2*d^4*e^2*p)*_Z^2+3*b^2*d^2*e*p^2*_Z-b^2*p^3))*a^2*d^2*e^7+2*sum(_R*ln(((-
4*a^3*e^13+6*a^2*b*d^3*e^10-2*b^3*d^9*e^4)*_R^3+(14*a^2*b*d*e^9*p-10*a*b^2*
d^4*e^6*p-4*b^3*d^7*e^3*p)*_R^2+(3*a*b^2*d^2*e^5*p^2+6*b^3*d^5*e^2*p^2)*_R+
3*a*b^2*e^4*p^3)*x+(-5*a^3*d*e^12+9*a^2*b*d^4*e^9-3*a*b^2*d^7*e^6-b^3*d^10*
e^3)*_R^3+(8*a^2*b*d^2*e^8*p-7*a*b^2*d^5*e^5*p-b^3*d^8*e^2*p)*_R^2+(-a^2*b*
e^7*p^2+5*a*b^2*d^3*e^4*p^2+5*b^3*d^6*e*p^2)*_R-3*a*b^2*d*e^3*p^3-3*b^3*d^4
*p^3),_R=RootOf((a^2*e^9-2*a*b*d^3*e^6+b^2*d^6*e^3)*_Z^3+(-6*a*b*d*e^5*p-3*
b^2*d^4*e^2*p)*_Z^2+3*b^2*d^2*e*p^2*_Z-b^2*p^3))*b^2*d^8*e-6*ln(e*x+d)*b^2*
d^6*p-6*a*d^3*e^3*b*p-2*I*Pi*a*b*d^3*e^3*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^
3+a)^p)*csgn(I*c)-8*sum(_R*ln(((-4*a^3*e^13+6*a^2*b*d^3*e^10-2*b^3*d^9*e^4)
*_R^3+(14*a^2*b*d*e^9*p-10*a*b^2*d^4*e^6*p-4*b^3*d^7*e^3*p)*_R^2+(3*a*b^2*d
^2*e^5*p^2+6*b^3*d^5*e^2*p^2)*_R+3*a*b^2*e^4*p^3)*x+(-5*a^3*d*e^12+9*a^2*b*
d^4*e^9-3*a*b^2*d^7*e^6-b^3*d^10*e^3)*_R^3+(8*a^2*b*d^2*e^8*p-7*a*b^2*d^5*e
^5*p-b^3*d^8*e^2*p)*_R^2+(-a^2*b*e^7*p^2+5*a*b^2*d^3*e^4*p^2+5*b^3*d^6*e*p^
2)*_R-3*a*b^2*d*e^3*p^3-3*b^3*d^4*p^3),_R=RootOf((a^2*e^9-2*a*b*d^3*e^6+b^2
*d^6*e^3)*_Z^3+(-6*a*b*d*e^5*p-3*b^2*d^4*e^2*p)*_Z^2+3*b^2*d^2*e*p^2*_Z-b^2
*p^3))*a*b*d^4*e^5*x+I*Pi*a^2*e^6*csgn(I*c*(b*x^3+a)^p)^3+I*Pi*b^2*d^6*csgn
(I*c*(b*x^3+a)^p)^3+6*b^2*d^5*e*p*x-4*sum(_R*ln(((-4*a^3*e^13+6*a^2*b*d^3*e
^10-2*b^3*d^9*e^4)*_R^3+(14*a^2*b*d*e^9*p-10*a*b^2*d^4*e^6*p-4*b^3*d^7*e^3*
p)*_R^2+(3*a*b^2*d^2*e^5*p^2+6*b^3*d^5*e^2*p^2)*_R+3*a*b^2*e^4*p^3)*x+(-5*a
^3*d*e^12+9*a^2*b*d^4*e^9-3*a*b^2*d^7*e^6-b^3*d...
```

Maxima [A]

time = 0.56, size = 500, normalized size = 1.28

$$\frac{1}{4}\left(\frac{2\sqrt{3}\left(2\,abd^3\left(\frac{a}{b}\right)^{\frac{1}{3}}e - 3\,abd^2\left(\frac{a}{b}\right)^{\frac{2}{3}}e^2 + a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}e^4\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\left(b^3d^6\left(\frac{a}{b}\right)^{\frac{2}{3}} - 2\,ab^2d^3\left(\frac{a}{b}\right)^{\frac{2}{3}}e^3 + a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}e^6\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{6\,d^2}{bd^4 - ade^3 + (bd^3e - ae^4)x} + \frac{\left(2\,b^2d^4\left(\frac{a}{b}\right)^{\frac{2}{3}} - 2\,abd^3e - 3\,abd^2\left(\frac{a}{b}\right)^{\frac{1}{3}}e^2 + 4\,abd\left(\frac{a}{b}\right)^{\frac{2}{3}}e^3 - a^2e^4\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b^3d^6\left(\frac{a}{b}\right)^{\frac{2}{3}} - 2\,ab^2d^3\left(\frac{a}{b}\right)^{\frac{2}{3}}e^3 + a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}e^6} - \frac{6\,(bd^4 + 2\,ade^3)\log(xe+d)}{b^2d^6 - 2\,abd^3e^3 + a^2e^6} + \frac{2\left(b^2d^4\left(\frac{a}{b}\right)^{\frac{2}{3}} + 2\,abd^3e + 3\,abd^2\left(\frac{a}{b}\right)^{\frac{1}{3}}e^2 + 2\,abd\left(\frac{a}{b}\right)^{\frac{2}{3}}e^3 + a^2e^4\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^3d^6\left(\frac{a}{b}\right)^{\frac{2}{3}} - 2\,ab^2d^3\left(\frac{a}{b}\right)^{\frac{2}{3}}e^3 + a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}e^6}\right)bpe^{(-1)} - \frac{e^{(-1)}\log\left((bx^3+a)^pc\right)}{2\,(xe+d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(3)*(2*a*b*d^3*(a/b)^(1/3)*e - 3*a*b*d^2*(a/b)^(2/3)*e^2 + a^2*(
a/b)^(1/3)*e^4)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^3*d
^6*(a/b)^(2/3) - 2*a*b^2*d^3*(a/b)^(2/3)*e^3 + a^2*b*(a/b)^(2/3)*e^6)*(a/b)
^(1/3)) + 6*d^2/(b*d^4 - a*d*e^3 + (b*d^3*e - a*e^4)*x) + (2*b^2*d^4*(a/b)^
(2/3) - 2*a*b*d^3*e - 3*a*b*d^2*(a/b)^(1/3)*e^2 + 4*a*b*d*(a/b)^(2/3)*e^3 -
 a^2*e^4)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*d^6*(a/b)^(2/3) - 2*a
*b^2*d^3*(a/b)^(2/3)*e^3 + a^2*b*(a/b)^(2/3)*e^6) - 6*(b*d^4 + 2*a*d*e^3)*l
og(x*e + d)/(b^2*d^6 - 2*a*b*d^3*e^3 + a^2*e^6) + 2*(b^2*d^4*(a/b)^(2/3) +
2*a*b*d^3*e + 3*a*b*d^2*(a/b)^(1/3)*e^2 + 2*a*b*d*(a/b)^(2/3)*e^3 + a^2*e^4
)*log(x + (a/b)^(1/3))/(b^3*d^6*(a/b)^(2/3) - 2*a*b^2*d^3*(a/b)^(2/3)*e^3 +
 a^2*b*(a/b)^(2/3)*e^6))*b*p*e^(-1) - 1/2*e^(-1)*log((b*x^3 + a)^p*c)/(x*e
+ d)^2
```

Fricas [C] Result contains complex when optimal does not.
time = 4.72, size = 12591, normalized size = 32.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(24*b^2*d^5*p*x*e + 24*b^2*d^6*p - 24*a*b*d^2*p*x*e^4 - 24*a*b*d^3*p*e
^3 + 2*(b^2*d^6*x^2*e^3 + 2*b^2*d^7*x*e^2 + b^2*d^8*e - 2*a*b*d^3*x^2*e^6 -
 4*a*b*d^4*x*e^5 - 2*a*b*d^5*e^4 + a^2*x^2*e^9 + 2*a^2*d*x*e^8 + a^2*d^2*e^
7)*((b^2*d^2*p^2/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8) - (b^2*d^4*p + 2*a
*b*d*p*e^3)^2/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^2)*(-I*sqrt(3) + 1)/(-3
/16*(b^2*d^4*p + 2*a*b*d*p*e^3)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 +
 a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^
3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*
e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*p*e^3)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2
*e^7)^3)^(1/3) - 4*(-3/16*(b^2*d^4*p + 2*a*b*d*p*e^3)*b^2*d^2*p^2/((b^2*d^6
*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/
16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)
*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*p*e^3)^3/(b^2*d^6*e
 - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3)*(I*sqrt(3) + 1) + 4*(b^2*d^4*p + 2*a*b
*d*p*e^3)/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7))*log(8*b^2*d^3*p^2*x*e - 7*
b^2*d^4*p^2 + a*b*p^2*x*e^4 - 2*a*b*d*p^2*e^3 - 3/16*(b^2*d^8*e^2 - 2*a*b*d
^5*e^5 + a^2*d^2*e^8)*((b^2*d^2*p^2/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)
```

```
 - (b^2*d^4*p + 2*a*b*d*p*e^3)^2/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^2)*(
-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p + 2*a*b*d*p*e^3)*b^2*d^2*p^2/((b^2*d^6*e^
2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*
b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*
b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*p*e^3)^3/(b^2*d^6*e -
2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3) - 4*(-3/16*(b^2*d^4*p + 2*a*b*d*p*e^3)*b^
2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e
^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/1
6*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*
p*e^3)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3)*(I*sqrt(3) + 1) + 4
*(b^2*d^4*p + 2*a*b*d*p*e^3)/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7))^2 + 1/4
*(10*b^2*d^6*p*e + 16*a*b*d^3*p*e^4 + a^2*p*e^7)*((b^2*d^2*p^2/(b^2*d^6*e^2
 - 2*a*b*d^3*e^5 + a^2*e^8) - (b^2*d^4*p + 2*a*b*d*p*e^3)^2/(b^2*d^6*e - 2*
a*b*d^3*e^4 + a^2*e^7)^2)*(-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p + 2*a*b*d*p*e^
3)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*
d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9)
 + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a
*b*d*p*e^3)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3) - 4*(-3/16*(b^
2*d^4*p + 2*a*b*d*p*e^3)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^
8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a
*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4
 + 1/8*(b^2*d^4*p + 2*a*b*d*p*e^3)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3
)^(1/3)*(I*sqrt(3) + 1) + 4*(b^2*d^4*p + 2*a*b*d*p*e^3)/(b^2*d^6*e - 2*a*b*
d^3*e^4 + a^2*e^7))) + (12*b^2*d^4*p*x^2*e^2 + 24*b^2*d^5*p*x*e + 12*b^2*d^
6*p + 24*a*b*d*p*x^2*e^5 + 48*a*b*d^2*p*x*e^4 + 24*a*b*d^3*p*e^3 - (b^2*d^6
*x^2*e^3 + 2*b^2*d^7*x*e^2 + b^2*d^8*e - 2*a*b*d^3*x^2*e^6 - 4*a*b*d^4*x*e^
5 - 2*a*b*d^5*e^4 + a^2*x^2*e^9 + 2*a^2*d*x*e^8 + a^2*d^2*e^7)*((b^2*d^2*p^
2/(b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8) - (b^2*d^4*p + 2*a*b*d*p*e^3)^2/(
b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^2)*(-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p
 + 2*a*b*d*p*e^3)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*
d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e
^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b
^2*d^4*p + 2*a*b*d*p*e^3)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3)
 - 4*(-3/16*(b^2*d^4*p + 2*a*b*d*p*e^3)*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^
3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2
*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) + 1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d
^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b*d*p*e^3)^3/(b^2*d^6*e - 2*a*b*d^3*e^
4 + a^2*e^7)^3)^(1/3)*(I*sqrt(3) + 1) + 4*(b^2*d^4*p + 2*a*b*d*p*e^3)/(b^2*
d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)) + sqrt(3)*(b^2*d^6*x^2*e^3 + 2*b^2*d^7*x*
e^2 + b^2*d^8*e - 2*a*b*d^3*x^2*e^6 - 4*a*b*d^4*x*e^5 - 2*a*b*d^5*e^4 + a^2
*x^2*e^9 + 2*a^2*d*x*e^8 + a^2*d^2*e^7)*sqrt(-(16*b^4*d^8*p^2 - 320*a*b^3*d
^5*p^2*e^3 - 128*a^2*b^2*d^2*p^2*e^6 + (b^4*d^12*e^2 - 4*a*b^3*d^9*e^5 + 6*
a^2*b^2*d^6*e^8 - 4*a^3*b*d^3*e^11 + a^4*e^14)*((b^2*d^2*p^2/(b^2*d^6*e^2 -
 2*a*b*d^3*e^5 + a^2*e^8) - (b^2*d^4*p + 2*a*b*d*p*e^3)^2/(b^2*d^6*e - 2*a*
b*d^3*e^4 + a^2*e^7)^2)*(-I*sqrt(3) + 1)/(-3/16*(b^2*d^4*p + 2*a*b*d*p*e^3)
```

```
*b^2*d^2*p^2/((b^2*d^6*e^2 - 2*a*b*d^3*e^5 + a^2*e^8)*(b^2*d^6*e - 2*a*b*d^
3*e^4 + a^2*e^7)) + 1/16*b^2*p^3/(b^2*d^6*e^3 - 2*a*b*d^3*e^6 + a^2*e^9) +
1/16*(8*b*d^3 + a*e^3)*a*b^2*p^3/(b*d^3 - a*e^3)^4 + 1/8*(b^2*d^4*p + 2*a*b
*d*p*e^3)^3/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7)^3)^(1/3) - 4*(-3/16*(b^2*
d^4*p + 2*a*b*d*p*e^3)*b^2*d^2*p^2/((b^2*d^6*e^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 790 vs. 2(325) = 650.
time = 3.25, size = 790, normalized size = 2.02

$$-\frac{\left(2\,ab^5d^9pe^2-3\,ab^5d^8p\left(-\frac{a}{b}\right)^{\frac{1}{3}}e^3-3\,a^2b^4d^6pe^5+6\,a^2b^4d^5p\left(-\frac{a}{b}\right)^{\frac{1}{3}}e^6-3\,a^3b^3d^2p\left(-\frac{a}{b}\right)^{\frac{1}{3}}e^9+a^4b^2pe^{11}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{2\,(ab^5d^{12}e^2-4\,a^2b^4d^9e^5+6\,a^3b^3d^6e^8-4\,a^4b^2d^3e^{11}+a^5be^{14})}+\frac{3\left(2\,(-ab^2)^{\frac{1}{3}}bdp-(-ab^2)^{\frac{2}{3}}pe\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{2\left(\sqrt{3}\,b^2d^4-2\,\sqrt{3}\,(-ab^2)^{\frac{1}{3}}bd^3e+2\,\sqrt{3}\,abde^3+3\,\sqrt{3}\,(-ab^2)^{\frac{2}{3}}d^2e^2-\sqrt{3}\,(-ab^2)^{\frac{1}{3}}ae^4\right)}+\frac{\left(2\,(-ab^2)^{\frac{1}{3}}bd^3p-3\,(-ab^2)^{\frac{2}{3}}d^2pe+(-ab^2)^{\frac{1}{3}}ape^3\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{4\,(b^2d^6-2\,abd^3e^3+a^2e^6)}+\frac{(b^2d^4p+2\,abdpe^3)\log\left(|bx^3+a|\right)}{2\,(b^2d^6e-2\,abd^3e^4+a^2e^7)}-\frac{3\,b^2d^4px^2e^2\log\left(xe+d\right)+6\,b^2d^5pxe\log\left(xe+d\right)-3\,b^2d^5pxe+b^2d^6p\log\left(bx^3+a\right)+3\,b^2d^6p\log\left(xe+d\right)-3\,b^2d^6p+b^2d^6\log\left(c\right)-2\,abd^3pe^3\log\left(bx^3+a\right)+6\,abdpx^2e^5\log\left(xe+d\right)+12\,abd^2pxe^4\log\left(xe+d\right)+6\,abd^3pe^3\log\left(xe+d\right)+3\,abd^2pxe^4+3\,abd^3pe^3-2\,abd^3e^3\log\left(c\right)+a^2pe^6\log\left(bx^3+a\right)+a^2e^6\log\left(c\right)}{2\,(b^2d^6x^2e^3+2\,b^2d^7xe^2+b^2d^8e-2\,abd^3x^2e^6-4\,abd^4xe^5-2\,abd^5e^4+a^2x^2e^9+2\,a^2dxe^8+a^2d^2e^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*a*b^5*d^9*p*e^2 - 3*a*b^5*d^8*p*(-a/b)^(1/3)*e^3 - 3*a^2*b^4*d^6*p*
e^5 + 6*a^2*b^4*d^5*p*(-a/b)^(1/3)*e^6 - 3*a^3*b^3*d^2*p*(-a/b)^(1/3)*e^9 +
 a^4*b^2*p*e^11)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5*d^12*e^2 -
4*a^2*b^4*d^9*e^5 + 6*a^3*b^3*d^6*e^8 - 4*a^4*b^2*d^3*e^11 + a^5*b*e^14) +
3/2*(2*(-a*b^2)^(1/3)*b*d*p - (-a*b^2)^(2/3)*p*e)*arctan(1/3*sqrt(3)*(2*x +
 (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*b^2*d^4 - 2*sqrt(3)*(-a*b^2)^(1/3)*b*
d^3*e + 2*sqrt(3)*a*b*d*e^3 + 3*sqrt(3)*(-a*b^2)^(2/3)*d^2*e^2 - sqrt(3)*(-
a*b^2)^(1/3)*a*e^4) + 1/4*(2*(-a*b^2)^(1/3)*b*d^3*p - 3*(-a*b^2)^(2/3)*d^2*
p*e + (-a*b^2)^(1/3)*a*p*e^3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(b^2
*d^6 - 2*a*b*d^3*e^3 + a^2*e^6) + 1/2*(b^2*d^4*p + 2*a*b*d*p*e^3)*log(abs(b
*x^3 + a))/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7) - 1/2*(3*b^2*d^4*p*x^2*e^2
*log(x*e + d) + 6*b^2*d^5*p*x*e*log(x*e + d) - 3*b^2*d^5*p*x*e + b^2*d^6*p*
log(b*x^3 + a) + 3*b^2*d^6*p*log(x*e + d) - 3*b^2*d^6*p + b^2*d^6*log(c) -
2*a*b*d^3*p*e^3*log(b*x^3 + a) + 6*a*b*d*p*x^2*e^5*log(x*e + d) + 12*a*b*d^
2*p*x*e^4*log(x*e + d) + 6*a*b*d^3*p*e^3*log(x*e + d) + 3*a*b*d^2*p*x*e^4 +
 3*a*b*d^3*p*e^3 - 2*a*b*d^3*e^3*log(c) + a^2*p*e^6*log(b*x^3 + a) + a^2*e^
6*log(c))/(b^2*d^6*x^2*e^3 + 2*b^2*d^7*x*e^2 + b^2*d^8*e - 2*a*b*d^3*x^2*e^
6 - 4*a*b*d^4*x*e^5 - 2*a*b*d^5*e^4 + a^2*x^2*e^9 + 2*a^2*d*x*e^8 + a^2*d^2
*e^7)
```

Mupad [B]
time = 0.88, size = 2227, normalized size = 5.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)/(d + e*x)^3,x)
[Out] symsum(log(-(27*a*b^6*d^4*p^3 + 216*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3
*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^
2*e*p^2*z + b^2*p^3, z, k)^3*a^2*b^5*d^7*e^6 - 648*root(16*a*b*d^3*e^6*z^3
- 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p
*z^2 - 6*b^2*d^2*e*p^2*z + b^2*p^3, z, k)^3*a^3*b^4*d^4*e^9 + 72*root(16*a*
b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12
*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*z + b^2*p^3, z, k)^3*a*b^6*d^10*e^3 +
360*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*
e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*z + b^2*p^3, z, k)^3*a^4
*b^3*d*e^12 + 18*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^
3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*z + b^2*p^3
, z, k)*a^3*b^4*e^7*p^2 + 288*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 -
 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^
2*z + b^2*p^3, z, k)^3*a^4*b^3*e^13*x + 27*a^2*b^5*d*e^3*p^3 - 27*a^2*b^5*e
^4*p^3*x + 36*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 +
 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*z + b^2*p^3, z
, k)^2*a*b^6*d^8*e^2*p + 144*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 -
8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2
*z + b^2*p^3, z, k)^3*a*b^6*d^9*e^4*x - 90*root(16*a*b*d^3*e^6*z^3 - 8*b^2*
d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6
*b^2*d^2*e*p^2*z + b^2*p^3, z, k)*a^2*b^5*d^3*e^4*p^2 + 252*root(16*a*b*d^3
*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*
d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*z + b^2*p^3, z, k)^2*a^2*b^5*d^5*e^5*p - 28
8*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^
5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*z + b^2*p^3, z, k)^2*a^3*b
^4*d^2*e^8*p - 432*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*
z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*z + b^2*p
^3, z, k)^3*a^3*b^4*d^3*e^10*x - 90*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3
*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^
2*e*p^2*z + b^2*p^3, z, k)*a*b^6*d^6*e*p^2 - 54*root(16*a*b*d^3*e^6*z^3 - 8
*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^
2 - 6*b^2*d^2*e*p^2*z + b^2*p^3, z, k)*a^2*b^5*d^2*e^5*p^2*x + 360*root(16*
a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 +
12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*z + b^2*p^3, z, k)^2*a^2*b^5*d^4*e^6
*p*x - 108*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24
*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*z + b^2*p^3, z, k
)*a*b^6*d^5*e^2*p^2*x + 144*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8
```

```
*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*
z + b^2*p^3, z, k)^2*a*b^6*d^7*e^3*p*x - 504*root(16*a*b*d^3*e^6*z^3 - 8*b^
2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 -
 6*b^2*d^2*e*p^2*z + b^2*p^3, z, k)^2*a^3*b^4*d*e^9*p*x)/(8*a^2*e^8 + 8*b^2
*d^6*e^2 - 16*a*b*d^3*e^5))*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8
*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*
z + b^2*p^3, z, k), k, 1, 3) - log(c*(a + b*x^3)^p)/(2*(d^2*e + e^3*x^2 + 2
*d*e^2*x)) - (3*b*d^2*p)/(2*a*d*e^4 - 2*b*d^4*e + 2*a*e^5*x - 2*b*d^3*e^2*x
) - (3*b^2*d^4*p*log(d + e*x))/(2*a^2*e^7 + 2*b^2*d^6*e - 4*a*b*d^3*e^4) -
(6*a*b*d*e^3*p*log(d + e*x))/(2*a^2*e^7 + 2*b^2*d^6*e - 4*a*b*d^3*e^4)
```

3.198 $\int (d+ex)^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right) \, dx$

Optimal. Leaf size=139

$$\frac{be(6a^2d^2-4abde+b^2e^2)\,px}{4a^3}+\frac{be^2(4ad-be)px^2}{8a^2}+\frac{be^3px^3}{12a}+\frac{(d+ex)^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4e}+\frac{d^4p\log(x)}{4e}-\frac{(ad-be)}{}$$

```
[Out] 1/4*b*e*(6*a^2*d^2-4*a*b*d*e+b^2*e^2)*p*x/a^3+1/8*b*e^2*(4*a*d-b*e)*p*x^2/a
^2+1/12*b*e^3*p*x^3/a+1/4*(e*x+d)^4*ln(c*(a+b/x)^p)/e+1/4*d^4*p*ln(x)/e-1/4
*(a*d-b*e)^4*p*ln(a*x+b)/a^4/e
```

Rubi [A]

time = 0.09, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2513, 528, 84}

$$-\frac{p(ad-be)^4\log(ax+b)}{4a^4e}+\frac{be^2px^2(4ad-be)}{8a^2}+\frac{bepx(6a^2d^2-4abde+b^2e^2)}{4a^3}+\frac{(d+ex)^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4e}+\frac{be^3px^3}{12a}+\frac{d^4p\log(x)}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*Log[c*(a + b/x)^p],x]
```

```
[Out] (b*e*(6*a^2*d^2 - 4*a*b*d*e + b^2*e^2)*p*x)/(4*a^3) + (b*e^2*(4*a*d - b*e)*
p*x^2)/(8*a^2) + (b*e^3*p*x^3)/(12*a) + ((d + e*x)^4*Log[c*(a + b/x)^p])/(4
*e) + (d^4*p*Log[x])/(4*e) - ((a*d - b*e)^4*p*Log[b + a*x])/(4*a^4*e)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx &= \frac{(d+ex)^4 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4e} + \frac{(bp)\int \frac{(d+ex)^4}{\left(a+\frac{b}{x}\right)x^2}\,dx}{4e} \\ &= \frac{(d+ex)^4 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4e} + \frac{(bp)\int \frac{(d+ex)^4}{x(b+ax)}\,dx}{4e} \\ &= \frac{(d+ex)^4 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4e} + \frac{(bp)\int \left(\frac{e^2(6a^2d^2-4abde+b^2e^2)}{a^3} + \frac{d^4}{bx} + \frac{e^3(4ad-be)}{a^2}\right.}{4e} \\ &= \frac{be(6a^2d^2-4abde+b^2e^2)\,px}{4a^3} + \frac{be^2(4ad-be)px^2}{8a^2} + \frac{be^3px^3}{12a} + \frac{(d+ex)^4}{}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 114, normalized size = 0.82

$$\frac{\frac{be^2px(6b^2e^2-3abe(8d+ex)+2a^2(18d^2+6dex+e^2x^2))}{6a^3} + (d+ex)^4\log\left(c\left(a+\frac{b}{x}\right)^p\right) + d^4p\log(x) - \frac{(ad-be)^4p\log(b+ax)}{a^4}}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*Log[c*(a + b/x)^p],x]
```

```
[Out] ((b*e^2*p*x*(6*b^2*e^2 - 3*a*b*e*(8*d + e*x) + 2*a^2*(18*d^2 + 6*d*e*x + e^
2*x^2)))/(6*a^3) + (d + e*x)^4*Log[c*(a + b/x)^p] + d^4*p*Log[x] - ((a*d -
b*e)^4*p*Log[b + a*x])/a^4)/(4*e)
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int (ex+d)^3 \ln\left(c\left(a+\frac{b}{x}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*ln(c*(a+b/x)^p),x)
```

```
[Out] int((e*x+d)^3*ln(c*(a+b/x)^p),x)
```

Maxima [A]
time = 0.36, size = 160, normalized size = 1.15

$$\frac{1}{24}\,bp\left(\frac{2\,a^2x^3e^3 + 3\,(4\,a^2de^2 - abe^3)x^2 + 6\,(6\,a^2d^2e - 4\,abde^2 + b^2e^3)x}{a^3} + \frac{6\,(4\,a^3d^3 - 6\,a^2bd^2e + 4\,ab^2de^2 - b^3e^3)\log(ax+b)}{a^4}\right) + \frac{1}{4}\,(x^4e^3 + 4\,dx^3e^2 + 6\,d^2x^2e + 4\,d^3x)\log\left(\left(a+\frac{b}{x}\right)^p c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(a+b/x)^p),x, algorithm="maxima")
```

```
[Out] 1/24*b*p*((2*a^2*x^3*e^3 + 3*(4*a^2*d*e^2 - a*b*e^3)*x^2 + 6*(6*a^2*d^2*e -
 4*a*b*d*e^2 + b^2*e^3)*x)/a^3 + 6*(4*a^3*d^3 - 6*a^2*b*d^2*e + 4*a*b^2*d*e
^2 - b^3*e^3)*log(a*x + b)/a^4) + 1/4*(x^4*e^3 + 4*d*x^3*e^2 + 6*d^2*x^2*e
+ 4*d^3*x)*log((a + b/x)^p*c)
```

Fricas [A]
time = 0.39, size = 236, normalized size = 1.70

$$\frac{36\,a^3bd^2pxe + (2\,a^3bpx^3 - 3\,a^2b^2px^2 + 6\,ab^3px)e^3 + 12\,(a^3bdpx^2 - 2\,a^2b^2dpx)e^2 + 6\,(4\,a^3bd^3p - 6\,a^2b^2d^2pe + 4\,ab^3dpe^2 - b^4pe^3)\log\left(ax + b\right) + 6\,(a^4x^4e^3 + 4\,a^4dx^3e^2 + 6\,a^4d^2x^2e + 4\,a^4d^3x)\log\left(c\right) + 6\,(a^4px^4e^3 + 4\,a^4dpx^3e^2 + 6\,a^4d^2px^2e + 4\,a^4d^3px)\log\left(\frac{ax+b}{x}\right)}{24\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(c*(a+b/x)^p),x, algorithm="fricas")
```

```
[Out] 1/24*(36*a^3*b*d^2*p*x*e + (2*a^3*b*p*x^3 - 3*a^2*b^2*p*x^2 + 6*a*b^3*p*x)*
e^3 + 12*(a^3*b*d*p*x^2 - 2*a^2*b^2*d*p*x)*e^2 + 6*(4*a^3*b*d^3*p - 6*a^2*b
^2*d^2*p*e + 4*a*b^3*d*p*e^2 - b^4*p*e^3)*log(a*x + b) + 6*(a^4*x^4*e^3 + 4
*a^4*d*x^3*e^2 + 6*a^4*d^2*x^2*e + 4*a^4*d^3*x)*log(c) + 6*(a^4*p*x^4*e^3 +
 4*a^4*d*p*x^3*e^2 + 6*a^4*d^2*p*x^2*e + 4*a^4*d^3*p*x)*log((a*x + b)/x))/a
^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(128) = 256.
time = 1.71, size = 355, normalized size = 2.55

$$\begin{cases} d^3x\log\left(c\left(a + \frac{b}{x}\right)^p\right) + \frac{3d^2ex^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2} + de^2x^3\log\left(c\left(a + \frac{b}{x}\right)^p\right) + \frac{e^3x^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4} + \frac{bd^3p\log\left(x+\frac{b}{a}\right)}{a} + \frac{3bd^2epx}{2a} + \frac{bde^2px^2}{2a} + \frac{be^3px^3}{12a} - \frac{3b^2d^2ep\log\left(x+\frac{b}{a}\right)}{2a^2} - \frac{b^2de^2px}{a^2} - \frac{b^2e^3px^2}{8a^2} + \frac{b^3de^2p\log\left(x+\frac{b}{a}\right)}{a^3} + \frac{b^3e^3px}{4a^3} - \frac{b^4e^3p\log\left(x+\frac{b}{a}\right)}{4a^4} & \text{for}\: a \neq 0 \\ d^3px + d^3x\log\left(c\left(\frac{b}{x}\right)^p\right) + \frac{3d^2epx^2}{4} + \frac{3d^2ex^2\log\left(c\left(\frac{b}{x}\right)^p\right)}{2} + \frac{de^2px^3}{3} + de^2x^3\log\left(c\left(\frac{b}{x}\right)^p\right) + \frac{e^3px^4}{16} + \frac{e^3x^4\log\left(c\left(\frac{b}{x}\right)^p\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*ln(c*(a+b/x)**p),x)
```

```
[Out] Piecewise((d**3*x*log(c*(a + b/x)**p) + 3*d**2*e*x**2*log(c*(a + b/x)**p)/2
 + d*e**2*x**3*log(c*(a + b/x)**p) + e**3*x**4*log(c*(a + b/x)**p)/4 + b*d*
*3*p*log(x + b/a)/a + 3*b*d**2*e*p*x/(2*a) + b*d*e**2*p*x**2/(2*a) + b*e**3
*p*x**3/(12*a) - 3*b**2*d**2*e*p*log(x + b/a)/(2*a**2) - b**2*d*e**2*p*x/a*
*2 - b**2*e**3*p*x**2/(8*a**2) + b**3*d*e**2*p*log(x + b/a)/a**3 + b**3*e**
3*p*x/(4*a**3) - b**4*e**3*p*log(x + b/a)/(4*a**4), Ne(a, 0)), (d**3*p*x +
d**3*x*log(c*(b/x)**p) + 3*d**2*e*p*x**2/4 + 3*d**2*e*x**2*log(c*(b/x)**p)/
2 + d*e**2*p*x**3/3 + d*e**2*x**3*log(c*(b/x)**p) + e**3*p*x**4/16 + e**3*x
**4*log(c*(b/x)**p)/4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1659 vs. 2(126) = 252.
time = 5.02, size = 1659, normalized size = 11.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3*log(c*(a+b/x)^p),x, algorithm="giac")

[Out] -1/24*(24*a^7*b^2*d^3*p*log(-a + (a*x + b)/x) - 36*a^6*b^3*d^2*p*e*log(-a + (a*x + b)/x) + 36*a^6*b^3*d^2*p*e - 96*(a*x + b)*a^6*b^2*d^3*p*log(-a + (a*x + b)/x)/x + 24*a^5*b^4*d*p*e^2*log(-a + (a*x + b)/x) + 144*(a*x + b)*a^5*b^3*d^2*p*e*log(-a + (a*x + b)/x)/x + 24*a^7*b^2*d^3*log(c) - 36*a^6*b^3*d^2*e*log(c) + 24*(a*x + b)*a^6*b^2*d^3*p*log((a*x + b)/x)/x - 72*(a*x + b)*a^5*b^3*d^2*p*e*log((a*x + b)/x)/x - 36*a^5*b^4*d*p*e^2 - 108*(a*x + b)*a^5*b^3*d^2*p*e/x + 144*(a*x + b)^2*a^5*b^2*d^3*p*log(-a + (a*x + b)/x)/x^2 - 6*a^4*b^5*p*e^3*log(-a + (a*x + b)/x) - 96*(a*x + b)*a^4*b^4*d*p*e^2*log(-a + (a*x + b)/x)/x - 216*(a*x + b)^2*a^4*b^3*d^2*p*e*log(-a + (a*x + b)/x)/x^2 - 72*(a*x + b)*a^6*b^2*d^3*log(c)/x + 24*a^5*b^4*d*e^2*log(c) + 72*(a*x + b)*a^5*b^3*d^2*e*log(c)/x - 72*(a*x + b)^2*a^5*b^2*d^3*p*log((a*x + b)/x)/x^2 + 72*(a*x + b)*a^4*b^4*d*p*e^2*log((a*x + b)/x)/x + 180*(a*x + b)^2*a^4*b^3*d^2*p*e*log((a*x + b)/x)/x^2 + 11*a^4*b^5*p*e^3 + 96*(a*x + b)*a^4*b^4*d*p*e^2/x + 108*(a*x + b)^2*a^4*b^3*d^2*p*e/x^2 - 96*(a*x + b)^3*a^4*b^2*d^3*p*log(-a + (a*x + b)/x)/x^3 + 24*(a*x + b)*a^3*b^5*p*e^3*log(-a + (a*x + b)/x)/x + 144*(a*x + b)^2*a^3*b^4*d*p*e^2*log(-a + (a*x + b)/x)/x^2 + 144*(a*x + b)^3*a^3*b^3*d^2*p*e*log(-a + (a*x + b)/x)/x^3 + 72*(a*x + b)^2*a^5*b^2*d^3*log(c)/x^2 - 6*a^4*b^5*e^3*log(c) - 24*(a*x + b)*a^4*b^4*d*e^2*log(c)/x - 36*(a*x + b)^2*a^4*b^3*d^2*e*log(c)/x^2 + 72*(a*x + b)^3*a^4*b^2*d^3*p*log((a*x + b)/x)/x^3 - 24*(a*x + b)*a^3*b^5*p*e^3*log((a*x + b)/x)/x - 144*(a*x + b)^2*a^3*b^4*d*p*e^2*log((a*x + b)/x)/x^2 - 144*(a*x + b)^3*a^3*b^3*d^2*p*e*log((a*x + b)/x)/x^3 - 26*(a*x + b)*a^3*b^5*p*e^3/x - 84*(a*x + b)^2*a^3*b^4*d*p*e^2/x^2 - 36*(a*x + b)^3*a^3*b^3*d^2*p*e/x^3 + 24*(a*x + b)^4*a^3*b^2*d^3*p*log(-a + (a*x + b)/x)/x^4 - 36*(a*x + b)^2*a^2*b^5*p*e^3*log(-a + (a*x + b)/x)/x^2 - 96*(a*x + b)^3*a^2*b^4*d*p*e^2*log(-a + (a*x + b)/x)/x^3 - 36*(a*x + b)^4*a^2*b^3*d^2*p*e*log(-a + (a*x + b)/x)/x^4 - 24*(a*x + b)^3*a^4*b^2*d^3*log(c)/x^3 - 24*(a*x + b)^4*a^3*b^2*d^3*p*log((a*x + b)/x)/x^4 + 36*(a*x + b)^2*a^2*b^5*p*e^3*log((a*x + b)/x)/x^2 + 96*(a*x + b)^3*a^2*b^4*d*p*e^2*log((a*x + b)/x)/x^3 + 36*(a*x + b)^4*a^2*b^3*d^2*p*e*log((a*x + b)/x)/x^4 + 21*(a*x + b)^2*a^2*b^5*p*e^3/x^2 + 24*(a*x + b)^3*a^2*b^4*d*p*e^2/x^3 + 24*(a*x + b)^3*a*b^5*p*e^3*log(-a + (a*x + b)/x)/x^3 + 24*(a*x + b)^4*a*b^4*d*p*e^2*log(-a + (a*x + b)/x)/x^4 - 24*(a*x + b)^3*a*b^5*p*e^3*log((a*x + b)/x)/x^3 - 24*(a*x + b)^4*a*b^4*d*p*e^2*log((a*x + b)/x)/x^4 - 6*(a*x + b)^3*a*b^5*p*e^3/x^3 - 6*(a*x + b)^4*b^5*p*e^3*log(-a + (a*x + b)/x)/x^4 + 6*(a*x + b)^4*b^5*p*e^3*log((a*x + b)/x)/x^4)/((a^8 - 4*(a*x + b)*a^7/x + 6*(a*x + b)^2*a^6/x^2 - 4*(a*x + b)^3*a^5/x^3 + (a*x + b)^4*a^4/x^4)*b)

Mupad [B]

time = 0.34, size = 184, normalized size = 1.32

$$x\left(\frac{b\left(\frac{b^2\,e^3\,p}{4\,a^2}-\frac{b\,d\,e^2\,p}{a}\right)}{a}+\frac{3\,b\,d^2\,e\,p}{2\,a}\right)+\ln\left(c\left(a+\frac{b}{x}\right)^p\right)\left(d^3\,x+\frac{3\,d^2\,e\,x^2}{2}+d\,e^2\,x^3+\frac{e^3\,x^4}{4}\right)-x^2\left(\frac{b^2\,e^3\,p}{8\,a^2}-\frac{b\,d\,e^2\,p}{2\,a}\right)-\frac{\ln\left(b+a\,x\right)\,\left(-4\,p\,a^3\,b\,d^3+6\,p\,a^2\,b^2\,d^2\,e-4\,p\,a\,b^3\,d\,e^2+p\,b^4\,e^3\right)}{4\,a^4}+\frac{b\,e^3\,p\,x^3}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)*(d + e*x)^3,x)
```

```
[Out] x*((b*((b^2*e^3*p)/(4*a^2) - (b*d*e^2*p)/a))/a + (3*b*d^2*e*p)/(2*a)) + log
(c*(a + b/x)^p)*(d^3*x + (e^3*x^4)/4 + (3*d^2*e*x^2)/2 + d*e^2*x^3) - x^2*(
(b^2*e^3*p)/(8*a^2) - (b*d*e^2*p)/(2*a)) - (log(b + a*x)*(b^4*e^3*p - 4*a^3
*b*d^3*p - 4*a*b^3*d*e^2*p + 6*a^2*b^2*d^2*e*p))/(4*a^4) + (b*e^3*p*x^3)/(1
2*a)
```

3.199 $\int (d+ex)^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right) \, dx$

Optimal. Leaf size=102

$$\frac{be(3ad-be)px}{3a^2}+\frac{be^2px^2}{6a}+\frac{(d+ex)^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e}+\frac{d^3p\log(x)}{3e}-\frac{(ad-be)^3p\log(b+ax)}{3a^3e}$$

```
[Out] 1/3*b*e*(3*a*d-b*e)*p*x/a^2+1/6*b*e^2*p*x^2/a+1/3*(e*x+d)^3*ln(c*(a+b/x)^p)
/e+1/3*d^3*p*ln(x)/e-1/3*(a*d-b*e)^3*p*ln(a*x+b)/a^3/e
```

Rubi [A]
time = 0.06, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2513, 528, 84}

$$-\frac{p(ad-be)^3\log(ax+b)}{3a^3e}+\frac{bepx(3ad-be)}{3a^2}+\frac{(d+ex)^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e}+\frac{be^2px^2}{6a}+\frac{d^3p\log(x)}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*Log[c*(a + b/x)^p],x]
```

```
[Out] (b*e*(3*a*d - b*e)*p*x)/(3*a^2) + (b*e^2*p*x^2)/(6*a) + ((d + e*x)^3*Log[c*
(a + b/x)^p])/(3*e) + (d^3*p*Log[x])/(3*e) - ((a*d - b*e)^3*p*Log[b + a*x])
/(3*a^3*e)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\int (d+ex)^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \frac{(d+ex)^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} + \frac{(bp)\int \frac{(d+ex)^3}{\left(a+\frac{b}{x}\right)x^2}\,dx}{3e}$$
$$= \frac{(d+ex)^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} + \frac{(bp)\int \frac{(d+ex)^3}{x(b+ax)}\,dx}{3e}$$
$$= \frac{(d+ex)^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} + \frac{(bp)\int \left(\frac{e^2(3ad-be)}{a^2}+\frac{d^3}{bx}+\frac{e^3x}{a}-\frac{(ad-be)^3}{a^2b(b+ax)}\right)}{3e}$$
$$= \frac{be(3ad-be)px}{3a^2} + \frac{be^2px^2}{6a} + \frac{(d+ex)^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} + \frac{d^3p\log(x)}{3e} -$$

Mathematica [A]
time = 0.06, size = 86, normalized size = 0.84

$$\frac{2a^3(d+ex)^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)+p(abe^2x(6ad-2be+aex)+2a^3d^3\log(x)-2(ad-be)^3\log(b+ax))}{6a^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*Log[c*(a + b/x)^p],x]
```

```
[Out] (2*a^3*(d + e*x)^3*Log[c*(a + b/x)^p] + p*(a*b*e^2*x*(6*a*d - 2*b*e + a*e*x
) + 2*a^3*d^3*Log[x] - 2*(a*d - b*e)^3*Log[b + a*x]))/(6*a^3*e)
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int (ex+d)^2 \ln\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*ln(c*(a+b/x)^p),x)
```

```
[Out] int((e*x+d)^2*ln(c*(a+b/x)^p),x)
```

Maxima [A]
time = 0.34, size = 101, normalized size = 0.99

$$\frac{1}{6}\,bp\left(\frac{ax^2e^2+2\,(3\,ade-be^2)x}{a^2}+\frac{2\,(3\,a^2d^2-3\,abde+b^2e^2)\log(ax+b)}{a^3}\right)+\frac{1}{3}\,(x^3e^2+3\,dx^2e+3\,d^2x)\log\left(\left(a+\frac{b}{x}\right)^p c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(a+b/x)^p),x, algorithm="maxima")
```

```
[Out] 1/6*b*p*((a*x^2*e^2 + 2*(3*a*d*e - b*e^2)*x)/a^2 + 2*(3*a^2*d^2 - 3*a*b*d*e
 + b^2*e^2)*log(a*x + b)/a^3) + 1/3*(x^3*e^2 + 3*d*x^2*e + 3*d^2*x)*log((a
+ b/x)^p*c)
```

Fricas [A]
time = 0.36, size = 153, normalized size = 1.50

$$\frac{6\,a^{2}bdpxe+\left(a^{2}bpx^{2}-2\,ab^{2}px\right)e^{2}+2\left(3\,a^{2}bd^{2}p-3\,ab^{2}dpe+b^{3}pe^{2}\right)\log\left(ax+b\right)+2\left(a^{3}x^{3}e^{2}+3\,a^{3}dx^{2}e+3\,a^{3}d^{2}x\right)\log\left(c\right)+2\left(a^{3}px^{3}e^{2}+3\,a^{3}dpx^{2}e+3\,a^{3}d^{2}px\right)\log\left(\frac{ax+b}{x}\right)}{6\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(a+b/x)^p),x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^2*b*d*p*x*e + (a^2*b*p*x^2 - 2*a*b^2*p*x)*e^2 + 2*(3*a^2*b*d^2*p -
 3*a*b^2*d*p*e + b^3*p*e^2)*log(a*x + b) + 2*(a^3*x^3*e^2 + 3*a^3*d*x^2*e +
 3*a^3*d^2*x)*log(c) + 2*(a^3*p*x^3*e^2 + 3*a^3*d*p*x^2*e + 3*a^3*d^2*p*x)*
log((a*x + b)/x))/a^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(88) = 176.
time = 0.99, size = 216, normalized size = 2.12

$$\begin{cases} d^{2}x\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)+dex^{2}\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)+\frac{e^{2}x^{3}\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{3}+\frac{bd^{2}p\log\left(x+\frac{b}{a}\right)}{a}+\frac{bdepx}{a}+\frac{be^{2}px^{2}}{6a}-\frac{b^{2}dep\log\left(x+\frac{b}{a}\right)}{a^{2}}-\frac{b^{2}e^{2}px}{3a^{2}}+\frac{b^{3}e^{2}p\log\left(x+\frac{b}{a}\right)}{3a^{3}} & \text{for}\ a\neq 0\\ d^{2}px+d^{2}x\log\left(c\left(\frac{b}{x}\right)^{p}\right)+\frac{depx^{2}}{2}+dex^{2}\log\left(c\left(\frac{b}{x}\right)^{p}\right)+\frac{e^{2}px^{3}}{9}+\frac{e^{2}x^{3}\log\left(c\left(\frac{b}{x}\right)^{p}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*ln(c*(a+b/x)**p),x)
```

```
[Out] Piecewise((d**2*x*log(c*(a + b/x)**p) + d*e*x**2*log(c*(a + b/x)**p) + e**2
*x**3*log(c*(a + b/x)**p)/3 + b*d**2*p*log(x + b/a)/a + b*d*e*p*x/a + b*e**
2*p*x**2/(6*a) - b**2*d*e*p*log(x + b/a)/a**2 - b**2*e**2*p*x/(3*a**2) + b*
*3*e**2*p*log(x + b/a)/(3*a**3), Ne(a, 0)), (d**2*p*x + d**2*x*log(c*(b/x)*
*p) + d*e*p*x**2/2 + d*e*x**2*log(c*(b/x)**p) + e**2*p*x**3/9 + e**2*x**3*l
og(c*(b/x)**p)/3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 918 vs. 2(92) = 184.
time = 3.87, size = 918, normalized size = 9.00

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(c*(a+b/x)^p),x, algorithm="giac")
```

```
[Out] -1/6*(6*a^5*b^2*d^2*p*log(-a + (a*x + b)/x) - 6*a^4*b^3*d*p*e*log(-a + (a*x
 + b)/x) + 6*a^4*b^3*d*p*e - 18*(a*x + b)*a^4*b^2*d^2*p*log(-a + (a*x + b)/
x)/x + 2*a^3*b^4*p*e^2*log(-a + (a*x + b)/x) + 18*(a*x + b)*a^3*b^3*d*p*e*l
```

```
og(-a + (a*x + b)/x)/x + 6*a^5*b^2*d^2*log(c) - 6*a^4*b^3*d*e*log(c) + 6*(a
*x + b)*a^4*b^2*d^2*p*log((a*x + b)/x)/x - 12*(a*x + b)*a^3*b^3*d*p*e*log((
a*x + b)/x)/x - 3*a^3*b^4*p*e^2 - 12*(a*x + b)*a^3*b^3*d*p*e/x + 18*(a*x +
b)^2*a^3*b^2*d^2*p*log(-a + (a*x + b)/x)/x^2 - 6*(a*x + b)*a^2*b^4*p*e^2*lo
g(-a + (a*x + b)/x)/x - 18*(a*x + b)^2*a^2*b^3*d*p*e*log(-a + (a*x + b)/x)/
x^2 - 12*(a*x + b)*a^4*b^2*d^2*log(c)/x + 2*a^3*b^4*e^2*log(c) + 6*(a*x + b
)*a^3*b^3*d*e*log(c)/x - 12*(a*x + b)^2*a^3*b^2*d^2*p*log((a*x + b)/x)/x^2
+ 6*(a*x + b)*a^2*b^4*p*e^2*log((a*x + b)/x)/x + 18*(a*x + b)^2*a^2*b^3*d*p
*e*log((a*x + b)/x)/x^2 + 5*(a*x + b)*a^2*b^4*p*e^2/x + 6*(a*x + b)^2*a^2*b
^3*d*p*e/x^2 - 6*(a*x + b)^3*a^2*b^2*d^2*p*log(-a + (a*x + b)/x)/x^3 + 6*(a
*x + b)^2*a*b^4*p*e^2*log(-a + (a*x + b)/x)/x^2 + 6*(a*x + b)^3*a*b^3*d*p*e
*log(-a + (a*x + b)/x)/x^3 + 6*(a*x + b)^2*a^3*b^2*d^2*log(c)/x^2 + 6*(a*x
+ b)^3*a^2*b^2*d^2*p*log((a*x + b)/x)/x^3 - 6*(a*x + b)^2*a*b^4*p*e^2*log((
a*x + b)/x)/x^2 - 6*(a*x + b)^3*a*b^3*d*p*e*log((a*x + b)/x)/x^3 - 2*(a*x +
 b)^2*a*b^4*p*e^2/x^2 - 2*(a*x + b)^3*b^4*p*e^2*log(-a + (a*x + b)/x)/x^3 +
 2*(a*x + b)^3*b^4*p*e^2*log((a*x + b)/x)/x^3)/((a^6 - 3*(a*x + b)*a^5/x +
3*(a*x + b)^2*a^4/x^2 - (a*x + b)^3*a^3/x^3)*b)
```

Mupad [B]
time = 0.32, size = 111, normalized size = 1.09

$$\ln\left(c\left(a+\frac{b}{x}\right)^p\right)\left(d^2\,x+d\,e\,x^2+\frac{e^2\,x^3}{3}\right)-x\left(\frac{b^2\,e^2\,p}{3\,a^2}-\frac{b\,d\,e\,p}{a}\right)+\frac{\ln\left(b+a\,x\right)\,\left(3\,p\,a^2\,b\,d^2-3\,p\,a\,b^2\,d\,e+p\,b^3\,e^2\right)}{3\,a^3}+\frac{b\,e^2\,p\,x^2}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)*(d + e*x)^2,x)
```

```
[Out] log(c*(a + b/x)^p)*(d^2*x + (e^2*x^3)/3 + d*e*x^2) - x*((b^2*e^2*p)/(3*a^2)
 - (b*d*e*p)/a) + (log(b + a*x)*(b^3*e^2*p + 3*a^2*b*d^2*p - 3*a*b^2*d*e*p)
)/(3*a^3) + (b*e^2*p*x^2)/(6*a)
```

3.200 $\int (d+ex)\log \left(c\left(a+\frac{b}{x}\right)^p\right) \, dx$

Optimal. Leaf size=78

$$\frac{bepx}{2a}+\frac{(d+ex)^2\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}+\frac{d^2p\log(x)}{2e}-\frac{(ad-be)^2p\log(b+ax)}{2a^2e}$$

```
[Out] 1/2*b*e*p*x/a+1/2*(e*x+d)^2*ln(c*(a+b/x)^p)/e+1/2*d^2*p*ln(x)/e-1/2*(a*d-b*
e)^2*p*ln(a*x+b)/a^2/e
```

Rubi [A]
time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2513, 528, 84}

$$-\frac{p(ad-be)^2\log(ax+b)}{2a^2e}+\frac{(d+ex)^2\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}+\frac{bepx}{2a}+\frac{d^2p\log(x)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*Log[c*(a + b/x)^p],x]
```

```
[Out] (b*e*p*x)/(2*a) + ((d + e*x)^2*Log[c*(a + b/x)^p])/(2*e) + (d^2*p*Log[x])/(
2*e) - ((a*d - b*e)^2*p*Log[b + a*x])/(2*a^2*e)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\int (d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \frac{(d+ex)^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}+\frac{(bp)\int\frac{(d+ex)^2}{\left(a+\frac{b}{x}\right)x^2}\,dx}{2e}$$
$$=\frac{(d+ex)^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}+\frac{(bp)\int\frac{(d+ex)^2}{x(b+ax)}\,dx}{2e}$$
$$=\frac{(d+ex)^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}+\frac{(bp)\int\left(\frac{e^2}{a}+\frac{d^2}{bx}-\frac{(ad-be)^2}{ab(b+ax)}\right)dx}{2e}$$
$$=\frac{bepx}{2a}+\frac{(d+ex)^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}+\frac{d^2p\log(x)}{2e}-\frac{(ad-be)^2p\log(b+a}{2a^2e}$$

Mathematica [A]
time = 0.03, size = 70, normalized size = 0.90

$$\frac{2abdp\log\left(a+\frac{b}{x}\right)+a^2x(2d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)+bp(aex+2ad\log(x)-be\log(b+ax))}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*Log[c*(a + b/x)^p],x]
```

```
[Out] (2*a*b*d*p*Log[a + b/x] + a^2*x*(2*d + e*x)*Log[c*(a + b/x)^p] + b*p*(a*e*x
 + 2*a*d*Log[x] - b*e*Log[b + a*x]))/(2*a^2)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (ex+d)\ln\left(c\left(a+\frac{b}{x}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*ln(c*(a+b/x)^p),x)
```

```
[Out] int((e*x+d)*ln(c*(a+b/x)^p),x)
```

Maxima [A]
time = 0.34, size = 58, normalized size = 0.74

$$\frac{1}{2}\,bp\left(\frac{xe}{a}+\frac{(2\,ad-be)\log(ax+b)}{a^2}\right)+\frac{1}{2}\,(x^2e+2\,dx)\log\left(\left(a+\frac{b}{x}\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(a+b/x)^p),x, algorithm="maxima")
```

```
[Out] 1/2*b*p*(x*e/a + (2*a*d - b*e)*log(a*x + b)/a^2) + 1/2*(x^2*e + 2*d*x)*log(
(a + b/x)^p*c)
```

Fricas [A]
time = 0.38, size = 85, normalized size = 1.09

$$\frac{abpxe + (2\,abdp - b^2pe)\log{(ax+b)} + (a^2x^2e + 2\,a^2dx)\log{(c)} + (a^2px^2e + 2\,a^2dpx)\log{\left(\frac{ax+b}{x}\right)}}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(a+b/x)^p),x, algorithm="fricas")
```

```
[Out] 1/2*(a*b*p*x*e + (2*a*b*d*p - b^2*p*e)*log(a*x + b) + (a^2*x^2*e + 2*a^2*d*
x)*log(c) + (a^2*p*x^2*e + 2*a^2*d*p*x)*log((a*x + b)/x))/a^2
```

Sympy [A]
time = 0.54, size = 112, normalized size = 1.44

$$\begin{cases} dx\log{\left(c\left(a+\frac{b}{x}\right)^p\right)} + \frac{ex^2\log{\left(c\left(a+\frac{b}{x}\right)^p\right)}}{2} + \frac{bdp\log{\left(x+\frac{b}{a}\right)}}{a} + \frac{bepx}{2a} - \frac{b^2ep\log{\left(x+\frac{b}{a}\right)}}{2a^2} & \text{for}\: a \neq 0 \\ dpx + dx\log{\left(c\left(\frac{b}{x}\right)^p\right)} + \frac{epx^2}{4} + \frac{ex^2\log{\left(c\left(\frac{b}{x}\right)^p\right)}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*ln(c*(a+b/x)**p),x)
```

```
[Out] Piecewise((d*x*log(c*(a + b/x)**p) + e*x**2*log(c*(a + b/x)**p)/2 + b*d*p*l
og(x + b/a)/a + b*e*p*x/(2*a) - b**2*e*p*log(x + b/a)/(2*a**2), Ne(a, 0)),
(d*p*x + d*x*log(c*(b/x)**p) + e*p*x**2/4 + e*x**2*log(c*(b/x)**p)/2, True)
)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 394 vs. 2(70) = 140.
time = 3.23, size = 394, normalized size = 5.05

$$-\frac{2\,a^3b^2dp\log{\left(-a+\frac{ax+b}{x}\right)} - a^2b^3pe\log{\left(-a+\frac{ax+b}{x}\right)} + a^2b^3pe - \frac{4\,(ax+b)a^2b^2dp\log{\left(-a+\frac{ax+b}{x}\right)}}{x} + \frac{2\,(ax+b)ab^3pe\log{\left(-a+\frac{ax+b}{x}\right)}}{x} + 2\,a^3b^2d\log{(c)} - a^2b^3e\log{(c)} + \frac{2\,(ax+b)a^2b^2dp\log{\left(\frac{ax+b}{x}\right)}}{x} - \frac{2\,(ax+b)ab^3pe\log{\left(\frac{ax+b}{x}\right)}}{x} - \frac{(ax+b)ab^3pe}{x} + \frac{2\,(ax+b)^2ab^2dp\log{\left(-a+\frac{ax+b}{x}\right)}}{x^2} - \frac{(ax+b)^2b^3pe\log{\left(-a+\frac{ax+b}{x}\right)}}{x^2} - \frac{2\,(ax+b)a^2b^2d\log(c)}{x} - \frac{2\,(ax+b)^2ab^2dp\log{\left(\frac{ax+b}{x}\right)}}{x^2} + \frac{(ax+b)^2b^3pe\log{\left(\frac{ax+b}{x}\right)}}{x^2}}{2\left(a^4 - \frac{2\,(ax+b)a^3}{x} + \frac{(ax+b)^2a^2}{x^2}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(c*(a+b/x)^p),x, algorithm="giac")
```

```
[Out] -1/2*(2*a^3*b^2*d*p*log(-a + (a*x + b)/x) - a^2*b^3*p*e*log(-a + (a*x + b)/
x) + a^2*b^3*p*e - 4*(a*x + b)*a^2*b^2*d*p*log(-a + (a*x + b)/x)/x + 2*(a*x
 + b)*a*b^3*p*e*log(-a + (a*x + b)/x)/x + 2*a^3*b^2*d*log(c) - a^2*b^3*e*lo
g(c) + 2*(a*x + b)*a^2*b^2*d*p*log((a*x + b)/x)/x - 2*(a*x + b)*a*b^3*p*e*l
og((a*x + b)/x)/x - (a*x + b)*a*b^3*p*e/x + 2*(a*x + b)^2*a*b^2*d*p*log(-a
+ (a*x + b)/x)/x^2 - (a*x + b)^2*b^3*p*e*log(-a + (a*x + b)/x)/x^2 - 2*(a*x
```

```
 + b)*a^2*b^2*d*log(c)/x - 2*(a*x + b)^2*a*b^2*d*p*log((a*x + b)/x)/x^2 + (
a*x + b)^2*b^3*p*e*log((a*x + b)/x)/x^2)/((a^4 - 2*(a*x + b)*a^3/x + (a*x +
 b)^2*a^2/x^2)*b)
```

Mupad [B]
time = 0.30, size = 57, normalized size = 0.73

$$\ln\left(c\left(a+\frac{b}{x}\right)^p\right)\left(\frac{e\,x^2}{2}+d\,x\right)-\frac{\ln\left(b+a\,x\right)\,\left(b^2\,e\,p-2\,a\,b\,d\,p\right)}{2\,a^2}+\frac{b\,e\,p\,x}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)*(d + e*x),x)
```

```
[Out] log(c*(a + b/x)^p)*(d*x + (e*x^2)/2) - (log(b + a*x)*(b^2*e*p - 2*a*b*d*p))
/(2*a^2) + (b*e*p*x)/(2*a)
```

3.201 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=113

$$\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e}+\frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e}-\frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e}-\frac{p\text{Li}_2\left(\frac{a(d+ex)}{ad-be}\right)}{e}+\frac{p\text{Li}_2(1}{e}$$

```
[Out] ln(c*(a+b/x)^p)*ln(e*x+d)/e+p*ln(-e*x/d)*ln(e*x+d)/e-p*ln(-e*(a*x+b)/(a*d-b
*e))*ln(e*x+d)/e-p*polylog(2,a*(e*x+d)/(a*d-b*e))/e+p*polylog(2,1+e*x/d)/e
```

Rubi [A]
time = 0.11, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2512, 266, 2463, 2441, 2352, 2440, 2438}

$$-\frac{p\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e}+\frac{p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e}+\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}-\frac{p\log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right)}{e}+\frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b/x)^p]*Log[d + e*x])/e + (p*Log[-((e*x)/d)]*Log[d + e*x])/e -
(p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e - (p*PolyLog[2, (a*(d
+ e*x))/(a*d - b*e)])/e + (p*PolyLog[2, 1 + (e*x)/d])/e
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{(bp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x}\right)x^2}\,dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{(bp)\int \left(\frac{\log(d+ex)}{bx} - \frac{a\log(d+ex)}{b(b+ax)}\right)dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{p\int \frac{\log(d+ex)}{x}\,dx}{e} - \frac{(ap)\int \frac{\log(d+ex)}{b+ax}\,dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+}{e}$$

Mathematica [A]
time = 0.02, size = 114, normalized size = 1.01

$$\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e}+\frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e}-\frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e}+\frac{p\text{Li}_2\left(\frac{d+ex}{d}\right)}{e}-\frac{p\text{Li}_2\left(\frac{a(d+ex)}{ad-be}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b/x)^p]*Log[d + e*x])/e + (p*Log[-((e*x)/d)]*Log[d + e*x])/e -
(p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e + (p*PolyLog[2, (d + e
*x)/d])/e - (p*PolyLog[2, (a*(d + e*x))/(a*d - b*e)])/e
```

Maple [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/(e*x+d),x)
```

```
[Out] int(ln(c*(a+b/x)^p)/(e*x+d),x)
```

Maxima [A]
time = 0.40, size = 167, normalized size = 1.48

$$bp\left(\frac{\log(xe+d)\log\left(a+\frac{b}{x}\right)}{b}-\frac{\log(xe+d)\log\left(-\frac{axe+ad}{ad-be}+1\right)+\text{Li}_2\left(\frac{axe+ad}{ad-be}\right)}{b}+\frac{\log(xe+d)\log\left(-\frac{xe+d}{d}+1\right)+\text{Li}_2\left(\frac{xe+d}{d}\right)}{b}\right)e^{(-1)}-pe^{(-1)}\log(xe+d)\log\left(a+\frac{b}{x}\right)+e^{(-1)}\log\left(\left(a+\frac{b}{x}\right)^pc\right)\log(xe+d)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] b*p*(log(x*e + d)*log(a + b/x)/b - (log(x*e + d)*log(-(a*x*e + a*d)/(a*d -
b*e) + 1) + dilog((a*x*e + a*d)/(a*d - b*e)))/b + (log(x*e + d)*log(-(x*e +
 d)/d + 1) + dilog((x*e + d)/d))/b)*e^(-1) - p*e^(-1)*log(x*e + d)*log(a +
b/x) + e^(-1)*log((a + b/x)^p*c)*log(x*e + d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x + b)/x)^p)/(x*e + d), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/(e*x+d),x)
```

```
[Out] Integral(log(c*(a + b/x)**p)/(d + e*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x)^p*c)/(x*e + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,\left(a+\frac{b}{x}\right)^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)/(d + e*x),x)
```

```
[Out] int(log(c*(a + b/x)^p)/(d + e*x), x)
```

3.202 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^2}\,dx$

Optimal. Leaf size=81

$$-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)}-\frac{p\log(x)}{de}+\frac{ap\log(b+ax)}{e(ad-be)}-\frac{bp\log(d+ex)}{d(ad-be)}$$

```
[Out] -ln(c*(a+b/x)^p)/e/(e*x+d)-p*ln(x)/d/e+a*p*ln(a*x+b)/e/(a*d-b*e)-b*p*ln(e*x
+d)/d/(a*d-b*e)
```

Rubi [A]
time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2513, 528, 84}

$$-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)}+\frac{ap\log(ax+b)}{e(ad-be)}-\frac{bp\log(d+ex)}{d(ad-be)}-\frac{p\log(x)}{de}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/(d + e*x)^2,x]
```

```
[Out] -(Log[c*(a + b/x)^p]/(e*(d + e*x))) - (p*Log[x])/(d*e) + (a*p*Log[b + a*x])
/(e*(a*d - b*e)) - (b*p*Log[d + e*x])/(d*(a*d - b*e))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^2}\,dx &= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)} - \frac{(bp)\int \frac{1}{\left(a+\frac{b}{x}\right)x^2(d+ex)}\,dx}{e} \\
&= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)} - \frac{(bp)\int \frac{1}{x(b+ax)(d+ex)}\,dx}{e} \\
&= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)} - \frac{(bp)\int \left(\frac{1}{bdx}+\frac{a^2}{b(-ad+be)(b+ax)}+\frac{e^2}{d(ad-be)(d+ex)}\right)dx}{e} \\
&= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)} - \frac{p\log(x)}{de} + \frac{ap\log(b+ax)}{e(ad-be)} - \frac{bp\log(d+ex)}{d(ad-be)}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 81, normalized size = 1.00

$$-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)} - \frac{p\log(x)}{de} + \frac{ap\log(b+ax)}{e(ad-be)} - \frac{bp\log(d+ex)}{d(ad-be)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/(d + e*x)^2,x]
[Out] -(Log[c*(a + b/x)^p]/(e*(d + e*x))) - (p*Log[x])/(d*e) + (a*p*Log[b + a*x])
/(e*(a*d - b*e)) - (b*p*Log[d + e*x])/(d*(a*d - b*e))
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{\left(ex+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/(e*x+d)^2,x)
[Out] int(ln(c*(a+b/x)^p)/(e*x+d)^2,x)
```

Maxima [A]
time = 0.33, size = 88, normalized size = 1.09

$$bp\left(\frac{a\log\left(ax+b\right)}{abd-b^2e} - \frac{e\log\left(xe+d\right)}{ad^2-bde} - \frac{\log\left(x\right)}{bd}\right)e^{(-1)} - \frac{e^{(-1)}\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{xe+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] b*p*(a*log(a*x + b)/(a*b*d - b^2*e) - e*log(x*e + d)/(a*d^2 - b*d*e) - log(
x)/(b*d))*e^(-1) - e^(-1)*log((a + b/x)^p*c)/(x*e + d)
```

Fricas [A]
time = 0.45, size = 152, normalized size = 1.88

$$\frac{(adpxe + ad^2p)\log(ax + b) - (bpxe^2 + bdpe)\log(xe + d) - (ad^2 - bde)\log(c) - (ad^2p - bpxe^2 + (adpx - bdp)e)\log(x) - (ad^2p - bdpe)\log\left(\frac{ax+b}{x}\right)}{ad^3e - bdxe^3 + (ad^2x - bd^2)e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] ((a*d*p*x*e + a*d^2*p)*log(a*x + b) - (b*p*x*e^2 + b*d*p*e)*log(x*e + d) -
(a*d^2 - b*d*e)*log(c) - (a*d^2*p - b*p*x*e^2 + (a*d*p*x - b*d*p)*e)*log(x)
 - (a*d^2*p - b*d*p*e)*log((a*x + b)/x))/(a*d^3*e - b*d*x*e^3 + (a*d^2*x -
b*d^2)*e^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 452 vs. 2(61) = 122.
time = 2.64, size = 452, normalized size = 5.58

$$\begin{cases} \frac{dp\log\left(\frac{d}{e}+x\right)}{d^2e+de^2x} + \frac{epx\log\left(\frac{d}{e}+x\right)}{d^2e+de^2x} + \frac{ex\log\left(c\left(\frac{b}{x}\right)^p\right)}{d^2e+de^2x} & \text{for}\: a = 0 \\ -\frac{dp}{d^2e+de^2x} + \frac{ex\log\left(c\left(\frac{b}{x}+\frac{be}{d}\right)^p\right)}{d^2e+de^2x} & \text{for}\: a = \frac{be}{d} \\ \frac{-\frac{a\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{b} + \frac{p}{x} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}}{e^2} & \text{for}\: d = 0 \\ \tilde{\infty}\left(x\log\left(c\left(a+\frac{b}{x}\right)^p\right) + \frac{bp\log(ax+b)}{a}\right) & \text{for}\: d = -ex \\ \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right) + \frac{bp\log(ax+b)}{a}}{d^2} & \text{for}\: e = 0 \\ \frac{adx\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{ad^3+ad^2ex-bd^2e-bde^2x} + \frac{bdp\log\left(x+\frac{b}{a}\right)}{ad^3+ad^2ex-bd^2e-bde^2x} - \frac{bdp\log\left(\frac{d}{e}+x\right)}{ad^3+ad^2ex-bd^2e-bde^2x} + \frac{bepx\log\left(x+\frac{b}{a}\right)}{ad^3+ad^2ex-bd^2e-bde^2x} - \frac{bepx\log\left(\frac{d}{e}+x\right)}{ad^3+ad^2ex-bd^2e-bde^2x} - \frac{bex\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{ad^3+ad^2ex-bd^2e-bde^2x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/(e*x+d)**2,x)
```

```
[Out] Piecewise((d*p*log(d/e + x)/(d**2*e + d*e**2*x) + e*p*x*log(d/e + x)/(d**2*
e + d*e**2*x) + e*x*log(c*(b/x)**p)/(d**2*e + d*e**2*x), Eq(a, 0)), (-d*p/(
d**2*e + d*e**2*x) + e*x*log(c*(b/x + b*e/d)**p)/(d**2*e + d*e**2*x), Eq(a,
 b*e/d)), ((-a*log(c*(a + b/x)**p)/b + p/x - log(c*(a + b/x)**p)/x)/e**2, E
q(d, 0)), (zoo*(x*log(c*(a + b/x)**p) + b*p*log(a*x + b)/a), Eq(d, -e*x)),
((x*log(c*(a + b/x)**p) + b*p*log(a*x + b)/a)/d**2, Eq(e, 0)), (a*d*x*log(c
*(a + b/x)**p)/(a*d**3 + a*d**2*e*x - b*d**2*e - b*d*e**2*x) + b*d*p*log(x
+ b/a)/(a*d**3 + a*d**2*e*x - b*d**2*e - b*d*e**2*x) - b*d*p*log(d/e + x)/(
a*d**3 + a*d**2*e*x - b*d**2*e - b*d*e**2*x) + b*e*p*x*log(x + b/a)/(a*d**3
 + a*d**2*e*x - b*d**2*e - b*d*e**2*x) - b*e*p*x*log(d/e + x)/(a*d**3 + a*d
**2*e*x - b*d**2*e - b*d*e**2*x) - b*e*x*log(c*(a + b/x)**p)/(a*d**3 + a*d*
*2*e*x - b*d**2*e - b*d*e**2*x), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(82) = 164.
time = 2.79, size = 192, normalized size = 2.37

$$-\frac{ab^{2}dp\log\left(-ad+be+\frac{(ax+b)d}{x}\right)-b^{3}pe\log\left(-ad+be+\frac{(ax+b)d}{x}\right)-\frac{(ax+b)b^{2}dp\log\left(-ad+be+\frac{(ax+b)d}{x}\right)}{x}+ab^{2}d\log\left(c\right)-b^{3}e\log\left(c\right)+\frac{(ax+b)b^{2}dp\log\left(\frac{ax+b}{x}\right)}{x}}{\left(a^{2}d^{3}-2\,abd^{2}e-\frac{(ax+b)ad^{3}}{x}+b^{2}de^{2}+\frac{(ax+b)bd^{2}e}{x}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -(a*b^2*d*p*log(-a*d + b*e + (a*x + b)*d/x) - b^3*p*e*log(-a*d + b*e + (a*x
 + b)*d/x) - (a*x + b)*b^2*d*p*log(-a*d + b*e + (a*x + b)*d/x)/x + a*b^2*d*
log(c) - b^3*e*log(c) + (a*x + b)*b^2*d*p*log((a*x + b)/x)/x)/((a^2*d^3 - 2
*a*b*d^2*e - (a*x + b)*a*d^3/x + b^2*d*e^2 + (a*x + b)*b*d^2*e/x)*b)
```

Mupad [B]
time = 0.53, size = 85, normalized size = 1.05

$$-\frac{\ln\left(c\,{\left(\frac{b+a\,x}{x}\right)}^p\right)}{x\,e^2+d\,e}-\frac{p\,\ln\left(x\right)}{d\,e}-\frac{a\,p\,\ln\left(b+a\,x\right)}{b\,e^2-a\,d\,e}-\frac{b\,p\,\ln\left(d+e\,x\right)}{a\,d^2-b\,d\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)/(d + e*x)^2,x)
```

```
[Out] - log(c*((b + a*x)/x)^p)/(d*e + e^2*x) - (p*log(x))/(d*e) - (a*p*log(b + a*
x))/(b*e^2 - a*d*e) - (b*p*log(d + e*x))/(a*d^2 - b*d*e)
```

3.203 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^3}\,dx$

Optimal. Leaf size=127

$$\frac{bp}{2d(ad-be)(d+ex)}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e(d+ex)^2}-\frac{p\log(x)}{2d^2e}+\frac{a^2p\log(b+ax)}{2e(ad-be)^2}-\frac{b(2ad-be)p\log(d+ex)}{2d^2(ad-be)^2}$$

```
[Out] 1/2*b*p/d/(a*d-b*e)/(e*x+d)-1/2*ln(c*(a+b/x)^p)/e/(e*x+d)^2-1/2*p*ln(x)/d^2
/e+1/2*a^2*p*ln(a*x+b)/e/(a*d-b*e)^2-1/2*b*(2*a*d-b*e)*p*ln(e*x+d)/d^2/(a*d
-b*e)^2
```

Rubi [A]
time = 0.08, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2513, 528, 84}

$$\frac{a^2p\log(ax+b)}{2e(ad-be)^2}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e(d+ex)^2}-\frac{bp(2ad-be)\log(d+ex)}{2d^2(ad-be)^2}+\frac{bp}{2d(d+ex)(ad-be)}-\frac{p\log(x)}{2d^2e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/(d + e*x)^3,x]
```

```
[Out] (b*p)/(2*d*(a*d - b*e)*(d + e*x)) - Log[c*(a + b/x)^p]/(2*e*(d + e*x)^2) -
(p*Log[x])/(2*d^2*e) + (a^2*p*Log[b + a*x])/(2*e*(a*d - b*e)^2) - (b*(2*a*d
 - b*e)*p*Log[d + e*x])/(2*d^2*(a*d - b*e)^2)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
```

```
&& (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^3}\,dx = -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e(d+ex)^2} - \frac{(bp)\int \frac{1}{\left(a+\frac{b}{x}\right)x^2(d+ex)^2}\,dx}{2e}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e(d+ex)^2} - \frac{(bp)\int \frac{1}{x(b+ax)(d+ex)^2}\,dx}{2e}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e(d+ex)^2} - \frac{(bp)\int \left(\frac{1}{bd^2x} - \frac{a^3}{b(-ad+be)^2(b+ax)} + \frac{e^2}{d(ad-be)(d+ex)^2} + \frac{e^2(2ad-be)}{d^2(ad-be)^2(d+ex}\right.}{2e}$$

$$= \frac{bp}{2d(ad-be)(d+ex)} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e(d+ex)^2} - \frac{p\log(x)}{2d^2e} + \frac{a^2p\log(b+ax)}{2e(ad-be)^2} - \frac{b(2ad-be}{2d^2(}$$

Mathematica [A]
time = 0.13, size = 113, normalized size = 0.89

$$\frac{\frac{bep}{d(ad-be)(d+ex)} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^2} - \frac{p\log(x)}{d^2} + \frac{a^2p\log(b+ax)}{(ad-be)^2} + \frac{be(-2ad+be)p\log(d+ex)}{d^2(ad-be)^2}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/(d + e*x)^3,x]
[Out] ((b*e*p)/(d*(a*d - b*e)*(d + e*x)) - Log[c*(a + b/x)^p]/(d + e*x)^2 - (p*Lo
g[x])/d^2 + (a^2*p*Log[b + a*x])/(a*d - b*e)^2 + (b*e*(-2*a*d + b*e)*p*Log[
d + e*x])/(d^2*(a*d - b*e)^2))/(2*e)
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/(e*x+d)^3,x)
[Out] int(ln(c*(a+b/x)^p)/(e*x+d)^3,x)
```

Maxima [A]
time = 0.43, size = 162, normalized size = 1.28

$$\frac{1}{2}\left(\frac{a^2\log(ax+b)}{a^2bd^2-2ab^2de+b^3e^2} - \frac{(2ade-be^2)\log(xe+d)}{a^2d^4-2abd^3e+b^2d^2e^2} + \frac{e}{ad^3-bd^2e+(ad^2e-bde^2)x} - \frac{\log(x)}{bd^2}\right)bpe^{(-1)} - \frac{e^{(-1)}\log\left(\left(a+\frac{b}{x}\right)^pc\right)}{2(xe+d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(a^2*log(a*x + b)/(a^2*b*d^2 - 2*a*b^2*d*e + b^3*e^2) - (2*a*d*e - b*e^
2)*log(x*e + d)/(a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2) + e/(a*d^3 - b*d^2*e
+ (a*d^2*e - b*d*e^2)*x) - log(x)/(b*d^2))*b*p*e^(-1) - 1/2*e^(-1)*log((a +
 b/x)^p*c)/(x*e + d)^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(121) = 242$.
time = 0.86, size = 423, normalized size = 3.33

$$\frac{abd^3pe - b^2dpxe^3 + (abd^2px - b^2d^2p)e^2 + (a^2d^2px^2e^2 + 2\,a^2d^3pxe + a^2d^4p)\log(ax+b) - (2\,abd^3pe - b^2px^2e^4 + 2\,(abdpx^2 - b^2dpx)e^3 + (4\,abd^2px - b^2d^2p)e^2)\log(xe+d) - (a^2d^4 - 2\,abd^3e + b^2d^2e^2)\log(c) - (a^2d^4p + b^2px^2e^4 - 2\,(abdpx^2 - b^2dpx)e^3 + (a^2d^2px^2 - 4\,abd^2px + b^2d^2p)e^2 + 2\,(a^2d^3px - abd^3p)e)\log(x) - (a^2d^4p - 2\,abd^3pe + b^2d^2pe^2)\log\left(\frac{ax+b}{x}\right)}{2\,(a^2d^6e + b^2d^2x^2e^5 - 2\,(abd^3x^2 - b^2d^3x)e^4 + (a^2d^4x^2 - 4\,abd^4x + b^2d^4)e^3 + 2\,(a^2d^5x - abd^5)e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a*b*d^3*p*e - b^2*d*p*x*e^3 + (a*b*d^2*p*x - b^2*d^2*p)*e^2 + (a^2*d^2
*p*x^2*e^2 + 2*a^2*d^3*p*x*e + a^2*d^4*p)*log(a*x + b) - (2*a*b*d^3*p*e - b
^2*p*x^2*e^4 + 2*(a*b*d*p*x^2 - b^2*d*p*x)*e^3 + (4*a*b*d^2*p*x - b^2*d^2*p
)*e^2)*log(x*e + d) - (a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2)*log(c) - (a^2*d
^4*p + b^2*p*x^2*e^4 - 2*(a*b*d*p*x^2 - b^2*d*p*x)*e^3 + (a^2*d^2*p*x^2 - 4
*a*b*d^2*p*x + b^2*d^2*p)*e^2 + 2*(a^2*d^3*p*x - a*b*d^3*p)*e)*log(x) - (a^
2*d^4*p - 2*a*b*d^3*p*e + b^2*d^2*p*e^2)*log((a*x + b)/x))/(a^2*d^6*e + b^2
*d^2*x^2*e^5 - 2*(a*b*d^3*x^2 - b^2*d^3*x)*e^4 + (a^2*d^4*x^2 - 4*a*b*d^4*x
 + b^2*d^4)*e^3 + 2*(a^2*d^5*x - a*b*d^5)*e^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3512 vs. $2(105) = 210$.
time = 48.70, size = 3512, normalized size = 27.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/(e*x+d)**3,x)
```

```
[Out] Piecewise(((d**2*p*log(d/e + x)/(2*d**4*e + 4*d**3*e**2*x + 2*d**2*e**3*x**2
) - d**2*p/(2*d**4*e + 4*d**3*e**2*x + 2*d**2*e**3*x**2) + 2*d*e*p*x*log(d/
e + x)/(2*d**4*e + 4*d**3*e**2*x + 2*d**2*e**3*x**2) - d*e*p*x/(2*d**4*e +
4*d**3*e**2*x + 2*d**2*e**3*x**2) + 2*d*e*x*log(c*(b/x)**p)/(2*d**4*e + 4*d
**3*e**2*x + 2*d**2*e**3*x**2) + e**2*p*x**2*log(d/e + x)/(2*d**4*e + 4*d**
3*e**2*x + 2*d**2*e**3*x**2) + e**2*x**2*log(c*(b/x)**p)/(2*d**4*e + 4*d**3
*e**2*x + 2*d**2*e**3*x**2), Eq(a, 0)), (-3*d**2*p/(4*d**4*e + 8*d**3*e**2*
x + 4*d**2*e**3*x**2) - 2*d*e*p*x/(4*d**4*e + 8*d**3*e**2*x + 4*d**2*e**3*x
**2) + 4*d*e*x*log(c*(b/x + b*e/d)**p)/(4*d**4*e + 8*d**3*e**2*x + 4*d**2*e
```

```
**3*x**2) + 2*e**2*x**2*log(c*(b/x + b*e/d)**p)/(4*d**4*e + 8*d**3*e**2*x +
 4*d**2*e**3*x**2), Eq(a, b*e/d)), ((a**2*log(c*(a + b/x)**p)/(2*b**2) - a*
p/(2*b*x) + p/(4*x**2) - log(c*(a + b/x)**p)/(2*x**2))/e**3, Eq(d, 0)), (zo
o*(x*log(c*(a + b/x)**p) + b*p*log(a*x + b)/a), Eq(d, -e*x)), ((x*log(c*(a
+ b/x)**p) + b*p*log(a*x + b)/a)/d**3, Eq(e, 0)), (2*a**2*d**3*x*log(c*(a +
 b/x)**p)/(2*a**2*d**6 + 4*a**2*d**5*e*x + 2*a**2*d**4*e**2*x**2 - 4*a*b*d*
*5*e - 8*a*b*d**4*e**2*x - 4*a*b*d**3*e**3*x**2 + 2*b**2*d**4*e**2 + 4*b**2
*d**3*e**3*x + 2*b**2*d**2*e**4*x**2) + a**2*d**2*e*x**2*log(c*(a + b/x)**p
)/(2*a**2*d**6 + 4*a**2*d**5*e*x + 2*a**2*d**4*e**2*x**2 - 4*a*b*d**5*e - 8
*a*b*d**4*e**2*x - 4*a*b*d**3*e**3*x**2 + 2*b**2*d**4*e**2 + 4*b**2*d**3*e*
*3*x + 2*b**2*d**2*e**4*x**2) + 2*a*b*d**3*p*log(x + b/a)/(2*a**2*d**6 + 4*
a**2*d**5*e*x + 2*a**2*d**4*e**2*x**2 - 4*a*b*d**5*e - 8*a*b*d**4*e**2*x -
4*a*b*d**3*e**3*x**2 + 2*b**2*d**4*e**2 + 4*b**2*d**3*e**3*x + 2*b**2*d**2*
e**4*x**2) - 2*a*b*d**3*p*log(d/e + x)/(2*a**2*d**6 + 4*a**2*d**5*e*x + 2*a
**2*d**4*e**2*x**2 - 4*a*b*d**5*e - 8*a*b*d**4*e**2*x - 4*a*b*d**3*e**3*x**
2 + 2*b**2*d**4*e**2 + 4*b**2*d**3*e**3*x + 2*b**2*d**2*e**4*x**2) + a*b*d*
*3*p/(2*a**2*d**6 + 4*a**2*d**5*e*x + 2*a**2*d**4*e**2*x**2 - 4*a*b*d**5*e
- 8*a*b*d**4*e**2*x - 4*a*b*d**3*e**3*x**2 + 2*b**2*d**4*e**2 + 4*b**2*d**3
*e**3*x + 2*b**2*d**2*e**4*x**2) + 4*a*b*d**2*e*p*x*log(x + b/a)/(2*a**2*d*
*6 + 4*a**2*d**5*e*x + 2*a**2*d**4*e**2*x**2 - 4*a*b*d**5*e - 8*a*b*d**4*e*
*2*x - 4*a*b*d**3*e**3*x**2 + 2*b**2*d**4*e**2 + 4*b**2*d**3*e**3*x + 2*b**
2*d**2*e**4*x**2) - 4*a*b*d**2*e*p*x*log(d/e + x)/(2*a**2*d**6 + 4*a**2*d**
5*e*x + 2*a**2*d**4*e**2*x**2 - 4*a*b*d**5*e - 8*a*b*d**4*e**2*x - 4*a*b*d*
*3*e**3*x**2 + 2*b**2*d**4*e**2 + 4*b**2*d**3*e**3*x + 2*b**2*d**2*e**4*x**
2) + a*b*d**2*e*p*x/(2*a**2*d**6 + 4*a**2*d**5*e*x + 2*a**2*d**4*e**2*x**2
- 4*a*b*d**5*e - 8*a*b*d**4*e**2*x - 4*a*b*d**3*e**3*x**2 + 2*b**2*d**4*e**
2 + 4*b**2*d**3*e**3*x + 2*b**2*d**2*e**4*x**2) - 4*a*b*d**2*e*x*log(c*(a +
 b/x)**p)/(2*a**2*d**6 + 4*a**2*d**5*e*x + 2*a**2*d**4*e**2*x**2 - 4*a*b*d*
*5*e - 8*a*b*d**4*e**2*x - 4*a*b*d**3*e**3*x**2 + 2*b**2*d**4*e**2 + 4*b**2
*d**3*e**3*x + 2*b**2*d**2*e**4*x**2) + 2*a*b*d*e**2*p*x**2*log(x + b/a)/(2
*a**2*d**6 + 4*a**2*d**5*e*x + 2*a**2*d**4*e**2*x**2 - 4*a*b*d**5*e - 8*a*b
*d**4*e**2*x - 4*a*b*d**3*e**3*x**2 + 2*b**2*d**4*e**2 + 4*b**2*d**3*e**3*x
 + 2*b**2*d**2*e**4*x**2) - 2*a*b*d*e**2*p*x**2*log(d/e + x)/(2*a**2*d**6 +
 4*a**2*d**5*e*x + 2*a**2*d**4*e**2*x**2 - 4*a*b*d**5*e - 8*a*b*d**4*e**2*x
 - 4*a*b*d**3*e**3*x**2 + 2*b**2*d**4*e**2 + 4*b**2*d**3*e**3*x + 2*b**2*d*
*2*e**4*x**2) - 2*a*b*d*e**2*x**2*log(c*(a + b/x)**p)/(2*a**2*d**6 + 4*a**2
*d**5*e*x + 2*a**2*d**4*e**2*x**2 - 4*a*b*d**5*e - 8*a*b*d**4*e**2*x - 4*a*
b*d**3*e**3*x**2 + 2*b**2*d**4*e**2 + 4*b**2*d**3*e**3*x + 2*b**2*d**2*e**4
*x**2) - b**2*d**2*e*p*log(x + b/a)/(2*a**2*d**6 + 4*a**2*d**5*e*x + 2*a**2
*d**4*e**2*x**2 - 4*a*b*d**5*e - 8*a*b*d**4*e**2*x - 4*a*b*d**3*e**3*x**2 +
 2*b**2*d**4*e**2 + 4*b**2*d**3*e**3*x + 2*b**2*d**2*e**4*x**2) + b**2*d**2
*e*p*log(d/e + x)/(2*a**2*d**6 + 4*a**2*d**5*e*x + 2*a**2*d**4*e**2*x**2 -
4*a*b*d**5*e - 8*a*b*d**4*e**2*x - 4*a*b*d**3*e**3*x**2 + 2*b**2*d**4*e**2
+ 4*b**2*d**3*e**3*x + 2*b**2*d**2*e**4*x**2) - b**2*d**2*e*p/(2*a**2*d**6
+ 4*a**2*d**5*e*x + 2*a**2*d**4*e**2*x**2 - 4*a*b*d**5*e - 8*a*b*d**4*e**2*
```

```
x - 4*a*b*d**3*e**3*x**2 + 2*b**2*d**4*e**2 + 4*b**2*d**3*e**3*x + 2*b**2*d
**2*e**4*x**2) - 2*b**2*d*e**2*p*x*log(x + b/a)/(2*a**2*d**6 + 4*a**2*d**5*
e*x + 2*a**2*d**4*e**2*x**2 - 4*a*b*d**5*e - 8*a*b*d**4*e**2*x - 4*a*b*d**3
*e**3*x**2 + 2*b**2*d**4*e**2 + 4*b**2*d**3*e**3*x + 2*b**2*d**2*e**4*x**2)
 + 2*b**2*d*e**2*p*x*log(d/e + x)/(2*a**2*d**6 + 4*a**2*d**5*e*x + 2*a**2*d
**4*e**2*x**2 - 4*a*b*d**5*e - 8*a*b*d**4*e**2*x - 4*a*b*d**3*e**3*x**2 + 2
*b**2*d**4*e**2 + 4*b**2*d**3*e**3*x + 2*b**2*d**2*e**4*x**2) - b**2*d*e**2
*p*x/(2*a**2*d**6 + 4*a**2*d**5*e*x + 2*a**2*d**4*e**2*x**2 - 4*a*b*d**5*e
- 8*a*b*d**4*e**2*x - 4*a*b*d**3*e**3*x**2 + 2*b**2*d**4*e**2 + 4*b**2*d**3
*e**3*x + 2*b**2*d**2*e**4*x**2) + 2*b**2*d*e**2*x*log(c*(a + b/x)**p)/(2*a
**2*d**6 + 4*a**2*d**5*e*x + 2*a**2*d**4*e**2*x**2 - 4*a*b*d**5*e - 8*a*b*d
**4*e**2*x - 4*a*b*d**3*e**3*x**2 + 2*b**2*d**4*e**2 + 4*b**2*d**3*e**3*x +
 2*b**2*d**2*e**4*x**2) - b**2*e**3*p*x**2*log(...
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 805 vs. $2(121) = 242$.
time = 4.52, size = 805, normalized size = 6.34

$$-\frac{2a^3b^2d^3p\log\left(-ad+be+\frac{(ax+b)d}{x}\right)-5a^2b^3d^2pe\log\left(-ad+be+\frac{(ax+b)d}{x}\right)-a^2b^3d^2pe-\frac{4(ax+b)a^2b^2d^3p\log\left(-ad+be+\frac{(ax+b)d}{x}\right)}{x}+4ab^4dpe^2\log\left(-ad+be+\frac{(ax+b)d}{x}\right)+\frac{6(ax+b)ab^3d^2pe\log\left(-ad+be+\frac{(ax+b)d}{x}\right)}{x}+2a^3b^2d^3\log(c)-5a^2b^3d^2e\log(c)+\frac{2(ax+b)a^2b^2d^3p\log\left(\frac{ax+b}{x}\right)}{x}-\frac{2(ax+b)ab^3d^2pe\log\left(\frac{ax+b}{x}\right)}{x}+2ab^4dpe^2+\frac{(ax+b)ab^3d^2pe}{x}+\frac{2(ax+b)^2ab^2d^3p\log\left(-ad+be+\frac{(ax+b)d}{x}\right)}{x^2}-b^5pe^3\log\left(-ad+be+\frac{(ax+b)d}{x}\right)-\frac{2(ax+b)b^4dpe^2\log\left(-ad+be+\frac{(ax+b)d}{x}\right)}{x}-\frac{(ax+b)^2b^3d^2pe\log\left(-ad+be+\frac{(ax+b)d}{x}\right)}{x^2}-\frac{2(ax+b)a^2b^2d^3\log(c)}{x}+4ab^4de^2\log(c)+\frac{4(ax+b)ab^3d^2e\log(c)}{x}-\frac{2(ax+b)^2ab^2d^3p\log\left(\frac{ax+b}{x}\right)}{x^2}+\frac{(ax+b)^2b^3d^2pe\log\left(\frac{ax+b}{x}\right)}{x^2}-b^5pe^3-\frac{(ax+b)b^4dpe^2}{x}-b^5e^3\log(c)-\frac{2(ax+b)b^4de^2\log(c)}{x}}{2\left(a^4d^6-4a^3bd^5e-\frac{2(ax+b)a^3d^6}{x}+6a^2b^2d^4e^2+\frac{6(ax+b)a^2bd^5e}{x}+\frac{(ax+b)^2a^2d^6}{x^2}-4ab^3d^3e^3-\frac{6(ax+b)ab^2d^4e^2}{x}-\frac{2(ax+b)^2abd^5e}{x^2}+b^4d^2e^4+\frac{2(ax+b)b^3d^3e^3}{x}+\frac{(ax+b)^2b^2d^4e^2}{x^2}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*a^3*b^2*d^3*p*log(-a*d + b*e + (a*x + b)*d/x) - 5*a^2*b^3*d^2*p*e*l
og(-a*d + b*e + (a*x + b)*d/x) - a^2*b^3*d^2*p*e - 4*(a*x + b)*a^2*b^2*d^3*
p*log(-a*d + b*e + (a*x + b)*d/x)/x + 4*a*b^4*d*p*e^2*log(-a*d + b*e + (a*x
 + b)*d/x) + 6*(a*x + b)*a*b^3*d^2*p*e*log(-a*d + b*e + (a*x + b)*d/x)/x +
2*a^3*b^2*d^3*log(c) - 5*a^2*b^3*d^2*e*log(c) + 2*(a*x + b)*a^2*b^2*d^3*p*l
og((a*x + b)/x)/x - 2*(a*x + b)*a*b^3*d^2*p*e*log((a*x + b)/x)/x + 2*a*b^4*
d*p*e^2 + (a*x + b)*a*b^3*d^2*p*e/x + 2*(a*x + b)^2*a*b^2*d^3*p*log(-a*d +
b*e + (a*x + b)*d/x)/x^2 - b^5*p*e^3*log(-a*d + b*e + (a*x + b)*d/x) - 2*(a
*x + b)*b^4*d*p*e^2*log(-a*d + b*e + (a*x + b)*d/x)/x - (a*x + b)^2*b^3*d^2
*p*e*log(-a*d + b*e + (a*x + b)*d/x)/x^2 - 2*(a*x + b)*a^2*b^2*d^3*log(c)/x
 + 4*a*b^4*d*e^2*log(c) + 4*(a*x + b)*a*b^3*d^2*e*log(c)/x - 2*(a*x + b)^2*
a*b^2*d^3*p*log((a*x + b)/x)/x^2 + (a*x + b)^2*b^3*d^2*p*e*log((a*x + b)/x)
/x^2 - b^5*p*e^3 - (a*x + b)*b^4*d*p*e^2/x - b^5*e^3*log(c) - 2*(a*x + b)*b
^4*d*e^2*log(c)/x)/((a^4*d^6 - 4*a^3*b*d^5*e - 2*(a*x + b)*a^3*d^6/x + 6*a^
2*b^2*d^4*e^2 + 6*(a*x + b)*a^2*b*d^5*e/x + (a*x + b)^2*a^2*d^6/x^2 - 4*a*b
^3*d^3*e^3 - 6*(a*x + b)*a*b^2*d^4*e^2/x - 2*(a*x + b)^2*a*b*d^5*e/x^2 + b^
4*d^2*e^4 + 2*(a*x + b)*b^3*d^3*e^3/x + (a*x + b)^2*b^2*d^4*e^2/x^2)*b)
```

Mupad [**B**]
time = 1.08, size = 217, normalized size = 1.71

$$\frac{a^2\,p\,\ln\left(b+a\,x\right)}{2\,a^2\,d^2\,e-4\,a\,b\,d\,e^2+2\,b^2\,e^3}-\frac{\ln\left(c\,{\left(\frac{b+a\,x}{x}\right)}^p\right)}{2\,\left(d^2\,e+2\,d\,e^2\,x+e^3\,x^2\right)}-\frac{p\,\ln\left(x\right)}{2\,d^2\,e}-\frac{b\,e\,p}{2\,b\,d^2\,e^2-2\,a\,d^3\,e+2\,b\,d\,e^3\,x-2\,a\,d^2\,e^2\,x}+\frac{b^2\,e\,p\,\ln\left(d+e\,x\right)}{2\,a^2\,d^4-4\,a\,b\,d^3\,e+2\,b^2\,d^2\,e^2}-\frac{2\,a\,b\,d\,p\,\ln\left(d+e\,x\right)}{2\,a^2\,d^4-4\,a\,b\,d^3\,e+2\,b^2\,d^2\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)/(d + e*x)^3,x)
```

```
[Out] (a^2*p*log(b + a*x))/(2*b^2*e^3 + 2*a^2*d^2*e - 4*a*b*d*e^2) - log(c*((b +
a*x)/x)^p)/(2*(d^2*e + e^3*x^2 + 2*d*e^2*x)) - (p*log(x))/(2*d^2*e) - (b*e*
p)/(2*b*d^2*e^2 - 2*a*d^3*e + 2*b*d*e^3*x - 2*a*d^2*e^2*x) + (b^2*e*p*log(d
 + e*x))/(2*a^2*d^4 + 2*b^2*d^2*e^2 - 4*a*b*d^3*e) - (2*a*b*d*p*log(d + e*x
))/(2*a^2*d^4 + 2*b^2*d^2*e^2 - 4*a*b*d^3*e)
```

3.204 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^4}\,dx$

Optimal. Leaf size=175

$$\frac{bp}{6d(ad-be)(d+ex)^2}+\frac{b(2ad-be)p}{3d^2(ad-be)^2(d+ex)}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e(d+ex)^3}-\frac{p\log(x)}{3d^3e}+\frac{a^3p\log(b+ax)}{3e(ad-be)^3}-\frac{b(3a^2d^2-3abde}{3d^3}$$

```
[Out] 1/6*b*p/d/(a*d-b*e)/(e*x+d)^2+1/3*b*(2*a*d-b*e)*p/d^2/(a*d-b*e)^2/(e*x+d)-1
/3*ln(c*(a+b/x)^p)/e/(e*x+d)^3-1/3*p*ln(x)/d^3/e+1/3*a^3*p*ln(a*x+b)/e/(a*d
-b*e)^3-1/3*b*(3*a^2*d^2-3*a*b*d*e+b^2*e^2)*p*ln(e*x+d)/d^3/(a*d-b*e)^3
```

Rubi [A]
time = 0.12, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2513, 528, 84}

$$\frac{a^3p\log(ax+b)}{3e(ad-be)^3}-\frac{bp(3a^2d^2-3abde+b^2e^2)\log(d+ex)}{3d^3(ad-be)^3}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e(d+ex)^3}+\frac{bp(2ad-be)}{3d^2(d+ex)(ad-be)^2}+\frac{bp}{6d(d+ex)^2(ad-be)}-\frac{p\log(x)}{3d^3e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/(d + e*x)^4,x]
```

```
[Out] (b*p)/(6*d*(a*d - b*e)*(d + e*x)^2) + (b*(2*a*d - b*e)*p)/(3*d^2*(a*d - b*e
)^2*(d + e*x)) - Log[c*(a + b/x)^p]/(3*e*(d + e*x)^3) - (p*Log[x])/(3*d^3*e
) + (a^3*p*Log[b + a*x])/(3*e*(a*d - b*e)^3) - (b*(3*a^2*d^2 - 3*a*b*d*e +
b^2*e^2)*p*Log[d + e*x])/(3*d^3*(a*d - b*e)^3)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
```

```
&& (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^4}\,dx &= -\frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{3e(d+ex)^3}-\frac{(bp)\int \frac{1}{\left(a+\frac{b}{x}\right)x^2(d+ex)^3}\,dx}{3e}\\
&= -\frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{3e(d+ex)^3}-\frac{(bp)\int \frac{1}{x(b+ax)(d+ex)^3}\,dx}{3e}\\
&= -\frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{3e(d+ex)^3}-\frac{(bp)\int \left(\frac{1}{bd^3x}+\frac{a^4}{b(-ad+be)^3(b+ax)}+\frac{e^2}{d(ad-be)(d+ex)^3}+\frac{e^2(2ad-be)}{d^2(ad-be)^2(d+ex}\right.}{3e}\\
&= \frac{bp}{6d(ad-be)(d+ex)^2}+\frac{b(2ad-be)p}{3d^2(ad-be)^2(d+ex)}-\frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{3e(d+ex)^3}-\frac{p\log(x)}{3d^3e}+\frac{a^3}{3}
\end{aligned}
$$

Mathematica [A]
time = 0.19, size = 164, normalized size = 0.94

$$
\frac{\frac{bep}{2d(ad-be)(d+ex)^2}+\frac{be(2ad-be)p}{d^2(ad-be)^2(d+ex)}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^3}-\frac{p\log(x)}{d^3}+\frac{a^3p\log(b+ax)}{(ad-be)^3}-\frac{be(3a^2d^2-3abde+b^2e^2)p\log(d+ex)}{d^3(ad-be)^3}}{3e}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/(d + e*x)^4,x]
```

```
[Out] ((b*e*p)/(2*d*(a*d - b*e)*(d + e*x)^2) + (b*e*(2*a*d - b*e)*p)/(d^2*(a*d -
b*e)^2*(d + e*x)) - Log[c*(a + b/x)^p]/(d + e*x)^3 - (p*Log[x])/d^3 + (a^3*
p*Log[b + a*x])/(a*d - b*e)^3 - (b*e*(3*a^2*d^2 - 3*a*b*d*e + b^2*e^2)*p*Lo
g[d + e*x])/(d^3*(a*d - b*e)^3))/(3*e)
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$
\int \frac{\ln \left(c\left(a+\frac{b}{x}\right)^p\right)}{(ex+d)^4}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/(e*x+d)^4,x)
```

```
[Out] int(ln(c*(a+b/x)^p)/(e*x+d)^4,x)
```

Maxima [A]
time = 0.40, size = 290, normalized size = 1.66

$$
\frac{1}{6}\left(\frac{2\,a^3\log(ax+b)}{a^3bd^3-3\,a^2b^2d^2e+3\,ab^3de^2-b^4e^3}-\frac{2\,(3\,a^2d^2e-3\,abde^2+b^2e^3)\log(xe+d)}{a^3d^6-3\,a^2bd^5e+3\,ab^2d^4e^2-b^3d^3e^3}+\frac{5\,ad^2e-3\,bde^2+2\,(2\,ade^2-be^3)x}{a^2d^6-2\,abd^5e+b^2d^4e^2+(a^2d^4e^2-2\,abd^3e^3+b^2d^2e^4)x^2+2\,(a^2d^5e-2\,abd^4e^2+b^2d^3e^3)x}-\frac{2\log(x)}{bd^3}\right)bpe^{(-1)}-\frac{e^{(-1)}\log\left(\left(a+\frac{b}{x}\right)^pc\right)}{3\,(xe+d)^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a^3*log(a*x + b)/(a^3*b*d^3 - 3*a^2*b^2*d^2*e + 3*a*b^3*d*e^2 - b^4*
e^3) - 2*(3*a^2*d^2*e - 3*a*b*d*e^2 + b^2*e^3)*log(x*e + d)/(a^3*d^6 - 3*a^
2*b*d^5*e + 3*a*b^2*d^4*e^2 - b^3*d^3*e^3) + (5*a*d^2*e - 3*b*d*e^2 + 2*(2*
a*d*e^2 - b*e^3)*x)/(a^2*d^6 - 2*a*b*d^5*e + b^2*d^4*e^2 + (a^2*d^4*e^2 - 2
*a*b*d^3*e^3 + b^2*d^2*e^4)*x^2 + 2*(a^2*d^5*e - 2*a*b*d^4*e^2 + b^2*d^3*e^
3)*x) - 2*log(x)/(b*d^3))*b*p*e^(-1) - 1/3*e^(-1)*log((a + b/x)^p*c)/(x*e +
 d)^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 814 vs. $2(169) = 338$.
time = 4.14, size = 814, normalized size = 4.65

$$\frac{5a^2bd^5pe + 2b^3dpx^2e^5 - (6ab^2d^2px^2 - 5b^3d^2px)e^4 + (4a^2bd^3px^2 - 14ab^2d^3px + 3b^3d^3p)e^3 + (9a^2bd^4px - 8ab^2d^4p)e^2 + 2(a^3d^3px^3e^3 + 3a^3d^4px^2e^2 + 3a^3d^5pxe + a^3d^6p)\log(ax+b) - 2(3a^2bd^5pe + b^3px^3e^6 - 3(ab^2dpx^3 - b^3dpx^2)e^5 + 3(a^2bd^2px^3 - 3ab^2d^2px^2 + b^3d^2px)e^4 + (9a^2bd^3px^2 - 9ab^2d^3px + b^3d^3p)e^3 + 3(3a^2bd^4px - ab^2d^4p)e^2)\log(xe+d) - 2(a^3d^6 - 3a^2bd^5e + 3ab^2d^4e^2 - b^3d^3e^3)\log(c) - 2(a^3d^6p - b^3px^3e^6 + 3(ab^2dpx^3 - b^3dpx^2)e^5 - 3(a^2bd^2px^3 - 3ab^2d^2px^2 + b^3d^2px)e^4 + (a^3d^3px^3 - 9a^2bd^3px^2 + 9ab^2d^3px - b^3d^3p)e^3 + 3(a^3d^4px^2 - 3a^2bd^4px + ab^2d^4p)e^2 + 3(a^3d^5px - a^2bd^5p)e)\log(x) - 2(a^3d^6p - 3a^2bd^5pe + 3ab^2d^4pe^2 - b^3d^3pe^3)\log\left(\frac{ax+b}{x}\right)}{6(a^3d^9e - b^3d^3x^3e^7 + 3(ab^2d^4x^3 - b^3d^4x^2)e^6 - 3(a^2bd^5x^3 - 3ab^2d^5x^2 + b^3d^5x)e^5 + (a^3d^6x^3 - 9a^2bd^6x^2 + 9ab^2d^6x - b^3d^6)e^4 + 3(a^3d^7x^2 - 3a^2bd^7x + ab^2d^7)e^3 + 3(a^3d^8x - a^2bd^8)e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(5*a^2*b*d^5*p*e + 2*b^3*d*p*x^2*e^5 - (6*a*b^2*d^2*p*x^2 - 5*b^3*d^2*p
*x)*e^4 + (4*a^2*b*d^3*p*x^2 - 14*a*b^2*d^3*p*x + 3*b^3*d^3*p)*e^3 + (9*a^2
*b*d^4*p*x - 8*a*b^2*d^4*p)*e^2 + 2*(a^3*d^3*p*x^3*e^3 + 3*a^3*d^4*p*x^2*e^
2 + 3*a^3*d^5*p*x*e + a^3*d^6*p)*log(a*x + b) - 2*(3*a^2*b*d^5*p*e + b^3*p*
x^3*e^6 - 3*(a*b^2*d*p*x^3 - b^3*d*p*x^2)*e^5 + 3*(a^2*b*d^2*p*x^3 - 3*a*b^
2*d^2*p*x^2 + b^3*d^2*p*x)*e^4 + (9*a^2*b*d^3*p*x^2 - 9*a*b^2*d^3*p*x + b^3
*d^3*p)*e^3 + 3*(3*a^2*b*d^4*p*x - a*b^2*d^4*p)*e^2)*log(x*e + d) - 2*(a^3*
d^6 - 3*a^2*b*d^5*e + 3*a*b^2*d^4*e^2 - b^3*d^3*e^3)*log(c) - 2*(a^3*d^6*p
- b^3*p*x^3*e^6 + 3*(a*b^2*d*p*x^3 - b^3*d*p*x^2)*e^5 - 3*(a^2*b*d^2*p*x^3
- 3*a*b^2*d^2*p*x^2 + b^3*d^2*p*x)*e^4 + (a^3*d^3*p*x^3 - 9*a^2*b*d^3*p*x^2
 + 9*a*b^2*d^3*p*x - b^3*d^3*p)*e^3 + 3*(a^3*d^4*p*x^2 - 3*a^2*b*d^4*p*x +
a*b^2*d^4*p)*e^2 + 3*(a^3*d^5*p*x - a^2*b*d^5*p)*e)*log(x) - 2*(a^3*d^6*p -
 3*a^2*b*d^5*p*e + 3*a*b^2*d^4*p*e^2 - b^3*d^3*p*e^3)*log((a*x + b)/x))/(a^
3*d^9*e - b^3*d^3*x^3*e^7 + 3*(a*b^2*d^4*x^3 - b^3*d^4*x^2)*e^6 - 3*(a^2*b*
d^5*x^3 - 3*a*b^2*d^5*x^2 + b^3*d^5*x)*e^5 + (a^3*d^6*x^3 - 9*a^2*b*d^6*x^2
 + 9*a*b^2*d^6*x - b^3*d^6)*e^4 + 3*(a^3*d^7*x^2 - 3*a^2*b*d^7*x + a*b^2*d^
7)*e^3 + 3*(a^3*d^8*x - a^2*b*d^8)*e^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/(e*x+d)**4,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1841 vs. $2(169) = 338$.

time = 5.62, size = 1841, normalized size = 10.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -1/6*(6*a^5*b^2*d^5*p*log(-a*d + b*e + (a*x + b)*d/x) - 24*a^4*b^3*d^4*p*e*
log(-a*d + b*e + (a*x + b)*d/x) - 6*a^4*b^3*d^4*p*e - 18*(a*x + b)*a^4*b^2*
d^5*p*log(-a*d + b*e + (a*x + b)*d/x)/x + 38*a^3*b^4*d^3*p*e^2*log(-a*d + b
*e + (a*x + b)*d/x) + 54*(a*x + b)*a^3*b^3*d^4*p*e*log(-a*d + b*e + (a*x +
b)*d/x)/x + 6*a^5*b^2*d^5*log(c) - 24*a^4*b^3*d^4*e*log(c) + 6*(a*x + b)*a^
4*b^2*d^5*p*log((a*x + b)/x)/x - 12*(a*x + b)*a^3*b^3*d^4*p*e*log((a*x + b)
/x)/x + 21*a^3*b^4*d^3*p*e^2 + 12*(a*x + b)*a^3*b^3*d^4*p*e/x + 18*(a*x + b
)^2*a^3*b^2*d^5*p*log(-a*d + b*e + (a*x + b)*d/x)/x^2 - 30*a^2*b^5*d^2*p*e^
3*log(-a*d + b*e + (a*x + b)*d/x) - 60*(a*x + b)*a^2*b^4*d^3*p*e^2*log(-a*d
 + b*e + (a*x + b)*d/x)/x - 36*(a*x + b)^2*a^2*b^3*d^4*p*e*log(-a*d + b*e +
 (a*x + b)*d/x)/x^2 - 12*(a*x + b)*a^4*b^2*d^5*log(c)/x + 38*a^3*b^4*d^3*e^
2*log(c) + 42*(a*x + b)*a^3*b^3*d^4*e*log(c)/x - 12*(a*x + b)^2*a^3*b^2*d^5
*p*log((a*x + b)/x)/x^2 + 6*(a*x + b)*a^2*b^4*d^3*p*e^2*log((a*x + b)/x)/x
+ 18*(a*x + b)^2*a^2*b^3*d^4*p*e*log((a*x + b)/x)/x^2 - 27*a^2*b^5*d^2*p*e^
3 - 31*(a*x + b)*a^2*b^4*d^3*p*e^2/x - 6*(a*x + b)^2*a^2*b^3*d^4*p*e/x^2 -
6*(a*x + b)^3*a^2*b^2*d^5*p*log(-a*d + b*e + (a*x + b)*d/x)/x^3 + 12*a*b^6*
d*p*e^4*log(-a*d + b*e + (a*x + b)*d/x) + 30*(a*x + b)*a*b^5*d^2*p*e^3*log(
-a*d + b*e + (a*x + b)*d/x)/x + 24*(a*x + b)^2*a*b^4*d^3*p*e^2*log(-a*d + b
*e + (a*x + b)*d/x)/x^2 + 6*(a*x + b)^3*a*b^3*d^4*p*e*log(-a*d + b*e + (a*x
 + b)*d/x)/x^3 + 6*(a*x + b)^2*a^3*b^2*d^5*log(c)/x^2 - 30*a^2*b^5*d^2*e^3*
log(c) - 54*(a*x + b)*a^2*b^4*d^3*e^2*log(c)/x - 18*(a*x + b)^2*a^2*b^3*d^4
*e*log(c)/x^2 + 6*(a*x + b)^3*a^2*b^2*d^5*p*log((a*x + b)/x)/x^3 - 6*(a*x +
 b)^2*a*b^4*d^3*p*e^2*log((a*x + b)/x)/x^2 - 6*(a*x + b)^3*a*b^3*d^4*p*e*lo
g((a*x + b)/x)/x^3 + 15*a*b^6*d*p*e^4 + 26*(a*x + b)*a*b^5*d^2*p*e^3/x + 10
*(a*x + b)^2*a*b^4*d^3*p*e^2/x^2 - 2*b^7*p*e^5*log(-a*d + b*e + (a*x + b)*d
/x) - 6*(a*x + b)*b^6*d*p*e^4*log(-a*d + b*e + (a*x + b)*d/x)/x - 6*(a*x +
b)^2*b^5*d^2*p*e^3*log(-a*d + b*e + (a*x + b)*d/x)/x^2 - 2*(a*x + b)^3*b^4*
d^3*p*e^2*log(-a*d + b*e + (a*x + b)*d/x)/x^3 + 12*a*b^6*d*e^4*log(c) + 30*
(a*x + b)*a*b^5*d^2*e^3*log(c)/x + 18*(a*x + b)^2*a*b^4*d^3*e^2*log(c)/x^2
+ 2*(a*x + b)^3*b^4*d^3*p*e^2*log((a*x + b)/x)/x^3 - 3*b^7*p*e^5 - 7*(a*x +
 b)*b^6*d*p*e^4/x - 4*(a*x + b)^2*b^5*d^2*p*e^3/x^2 - 2*b^7*e^5*log(c) - 6*
(a*x + b)*b^6*d*e^4*log(c)/x - 6*(a*x + b)^2*b^5*d^2*e^3*log(c)/x^2)/((a^6*
d^9 - 6*a^5*b*d^8*e - 3*(a*x + b)*a^5*d^9/x + 15*a^4*b^2*d^7*e^2 + 15*(a*x
+ b)*a^4*b*d^8*e/x + 3*(a*x + b)^2*a^4*d^9/x^2 - 20*a^3*b^3*d^6*e^3 - 30*(a
```

```
*x + b)*a^3*b^2*d^7*e^2/x - 12*(a*x + b)^2*a^3*b*d^8*e/x^2 - (a*x + b)^3*a^
3*d^9/x^3 + 15*a^2*b^4*d^5*e^4 + 30*(a*x + b)*a^2*b^3*d^6*e^3/x + 18*(a*x +
 b)^2*a^2*b^2*d^7*e^2/x^2 + 3*(a*x + b)^3*a^2*b*d^8*e/x^3 - 6*a*b^5*d^4*e^5
 - 15*(a*x + b)*a*b^4*d^5*e^4/x - 12*(a*x + b)^2*a*b^3*d^6*e^3/x^2 - 3*(a*x
 + b)^3*a*b^2*d^7*e^2/x^3 + b^6*d^3*e^6 + 3*(a*x + b)*b^5*d^4*e^5/x + 3*(a*
x + b)^2*b^4*d^5*e^4/x^2 + (a*x + b)^3*b^3*d^6*e^3/x^3)*b)
```

Mupad [B]
time = 1.85, size = 662, normalized size = 3.78

$$\frac{p\ln(d+ex)}{3d^3e}-\frac{3b^2e^2p}{2\left(3a^2d^5e+6a^2d^4e^2x+3a^2d^3e^3x^2-6abd^4e^2-12abd^3e^3x-6abd^2e^4x^2+3b^2d^3e^3+6b^2d^2e^4x+3b^2de^5x^2\right)}-\frac{p\ln(x)}{3d^3e}-\frac{a^3p\ln(b+ax)}{-3a^3d^3e+9a^2bd^2e^2-9ab^2de^3+3b^3e^4}-\frac{\ln\left(c\left(\frac{b+ax}{x}\right)^p\right)}{3\left(d^3e+3d^2e^2x+3de^3x^2+e^4x^3\right)}-\frac{b^2e^3px}{3a^2d^6e+6a^2d^5e^2x+3a^2d^4e^3x^2-6abd^5e^2-12abd^4e^3x-6abd^3e^4x^2+3b^2d^4e^3+6b^2d^3e^4x+3b^2d^2e^5x^2}-\frac{a^3d^3p\ln(d+ex)}{3a^3d^6e-9a^2bd^5e^2+9ab^2d^4e^3-3b^3d^3e^4}+\frac{5abdep}{2\left(3a^2d^5e+6a^2d^4e^2x+3a^2d^3e^3x^2-6abd^4e^2-12abd^3e^3x-6abd^2e^4x^2+3b^2d^3e^3+6b^2d^2e^4x+3b^2de^5x^2\right)}+\frac{2abde^2px}{3a^2d^6e+6a^2d^5e^2x+3a^2d^4e^3x^2-6abd^5e^2-12abd^4e^3x-6abd^3e^4x^2+3b^2d^4e^3+6b^2d^3e^4x+3b^2d^2e^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)/(d + e*x)^4,x)
```

```
[Out] (p*log(d + e*x))/(3*d^3*e) - (3*b^2*e^2*p)/(2*(3*a^2*d^5*e + 3*b^2*d^3*e^3
+ 6*a^2*d^4*e^2*x + 6*b^2*d^2*e^4*x + 3*b^2*d*e^5*x^2 + 3*a^2*d^3*e^3*x^2 -
 6*a*b*d^4*e^2 - 12*a*b*d^3*e^3*x - 6*a*b*d^2*e^4*x^2)) - (p*log(x))/(3*d^3
*e) - (a^3*p*log(b + a*x))/(3*b^3*e^4 - 3*a^3*d^3*e + 9*a^2*b*d^2*e^2 - 9*a
*b^2*d*e^3) - log(c*((b + a*x)/x)^p)/(3*(d^3*e + e^4*x^3 + 3*d^2*e^2*x + 3*
d*e^3*x^2)) - (b^2*e^3*p*x)/(3*a^2*d^6*e + 3*b^2*d^4*e^3 + 6*a^2*d^5*e^2*x
+ 6*b^2*d^3*e^4*x + 3*a^2*d^4*e^3*x^2 + 3*b^2*d^2*e^5*x^2 - 6*a*b*d^5*e^2 -
 12*a*b*d^4*e^3*x - 6*a*b*d^3*e^4*x^2) - (a^3*d^3*p*log(d + e*x))/(3*a^3*d^
6*e - 3*b^3*d^3*e^4 + 9*a*b^2*d^4*e^3 - 9*a^2*b*d^5*e^2) + (5*a*b*d*e*p)/(2
*(3*a^2*d^5*e + 3*b^2*d^3*e^3 + 6*a^2*d^4*e^2*x + 6*b^2*d^2*e^4*x + 3*b^2*d
*e^5*x^2 + 3*a^2*d^3*e^3*x^2 - 6*a*b*d^4*e^2 - 12*a*b*d^3*e^3*x - 6*a*b*d^2
*e^4*x^2)) + (2*a*b*d*e^2*p*x)/(3*a^2*d^6*e + 3*b^2*d^4*e^3 + 6*a^2*d^5*e^2
*x + 6*b^2*d^3*e^4*x + 3*a^2*d^4*e^3*x^2 + 3*b^2*d^2*e^5*x^2 - 6*a*b*d^5*e^
2 - 12*a*b*d^4*e^3*x - 6*a*b*d^3*e^4*x^2)
```

3.205 $\int \frac{\log\left(a+\frac{b}{x}\right)}{c+dx} \, dx$

Optimal. Leaf size=105

$$\frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)}{d}+\frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d}-\frac{\log\left(-\frac{d(b+ax)}{ac-bd}\right)\log(c+dx)}{d}-\frac{\text{Li}_2\left(\frac{a(c+dx)}{ac-bd}\right)}{d}+\frac{\text{Li}_2\left(1+\frac{dx}{c}\right)}{d}$$

```
[Out] ln(a+b/x)*ln(d*x+c)/d+ln(-d*x/c)*ln(d*x+c)/d-ln(-d*(a*x+b)/(a*c-b*d))*ln(d*
x+c)/d-polylog(2,a*(d*x+c)/(a*c-b*d))/d+polylog(2,1+d*x/c)/d
```

Rubi [A]
time = 0.12, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {2512, 266, 2463, 2441, 2352, 2440, 2438}

$$-\frac{\text{PolyLog}\left(2,\frac{a(c+dx)}{ac-bd}\right)}{d}+\frac{\text{PolyLog}\left(2,\frac{dx}{c}+1\right)}{d}+\frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)}{d}-\frac{\log(c+dx)\log\left(-\frac{d(ax+b)}{ac-bd}\right)}{d}+\frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Log[a + b/x]/(c + d*x),x]
```

```
[Out] (Log[a + b/x]*Log[c + d*x])/d + (Log[-((d*x)/c)]*Log[c + d*x])/d - (Log[-((
d*(b + a*x))/(a*c - b*d))]*Log[c + d*x])/d - PolyLog[2, (a*(c + d*x))/(a*c
- b*d)]/d + PolyLog[2, 1 + (d*x)/c]/d
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\log\left(a+\frac{b}{x}\right)}{c+dx}\,dx = \frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)}{d}+\frac{b\int \frac{\log(c+dx)}{\left(a+\frac{b}{x}\right)x^2}\,dx}{d}$$

$$= \frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)}{d}+\frac{b\int \left(\frac{\log(c+dx)}{bx}-\frac{a\log(c+dx)}{b(b+ax)}\right)dx}{d}$$

$$= \frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)}{d}+\frac{\int \frac{\log(c+dx)}{x}\,dx}{d}-\frac{a\int \frac{\log(c+dx)}{b+ax}\,dx}{d}$$

$$= \frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)}{d}+\frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d}-\frac{\log\left(-\frac{d(b+ax)}{ac-bd}\right)\log(c+dx)}{d}-\int \frac{\log}{c+}$$

$$= \frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)}{d}+\frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d}-\frac{\log\left(-\frac{d(b+ax)}{ac-bd}\right)\log(c+dx)}{d}+\frac{\mathrm{Li}_2(1-}{d}$$

$$= \frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)}{d}+\frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d}-\frac{\log\left(-\frac{d(b+ax)}{ac-bd}\right)\log(c+dx)}{d}-\frac{\mathrm{Li}_2\left(\frac{a(}{a}\right.}{d}$$

Mathematica [A]
time = 0.03, size = 80, normalized size = 0.76

$$\frac{\left(\log\left(a+\frac{b}{x}\right)+\log\left(-\frac{dx}{c}\right)-\log\left(\frac{d(b+ax)}{-ac+bd}\right)\right)\log(c+dx)-\text{Li}_2\left(\frac{a(c+dx)}{ac-bd}\right)+\text{Li}_2\left(1+\frac{dx}{c}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a + b/x]/(c + d*x),x]
```

```
[Out] ((Log[a + b/x] + Log[-((d*x)/c)] - Log[(d*(b + a*x))/(-(a*c) + b*d)])*Log[c
 + d*x] - PolyLog[2, (a*(c + d*x))/(a*c - b*d)] + PolyLog[2, 1 + (d*x)/c])/
d
```

Maple [A]
time = 2.63, size = 126, normalized size = 1.20

method	result	si
risch	$-\frac{\ln\left(a+\frac{b}{x}\right)\ln\left(-\frac{b}{ax}\right)}{d}-\frac{\text{dilog}\left(-\frac{b}{ax}\right)}{d}+\frac{\text{dilog}\left(\frac{-ca+bd+c\left(a+\frac{b}{x}\right)}{-ca+bd}\right)}{d}+\frac{\ln\left(a+\frac{b}{x}\right)\ln\left(\frac{-ca+bd+c\left(a+\frac{b}{x}\right)}{-ca+bd}\right)}{d}$	11
derivativedivides	$-b\left(\frac{\text{dilog}\left(-\frac{b}{ax}\right)+\ln\left(a+\frac{b}{x}\right)\ln\left(-\frac{b}{ax}\right)}{db}-\frac{\left(\frac{\text{dilog}\left(\frac{-ca+bd+c\left(a+\frac{b}{x}\right)}{-ca+bd}\right)}{c}+\frac{\ln\left(a+\frac{b}{x}\right)\ln\left(\frac{-ca+bd+c\left(a+\frac{b}{x}\right)}{-ca+bd}\right)}{c}\right)c}{db}\right)$	12
default	$-b\left(\frac{\text{dilog}\left(-\frac{b}{ax}\right)+\ln\left(a+\frac{b}{x}\right)\ln\left(-\frac{b}{ax}\right)}{db}-\frac{\left(\frac{\text{dilog}\left(\frac{-ca+bd+c\left(a+\frac{b}{x}\right)}{-ca+bd}\right)}{c}+\frac{\ln\left(a+\frac{b}{x}\right)\ln\left(\frac{-ca+bd+c\left(a+\frac{b}{x}\right)}{-ca+bd}\right)}{c}\right)c}{db}\right)$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a+b/x)/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -b*((dilog(-b/a/x)+ln(a+b/x)*ln(-b/a/x))/d/b-(dilog((-c*a+b*d+c*(a+b/x))/(-
a*c+b*d))/c+ln(a+b/x)*ln((-c*a+b*d+c*(a+b/x))/(-a*c+b*d))/c)*c/d/b)
```

Maxima [A]
time = 0.29, size = 82, normalized size = 0.78

$$-\frac{\log\left(\frac{dx}{c}+1\right)\log(x)+\text{Li}_2\left(-\frac{dx}{c}\right)}{d}+\frac{\log(ax+b)\log\left(\frac{adx+bd}{ac-bd}+1\right)+\text{Li}_2\left(-\frac{adx+bd}{ac-bd}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b/x)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -(log(d*x/c + 1)*log(x) + dilog(-d*x/c))/d + (log(a*x + b)*log((a*d*x + b*d
)/(a*c - b*d) + 1) + dilog(-(a*d*x + b*d)/(a*c - b*d)))/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b/x)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(log((a*x + b)/x)/(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(a+\frac{b}{x}\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a+b/x)/(d*x+c),x)
```

```
[Out] Integral(log(a + b/x)/(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b/x)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(log(a + b/x)/(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(a+\frac{b}{x}\right)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a + b/x)/(c + d*x),x)
```

```
[Out] int(log(a + b/x)/(c + d*x), x)
```

3.206 $\int (d+ex)^m \log \left(c(a+bx^3)^p\right)\,dx$

Optimal. Leaf size=301

$$\frac{\sqrt[3]{b}\,p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{\sqrt[3]{b}\,(d+ex)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)}{e\left(\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)(1+m)(2+m)}+\frac{\sqrt[3]{b}\,p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{\sqrt[3]{b}\,(d+ex}{\sqrt[3]{b}\,d+\sqrt[3]{-1}}\right.}{e\left(\sqrt[3]{b}\,d+\sqrt[3]{-1}\,\sqrt[3]{a}\,e\right)(1+m)(2+m)}$$

```
[Out] b^(1/3)*p*(e*x+d)^(2+m)*hypergeom([1, 2+m],[3+m],b^(1/3)*(e*x+d)/(b^(1/3)*d
-a^(1/3)*e))/e/(b^(1/3)*d-a^(1/3)*e)/(1+m)/(2+m)+b^(1/3)*p*(e*x+d)^(2+m)*hy
pergeom([1, 2+m],[3+m],b^(1/3)*(e*x+d)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))/e/
(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e)/(1+m)/(2+m)+b^(1/3)*p*(e*x+d)^(2+m)*hyperg
eom([1, 2+m],[3+m],b^(1/3)*(e*x+d)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))/e/(b^(
1/3)*d-(-1)^(2/3)*a^(1/3)*e)/(1+m)/(2+m)+(e*x+d)^(1+m)*ln(c*(b*x^3+a)^p)/e/
(1+m)
```

Rubi [A]
time = 0.52, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2513, 6857, 70}

$$\frac{(d+ex)^{m+1}\log\left(c(a+bx^3)^p\right)}{e(m+1)}+\frac{\sqrt[3]{b}\,p(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{\sqrt[3]{b}\,(d+ex)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)}{e(m+1)(m+2)\left(\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)}+\frac{\sqrt[3]{b}\,p(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{\sqrt[3]{b}\,(d+ex)}{\sqrt[3]{b}\,d+\sqrt[3]{-1}\,\sqrt[3]{a}\,e}\right)}{e(m+1)(m+2)\left(\sqrt[3]{-1}\,\sqrt[3]{a}\,e+\sqrt[3]{b}\,d\right)}+\frac{\sqrt[3]{b}\,p(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{\sqrt[3]{b}\,(d+ex)}{\sqrt[3]{b}\,d-(-1)^{2/3}\sqrt[3]{a}\,e}\right)}{e(m+1)(m+2)\left(\sqrt[3]{b}\,d-(-1)^{2/3}\sqrt[3]{a}\,e\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (b^(1/3)*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (b^(1/3)*(d
 + e*x))/(b^(1/3)*d - a^(1/3)*e)])/(e*(b^(1/3)*d - a^(1/3)*e)*(1 + m)*(2 +
m)) + (b^(1/3)*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (b^(1
/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)])/(e*(b^(1/3)*d + (-1)^(1
/3)*a^(1/3)*e)*(1 + m)*(2 + m)) + (b^(1/3)*p*(d + e*x)^(2 + m)*Hypergeometr
ic2F1[1, 2 + m, 3 + m, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*
e)])/(e*(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)*(1 + m)*(2 + m)) + ((d + e*x)^(1
 + m)*Log[c*(a + b*x^3)^p])/(e*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
```

```
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int (d+ex)^m \log\left(c\left(a+bx^3\right)^p\right)\,dx = \frac{(d+ex)^{1+m}\log\left(c\left(a+bx^3\right)^p\right)}{e(1+m)} - \frac{(3bp)\int \frac{x^2(d+ex)^{1+m}}{a+bx^3}\,dx}{e(1+m)}$$

$$= \frac{(d+ex)^{1+m}\log\left(c\left(a+bx^3\right)^p\right)}{e(1+m)} - \frac{(3bp)\int \left(\frac{(d+ex)^{1+m}}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{(d}{3b^{2/3}\left(-\sqrt[3]{\ }\right.}\right.}{e}$$

$$= \frac{(d+ex)^{1+m}\log\left(c\left(a+bx^3\right)^p\right)}{e(1+m)} - \frac{\left(\sqrt[3]{b}\,p\right)\int \frac{(d+ex)^{1+m}}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{e(1+m)} - \frac{\left(\sqrt[3]{b}\,p\right)\int -}{e}$$

$$= \frac{\sqrt[3]{b}\,p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{\sqrt[3]{b}\,(d+ex)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)}{e\left(\sqrt[3]{b}\,d-\sqrt[3]{a}\,e\right)(1+m)(2+m)} + \frac{\sqrt[3]{b}\,p(d+ex)^{2+}}{e\left(\sqrt[3]{b}\right.}$$

Mathematica [A]
time = 0.44, size = 239, normalized size = 0.79

$$\frac{(d+ex)^{1+m}\left(-\frac{\sqrt[3]{b}\,p(d+ex)\left(-\frac{{}_2F_1\left(1,2+m;3+m;\frac{\sqrt[3]{b}\,(d+ex)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}-\frac{{}_2F_1\left(1,2+m;3+m;\frac{\sqrt[3]{b}\,(d+ex)}{\sqrt[3]{b}\,d+\sqrt[3]{-1}\,\sqrt[3]{a}\,e}\right)}{\sqrt[3]{b}\,d+\sqrt[3]{-1}\,\sqrt[3]{a}\,e}-\frac{{}_2F_1\left(1,2+m;3+m;\frac{\sqrt[3]{b}\,(d+ex)}{\sqrt[3]{b}\,d-(-1)^{2/3}\sqrt[3]{a}\,e}\right)}{\sqrt[3]{b}\,d-(-1)^{2/3}\sqrt[3]{a}\,e}\right)}{2+m}+\log\left(c\left(a+bx^3\right)^p\right)\right)}{e(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*Log[c*(a + b*x^3)^p],x]
```

```
[Out] ((d + e*x)^(1 + m)*(-((b^(1/3)*p*(d + e*x)*(-(Hypergeometric2F1[1, 2 + m, 3
 + m, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)]/(b^(1/3)*d - a^(1/3)*e))
```

```
 - Hypergeometric2F1[1, 2 + m, 3 + m, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)
^(1/3)*a^(1/3)*e)]/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e) - Hypergeometric2F1[1
, 2 + m, 3 + m, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)]/(b^
(1/3)*d - (-1)^(2/3)*a^(1/3)*e)))/(2 + m)) + Log[c*(a + b*x^3)^p]))/(e*(1 +
 m))
```

Maple [**F**]
time = 0.20, size = 0, normalized size = 0.00

$$\int (ex+d)^m \ln\left(c\left(x^3 b+a\right)^p\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*ln(c*(b*x^3+a)^p),x)
```

```
[Out] int((e*x+d)^m*ln(c*(b*x^3+a)^p),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] (x*e + d)*e^(m*log(x*e + d) - 1)*log((b*x^3 + a)^p)/(m + 1) + integrate(((
m + 1)*log(c) - 3*p)*b*x^3*e - 3*b*d*p*x^2 + a*(m + 1)*e*log(c))*(x*e + d)^
m/(b*(m + 1)*x^3*e + a*(m + 1)*e), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] integral((x*e + d)^m*log((b*x^3 + a)^p*c), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*ln(c*(b*x**3+a)**p),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^m*log((b*x^3 + a)^p*c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\left(b\,x^3+a\right)^p\right)\,\left(d+e\,x\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)*(d + e*x)^m,x)
```

```
[Out] int(log(c*(a + b*x^3)^p)*(d + e*x)^m, x)
```

3.207 $\int (d+ex)^m \log \left(c\left(a+bx^2\right)^p\right) \, dx$

Optimal. Leaf size=205

$$\frac{\sqrt{b}\,p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{\sqrt{b}\,(d+ex)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)}{e\left(\sqrt{b}\,d-\sqrt{-a}\,e\right)(1+m)(2+m)}+\frac{\sqrt{b}\,p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{\sqrt{b}\,(d+ex)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)}{e\left(\sqrt{b}\,d+\sqrt{-a}\,e\right)(1+m)(2+m)}$$

```
[Out] (e*x+d)^(1+m)*ln(c*(b*x^2+a)^p)/e/(1+m)+p*(e*x+d)^(2+m)*hypergeom([1, 2+m],
[3+m],(e*x+d)*b^(1/2)/(-e*(-a)^(1/2)+d*b^(1/2)))*b^(1/2)/e/(1+m)/(2+m)/(-e*
(-a)^(1/2)+d*b^(1/2))+p*(e*x+d)^(2+m)*hypergeom([1, 2+m],[3+m],(e*x+d)*b^(1
/2)/(e*(-a)^(1/2)+d*b^(1/2)))*b^(1/2)/e/(1+m)/(2+m)/(e*(-a)^(1/2)+d*b^(1/2)
)
```

Rubi [A]
time = 0.17, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2513, 845, 70}

$$\frac{(d+ex)^{m+1}\log\left(c\left(a+bx^2\right)^p\right)}{e(m+1)}+\frac{\sqrt{b}\,p(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{\sqrt{b}\,(d+ex)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)}{e(m+1)(m+2)\left(\sqrt{b}\,d-\sqrt{-a}\,e\right)}+\frac{\sqrt{b}\,p(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{\sqrt{b}\,(d+ex)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)}{e(m+1)(m+2)\left(\sqrt{-a}\,e+\sqrt{b}\,d\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (Sqrt[b]*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (Sqrt[b]*(d
 + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/(e*(Sqrt[b]*d - Sqrt[-a]*e)*(1 + m)*(2
+ m)) + (Sqrt[b]*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (Sq
rt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/(e*(Sqrt[b]*d + Sqrt[-a]*e)*(1
+ m)*(2 + m)) + ((d + e*x)^(1 + m)*Log[c*(a + b*x^2)^p])/(e*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\int (d+ex)^m \log\left(c\left(a+bx^2\right)^p\right)\,dx = \frac{(d+ex)^{1+m}\log\left(c\left(a+bx^2\right)^p\right)}{e(1+m)} - \frac{(2bp)\int \frac{x(d+ex)^{1+m}}{a+bx^2}\,dx}{e(1+m)}$$

$$= \frac{(d+ex)^{1+m}\log\left(c\left(a+bx^2\right)^p\right)}{e(1+m)} - \frac{(2bp)\int\left(-\frac{(d+ex)^{1+m}}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)}+\frac{\cdots}{2\sqrt{b}\cdots}\right.}{e(1+m)}$$

$$= \frac{(d+ex)^{1+m}\log\left(c\left(a+bx^2\right)^p\right)}{e(1+m)} + \frac{\left(\sqrt{b}\,p\right)\int\frac{(d+ex)^{1+m}}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{e(1+m)} - \frac{\left(\sqrt{b}\,p\right)\int\cdots}{e\cdots}$$

$$= \frac{\sqrt{b}\,p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{\sqrt{b}\,(d+ex)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)}{e\left(\sqrt{b}\,d-\sqrt{-a}\,e\right)(1+m)(2+m)} + \frac{\sqrt{b}\,p(d+ex)^{\cdots}}{e\left(\sqrt{\cdots}\right.}$$

Mathematica [A]
time = 0.17, size = 176, normalized size = 0.86

$$\frac{(d+ex)^{1+m}\left(\frac{\sqrt{b}\,p(d+ex)\left(\left(\sqrt{b}\,d+\sqrt{-a}\,e\right){}_2F_1\left(1,2+m;3+m;\frac{\sqrt{b}\,(d+ex)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)+\left(\sqrt{b}\,d-\sqrt{-a}\,e\right){}_2F_1\left(1,2+m;3+m;\frac{\sqrt{b}\,(d+ex)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\right)}{(bd^2+ae^2)(2+m)}+\log\left(c\left(a+bx^2\right)^p\right)\right)}{e(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*Log[c*(a + b*x^2)^p],x]
```

```
[Out] ((d + e*x)^(1 + m)*((Sqrt[b]*p*(d + e*x)*((Sqrt[b]*d + Sqrt[-a]*e)*Hypergeo
metric2F1[1, 2 + m, 3 + m, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)] +
(Sqrt[b]*d - Sqrt[-a]*e)*Hypergeometric2F1[1, 2 + m, 3 + m, (Sqrt[b]*(d + e
*x))/(Sqrt[b]*d + Sqrt[-a]*e)]))/((b*d^2 + a*e^2)*(2 + m)) + Log[c*(a + b*x
^2)^p]))/(e*(1 + m))
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int (ex+d)^m \ln\left(c\left(b\,x^2+a\right)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*ln(c*(b*x^2+a)^p),x)
```

```
[Out] int((e*x+d)^m*ln(c*(b*x^2+a)^p),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] (p*x*e + d*p)*e^(m*log(x*e + d) - 1)*log(b*x^2 + a)/(m + 1) + integrate(((
m + 1)*log(c) - 2*p)*b*x^2*e - 2*b*d*p*x + a*(m + 1)*e*log(c))*(x*e + d)^m/
(b*(m + 1)*x^2*e + a*(m + 1)*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] integral((x*e + d)^m*log((b*x^2 + a)^p*c), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*ln(c*(b*x**2+a)**p),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^m*log((b*x^2 + a)^p*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln \left(c\left(b\,x^2+a\right)^p\right)\,\left(d+e\,x\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)*(d + e*x)^m,x)
[Out] int(log(c*(a + b*x^2)^p)*(d + e*x)^m, x)
```

3.208 $\int (d+ex)^m \log \left(c(a+bx)^p\right) \, dx$

Optimal. Leaf size=89

$$\frac{bp(d+ex)^{2+m} {}_2F_1\left(1,2+m;3+m;\frac{b(d+ex)}{bd-ae}\right)}{e(bd-ae)(1+m)(2+m)}+\frac{(d+ex)^{1+m}\log\left(c(a+bx)^p\right)}{e(1+m)}$$

```
[Out] b*p*(e*x+d)^(2+m)*hypergeom([1, 2+m],[3+m],b*(e*x+d)/(-a*e+b*d))/e/(-a*e+b*
d)/(1+m)/(2+m)+(e*x+d)^(1+m)*ln(c*(b*x+a)^p)/e/(1+m)
```

Rubi [A]
time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2442, 70}

$$\frac{(d+ex)^{m+1}\log\left(c(a+bx)^p\right)}{e(m+1)}+\frac{bp(d+ex)^{m+2} {}_2F_1\left(1,m+2;m+3;\frac{b(d+ex)}{bd-ae}\right)}{e(m+1)(m+2)(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*Log[c*(a + b*x)^p],x]
```

```
[Out] (b*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (b*(d + e*x))/(b*
d - a*e)])/(e*(b*d - a*e)*(1 + m)*(2 + m)) + ((d + e*x)^(1 + m)*Log[c*(a +
b*x)^p])/(e*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int (d+ex)^m \log \left(c(a+bx)^p\right) \, dx = \frac{(d+ex)^{1+m} \log \left(c(a+bx)^p\right)}{e(1+m)} - \frac{(bp) \int \frac{(d+ex)^{1+m}}{a+bx} \, dx}{e(1+m)}$$
$$= \frac{bp(d+ex)^{2+m} \, {}_2F_1\left(1,2+m;3+m;\frac{b(d+ex)}{bd-ae}\right)}{e(bd-ae)(1+m)(2+m)} + \frac{(d+ex)^{1+m} \log \left(c(a+bx)^p\right)}{e(1+m)}$$

Mathematica [A]
time = 0.04, size = 77, normalized size = 0.87

$$\frac{(d+ex)^{1+m} \left(\frac{bp(d+ex) \, {}_2F_1\left(1,2+m;3+m;\frac{b(d+ex)}{bd-ae}\right)}{(bd-ae)(2+m)} + \log \left(c(a+bx)^p\right)\right)}{e(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*Log[c*(a + b*x)^p],x]
```

```
[Out] ((d + e*x)^(1 + m)*((b*p*(d + e*x)*Hypergeometric2F1[1, 2 + m, 3 + m, (b*(d
 + e*x))/(b*d - a*e)])/((b*d - a*e)*(2 + m)) + Log[c*(a + b*x)^p]))/(e*(1 +
 m))
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int (ex+d)^m \ln \left(c(bx+a)^p\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*ln(c*(b*x+a)^p),x)
```

```
[Out] int((e*x+d)^m*ln(c*(b*x+a)^p),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(b*x+a)^p),x, algorithm="maxima")
```

```
[Out] (x*e + d)*e^(m*log(x*e + d) - 1)*log((b*x + a)^p)/(m + 1) + integrate((((m
+ 1)*log(c) - p)*b*x*e + a*(m + 1)*e*log(c) - b*d*p)*(x*e + d)^m/(b*(m + 1)
*x*e + a*(m + 1)*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(b*x+a)^p),x, algorithm="fricas")
[Out] integral((x*e + d)^m*log((b*x + a)^p*c), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*ln(c*(b*x+a)**p),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(b*x+a)^p),x, algorithm="giac")
[Out] integrate((x*e + d)^m*log((b*x + a)^p*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\,{\left(a+b\,x\right)}^p\right)\,{\left(d+e\,x\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x)^p)*(d + e*x)^m,x)
[Out] int(log(c*(a + b*x)^p)*(d + e*x)^m, x)
```

3.209 $\int (d+ex)^m \log \left(c\left(a+\frac{b}{x}\right)^p\right) \, dx$

Optimal. Leaf size=135

$$\frac{ap(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{a(d+ex)}{ad-be}\right)}{e(ad-be)(1+m)(2+m)}-\frac{p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;1+\frac{ex}{d}\right)}{de\left(2+3m+m^2\right)}+\frac{(d+ex)^{1+m}\log\left(\right.}{e(1+m}$$

```
[Out] a*p*(e*x+d)^(2+m)*hypergeom([1, 2+m],[3+m],a*(e*x+d)/(a*d-b*e))/e/(a*d-b*e)
/(1+m)/(2+m)-p*(e*x+d)^(2+m)*hypergeom([1, 2+m],[3+m],1+e*x/d)/d/e/(m^2+3*m
+2)+(e*x+d)^(1+m)*ln(c*(a+b/x)^p)/e/(1+m)
```

Rubi [A]
time = 0.07, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2513, 528, 88, 67, 70}

$$\frac{(d+ex)^{m+1}\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(m+1)}+\frac{ap(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{a(d+ex)}{ad-be}\right)}{e(m+1)(m+2)(ad-be)}-\frac{p(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{ex}{d}+1\right)}{de\left(m^2+3m+2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*Log[c*(a + b/x)^p],x]
```

```
[Out] (a*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (a*(d + e*x))/(a*
d - b*e)])/(e*(a*d - b*e)*(1 + m)*(2 + m)) - (p*(d + e*x)^(2 + m)*Hypergeom
etric2F1[1, 2 + m, 3 + m, 1 + (e*x)/d])/(d*e*(2 + 3*m + m^2)) + ((d + e*x)^
(1 + m)*Log[c*(a + b/x)^p])/(e*(1 + m))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
p}, x] && !IntegerQ[p]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] || !I
ntegerQ[p])
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\int (d+ex)^m \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx = \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(1+m)} + \frac{(bp)\int \frac{(d+ex)^{1+m}}{\left(a+\frac{b}{x}\right)x^2}\,dx}{e(1+m)}$$

$$= \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(1+m)} + \frac{(bp)\int \frac{(d+ex)^{1+m}}{x(b+ax)}\,dx}{e(1+m)}$$

$$= \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(1+m)} + \frac{p\int \frac{(d+ex)^{1+m}}{x}\,dx}{e(1+m)} - \frac{(ap)\int \frac{(d+ex)^{1+m}}{b+ax}\,dx}{e(1+m)}$$

$$= \frac{ap(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{a(d+ex)}{ad-be}\right)}{e(ad-be)(1+m)(2+m)} - \frac{p(d+ex)^{2+m}\,{}_2F_1(1,2}{de\,(2+3}$$

Mathematica [A]
time = 0.06, size = 123, normalized size = 0.91

$$\frac{(d+ex)^{1+m}\left(-adp(d+ex)\,{}_2F_1\left(1,2+m;3+m;\frac{a(d+ex)}{ad-be}\right)+(ad-be)\left(p(d+ex)\,{}_2F_1\left(1,2+m;3+m;1+\frac{ex}{d}\right)-d(2+m)\log\left(c\left(a+\frac{b}{x}\right)^p\right)\right)\right)}{de(-ad+be)(1+m)(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*Log[c*(a + b/x)^p],x]
[Out] ((d + e*x)^(1 + m)*(-(a*d*p*(d + e*x)*Hypergeometric2F1[1, 2 + m, 3 + m, (a
*(d + e*x))/(a*d - b*e)]) + (a*d - b*e)*(p*(d + e*x)*Hypergeometric2F1[1, 2
```

```
 + m, 3 + m, 1 + (e*x)/d] - d*(2 + m)*Log[c*(a + b/x)^p])))/(d*e*(-(a*d) +
b*e)*(1 + m)*(2 + m))
```

Maple [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int (ex+d)^m \ln \left(c\left(a+\frac{b}{x}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*ln(c*(a+b/x)^p),x)
```

```
[Out] int((e*x+d)^m*ln(c*(a+b/x)^p),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(a+b/x)^p),x, algorithm="maxima")
```

```
[Out] ((x*e + d)*(x*e + d)^m*log((a*x + b)^p) - (x*e + d)*(x*e + d)^m*log(x^p))*e
^(-1)/(m + 1) + integrate((a*(m + 1)*x^2*e*log(c) + ((m + 1)*log(c) + p)*b*
x*e + b*d*p)*(x*e + d)^m/(a*(m + 1)*x^2*e + b*(m + 1)*x*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(a+b/x)^p),x, algorithm="fricas")
```

```
[Out] integral((x*e + d)^m*log(c*((a*x + b)/x)^p), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*ln(c*(a+b/x)**p),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(a+b/x)^p),x, algorithm="giac")
[Out] integrate((x*e + d)^m*log((a + b/x)^p*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\,\left(a+\frac{b}{x}\right)^p\right)\,{\left(d+e\,x\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)*(d + e*x)^m,x)
[Out] int(log(c*(a + b/x)^p)*(d + e*x)^m, x)
```

3.210 $\int (d+ex)^m \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)\, dx$

Optimal. Leaf size=257

$$\frac{\sqrt{-a}\, p(d+ex)^{2+m}\, {}_2F_1\left(1,2+m;3+m;\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d-\sqrt{b}\,e}\right)}{e\left(\sqrt{-a}\,d-\sqrt{b}\,e\right)(1+m)(2+m)}+\frac{\sqrt{-a}\, p(d+ex)^{2+m}\, {}_2F_1\left(1,2+m;3+m;\frac{\sqrt{-a}\,(}{\sqrt{-a}\,d-}\right.}{e\left(\sqrt{-a}\,d+\sqrt{b}\,e\right)(1+m)(2+m)}$$

```
[Out] -2*p*(e*x+d)^(2+m)*hypergeom([1, 2+m],[3+m],1+e*x/d)/d/e/(m^2+3*m+2)+(e*x+d
)^(1+m)*ln(c*(a+b/x^2)^p)/e/(1+m)+p*(e*x+d)^(2+m)*hypergeom([1, 2+m],[3+m],
(e*x+d)*(-a)^(1/2)/(d*(-a)^(1/2)-e*b^(1/2)))*(-a)^(1/2)/e/(1+m)/(2+m)/(d*(-
a)^(1/2)-e*b^(1/2))+p*(e*x+d)^(2+m)*hypergeom([1, 2+m],[3+m],(e*x+d)*(-a)^(
1/2)/(d*(-a)^(1/2)+e*b^(1/2)))*(-a)^(1/2)/e/(1+m)/(2+m)/(d*(-a)^(1/2)+e*b^(
1/2))
```

Rubi [A]
time = 0.36, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2513, 1584, 975, 67, 845, 70}

$$\frac{(d+ex)^{m+1}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(m+1)}+\frac{\sqrt{-a}\,p(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d-\sqrt{b}\,e}\right)}{e(m+1)(m+2)\left(\sqrt{-a}\,d-\sqrt{b}\,e\right)}+\frac{\sqrt{-a}\,p(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right)}{e(m+1)(m+2)\left(\sqrt{-a}\,d+\sqrt{b}\,e\right)}-\frac{2p(d+ex)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{ex}{d}+1\right)}{de\left(m^2+3m+2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*Log[c*(a + b/x^2)^p],x]
```

```
[Out] (Sqrt[-a]*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (Sqrt[-a]*
(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/(e*(Sqrt[-a]*d - Sqrt[b]*e)*(1 + m)*(
2 + m)) + (Sqrt[-a]*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m,
(Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)])/(e*(Sqrt[-a]*d + Sqrt[b]*e)
*(1 + m)*(2 + m)) - (2*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 +
m, 1 + (e*x)/d])/(d*e*(2 + 3*m + m^2)) + ((d + e*x)^(1 + m)*Log[c*(a + b/x^
2)^p])/(e*(1 + m))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
```

```
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 1584

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))
^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x]
/; FreeQ[{a, c, d, e, m, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\int (d+ex)^m \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx = \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(1+m)} + \frac{(2bp)\int\frac{(d+ex)^{1+m}}{\left(a+\frac{b}{x^2}\right)x^3}\,dx}{e(1+m)}$$

$$= \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(1+m)} + \frac{(2bp)\int\frac{(d+ex)^{1+m}}{x(b+ax^2)}\,dx}{e(1+m)}$$

$$= \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(1+m)} + \frac{(2bp)\int\left(\frac{(d+ex)^{1+m}}{bx} - \frac{ax(d+ex)^{1+m}}{b(b+ax^2)}\right)dx}{e(1+m)}$$

$$= \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(1+m)} + \frac{(2p)\int\frac{(d+ex)^{1+m}}{x}\,dx}{e(1+m)} - \frac{(2ap)\int\frac{x(d+ex)^{1\text{-}}}{b+ax^2}}{e(1+m)}$$

$$= -\frac{2p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;1+\frac{ex}{d}\right)}{de\,(2+3m+m^2)} + \frac{(d+ex)^{1+m}\log\left(c(a\right.}{e(1+m)}$$

$$= -\frac{2p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;1+\frac{ex}{d}\right)}{de\,(2+3m+m^2)} + \frac{(d+ex)^{1+m}\log\left(c(a\right.}{e(1+m)}$$

$$= \frac{\sqrt{-a}\,p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d-\sqrt{b}\,e}\right)}{e\left(\sqrt{-a}\,d-\sqrt{b}\,e\right)(1+m)(2+m)} + \frac{\sqrt{-a}\,p(d}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.70, size = 310, normalized size = 1.21

$$\frac{(d+ex)^m\left(2dp\left(1+\frac{d}{ex}\right)^{-m}{}_2F_1\left(-m,-m;1-m;-\frac{d}{ex}\right) - \frac{\left(\sqrt{a}\,d+i\sqrt{b}\,e\right)p\left(\frac{\sqrt{a}\,(d+ex)}{e\left(-i\sqrt{b}+\sqrt{a}\,x\right)}\right)^{-m}{}_2F_1\left(-m,-m;1-m;\frac{\sqrt{a}\,d+i\sqrt{b}\,e}{i\sqrt{b}\,e-\sqrt{a}\,ex}\right)}{\sqrt{a}} - \frac{\left(\sqrt{a}\,d-i\sqrt{b}\,e\right)p\left(\frac{\sqrt{a}\,(d+ex)}{e\left(i\sqrt{b}+\sqrt{a}\,x\right)}\right)^{-m}{}_2F_1\left(-m,-m;1-m;-\frac{\sqrt{a}\,d-i\sqrt{b}\,e}{i\sqrt{b}\,e+\sqrt{a}\,ex}\right)}{\sqrt{a}} + m(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\right)}{em(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*Log[c*(a + b/x^2)^p],x]
```

```
[Out] ((d + e*x)^m*((2*d*p*Hypergeometric2F1[-m, -m, 1 - m, -(d/(e*x))])/(1 + d/(
e*x))^m - ((Sqrt[a]*d + I*Sqrt[b]*e)*p*Hypergeometric2F1[-m, -m, 1 - m, (Sq
rt[a]*d + I*Sqrt[b]*e)/(I*Sqrt[b]*e - Sqrt[a]*e*x)])/(Sqrt[a]*((Sqrt[a]*(d
+ e*x))/(e*((-I)*Sqrt[b] + Sqrt[a]*x)))^m) - ((Sqrt[a]*d - I*Sqrt[b]*e)*p*H
ypergeometric2F1[-m, -m, 1 - m, -((Sqrt[a]*d - I*Sqrt[b]*e)/(I*Sqrt[b]*e +
Sqrt[a]*e*x))])/(Sqrt[a]*((Sqrt[a]*(d + e*x))/(e*(I*Sqrt[b] + Sqrt[a]*x)))^
m) + m*(d + e*x)*Log[c*(a + b/x^2)^p]))/(e*m*(1 + m))
```

Maple [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int (ex+d)^m \ln \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*ln(c*(a+b/x^2)^p),x)
```

```
[Out] int((e*x+d)^m*ln(c*(a+b/x^2)^p),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(a+b/x^2)^p),x, algorithm="maxima")
```

```
[Out] (p*x*e + d*p)*e^(m*log(x*e + d) - 1)*log(a*x^2 + b)/(m + 1) - integrate(-((
(m + 1)*log(c) - 2*p)*a*x^2*e - 2*a*d*p*x + b*(m + 1)*e*log(c) - 2*(a*(m +
1)*x^2*e + b*(m + 1)*e)*log(x^p))*(x*e + d)^m/(a*(m + 1)*x^2*e + b*(m + 1)*
e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(a+b/x^2)^p),x, algorithm="fricas")
```

```
[Out] integral((x*e + d)^m*log(c*((a*x^2 + b)/x^2)^p), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*ln(c*(a+b/x**2)**p),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*log(c*(a+b/x^2)^p),x, algorithm="giac")
[Out] integrate((x*e + d)^m*log((a + b/x^2)^p*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,{\left(d+e\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x^2)^p)*(d + e*x)^m,x)
[Out] int(log(c*(a + b/x^2)^p)*(d + e*x)^m, x)
```

3.211 $\int (f+gx)^m \log\left(c(d+ex^n)^p\right) dx$

Optimal. Leaf size=23

$$\text{Int}\left((f+gx)^m \log\left(c(d+ex^n)^p\right), x\right)$$

```
[Out] Unintegrable((g*x+f)^m*ln(c*(d+e*x^n)^p),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (f+gx)^m \log\left(c(d+ex^n)^p\right) dx$$

Verification is not applicable to the result.

```
[In] Int[(f + g*x)^m*Log[c*(d + e*x^n)^p],x]
```

```
[Out] Defer[Int][(f + g*x)^m*Log[c*(d + e*x^n)^p], x]
```

Rubi steps

$$\int (f+gx)^m \log\left(c(d+ex^n)^p\right) dx = \int (f+gx)^m \log\left(c(d+ex^n)^p\right) dx$$

Mathematica [A]
time = 0.36, size = 0, normalized size = 0.00

$$\int (f+gx)^m \log\left(c(d+ex^n)^p\right) dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f + g*x)^m*Log[c*(d + e*x^n)^p],x]
```

```
[Out] Integrate[(f + g*x)^m*Log[c*(d + e*x^n)^p], x]
```

Maple [A]
time = 0.26, size = 0, normalized size = 0.00

$$\int (gx+f)^m \ln\left(c(d+e\,x^n)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^m*ln(c*(d+e*x^n)^p),x)
```

```
[Out] int((g*x+f)^m*ln(c*(d+e*x^n)^p),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^m*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] (g*x + f)*(g*x + f)^m*log((d + e^(n*log(x) + 1))^p)/(g*(m + 1)) + integrate
((d*g*(m + 1)*x*log(c) - (f*n*p*e + (g*n*p - g*(m + 1)*log(c))*x*e)*x^n)*(g
*x + f)^m/(d*g*(m + 1)*x + g*(m + 1)*x*e^(n*log(x) + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^m*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] integral((g*x + f)^m*log((x^n*e + d)^p*c), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**m*ln(c*(d+e*x**n)**p),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^m*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Simplification assuming sageVARf near 0Unable to divide,
```

```
perhaps due to rounding error%%%{-1,[0,0,6,3,6,0,2,2,0,1,0]%%%}+%%%{1,[0,0,
6,2,6,1,2
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \ln \left(c\left(d+e\,x^n\right)^p\right)\,\left(f+g\,x\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)*(f + g*x)^m,x)
[Out] int(log(c*(d + e*x^n)^p)*(f + g*x)^m, x)
```

3.212 $\int (f+gx)^3 \log \left(c(d+ex^n)^p\right) \, dx$

Optimal. Leaf size=234

$$-\frac{ef^3npx^{1+n}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1+n)}-\frac{3ef^2gnpx^{2+n}\,{}_2F_1\left(1,\frac{2+n}{n};2\left(1+\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{2d(2+n)}-\frac{efg^2npx^{3+n}\,{}_2F_1\left(1,\frac{3+n}{n}\right.}{d(3+n)}$$

```
[Out] -e*f^3*n*p*x^(1+n)*hypergeom([1, 1+1/n],[2+1/n],-e*x^n/d)/d/(1+n)-3/2*e*f^2
*g*n*p*x^(2+n)*hypergeom([1, (2+n)/n],[2+2/n],-e*x^n/d)/d/(2+n)-e*f*g^2*n*p
*x^(3+n)*hypergeom([1, (3+n)/n],[2+3/n],-e*x^n/d)/d/(3+n)-1/4*e*g^3*n*p*x^(
4+n)*hypergeom([1, (4+n)/n],[2+4/n],-e*x^n/d)/d/(4+n)-1/4*f^4*p*ln(d+e*x^n)
/g+1/4*(g*x+f)^4*ln(c*(d+e*x^n)^p)/g
```

Rubi [A]

time = 0.17, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2513, 1858, 266, 371}

$$\frac{(f+gx)^4\log\left(c(d+ex^n)^p\right)}{4g}-\frac{f^4p\log\left(d+ex^n\right)}{4g}-\frac{ef^3npx^{n+1}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(n+1)}-\frac{3ef^2gnpx^{n+2}\,{}_2F_1\left(1,\frac{n+2}{n};2\left(1+\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{2d(n+2)}-\frac{efg^2npx^{n+3}\,{}_2F_1\left(1,\frac{n+3}{n};2+\frac{3}{n};-\frac{ex^n}{d}\right)}{d(n+3)}-\frac{eg^3npx^{n+4}\,{}_2F_1\left(1,\frac{n+4}{n};2\left(1+\frac{2}{n}\right);-\frac{ex^n}{d}\right)}{4d(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -((e*f^3*n*p*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^
n)/d)])/(d*(1 + n))) - (3*e*f^2*g*n*p*x^(2 + n)*Hypergeometric2F1[1, (2 + n
)/n, 2*(1 + n^(-1)), -((e*x^n)/d)])/(2*d*(2 + n)) - (e*f*g^2*n*p*x^(3 + n)*
Hypergeometric2F1[1, (3 + n)/n, 2 + 3/n, -((e*x^n)/d)])/(d*(3 + n)) - (e*g^
3*n*p*x^(4 + n)*Hypergeometric2F1[1, (4 + n)/n, 2*(1 + 2/n), -((e*x^n)/d)])
/(4*d*(4 + n)) - (f^4*p*Log[d + e*x^n])/(4*g) + ((f + g*x)^4*Log[c*(d + e*x
^n)^p])/(4*g)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1858

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
```

```
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\begin{aligned}
\int (f+gx)^3\log\left(c(d+ex^n)^p\right)\,dx &= \frac{(f+gx)^4\log\left(c(d+ex^n)^p\right)}{4g}-\frac{(enp)\int\frac{x^{-1+n}(f+gx)^4}{d+ex^n}\,dx}{4g}\\
&= \frac{(f+gx)^4\log\left(c(d+ex^n)^p\right)}{4g}-\frac{(enp)\int\left(\frac{f^4x^{-1+n}}{d+ex^n}+\frac{4f^3gx^n}{d+ex^n}+\frac{6f^2g^2x^{1+n}}{d+ex^n}+4\right.}{4g}\\
&= \frac{(f+gx)^4\log\left(c(d+ex^n)^p\right)}{4g}-\left(ef^3np\right)\int\frac{x^n}{d+ex^n}\,dx-\frac{\left(ef^4np\right)\int\frac{x^{-1+n}}{d+ex^n}}{4g}\\
&= -\frac{ef^3npx^{1+n}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1+n)}-\frac{3ef^2gnpx^{2+n}\,{}_2F_1\left(1,\frac{2+n}{n};2(1\right.}{2d(2+n)}
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 245, normalized size = 1.05

$$\frac{1}{48}x\left(-48f^3np-36f^2gnpx-16fg^2npx^2-3g^3npx^3+48f^3np\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right)+36f^2gnpx\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{ex^n}{d}\right)+16fg^2npx^2\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{ex^n}{d}\right)+3g^3npx^3\,{}_2F_1\left(1,\frac{4}{n};\frac{4+n}{n};-\frac{ex^n}{d}\right)+48f^3\log\left(c(d+ex^n)^p\right)+72f^2gx\log\left(c(d+ex^n)^p\right)+48fg^2x^2\log\left(c(d+ex^n)^p\right)+12g^3x^3\log\left(c(d+ex^n)^p\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3*Log[c*(d + e*x^n)^p],x]
```

```
[Out] (x*(-48*f^3*n*p - 36*f^2*g*n*p*x - 16*f*g^2*n*p*x^2 - 3*g^3*n*p*x^3 + 48*f^
3*n*p*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)] + 36*f^2*g*n*p
*x*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((e*x^n)/d)] + 16*f*g^2*n*p*x^2*Hy
pergeometric2F1[1, 3/n, (3 + n)/n, -((e*x^n)/d)] + 3*g^3*n*p*x^3*Hypergeome
tric2F1[1, 4/n, (4 + n)/n, -((e*x^n)/d)] + 48*f^3*Log[c*(d + e*x^n)^p] + 72
*f^2*g*x*Log[c*(d + e*x^n)^p] + 48*f*g^2*x^2*Log[c*(d + e*x^n)^p] + 12*g^3*
x^3*Log[c*(d + e*x^n)^p]))/48
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int (gx+f)^3\ln\left(c(d+e\,x^n)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*ln(c*(d+e*x^n)^p),x)
```

```
[Out] int((g*x+f)^3*ln(c*(d+e*x^n)^p),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] -1/16*(g^3*n*p - 4*g^3*log(c))*x^4 - 1/3*(f*g^2*n*p - 3*f*g^2*log(c))*x^3 -
 3/4*(f^2*g*n*p - 2*f^2*g*log(c))*x^2 - (f^3*n*p - f^3*log(c))*x + 1/4*(g^3
*x^4 + 4*f*g^2*x^3 + 6*f^2*g*x^2 + 4*f^3*x)*log((d + e^(n*log(x) + 1))^p) +
 integrate(1/4*(d*g^3*n*p*x^3 + 4*d*f*g^2*n*p*x^2 + 6*d*f^2*g*n*p*x + 4*d*f
^3*n*p)/(d + e^(n*log(x) + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] integral((g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3)*log((x^n*e + d)^p*c), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 18.16, size = 415, normalized size = 1.77

$$f^3 x \log\left(c(d+ex^n)^p\right) + \frac{f^3 p x \Phi\left(\frac{dx^{-n}e^{i\pi}}{e}, 1, \frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{3f^2 g x^2 \log\left(c(d+ex^n)^p\right)}{2} + f g^2 x^3 \log\left(c(d+ex^n)^p\right) + \frac{g^3 x^4 \log\left(c(d+ex^n)^p\right)}{4} - \frac{3ef^2 g p x^2 x^n \Phi\left(\frac{ex^n e^{i\pi}}{d}, 1, 1+\frac{2}{n}\right)\Gamma\left(1+\frac{2}{n}\right)}{2d\Gamma\left(2+\frac{2}{n}\right)} - \frac{3ef^2 g p x^2 x^n \Phi\left(\frac{ex^n e^{i\pi}}{d}, 1, 1+\frac{2}{n}\right)\Gamma\left(1+\frac{2}{n}\right)}{dn\Gamma\left(2+\frac{2}{n}\right)} - \frac{efg^2 p x^3 x^n \Phi\left(\frac{ex^n e^{i\pi}}{d}, 1, 1+\frac{3}{n}\right)\Gamma\left(1+\frac{3}{n}\right)}{d\Gamma\left(2+\frac{3}{n}\right)} - \frac{3efg^2 p x^3 x^n \Phi\left(\frac{ex^n e^{i\pi}}{d}, 1, 1+\frac{3}{n}\right)\Gamma\left(1+\frac{3}{n}\right)}{dn\Gamma\left(2+\frac{3}{n}\right)} - \frac{eg^3 p x^4 x^n \Phi\left(\frac{ex^n e^{i\pi}}{d}, 1, 1+\frac{4}{n}\right)\Gamma\left(1+\frac{4}{n}\right)}{4d\Gamma\left(2+\frac{4}{n}\right)} - \frac{eg^3 p x^4 x^n \Phi\left(\frac{ex^n e^{i\pi}}{d}, 1, 1+\frac{4}{n}\right)\Gamma\left(1+\frac{4}{n}\right)}{dn\Gamma\left(2+\frac{4}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*ln(c*(d+e*x**n)**p),x)
```

```
[Out] f**3*x*log(c*(d + e*x**n)**p) + f**3*p*x*lerchphi(d*exp_polar(I*pi)/(e*x**n
), 1, exp_polar(I*pi)/n)*gamma(1/n)/(n*gamma(1 + 1/n)) + 3*f**2*g*x**2*log(
c*(d + e*x**n)**p)/2 + f*g**2*x**3*log(c*(d + e*x**n)**p) + g**3*x**4*log(c
*(d + e*x**n)**p)/4 - 3*e*f**2*g*p*x**2*x**n*lerchphi(e*x**n*exp_polar(I*pi
)/d, 1, 1 + 2/n)*gamma(1 + 2/n)/(2*d*gamma(2 + 2/n)) - 3*e*f**2*g*p*x**2*x*
*n*lerchphi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 2/n)*gamma(1 + 2/n)/(d*n*gamma
(2 + 2/n)) - e*f*g**2*p*x**3*x**n*lerchphi(e*x**n*exp_polar(I*pi)/d, 1, 1 +
 3/n)*gamma(1 + 3/n)/(d*gamma(2 + 3/n)) - 3*e*f*g**2*p*x**3*x**n*lerchphi(e
*x**n*exp_polar(I*pi)/d, 1, 1 + 3/n)*gamma(1 + 3/n)/(d*n*gamma(2 + 3/n)) -
```

```
e*g**3*p*x**4*x**n*lerchphi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 4/n)*gamma(1 +
 4/n)/(4*d*gamma(2 + 4/n)) - e*g**3*p*x**4*x**n*lerchphi(e*x**n*exp_polar(I
*pi)/d, 1, 1 + 4/n)*gamma(1 + 4/n)/(d*n*gamma(2 + 4/n))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*log(c*(d+e*x^n)^p),x, algorithm="giac")
[Out] integrate((g*x + f)^3*log((x^n*e + d)^p*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\,{\left(d+e\,x^n\right)}^p\right)\,{\left(f+g\,x\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)*(f + g*x)^3,x)
[Out] int(log(c*(d + e*x^n)^p)*(f + g*x)^3, x)
```

3.213 $\int (f+gx)^2 \log \left(c(d+ex^n)^p\right) \, dx$

Optimal. Leaf size=181

$$-\frac{ef^2npx^{1+n}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1+n)}-\frac{efgnpx^{2+n}\,{}_2F_1\left(1,\frac{2+n}{n};2\left(1+\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{d(2+n)}-\frac{eg^2npx^{3+n}\,{}_2F_1\left(1,\frac{3+n}{n};2-\right.}{3d(3+n)}$$

```
[Out] -e*f^2*n*p*x^(1+n)*hypergeom([1, 1+1/n],[2+1/n],-e*x^n/d)/d/(1+n)-e*f*g*n*p
*x^(2+n)*hypergeom([1, (2+n)/n],[2+2/n],-e*x^n/d)/d/(2+n)-1/3*e*g^2*n*p*x^(
3+n)*hypergeom([1, (3+n)/n],[2+3/n],-e*x^n/d)/d/(3+n)-1/3*f^3*p*ln(d+e*x^n)
/g+1/3*(g*x+f)^3*ln(c*(d+e*x^n)^p)/g
```

Rubi [A]
time = 0.12, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2513, 1858, 266, 371}

$$\frac{(f+gx)^3\log\left(c(d+ex^n)^p\right)}{3g}-\frac{f^3p\log\left(d+ex^n\right)}{3g}-\frac{ef^2npx^{n+1}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(n+1)}-\frac{efgnpx^{n+2}\,{}_2F_1\left(1,\frac{n+2}{n};2\left(1+\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{d(n+2)}-\frac{eg^2npx^{n+3}\,{}_2F_1\left(1,\frac{n+3}{n};2+\frac{3}{n};-\frac{ex^n}{d}\right)}{3d(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -((e*f^2*n*p*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^
n)/d)])/(d*(1 + n))) - (e*f*g*n*p*x^(2 + n)*Hypergeometric2F1[1, (2 + n)/n,
 2*(1 + n^(-1)), -((e*x^n)/d)])/(d*(2 + n)) - (e*g^2*n*p*x^(3 + n)*Hypergeo
metric2F1[1, (3 + n)/n, 2 + 3/n, -((e*x^n)/d)])/(3*d*(3 + n)) - (f^3*p*Log[
d + e*x^n])/(3*g) + ((f + g*x)^3*Log[c*(d + e*x^n)^p])/(3*g)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1858

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\int (f+gx)^2 \log \left(c(d+ex^n)^p\right) \, dx = \frac{(f+gx)^3 \log \left(c(d+ex^n)^p\right)}{3g} - \frac{(enp)\int \frac{x^{-1+n}(f+gx)^3}{d+ex^n} \, dx}{3g}$$

$$= \frac{(f+gx)^3 \log \left(c(d+ex^n)^p\right)}{3g} - \frac{(enp)\int \left(\frac{f^3x^{-1+n}}{d+ex^n} + \frac{3f^2gx^n}{d+ex^n} + \frac{3fg^2x^{1+n}}{d+ex^n} + \frac{g^3 \cdots}{d \cdots}\right.}{3g}$$

$$= \frac{(f+gx)^3 \log \left(c(d+ex^n)^p\right)}{3g} - (ef^2np)\int \frac{x^n}{d+ex^n} \, dx - \frac{(ef^3np)\int \frac{x^{-1+n}}{d+ex^n}}{3g}$$

$$= -\frac{ef^2npx^{1+n}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1+n)} - \frac{efgnpx^{2+n}\,{}_2F_1\left(1,\frac{2+n}{n};2(1+\cdots\right.}{d(2+n)}$$

Mathematica [A]
time = 0.07, size = 173, normalized size = 0.96

$$\frac{1}{18}x\left(-18f^2np - 9fgnpx - 2g^2npx^2 + 18f^2np\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{ex^n}{d}\right) + 9fgnpx\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{ex^n}{d}\right) + 2g^2npx^2\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{ex^n}{d}\right) + 18f^2\log\left(c(d+ex^n)^p\right) + 18fgx\log\left(c(d+ex^n)^p\right) + 6g^2x^2\log\left(c(d+ex^n)^p\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2*Log[c*(d + e*x^n)^p],x]
```

```
[Out] (x*(-18*f^2*n*p - 9*f*g*n*p*x - 2*g^2*n*p*x^2 + 18*f^2*n*p*Hypergeometric2F
1[1, n^(-1), 1 + n^(-1), -((e*x^n)/d)] + 9*f*g*n*p*x*Hypergeometric2F1[1, 2
/n, (2 + n)/n, -((e*x^n)/d)] + 2*g^2*n*p*x^2*Hypergeometric2F1[1, 3/n, (3 +
 n)/n, -((e*x^n)/d)] + 18*f^2*Log[c*(d + e*x^n)^p] + 18*f*g*x*Log[c*(d + e*
x^n)^p] + 6*g^2*x^2*Log[c*(d + e*x^n)^p]))/18
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int (gx+f)^2 \ln \left(c(d+e\,x^n)^p\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*ln(c*(d+e*x^n)^p),x)
```

```
[Out] int((g*x+f)^2*ln(c*(d+e*x^n)^p),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] -1/9*(g^2*n*p - 3*g^2*log(c))*x^3 - 1/2*(f*g*n*p - 2*f*g*log(c))*x^2 - (f^2
*n*p - f^2*log(c))*x + 1/3*(g^2*x^3 + 3*f*g*x^2 + 3*f^2*x)*log((d + e^(n*lo
g(x) + 1))^p) + integrate(1/3*(d*g^2*n*p*x^2 + 3*d*f*g*n*p*x + 3*d*f^2*n*p)
/(d + e^(n*log(x) + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] integral((g^2*x^2 + 2*f*g*x + f^2)*log((x^n*e + d)^p*c), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 10.69, size = 284, normalized size = 1.57

$$f^2x\log\left(c(d+ex^n)^p\right)+\frac{f^2px\Phi\left(\frac{dx^{-n}e^{i\pi}}{e},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n\Gamma\left(1+\frac{1}{n}\right)}+fgx^2\log\left(c(d+ex^n)^p\right)+\frac{g^2x^3\log\left(c(d+ex^n)^p\right)}{3}-\frac{efgpx^2x^n\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{2}{n}\right)\Gamma\left(1+\frac{2}{n}\right)}{d\Gamma\left(2+\frac{2}{n}\right)}-\frac{2efgpx^2x^n\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{2}{n}\right)\Gamma\left(1+\frac{2}{n}\right)}{dn\Gamma\left(2+\frac{2}{n}\right)}-\frac{eg^2px^3x^n\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{3}{n}\right)\Gamma\left(1+\frac{3}{n}\right)}{3d\Gamma\left(2+\frac{3}{n}\right)}-\frac{eg^2px^3x^n\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{3}{n}\right)\Gamma\left(1+\frac{3}{n}\right)}{dn\Gamma\left(2+\frac{3}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*ln(c*(d+e*x**n)**p),x)
```

```
[Out] f**2*x*log(c*(d + e*x**n)**p) + f**2*p*x*lerchphi(d*exp_polar(I*pi)/(e*x**n
), 1, exp_polar(I*pi)/n)*gamma(1/n)/(n*gamma(1 + 1/n)) + f*g*x**2*log(c*(d
+ e*x**n)**p) + g**2*x**3*log(c*(d + e*x**n)**p)/3 - e*f*g*p*x**2*x**n*lerc
hphi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 2/n)*gamma(1 + 2/n)/(d*gamma(2 + 2/n)
) - 2*e*f*g*p*x**2*x**n*lerchphi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 2/n)*gamm
a(1 + 2/n)/(d*n*gamma(2 + 2/n)) - e*g**2*p*x**3*x**n*lerchphi(e*x**n*exp_po
lar(I*pi)/d, 1, 1 + 3/n)*gamma(1 + 3/n)/(3*d*gamma(2 + 3/n)) - e*g**2*p*x**
3*x**n*lerchphi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 3/n)*gamma(1 + 3/n)/(d*n*g
amma(2 + 3/n))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*log(c*(d+e*x^n)^p),x, algorithm="giac")
[Out] integrate((g*x + f)^2*log((x^n*e + d)^p*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\left(d+e\,x^n\right)^p\right)\,\left(f+g\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)*(f + g*x)^2,x)
[Out] int(log(c*(d + e*x^n)^p)*(f + g*x)^2, x)
```

3.214 $\int (f+gx)\log\left(c(d+ex^n)^p\right)\,dx$

Optimal. Leaf size=132

$$-\frac{efnpx^{1+n}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1+n)}-\frac{egnpx^{2+n}\,{}_2F_1\left(1,\frac{2+n}{n};2\left(1+\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{2d(2+n)}-\frac{f^2p\log\left(d+ex^n\right)}{2g}+\frac{(f+gx)}{}$$

```
[Out] -e*f*n*p*x^(1+n)*hypergeom([1, 1+1/n],[2+1/n],-e*x^n/d)/d/(1+n)-1/2*e*g*n*p
*x^(2+n)*hypergeom([1, (2+n)/n],[2+2/n],-e*x^n/d)/d/(2+n)-1/2*f^2*p*ln(d+e*
x^n)/g+1/2*(g*x+f)^2*ln(c*(d+e*x^n)^p)/g
```

Rubi [A]
time = 0.09, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2513, 1858, 266, 371}

$$\frac{(f+gx)^2\log\left(c(d+ex^n)^p\right)}{2g}-\frac{f^2p\log\left(d+ex^n\right)}{2g}-\frac{efnpx^{n+1}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(n+1)}-\frac{egnpx^{n+2}\,{}_2F_1\left(1,\frac{n+2}{n};2\left(1+\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{2d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -((e*f*n*p*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^n)
/d)])/(d*(1 + n))) - (e*g*n*p*x^(2 + n)*Hypergeometric2F1[1, (2 + n)/n, 2*(
1 + n^(-1)), -((e*x^n)/d)])/(2*d*(2 + n)) - (f^2*p*Log[d + e*x^n])/(2*g) +
((f + g*x)^2*Log[c*(d + e*x^n)^p])/(2*g)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1858

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) && !IGtQ[m, 0]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\begin{aligned}\int (f+gx)\log\left(c(d+ex^n)^p\right)\,dx &= \frac{(f+gx)^2\log\left(c(d+ex^n)^p\right)}{2g}-\frac{(enp)\int \frac{x^{-1+n}(f+gx)^2}{d+ex^n}\,dx}{2g}\\ &= \frac{(f+gx)^2\log\left(c(d+ex^n)^p\right)}{2g}-\frac{(enp)\int\left(\frac{f^2x^{-1+n}}{d+ex^n}+\frac{2fgx^n}{d+ex^n}+\frac{g^2x^{1+n}}{d+ex^n}\right)dx}{2g}\\ &= \frac{(f+gx)^2\log\left(c(d+ex^n)^p\right)}{2g}-(efnp)\int\frac{x^n}{d+ex^n}\,dx-\frac{(ef^2np)\int\frac{x^{-1+n}}{d+ex^n}\,dx}{2g}\\ &= -\frac{efnpx^{1+n}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1+n)}-\frac{egnpx^{2+n}\,{}_2F_1\left(1,\frac{2+n}{n};2\left(1+\frac{1}{n}\right)\ldots\right.}{2d(2+n)}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 130, normalized size = 0.98

$$-\frac{efnpx^{1+n}\,{}_2F_1\left(1,\frac{1+n}{n};1+\frac{1+n}{n};-\frac{ex^n}{d}\right)}{d(1+n)}-\frac{egnpx^{2+n}\,{}_2F_1\left(1,\frac{2+n}{n};1+\frac{2+n}{n};-\frac{ex^n}{d}\right)}{2d(2+n)}+fx\log\left(c(d+ex^n)^p\right)+\frac{1}{2}gx^2\log\left(c(d+ex^n)^p\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -((e*f*n*p*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/n, 1 + (1 + n)/n, -((e*x^
n)/d)])/(d*(1 + n))) - (e*g*n*p*x^(2 + n)*Hypergeometric2F1[1, (2 + n)/n, 1
 + (2 + n)/n, -((e*x^n)/d)])/(2*d*(2 + n)) + f*x*Log[c*(d + e*x^n)^p] + (g*
x^2*Log[c*(d + e*x^n)^p])/2
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (gx+f)\ln\left(c(d+e\,x^n)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*ln(c*(d+e*x^n)^p),x)
```

```
[Out] int((g*x+f)*ln(c*(d+e*x^n)^p),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] -1/4*(g*n*p - 2*g*log(c))*x^2 - (f*n*p - f*log(c))*x + 1/2*(g*x^2 + 2*f*x)*
log((d + e^(n*log(x) + 1))^p) + integrate(1/2*(d*g*n*p*x + 2*d*f*n*p)/(d +
e^(n*log(x) + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] integral((g*x + f)*log((x^n*e + d)^p*c), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 6.30, size = 162, normalized size = 1.23

$$fx\log\left(c(d+ex^n)^p\right)+\frac{fpx\Phi\left(\frac{dx^{-n}e^{i\pi}}{e},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n\Gamma\left(1+\frac{1}{n}\right)}+\frac{gx^2\log\left(c(d+ex^n)^p\right)}{2}-\frac{egpx^2x^n\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{2}{n}\right)\Gamma\left(1+\frac{2}{n}\right)}{2d\Gamma\left(2+\frac{2}{n}\right)}-\frac{egpx^2x^n\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{2}{n}\right)\Gamma\left(1+\frac{2}{n}\right)}{dn\Gamma\left(2+\frac{2}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*ln(c*(d+e*x**n)**p),x)
```

```
[Out] f*x*log(c*(d + e*x**n)**p) + f*p*x*lerchphi(d*exp_polar(I*pi)/(e*x**n), 1,
exp_polar(I*pi)/n)*gamma(1/n)/(n*gamma(1 + 1/n)) + g*x**2*log(c*(d + e*x**n
)**p)/2 - e*g*p*x**2*x**n*lerchphi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 2/n)*ga
mma(1 + 2/n)/(2*d*gamma(2 + 2/n)) - e*g*p*x**2*x**n*lerchphi(e*x**n*exp_pol
ar(I*pi)/d, 1, 1 + 2/n)*gamma(1 + 2/n)/(d*n*gamma(2 + 2/n))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)*log((x^n*e + d)^p*c), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \ln \left(c \left(d+e x^n\right)^p\right) (f+g x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)*(f + g*x),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)*(f + g*x), x)
```

3.215 $\int \log(c(d+ex^n)^p)\,dx$

Optimal. Leaf size=54

$$-\frac{enpx^{1+n}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1+n)}+x\log\left(c(d+ex^n)^p\right)$$

```
[Out] -e*n*p*x^(1+n)*hypergeom([1, 1+1/n],[2+1/n],-e*x^n/d)/d/(1+n)+x*ln(c*(d+e*x
^n)^p)
```

Rubi [A]
time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2498, 371}

$$x\log\left(c(d+ex^n)^p\right)-\frac{enpx^{n+1}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p],x]
```

```
[Out] -((e*n*p*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^n)/d
)])/(d*(1 + n))) + x*Log[c*(d + e*x^n)^p]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \log\left(c(d+ex^n)^p\right)\,dx&=x\log\left(c(d+ex^n)^p\right)-(enp)\int\frac{x^n}{d+ex^n}\,dx\\&=-\frac{enpx^{1+n}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1+n)}+x\log\left(c(d+ex^n)^p\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 52, normalized size = 0.96

$$x\left(-\frac{enpx^n\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1+n)}+\log\left(c(d+ex^n)^p\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p],x]
```

```
[Out] x*(-((e*n*p*x^n*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^n)/d)])
/(d*(1 + n))) + Log[c*(d + e*x^n)^p])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \ln\left(c(d+e\,x^n)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p),x)
```

```
[Out] int(ln(c*(d+e*x^n)^p),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] d*n*p*integrate(1/(d + e^(n*log(x) + 1)), x) - (n*p - log(c))*x + x*log((d
+ e^(n*log(x) + 1))^p)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] integral(log((x^n*e + d)^p*c), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.93, size = 48, normalized size = 0.89

$$x\log\left(c(d+ex^n)^p\right)+\frac{px\Phi\left(\frac{dx^{-n}e^{i\pi}}{e},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n\Gamma\left(1+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p),x)
```

```
[Out] x*log(c*(d + e*x**n)**p) + p*x*lerchphi(d*exp_polar(I*pi)/(e*x**n), 1, exp_
polar(I*pi)/n)*gamma(1/n)/(n*gamma(1 + 1/n))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate(log((x^n*e + d)^p*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(c\,\left(d+e\,x^n\right)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p),x)
```

```
[Out] int(log(c*(d + e*x^n)^p), x)
```

3.216 $\int \frac{\log(c(d+ex^n)^p)}{f+gx} dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\log\left(c(d+ex^n)^p\right)}{f+gx},x\right)$$

```
[Out] Unintegrable(ln(c*(d+e*x^n)^p)/(g*x+f),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{f+gx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[c*(d + e*x^n)^p]/(f + g*x),x]
[Out] Defer[Int][Log[c*(d + e*x^n)^p]/(f + g*x), x]
```

Rubi steps

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{f+gx}\,dx = \int \frac{\log\left(c(d+ex^n)^p\right)}{f+gx}\,dx$$

Mathematica [A]
time = 1.24, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{f+gx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(f + g*x),x]
[Out] Integrate[Log[c*(d + e*x^n)^p]/(f + g*x), x]
```

Maple [A]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c(d+e\,x^n)^p\right)}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)/(g*x+f),x)
```

```
[Out] int(ln(c*(d+e*x^n)^p)/(g*x+f),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f),x, algorithm="maxima")
```

```
[Out] integrate(log((x^n*e + d)^p*c)/(g*x + f), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f),x, algorithm="fricas")
```

```
[Out] integral(log((x^n*e + d)^p*c)/(g*x + f), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/(g*x+f),x)
```

```
[Out] Integral(log(c*(d + e*x**n)**p)/(f + g*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate(log((x^n*e + d)^p*c)/(g*x + f), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)}{f+g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/(f + g*x),x)
[Out] int(log(c*(d + e*x^n)^p)/(f + g*x), x)
```

3.217 $\int \frac{\log(c(d+ex^n)^p)}{(f+gx)^2} dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\log(c(d+ex^n)^p)}{(f+gx)^2},x\right)$$

```
[Out] Unintegrable(ln(c*(d+e*x^n)^p)/(g*x+f)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log(c(d+ex^n)^p)}{(f+gx)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[Log[c*(d + e*x^n)^p]/(f + g*x)^2,x]
[Out] Defer[Int][Log[c*(d + e*x^n)^p]/(f + g*x)^2, x]
```

Rubi steps

$$\int \frac{\log(c(d+ex^n)^p)}{(f+gx)^2} dx = \int \frac{\log(c(d+ex^n)^p)}{(f+gx)^2} dx$$

Mathematica [A]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\log(c(d+ex^n)^p)}{(f+gx)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(f + g*x)^2,x]
[Out] Integrate[Log[c*(d + e*x^n)^p]/(f + g*x)^2, x]
```

Maple [A]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\ln(c(d+e\,x^n)^p)}{(gx+f)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)/(g*x+f)^2,x)
```

```
[Out] int(ln(c*(d+e*x^n)^p)/(g*x+f)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f)^2,x, algorithm="maxima")
```

```
[Out] -d*n*p*integrate(1/(d*g^2*x^2 + d*f*g*x + (g^2*x^2*e + f*g*x*e)*x^n), x) -
n*p*log(g*x + f)/(f*g) - (f*log(c) + f*log((d + e^(n*log(x) + 1))^p) - (g*n
*p*x + f*n*p)*log(x))/(f*g^2*x + f^2*g)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] integral(log((x^n*e + d)^p*c)/(g^2*x^2 + 2*f*g*x + f^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{(f+gx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/(g*x+f)**2,x)
```

```
[Out] Integral(log(c*(d + e*x**n)**p)/(f + g*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((x^n*e + d)^p*c)/(g*x + f)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln \left(c \left(d+e\,x^n\right)^p\right)}{\left(f+g\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/(f + g*x)^2,x)
[Out] int(log(c*(d + e*x^n)^p)/(f + g*x)^2, x)
```

3.218 $\int \frac{\log(c(d+ex^n)^p)}{(f+gx)^3}\,dx$

Optimal. Leaf size=23

$$\mathrm{Int}\left(\frac{\log\left(c(d+ex^n)^p\right)}{(f+gx)^3},x\right)$$

```
[Out] Unintegrable(ln(c*(d+e*x^n)^p)/(g*x+f)^3,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{(f+gx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[c*(d + e*x^n)^p]/(f + g*x)^3,x]
```

```
[Out] Defer[Int][Log[c*(d + e*x^n)^p]/(f + g*x)^3, x]
```

Rubi steps

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{(f+gx)^3}\,dx = \int \frac{\log\left(c(d+ex^n)^p\right)}{(f+gx)^3}\,dx$$

Mathematica [A]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{(f+gx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(f + g*x)^3,x]
```

```
[Out] Integrate[Log[c*(d + e*x^n)^p]/(f + g*x)^3, x]
```

Maple [A]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c(d+e\,x^n)^p\right)}{(gx+f)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)/(g*x+f)^3,x)
```

```
[Out] int(ln(c*(d+e*x^n)^p)/(g*x+f)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f)^3,x, algorithm="maxima")
```

```
[Out] -d*n*p*integrate(1/2/(d*g^3*x^3 + 2*d*f*g^2*x^2 + d*f^2*g*x + (g^3*x^3*e +
2*f*g^2*x^2*e + f^2*g*x*e)*x^n), x) + 1/2*(f*g*n*p*x + f^2*n*p - f^2*log(c)
 - f^2*log((d + e^(n*log(x) + 1))^p) + (g^2*n*p*x^2 + 2*f*g*n*p*x + f^2*n*p
)*log(x))/(f^2*g^3*x^2 + 2*f^3*g^2*x + f^4*g) - 1/2*n*p*log(g*x + f)/(f^2*g
)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f)^3,x, algorithm="fricas")
```

```
[Out] integral(log((x^n*e + d)^p*c)/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{(f+gx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/(g*x+f)**3,x)
```

```
[Out] Integral(log(c*(d + e*x**n)**p)/(f + g*x)**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f)^3,x, algorithm="giac")
```

```
[Out] integrate(log((x^n*e + d)^p*c)/(g*x + f)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln \left(c\left(d+e\,x^n\right)^p\right)}{\left(f+g\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/(f + g*x)^3,x)
[Out] int(log(c*(d + e*x^n)^p)/(f + g*x)^3, x)
```

3.219 $\int \frac{x^3 \log(c(a+bx)^p)}{d+ex} \, dx$

Optimal. Leaf size=250

$$-\frac{d^2px}{e^3}-\frac{adpx}{2be^2}-\frac{a^2px}{3b^2e}+\frac{dpx^2}{4e^2}+\frac{apx^2}{6be}-\frac{px^3}{9e}+\frac{a^2dp\log(a+bx)}{2b^2e^2}+\frac{a^3p\log(a+bx)}{3b^3e}-\frac{dx^2\log\left(c(a+bx)^p\right)}{2e^2}+\frac{x^3\log(\ldots}{}$$

```
[Out] -d^2*p*x/e^3-1/2*a*d*p*x/b/e^2-1/3*a^2*p*x/b^2/e+1/4*d*p*x^2/e^2+1/6*a*p*x^
2/b/e-1/9*p*x^3/e+1/2*a^2*d*p*ln(b*x+a)/b^2/e^2+1/3*a^3*p*ln(b*x+a)/b^3/e-1
/2*d*x^2*ln(c*(b*x+a)^p)/e^2+1/3*x^3*ln(c*(b*x+a)^p)/e+d^2*(b*x+a)*ln(c*(b*
x+a)^p)/b/e^3-d^3*ln(c*(b*x+a)^p)*ln(b*(e*x+d)/(-a*e+b*d))/e^4-d^3*p*polylo
g(2,-e*(b*x+a)/(-a*e+b*d))/e^4
```

Rubi [A]

time = 0.17, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {45, 2463, 2436, 2332, 2442, 2441, 2440, 2438}

$$-\frac{d^3p\,\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e^4}+\frac{a^3p\log(a+bx)}{3b^3e}+\frac{a^2dp\log(a+bx)}{2b^2e^2}-\frac{a^2px}{3b^2e}-\frac{d^3\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^4}+\frac{d^2(a+bx)\log\left(c(a+bx)^p\right)}{be^3}-\frac{dx^2\log\left(c(a+bx)^p\right)}{2e^2}+\frac{x^3\log\left(c(a+bx)^p\right)}{3e}-\frac{adpx}{2be^2}+\frac{apx^2}{6be}-\frac{d^2px}{e^3}+\frac{dpx^2}{4e^2}-\frac{px^3}{9e}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Log[c*(a + b*x)^p])/(d + e*x),x]
```

```
[Out] -((d^2*p*x)/e^3) - (a*d*p*x)/(2*b*e^2) - (a^2*p*x)/(3*b^2*e) + (d*p*x^2)/(4
*e^2) + (a*p*x^2)/(6*b*e) - (p*x^3)/(9*e) + (a^2*d*p*Log[a + b*x])/(2*b^2*e
^2) + (a^3*p*Log[a + b*x])/(3*b^3*e) - (d*x^2*Log[c*(a + b*x)^p])/(2*e^2) +
 (x^3*Log[c*(a + b*x)^p])/(3*e) + (d^2*(a + b*x)*Log[c*(a + b*x)^p])/(b*e^3
) - (d^3*Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e^4 - (d^3*p*Po
lyLog[2, -((e*(a + b*x))/(b*d - a*e))])/e^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\int \frac{x^3 \log (c(a+bx)^p)}{d+ex}\, dx = \int \left(\frac{d^2 \log (c(a+bx)^p)}{e^3} - \frac{dx \log (c(a+bx)^p)}{e^2} + \frac{x^2 \log (c(a+bx)^p)}{e} - \frac{d^3 \log (c(a\right.}{e^3(d+}$$

$$= \frac{d^2 \int \log (c(a+bx)^p)\, dx}{e^3} - \frac{d^3 \int \frac{\log(c(a+bx)^p)}{d+ex}\, dx}{e^3} - \frac{d \int x \log (c(a+bx)^p)\, dx}{e^2} + \frac{\int x^2}{}$$

$$= -\frac{dx^2 \log (c(a+bx)^p)}{2e^2} + \frac{x^3 \log (c(a+bx)^p)}{3e} - \frac{d^3 \log (c(a+bx)^p) \log \left(\frac{b(d+ex)}{bd-ae}\right)}{e^4} +$$

$$= -\frac{d^2px}{e^3} - \frac{dx^2 \log (c(a+bx)^p)}{2e^2} + \frac{x^3 \log (c(a+bx)^p)}{3e} + \frac{d^2(a+bx) \log (c(a+bx)^p)}{be^3}$$

$$= -\frac{d^2px}{e^3} - \frac{adpx}{2be^2} - \frac{a^2px}{3b^2e} + \frac{dpx^2}{4e^2} + \frac{apx^2}{6be} - \frac{px^3}{9e} + \frac{a^2dp \log(a+bx)}{2b^2e^2} + \frac{a^3p \log(a+}{3b^3e}$$

Mathematica [A]
time = 0.13, size = 183, normalized size = 0.73

$$\frac{6a^2e^2(3bd+2ae)p\log(a+bx)+b\left(-epx(12a^2e^2-6abe(-3d+ex)+b^2(36d^2-9dex+4e^2x^2))+6b\log(c(a+bx)^p)\left(6ad^2e+bex(6d^2-3dex+2e^2x^2)-6bd^3\log\left(\frac{b(d+ex)}{bd-ae}\right)\right)\right)-36b^3d^3p\text{Li}_2\left(\frac{e(a+bx)}{-bd+ae}\right)}{36b^3e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Log[c*(a + b*x)^p])/(d + e*x),x]
```

```
[Out] (6*a^2*e^2*(3*b*d + 2*a*e)*p*Log[a + b*x] + b*(-(e*p*x*(12*a^2*e^2 - 6*a*b*
e*(-3*d + e*x) + b^2*(36*d^2 - 9*d*e*x + 4*e^2*x^2))) + 6*b*Log[c*(a + b*x)
^p]*(6*a*d^2*e + b*e*x*(6*d^2 - 3*d*e*x + 2*e^2*x^2) - 6*b*d^3*Log[(b*(d +
e*x))/(b*d - a*e)])) - 36*b^3*d^3*p*PolyLog[2, (e*(a + b*x))/(-(b*d) + a*e)
])/(36*b^3*e^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.48, size = 919, normalized size = 3.68

method	result
risch	$\frac{p\,a^3 \ln((ex+d)b+ae-bd)}{3b^3e} + \frac{p\,d^3 \ln(ex+d) \ln\left(\frac{(ex+d)b+ae-bd}{ae-bd}\right)}{e^4} + \frac{i\pi\, \text{csgn}(i(bx+a)^p)\text{csgn}(ic(bx+a)^p)\text{csgn}(ic)d\,x^2}{4e^2} + \frac{i\pi\, \text{csgn}(i(bx+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(b*x+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*p/b^3/e*a^3*ln((e*x+d)*b+a*e-b*d)+p/e^4*d^3*ln(e*x+d)*ln(((e*x+d)*b+a*e
-b*d)/(a*e-b*d))-d^2*p*x/e^3+1/2*p/b^2/e^2*a^2*ln((e*x+d)*b+a*e-b*d)*d+p/b/
e^3*a*ln((e*x+d)*b+a*e-b*d)*d^2+1/6*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)
^p)^2/e*x^3+1/4*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)/e^2*d*
```

```
x^2+1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)*d^3/e^4*ln(e*x
+d)-1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)/e^3*x*d^2+1/6*
I*Pi*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)/e*x^3+1/3*ln(c)/e*x^3-2/3*p/b/e^3*a*d^
2-1/3*p/b^2/e^2*a^2*d-1/2*I*Pi*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)*d^3/e^4*ln(e
*x+d)-1/6*I*Pi*csgn(I*c*(b*x+a)^p)^3/e*x^3-49/36*p/e^4*d^3+1/2*I*Pi*csgn(I*
(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2/e^3*x*d^2-ln((b*x+a)^p)*d^3/e^4*ln(e*x+d)+
1/4*I*Pi*csgn(I*c*(b*x+a)^p)^3/e^2*d*x^2+p/e^4*d^3*dilog(((e*x+d)*b+a*e-b*d
)/(a*e-b*d))+1/2*I*Pi*csgn(I*c*(b*x+a)^p)^3*d^3/e^4*ln(e*x+d)+ln((b*x+a)^p)
/e^3*x*d^2-1/2*ln(c)/e^2*d*x^2+ln(c)/e^3*x*d^2-ln(c)*d^3/e^4*ln(e*x+d)-1/2*
ln((b*x+a)^p)/e^2*d*x^2-1/6*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn
(I*c)/e*x^3+1/2*I*Pi*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)/e^3*x*d^2-1/2*I*Pi*csg
n(I*c*(b*x+a)^p)^3/e^3*x*d^2-1/3*a^2*p*x/e/b^2+1/6*a*p*x^2/b/e+1/3*ln((b*x+
a)^p)/e*x^3-1/9*p*x^3/e+1/4*d*p*x^2/e^2-1/4*I*Pi*csgn(I*c*(b*x+a)^p)^2*csgn
(I*c)/e^2*d*x^2-1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2*d^3/e^4*ln
(e*x+d)-1/2*a*d*p*x/b/e^2-1/4*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2/
e^2*d*x^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(x^3*log((b*x + a)^p*c)/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="fricas")
[Out] integral(x^3*log((b*x + a)^p*c)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \log \left(c(a+bx)^p\right)}{d+ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(b*x+a)**p)/(e*x+d),x)
```

```
[Out] Integral(x**3*log(c*(a + b*x)**p)/(d + e*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(x^3*log((b*x + a)^p*c)/(x*e + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\ln\left(c\,{\left(a+b\,x\right)}^p\right)}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*log(c*(a + b*x)^p))/(d + e*x),x)
[Out] int((x^3*log(c*(a + b*x)^p))/(d + e*x), x)
```

3.220 $\int \frac{x^2\log(c(a+bx)^p)}{d+ex}\,dx$

Optimal. Leaf size=159

$$\frac{dpx}{e^2}+\frac{apx}{2be}-\frac{px^2}{4e}-\frac{a^2p\log(a+bx)}{2b^2e}+\frac{x^2\log\left(c(a+bx)^p\right)}{2e}-\frac{d(a+bx)\log\left(c(a+bx)^p\right)}{be^2}+\frac{d^2\log\left(c(a+bx)^p\right)\log\left(\right.}{e^3}$$

```
[Out] d*p*x/e^2+1/2*a*p*x/b/e-1/4*p*x^2/e-1/2*a^2*p*ln(b*x+a)/b^2/e+1/2*x^2*ln(c*
(b*x+a)^p)/e-d*(b*x+a)*ln(c*(b*x+a)^p)/b/e^2+d^2*ln(c*(b*x+a)^p)*ln(b*(e*x+
d)/(-a*e+b*d))/e^3+d^2*p*polylog(2,-e*(b*x+a)/(-a*e+b*d))/e^3
```

Rubi [A]

time = 0.11, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {45, 2463, 2436, 2332, 2442, 2441, 2440, 2438}

$$\frac{d^2p\,\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e^3}-\frac{a^2p\log(a+bx)}{2b^2e}+\frac{d^2\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^3}-\frac{d(a+bx)\log\left(c(a+bx)^p\right)}{be^2}+\frac{x^2\log\left(c(a+bx)^p\right)}{2e}+\frac{apx}{2be}+\frac{dpx}{e^2}-\frac{px^2}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Log[c*(a + b*x)^p])/(d + e*x),x]
```

```
[Out] (d*p*x)/e^2 + (a*p*x)/(2*b*e) - (p*x^2)/(4*e) - (a^2*p*Log[a + b*x])/(2*b^2
*e) + (x^2*Log[c*(a + b*x)^p])/(2*e) - (d*(a + b*x)*Log[c*(a + b*x)^p])/(b*
e^2) + (d^2*Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e^3 + (d^2*p
*PolyLog[2, -((e*(a + b*x))/(b*d - a*e))])/e^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\int \frac{x^2 \log (c(a+bx)^p)}{d+ex} dx = \int \left(-\frac{d \log (c(a+bx)^p)}{e^2} + \frac{x \log (c(a+bx)^p)}{e} + \frac{d^2 \log (c(a+bx)^p)}{e^2(d+ex)}\right) dx$$

$$= -\frac{d \int \log (c(a+bx)^p)\, dx}{e^2} + \frac{d^2 \int \frac{\log(c(a+bx)^p)}{d+ex} dx}{e^2} + \frac{\int x \log (c(a+bx)^p)\, dx}{e}$$

$$= \frac{x^2 \log (c(a+bx)^p)}{2e} + \frac{d^2 \log (c(a+bx)^p) \log \left(\frac{b(d+ex)}{bd-ae}\right)}{e^3} - \frac{d\text{Subst}\left(\int \log (cx^p)\, dx,\right.}{be^2}$$

$$= \frac{dpx}{e^2} + \frac{x^2 \log (c(a+bx)^p)}{2e} - \frac{d(a+bx) \log (c(a+bx)^p)}{be^2} + \frac{d^2 \log (c(a+bx)^p) \log}{e^3}$$

$$= \frac{dpx}{e^2} + \frac{apx}{2be} - \frac{px^2}{4e} - \frac{a^2 p \log(a+bx)}{2b^2 e} + \frac{x^2 \log (c(a+bx)^p)}{2e} - \frac{d(a+bx) \log (c(a}{be^2}$$

Mathematica [A]
time = 0.07, size = 127, normalized size = 0.80

$$\frac{bepx(4bd+2ae-bex) - 2a^2e^2p \log(a+bx) + b \log (c(a+bx)^p) \left(-4ade + 2bex(-2d+ex) + 4bd^2 \log \left(\frac{b(d+ex)}{bd-ae}\right)\right) + 4b^2d^2p\text{Li}_2\left(\frac{e(a+bx)}{-bd+ae}\right)}{4b^2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Log[c*(a + b*x)^p])/(d + e*x),x]
```

```
[Out] (b*e*p*x*(4*b*d + 2*a*e - b*e*x) - 2*a^2*e^2*p*Log[a + b*x] + b*Log[c*(a +
b*x)^p]*(-4*a*d*e + 2*b*e*x*(-2*d + e*x) + 4*b*d^2*Log[(b*(d + e*x))/(b*d -
 a*e)]) + 4*b^2*d^2*p*PolyLog[2, (e*(a + b*x))/(-(b*d) + a*e)])/(4*b^2*e^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.43, size = 666, normalized size = 4.19

method	result
risch	$\frac{pad}{2b\,e^2} + \frac{dpx}{e^2} - \frac{\ln(c)dx}{e^2} + \frac{\ln(c)d^2\ln(ex+d)}{e^3} - \frac{pa\ln((ex+d)b+ae-bd)d}{b\,e^2} + \frac{i\pi\,\text{csgn}(i(bx+a)^p)\text{csgn}(ic(bx+a)^p)^2 d^2\ln(ex+d)}{2e^3} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(b*x+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*p/b/e^2*a*d+d*p*x/e^2-1/4*I*Pi*csgn(I*c*(b*x+a)^p)^3/e*x^2-ln(c)/e^2*d*
x+ln(c)*d^2/e^3*ln(e*x+d)+1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*cs
gn(I*c)/e^2*d*x-1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)*d^
2/e^3*ln(e*x+d)-p/b/e^2*a*ln((e*x+d)*b+a*e-b*d)*d-1/2*p/b^2/e*a^2*ln((e*x+d
)*b+a*e-b*d)-1/2*I*Pi*csgn(I*c*(b*x+a)^p)^3*d^2/e^3*ln(e*x+d)+1/4*I*Pi*csgn
```

```
(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2/e*x^2+1/2*ln(c)/e*x^2+1/2*I*Pi*csgn(I*(
b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2*d^2/e^3*ln(e*x+d)+5/4*p/e^3*d^2+1/4*I*Pi*cs
gn(I*c*(b*x+a)^p)^2*csgn(I*c)/e*x^2-p/e^3*d^2*ln(e*x+d)*ln(((e*x+d)*b+a*e-b
*d)/(a*e-b*d))+1/2*ln((b*x+a)^p)/e*x^2+1/2*I*Pi*csgn(I*c*(b*x+a)^p)^2*csgn(
I*c)*d^2/e^3*ln(e*x+d)+1/2*I*Pi*csgn(I*c*(b*x+a)^p)^3/e^2*d*x-ln((b*x+a)^p)
/e^2*d*x+ln((b*x+a)^p)*d^2/e^3*ln(e*x+d)-p/e^3*d^2*dilog(((e*x+d)*b+a*e-b*d
)/(a*e-b*d))-1/4*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)/e*x^2
-1/4*p*x^2/e+1/2*a*p*x/b/e-1/2*I*Pi*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)/e^2*d*x
-1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2/e^2*d*x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(x^2*log((b*x + a)^p*c)/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x^2*log((b*x + a)^p*c)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \log \left(c(a+bx)^p\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(b*x+a)**p)/(e*x+d),x)
```

```
[Out] Integral(x**2*log(c*(a + b*x)**p)/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(x^2*log((b*x + a)^p*c)/(x*e + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\ \ln\left(c\,{\left(a+b\,x\right)}^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*log(c*(a + b*x)^p))/(d + e*x),x)
```

```
[Out] int((x^2*log(c*(a + b*x)^p))/(d + e*x), x)
```

3.221 $\int \frac{x\log(c(a+bx)^p)}{d+ex}\,dx$

Optimal. Leaf size=91

$$-\frac{px}{e}+\frac{(a+bx)\log\left(c(a+bx)^p\right)}{be}-\frac{d\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^2}-\frac{dp\text{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{e^2}$$

```
[Out] -p*x/e+(b*x+a)*ln(c*(b*x+a)^p)/b/e-d*ln(c*(b*x+a)^p)*ln(b*(e*x+d)/(-a*e+b*d
))/e^2-d*p*polylog(2,-e*(b*x+a)/(-a*e+b*d))/e^2
```

Rubi [A]

time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {45, 2463, 2436, 2332, 2441, 2440, 2438}

$$-\frac{dp\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e^2}-\frac{d\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^2}+\frac{(a+bx)\log\left(c(a+bx)^p\right)}{be}-\frac{px}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[c*(a + b*x)^p])/(d + e*x),x]
```

```
[Out] -((p*x)/e) + ((a + b*x)*Log[c*(a + b*x)^p])/(b*e) - (d*Log[c*(a + b*x)^p]*L
og[(b*(d + e*x))/(b*d - a*e)])/e^2 - (d*p*PolyLog[2, -((e*(a + b*x))/(b*d -
 a*e))])/e^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\int \frac{x\log\left(c(a+bx)^p\right)}{d+ex}\,dx = \int \left(\frac{\log\left(c(a+bx)^p\right)}{e} - \frac{d\log\left(c(a+bx)^p\right)}{e(d+ex)}\right)\,dx$$

$$= \frac{\int \log\left(c(a+bx)^p\right)\,dx}{e} - \frac{d\int \frac{\log(c(a+bx)^p)}{d+ex}\,dx}{e}$$

$$= -\frac{d\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^2} + \frac{\mathrm{Subst}\left(\int \log\left(cx^p\right)\,dx, x, a+bx\right)}{be} + \frac{(bdp)\int \frac{\log}{}}{}$$

$$= -\frac{px}{e} + \frac{(a+bx)\log\left(c(a+bx)^p\right)}{be} - \frac{d\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^2} + \frac{(dp)\mathrm{Subst}\left(\right.}{}$$

$$= -\frac{px}{e} + \frac{(a+bx)\log\left(c(a+bx)^p\right)}{be} - \frac{d\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^2} - \frac{dp\mathrm{Li}_2\left(-\frac{e(}{bd}\right.}{e^2}$$

Mathematica [A]

time = 0.03, size = 79, normalized size = 0.87

$$\frac{-bepx + \log\left(c(a+bx)^p\right)\left(ae + bex - bd\log\left(\frac{b(d+ex)}{bd-ae}\right)\right) - bdp\text{Li}_2\left(\frac{e(a+bx)}{-bd+ae}\right)}{be^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[c*(a + b*x)^p])/(d + e*x),x]
```

```
[Out] (-(b*e*p*x) + Log[c*(a + b*x)^p]*(a*e + b*e*x - b*d*Log[(b*(d + e*x))/(b*d
- a*e)]) - b*d*p*PolyLog[2, (e*(a + b*x))/(-(b*d) + a*e)])/(b*e^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.49, size = 427, normalized size = 4.69

method	result
risch	$\frac{\ln((bx+a)^p)x}{e} - \frac{\ln((bx+a)^p)d\ln(ex+d)}{e^2} - \frac{px}{e} - \frac{pd}{e^2} + \frac{pa\ln((ex+d)b+ae-bd)}{be} + \frac{pd\,\text{dilog}\left(\frac{(ex+d)b+ae-bd}{ae-bd}\right)}{e^2} + \frac{pd\ln(ex+d)\ln(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(b*x+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] ln((b*x+a)^p)/e*x-ln((b*x+a)^p)*d/e^2*ln(e*x+d)-p*x/e-p/e^2*d+p/b/e*a*ln((e
*x+d)*b+a*e-b*d)+p/e^2*d*dilog(((e*x+d)*b+a*e-b*d)/(a*e-b*d))+p/e^2*d*ln(e*
x+d)*ln(((e*x+d)*b+a*e-b*d)/(a*e-b*d))+1/2*I*Pi*csgn(I*c*(b*x+a)^p)^3*d/e^2
*ln(e*x+d)-1/2*I*Pi*csgn(I*c*(b*x+a)^p)^3/e*x-1/2*I*Pi*csgn(I*c*(b*x+a)^p)^
2*csgn(I*c)*d/e^2*ln(e*x+d)-1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*
csgn(I*c)/e*x-1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2*d/e^2*ln(e*x
+d)+1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2/e*x+1/2*I*Pi*csgn(I*(b
*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)*d/e^2*ln(e*x+d)+1/2*I*Pi*csgn(I*c*(b
*x+a)^p)^2*csgn(I*c)/e*x+ln(c)/e*x-ln(c)*d/e^2*ln(e*x+d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(x*log((b*x + a)^p*c)/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="fricas")
[Out] integral(x*log((b*x + a)^p*c)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \log \left(c(a+bx)^p\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(b*x+a)**p)/(e*x+d),x)
[Out] Integral(x*log(c*(a + b*x)**p)/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(x*log((b*x + a)^p*c)/(x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\ln\left(c\,{\left(a+b\,x\right)}^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(c*(a + b*x)^p))/(d + e*x),x)
[Out] int((x*log(c*(a + b*x)^p))/(d + e*x), x)
```

3.222 $\int \frac{\log(c(a+bx)^p)}{d+ex}\,dx$

Optimal. Leaf size=58

$$\frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e}+\frac{p\text{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{e}$$

```
[Out] ln(c*(b*x+a)^p)*ln(b*(e*x+d)/(-a*e+b*d))/e+p*polylog(2,-e*(b*x+a)/(-a*e+b*d
))/e
```

Rubi [A]
time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2441, 2440, 2438}

$$\frac{p\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e}+\frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e + (p*PolyLog[2, -((e*
(a + b*x))/(b*d - a*e))])/e
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rubi steps

$$\int \frac{\log(c(a+bx)^p)}{d+ex}\,dx = \frac{\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e} - \frac{(bp)\int \frac{\log\left(\frac{b(d+ex)}{bd-ae}\right)}{a+bx}\,dx}{e}$$
$$= \frac{\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e} - \frac{p\text{Subst}\left(\int \frac{\log\left(1+\frac{ex}{bd-ae}\right)}{x}\,dx, x, a+bx\right)}{e}$$
$$= \frac{\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e} + \frac{p\text{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{e}$$

Mathematica [A]
time = 0.00, size = 57, normalized size = 0.98

$$\frac{\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e} + \frac{p\text{Li}_2\left(\frac{e(a+bx)}{-bd+ae}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x)^p]/(d + e*x),x]
[Out] (Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e + (p*PolyLog[2, (e*(a
 + b*x))/(-(b*d) + a*e)])/e
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.41, size = 242, normalized size = 4.17

method	result
risch	$\frac{\ln(ex+d)\ln((bx+a)^p)}{e} - \frac{p\,\text{dilog}\left(\frac{(ex+d)b+ae-bd}{ae-bd}\right)}{e} - \frac{p\ln(ex+d)\ln\left(\frac{(ex+d)b+ae-bd}{ae-bd}\right)}{e} + \frac{i\ln(ex+d)\pi\,\text{csgn}(i(bx+a)^p)\text{csgn}(ic(bx+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
[Out] ln(e*x+d)/e*ln((b*x+a)^p)-1/e*p*dilog(((e*x+d)*b+a*e-b*d)/(a*e-b*d))-1/e*p*
ln(e*x+d)*ln(((e*x+d)*b+a*e-b*d)/(a*e-b*d))+1/2*I*ln(e*x+d)/e*Pi*csgn(I*(b*
x+a)^p)*csgn(I*c*(b*x+a)^p)^2-1/2*I*ln(e*x+d)/e*Pi*csgn(I*(b*x+a)^p)*csgn(I
*c*(b*x+a)^p)*csgn(I*c)-1/2*I*ln(e*x+d)/e*Pi*csgn(I*c*(b*x+a)^p)^3+1/2*I*ln
(e*x+d)/e*Pi*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)+ln(e*x+d)/e*ln(c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(59) = 118.
time = 0.28, size = 123, normalized size = 2.12

$$bp\left(\frac{\log(bx+a)\log(xe+d)}{b} - \frac{\log(xe+d)\log\left(-\frac{bxe+bd}{bd-ae}+1\right) + \text{Li}_2\left(\frac{bxe+bd}{bd-ae}\right)}{b}\right)e^{(-1)} - pe^{(-1)}\log(bx+a)\log(xe+d) + e^{(-1)}\log((bx+a)^p c)\log(xe+d)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] b*p*(log(b*x + a)*log(x*e + d)/b - (log(x*e + d)*log(-(b*x*e + b*d)/(b*d -
a*e) + 1) + dilog((b*x*e + b*d)/(b*d - a*e)))/b)*e^(-1) - p*e^(-1)*log(b*x
+ a)*log(x*e + d) + e^(-1)*log((b*x + a)^p*c)*log(x*e + d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x + a)^p*c)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c(a+bx)^{p}\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x+a)**p)/(e*x+d),x)
```

```
[Out] Integral(log(c*(a + b*x)**p)/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x + a)^p*c)/(x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(c\,{\left(a+b\,x\right)}^{p}\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x)^p)/(d + e*x),x)
```

```
[Out] int(log(c*(a + b*x)^p)/(d + e*x), x)
```

3.223 $\int \frac{\log(c(a+bx)^p)}{x(d+ex)}\,dx$

Optimal. Leaf size=97

$$\frac{\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d}-\frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d}-\frac{p\text{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{d}+\frac{p\text{Li}_2\left(1+\frac{bx}{a}\right)}{d}$$

```
[Out] ln(-b*x/a)*ln(c*(b*x+a)^p)/d-ln(c*(b*x+a)^p)*ln(b*(e*x+d)/(-a*e+b*d))/d-p*p
olylog(2,-e*(b*x+a)/(-a*e+b*d))/d+p*polylog(2,1+b*x/a)/d
```

Rubi [A]
time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {36, 29, 31, 2463, 2441, 2352, 2440, 2438}

$$-\frac{p\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d}+\frac{p\text{PolyLog}\left(2,\frac{bx}{a}+1\right)}{d}-\frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d}+\frac{\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x)^p]/(x*(d + e*x)),x]
```

```
[Out] (Log[-((b*x)/a)]*Log[c*(a + b*x)^p])/d - (Log[c*(a + b*x)^p]*Log[(b*(d + e*
x))/(b*d - a*e)])/d - (p*PolyLog[2, -((e*(a + b*x))/(b*d - a*e))])/d + (p*P
olyLog[2, 1 + (b*x)/a])/d
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(c(a+bx)^p\right)}{x(d+ex)}\,dx &= \int \left(\frac{\log\left(c(a+bx)^p\right)}{dx} - \frac{e\log\left(c(a+bx)^p\right)}{d(d+ex)}\right)dx \\
&= \frac{\int \frac{\log(c(a+bx)^p)}{x}\,dx}{d} - \frac{e\int \frac{\log(c(a+bx)^p)}{d+ex}\,dx}{d} \\
&= \frac{\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d} - \frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} - \frac{(bp)\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx}{d} + \frac{(b
\\
&= \frac{\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d} - \frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} + \frac{p\text{Li}_2\left(1+\frac{bx}{a}\right)}{d} + \frac{p\text{Subs}
\\
&= \frac{\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d} - \frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} - \frac{p\text{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{d} + \frac{p\text{Li}
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 98, normalized size = 1.01

$$\frac{\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d}-\frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d}+\frac{p\text{Li}_2\left(\frac{a+bx}{a}\right)}{d}-\frac{p\text{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x)^p]/(x*(d + e*x)),x]
```

```
[Out] (Log[-((b*x)/a)]*Log[c*(a + b*x)^p])/d - (Log[c*(a + b*x)^p]*Log[(b*(d + e*
x))/(b*d - a*e)])/d + (p*PolyLog[2, (a + b*x)/a])/d - (p*PolyLog[2, -((e*(a
 + b*x))/(b*d - a*e))])/d
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.45, size = 420, normalized size = 4.33

method	result
risch	$-\frac{\ln((bx+a)^p)\ln(ex+d)}{d}+\frac{\ln((bx+a)^p)\ln(x)}{d}-\frac{p\,\mathrm{dilog}\left(\frac{bx+a}{a}\right)}{d}-\frac{p\ln(x)\ln\left(\frac{bx+a}{a}\right)}{d}+\frac{p\,\mathrm{dilog}\left(\frac{(ex+d)b+ae-bd}{ae-bd}\right)}{d}+\frac{p\ln(ex+d}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x+a)^p)/x/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -ln((b*x+a)^p)/d*ln(e*x+d)+ln((b*x+a)^p)/d*ln(x)-p/d*dilog(1/a*(b*x+a))-p/d
*ln(x)*ln(1/a*(b*x+a))+p/d*dilog(((e*x+d)*b+a*e-b*d)/(a*e-b*d))+p/d*ln(e*x+
d)*ln(((e*x+d)*b+a*e-b*d)/(a*e-b*d))-1/2*I*Pi*csgn(I*c*(b*x+a)^p)^2*csgn(I*
c)/d*ln(e*x+d)+1/2*I*Pi*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)/d*ln(x)+1/2*I*Pi*cs
gn(I*c*(b*x+a)^p)^3/d*ln(e*x+d)-1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)
^p)*csgn(I*c)/d*ln(x)+1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I
*c)/d*ln(e*x+d)-1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2/d*ln(e*x+d
)-1/2*I*Pi*csgn(I*c*(b*x+a)^p)^3/d*ln(x)+1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*
c*(b*x+a)^p)^2/d*ln(x)-ln(c)/d*ln(e*x+d)+ln(c)/d*ln(x)
```

Maxima [A]
time = 0.32, size = 129, normalized size = 1.33

$$-bp\left(\frac{\log\left(\frac{bx}{a}+1\right)\log\left(x\right)+\text{Li}_2\left(-\frac{bx}{a}\right)}{bd}-\frac{\log\left(xe+d\right)\log\left(-\frac{bxe+bd}{bd-ae}+1\right)+\text{Li}_2\left(\frac{bxe+bd}{bd-ae}\right)}{bd}\right)-\left(\frac{\log\left(xe+d\right)}{d}-\frac{\log\left(x\right)}{d}\right)\log\left((bx+a)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] -b*p*((log(b*x/a + 1)*log(x) + dilog(-b*x/a))/(b*d) - (log(x*e + d)*log(-(b
*x*e + b*d)/(b*d - a*e) + 1) + dilog((b*x*e + b*d)/(b*d - a*e)))/(b*d)) - (
log(x*e + d)/d - log(x)/d)*log((b*x + a)^p*c)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/x/(e*x+d),x, algorithm="fricas")
[Out] integral(log((b*x + a)^p*c)/(x^2*e + d*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(c(a+bx)^p\right)}{x\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x+a)**p)/x/(e*x+d),x)
[Out] Integral(log(c*(a + b*x)**p)/(x*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/x/(e*x+d),x, algorithm="giac")
[Out] integrate(log((b*x + a)^p*c)/((x*e + d)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(a+b\,x\right)}^p\right)}{x\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x)^p)/(x*(d + e*x)),x)
[Out] int(log(c*(a + b*x)^p)/(x*(d + e*x)), x)
```

3.224 $\int \frac{\log(c(a+bx)^p)}{x^2(d+ex)}\,dx$

Optimal. Leaf size=146

$$\frac{bp\log(x)}{ad}-\frac{bp\log(a+bx)}{ad}-\frac{\log\left(c(a+bx)^p\right)}{dx}-\frac{e\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d^2}+\frac{e\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d^2}+$$

[Out] b*p*ln(x)/a/d-b*p*ln(b*x+a)/a/d-ln(c*(b*x+a)^p)/d/x-e*ln(-b*x/a)*ln(c*(b*x+a)^p)/d^2+e*ln(c*(b*x+a)^p)*ln(b*(e*x+d)/(-a*e+b*d))/d^2+e*p*polylog(2,-e*(b*x+a)/(-a*e+b*d))/d^2-e*p*polylog(2,1+b*x/a)/d^2

Rubi [A]
time = 0.11, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {46, 2463, 2442, 36, 29, 31, 2441, 2352, 2440, 2438}

$$\frac{ep\,\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d^2}-\frac{ep\,\text{PolyLog}\left(2,\frac{bx}{a}+1\right)}{d^2}-\frac{e\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d^2}+\frac{e\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d^2}-\frac{\log\left(c(a+bx)^p\right)}{dx}+\frac{bp\log(x)}{ad}-\frac{bp\log(a+bx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x)^p]/(x^2*(d + e*x)),x]
```

```
[Out] (b*p*Log[x])/(a*d) - (b*p*Log[a + b*x])/(a*d) - Log[c*(a + b*x)^p]/(d*x) -
(e*Log[-((b*x)/a)]*Log[c*(a + b*x)^p])/d^2 + (e*Log[c*(a + b*x)^p]*Log[(b*(
d + e*x))/(b*d - a*e)])/d^2 + (e*p*PolyLog[2, -((e*(a + b*x))/(b*d - a*e))]
)/d^2 - (e*p*PolyLog[2, 1 + (b*x)/a])/d^2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
```

```
n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\int \frac{\log(c(a+bx)^p)}{x^2(d+ex)}\,dx = \int \left(\frac{\log(c(a+bx)^p)}{dx^2} - \frac{e\log(c(a+bx)^p)}{d^2x} + \frac{e^2\log(c(a+bx)^p)}{d^2(d+ex)}\right)dx$$

$$= \frac{\int \frac{\log(c(a+bx)^p)}{x^2}\,dx}{d} - \frac{e\int \frac{\log(c(a+bx)^p)}{x}\,dx}{d^2} + \frac{e^2\int \frac{\log(c(a+bx)^p)}{d+ex}\,dx}{d^2}$$

$$= -\frac{\log(c(a+bx)^p)}{dx} - \frac{e\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^p)}{d^2} + \frac{e\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d^2}$$

$$= -\frac{\log(c(a+bx)^p)}{dx} - \frac{e\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^p)}{d^2} + \frac{e\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d^2}$$

$$= \frac{bp\log(x)}{ad} - \frac{bp\log(a+bx)}{ad} - \frac{\log(c(a+bx)^p)}{dx} - \frac{e\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^p)}{d^2} + \frac{e\,\mathrm{lo}}{}$$

Mathematica [A]
time = 0.04, size = 139, normalized size = 0.95

$$\frac{bdpx\log(x) - bdpx\log(a+bx) - ad\log(c(a+bx)^p) - aex\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^p) + aex\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right) + aepx\mathrm{Li}_2\left(\frac{e(a+bx)}{-bd+ae}\right) - aepx\mathrm{Li}_2\left(1+\frac{bx}{a}\right)}{ad^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x)^p]/(x^2*(d + e*x)),x]
```

```
[Out] (b*d*p*x*Log[x] - b*d*p*x*Log[a + b*x] - a*d*Log[c*(a + b*x)^p] - a*e*x*Log
[-((b*x)/a)]*Log[c*(a + b*x)^p] + a*e*x*Log[c*(a + b*x)^p]*Log[(b*(d + e*x)
)/(b*d - a*e)] + a*e*p*x*PolyLog[2, (e*(a + b*x))/(-(b*d) + a*e)] - a*e*p*x
*PolyLog[2, 1 + (b*x)/a])/(a*d^2*x)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.48, size = 615, normalized size = 4.21

method	result
risch	$\frac{\ln((bx+a)^p)e\ln(ex+d)}{d^2} - \frac{\ln((bx+a)^p)}{dx} - \frac{\ln((bx+a)^p)e\ln(x)}{d^2} - \frac{pe\,\mathrm{dilog}\left(\frac{(ex+d)b+ae-bd}{ae-bd}\right)}{d^2} - \frac{pe\ln(ex+d)\ln\left(\frac{(ex+d)b+ae-bd}{ae-bd}\right)}{d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x+a)^p)/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] ln((b*x+a)^p)*e/d^2*ln(e*x+d)-ln((b*x+a)^p)/d/x-ln((b*x+a)^p)*e/d^2*ln(x)-p
*e/d^2*dilog(((e*x+d)*b+a*e-b*d)/(a*e-b*d))-p*e/d^2*ln(e*x+d)*ln(((e*x+d)*b
+a*e-b*d)/(a*e-b*d))+b*p*ln(x)/a/d-b*p*ln(b*x+a)/a/d+p*e/d^2*dilog(1/a*(b*x
+a))+p*e/d^2*ln(x)*ln(1/a*(b*x+a))+1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x
+a)^p)*csgn(I*c)/d/x-1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2/d/x+1
```

```
/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)*e/d^2*ln(x)-1/2*I*P
i*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)*e/d^2*ln(x)+1/2*I*Pi*csgn(I*c*(b*x+a)^p)^
3/d/x-1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2*e/d^2*ln(x)+1/2*I*Pi
*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2*e/d^2*ln(e*x+d)+1/2*I*Pi*csgn(I*c*
(b*x+a)^p)^2*csgn(I*c)*e/d^2*ln(e*x+d)+1/2*I*Pi*csgn(I*c*(b*x+a)^p)^3*e/d^2
*ln(x)-1/2*I*Pi*csgn(I*c*(b*x+a)^p)^3*e/d^2*ln(e*x+d)-1/2*I*Pi*csgn(I*c*(b*
x+a)^p)^2*csgn(I*c)/d/x-1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn
(I*c)*e/d^2*ln(e*x+d)+ln(c)*e/d^2*ln(e*x+d)-ln(c)/d/x-ln(c)*e/d^2*ln(x)
```

Maxima [A]
time = 0.36, size = 166, normalized size = 1.14

$$bp\left(\frac{\left(\log\left(\frac{bx}{a}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{bx}{a}\right)\right)e}{bd^2}-\frac{\left(\log\left(xe+d\right)\log\left(-\frac{bxe+bd}{bd-ae}+1\right)+\mathrm{Li}_2\left(\frac{bxe+bd}{bd-ae}\right)\right)e}{bd^2}-\frac{\log\left(bx+a\right)}{ad}+\frac{\log\left(x\right)}{ad}\right)+\left(\frac{e\log\left(xe+d\right)}{d^2}-\frac{e\log\left(x\right)}{d^2}-\frac{1}{dx}\right)\log\left(\left(bx+a\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] b*p*((log(b*x/a + 1)*log(x) + dilog(-b*x/a))*e/(b*d^2) - (log(x*e + d)*log(
-(b*x*e + b*d)/(b*d - a*e) + 1) + dilog((b*x*e + b*d)/(b*d - a*e)))*e/(b*d^
2) - log(b*x + a)/(a*d) + log(x)/(a*d)) + (e*log(x*e + d)/d^2 - e*log(x)/d^
2 - 1/(d*x))*log((b*x + a)^p*c)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x + a)^p*c)/(x^3*e + d*x^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x+a)**p)/x**2/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x + a)^p*c)/((x*e + d)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(a+b\,x\right)}^p\right)}{x^2\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x)^p)/(x^2*(d + e*x)),x)
```

```
[Out] int(log(c*(a + b*x)^p)/(x^2*(d + e*x)), x)
```

3.225 $\int \frac{\log(c(a+bx)^p)}{x^3(d+ex)}\,dx$

Optimal. Leaf size=227

$$-\frac{bp}{2adx}-\frac{b^2p\log(x)}{2a^2d}-\frac{bep\log(x)}{ad^2}+\frac{b^2p\log(a+bx)}{2a^2d}+\frac{bep\log(a+bx)}{ad^2}-\frac{\log\left(c(a+bx)^p\right)}{2dx^2}+\frac{e\log\left(c(a+bx)^p\right)}{d^2x}+\frac{e^2 \mathrm{l}}{}$$

```
[Out] -1/2*b*p/a/d/x-1/2*b^2*p*ln(x)/a^2/d-b*e*p*ln(x)/a/d^2+1/2*b^2*p*ln(b*x+a)/
a^2/d+b*e*p*ln(b*x+a)/a/d^2-1/2*ln(c*(b*x+a)^p)/d/x^2+e*ln(c*(b*x+a)^p)/d^2
/x+e^2*ln(-b*x/a)*ln(c*(b*x+a)^p)/d^3-e^2*ln(c*(b*x+a)^p)*ln(b*(e*x+d)/(-a*
e+b*d))/d^3-e^2*p*polylog(2,-e*(b*x+a)/(-a*e+b*d))/d^3+e^2*p*polylog(2,1+b*
x/a)/d^3
```

Rubi [A]
time = 0.15, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {46, 2463, 2442, 36, 29, 31, 2441, 2352, 2440, 2438}

$$-\frac{e^2p\,\mathrm{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d^3}+\frac{e^2p\,\mathrm{PolyLog}\left(2,\frac{bx}{a}+1\right)}{d^3}-\frac{b^2p\log(x)}{2a^2d}+\frac{b^2p\log(a+bx)}{2a^2d}+\frac{e^2\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d^3}-\frac{e^2\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d^3}+\frac{e\log\left(c(a+bx)^p\right)}{d^2x}-\frac{\log\left(c(a+bx)^p\right)}{2dx^2}-\frac{bep\log(x)}{ad^2}+\frac{bep\log(a+bx)}{ad^2}-\frac{bp}{2adx}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x)^p]/(x^3*(d + e*x)),x]
```

```
[Out] -1/2*(b*p)/(a*d*x) - (b^2*p*Log[x])/(2*a^2*d) - (b*e*p*Log[x])/(a*d^2) + (b
^2*p*Log[a + b*x])/(2*a^2*d) + (b*e*p*Log[a + b*x])/(a*d^2) - Log[c*(a + b*
x)^p]/(2*d*x^2) + (e*Log[c*(a + b*x)^p])/(d^2*x) + (e^2*Log[-((b*x)/a)]*Log
[c*(a + b*x)^p])/d^3 - (e^2*Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e
)])/d^3 - (e^2*p*PolyLog[2, -((e*(a + b*x))/(b*d - a*e))])/d^3 + (e^2*p*Pol
yLog[2, 1 + (b*x)/a])/d^3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(c(a+bx)^p\right)}{x^3(d+ex)}\,dx &= \int \left(\frac{\log\left(c(a+bx)^p\right)}{dx^3} - \frac{e\log\left(c(a+bx)^p\right)}{d^2x^2} + \frac{e^2\log\left(c(a+bx)^p\right)}{d^3x} - \frac{e^3\log\left(c(a+bx)^p\right)}{d^3(d+ex)}\right.\\
&= \frac{\int \frac{\log(c(a+bx)^p)}{x^3}\,dx}{d} - \frac{e\int \frac{\log(c(a+bx)^p)}{x^2}\,dx}{d^2} + \frac{e^2\int \frac{\log(c(a+bx)^p)}{x}\,dx}{d^3} - \frac{e^3\int \frac{\log(c(a+bx)^p)}{d+ex}\,dx}{d^3}\\
&= -\frac{\log\left(c(a+bx)^p\right)}{2dx^2} + \frac{e\log\left(c(a+bx)^p\right)}{d^2x} + \frac{e^2\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d^3} - \frac{e^2\log\left(c(a\right.}{}\\
&= -\frac{\log\left(c(a+bx)^p\right)}{2dx^2} + \frac{e\log\left(c(a+bx)^p\right)}{d^2x} + \frac{e^2\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d^3} - \frac{e^2\log\left(c(a\right.}{}\\
&= -\frac{bp}{2adx} - \frac{b^2p\log(x)}{2a^2d} - \frac{bep\log(x)}{ad^2} + \frac{b^2p\log(a+bx)}{2a^2d} + \frac{bep\log(a+bx)}{ad^2} - \frac{\log\left(c(a+\right.}{2dx^2}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 188, normalized size = 0.83

$$-\frac{\frac{2bdep(\log(x)-\log(a+bx))}{a}+\frac{bd^2p(a+bx\log(x)-bx\log(a+bx))}{a^2x}+\frac{d^2\log(c(a+bx)^p)}{x^2}-\frac{2de\log(c(a+bx)^p)}{x}-2e^2\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)+2e^2\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)+2e^2p\text{Li}_2\left(\frac{e(a+bx)}{-bd+ae}\right)-2e^2p\text{Li}_2\left(1+\frac{bx}{a}\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x)^p]/(x^3*(d + e*x)),x]
```

```
[Out] -1/2*((2*b*d*e*p*(Log[x] - Log[a + b*x]))/a + (b*d^2*p*(a + b*x*Log[x] - b*
x*Log[a + b*x]))/(a^2*x) + (d^2*Log[c*(a + b*x)^p])/x^2 - (2*d*e*Log[c*(a +
 b*x)^p])/x - 2*e^2*Log[-((b*x)/a)]*Log[c*(a + b*x)^p] + 2*e^2*Log[c*(a + b
*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)] + 2*e^2*p*PolyLog[2, (e*(a + b*x))/(-
(b*d) + a*e)] - 2*e^2*p*PolyLog[2, 1 + (b*x)/a])/d^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.48, size = 850, normalized size = 3.74

method	result
risch	$-\frac{bep\ln(x)}{a\,d^2}+\frac{bep\ln(bx+a)}{a\,d^2}-\frac{p\,e^2\,\text{dilog}\left(\frac{bx+a}{a}\right)}{d^3}+\frac{p\,e^2\,\text{dilog}\left(\frac{(ex+d)b+ae-bd}{ae-bd}\right)}{d^3}+\frac{b^2p\ln(bx+a)}{2a^2d}+\frac{\ln((bx+a)^p)e}{d^2x}+\frac{i\pi\text{csgn}(ic($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x+a)^p)/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -b*e*p*ln(x)/a/d^2+b*e*p*ln(b*x+a)/a/d^2-1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*
c*(b*x+a)^p)*csgn(I*c)*e^2/d^3*ln(x)+1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b
```

```
*x+a)^p)*csgn(I*c)*e^2/d^3*ln(e*x+d)+1/2*b^2*p*ln(b*x+a)/a^2/d+1/4*I*Pi*csg
n(I*c*(b*x+a)^p)^3/d/x^2+ln((b*x+a)^p)*e/d^2/x-p*e^2/d^3*dilog(1/a*(b*x+a))
+p*e^2/d^3*dilog(((e*x+d)*b+a*e-b*d)/(a*e-b*d))-ln((b*x+a)^p)*e^2/d^3*ln(e*
x+d)+ln((b*x+a)^p)*e^2/d^3*ln(x)-1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a
)^p)*csgn(I*c)*e/d^2/x-ln(c)*e^2/d^3*ln(e*x+d)+ln(c)*e^2/d^3*ln(x)+ln(c)*e/
d^2/x+1/2*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2*e^2/d^3*ln(x)-1/2*ln
(c)/d/x^2-1/2*I*Pi*csgn(I*c*(b*x+a)^p)^3*e/d^2/x+1/2*I*Pi*csgn(I*(b*x+a)^p)
*csgn(I*c*(b*x+a)^p)^2*e/d^2/x-1/2*I*Pi*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)*e^2
/d^3*ln(e*x+d)-p*e^2/d^3*ln(x)*ln(1/a*(b*x+a))+p*e^2/d^3*ln(e*x+d)*ln(((e*x
+d)*b+a*e-b*d)/(a*e-b*d))-1/2*I*Pi*csgn(I*c*(b*x+a)^p)^3*e^2/d^3*ln(x)+1/4*
I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)*csgn(I*c)/d/x^2-1/2*b*p/x/a/d-1/
2*b^2*p*ln(x)/a^2/d-1/4*I*Pi*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)/d/x^2-1/2*I*Pi
*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^p)^2*e^2/d^3*ln(e*x+d)-1/2*ln((b*x+a)^p
)/d/x^2+1/2*I*Pi*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)*e/d^2/x+1/2*I*Pi*csgn(I*c*
(b*x+a)^p)^3*e^2/d^3*ln(e*x+d)-1/4*I*Pi*csgn(I*(b*x+a)^p)*csgn(I*c*(b*x+a)^
p)^2/d/x^2+1/2*I*Pi*csgn(I*c*(b*x+a)^p)^2*csgn(I*c)*e^2/d^3*ln(x)
```

Maxima [A]
time = 0.35, size = 220, normalized size = 0.97

$$\frac{1}{2}\left(2\left(\frac{\log(bx+a)}{ad^2}-\frac{\log(x)}{ad^2}\right)e+\frac{b\log(bx+a)}{a^2d}-\frac{b\log(x)}{a^2d}-\frac{2\left(\log\left(\frac{bx}{a}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{bx}{a}\right)\right)e^2}{bd^3}+\frac{2\left(\log(xe+d)\log\left(-\frac{bxe+bd}{bd-ae}+1\right)+\mathrm{Li}_2\left(\frac{bxe+bd}{bd-ae}\right)\right)e^2}{bd^3}-\frac{1}{adx}\right)bp-\frac{1}{2}\left(\frac{2e^2\log(xe+d)}{d^3}-\frac{2e^2\log(x)}{d^3}-\frac{2xe-d}{d^2x^2}\right)\log\left((bx+a)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(log(b*x + a)/(a*d^2) - log(x)/(a*d^2))*e + b*log(b*x + a)/(a^2*d) -
 b*log(x)/(a^2*d) - 2*(log(b*x/a + 1)*log(x) + dilog(-b*x/a))*e^2/(b*d^3) +
 2*(log(x*e + d)*log(-(b*x*e + b*d)/(b*d - a*e) + 1) + dilog((b*x*e + b*d)/
(b*d - a*e)))*e^2/(b*d^3) - 1/(a*d*x))*b*p - 1/2*(2*e^2*log(x*e + d)/d^3 -
2*e^2*log(x)/d^3 - (2*x*e - d)/(d^2*x^2))*log((b*x + a)^p*c)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x + a)^p*c)/(x^4*e + d*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c(a+bx)^p\right)}{x^3\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x+a)**p)/x**3/(e*x+d),x)
```

```
[Out] Integral(log(c*(a + b*x)**p)/(x**3*(d + e*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x+a)^p)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x + a)^p*c)/((x*e + d)*x^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(a+b\,x\right)}^p\right)}{x^3\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x)^p)/(x^3*(d + e*x)),x)
```

```
[Out] int(log(c*(a + b*x)^p)/(x^3*(d + e*x)), x)
```

3.226 $\int \frac{x^3 \log\left(c\left(a+bx^2\right)^p\right)}{d+ex} \, dx$

Optimal. Leaf size=394

$$-\frac{2d^2px}{e^3}+\frac{2apx}{3be}+\frac{dpx^2}{2e^2}-\frac{2px^3}{9e}+\frac{2\sqrt{a}\,d^2p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,e^3}-\frac{2a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3b^{3/2}e}+\frac{d^3p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right.}{e^4}$$

```
[Out] -2*d^2*p*x/e^3+2/3*a*p*x/b/e+1/2*d*p*x^2/e^2-2/9*p*x^3/e-2/3*a^(3/2)*p*arct
an(x*b^(1/2)/a^(1/2))/b^(3/2)/e+d^2*x*ln(c*(b*x^2+a)^p)/e^3+1/3*x^3*ln(c*(b
*x^2+a)^p)/e-1/2*d*(b*x^2+a)*ln(c*(b*x^2+a)^p)/b/e^2-d^3*ln(e*x+d)*ln(c*(b*
x^2+a)^p)/e^4+d^3*p*ln(e*x+d)*ln(e*((-a)^(1/2)-x*b^(1/2))/(e*(-a)^(1/2)+d*b
^(1/2)))/e^4+d^3*p*ln(e*x+d)*ln(-e*((-a)^(1/2)+x*b^(1/2))/(-e*(-a)^(1/2)+d*
b^(1/2)))/e^4+d^3*p*polylog(2,(e*x+d)*b^(1/2)/(-e*(-a)^(1/2)+d*b^(1/2)))/e^
4+d^3*p*polylog(2,(e*x+d)*b^(1/2)/(e*(-a)^(1/2)+d*b^(1/2)))/e^4+2*d^2*p*arc
tan(x*b^(1/2)/a^(1/2))*a^(1/2)/e^3/b^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 15, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {2516, 2498, 327, 211, 2504, 2436, 2332, 2505, 308, 2512, 266, 2463, 2441, 2440, 2438}

$$\frac{d^3p\,\text{PolyLog}\left(2,\frac{\sqrt{b}\,(d+ex)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)}{e^4}+\frac{d^3p\,\text{PolyLog}\left(2,\frac{\sqrt{b}\,(d+ex)}{\sqrt{-a}\,e+\sqrt{b}\,d}\right)}{e^4}-\frac{2a^{3/2}p\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3b^{3/2}e}+\frac{2\sqrt{a}\,d^2p\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,e^3}-\frac{d^3\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e^4}+\frac{d^2x\log\left(c\left(a+bx^2\right)^p\right)}{e^3}-\frac{d\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{2be^2}+\frac{x^3\log\left(c\left(a+bx^2\right)^p\right)}{3e}+\frac{d^3p\log(d+ex)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{-a}\,e+\sqrt{b}\,d}\right)}{e^4}+\frac{d^3p\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)}{e^4}+\frac{2apx}{3be}-\frac{2d^2px}{e^3}+\frac{dpx^2}{2e^2}-\frac{2px^3}{9e}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Log[c*(a + b*x^2)^p])/(d + e*x),x]
```

```
[Out] (-2*d^2*p*x)/e^3 + (2*a*p*x)/(3*b*e) + (d*p*x^2)/(2*e^2) - (2*p*x^3)/(9*e)
+ (2*Sqrt[a]*d^2*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*e^3) - (2*a^(3/2)*
p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3*b^(3/2)*e) + (d^3*p*Log[(e*(Sqrt[-a] - Sq
rt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])/e^4 + (d^3*p*Log[-((e*(Sq
rt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d + e*x])/e^4 + (d^2*x*
Log[c*(a + b*x^2)^p])/e^3 + (x^3*Log[c*(a + b*x^2)^p])/(3*e) - (d*(a + b*x^
2)*Log[c*(a + b*x^2)^p])/(2*b*e^2) - (d^3*Log[d + e*x]*Log[c*(a + b*x^2)^p]
)/e^4 + (d^3*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/e^
4 + (d^3*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/e^4
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{x^3 \log\left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \left(\frac{d^2 \log\left(c(a+bx^2)^p\right)}{e^3} - \frac{dx \log\left(c(a+bx^2)^p\right)}{e^2} + \frac{x^2 \log\left(c(a+bx^2)^p\right)}{e} - \frac{d^3 \log}{e}\right.$$

$$= \frac{d^2 \int \log\left(c(a+bx^2)^p\right)\,dx}{e^3} - \frac{d^3 \int \frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx}{e^3} - \frac{d \int x \log\left(c(a+bx^2)^p\right)\,dx}{e^2} +$$

$$= \frac{d^2 x \log\left(c(a+bx^2)^p\right)}{e^3} + \frac{x^3 \log\left(c(a+bx^2)^p\right)}{3e} - \frac{d^3 \log(d+ex)\log\left(c(a+bx^2)^p\right)}{e^4}$$

$$= -\frac{2d^2px}{e^3} + \frac{d^2 x \log\left(c(a+bx^2)^p\right)}{e^3} + \frac{x^3 \log\left(c(a+bx^2)^p\right)}{3e} - \frac{d^3 \log(d+ex)\log\left(c(\right.}{e^4}$$

$$= -\frac{2d^2px}{e^3} + \frac{2apx}{3be} + \frac{dpx^2}{2e^2} - \frac{2px^3}{9e} + \frac{2\sqrt{a}\,d^2 p \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,e^3} + \frac{d^2 x \log\left(c(a+b\right.}{e^3}$$

$$= -\frac{2d^2px}{e^3} + \frac{2apx}{3be} + \frac{dpx^2}{2e^2} - \frac{2px^3}{9e} + \frac{2\sqrt{a}\,d^2 p \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,e^3} - \frac{2a^{3/2} p \tan^{-1}\left(\right.}{3b^{3/2}e}$$

$$= -\frac{2d^2px}{e^3} + \frac{2apx}{3be} + \frac{dpx^2}{2e^2} - \frac{2px^3}{9e} + \frac{2\sqrt{a}\,d^2 p \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,e^3} - \frac{2a^{3/2} p \tan^{-1}\left(\right.}{3b^{3/2}e}$$

$$= -\frac{2d^2px}{e^3} + \frac{2apx}{3be} + \frac{dpx^2}{2e^2} - \frac{2px^3}{9e} + \frac{2\sqrt{a}\,d^2 p \tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,e^3} - \frac{2a^{3/2} p \tan^{-1}\left(\right.}{3b^{3/2}e}$$

Mathematica [A]
time = 0.23, size = 338, normalized size = 0.86

$$\frac{-4e^3p\left(-\frac{3ax}{b}+x^3+\frac{3a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}\right)-36d^2ep\left(x-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}\right)+18d^2ex\log\left(c(a+bx^2)^p\right)+6e^3x^3\log\left(c(a+bx^2)^p\right)-18d^3\log(d+ex)\log\left(c(a+bx^2)^p\right)+9de^2\left(px^2-\frac{(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b}\right)+18d^3p\left(\left(\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)+\log\left(\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{-\sqrt{b}\,d+\sqrt{-a}\,e}\right)\right)\log(d+ex)+\mathrm{Li}_2\left(\frac{\sqrt{b}\,(d+ex)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)+\mathrm{Li}_2\left(\frac{\sqrt{b}\,(d+ex)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\right)}{18e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Log[c*(a + b*x^2)^p])/(d + e*x),x]
```

```
[Out] (-4*e^3*p*((-3*a*x)/b + x^3 + (3*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/
2)) - 36*d^2*e*p*(x - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b]) + 18*d
^2*e*x*Log[c*(a + b*x^2)^p] + 6*e^3*x^3*Log[c*(a + b*x^2)^p] - 18*d^3*Log[d
```

```
 + e*x]*Log[c*(a + b*x^2)^p] + 9*d*e^2*(p*x^2 - ((a + b*x^2)*Log[c*(a + b*x
^2)^p])/b) + 18*d^3*p*((Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a
]*e)] + Log[(e*(Sqrt[-a] + Sqrt[b]*x))/(-(Sqrt[b]*d) + Sqrt[-a]*e)])*Log[d
+ e*x] + PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)] + PolyLog
[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)]))/(18*e^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.50, size = 1083, normalized size = 2.75

method	result	size
risch	Expression too large to display	1083

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(b*x^2+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*p/b/e^2*a*d-2/3/b*p/e*a^2/(b*a)^(1/2)*arctan(1/2*(2*(e*x+d)*b-2*b*d)/e/
(b*a)^(1/2))-1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)/e
^3*x*d^2-2*d^2*p*x/e^3+p/e^4*d^3*dilog((e*(-b*a)^(1/2)-(e*x+d)*b+b*d)/(e*(-
b*a)^(1/2)+b*d))+p/e^4*d^3*dilog((e*(-b*a)^(1/2)+(e*x+d)*b-b*d)/(e*(-b*a)^(
1/2)-b*d))+1/3*ln(c)/e*x^3-49/18*p/e^4*d^3+1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csg
n(I*c*(b*x^2+a)^p)*csgn(I*c)*d^3/e^4*ln(e*x+d)+1/4*I*Pi*csgn(I*(b*x^2+a)^p)
*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)/e^2*d*x^2-1/2*ln(c)/e^2*d*x^2+ln(c)/e^3*x*
d^2-ln(c)*d^3/e^4*ln(e*x+d)+2*p/e^3*a/(b*a)^(1/2)*arctan(1/2*(2*(e*x+d)*b-2
*b*d)/e/(b*a)^(1/2))*d^2-1/2*ln((b*x^2+a)^p)/e^2*d*x^2+ln((b*x^2+a)^p)/e^3*
x*d^2-ln((b*x^2+a)^p)*d^3/e^4*ln(e*x+d)+1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I
*c*(b*x^2+a)^p)^2/e^3*x*d^2+p/e^4*d^3*ln(e*x+d)*ln((e*(-b*a)^(1/2)-(e*x+d)*
b+b*d)/(e*(-b*a)^(1/2)+b*d))+p/e^4*d^3*ln(e*x+d)*ln((e*(-b*a)^(1/2)+(e*x+d)
*b-b*d)/(e*(-b*a)^(1/2)-b*d))+1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)/e^
3*x*d^2-1/6*I*Pi*csgn(I*c*(b*x^2+a)^p)^3/e*x^3+1/4*I*Pi*csgn(I*c*(b*x^2+a)^
p)^3/e^2*d*x^2-1/4*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2/e^2*d*x
^2-1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2*d^3/e^4*ln(e*x+d)-1
/6*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)/e*x^3+2/3*a*p*x
/b/e+1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^3*d^3/e^4*ln(e*x+d)-1/2*I*Pi*csgn(I*c*(
b*x^2+a)^p)^3/e^3*x*d^2-1/4*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)/e^2*d*x^
2+1/6*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2/e*x^3-2/9*p*x^3/e+1/
2*d*p*x^2/e^2-1/2/b*p/e^2*a*d*ln((e*x+d)^2*b-2*d*(e*x+d)*b+a*e^2+b*d^2)+1/3
*ln((b*x^2+a)^p)/e*x^3+1/6*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)/e*x^3-1/2
*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)*d^3/e^4*ln(e*x+d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(x^3*log((b*x^2 + a)^p*c)/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x^3*log((b*x^2 + a)^p*c)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \log \left(c(a+bx^2)^p\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(b*x**2+a)**p)/(e*x+d),x)
```

```
[Out] Integral(x**3*log(c*(a + b*x**2)**p)/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(x^3*log((b*x^2 + a)^p*c)/(x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*log(c*(a + b*x^2)^p))/(d + e*x),x)
```

```
[Out] int((x^3*log(c*(a + b*x^2)^p))/(d + e*x), x)
```

3.227 $\int \frac{x^2 \log\left(c(a+bx^2)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=313

$$\frac{2dpx}{e^2}-\frac{px^2}{2e}-\frac{2\sqrt{a}\,dp\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,e^2}-\frac{d^2p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{e^3}-\frac{d^2p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)}{e^3}$$

```
[Out] 2*d*p*x/e^2-1/2*p*x^2/e-d*x*ln(c*(b*x^2+a)^p)/e^2+1/2*(b*x^2+a)*ln(c*(b*x^2
+a)^p)/b/e+d^2*ln(e*x+d)*ln(c*(b*x^2+a)^p)/e^3-d^2*p*ln(e*x+d)*ln(e*((-a)^(
1/2)-x*b^(1/2))/(e*(-a)^(1/2)+d*b^(1/2)))/e^3-d^2*p*ln(e*x+d)*ln(-e*((-a)^(
1/2)+x*b^(1/2))/(-e*(-a)^(1/2)+d*b^(1/2)))/e^3-d^2*p*polylog(2,(e*x+d)*b^(1
/2)/(-e*(-a)^(1/2)+d*b^(1/2)))/e^3-d^2*p*polylog(2,(e*x+d)*b^(1/2)/(e*(-a)^
(1/2)+d*b^(1/2)))/e^3-2*d*p*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/e^2/b^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {2516, 2498, 327, 211, 2504, 2436, 2332, 2512, 266, 2463, 2441, 2440, 2438}

$$-\frac{d^2p\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)}{e^3}-\frac{d^2p\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{-a}\,e+\sqrt{b}\,d}\right)}{e^3}-\frac{2\sqrt{a}\,dp\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,e^2}+\frac{d^2\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e^3}-\frac{dx\log\left(c(a+bx^2)^p\right)}{e^2}+\frac{(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2be}-\frac{d^2p\log(d+ex)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{-a}\,e+\sqrt{b}\,d}\right)}{e^3}-\frac{d^2p\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)}{e^3}+\frac{2dpx}{e^2}-\frac{px^2}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Log[c*(a + b*x^2)^p])/(d + e*x),x]
```

```
[Out] (2*d*p*x)/e^2 - (p*x^2)/(2*e) - (2*Sqrt[a]*d*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(Sqrt[b]*e^2) - (d^2*p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a
]*e)]*Log[d + e*x])/e^3 - (d^2*p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*
d - Sqrt[-a]*e))]*Log[d + e*x])/e^3 - (d*x*Log[c*(a + b*x^2)^p])/e^2 + ((a
+ b*x^2)*Log[c*(a + b*x^2)^p])/(2*b*e) + (d^2*Log[d + e*x]*Log[c*(a + b*x^2
)^p])/e^3 - (d^2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)]
)/e^3 - (d^2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/e^
3
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{x^2 \log \left(c(a+bx^2)^p\right)}{d+ex} \, dx = \int \left(-\frac{d \log \left(c(a+bx^2)^p\right)}{e^2} + \frac{x \log \left(c(a+bx^2)^p\right)}{e} + \frac{d^2 \log \left(c(a+bx^2)^p\right)}{e^2(d+ex)}\right) \, dx$$

$$= -\frac{d \int \log \left(c(a+bx^2)^p\right) \, dx}{e^2} + \frac{d^2 \int \frac{\log(c(a+bx^2)^p)}{d+ex} \, dx}{e^2} + \frac{\int x \log \left(c(a+bx^2)^p\right) \, dx}{e}$$

$$= -\frac{dx \log \left(c(a+bx^2)^p\right)}{e^2} + \frac{d^2 \log(d+ex) \log \left(c(a+bx^2)^p\right)}{e^3} + \frac{\text{Subst}\left(\int \log \left(c(a+\right.\right.}{2e}$$

$$= \frac{2dpx}{e^2} - \frac{dx \log \left(c(a+bx^2)^p\right)}{e^2} + \frac{d^2 \log(d+ex) \log \left(c(a+bx^2)^p\right)}{e^3} + \text{Subst}\left(\int \log \left(\right.\right.$$

$$= \frac{2dpx}{e^2} - \frac{px^2}{2e} - \frac{2\sqrt{a}\, dp \tan^{-1}\left(\frac{\sqrt{b}\, x}{\sqrt{a}}\right)}{\sqrt{b}\, e^2} - \frac{dx \log \left(c(a+bx^2)^p\right)}{e^2} + \frac{(a+bx^2) \log \left(\right.}{2be}$$

$$= \frac{2dpx}{e^2} - \frac{px^2}{2e} - \frac{2\sqrt{a}\, dp \tan^{-1}\left(\frac{\sqrt{b}\, x}{\sqrt{a}}\right)}{\sqrt{b}\, e^2} - \frac{d^2 p \log \left(\frac{e\left(\sqrt{-a}-\sqrt{b}\, x\right)}{\sqrt{b}\, d+\sqrt{-a}\, e}\right) \log(d+ex}{e^3}$$

$$= \frac{2dpx}{e^2} - \frac{px^2}{2e} - \frac{2\sqrt{a}\, dp \tan^{-1}\left(\frac{\sqrt{b}\, x}{\sqrt{a}}\right)}{\sqrt{b}\, e^2} - \frac{d^2 p \log \left(\frac{e\left(\sqrt{-a}-\sqrt{b}\, x\right)}{\sqrt{b}\, d+\sqrt{-a}\, e}\right) \log(d+ex}{e^3}$$

$$= \frac{2dpx}{e^2} - \frac{px^2}{2e} - \frac{2\sqrt{a}\, dp \tan^{-1}\left(\frac{\sqrt{b}\, x}{\sqrt{a}}\right)}{\sqrt{b}\, e^2} - \frac{d^2 p \log \left(\frac{e\left(\sqrt{-a}-\sqrt{b}\, x\right)}{\sqrt{b}\, d+\sqrt{-a}\, e}\right) \log(d+ex}{e^3}$$

Mathematica [A]
time = 0.11, size = 271, normalized size = 0.87

$$\frac{-e^2px^2 + 4dep\left(x - \frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\, x}{\sqrt{a}}\right)}{\sqrt{b}}\right) - 2dex \log \left(c(a+bx^2)^p\right) + \frac{e^2(a+bx^2)\log(c(a+bx^2)^p)}{b} + 2d^2 \log(d+ex) \log \left(c(a+bx^2)^p\right) - 2d^2p\left(\left(\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\, x\right)}{\sqrt{b}\, d+\sqrt{-a}\, e}\right) + \log\left(\frac{e\left(\sqrt{-a}+\sqrt{b}\, x\right)}{-\sqrt{b}\, d+\sqrt{-a}\, e}\right)\right)\log(d+ex) + \text{Li}_2\left(\frac{\sqrt{b}\,(d+ex)}{\sqrt{b}\, d-\sqrt{-a}\, e}\right) + \text{Li}_2\left(\frac{\sqrt{b}\,(d+ex)}{\sqrt{b}\, d+\sqrt{-a}\, e}\right)\right)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Log[c*(a + b*x^2)^p])/(d + e*x),x]
```

```
[Out] (-(e^2*p*x^2) + 4*d*e*p*(x - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b])
 - 2*d*e*x*Log[c*(a + b*x^2)^p] + (e^2*(a + b*x^2)*Log[c*(a + b*x^2)^p])/b
+ 2*d^2*Log[d + e*x]*Log[c*(a + b*x^2)^p] - 2*d^2*p*((Log[(e*(Sqrt[-a] - Sq
```

```
rt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)] + Log[(e*(Sqrt[-a] + Sqrt[b]*x))/(-(Sqr
t[b]*d) + Sqrt[-a]*e)])*Log[d + e*x] + PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt
[b]*d - Sqrt[-a]*e)] + PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]
*e)]))/(2*e^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.48, size = 825, normalized size = 2.64

method	result
risch	$\frac{i\pi\,\mathrm{csgn}\left(i\left(b\,x^2+a\right)^p\right)\mathrm{csgn}\left(ic\left(b\,x^2+a\right)^p\right)^2 d^2 \ln(ex+d)}{2e^3} - \frac{i\pi \mathrm{csgn}\left(ic\left(b\,x^2+a\right)^p\right)^2 \mathrm{csgn}(ic) dx}{2e^2} + \frac{2dpx}{e^2} - \frac{p\,d^2\,\mathrm{dilog}\left(\frac{e\sqrt{-ba}-(ex+}{e\sqrt{-ba}+}\right.}{e^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(b*x^2+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d*p*x/e^2-p/e^3*d^2*ln(e*x+d)*ln((e*(-b*a)^(1/2)-(e*x+d)*b+b*d)/(e*(-b*a)
^(1/2)+b*d))-p/e^3*d^2*ln(e*x+d)*ln((e*(-b*a)^(1/2)+(e*x+d)*b-b*d)/(e*(-b*a
)^(1/2)-b*d))-ln(c)/e^2*d*x+ln(c)*d^2/e^3*ln(e*x+d)-1/4*I*Pi*csgn(I*(b*x^2+
a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)/e*x^2-2*p/e^2*a*d/(b*a)^(1/2)*arctan(
1/2*(2*(e*x+d)*b-2*b*d)/e/(b*a)^(1/2))-ln((b*x^2+a)^p)/e^2*d*x+ln((b*x^2+a)
^p)*d^2/e^3*ln(e*x+d)+1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^3/e^2*d*x+1/2*ln(c)/e*
x^2-1/4*I*Pi*csgn(I*c*(b*x^2+a)^p)^3/e*x^2+1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csg
n(I*c*(b*x^2+a)^p)*csgn(I*c)/e^2*d*x+1/2/b*p/e*a*ln((e*x+d)^2*b-2*d*(e*x+d)
*b+a*e^2+b*d^2)-1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c
)*d^2/e^3*ln(e*x+d)+5/2*p/e^3*d^2-p/e^3*d^2*dilog((e*(-b*a)^(1/2)-(e*x+d)*b
+b*d)/(e*(-b*a)^(1/2)+b*d))-p/e^3*d^2*dilog((e*(-b*a)^(1/2)+(e*x+d)*b-b*d)/
(e*(-b*a)^(1/2)-b*d))-1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^3*d^2/e^3*ln(e*x+d)-1/
2*p*x^2/e+1/2*ln((b*x^2+a)^p)/e*x^2-1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(
b*x^2+a)^p)^2/e^2*d*x+1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)*d^2/e^3*ln
(e*x+d)+1/4*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)/e*x^2+1/4*I*Pi*csgn(I*(b
*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2/e*x^2+1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(
I*c*(b*x^2+a)^p)^2*d^2/e^3*ln(e*x+d)-1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(
I*c)/e^2*d*x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(x^2*log((b*x^2 + a)^p*c)/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="fricas")
[Out] integral(x^2*log((b*x^2 + a)^p*c)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \log \left(c(a+bx^2)^p\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(b*x**2+a)**p)/(e*x+d),x)
[Out] Integral(x**2*log(c*(a + b*x**2)**p)/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(x^2*log((b*x^2 + a)^p*c)/(x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*log(c*(a + b*x^2)^p))/(d + e*x),x)
[Out] int((x^2*log(c*(a + b*x^2)^p))/(d + e*x), x)
```

3.228 $\int \frac{x \log\left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=256

$$-\frac{2px}{e}+\frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,e}+\frac{dp\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{e^2}+\frac{dp\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)\log(d}{e^2}$$

```
[Out] -2*p*x/e+x*ln(c*(b*x^2+a)^p)/e-d*ln(e*x+d)*ln(c*(b*x^2+a)^p)/e^2+d*p*ln(e*x
+d)*ln(e*((-a)^(1/2)-x*b^(1/2))/(e*(-a)^(1/2)+d*b^(1/2)))/e^2+d*p*ln(e*x+d)
*ln(-e*((-a)^(1/2)+x*b^(1/2))/(-e*(-a)^(1/2)+d*b^(1/2)))/e^2+d*p*polylog(2,
(e*x+d)*b^(1/2)/(-e*(-a)^(1/2)+d*b^(1/2)))/e^2+d*p*polylog(2,(e*x+d)*b^(1/2
)/(e*(-a)^(1/2)+d*b^(1/2)))/e^2+2*p*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/e/b^(
1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2516, 2498, 327, 211, 2512, 266, 2463, 2441, 2440, 2438}

$$\frac{dp\,\text{PolyLog}\left(2,\frac{\sqrt{b}\,(d+ex)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)}{e^2}+\frac{dp\,\text{PolyLog}\left(2,\frac{\sqrt{b}\,(d+ex)}{\sqrt{-a}\,e+\sqrt{b}\,d}\right)}{e^2}+\frac{2\sqrt{a}\,p\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,e}-\frac{d\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e^2}+\frac{x\log\left(c\left(a+bx^2\right)^p\right)}{e}+\frac{dp\log(d+ex)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{-a}\,e+\sqrt{b}\,d}\right)}{e^2}+\frac{dp\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)}{e^2}-\frac{2px}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[c*(a + b*x^2)^p])/(d + e*x),x]
```

```
[Out] (-2*p*x)/e + (2*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*e) + (d*p*L
og[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])/e^2 +
 (d*p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d + e
*x])/e^2 + (x*Log[c*(a + b*x^2)^p])/e - (d*Log[d + e*x]*Log[c*(a + b*x^2)^p
])/e^2 + (d*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/e^2
 + (d*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/e^2
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{x \log\left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx = \int \left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{e} - \frac{d\log\left(c\left(a+bx^2\right)^p\right)}{e(d+ex)}\right)dx$$

$$= \frac{\int \log\left(c\left(a+bx^2\right)^p\right)\,dx}{e} - \frac{d\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx}{e}$$

$$= \frac{x\log\left(c\left(a+bx^2\right)^p\right)}{e} - \frac{d\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e^2} + \frac{(2bdp)\int \frac{x\log(d+ex)}{a+bx^2}\,dx}{e^2} -$$

$$= -\frac{2px}{e} + \frac{x\log\left(c\left(a+bx^2\right)^p\right)}{e} - \frac{d\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e^2} + \frac{(2bdp)\int\left(-\frac{}{2}\right.}{}$$

$$= -\frac{2px}{e} + \frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,e} + \frac{x\log\left(c\left(a+bx^2\right)^p\right)}{e} - \frac{d\log(d+ex)\log\left(c(a+\right.}{e^2}$$

$$= -\frac{2px}{e} + \frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,e} + \frac{dp\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{e^2} + \frac{dp}{}$$

$$= -\frac{2px}{e} + \frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,e} + \frac{dp\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{e^2} + \frac{dp}{}$$

$$= -\frac{2px}{e} + \frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,e} + \frac{dp\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{e^2} + \frac{dp}{}$$

Mathematica [A]

time = 0.08, size = 225, normalized size = 0.88

$$\frac{-2ep\left(x-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}\right)+ex\log\left(c\left(a+bx^2\right)^p\right)-d\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)+dp\left(\left(\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)+\log\left(\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{-\sqrt{b}\,d+\sqrt{-a}\,e}\right)\right)\log(d+ex)+\text{Li}_2\left(\frac{\sqrt{b}\,(d+ex)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)+\text{Li}_2\left(\frac{\sqrt{b}\,(d+ex)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[c*(a + b*x^2)^p])/(d + e*x),x]
```

```
[Out] (-2*e*p*(x - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b]) + e*x*Log[c*(a
+ b*x^2)^p] - d*Log[d + e*x]*Log[c*(a + b*x^2)^p] + d*p*((Log[(e*(Sqrt[-a]
- Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)] + Log[(e*(Sqrt[-a] + Sqrt[b]*x))/(-
(Sqrt[b]*d) + Sqrt[-a]*e)])*Log[d + e*x] + PolyLog[2, (Sqrt[b]*(d + e*x))/(
Sqrt[b]*d - Sqrt[-a]*e)] + PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt
[-a]*e)]))/e^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.51, size = 576, normalized size = 2.25

method	result
risch	$\frac{\ln\left(\left(b\,x^{2}+a\right)^{p}\right)x}{e}-\frac{\ln\left(\left(b\,x^{2}+a\right)^{p}\right)d\ln\left(ex+d\right)}{e^{2}}-\frac{2px}{e}-\frac{2pd}{e^{2}}+\frac{2pa\arctan\left(\frac{2(ex+d)b-2bd}{2e\sqrt{ba}}\right)}{e\sqrt{ba}}+\frac{pd\ln\left(ex+d\right)\ln\left(\frac{e\sqrt{-ba}-(ex+d}{e\sqrt{-ba}+b}\right.}{e^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(b*x^2+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] ln((b*x^2+a)^p)/e*x-ln((b*x^2+a)^p)*d/e^2*ln(e*x+d)-2*p*x/e-2*p/e^2*d+2*p/e
*a/(b*a)^(1/2)*arctan(1/2*(2*(e*x+d)*b-2*b*d)/e/(b*a)^(1/2))+p/e^2*d*ln(e*x
+d)*ln((e*(-b*a)^(1/2)-(e*x+d)*b+b*d)/(e*(-b*a)^(1/2)+b*d))+p/e^2*d*ln(e*x+
d)*ln((e*(-b*a)^(1/2)+(e*x+d)*b-b*d)/(e*(-b*a)^(1/2)-b*d))+p/e^2*d*dilog((e
*(-b*a)^(1/2)-(e*x+d)*b+b*d)/(e*(-b*a)^(1/2)+b*d))+p/e^2*d*dilog((e*(-b*a)^
(1/2)+(e*x+d)*b-b*d)/(e*(-b*a)^(1/2)-b*d))+1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^2
*csgn(I*c)/e*x-1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^3/e*x-1/2*I*Pi*csgn(I*(b*x^2+
a)^p)*csgn(I*c*(b*x^2+a)^p)^2*d/e^2*ln(e*x+d)+1/2*I*Pi*csgn(I*(b*x^2+a)^p)*
csgn(I*c*(b*x^2+a)^p)*csgn(I*c)*d/e^2*ln(e*x+d)-1/2*I*Pi*csgn(I*(b*x^2+a)^p
)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)/e*x-1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn
(I*c)*d/e^2*ln(e*x+d)+1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^3*d/e^2*ln(e*x+d)+1/2*
I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2/e*x+ln(c)/e*x-ln(c)*d/e^2*
ln(e*x+d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(x*log((b*x^2 + a)^p*c)/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="fricas")
[Out] integral(x*log((b*x^2 + a)^p*c)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \log \left(c(a+bx^2)^p\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(b*x**2+a)**p)/(e*x+d),x)
[Out] Integral(x*log(c*(a + b*x**2)**p)/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(x*log((b*x^2 + a)^p*c)/(x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(c*(a + b*x^2)^p))/(d + e*x),x)
[Out] int((x*log(c*(a + b*x^2)^p))/(d + e*x), x)
```

3.229 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=201

$$-\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{e}-\frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)\log(d+ex)}{e}+\frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e}$$

```
[Out] ln(e*x+d)*ln(c*(b*x^2+a)^p)/e-p*ln(e*x+d)*ln(e*((-a)^(1/2)-x*b^(1/2))/(e*(-
a)^(1/2)+d*b^(1/2)))/e-p*ln(e*x+d)*ln(-e*((-a)^(1/2)+x*b^(1/2))/(-e*(-a)^(1
/2)+d*b^(1/2)))/e-p*polylog(2,(e*x+d)*b^(1/2)/(-e*(-a)^(1/2)+d*b^(1/2)))/e-
p*polylog(2,(e*x+d)*b^(1/2)/(e*(-a)^(1/2)+d*b^(1/2)))/e
```

Rubi [A]
time = 0.13, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2512, 266, 2463, 2441, 2440, 2438}

$$-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{b}\,(d+ex)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)}{e}-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{b}\,(d+ex)}{\sqrt{-a}\,e+\sqrt{b}\,d}\right)}{e}+\frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e}-\frac{p\log(d+ex)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{-a}\,e+\sqrt{b}\,d}\right)}{e}-\frac{p\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/(d + e*x),x]
```

```
[Out] -((p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])
/e) - (p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d
+ e*x])/e + (Log[d + e*x]*Log[c*(a + b*x^2)^p])/e - (p*PolyLog[2, (Sqrt[b]*
(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/e - (p*PolyLog[2, (Sqrt[b]*(d + e*x))
/(Sqrt[b]*d + Sqrt[-a]*e)])/e
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{d+ex}\,dx = \frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e} - \frac{(2bp)\int \frac{x\log(d+ex)}{a+bx^2}\,dx}{e}$$

$$= \frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e} - \frac{(2bp)\int \left(-\frac{\log(d+ex)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\log(d+ex)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{\right.}}\right.}{e}$$

$$= \frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e} + \frac{\left(\sqrt{b}\,p\right)\int \frac{\log(d+ex)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{e} - \frac{\left(\sqrt{b}\,p\right)\int \frac{\log(d+ex)}{\sqrt{-a}+\sqrt{}}}{e}$$

$$= -\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)\log(d+ex)}{e}$$

$$= -\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)\log(d+ex)}{e}$$

$$= -\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)\log(d+ex)}{e}$$

Mathematica [A]
time = 0.02, size = 201, normalized size = 1.00

$$-\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e} - \frac{p\text{Li}_2\left(\frac{\sqrt{b}\,(d+ex)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)}{e} - \frac{p\text{Li}_2\left(\frac{\sqrt{b}\,(d+ex)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/(d + e*x),x]
```

```
[Out] -((p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])
/e) - (p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d
+ e*x])/e + (Log[d + e*x]*Log[c*(a + b*x^2)^p])/e - (p*PolyLog[2, (Sqrt[b]*
(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/e - (p*PolyLog[2, (Sqrt[b]*(d + e*x))
/(Sqrt[b]*d + Sqrt[-a]*e)])/e
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.37, size = 366, normalized size = 1.82

method	result
risch	$\frac{\ln(ex+d)\ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{e}-\frac{p\ln(ex+d)\ln\left(\frac{e\sqrt{-ba}-(ex+d)b+bd}{e\sqrt{-ba}+bd}\right)}{e}-\frac{p\ln(ex+d)\ln\left(\frac{e\sqrt{-ba}+(ex+d)b-bd}{e\sqrt{-ba}-bd}\right)}{e}-\frac{p\,\mathrm{dilog}\left(\frac{e\sqrt{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(c*(b*x^2+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)

[Out] ln(e*x+d)/e*ln((b*x^2+a)^p)-p/e*ln(e*x+d)*ln((e*(-b*a)^(1/2)-(e*x+d)*b+b*d)/(e*(-b*a)^(1/2)+b*d))-p/e*ln(e*x+d)*ln((e*(-b*a)^(1/2)+(e*x+d)*b-b*d)/(e*(-b*a)^(1/2)-b*d))-p/e*dilog((e*(-b*a)^(1/2)-(e*x+d)*b+b*d)/(e*(-b*a)^(1/2)+b*d))-p/e*dilog((e*(-b*a)^(1/2)+(e*x+d)*b-b*d)/(e*(-b*a)^(1/2)-b*d))+1/2*I*ln(e*x+d)/e*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-1/2*I*ln(e*x+d)/e*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-1/2*I*ln(e*x+d)/e*Pi*csgn(I*c*(b*x^2+a)^p)^3+1/2*I*ln(e*x+d)/e*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+ln(e*x+d)/e*ln(c)

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="maxima")

[Out] integrate(log((b*x^2 + a)^p*c)/(x*e + d), x)

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="fricas")

[Out] integral(log((b*x^2 + a)^p*c)/(x*e + d), x)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(c*(b*x**2+a)**p)/(e*x+d),x)

```
[Out] Integral(log(c*(a + b*x**2)**p)/(d + e*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(log((b*x^2 + a)^p*c)/(x*e + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)/(d + e*x),x)
[Out] int(log(c*(a + b*x^2)^p)/(d + e*x), x)
```

3.230 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x(d+ex)}\,dx$

Optimal. Leaf size=247

$$\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{d}+\frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)\log(d+ex)}{d}+\frac{\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2\right)}{2d}$$

```
[Out] 1/2*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)/d-ln(e*x+d)*ln(c*(b*x^2+a)^p)/d+p*ln(e*x
+d)*ln(e*((-a)^(1/2)-x*b^(1/2))/(e*(-a)^(1/2)+d*b^(1/2)))/d+p*ln(e*x+d)*ln(
-e*((-a)^(1/2)+x*b^(1/2))/(-e*(-a)^(1/2)+d*b^(1/2)))/d+1/2*p*polylog(2,1+b*
x^2/a)/d+p*polylog(2,(e*x+d)*b^(1/2)/(-e*(-a)^(1/2)+d*b^(1/2)))/d+p*polylog
(2,(e*x+d)*b^(1/2)/(e*(-a)^(1/2)+d*b^(1/2)))/d
```

Rubi [A]
time = 0.21, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2516, 2504, 2441, 2352, 2512, 266, 2463, 2440, 2438}

$$\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{b}\,(d+ex)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)}{d}+\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{b}\,(d+ex)}{\sqrt{-a}\,e+\sqrt{b}\,d}\right)}{d}+\frac{p\,\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{2d}-\frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{d}+\frac{\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2d}+\frac{p\log(d+ex)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{-a}\,e+\sqrt{b}\,d}\right)}{d}+\frac{p\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/(x*(d + e*x)),x]
```

```
[Out] (p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])/d
 + (p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d + e
*x])/d + (Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/(2*d) - (Log[d + e*x]*Log
[c*(a + b*x^2)^p])/d + (p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[
-a]*e)])/d + (p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/d
 + (p*PolyLog[2, 1 + (b*x^2)/a])/(2*d)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x(d+ex)}\,dx &= \int \left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{dx} - \frac{e\log\left(c\left(a+bx^2\right)^p\right)}{d(d+ex)}\right)dx\\
&= \frac{\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x}\,dx}{d} - \frac{e\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx}{d}\\
&= -\frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{d} + \frac{\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}\,dx, x, x^2\right)}{2d} + \frac{(2bp)\int \frac{x\log(d\cdots}{a+b\cdots}}{d}\\
&= \frac{\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2d} - \frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{d} - \frac{(bp)\text{Subst}\left(\int \frac{\log(\cdots}{a\cdots}\right.}{2d}\\
&= \frac{\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2d} - \frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{d} + \frac{p\text{Li}_2\left(1+\frac{bx^2}{a}\right)}{2d} - \cdots\\
&= \frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)\log(d+ex)}{d} - \cdots\\
&= \frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)\log(d+ex)}{d} - \cdots\\
&= \frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)\log(d+ex)}{d} - \cdots
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 232, normalized size = 0.94

$$
-\frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{d}+\frac{p\left(\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)+\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)\log(d+ex)+\text{Li}_2\left(\frac{\sqrt{b}\,(d+ex)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)+\text{Li}_2\left(\frac{\sqrt{b}\,(d+ex)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\right)}{d}+\frac{\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)+p\text{Li}_2\left(\frac{a+bx^2}{a}\right)}{2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/(x*(d + e*x)),x]
```

```
[Out] -((Log[d + e*x]*Log[c*(a + b*x^2)^p])/d) + (p*(Log[(e*(Sqrt[-a] - Sqrt[b]*x
))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x] + Log[-((e*(Sqrt[-a] + Sqrt[b]*x)
```

)/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d + e*x] + PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)] + PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)]))/d + (Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p] + p*PolyLog[2, (a + b*x^2)/a])/(2*d)

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.42, size = 624, normalized size = 2.53

method	result
risch	$-\frac{\ln\left(\left(b\,x^{2}+a\right)^{p}\right)\ln\left(ex+d\right)}{d}+\frac{\ln\left(\left(b\,x^{2}+a\right)^{p}\right)\ln\left(x\right)}{d}-\frac{p\ln\left(x\right)\ln\left(\frac{-bx+\sqrt{-ba}}{\sqrt{-ba}}\right)}{d}-\frac{p\ln\left(x\right)\ln\left(\frac{bx+\sqrt{-ba}}{\sqrt{-ba}}\right)}{d}-\frac{p\,\mathrm{dilog}\left(\frac{-bx+}{\sqrt{}}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(c*(b*x^2+a)^p)/x/(e*x+d),x,method=_RETURNVERBOSE)

[Out] -ln((b*x^2+a)^p)/d*ln(e*x+d)+ln((b*x^2+a)^p)/d*ln(x)-p/d*ln(x)*ln((-b*x+(-b*a)^(1/2))/(-b*a)^(1/2))-p/d*ln(x)*ln((b*x+(-b*a)^(1/2))/(-b*a)^(1/2))-p/d*dilog((-b*x+(-b*a)^(1/2))/(-b*a)^(1/2))-p/d*dilog((b*x+(-b*a)^(1/2))/(-b*a)^(1/2))+p/d*ln(e*x+d)*ln((e*(-b*a)^(1/2)-(e*x+d)*b+b*d)/(e*(-b*a)^(1/2)+b*d))+p/d*ln(e*x+d)*ln((e*(-b*a)^(1/2)+(e*x+d)*b-b*d)/(e*(-b*a)^(1/2)-b*d))+p/d*dilog((e*(-b*a)^(1/2)-(e*x+d)*b+b*d)/(e*(-b*a)^(1/2)+b*d))+p/d*dilog((e*(-b*a)^(1/2)+(e*x+d)*b-b*d)/(e*(-b*a)^(1/2)-b*d))-1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)/d*ln(e*x+d)+1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)/d*ln(e*x+d)-1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2/d*ln(e*x+d)-1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^3/d*ln(x)-1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)/d*ln(x)+1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2/d*ln(x)+1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^3/d*ln(e*x+d)+1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)/d*ln(x)-ln(c)/d*ln(e*x+d)+ln(c)/d*ln(x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c*(b*x^2+a)^p)/x/(e*x+d),x, algorithm="maxima")

[Out] integrate(log((b*x^2 + a)^p*c)/((x*e + d)*x), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)/(x^2*e + d*x), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/x/(e*x+d),x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)/(x*(d + e*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)/((x*e + d)*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{x\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)/(x*(d + e*x)),x)
```

```
[Out] int(log(c*(a + b*x^2)^p)/(x*(d + e*x)), x)
```

3.231 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^2(d+ex)}\,dx$

Optimal. Leaf size=306

$$\frac{2\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{ep\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{d^2}-\frac{ep\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)\log(d+ex)}{d^2}-\frac{\text{lo}}{}$$

```
[Out] -ln(c*(b*x^2+a)^p)/d/x-1/2*e*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)/d^2+e*ln(e*x+d)
*ln(c*(b*x^2+a)^p)/d^2-e*p*ln(e*x+d)*ln(e*((-a)^(1/2)-x*b^(1/2))/(e*(-a)^(1
/2)+d*b^(1/2)))/d^2-e*p*ln(e*x+d)*ln(-e*((-a)^(1/2)+x*b^(1/2))/(-e*(-a)^(1/
2)+d*b^(1/2)))/d^2-1/2*e*p*polylog(2,1+b*x^2/a)/d^2-e*p*polylog(2,(e*x+d)*b
^(1/2)/(-e*(-a)^(1/2)+d*b^(1/2)))/d^2-e*p*polylog(2,(e*x+d)*b^(1/2)/(e*(-a)
^(1/2)+d*b^(1/2)))/d^2+2*p*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/d/a^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {2516, 2505, 211, 2504, 2441, 2352, 2512, 266, 2463, 2440, 2438}

$$-\frac{ep\,\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{2d^2}-\frac{ep\,\text{PolyLog}\left(2,\frac{\sqrt{b}\,(d+ex)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)}{d^2}-\frac{ep\,\text{PolyLog}\left(2,\frac{\sqrt{b}\,(d+ex)}{\sqrt{-a}\,e+\sqrt{b}\,d}\right)}{d^2}+\frac{2\sqrt{b}\,p\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{e\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2d^2}+\frac{e\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{d^2}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{dx}-\frac{ep\log(d+ex)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{-a}\,e+\sqrt{b}\,d}\right)}{d^2}-\frac{ep\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/(x^2*(d + e*x)),x]
```

```
[Out] (2*Sqrt[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*d) - (e*p*Log[(e*(Sqrt[-
a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])/d^2 - (e*p*Log[-((
e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d + e*x])/d^2 - Lo
g[c*(a + b*x^2)^p]/(d*x) - (e*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/(2*d^
2) + (e*Log[d + e*x]*Log[c*(a + b*x^2)^p])/d^2 - (e*p*PolyLog[2, (Sqrt[b]*(
d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/d^2 - (e*p*PolyLog[2, (Sqrt[b]*(d + e*
x))/(Sqrt[b]*d + Sqrt[-a]*e)])/d^2 - (e*p*PolyLog[2, 1 + (b*x^2)/a])/(2*d^2
)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^2(d+ex)}\,dx = \int \left(\frac{\log\left(c(a+bx^2)^p\right)}{dx^2} - \frac{e\log\left(c(a+bx^2)^p\right)}{d^2x} + \frac{e^2\log\left(c(a+bx^2)^p\right)}{d^2(d+ex)}\right)dx$$

$$= \frac{\int \frac{\log\left(c(a+bx^2)^p\right)}{x^2}\,dx}{d} - \frac{e\int \frac{\log\left(c(a+bx^2)^p\right)}{x}\,dx}{d^2} + \frac{e^2\int \frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx}{d^2}$$

$$= -\frac{\log\left(c(a+bx^2)^p\right)}{dx} + \frac{e\log(d+ex)\log\left(c(a+bx^2)^p\right)}{d^2} - \frac{\text{eSubst}\left(\int \frac{\log(c(a+bx)^p)}{x}\,dx,\right.}{2d^2}$$

$$= \frac{2\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,d} - \frac{\log\left(c(a+bx^2)^p\right)}{dx} - \frac{e\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{2d^2} + \frac{e\,\mathrm{l}}{}$$

$$= \frac{2\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,d} - \frac{\log\left(c(a+bx^2)^p\right)}{dx} - \frac{e\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{2d^2} + \frac{e\,\mathrm{l}}{}$$

$$= \frac{2\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,d} - \frac{ep\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{e(\sqrt{}}{\sqrt{}}\right.}{}$$

$$= \frac{2\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,d} - \frac{ep\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{e(\sqrt{}}{\sqrt{}}\right.}{}$$

$$= \frac{2\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,d} - \frac{ep\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{e(\sqrt{}}{\sqrt{}}\right.}{}$$

Mathematica [A]
time = 0.13, size = 268, normalized size = 0.88

$$-\frac{-\frac{4\sqrt{b}\,dp\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{2d\log\left(c(a+bx^2)^p\right)}{x} - 2e\log(d+ex)\log\left(c(a+bx^2)^p\right) + 2ep\left(\left(\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right) + \log\left(\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{-\sqrt{b}\,d+\sqrt{-a}\,e}\right)\right)\log(d+ex) + \mathrm{Li}_2\left(\frac{\sqrt{b}\,(d+ex)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right) + \mathrm{Li}_2\left(\frac{\sqrt{b}\,(d+ex)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\right) + e\left(\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right) + p\mathrm{Li}_2\left(1+\frac{bx^2}{a}\right)\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/(x^2*(d + e*x)),x]
```

```
[Out] -1/2*((-4*Sqrt[b]*d*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] + (2*d*Log[c*(a
+ b*x^2)^p])/x - 2*e*Log[d + e*x]*Log[c*(a + b*x^2)^p] + 2*e*p*((Log[(e*(Sq
```

```
rt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)] + Log[(e*(Sqrt[-a] + Sqrt[b]
*x))/(-(Sqrt[b]*d) + Sqrt[-a]*e)])*Log[d + e*x] + PolyLog[2, (Sqrt[b]*(d +
e*x))/(Sqrt[b]*d - Sqrt[-a]*e)] + PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d
 + Sqrt[-a]*e)]) + e*(Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p] + p*PolyLog[2,
 1 + (b*x^2)/a]))/d^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.52, size = 831, normalized size = 2.72

method	result
risch	$\frac{pe\ln(x)\ln\left(\frac{bx+\sqrt{-ba}}{\sqrt{-ba}}\right)}{d^2}+\frac{i\pi\operatorname{csgn}(ic(b\,x^2+a)^p)^2\operatorname{csgn}(ic)e\ln(ex+d)}{2d^2}-\frac{i\pi\operatorname{csgn}(i(b\,x^2+a)^p)\operatorname{csgn}(ic(b\,x^2+a)^p)^2e\ln(x)}{2d^2}+\ln(($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] p*e/d^2*dilog((-b*x+(-b*a)^(1/2))/(-b*a)^(1/2))+p*e/d^2*dilog((b*x+(-b*a)^(
1/2))/(-b*a)^(1/2))+p*e/d^2*ln(x)*ln((b*x+(-b*a)^(1/2))/(-b*a)^(1/2))-1/2*I
*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)*e/d^2*ln(e*x+d)+ln(
(b*x^2+a)^p)*e/d^2*ln(e*x+d)-ln((b*x^2+a)^p)*e/d^2*ln(x)+1/2*I*Pi*csgn(I*(b
*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)*e/d^2*ln(x)-p*e/d^2*dilog((e*(-b
*a)^(1/2)-(e*x+d)*b+b*d)/(e*(-b*a)^(1/2)+b*d))-p*e/d^2*dilog((e*(-b*a)^(1/2
)+(e*x+d)*b-b*d)/(e*(-b*a)^(1/2)-b*d))+1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csg
n(I*c)*e/d^2*ln(e*x+d)+ln(c)*e/d^2*ln(e*x+d)-ln(c)*e/d^2*ln(x)-ln(c)/d/x+1/
2*I*Pi*csgn(I*c*(b*x^2+a)^p)^3/d/x-1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b
*x^2+a)^p)^2*e/d^2*ln(x)-1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)
^2/d/x+2*p*b/d/(b*a)^(1/2)*arctan(b*x/(b*a)^(1/2))+p*e/d^2*ln(x)*ln((-b*x+(
-b*a)^(1/2))/(-b*a)^(1/2))-1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^3*e/d^2*ln(e*x+d)
-1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)/d/x+1/2*I*Pi*csgn(I*(b*x^2+a)^p
)*csgn(I*c*(b*x^2+a)^p)^2*e/d^2*ln(e*x+d)-p*e/d^2*ln(e*x+d)*ln((e*(-b*a)^(1
/2)+(e*x+d)*b-b*d)/(e*(-b*a)^(1/2)-b*d))+1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^3*e
/d^2*ln(x)+1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)/d/x
-p*e/d^2*ln(e*x+d)*ln((e*(-b*a)^(1/2)-(e*x+d)*b+b*d)/(e*(-b*a)^(1/2)+b*d))-
ln((b*x^2+a)^p)/d/x-1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)*e/d^2*ln(x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)/((x*e + d)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)/(x^3*e + d*x^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/x**2/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)/((x*e + d)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{x^2\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)/(x^2*(d + e*x)),x)
```

```
[Out] int(log(c*(a + b*x^2)^p)/(x^2*(d + e*x)), x)
```

3.232 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^3(d+ex)}\,dx$

Optimal. Leaf size=371

$$-\frac{2\sqrt{b}\,ep\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,d^2}+\frac{bp\log(x)}{ad}+\frac{e^2p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{d^3}+\frac{e^2p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)}{d^3}$$

```
[Out] b*p*ln(x)/a/d-1/2*b*p*ln(b*x^2+a)/a/d-1/2*ln(c*(b*x^2+a)^p)/d/x^2+e*ln(c*(b
*x^2+a)^p)/d^2/x+1/2*e^2*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)/d^3-e^2*ln(e*x+d)*l
n(c*(b*x^2+a)^p)/d^3+e^2*p*ln(e*x+d)*ln(e*((-a)^(1/2)-x*b^(1/2))/(e*(-a)^(1
/2)+d*b^(1/2)))/d^3+e^2*p*ln(e*x+d)*ln(-e*((-a)^(1/2)+x*b^(1/2))/(-e*(-a)^(
1/2)+d*b^(1/2)))/d^3+1/2*e^2*p*polylog(2,1+b*x^2/a)/d^3+e^2*p*polylog(2,(e*
x+d)*b^(1/2)/(-e*(-a)^(1/2)+d*b^(1/2)))/d^3+e^2*p*polylog(2,(e*x+d)*b^(1/2)
/(e*(-a)^(1/2)+d*b^(1/2)))/d^3-2*e*p*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/d^2/
a^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 15, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {2516, 2504, 2442, 36, 29, 31, 2505, 211, 2441, 2352, 2512, 266, 2463, 2440, 2438}

$$\frac{e^2p\,\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{2d^3}+\frac{e^2p\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)}{d^3}+\frac{e^2p\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{-a}\,e+\sqrt{b}\,d}\right)}{d^3}-\frac{2\sqrt{b}\,ep\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,d^2}+\frac{e^2\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2d^3}-\frac{e^2\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{d^3}+\frac{e\log\left(c\left(a+bx^2\right)^p\right)}{d^2x}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{2dx^2}+\frac{e^2p\log(d+ex)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{-a}\,e+\sqrt{b}\,d}\right)}{d^3}+\frac{e^2p\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right)}{d^3}-\frac{bp\log\left(a+bx^2\right)}{2ad}+\frac{bp\log(x)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^p]/(x^3*(d + e*x)),x]
```

```
[Out] (-2*Sqrt[b]*e*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*d^2) + (b*p*Log[x])/(
a*d) + (e^2*p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[
d + e*x])/d^3 + (e^2*p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-
a]*e))]*Log[d + e*x])/d^3 - (b*p*Log[a + b*x^2])/(2*a*d) - Log[c*(a + b*x^2
)^p]/(2*d*x^2) + (e*Log[c*(a + b*x^2)^p])/(d^2*x) + (e^2*Log[-((b*x^2)/a)]*
Log[c*(a + b*x^2)^p])/(2*d^3) - (e^2*Log[d + e*x]*Log[c*(a + b*x^2)^p])/d^3
 + (e^2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/d^3 + (
e^2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/d^3 + (e^2*
p*PolyLog[2, 1 + (b*x^2)/a])/(2*d^3)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
```

```
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^3(d+ex)}\,dx = \int \left(\frac{\log\left(c(a+bx^2)^p\right)}{dx^3} - \frac{e\log\left(c(a+bx^2)^p\right)}{d^2x^2} + \frac{e^2\log\left(c(a+bx^2)^p\right)}{d^3x} - \frac{e^3\log\left(c(a\right.}{d^3(d+}\right.$$

$$= \frac{\int \frac{\log(c(a+bx^2)^p)}{x^3}\,dx}{d} - \frac{e\int \frac{\log(c(a+bx^2)^p)}{x^2}\,dx}{d^2} + \frac{e^2\int \frac{\log(c(a+bx^2)^p)}{x}\,dx}{d^3} - \frac{e^3\int \frac{\log(c(a+bx^2)^{}}{d+ex}}{d^3}$$

$$= \frac{e\log\left(c(a+bx^2)^p\right)}{d^2x} - \frac{e^2\log(d+ex)\log\left(c(a+bx^2)^p\right)}{d^3} + \frac{\mathrm{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x^2}\,dx,\right.}{2d}$$

$$= -\frac{2\sqrt{b}\,ep\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,d^2} - \frac{\log\left(c(a+bx^2)^p\right)}{2dx^2} + \frac{e\log\left(c(a+bx^2)^p\right)}{d^2x} + \frac{e^2\log\left(-\frac{b}{}\right.}{}$$

$$= -\frac{2\sqrt{b}\,ep\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,d^2} - \frac{\log\left(c(a+bx^2)^p\right)}{2dx^2} + \frac{e\log\left(c(a+bx^2)^p\right)}{d^2x} + \frac{e^2\log\left(-\frac{b}{}\right.}{}$$

$$= -\frac{2\sqrt{b}\,ep\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,d^2} + \frac{bp\log(x)}{ad} + \frac{e^2p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{d^3} -$$

$$= -\frac{2\sqrt{b}\,ep\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,d^2} + \frac{bp\log(x)}{ad} + \frac{e^2p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{d^3} -$$

$$= -\frac{2\sqrt{b}\,ep\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,d^2} + \frac{bp\log(x)}{ad} + \frac{e^2p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\log(d+ex)}{d^3} -$$

Mathematica [A]
time = 0.14, size = 320, normalized size = 0.86

$$\frac{-\frac{4\sqrt{b}\,dep\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{bd^2p(2\log(x)-\log(a+bx^2))}{a} - \frac{d^2\log(c(a+bx^2)^p)}{x^2} + \frac{2de\log(c(a+bx^2)^p)}{x} - 2e^2\log(d+ex)\log\left(c(a+bx^2)^p\right) + 2e^2p\left(\left(\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right) + \log\left(\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{-\sqrt{b}\,d+\sqrt{-a}\,e}\right)\right)\log(d+ex) + \mathrm{Li}_2\left(\frac{\sqrt{b}\,(d+ex)}{\sqrt{b}\,d-\sqrt{-a}\,e}\right) + \mathrm{Li}_2\left(\frac{\sqrt{b}\,(d+ex)}{\sqrt{b}\,d+\sqrt{-a}\,e}\right)\right) + e^2\left(\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right) + p\mathrm{Li}_2\left(1+\frac{bx^2}{a}\right)\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^p]/(x^3*(d + e*x)),x]
```

```
[Out] ((-4*Sqrt[b]*d*e*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] + (b*d^2*p*(2*Log[x
] - Log[a + b*x^2]))/a - (d^2*Log[c*(a + b*x^2)^p])/x^2 + (2*d*e*Log[c*(a +
```

```
b*x^2)^p])/x - 2*e^2*Log[d + e*x]*Log[c*(a + b*x^2)^p] + 2*e^2*p*((Log[(e*
(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)] + Log[(e*(Sqrt[-a] + Sqrt
[b]*x))/(-(Sqrt[b]*d) + Sqrt[-a]*e)])*Log[d + e*x] + PolyLog[2, (Sqrt[b]*(d
 + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)] + PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b
]*d + Sqrt[-a]*e)]) + e^2*(Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p] + p*PolyL
og[2, 1 + (b*x^2)/a]))/(2*d^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.40, size = 1071, normalized size = 2.89

method	result	size
risch	Expression too large to display	1071

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^p)/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)/d/x^2-1/2*I*Pi
*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)*e/d^2/x-p*e^2/d^3*dilo
g((-b*x+(-b*a)^(1/2))/(-b*a)^(1/2))-p*e^2/d^3*dilog((b*x+(-b*a)^(1/2))/(-b*
a)^(1/2))+p*e^2/d^3*dilog((e*(-b*a)^(1/2)-(e*x+d)*b+b*d)/(e*(-b*a)^(1/2)+b*
d))+p*e^2/d^3*dilog((e*(-b*a)^(1/2)+(e*x+d)*b-b*d)/(e*(-b*a)^(1/2)-b*d))+1/
2*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)*e^2/d^3*ln(x)+1/2*I*Pi*csgn(I*(b*x
^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)*e^2/d^3*ln(e*x+d)-1/2*I*Pi*csgn(I*
(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)*e^2/d^3*ln(x)-ln(c)*e^2/d^3*ln
(e*x+d)+ln(c)*e^2/d^3*ln(x)+ln(c)*e/d^2/x-1/2*ln(c)/d/x^2+1/2*I*Pi*csgn(I*c
*(b*x^2+a)^p)^3*e^2/d^3*ln(e*x+d)-ln((b*x^2+a)^p)*e^2/d^3*ln(e*x+d)+ln((b*x
^2+a)^p)*e^2/d^3*ln(x)+ln((b*x^2+a)^p)*e/d^2/x+p*e^2/d^3*ln(e*x+d)*ln((e*(-
b*a)^(1/2)+(e*x+d)*b-b*d)/(e*(-b*a)^(1/2)-b*d))-1/2*b*p*ln(b*x^2+a)/a/d-1/2
*ln((b*x^2+a)^p)/d/x^2+1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)*e/d^2/x+1
/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2*e^2/d^3*ln(x)-1/2*I*Pi*
csgn(I*c*(b*x^2+a)^p)^3*e^2/d^3*ln(x)-1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c
*(b*x^2+a)^p)^2*e^2/d^3*ln(e*x+d)+b*p*ln(x)/a/d-1/2*I*Pi*csgn(I*c*(b*x^2+a)
^p)^3*e/d^2/x+1/4*I*Pi*csgn(I*c*(b*x^2+a)^p)^3/d/x^2-p*e^2/d^3*ln(x)*ln((-b
*x+(-b*a)^(1/2))/(-b*a)^(1/2))-p*e^2/d^3*ln(x)*ln((b*x+(-b*a)^(1/2))/(-b*a)
^(1/2))+p*e^2/d^3*ln(e*x+d)*ln((e*(-b*a)^(1/2)-(e*x+d)*b+b*d)/(e*(-b*a)^(1/
2)+b*d))-1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)*e^2/d^3*ln(e*x+d)-1/4*I
*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2/d/x^2-1/4*I*Pi*csgn(I*c*(b*
x^2+a)^p)^2*csgn(I*c)/d/x^2-2*p*b/d^2*e/(b*a)^(1/2)*arctan(b*x/(b*a)^(1/2))
+1/2*I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2*e/d^2/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)/((x*e + d)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)/(x^4*e + d*x^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**p)/x**3/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^p)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)/((x*e + d)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{x^3\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^p)/(x^3*(d + e*x)),x)
```

```
[Out] int(log(c*(a + b*x^2)^p)/(x^3*(d + e*x)), x)
```

3.233 $\int \frac{x^3 \log\left(c\left(a+bx^3\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=692

$$-\frac{3d^2px}{e^3}+\frac{3dpx^2}{4e^2}-\frac{px^3}{3e}-\frac{\sqrt{3}\,\sqrt[3]{a}\,d^2p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt[3]{b}\,e^3}+\frac{\sqrt{3}\,a^{2/3}dp\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2b^{2/3}e^2}+\frac{\sqrt[3]{a}\,d^2p\log\left(\sqrt[3]{\cdots}\right.}{\sqrt[3]{b}\cdots}$$

```
[Out] -3*d^2*p*x/e^3+3/4*d*p*x^2/e^2-1/3*p*x^3/e+a^(1/3)*d^2*p*ln(a^(1/3)+b^(1/3)
*x)/b^(1/3)/e^3+1/2*a^(2/3)*d*p*ln(a^(1/3)+b^(1/3)*x)/b^(2/3)/e^2+d^3*p*ln(
-e*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-a^(1/3)*e))*ln(e*x+d)/e^4+d^3*p*ln(-e*((-
1)^(2/3)*a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))*ln(e*x+d)/e^4
+d^3*p*ln((-1)^(1/3)*e*(a^(1/3)+(-1)^(2/3)*b^(1/3)*x)/(b^(1/3)*d+(-1)^(1/3)
*a^(1/3)*e))*ln(e*x+d)/e^4-1/2*a^(1/3)*d^2*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b
^(2/3)*x^2)/b^(1/3)/e^3-1/4*a^(2/3)*d*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3
)*x^2)/b^(2/3)/e^2+d^2*x*ln(c*(b*x^3+a)^p)/e^3-1/2*d*x^2*ln(c*(b*x^3+a)^p)/
e^2+1/3*(b*x^3+a)*ln(c*(b*x^3+a)^p)/b/e-d^3*ln(e*x+d)*ln(c*(b*x^3+a)^p)/e^4
+d^3*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-a^(1/3)*e))/e^4+d^3*p*polylog(2
,b^(1/3)*(e*x+d)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))/e^4+d^3*p*polylog(2,b^(1
/3)*(e*x+d)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))/e^4-a^(1/3)*d^2*p*arctan(1/3*
(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/b^(1/3)/e^3+1/2*a^(2/3)*d*p*
arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/b^(2/3)/e^2
```

Rubi [A]
time = 0.58, antiderivative size = 692, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 20, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {2516, 2498, 327, 206, 31, 648, 631, 210, 642, 2505, 298, 2504, 2436, 2332, 2512, 266, 2463, 2441, 2440, 2438}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(x^3*Log[c*(a + b*x^3)^p])/(d + e*x),x]
```

```
[Out] (-3*d^2*p*x)/e^3 + (3*d*p*x^2)/(4*e^2) - (p*x^3)/(3*e) - (Sqrt[3]*a^(1/3)*d
^2*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(b^(1/3)*e^3) + (Sq
rt[3]*a^(2/3)*d*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2*b^(
2/3)*e^2) + (a^(1/3)*d^2*p*Log[a^(1/3) + b^(1/3)*x])/(b^(1/3)*e^3) + (a^(2/
3)*d*p*Log[a^(1/3) + b^(1/3)*x])/(2*b^(2/3)*e^2) + (d^3*p*Log[-((e*(a^(1/3)
 + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/e^4 + (d^3*p*Log[-((
e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e))]*Lo
g[d + e*x])/e^4 + (d^3*p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)
)/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/e^4 - (a^(1/3)*d^2*p*Lo
```

```
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)*e^3) - (a^(2/3)*d*
p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(4*b^(2/3)*e^2) + (d^2*x*
Log[c*(a + b*x^3)^p])/e^3 - (d*x^2*Log[c*(a + b*x^3)^p])/(2*e^2) + ((a + b*
x^3)*Log[c*(a + b*x^3)^p])/(3*b*e) - (d^3*Log[d + e*x]*Log[c*(a + b*x^3)^p]
)/e^4 + (d^3*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)])/e^4
 + (d^3*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)
])/e^4 + (d^3*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1
/3)*e)])/e^4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
```

```
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{x^3 \log\left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \left(\frac{d^2\log\left(c(a+bx^3)^p\right)}{e^3} - \frac{dx\log\left(c(a+bx^3)^p\right)}{e^2} + \frac{x^2\log\left(c(a+bx^3)^p\right)}{e} - \frac{d^3\log\ldots}{e\ldots}\right.$$

$$= \frac{d^2\int \log\left(c(a+bx^3)^p\right)\,dx}{e^3} - \frac{d^3\int \frac{\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx}{e^3} - \frac{d\int x\log\left(c(a+bx^3)^p\right)\,dx}{e^2} + \ldots$$

$$= \frac{d^2x\log\left(c(a+bx^3)^p\right)}{e^3} - \frac{dx^2\log\left(c(a+bx^3)^p\right)}{2e^2} - \frac{d^3\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e^4}\ldots$$

$$= -\frac{3d^2px}{e^3} + \frac{3dpx^2}{4e^2} + \frac{d^2x\log\left(c(a+bx^3)^p\right)}{e^3} - \frac{dx^2\log\left(c(a+bx^3)^p\right)}{2e^2} - \frac{d^3\log(d+\ldots}{\ldots}$$

$$= -\frac{3d^2px}{e^3} + \frac{3dpx^2}{4e^2} - \frac{px^3}{3e} + \frac{d^2x\log\left(c(a+bx^3)^p\right)}{e^3} - \frac{dx^2\log\left(c(a+bx^3)^p\right)}{2e^2} + \frac{(a+\ldots}{\ldots}$$

$$= -\frac{3d^2px}{e^3} + \frac{3dpx^2}{4e^2} - \frac{px^3}{3e} + \frac{\sqrt[3]{a}\,d^2p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,e^3} + \frac{a^{2/3}dp\log\left(\sqrt[3]{a}+\sqrt[3]{b}\ldots\right.}{2b^{2/3}e^2}$$

$$= -\frac{3d^2px}{e^3} + \frac{3dpx^2}{4e^2} - \frac{px^3}{3e} + \frac{\sqrt[3]{a}\,d^2p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,e^3} + \frac{a^{2/3}dp\log\left(\sqrt[3]{a}+\sqrt[3]{b}\ldots\right.}{2b^{2/3}e^2}$$

$$= -\frac{3d^2px}{e^3} + \frac{3dpx^2}{4e^2} - \frac{px^3}{3e} - \frac{\sqrt{3}\,\sqrt[3]{a}\,d^2p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt[3]{b}\,e^3} + \frac{\sqrt{3}\,a^{2/3}dp\tan^{-\ldots}}{2b^{2\ldots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.44, size = 497, normalized size = 0.72

$$-\frac{9de^2px^2\left(-1+{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a}\right)\right)+\frac{6d^2ep\left(6\sqrt[3]{b}\,x-2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+\sqrt[3]{a}\left(2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\right)\right)}{\sqrt[3]{b}}-12d^2ex\log\left(c(a+bx^3)^p\right)+6de^2x^2\log\left(c(a+bx^3)^p\right)+12d^3\log(d+ex)\log\left(c(a+bx^3)^p\right)+\frac{4e^3\left(bpx^3-(a+bx^3)\log\left(c(a+bx^3)^p\right)\right)}{b}-12d^3p\left(\log\left(\frac{e\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d+\sqrt[3]{-1}\sqrt[3]{a}\,e}\right)\log(d+ex)+\log\left(\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{-\sqrt[3]{b}\,d+\sqrt[3]{a}\,e}\right)\log(d+ex)+\log\left(\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{-\sqrt[3]{b}\,d+(-1)^{2/3}\sqrt[3]{a}\,e}\right)\log(d+ex)+\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)+\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}\,d+\sqrt[3]{-1}\sqrt[3]{a}\,e}\right)+\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}\,d-(-1)^{2/3}\sqrt[3]{a}\,e}\right)\right)}{12e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Log[c*(a + b*x^3)^p])/(d + e*x),x]
```

```
[Out] -1/12*(9*d*e^2*p*x^2*(-1 + Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)]) +
(6*d^2*e*p*(6*b^(1/3)*x - 2*a^(1/3)*Log[a^(1/3) + b^(1/3)*x] + a^(1/3)*(2*S
```

```
qrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])))/b^(1/3) - 12*d^2*e*x*Log[c*(a + b*x^3)^p] + 6*d
*e^2*x^2*Log[c*(a + b*x^3)^p] + 12*d^3*Log[d + e*x]*Log[c*(a + b*x^3)^p] +
(4*e^3*(b*p*x^3 - (a + b*x^3)*Log[c*(a + b*x^3)^p]))/b - 12*d^3*p*(Log[(e*(
(-1)^(1/3)*a^(1/3) - b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d
+ e*x] + Log[(e*(a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*d) + a^(1/3)*e)]*Log[d +
e*x] + Log[(e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*d) + (-1)^(2/3)*
a^(1/3)*e)]*Log[d + e*x] + PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1
/3)*e)] + PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)
] + PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)]))/e^
4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.63, size = 912, normalized size = 1.32

method	result	size
risch	Expression too large to display	912

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(b*x^3+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -3*d^2*p*x/e^3+1/3*ln((b*x^3+a)^p)/e*x^3+1/3*ln(c)/e*x^3-49/12*p/e^4*d^3-1/
2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)/e^3*x*d^2+1/2*I*
Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)*d^3/e^4*ln(e*x+d)+1/
4*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)/e^2*d*x^2+1/6*p/
b/e*sum((2*_R^2-7*_R*d+11*d^2)/(_R^2-2*_R*d+d^2)*ln(e*x-_R+d),_R=RootOf(_Z^
3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3))*a+p/e^4*d^3*sum(ln(e*x+d)*ln((-e*x+
_R1-d)/_R1)+dilog((-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2
+a*e^3-b*d^3))+1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)/e^3*x*d^2+1/6*I*P
i*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2/e*x^3-1/2*ln(c)/e^2*d*x^2+ln(
c)/e^3*x*d^2-ln(c)*d^3/e^4*ln(e*x+d)-1/2*ln((b*x^3+a)^p)/e^2*d*x^2+ln((b*x^
3+a)^p)/e^3*x*d^2-ln((b*x^3+a)^p)*d^3/e^4*ln(e*x+d)+1/6*I*Pi*csgn(I*c*(b*x^
3+a)^p)^2*csgn(I*c)/e*x^3+1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^3*d^3/e^4*ln(e*x+d
)+1/4*I*Pi*csgn(I*c*(b*x^3+a)^p)^3/e^2*d*x^2-1/2*I*Pi*csgn(I*(b*x^3+a)^p)*c
sgn(I*c*(b*x^3+a)^p)^2*d^3/e^4*ln(e*x+d)-1/6*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(
I*c*(b*x^3+a)^p)*csgn(I*c)/e*x^3-1/6*I*Pi*csgn(I*c*(b*x^3+a)^p)^3/e*x^3+1/2
*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2/e^3*x*d^2-1/3*p*x^3/e+3/4
*d*p*x^2/e^2-1/4*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2/e^2*d*x^2
-1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^3/e^3*x*d^2-1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^
2*csgn(I*c)*d^3/e^4*ln(e*x+d)-1/4*I*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)/e^
2*d*x^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(x^3*log((b*x^3 + a)^p*c)/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x^3*log((b*x^3 + a)^p*c)/(x*e + d), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(b*x**3+a)**p)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(x^3*log((b*x^3 + a)^p*c)/(x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\ln\left(c\,{\left(b\,x^3+a\right)}^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*log(c*(a + b*x^3)^p))/(d + e*x),x)
```

```
[Out] int((x^3*log(c*(a + b*x^3)^p))/(d + e*x), x)
```

3.234 $\int \frac{x^2 \log\left(c(a+bx^3)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=643

$$\frac{3dpx}{e^2}-\frac{3px^2}{4e}+\frac{\sqrt{3}\,\sqrt[3]{a}\,dp\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt[3]{b}\,e^2}-\frac{\sqrt{3}\,a^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2b^{2/3}e}-\frac{\sqrt[3]{a}\,dp\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right.}{\sqrt[3]{b}\,e^2}$$

```
[Out] 3*d*p*x/e^2-3/4*p*x^2/e-a^(1/3)*d*p*ln(a^(1/3)+b^(1/3)*x)/b^(1/3)/e^2-1/2*a
^(2/3)*p*ln(a^(1/3)+b^(1/3)*x)/b^(2/3)/e-d^2*p*ln(-e*(a^(1/3)+b^(1/3)*x)/(b
^(1/3)*d-a^(1/3)*e))*ln(e*x+d)/e^3-d^2*p*ln(-e*((-1)^(2/3)*a^(1/3)+b^(1/3)*
x)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))*ln(e*x+d)/e^3-d^2*p*ln((-1)^(1/3)*e*(a
^(1/3)+(-1)^(2/3)*b^(1/3)*x)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))*ln(e*x+d)/e^
3+1/2*a^(1/3)*d*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(1/3)/e^2+1/4
*a^(2/3)*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(2/3)/e-d*x*ln(c*(b*
x^3+a)^p)/e^2+1/2*x^2*ln(c*(b*x^3+a)^p)/e+d^2*ln(e*x+d)*ln(c*(b*x^3+a)^p)/e
^3-d^2*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-a^(1/3)*e))/e^3-d^2*p*polylog
(2,b^(1/3)*(e*x+d)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))/e^3-d^2*p*polylog(2,b^
(1/3)*(e*x+d)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))/e^3+a^(1/3)*d*p*arctan(1/3*
(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/b^(1/3)/e^2-1/2*a^(2/3)*p*ar
ctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/b^(2/3)/e
```

Rubi [A]
time = 0.47, antiderivative size = 643, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 17, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {2516, 2498, 327, 206, 31, 648, 631, 210, 642, 2505, 298, 2512, 266, 2463, 2441, 2440, 2438}

$$-\frac{d^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)}{e^3}-\frac{d^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\,\sqrt[3]{a}\,e+\sqrt[3]{b}\,d}\right)}{e^3}-\frac{d^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}\,d-(-1)^{2/3}\sqrt[3]{a}\,e}\right)}{e^3}-\frac{\sqrt{3}\,a^{2/3}p\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2b^{2/3}e}+\frac{\sqrt[3]{a}\,dp\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{2\sqrt[3]{b}\,e^2}+\frac{a^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{4b^{2/3}e}-\frac{a^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2b^{2/3}e}+\frac{\sqrt{3}\,\sqrt[3]{a}\,dp\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt[3]{b}\,e^2}+\frac{d^2\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e^3}-\frac{dx\log\left(c(a+bx^3)^p\right)}{e^2}+\frac{x^2\log\left(c(a+bx^3)^p\right)}{2e}-\frac{d^2p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)}{e^3}-\frac{d^2p\log(d+ex)\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-(-1)^{2/3}\sqrt[3]{a}\,e}\right)}{e^3}-\frac{d^2p\log(d+ex)\log\left(\frac{\sqrt[3]{-1}\,e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x\right)}{\sqrt[3]{-1}\,\sqrt[3]{a}\,e+\sqrt[3]{b}\,d}\right)}{e^3}-\frac{\sqrt[3]{a}\,dp\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,e^2}+\frac{3dpx}{e^2}-\frac{3px^2}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Log[c*(a + b*x^3)^p])/(d + e*x),x]
```

```
[Out] (3*d*p*x)/e^2 - (3*p*x^2)/(4*e) + (Sqrt[3]*a^(1/3)*d*p*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(b^(1/3)*e^2) - (Sqrt[3]*a^(2/3)*p*ArcTan[(a
^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2*b^(2/3)*e) - (a^(1/3)*d*p*Log[
a^(1/3) + b^(1/3)*x])/(b^(1/3)*e^2) - (a^(2/3)*p*Log[a^(1/3) + b^(1/3)*x])/
(2*b^(2/3)*e) - (d^2*p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)
*e))]*Log[d + e*x])/e^3 - (d^2*p*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))
/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e))]*Log[d + e*x])/e^3 - (d^2*p*Log[((-1)^
(1/3)*e*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e
)]*Log[d + e*x])/e^3 + (a^(1/3)*d*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/(2*b^(1/3)*e^2) + (a^(2/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
```

```
2/3)*x^2])/(4*b^(2/3)*e) - (d*x*Log[c*(a + b*x^3)^p])/e^2 + (x^2*Log[c*(a +
 b*x^3)^p])/(2*e) + (d^2*Log[d + e*x]*Log[c*(a + b*x^3)^p])/e^3 - (d^2*p*Po
lyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)])/e^3 - (d^2*p*PolyLog
[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)])/e^3 - (d^2*p*P
olyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)])/e^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
```

```
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{x^2 \log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \left(-\frac{d \log \left(c(a+bx^3)^p\right)}{e^2} + \frac{x \log \left(c(a+bx^3)^p\right)}{e} + \frac{d^2 \log \left(c(a+bx^3)^p\right)}{e^2(d+ex)}\right) dx$$

$$= -\frac{d \int \log \left(c(a+bx^3)^p\right)\,dx}{e^2} + \frac{d^2 \int \frac{\log \left(c(a+bx^3)^p\right)}{d+ex}\,dx}{e^2} + \frac{\int x \log \left(c(a+bx^3)^p\right)\,dx}{e}$$

$$= -\frac{dx \log \left(c(a+bx^3)^p\right)}{e^2} + \frac{x^2 \log \left(c(a+bx^3)^p\right)}{2e} + \frac{d^2 \log(d+ex) \log \left(c(a+bx^3)^p\right.}{e^3}$$

$$= \frac{3dpx}{e^2} - \frac{3px^2}{4e} - \frac{dx \log \left(c(a+bx^3)^p\right)}{e^2} + \frac{x^2 \log \left(c(a+bx^3)^p\right)}{2e} + \frac{d^2 \log(d+ex)\,\mathrm{l}}{e}$$

$$= \frac{3dpx}{e^2} - \frac{3px^2}{4e} - \frac{dx \log \left(c(a+bx^3)^p\right)}{e^2} + \frac{x^2 \log \left(c(a+bx^3)^p\right)}{2e} + \frac{d^2 \log(d+ex)\,\mathrm{l}}{e}$$

$$= \frac{3dpx}{e^2} - \frac{3px^2}{4e} - \frac{\sqrt[3]{a}\, dp \log \left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\, e^2} - \frac{a^{2/3} p \log \left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2b^{2/3}e} - \frac{d^2 p\,\mathrm{l}}{}$$

$$= \frac{3dpx}{e^2} - \frac{3px^2}{4e} - \frac{\sqrt[3]{a}\, dp \log \left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\, e^2} - \frac{a^{2/3} p \log \left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2b^{2/3}e} - \frac{d^2 p\,\mathrm{l}}{}$$

$$= \frac{3dpx}{e^2} - \frac{3px^2}{4e} + \frac{\sqrt{3}\,\sqrt[3]{a}\, dp \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt[3]{b}\, e^2} - \frac{\sqrt{3}\, a^{2/3} p \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{}}{\sqrt{3}\,\sqrt[3]{}}\right.}{2b^{2/3}e}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.28, size = 504, normalized size = 0.78

$$-\frac{-12depx+3e^2px^2-\frac{4\sqrt{3}\,\sqrt[3]{a}\,dep\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}-3e^2px^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a}\right)+\frac{4\sqrt[3]{a}\,dep\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}+4d^2p\log\left(\frac{e\left(\sqrt[3]{-1}\,\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d+\sqrt[3]{-1}\,\sqrt[3]{a}\,e}\right)\log(d+ex)+4d^2p\log\left(\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{-\sqrt[3]{b}\,d+\sqrt[3]{a}\,e}\right)\log(d+ex)+4d^2p\log\left(\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{-\sqrt[3]{b}\,d+(-1)^{2/3}\sqrt[3]{a}\,e}\right)\log(d+ex)-\frac{2\sqrt[3]{a}\,dep\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[3]{b}}+4dex\log\left(c(a+bx^3)^p\right)-2e^2x^2\log\left(c(a+bx^3)^p\right)-4d^2\log(d+ex)\log\left(c(a+bx^3)^p\right)+4d^2p\,\mathrm{Li}_2\left(\frac{\sqrt[3]{b}\,(d+ex)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)+4d^2p\,\mathrm{Li}_2\left(\frac{\sqrt[3]{b}\,(d+ex)}{\sqrt[3]{b}\,d+\sqrt[3]{-1}\,\sqrt[3]{a}\,e}\right)+4d^2p\,\mathrm{Li}_2\left(\frac{\sqrt[3]{b}\,(d+ex)}{\sqrt[3]{b}\,d-(-1)^{2/3}\sqrt[3]{a}\,e}\right)}{4e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Log[c*(a + b*x^3)^p])/(d + e*x),x]
```

```
[Out] -1/4*(-12*d*e*p*x + 3*e^2*p*x^2 - (4*Sqrt[3]*a^(1/3)*d*e*p*ArcTan[(1 - (2*b
^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) - 3*e^2*p*x^2*Hypergeometric2F1[2/3, 1
```

```
, 5/3, -((b*x^3)/a)] + (4*a^(1/3)*d*e*p*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) +
 4*d^2*p*Log[(e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a
^(1/3)*e)]*Log[d + e*x] + 4*d^2*p*Log[(e*(a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*
d) + a^(1/3)*e)]*Log[d + e*x] + 4*d^2*p*Log[(e*((-1)^(2/3)*a^(1/3) + b^(1/3
)*x))/(-(b^(1/3)*d) + (-1)^(2/3)*a^(1/3)*e)]*Log[d + e*x] - (2*a^(1/3)*d*e*
p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3) + 4*d*e*x*Log[c*(
a + b*x^3)^p] - 2*e^2*x^2*Log[c*(a + b*x^3)^p] - 4*d^2*Log[d + e*x]*Log[c*(
a + b*x^3)^p] + 4*d^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)
*e)] + 4*d^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/
3)*e)] + 4*d^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(
1/3)*e)])/e^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.61, size = 704, normalized size = 1.09

method	result
risch	$\frac{\ln((x^3b+a)^p)x^2}{2e} - \frac{\ln((x^3b+a)^p)dx}{e^2} + \frac{\ln((x^3b+a)^p)d^2\ln(ex+d)}{e^3} - \frac{pd^2\left(\sum_{\_R1=\mathrm{RootOf}(\_Z^3b-3\_Z^2bd+3\_Zbd^2+e^3a-bd^3)}\left(\ln(e\right.\right.}{e^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(b*x^3+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln((b*x^3+a)^p)/e*x^2-ln((b*x^3+a)^p)/e^2*d*x+ln((b*x^3+a)^p)*d^2/e^3*l
n(e*x+d)-p/e^3*d^2*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((-e*x+_R1-d)/_R
1),_R1=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3))-3/4*p*x^2/e+3*d*p*
x/e^2+15/4*p/e^3*d^2+1/2*p/b*sum((_R-3*d)/(_R^2-2*_R*d+d^2)*ln(e*x-_R+d),_R
=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3))*a-1/2*I*Pi*csgn(I*(b*x^3
+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)*d^2/e^3*ln(e*x+d)-1/4*I*Pi*csgn(I*(b
*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)/e*x^2+1/2*I*Pi*csgn(I*(b*x^3+a)^
p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)/e^2*d*x-1/4*I*Pi*csgn(I*c*(b*x^3+a)^p)^3
/e*x^2+1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)*d^2/e^3*ln(e*x+d)-1/2*I*P
i*csgn(I*c*(b*x^3+a)^p)^3*d^2/e^3*ln(e*x+d)+1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^
3/e^2*d*x+1/4*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2/e*x^2-1/2*I*
Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2/e^2*d*x-1/2*I*Pi*csgn(I*c*(b
*x^3+a)^p)^2*csgn(I*c)/e^2*d*x+1/4*I*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)/e
*x^2+1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2*d^2/e^3*ln(e*x+d)
+1/2*ln(c)/e*x^2-ln(c)/e^2*d*x+ln(c)*d^2/e^3*ln(e*x+d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(x^2*log((b*x^3 + a)^p*c)/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="fricas")
[Out] integral(x^2*log((b*x^3 + a)^p*c)/(x*e + d), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(b*x**3+a)**p)/(e*x+d),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(x^2*log((b*x^3 + a)^p*c)/(x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \ln\left(c\,{\left(b\,x^3+a\right)}^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*log(c*(a + b*x^3)^p))/(d + e*x),x)
[Out] int((x^2*log(c*(a + b*x^3)^p))/(d + e*x), x)
```

3.235 $\int \frac{x \log\left(c\left(a+bx^3\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=457

$$-\frac{3px}{e}-\frac{\sqrt{3}\,\sqrt[3]{a}\,p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt[3]{b}\,e}+\frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,e}+\frac{dp\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)\log(d+ex)}{e^2}+$$

```
[Out] -3*p*x/e+a^(1/3)*p*ln(a^(1/3)+b^(1/3)*x)/b^(1/3)/e+d*p*ln(-e*(a^(1/3)+b^(1/
3)*x)/(b^(1/3)*d-a^(1/3)*e))*ln(e*x+d)/e^2+d*p*ln(-e*((-1)^(2/3)*a^(1/3)+b^
(1/3)*x)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))*ln(e*x+d)/e^2+d*p*ln((-1)^(1/3)*
e*(a^(1/3)+(-1)^(2/3)*b^(1/3)*x)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))*ln(e*x+d
)/e^2-1/2*a^(1/3)*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(1/3)/e+x*l
n(c*(b*x^3+a)^p)/e-d*ln(e*x+d)*ln(c*(b*x^3+a)^p)/e^2+d*p*polylog(2,b^(1/3)*
(e*x+d)/(b^(1/3)*d-a^(1/3)*e))/e^2+d*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d
+(-1)^(1/3)*a^(1/3)*e))/e^2+d*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-(-1)^(
2/3)*a^(1/3)*e))/e^2-a^(1/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(
1/2))*3^(1/2)/b^(1/3)/e
```

Rubi [A]
time = 0.37, antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {2516, 2498, 327, 206, 31, 648, 631, 210, 642, 2512, 266, 2463, 2441, 2440, 2438}

$$\frac{dp\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)}{e^2}+\frac{dp\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}\,e+\sqrt[3]{b}\,d}\right)}{e^2}+\frac{dp\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}\,d-(-1)^{2/3}\sqrt[3]{a}\,e}\right)}{e^2}-\frac{\sqrt[3]{a}\,p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{2\sqrt[3]{b}\,e}-\frac{\sqrt{3}\sqrt[3]{a}\,p\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}\,e}-\frac{d\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e^2}+\frac{x\log\left(c\left(a+bx^3\right)^p\right)}{e}+\frac{dp\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)}{e^2}+\frac{dp\log(d+ex)\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-(-1)^{2/3}\sqrt[3]{a}\,e}\right)}{e^2}+\frac{dp\log(d+ex)\log\left(\frac{\sqrt[3]{-1}\,e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x\right)}{\sqrt[3]{-1}\sqrt[3]{a}\,e+\sqrt[3]{b}\,d}\right)}{e^2}+\frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,e}-\frac{3px}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[c*(a + b*x^3)^p])/(d + e*x),x]
```

```
[Out] (-3*p*x)/e - (Sqrt[3]*a^(1/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(
1/3))])/(b^(1/3)*e) + (a^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/(b^(1/3)*e) + (d
*p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/
e^2 + (d*p*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/
3)*a^(1/3)*e))]*Log[d + e*x])/e^2 + (d*p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^
(2/3)*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/e^2 - (
a^(1/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)*e) + (
x*Log[c*(a + b*x^3)^p])/e - (d*Log[d + e*x]*Log[c*(a + b*x^3)^p])/e^2 + (d*
p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)])/e^2 + (d*p*PolyL
og[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)])/e^2 + (d*p*P
olyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)])/e^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
```

```
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{x\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \left(\frac{\log\left(c(a+bx^3)^p\right)}{e}-\frac{d\log\left(c(a+bx^3)^p\right)}{e(d+ex)}\right)\,dx$$

$$= \frac{\int \log\left(c(a+bx^3)^p\right)\,dx}{e}-\frac{d\int \frac{\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx}{e}$$

$$= \frac{x\log\left(c(a+bx^3)^p\right)}{e}-\frac{d\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e^2}+\frac{(3bdp)\int \frac{x^2\log(d+ex)}{a+bx^3}\,dx}{e^2}$$

$$= -\frac{3px}{e}+\frac{x\log\left(c(a+bx^3)^p\right)}{e}-\frac{d\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e^2}+\frac{(3bdp)\int \Bigg(}{}$$

$$= -\frac{3px}{e}+\frac{x\log\left(c(a+bx^3)^p\right)}{e}-\frac{d\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e^2}+\frac{\left(\sqrt[3]{b}\,dp\right)\int}{e^2}$$

$$= -\frac{3px}{e}+\frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,e}+\frac{dp\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)\log(d+ex)}{e^2}+$$

$$= -\frac{3px}{e}+\frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,e}+\frac{dp\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)\log(d+ex)}{e^2}+$$

$$= -\frac{3px}{e}-\frac{\sqrt{3}\,\sqrt[3]{a}\,p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt[3]{b}\,e}+\frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,e}+\frac{dp\log\Bigg(}{}$$

Mathematica [A]

time = 0.15, size = 430, normalized size = 0.94

$$\frac{-6epx-\frac{2\sqrt{3}\,\sqrt[3]{a}\,ep\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}+\frac{2\sqrt[3]{a}\,ep\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}+2dp\log\left(\frac{e\left(\sqrt[3]{-1}\,\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d+\sqrt[3]{-1}\,\sqrt[3]{a}\,e}\right)\log(d+ex)+2dp\log\left(\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{-\sqrt[3]{b}\,d+\sqrt[3]{a}\,e}\right)\log(d+ex)+2dp\log\left(\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{-\sqrt[3]{b}\,d+(-1)^{2/3}\sqrt[3]{a}\,e}\right)\log(d+ex)-\frac{\sqrt[3]{a}\,ep\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[3]{b}}+2ex\log\left(c(a+bx^3)^p\right)-2d\log(d+ex)\log\left(c(a+bx^3)^p\right)+2dp\mathrm{Li}_2\left(\frac{\sqrt[3]{b}\,(d+ex)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)+2dp\mathrm{Li}_2\left(\frac{\sqrt[3]{b}\,(d+ex)}{\sqrt[3]{b}\,d+\sqrt[3]{-1}\,\sqrt[3]{a}\,e}\right)+2dp\mathrm{Li}_2\left(\frac{\sqrt[3]{b}\,(d+ex)}{\sqrt[3]{b}\,d-(-1)^{2/3}\sqrt[3]{a}\,e}\right)}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[c*(a + b*x^3)^p])/(d + e*x),x]
```

```
[Out] (-6*e*p*x - (2*Sqrt[3]*a^(1/3)*e*p*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[
3]])/b^(1/3) + (2*a^(1/3)*e*p*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + 2*d*p*Log
[(e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*L
og[d + e*x] + 2*d*p*Log[(e*(a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*d) + a^(1/3)*e
)]*Log[d + e*x] + 2*d*p*Log[(e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(-(b^(1/3)
*d) + (-1)^(2/3)*a^(1/3)*e)]*Log[d + e*x] - (a^(1/3)*e*p*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3) + 2*e*x*Log[c*(a + b*x^3)^p] - 2*d*Lo
g[d + e*x]*Log[c*(a + b*x^3)^p] + 2*d*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(
1/3)*d - a^(1/3)*e)] + 2*d*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-
1)^(1/3)*a^(1/3)*e)] + 2*d*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-
1)^(2/3)*a^(1/3)*e)])/(2*e^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.72, size = 500, normalized size = 1.09

method	result
risch	$\frac{\ln\left(\left(x^3 b+a\right)^p\right) x}{e}-\frac{\ln\left(\left(x^3 b+a\right)^p\right) d \ln(ex+d)}{e^2}-\frac{3px}{e}-\frac{3pd}{e^2}+\frac{pe\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^3 b-3\_Z^2 bd+3\_Z b d^2+e^3 a-b d^3\right)}\frac{\ln\left(ex-\_R+\right.}{\_R^2-2\_R d}\right.}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(b*x^3+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] ln((b*x^3+a)^p)/e*x-ln((b*x^3+a)^p)*d/e^2*ln(e*x+d)-3*p*x/e-3*p/e^2*d+1/b*p
*e*sum(1/(_R^2-2*_R*d+d^2)*ln(e*x-_R+d),_R=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*
d^2+a*e^3-b*d^3))*a+p/e^2*d*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((-e*x+
_R1-d)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3))+1/2*I*Pi*
csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)*d/e^2*ln(e*x+d)+1/2*I*P
i*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)/e*x-1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^3/e*
x+1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2/e*x+1/2*I*Pi*csgn(I*
c*(b*x^3+a)^p)^3*d/e^2*ln(e*x+d)-1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x
^3+a)^p)^2*d/e^2*ln(e*x+d)-1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)*d/e^2
*ln(e*x+d)-1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)/e*x
+ln(c)/e*x-ln(c)*d/e^2*ln(e*x+d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(x*log((b*x^3 + a)^p*c)/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="fricas")
[Out] integral(x*log((b*x^3 + a)^p*c)/(x*e + d), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(b*x**3+a)**p)/(e*x+d),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(x*log((b*x^3 + a)^p*c)/(x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\ln\left(c\,{\left(b\,x^3+a\right)}^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(c*(a + b*x^3)^p))/(d + e*x),x)
[Out] int((x*log(c*(a + b*x^3)^p))/(d + e*x), x)
```

3.236 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=308

$$-\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)\log(d+ex)}{e}-\frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-(-1)^{2/3}\sqrt[3]{a}\,e}\right)\log(d+ex)}{e}-\frac{p\log\left(\frac{\sqrt[3]{-1}\,e\left(\sqrt[3]{a}+(-\cdots\right.}{\sqrt[3]{b}\,d+\sqrt[3]{-1}\cdots}\right.}{\cdots}$$

```
[Out] -p*ln(-e*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-a^(1/3)*e))*ln(e*x+d)/e-p*ln(-e*((-
1)^(2/3)*a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))*ln(e*x+d)/e-p
*ln((-1)^(1/3)*e*(a^(1/3)+(-1)^(2/3)*b^(1/3)*x)/(b^(1/3)*d+(-1)^(1/3)*a^(1/
3)*e))*ln(e*x+d)/e+ln(e*x+d)*ln(c*(b*x^3+a)^p)/e-p*polylog(2,b^(1/3)*(e*x+d
)/(b^(1/3)*d-a^(1/3)*e))/e-p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d+(-1)^(1/3
)*a^(1/3)*e))/e-p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e
))/e
```

Rubi [A]
time = 0.22, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2512, 266, 2463, 2441, 2440, 2438}

$$-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}\,(d+ex)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)}{e}-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}\,(d+ex)}{\sqrt[3]{-1}\,\sqrt[3]{a}\,e+\sqrt[3]{b}\,d}\right)}{e}-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}\,(d+ex)}{\sqrt[3]{b}\,d-(-1)^{2/3}\sqrt[3]{a}\,e}\right)}{e}+\frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e}-\frac{p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)}{e}-\frac{p\log(d+ex)\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-(-1)^{2/3}\sqrt[3]{a}\,e}\right)}{e}-\frac{p\log(d+ex)\log\left(\frac{\sqrt[3]{-1}\,e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x\right)}{\sqrt[3]{-1}\,\sqrt[3]{a}\,e+\sqrt[3]{b}\,d}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/(d + e*x),x]
```

```
[Out] -((p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x]
)/e) - (p*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3
)*a^(1/3)*e))]*Log[d + e*x])/e - (p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)
*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/e + (Log[d +
 e*x]*Log[c*(a + b*x^3)^p])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*
d - a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1
/3)*a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2
/3)*a^(1/3)*e)])/e
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx = \frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e} - \frac{(3bp)\int \frac{x^2\log(d+ex)}{a+bx^3}\,dx}{e}$$

$$= \frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e} - \frac{(3bp)\int \left(\frac{\log(d+ex)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{\log(d+ex)}{3b^{2/3}\left(-\sqrt[3]{-1}\,\sqrt[3]{a}+\sqrt[3]{\cdots}\right.}\right.}{e}$$

$$= \frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e} - \frac{\left(\sqrt[3]{b}\,p\right)\int \frac{\log(d+ex)}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{e} - \frac{\left(\sqrt[3]{b}\,p\right)\int \frac{\log(d+ex}{-\sqrt[3]{-1}\,\sqrt[3]{a}\cdots}}{e}$$

$$= -\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-(-1)^{2/3}\sqrt[3]{a}\,e}\right)\log(d+e}{e}$$

$$= -\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-(-1)^{2/3}\sqrt[3]{a}\,e}\right)\log(d+e}{e}$$

$$= -\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-(-1)^{2/3}\sqrt[3]{a}\,e}\right)\log(d+e}{e}$$

Mathematica [A]
time = 0.04, size = 313, normalized size = 1.02

$$-\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e}-\frac{p\log\left(-\frac{(-1)^{2/3}e\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e}-\frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{e}+\frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e}-\frac{p\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e}-\frac{p\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e}-\frac{p\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/(d + e*x),x]
```

```
[Out] -((p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x]
)/e) - (p*Log[-(((-1)^(2/3)*e*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/(b^(1/3)*d
- (-1)^(2/3)*a^(1/3)*e))]*Log[d + e*x])/e - (p*Log[((-1)^(1/3)*e*(a^(1/3) +
 (-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/e
 + (Log[d + e*x]*Log[c*(a + b*x^3)^p])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x)
)/(b^(1/3)*d - a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*
d + (-1)^(1/3)*a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*
d - (-1)^(2/3)*a^(1/3)*e)])/e
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.34, size = 261, normalized size = 0.85

method	result
risch	$\frac{\ln(ex+d)\ln\left(\left(x^{3}b+a\right)^{p}\right)}{e}-\frac{p\left(\sum_{\_R1=\mathrm{RootOf}\left(\_Z^{3}b-3\_Z^{2}bd+3\_Zb\,d^{2}+e^{3}a-b\,d^{3}\right)}\left(\ln(ex+d)\ln\left(\frac{-ex+\_R1-d}{\_R1}\right)+\mathrm{dilog}\left(\frac{-ex+\_R1-d}{\_R1}\right)\right)\right)}{e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] ln(e*x+d)/e*ln((b*x^3+a)^p)-p/e*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((-
e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3))+1/2*I
*ln(e*x+d)/e*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2-1/2*I*ln(e*x+d)
/e*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)-1/2*I*ln(e*x+d)/e
*Pi*csgn(I*c*(b*x^3+a)^p)^3+1/2*I*ln(e*x+d)/e*Pi*csgn(I*c*(b*x^3+a)^p)^2*cs
gn(I*c)+ln(e*x+d)/e*ln(c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((b*x^3 + a)^p*c)/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x^3 + a)^p*c)/(x*e + d), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x^3 + a)^p*c)/(x*e + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(b\,x^3+a\right)}^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)/(d + e*x),x)
```

```
[Out] int(log(c*(a + b*x^3)^p)/(d + e*x), x)
```

3.237 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x(d+ex)}\,dx$

Optimal. Leaf size=352

$$\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)\log(d+ex)}{d}+\frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-(-1)^{2/3}\sqrt[3]{a}\,e}\right)\log(d+ex)}{d}+\frac{p\log\left(\frac{\sqrt[3]{-1}\,e\left(\sqrt[3]{a}+(-\right.}{\sqrt[3]{b}\,d+\sqrt[3]{-1}\right.}$$

```
[Out] p*ln(-e*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-a^(1/3)*e))*ln(e*x+d)/d+p*ln(-e*((-1
)^(2/3)*a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))*ln(e*x+d)/d+p*
ln((-1)^(1/3)*e*(a^(1/3)+(-1)^(2/3)*b^(1/3)*x)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3
)*e))*ln(e*x+d)/d+1/3*ln(-b*x^3/a)*ln(c*(b*x^3+a)^p)/d-ln(e*x+d)*ln(c*(b*x^
3+a)^p)/d+p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-a^(1/3)*e))/d+p*polylog(2,
b^(1/3)*(e*x+d)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))/d+p*polylog(2,b^(1/3)*(e*
x+d)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))/d+1/3*p*polylog(2,1+b*x^3/a)/d
```

Rubi [A]

time = 0.33, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2516, 2504, 2441, 2352, 2512, 266, 2463, 2440, 2438}

$$\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d}+\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{d}+\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d}+\frac{p\,\text{PolyLog}\left(2,\frac{bx^3}{a}+1\right)}{3d}-\frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{d}+\frac{\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right)}{3d}+\frac{p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d}+\frac{p\log(d+ex)\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d}+\frac{p\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/(x*(d + e*x)),x]
```

```
[Out] (p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/
d + (p*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3)*a
^(1/3)*e))]*Log[d + e*x])/d + (p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)*b^
(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/d + (Log[-((b*x
^3)/a)]*Log[c*(a + b*x^3)^p])/(3*d) - (Log[d + e*x]*Log[c*(a + b*x^3)^p])/d
 + (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)])/d + (p*PolyL
og[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)])/d + (p*PolyL
og[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)])/d + (p*PolyL
og[2, 1 + (b*x^3)/a])/(3*d)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x(d+ex)}\,dx = \int \left(\frac{\log\left(c(a+bx^3)^p\right)}{dx} - \frac{e\log\left(c(a+bx^3)^p\right)}{d(d+ex)}\right)dx$$

$$= \frac{\int \frac{\log\left(c(a+bx^3)^p\right)}{x}\,dx}{d} - \frac{e\int \frac{\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx}{d}$$

$$= -\frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{d} + \frac{\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}\,dx, x, x^3\right)}{3d} + \frac{(3bp)\int \frac{x^2\log(}{a+b}}{d}$$

$$= \frac{\log\left(-\frac{bx^3}{a}\right)\log\left(c(a+bx^3)^p\right)}{3d} - \frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{d} - \frac{(bp)\text{Subst}\left(\int \frac{\log(}{a}\right.}{3d}$$

$$= \frac{\log\left(-\frac{bx^3}{a}\right)\log\left(c(a+bx^3)^p\right)}{3d} - \frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{d} + \frac{p\text{Li}_2\left(1+\frac{bx^3}{a}\right)}{3d} +$$

$$= \frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-(-1)^{2/3}\sqrt[3]{a}\,e}\right)\log(d+e}{d}$$

$$= \frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-(-1)^{2/3}\sqrt[3]{a}\,e}\right)\log(d+e}{d}$$

$$= \frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-(-1)^{2/3}\sqrt[3]{a}\,e}\right)\log(d+e}{d}$$

Mathematica [A]
time = 0.05, size = 358, normalized size = 1.02

$$\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{(-1)^{2/3}e\left(\sqrt[3]{a}-\sqrt[3]{-1}\,\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-(-1)^{2/3}\sqrt[3]{a}\,e}\right)\log(d+ex)}{d} + \frac{p\log\left(\frac{\sqrt[3]{-1}\,e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d+\sqrt[3]{-1}\,\sqrt[3]{a}\,e}\right)\log(d+ex)}{d} + \frac{\log\left(-\frac{bx^3}{a}\right)\log\left(c(a+bx^3)^p\right)}{3d} - \frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{d} + \frac{p\text{Li}_2\left(\frac{\sqrt[3]{b}\,(d+ex)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)}{d} + \frac{p\text{Li}_2\left(\frac{\sqrt[3]{b}\,(d+ex)}{\sqrt[3]{b}\,d+\sqrt[3]{-1}\,\sqrt[3]{a}\,e}\right)}{d} + \frac{p\text{Li}_2\left(\frac{\sqrt[3]{b}\,(d+ex)}{\sqrt[3]{b}\,d-(-1)^{2/3}\sqrt[3]{a}\,e}\right)}{d} + \frac{p\text{Li}_2\left(\frac{a+bx^3}{a}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/(x*(d + e*x)),x]
```

```
[Out] (p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/
d + (p*Log[-(((-1)^(2/3)*e*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/(b^(1/3)*d - (
-1)^(2/3)*a^(1/3)*e))]*Log[d + e*x])/d + (p*Log[((-1)^(1/3)*e*(a^(1/3) + (-
1)^(2/3)*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/d +
(Log[-((b*x^3)/a)]*Log[c*(a + b*x^3)^p])/(3*d) - (Log[d + e*x]*Log[c*(a + b
*x^3)^p])/d + (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)])/d
 + (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)])/d
 + (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)])/d
 + (p*PolyLog[2, (a + b*x^3)/a])/(3*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.63, size = 461, normalized size = 1.31

method	result
risch	$-\frac{\ln\left(\left(x^{3}b+a\right)^{p}\right)\ln\left(ex+d\right)}{d}+\frac{\ln\left(\left(x^{3}b+a\right)^{p}\right)\ln\left(x\right)}{d}-\frac{p\left(\sum_{\_R1=\mathrm{RootOf}\left(\_Z^{3}b+a\right)}\left(\ln\left(x\right)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)\right)}{d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/x/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -ln((b*x^3+a)^p)/d*ln(e*x+d)+ln((b*x^3+a)^p)/d*ln(x)-p/d*sum(ln(x)*ln((_R1-
x)/_R1)+dilog((_R1-x)/_R1),_R1=RootOf(_Z^3*b+a))+p/d*sum(ln(e*x+d)*ln((-e*x
+_R1-d)/_R1)+dilog((-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^
2+a*e^3-b*d^3))-1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2/d*ln(e
*x+d)+1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2/d*ln(x)+1/2*I*Pi
*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)/d*ln(e*x+d)-1/2*I*Pi*c
sgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)/d*ln(x)+1/2*I*Pi*csgn(I*
c*(b*x^3+a)^p)^3/d*ln(e*x+d)-1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^3/d*ln(x)-1/2*I
*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)/d*ln(e*x+d)+1/2*I*Pi*csgn(I*c*(b*x^3+
a)^p)^2*csgn(I*c)/d*ln(x)-ln(c)/d*ln(e*x+d)+ln(c)/d*ln(x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((b*x^3 + a)^p*c)/((x*e + d)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x^3 + a)^p*c)/(x^2*e + d*x), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/x/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x^3 + a)^p*c)/((x*e + d)*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(b\,x^3+a\right)}^p\right)}{x\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)/(x*(d + e*x)),x)
```

```
[Out] int(log(c*(a + b*x^3)^p)/(x*(d + e*x)), x)
```

3.238 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^2(d+ex)}\,dx$

Optimal. Leaf size=510

$$-\frac{\sqrt{3}\,\sqrt[3]{b}\,p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt[3]{a}\,d}-\frac{\sqrt[3]{b}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a}\,d}-\frac{ep\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)\log(d+ex)}{d^2}-\frac{ep\log}{}$$

```
[Out] -b^(1/3)*p*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/d-e*p*ln(-e*(a^(1/3)+b^(1/3)*x)/(b
^(1/3)*d-a^(1/3)*e))*ln(e*x+d)/d^2-e*p*ln(-e*((-1)^(2/3)*a^(1/3)+b^(1/3)*x)
/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))*ln(e*x+d)/d^2-e*p*ln((-1)^(1/3)*e*(a^(1/
3)+(-1)^(2/3)*b^(1/3)*x)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))*ln(e*x+d)/d^2+1/
2*b^(1/3)*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/d-ln(c*(b*x^3
+a)^p)/d/x-1/3*e*ln(-b*x^3/a)*ln(c*(b*x^3+a)^p)/d^2+e*ln(e*x+d)*ln(c*(b*x^3
+a)^p)/d^2-e*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-a^(1/3)*e))/d^2-e*p*pol
ylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))/d^2-e*p*polylog(2,
b^(1/3)*(e*x+d)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))/d^2-1/3*e*p*polylog(2,1+b
*x^3/a)/d^2-b^(1/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(
1/2)/a^(1/3)/d
```

Rubi [A]

time = 0.40, antiderivative size = 510, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 16, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {2516, 2505, 298, 31, 648, 631, 210, 642, 2504, 2441, 2352, 2512, 266, 2463, 2440, 2438}

$$-\frac{ep\,\text{PolyLog}\left(2,\frac{bx^3}{a}+1\right)}{3d^2}-\frac{ep\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d^2}-\frac{ep\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{d^2}-\frac{ep\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d^2}+\frac{\sqrt[3]{b}\,p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{a}\,d}-\frac{\sqrt{3}\sqrt[3]{b}\,p\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}\,d}-\frac{e\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right)}{3d^2}+\frac{e\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{d^2}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{dx}-\frac{ep\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d^2}-\frac{ep\log(d+ex)\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d^2}-\frac{ep\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{d^2}-\frac{\sqrt[3]{b}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/(x^2*(d + e*x)),x]
```

```
[Out] -((Sqrt[3]*b^(1/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(a^
(1/3)*d)) - (b^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/(a^(1/3)*d) - (e*p*Log[-((
e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/d^2 - (e*p
*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)
*e))]*Log[d + e*x])/d^2 - (e*p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)*b^(1
/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/d^2 + (b^(1/3)*p*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*a^(1/3)*d) - Log[c*(a +
b*x^3)^p]/(d*x) - (e*Log[-((b*x^3)/a)]*Log[c*(a + b*x^3)^p])/(3*d^2) + (e*L
og[d + e*x]*Log[c*(a + b*x^3)^p])/d^2 - (e*p*PolyLog[2, (b^(1/3)*(d + e*x))
/(b^(1/3)*d - a^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/
3)*d + (-1)^(1/3)*a^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b
```

```
^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)])/d^2 - (e*p*PolyLog[2, 1 + (b*x^3)/a])/(3
*d^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^2(d+ex)}\,dx = \int \left(\frac{\log\left(c(a+bx^3)^p\right)}{dx^2} - \frac{e\log\left(c(a+bx^3)^p\right)}{d^2x} + \frac{e^2\log\left(c(a+bx^3)^p\right)}{d^2(d+ex)}\right)dx$$

$$= \frac{\int \frac{\log(c(a+bx^3)^p)}{x^2}\,dx}{d} - \frac{e\int \frac{\log(c(a+bx^3)^p)}{x}\,dx}{d^2} + \frac{e^2\int \frac{\log(c(a+bx^3)^p)}{d+ex}\,dx}{d^2}$$

$$= -\frac{\log\left(c(a+bx^3)^p\right)}{dx} + \frac{e\log(d+ex)\log\left(c(a+bx^3)^p\right)}{d^2} - \frac{e\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}\,dx, x\right.}{3d^2}$$

$$= -\frac{\log\left(c(a+bx^3)^p\right)}{dx} - \frac{e\log\left(-\frac{bx^3}{a}\right)\log\left(c(a+bx^3)^p\right)}{3d^2} + \frac{e\log(d+ex)\log\left(c(a+bx^3\right.}{d^2}$$

$$= -\frac{\sqrt[3]{b}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a}\,d} - \frac{\log\left(c(a+bx^3)^p\right)}{dx} - \frac{e\log\left(-\frac{bx^3}{a}\right)\log\left(c(a+bx^3)^p\right)}{3d^2} +$$

$$= -\frac{\sqrt[3]{b}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a}\,d} - \frac{ep\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{e}{\cdots}\right.}{}$$

$$= -\frac{\sqrt{3}\,\sqrt[3]{b}\,p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt[3]{a}\,d} - \frac{\sqrt[3]{b}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a}\,d} - \frac{ep\log\left(-\frac{e\left(\sqrt[3]{a}+\cdots\right.}{\sqrt[3]{b}\,d-\cdots}\right.}{d\cdots}$$

$$= -\frac{\sqrt{3}\,\sqrt[3]{b}\,p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt[3]{a}\,d} - \frac{\sqrt[3]{b}\,p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a}\,d} - \frac{ep\log\left(-\frac{e\left(\sqrt[3]{a}+\cdots\right.}{\sqrt[3]{b}\,d-\cdots}\right.}{d\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.05, size = 424, normalized size = 0.83

$$\frac{3bpx^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a}\right)}{2ad} - \frac{ep\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{(-1)^{2/3}e\left(\sqrt[3]{a}-\sqrt[3]{-1}\,\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-(-1)^{2/3}\sqrt[3]{a}\,e}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(\frac{\sqrt[3]{-1}\,e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d+\sqrt[3]{-1}\,\sqrt[3]{a}\,e}\right)\log(d+ex)}{d^2} - \frac{\log\left(c(a+bx^3)^p\right)}{dx} - \frac{e\log\left(-\frac{bx^3}{a}\right)\log\left(c(a+bx^3)^p\right)}{3d^2} + \frac{e\log(d+ex)\log\left(c(a+bx^3)^p\right)}{d^2} - \frac{ep\text{Li}_2\left(\frac{\sqrt[3]{b}\,(d+ex)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\right)}{d^2} - \frac{ep\text{Li}_2\left(\frac{\sqrt[3]{b}\,(d+ex)}{\sqrt[3]{b}\,d+\sqrt[3]{-1}\,\sqrt[3]{a}\,e}\right)}{d^2} - \frac{ep\text{Li}_2\left(\frac{\sqrt[3]{b}\,(d+ex)}{\sqrt[3]{b}\,d-(-1)^{2/3}\sqrt[3]{a}\,e}\right)}{d^2} - \frac{ep\text{Li}_2\left(\frac{a+bx^3}{a}\right)}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/(x^2*(d + e*x)),x]
```

```
[Out] (3*b*p*x^2*Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)])/(2*a*d) - (e*p*Log
[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/d^2 -
```

```
(e*p*Log[-(((-1)^(2/3)*e*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/(b^(1/3)*d - (-1
)^(2/3)*a^(1/3)*e))]*Log[d + e*x])/d^2 - (e*p*Log[((-1)^(1/3)*e*(a^(1/3) +
(-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/d^
2 - Log[c*(a + b*x^3)^p]/(d*x) - (e*Log[-((b*x^3)/a)]*Log[c*(a + b*x^3)^p])
/(3*d^2) + (e*Log[d + e*x]*Log[c*(a + b*x^3)^p])/d^2 - (e*p*PolyLog[2, (b^(
1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (b^(1/3)*(d
 + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (b^(1/3
)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (a
+ b*x^3)/a])/(3*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.62, size = 732, normalized size = 1.44

method	result
risch	$\frac{\ln((x^3b+a)^p)e\ln(ex+d)}{d^2} - \frac{\ln((x^3b+a)^p)}{dx} - \frac{\ln((x^3b+a)^p)e\ln(x)}{d^2} - \frac{pe\left(\sum_{\_R1=\mathrm{RootOf}(\_Z^3b-3\_Z^2bd+3\_Zbd^2+e^3a-bd^3)}\left(\ln(e\right.\right.}{d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] ln((b*x^3+a)^p)*e/d^2*ln(e*x+d)-ln((b*x^3+a)^p)/d/x-ln((b*x^3+a)^p)*e/d^2*l
n(x)-p*e/d^2*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((-e*x+_R1-d)/_R1),_R1
=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3))-p/d/(a/b)^(1/3)*ln(x+(a/
b)^(1/3))+1/2*p/d/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+p/d*3^(1/2)
/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+p*e/d^2*sum(ln(x)*ln((
_R1-x)/_R1)+dilog((_R1-x)/_R1),_R1=RootOf(_Z^3*b+a))-1/2*I*Pi*csgn(I*(b*x^3
+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)*e/d^2*ln(e*x+d)+1/2*I*Pi*csgn(I*(b*x
^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)*e/d^2*ln(x)+1/2*I*Pi*csgn(I*c*(b*x
^3+a)^p)^2*csgn(I*c)*e/d^2*ln(e*x+d)+1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*
(b*x^3+a)^p)*csgn(I*c)/d/x-1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^
p)^2/d/x+1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^3*e/d^2*ln(x)-1/2*I*Pi*csgn(I*c*(b*
x^3+a)^p)^2*csgn(I*c)*e/d^2*ln(x)-1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^3*e/d^2*ln
(e*x+d)+1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^3/d/x-1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)
^2*csgn(I*c)/d/x+1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2*e/d^2
*ln(e*x+d)-1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2*e/d^2*ln(x)
+ln(c)*e/d^2*ln(e*x+d)-ln(c)/d/x-ln(c)*e/d^2*ln(x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((b*x^3 + a)^p*c)/((x*e + d)*x^2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x^3 + a)^p*c)/(x^3*e + d*x^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/x**2/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x^3 + a)^p*c)/((x*e + d)*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(b\,x^3+a\right)}^p\right)}{x^2\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)/(x^2*(d + e*x)),x)
```

```
[Out] int(log(c*(a + b*x^3)^p)/(x^2*(d + e*x)), x)
```

3.239 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^3(d+ex)}\,dx$

Optimal. Leaf size=674

$$-\frac{\sqrt{3}\,b^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2a^{2/3}d}+\frac{\sqrt{3}\,\sqrt[3]{b}\,ep\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt[3]{a}\,d^2}+\frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2a^{2/3}d}+\frac{\sqrt[3]{b}\,ep\log\left(\right.}{\sqrt[3]{}}$$

```
[Out] 1/2*b^(2/3)*p*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/d+b^(1/3)*e*p*ln(a^(1/3)+b^(1/3
)*x)/a^(1/3)/d^2+e^2*p*ln(-e*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-a^(1/3)*e))*ln(
e*x+d)/d^3+e^2*p*ln(-e*((-1)^(2/3)*a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-(-1)^(2/3)
*a^(1/3)*e))*ln(e*x+d)/d^3+e^2*p*ln((-1)^(1/3)*e*(a^(1/3)+(-1)^(2/3)*b^(1/3
)*x)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))*ln(e*x+d)/d^3-1/4*b^(2/3)*p*ln(a^(2/
3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/d-1/2*b^(1/3)*e*p*ln(a^(2/3)-a^(1
/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/d^2-1/2*ln(c*(b*x^3+a)^p)/d/x^2+e*ln(c*(
b*x^3+a)^p)/d^2/x+1/3*e^2*ln(-b*x^3/a)*ln(c*(b*x^3+a)^p)/d^3-e^2*ln(e*x+d)*
ln(c*(b*x^3+a)^p)/d^3+e^2*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-a^(1/3)*e)
)/d^3+e^2*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))/d^3
+e^2*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))/d^3+1/3*
e^2*p*polylog(2,1+b*x^3/a)/d^3-1/2*b^(2/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*
x)/a^(1/3)*3^(1/2))*3^(1/2)/a^(2/3)/d+b^(1/3)*e*p*arctan(1/3*(a^(1/3)-2*b^(
1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/a^(1/3)/d^2
```

Rubi [A]
time = 0.48, antiderivative size = 674, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 17, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {2516, 2505, 206, 31, 648, 631, 210, 642, 298, 2504, 2441, 2352, 2512, 266, 2463, 2440, 2438}

$$\frac{e^2p\,\text{PolyLog}\left(2,\frac{bx^3}{a}+1\right)}{3d^3}+\frac{e^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d^3}+\frac{e^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{d^3}+\frac{e^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d^3}-\frac{\sqrt{3}\,b^{2/3}p\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{2/3}d}-\frac{\sqrt[3]{b}\,ep\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{a}d^2}-\frac{b^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4a^{2/3}d}+\frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a^{2/3}d}+\frac{\sqrt{3}\sqrt[3]{b}\,ep\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}d^2}+\frac{e^2\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right)}{3d^3}-\frac{e^2\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{d^3}+\frac{e\log\left(c\left(a+bx^3\right)^p\right)}{d^2x}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2dx^2}+\frac{e^2p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d^3}+\frac{e^2p\log(d+ex)\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d^3}+\frac{e^2p\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{d^3}+\frac{\sqrt[3]{b}\,ep\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}d^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^3)^p]/(x^3*(d + e*x)),x]
```

```
[Out] -1/2*(Sqrt[3]*b^(2/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/
(a^(2/3)*d) + (Sqrt[3]*b^(1/3)*e*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(a^(1/3)*d^2) + (b^(2/3)*p*Log[a^(1/3) + b^(1/3)*x])/(2*a^(2/3)*
d) + (b^(1/3)*e*p*Log[a^(1/3) + b^(1/3)*x])/(a^(1/3)*d^2) + (e^2*p*Log[-((e
*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/d^3 + (e^2*
p*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3
)*e))]*Log[d + e*x])/d^3 + (e^2*p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)*b
^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/d^3 - (b^(2/3)
*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(4*a^(2/3)*d) - (b^(1/3)
```

```
*e*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*a^(1/3)*d^2) - Log[
c*(a + b*x^3)^p]/(2*d*x^2) + (e*Log[c*(a + b*x^3)^p])/(d^2*x) + (e^2*Log[-(
(b*x^3)/a)]*Log[c*(a + b*x^3)^p])/(3*d^3) - (e^2*Log[d + e*x]*Log[c*(a + b*
x^3)^p])/d^3 + (e^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e
)])/d^3 + (e^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(
1/3)*e)])/d^3 + (e^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/
3)*a^(1/3)*e)])/d^3 + (e^2*p*PolyLog[2, 1 + (b*x^3)/a])/(3*d^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^3(d+ex)}\,dx = \int \left(\frac{\log\left(c(a+bx^3)^p\right)}{dx^3} - \frac{e\log\left(c(a+bx^3)^p\right)}{d^2x^2} + \frac{e^2\log\left(c(a+bx^3)^p\right)}{d^3x} - \frac{e^3\log\left(c(a\ \cdots\right.}{d^3(d+\ \cdots}\right.$$

$$= \frac{\int \frac{\log\left(c(a+bx^3)^p\right)}{x^3}\,dx}{d} - \frac{e\int \frac{\log\left(c(a+bx^3)^p\right)}{x^2}\,dx}{d^2} + \frac{e^2\int \frac{\log\left(c(a+bx^3)^p\right)}{x}\,dx}{d^3} - \frac{e^3\int \frac{\log\left(c(a+bx^3)^p\right)}{d+ex}\ \cdots}{d^3}$$

$$= -\frac{\log\left(c(a+bx^3)^p\right)}{2dx^2} + \frac{e\log\left(c(a+bx^3)^p\right)}{d^2x} - \frac{e^2\log(d+ex)\log\left(c(a+bx^3)^p\right)}{d^3} + \frac{e^2\ \cdots}{\cdots}$$

$$= -\frac{\log\left(c(a+bx^3)^p\right)}{2dx^2} + \frac{e\log\left(c(a+bx^3)^p\right)}{d^2x} + \frac{e^2\log\left(-\frac{bx^3}{a}\right)\log\left(c(a+bx^3)^p\right)}{3d^3} - \frac{e^2\ \cdots}{\cdots}$$

$$= \frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2a^{2/3}d} + \frac{\sqrt[3]{b}\,ep\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a}\,d^2} - \frac{\log\left(c(a+bx^3)^p\right)}{2dx^2} + \frac{e\log\ \cdots}{\cdots}$$

$$= \frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2a^{2/3}d} + \frac{\sqrt[3]{b}\,ep\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a}\,d^2} + \frac{e^2p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}\,d-\sqrt[3]{a}\,e}\ \cdots\right.}{d^3}$$

$$= -\frac{\sqrt{3}\,b^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2a^{2/3}d} + \frac{\sqrt{3}\,\sqrt[3]{b}\,ep\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt[3]{a}\,d^2} + \frac{b^{2/3}p\log\left(\ \cdots\right.}{2\ \cdots}$$

$$= -\frac{\sqrt{3}\,b^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2a^{2/3}d} + \frac{\sqrt{3}\,\sqrt[3]{b}\,ep\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt[3]{a}\,d^2} + \frac{b^{2/3}p\log\left(\ \cdots\right.}{2\ \cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.27, size = 542, normalized size = 0.80

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^3)^p]/(x^3*(d + e*x)),x]
```

```
[Out] ((-6*Sqrt[3]*b^(2/3)*d^2*p*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(
2/3) - (18*b*d*e*p*x^2*Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)])/a + (6
```

```
*b^(2/3)*d^2*p*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + 12*e^2*p*Log[(e*((-1)^(1
/3)*a^(1/3) - b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x]
+ 12*e^2*p*Log[(e*(a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*d) + a^(1/3)*e)]*Log[d
+ e*x] + 12*e^2*p*Log[(e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*d) +
(-1)^(2/3)*a^(1/3)*e)]*Log[d + e*x] - (3*b^(2/3)*d^2*p*Log[a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3) - (6*d^2*Log[c*(a + b*x^3)^p])/x^2 + (1
2*d*e*Log[c*(a + b*x^3)^p])/x + 4*e^2*Log[-((b*x^3)/a)]*Log[c*(a + b*x^3)^p
] - 12*e^2*Log[d + e*x]*Log[c*(a + b*x^3)^p] + 12*e^2*p*PolyLog[2, (b^(1/3)
*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)] + 12*e^2*p*PolyLog[2, (b^(1/3)*(d + e*
x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)] + 12*e^2*p*PolyLog[2, (b^(1/3)*(d +
 e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)] + 4*e^2*p*PolyLog[2, 1 + (b*x^3)
/a])/(12*d^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.61, size = 1025, normalized size = 1.52

method	result	size
risch	Expression too large to display	1025

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^3+a)^p)/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)*e^2/d^3*ln(e*x+d)+p*e^2/d^3*sum
(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*b-3
*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3))-1/2*ln((b*x^3+a)^p)/d/x^2-1/2*I*Pi*csgn(
I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)*e/d^2/x+1/2*I*Pi*csgn(I*(b*x
^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)*e^2/d^3*ln(e*x+d)+1/2*p/d/(a/b)^(2
/3)*ln(x+(a/b)^(1/3))-1/4*p/d/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))
-p*e^2/d^3*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R1),_R1=RootOf(_Z^3*b+a
))-1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)*csgn(I*c)*e^2/d^3*ln(
x)-ln(c)*e^2/d^3*ln(e*x+d)+ln(c)*e^2/d^3*ln(x)+ln(c)*e/d^2/x+1/2*I*Pi*csgn(
I*c*(b*x^3+a)^p)^2*csgn(I*c)*e^2/d^3*ln(x)+p/d^2*e/(a/b)^(1/3)*ln(x+(a/b)^(
1/3))-1/2*p/d^2*e/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/2*ln(c)/d
/x^2-ln((b*x^3+a)^p)*e^2/d^3*ln(e*x+d)+ln((b*x^3+a)^p)*e^2/d^3*ln(x)+ln((b*
x^3+a)^p)*e/d^2/x-p/d^2*e*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(
1/3)*x-1))+1/2*p/d/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-
1))-1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2*e^2/d^3*ln(e*x+d)-
1/4*I*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)/d/x^2-1/4*I*Pi*csgn(I*(b*x^3+a)^
p)*csgn(I*c*(b*x^3+a)^p)^2/d/x^2+1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x
^3+a)^p)^2*e/d^2/x+1/4*I*Pi*csgn(I*c*(b*x^3+a)^p)^3/d/x^2+1/2*I*Pi*csgn(I*c
*(b*x^3+a)^p)^3*e^2/d^3*ln(e*x+d)+1/4*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*
x^3+a)^p)*csgn(I*c)/d/x^2+1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^2*csgn(I*c)*e/d^2/
x+1/2*I*Pi*csgn(I*(b*x^3+a)^p)*csgn(I*c*(b*x^3+a)^p)^2*e^2/d^3*ln(x)-1/2*I*
Pi*csgn(I*c*(b*x^3+a)^p)^3*e^2/d^3*ln(x)-1/2*I*Pi*csgn(I*c*(b*x^3+a)^p)^3*e
/d^2/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^3/(e*x+d),x, algorithm="maxima")
[Out] integrate(log((b*x^3 + a)^p*c)/((x*e + d)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^3/(e*x+d),x, algorithm="fricas")
[Out] integral(log((b*x^3 + a)^p*c)/(x^4*e + d*x^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**3+a)**p)/x**3/(e*x+d),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^3+a)^p)/x^3/(e*x+d),x, algorithm="giac")
[Out] integrate(log((b*x^3 + a)^p*c)/((x*e + d)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(b\,x^3+a\right)}^p\right)}{x^3\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^3)^p)/(x^3*(d + e*x)),x)
[Out] int(log(c*(a + b*x^3)^p)/(x^3*(d + e*x)), x)
```

3.240 $\int \frac{x^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex} \, dx$

Optimal. Leaf size=297

$$-\frac{bdpx}{2ae^2}-\frac{b^2px}{3a^2e}+\frac{bpx^2}{6ae}+\frac{d^2x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3}-\frac{dx^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e^2}+\frac{x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e}+\frac{bd^2p\log(b+ax)}{ae^3}+\frac{b^2dp}{}$$

```
[Out] -1/2*b*d*p*x/a/e^2-1/3*b^2*p*x/a^2/e+1/6*b*p*x^2/a/e+d^2*x*ln(c*(a+b/x)^p)/
e^3-1/2*d*x^2*ln(c*(a+b/x)^p)/e^2+1/3*x^3*ln(c*(a+b/x)^p)/e+b*d^2*p*ln(a*x+
b)/a/e^3+1/2*b^2*d*p*ln(a*x+b)/a^2/e^2+1/3*b^3*p*ln(a*x+b)/a^3/e-d^3*ln(c*(
a+b/x)^p)*ln(e*x+d)/e^4-d^3*p*ln(-e*x/d)*ln(e*x+d)/e^4+d^3*p*ln(-e*(a*x+b)/
(a*d-b*e))*ln(e*x+d)/e^4+d^3*p*polylog(2,a*(e*x+d)/(a*d-b*e))/e^4-d^3*p*pol
ylog(2,1+e*x/d)/e^4
```

Rubi [A]
time = 0.22, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {2516, 2498, 269, 31, 2505, 199, 45, 2512, 266, 2463, 2441, 2352, 2440, 2438}

$$\frac{d^3p\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e^4}-\frac{d^3p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^4}+\frac{b^3p\log(ax+b)}{3a^3e}+\frac{b^2dp\log(ax+b)}{2a^2e^2}-\frac{b^2px}{3a^2e}-\frac{d^3\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^4}+\frac{d^2x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3}-\frac{dx^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e^2}+\frac{x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e}+\frac{d^3p\log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right)}{e^4}+\frac{bd^2p\log(ax+b)}{ae^3}-\frac{bdpx}{2ae^2}+\frac{bpx^2}{6ae}-\frac{d^3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Log[c*(a + b/x)^p])/(d + e*x),x]
```

```
[Out] -1/2*(b*d*p*x)/(a*e^2) - (b^2*p*x)/(3*a^2*e) + (b*p*x^2)/(6*a*e) + (d^2*x*L
og[c*(a + b/x)^p])/e^3 - (d*x^2*Log[c*(a + b/x)^p])/(2*e^2) + (x^3*Log[c*(a
 + b/x)^p])/(3*e) + (b*d^2*p*Log[b + a*x])/(a*e^3) + (b^2*d*p*Log[b + a*x])
/(2*a^2*e^2) + (b^3*p*Log[b + a*x])/(3*a^3*e) - (d^3*Log[c*(a + b/x)^p]*Log
[d + e*x])/e^4 - (d^3*p*Log[-((e*x)/d)]*Log[d + e*x])/e^4 + (d^3*p*Log[-((e
*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e^4 + (d^3*p*PolyLog[2, (a*(d + e*x
))/(a*d - b*e)])/e^4 - (d^3*p*PolyLog[2, 1 + (e*x)/d])/e^4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \int \left(\frac{d^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3} - \frac{dx \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2} + \frac{x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e} - \frac{d^3 \log \left(c(a\right.}{e^3(d+}\right.$$

$$= \frac{d^2 \int \log \left(c\left(a+\frac{b}{x}\right)^p\right)\,dx}{e^3} - \frac{d^3 \int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx}{e^3} - \frac{d \int x \log \left(c\left(a+\frac{b}{x}\right)^p\right)\,dx}{e^2} + \int$$

$$= \frac{d^2 x \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3} - \frac{dx^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{2e^2} + \frac{x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} - \frac{d^3 \log \left(c\left(a+\right.\right.}{}$$

$$= \frac{d^2 x \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3} - \frac{dx^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{2e^2} + \frac{x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} - \frac{d^3 \log \left(c\left(a+\right.\right.}{}$$

$$= \frac{d^2 x \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3} - \frac{dx^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{2e^2} + \frac{x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} + \frac{bd^2 p \log(b+}{ae^3}$$

$$= -\frac{bdpx}{2ae^2} - \frac{b^2 px}{3a^2 e} + \frac{bpx^2}{6ae} + \frac{d^2 x \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3} - \frac{dx^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{2e^2} + \frac{x^3 \log \left(c\right.}{3}$$

$$= -\frac{bdpx}{2ae^2} - \frac{b^2 px}{3a^2 e} + \frac{bpx^2}{6ae} + \frac{d^2 x \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3} - \frac{dx^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{2e^2} + \frac{x^3 \log \left(c\right.}{3}$$

$$= -\frac{bdpx}{2ae^2} - \frac{b^2 px}{3a^2 e} + \frac{bpx^2}{6ae} + \frac{d^2 x \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3} - \frac{dx^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{2e^2} + \frac{x^3 \log \left(c\right.}{3}$$

Mathematica [A]
time = 0.15, size = 344, normalized size = 1.16

$$\frac{-6a^2bd^2ep - 3a^2bde^2px - 2ab^2e^3px + a^2be^3px^2 + 6a^3d^2ex\log\left(c\left(a+\frac{b}{x}\right)^p\right) - 3a^3de^2x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right) + 2a^3e^3x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right) + 6a^2bd^2ep\log\left(\frac{b}{a}+x\right) + 3ab^2de^2p\log(b+ax) + 2b^3e^3p\log(b+ax) - 6a^3d^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex) - 6a^3d^3p\log(x)\log(d+ex) + 6a^3d^3p\log\left(\frac{b}{a}+x\right)\log(d+ex) - 6a^3d^3p\log\left(\frac{b}{a}+x\right)\log\left(\frac{a(d+ex)}{ad-be}\right) + 6a^3d^3p\log(x)\log\left(1+\frac{ex}{d}\right) + 6a^3d^3p\text{Li}_2\left(-\frac{ex}{d}\right) - 6a^3d^3p\text{Li}_2\left(\frac{e(b+ax)}{-ad+be}\right)}{6a^3e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Log[c*(a + b/x)^p])/(d + e*x),x]
```

```
[Out] (-6*a^2*b*d^2*e*p - 3*a^2*b*d*e^2*p*x - 2*a*b^2*e^3*p*x + a^2*b*e^3*p*x^2 +
 6*a^3*d^2*e*x*Log[c*(a + b/x)^p] - 3*a^3*d*e^2*x^2*Log[c*(a + b/x)^p] + 2*
a^3*e^3*x^3*Log[c*(a + b/x)^p] + 6*a^2*b*d^2*e*p*Log[b/a + x] + 3*a*b^2*d*e
^2*p*Log[b + a*x] + 2*b^3*e^3*p*Log[b + a*x] - 6*a^3*d^3*Log[c*(a + b/x)^p]
*Log[d + e*x] - 6*a^3*d^3*p*Log[x]*Log[d + e*x] + 6*a^3*d^3*p*Log[b/a + x]*
Log[d + e*x] - 6*a^3*d^3*p*Log[b/a + x]*Log[(a*(d + e*x))/(a*d - b*e)] + 6*
a^3*d^3*p*Log[x]*Log[1 + (e*x)/d] + 6*a^3*d^3*p*PolyLog[2, -((e*x)/d)] - 6*
a^3*d^3*p*PolyLog[2, (e*(b + a*x))/(-(a*d) + b*e)])/(6*a^3*e^4)
```

Maple [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{x^3 \ln \left(c\left(a+\frac{b}{x}\right)^p\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(a+b/x)^p)/(e*x+d),x)
```

```
[Out] int(x^3*ln(c*(a+b/x)^p)/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(x^3*log((a + b/x)^p*c)/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x^3*log(c*((a*x + b)/x)^p)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(a+b/x)**p)/(e*x+d),x)
```

```
[Out] Integral(x**3*log(c*(a + b/x)**p)/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(x^3*log((a + b/x)^p*c)/(x*e + d), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \ln \left(c \left(a+\frac{b}{x}\right)^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*log(c*(a + b/x)^p))/(d + e*x),x)
[Out] int((x^3*log(c*(a + b/x)^p))/(d + e*x), x)
```

3.241 $\int \frac{x^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=219

$$\frac{bpx}{2ae}-\frac{dx\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}-\frac{bdp\log(b+ax)}{ae^2}-\frac{b^2p\log(b+ax)}{2a^2e}+\frac{d^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+$$

```
[Out] 1/2*b*p*x/a/e-d*x*ln(c*(a+b/x)^p)/e^2+1/2*x^2*ln(c*(a+b/x)^p)/e-b*d*p*ln(a*
x+b)/a/e^2-1/2*b^2*p*ln(a*x+b)/a^2/e+d^2*ln(c*(a+b/x)^p)*ln(e*x+d)/e^3+d^2*
p*ln(-e*x/d)*ln(e*x+d)/e^3-d^2*p*ln(-e*(a*x+b)/(a*d-b*e))*ln(e*x+d)/e^3-d^2
*p*polylog(2,a*(e*x+d)/(a*d-b*e))/e^3+d^2*p*polylog(2,1+e*x/d)/e^3
```

Rubi [A]
time = 0.19, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {2516, 2498, 269, 31, 2505, 199, 45, 2512, 266, 2463, 2441, 2352, 2440, 2438}

$$-\frac{d^2p\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e^3}+\frac{d^2p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^3}-\frac{b^2p\log(ax+b)}{2a^2e}+\frac{d^2\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3}-\frac{dx\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}-\frac{d^2p\log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right)}{e^3}-\frac{bdp\log(ax+b)}{ae^2}+\frac{bpx}{2ae}+\frac{d^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Log[c*(a + b/x)^p])/(d + e*x),x]
```

```
[Out] (b*p*x)/(2*a*e) - (d*x*Log[c*(a + b/x)^p])/e^2 + (x^2*Log[c*(a + b/x)^p])/(
2*e) - (b*d*p*Log[b + a*x])/(a*e^2) - (b^2*p*Log[b + a*x])/(2*a^2*e) + (d^2
*Log[c*(a + b/x)^p]*Log[d + e*x])/e^3 + (d^2*p*Log[-((e*x)/d)]*Log[d + e*x]
)/e^3 - (d^2*p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e^3 - (d^2*p
*PolyLog[2, (a*(d + e*x))/(a*d - b*e)])/e^3 + (d^2*p*PolyLog[2, 1 + (e*x)/d
])/e^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \int \left(-\frac{d\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2} + \frac{x\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e} + \frac{d^2\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2(d+ex)}\right) dx$$

$$= -\frac{d\int \log \left(c\left(a+\frac{b}{x}\right)^p\right)\,dx}{e^2} + \frac{d^2\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}dx}{e^2} + \frac{\int x\log \left(c\left(a+\frac{b}{x}\right)^p\right)\,dx}{e}$$

$$= -\frac{dx\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2} + \frac{x^2\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{2e} + \frac{d^2\log \left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^3} + $$

$$= -\frac{dx\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2} + \frac{x^2\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{2e} + \frac{d^2\log \left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^3} + $$

$$= -\frac{dx\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2} + \frac{x^2\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{2e} - \frac{bdp\log(b+ax)}{ae^2} + \frac{d^2\log \left(c\left(a+\frac{b}{x}\right)\right.}{e^3}$$

$$= \frac{bpx}{2ae} - \frac{dx\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2} + \frac{x^2\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{2e} - \frac{bdp\log(b+ax)}{ae^2} - \frac{b^2p\log(b+}{2a^2e}$$

$$= \frac{bpx}{2ae} - \frac{dx\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2} + \frac{x^2\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{2e} - \frac{bdp\log(b+ax)}{ae^2} - \frac{b^2p\log(b+}{2a^2e}$$

$$= \frac{bpx}{2ae} - \frac{dx\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2} + \frac{x^2\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{2e} - \frac{bdp\log(b+ax)}{ae^2} - \frac{b^2p\log(b+}{2a^2e}$$

Mathematica [A]
time = 0.08, size = 265, normalized size = 1.21

$$\frac{2abdep + abe^2px - 2a^2dex\log\left(c\left(a+\frac{b}{x}\right)^p\right) + a^2e^2x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right) - 2abdep\log\left(\frac{b}{a}+x\right) - b^2e^2p\log(b+ax) + 2a^2d^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex) + 2a^2d^2p\log(x)\log(d+ex) - 2a^2d^2p\log\left(\frac{b}{a}+x\right)\log(d+ex) + 2a^2d^2p\log\left(\frac{b}{a}+x\right)\log\left(\frac{a(d+ex)}{ad-be}\right) - 2a^2d^2p\log(x)\log\left(1+\frac{ex}{d}\right) - 2a^2d^2p\text{Li}_2\left(-\frac{ex}{d}\right) + 2a^2d^2p\text{Li}_2\left(\frac{e(b+ax)}{-ad+be}\right)}{2a^2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Log[c*(a + b/x)^p])/(d + e*x),x]
```

```
[Out] (2*a*b*d*e*p + a*b*e^2*p*x - 2*a^2*d*e*x*Log[c*(a + b/x)^p] + a^2*e^2*x^2*L
og[c*(a + b/x)^p] - 2*a*b*d*e*p*Log[b/a + x] - b^2*e^2*p*Log[b + a*x] + 2*a
^2*d^2*Log[c*(a + b/x)^p]*Log[d + e*x] + 2*a^2*d^2*p*Log[x]*Log[d + e*x] -
2*a^2*d^2*p*Log[b/a + x]*Log[d + e*x] + 2*a^2*d^2*p*Log[b/a + x]*Log[(a*(d
+ e*x))/(a*d - b*e)] - 2*a^2*d^2*p*Log[x]*Log[1 + (e*x)/d] - 2*a^2*d^2*p*Po
lyLog[2, -((e*x)/d)] + 2*a^2*d^2*p*PolyLog[2, (e*(b + a*x))/(-(a*d) + b*e)]
)/(2*a^2*e^3)
```

Maple [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{x^2 \ln \left(c\left(a+\frac{b}{x}\right)^p\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(a+b/x)^p)/(e*x+d),x)
```

```
[Out] int(x^2*ln(c*(a+b/x)^p)/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(x^2*log((a + b/x)^p*c)/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x^2*log(c*((a*x + b)/x)^p)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(a+b/x)**p)/(e*x+d),x)
```

```
[Out] Integral(x**2*log(c*(a + b/x)**p)/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(x^2*log((a + b/x)^p*c)/(x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\ln\left(c\,{\left(a+\frac{b}{x}\right)}^p\right)}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*log(c*(a + b/x)^p))/(d + e*x),x)
[Out] int((x^2*log(c*(a + b/x)^p))/(d + e*x), x)
```

3.242 $\int \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=151

$$\frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}+\frac{bp\log(b+ax)}{ae}-\frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^2}-\frac{dp\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^2}+\frac{dp\log\left(-\frac{e(b+ax)}{ad-be}\right.}{e}$$

```
[Out] x*ln(c*(a+b/x)^p)/e+b*p*ln(a*x+b)/a/e-d*ln(c*(a+b/x)^p)*ln(e*x+d)/e^2-d*p*l
n(-e*x/d)*ln(e*x+d)/e^2+d*p*ln(-e*(a*x+b)/(a*d-b*e))*ln(e*x+d)/e^2+d*p*poly
log(2,a*(e*x+d)/(a*d-b*e))/e^2-d*p*polylog(2,1+e*x/d)/e^2
```

Rubi [A]

time = 0.15, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {2516, 2498, 269, 31, 2512, 266, 2463, 2441, 2352, 2440, 2438}

$$\frac{dp\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e^2}-\frac{dp\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^2}-\frac{d\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2}+\frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}+\frac{dp\log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right)}{e^2}+\frac{bp\log(ax+b)}{ae}-\frac{dp\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[c*(a + b/x)^p])/(d + e*x),x]
```

```
[Out] (x*Log[c*(a + b/x)^p])/e + (b*p*Log[b + a*x])/(a*e) - (d*Log[c*(a + b/x)^p]
*Log[d + e*x])/e^2 - (d*p*Log[-((e*x)/d)]*Log[d + e*x])/e^2 + (d*p*Log[-((e
*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e^2 + (d*p*PolyLog[2, (a*(d + e*x))
/(a*d - b*e)])/e^2 - (d*p*PolyLog[2, 1 + (e*x)/d])/e^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \int \left(\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)}\right)dx$$

$$= \frac{\int \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx}{e} - \frac{d\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx}{e}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^2} - \frac{(bdp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x}\right)x^2}\,dx}{e^2} + \frac{(bp)\int}{}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^2} - \frac{(bdp)\int\left(\frac{\log(d+ex)}{bx} - \frac{a\log(d+ex)}{b(b+ax)}\right.}{e^2}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e} + \frac{bp\log(b+ax)}{ae} - \frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^2} - \frac{(dp)\int \frac{\log(d+}{x}}{e^2}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e} + \frac{bp\log(b+ax)}{ae} - \frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^2} - \frac{dp\log\left(-\frac{ex}{d}\right.}{e}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e} + \frac{bp\log(b+ax)}{ae} - \frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^2} - \frac{dp\log\left(-\frac{ex}{d}\right.}{e}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e} + \frac{bp\log(b+ax)}{ae} - \frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^2} - \frac{dp\log\left(-\frac{ex}{d}\right.}{e}$$

Mathematica [A]
time = 0.04, size = 149, normalized size = 0.99

$$\frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e} + \frac{bp\left(\frac{\log\left(a+\frac{b}{x}\right)}{a} + \frac{\log(x)}{a}\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^2} - \frac{dp\left(\log\left(-\frac{ex}{d}\right)\log(d+ex) - \log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex) + \text{Li}_2\left(\frac{d+ex}{d}\right) - \text{Li}_2\left(\frac{a(d+ex)}{ad-be}\right)\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[c*(a + b/x)^p])/(d + e*x),x]
```

```
[Out] (x*Log[c*(a + b/x)^p])/e + (b*p*(Log[a + b/x]/a + Log[x]/a))/e - (d*Log[c*(
a + b/x)^p]*Log[d + e*x])/e^2 - (d*p*(Log[-((e*x)/d)]*Log[d + e*x] - Log[-(
(e*(b + a*x))/(a*d - b*e))]*Log[d + e*x] + PolyLog[2, (d + e*x)/d] - PolyLo
g[2, (a*(d + e*x))/(a*d - b*e)]))/e^2
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{x\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(a+b/x)^p)/(e*x+d),x)
```

```
[Out] int(x*ln(c*(a+b/x)^p)/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(x*log((a + b/x)^p*c)/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x*log(c*((a*x + b)/x)^p)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \log \left(c \left(a + \frac{b}{x}\right)^{p}\right)}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(a+b/x)**p)/(e*x+d),x)
```

```
[Out] Integral(x*log(c*(a + b/x)**p)/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(x*log((a + b/x)^p*c)/(x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \ln \left(c \left(a + \frac{b}{x}\right)^p\right)}{d + e\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(c*(a + b/x)^p))/(d + e*x),x)
[Out] int((x*log(c*(a + b/x)^p))/(d + e*x), x)
```

3.243 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=113

$$\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e}+\frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e}-\frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e}-\frac{p\text{Li}_2\left(\frac{a(d+ex)}{ad-be}\right)}{e}+\frac{p\text{Li}_2(}{}$$

[Out] ln(c*(a+b/x)^p)*ln(e*x+d)/e+p*ln(-e*x/d)*ln(e*x+d)/e-p*ln(-e*(a*x+b)/(a*d-b*e))*ln(e*x+d)/e-p*polylog(2,a*(e*x+d)/(a*d-b*e))/e+p*polylog(2,1+e*x/d)/e

Rubi [A]
time = 0.10, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2512, 266, 2463, 2441, 2352, 2440, 2438}

$$-\frac{p\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e}+\frac{p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e}+\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}-\frac{p\log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right)}{e}+\frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e}$$

Antiderivative was successfully verified.

[In] Int[Log[c*(a + b/x)^p]/(d + e*x),x]

[Out] (Log[c*(a + b/x)^p]*Log[d + e*x])/e + (p*Log[-((e*x)/d)]*Log[d + e*x])/e - (p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e - (p*PolyLog[2, (a*(d + e*x))/(a*d - b*e)])/e + (p*PolyLog[2, 1 + (e*x)/d])/e

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx &= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e}+\frac{(bp)\int\frac{\log(d+ex)}{\left(a+\frac{b}{x}\right)x^2}\,dx}{e}\\
&= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e}+\frac{(bp)\int\left(\frac{\log(d+ex)}{bx}-\frac{a\log(d+ex)}{b(b+ax)}\right)dx}{e}\\
&= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e}+\frac{p\int\frac{\log(d+ex)}{x}\,dx}{e}-\frac{(ap)\int\frac{\log(d+ex)}{b+ax}\,dx}{e}\\
&= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e}+\frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e}-\frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+e}{e}\\
&= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e}+\frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e}-\frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+e}{e}\\
&= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e}+\frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e}-\frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+e}{e}
\end{aligned}
$$

Mathematica [**A**]
time = 0.01, size = 114, normalized size = 1.01

$$\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e}+\frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e}-\frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e}+\frac{p\text{Li}_2\left(\frac{d+ex}{d}\right)}{e}-\frac{p\text{Li}_2\left(\frac{a(d+ex)}{ad-be}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b/x)^p]*Log[d + e*x])/e + (p*Log[-((e*x)/d)]*Log[d + e*x])/e -
(p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e + (p*PolyLog[2, (d + e
*x)/d])/e - (p*PolyLog[2, (a*(d + e*x))/(a*d - b*e)])/e
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/(e*x+d),x)
```

```
[Out] int(ln(c*(a+b/x)^p)/(e*x+d),x)
```

Maxima [**A**]
time = 0.30, size = 167, normalized size = 1.48

$$bp\left(\frac{\log(xe+d)\log\left(a+\frac{b}{x}\right)}{b}-\frac{\log(xe+d)\log\left(-\frac{axe+ad}{ad-be}+1\right)+\text{Li}_2\left(\frac{axe+ad}{ad-be}\right)}{b}+\frac{\log(xe+d)\log\left(-\frac{xe+d}{d}+1\right)+\text{Li}_2\left(\frac{xe+d}{d}\right)}{b}\right)e^{(-1)}-pe^{(-1)}\log(xe+d)\log\left(a+\frac{b}{x}\right)+e^{(-1)}\log\left(\left(a+\frac{b}{x}\right)^p c\right)\log(xe+d)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] b*p*(log(x*e + d)*log(a + b/x)/b - (log(x*e + d)*log(-(a*x*e + a*d)/(a*d -
b*e) + 1) + dilog((a*x*e + a*d)/(a*d - b*e)))/b + (log(x*e + d)*log(-(x*e +
 d)/d + 1) + dilog((x*e + d)/d))/b)*e^(-1) - p*e^(-1)*log(x*e + d)*log(a +
b/x) + e^(-1)*log((a + b/x)^p*c)*log(x*e + d)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x + b)/x)^p)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/(e*x+d),x)
[Out] Integral(log(c*(a + b/x)**p)/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(log((a + b/x)^p*c)/(x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,\left(a+\frac{b}{x}\right)^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)/(d + e*x),x)
[Out] int(log(c*(a + b/x)^p)/(d + e*x), x)
```

3.244 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x(d+ex)}\,dx$

Optimal. Leaf size=159

$$-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d}-\frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d}+\frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d$$

```
[Out] -ln(c*(a+b/x)^p)*ln(-b/a/x)/d-ln(c*(a+b/x)^p)*ln(e*x+d)/d-p*ln(-e*x/d)*ln(e
*x+d)/d+p*ln(-e*(a*x+b)/(a*d-b*e))*ln(e*x+d)/d-p*polylog(2,1+b/a/x)/d+p*pol
ylog(2,a*(e*x+d)/(a*d-b*e))/d-p*polylog(2,1+e*x/d)/d
```

Rubi [A]
time = 0.17, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2516, 2504, 2441, 2352, 2512, 266, 2463, 2440, 2438}

$$\frac{p\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{d}-\frac{p\text{PolyLog}\left(2,\frac{b}{ax}+1\right)}{d}-\frac{p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d}-\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d}-\frac{\log\left(-\frac{b}{ax}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d}+\frac{p\log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right)}{d}-\frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/(x*(d + e*x)),x]
```

```
[Out] -((Log[c*(a + b/x)^p]*Log[-(b/(a*x))])/d) - (Log[c*(a + b/x)^p]*Log[d + e*x
])/d - (p*Log[-((e*x)/d)]*Log[d + e*x])/d + (p*Log[-((e*(b + a*x))/(a*d - b
*e))]*Log[d + e*x])/d - (p*PolyLog[2, 1 + b/(a*x)])/d + (p*PolyLog[2, (a*(d
 + e*x))/(a*d - b*e)])/d - (p*PolyLog[2, 1 + (e*x)/d])/d
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x(d+ex)}\,dx = \int \left(\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{dx} - \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d(d+ex)}\right)dx$$

$$= \frac{\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx}{d} - \frac{e\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d} - \frac{\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}\,dx, x, \frac{1}{x}\right)}{d} - \frac{(bp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x}\right)x^2}\,dx}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d} - \frac{(bp)\int \left(\frac{\log(d+ex)}{bx} - \frac{a\,l}{}\right.}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d} - \frac{p\text{Li}_2\left(1+\frac{b}{ax}\right)}{d} - \frac{p\int \frac{1}{}}{}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d} - \frac{p\log\left(-\frac{ex}{d}\right)\log(d+e}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d} - \frac{p\log\left(-\frac{ex}{d}\right)\log(d+e}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d} - \frac{p\log\left(-\frac{ex}{d}\right)\log(d+e}{d}$$

Mathematica [A]
time = 0.04, size = 139, normalized size = 0.87

$$-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right) + \log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex) + p\log\left(-\frac{ex}{d}\right)\log(d+ex) - p\log\left(\frac{e(b+ax)}{-ad+be}\right)\log(d+ex) + p\text{Li}_2\left(1+\frac{b}{ax}\right) - p\text{Li}_2\left(\frac{a(d+ex)}{ad-be}\right) + p\text{Li}_2\left(1+\frac{ex}{d}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/(x*(d + e*x)),x]
[Out] -((Log[c*(a + b/x)^p]*Log[-(b/(a*x))] + Log[c*(a + b/x)^p]*Log[d + e*x] + p
*Log[-((e*x)/d)]*Log[d + e*x] - p*Log[(e*(b + a*x))/(-(a*d) + b*e)]*Log[d +
 e*x] + p*PolyLog[2, 1 + b/(a*x)] - p*PolyLog[2, (a*(d + e*x))/(a*d - b*e)]
 + p*PolyLog[2, 1 + (e*x)/d])/d)
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{x\left(ex+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/x/(e*x+d),x)
```

```
[Out] int(ln(c*(a+b/x)^p)/x/(e*x+d),x)
```

Maxima [A]
time = 0.33, size = 188, normalized size = 1.18

$$-\frac{1}{2}bp\left(\frac{2\log\left(xe+d\right)\log\left(x\right)-\log\left(x\right)^{2}}{bd}+\frac{2\left(\log\left(\frac{ax}{b}+1\right)\log\left(x\right)+\mathrm{Li}_{2}\left(-\frac{ax}{b}\right)\right)}{bd}-\frac{2\left(\log\left(x\right)\log\left(\frac{xe}{d}+1\right)+\mathrm{Li}_{2}\left(-\frac{xe}{d}\right)\right)}{bd}-\frac{2\left(\log\left(xe+d\right)\log\left(-\frac{axe+ad}{ad-be}+1\right)+\mathrm{Li}_{2}\left(\frac{axe+ad}{ad-be}\right)\right)}{bd}\right)-\left(\frac{\log\left(xe+d\right)}{d}-\frac{\log\left(x\right)}{d}\right)\log\left(\left(a+\frac{b}{x}\right)^{p}c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/2*b*p*((2*log(x*e + d)*log(x) - log(x)^2)/(b*d) + 2*(log(a*x/b + 1)*log(
x) + dilog(-a*x/b))/(b*d) - 2*(log(x)*log(x*e/d + 1) + dilog(-x*e/d))/(b*d)
 - 2*(log(x*e + d)*log(-(a*x*e + a*d)/(a*d - b*e) + 1) + dilog((a*x*e + a*d
)/(a*d - b*e)))/(b*d)) - (log(x*e + d)/d - log(x)/d)*log((a + b/x)^p*c)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x + b)/x)^p)/(x^2*e + d*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/x/(e*x+d),x)
```

```
[Out] Integral(log(c*(a + b/x)**p)/(x*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x)^p*c)/((x*e + d)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{x\ (d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)/(x*(d + e*x)),x)
[Out] int(log(c*(a + b/x)^p)/(x*(d + e*x)), x)
```

3.245 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2(d+ex)}\,dx$

Optimal. Leaf size=198

$$\frac{p}{dx}-\frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd}+\frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^2}+\frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^2}+\frac{ep\log\left(-\frac{ex}{d}\right)\log(\cdots}{d^2}$$

```
[Out] p/d/x-(a+b/x)*ln(c*(a+b/x)^p)/b/d+e*ln(c*(a+b/x)^p)*ln(-b/a/x)/d^2+e*ln(c*(
a+b/x)^p)*ln(e*x+d)/d^2+e*p*ln(-e*x/d)*ln(e*x+d)/d^2-e*p*ln(-e*(a*x+b)/(a*d
-b*e))*ln(e*x+d)/d^2+e*p*polylog(2,1+b/a/x)/d^2-e*p*polylog(2,a*(e*x+d)/(a*
d-b*e))/d^2+e*p*polylog(2,1+e*x/d)/d^2
```

Rubi [A]
time = 0.20, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {2516, 2504, 2436, 2332, 2441, 2352, 2512, 266, 2463, 2440, 2438}

$$\frac{ep\,\text{PolyLog}\left(2,\frac{b}{ax}+1\right)}{d^2}-\frac{ep\,\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{d^2}+\frac{ep\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d^2}+\frac{e\log\left(-\frac{b}{ax}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^2}+\frac{e\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^2}-\frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd}-\frac{ep\log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right)}{d^2}+\frac{ep\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^2}+\frac{p}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/(x^2*(d + e*x)),x]
```

```
[Out] p/(d*x) - ((a + b/x)*Log[c*(a + b/x)^p])/(b*d) + (e*Log[c*(a + b/x)^p]*Log[
-(b/(a*x))])/d^2 + (e*Log[c*(a + b/x)^p]*Log[d + e*x])/d^2 + (e*p*Log[-((e*
x)/d)]*Log[d + e*x])/d^2 - (e*p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e
*x])/d^2 + (e*p*PolyLog[2, 1 + b/(a*x)])/d^2 - (e*p*PolyLog[2, (a*(d + e*x)
)/(a*d - b*e)])/d^2 + (e*p*PolyLog[2, 1 + (e*x)/d])/d^2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2(d+ex)}\,dx = \int \left(\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{dx^2} - \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^2x} + \frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^2(d+ex)}\right)dx$$

$$= \frac{\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2}\,dx}{d} - \frac{e\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx}{d^2} + \frac{e^2\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx}{d^2}$$

$$= \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^2} - \frac{\mathrm{Subst}\left(\int \log\left(c(a+bx)^p\right)\,dx, x, \frac{1}{x}\right)}{d} + \frac{e\mathrm{Subst}\left(\int \frac{\log(c}{}\right.}{}$$

$$= \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^2} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^2} - \frac{\mathrm{Subst}\left(\int \log\left(cx^p\right)\,dx\right.}{bd}$$

$$= \frac{p}{dx} - \frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^2} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log}{d^2}$$

$$= \frac{p}{dx} - \frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^2} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log}{d^2}$$

$$= \frac{p}{dx} - \frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^2} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log}{d^2}$$

$$= \frac{p}{dx} - \frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^2} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log}{d^2}$$

Mathematica [A]
time = 0.07, size = 166, normalized size = 0.84

$$\frac{\frac{dp}{x} - \frac{d\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{b} + e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right) + e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex) + ep\mathrm{Li}_2\left(1+\frac{b}{ax}\right) + ep\left(\left(\log\left(-\frac{ex}{d}\right) - \log\left(\frac{e(b+ax)}{-ad+be}\right)\right)\log(d+ex) - \mathrm{Li}_2\left(\frac{a(d+ex)}{ad-be}\right) + \mathrm{Li}_2\left(1+\frac{ex}{d}\right)\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/(x^2*(d + e*x)),x]
```

```
[Out] ((d*p)/x - (d*(a + b/x)*Log[c*(a + b/x)^p])/b + e*Log[c*(a + b/x)^p]*Log[-(
b/(a*x))] + e*Log[c*(a + b/x)^p]*Log[d + e*x] + e*p*PolyLog[2, 1 + b/(a*x)]
 + e*p*((Log[-((e*x)/d)] - Log[(e*(b + a*x))/(-(a*d) + b*e)])*Log[d + e*x]
- PolyLog[2, (a*(d + e*x))/(a*d - b*e)] + PolyLog[2, 1 + (e*x)/d]))/d^2
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x^{2} \left(e x +d \right)}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/x^2/(e*x+d),x)
```

```
[Out] int(ln(c*(a+b/x)^p)/x^2/(e*x+d),x)
```

Maxima [A]
time = 0.34, size = 246, normalized size = 1.24

$$\frac{1}{2} \, b p {\left(\frac{2 \, {\left(\log\left(\frac{a x}{b} + 1\right) \log\left(x\right) + {\rm Li}_2\left(-\frac{a x}{b}\right)\right)} e}{b d^{2}} - \frac{2 \, {\left(\log\left(x\right) \log\left(\frac{x e}{d} + 1\right) + {\rm Li}_2\left(-\frac{x e}{d}\right)\right)} e}{b d^{2}} - \frac{2 \, {\left(\log\left(x e + d\right) \log\left(-\frac{a x e + a d}{a d - b e} + 1\right) + {\rm Li}_2\left(\frac{a x e + a d}{a d - b e}\right)\right)} e}{b d^{2}} - \frac{2 \, a \log\left(a x + b\right)}{b^{2} d} + \frac{2 \, a \log\left(x\right)}{b^{2} d} + \frac{2 \, e \log\left(x e + d\right) \log\left(x\right) - e \log\left(x\right)^{2}}{b d^{2}} + \frac{2}{b d x}\right)} + {\left(\frac{e \log\left(x e + d\right)}{d^{2}} - \frac{e \log\left(x\right)}{d^{2}} - \frac{1}{d x}\right)} \log\left({\left(a + \frac{b}{x}\right)}^{p} c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*b*p*(2*(log(a*x/b + 1)*log(x) + dilog(-a*x/b))*e/(b*d^2) - 2*(log(x)*lo
g(x*e/d + 1) + dilog(-x*e/d))*e/(b*d^2) - 2*(log(x*e + d)*log(-(a*x*e + a*d
)/(a*d - b*e) + 1) + dilog((a*x*e + a*d)/(a*d - b*e)))*e/(b*d^2) - 2*a*log(
a*x + b)/(b^2*d) + 2*a*log(x)/(b^2*d) + (2*e*log(x*e + d)*log(x) - e*log(x)
^2)/(b*d^2) + 2/(b*d*x)) + (e*log(x*e + d)/d^2 - e*log(x)/d^2 - 1/(d*x))*lo
g((a + b/x)^p*c)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x + b)/x)^p)/(x^3*e + d*x^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/x**2/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x)^p*c)/((x*e + d)*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)/(x^2*(d + e*x)),x)
```

```
[Out] int(log(c*(a + b/x)^p)/(x^2*(d + e*x)), x)
```

3.246 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3(d+ex)}\,dx$

Optimal. Leaf size=287

$$\frac{p}{4dx^2}-\frac{ap}{2bdx}-\frac{ep}{d^2x}+\frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2d}+\frac{e\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd^2}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2dx^2}-\frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\right.}{d^3}$$

```
[Out] 1/4*p/d/x^2-1/2*a*p/b/d/x-e*p/d^2/x+1/2*a^2*p*ln(a+b/x)/b^2/d+e*(a+b/x)*ln(
c*(a+b/x)^p)/b/d^2-1/2*ln(c*(a+b/x)^p)/d/x^2-e^2*ln(c*(a+b/x)^p)*ln(-b/a/x)
/d^3-e^2*ln(c*(a+b/x)^p)*ln(e*x+d)/d^3-e^2*p*ln(-e*x/d)*ln(e*x+d)/d^3+e^2*p
*ln(-e*(a*x+b)/(a*d-b*e))*ln(e*x+d)/d^3-e^2*p*polylog(2,1+b/a/x)/d^3+e^2*p*
polylog(2,a*(e*x+d)/(a*d-b*e))/d^3-e^2*p*polylog(2,1+e*x/d)/d^3
```

Rubi [A]
time = 0.23, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {2516, 2504, 2442, 45, 2436, 2332, 2441, 2352, 2512, 266, 2463, 2440, 2438}

$$-\frac{e^2p\text{PolyLog}\left(2,\frac{b}{ax}+1\right)}{d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{d^3}-\frac{e^2p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d^3}+\frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2d}-\frac{e^2\log\left(-\frac{b}{ax}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^3}-\frac{e^2\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^3}+\frac{e\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd^2}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2dx^2}+\frac{e^2p\log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right)}{d^3}-\frac{ap}{2bdx}-\frac{e^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^3}-\frac{ep}{d^2x}+\frac{p}{4dx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x)^p]/(x^3*(d + e*x)),x]
```

```
[Out] p/(4*d*x^2) - (a*p)/(2*b*d*x) - (e*p)/(d^2*x) + (a^2*p*Log[a + b/x])/(2*b^2
*d) + (e*(a + b/x)*Log[c*(a + b/x)^p])/(b*d^2) - Log[c*(a + b/x)^p]/(2*d*x^
2) - (e^2*Log[c*(a + b/x)^p]*Log[-(b/(a*x))])/d^3 - (e^2*Log[c*(a + b/x)^p]
*Log[d + e*x])/d^3 - (e^2*p*Log[-((e*x)/d)]*Log[d + e*x])/d^3 + (e^2*p*Log[
-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/d^3 - (e^2*p*PolyLog[2, 1 + b/(
a*x)])/d^3 + (e^2*p*PolyLog[2, (a*(d + e*x))/(a*d - b*e)])/d^3 - (e^2*p*Pol
yLog[2, 1 + (e*x)/d])/d^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3(d+ex)}\,dx = \int\left(\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{dx^3} - \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^2x^2} + \frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^3x} - \frac{e^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^3(d+ex)}\right.$$

$$= \frac{\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3}\,dx}{d} - \frac{e\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2}\,dx}{d^2} + \frac{e^2\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx}{d^3} - \frac{e^3\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx}{d^3}$$

$$= -\frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^3} - \frac{\text{Subst}\left(\int x\log\left(c(a+bx)^p\right)\,dx, x, \frac{1}{x}\right)}{d} + \frac{e\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2dx^2} - \frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^3} - \frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^3} +$$

$$= -\frac{ep}{d^2x} + \frac{e\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd^2} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2dx^2} - \frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^3}$$

$$= \frac{p}{4dx^2} - \frac{ap}{2bdx} - \frac{ep}{d^2x} + \frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2d} + \frac{e\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd^2} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right.}{2dx^2}$$

$$= \frac{p}{4dx^2} - \frac{ap}{2bdx} - \frac{ep}{d^2x} + \frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2d} + \frac{e\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd^2} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right.}{2dx^2}$$

$$= \frac{p}{4dx^2} - \frac{ap}{2bdx} - \frac{ep}{d^2x} + \frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2d} + \frac{e\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd^2} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right.}{2dx^2}$$

Mathematica [A]
time = 0.15, size = 241, normalized size = 0.84

$$-\frac{\frac{4dep}{x} - \frac{d^2p\left(b(b-2ax)+2a^2x^2\log\left(a+\frac{b}{x}\right)\right)}{b^2x^2} - \frac{4de\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{b} + \frac{2d^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2} + 4e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right) + 4e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex) + 4e^2p\text{Li}_2\left(1+\frac{b}{ax}\right) + 4e^2p\left(\left(\log\left(-\frac{ex}{d}\right) - \log\left(\frac{e(b+ax)}{-ad+be}\right)\right)\log(d+ex) - \text{Li}_2\left(\frac{a(d+ex)}{ad-be}\right) + \text{Li}_2\left(1+\frac{ex}{d}\right)\right)}{4d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x)^p]/(x^3*(d + e*x)),x]
```

```
[Out] -1/4*((4*d*e*p)/x - (d^2*p*(b*(b - 2*a*x) + 2*a^2*x^2*Log[a + b/x]))/(b^2*x
^2) - (4*d*e*(a + b/x)*Log[c*(a + b/x)^p])/b + (2*d^2*Log[c*(a + b/x)^p])/x
^2 + 4*e^2*Log[c*(a + b/x)^p]*Log[-(b/(a*x))] + 4*e^2*Log[c*(a + b/x)^p]*Lo
g[d + e*x] + 4*e^2*p*PolyLog[2, 1 + b/(a*x)] + 4*e^2*p*((Log[-((e*x)/d)] -
Log[(e*(b + a*x))/(-(a*d) + b*e)])*Log[d + e*x] - PolyLog[2, (a*(d + e*x))/
(a*d - b*e)] + PolyLog[2, 1 + (e*x)/d]))/d^3
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3\left(ex+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x)^p)/x^3/(e*x+d),x)
```

```
[Out] int(ln(c*(a+b/x)^p)/x^3/(e*x+d),x)
```

Maxima [A]
time = 0.33, size = 311, normalized size = 1.08

$$\frac{1}{4}\left(4\left(\frac{a\log\left(ax+b\right)}{b^2d^2}-\frac{a\log\left(x\right)}{b^2d^2}-\frac{1}{bd^2x}\right)e+\frac{2\,a^2\log\left(ax+b\right)}{b^3d}-\frac{2\,a^2\log\left(x\right)}{b^3d}-\frac{4\left(\log\left(\frac{ax}{b}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{ax}{b}\right)\right)e^2}{bd^3}+\frac{4\left(\log\left(x\right)\log\left(\frac{xe}{d}+1\right)+\mathrm{Li}_2\left(-\frac{xe}{d}\right)\right)e^2}{bd^3}+\frac{4\left(\log\left(xe+d\right)\log\left(-\frac{axe+ad}{ad-be}+1\right)+\mathrm{Li}_2\left(\frac{axe+ad}{ad-be}\right)\right)e^2}{bd^3}-\frac{2\left(2\,e^2\log\left(xe+d\right)\log\left(x\right)-e^2\log\left(x\right)^2\right)}{bd^3}-\frac{2\,ax-b}{b^2dx^2}\right)bp-\frac{1}{2}\left(\frac{2\,e^2\log\left(xe+d\right)}{d^3}-\frac{2\,e^2\log\left(x\right)}{d^3}-\frac{2\,xe-d}{d^2x^2}\right)\log\left(\left(a+\frac{b}{x}\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(a*log(a*x + b)/(b^2*d^2) - a*log(x)/(b^2*d^2) - 1/(b*d^2*x))*e + 2*
a^2*log(a*x + b)/(b^3*d) - 2*a^2*log(x)/(b^3*d) - 4*(log(a*x/b + 1)*log(x)
+ dilog(-a*x/b))*e^2/(b*d^3) + 4*(log(x)*log(x*e/d + 1) + dilog(-x*e/d))*e^
2/(b*d^3) + 4*(log(x*e + d)*log(-(a*x*e + a*d)/(a*d - b*e) + 1) + dilog((a*
x*e + a*d)/(a*d - b*e)))*e^2/(b*d^3) - 2*(2*e^2*log(x*e + d)*log(x) - e^2*l
og(x)^2)/(b*d^3) - (2*a*x - b)/(b^2*d*x^2))*b*p - 1/2*(2*e^2*log(x*e + d)/d
^3 - 2*e^2*log(x)/d^3 - (2*x*e - d)/(d^2*x^2))*log((a + b/x)^p*c)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x + b)/x)^p)/(x^4*e + d*x^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x)**p)/x**3/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x)^p)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x)^p*c)/((x*e + d)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(a+\frac{b}{x}\right)}^p\right)}{x^3\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x)^p)/(x^3*(d + e*x)),x)
```

```
[Out] int(log(c*(a + b/x)^p)/(x^3*(d + e*x)), x)
```

3.247 $\int \frac{x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=421

$$\frac{2bpx}{3ae}+\frac{2\sqrt{b}\,d^2p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,e^3}-\frac{2b^{3/2}p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{3a^{3/2}e}+\frac{d^2x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3}-\frac{dx^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e^2}+\frac{x^3\log\cdots}{\cdots}$$

```
[Out] 2/3*b*p*x/a/e-2/3*b^(3/2)*p*arctan(x*a^(1/2)/b^(1/2))/a^(3/2)/e+d^2*x*ln(c*
(a+b/x^2)^p)/e^3-1/2*d*x^2*ln(c*(a+b/x^2)^p)/e^2+1/3*x^3*ln(c*(a+b/x^2)^p)/
e-d^3*ln(c*(a+b/x^2)^p)*ln(e*x+d)/e^4-2*d^3*p*ln(-e*x/d)*ln(e*x+d)/e^4-1/2*
b*d*p*ln(a*x^2+b)/a/e^2+d^3*p*ln(e*x+d)*ln(-e*(x*(-a)^(1/2)+b^(1/2))/(d*(-a
)^(1/2)-e*b^(1/2)))/e^4+d^3*p*ln(e*x+d)*ln(e*(-x*(-a)^(1/2)+b^(1/2))/(d*(-a
)^(1/2)+e*b^(1/2)))/e^4-2*d^3*p*polylog(2,1+e*x/d)/e^4+d^3*p*polylog(2,(e*x
+d)*(-a)^(1/2)/(d*(-a)^(1/2)-e*b^(1/2)))/e^4+d^3*p*polylog(2,(e*x+d)*(-a)^(
1/2)/(d*(-a)^(1/2)+e*b^(1/2)))/e^4+2*d^2*p*arctan(x*a^(1/2)/b^(1/2))*b^(1/2
)/e^3/a^(1/2)
```

Rubi [A]

time = 0.39, antiderivative size = 421, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 14, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {2516, 2498, 269, 211, 2505, 266, 199, 327, 2512, 2463, 2441, 2352, 2440, 2438}

$$\frac{d^3p\,\text{PolyLog}\left(2,\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d-\sqrt{b}\,e}\right)}{e^4}+\frac{d^3p\,\text{PolyLog}\left(2,\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right)}{e^4}-\frac{2d^3p\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^4}-\frac{2b^{3/2}p\,\text{ArcTan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{3a^{3/2}e}+\frac{2\sqrt{b}\,d^2p\,\text{ArcTan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,e^3}-\frac{d^3\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^4}+\frac{d^2x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3}-\frac{dx^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e^2}+\frac{x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3e}+\frac{d^3p\log(d+ex)\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}\,x\right)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right)}{e^4}+\frac{d^3p\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}\,x+\sqrt{b}\right)}{\sqrt{-a}\,d-\sqrt{b}\,e}\right)}{e^4}-\frac{bdp\log\left(ax^2+b\right)}{2ae^2}+\frac{2bpx}{3ae}-\frac{2d^3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Log[c*(a + b/x^2)^p])/(d + e*x),x]
```

```
[Out] (2*b*p*x)/(3*a*e) + (2*Sqrt[b]*d^2*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(Sqrt[a]*
e^3) - (2*b^(3/2)*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(3*a^(3/2)*e) + (d^2*x*Log
[c*(a + b/x^2)^p])/e^3 - (d*x^2*Log[c*(a + b/x^2)^p])/(2*e^2) + (x^3*Log[c*
(a + b/x^2)^p])/(3*e) - (d^3*Log[c*(a + b/x^2)^p]*Log[d + e*x])/e^4 - (2*d^
3*p*Log[-((e*x)/d)]*Log[d + e*x])/e^4 + (d^3*p*Log[(e*(Sqrt[b] - Sqrt[-a]*x
))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x])/e^4 + (d^3*p*Log[-((e*(Sqrt[b] +
 Sqrt[-a]*x))/(Sqrt[-a]*d - Sqrt[b]*e))]*Log[d + e*x])/e^4 - (b*d*p*Log[b +
 a*x^2])/(2*a*e^2) + (d^3*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - S
qrt[b]*e)])/e^4 + (d^3*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt
[b]*e)])/e^4 - (2*d^3*p*PolyLog[2, 1 + (e*x)/d])/e^4
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \int \left(\frac{d^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{dx \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2} + \frac{x^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d^3 \log\left(c\right.}{e^3(d}\right.$$

$$= \frac{d^2 \int \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx}{e^3} - \frac{d^3 \int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx}{e^3} - \frac{d \int x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx}{e^2} +$$

$$= \frac{d^2 x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{dx^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e^2} + \frac{x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3e} - \frac{d^3 \log\left(c\left(a\right.\right.}{}$$

$$= \frac{d^2 x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{dx^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e^2} + \frac{x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3e} - \frac{d^3 \log\left(c\left(a\right.\right.}{}$$

$$= \frac{2bpx}{3ae} + \frac{2\sqrt{b}\,d^2 p \tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,e^3} + \frac{d^2 x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{dx^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e^2}$$

$$= \frac{2bpx}{3ae} + \frac{2\sqrt{b}\,d^2 p \tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,e^3} - \frac{2b^{3/2} p \tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{3a^{3/2} e} + \frac{d^2 x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right.}{e^3}$$

$$= \frac{2bpx}{3ae} + \frac{2\sqrt{b}\,d^2 p \tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,e^3} - \frac{2b^{3/2} p \tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{3a^{3/2} e} + \frac{d^2 x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right.}{e^3}$$

$$= \frac{2bpx}{3ae} + \frac{2\sqrt{b}\,d^2 p \tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,e^3} - \frac{2b^{3/2} p \tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{3a^{3/2} e} + \frac{d^2 x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right.}{e^3}$$

$$= \frac{2bpx}{3ae} + \frac{2\sqrt{b}\,d^2 p \tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,e^3} - \frac{2b^{3/2} p \tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{3a^{3/2} e} + \frac{d^2 x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right.}{e^3}$$

$$= \frac{2bpx}{3ae} + \frac{2\sqrt{b}\,d^2 p \tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,e^3} - \frac{2b^{3/2} p \tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{3a^{3/2} e} + \frac{d^2 x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right.}{e^3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.31, size = 375, normalized size = 0.89

$$-\frac{\frac{12\sqrt{b}\,d^2 ep \tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}\,x}\right)}{\sqrt{a}} - \frac{4be^3px\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{b}{ax^2}\right)}{a} - 6d^2ex\log\left(c\left(a+\frac{b}{x^2}\right)^p\right) + 3de^2x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right) - 2e^3x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right) + \frac{3bde^2p\left(\log\left(a+\frac{b}{x^2}\right)+2\log(x)\right)}{a} + 6d^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex) + 6d^3p\left(2\log\left(-\frac{ex}{d}\right)\log(d+ex) - \log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}\,x\right)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right)\log(d+ex) - \log\left(\frac{e\left(\sqrt{b}+\sqrt{-a}\,x\right)}{-\sqrt{-a}\,d+\sqrt{b}\,e}\right)\log(d+ex) - \mathrm{Li}_2\left(\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d-\sqrt{b}\,e}\right) - \mathrm{Li}_2\left(\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right) + 2\mathrm{Li}_2\left(1+\frac{ex}{d}\right)\right)}{6e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Log[c*(a + b/x^2)^p])/(d + e*x),x]
```

```
[Out] -1/6*((12*Sqrt[b]*d^2*e*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)])/Sqrt[a] - (4*b*e^3*p
*x*Hypergeometric2F1[-1/2, 1, 1/2, -(b/(a*x^2))])/a - 6*d^2*e*x*Log[c*(a +
b/x^2)^p] + 3*d*e^2*x^2*Log[c*(a + b/x^2)^p] - 2*e^3*x^3*Log[c*(a + b/x^2)^
p] + (3*b*d*e^2*p*(Log[a + b/x^2] + 2*Log[x]))/a + 6*d^3*Log[c*(a + b/x^2)^
p]*Log[d + e*x] + 6*d^3*p*(2*Log[-((e*x)/d)]*Log[d + e*x] - Log[(e*(Sqrt[b]
 - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x] - Log[(e*(Sqrt[b] +
Sqrt[-a]*x))/(-(Sqrt[-a]*d) + Sqrt[b]*e)]*Log[d + e*x] - PolyLog[2, (Sqrt[-
a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)] - PolyLog[2, (Sqrt[-a]*(d + e*x))/(
Sqrt[-a]*d + Sqrt[b]*e)] + 2*PolyLog[2, 1 + (e*x)/d]))/e^4
```

Maple [**F**]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \ln \left(c \left(a +\frac{b}{x^{2}}\right)^{p}\right)}{e x +d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(a+b/x^2)^p)/(e*x+d),x)
```

```
[Out] int(x^3*ln(c*(a+b/x^2)^p)/(e*x+d),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(x^3*log((a + b/x^2)^p*c)/(x*e + d), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x^3*log(c*((a*x^2 + b)/x^2)^p)/(x*e + d), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \log{\left(c \left(a + \frac{b}{x^{2}}\right)^{p} \right)}}{d + e x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(a+b/x**2)**p)/(e*x+d),x)
```

```
[Out] Integral(x**3*log(c*(a + b/x**2)**p)/(d + e*x), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\ln\left(c\,{\left(a+\frac{b}{x^2}\right)}^p\right)}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*log(c*(a + b/x^2)^p))/(d + e*x),x)
```

```
[Out] int((x^3*log(c*(a + b/x^2)^p))/(d + e*x), x)
```

3.248 $\int \frac{x^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} \, dx$

Optimal. Leaf size=353

$$-\frac{2\sqrt{b}\, dp \tan^{-1}\left(\frac{\sqrt{a}\, x}{\sqrt{b}}\right)}{\sqrt{a}\, e^2}-\frac{dx \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2}+\frac{x^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e}+\frac{d^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^3}+\frac{2d^2 p}{}$$

```
[Out] -d*x*ln(c*(a+b/x^2)^p)/e^2+1/2*x^2*ln(c*(a+b/x^2)^p)/e+d^2*ln(c*(a+b/x^2)^p
)*ln(e*x+d)/e^3+2*d^2*p*ln(-e*x/d)*ln(e*x+d)/e^3+1/2*b*p*ln(a*x^2+b)/a/e-d^
2*p*ln(e*x+d)*ln(-e*(x*(-a)^(1/2)+b^(1/2))/(d*(-a)^(1/2)-e*b^(1/2)))/e^3-d^
2*p*ln(e*x+d)*ln(e*(-x*(-a)^(1/2)+b^(1/2))/(d*(-a)^(1/2)+e*b^(1/2)))/e^3+2*
d^2*p*polylog(2,1+e*x/d)/e^3-d^2*p*polylog(2,(e*x+d)*(-a)^(1/2)/(d*(-a)^(1/
2)-e*b^(1/2)))/e^3-d^2*p*polylog(2,(e*x+d)*(-a)^(1/2)/(d*(-a)^(1/2)+e*b^(1/
2)))/e^3-2*d*p*arctan(x*a^(1/2)/b^(1/2))*b^(1/2)/e^2/a^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {2516, 2498, 269, 211, 2505, 266, 2512, 2463, 2441, 2352, 2440, 2438}

$$-\frac{d^2 p \text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}\,d-\sqrt{b}\,e}\right)}{e^3}-\frac{d^2 p \text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right)}{e^3}+\frac{2d^2 p \text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^3}-\frac{2\sqrt{b}\, dp \text{ArcTan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\, e^2}+\frac{d^2 \log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3}-\frac{dx \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2}+\frac{x^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e}-\frac{d^2 p \log(d+ex)\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}\,x\right)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right)}{e^3}-\frac{d^2 p \log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}\,x+\sqrt{b}\right)}{\sqrt{-a}\,d-\sqrt{b}\,e}\right)}{e^3}+\frac{bp\log\left(ax^2+b\right)}{2ae}+\frac{2d^2 p \log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Log[c*(a + b/x^2)^p])/(d + e*x),x]
```

```
[Out] (-2*Sqrt[b]*d*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(Sqrt[a]*e^2) - (d*x*Log[c*(a
+ b/x^2)^p])/e^2 + (x^2*Log[c*(a + b/x^2)^p])/(2*e) + (d^2*Log[c*(a + b/x^2
)^p]*Log[d + e*x])/e^3 + (2*d^2*p*Log[-((e*x)/d)]*Log[d + e*x])/e^3 - (d^2*
p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x])/e^
3 - (d^2*p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqrt[-a]*d - Sqrt[b]*e))]*Log[
d + e*x])/e^3 + (b*p*Log[b + a*x^2])/(2*a*e) - (d^2*p*PolyLog[2, (Sqrt[-a]*
(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/e^3 - (d^2*p*PolyLog[2, (Sqrt[-a]*(d
+ e*x))/(Sqrt[-a]*d + Sqrt[b]*e)])/e^3 + (2*d^2*p*PolyLog[2, 1 + (e*x)/d])/
e^3
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \int \left(-\frac{d \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2} + \frac{x \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} + \frac{d^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2(d+ex)}\right) dx$$

$$= -\frac{d \int \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx}{e^2} + \frac{d^2 \int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} dx}{e^2} + \frac{\int x \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx}{e}$$

$$= -\frac{dx \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2} + \frac{x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e} + \frac{d^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) \log(d+ex)}{e^3} +$$

$$= -\frac{dx \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2} + \frac{x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e} + \frac{d^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) \log(d+ex)}{e^3} +$$

$$= -\frac{2\sqrt{b}\, dp \tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\, e^2} - \frac{dx \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2} + \frac{x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e} + \frac{d^2 \log \big(}{}$$

$$= -\frac{2\sqrt{b}\, dp \tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\, e^2} - \frac{dx \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2} + \frac{x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e} + \frac{d^2 \log \big(}{}$$

$$= -\frac{2\sqrt{b}\, dp \tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\, e^2} - \frac{dx \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2} + \frac{x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e} + \frac{d^2 \log \big(}{}$$

$$= -\frac{2\sqrt{b}\, dp \tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\, e^2} - \frac{dx \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2} + \frac{x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e} + \frac{d^2 \log \big(}{}$$

$$= -\frac{2\sqrt{b}\, dp \tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\, e^2} - \frac{dx \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2} + \frac{x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e} + \frac{d^2 \log \big(}{}$$

$$= -\frac{2\sqrt{b}\, dp \tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\, e^2} - \frac{dx \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2} + \frac{x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e} + \frac{d^2 \log \big(}{}$$

Mathematica [A]
time = 0.16, size = 319, normalized size = 0.90

$$\frac{\frac{4\sqrt{b}\, dep \tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}\,x}\right)}{\sqrt{a}} - 2dex \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) + e^2x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) + \frac{be^2p\left(\log\left(a+\frac{b}{x^2}\right)+2\log(x)\right)}{a} + 2d^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) \log(d+ex) + 2d^2p\left(2\log\left(-\frac{ex}{d}\right)\log(d+ex) - \log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}\,x\right)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right)\log(d+ex) - \log\left(\frac{e\left(\sqrt{b}+\sqrt{-a}\,x\right)}{-\sqrt{-a}\,d+\sqrt{b}\,e}\right)\log(d+ex) - \mathrm{Li}_2\left(\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d-\sqrt{b}\,e}\right) - \mathrm{Li}_2\left(\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right) + 2\mathrm{Li}_2\left(1+\frac{ex}{d}\right)\right)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Log[c*(a + b/x^2)^p])/(d + e*x),x]
```

```
[Out] ((4*Sqrt[b]*d*e*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)])/Sqrt[a] - 2*d*e*x*Log[c*(a +
 b/x^2)^p] + e^2*x^2*Log[c*(a + b/x^2)^p] + (b*e^2*p*(Log[a + b/x^2] + 2*Lo
g[x]))/a + 2*d^2*Log[c*(a + b/x^2)^p]*Log[d + e*x] + 2*d^2*p*(2*Log[-((e*x)
/d)]*Log[d + e*x] - Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)
]*Log[d + e*x] - Log[(e*(Sqrt[b] + Sqrt[-a]*x))/(-(Sqrt[-a]*d) + Sqrt[b]*e)
]*Log[d + e*x] - PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)]
- PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)] + 2*PolyLog[2,
1 + (e*x)/d]))/(2*e^3)
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{x^2 \ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(a+b/x^2)^p)/(e*x+d),x)
```

```
[Out] int(x^2*ln(c*(a+b/x^2)^p)/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(x^2*log((a + b/x^2)^p*c)/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x^2*log(c*((a*x^2 + b)/x^2)^p)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(a+b/x**2)**p)/(e*x+d),x)
```

```
[Out] Integral(x**2*log(c*(a + b/x**2)**p)/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(x^2*log((a + b/x^2)^p*c)/(x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\ln\left(c\,{\left(a+\frac{b}{x^2}\right)}^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*log(c*(a + b/x^2)^p))/(d + e*x),x)
```

```
[Out] int((x^2*log(c*(a + b/x^2)^p))/(d + e*x), x)
```

3.249 $\int \frac{x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=291

$$\frac{2\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,e}+\frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e}-\frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2}-\frac{2dp\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^2}+\frac{dp}{}$$

```
[Out] x*ln(c*(a+b/x^2)^p)/e-d*ln(c*(a+b/x^2)^p)*ln(e*x+d)/e^2-2*d*p*ln(-e*x/d)*ln
(e*x+d)/e^2+d*p*ln(e*x+d)*ln(-e*(x*(-a)^(1/2)+b^(1/2))/(d*(-a)^(1/2)-e*b^(1
/2)))/e^2+d*p*ln(e*x+d)*ln(e*(-x*(-a)^(1/2)+b^(1/2))/(d*(-a)^(1/2)+e*b^(1/2
)))/e^2-2*d*p*polylog(2,1+e*x/d)/e^2+d*p*polylog(2,(e*x+d)*(-a)^(1/2)/(d*(-
a)^(1/2)-e*b^(1/2)))/e^2+d*p*polylog(2,(e*x+d)*(-a)^(1/2)/(d*(-a)^(1/2)+e*b
^(1/2)))/e^2+2*p*arctan(x*a^(1/2)/b^(1/2))*b^(1/2)/e/a^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {2516, 2498, 269, 211, 2512, 266, 2463, 2441, 2352, 2440, 2438}

$$\frac{dp\text{PolyLog}\left(2,\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d-\sqrt{b}\,e}\right)}{e^2}+\frac{dp\text{PolyLog}\left(2,\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right)}{e^2}-\frac{2dp\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^2}+\frac{2\sqrt{b}\,p\text{ArcTan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,e}-\frac{d\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2}+\frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e}+\frac{dp\log(d+ex)\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}\,x\right)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right)}{e^2}+\frac{dp\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}\,x+\sqrt{b}\right)}{\sqrt{-a}\,d-\sqrt{b}\,e}\right)}{e^2}-\frac{2dp\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[c*(a + b/x^2)^p])/(d + e*x),x]
```

```
[Out] (2*Sqrt[b]*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(Sqrt[a]*e) + (x*Log[c*(a + b/x^2
)^p])/e - (d*Log[c*(a + b/x^2)^p]*Log[d + e*x])/e^2 - (2*d*p*Log[-((e*x)/d)
]*Log[d + e*x])/e^2 + (d*p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqr
t[b]*e)]*Log[d + e*x])/e^2 + (d*p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqrt[-a
]*d - Sqrt[b]*e))]*Log[d + e*x])/e^2 + (d*p*PolyLog[2, (Sqrt[-a]*(d + e*x))
/(Sqrt[-a]*d - Sqrt[b]*e)])/e^2 + (d*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqr
t[-a]*d + Sqrt[b]*e)])/e^2 - (2*d*p*PolyLog[2, 1 + (e*x)/d])/e^2
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \int \left(\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(d+ex)}\right)dx$$

$$= \frac{\int \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx}{e} - \frac{d\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx}{e}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2} - \frac{(2bdp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x^2}\right)x^3}\,dx}{e^2} + \frac{(2b}{}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2} - \frac{(2bdp)\int \left(\frac{\log(d+ex)}{bx} - \frac{ax\log(}{b(b+}\right.}{e^2}$$

$$= \frac{2\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,e} + \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2} - \frac{(}{}$$

$$= \frac{2\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,e} + \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2} - \frac{2}{}$$

$$= \frac{2\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,e} + \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2} - \frac{2}{}$$

$$= \frac{2\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,e} + \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2} - \frac{2}{}$$

$$= \frac{2\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,e} + \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2} - \frac{2}{}$$

$$= \frac{2\sqrt{b}\,p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,e} + \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2} - \frac{2}{}$$

Mathematica [A]
time = 0.12, size = 271, normalized size = 0.93

$$\frac{-\frac{2\sqrt{b}\,ep\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}\,x}\right)}{\sqrt{a}} + ex\log\left(c\left(a+\frac{b}{x^2}\right)^p\right) - d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex) - 2dp\log\left(-\frac{ex}{d}\right)\log(d+ex) + dp\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}\,x\right)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right)\log(d+ex) + dp\log\left(\frac{e\left(\sqrt{b}+\sqrt{-a}\,x\right)}{-\sqrt{-a}\,d+\sqrt{b}\,e}\right)\log(d+ex) + dp\text{Li}_2\left(\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d-\sqrt{b}\,e}\right) + dp\text{Li}_2\left(\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right) - 2dp\text{Li}_2\left(1+\frac{ex}{d}\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[c*(a + b/x^2)^p])/(d + e*x),x]
```

```
[Out] ((-2*Sqrt[b]*e*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)])/Sqrt[a] + e*x*Log[c*(a + b/x^
2)^p] - d*Log[c*(a + b/x^2)^p]*Log[d + e*x] - 2*d*p*Log[-((e*x)/d)]*Log[d +
 e*x] + d*p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d
+ e*x] + d*p*Log[(e*(Sqrt[b] + Sqrt[-a]*x))/(-(Sqrt[-a]*d) + Sqrt[b]*e)]*Lo
g[d + e*x] + d*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)]
+ d*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)] - 2*d*p*Pol
yLog[2, 1 + (e*x)/d])/e^2
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{x \ln \left(c \left(a+\frac{b}{x^2}\right)^p\right)}{e x+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(a+b/x^2)^p)/(e*x+d),x)
```

```
[Out] int(x*ln(c*(a+b/x^2)^p)/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(x*log((a + b/x^2)^p*c)/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x*log(c*((a*x^2 + b)/x^2)^p)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)}{d+e x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(a+b/x**2)**p)/(e*x+d),x)
```

```
[Out] Integral(x*log(c*(a + b/x**2)**p)/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(x*log((a + b/x^2)^p*c)/(x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\ln\left(c\,{\left(a+\frac{b}{x^2}\right)}^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(c*(a + b/x^2)^p))/(d + e*x),x)
```

```
[Out] int((x*log(c*(a + b/x^2)^p))/(d + e*x), x)
```

3.250 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=241

$$\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e}+\frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e}-\frac{p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}\,x\right)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right)\log(d+ex)}{e}-\frac{p\log\left(-\frac{e\left(\right.}{}\right.}{}$$

```
[Out] ln(c*(a+b/x^2)^p)*ln(e*x+d)/e+2*p*ln(-e*x/d)*ln(e*x+d)/e-p*ln(e*x+d)*ln(-e*
(x*(-a)^(1/2)+b^(1/2))/(d*(-a)^(1/2)-e*b^(1/2)))/e-p*ln(e*x+d)*ln(e*(-x*(-a
)^(1/2)+b^(1/2))/(d*(-a)^(1/2)+e*b^(1/2)))/e+2*p*polylog(2,1+e*x/d)/e-p*pol
ylog(2,(e*x+d)*(-a)^(1/2)/(d*(-a)^(1/2)-e*b^(1/2)))/e-p*polylog(2,(e*x+d)*(
-a)^(1/2)/(d*(-a)^(1/2)+e*b^(1/2)))/e
```

Rubi [A]

time = 0.22, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2512, 266, 2463, 2441, 2352, 2440, 2438}

$$-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d-\sqrt{b}\,e}\right)}{e}-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right)}{e}+\frac{2p\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e}+\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e}-\frac{p\log(d+ex)\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}\,x\right)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right)}{e}-\frac{p\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}\,x+\sqrt{b}\right)}{\sqrt{-a}\,d-\sqrt{b}\,e}\right)}{e}+\frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^2)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b/x^2)^p]*Log[d + e*x])/e + (2*p*Log[-((e*x)/d)]*Log[d + e*x])/
e - (p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x
])/e - (p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqrt[-a]*d - Sqrt[b]*e))]*Log[d
 + e*x])/e - (p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/
e - (p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)])/e + (2*p*
PolyLog[2, 1 + (e*x)/d])/e
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e} + \frac{(2bp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x^2}\right)x^3}\,dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e} + \frac{(2bp)\int \left(\frac{\log(d+ex)}{bx} - \frac{ax\log(d+ex)}{b(b+ax^2)}\right)dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e} + \frac{(2p)\int \frac{\log(d+ex)}{x}\,dx}{e} - \frac{(2ap)\int \frac{x\log(d+ex)}{b+ax^2}\,dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e} + \frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - (2p)\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx -$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e} + \frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} + \frac{2p\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{e} + \frac{(\sqrt{-}}{}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e} + \frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}\,x\right)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right.}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e} + \frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}\,x\right)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right.}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e} + \frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}\,x\right)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right.}{e}$$

Mathematica [A]

time = 0.05, size = 242, normalized size = 1.00

$$\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e} + \frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}\,x\right)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}\,x\right)}{\sqrt{-a}\,d-\sqrt{b}\,e}\right)\log(d+ex)}{e} + \frac{2p\operatorname{Li}_2\left(\frac{d+ex}{d}\right)}{e} - \frac{p\operatorname{Li}_2\left(\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d-\sqrt{b}\,e}\right)}{e} - \frac{p\operatorname{Li}_2\left(\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^2)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b/x^2)^p]*Log[d + e*x])/e + (2*p*Log[-((e*x)/d)]*Log[d + e*x])/
e - (p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x
])/e - (p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqrt[-a]*d - Sqrt[b]*e))]*Log[d
 + e*x])/e + (2*p*PolyLog[2, (d + e*x)/d])/e - (p*PolyLog[2, (Sqrt[-a]*(d +
```

```
e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/e - (p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(S
qrt[-a]*d + Sqrt[b]*e)])/e
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^2)^p)/(e*x+d),x)
[Out] int(ln(c*(a+b/x^2)^p)/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(log((a + b/x^2)^p*c)/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="fricas")
[Out] integral(log(c*((a*x^2 + b)/x^2)^p)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**2)**p)/(e*x+d),x)
[Out] Integral(log(c*(a + b/x**2)**p)/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x^2)^p*c)/(x*e + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x^2)^p)/(d + e*x),x)
```

```
[Out] int(log(c*(a + b/x^2)^p)/(d + e*x), x)
```

3.251 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x(d+ex)}\,dx$

Optimal. Leaf size=287

$$-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d}-\frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d}+\frac{p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}\right.}{\sqrt{-a}\,d+\sqrt{b}\right.}}{d}$$

```
[Out] -1/2*ln(c*(a+b/x^2)^p)*ln(-b/a/x^2)/d-ln(c*(a+b/x^2)^p)*ln(e*x+d)/d-2*p*ln(
-e*x/d)*ln(e*x+d)/d+p*ln(e*x+d)*ln(-e*(x*(-a)^(1/2)+b^(1/2))/(d*(-a)^(1/2)-
e*b^(1/2)))/d+p*ln(e*x+d)*ln(e*(-x*(-a)^(1/2)+b^(1/2))/(d*(-a)^(1/2)+e*b^(1
/2)))/d-1/2*p*polylog(2,1+b/a/x^2)/d-2*p*polylog(2,1+e*x/d)/d+p*polylog(2,(
e*x+d)*(-a)^(1/2)/(d*(-a)^(1/2)-e*b^(1/2)))/d+p*polylog(2,(e*x+d)*(-a)^(1/2
)/(d*(-a)^(1/2)+e*b^(1/2)))/d
```

Rubi [A]
time = 0.30, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2516, 2504, 2441, 2352, 2512, 266, 2463, 2440, 2438}

$$\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d-\sqrt{b}\,e}\right)}{d}+\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right)}{d}-\frac{p\,\text{PolyLog}\left(2,\frac{b}{ax^2}+1\right)}{2d}-\frac{2p\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d}-\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d}-\frac{\log\left(-\frac{b}{ax^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2d}+\frac{p\log(d+ex)\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}\,x\right)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right)}{d}+\frac{p\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}\,x+\sqrt{b}\right)}{\sqrt{-a}\,d-\sqrt{b}\,e}\right)}{d}-\frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^2)^p]/(x*(d + e*x)),x]
```

```
[Out] -1/2*(Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))])/d - (Log[c*(a + b/x^2)^p]*Log
[d + e*x])/d - (2*p*Log[-((e*x)/d)]*Log[d + e*x])/d + (p*Log[(e*(Sqrt[b] -
Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x])/d + (p*Log[-((e*(Sqrt[
b] + Sqrt[-a]*x))/(Sqrt[-a]*d - Sqrt[b]*e))]*Log[d + e*x])/d - (p*PolyLog[2
, 1 + b/(a*x^2)])/(2*d) + (p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d -
Sqrt[b]*e)])/d + (p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e
)])/d - (2*p*PolyLog[2, 1 + (e*x)/d])/d
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x(d+ex)}\,dx = \int \left(\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx} - \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d(d+ex)}\right)dx$$

$$= \frac{\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx}{d} - \frac{e\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d} - \frac{\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}\,dx, x, \frac{1}{x^2}\right)}{2d} - \frac{(2bp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x^2}\right)x^3}}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d} + \frac{(bp)\text{Subst}\left(\int \frac{\log\left(-\right.}{a+b}\right.}{2d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d} - \frac{p\text{Li}_2\left(1+\frac{b}{ax^2}\right)}{2d} - \frac{(2}{}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d} - \frac{2p\log\left(-\frac{ex}{d}\right)\log(d}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d} - \frac{2p\log\left(-\frac{ex}{d}\right)\log(d}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d} - \frac{2p\log\left(-\frac{ex}{d}\right)\log(d}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d} - \frac{2p\log\left(-\frac{ex}{d}\right)\log(d}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d} - \frac{2p\log\left(-\frac{ex}{d}\right)\log(d}{d}$$

Mathematica [A]

time = 0.08, size = 264, normalized size = 0.92

$$-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)+2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)+4p\log\left(-\frac{ex}{d}\right)\log(d+ex)-2p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}\,x\right)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right)\log(d+ex)-2p\log\left(\frac{e\left(\sqrt{b}+\sqrt{-a}\,x\right)}{-\sqrt{-a}\,d+\sqrt{b}\,e}\right)\log(d+ex)+p\text{Li}_2\left(1+\frac{b}{ax^2}\right)-2p\text{Li}_2\left(\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d-\sqrt{b}\,e}\right)-2p\text{Li}_2\left(\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right)+4p\text{Li}_2\left(1+\frac{ex}{d}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^2)^p]/(x*(d + e*x)),x]
```

```
[Out] -1/2*(Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))] + 2*Log[c*(a + b/x^2)^p]*Log[d
 + e*x] + 4*p*Log[-((e*x)/d)]*Log[d + e*x] - 2*p*Log[(e*(Sqrt[b] - Sqrt[-a]
*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x] - 2*p*Log[(e*(Sqrt[b] + Sqrt[-a
]*x))/(-(Sqrt[-a]*d) + Sqrt[b]*e)]*Log[d + e*x] + p*PolyLog[2, 1 + b/(a*x^2
)] - 2*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)] - 2*p*Po
lyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)] + 4*p*PolyLog[2, 1
+ (e*x)/d])/d
```

Maple [**F**]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x\left(ex+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^2)^p)/x/(e*x+d),x)
```

```
[Out] int(ln(c*(a+b/x^2)^p)/x/(e*x+d),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((a + b/x^2)^p*c)/((x*e + d)*x), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x^2 + b)/x^2)^p)/(x^2*e + d*x), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**2)**p)/x/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x^2)^p*c)/((x*e + d)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x^2)^p)/(x*(d + e*x)),x)
```

```
[Out] int(log(c*(a + b/x^2)^p)/(x*(d + e*x)), x)
```

3.252 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2(d+ex)}\,dx$

Optimal. Leaf size=357

$$\frac{2p}{dx}+\frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,d}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx}+\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^2}+\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+e$$

```
[Out] 2*p/d/x-ln(c*(a+b/x^2)^p)/d/x+1/2*e*ln(c*(a+b/x^2)^p)*ln(-b/a/x^2)/d^2+e*ln
(c*(a+b/x^2)^p)*ln(e*x+d)/d^2+2*e*p*ln(-e*x/d)*ln(e*x+d)/d^2-e*p*ln(e*x+d)*
ln(-e*(x*(-a)^(1/2)+b^(1/2))/(d*(-a)^(1/2)-e*b^(1/2)))/d^2-e*p*ln(e*x+d)*ln
(e*(-x*(-a)^(1/2)+b^(1/2))/(d*(-a)^(1/2)+e*b^(1/2)))/d^2+1/2*e*p*polylog(2,
1+b/a/x^2)/d^2+2*e*p*polylog(2,1+e*x/d)/d^2-e*p*polylog(2,(e*x+d)*(-a)^(1/2
)/(d*(-a)^(1/2)-e*b^(1/2)))/d^2-e*p*polylog(2,(e*x+d)*(-a)^(1/2)/(d*(-a)^(1
/2)+e*b^(1/2)))/d^2+2*p*arctan(x*a^(1/2)/b^(1/2))*a^(1/2)/d/b^(1/2)
```

Rubi [A]

time = 0.32, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {2516, 2505, 269, 331, 211, 2504, 2441, 2352, 2512, 266, 2463, 2440, 2438}

$$\frac{ep\text{PolyLog}\left(2,\frac{b}{ax^2}+1\right)}{2d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d-\sqrt{b}\,e}\right)}{d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right)}{d^2}+\frac{2ep\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d^2}+\frac{2\sqrt{a}\,p\text{ArcTan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,d}+\frac{e\log\left(-\frac{b}{ax^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2d^2}+\frac{e\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx}-\frac{ep\log(d+ex)\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}\,x\right)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right)}{d^2}-\frac{ep\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}\,x+\sqrt{b}\right)}{\sqrt{-a}\,d-\sqrt{b}\,e}\right)}{d^2}+\frac{2ep\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^2}+\frac{2p}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^2)^p]/(x^2*(d + e*x)),x]
```

```
[Out] (2*p)/(d*x) + (2*Sqrt[a]*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(Sqrt[b]*d) - Log[c
*(a + b/x^2)^p]/(d*x) + (e*Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))])/(2*d^2)
+ (e*Log[c*(a + b/x^2)^p]*Log[d + e*x])/d^2 + (2*e*p*Log[-((e*x)/d)]*Log[d
+ e*x])/d^2 - (e*p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]
*Log[d + e*x])/d^2 - (e*p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqrt[-a]*d - Sq
rt[b]*e))]*Log[d + e*x])/d^2 + (e*p*PolyLog[2, 1 + b/(a*x^2)])/(2*d^2) - (e
*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/d^2 - (e*p*Po
lyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)])/d^2 + (2*e*p*PolyL
og[2, 1 + (e*x)/d])/d^2
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2(d+ex)}\,dx = \int \left(\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx^2} - \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x} + \frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2(d+ex)}\right)dx$$

$$= \frac{\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2}\,dx}{d} - \frac{e\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx}{d^2} + \frac{e^2\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx}{d^2}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d^2} + \frac{e\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}\,dx, x, \right.}{2d^2}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^2} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d^2}$$

$$= \frac{2p}{dx} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^2} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+}{d^2}$$

$$= \frac{2p}{dx} + \frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^2} +$$

$$= \frac{2p}{dx} + \frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^2} +$$

$$= \frac{2p}{dx} + \frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^2} +$$

$$= \frac{2p}{dx} + \frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^2} +$$

$$= \frac{2p}{dx} + \frac{2\sqrt{a}\,p\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^2} +$$

Mathematica [A]

time = 0.15, size = 320, normalized size = 0.90

$$\frac{4dp\left(\frac{1}{x} - \frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}\,x}\right)}{\sqrt{b}}\right) - \frac{2d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x} + 2e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex) + e\left(\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right) + p\text{Li}_2\left(1+\frac{b}{ax^2}\right)\right) + 2ep\left(2\log\left(-\frac{ex}{d}\right)\log(d+ex) - \log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}\,x\right)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right)\log(d+ex) - \log\left(\frac{e\left(\sqrt{b}+\sqrt{-a}\,x\right)}{-\sqrt{-a}\,d+\sqrt{b}\,e}\right)\log(d+ex) - \text{Li}_2\left(\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d-\sqrt{b}\,e}\right) - \text{Li}_2\left(\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right) + 2\text{Li}_2\left(1+\frac{ex}{d}\right)\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^2)^p]/(x^2*(d + e*x)),x]
```

```
[Out] (4*d*p*(x^(-1) - (Sqrt[a]*ArcTan[Sqrt[b]/(Sqrt[a]*x)])/Sqrt[b]) - (2*d*Log[
c*(a + b/x^2)^p])/x + 2*e*Log[c*(a + b/x^2)^p]*Log[d + e*x] + e*(Log[c*(a +
 b/x^2)^p]*Log[-(b/(a*x^2))] + p*PolyLog[2, 1 + b/(a*x^2)]) + 2*e*p*(2*Log[
-((e*x)/d)]*Log[d + e*x] - Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqr
t[b]*e)]*Log[d + e*x] - Log[(e*(Sqrt[b] + Sqrt[-a]*x))/(-(Sqrt[-a]*d) + Sqr
t[b]*e)]*Log[d + e*x] - PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[
b]*e)] - PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)] + 2*Poly
Log[2, 1 + (e*x)/d]))/(2*d^2)
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2\left(ex+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^2)^p)/x^2/(e*x+d),x)
```

```
[Out] int(ln(c*(a+b/x^2)^p)/x^2/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((a + b/x^2)^p*c)/((x*e + d)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x^2 + b)/x^2)^p)/(x^3*e + d*x^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**2)**p)/x**2/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x^2)^p*c)/((x*e + d)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(a+\frac{b}{x^2}\right)}^p\right)}{x^2\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x^2)^p)/(x^2*(d + e*x)),x)
```

```
[Out] int(log(c*(a + b/x^2)^p)/(x^2*(d + e*x)), x)
```

3.253 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3(d+ex)}\,dx$

Optimal. Leaf size=414

$$\frac{p}{2dx^2}-\frac{2ep}{d^2x}-\frac{2\sqrt{a}\,ep\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,d^2}-\frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2bd}+\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x}-\frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\text{lo}}{2d^3}$$

```
[Out] 1/2*p/d/x^2-2*e*p/d^2/x-1/2*(a+b/x^2)*ln(c*(a+b/x^2)^p)/b/d+e*ln(c*(a+b/x^2
)^p)/d^2/x-1/2*e^2*ln(c*(a+b/x^2)^p)*ln(-b/a/x^2)/d^3-e^2*ln(c*(a+b/x^2)^p)
*ln(e*x+d)/d^3-2*e^2*p*ln(-e*x/d)*ln(e*x+d)/d^3+e^2*p*ln(e*x+d)*ln(-e*(x*(-
a)^(1/2)+b^(1/2))/(d*(-a)^(1/2)-e*b^(1/2)))/d^3+e^2*p*ln(e*x+d)*ln(e*(-x*(-
a)^(1/2)+b^(1/2))/(d*(-a)^(1/2)+e*b^(1/2)))/d^3-1/2*e^2*p*polylog(2,1+b/a/x
^2)/d^3-2*e^2*p*polylog(2,1+e*x/d)/d^3+e^2*p*polylog(2,(e*x+d)*(-a)^(1/2)/(
d*(-a)^(1/2)-e*b^(1/2)))/d^3+e^2*p*polylog(2,(e*x+d)*(-a)^(1/2)/(d*(-a)^(1/
2)+e*b^(1/2)))/d^3-2*e*p*arctan(x*a^(1/2)/b^(1/2))*a^(1/2)/d^2/b^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 15, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {2516, 2504, 2436, 2332, 2505, 269, 331, 211, 2441, 2352, 2512, 266, 2463, 2440, 2438}

$$-\frac{e^2p\text{PolyLog}\left(2,\frac{b}{ax^2}+1\right)}{2d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d-\sqrt{b}\,e}\right)}{d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right)}{d^3}-\frac{2e^2p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d^3}-\frac{2\sqrt{a}\,ep\text{ArcTan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,d^2}-\frac{e^2\log\left(-\frac{b}{ax^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2d^3}-\frac{e^2\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^3}+\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x}-\frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2bd}+\frac{e^2p\log(d+ex)\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}\,x\right)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right)}{d^3}+\frac{e^2p\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}\,x+\sqrt{b}\right)}{\sqrt{-a}\,d-\sqrt{b}\,e}\right)}{d^3}-\frac{2e^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^3}-\frac{2ep}{d^2x}+\frac{p}{2dx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^2)^p]/(x^3*(d + e*x)),x]
```

```
[Out] p/(2*d*x^2) - (2*e*p)/(d^2*x) - (2*Sqrt[a]*e*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])
/(Sqrt[b]*d^2) - ((a + b/x^2)*Log[c*(a + b/x^2)^p])/(2*b*d) + (e*Log[c*(a +
 b/x^2)^p])/(d^2*x) - (e^2*Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))])/(2*d^3)
- (e^2*Log[c*(a + b/x^2)^p]*Log[d + e*x])/d^3 - (2*e^2*p*Log[-((e*x)/d)]*Lo
g[d + e*x])/d^3 + (e^2*p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[
b]*e)]*Log[d + e*x])/d^3 + (e^2*p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqrt[-a
]*d - Sqrt[b]*e))]*Log[d + e*x])/d^3 - (e^2*p*PolyLog[2, 1 + b/(a*x^2)])/(2
*d^3) + (e^2*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/d
^3 + (e^2*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)])/d^3
- (2*e^2*p*PolyLog[2, 1 + (e*x)/d])/d^3
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3(d+ex)}\,dx = \int \left(\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx^3} - \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x^2} + \frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^3x} - \frac{e^3\log\left(c\left(a+\frac{b}{x^2} \cdots\right.\right.}{d^3(d+ex)} \cdots\right.$$

$$= \frac{\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3}\,dx}{d} - \frac{e\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2}\,dx}{d^2} + \frac{e^2\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx}{d^3} - \frac{e^3\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^{\cdots}\right.}{d+ex} \cdots}{d^3}$$

$$= \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x} - \frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d^3} - \frac{\text{Subst}\left(\int \log\left(c(a+bx)^p\right)\,dx \cdots\right.}{2d}$$

$$= \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x} - \frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^3} - \frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+e \cdots}{d^3}$$

$$= \frac{p}{2dx^2} - \frac{2ep}{d^2x} - \frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2bd} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x} - \frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p \cdots\right.}{2d^3}$$

$$= \frac{p}{2dx^2} - \frac{2ep}{d^2x} - \frac{2\sqrt{a}\,ep\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,d^2} - \frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2bd} + \frac{e\log\left(c\left(a+ \cdots\right.\right.}{d^2x}$$

$$= \frac{p}{2dx^2} - \frac{2ep}{d^2x} - \frac{2\sqrt{a}\,ep\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,d^2} - \frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2bd} + \frac{e\log\left(c\left(a+ \cdots\right.\right.}{d^2x}$$

$$= \frac{p}{2dx^2} - \frac{2ep}{d^2x} - \frac{2\sqrt{a}\,ep\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,d^2} - \frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2bd} + \frac{e\log\left(c\left(a+ \cdots\right.\right.}{d^2x}$$

$$= \frac{p}{2dx^2} - \frac{2ep}{d^2x} - \frac{2\sqrt{a}\,ep\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,d^2} - \frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2bd} + \frac{e\log\left(c\left(a+ \cdots\right.\right.}{d^2x}$$

$$= \frac{p}{2dx^2} - \frac{2ep}{d^2x} - \frac{2\sqrt{a}\,ep\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,d^2} - \frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2bd} + \frac{e\log\left(c\left(a+ \cdots\right.\right.}{d^2x}$$

Mathematica [A]
time = 0.20, size = 364, normalized size = 0.88

$$\frac{4dep\left(-\frac{1}{x}+\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}\,x}\right)}{\sqrt{b}}\right)+\frac{2de\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}+d^2\left(\frac{p}{x^2}-\frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{b}\right)-2e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)-e^2\left(\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)+p\text{Li}_2\left(1+\frac{b}{ax^2}\right)\right)-2e^2p\left(2\log\left(-\frac{ex}{d}\right)\log(d+ex)-\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}\,x\right)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right)\log(d+ex)-\log\left(\frac{e\left(\sqrt{b}+\sqrt{-a}\,x\right)}{-\sqrt{-a}\,d+\sqrt{b}\,e}\right)\log(d+ex)-\text{Li}_2\left(\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d-\sqrt{b}\,e}\right)-\text{Li}_2\left(\frac{\sqrt{-a}\,(d+ex)}{\sqrt{-a}\,d+\sqrt{b}\,e}\right)+2\text{Li}_2\left(1+\frac{ex}{d}\right)\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^2)^p]/(x^3*(d + e*x)),x]
```

```
[Out] (4*d*e*p*(-x^(-1) + (Sqrt[a]*ArcTan[Sqrt[b]/(Sqrt[a]*x)])/Sqrt[b]) + (2*d*e
*Log[c*(a + b/x^2)^p])/x + d^2*(p/x^2 - ((a + b/x^2)*Log[c*(a + b/x^2)^p])/
b) - 2*e^2*Log[c*(a + b/x^2)^p]*Log[d + e*x] - e^2*(Log[c*(a + b/x^2)^p]*Lo
g[-(b/(a*x^2))] + p*PolyLog[2, 1 + b/(a*x^2)]) - 2*e^2*p*(2*Log[-((e*x)/d)]
*Log[d + e*x] - Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Lo
g[d + e*x] - Log[(e*(Sqrt[b] + Sqrt[-a]*x))/(-(Sqrt[-a]*d) + Sqrt[b]*e)]*Lo
g[d + e*x] - PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)] - Po
lyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)] + 2*PolyLog[2, 1 +
(e*x)/d]))/(2*d^3)
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3\left(ex+d\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^2)^p)/x^3/(e*x+d),x)
```

```
[Out] int(ln(c*(a+b/x^2)^p)/x^3/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((a + b/x^2)^p*c)/((x*e + d)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x^2 + b)/x^2)^p)/(x^4*e + d*x^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**2)**p)/x**3/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^2)^p)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x^2)^p*c)/((x*e + d)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x^2)^p)/(x^3*(d + e*x)),x)
```

```
[Out] int(log(c*(a + b/x^2)^p)/(x^3*(d + e*x)), x)
```

3.254 $\int \frac{x^3 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=714

$$-\frac{\sqrt{3}\,\sqrt[3]{b}\,d^2p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt[3]{a}\,e^3}+\frac{\sqrt{3}\,b^{2/3}dp\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{2a^{2/3}e^2}+\frac{d^2x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3}-\frac{dx^2\log\left(c(a-\right.}{2e^2}$$

```
[Out] d^2*x*ln(c*(a+b/x^3)^p)/e^3-1/2*d*x^2*ln(c*(a+b/x^3)^p)/e^2+1/3*x^3*ln(c*(a
+b/x^3)^p)/e+b^(1/3)*d^2*p*ln(b^(1/3)+a^(1/3)*x)/a^(1/3)/e^3+1/2*b^(2/3)*d*
p*ln(b^(1/3)+a^(1/3)*x)/a^(2/3)/e^2-d^3*ln(c*(a+b/x^3)^p)*ln(e*x+d)/e^4-3*d
^3*p*ln(-e*x/d)*ln(e*x+d)/e^4+d^3*p*ln(-e*(b^(1/3)+a^(1/3)*x)/(a^(1/3)*d-b^
(1/3)*e))*ln(e*x+d)/e^4+d^3*p*ln(-e*((-1)^(2/3)*b^(1/3)+a^(1/3)*x)/(a^(1/3)
*d-(-1)^(2/3)*b^(1/3)*e))*ln(e*x+d)/e^4+d^3*p*ln((-1)^(1/3)*e*(b^(1/3)+(-1)
^(2/3)*a^(1/3)*x)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))*ln(e*x+d)/e^4-1/2*b^(1/
3)*d^2*p*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/a^(1/3)/e^3-1/4*b^(2/3)*
d*p*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/a^(2/3)/e^2+1/3*b*p*ln(a*x^3+
b)/a/e+d^3*p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-b^(1/3)*e))/e^4+d^3*p*pol
ylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))/e^4+d^3*p*polylog(
2,a^(1/3)*(e*x+d)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3)*e))/e^4-3*d^3*p*polylog(2,1
+e*x/d)/e^4-b^(1/3)*d^2*p*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)/b^(1/3)*3^(1/2))
*3^(1/2)/a^(1/3)/e^3+1/2*b^(2/3)*d*p*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)/b^(1/
3)*3^(1/2))*3^(1/2)/a^(2/3)/e^2
```

Rubi [A]

time = 0.64, antiderivative size = 714, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 18, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {2516, 2498, 269, 206, 31, 648, 631, 210, 642, 2505, 298, 266, 2512, 2463, 2441, 2352, 2440, 2438}

Antiderivative was successfully verified.

```
[In] Int[(x^3*Log[c*(a + b/x^3)^p])/(d + e*x),x]
```

```
[Out] -((Sqrt[3]*b^(1/3)*d^2*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))])
/(a^(1/3)*e^3)) + (Sqrt[3]*b^(2/3)*d*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt
[3]*b^(1/3))])/(2*a^(2/3)*e^2) + (d^2*x*Log[c*(a + b/x^3)^p])/e^3 - (d*x^2*
Log[c*(a + b/x^3)^p])/(2*e^2) + (x^3*Log[c*(a + b/x^3)^p])/(3*e) + (b^(1/3)
*d^2*p*Log[b^(1/3) + a^(1/3)*x])/(a^(1/3)*e^3) + (b^(2/3)*d*p*Log[b^(1/3) +
 a^(1/3)*x])/(2*a^(2/3)*e^2) - (d^3*Log[c*(a + b/x^3)^p]*Log[d + e*x])/e^4
- (3*d^3*p*Log[-((e*x)/d)]*Log[d + e*x])/e^4 + (d^3*p*Log[-((e*(b^(1/3) + a
^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*x])/e^4 + (d^3*p*Log[-((e*((
```

```
-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d
+ e*x])/e^4 + (d^3*p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a
^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/e^4 - (b^(1/3)*d^2*p*Log[b^
(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(2*a^(1/3)*e^3) - (b^(2/3)*d*p*Lo
g[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(4*a^(2/3)*e^2) + (b*p*Log[b
+ a*x^3])/(3*a*e) + (d^3*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1
/3)*e)])/e^4 + (d^3*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3
)*b^(1/3)*e)])/e^4 + (d^3*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1
)^(2/3)*b^(1/3)*e)])/e^4 - (3*d^3*p*PolyLog[2, 1 + (e*x)/d])/e^4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{x^3 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \int \left(\frac{d^2 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{dx \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2} + \frac{x^2 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} - \frac{d^3 \log \left(\cdots\right.}{e^3(\cdots}\right.$$

$$= \frac{d^2 \int \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)\,dx}{e^3} - \frac{d^3 \int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx}{e^3} - \frac{d \int x \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)\,dx}{e^2} + \cdots$$

$$= \frac{d^2 x \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{dx^2 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e^2} + \frac{x^3 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3e} - \frac{d^3 \log \left(c\left(\cdots\right.\right.}{\cdots}$$

$$= \frac{d^2 x \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{dx^2 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e^2} + \frac{x^3 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3e} - \frac{d^3 \log \left(c\left(\cdots\right.\right.}{\cdots}$$

$$= \frac{d^2 x \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{dx^2 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e^2} + \frac{x^3 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3e} - \frac{d^3 \log \left(c\left(\cdots\right.\right.}{\cdots}$$

$$= \frac{d^2 x \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{dx^2 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e^2} + \frac{x^3 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3e} + \frac{\sqrt[3]{b}\, d^2 p \,\mathrm{lo}\cdots}{\cdots}$$

$$= \frac{d^2 x \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{dx^2 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e^2} + \frac{x^3 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3e} + \frac{\sqrt[3]{b}\, d^2 p \,\mathrm{lo}\cdots}{\cdots}$$

$$= -\frac{\sqrt{3}\,\sqrt[3]{b}\,d^2 p \tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt[3]{a}\,e^3} + \frac{\sqrt{3}\,b^{2/3} d p \tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{2a^{2/3}e^2} + \frac{d^2 x \,\mathrm{lo}\cdots}{\cdots}$$

$$= -\frac{\sqrt{3}\,\sqrt[3]{b}\,d^2 p \tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt[3]{a}\,e^3} + \frac{\sqrt{3}\,b^{2/3} d p \tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{2a^{2/3}e^2} + \frac{d^2 x \,\mathrm{lo}\cdots}{\cdots}$$

$$= -\frac{\sqrt{3}\,\sqrt[3]{b}\,d^2 p \tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt[3]{a}\,e^3} + \frac{\sqrt{3}\,b^{2/3} d p \tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{2a^{2/3}e^2} + \frac{d^2 x \,\mathrm{lo}\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.29, size = 505, normalized size = 0.71

$$\frac{9bde^2px\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};-\frac{b}{ax^3}\right) - 9bd^2ep\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{b}{ax^3}\right) + x^2\left(2be^3p\log\left(a+\frac{b}{x^3}\right) + 6ad^2ex\log\left(c\left(a+\frac{b}{x^3}\right)^p\right) - 3ade^2x^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right) + 2ae^3x^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right) + 6be^3p\log(x) - 6ad^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex) - 18ad^3p\log\left(-\frac{ex}{d}\right)\log(d+ex) + 6ad^3p\log\left(\frac{e\left(\sqrt[3]{-1}\,\sqrt[3]{b}-\sqrt[3]{a}\,x\right)}{\sqrt[3]{a}\,d+\sqrt[3]{-1}\,\sqrt[3]{b}\,e}\right)\log(d+ex) + 6ad^3p\log\left(\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{-\sqrt[3]{a}\,d+\sqrt[3]{b}\,e}\right)\log(d+ex) + 6ad^3p\log\left(\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{-\sqrt[3]{a}\,d+(-1)^{2/3}\sqrt[3]{b}\,e}\right)\log(d+ex) + 6ad^3p\,\mathrm{Li}_2\left(\frac{\sqrt[3]{a}\,(d+ex)}{\sqrt[3]{a}\,d-\sqrt[3]{b}\,e}\right) + 6ad^3p\,\mathrm{Li}_2\left(\frac{\sqrt[3]{a}\,(d+ex)}{\sqrt[3]{a}\,d+\sqrt[3]{-1}\,\sqrt[3]{b}\,e}\right) + 6ad^3p\,\mathrm{Li}_2\left(\frac{\sqrt[3]{a}\,(d+ex)}{\sqrt[3]{a}\,d-(-1)^{2/3}\sqrt[3]{b}\,e}\right) - 18ad^3p\,\mathrm{Li}_2\left(1+\frac{ex}{d}\right)\right)}{6ae^4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Log[c*(a + b/x^3)^p])/(d + e*x),x]
```

```
[Out] (9*b*d*e^2*p*x*Hypergeometric2F1[1/3, 1, 4/3, -(b/(a*x^3))] - 9*b*d^2*e*p*H
ypergeometric2F1[2/3, 1, 5/3, -(b/(a*x^3))] + x^2*(2*b*e^3*p*Log[a + b/x^3]
 + 6*a*d^2*e*x*Log[c*(a + b/x^3)^p] - 3*a*d*e^2*x^2*Log[c*(a + b/x^3)^p] +
2*a*e^3*x^3*Log[c*(a + b/x^3)^p] + 6*b*e^3*p*Log[x] - 6*a*d^3*Log[c*(a + b/
x^3)^p]*Log[d + e*x] - 18*a*d^3*p*Log[-((e*x)/d)]*Log[d + e*x] + 6*a*d^3*p*
Log[(e*((-1)^(1/3)*b^(1/3) - a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)
]*Log[d + e*x] + 6*a*d^3*p*Log[(e*(b^(1/3) + a^(1/3)*x))/(-(a^(1/3)*d) + b^
(1/3)*e)]*Log[d + e*x] + 6*a*d^3*p*Log[(e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))
/(-(a^(1/3)*d) + (-1)^(2/3)*b^(1/3)*e)]*Log[d + e*x] + 6*a*d^3*p*PolyLog[2,
 (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)] + 6*a*d^3*p*PolyLog[2, (a^(1/
3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)] + 6*a*d^3*p*PolyLog[2, (a
^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)] - 18*a*d^3*p*PolyLog[
2, 1 + (e*x)/d]))/(6*a*e^4*x^2)
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \ln \left(c \left(a+\frac{b}{x^{3}}\right)^{p}\right)}{e x +d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(a+b/x^3)^p)/(e*x+d),x)
```

```
[Out] int(x^3*ln(c*(a+b/x^3)^p)/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(x^3*log((a + b/x^3)^p*c)/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x^3*log(c*((a*x^3 + b)/x^3)^p)/(x*e + d), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(a+b/x**3)**p)/(e*x+d),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(x^3*log((a + b/x^3)^p*c)/(x*e + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\ln\left(c\,{\left(a+\frac{b}{x^3}\right)}^p\right)}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*log(c*(a + b/x^3)^p))/(d + e*x),x)
[Out] int((x^3*log(c*(a + b/x^3)^p))/(d + e*x), x)
```

3.255 $\int \frac{x^2 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=666

$$\frac{\sqrt{3}\,\sqrt[3]{b}\,dp\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt[3]{a}\,e^2}-\frac{\sqrt{3}\,b^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{2a^{2/3}e}-\frac{dx\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e}$$

```
[Out] -d*x*ln(c*(a+b/x^3)^p)/e^2+1/2*x^2*ln(c*(a+b/x^3)^p)/e-b^(1/3)*d*p*ln(b^(1/
3)+a^(1/3)*x)/a^(1/3)/e^2-1/2*b^(2/3)*p*ln(b^(1/3)+a^(1/3)*x)/a^(2/3)/e+d^2
*ln(c*(a+b/x^3)^p)*ln(e*x+d)/e^3+3*d^2*p*ln(-e*x/d)*ln(e*x+d)/e^3-d^2*p*ln(
-e*(b^(1/3)+a^(1/3)*x)/(a^(1/3)*d-b^(1/3)*e))*ln(e*x+d)/e^3-d^2*p*ln(-e*((-
1)^(2/3)*b^(1/3)+a^(1/3)*x)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3)*e))*ln(e*x+d)/e^3
-d^2*p*ln((-1)^(1/3)*e*(b^(1/3)+(-1)^(2/3)*a^(1/3)*x)/(a^(1/3)*d+(-1)^(1/3)
*b^(1/3)*e))*ln(e*x+d)/e^3+1/2*b^(1/3)*d*p*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(
2/3)*x^2)/a^(1/3)/e^2+1/4*b^(2/3)*p*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^
2)/a^(2/3)/e-d^2*p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-b^(1/3)*e))/e^3-d^2
*p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))/e^3-d^2*p*po
lylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3)*e))/e^3+3*d^2*p*polyl
og(2,1+e*x/d)/e^3+b^(1/3)*d*p*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)/b^(1/3)*3^(1
/2))*3^(1/2)/a^(1/3)/e^2-1/2*b^(2/3)*p*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)/b^(
1/3)*3^(1/2))*3^(1/2)/a^(2/3)/e
```

Rubi [A]
time = 0.49, antiderivative size = 666, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 18, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {2516, 2498, 269, 206, 31, 648, 631, 210, 642, 2505, 298, 2512, 266, 2463, 2441, 2352, 2440, 2438}

$$-\frac{d^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}\,(d+ex)}{\sqrt[3]{a}\,d-\sqrt[3]{b}\,e}\right)}{e^3}-\frac{d^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}\,(d+ex)}{\sqrt[3]{a}\,d+\sqrt[3]{-1}\,\sqrt[3]{b}\,e}\right)}{e^3}-\frac{d^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}\,(d+ex)}{\sqrt[3]{a}\,d-(-1)^{2/3}\sqrt[3]{b}\,e}\right)}{e^3}+\frac{3d^2p\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^3}-\frac{\sqrt{3}\,b^{2/3}p\,\text{ArcTan}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{2a^{2/3}e}+\frac{\sqrt[3]{b}\,dp\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{2\sqrt[3]{a}\,e^2}+\frac{b^{2/3}p\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{4a^{2/3}e}-\frac{b^{2/3}p\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{2a^{2/3}e}+\frac{\sqrt{3}\,\sqrt[3]{b}\,dp\,\text{ArcTan}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt[3]{a}\,e^2}+\frac{d^2\log(d+ex)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3}-\frac{dx\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e}-\frac{d^2p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{\sqrt[3]{a}\,d-\sqrt[3]{b}\,e}\right)}{e^3}-\frac{d^2p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}\,x+(-1)^{2/3}\sqrt[3]{b}\right)}{\sqrt[3]{a}\,d-(-1)^{2/3}\sqrt[3]{b}\,e}\right)}{e^3}-\frac{d^2p\log(d+ex)\log\left(\frac{\sqrt[3]{-1}\,e\left((-1)^{2/3}\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{\sqrt[3]{a}\,d+\sqrt[3]{-1}\,\sqrt[3]{b}\,e}\right)}{e^3}-\frac{\sqrt[3]{b}\,dp\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{\sqrt[3]{a}\,e^2}+\frac{3d^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Log[c*(a + b/x^3)^p])/(d + e*x),x]
```

```
[Out] (Sqrt[3]*b^(1/3)*d*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))])/(a^
(1/3)*e^2) - (Sqrt[3]*b^(2/3)*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(
1/3))])/(2*a^(2/3)*e) - (d*x*Log[c*(a + b/x^3)^p])/e^2 + (x^2*Log[c*(a + b/
x^3)^p])/(2*e) - (b^(1/3)*d*p*Log[b^(1/3) + a^(1/3)*x])/(a^(1/3)*e^2) - (b^
(2/3)*p*Log[b^(1/3) + a^(1/3)*x])/(2*a^(2/3)*e) + (d^2*Log[c*(a + b/x^3)^p]
*Log[d + e*x])/e^3 + (3*d^2*p*Log[-((e*x)/d)]*Log[d + e*x])/e^3 - (d^2*p*Lo
g[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*x])/e^3 -
 (d^2*p*Log[-((e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*
b^(1/3)*e))]*Log[d + e*x])/e^3 - (d^2*p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(
```

```
2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/e^3 + (b
^(1/3)*d*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(2*a^(1/3)*e^2)
+ (b^(2/3)*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(4*a^(2/3)*e)
- (d^2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/e^3 - (d^
2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/e^3
 - (d^2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)
])/e^3 + (3*d^2*p*PolyLog[2, 1 + (e*x)/d])/e^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \int \left(-\frac{d \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2}+\frac{x \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e}+\frac{d^2 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2(d+ex)}\right) dx$$

$$= -\frac{d \int \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)\,dx}{e^2}+\frac{d^2 \int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx}{e^2}+\frac{\int x \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)\,dx}{e}$$

$$= -\frac{dx \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2}+\frac{x^2 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e}+\frac{d^2 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^3}+$$

$$= -\frac{dx \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2}+\frac{x^2 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e}+\frac{d^2 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^3}+$$

$$= -\frac{dx \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2}+\frac{x^2 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e}+\frac{d^2 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^3}+$$

$$= -\frac{dx \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2}+\frac{x^2 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e}-\frac{\sqrt[3]{b}\,dp \log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{\sqrt[3]{a}\,e^2}-\frac{b^{2/3}p}{}$$

$$= -\frac{dx \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2}+\frac{x^2 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e}-\frac{\sqrt[3]{b}\,dp \log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{\sqrt[3]{a}\,e^2}-\frac{b^{2/3}p}{}$$

$$= \frac{\sqrt{3}\,\sqrt[3]{b}\,dp \tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt[3]{a}\,e^2}-\frac{\sqrt{3}\,b^{2/3}p \tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{2a^{2/3}e}-\frac{dx \log\left(c\left(a\right.\right.}{e^2}$$

$$= \frac{\sqrt{3}\,\sqrt[3]{b}\,dp \tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt[3]{a}\,e^2}-\frac{\sqrt{3}\,b^{2/3}p \tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{2a^{2/3}e}-\frac{dx \log\left(c\left(a\right.\right.}{e^2}$$

$$= \frac{\sqrt{3}\,\sqrt[3]{b}\,dp \tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt[3]{a}\,e^2}-\frac{\sqrt{3}\,b^{2/3}p \tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{2a^{2/3}e}-\frac{dx \log\left(c\left(a\right.\right.}{e^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.17, size = 443, normalized size = 0.67

$$\frac{-3be^2px\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};-\frac{b}{ax^3}\right)+3bdep\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{b}{ax^3}\right)+ax^2\left(-2dex\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)+e^2x^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)+2d^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)+6d^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)-2d^2p\log\left(\frac{e\left(\sqrt[3]{-1}\,\sqrt[3]{b}-\sqrt[3]{a}\,x\right)}{\sqrt[3]{a}\,d+\sqrt[3]{-1}\,\sqrt[3]{b}\,e}\right)\log(d+ex)-2d^2p\log\left(\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{-\sqrt[3]{a}\,d+\sqrt[3]{b}\,e}\right)\log(d+ex)-2d^2p\log\left(\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{-\sqrt[3]{a}\,d+(-1)^{2/3}\sqrt[3]{b}\,e}\right)\log(d+ex)-2d^2p\,\mathrm{Li}_2\left(\frac{\sqrt[3]{a}\,(d+ex)}{\sqrt[3]{a}\,d-\sqrt[3]{b}\,e}\right)-2d^2p\,\mathrm{Li}_2\left(\frac{\sqrt[3]{a}\,(d+ex)}{\sqrt[3]{a}\,d+\sqrt[3]{-1}\,\sqrt[3]{b}\,e}\right)-2d^2p\,\mathrm{Li}_2\left(\frac{\sqrt[3]{a}\,(d+ex)}{\sqrt[3]{a}\,d-(-1)^{2/3}\sqrt[3]{b}\,e}\right)+6d^2p\,\mathrm{Li}_2\left(1+\frac{ex}{d}\right)\right)}{2ae^3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Log[c*(a + b/x^3)^p])/(d + e*x),x]
```

```
[Out] (-3*b*e^2*p*x*Hypergeometric2F1[1/3, 1, 4/3, -(b/(a*x^3))] + 3*b*d*e*p*Hype
rgeometric2F1[2/3, 1, 5/3, -(b/(a*x^3))] + a*x^2*(-2*d*e*x*Log[c*(a + b/x^3
)^p] + e^2*x^2*Log[c*(a + b/x^3)^p] + 2*d^2*Log[c*(a + b/x^3)^p]*Log[d + e*
x] + 6*d^2*p*Log[-((e*x)/d)]*Log[d + e*x] - 2*d^2*p*Log[(e*((-1)^(1/3)*b^(1
/3) - a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x] - 2*d^2*
p*Log[(e*(b^(1/3) + a^(1/3)*x))/(-(a^(1/3)*d) + b^(1/3)*e)]*Log[d + e*x] -
2*d^2*p*Log[(e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(-(a^(1/3)*d) + (-1)^(2/3)
*b^(1/3)*e)]*Log[d + e*x] - 2*d^2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)
*d - b^(1/3)*e)] - 2*d^2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)
^(1/3)*b^(1/3)*e)] - 2*d^2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-
1)^(2/3)*b^(1/3)*e)] + 6*d^2*p*PolyLog[2, 1 + (e*x)/d]))/(2*a*e^3*x^2)
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{x^2 \ln \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(a+b/x^3)^p)/(e*x+d),x)
```

```
[Out] int(x^2*ln(c*(a+b/x^3)^p)/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(x^2*log((a + b/x^3)^p*c)/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x^2*log(c*((a*x^3 + b)/x^3)^p)/(x*e + d), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(a+b/x**3)**p)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(x^2*log((a + b/x^3)^p*c)/(x*e + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\ln\left(c\,{\left(a+\frac{b}{x^3}\right)}^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*log(c*(a + b/x^3)^p))/(d + e*x),x)
```

```
[Out] int((x^2*log(c*(a + b/x^3)^p))/(d + e*x), x)
```

3.256 $\int \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=488

$$-\frac{\sqrt{3}\,\sqrt[3]{b}\,p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt[3]{a}\,e}+\frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e}+\frac{\sqrt[3]{b}\,p\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{\sqrt[3]{a}\,e}-\frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+$$

```
[Out] x*ln(c*(a+b/x^3)^p)/e+b^(1/3)*p*ln(b^(1/3)+a^(1/3)*x)/a^(1/3)/e-d*ln(c*(a+b
/x^3)^p)*ln(e*x+d)/e^2-3*d*p*ln(-e*x/d)*ln(e*x+d)/e^2+d*p*ln(-e*(b^(1/3)+a^
(1/3)*x)/(a^(1/3)*d-b^(1/3)*e))*ln(e*x+d)/e^2+d*p*ln(-e*((-1)^(2/3)*b^(1/3)
+a^(1/3)*x)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3)*e))*ln(e*x+d)/e^2+d*p*ln((-1)^(1/
3)*e*(b^(1/3)+(-1)^(2/3)*a^(1/3)*x)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))*ln(e*
x+d)/e^2-1/2*b^(1/3)*p*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/a^(1/3)/e+
d*p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-b^(1/3)*e))/e^2+d*p*polylog(2,a^(1
/3)*(e*x+d)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))/e^2+d*p*polylog(2,a^(1/3)*(e*
x+d)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3)*e))/e^2-3*d*p*polylog(2,1+e*x/d)/e^2-b^(
1/3)*p*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)/b^(1/3)*3^(1/2))*3^(1/2)/a^(1/3)/e
```

Rubi [A]
time = 0.40, antiderivative size = 488, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 16, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {2516, 2498, 269, 206, 31, 648, 631, 210, 642, 2512, 266, 2463, 2441, 2352, 2440, 2438}

$$\frac{dp\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^2}+\frac{dp\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^2}+\frac{dp\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^2}-\frac{3dp\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^2}-\frac{\sqrt[3]{b}\,p\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{2\sqrt[3]{a}\,e}-\frac{\sqrt{3}\sqrt[3]{b}\,p\,\text{ArcTan}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}\,e}-\frac{d\log(d+ex)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2}+\frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e}+\frac{dp\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^2}+\frac{dp\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}x+(-1)^{2/3}\sqrt[3]{b}\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^2}+\frac{dp\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left((-1)^{2/3}\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^2}+\frac{\sqrt[3]{b}\,p\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}\,e}-\frac{3dp\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[c*(a + b/x^3)^p])/(d + e*x),x]
```

```
[Out] -((Sqrt[3]*b^(1/3)*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))])/(a^
(1/3)*e)) + (x*Log[c*(a + b/x^3)^p])/e + (b^(1/3)*p*Log[b^(1/3) + a^(1/3)*x
])/(a^(1/3)*e) - (d*Log[c*(a + b/x^3)^p]*Log[d + e*x])/e^2 - (3*d*p*Log[-((
e*x)/d)]*Log[d + e*x])/e^2 + (d*p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*
d - b^(1/3)*e))]*Log[d + e*x])/e^2 + (d*p*Log[-((e*((-1)^(2/3)*b^(1/3) + a^
(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x])/e^2 + (d*p*Log
[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^
(1/3)*e)]*Log[d + e*x])/e^2 - (b^(1/3)*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x +
a^(2/3)*x^2])/(2*a^(1/3)*e) + (d*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*
d - b^(1/3)*e)])/e^2 + (d*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1
)^(1/3)*b^(1/3)*e)])/e^2 + (d*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d -
 (-1)^(2/3)*b^(1/3)*e)])/e^2 - (3*d*p*PolyLog[2, 1 + (e*x)/d])/e^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
```

```
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \int \left(\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e(d+ex)}\right) dx$$

$$= \frac{\int \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\,dx}{e} - \frac{d\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx}{e}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^2} - \frac{(3bdp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x^3}\right)x^4}\,dx}{e^2} + \frac{(3b}{}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^2} - \frac{(3bdp)\int \left(\frac{\log(d+ex)}{bx} - \frac{ax^2\log}{b(b+}\right.}{e^2}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^2} - \frac{(3dp)\int \frac{\log(d+ex)}{x}\,dx}{e^2} + \frac{(3ad}{}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} + \frac{\sqrt[3]{b}\,p\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{\sqrt[3]{a}\,e} - \frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^2} -$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} + \frac{\sqrt[3]{b}\,p\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{\sqrt[3]{a}\,e} - \frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^2} -$$

$$= -\frac{\sqrt{3}\,\sqrt[3]{b}\,p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt[3]{a}\,e} + \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} + \frac{\sqrt[3]{b}\,p\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{\sqrt[3]{a}\,e}$$

$$= -\frac{\sqrt{3}\,\sqrt[3]{b}\,p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt[3]{a}\,e} + \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} + \frac{\sqrt[3]{b}\,p\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{\sqrt[3]{a}\,e}$$

$$= -\frac{\sqrt{3}\,\sqrt[3]{b}\,p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt[3]{a}\,e} + \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} + \frac{\sqrt[3]{b}\,p\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{\sqrt[3]{a}\,e}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order

4 in optimal.
time = 0.10, size = 403, normalized size = 0.83

$$-\frac{3bp\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{b}{ax^3}\right)}{2aex^2}+\frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e}-\frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^2}-\frac{3dp\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^2}+\frac{dp\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{\sqrt[3]{a}\,d-\sqrt[3]{b}\,e}\right)\log(d+ex)}{e^2}+\frac{dp\log\left(-\frac{(-1)^{2/3}e\left(\sqrt[3]{b}-\sqrt[3]{-1}\,\sqrt[3]{a}\,x\right)}{\sqrt[3]{a}\,d-(-1)^{2/3}\sqrt[3]{b}\,e}\right)\log(d+ex)}{e^2}+\frac{dp\log\left(\frac{\sqrt[3]{-1}\,e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\,x\right)}{\sqrt[3]{a}\,d+\sqrt[3]{-1}\,\sqrt[3]{b}\,e}\right)\log(d+ex)}{e^2}-\frac{3dp\operatorname{Li}_2\left(\frac{d+ex}{d}\right)}{e^2}+\frac{dp\operatorname{Li}_2\left(\frac{\sqrt[3]{a}\,(d+ex)}{\sqrt[3]{a}\,d-\sqrt[3]{b}\,e}\right)}{e^2}+\frac{dp\operatorname{Li}_2\left(\frac{\sqrt[3]{a}\,(d+ex)}{\sqrt[3]{a}\,d+\sqrt[3]{-1}\,\sqrt[3]{b}\,e}\right)}{e^2}+\frac{dp\operatorname{Li}_2\left(\frac{\sqrt[3]{a}\,(d+ex)}{\sqrt[3]{a}\,d-(-1)^{2/3}\sqrt[3]{b}\,e}\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[c*(a + b/x^3)^p])/(d + e*x),x]
```

```
[Out] (-3*b*p*Hypergeometric2F1[2/3, 1, 5/3, -(b/(a*x^3))])/(2*a*e*x^2) + (x*Log[
c*(a + b/x^3)^p])/e - (d*Log[c*(a + b/x^3)^p]*Log[d + e*x])/e^2 - (3*d*p*Lo
g[-((e*x)/d)]*Log[d + e*x])/e^2 + (d*p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(
1/3)*d - b^(1/3)*e))]*Log[d + e*x])/e^2 + (d*p*Log[-(((-1)^(2/3)*e*(b^(1/3)
 - (-1)^(1/3)*a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x]
)/e^2 + (d*p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d
 + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/e^2 - (3*d*p*PolyLog[2, (d + e*x)/d
])/e^2 + (d*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/e^2
+ (d*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/
e^2 + (d*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e
)])/e^2
```

Maple [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{x\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(a+b/x^3)^p)/(e*x+d),x)
```

```
[Out] int(x*ln(c*(a+b/x^3)^p)/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(x*log((a + b/x^3)^p*c)/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x*log(c*((a*x^3 + b)/x^3)^p)/(x*e + d), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(a+b/x**3)**p)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(x*log((a + b/x^3)^p*c)/(x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\ln\left(c\,{\left(a+\frac{b}{x^3}\right)}^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(c*(a + b/x^3)^p))/(d + e*x),x)
```

```
[Out] int((x*log(c*(a + b/x^3)^p))/(d + e*x), x)
```

3.257 $\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx$

Optimal. Leaf size=344

$$\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e}+\frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e}-\frac{p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{\sqrt[3]{a}\,d-\sqrt[3]{b}\,e}\right)\log(d+ex)}{e}-\frac{p\log\left(-\frac{e(\ldots}{\ldots}\right.}{\ldots}$$

```
[Out] ln(c*(a+b/x^3)^p)*ln(e*x+d)/e+3*p*ln(-e*x/d)*ln(e*x+d)/e-p*ln(-e*(b^(1/3)+a
^(1/3)*x)/(a^(1/3)*d-b^(1/3)*e))*ln(e*x+d)/e-p*ln(-e*((-1)^(2/3)*b^(1/3)+a^
(1/3)*x)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3)*e))*ln(e*x+d)/e-p*ln((-1)^(1/3)*e*(b
^(1/3)+(-1)^(2/3)*a^(1/3)*x)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))*ln(e*x+d)/e-
p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-b^(1/3)*e))/e-p*polylog(2,a^(1/3)*(e
*x+d)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))/e-p*polylog(2,a^(1/3)*(e*x+d)/(a^(1
/3)*d-(-1)^(2/3)*b^(1/3)*e))/e+3*p*polylog(2,1+e*x/d)/e
```

Rubi [A]

time = 0.28, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2512, 266, 2463, 2441, 2352, 2440, 2438}

$$-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}\,d-\sqrt[3]{b}\,e}\right)}{e}-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}\,d+\sqrt[3]{-1}\sqrt[3]{b}\,e}\right)}{e}-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}\,d-(-1)^{2/3}\sqrt[3]{b}\,e}\right)}{e}+\frac{3p\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e}+\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e}-\frac{p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{\sqrt[3]{a}\,d-\sqrt[3]{b}\,e}\right)}{e}-\frac{p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}\,x+(-1)^{2/3}\sqrt[3]{b}\right)}{\sqrt[3]{a}\,d-(-1)^{2/3}\sqrt[3]{b}\,e}\right)}{e}-\frac{p\log(d+ex)\log\left(\frac{\sqrt[3]{-1}\,e\left((-1)^{2/3}\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{\sqrt[3]{a}\,d+\sqrt[3]{-1}\sqrt[3]{b}\,e}\right)}{e}+\frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^3)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b/x^3)^p]*Log[d + e*x])/e + (3*p*Log[-((e*x)/d)]*Log[d + e*x])/
e - (p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*
x])/e - (p*Log[-((e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/
3)*b^(1/3)*e))]*Log[d + e*x])/e - (p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3
)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/e - (p*Poly
Log[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/e - (p*PolyLog[2, (a^(
1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/e - (p*PolyLog[2, (a^(
1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)])/e + (3*p*PolyLog[2, 1
+ (e*x)/d])/e
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e} + \frac{(3bp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x^3}\right)x^4}\,dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e} + \frac{(3bp)\int \left(\frac{\log(d+ex)}{bx} - \frac{ax^2\log(d+ex)}{b(b+ax^3)}\right)dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e} + \frac{(3p)\int \frac{\log(d+ex)}{x}\,dx}{e} - \frac{(3ap)\int \frac{x^2\log(d+ex)}{b+ax^3}\,dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e} + \frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - (3p)\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx -$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e} + \frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} + \frac{3p\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{e} - \frac{\left(\sqrt[3]{a}\right.}{}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e} + \frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{\sqrt[3]{a}\,d-\sqrt[3]{b}\,e}\right.}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e} + \frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{\sqrt[3]{a}\,d-\sqrt[3]{b}\,e}\right.}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e} + \frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{\sqrt[3]{a}\,d-\sqrt[3]{b}\,e}\right.}{e}$$

Mathematica [A]
time = 0.09, size = 350, normalized size = 1.02

$$\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e} + \frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{\sqrt[3]{a}\,d-\sqrt[3]{b}\,e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{(-1)^{2/3}e\left(\sqrt[3]{b}-\sqrt[3]{-1}\,\sqrt[3]{a}\,x\right)}{\sqrt[3]{a}\,d-(-1)^{2/3}\sqrt[3]{b}\,e}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{\sqrt[3]{-1}\,e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\,x\right)}{\sqrt[3]{a}\,d+\sqrt[3]{-1}\,\sqrt[3]{b}\,e}\right)\log(d+ex)}{e} + \frac{3p\operatorname{Li}_2\left(\frac{d+ex}{d}\right)}{e} - \frac{p\operatorname{Li}_2\left(\frac{\sqrt[3]{a}\,(d+ex)}{\sqrt[3]{a}\,d-\sqrt[3]{b}\,e}\right)}{e} - \frac{p\operatorname{Li}_2\left(\frac{\sqrt[3]{a}\,(d+ex)}{\sqrt[3]{a}\,d+\sqrt[3]{-1}\,\sqrt[3]{b}\,e}\right)}{e} - \frac{p\operatorname{Li}_2\left(\frac{\sqrt[3]{a}\,(d+ex)}{\sqrt[3]{a}\,d-(-1)^{2/3}\sqrt[3]{b}\,e}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^3)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b/x^3)^p]*Log[d + e*x])/e + (3*p*Log[-((e*x)/d)]*Log[d + e*x])/
e - (p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*
x])/e - (p*Log[-(((-1)^(2/3)*e*(b^(1/3) - (-1)^(1/3)*a^(1/3)*x))/(a^(1/3)*d
 - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x])/e - (p*Log[((-1)^(1/3)*e*(b^(1/3)
```

```
+ (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/
e + (3*p*PolyLog[2, (d + e*x)/d])/e - (p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^
(1/3)*d - b^(1/3)*e)])/e - (p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (
-1)^(1/3)*b^(1/3)*e)])/e - (p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (
-1)^(2/3)*b^(1/3)*e)])/e
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^3)^p)/(e*x+d),x)
[Out] int(ln(c*(a+b/x^3)^p)/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(log((a + b/x^3)^p*c)/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="fricas")
[Out] integral(log(c*((a*x^3 + b)/x^3)^p)/(x*e + d), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**3)**p)/(e*x+d),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x^3)^p*c)/(x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x^3)^p)/(d + e*x),x)
```

```
[Out] int(log(c*(a + b/x^3)^p)/(d + e*x), x)
```

3.258 $\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x(d+ex)}\,dx$

Optimal. Leaf size=388

$$-\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d}-\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d}-\frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d}+\frac{p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}\right.}{\sqrt[3]{a}\,d-\sqrt[3]{b}}\right.}{d}$$

```
[Out] -1/3*ln(c*(a+b/x^3)^p)*ln(-b/a/x^3)/d-ln(c*(a+b/x^3)^p)*ln(e*x+d)/d-3*p*ln(
-e*x/d)*ln(e*x+d)/d+p*ln(-e*(b^(1/3)+a^(1/3)*x)/(a^(1/3)*d-b^(1/3)*e))*ln(e
*x+d)/d+p*ln(-e*((-1)^(2/3)*b^(1/3)+a^(1/3)*x)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3
)*e))*ln(e*x+d)/d+p*ln((-1)^(1/3)*e*(b^(1/3)+(-1)^(2/3)*a^(1/3)*x)/(a^(1/3)
*d+(-1)^(1/3)*b^(1/3)*e))*ln(e*x+d)/d-1/3*p*polylog(2,1+b/a/x^3)/d+p*polylo
g(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-b^(1/3)*e))/d+p*polylog(2,a^(1/3)*(e*x+d)/(a
^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))/d+p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-(-
1)^(2/3)*b^(1/3)*e))/d-3*p*polylog(2,1+e*x/d)/d
```

Rubi [A]
time = 0.37, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2516, 2504, 2441, 2352, 2512, 266, 2463, 2440, 2438}

$$\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d}+\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d}+\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d}-\frac{p\,\text{PolyLog}\left(2,\frac{b}{ax^3}+1\right)}{3d}-\frac{3p\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d}-\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d}-\frac{\log\left(-\frac{b}{ax^3}\right)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3d}+\frac{p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d}+\frac{p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}x+(-1)^{2/3}\sqrt[3]{b}\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d}+\frac{p\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left((-1)^{2/3}\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d}-\frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^3)^p]/(x*(d + e*x)),x]
```

```
[Out] -1/3*(Log[c*(a + b/x^3)^p]*Log[-(b/(a*x^3))])/d - (Log[c*(a + b/x^3)^p]*Log
[d + e*x])/d - (3*p*Log[-((e*x)/d)]*Log[d + e*x])/d + (p*Log[-((e*(b^(1/3)
+ a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*x])/d + (p*Log[-((e*((-1)
^(2/3)*b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e
*x])/d + (p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d
+ (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/d - (p*PolyLog[2, 1 + b/(a*x^3)])/(3
*d) + (p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/d + (p*Po
lyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/d + (p*Po
lyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)])/d - (3*p*
PolyLog[2, 1 + (e*x)/d])/d
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x(d+ex)}\,dx = \int \left(\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} - \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d(d+ex)}\right)dx$$

$$= \frac{\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x}\,dx}{d} - \frac{e\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} - \frac{\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}\,dx, x, \frac{1}{x^3}\right)}{3d} - \frac{(3bp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x^3}\right)x^4}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} + \frac{(bp)\text{Subst}\left(\int \frac{\log\left(-\right.}{a+b}\right.}{3d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} - \frac{p\text{Li}_2\left(1+\frac{b}{ax^3}\right)}{3d} - \frac{(3}{}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} - \frac{3p\log\left(-\frac{ex}{d}\right)\log(d}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} - \frac{3p\log\left(-\frac{ex}{d}\right)\log(d}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} - \frac{3p\log\left(-\frac{ex}{d}\right)\log(d}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} - \frac{3p\log\left(-\frac{ex}{d}\right)\log(d}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} - \frac{3p\log\left(-\frac{ex}{d}\right)\log(d}{d}$$

Mathematica [A]
time = 0.09, size = 395, normalized size = 1.02

$$-\frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right) \log \left(-\frac{b}{a x^3}\right)}{3 d}-\frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right) \log (d+e x)}{d}-\frac{3 p \log \left(-\frac{e x}{d}\right) \log (d+e x)}{d}+\frac{p \log \left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a} x\right)}{\sqrt[3]{a} d-\sqrt[3]{b} e}\right) \log (d+e x)}{d}+\frac{p \log \left(-\frac{(-1)^{2/3} e\left(\sqrt[3]{b}-\sqrt[3]{-1} \sqrt[3]{a} x\right)}{\sqrt[3]{a} d-(-1)^{2/3} \sqrt[3]{b} e}\right) \log (d+e x)}{d}+\frac{p \log \left(\frac{\sqrt[3]{-1} e\left(\sqrt[3]{b}+(-1)^{2/3} \sqrt[3]{a} x\right)}{\sqrt[3]{a} d+\sqrt[3]{-1} \sqrt[3]{b} e}\right) \log (d+e x)}{d}-\frac{p \operatorname{Li}_2\left(\frac{a+\frac{b}{x^3}}{a}\right)}{3 d}-\frac{3 p \operatorname{Li}_2\left(\frac{d+e x}{d}\right)}{d}+\frac{p \operatorname{Li}_2\left(\frac{\sqrt[3]{a}(d+e x)}{\sqrt[3]{a} d-\sqrt[3]{b} e}\right)}{d}+\frac{p \operatorname{Li}_2\left(\frac{\sqrt[3]{a}(d+e x)}{\sqrt[3]{a} d+\sqrt[3]{-1} \sqrt[3]{b} e}\right)}{d}+\frac{p \operatorname{Li}_2\left(\frac{\sqrt[3]{a}(d+e x)}{\sqrt[3]{a} d-(-1)^{2/3} \sqrt[3]{b} e}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^3)^p]/(x*(d + e*x)),x]
```

```
[Out] -1/3*(Log[c*(a + b/x^3)^p]*Log[-(b/(a*x^3))])/d - (Log[c*(a + b/x^3)^p]*Log
[d + e*x])/d - (3*p*Log[-((e*x)/d)]*Log[d + e*x])/d + (p*Log[-((e*(b^(1/3)
+ a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*x])/d + (p*Log[-(((-1)^(2
/3)*e*(b^(1/3) - (-1)^(1/3)*a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))
]*Log[d + e*x])/d + (p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/
(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/d - (p*PolyLog[2, (a + b/
x^3)/a])/(3*d) - (3*p*PolyLog[2, (d + e*x)/d])/d + (p*PolyLog[2, (a^(1/3)*(
d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/d + (p*PolyLog[2, (a^(1/3)*(d + e*x))/(
a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/d + (p*PolyLog[2, (a^(1/3)*(d + e*x))/(
a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)])/d
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x\,(e x+d)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^3)^p)/x/(e*x+d),x)
```

```
[Out] int(ln(c*(a+b/x^3)^p)/x/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((a + b/x^3)^p*c)/((x*e + d)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x^3 + b)/x^3)^p)/(x^2*e + d*x), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**3)**p)/x/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x^3)^p*c)/((x*e + d)*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x^3)^p)/(x*(d + e*x)),x)
```

```
[Out] int(log(c*(a + b/x^3)^p)/(x*(d + e*x)), x)
```

3.259 $\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2(d+ex)}\,dx$

Optimal. Leaf size=557

$$\frac{3p}{dx}-\frac{\sqrt{3}\,\sqrt[3]{a}\,p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt[3]{b}\,d}-\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx}+\frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^2}-\frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{b}+\sqrt[3]{\ldots}\right.}{\sqrt[3]{b}\,d}$$

```
[Out] 3*p/d/x-ln(c*(a+b/x^3)^p)/d/x+1/3*e*ln(c*(a+b/x^3)^p)*ln(-b/a/x^3)/d^2-a^(1
/3)*p*ln(b^(1/3)+a^(1/3)*x)/b^(1/3)/d+e*ln(c*(a+b/x^3)^p)*ln(e*x+d)/d^2+3*e
*p*ln(-e*x/d)*ln(e*x+d)/d^2-e*p*ln(-e*(b^(1/3)+a^(1/3)*x)/(a^(1/3)*d-b^(1/3
)*e))*ln(e*x+d)/d^2-e*p*ln(-e*((-1)^(2/3)*b^(1/3)+a^(1/3)*x)/(a^(1/3)*d-(-1
)^(2/3)*b^(1/3)*e))*ln(e*x+d)/d^2-e*p*ln((-1)^(1/3)*e*(b^(1/3)+(-1)^(2/3)*a
^(1/3)*x)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))*ln(e*x+d)/d^2+1/2*a^(1/3)*p*ln(
b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/b^(1/3)/d+1/3*e*p*polylog(2,1+b/a/x^
3)/d^2-e*p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-b^(1/3)*e))/d^2-e*p*polylog
(2,a^(1/3)*(e*x+d)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))/d^2-e*p*polylog(2,a^(1
/3)*(e*x+d)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3)*e))/d^2+3*e*p*polylog(2,1+e*x/d)/
d^2-a^(1/3)*p*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)/b^(1/3)*3^(1/2))*3^(1/2)/b^(
1/3)/d
```

Rubi [A]
time = 0.46, antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 18, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {2516, 2505, 269, 331, 298, 31, 648, 631, 210, 642, 2504, 2441, 2352, 2512, 266, 2463, 2440, 2438}

$$\frac{ep\text{PolyLog}\left(2,\frac{b}{ax^3}+1\right)}{3d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d^2}-\frac{ep\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d^2}+\frac{3ep\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d^2}+\frac{\sqrt[3]{a}\,p\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{2\sqrt[3]{b}d}-\frac{\sqrt{3}\sqrt[3]{a}\,p\text{ArcTan}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{b}d}+\frac{e\log\left(-\frac{b}{ax^3}\right)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3d^2}+\frac{e\log(d+ex)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2}-\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx}-\frac{ep\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d^2}-\frac{ep\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}x+(-1)^{2/3}\sqrt[3]{b}\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d^2}-\frac{ep\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left((-1)^{2/3}\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d^2}-\frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{b}d}+\frac{3ep\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^2}+\frac{3p}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^3)^p]/(x^2*(d + e*x)),x]
```

```
[Out] (3*p)/(d*x) - (Sqrt[3]*a^(1/3)*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^
(1/3))])/(b^(1/3)*d) - Log[c*(a + b/x^3)^p]/(d*x) + (e*Log[c*(a + b/x^3)^p]
*Log[-(b/(a*x^3))])/(3*d^2) - (a^(1/3)*p*Log[b^(1/3) + a^(1/3)*x])/(b^(1/3)
*d) + (e*Log[c*(a + b/x^3)^p]*Log[d + e*x])/d^2 + (3*e*p*Log[-((e*x)/d)]*Lo
g[d + e*x])/d^2 - (e*p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)
*e))]*Log[d + e*x])/d^2 - (e*p*Log[-((e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(
a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x])/d^2 - (e*p*Log[((-1)^(1/3
)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*L
og[d + e*x])/d^2 + (a^(1/3)*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2
])/(2*b^(1/3)*d) + (e*p*PolyLog[2, 1 + b/(a*x^3)])/(3*d^2) - (e*p*PolyLog[2
, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (a^(1
```

```
/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (
a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)])/d^2 + (3*e*p*PolyLo
g[2, 1 + (e*x)/d])/d^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2(d+ex)}\,dx = \int \left(\frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx^2} - \frac{e\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x} + \frac{e^2\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2(d+ex)}\right)dx$$

$$= \frac{\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2}dx}{d} - \frac{e\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x}dx}{d^2} + \frac{e^2\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}dx}{d^2}$$

$$= -\frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} + \frac{e\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d^2} + \frac{e\mathrm{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}dx, x,\right.}{3d^2}$$

$$= -\frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} + \frac{e\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^2} + \frac{e\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+e}{d^2}$$

$$= \frac{3p}{dx} - \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} + \frac{e\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^2} + \frac{e\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d}{d^2}$$

$$= \frac{3p}{dx} - \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} + \frac{e\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^2} + \frac{e\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d}{d^2}$$

$$= \frac{3p}{dx} - \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} + \frac{e\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^2} - \frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right.}{\sqrt[3]{b}\,d}$$

$$= \frac{3p}{dx} - \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} + \frac{e\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^2} - \frac{\sqrt[3]{a}\,p\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right.}{\sqrt[3]{b}\,d}$$

$$= \frac{3p}{dx} - \frac{\sqrt{3}\,\sqrt[3]{a}\,p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt[3]{b}\,d} - \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} + \frac{e\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)\log}{3d^2}$$

$$= \frac{3p}{dx} - \frac{\sqrt{3}\,\sqrt[3]{a}\,p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt[3]{b}\,d} - \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} + \frac{e\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)\log}{3d^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.16, size = 429, normalized size = 0.77

$$\frac{9bdp\,{}_2F_1\left(1,\frac{4}{3};\frac{7}{3};-\frac{b}{ax^3}\right) + 4ax^3\left(-3d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right) + ex\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right) + 3ex\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex) + 9epx\log\left(-\frac{ex}{d}\right)\log(d+ex) - 3epx\log\left(\frac{e\left(\sqrt[3]{-1}\,\sqrt[3]{b}-\sqrt[3]{a}\,x\right)}{\sqrt[3]{a}\,d+\sqrt[3]{-1}\,\sqrt[3]{b}\,e}\right)\log(d+ex) - 3epx\log\left(\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{-\sqrt[3]{a}\,d+\sqrt[3]{b}\,e}\right)\log(d+ex) - 3epx\log\left(\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{-\sqrt[3]{a}\,d+(-1)^{2/3}\sqrt[3]{b}\,e}\right)\log(d+ex) + epx\mathrm{Li}_2\left(1+\frac{b}{ax^3}\right) - 3epx\mathrm{Li}_2\left(\frac{\sqrt[3]{a}\,(d+ex)}{\sqrt[3]{a}\,d-\sqrt[3]{b}\,e}\right) - 3epx\mathrm{Li}_2\left(\frac{\sqrt[3]{a}\,(d+ex)}{\sqrt[3]{a}\,d+\sqrt[3]{-1}\,\sqrt[3]{b}\,e}\right) - 3epx\mathrm{Li}_2\left(\frac{\sqrt[3]{a}\,(d+ex)}{\sqrt[3]{a}\,d-(-1)^{2/3}\sqrt[3]{b}\,e}\right) + 9epx\mathrm{Li}_2\left(1+\frac{ex}{d}\right)\right)}{12ad^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^3)^p]/(x^2*(d + e*x)),x]
```

```
[Out] (9*b*d*p*Hypergeometric2F1[1, 4/3, 7/3, -(b/(a*x^3))] + 4*a*x^3*(-3*d*Log[c
*(a + b/x^3)^p] + e*x*Log[c*(a + b/x^3)^p]*Log[-(b/(a*x^3))] + 3*e*x*Log[c*
(a + b/x^3)^p]*Log[d + e*x] + 9*e*p*x*Log[-((e*x)/d)]*Log[d + e*x] - 3*e*p*
x*Log[(e*((-1)^(1/3)*b^(1/3) - a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*
e)]*Log[d + e*x] - 3*e*p*x*Log[(e*(b^(1/3) + a^(1/3)*x))/(-(a^(1/3)*d) + b^
(1/3)*e)]*Log[d + e*x] - 3*e*p*x*Log[(e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(
-(a^(1/3)*d) + (-1)^(2/3)*b^(1/3)*e)]*Log[d + e*x] + e*p*x*PolyLog[2, 1 + b
/(a*x^3)] - 3*e*p*x*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)]
 - 3*e*p*x*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e
)] - 3*e*p*x*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)
*e)] + 9*e*p*x*PolyLog[2, 1 + (e*x)/d]))/(12*a*d^2*x^4)
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x^{3}}\right)^{p}\right)}{x^{2}\left(ex+d\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^3)^p)/x^2/(e*x+d),x)
```

```
[Out] int(ln(c*(a+b/x^3)^p)/x^2/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((a + b/x^3)^p*c)/((x*e + d)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x^3 + b)/x^3)^p)/(x^3*e + d*x^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**3)**p)/x**2/(e*x+d),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/x^2/(e*x+d),x, algorithm="giac")
[Out] integrate(log((a + b/x^3)^p*c)/((x*e + d)*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x^3)^p)/(x^2*(d + e*x)),x)
[Out] int(log(c*(a + b/x^3)^p)/(x^2*(d + e*x)), x)
```

3.260 $\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^3(d+ex)}\,dx$

Optimal. Leaf size=737

$$\frac{3p}{4dx^2}-\frac{3ep}{d^2x}-\frac{\sqrt{3}\,a^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{2b^{2/3}d}+\frac{\sqrt{3}\,\sqrt[3]{a}\,ep\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt[3]{b}\,d^2}-\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2}+\frac{e\log\left(c\left(a\right.\right.}{d^2}$$

```
[Out] 3/4*p/d/x^2-3*e*p/d^2/x-1/2*ln(c*(a+b/x^3)^p)/d/x^2+e*ln(c*(a+b/x^3)^p)/d^2
/x-1/3*e^2*ln(c*(a+b/x^3)^p)*ln(-b/a/x^3)/d^3+1/2*a^(2/3)*p*ln(b^(1/3)+a^(1
/3)*x)/b^(2/3)/d+a^(1/3)*e*p*ln(b^(1/3)+a^(1/3)*x)/b^(1/3)/d^2-e^2*ln(c*(a+
b/x^3)^p)*ln(e*x+d)/d^3-3*e^2*p*ln(-e*x/d)*ln(e*x+d)/d^3+e^2*p*ln(-e*(b^(1/
3)+a^(1/3)*x)/(a^(1/3)*d-b^(1/3)*e))*ln(e*x+d)/d^3+e^2*p*ln(-e*((-1)^(2/3)*
b^(1/3)+a^(1/3)*x)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3)*e))*ln(e*x+d)/d^3+e^2*p*ln
((-1)^(1/3)*e*(b^(1/3)+(-1)^(2/3)*a^(1/3)*x)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*
e))*ln(e*x+d)/d^3-1/4*a^(2/3)*p*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/b
^(2/3)/d-1/2*a^(1/3)*e*p*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/b^(1/3)/
d^2-1/3*e^2*p*polylog(2,1+b/a/x^3)/d^3+e^2*p*polylog(2,a^(1/3)*(e*x+d)/(a^(
1/3)*d-b^(1/3)*e))/d^3+e^2*p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d+(-1)^(1/3
)*b^(1/3)*e))/d^3+e^2*p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-(-1)^(2/3)*b^(
1/3)*e))/d^3-3*e^2*p*polylog(2,1+e*x/d)/d^3-1/2*a^(2/3)*p*arctan(1/3*(b^(1/
3)-2*a^(1/3)*x)/b^(1/3)*3^(1/2))*3^(1/2)/b^(2/3)/d+a^(1/3)*e*p*arctan(1/3*(
b^(1/3)-2*a^(1/3)*x)/b^(1/3)*3^(1/2))*3^(1/2)/b^(1/3)/d^2
```

Rubi [A]

time = 0.56, antiderivative size = 737, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 19, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.826$, Rules used = {2516, 2505, 269, 331, 206, 31, 648, 631, 210, 642, 298, 2504, 2441, 2352, 2512, 266, 2463, 2440, 2438}

$$-\frac{e^2p\text{PolyLog}\left(2,\frac{b}{ax^3}+1\right)}{3d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d^3}+\frac{e^2p\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d^3}-\frac{3e^2p\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d^3}-\frac{\sqrt{3}\,a^{2/3}p\text{ArcTan}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{2b^{2/3}d}-\frac{\sqrt[3]{a}\,ep\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{2\sqrt[3]{b}d^2}-\frac{a^{2/3}p\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{4b^{2/3}d}+\frac{a^{2/3}p\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{2b^{2/3}d}+\frac{\sqrt{3}\sqrt[3]{a}\,ep\text{ArcTan}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{b}d^2}-\frac{e^2\log\left(-\frac{b}{ax^3}\right)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3d^3}-\frac{e^2\log(d+ex)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^3}+\frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x}-\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2}+\frac{e^2p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d^3}+\frac{e^2p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}x+(-1)^{2/3}\sqrt[3]{b}\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d^3}+\frac{e^2p\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left((-1)^{2/3}\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d^3}+\frac{\sqrt[3]{a}\,ep\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{b}d^2}-\frac{3e^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^3}-\frac{3ep}{d^2x}+\frac{3p}{4dx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b/x^3)^p]/(x^3*(d + e*x)),x]
```

```
[Out] (3*p)/(4*d*x^2) - (3*e*p)/(d^2*x) - (Sqrt[3]*a^(2/3)*p*ArcTan[(b^(1/3) - 2*
a^(1/3)*x)/(Sqrt[3]*b^(1/3))])/(2*b^(2/3)*d) + (Sqrt[3]*a^(1/3)*e*p*ArcTan[
(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))])/(b^(1/3)*d^2) - Log[c*(a + b/x^
3)^p]/(2*d*x^2) + (e*Log[c*(a + b/x^3)^p])/(d^2*x) - (e^2*Log[c*(a + b/x^3)
^p]*Log[-(b/(a*x^3))])/(3*d^3) + (a^(2/3)*p*Log[b^(1/3) + a^(1/3)*x])/(2*b^
(2/3)*d) + (a^(1/3)*e*p*Log[b^(1/3) + a^(1/3)*x])/(b^(1/3)*d^2) - (e^2*Log[
c*(a + b/x^3)^p]*Log[d + e*x])/d^3 - (3*e^2*p*Log[-((e*x)/d)]*Log[d + e*x])
/d^3 + (e^2*p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log
```

```
[d + e*x])/d^3 + (e^2*p*Log[-((e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(a^(1/3)
*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x])/d^3 + (e^2*p*Log[((-1)^(1/3)*e*(
b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d
+ e*x])/d^3 - (a^(2/3)*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(4
*b^(2/3)*d) - (a^(1/3)*e*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/
(2*b^(1/3)*d^2) - (e^2*p*PolyLog[2, 1 + b/(a*x^3)])/(3*d^3) + (e^2*p*PolyLo
g[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/d^3 + (e^2*p*PolyLog[2,
(a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/d^3 + (e^2*p*PolyL
og[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)])/d^3 - (3*e^2
*p*PolyLog[2, 1 + (e*x)/d])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
```

```
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\int \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^3(d+ex)}\,dx = \int \left(\frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx^3} - \frac{e\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x^2} + \frac{e^2\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^3x} - \frac{e^3\log \left(c\left(a+\frac{b}{x^3}\right.\right.}{d^3(d+ex)}\right.$$

$$= \frac{\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^3}\,dx}{d} - \frac{e\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2}\,dx}{d^2} + \frac{e^2\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x}\,dx}{d^3} - \frac{e^3\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^{p}\right.}{d+ex}}{d^3}$$

$$= -\frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2} + \frac{e\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x} - \frac{e^2\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d^3} - \frac{e^2\text{Subs}}{}$$

$$= -\frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2} + \frac{e\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x} - \frac{e^2\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^3} - \frac{e^2\log\left(\right.}{}$$

$$= \frac{3p}{4dx^2} - \frac{3ep}{d^2x} - \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2} + \frac{e\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x} - \frac{e^2\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right.}{3d^3}$$

$$= \frac{3p}{4dx^2} - \frac{3ep}{d^2x} - \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2} + \frac{e\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x} - \frac{e^2\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right.}{3d^3}$$

$$= \frac{3p}{4dx^2} - \frac{3ep}{d^2x} - \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2} + \frac{e\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x} - \frac{e^2\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right.}{3d^3}$$

$$= \frac{3p}{4dx^2} - \frac{3ep}{d^2x} - \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2} + \frac{e\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x} - \frac{e^2\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right.}{3d^3}$$

$$= \frac{3p}{4dx^2} - \frac{3ep}{d^2x} - \frac{\sqrt{3}\,a^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{2b^{2/3}d} + \frac{\sqrt{3}\,\sqrt[3]{a}\,ep\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt[3]{b}\,d^2}$$

$$= \frac{3p}{4dx^2} - \frac{3ep}{d^2x} - \frac{\sqrt{3}\,a^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{2b^{2/3}d} + \frac{\sqrt{3}\,\sqrt[3]{a}\,ep\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt[3]{b}\,d^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.21, size = 520, normalized size = 0.71

$$\frac{-45bdepx\,{}_2F_1\left(1,\frac{4}{3};\frac{7}{3};-\frac{b}{ax^3}\right)+18bd^2p\,{}_2F_1\left(1,\frac{5}{3};\frac{8}{3};-\frac{b}{ax^3}\right)-10ax^3\left(3d^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)-6dex\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)+2e^2x^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)+6e^2x^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)+18e^2px^2\log\left(-\frac{ex}{d}\right)\log(d+ex)-6e^2px^2\log\left(\frac{e\left(\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\,x\right)}{\sqrt[3]{a}\,d+\sqrt[3]{-1}\sqrt[3]{b}\,e}\right)\log(d+ex)-6e^2px^2\log\left(\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{-\sqrt[3]{a}\,d+\sqrt[3]{b}\,e}\right)\log(d+ex)-6e^2px^2\log\left(\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{-\sqrt[3]{a}\,d+(-1)^{2/3}\sqrt[3]{b}\,e}\right)\log(d+ex)+2e^2px^2\text{Li}_2\left(1+\frac{b}{ax^3}\right)-6e^2px^2\text{Li}_2\left(\frac{\sqrt[3]{a}\,(d+ex)}{\sqrt[3]{a}\,d-\sqrt[3]{b}\,e}\right)-6e^2px^2\text{Li}_2\left(\frac{\sqrt[3]{a}\,(d+ex)}{\sqrt[3]{a}\,d+\sqrt[3]{-1}\sqrt[3]{b}\,e}\right)-6e^2px^2\text{Li}_2\left(\frac{\sqrt[3]{a}\,(d+ex)}{\sqrt[3]{a}\,d-(-1)^{2/3}\sqrt[3]{b}\,e}\right)+18e^2px^2\text{Li}_2\left(1+\frac{ex}{d}\right)\right)}{60ad^3x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b/x^3)^p]/(x^3*(d + e*x)),x]
```

```
[Out] (-45*b*d*e*p*x*Hypergeometric2F1[1, 4/3, 7/3, -(b/(a*x^3))] + 18*b*d^2*p*Hy
pergeometric2F1[1, 5/3, 8/3, -(b/(a*x^3))] - 10*a*x^3*(3*d^2*Log[c*(a + b/x
^3)^p] - 6*d*e*x*Log[c*(a + b/x^3)^p] + 2*e^2*x^2*Log[c*(a + b/x^3)^p]*Log[
-(b/(a*x^3))] + 6*e^2*x^2*Log[c*(a + b/x^3)^p]*Log[d + e*x] + 18*e^2*p*x^2*
Log[-((e*x)/d)]*Log[d + e*x] - 6*e^2*p*x^2*Log[(e*((-1)^(1/3)*b^(1/3) - a^(
1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x] - 6*e^2*p*x^2*Log
[(e*(b^(1/3) + a^(1/3)*x))/(-(a^(1/3)*d) + b^(1/3)*e)]*Log[d + e*x] - 6*e^2
*p*x^2*Log[(e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(-(a^(1/3)*d) + (-1)^(2/3)*
b^(1/3)*e)]*Log[d + e*x] + 2*e^2*p*x^2*PolyLog[2, 1 + b/(a*x^3)] - 6*e^2*p*
x^2*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)] - 6*e^2*p*x^2*P
olyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)] - 6*e^2*p
*x^2*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)] + 1
8*e^2*p*x^2*PolyLog[2, 1 + (e*x)/d]))/(60*a*d^3*x^5)
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^3\left(ex+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a+b/x^3)^p)/x^3/(e*x+d),x)
```

```
[Out] int(ln(c*(a+b/x^3)^p)/x^3/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((a + b/x^3)^p*c)/((x*e + d)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x^3 + b)/x^3)^p)/(x^4*e + d*x^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a+b/x**3)**p)/x**3/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a+b/x^3)^p)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x^3)^p*c)/((x*e + d)*x^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(a+\frac{b}{x^3}\right)}^p\right)}{x^3\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b/x^3)^p)/(x^3*(d + e*x)),x)
```

```
[Out] int(log(c*(a + b/x^3)^p)/(x^3*(d + e*x)), x)
```

3.261 $\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{f+gx^2}\,dx$

Optimal. Leaf size=749

$$\frac{3p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}}-\frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\left(i\sqrt[3]{e}\,\sqrt{f}+\sqrt[3]{d}\,\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}\right)}{\sqrt{f}\,\sqrt{g}}-\frac{p\tan}{}$$

```
[Out] arctan(x*g^(1/2)/f^(1/2))*ln(c*(e*x^3+d)^p)/f^(1/2)/g^(1/2)+3*p*arctan(x*g^
(1/2)/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)-p*arctan
(x*g^(1/2)/f^(1/2))*ln(2*(d^(1/3)+e^(1/3)*x)*f^(1/2)*g^(1/2)/(I*e^(1/3)*f^(
1/2)+d^(1/3)*g^(1/2))/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)-p*arctan(x*g^(
1/2)/f^(1/2))*ln(-2*I*((-1)^(2/3)*d^(1/3)+e^(1/3)*x)*f^(1/2)*g^(1/2)/(e^(1/
3)*f^(1/2)+(-1)^(1/6)*d^(1/3)*g^(1/2))/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/
2)-p*arctan(x*g^(1/2)/f^(1/2))*ln(2*(-1)^(5/6)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*
x)*f^(1/2)*g^(1/2)/(e^(1/3)*f^(1/2)+(-1)^(5/6)*d^(1/3)*g^(1/2))/(f^(1/2)-I*
x*g^(1/2)))/f^(1/2)/g^(1/2)-3/2*I*p*polylog(2,1-2*f^(1/2)/(f^(1/2)-I*x*g^(1
/2)))/f^(1/2)/g^(1/2)+1/2*I*p*polylog(2,1-2*(d^(1/3)+e^(1/3)*x)*f^(1/2)*g^(
1/2)/(I*e^(1/3)*f^(1/2)+d^(1/3)*g^(1/2))/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(
1/2)+1/2*I*p*polylog(2,1+2*I*((-1)^(2/3)*d^(1/3)+e^(1/3)*x)*f^(1/2)*g^(1/2)
/(e^(1/3)*f^(1/2)+(-1)^(1/6)*d^(1/3)*g^(1/2))/(f^(1/2)-I*x*g^(1/2)))/f^(1/2
)/g^(1/2)+1/2*I*p*polylog(2,1-2*(-1)^(5/6)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)*f
^(1/2)*g^(1/2)/(e^(1/3)*f^(1/2)+(-1)^(5/6)*d^(1/3)*g^(1/2))/(f^(1/2)-I*x*g^
(1/2)))/f^(1/2)/g^(1/2)
```

Rubi [A]

time = 0.75, antiderivative size = 749, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {211, 2520, 12, 266, 6857, 4966, 2449, 2352, 2497}

$$\frac{ip\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(\sqrt[3]{d}\,\sqrt{g}+i\sqrt[3]{e}\,\sqrt{f}\right)}\right)}{2\sqrt{f}\,\sqrt{g}}+\frac{ip\,\text{PolyLog}\left(2,1+\frac{2i\sqrt{f}\,\sqrt{g}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(\sqrt[6]{-1}\,\sqrt[3]{d}\,\sqrt{g}+\sqrt[3]{e}\,\sqrt{f}\right)}\right)}{2\sqrt{f}\,\sqrt{g}}+\frac{ip\,\text{PolyLog}\left(2,1-\frac{2(-1)^{5/6}\sqrt{f}\,\sqrt{g}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left((-1)^{5/6}\sqrt[3]{d}\,\sqrt{g}+\sqrt[3]{e}\,\sqrt{f}\right)}\right)}{2\sqrt{f}\,\sqrt{g}}-\frac{3ip\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{2\sqrt{f}\,\sqrt{g}}+\frac{\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt{f}\,\sqrt{g}}-\frac{p\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(\sqrt[3]{d}\,\sqrt{g}+i\sqrt[3]{e}\,\sqrt{f}\right)}\right)}{\sqrt{f}\,\sqrt{g}}-\frac{p\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2i\sqrt{f}\,\sqrt{g}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(\sqrt[6]{-1}\,\sqrt[3]{d}\,\sqrt{g}+\sqrt[3]{e}\,\sqrt{f}\right)}\right)}{\sqrt{f}\,\sqrt{g}}-\frac{p\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2(-1)^{5/6}\sqrt{f}\,\sqrt{g}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left((-1)^{5/6}\sqrt[3]{d}\,\sqrt{g}+\sqrt[3]{e}\,\sqrt{f}\right)}\right)}{\sqrt{f}\,\sqrt{g}}+\frac{3p\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^3)^p]/(f + g*x^2),x]
```

```
[Out] (3*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/
(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(
d^(1/3) + e^(1/3)*x))/((I*e^(1/3)*Sqrt[f] + d^(1/3)*Sqrt[g])*(Sqrt[f] - I*S
qrt[g]*x))])/(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[((-2*I)
*Sqrt[f]*Sqrt[g]*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/((e^(1/3)*Sqrt[f] + (-1)
^(1/6)*d^(1/3)*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) - (p*A
rcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*(-1)^(5/6)*Sqrt[f]*Sqrt[g]*(d^(1/3) + (-1
)^(2/3)*e^(1/3)*x))/((e^(1/3)*Sqrt[f] + (-1)^(5/6)*d^(1/3)*Sqrt[g])*(Sqrt[f
```

```
] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*
(d + e*x^3)^p])/(Sqrt[f]*Sqrt[g]) - (((3*I)/2)*p*PolyLog[2, 1 - (2*Sqrt[f])
/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 - (2*S
qrt[f]*Sqrt[g]*(d^(1/3) + e^(1/3)*x))/((I*e^(1/3)*Sqrt[f] + d^(1/3)*Sqrt[g]
)*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 + ((
2*I)*Sqrt[f]*Sqrt[g]*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/((e^(1/3)*Sqrt[f] +
(-1)^(1/6)*d^(1/3)*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) +
((I/2)*p*PolyLog[2, 1 - (2*(-1)^(5/6)*Sqrt[f]*Sqrt[g]*(d^(1/3) + (-1)^(2/3)
*e^(1/3)*x))/((e^(1/3)*Sqrt[f] + (-1)^(5/6)*d^(1/3)*Sqrt[g])*(Sqrt[f] - I*S
qrt[g]*x))])/(Sqrt[f]*Sqrt[g])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
```

```
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\log\left(c(d+ex^3)^p\right)}{f+gx^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^3)^p\right)}{\sqrt{f}\,\sqrt{g}} - (3ep)\int \frac{x^2\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}\,\sqrt{g}\,(d+ex^3)}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^3)^p\right)}{\sqrt{f}\,\sqrt{g}} - \frac{(3ep)\int \frac{x^2\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{d+ex^3}\,dx}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^3)^p\right)}{\sqrt{f}\,\sqrt{g}} - \frac{(3ep)\int \left(\frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{3e^{2/3}\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}}{\sqrt{f}}\right.}{3e^{2/3}\left(-\sqrt[3]{-1}\,\sqrt[3]{d}\right.}\right.}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^3)^p\right)}{\sqrt{f}\,\sqrt{g}} - \frac{\left(\sqrt[3]{e}\,p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt[3]{d}+\sqrt[3]{e}\,x}\,dx}{\sqrt{f}\,\sqrt{g}} - \frac{\left(\sqrt[3]{e}\,p\right)\int \frac{\tan^{-1}}{-\sqrt[3]{-1}}}{\sqrt{f}}$$

$$= \frac{3p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\,\sqrt{g}\,(}{\left(i\sqrt[3]{e}\,\sqrt{f}+\sqrt[3]{d}\,\sqrt{\right.}}\right.}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{3p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\,\sqrt{g}\,(}{\left(i\sqrt[3]{e}\,\sqrt{f}+\sqrt[3]{d}\,\sqrt{\right.}}\right.}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{3p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\,\sqrt{g}\,(}{\left(i\sqrt[3]{e}\,\sqrt{f}+\sqrt[3]{d}\,\sqrt{\right.}}\right.}{\sqrt{f}\,\sqrt{g}}$$

<u>Mathematica</u> [A]
time = 0.41, size = 867, normalized size = 1.16

$$\frac{-p\log\left(\frac{\sqrt{g}\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\sqrt[3]{e}\,\sqrt{-f}+\sqrt[3]{d}\,\sqrt{g}}\right)\log\left(\sqrt{-f}-\sqrt{g}\,x\right)-p\log\left(\frac{\sqrt{g}\left(-\sqrt[3]{-1}\,\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\sqrt[3]{e}\,\sqrt{-f}-\sqrt[3]{-1}\,\sqrt[3]{d}\,\sqrt{g}}\right)\log\left(\sqrt{-f}-\sqrt{g}\,x\right)-p\log\left(\frac{\sqrt{g}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\sqrt[3]{e}\,\sqrt{-f}+(-1)^{2/3}\sqrt[3]{d}\,\sqrt{g}}\right)\log\left(\sqrt{-f}-\sqrt{g}\,x\right)+p\log\left(-\frac{\sqrt{g}\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\sqrt[3]{e}\,\sqrt{-f}-\sqrt[3]{d}\,\sqrt{g}}\right)\log\left(\sqrt{-f}+\sqrt{g}\,x\right)+p\log\left(\frac{\sqrt{g}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{-\sqrt[3]{e}\,\sqrt{-f}+(-1)^{2/3}\sqrt[3]{d}\,\sqrt{g}}\right)\log\left(\sqrt{-f}+\sqrt{g}\,x\right)+p\log\left(\frac{\sqrt[3]{-1}\,\sqrt{g}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}\,x\right)}{\sqrt[3]{e}\,\sqrt{-f}+\sqrt[3]{-1}\,\sqrt[3]{d}\,\sqrt{g}}\right)\log\left(\sqrt{-f}+\sqrt{g}\,x\right)+\log\left(\sqrt{-f}-\sqrt{g}\,x\right)\log\left(c(d+ex^3)^p\right)-\log\left(\sqrt{-f}+\sqrt{g}\,x\right)\log\left(c(d+ex^3)^p\right)-p\,\mathrm{Li}_2\left(\frac{\sqrt[3]{e}\left(\sqrt{-f}-\sqrt{g}\,x\right)}{\sqrt[3]{e}\,\sqrt{-f}+\sqrt[3]{d}\,\sqrt{g}}\right)-p\,\mathrm{Li}_2\left(\frac{\sqrt[3]{e}\left(\sqrt{-f}-\sqrt{g}\,x\right)}{\sqrt[3]{e}\,\sqrt{-f}-\sqrt[3]{-1}\,\sqrt[3]{d}\,\sqrt{g}}\right)-p\,\mathrm{Li}_2\left(\frac{\sqrt[3]{e}\left(\sqrt{-f}-\sqrt{g}\,x\right)}{\sqrt[3]{e}\,\sqrt{-f}+(-1)^{2/3}\sqrt[3]{d}\,\sqrt{g}}\right)+p\,\mathrm{Li}_2\left(\frac{\sqrt[3]{e}\left(\sqrt{-f}+\sqrt{g}\,x\right)}{\sqrt[3]{e}\,\sqrt{-f}-\sqrt[3]{d}\,\sqrt{g}}\right)+p\,\mathrm{Li}_2\left(\frac{\sqrt[3]{e}\left(\sqrt{-f}+\sqrt{g}\,x\right)}{\sqrt[3]{e}\,\sqrt{-f}+\sqrt[3]{-1}\,\sqrt[3]{d}\,\sqrt{g}}\right)+p\,\mathrm{Li}_2\left(\frac{\sqrt[3]{e}\left(\sqrt{-f}+\sqrt{g}\,x\right)}{\sqrt[3]{e}\,\sqrt{-f}-(-1)^{2/3}\sqrt[3]{d}\,\sqrt{g}}\right)}{2\sqrt{-f}\,\sqrt{g}}$$

Antiderivative was successfully verified.

[In] Integrate[Log[c*(d + e*x^3)^p]/(f + g*x^2),x]

[Out] (-(p*Log[(Sqrt[g]*(d^(1/3) + e^(1/3)*x))/(e^(1/3)*Sqrt[-f] + d^(1/3)*Sqrt[g
])]*Log[Sqrt[-f] - Sqrt[g]*x]) - p*Log[(Sqrt[g]*(-((-1)^(1/3)*d^(1/3)) + e^
(1/3)*x))/(e^(1/3)*Sqrt[-f] - (-1)^(1/3)*d^(1/3)*Sqrt[g])]*Log[Sqrt[-f] - S
qrt[g]*x] - p*Log[(Sqrt[g]*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/(e^(1/3)*Sqrt[
-f] + (-1)^(2/3)*d^(1/3)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*x] + p*Log[-((Sqr
t[g]*(d^(1/3) + e^(1/3)*x))/(e^(1/3)*Sqrt[-f] - d^(1/3)*Sqrt[g]))]*Log[Sqrt
[-f] + Sqrt[g]*x] + p*Log[(Sqrt[g]*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/(-(e^(
1/3)*Sqrt[-f]) + (-1)^(2/3)*d^(1/3)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*x] + p
*Log[((-1)^(1/3)*Sqrt[g]*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/(e^(1/3)*Sqrt[-f
] + (-1)^(1/3)*d^(1/3)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*x] + Log[Sqrt[-f] -
 Sqrt[g]*x]*Log[c*(d + e*x^3)^p] - Log[Sqrt[-f] + Sqrt[g]*x]*Log[c*(d + e*x
^3)^p] - p*PolyLog[2, (e^(1/3)*(Sqrt[-f] - Sqrt[g]*x))/(e^(1/3)*Sqrt[-f] +
d^(1/3)*Sqrt[g])] - p*PolyLog[2, (e^(1/3)*(Sqrt[-f] - Sqrt[g]*x))/(e^(1/3)*
Sqrt[-f] - (-1)^(1/3)*d^(1/3)*Sqrt[g])] - p*PolyLog[2, (e^(1/3)*(Sqrt[-f] -
 Sqrt[g]*x))/(e^(1/3)*Sqrt[-f] + (-1)^(2/3)*d^(1/3)*Sqrt[g])] + p*PolyLog[2
, (e^(1/3)*(Sqrt[-f] + Sqrt[g]*x))/(e^(1/3)*Sqrt[-f] - d^(1/3)*Sqrt[g])] +
p*PolyLog[2, (e^(1/3)*(Sqrt[-f] + Sqrt[g]*x))/(e^(1/3)*Sqrt[-f] + (-1)^(1/3
)*d^(1/3)*Sqrt[g])] + p*PolyLog[2, (e^(1/3)*(Sqrt[-f] + Sqrt[g]*x))/(e^(1/3
)*Sqrt[-f] - (-1)^(2/3)*d^(1/3)*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g])

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.83, size = 327, normalized size = 0.44

method	result
risch	$\frac{\left(\ln\left(\left(e\,x^3+d\right)^p\right)-p\ln\left(e\,x^3+d\right)\right)\arctan\left(\frac{xg}{\sqrt{fg}}\right)}{\sqrt{fg}}+\frac{p\left(\sum_{\_\alpha=\mathrm{RootOf}\left(g\_Z^2+f\right)}\frac{\ln(x-\_\alpha)\ln\left(e\,x^3+d\right)-\left(\sum_{\_R1=\mathrm{RootOf}\left(\_Z^3 eg+3\_\alpha\_\cdots\right.}\cdots\right.}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(c*(e*x^3+d)^p)/(g*x^2+f),x,method=_RETURNVERBOSE)

[Out] (ln((e*x^3+d)^p)-p*ln(e*x^3+d))/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))+1/2*p/g
sum(1/_alpha(ln(x-_alpha)*ln(e*x^3+d)-sum(ln(x-_alpha)*ln((_R1-x+_alpha)/
_R1)+dilog((_R1-x+_alpha)/_R1),_R1=RootOf(_Z^3*e*g+3*_Z^2*_alpha*e*g-3*_Z*e
*f-_alpha*e*f+d*g))),_alpha=RootOf(_Z^2*g+f))+1/2*I/(f*g)^(1/2)*arctan(x*g/
(f*g)^(1/2))*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-1/2*I/(f*g)^(1/
2)*arctan(x*g/(f*g)^(1/2))*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csg
n(I*c)-1/2*I/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))*Pi*csgn(I*c*(e*x^3+d)^p)^3
+1/2*I/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(
I*c)+1/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))*ln(c)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)/(g*x^2+f),x, algorithm="maxima")
[Out] integrate(log((x^3*e + d)^p*c)/(g*x^2 + f), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)/(g*x^2+f),x, algorithm="fricas")
[Out] integral(log((x^3*e + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**3+d)**p)/(g*x**2+f),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^3+d)^p)/(g*x^2+f),x, algorithm="giac")
[Out] integrate(log((x^3*e + d)^p*c)/(g*x^2 + f), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(e\,x^3+d\right)}^p\right)}{g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^3)^p)/(f + g*x^2),x)
[Out] int(log(c*(d + e*x^3)^p)/(f + g*x^2), x)
```

3.262 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$

Optimal. Leaf size=533

$$\frac{2p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}}-\frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(i\sqrt{e}\,\sqrt{f}-\sqrt{-d}\,\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}\right)}{\sqrt{f}\,\sqrt{g}}-$$

```
[Out] arctan(x*g^(1/2)/f^(1/2))*ln(c*(e*x^2+d)^p)/f^(1/2)/g^(1/2)+2*p*arctan(x*g^
(1/2)/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)-p*arctan
(x*g^(1/2)/f^(1/2))*ln(-2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I
*x*g^(1/2))/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)-p*arcta
n(x*g^(1/2)/f^(1/2))*ln(2*((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I
*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)-I*p*pol
ylog(2,1-2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)+1/2*I*p*polylog(2
,1+2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2
)*f^(1/2)-(-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)+1/2*I*p*polylog(2,1-2*((-d)^
(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(
-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 533, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {211, 2520, 12, 5048, 4966, 2449, 2352, 2497}

$$\frac{ip\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(-\sqrt{-d}\,\sqrt{g}+i\sqrt{e}\,\sqrt{f}\right)}\right)}{2\sqrt{f}\,\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(\sqrt{-d}\,\sqrt{g}+i\sqrt{e}\,\sqrt{f}\right)}\right)}{2\sqrt{f}\,\sqrt{g}}-\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}}+\frac{\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\,\sqrt{g}}-\frac{p\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(-\sqrt{-d}\,\sqrt{g}+i\sqrt{e}\,\sqrt{f}\right)}\right)}{\sqrt{f}\,\sqrt{g}}-\frac{p\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(\sqrt{-d}\,\sqrt{g}+i\sqrt{e}\,\sqrt{f}\right)}\right)}{\sqrt{f}\,\sqrt{g}}+\frac{2p\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(f + g*x^2),x]
```

```
[Out] (2*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/
(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*
(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*S
qrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[
g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + (ArcTan[(Sqrt[g]*x)/Sqrt
[f]]*Log[c*(d + e*x^2)^p])/(Sqrt[f]*Sqrt[g]) - (I*p*PolyLog[2, 1 - (2*Sqrt[
f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 + (
2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sq
rt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1
 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d
]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
```

```
/; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\,\sqrt{g}} - (2ep)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}\,\sqrt{g}\,(d+ex^2)}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\,\sqrt{g}} - \frac{(2ep)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{d+ex^2}\,dx}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\,\sqrt{g}} - \frac{(2ep)\int \left(-\frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}\,x\right)} + \frac{\tan^{-1}\left(\cdots\right.}{2\sqrt{e}\left(\sqrt{-\cdots\right.}\cdots}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\,\sqrt{g}} + \frac{(\sqrt{e}\,p)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{-d}-\sqrt{e}\,x}\,dx}{\sqrt{f}\,\sqrt{g}} - \frac{(\sqrt{e}\,p)\int \frac{\tan^{-1}\left(\cdots\right.}{\sqrt{-\cdots}}}{\sqrt{f}\,\sqrt{\cdots}}$$

$$= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\,\sqrt{\cdots}}{\left(i\sqrt{e}\,\sqrt{f}-\sqrt{-\cdots\right.}}\cdots}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\,\sqrt{\cdots}}{\left(i\sqrt{e}\,\sqrt{f}-\sqrt{-\cdots\right.}}\cdots}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\,\sqrt{\cdots}}{\left(i\sqrt{e}\,\sqrt{f}-\sqrt{-\cdots\right.}}\cdots}{\sqrt{f}\,\sqrt{g}}$$

Mathematica [A]
time = 0.25, size = 564, normalized size = 1.06

$$-\frac{i\left(p\log\left(\frac{\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{i\sqrt{e}\,\sqrt{f}+\sqrt{-d}\,\sqrt{g}}\right)\log\left(1-\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)+p\log\left(\frac{\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{-i\sqrt{e}\,\sqrt{f}+\sqrt{-d}\,\sqrt{g}}\right)\log\left(1-\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)-p\log\left(\frac{\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{-i\sqrt{e}\,\sqrt{f}+\sqrt{-d}\,\sqrt{g}}\right)\log\left(1+\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)-p\log\left(\frac{\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{i\sqrt{e}\,\sqrt{f}+\sqrt{-d}\,\sqrt{g}}\right)\log\left(1+\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)+2i\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)+p\text{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}\,x\right)}{\sqrt{e}\,\sqrt{f}-i\sqrt{-d}\,\sqrt{g}}\right)+p\text{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}\,x\right)}{\sqrt{e}\,\sqrt{f}+i\sqrt{-d}\,\sqrt{g}}\right)-p\text{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}+i\sqrt{g}\,x\right)}{\sqrt{e}\,\sqrt{f}-i\sqrt{-d}\,\sqrt{g}}\right)-p\text{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}+i\sqrt{g}\,x\right)}{\sqrt{e}\,\sqrt{f}+i\sqrt{-d}\,\sqrt{g}}\right)\right)}{2\sqrt{f}\,\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^2),x]
```

```
[Out] ((-1/2*I)*(p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt
[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + p*Log[(Sqrt[g]*(Sqrt[-d] +
Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x
)/Sqrt[f]] - p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] +
 Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] - p*Log[(Sqrt[g]*(Sqrt[-
d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]
*x)/Sqrt[f]] + (2*I)*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p] + p*P
olyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*S
qrt[g])] + p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f]
+ I*Sqrt[-d]*Sqrt[g])] - p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sq
rt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] - p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*S
qrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])]))/(Sqrt[f]*Sqrt[g])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.90, size = 504, normalized size = 0.95

method	result
risch	$\frac{\left(\ln\left(\left(e\,x^2+d\right)^p\right)-p\ln\left(e\,x^2+d\right)\right)\arctan\left(\frac{xg}{\sqrt{fg}}\right)}{\sqrt{fg}}+\frac{p\left(\sum_{\_\alpha=\mathrm{RootOf}\left(g\_Z^2+f\right)}\frac{\ln(x-\_\alpha)\ln\left(e\,x^2+d\right)-\ln(x-\_\alpha)\left(\ln\left(\frac{\mathrm{RootOf}\left(e\_Z^2g+\right.}{\mathrm{RootOf}\left(e\_Z\right.}\right.\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/(g*x^2+f),x,method=_RETURNVERBOSE)
```

```
[Out] (ln((e*x^2+d)^p)-p*ln(e*x^2+d))/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))+1/2*p/g
*sum(1/_alpha*(ln(x-_alpha)*ln(e*x^2+d)-ln(x-_alpha)*(ln((RootOf(_Z^2*e*g+2
*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d
*g-e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x+_al
pha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)))-dilog((RootOf(_Z^2*
e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*
e*g+d*g-e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=
2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2))),_alpha=Root
Of(_Z^2*g+f))+1/2*I/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))*Pi*csgn(I*(e*x^2+d)
^p)*csgn(I*c*(e*x^2+d)^p)^2-1/2*I/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))*Pi*cs
gn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/2*I/(f*g)^(1/2)*arctan(
x*g/(f*g)^(1/2))*Pi*csgn(I*c*(e*x^2+d)^p)^3+1/2*I/(f*g)^(1/2)*arctan(x*g/(f
*g)^(1/2))*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+1/(f*g)^(1/2)*arctan(x*g/(f
*g)^(1/2))*ln(c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
[Out] integrate(log((x^2*e + d)^p*c)/(g*x^2 + f), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
[Out] integral(log((x^2*e + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
[Out] Integral(log(c*(d + e*x**2)**p)/(f + g*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
[Out] integrate(log((x^2*e + d)^p*c)/(g*x^2 + f), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln \left(c\,{\left(e\,x^2+d\right)}^p\right)}{g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)/(f + g*x^2),x)
[Out] int(log(c*(d + e*x^2)^p)/(f + g*x^2), x)
```

3.263 $\int \frac{\log(c(d+ex)^p)}{f+gx^2}\,dx$

Optimal. Leaf size=229

$$\frac{\log\left(c(d+ex)^p\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}\,x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\,\sqrt{g}}-\frac{\log\left(c(d+ex)^p\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}\,x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\,\sqrt{g}}-\frac{p\mathrm{Li}_2\left(-\frac{\sqrt{g}\,(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\,\sqrt{g}}$$

```
[Out] 1/2*ln(c*(e*x+d)^p)*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))/(
-f)^(1/2)/g^(1/2)-1/2*ln(c*(e*x+d)^p)*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(
1/2)-d*g^(1/2)))/(-f)^(1/2)/g^(1/2)-1/2*p*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f
)^(1/2)-d*g^(1/2)))/(-f)^(1/2)/g^(1/2)+1/2*p*polylog(2,(e*x+d)*g^(1/2)/(e*(
-f)^(1/2)+d*g^(1/2)))/(-f)^(1/2)/g^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2456, 2441, 2440, 2438}

$$-\frac{p\text{PolyLog}\left(2,-\frac{\sqrt{g}\,(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\,\sqrt{g}}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt{g}\,(d+ex)}{d\sqrt{g}+e\sqrt{-f}}\right)}{2\sqrt{-f}\,\sqrt{g}}+\frac{\log\left(c(d+ex)^p\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}\,x\right)}{d\sqrt{g}+e\sqrt{-f}}\right)}{2\sqrt{-f}\,\sqrt{g}}-\frac{\log\left(c(d+ex)^p\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}\,x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\,\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x)^p]/(f + g*x^2),x]
```

```
[Out] (Log[c*(d + e*x)^p]*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])
])/(2*Sqrt[-f]*Sqrt[g]) - (Log[c*(d + e*x)^p]*Log[(e*(Sqrt[-f] + Sqrt[g]*x)
)/(e*Sqrt[-f] - d*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g]) - (p*PolyLog[2, -((Sqrt[g
]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*Sqrt[-f]*Sqrt[g]) + (p*PolyLog[
2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rubi steps

$$\int \frac{\log(c(d+ex)^p)}{f+gx^2}\,dx = \int \left(\frac{\sqrt{-f}\,\log(c(d+ex)^p)}{2f\left(\sqrt{-f}-\sqrt{g}\,x\right)}+\frac{\sqrt{-f}\,\log(c(d+ex)^p)}{2f\left(\sqrt{-f}+\sqrt{g}\,x\right)}\right)dx$$

$$= -\frac{\int \frac{\log(c(d+ex)^p)}{\sqrt{-f}-\sqrt{g}\,x}\,dx}{2\sqrt{-f}} - \frac{\int \frac{\log(c(d+ex)^p)}{\sqrt{-f}+\sqrt{g}\,x}\,dx}{2\sqrt{-f}}$$

$$= \frac{\log(c(d+ex)^p)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}\,x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\,\sqrt{g}} - \frac{\log(c(d+ex)^p)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}\,x\right)}{e\sqrt{-f}-d\sqrt{g}}\right.}{2\sqrt{-f}\,\sqrt{g}}$$

$$= \frac{\log(c(d+ex)^p)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}\,x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\,\sqrt{g}} - \frac{\log(c(d+ex)^p)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}\,x\right)}{e\sqrt{-f}-d\sqrt{g}}\right.}{2\sqrt{-f}\,\sqrt{g}}$$

$$= \frac{\log(c(d+ex)^p)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}\,x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\,\sqrt{g}} - \frac{\log(c(d+ex)^p)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}\,x\right)}{e\sqrt{-f}-d\sqrt{g}}\right.}{2\sqrt{-f}\,\sqrt{g}}$$

Mathematica [A]
time = 0.08, size = 178, normalized size = 0.78

$$\frac{\log(c(d+ex)^p)\left(\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}\,x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)-\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}\,x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)\right)-p\text{Li}_2\left(-\frac{\sqrt{g}\,(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)+p\text{Li}_2\left(\frac{\sqrt{g}\,(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\,\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x)^p]/(f + g*x^2),x]
```

```
[Out] (Log[c*(d + e*x)^p]*(Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g]
)] - Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])]) - p*PolyLog[
2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))] + p*PolyLog[2, (Sqrt[g]
*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.56, size = 419, normalized size = 1.83

method	result
risch	$\frac{(\ln((ex+d)^p)-p\ln(ex+d))\arctan\left(\frac{2g(ex+d)-2dg}{2e\sqrt{fg}}\right)}{\sqrt{fg}}+\frac{p\ln(ex+d)\ln\left(\frac{e\sqrt{-fg}-g(ex+d)+dg}{e\sqrt{-fg}+dg}\right)}{2\sqrt{-fg}}-\frac{p\ln(ex+d)\ln\left(\frac{e\sqrt{-fg}+g(ex}{e\sqrt{-fg}}\right.}{2\sqrt{-fg}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x+d)^p)/(g*x^2+f),x,method=_RETURNVERBOSE)
```

```
[Out] (ln((e*x+d)^p)-p*ln(e*x+d))/(f*g)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/(f
*g)^(1/2))+1/2*p*ln(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(
e*(-f*g)^(1/2)+d*g))-1/2*p*ln(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x
+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+1/2*p/(-f*g)^(1/2)*dilog((e*(-f*g)^(1/2)-g*(
e*x+d)+d*g)/(e*(-f*g)^(1/2)+d*g))-1/2*p/(-f*g)^(1/2)*dilog((e*(-f*g)^(1/2)+
g*(e*x+d)-d*g)/(e*(-f*g)^(1/2)-d*g))+1/2*I/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/
2))*Pi*csgn(I*(e*x+d)^p)*csgn(I*c*(e*x+d)^p)^2-1/2*I/(f*g)^(1/2)*arctan(x*g
/(f*g)^(1/2))*Pi*csgn(I*(e*x+d)^p)*csgn(I*c*(e*x+d)^p)*csgn(I*c)-1/2*I/(f*g
)^(1/2)*arctan(x*g/(f*g)^(1/2))*Pi*csgn(I*c*(e*x+d)^p)^3+1/2*I/(f*g)^(1/2)*
arctan(x*g/(f*g)^(1/2))*Pi*csgn(I*c*(e*x+d)^p)^2*csgn(I*c)+1/(f*g)^(1/2)*ar
ctan(x*g/(f*g)^(1/2))*ln(c)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.52, size = 311, normalized size = 1.36

$$\frac{\left(2\arctan\left(\frac{gx}{\sqrt{fg}}\right)e^{(-1)}\log(xe+d)+\left(\arctan\left(\frac{(xe^2+de)\sqrt{f}\sqrt{g}}{d^2g+fe^2},\frac{dgxe+d^2g}{d^2g+fe^2}\right)\log(gx^2+f)-\arctan\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{gx^2e^2+2\,dgxe+d^2g}{d^2g+fe^2}\right)+i\,\mathrm{Li}_2\left(-\frac{dgxe+(i\,xe^2-i\,de)\sqrt{f}\sqrt{g}+fe^2}{d^2g+2i\,d\sqrt{f}\sqrt{g}\,e-fe^2}\right)-i\,\mathrm{Li}_2\left(-\frac{dgxe-(i\,xe^2-i\,de)\sqrt{f}\sqrt{g}+fe^2}{d^2g-2i\,d\sqrt{f}\sqrt{g}\,e-fe^2}\right)\right)e^{(-1)}\right)pe}{2\sqrt{fg}}-\frac{p\arctan\left(\frac{gx}{\sqrt{fg}}\right)\log(xe+d)}{\sqrt{fg}}+\frac{\arctan\left(\frac{gx}{\sqrt{fg}}\right)\log((xe+d)^pc)}{\sqrt{fg}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] 1/2*(2*arctan(g*x/sqrt(f*g))*e^(-1)*log(x*e + d) + (arctan2((x*e^2 + d*e)*s
qrt(f)*sqrt(g)/(d^2*g + f*e^2), (d*g*x*e + d^2*g)/(d^2*g + f*e^2))*log(g*x^
2 + f) - arctan(sqrt(g)*x/sqrt(f))*log((g*x^2*e^2 + 2*d*g*x*e + d^2*g)/(d^2
*g + f*e^2)) + I*dilog(-(d*g*x*e + (I*x*e^2 - I*d*e)*sqrt(f)*sqrt(g) + f*e^
2)/(d^2*g + 2*I*d*sqrt(f)*sqrt(g)*e - f*e^2)) - I*dilog(-(d*g*x*e - (I*x*e^
2 - I*d*e)*sqrt(f)*sqrt(g) + f*e^2)/(d^2*g - 2*I*d*sqrt(f)*sqrt(g)*e - f*e^
```

```
2)))*e^(-1))*p*e/sqrt(f*g) - p*arctan(g*x/sqrt(f*g))*log(x*e + d)/sqrt(f*g)
 + arctan(g*x/sqrt(f*g))*log((x*e + d)^p*c)/sqrt(f*g)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x+d)^p)/(g*x^2+f),x, algorithm="fricas")
[Out] integral(log((x*e + d)^p*c)/(g*x^2 + f), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(c(d+ex)^p\right)}{f+gx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x+d)**p)/(g*x**2+f),x)
[Out] Integral(log(c*(d + e*x)**p)/(f + g*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x+d)^p)/(g*x^2+f),x, algorithm="giac")
[Out] integrate(log((x*e + d)^p*c)/(g*x^2 + f), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln \left(c\,{\left(d+e\,x\right)}^p\right)}{g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x)^p)/(f + g*x^2),x)
[Out] int(log(c*(d + e*x)^p)/(f + g*x^2), x)
```

3.264 $\int \frac{\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f+gx^2}\,dx$

Optimal. Leaf size=360

$$\frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\,\sqrt{g}}+\frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}}-\frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\,\sqrt{\cdots}}{\left(id\sqrt{f}+e\sqrt{g}\cdots\right.}\right.}{\sqrt{f}\,\sqrt{g}}$$

```
[Out] arctan(x*g^(1/2)/f^(1/2))*ln(c*(d+e/x)^p)/f^(1/2)/g^(1/2)+p*arctan(x*g^(1/2
)/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)-p*arctan(x*g
^(1/2)/f^(1/2))*ln(2*(d*x+e)*f^(1/2)*g^(1/2)/(I*d*f^(1/2)+e*g^(1/2))/(f^(1/
2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)+1/2*I*p*polylog(2,-I*x*g^(1/2)/f^(1/2))/f^
(1/2)/g^(1/2)-1/2*I*p*polylog(2,I*x*g^(1/2)/f^(1/2))/f^(1/2)/g^(1/2)-1/2*I*
p*polylog(2,1-2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)+1/2*I*p*poly
log(2,1-2*(d*x+e)*f^(1/2)*g^(1/2)/(I*d*f^(1/2)+e*g^(1/2))/(f^(1/2)-I*x*g^(1
/2)))/f^(1/2)/g^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {211, 2520, 12, 266, 6820, 4996, 4940, 2438, 4966, 2449, 2352, 2497}

$$\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\,\sqrt{g}\,(dx+e)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(e\sqrt{g}+id\sqrt{f}\right)}\right)}{2\sqrt{f}\,\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,-\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)}{2\sqrt{f}\,\sqrt{g}}-\frac{ip\text{PolyLog}\left(2,\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)}{2\sqrt{f}\,\sqrt{g}}-\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{2\sqrt{f}\,\sqrt{g}}+\frac{\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\,\sqrt{g}}-\frac{p\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\,\sqrt{g}\,(dx+e)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(e\sqrt{g}+id\sqrt{f}\right)}\right)}{\sqrt{f}\,\sqrt{g}}+\frac{p\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e/x)^p]/(f + g*x^2),x]
```

```
[Out] (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e/x)^p])/(Sqrt[f]*Sqrt[g]) + (p*Arc
Tan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]
*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(e + d*x)
)/((I*d*Sqrt[f] + e*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) +
 ((I/2)*p*PolyLog[2, ((-I)*Sqrt[g]*x)/Sqrt[f]])/(Sqrt[f]*Sqrt[g]) - ((I/2)*
p*PolyLog[2, (I*Sqrt[g]*x)/Sqrt[f]])/(Sqrt[f]*Sqrt[g]) - ((I/2)*p*PolyLog[2
, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*Po
lyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(e + d*x))/((I*d*Sqrt[f] + e*Sqrt[g])*(Sqrt
[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f+gx^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\,\sqrt{g}} + (ep)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}\,\sqrt{g}\,\left(d+\frac{e}{x}\right)x^2}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\,\sqrt{g}} + \frac{(ep)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\left(d+\frac{e}{x}\right)x^2}\,dx}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\,\sqrt{g}} + \frac{(ep)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{x(e+dx)}\,dx}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\,\sqrt{g}} + \frac{(ep)\int \left(\frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{ex} - \frac{d\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{e(e+dx)}\right)dx}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\,\sqrt{g}} + \frac{p\int \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{x}\,dx}{\sqrt{f}\,\sqrt{g}} - \frac{(dp)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{e+dx}\,dx}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\,\sqrt{g}} + \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\tan^{-1}}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\,\sqrt{g}} + \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\tan^{-1}}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\,\sqrt{g}} + \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\tan^{-1}}{}$$

Mathematica [A]

time = 0.16, size = 373, normalized size = 1.04

$$\frac{\log\left(c\left(d+\frac{e}{x}\right)^p\right)\log\left(\sqrt{-f}-\sqrt{g}\,x\right)+p\log\left(\frac{\sqrt{g}\,x}{\sqrt{-f}}\right)\log\left(\sqrt{-f}-\sqrt{g}\,x\right)-p\log\left(\frac{\sqrt{g}\,(e+dx)}{d\sqrt{-f}+e\sqrt{g}}\right)\log\left(\sqrt{-f}-\sqrt{g}\,x\right)-\log\left(c\left(d+\frac{e}{x}\right)^p\right)\log\left(\sqrt{-f}+\sqrt{g}\,x\right)-p\log\left(\frac{f\sqrt{g}\,x}{(-f)^{3/2}}\right)\log\left(\sqrt{-f}+\sqrt{g}\,x\right)+p\log\left(-\frac{\sqrt{g}\,(e+dx)}{d\sqrt{-f}-e\sqrt{g}}\right)\log\left(\sqrt{-f}+\sqrt{g}\,x\right)-p\mathrm{Li}_2\left(\frac{d\left(\sqrt{-f}-\sqrt{g}\,x\right)}{d\sqrt{-f}+e\sqrt{g}}\right)+p\mathrm{Li}_2\left(\frac{d\left(\sqrt{-f}+\sqrt{g}\,x\right)}{d\sqrt{-f}-e\sqrt{g}}\right)-p\mathrm{Li}_2\left(1+\frac{\sqrt{g}\,x}{\sqrt{-f}}\right)+p\mathrm{Li}_2\left(1+\frac{f\sqrt{g}\,x}{(-f)^{3/2}}\right)}{2\sqrt{-f}\,\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e/x)^p]/(f + g*x^2),x]
```

```
[Out] (Log[c*(d + e/x)^p]*Log[Sqrt[-f] - Sqrt[g]*x] + p*Log[(Sqrt[g]*x)/Sqrt[-f]]
*Log[Sqrt[-f] - Sqrt[g]*x] - p*Log[(Sqrt[g]*(e + d*x))/(d*Sqrt[-f] + e*Sqrt
[g])]*Log[Sqrt[-f] - Sqrt[g]*x] - Log[c*(d + e/x)^p]*Log[Sqrt[-f] + Sqrt[g]
*x] - p*Log[(f*Sqrt[g]*x)/(-f)^(3/2)]*Log[Sqrt[-f] + Sqrt[g]*x] + p*Log[-((
Sqrt[g]*(e + d*x))/(d*Sqrt[-f] - e*Sqrt[g]))]*Log[Sqrt[-f] + Sqrt[g]*x] - p
*PolyLog[2, (d*(Sqrt[-f] - Sqrt[g]*x))/(d*Sqrt[-f] + e*Sqrt[g])] + p*PolyLo
g[2, (d*(Sqrt[-f] + Sqrt[g]*x))/(d*Sqrt[-f] - e*Sqrt[g])] - p*PolyLog[2, 1
+ (Sqrt[g]*x)/Sqrt[-f]] + p*PolyLog[2, 1 + (f*Sqrt[g]*x)/(-f)^(3/2)])/(2*Sq
rt[-f]*Sqrt[g])
```

Maple [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(d+\frac{e}{x}\right)^p\right)}{g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e/x)^p)/(g*x^2+f),x)
```

```
[Out] int(ln(c*(d+e/x)^p)/(g*x^2+f),x)
```

Maxima [A]
time = 0.55, size = 381, normalized size = 1.06

$$\frac{\left(4\arctan\left(\frac{gx}{\sqrt{fg}}\right)e^{(-1)}\log\left(d+\frac{e}{x}\right)-\left(\left(\pi-2\arctan\left(\frac{(d^2x+de)\sqrt{f}\sqrt{g}}{d^2f+ge^2},\frac{dgxe+ge^2}{d^2f+ge^2}\right)\right)\log\left(gx^2+f\right)-4\arctan\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)+2\arctan\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{d^2gx^2+2dgxe+ge^2}{d^2f+ge^2}\right)+2i\,\mathrm{Li}_2\left(\frac{i\sqrt{g}\,x+\sqrt{f}}{\sqrt{f}}\right)-2i\,\mathrm{Li}_2\left(-\frac{i\sqrt{g}\,x-\sqrt{f}}{\sqrt{f}}\right)+2i\,\mathrm{Li}_2\left(\frac{dgxe+d^2f-(i\,d^2x-i\,de)\sqrt{f}\sqrt{g}}{d^2f+2i\,d\sqrt{f}\sqrt{g}\,e-ge^2}\right)-2i\,\mathrm{Li}_2\left(\frac{dgxe+d^2f+(i\,d^2x-i\,de)\sqrt{f}\sqrt{g}}{d^2f-2i\,d\sqrt{f}\sqrt{g}\,e-ge^2}\right)\right)e^{(-1)}\right)pe}{4\sqrt{fg}}-\frac{p\arctan\left(\frac{gx}{\sqrt{fg}}\right)\log\left(d+\frac{e}{x}\right)}{\sqrt{fg}}+\frac{\arctan\left(\frac{gx}{\sqrt{fg}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{fg}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/x)^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] 1/4*(4*arctan(g*x/sqrt(f*g))*e^(-1)*log(d + e/x) - ((pi - 2*arctan2((d^2*x
+ d*e)*sqrt(f)*sqrt(g)/(d^2*f + g*e^2), (d*g*x*e + g*e^2)/(d^2*f + g*e^2)))
*log(g*x^2 + f) - 4*arctan(sqrt(g)*x/sqrt(f))*log(sqrt(g)*x/sqrt(f)) + 2*ar
ctan(sqrt(g)*x/sqrt(f))*log((d^2*g*x^2 + 2*d*g*x*e + g*e^2)/(d^2*f + g*e^2)
) + 2*I*dilog((I*sqrt(g)*x + sqrt(f))/sqrt(f)) - 2*I*dilog(-(I*sqrt(g)*x -
sqrt(f))/sqrt(f)) + 2*I*dilog((d*g*x*e + d^2*f - (I*d^2*x - I*d*e)*sqrt(f)*
sqrt(g))/(d^2*f + 2*I*d*sqrt(f)*sqrt(g)*e - g*e^2)) - 2*I*dilog((d*g*x*e +
d^2*f + (I*d^2*x - I*d*e)*sqrt(f)*sqrt(g))/(d^2*f - 2*I*d*sqrt(f)*sqrt(g)*e
 - g*e^2)))*e^(-1))*p*e/sqrt(f*g) - p*arctan(g*x/sqrt(f*g))*log(d + e/x)/sq
rt(f*g) + arctan(g*x/sqrt(f*g))*log(c*(d + e/x)^p)/sqrt(f*g)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/x)^p)/(g*x^2+f),x, algorithm="fricas")
[Out] integral(log(c*((d*x + e)/x)^p)/(g*x^2 + f), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(c\left(d+\frac{e}{x}\right)^{p}\right)}{f+gx^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e/x)**p)/(g*x**2+f),x)
[Out] Integral(log(c*(d + e/x)**p)/(f + g*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/x)^p)/(g*x^2+f),x, algorithm="giac")
[Out] integrate(log(c*(d + e/x)^p)/(g*x^2 + f), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln \left(c\,\left(d+\frac{e}{x}\right)^{p}\right)}{g\,x^{2}+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e/x)^p)/(f + g*x^2),x)
[Out] int(log(c*(d + e/x)^p)/(f + g*x^2), x)
```

3.265 $\int \frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f+gx^2}\,dx$

Optimal. Leaf size=597

$$\frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\,\sqrt{g}}+\frac{2p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}}-\frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{}}{\left(i\sqrt{-d}\,\sqrt{}\right.}\right.}{\sqrt{f}\,\sqrt{}}$$

[Out] arctan(x*g^(1/2)/f^(1/2))*ln(c*(d+e/x^2)^p)/f^(1/2)/g^(1/2)+2*p*arctan(x*g^(1/2)/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)-p*arctan(x*g^(1/2)/f^(1/2))*ln(-2*(-x*(-d)^(1/2)+e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*(-d)^(1/2)*f^(1/2)-e^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)-p*arctan(x*g^(1/2)/f^(1/2))*ln(2*(x*(-d)^(1/2)+e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*(-d)^(1/2)*f^(1/2)+e^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)+I*p*polylog(2,-I*x*g^(1/2)/f^(1/2))/f^(1/2)/g^(1/2)-I*p*polylog(2,I*x*g^(1/2)/f^(1/2))/f^(1/2)/g^(1/2)-I*p*polylog(2,1-2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)+1/2*I*p*polylog(2,1+2*(-x*(-d)^(1/2)+e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*(-d)^(1/2)*f^(1/2)-e^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)+1/2*I*p*polylog(2,1-2*(x*(-d)^(1/2)+e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*(-d)^(1/2)*f^(1/2)+e^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)

Rubi [A]
time = 0.57, antiderivative size = 597, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$,
Rules used = {211, 2520, 12, 266, 6820, 5048, 4940, 2438, 4966, 2449, 2352, 2497}

$$\frac{ip\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{e}-\sqrt{-d}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(-\sqrt{e}\sqrt{g}+i\sqrt{-d}\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}\,x+\sqrt{e}\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(\sqrt{e}\sqrt{g}+i\sqrt{-d}\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,-\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}-\frac{ip\text{PolyLog}\left(2,\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}-\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\sqrt{g}}+\frac{\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}}-\frac{p\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{e}-\sqrt{-d}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(-\sqrt{e}\sqrt{g}+i\sqrt{-d}\sqrt{f}\right)}\right)}{\sqrt{f}\sqrt{g}}-\frac{p\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}\,x+\sqrt{e}\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(\sqrt{e}\sqrt{g}+i\sqrt{-d}\sqrt{f}\right)}\right)}{\sqrt{f}\sqrt{g}}+\frac{2p\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\sqrt{g}}$$

Antiderivative was successfully verified.

[In] Int[Log[c*(d + e/x^2)^p]/(f + g*x^2),x]

[Out] (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e/x^2)^p])/(Sqrt[f]*Sqrt[g]) + (2*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[e] - Sqrt[-d]*x))/((I*Sqrt[-d]*Sqrt[f] - Sqrt[e]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(Sqrt[e] + Sqrt[-d]*x))/((I*Sqrt[-d]*Sqrt[f] + Sqrt[e]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + (I*p*PolyLog[2, ((-I)*Sqrt[g]*x)/Sqrt[f]])/(Sqrt[f]*Sqrt[g]) - (I*p*PolyLog[2, (I*Sqrt[g]*x)/Sqrt[f]])/(Sqrt[f]*Sqrt[g]) - (I*p*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[e] - Sqrt[-d]*x))/((I*Sqrt[-d]*Sqrt[f] - Sqrt[e]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]

```
*x))])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt
[e] + Sqrt[-d]*x))/((I*Sqrt[-d]*Sqrt[f] + Sqrt[e]*Sqrt[g])*(Sqrt[f] - I*Sqr
t[g]*x))])/(Sqrt[f]*Sqrt[g])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
```

```
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f+gx^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\,\sqrt{g}} + (2ep)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}\,\sqrt{g}\,\left(d+\frac{e}{x^2}\right)x^3}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\,\sqrt{g}} + \frac{(2ep)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\left(d+\frac{e}{x^2}\right)x^3}\,dx}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\,\sqrt{g}} + \frac{(2ep)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{x(e+dx^2)}\,dx}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\,\sqrt{g}} + \frac{(2ep)\int \left(\frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{ex} - \frac{dx\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{e(e+dx^2)}\right) dx}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\,\sqrt{g}} + \frac{(2p)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{x}\,dx}{\sqrt{f}\,\sqrt{g}} - \frac{(2dp)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{e+dx^2}}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\,\sqrt{g}} + \frac{(ip)\int \frac{\log\left(1-\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)}{x}\,dx}{\sqrt{f}\,\sqrt{g}} - \frac{(ip)\int \frac{\log\left(1+\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)}{x}}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\,\sqrt{g}} + \frac{ip\text{Li}_2\left(-\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{ip\text{Li}_2\left(\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}\,\sqrt{g}} + \frac{\left(\sqrt{-d}\,p\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\,\sqrt{g}} + \frac{2p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\tan}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\,\sqrt{g}} + \frac{2p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\tan}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{} + \frac{2p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{} - \frac{p\tan}{}$$

Mathematica [A]
time = 0.27, size = 706, normalized size = 1.18

$$\frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)\log\left(\sqrt{-f}-\sqrt{g}\,x\right)+2p\log\left(\frac{\sqrt{g}\,x}{\sqrt{-f}}\right)\log\left(\sqrt{-f}-\sqrt{g}\,x\right)-p\log\left(\frac{\sqrt{g}\left(-\sqrt{e}+\sqrt{-d}\,x\right)}{\sqrt{-d}\,\sqrt{-f}-\sqrt{e}\,\sqrt{g}}\right)\log\left(\sqrt{-f}-\sqrt{g}\,x\right)-p\log\left(\frac{\sqrt{g}\left(\sqrt{e}+\sqrt{-d}\,x\right)}{\sqrt{-d}\,\sqrt{-f}+\sqrt{e}\,\sqrt{g}}\right)\log\left(\sqrt{-f}-\sqrt{g}\,x\right)-\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)\log\left(\sqrt{-f}+\sqrt{g}\,x\right)-2p\log\left(\frac{f\sqrt{g}\,x}{(-f)^{3/2}}\right)\log\left(\sqrt{-f}+\sqrt{g}\,x\right)+p\log\left(\frac{\sqrt{g}\left(\sqrt{e}-\sqrt{-d}\,x\right)}{\sqrt{-d}\,\sqrt{-f}+\sqrt{e}\,\sqrt{g}}\right)\log\left(\sqrt{-f}+\sqrt{g}\,x\right)+p\log\left(-\frac{\sqrt{g}\left(\sqrt{e}+\sqrt{-d}\,x\right)}{\sqrt{-d}\,\sqrt{-f}-\sqrt{e}\,\sqrt{g}}\right)\log\left(\sqrt{-f}+\sqrt{g}\,x\right)-p\mathrm{Li}_2\left(\frac{\sqrt{-d}\left(\sqrt{-f}-\sqrt{g}\,x\right)}{\sqrt{-d}\,\sqrt{-f}-\sqrt{e}\,\sqrt{g}}\right)-p\mathrm{Li}_2\left(\frac{\sqrt{-d}\left(\sqrt{-f}-\sqrt{g}\,x\right)}{\sqrt{-d}\,\sqrt{-f}+\sqrt{e}\,\sqrt{g}}\right)+p\mathrm{Li}_2\left(\frac{\sqrt{-d}\left(\sqrt{-f}+\sqrt{g}\,x\right)}{\sqrt{-d}\,\sqrt{-f}-\sqrt{e}\,\sqrt{g}}\right)+p\mathrm{Li}_2\left(\frac{\sqrt{-d}\left(\sqrt{-f}+\sqrt{g}\,x\right)}{\sqrt{-d}\,\sqrt{-f}+\sqrt{e}\,\sqrt{g}}\right)-2p\mathrm{Li}_2\left(1+\frac{\sqrt{g}\,x}{\sqrt{-f}}\right)+2p\mathrm{Li}_2\left(1+\frac{f\sqrt{g}\,x}{(-f)^{3/2}}\right)}{2\sqrt{-f}\,\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e/x^2)^p]/(f + g*x^2),x]
```

```
[Out] (Log[c*(d + e/x^2)^p]*Log[Sqrt[-f] - Sqrt[g]*x] + 2*p*Log[(Sqrt[g]*x)/Sqrt[
-f]]*Log[Sqrt[-f] - Sqrt[g]*x] - p*Log[(Sqrt[g]*(-Sqrt[e] + Sqrt[-d]*x))/(S
qrt[-d]*Sqrt[-f] - Sqrt[e]*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*x] - p*Log[(Sqr
t[g]*(Sqrt[e] + Sqrt[-d]*x))/(Sqrt[-d]*Sqrt[-f] + Sqrt[e]*Sqrt[g])]*Log[Sqr
t[-f] - Sqrt[g]*x] - Log[c*(d + e/x^2)^p]*Log[Sqrt[-f] + Sqrt[g]*x] - 2*p*L
og[(f*Sqrt[g]*x)/(-f)^(3/2)]*Log[Sqrt[-f] + Sqrt[g]*x] + p*Log[(Sqrt[g]*(Sq
rt[e] - Sqrt[-d]*x))/(Sqrt[-d]*Sqrt[-f] + Sqrt[e]*Sqrt[g])]*Log[Sqrt[-f] +
Sqrt[g]*x] + p*Log[-((Sqrt[g]*(Sqrt[e] + Sqrt[-d]*x))/(Sqrt[-d]*Sqrt[-f] -
Sqrt[e]*Sqrt[g]))]*Log[Sqrt[-f] + Sqrt[g]*x] - p*PolyLog[2, (Sqrt[-d]*(Sqrt
[-f] - Sqrt[g]*x))/(Sqrt[-d]*Sqrt[-f] - Sqrt[e]*Sqrt[g])] - p*PolyLog[2, (S
qrt[-d]*(Sqrt[-f] - Sqrt[g]*x))/(Sqrt[-d]*Sqrt[-f] + Sqrt[e]*Sqrt[g])] + p*
PolyLog[2, (Sqrt[-d]*(Sqrt[-f] + Sqrt[g]*x))/(Sqrt[-d]*Sqrt[-f] - Sqrt[e]*S
qrt[g])] + p*PolyLog[2, (Sqrt[-d]*(Sqrt[-f] + Sqrt[g]*x))/(Sqrt[-d]*Sqrt[-f
] + Sqrt[e]*Sqrt[g])] - 2*p*PolyLog[2, 1 + (Sqrt[g]*x)/Sqrt[-f]] + 2*p*Poly
Log[2, 1 + (f*Sqrt[g]*x)/(-f)^(3/2)])/(2*Sqrt[-f]*Sqrt[g])
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e/x^2)^p)/(g*x^2+f),x)
```

```
[Out] int(ln(c*(d+e/x^2)^p)/(g*x^2+f),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/x^2)^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] integrate(log(c*(d + e/x^2)^p)/(g*x^2 + f), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/x^2)^p)/(g*x^2+f),x, algorithm="fricas")
[Out] integral(log(c*((d*x^2 + e)/x^2)^p)/(g*x^2 + f), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e/x**2)**p)/(g*x**2+f),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/x^2)^p)/(g*x^2+f),x, algorithm="giac")
[Out] integrate(log(c*(d + e/x^2)^p)/(g*x^2 + f), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e/x^2)^p)/(f + g*x^2),x)
[Out] int(log(c*(d + e/x^2)^p)/(f + g*x^2), x)
```

3.266 $\int \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f+gx^2}\,dx$

Optimal. Leaf size=541

$$-\frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt{-\sqrt{-f}}-\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt{-\sqrt{-f}}+d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{-f}-\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt[4]{-f}+d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}-$$

```
[Out] 1/2*ln(c*(d+e*x^(1/2))^p)*ln(e*((-f)^(1/4)-g^(1/4)*x^(1/2))/(e*(-f)^(1/4)+d
*g^(1/4)))/(-f)^(1/2)/g^(1/2)+1/2*ln(c*(d+e*x^(1/2))^p)*ln(e*((-f)^(1/4)+g^
(1/4)*x^(1/2))/(e*(-f)^(1/4)-d*g^(1/4)))/(-f)^(1/2)/g^(1/2)-1/2*ln(c*(d+e*x
^(1/2))^p)*ln(e*(g^(1/4)*x^(1/2)+(-(-f)^(1/2))^(1/2))/(-d*g^(1/4)+e*(-(-f)^
(1/2))^(1/2)))/(-f)^(1/2)/g^(1/2)-1/2*ln(c*(d+e*x^(1/2))^p)*ln(e*(-g^(1/4)*
x^(1/2)+(-(-f)^(1/2))^(1/2))/(d*g^(1/4)+e*(-(-f)^(1/2))^(1/2)))/(-f)^(1/2)/
g^(1/2)+1/2*p*polylog(2,-g^(1/4)*(d+e*x^(1/2))/(e*(-f)^(1/4)-d*g^(1/4)))/(-
f)^(1/2)/g^(1/2)+1/2*p*polylog(2,g^(1/4)*(d+e*x^(1/2))/(e*(-f)^(1/4)+d*g^(1
/4)))/(-f)^(1/2)/g^(1/2)-1/2*p*polylog(2,-g^(1/4)*(d+e*x^(1/2))/(-d*g^(1/4)
+e*(-(-f)^(1/2))^(1/2)))/(-f)^(1/2)/g^(1/2)-1/2*p*polylog(2,g^(1/4)*(d+e*x^
(1/2))/(d*g^(1/4)+e*(-(-f)^(1/2))^(1/2)))/(-f)^(1/2)/g^(1/2)
```

Rubi [A]
time = 0.57, antiderivative size = 541, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2522, 281, 211, 2463, 266, 2441, 2440, 2438}

$$-\frac{p\text{PolyLog}\left(2,-\frac{\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{e\sqrt{-\sqrt{-f}}-d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{p\text{PolyLog}\left(2,-\frac{\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{e\sqrt[4]{-f}-d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{d\sqrt[4]{g}+e\sqrt{-\sqrt{-f}}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{d\sqrt[4]{g}+e\sqrt[4]{-f}}\right)}{2\sqrt{-f}\sqrt{g}}-\frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt{-\sqrt{-f}}-\sqrt[4]{g}\sqrt{x}\right)}{d\sqrt[4]{g}+e\sqrt{-\sqrt{-f}}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{-f}-\sqrt[4]{g}\sqrt{x}\right)}{d\sqrt[4]{g}+e\sqrt[4]{-f}}\right)}{2\sqrt{-f}\sqrt{g}}-\frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt{-\sqrt{-f}}+\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt{-\sqrt{-f}}-d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{-f}+\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt[4]{-f}-d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*Sqrt[x])^p]/(f + g*x^2),x]
```

```
[Out] -1/2*(Log[c*(d + e*Sqrt[x])^p]*Log[(e*(Sqrt[-Sqrt[-f]] - g^(1/4)*Sqrt[x]))/
(e*Sqrt[-Sqrt[-f]] + d*g^(1/4))])/(Sqrt[-f]*Sqrt[g]) + (Log[c*(d + e*Sqrt[x
])^p]*Log[(e*((-f)^(1/4) - g^(1/4)*Sqrt[x]))/(e*(-f)^(1/4) + d*g^(1/4))])/(
2*Sqrt[-f]*Sqrt[g]) - (Log[c*(d + e*Sqrt[x])^p]*Log[(e*(Sqrt[-Sqrt[-f]] + g
^(1/4)*Sqrt[x]))/(e*Sqrt[-Sqrt[-f]] - d*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g]) + (
Log[c*(d + e*Sqrt[x])^p]*Log[(e*((-f)^(1/4) + g^(1/4)*Sqrt[x]))/(e*(-f)^(1/
4) - d*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g]) - (p*PolyLog[2, -((g^(1/4)*(d + e*Sq
rt[x]))/(e*Sqrt[-Sqrt[-f]] - d*g^(1/4)))])/(2*Sqrt[-f]*Sqrt[g]) + (p*PolyLo
g[2, -((g^(1/4)*(d + e*Sqrt[x]))/(e*(-f)^(1/4) - d*g^(1/4)))])/(2*Sqrt[-f]*
Sqrt[g]) - (p*PolyLog[2, (g^(1/4)*(d + e*Sqrt[x]))/(e*Sqrt[-Sqrt[-f]] + d*g
^(1/4))])/(2*Sqrt[-f]*Sqrt[g]) + (p*PolyLog[2, (g^(1/4)*(d + e*Sqrt[x]))/(e
*(-f)^(1/4) + d*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2522

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Su
bst[Int[x^(k - 1)*(f + g*x^(k*s))^r*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x],
```

```
 x, x^(1/k)], x] /; IntegerQ[k*s]] /; FreeQ[{a, b, c, d, e, f, g, n, p, q,
r, s}, x] && FractionQ[n]
```

Rubi steps

$$\int \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f+gx^2}\,dx = 2\text{Subst}\left(\int \frac{x\log\left(c(d+ex)^p\right)}{f+gx^4}\,dx, x, \sqrt{x}\right)$$

$$= 2\text{Subst}\left(\int \left(-\frac{\sqrt{g}\,x\log\left(c(d+ex)^p\right)}{2\sqrt{-f}\left(\sqrt{-f}\sqrt{g}-gx^2\right)} - \frac{\sqrt{g}\,x\log\left(c(d+ex)^p\right)}{2\sqrt{-f}\left(\sqrt{-f}\sqrt{g}+gx^2\right)}\right)\right.$$

$$= -\frac{\sqrt{g}\,\text{Subst}\left(\int \frac{x\log(c(d+ex)^p)}{\sqrt{-f}\sqrt{g}-gx^2}\,dx, x, \sqrt{x}\right)}{\sqrt{-f}} - \frac{\sqrt{g}\,\text{Subst}\left(\int \frac{x\log(c(d+ex)^p)}{\sqrt{-f}\sqrt{g}+gx^2}\,dx\right.}{\sqrt{-f}}$$

$$= -\frac{\sqrt{g}\,\text{Subst}\left(\int \left(-\frac{\log(c(d+ex)^p)}{2g^{3/4}\left(\sqrt{-\sqrt{-f}}-\sqrt[4]{g}\,x\right)} + \frac{\log(c(d+ex)^p)}{2g^{3/4}\left(\sqrt{-\sqrt{-f}}+\sqrt[4]{g}\,x\right)}\right)\,d\right.}{\sqrt{-f}}$$

$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt{-\sqrt{-f}}-\sqrt[4]{g}\,x}\,dx, x, \sqrt{x}\right)}{2\sqrt{-f}\,\sqrt[4]{g}} - \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt[4]{-f}-\sqrt[4]{g}\,x}\,dx, x, \sqrt{x}\right.}{2\sqrt{-f}\,\sqrt[4]{g}}$$

$$= -\frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt{-\sqrt{-f}}-\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt{-\sqrt{-f}}+d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{2}$$

$$= -\frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt{-\sqrt{-f}}-\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt{-\sqrt{-f}}+d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{2}$$

$$= -\frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt{-\sqrt{-f}}-\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt{-\sqrt{-f}}+d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.20, size = 422, normalized size = 0.78

$$\frac{\log \left(c\left(d+e\sqrt{x}\right)^p\right) \log \left(\frac{e\left(\sqrt[4]{-f}-\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt[4]{-f}+d\sqrt[4]{g}}\right)-\log \left(c\left(d+e\sqrt{x}\right)^p\right) \log \left(\frac{e\left(\sqrt[4]{-f}-i\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt[4]{-f}+id\sqrt[4]{g}}\right)-\log \left(c\left(d+e\sqrt{x}\right)^p\right) \log \left(\frac{e\left(\sqrt[4]{-f}+i\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt[4]{-f}-id\sqrt[4]{g}}\right)+\log \left(c\left(d+e\sqrt{x}\right)^p\right) \log \left(\frac{e\left(\sqrt[4]{-f}+\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt[4]{-f}-d\sqrt[4]{g}}\right)+p\text{Li}_2\left(-\frac{\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{e\sqrt[4]{-f}-d\sqrt[4]{g}}\right)-p\text{Li}_2\left(\frac{i\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{e\sqrt[4]{-f}+id\sqrt[4]{g}}\right)-p\text{Li}_2\left(\frac{\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{ie\sqrt[4]{-f}+d\sqrt[4]{g}}\right)+p\text{Li}_2\left(\frac{\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{e\sqrt[4]{-f}+d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*Sqrt[x])^p]/(f + g*x^2),x]
```

```
[Out] (Log[c*(d + e*Sqrt[x])^p]*Log[(e*((-f)^(1/4) - g^(1/4)*Sqrt[x]))/(e*(-f)^(1
/4) + d*g^(1/4))] - Log[c*(d + e*Sqrt[x])^p]*Log[(e*((-f)^(1/4) - I*g^(1/4)
*Sqrt[x]))/(e*(-f)^(1/4) + I*d*g^(1/4))] - Log[c*(d + e*Sqrt[x])^p]*Log[(e*
((-f)^(1/4) + I*g^(1/4)*Sqrt[x]))/(e*(-f)^(1/4) - I*d*g^(1/4))] + Log[c*(d
+ e*Sqrt[x])^p]*Log[(e*((-f)^(1/4) + g^(1/4)*Sqrt[x]))/(e*(-f)^(1/4) - d*g^
(1/4))] + p*PolyLog[2, -((g^(1/4)*(d + e*Sqrt[x]))/(e*(-f)^(1/4) - d*g^(1/4
)))] - p*PolyLog[2, (I*g^(1/4)*(d + e*Sqrt[x]))/(e*(-f)^(1/4) + I*d*g^(1/4)
)] - p*PolyLog[2, (g^(1/4)*(d + e*Sqrt[x]))/(I*e*(-f)^(1/4) + d*g^(1/4))] +
 p*PolyLog[2, (g^(1/4)*(d + e*Sqrt[x]))/(e*(-f)^(1/4) + d*g^(1/4))])/(2*Sqr
t[-f]*Sqrt[g])
```

Maple [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(c\left(d+e\sqrt{x}\right)^p\right)}{g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^(1/2))^p)/(g*x^2+f),x)
```

```
[Out] int(ln(c*(d+e*x^(1/2))^p)/(g*x^2+f),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^(1/2))^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] integrate(log((sqrt(x)*e + d)^p*c)/(g*x^2 + f), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^(1/2))^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log((sqrt(x)*e + d)^p*c)/(g*x^2 + f), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**(1/2))**p)/(g*x**2+f),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^(1/2))^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(log((sqrt(x)*e + d)^p*c)/(g*x^2 + f), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(d+e\sqrt{x}\right)^p\right)}{g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^(1/2))^p)/(f + g*x^2),x)
```

```
[Out] int(log(c*(d + e*x^(1/2))^p)/(f + g*x^2), x)
```

3.267 $\int \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f+gx^2}\,dx$

Optimal. Leaf size=561

$$-\frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{g}-\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}-\frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(-\frac{e\left(\sqrt[4]{g}+\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}$$

```
[Out] 1/2*ln(c*(d+e/x^(1/2))^p)*ln(e*(g^(1/4)-(-f)^(1/4)/x^(1/2))/(d*(-f)^(1/4)+e
*g^(1/4)))/(-f)^(1/2)/g^(1/2)+1/2*ln(c*(d+e/x^(1/2))^p)*ln(-e*(g^(1/4)+(-f)
^(1/4)/x^(1/2))/(d*(-f)^(1/4)-e*g^(1/4)))/(-f)^(1/2)/g^(1/2)-1/2*ln(c*(d+e/
x^(1/2))^p)*ln(e*(g^(1/4)-(-(-f)^(1/2))^(1/2)/x^(1/2))/(e*g^(1/4)+d*(-(-f)^
(1/2))^(1/2)))/(-f)^(1/2)/g^(1/2)-1/2*ln(c*(d+e/x^(1/2))^p)*ln(-e*(g^(1/4)+
(-(-f)^(1/2))^(1/2)/x^(1/2))/(-e*g^(1/4)+d*(-(-f)^(1/2))^(1/2)))/(-f)^(1/2)
/g^(1/2)+1/2*p*polylog(2,(-f)^(1/4)*(d+e/x^(1/2))/(d*(-f)^(1/4)-e*g^(1/4)))
/(-f)^(1/2)/g^(1/2)+1/2*p*polylog(2,(-f)^(1/4)*(d+e/x^(1/2))/(d*(-f)^(1/4)+
e*g^(1/4)))/(-f)^(1/2)/g^(1/2)-1/2*p*polylog(2,(d+e/x^(1/2))*(-(-f)^(1/2))^
(1/2)/(-e*g^(1/4)+d*(-(-f)^(1/2))^(1/2)))/(-f)^(1/2)/g^(1/2)-1/2*p*polylog(
2,(d+e/x^(1/2))*(-(-f)^(1/2))^(1/2)/(e*g^(1/4)+d*(-(-f)^(1/2))^(1/2)))/(-f)
^(1/2)/g^(1/2)
```

Rubi [A]
time = 0.84, antiderivative size = 561, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2522, 2525, 269, 281, 211, 2463, 266, 2441, 2440, 2438}

$$-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{-\sqrt{-f}}\left(d+\frac{e}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[4]{-f}\left(d+\frac{e}{\sqrt{x}}\right)}{d\sqrt[4]{-f}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{-\sqrt{-f}}\left(d+\frac{e}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[4]{-f}\left(d+\frac{e}{\sqrt{x}}\right)}{d\sqrt[4]{-f}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}-\frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{g}-\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}-\frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(-\frac{e\left(\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}+\sqrt[4]{g}\right)}{d\sqrt{-\sqrt{-f}}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{g}-\frac{\sqrt[4]{-f}}{\sqrt{x}}\right)}{d\sqrt[4]{-f}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(-\frac{e\left(\frac{\sqrt[4]{-f}}{\sqrt{x}}+\sqrt[4]{g}\right)}{d\sqrt[4]{-f}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e/Sqrt[x])^p]/(f + g*x^2),x]
```

```
[Out] -1/2*(Log[c*(d + e/Sqrt[x])^p]*Log[(e*(g^(1/4) - Sqrt[-Sqrt[-f]]/Sqrt[x]))/
(d*Sqrt[-Sqrt[-f]] + e*g^(1/4))])/(Sqrt[-f]*Sqrt[g]) - (Log[c*(d + e/Sqrt[x
])^p]*Log[-((e*(g^(1/4) + Sqrt[-Sqrt[-f]]/Sqrt[x]))/(d*Sqrt[-Sqrt[-f]] - e*
g^(1/4)))])/(2*Sqrt[-f]*Sqrt[g]) + (Log[c*(d + e/Sqrt[x])^p]*Log[(e*(g^(1/4
) - (-f)^(1/4)/Sqrt[x]))/(d*(-f)^(1/4) + e*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g])
+ (Log[c*(d + e/Sqrt[x])^p]*Log[-((e*(g^(1/4) + (-f)^(1/4)/Sqrt[x]))/(d*(-f
```

```
)^(1/4) - e*g^(1/4)))])/(2*Sqrt[-f]*Sqrt[g]) - (p*PolyLog[2, (Sqrt[-Sqrt[-f
]]*(d + e/Sqrt[x]))/(d*Sqrt[-Sqrt[-f]] - e*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g])
+ (p*PolyLog[2, ((-f)^(1/4)*(d + e/Sqrt[x]))/(d*(-f)^(1/4) - e*g^(1/4))])/(
2*Sqrt[-f]*Sqrt[g]) - (p*PolyLog[2, (Sqrt[-Sqrt[-f]]*(d + e/Sqrt[x]))/(d*Sq
rt[-Sqrt[-f]] + e*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g]) + (p*PolyLog[2, ((-f)^(1/
4)*(d + e/Sqrt[x]))/(d*(-f)^(1/4) + e*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2522

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Su
bst[Int[x^(k - 1)*(f + g*x^(k*s))^r*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x],
 x, x^(1/k)], x] /; IntegerQ[k*s]] /; FreeQ[{a, b, c, d, e, f, g, n, p, q,
r, s}, x] && FractionQ[n]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f+gx^2}\,dx = 2\text{Subst}\left(\int \frac{x\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f+gx^4}\,dx, x, \sqrt{x}\right)$$

$$= -\left(2\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{\left(f+\frac{g}{x^4}\right)x^3}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\text{Subst}\left(\int \left(-\frac{fx\log\left(c(d+ex)^p\right)}{2\sqrt{-f}\sqrt{g}\left(\sqrt{-f}\sqrt{g}-fx^2\right)} - \frac{fx\log\left(c(d+\right.}{2\sqrt{-f}\sqrt{g}\left(\sqrt{-f}\right.}\right.\right.\right.$$

$$= -\frac{\sqrt{-f}\,\text{Subst}\left(\int \frac{x\log(c(d+ex)^p)}{\sqrt{-f}\sqrt{g}-fx^2}\,dx, x, \frac{1}{\sqrt{x}}\right)}{\sqrt{g}} - \frac{\sqrt{-f}\,\text{Subst}\left(\int \frac{x\log(c(d+e}{\sqrt{-f}\sqrt{g}}\right.}{\sqrt{g}}$$

$$= -\frac{\sqrt{-f}\,\text{Subst}\left(\int \left(\frac{\sqrt{-\sqrt{-f}}\log(c(d+ex)^p)}{2f\left(\sqrt[4]{g}-\sqrt{-\sqrt{-f}}\,x\right)} - \frac{\sqrt{-\sqrt{-f}}\log(c(d+ex)^p)}{2f\left(\sqrt[4]{g}+\sqrt{-\sqrt{-f}}\,x\right)}\right)dx\right.}{\sqrt{g}}$$

$$= -\frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt[4]{g}-\sqrt{-\sqrt{-f}}\,x}\,dx, x, \frac{1}{\sqrt{x}}\right)}{2\sqrt{-\sqrt{-f}}\sqrt{g}} + \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt[4]{g}+\sqrt{-\sqrt{-f}}\,x}\right.}{2\sqrt{-\sqrt{-f}}\sqrt{g}}$$

$$= -\frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{g}-\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right.}{}$$

$$= -\frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{g}-\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right.}{}$$

$$= -\frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{g}-\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.39, size = 912, normalized size = 1.63

$$\frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(-\sqrt[4]{-f}-\sqrt[4]{g}\sqrt{x}\right)-p\log\left(-\frac{\sqrt[4]{g}\left(e+d\sqrt{x}\right)}{d\sqrt[4]{-f}-e\sqrt[4]{g}}\right)\log\left(-\sqrt[4]{-f}-\sqrt[4]{g}\sqrt{x}\right)-\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(-i\sqrt[4]{-f}-\sqrt[4]{g}\sqrt{x}\right)+p\log\left(\frac{i\sqrt[4]{g}\left(e+d\sqrt{x}\right)}{d\sqrt[4]{-f}+ie\sqrt[4]{g}}\right)\log\left(-i\sqrt[4]{-f}-\sqrt[4]{g}\sqrt{x}\right)-\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(i\sqrt[4]{-f}-\sqrt[4]{g}\sqrt{x}\right)+p\log\left(\frac{\sqrt[4]{g}\left(e+d\sqrt{x}\right)}{id\sqrt[4]{-f}+e\sqrt[4]{g}}\right)\log\left(i\sqrt[4]{-f}-\sqrt[4]{g}\sqrt{x}\right)+\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(\sqrt[4]{-f}-\sqrt[4]{g}\sqrt{x}\right)-p\log\left(\frac{\sqrt[4]{g}\left(e+d\sqrt{x}\right)}{d\sqrt[4]{-f}+e\sqrt[4]{g}}\right)\log\left(\sqrt[4]{-f}-\sqrt[4]{g}\sqrt{x}\right)-p\log\left(i\sqrt[4]{-f}-\sqrt[4]{g}\sqrt{x}\right)\log\left(-\frac{i\sqrt[4]{g}\sqrt{x}}{\sqrt[4]{-f}}\right)-p\log\left(-i\sqrt[4]{-f}-\sqrt[4]{g}\sqrt{x}\right)\log\left(\frac{i\sqrt[4]{g}\sqrt{x}}{\sqrt[4]{-f}}\right)+p\log\left(\sqrt[4]{-f}-\sqrt[4]{g}\sqrt{x}\right)\log\left(\frac{\sqrt[4]{g}\sqrt{x}}{\sqrt[4]{-f}}\right)+p\log\left(-\sqrt[4]{-f}-\sqrt[4]{g}\sqrt{x}\right)\log\left(\frac{f\sqrt[4]{g}\sqrt{x}}{(-f)^{5/4}}\right)-p\,\mathrm{Li}_2\left(\frac{d\left(\sqrt[4]{-f}-\sqrt[4]{g}\sqrt{x}\right)}{d\sqrt[4]{-f}+e\sqrt[4]{g}}\right)+p\,\mathrm{Li}_2\left(\frac{d\left(\sqrt[4]{-f}-i\sqrt[4]{g}\sqrt{x}\right)}{d\sqrt[4]{-f}+ie\sqrt[4]{g}}\right)+p\,\mathrm{Li}_2\left(\frac{d\left(\sqrt[4]{-f}+i\sqrt[4]{g}\sqrt{x}\right)}{d\sqrt[4]{-f}-ie\sqrt[4]{g}}\right)-p\,\mathrm{Li}_2\left(\frac{d\left(\sqrt[4]{-f}+\sqrt[4]{g}\sqrt{x}\right)}{d\sqrt[4]{-f}-e\sqrt[4]{g}}\right)-p\,\mathrm{Li}_2\left(1-\frac{i\sqrt[4]{g}\sqrt{x}}{\sqrt[4]{-f}}\right)-p\,\mathrm{Li}_2\left(1+\frac{i\sqrt[4]{g}\sqrt{x}}{\sqrt[4]{-f}}\right)+p\,\mathrm{Li}_2\left(1+\frac{\sqrt[4]{g}\sqrt{x}}{\sqrt[4]{-f}}\right)+p\,\mathrm{Li}_2\left(1+\frac{f\sqrt[4]{g}\sqrt{x}}{(-f)^{5/4}}\right)}{2\sqrt{-f}\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e/Sqrt[x])^p]/(f + g*x^2),x]
```

```
[Out] (Log[c*(d + e/Sqrt[x])^p]*Log[-(-f)^(1/4) - g^(1/4)*Sqrt[x]] - p*Log[-((g^(
1/4)*(e + d*Sqrt[x]))/(d*(-f)^(1/4) - e*g^(1/4)))]*Log[-(-f)^(1/4) - g^(1/4
)*Sqrt[x]] - Log[c*(d + e/Sqrt[x])^p]*Log[(-I)*(-f)^(1/4) - g^(1/4)*Sqrt[x]
] + p*Log[(I*g^(1/4)*(e + d*Sqrt[x]))/(d*(-f)^(1/4) + I*e*g^(1/4))]*Log[(-I
)*(-f)^(1/4) - g^(1/4)*Sqrt[x]] - Log[c*(d + e/Sqrt[x])^p]*Log[I*(-f)^(1/4)
 - g^(1/4)*Sqrt[x]] + p*Log[(g^(1/4)*(e + d*Sqrt[x]))/(I*d*(-f)^(1/4) + e*g
^(1/4))]*Log[I*(-f)^(1/4) - g^(1/4)*Sqrt[x]] + Log[c*(d + e/Sqrt[x])^p]*Log
[(-f)^(1/4) - g^(1/4)*Sqrt[x]] - p*Log[(g^(1/4)*(e + d*Sqrt[x]))/(d*(-f)^(1
/4) + e*g^(1/4))]*Log[(-f)^(1/4) - g^(1/4)*Sqrt[x]] - p*Log[I*(-f)^(1/4) -
g^(1/4)*Sqrt[x]]*Log[((-I)*g^(1/4)*Sqrt[x])/(-f)^(1/4)] - p*Log[(-I)*(-f)^(
1/4) - g^(1/4)*Sqrt[x]]*Log[(I*g^(1/4)*Sqrt[x])/(-f)^(1/4)] + p*Log[(-f)^(1
/4) - g^(1/4)*Sqrt[x]]*Log[(g^(1/4)*Sqrt[x])/(-f)^(1/4)] + p*Log[-(-f)^(1/4
) - g^(1/4)*Sqrt[x]]*Log[(f*g^(1/4)*Sqrt[x])/(-f)^(5/4)] - p*PolyLog[2, (d*
((-f)^(1/4) - g^(1/4)*Sqrt[x]))/(d*(-f)^(1/4) + e*g^(1/4))] + p*PolyLog[2,
(d*((-f)^(1/4) - I*g^(1/4)*Sqrt[x]))/(d*(-f)^(1/4) + I*e*g^(1/4))] + p*Poly
Log[2, (d*((-f)^(1/4) + I*g^(1/4)*Sqrt[x]))/(d*(-f)^(1/4) - I*e*g^(1/4))] -
 p*PolyLog[2, (d*((-f)^(1/4) + g^(1/4)*Sqrt[x]))/(d*(-f)^(1/4) - e*g^(1/4))
] - p*PolyLog[2, 1 - (I*g^(1/4)*Sqrt[x])/(-f)^(1/4)] - p*PolyLog[2, 1 + (I*
g^(1/4)*Sqrt[x])/(-f)^(1/4)] + p*PolyLog[2, 1 + (g^(1/4)*Sqrt[x])/(-f)^(1/4
)] + p*PolyLog[2, 1 + (f*g^(1/4)*Sqrt[x])/(-f)^(5/4)])/(2*Sqrt[-f]*Sqrt[g])
```

Maple [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{p}\right)}{g\,x^{2}+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e/x^(1/2))^p)/(g*x^2+f),x)
```

```
[Out] int(ln(c*(d+e/x^(1/2))^p)/(g*x^2+f),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/x^(1/2))^p)/(g*x^2+f),x, algorithm="maxima")
[Out] integrate(log(c*(d + e/sqrt(x))^p)/(g*x^2 + f), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/x^(1/2))^p)/(g*x^2+f),x, algorithm="fricas")
[Out] integral(log(c*((d*x + sqrt(x)*e)/x)^p)/(g*x^2 + f), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e/x**(1/2))**p)/(g*x**2+f),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/x^(1/2))^p)/(g*x^2+f),x, algorithm="giac")
[Out] integrate(log(c*(d + e/sqrt(x))^p)/(g*x^2 + f), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(d+\frac{e}{\sqrt{x}}\right)}^p\right)}{g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e/x^(1/2))^p)/(f + g*x^2),x)
[Out] int(log(c*(d + e/x^(1/2))^p)/(f + g*x^2), x)
```

3.268 $\int (f+gx^2)^3 \log\left(c(d+ex^2)^p\right)\,dx$

Optimal. Leaf size=338

$$-2f^3px+\frac{2df^2gpx}{e}-\frac{6d^2fg^2px}{5e^2}+\frac{2d^3g^3px}{7e^3}-\frac{2}{3}f^2gpx^3+\frac{2dfg^2px^3}{5e}-\frac{2d^2g^3px^3}{21e^2}-\frac{6}{25}fg^2px^5+\frac{2dg^3px^5}{35e}-\frac{2}{49}g^3px^7+$$

```
[Out] -2*f^3*p*x+2*d*f^2*g*p*x/e-6/5*d^2*f*g^2*p*x/e^2+2/7*d^3*g^3*p*x/e^3-2/3*f^
2*g*p*x^3+2/5*d*f*g^2*p*x^3/e-2/21*d^2*g^3*p*x^3/e^2-6/25*f*g^2*p*x^5+2/35*
d*g^3*p*x^5/e-2/49*g^3*p*x^7-2*d^(3/2)*f^2*g*p*arctan(x*e^(1/2)/d^(1/2))/e^
(3/2)+6/5*d^(5/2)*f*g^2*p*arctan(x*e^(1/2)/d^(1/2))/e^(5/2)-2/7*d^(7/2)*g^3
*p*arctan(x*e^(1/2)/d^(1/2))/e^(7/2)+f^3*x*ln(c*(e*x^2+d)^p)+f^2*g*x^3*ln(c
*(e*x^2+d)^p)+3/5*f*g^2*x^5*ln(c*(e*x^2+d)^p)+1/7*g^3*x^7*ln(c*(e*x^2+d)^p)
+2*f^3*p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2521, 2498, 327, 211, 2505, 308}

$$-\frac{2d^{3/2}f^2gp\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{3/2}}+\frac{6d^{5/2}fg^2p\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{5e^{5/2}}-\frac{2d^{7/2}g^3p\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{7e^{7/2}}+\frac{2\sqrt{d}\,f^3p\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}+f^3x\log\left(c(d+ex^2)^p\right)+f^2gx^3\log\left(c(d+ex^2)^p\right)+\frac{3}{5}fg^2x^5\log\left(c(d+ex^2)^p\right)+\frac{1}{7}g^3x^7\log\left(c(d+ex^2)^p\right)+\frac{2d^3g^3px}{7e^3}-\frac{6d^2fg^2px}{5e^2}-\frac{2d^2g^3px^3}{21e^2}+\frac{2df^2gpx}{e}+\frac{2dfg^2px^3}{5e}+\frac{2dg^3px^5}{35e}-2f^3px-\frac{2}{3}f^2gpx^3-\frac{6}{25}fg^2px^5-\frac{2}{49}g^3px^7$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^2)^3*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f^3*p*x + (2*d*f^2*g*p*x)/e - (6*d^2*f*g^2*p*x)/(5*e^2) + (2*d^3*g^3*p*x
)/(7*e^3) - (2*f^2*g*p*x^3)/3 + (2*d*f*g^2*p*x^3)/(5*e) - (2*d^2*g^3*p*x^3)
/(21*e^2) - (6*f*g^2*p*x^5)/25 + (2*d*g^3*p*x^5)/(35*e) - (2*g^3*p*x^7)/49
+ (2*Sqrt[d]*f^3*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(3/2)*f^2*g*
p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e^(3/2) + (6*d^(5/2)*f*g^2*p*ArcTan[(Sqrt[e]
*x)/Sqrt[d]])/(5*e^(5/2)) - (2*d^(7/2)*g^3*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(
7*e^(7/2)) + f^3*x*Log[c*(d + e*x^2)^p] + f^2*g*x^3*Log[c*(d + e*x^2)^p] +
(3*f*g^2*x^5*Log[c*(d + e*x^2)^p])/5 + (g^3*x^7*Log[c*(d + e*x^2)^p])/7
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$\int (f+gx^2)^3 \log \left(c(d+ex^2)^p\right) dx = \int \left(f^3 \log \left(c(d+ex^2)^p\right) + 3f^2gx^2 \log \left(c(d+ex^2)^p\right) + 3fg^2x^4 \log \left(c(d\right.\right.$$

$$= f^3 \int \log \left(c(d+ex^2)^p\right) dx + (3f^2g) \int x^2 \log \left(c(d+ex^2)^p\right) dx + (3fg$$

$$= f^3x \log \left(c(d+ex^2)^p\right) + f^2gx^3 \log \left(c(d+ex^2)^p\right) + \frac{3}{5}fg^2x^5 \log \left(c(d+e\right.$$

$$= -2f^3px + f^3x \log \left(c(d+ex^2)^p\right) + f^2gx^3 \log \left(c(d+ex^2)^p\right) + \frac{3}{5}fg^2x^5 \text{lo}$$

$$= -2f^3px + \frac{2df^2gpx}{e} - \frac{6d^2fg^2px}{5e^2} + \frac{2d^3g^3px}{7e^3} - \frac{2}{3}f^2gpx^3 + \frac{2dfg^2px^3}{5e} -$$

$$= -2f^3px + \frac{2df^2gpx}{e} - \frac{6d^2fg^2px}{5e^2} + \frac{2d^3g^3px}{7e^3} - \frac{2}{3}f^2gpx^3 + \frac{2dfg^2px^3}{5e} -$$

Mathematica [A]
time = 0.19, size = 215, normalized size = 0.64

$$-\frac{2px(-525d^3g^3+35d^2eg^2(63f+5gx^2)-105de^2g(35f^2+7fgx^2+g^2x^4)+e^3(3675f^3+1225f^2gx^2+441fg^2x^4+75g^3x^6))}{3675e^3} - \frac{2\sqrt{d}\,(-35e^3f^3+35de^2f^2g-21d^2efg^2+5d^3g^3)\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{35e^{7/2}} + \frac{1}{35}x(35f^3+35f^2gx^2+21fg^2x^4+5g^3x^6)\log\left(c(d+ex^2)^p\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^2)^3*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (-2*p*x*(-525*d^3*g^3 + 35*d^2*e*g^2*(63*f + 5*g*x^2) - 105*d*e^2*g*(35*f^2
 + 7*f*g*x^2 + g^2*x^4) + e^3*(3675*f^3 + 1225*f^2*g*x^2 + 441*f*g^2*x^4 +
75*g^3*x^6)))/(3675*e^3) - (2*Sqrt[d]*(-35*e^3*f^3 + 35*d*e^2*f^2*g - 21*d^
2*e*f*g^2 + 5*d^3*g^3)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(35*e^(7/2)) + (x*(35
*f^3 + 35*f^2*g*x^2 + 21*f*g^2*x^4 + 5*g^3*x^6)*Log[c*(d + e*x^2)^p])/35
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.65, size = 995, normalized size = 2.94

method	result
risch	$-\frac{\sqrt{-ed}\;p\ln\left(\sqrt{-ed}\;x-d\right)d^3g^3}{7e^4} + \left(\frac{1}{7}g^3x^7 + \frac{3}{5}f\,g^2x^5 + f^2g\,x^3 + f^3x\right)\ln\left(\left(e\,x^2+d\right)^p\right) + \frac{i\pi\,g^3x^7\text{csgn}(ic(e}{1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^3*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] -1/7/e^4*(-e*d)^(1/2)*p*ln((-e*d)^(1/2)*x-d)*d^3*g^3+1/14*I*Pi*g^3*x^7*csgn
(I*c*(e*x^2+d)^p)^2*csgn(I*c)-3/10*I*Pi*f*g^2*x^5*csgn(I*c*(e*x^2+d)^p)^3+1
```

/14*I*Pi*g^3*x^7*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/2*I*Pi*f^3*c
sgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*x+1/2*I*Pi*f^3*csgn(I*c*(e*x^2+d
)^p)^2*csgn(I*c)*x-1/2*I*Pi*f^2*g*x^3*csgn(I*c*(e*x^2+d)^p)^3+(1/7*g^3*x^7+
3/5*f*g^2*x^5+f^2*g*x^3+f^3*x)*ln((e*x^2+d)^p)+1/e*(-e*d)^(1/2)*p*ln((-e*d)
^(1/2)*x-d)*f^3-1/e*(-e*d)^(1/2)*p*ln(-(-e*d)^(1/2)*x-d)*f^3+2/35*d*g^3*p*x
^5/e+2*d*f^2*g*p*x/e-6/5*d^2*f*g^2*p*x/e^2+2/5*d*f*g^2*p*x^3/e-1/e^2*(-e*d)
^(1/2)*p*ln((-e*d)^(1/2)*x-d)*d*f^2*g-3/5/e^3*(-e*d)^(1/2)*p*ln(-(-e*d)^(1/
2)*x-d)*d^2*f*g^2+1/e^2*(-e*d)^(1/2)*p*ln(-(-e*d)^(1/2)*x-d)*d*f^2*g+3/5/e^
3*(-e*d)^(1/2)*p*ln((-e*d)^(1/2)*x-d)*d^2*f*g^2-1/14*I*Pi*g^3*x^7*csgn(I*(e
*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+3/10*I*Pi*f*g^2*x^5*csgn(I*c*(e*
x^2+d)^p)^2*csgn(I*c)-1/14*I*Pi*g^3*x^7*csgn(I*c*(e*x^2+d)^p)^3-1/2*I*Pi*f^
3*csgn(I*c*(e*x^2+d)^p)^3*x-1/2*I*Pi*f^2*g*x^3*csgn(I*(e*x^2+d)^p)*csgn(I*c
*(e*x^2+d)^p)*csgn(I*c)-3/10*I*Pi*f*g^2*x^5*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e
*x^2+d)^p)*csgn(I*c)+3/5*ln(c)*f*g^2*x^5+ln(c)*f^2*g*x^3-2/3*f^2*g*p*x^3-6/
25*f*g^2*p*x^5-2*f^3*p*x-2/49*g^3*p*x^7+1/7/e^4*(-e*d)^(1/2)*p*ln(-(-e*d)^(
1/2)*x-d)*d^3*g^3+1/2*I*Pi*f^2*g*x^3*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/2*
I*Pi*f^3*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*x+3/10*I*Pi*f*
g^2*x^5*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/2*I*Pi*f^2*g*x^3*csgn
(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+2/7*d^3*g^3*p*x/e^3-2/21*d^2*g^3*p*
x^3/e^2+1/7*ln(c)*g^3*x^7+ln(c)*f^3*x

Maxima [A]
time = 0.51, size = 217, normalized size = 0.64

$$-\frac{2}{3675}\left(\frac{105\left(5d^4g^3-21d^3fg^2e+35d^2f^2ge^2-35df^3e^3\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{\sqrt{d}}+\left(75g^3x^7e^3-21\left(5dg^3e^2-21fg^2e^3\right)x^5+35\left(5d^2g^3e-21dfg^2e^2+35f^2ge^3\right)x^3-105\left(5d^3g^3-21d^2fg^2e+35df^2ge^2-35f^3e^3\right)x\right)e^{(-4)}\right)pe+\frac{1}{35}\left(5g^3x^7+21fg^2x^5+35f^2gx^3+35f^3x\right)\log\left(\left(x^2e+d\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^2+f)^3*log(c*(e*x^2+d)^p),x, algorithm="maxima")

[Out] -2/3675*(105*(5*d^4*g^3 - 21*d^3*f*g^2*e + 35*d^2*f^2*g*e^2 - 35*d*f^3*e^3)
*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/sqrt(d) + (75*g^3*x^7*e^3 - 21*(5*d*g^3
*e^2 - 21*f*g^2*e^3)*x^5 + 35*(5*d^2*g^3*e - 21*d*f*g^2*e^2 + 35*f^2*g*e^3)
x^3 - 105(5*d^3*g^3 - 21*d^2*f*g^2*e + 35*d*f^2*g*e^2 - 35*f^3*e^3)*x)*e^
(-4))*p*e + 1/35*(5*g^3*x^7 + 21*f*g^2*x^5 + 35*f^2*g*x^3 + 35*f^3*x)*log((
x^2*e + d)^p*c)

Fricas [A]
time = 0.41, size = 553, normalized size = 1.64

$$\left[\frac{1}{3675}\left(1050d^3g^3px+105\left(5g^3px^7+21fg^2px^5+35f^2gpx^3+35f^3px\right)e^3\log\left(x^2e+d\right)+105\left(5g^3x^7+21fg^2x^5+35f^2gx^3+35f^3x\right)e^3\log\left(c\right)-105\left(5d^3g^3p-21d^2fg^2pe+35df^2gpe^2-35f^3pe^3\right)\sqrt{-de^{(-1)}}\log\left(\frac{x^2e+2\sqrt{-de^{(-1)}}xe-d}{x^2e+d}\right)-2\left(75g^3px^7+441fg^2px^5+1225f^2gpx^3+3675f^3px\right)e^3+210\left(dg^3px^5+7dfg^2px^3+35df^2gpx\right)e^2-70\left(5d^2g^3px^3+63d^2fg^2px\right)e\right)e^{(-3)}, \frac{1}{3675}\left(1050d^3g^3px-210\left(5d^3g^3p-21d^2fg^2pe+35df^2gpe^2-35f^3pe^3\right)\sqrt{d}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}+105\left(5g^3px^7+21fg^2px^5+35f^2gpx^3+35f^3px\right)e^3\log\left(x^2e+d\right)+105\left(5g^3x^7+21fg^2x^5+35f^2gx^3+35f^3x\right)e^3\log\left(c\right)-2\left(75g^3px^7+441fg^2px^5+1225f^2gpx^3+3675f^3px\right)e^3+210\left(dg^3px^5+7dfg^2px^3+35df^2gpx\right)e^2-70\left(5d^2g^3px^3+63d^2fg^2px\right)e\right)e^{(-3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^2+f)^3*log(c*(e*x^2+d)^p),x, algorithm="fricas")

[Out] [1/3675*(1050*d^3*g^3*p*x + 105*(5*g^3*p*x^7 + 21*f*g^2*p*x^5 + 35*f^2*g*p*
x^3 + 35*f^3*p*x)*e^3*log(x^2*e + d) + 105*(5*g^3*x^7 + 21*f*g^2*x^5 + 35*f

```
^2*g*x^3 + 35*f^3*x)*e^3*log(c) - 105*(5*d^3*g^3*p - 21*d^2*f*g^2*p*e + 35*
d*f^2*g*p*e^2 - 35*f^3*p*e^3)*sqrt(-d*e^(-1))*log((x^2*e + 2*sqrt(-d*e^(-1)
)*x*e - d)/(x^2*e + d)) - 2*(75*g^3*p*x^7 + 441*f*g^2*p*x^5 + 1225*f^2*g*p*
x^3 + 3675*f^3*p*x)*e^3 + 210*(d*g^3*p*x^5 + 7*d*f*g^2*p*x^3 + 35*d*f^2*g*p
*x)*e^2 - 70*(5*d^2*g^3*p*x^3 + 63*d^2*f*g^2*p*x)*e)*e^(-3), 1/3675*(1050*d
^3*g^3*p*x - 210*(5*d^3*g^3*p - 21*d^2*f*g^2*p*e + 35*d*f^2*g*p*e^2 - 35*f^
3*p*e^3)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2) + 105*(5*g^3*p*x^7 + 21
*f*g^2*p*x^5 + 35*f^2*g*p*x^3 + 35*f^3*p*x)*e^3*log(x^2*e + d) + 105*(5*g^3
*x^7 + 21*f*g^2*x^5 + 35*f^2*g*x^3 + 35*f^3*x)*e^3*log(c) - 2*(75*g^3*p*x^7
 + 441*f*g^2*p*x^5 + 1225*f^2*g*p*x^3 + 3675*f^3*p*x)*e^3 + 210*(d*g^3*p*x^
5 + 7*d*f*g^2*p*x^3 + 35*d*f^2*g*p*x)*e^2 - 70*(5*d^2*g^3*p*x^3 + 63*d^2*f*
g^2*p*x)*e)*e^(-3)]
```

Sympy [A]
time = 133.19, size = 697, normalized size = 2.06

$$\begin{cases} \left(f^3x + f^2gx^3 + \frac{3fg^2x^5}{5} + \frac{g^3x^7}{7}\right)\log(0^pc) & \text{for } d = 0 \wedge e = 0 \\ -2f^3px + f^3x\log\left(c(ex^2)^p\right) - \frac{2f^2gpx^3}{3} + f^2gx^3\log\left(c(ex^2)^p\right) - \frac{6fg^2px^5}{25} + \frac{3fg^2x^5\log\left(c(ex^2)^p\right)}{5} - \frac{2g^3px^7}{49} + \frac{g^3x^7\log\left(c(ex^2)^p\right)}{7} & \text{for } d = 0 \\ \left(f^3x + f^2gx^3 + \frac{3fg^2x^5}{5} + \frac{g^3x^7}{7}\right)\log(cd^p) & \text{for } e = 0 \\ -\frac{2d^4g^3p\log\left(x-\sqrt{-\frac{d}{e}}\right)}{7e^4\sqrt{-\frac{d}{e}}} + \frac{d^4g^3\log\left(c(d+ex^2)^p\right)}{7e^4\sqrt{-\frac{d}{e}}} + \frac{6d^3fg^2p\log\left(x-\sqrt{-\frac{d}{e}}\right)}{5e^3\sqrt{-\frac{d}{e}}} - \frac{3d^3fg^2\log\left(c(d+ex^2)^p\right)}{5e^3\sqrt{-\frac{d}{e}}} + \frac{2d^3g^3px}{7e^3} - \frac{2d^2f^2gp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{e^2\sqrt{-\frac{d}{e}}} + \frac{d^2f^2g\log\left(c(d+ex^2)^p\right)}{e^2\sqrt{-\frac{d}{e}}} - \frac{6d^2fg^2px}{5e^2} - \frac{2d^2g^3px^3}{21e^2} + \frac{2df^3p\log\left(x-\sqrt{-\frac{d}{e}}\right)}{e\sqrt{-\frac{d}{e}}} - \frac{df^3\log\left(c(d+ex^2)^p\right)}{e\sqrt{-\frac{d}{e}}} + \frac{2df^2gpx}{e} + \frac{2dfg^2px^3}{5e} + \frac{2dg^3px^5}{35e} - 2f^3px + f^3x\log\left(c(d+ex^2)^p\right) - \frac{2f^2gpx^3}{3} + f^2gx^3\log\left(c(d+ex^2)^p\right) - \frac{6fg^2px^5}{25} + \frac{3fg^2x^5\log\left(c(d+ex^2)^p\right)}{5} - \frac{2g^3px^7}{49} + \frac{g^3x^7\log\left(c(d+ex^2)^p\right)}{7} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**3*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise(((f**3*x + f**2*g*x**3 + 3*f*g**2*x**5/5 + g**3*x**7/7)*log(0**p*
c), Eq(d, 0) & Eq(e, 0)), (-2*f**3*p*x + f**3*x*log(c*(e*x**2)**p) - 2*f**2
*g*p*x**3/3 + f**2*g*x**3*log(c*(e*x**2)**p) - 6*f*g**2*p*x**5/25 + 3*f*g**
2*x**5*log(c*(e*x**2)**p)/5 - 2*g**3*p*x**7/49 + g**3*x**7*log(c*(e*x**2)**
p)/7, Eq(d, 0)), ((f**3*x + f**2*g*x**3 + 3*f*g**2*x**5/5 + g**3*x**7/7)*lo
g(c*d**p), Eq(e, 0)), (-2*d**4*g**3*p*log(x - sqrt(-d/e))/(7*e**4*sqrt(-d/e
)) + d**4*g**3*log(c*(d + e*x**2)**p)/(7*e**4*sqrt(-d/e)) + 6*d**3*f*g**2*p
*log(x - sqrt(-d/e))/(5*e**3*sqrt(-d/e)) - 3*d**3*f*g**2*log(c*(d + e*x**2)
**p)/(5*e**3*sqrt(-d/e)) + 2*d**3*g**3*p*x/(7*e**3) - 2*d**2*f**2*g*p*log(x
 - sqrt(-d/e))/(e**2*sqrt(-d/e)) + d**2*f**2*g*log(c*(d + e*x**2)**p)/(e**2
*sqrt(-d/e)) - 6*d**2*f*g**2*p*x/(5*e**2) - 2*d**2*g**3*p*x**3/(21*e**2) +
2*d*f**3*p*log(x - sqrt(-d/e))/(e*sqrt(-d/e)) - d*f**3*log(c*(d + e*x**2)**
p)/(e*sqrt(-d/e)) + 2*d*f**2*g*p*x/e + 2*d*f*g**2*p*x**3/(5*e) + 2*d*g**3*p
*x**5/(35*e) - 2*f**3*p*x + f**3*x*log(c*(d + e*x**2)**p) - 2*f**2*g*p*x**3
/3 + f**2*g*x**3*log(c*(d + e*x**2)**p) - 6*f*g**2*p*x**5/25 + 3*f*g**2*x**
5*log(c*(d + e*x**2)**p)/5 - 2*g**3*p*x**7/49 + g**3*x**7*log(c*(d + e*x**2
)**p)/7, True))
```

Giac [A]
time = 5.89, size = 309, normalized size = 0.91

$$-\frac{2\,(5\,d^4g^3p - 21\,d^3fg^2pe + 35\,d^2f^2gpe^2 - 35\,df^3pe^3)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{7}{2}\right)}}{35\,\sqrt{d}} + \frac{1}{3675}\,(525\,g^3px^7e^3\log(x^2e+d) - 150\,g^3px^7e^3 + 525\,g^3x^7e^3\log(c) + 210\,dg^3px^5e^2 + 2205\,fg^2px^5e^3\log(x^2e+d) - 882\,fg^2px^5e^3 - 350\,d^2g^3px^3e + 2205\,fg^2x^5e^3\log(c) + 1470\,dfg^2px^3e^2 + 3675\,f^2gpx^3e^3\log(x^2e+d) + 1050\,d^3g^3px - 2450\,f^2gpx^3e^3 - 4410\,d^2fg^2pxe + 3675\,f^2gx^3e^3\log(c) + 7350\,df^2gpxe^2 + 3675\,f^3pxe^3\log(x^2e+d) - 7350\,f^3pxe^3 + 3675\,f^3xe^3\log(c))e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^3*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] -2/35*(5*d^4*g^3*p - 21*d^3*f*g^2*p*e + 35*d^2*f^2*g*p*e^2 - 35*d*f^3*p*e^3
)*arctan(x*e^(1/2)/sqrt(d))*e^(-7/2)/sqrt(d) + 1/3675*(525*g^3*p*x^7*e^3*lo
g(x^2*e + d) - 150*g^3*p*x^7*e^3 + 525*g^3*x^7*e^3*log(c) + 210*d*g^3*p*x^5
*e^2 + 2205*f*g^2*p*x^5*e^3*log(x^2*e + d) - 882*f*g^2*p*x^5*e^3 - 350*d^2*
g^3*p*x^3*e + 2205*f*g^2*x^5*e^3*log(c) + 1470*d*f*g^2*p*x^3*e^2 + 3675*f^2
*g*p*x^3*e^3*log(x^2*e + d) + 1050*d^3*g^3*p*x - 2450*f^2*g*p*x^3*e^3 - 441
0*d^2*f*g^2*p*x*e + 3675*f^2*g*x^3*e^3*log(c) + 7350*d*f^2*g*p*x*e^2 + 3675
*f^3*p*x*e^3*log(x^2*e + d) - 7350*f^3*p*x*e^3 + 3675*f^3*x*e^3*log(c))*e^(
-3)
```

Mupad [B]
time = 0.38, size = 298, normalized size = 0.88

$$x^3\left(\frac{d\left(\frac{6\,f\,g^2\,p}{5}-\frac{2\,d\,g^3\,p}{7\,e}\right)}{3\,e}-\frac{2\,f^2\,g\,p}{3}\right)-x\left(2\,f^3\,p+\frac{d\left(\frac{d\left(\frac{6\,f\,g^2\,p}{5}-\frac{2\,d\,g^3\,p}{7\,e}\right)}{e}-2\,f^2\,g\,p\right)}{e}\right)-x^5\left(\frac{6\,f\,g^2\,p}{25}-\frac{2\,d\,g^3\,p}{35\,e}\right)+\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(f^3\,x+f^2\,g\,x^3+\frac{3\,f\,g^2\,x^5}{5}+\frac{g^3\,x^7}{7}\right)-\frac{2\,g^3\,p\,x^7}{49}-\frac{2\,\sqrt{d}\,p\,\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e}\,p\,x\,\left(5\,d^3\,g^3-21\,d^2\,e\,f\,g^2+35\,d\,e^2\,f^2\,g-35\,e^3\,f^3\right)}{5\,p\,d^4\,g^3-21\,p\,d^3\,e\,f\,g^2+35\,p\,d^2\,e^2\,f^2\,g-35\,p\,d\,e^3\,f^3}\right)\,\left(5\,d^3\,g^3-21\,d^2\,e\,f\,g^2+35\,d\,e^2\,f^2\,g-35\,e^3\,f^3\right)}{35\,e^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)*(f + g*x^2)^3,x)
```

```
[Out] x^3*((d*((6*f*g^2*p)/5 - (2*d*g^3*p)/(7*e)))/(3*e) - (2*f^2*g*p)/3) - x*(2*
f^3*p + (d*((d*((6*f*g^2*p)/5 - (2*d*g^3*p)/(7*e)))/e - 2*f^2*g*p))/e) - x^
5*((6*f*g^2*p)/25 - (2*d*g^3*p)/(35*e)) + log(c*(d + e*x^2)^p)*(f^3*x + (g^
3*x^7)/7 + f^2*g*x^3 + (3*f*g^2*x^5)/5) - (2*g^3*p*x^7)/49 - (2*d^(1/2)*p*a
tan((d^(1/2)*e^(1/2)*p*x*(5*d^3*g^3 - 35*e^3*f^3 + 35*d*e^2*f^2*g - 21*d^2*
e*f*g^2))/(5*d^4*g^3*p - 35*d*e^3*f^3*p - 21*d^3*e*f*g^2*p + 35*d^2*e^2*f^2
*g*p))*(5*d^3*g^3 - 35*e^3*f^3 + 35*d*e^2*f^2*g - 21*d^2*e*f*g^2))/(35*e^(7
/2))
```

3.269 $\int (f+gx^2)^2 \log \left(c(d+ex^2)^p\right) \, dx$

Optimal. Leaf size=221

$$-2f^2px+\frac{4dfgpx}{3e}-\frac{2d^2g^2px}{5e^2}-\frac{4}{9}fgpx^3+\frac{2dg^2px^3}{15e}-\frac{2}{25}g^2px^5+\frac{2\sqrt{d}\,f^2p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{4d^{3/2}fgp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3e^{3/2}}$$

```
[Out] -2*f^2*p*x+4/3*d*f*g*p*x/e-2/5*d^2*g^2*p*x/e^2-4/9*f*g*p*x^3+2/15*d*g^2*p*x
^3/e-2/25*g^2*p*x^5-4/3*d^(3/2)*f*g*p*arctan(x*e^(1/2)/d^(1/2))/e^(3/2)+2/5
*d^(5/2)*g^2*p*arctan(x*e^(1/2)/d^(1/2))/e^(5/2)+f^2*x*ln(c*(e*x^2+d)^p)+2/
3*f*g*x^3*ln(c*(e*x^2+d)^p)+1/5*g^2*x^5*ln(c*(e*x^2+d)^p)+2*f^2*p*arctan(x*
e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2521, 2498, 327, 211, 2505, 308}

$$-\frac{4d^{3/2}fgp\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2d^{5/2}g^2p\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{5e^{5/2}}+\frac{2\sqrt{d}\,f^2p\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}+f^2x\log\left(c(d+ex^2)^p\right)+\frac{2}{3}fgx^3\log\left(c(d+ex^2)^p\right)+\frac{1}{5}g^2x^5\log\left(c(d+ex^2)^p\right)-\frac{2d^2g^2px}{5e^2}+\frac{4dfgpx}{3e}+\frac{2dg^2px^3}{15e}-2f^2px-\frac{4}{9}fgpx^3-\frac{2}{25}g^2px^5$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f^2*p*x + (4*d*f*g*p*x)/(3*e) - (2*d^2*g^2*p*x)/(5*e^2) - (4*f*g*p*x^3)/
9 + (2*d*g^2*p*x^3)/(15*e) - (2*g^2*p*x^5)/25 + (2*Sqrt[d]*f^2*p*ArcTan[(Sq
rt[e]*x)/Sqrt[d]])/Sqrt[e] - (4*d^(3/2)*f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/
(3*e^(3/2)) + (2*d^(5/2)*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*e^(5/2)) + f
^2*x*Log[c*(d + e*x^2)^p] + (2*f*g*x^3*Log[c*(d + e*x^2)^p])/3 + (g^2*x^5*L
og[c*(d + e*x^2)^p])/5
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$\int (f+gx^2)^2 \log\left(c\left(d+ex^2\right)^p\right)\,dx = \int \left(f^2\log\left(c\left(d+ex^2\right)^p\right)+2fgx^2\log\left(c\left(d+ex^2\right)^p\right)+g^2x^4\log\left(c(d+\right.\right.$$

$$= f^2\int \log\left(c\left(d+ex^2\right)^p\right)\,dx+(2fg)\int x^2\log\left(c\left(d+ex^2\right)^p\right)\,dx+g^2\int$$

$$= f^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}g^2x^5\log\left(c(d+\right.$$

$$= -2f^2px+f^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}g^2x^5$$

$$= -2f^2px+\frac{4dfgpx}{3e}-\frac{2d^2g^2px}{5e^2}-\frac{4}{9}fgpx^3+\frac{2dg^2px^3}{15e}-\frac{2}{25}g^2px^5+\frac{2\sqrt{}}{}$$

$$= -2f^2px+\frac{4dfgpx}{3e}-\frac{2d^2g^2px}{5e^2}-\frac{4}{9}fgpx^3+\frac{2dg^2px^3}{15e}-\frac{2}{25}g^2px^5+\frac{2\sqrt{}}{}$$

Mathematica [A]
time = 0.09, size = 151, normalized size = 0.68

$$\frac{30\sqrt{d}\,(15e^2f^2-10defg+3d^2g^2)\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)+\sqrt{e}\,x(-2p(45d^2g^2-15deg(10f+gx^2)+e^2(225f^2+50fgx^2+9g^2x^4))+15e^2(15f^2+10fgx^2+3g^2x^4)\log\left(c(d+ex^2)^p\right))}{225e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (30*Sqrt[d]*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*p*ArcTan[(Sqrt[e]*x)/Sqrt
[d]] + Sqrt[e]*x*(-2*p*(45*d^2*g^2 - 15*d*e*g*(10*f + g*x^2) + e^2*(225*f^2
 + 50*f*g*x^2 + 9*g^2*x^4)) + 15*e^2*(15*f^2 + 10*f*g*x^2 + 3*g^2*x^4)*Log[
c*(d + e*x^2)^p]))/(225*e^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.71, size = 686, normalized size = 3.10

method	result
risch	$-\frac{i\pi g^2x^5\operatorname{csgn}(i(e\,x^2+d)^p)\operatorname{csgn}(ic(e\,x^2+d)^p)\operatorname{csgn}(ic)}{10}+\frac{i\pi f g\,x^3\operatorname{csgn}(i(e\,x^2+d)^p)\operatorname{csgn}(ic(e\,x^2+d)^p)^2}{3}+\frac{i\pi f g\,x^3\operatorname{csgn}(ic(e\,x^2+d)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] (1/5*g^2*x^5+2/3*f*g*x^3+f^2*x)*ln((e*x^2+d)^p)+ln(c)*f^2*x+1/5*ln(c)*g^2*x
^5-2/3/e^2*(-e*d)^(1/2)*p*ln(-(-e*d)^(1/2)*x+d)*d*f*g+2/3/e^2*(-e*d)^(1/2)*
p*ln((-e*d)^(1/2)*x+d)*d*f*g+1/e*(-e*d)^(1/2)*p*ln(-(-e*d)^(1/2)*x+d)*f^2-1
/e*(-e*d)^(1/2)*p*ln((-e*d)^(1/2)*x+d)*f^2-1/10*I*Pi*g^2*x^5*csgn(I*c*(e*x^
2+d)^p)^3-1/2*I*Pi*f^2*csgn(I*c*(e*x^2+d)^p)^3*x-4/9*f*g*p*x^3-2*f^2*p*x-2/
25*g^2*p*x^5-2/5*d^2*g^2*p*x/e^2+2/15*d*g^2*p*x^3/e+1/5/e^3*(-e*d)^(1/2)*p*
ln(-(-e*d)^(1/2)*x+d)*d^2*g^2-1/5/e^3*(-e*d)^(1/2)*p*ln((-e*d)^(1/2)*x+d)*d
^2*g^2+1/10*I*Pi*g^2*x^5*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/10*I
*Pi*g^2*x^5*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/3*I*Pi*f*g*x^3*csgn(I*c*(e*
x^2+d)^p)^3+1/2*I*Pi*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*x+1/2*
I*Pi*f^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*x-1/3*I*Pi*f*g*x^3*csgn(I*(e*x^2
+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+4/3*d*f*g*p*x/e+2/3*ln(c)*f*g*x^3-1/
10*I*Pi*g^2*x^5*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/3*I*P
i*f*g*x^3*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/3*I*Pi*f*g*x^3*csgn
(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/2*I*Pi*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*
x^2+d)^p)*csgn(I*c)*x
```

Maxima [A]
time = 0.57, size = 146, normalized size = 0.66

$$\frac{2}{225}\left(\frac{15\left(3\,d^3g^2-10\,d^2fge+15\,df^2e^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{7}{2}\right)}}{\sqrt{d}}-\left(9\,g^2x^5e^2-5\left(3\,dg^2e-10\,fge^2\right)x^3+15\left(3\,d^2g^2-10\,dfge+15\,f^2e^2\right)x\right)e^{(-3)}\right)pe+\frac{1}{15}\left(3\,g^2x^5+10\,fgx^3+15\,f^2x\right)\log\left(\left(x^2e+d\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")

[Out] 2/225*(15*(3*d^3*g^2 - 10*d^2*f*g*e + 15*d*f^2*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-7/2)/sqrt(d) - (9*g^2*x^5*e^2 - 5*(3*d*g^2*e - 10*f*g*e^2)*x^3 + 15*(3*d^2*g^2 - 10*d*f*g*e + 15*f^2*e^2)*x)*e^(-3))*p*e + 1/15*(3*g^2*x^5 + 10*f*g*x^3 + 15*f^2*x)*log((x^2*e + d)^p*c)

Fricas [**A**]
time = 0.40, size = 373, normalized size = 1.69

$$\left[-\frac{1}{225}\left(90\,d^2g^2px - 15\left(3\,g^2px^5 + 10\,fgpx^3 + 15\,f^2px\right)e^2\log\left(x^2e + d\right) - 15\left(3\,g^2x^5 + 10\,fgx^3 + 15\,f^2x\right)e^2\log\left(c\right) - 15\left(3\,d^2g^2p - 10\,dfgpe + 15\,f^2pe^2\right)\sqrt{-de^{(-1)}}\log\left(\frac{x^2e + 2\,\sqrt{-de^{(-1)}}xe - d}{x^2e + d}\right) + 2\left(9\,g^2px^5 + 50\,fgpx^3 + 225\,f^2px\right)e^2 - 30\left(dg^2px^3 + 10\,dfgpx\right)e\right)e^{(-2)}, -\frac{1}{225}\left(90\,d^2g^2px - 30\left(3\,d^2g^2p - 10\,dfgpe + 15\,f^2pe^2\right)\sqrt{d}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)} - 15\left(3\,g^2px^5 + 10\,fgpx^3 + 15\,f^2px\right)e^2\log\left(x^2e + d\right) - 15\left(3\,g^2x^5 + 10\,fgx^3 + 15\,f^2x\right)e^2\log\left(c\right) + 2\left(9\,g^2px^5 + 50\,fgpx^3 + 225\,f^2px\right)e^2 - 30\left(dg^2px^3 + 10\,dfgpx\right)e\right)e^{(-2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")

[Out] [-1/225*(90*d^2*g^2*p*x - 15*(3*g^2*p*x^5 + 10*f*g*p*x^3 + 15*f^2*p*x)*e^2*log(x^2*e + d) - 15*(3*g^2*x^5 + 10*f*g*x^3 + 15*f^2*x)*e^2*log(c) - 15*(3*d^2*g^2*p - 10*d*f*g*p*e + 15*f^2*p*e^2)*sqrt(-d*e^(-1))*log((x^2*e + 2*sqrt(-d*e^(-1))*x*e - d)/(x^2*e + d)) + 2*(9*g^2*p*x^5 + 50*f*g*p*x^3 + 225*f^2*p*x)*e^2 - 30*(d*g^2*p*x^3 + 10*d*f*g*p*x)*e)*e^(-2), -1/225*(90*d^2*g^2*p*x - 30*(3*d^2*g^2*p - 10*d*f*g*p*e + 15*f^2*p*e^2)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2) - 15*(3*g^2*p*x^5 + 10*f*g*p*x^3 + 15*f^2*p*x)*e^2*log(x^2*e + d) - 15*(3*g^2*x^5 + 10*f*g*x^3 + 15*f^2*x)*e^2*log(c) + 2*(9*g^2*p*x^5 + 50*f*g*p*x^3 + 225*f^2*p*x)*e^2 - 30*(d*g^2*p*x^3 + 10*d*f*g*p*x)*e)*e^(-2)]

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(231) = 462$.
time = 35.03, size = 478, normalized size = 2.16

$$\begin{cases} \left(f^2x + \frac{2fgx^3}{3} + \frac{g^2x^5}{5}\right)\log\left(0^pc\right) & \text{for}\ d = 0 \wedge e = 0 \\ -2f^2px + f^2x\log\left(c\left(ex^2\right)^p\right) - \frac{4fgpx^3}{9} + \frac{2fgx^3\log\left(c\left(ex^2\right)^p\right)}{3} - \frac{2g^2px^5}{25} + \frac{g^2x^5\log\left(c\left(ex^2\right)^p\right)}{5} & \text{for}\ d = 0 \\ \left(f^2x + \frac{2fgx^3}{3} + \frac{g^2x^5}{5}\right)\log\left(cd^p\right) & \text{for}\ e = 0 \\ \frac{2d^3g^2p\log\left(x - \sqrt{-\frac{d}{e}}\right)}{5e^3\sqrt{-\frac{d}{e}}} - \frac{d^3g^2\log\left(c\left(d + ex^2\right)^p\right)}{5e^3\sqrt{-\frac{d}{e}}} - \frac{4d^2fgp\log\left(x - \sqrt{-\frac{d}{e}}\right)}{3e^2\sqrt{-\frac{d}{e}}} + \frac{2d^2fg\log\left(c\left(d + ex^2\right)^p\right)}{3e^2\sqrt{-\frac{d}{e}}} - \frac{2d^2g^2px}{5e^2} + \frac{2df^2p\log\left(x - \sqrt{-\frac{d}{e}}\right)}{e\sqrt{-\frac{d}{e}}} - \frac{df^2\log\left(c\left(d + ex^2\right)^p\right)}{e\sqrt{-\frac{d}{e}}} + \frac{4dfgpx}{3e} + \frac{2dg^2px^3}{15e} - 2f^2px + f^2x\log\left(c\left(d + ex^2\right)^p\right) - \frac{4fgpx^3}{9} + \frac{2fgx^3\log\left(c\left(d + ex^2\right)^p\right)}{3} - \frac{2g^2px^5}{25} + \frac{g^2x^5\log\left(c\left(d + ex^2\right)^p\right)}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p),x)

[Out] Piecewise(((f**2*x + 2*f*g*x**3/3 + g**2*x**5/5)*log(0**p*c), Eq(d, 0) & Eq(e, 0)), (-2*f**2*p*x + f**2*x*log(c*(e*x**2)**p) - 4*f*g*p*x**3/9 + 2*f*g*x**3*log(c*(e*x**2)**p)/3 - 2*g**2*p*x**5/25 + g**2*x**5*log(c*(e*x**2)**p)/5, Eq(d, 0)), ((f**2*x + 2*f*g*x**3/3 + g**2*x**5/5)*log(c*d**p), Eq(e, 0)), (2*d**3*g**2*p*log(x - sqrt(-d/e))/(5*e**3*sqrt(-d/e)) - d**3*g**2*log(c*(d + e*x**2)**p)/(5*e**3*sqrt(-d/e)) - 4*d**2*f*g*p*log(x - sqrt(-d/e))/(3*e**2*sqrt(-d/e)) + 2*d**2*f*g*log(c*(d + e*x**2)**p)/(3*e**2*sqrt(-d/e)) -

```
 2*d**2*g**2*p*x/(5*e**2) + 2*d*f**2*p*log(x - sqrt(-d/e))/(e*sqrt(-d/e)) -
 d*f**2*log(c*(d + e*x**2)**p)/(e*sqrt(-d/e)) + 4*d*f*g*p*x/(3*e) + 2*d*g**
2*p*x**3/(15*e) - 2*f**2*p*x + f**2*x*log(c*(d + e*x**2)**p) - 4*f*g*p*x**3
/9 + 2*f*g*x**3*log(c*(d + e*x**2)**p)/3 - 2*g**2*p*x**5/25 + g**2*x**5*log
(c*(d + e*x**2)**p)/5, True))
```

Giac [A]
time = 3.93, size = 201, normalized size = 0.91

$$\frac{2\left(3\,d^{3}g^{2}p-10\,d^{2}fgpe+15\,df^{2}pe^{2}\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{15\,\sqrt{d}}+\frac{1}{225}\left(45\,g^{2}px^{5}e^{2}\log\left(x^{2}e+d\right)-18\,g^{2}px^{5}e^{2}+45\,g^{2}x^{5}e^{2}\log\left(c\right)+30\,dg^{2}px^{3}e+150\,fgpx^{3}e^{2}\log\left(x^{2}e+d\right)-100\,fgpx^{3}e^{2}+150\,fgx^{3}e^{2}\log\left(c\right)-90\,d^{2}g^{2}px+300\,dfgpxe+225\,f^{2}pxe^{2}\log\left(x^{2}e+d\right)-450\,f^{2}pxe^{2}+225\,f^{2}xe^{2}\log\left(c\right)\right)e^{\left(-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 2/15*(3*d^3*g^2*p - 10*d^2*f*g*p*e + 15*d*f^2*p*e^2)*arctan(x*e^(1/2)/sqrt(
d))*e^(-5/2)/sqrt(d) + 1/225*(45*g^2*p*x^5*e^2*log(x^2*e + d) - 18*g^2*p*x^
5*e^2 + 45*g^2*x^5*e^2*log(c) + 30*d*g^2*p*x^3*e + 150*f*g*p*x^3*e^2*log(x^
2*e + d) - 100*f*g*p*x^3*e^2 + 150*f*g*x^3*e^2*log(c) - 90*d^2*g^2*p*x + 30
0*d*f*g*p*x*e + 225*f^2*p*x*e^2*log(x^2*e + d) - 450*f^2*p*x*e^2 + 225*f^2*
x*e^2*log(c))*e^(-2)
```

Mupad [B]
time = 0.34, size = 193, normalized size = 0.87

$$\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(f^2\,x+\frac{2\,f\,g\,x^3}{3}+\frac{g^2\,x^5}{5}\right)-x\,\left(2\,f^2\,p-\frac{d\,\left(\frac{4\,f\,g\,p}{3}-\frac{2\,d\,g^2\,p}{5\,e}\right)}{e}\right)-x^3\,\left(\frac{4\,f\,g\,p}{9}-\frac{2\,d\,g^2\,p}{15\,e}\right)-\frac{2\,g^2\,p\,x^5}{25}+\frac{2\,\sqrt{d}\,p\,\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e}\,p\,x\,\left(3\,d^2\,g^2-10\,d\,e\,f\,g+15\,e^2\,f^2\right)}{3\,p\,d^3\,g^2-10\,p\,d^2\,e\,f\,g+15\,p\,d\,e^2\,f^2}\right)\,\left(3\,d^2\,g^2-10\,d\,e\,f\,g+15\,e^2\,f^2\right)}{15\,e^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)*(f + g*x^2)^2,x)
```

```
[Out] log(c*(d + e*x^2)^p)*(f^2*x + (g^2*x^5)/5 + (2*f*g*x^3)/3) - x*(2*f^2*p - (
d*((4*f*g*p)/3 - (2*d*g^2*p)/(5*e)))/e) - x^3*((4*f*g*p)/9 - (2*d*g^2*p)/(1
5*e)) - (2*g^2*p*x^5)/25 + (2*d^(1/2)*p*atan((d^(1/2)*e^(1/2)*p*x*(3*d^2*g^
2 + 15*e^2*f^2 - 10*d*e*f*g))/(3*d^3*g^2*p + 15*d*e^2*f^2*p - 10*d^2*e*f*g*
p))*(3*d^2*g^2 + 15*e^2*f^2 - 10*d*e*f*g))/(15*e^(5/2))
```

3.270 $\int (f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx$

Optimal. Leaf size=117

$$-2fpx+\frac{2dgpx}{3e}-\frac{2}{9}gpx^3+\frac{2\sqrt{d}\,fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{2d^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3e^{3/2}}+fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log$$

```
[Out] -2*f*p*x+2/3*d*g*p*x/e-2/9*g*p*x^3-2/3*d^(3/2)*g*p*arctan(x*e^(1/2)/d^(1/2)
)/e^(3/2)+f*x*ln(c*(e*x^2+d)^p)+1/3*g*x^3*ln(c*(e*x^2+d)^p)+2*f*p*arctan(x*
e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2521, 2498, 327, 211, 2505, 308}

$$-\frac{2d^{3/2}gp\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2\sqrt{d}\,fp\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}+fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{2dgpx}{3e}-2fpx-\frac{2}{9}gpx^3$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f*p*x + (2*d*g*p*x)/(3*e) - (2*g*p*x^3)/9 + (2*Sqrt[d]*f*p*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^
(3/2)) + f*x*Log[c*(d + e*x^2)^p] + (g*x^3*Log[c*(d + e*x^2)^p])/3
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)\,dx &= \int \left(f\log\left(c\left(d+ex^2\right)^p\right)+gx^2\log\left(c\left(d+ex^2\right)^p\right)\right)\,dx\\
&= f\int \log\left(c\left(d+ex^2\right)^p\right)\,dx+g\int x^2\log\left(c\left(d+ex^2\right)^p\right)\,dx\\
&= fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log\left(c\left(d+ex^2\right)^p\right)-(2efp)\int\frac{x^2}{d+ex^2}\,dx -\\
&= -2fpx+fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log\left(c\left(d+ex^2\right)^p\right)+(2dfp)\int\frac{}{d-}\\
&= -2fpx+\frac{2dgpx}{3e}-\frac{2}{9}gpx^3+\frac{2\sqrt{d}\,fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}+fx\log\left(c\left(d+ex\right.\right.\\
&= -2fpx+\frac{2dgpx}{3e}-\frac{2}{9}gpx^3+\frac{2\sqrt{d}\,fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{2d^{3/2}gp\tan^{-1}\left(\right.}{3e^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 117, normalized size = 1.00

$$-2fpx+\frac{2dgpx}{3e}-\frac{2}{9}gpx^3+\frac{2\sqrt{d}\,fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{2d^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3e^{3/2}}+fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log\left(c\left(d+ex^2\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f*p*x + (2*d*g*p*x)/(3*e) - (2*g*p*x^3)/9 + (2*Sqrt[d]*f*p*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^
(3/2)) + f*x*Log[c*(d + e*x^2)^p] + (g*x^3*Log[c*(d + e*x^2)^p])/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.37, size = 416, normalized size = 3.56

method	result
risch	$\left(\frac{1}{3} g\,x^{3}+f x\right) \ln \left(\left(e\,x^{2}+d\right)^{p}\right)+\frac{i \pi g\,x^{3} \operatorname{csgn}\left(i \left(e\,x^{2}+d\right)^{p}\right) \operatorname{csgn}\left(i c \left(e\,x^{2}+d\right)^{p}\right)^{2}}{6}+\frac{i \pi g\,x^{3} \operatorname{csgn}\left(i c \left(e\,x^{2}+d\right)^{p}\right)^{2} \operatorname{csgn}\left(i c\right)}{6}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] (1/3*g*x^3+f*x)*ln((e*x^2+d)^p)+1/6*I*Pi*g*x^3*csgn(I*(e*x^2+d)^p)*csgn(I*c
*(e*x^2+d)^p)^2+1/6*I*Pi*g*x^3*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/2*I*Pi*f
*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*x-1/6*I*Pi*g*x^3*csgn(
I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/2*I*Pi*f*csgn(I*c*(e*x^2+d
)^p)^3*x-1/6*I*Pi*g*x^3*csgn(I*c*(e*x^2+d)^p)^3+1/2*I*Pi*f*csgn(I*c*(e*x^2+
d)^p)^2*csgn(I*c)*x+1/2*I*Pi*f*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*
x+1/3*ln(c)*g*x^3-2/9*g*p*x^3+1/3/e^2*(-e*d)^(1/2)*p*ln(-(-e*d)^(1/2)*x-d)*
d*g-1/e*(-e*d)^(1/2)*p*ln(-(-e*d)^(1/2)*x-d)*f-1/3/e^2*(-e*d)^(1/2)*p*ln((-
e*d)^(1/2)*x-d)*d*g+1/e*(-e*d)^(1/2)*p*ln((-e*d)^(1/2)*x-d)*f+ln(c)*f*x+2/3
*d*g*p*x/e-2*f*p*x
```

Maxima [A]
time = 0.55, size = 82, normalized size = 0.70

$$-\frac{2}{9}\left(\frac{3\left(d^{2} g-3\,d f e\right) \arctan\left(\frac{x e^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{5}{2}\right)}}{\sqrt{d}}+\left(g x^{3} e-3\left(d g-3\,f e\right) x\right) e^{\left(-2\right)}\right) p e+\frac{1}{3}\left(g x^{3}+3\,f x\right) \log\left(\left(x^{2} e+d\right)^{p} c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] -2/9*(3*(d^2*g - 3*d*f*e)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/sqrt(d) + (g*x
^3*e - 3*(d*g - 3*f*e)*x)*e^(-2))*p*e + 1/3*(g*x^3 + 3*f*x)*log((x^2*e + d)
^p*c)
```

Fricas [A]
time = 0.36, size = 217, normalized size = 1.85

$$\left[\frac{1}{9}\left(6\,d g p x+3\left(g p x^{3}+3\,f p x\right) e \log\left(x^{2} e+d\right)+3\left(g x^{3}+3\,f x\right) e \log\left(c\right)-3\left(d g p-3\,f p e\right) \sqrt{-d e^{\left(-1\right)}} \log\left(\frac{x^{2} e+2\,\sqrt{-d e^{\left(-1\right)}}\,x e-d}{x^{2} e+d}\right)-2\left(g p x^{3}+9\,f p x\right) e\right) e^{\left(-1\right)}, \frac{1}{9}\left(6\,d g p x-6\left(d g p-3\,f p e\right) \sqrt{d} \arctan\left(\frac{x e^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{1}{2}\right)}+3\left(g p x^{3}+3\,f p x\right) e \log\left(x^{2} e+d\right)+3\left(g x^{3}+3\,f x\right) e \log\left(c\right)-2\left(g p x^{3}+9\,f p x\right) e\right) e^{\left(-1\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [1/9*(6*d*g*p*x + 3*(g*p*x^3 + 3*f*p*x)*e*log(x^2*e + d) + 3*(g*x^3 + 3*f*x
)*e*log(c) - 3*(d*g*p - 3*f*p*e)*sqrt(-d*e^(-1))*log((x^2*e + 2*sqrt(-d*e^(
-1))*x*e - d)/(x^2*e + d)) - 2*(g*p*x^3 + 9*f*p*x)*e)*e^(-1), 1/9*(6*d*g*p*
x - 6*(d*g*p - 3*f*p*e)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2) + 3*(g*p
*x^3 + 3*f*p*x)*e*log(x^2*e + d) + 3*(g*x^3 + 3*f*x)*e*log(c) - 2*(g*p*x^3
+ 9*f*p*x)*e)*e^(-1)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(121) = 242$.
time = 8.81, size = 260, normalized size = 2.22

$$\begin{cases} \left(fx + \frac{gx^3}{3}\right)\log\left(0^p c\right) & \text{for}\ d = 0 \wedge e = 0 \\ -2fpx + fx\log\left(c(ex^2)^p\right) - \frac{2gpx^3}{9} + \frac{gx^3\log\left(c(ex^2)^p\right)}{3} & \text{for}\ d = 0 \\ \left(fx + \frac{gx^3}{3}\right)\log\left(cd^p\right) & \text{for}\ e = 0 \\ -\frac{2d^2gp\log\left(x - \sqrt{-\frac{d}{e}}\right)}{3e^2\sqrt{-\frac{d}{e}}} + \frac{d^2g\log\left(c(d+ex^2)^p\right)}{3e^2\sqrt{-\frac{d}{e}}} + \frac{2dfp\log\left(x - \sqrt{-\frac{d}{e}}\right)}{e\sqrt{-\frac{d}{e}}} - \frac{df\log\left(c(d+ex^2)^p\right)}{e\sqrt{-\frac{d}{e}}} + \frac{2dgpx}{3e} - 2fpx + fx\log\left(c(d+ex^2)^p\right) - \frac{2gpx^3}{9} + \frac{gx^3\log\left(c(d+ex^2)^p\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise(((f*x + g*x**3/3)*log(0**p*c), Eq(d, 0) & Eq(e, 0)), (-2*f*p*x +
f*x*log(c*(e*x**2)**p) - 2*g*p*x**3/9 + g*x**3*log(c*(e*x**2)**p)/3, Eq(d,
0)), ((f*x + g*x**3/3)*log(c*d**p), Eq(e, 0)), (-2*d**2*g*p*log(x - sqrt(-d
/e))/(3*e**2*sqrt(-d/e)) + d**2*g*log(c*(d + e*x**2)**p)/(3*e**2*sqrt(-d/e)
) + 2*d*f*p*log(x - sqrt(-d/e))/(e*sqrt(-d/e)) - d*f*log(c*(d + e*x**2)**p)
/(e*sqrt(-d/e)) + 2*d*g*p*x/(3*e) - 2*f*p*x + f*x*log(c*(d + e*x**2)**p) -
2*g*p*x**3/9 + g*x**3*log(c*(d + e*x**2)**p)/3, True))
```

Giac [A]
time = 4.79, size = 109, normalized size = 0.93

$$-\frac{2\,(d^2gp - 3\,dfpe)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{3}{2}\right)}}{3\,\sqrt{d}} + \frac{1}{9}\,(3\,gpx^3e\log\left(x^2e + d\right) - 2\,gpx^3e + 3\,gx^3e\log\left(c\right) + 9\,fpxe\log\left(x^2e + d\right) + 6\,dgpx - 18\,fpxe + 9\,fxe\log\left(c\right))e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] -2/3*(d^2*g*p - 3*d*f*p*e)*arctan(x*e^(1/2)/sqrt(d))*e^(-3/2)/sqrt(d) + 1/9
*(3*g*p*x^3*e*log(x^2*e + d) - 2*g*p*x^3*e + 3*g*x^3*e*log(c) + 9*f*p*x*e*l
og(x^2*e + d) + 6*d*g*p*x - 18*f*p*x*e + 9*f*x*e*log(c))*e^(-1)
```

Mupad [B]
time = 0.32, size = 97, normalized size = 0.83

$$\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(\frac{g\,x^3}{3}+f\,x\right)-x\,\left(2\,f\,p-\frac{2\,d\,g\,p}{3\,e}\right)-\frac{2\,g\,p\,x^3}{9}-\frac{2\,\sqrt{d}\,p\,\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e}\,p\,x\,\left(d\,g-3\,e\,f\right)}{d^2\,g\,p-3\,d\,e\,f\,p}\right)\,\left(d\,g-3\,e\,f\right)}{3\,e^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)*(f + g*x^2),x)
```

```
[Out] log(c*(d + e*x^2)^p)*(f*x + (g*x^3)/3) - x*(2*f*p - (2*d*g*p)/(3*e)) - (2*g
*p*x^3)/9 - (2*d^(1/2)*p*atan((d^(1/2)*e^(1/2)*p*x*(d*g - 3*e*f))/(d^2*g*p
- 3*d*e*f*p))*(d*g - 3*e*f))/(3*e^(3/2))
```

3.271 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$

Optimal. Leaf size=533

$$\frac{2p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}}-\frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(i\sqrt{e}\,\sqrt{f}-\sqrt{-d}\,\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}\right)}{\sqrt{f}\,\sqrt{g}}-\frac{p}{}$$

```
[Out] arctan(x*g^(1/2)/f^(1/2))*ln(c*(e*x^2+d)^p)/f^(1/2)/g^(1/2)+2*p*arctan(x*g^
(1/2)/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)-p*arctan
(x*g^(1/2)/f^(1/2))*ln(-2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I
*x*g^(1/2))/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)-p*arcta
n(x*g^(1/2)/f^(1/2))*ln(2*((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I
*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)-I*p*pol
ylog(2,1-2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)+1/2*I*p*polylog(2
,1+2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2
)*f^(1/2)-(-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)+1/2*I*p*polylog(2,1-2*((-d)^
(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(
-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 533, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$,
Rules used = {211, 2520, 12, 5048, 4966, 2449, 2352, 2497}

$$\frac{ip\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}-\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}}+\frac{\text{ArcTan}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}}-\frac{p\text{ArcTan}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{\sqrt{f}\sqrt{g}}-\frac{p\text{ArcTan}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{\sqrt{f}\sqrt{g}}+\frac{2p\text{ArcTan}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(f + g*x^2),x]
```

```
[Out] (2*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/
(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*
(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*S
qrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[
g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + (ArcTan[(Sqrt[g]*x)/Sqrt
[f]]*Log[c*(d + e*x^2)^p])/(Sqrt[f]*Sqrt[g]) - (I*p*PolyLog[2, 1 - (2*Sqrt[
f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 + (
2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sq
rt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1
 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d
]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
```

```
/; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\,\sqrt{g}} - (2ep)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}\,\sqrt{g}\,(d+ex^2)}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\,\sqrt{g}} - \frac{(2ep)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{d+ex^2}\,dx}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\,\sqrt{g}} - \frac{(2ep)\int \left(-\frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}\,x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{\ldots}}{\sqrt{\ldots}}\right.}{2\sqrt{e}\left(\sqrt{-d}\ldots\right.}\right.}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\,\sqrt{g}} + \frac{\left(\sqrt{e}\,p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{-d}-\sqrt{e}\,x}\,dx}{\sqrt{f}\,\sqrt{g}} - \frac{\left(\sqrt{e}\,p\right)\int \frac{\tan^{-1}\left(\ldots\right.}{\sqrt{-d}\ldots}}{\sqrt{f}\,\sqrt{g}\ldots}$$

$$= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\,\sqrt{g}\ldots}{\left(i\sqrt{e}\,\sqrt{f}-\sqrt{-d}\ldots\right.}\right.}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\,\sqrt{g}\ldots}{\left(i\sqrt{e}\,\sqrt{f}-\sqrt{-d}\ldots\right.}\right.}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\,\sqrt{g}\ldots}{\left(i\sqrt{e}\,\sqrt{f}-\sqrt{-d}\ldots\right.}\right.}{\sqrt{f}\,\sqrt{g}}$$

Mathematica [A]
time = 0.19, size = 564, normalized size = 1.06

$$-\frac{i\left(p\log\left(\frac{\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{i\sqrt{e}\,\sqrt{f}+\sqrt{-d}\,\sqrt{g}}\right)\log\left(1-\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)+p\log\left(\frac{\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{-i\sqrt{e}\,\sqrt{f}+\sqrt{-d}\,\sqrt{g}}\right)\log\left(1-\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)-p\log\left(\frac{\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{-i\sqrt{e}\,\sqrt{f}+\sqrt{-d}\,\sqrt{g}}\right)\log\left(1+\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)-p\log\left(\frac{\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{i\sqrt{e}\,\sqrt{f}+\sqrt{-d}\,\sqrt{g}}\right)\log\left(1+\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)+2i\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)+p\text{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}\,x\right)}{\sqrt{e}\,\sqrt{f}-i\sqrt{-d}\,\sqrt{g}}\right)+p\text{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}\,x\right)}{\sqrt{e}\,\sqrt{f}+i\sqrt{-d}\,\sqrt{g}}\right)-p\text{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}+i\sqrt{g}\,x\right)}{\sqrt{e}\,\sqrt{f}-i\sqrt{-d}\,\sqrt{g}}\right)-p\text{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}+i\sqrt{g}\,x\right)}{\sqrt{e}\,\sqrt{f}+i\sqrt{-d}\,\sqrt{g}}\right)\right)}{2\sqrt{f}\,\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^2),x]
```

```
[Out] ((-1/2*I)*(p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt
[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + p*Log[(Sqrt[g]*(Sqrt[-d] +
Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x
)/Sqrt[f]] - p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] +
 Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] - p*Log[(Sqrt[g]*(Sqrt[-
d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]
*x)/Sqrt[f]] + (2*I)*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p] + p*P
olyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*S
qrt[g])] + p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f]
+ I*Sqrt[-d]*Sqrt[g])] - p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sq
rt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] - p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*S
qrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])]))/(Sqrt[f]*Sqrt[g])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.34, size = 504, normalized size = 0.95

method	result
risch	$\frac{\left(\ln\left(\left(e\,x^{2}+d\right)^{p}\right)-p\ln\left(e\,x^{2}+d\right)\right)\arctan\left(\frac{xg}{\sqrt{fg}}\right)}{\sqrt{fg}}+\frac{p\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^{2}g+f\right)}\frac{\ln(x-\_\alpha)\ln\left(e\,x^{2}+d\right)-\ln(x-\_\alpha)\left(\ln\left(\frac{\mathrm{RootOf}\left(e\_Z^{2}\right.}{\mathrm{RootOf}\left(e\_\right.}\right.\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/(g*x^2+f),x,method=_RETURNVERBOSE)
```

```
[Out] (ln((e*x^2+d)^p)-p*ln(e*x^2+d))/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))+1/2*p/g
*sum(1/_alpha*(ln(x-_alpha)*ln(e*x^2+d)-ln(x-_alpha)*(ln((RootOf(_Z^2*e*g+2
*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d
*g-e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x+_al
pha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)))-dilog((RootOf(_Z^2*
e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*
e*g+d*g-e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=
2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2))),_alpha=Root
Of(_Z^2*g+f))+1/2*I/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))*Pi*csgn(I*(e*x^2+d)
^p)*csgn(I*c*(e*x^2+d)^p)^2-1/2*I/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))*Pi*cs
gn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/2*I/(f*g)^(1/2)*arctan(
x*g/(f*g)^(1/2))*Pi*csgn(I*c*(e*x^2+d)^p)^3+1/2*I/(f*g)^(1/2)*arctan(x*g/(f
*g)^(1/2))*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+1/(f*g)^(1/2)*arctan(x*g/(f
*g)^(1/2))*ln(c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
[Out] integrate(log((x^2*e + d)^p*c)/(g*x^2 + f), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
[Out] integral(log((x^2*e + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
[Out] Integral(log(c*(d + e*x**2)**p)/(f + g*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
[Out] integrate(log((x^2*e + d)^p*c)/(g*x^2 + f), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln \left(c\,{\left(e\,x^2+d\right)}^p\right)}{g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)/(f + g*x^2),x)
[Out] int(log(c*(d + e*x^2)^p)/(f + g*x^2), x)
```

3.272 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx$

Optimal. Leaf size=751

$$\frac{\sqrt{d}\,\sqrt{e}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{f(ef-dg)}-\frac{ep\log\left(\sqrt{-f}-\sqrt{g}\,x\right)}{2\sqrt{-f}\,\sqrt{g}\,(ef-dg)}+\frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{f^{3/2}\sqrt{g}}-\frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right.}{}$$

```
[Out] p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)*e^(1/2)/f/(-d*g+e*f)+1/2*arctan(x*g^(1/
2)/f^(1/2))*ln(c*(e*x^2+d)^p)/f^(3/2)/g^(1/2)+p*arctan(x*g^(1/2)/f^(1/2))*l
n(2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(3/2)/g^(1/2)-1/2*p*arctan(x*g^(1/2)/f
^(1/2))*ln(-2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/
(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2)))/f^(3/2)/g^(1/2)-1/2*p*arctan(x*g^(1
/2)/f^(1/2))*ln(2*((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/
2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))/f^(3/2)/g^(1/2)-1/2*I*p*polylog
(2,1-2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(3/2)/g^(1/2)+1/4*I*p*polylog(2,1+2
*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^
(1/2)-(-d)^(1/2)*g^(1/2)))/f^(3/2)/g^(1/2)+1/4*I*p*polylog(2,1-2*((-d)^(1/2
)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^
(1/2)*g^(1/2)))/f^(3/2)/g^(1/2)-1/2*e*p*ln((-f)^(1/2)-x*g^(1/2))/(-d*g+e*f)
/(-f)^(1/2)/g^(1/2)+1/2*e*p*ln((-f)^(1/2)+x*g^(1/2))/(-d*g+e*f)/(-f)^(1/2)/
g^(1/2)-1/4*ln(c*(e*x^2+d)^p)/f/g^(1/2)/((-f)^(1/2)-x*g^(1/2))+1/4*ln(c*(e*
x^2+d)^p)/f/g^(1/2)/((-f)^(1/2)+x*g^(1/2))
```

Rubi [A]

time = 0.73, antiderivative size = 751, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {2521, 2513, 815, 649, 211, 266, 2520, 12, 5048, 4966, 2449, 2352, 2497}

$$\frac{ip\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(-\sqrt{-d}\,\sqrt{g}+i\sqrt{e}\,\sqrt{f}\right)}\right)}{4f^{3/2}\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(\sqrt{-d}\,\sqrt{g}+i\sqrt{e}\,\sqrt{f}\right)}\right)}{4f^{3/2}\sqrt{g}}-\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{2f^{3/2}\sqrt{g}}+\frac{\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^{3/2}\sqrt{g}}-\frac{p\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(-\sqrt{-d}\,\sqrt{g}+i\sqrt{e}\,\sqrt{f}\right)}\right)}{2f^{3/2}\sqrt{g}}-\frac{p\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(\sqrt{-d}\,\sqrt{g}+i\sqrt{e}\,\sqrt{f}\right)}\right)}{2f^{3/2}\sqrt{g}}+\frac{\sqrt{d}\,\sqrt{e}\,p\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{f(ef-dg)}+\frac{p\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{f^{3/2}\sqrt{g}}-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}\,x\right)}+\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}\,x\right)}-\frac{ep\log\left(\sqrt{-f}-\sqrt{g}\,x\right)}{2\sqrt{-f}\,\sqrt{g}\,(ef-dg)}+\frac{ep\log\left(\sqrt{-f}+\sqrt{g}\,x\right)}{2\sqrt{-f}\,\sqrt{g}\,(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(f + g*x^2)^2,x]
```

```
[Out] (Sqrt[d]*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(f*(e*f - d*g)) - (e*p*Log[
Sqrt[-f] - Sqrt[g]*x])/(2*Sqrt[-f]*Sqrt[g]*(e*f - d*g)) + (p*ArcTan[(Sqrt[g
]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(f^(3/2)*Sqrt[g]) -
 (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]
*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(2*
f^(3/2)*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(S
qrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*
Sqrt[g]*x))])/(2*f^(3/2)*Sqrt[g]) + (e*p*Log[Sqrt[-f] + Sqrt[g]*x])/(2*Sqrt
```

```
[-f]*Sqrt[g]*(e*f - d*g)) - Log[c*(d + e*x^2)^p]/(4*f*Sqrt[g]*(Sqrt[-f] - S
qrt[g]*x)) + Log[c*(d + e*x^2)^p]/(4*f*Sqrt[g]*(Sqrt[-f] + Sqrt[g]*x)) + (A
rcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/(2*f^(3/2)*Sqrt[g]) - ((I/
2)*p*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(f^(3/2)*Sqrt[g])
 + ((I/4)*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*S
qrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(f^(3/2)*Sqrt
[g]) + ((I/4)*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(
(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(f^(3/2)*
Sqrt[g])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \int \left(-\frac{g\log\left(c(d+ex^2)^p\right)}{4f\left(\sqrt{-f}\,\sqrt{g}-gx\right)^2}-\frac{g\log\left(c(d+ex^2)^p\right)}{4f\left(\sqrt{-f}\,\sqrt{g}+gx\right)^2}-\frac{g\log\left(c(d+ex^2)^p\right)}{2f\left(-fg-g^2x^2\right)}\right)$$

$$= -\frac{g\int \frac{\log(c(d+ex^2)^p)}{\left(\sqrt{-f}\,\sqrt{g}-gx\right)^2}\,dx}{4f}-\frac{g\int \frac{\log(c(d+ex^2)^p)}{\left(\sqrt{-f}\,\sqrt{g}+gx\right)^2}\,dx}{4f}-\frac{g\int \frac{\log(c(d+ex^2)^p)}{-fg-g^2x^2}\,dx}{2f}$$

$$= -\frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}\,x\right)}+\frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}\,x\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\right.}{2f^{3/2}\sqrt{g}}$$

$$= -\frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}\,x\right)}+\frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}\,x\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\right.}{2f^{3/2}\sqrt{g}}$$

$$= -\frac{ep\log\left(\sqrt{-f}-\sqrt{g}\,x\right)}{2\sqrt{-f}\,\sqrt{g}\,(ef-dg)}+\frac{ep\log\left(\sqrt{-f}+\sqrt{g}\,x\right)}{2\sqrt{-f}\,\sqrt{g}\,(ef-dg)}-\frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}\,x\right.}$$

$$= -\frac{ep\log\left(\sqrt{-f}-\sqrt{g}\,x\right)}{2\sqrt{-f}\,\sqrt{g}\,(ef-dg)}+\frac{ep\log\left(\sqrt{-f}+\sqrt{g}\,x\right)}{2\sqrt{-f}\,\sqrt{g}\,(ef-dg)}-\frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}\,x\right.}$$

$$= \frac{\sqrt{d}\,\sqrt{e}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{f(ef-dg)}-\frac{ep\log\left(\sqrt{-f}-\sqrt{g}\,x\right)}{2\sqrt{-f}\,\sqrt{g}\,(ef-dg)}+\frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\right.}{f^{3/2}\sqrt{g}}$$

$$= \frac{\sqrt{d}\,\sqrt{e}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{f(ef-dg)}-\frac{ep\log\left(\sqrt{-f}-\sqrt{g}\,x\right)}{2\sqrt{-f}\,\sqrt{g}\,(ef-dg)}+\frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\right.}{f^{3/2}\sqrt{g}}$$

$$= \frac{\sqrt{d}\,\sqrt{e}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{f(ef-dg)}-\frac{ep\log\left(\sqrt{-f}-\sqrt{g}\,x\right)}{2\sqrt{-f}\,\sqrt{g}\,(ef-dg)}+\frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\right.}{f^{3/2}\sqrt{g}}$$

Mathematica [**A**]
time = 2.60, size = 1236, normalized size = 1.65

$$\frac{1}{2}\left(\frac{x\left(-p\log\left(d+ex^2\right)+\log\left(c\left(d+ex^2\right)^p\right)\right)}{f\left(f+gx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\left(-p\log\left(d+ex^2\right)+\log\left(c\left(d+ex^2\right)^p\right)\right)}{f^{3/2}\sqrt{g}}+\frac{1}{2}p\left(\frac{i\left(\frac{\log\left(-\frac{i\sqrt{d}}{\sqrt{e}}+x\right)}{\sqrt{f}+i\sqrt{g}\,x}+\frac{\sqrt{e}\left(-\log\left(i\sqrt{d}-\sqrt{e}\,x\right)+\log\left(i\sqrt{f}-\sqrt{g}\,x\right)\right)}{\sqrt{e}\sqrt{f}-\sqrt{d}\sqrt{g}}\right)}{f\sqrt{g}}+\frac{i\left(\frac{\log\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)}{\sqrt{f}+i\sqrt{g}\,x}+\frac{\sqrt{e}\left(-\log\left(i\sqrt{d}+\sqrt{e}\,x\right)+\log\left(i\sqrt{f}-\sqrt{g}\,x\right)\right)}{\sqrt{e}\sqrt{f}+\sqrt{d}\sqrt{g}}\right)}{f\sqrt{g}}+\frac{-i\left(\sqrt{e}\sqrt{f}+\sqrt{d}\sqrt{g}\right)\log\left(-\frac{i\sqrt{d}}{\sqrt{e}}+x\right)+\sqrt{e}\left(i\sqrt{f}+\sqrt{g}\,x\right)\left(\log\left(i\sqrt{d}-\sqrt{e}\,x\right)-\log\left(i\sqrt{f}+\sqrt{g}\,x\right)\right)}{f\left(\sqrt{e}\sqrt{f}+\sqrt{d}\sqrt{g}\right)\sqrt{g}\left(\sqrt{f}-i\sqrt{g}\,x\right)}-\frac{-\frac{\log\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)}{i\sqrt{f}+\sqrt{g}\,x}-\frac{i\sqrt{e}\left(\log\left(i\sqrt{d}+\sqrt{e}\,x\right)-\log\left(i\sqrt{f}+\sqrt{g}\,x\right)\right)}{\sqrt{e}\sqrt{f}-\sqrt{d}\sqrt{g}}}{f\sqrt{g}}+2\left(\frac{x}{f^2+fgx^2}+\frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{f^{3/2}\sqrt{g}}\right)\left(-\log\left(-\frac{i\sqrt{d}}{\sqrt{e}}+x\right)-\log\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)+\log\left(d+ex^2\right)\right)+\frac{i\left(\log\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)\log\left(\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}\,x\right)}{\sqrt{e}\sqrt{f}-\sqrt{d}\sqrt{g}}\right)+\text{Li}_2\left(-\frac{\sqrt{g}\left(\sqrt{d}-i\sqrt{e}\,x\right)}{\sqrt{e}\sqrt{f}-\sqrt{d}\sqrt{g}}\right)\right)}{f^{3/2}\sqrt{g}}-\frac{i\left(\log\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)\log\left(\frac{\sqrt{e}\left(\sqrt{f}+i\sqrt{g}\,x\right)}{\sqrt{e}\sqrt{f}+\sqrt{d}\sqrt{g}}\right)+\text{Li}_2\left(\frac{\sqrt{g}\left(\sqrt{d}-i\sqrt{e}\,x\right)}{\sqrt{e}\sqrt{f}+\sqrt{d}\sqrt{g}}\right)\right)}{f^{3/2}\sqrt{g}}-\frac{i\left(\log\left(-\frac{i\sqrt{d}}{\sqrt{e}}+x\right)\log\left(\frac{\sqrt{e}\left(\sqrt{f}+i\sqrt{g}\,x\right)}{\sqrt{e}\sqrt{f}-\sqrt{d}\sqrt{g}}\right)+\text{Li}_2\left(-\frac{\sqrt{g}\left(\sqrt{d}+i\sqrt{e}\,x\right)}{\sqrt{e}\sqrt{f}-\sqrt{d}\sqrt{g}}\right)\right)}{f^{3/2}\sqrt{g}}+\frac{i\left(\log\left(-\frac{i\sqrt{d}}{\sqrt{e}}+x\right)\log\left(\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}\,x\right)}{\sqrt{e}\sqrt{f}+\sqrt{d}\sqrt{g}}\right)+\text{Li}_2\left(\frac{\sqrt{g}\left(\sqrt{d}+i\sqrt{e}\,x\right)}{\sqrt{e}\sqrt{f}+\sqrt{d}\sqrt{g}}\right)\right)}{f^{3/2}\sqrt{g}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^2)^2,x]
```

```
[Out] ((x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]))/(f*(f + g*x^2)) + (ArcTan
[(Sqrt[g]*x)/Sqrt[f]]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]))/(f^(3/2
)*Sqrt[g]) + (p*((I*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]/(Sqrt[f] + I*Sqrt[g]*x
) + (Sqrt[e]*(-Log[I*Sqrt[d] - Sqrt[e]*x] + Log[I*Sqrt[f] - Sqrt[g]*x]))/(S
qrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g])))/(f*Sqrt[g]) + (I*(Log[(I*Sqrt[d])/Sqrt[
e] + x]/(Sqrt[f] + I*Sqrt[g]*x) + (Sqrt[e]*(-Log[I*Sqrt[d] + Sqrt[e]*x] + L
og[I*Sqrt[f] - Sqrt[g]*x]))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])))/(f*Sqrt[g
]) + ((-I)*(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])*Log[((-I)*Sqrt[d])/Sqrt[e] +
 x] + Sqrt[e]*(I*Sqrt[f] + Sqrt[g]*x)*(Log[I*Sqrt[d] - Sqrt[e]*x] - Log[I*S
qrt[f] + Sqrt[g]*x]))/(f*(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])*Sqrt[g]*(Sqrt[
f] - I*Sqrt[g]*x)) - (-(Log[(I*Sqrt[d])/Sqrt[e] + x]/(I*Sqrt[f] + Sqrt[g]*x
)) - (I*Sqrt[e]*(Log[I*Sqrt[d] + Sqrt[e]*x] - Log[I*Sqrt[f] + Sqrt[g]*x]))/
(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g]))/(f*Sqrt[g]) + 2*(x/(f^2 + f*g*x^2) + A
rcTan[(Sqrt[g]*x)/Sqrt[f]]/(f^(3/2)*Sqrt[g]))*(-Log[((-I)*Sqrt[d])/Sqrt[e]
+ x] - Log[(I*Sqrt[d])/Sqrt[e] + x] + Log[d + e*x^2]) + (I*(Log[(I*Sqrt[d])
/Sqrt[e] + x]*Log[(Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - Sqrt
[d]*Sqrt[g])] + PolyLog[2, -((Sqrt[g]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[e]*Sqr
t[f] - Sqrt[d]*Sqrt[g]))]))/(f^(3/2)*Sqrt[g]) - (I*(Log[(I*Sqrt[d])/Sqrt[e]
 + x]*Log[(Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt
[g])] + PolyLog[2, (Sqrt[g]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] + Sqr
t[d]*Sqrt[g])]))/(f^(3/2)*Sqrt[g]) - (I*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Lo
g[(Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g])] +
PolyLog[2, -((Sqrt[g]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*S
qrt[g]))]))/(f^(3/2)*Sqrt[g]) + (I*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sq
rt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])] + PolyL
og[2, (Sqrt[g]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])
]))/(f^(3/2)*Sqrt[g])))/2)/2
```

Maple [**F**]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)}{\left(g\,x^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

```
[Out] int(ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
[Out] integrate(log((x^2*e + d)^p*c)/(g*x^2 + f)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
[Out] integral(log((x^2*e + d)^p*c)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
[Out] integrate(log((x^2*e + d)^p*c)/(g*x^2 + f)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{{\left(g\,x^2+f\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)/(f + g*x^2)^2,x)
[Out] int(log(c*(d + e*x^2)^p)/(f + g*x^2)^2, x)
```

3.273 $\int (f+gx^2)^2 \log^2\left(c(d+ex^2)^p\right)\,dx$

Optimal. Leaf size=945

$$8f^2p^2x-\frac{64dfgp^2x}{9e}+\frac{184d^2g^2p^2x}{75e^2}+\frac{16}{27}fgp^2x^3-\frac{64dg^2p^2x^3}{225e}+\frac{8}{125}g^2p^2x^5-\frac{8\sqrt{d}\,f^2p^2\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{64d^{3/2}fgp}{}$$

```
[Out] 1/5*g^2*x^5*ln(c*(e*x^2+d)^p)^2+f^2*x*ln(c*(e*x^2+d)^p)^2+184/75*d^2*g^2*p^
2*x/e^2-64/225*d*g^2*p^2*x^3/e-64/9*d*f*g*p^2*x/e+8/3*d*f*g*p*x*ln(c*(e*x^2
+d)^p)/e-8/3*d^(3/2)*f*g*p*arctan(x*e^(1/2)/d^(1/2))*ln(c*(e*x^2+d)^p)/e^(3
/2)-16/3*d^(3/2)*f*g*p^2*arctan(x*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*
x*e^(1/2)))/e^(3/2)-8/3*I*d^(3/2)*f*g*p^2*arctan(x*e^(1/2)/d^(1/2))^2/e^(3/
2)-8/3*I*d^(3/2)*f*g*p^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))/e^(3/
2)+64/9*d^(3/2)*f*g*p^2*arctan(x*e^(1/2)/d^(1/2))/e^(3/2)-4/5*d^2*g^2*p*x*l
n(c*(e*x^2+d)^p)/e^2+4/15*d*g^2*p*x^3*ln(c*(e*x^2+d)^p)/e+4/5*d^(5/2)*g^2*p
*arctan(x*e^(1/2)/d^(1/2))*ln(c*(e*x^2+d)^p)/e^(5/2)+8/5*d^(5/2)*g^2*p^2*ar
ctan(x*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))/e^(5/2)+4*f^2*p
*arctan(x*e^(1/2)/d^(1/2))*ln(c*(e*x^2+d)^p)*d^(1/2)/e^(1/2)+8*f^2*p^2*arct
an(x*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))*d^(1/2)/e^(1/2)+4
/5*I*d^(5/2)*g^2*p^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))/e^(5/2)+4
*I*f^2*p^2*arctan(x*e^(1/2)/d^(1/2))^2*d^(1/2)/e^(1/2)+4*I*f^2*p^2*polylog(
2,1-2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))*d^(1/2)/e^(1/2)+4/5*I*d^(5/2)*g^2*p^2*
arctan(x*e^(1/2)/d^(1/2))^2/e^(5/2)+8*f^2*p^2*x+8/125*g^2*p^2*x^5+16/27*f*g
*p^2*x^3-4*f^2*p*x*ln(c*(e*x^2+d)^p)-4/25*g^2*p*x^5*ln(c*(e*x^2+d)^p)+2/3*f
*g*x^3*ln(c*(e*x^2+d)^p)^2-184/75*d^(5/2)*g^2*p^2*arctan(x*e^(1/2)/d^(1/2))
/e^(5/2)-8/9*f*g*p*x^3*ln(c*(e*x^2+d)^p)-8*f^2*p^2*arctan(x*e^(1/2)/d^(1/2)
)*d^(1/2)/e^(1/2)
```

Rubi [A]
time = 0.79, antiderivative size = 945, normalized size of antiderivative = 1.00, number of steps used = 50, number of rules used = 15, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {2521, 2500, 2526, 2498, 327, 211, 2520, 12, 5040, 4964, 2449, 2352, 2507, 2505, 308}

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^2)^2*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] 8*f^2*p^2*x - (64*d*f*g*p^2*x)/(9*e) + (184*d^2*g^2*p^2*x)/(75*e^2) + (16*f
*g*p^2*x^3)/27 - (64*d*g^2*p^2*x^3)/(225*e) + (8*g^2*p^2*x^5)/125 - (8*Sqrt
[d]*f^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + (64*d^(3/2)*f*g*p^2*ArcT
an[(Sqrt[e]*x)/Sqrt[d]])/(9*e^(3/2)) - (184*d^(5/2)*g^2*p^2*ArcTan[(Sqrt[e]
*x)/Sqrt[d]])/(75*e^(5/2)) + ((4*I)*Sqrt[d]*f^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt
```

```
[d]]^2)/Sqrt[e] - (((8*I)/3)*d^(3/2)*f*g*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)
/e^(3/2) + (((4*I)/5)*d^(5/2)*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e^(5/2
) + (8*Sqrt[d]*f^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d]
 + I*Sqrt[e]*x)])/Sqrt[e] - (16*d^(3/2)*f*g*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/(3*e^(3/2)) + (8*d^(5/2)*g^2*p^2
*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/(5*e
^(5/2)) - 4*f^2*p*x*Log[c*(d + e*x^2)^p] + (8*d*f*g*p*x*Log[c*(d + e*x^2)^p
])/(3*e) - (4*d^2*g^2*p*x*Log[c*(d + e*x^2)^p])/(5*e^2) - (8*f*g*p*x^3*Log[
c*(d + e*x^2)^p])/9 + (4*d*g^2*p*x^3*Log[c*(d + e*x^2)^p])/(15*e) - (4*g^2*
p*x^5*Log[c*(d + e*x^2)^p])/25 + (4*Sqrt[d]*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d
]]*Log[c*(d + e*x^2)^p])/Sqrt[e] - (8*d^(3/2)*f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt
[d]]*Log[c*(d + e*x^2)^p])/(3*e^(3/2)) + (4*d^(5/2)*g^2*p*ArcTan[(Sqrt[e]*x
)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/(5*e^(5/2)) + f^2*x*Log[c*(d + e*x^2)^p]^2
 + (2*f*g*x^3*Log[c*(d + e*x^2)^p]^2)/3 + (g^2*x^5*Log[c*(d + e*x^2)^p]^2)/
5 + ((4*I)*Sqrt[d]*f^2*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*
x)])/Sqrt[e] - (((8*I)/3)*d^(3/2)*f*g*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[
d] + I*Sqrt[e]*x)])/e^(3/2) + (((4*I)/5)*d^(5/2)*g^2*p^2*PolyLog[2, 1 - (2*
Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/e^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2500

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[x^n*(
(a + b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
```

```
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(f+gx^2\right)^2 \log^2 \left(c\left(d+ex^2\right)^p\right)\, dx = \int \left(f^2 \log^2 \left(c\left(d+ex^2\right)^p\right) + 2fgx^2 \log^2 \left(c\left(d+ex^2\right)^p\right) + g^2x^4 \log^2 \left(c(d\right.\right.$$

$$= f^2 \int \log^2 \left(c\left(d+ex^2\right)^p\right)\, dx + (2fg) \int x^2 \log^2 \left(c\left(d+ex^2\right)^p\right)\, dx + g^2$$

$$= f^2x \log^2 \left(c\left(d+ex^2\right)^p\right) + \frac{2}{3}fgx^3 \log^2 \left(c\left(d+ex^2\right)^p\right) + \frac{1}{5}g^2x^5 \log^2 \left(c(d\right.$$

$$= f^2x \log^2 \left(c\left(d+ex^2\right)^p\right) + \frac{2}{3}fgx^3 \log^2 \left(c\left(d+ex^2\right)^p\right) + \frac{1}{5}g^2x^5 \log^2 \left(c(d\right.$$

$$= f^2x \log^2 \left(c\left(d+ex^2\right)^p\right) + \frac{2}{3}fgx^3 \log^2 \left(c\left(d+ex^2\right)^p\right) + \frac{1}{5}g^2x^5 \log^2 \left(c(d\right.$$

$$= -4f^2px \log \left(c\left(d+ex^2\right)^p\right) + \frac{8dfgpx \log \left(c\left(d+ex^2\right)^p\right)}{3e} - \frac{4d^2g^2px \log (}{5e}$$

$$= 8f^2p^2x - \frac{16dfgp^2x}{3e} + \frac{8d^2g^2p^2x}{5e^2} - 4f^2px \log \left(c\left(d+ex^2\right)^p\right) + \frac{8dfgpx}{}$$

$$= 8f^2p^2x - \frac{64dfgp^2x}{9e} + \frac{184d^2g^2p^2x}{75e^2} + \frac{16}{27}fgp^2x^3 - \frac{64dg^2p^2x^3}{225e} + \frac{8}{125}g^2$$

$$= 8f^2p^2x - \frac{64dfgp^2x}{9e} + \frac{184d^2g^2p^2x}{75e^2} + \frac{16}{27}fgp^2x^3 - \frac{64dg^2p^2x^3}{225e} + \frac{8}{125}g^2$$

$$= 8f^2p^2x - \frac{64dfgp^2x}{9e} + \frac{184d^2g^2p^2x}{75e^2} + \frac{16}{27}fgp^2x^3 - \frac{64dg^2p^2x^3}{225e} + \frac{8}{125}g^2$$

$$= 8f^2p^2x - \frac{64dfgp^2x}{9e} + \frac{184d^2g^2p^2x}{75e^2} + \frac{16}{27}fgp^2x^3 - \frac{64dg^2p^2x^3}{225e} + \frac{8}{125}g^2$$

Mathematica [A]

time = 0.35, size = 435, normalized size = 0.46

$$\frac{900i\sqrt{d}\,(15e^2f^2-10defg+3d^2g^2)\,p^2\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)^2+60\sqrt{d}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(-2(225e^2f^2-200defg+69d^2g^2)\,p+30(15e^2f^2-10defg+3d^2g^2)\,p\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\,x}\right)+15(15e^2f^2-10defg+3d^2g^2)\log\left(c(d+ex^2)^p\right)\right)+\sqrt{e}\,x\left(8p^2(1035d^2g^2-120deg(25f+gx^2)+e^2(3375f^2+250fgx^2+27g^2x^4))-60p(45d^2g^2-15deg(10f+gx^2)+e^2(225f^2+50fgx^2+9g^2x^4))\log\left(c(d+ex^2)^p\right)+225e^2(15f^2+10fgx^2+3g^2x^4)\log^2\left(c(d+ex^2)^p\right)\right)+900i\sqrt{d}\,(15e^2f^2-10defg+3d^2g^2)\,p^2\mathrm{Li}_2\left(\frac{i\sqrt{d}+\sqrt{e}\,x}{-i\sqrt{d}+\sqrt{e}\,x}\right)}{3375e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^2)^2*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] ((900*I)*Sqrt[d]*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*p^2*ArcTan[(Sqrt[e]*
x)/Sqrt[d]]^2 + 60*Sqrt[d]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-2*(225*e^2*f^2 -
 200*d*e*f*g + 69*d^2*g^2)*p + 30*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*p*L
og[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)] + 15*(15*e^2*f^2 - 10*d*e*f*g + 3*d
^2*g^2)*Log[c*(d + e*x^2)^p]) + Sqrt[e]*x*(8*p^2*(1035*d^2*g^2 - 120*d*e*g*
(25*f + g*x^2) + e^2*(3375*f^2 + 250*f*g*x^2 + 27*g^2*x^4)) - 60*p*(45*d^2*
g^2 - 15*d*e*g*(10*f + g*x^2) + e^2*(225*f^2 + 50*f*g*x^2 + 9*g^2*x^4))*Log
[c*(d + e*x^2)^p] + 225*e^2*(15*f^2 + 10*f*g*x^2 + 3*g^2*x^4)*Log[c*(d + e*
x^2)^p]^2) + (900*I)*Sqrt[d]*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*p^2*Poly
Log[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)])/(3375*e^(5/2))
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \left(g\,x^2+f\right)^2 \ln\left(c\left(e\,x^2+d\right)^p\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(3*g^2*p^2*x^5 + 10*f*g*p^2*x^3 + 15*f^2*p^2*x)*log(x^2*e + d)^2 + int
egrate(1/15*(15*g^2*x^6*e*log(c)^2 + 15*(d*g^2*log(c)^2 + 2*f*g*e*log(c)^2)
*x^4 + 15*d*f^2*log(c)^2 + 15*(2*d*f*g*log(c)^2 + f^2*e*log(c)^2)*x^2 - 2*(
3*(2*g^2*p^2 - 5*g^2*p*log(c))*x^6*e - 5*(3*d*g^2*p*log(c) - 2*(2*f*g*p^2 -
 3*f*g*p*log(c))*e)*x^4 - 15*d*f^2*p*log(c) - 15*(2*d*f*g*p*log(c) - (2*f^2
*p^2 - f^2*p*log(c))*e)*x^2)*log(x^2*e + d))/(x^2*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral((g^2*x^4 + 2*f*g*x^2 + f^2)*log((x^2*e + d)^p*c)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(f+gx^2\right)^2\log \left(c\left(d+ex^2\right)^p\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Integral((f + g*x**2)**2*log(c*(d + e*x**2)**p)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((g*x^2 + f)^2*log((x^2*e + d)^p*c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\left(e\,x^2+d\right)^p\right)^2\,\left(g\,x^2+f\right)^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^2*(f + g*x^2)^2,x)
```

```
[Out] int(log(c*(d + e*x^2)^p)^2*(f + g*x^2)^2, x)
```

3.274 $\int (f+gx^2)\log^2\left(c(d+ex^2)^p\right)\,dx$

Optimal. Leaf size=548

$$8fp^2x-\frac{32dgp^2x}{9e}+\frac{8}{27}gp^2x^3-\frac{8\sqrt{d}\,fp^2\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{32d^{3/2}gp^2\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{9e^{3/2}}+\frac{4i\sqrt{d}\,fp^2\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right.}{\sqrt{e}}$$

[Out] 8*f*p^2*x-32/9*d*g*p^2*x/e+8/27*g*p^2*x^3+32/9*d^(3/2)*g*p^2*arctan(x*e^(1/
2)/d^(1/2))/e^(3/2)-4/3*I*d^(3/2)*g*p^2*arctan(x*e^(1/2)/d^(1/2))^2/e^(3/2)
-4*f*p*x*ln(c*(e*x^2+d)^p)+4/3*d*g*p*x*ln(c*(e*x^2+d)^p)/e-4/9*g*p*x^3*ln(c
*(e*x^2+d)^p)-4/3*d^(3/2)*g*p*arctan(x*e^(1/2)/d^(1/2))*ln(c*(e*x^2+d)^p)/e
^(3/2)+f*x*ln(c*(e*x^2+d)^p)^2+1/3*g*x^3*ln(c*(e*x^2+d)^p)^2-8/3*d^(3/2)*g*
p^2*arctan(x*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))/e^(3/2)+4
*I*f*p^2*arctan(x*e^(1/2)/d^(1/2))^2*d^(1/2)/e^(1/2)-8*f*p^2*arctan(x*e^(1/
2)/d^(1/2))*d^(1/2)/e^(1/2)+4*I*f*p^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x*e^
(1/2)))*d^(1/2)/e^(1/2)+4*f*p*arctan(x*e^(1/2)/d^(1/2))*ln(c*(e*x^2+d)^p)*d
^(1/2)/e^(1/2)+8*f*p^2*arctan(x*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x*
e^(1/2)))*d^(1/2)/e^(1/2)-4/3*I*d^(3/2)*g*p^2*polylog(2,1-2*d^(1/2)/(d^(1/2
)+I*x*e^(1/2)))/e^(3/2)

Rubi [A]
time = 0.46, antiderivative size = 548, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {2521, 2500, 2526, 2498, 327, 211, 2520, 12, 5040, 4964, 2449, 2352, 2507, 2505, 308}

$$-\frac{4id^{3/2}gp^2\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\,x}\right)}{3e^{3/2}}+\frac{4i\sqrt{d}\,fp^2\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\,x}\right)}{\sqrt{e}}-\frac{4d^{3/2}gp\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\log\left(c(d+ex^2)^p\right)}{3e^{3/2}}+\frac{4\sqrt{d}\,fp\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{e}}-\frac{4id^{3/2}gp^2\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)^2}{3e^{3/2}}+\frac{32d^{3/2}gp^2\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{9e^{3/2}}-\frac{8d^{3/2}gp^2\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\,x}\right)}{3e^{3/2}}+\frac{4i\sqrt{d}\,fp^2\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)^2}{\sqrt{e}}-\frac{8\sqrt{d}\,fp^2\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{8\sqrt{d}\,fp^2\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\,x}\right)}{\sqrt{e}}+fx\log^2\left(c(d+ex^2)^p\right)-4fpx\log\left(c(d+ex^2)^p\right)+\frac{4dgpx\log\left(c(d+ex^2)^p\right)}{3e}+\frac{1}{3}gx^3\log^2\left(c(d+ex^2)^p\right)-\frac{4}{9}gpx^3\log\left(c(d+ex^2)^p\right)-\frac{32dgp^2x}{9e}+8fp^2x+\frac{8}{27}gp^2x^3$$

Antiderivative was successfully verified.

[In] Int[(f + g*x^2)*Log[c*(d + e*x^2)^p]^2,x]

[Out] 8*f*p^2*x - (32*d*g*p^2*x)/(9*e) + (8*g*p^2*x^3)/27 - (8*Sqrt[d]*f*p^2*ArcT
an[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + (32*d^(3/2)*g*p^2*ArcTan[(Sqrt[e]*x)/Sqr
t[d]])/(9*e^(3/2)) + ((4*I)*Sqrt[d]*f*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/Sq
rt[e] - (((4*I)/3)*d^(3/2)*g*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e^(3/2) + (
8*Sqrt[d]*f*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sq
rt[e]*x)])/Sqrt[e] - (8*d^(3/2)*g*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sq
rt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/(3*e^(3/2)) - 4*f*p*x*Log[c*(d + e*x^2)^p]
 + (4*d*g*p*x*Log[c*(d + e*x^2)^p])/(3*e) - (4*g*p*x^3*Log[c*(d + e*x^2)^p]
)/9 + (4*Sqrt[d]*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/Sqrt
[e] - (4*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/(3*e
^(3/2)) + f*x*Log[c*(d + e*x^2)^p]^2 + (g*x^3*Log[c*(d + e*x^2)^p]^2)/3 + (
(4*I)*Sqrt[d]*f*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sq

```
rt[e] - (((4*I)/3)*d^(3/2)*g*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sq
rt[e]*x)])/e^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2500

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[x^n*(
```

```
(a + b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
```

```
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(f+gx^2\right)\log^2\left(c\left(d+ex^2\right)^p\right)\,dx = \int \left(f\log^2\left(c\left(d+ex^2\right)^p\right)+gx^2\log^2\left(c\left(d+ex^2\right)^p\right)\right)\,dx$$

$$= f\int \log^2\left(c\left(d+ex^2\right)^p\right)\,dx + g\int x^2\log^2\left(c\left(d+ex^2\right)^p\right)\,dx$$

$$= fx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log^2\left(c\left(d+ex^2\right)^p\right)-(4efp)\int \frac{x^2\log\left(c\right.}{d}$$

$$= fx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log^2\left(c\left(d+ex^2\right)^p\right)-(4efp)\int \left(\frac{\log\left(c\right.}{}\right.$$

$$= fx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log^2\left(c\left(d+ex^2\right)^p\right)-(4fp)\int \log\left(c\left(d\right.\right.$$

$$= -4fpx\log\left(c\left(d+ex^2\right)^p\right)+\frac{4dgpx\log\left(c\left(d+ex^2\right)^p\right)}{3e}-\frac{4}{9}gpx^3\log\left(c\left(d\right.\right.$$

$$= 8fp^2x-\frac{8dgp^2x}{3e}-4fpx\log\left(c\left(d+ex^2\right)^p\right)+\frac{4dgpx\log\left(c\left(d+ex^2\right)^p\right)}{3e}$$

$$= 8fp^2x-\frac{32dgp^2x}{9e}+\frac{8}{27}gp^2x^3-\frac{8\sqrt{d}\,fp^2\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{8d^{3/2}gp^2}{}$$

$$= 8fp^2x-\frac{32dgp^2x}{9e}+\frac{8}{27}gp^2x^3-\frac{8\sqrt{d}\,fp^2\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{32d^{3/2}gp^2}{}$$

$$= 8fp^2x-\frac{32dgp^2x}{9e}+\frac{8}{27}gp^2x^3-\frac{8\sqrt{d}\,fp^2\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{32d^{3/2}gp^2}{}$$

$$= 8fp^2x-\frac{32dgp^2x}{9e}+\frac{8}{27}gp^2x^3-\frac{8\sqrt{d}\,fp^2\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{32d^{3/2}gp^2}{}$$

Mathematica [A]

time = 0.19, size = 281, normalized size = 0.51

$$\frac{-36i\sqrt{d}\,(-3ef+dg)p^2\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)^2-12\sqrt{d}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(2(9ef-4dg)p+6(-3ef+dg)p\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\,x}\right)+(-9ef+3dg)\log\left(c(d+ex^2)^p\right)\right)+\sqrt{e}\,x\left(8p^2(27ef-12dg+egx^2)-12p(9ef-3dg+egx^2)\log\left(c(d+ex^2)^p\right)+9e(3f+gx^2)\log^2\left(c(d+ex^2)^p\right)\right)-36i\sqrt{d}\,(-3ef+dg)p^2\mathrm{Li}_2\left(\frac{i\sqrt{d}+\sqrt{e}\,x}{-i\sqrt{d}+\sqrt{e}\,x}\right)}{27e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^2)*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] ((-36*I)*Sqrt[d]*(-3*e*f + d*g)*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2 - 12*Sqrt
[d]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(2*(9*e*f - 4*d*g)*p + 6*(-3*e*f + d*g)*p
*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)] + (-9*e*f + 3*d*g)*Log[c*(d + e*x
^2)^p]) + Sqrt[e]*x*(8*p^2*(27*e*f - 12*d*g + e*g*x^2) - 12*p*(9*e*f - 3*d*
g + e*g*x^2)*Log[c*(d + e*x^2)^p] + 9*e*(3*f + g*x^2)*Log[c*(d + e*x^2)^p]^
2) - (36*I)*Sqrt[d]*(-3*e*f + d*g)*p^2*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x)/(
(-I)*Sqrt[d] + Sqrt[e]*x)])/(27*e^(3/2))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \left(g\,x^2+f\right)\ln\left(c\left(e\,x^2+d\right)^p\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int((g*x^2+f)*ln(c*(e*x^2+d)^p)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(g*p^2*x^3 + 3*f*p^2*x)*log(x^2*e + d)^2 + integrate(1/3*(3*g*x^4*e*log
(c)^2 + 3*d*f*log(c)^2 + 3*(d*g*log(c)^2 + f*e*log(c)^2)*x^2 - 2*((2*g*p^2
- 3*g*p*log(c))*x^4*e - 3*d*f*p*log(c) - 3*(d*g*p*log(c) - (2*f*p^2 - f*p*l
og(c))*e)*x^2)*log(x^2*e + d))/(x^2*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral((g*x^2 + f)*log((x^2*e + d)^p*c)^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(f+gx^{2}\right)\log \left(c\left(d+ex^{2}\right)^{p}\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)**2,x)
[Out] Integral((f + g*x**2)*log(c*(d + e*x**2)**p)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
[Out] integrate((g*x^2 + f)*log((x^2*e + d)^p*c)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln \left(c\,{\left(e\,x^2+d\right)}^p\right)^2\,\left(g\,x^2+f\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^2*(f + g*x^2),x)
[Out] int(log(c*(d + e*x^2)^p)^2*(f + g*x^2), x)
```

3.275 $\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^2} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^2},x\right)$$

```
[Out] Unintegrable(ln(c*(e*x^2+d)^p)^2/(g*x^2+f),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^2} dx$$

Verification is not applicable to the result.

```
[In] Int[Log[c*(d + e*x^2)^p]^2/(f + g*x^2),x]
[Out] Defer[Int][Log[c*(d + e*x^2)^p]^2/(f + g*x^2), x]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^2} dx = \int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^2} dx$$

Mathematica [A]
time = 1.97, size = 0, normalized size = 0.00

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^2),x]
[Out] Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^2), x]
```

Maple [A]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)^2}{g\,x^2+f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)^2/(g*x^2+f),x)
```

```
[Out] int(ln(c*(e*x^2+d)^p)^2/(g*x^2+f),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] integrate(log((x^2*e + d)^p*c)^2/(g*x^2 + f), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log((x^2*e + d)^p*c)^2/(g*x^2 + f), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c(d+ex^2)^p\right)^2}{f+gx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)**2/(g*x**2+f),x)
```

```
[Out] Integral(log(c*(d + e*x**2)**p)**2/(f + g*x**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(log((x^2*e + d)^p*c)^2/(g*x^2 + f), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)^2}{g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^2/(f + g*x^2),x)
[Out] int(log(c*(d + e*x^2)^p)^2/(f + g*x^2), x)
```

3.276 $\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2},x\right)$$

```
[Out] Unintegrable(ln(c*(e*x^2+d)^p)^2/(g*x^2+f)^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[c*(d + e*x^2)^p]^2/(f + g*x^2)^2,x]
[Out] Defer[Int][Log[c*(d + e*x^2)^p]^2/(f + g*x^2)^2, x]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx = \int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx$$

Mathematica [A]
time = 5.45, size = 0, normalized size = 0.00

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^2)^2,x]
[Out] Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^2)^2, x]
```

Maple [A]
time = 1.60, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)^2}{\left(g\,x^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)^2/(g*x^2+f)^2,x)
```

```
[Out] int(ln(c*(e*x^2+d)^p)^2/(g*x^2+f)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] integrate(log((x^2*e + d)^p*c)^2/(g*x^2 + f)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] integral(log((x^2*e + d)^p*c)^2/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)**2/(g*x**2+f)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((x^2*e + d)^p*c)^2/(g*x^2 + f)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)^2}{\left(g\,x^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^2/(f + g*x^2)^2,x)
[Out] int(log(c*(d + e*x^2)^p)^2/(f + g*x^2)^2, x)
```

3.277 $\int (f+gx^2)\log^3\left(c(d+ex^2)^p\right)\,dx$

Optimal. Leaf size=683

$$-48fp^3x+\frac{208dgp^3x}{9e}-\frac{16}{27}gp^3x^3+\frac{48\sqrt{d}\,fp^3\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{208d^{3/2}gp^3\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{9e^{3/2}}-\frac{24i\sqrt{d}\,fp^3\tan^{-1}\left(\right.}{\sqrt{e}}$$

```
[Out] -48*f*p^3*x+208/9*d*g*p^3*x/e-16/27*g*p^3*x^3-208/9*d^(3/2)*g*p^3*arctan(x*
e^(1/2)/d^(1/2))/e^(3/2)-24*I*f*p^3*arctan(x*e^(1/2)/d^(1/2))^2*d^(1/2)/e^(
1/2)+24*f*p^2*x*ln(c*(e*x^2+d)^p)-32/3*d*g*p^2*x*ln(c*(e*x^2+d)^p)/e+8/9*g*
p^2*x^3*ln(c*(e*x^2+d)^p)+32/3*d^(3/2)*g*p^2*arctan(x*e^(1/2)/d^(1/2))*ln(c
*(e*x^2+d)^p)/e^(3/2)-6*f*p*x*ln(c*(e*x^2+d)^p)^2+2*d*g*p*x*ln(c*(e*x^2+d)^
p)^2/e-2/3*g*p*x^3*ln(c*(e*x^2+d)^p)^2+f*x*ln(c*(e*x^2+d)^p)^3+1/3*g*x^3*ln
(c*(e*x^2+d)^p)^3+64/3*d^(3/2)*g*p^3*arctan(x*e^(1/2)/d^(1/2))*ln(2*d^(1/2)
/(d^(1/2)+I*x*e^(1/2)))/e^(3/2)+32/3*I*d^(3/2)*g*p^3*polylog(2,(-d^(1/2)+I*
x*e^(1/2))/(d^(1/2)+I*x*e^(1/2)))/e^(3/2)+48*f*p^3*arctan(x*e^(1/2)/d^(1/2)
)*d^(1/2)/e^(1/2)+32/3*I*d^(3/2)*g*p^3*arctan(x*e^(1/2)/d^(1/2))^2/e^(3/2)-
24*f*p^2*arctan(x*e^(1/2)/d^(1/2))*ln(c*(e*x^2+d)^p)*d^(1/2)/e^(1/2)-48*f*p
^3*arctan(x*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))*d^(1/2)/e^
(1/2)-24*I*f*p^3*polylog(2,(-d^(1/2)+I*x*e^(1/2))/(d^(1/2)+I*x*e^(1/2)))*d^
(1/2)/e^(1/2)-2*d*(d*g-3*e*f)*p*Unintegrable(ln(c*(e*x^2+d)^p)^2/(e*x^2+d),
x)/e
```

Rubi [A]
time = 0.90, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (f+gx^2)\log^3\left(c(d+ex^2)^p\right)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(f + g*x^2)*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] -48*f*p^3*x + (208*d*g*p^3*x)/(9*e) - (16*g*p^3*x^3)/27 + (48*Sqrt[d]*f*p^3
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (208*d^(3/2)*g*p^3*ArcTan[(Sqrt[e]*
x)/Sqrt[d]])/(9*e^(3/2)) - ((24*I)*Sqrt[d]*f*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]
]^2)/Sqrt[e] + (((32*I)/3)*d^(3/2)*g*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e^(
3/2) - (48*Sqrt[d]*f*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[
d] + I*Sqrt[e]*x)])/Sqrt[e] + (64*d^(3/2)*g*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/(3*e^(3/2)) + 24*f*p^2*x*Log[c*(
d + e*x^2)^p] - (32*d*g*p^2*x*Log[c*(d + e*x^2)^p])/(3*e) + (8*g*p^2*x^3*Lo
g[c*(d + e*x^2)^p])/9 - (24*Sqrt[d]*f*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c
*(d + e*x^2)^p])/Sqrt[e] + (32*d^(3/2)*g*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Lo
```

```
g[c*(d + e*x^2)^p])/(3*e^(3/2)) - 6*f*p*x*Log[c*(d + e*x^2)^p]^2 + (2*d*g*p
*x*Log[c*(d + e*x^2)^p]^2)/e - (2*g*p*x^3*Log[c*(d + e*x^2)^p]^2)/3 + f*x*L
og[c*(d + e*x^2)^p]^3 + (g*x^3*Log[c*(d + e*x^2)^p]^3)/3 - ((24*I)*Sqrt[d]*
f*p^3*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] + (((32*
I)/3)*d^(3/2)*g*p^3*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/e^
(3/2) + 6*d*f*p*Defer[Int][Log[c*(d + e*x^2)^p]^2/(d + e*x^2), x] - (2*d^2*
g*p*Defer[Int][Log[c*(d + e*x^2)^p]^2/(d + e*x^2), x])/e
```

Rubi steps

$$\int \left(f+gx^2\right)\log^3\left(c\left(d+ex^2\right)^p\right)\,dx=\int \left(f\log^3\left(c\left(d+ex^2\right)^p\right)+gx^2\log^3\left(c\left(d+ex^2\right)^p\right)\right)\,dx$$

$$=f\int \log^3\left(c\left(d+ex^2\right)^p\right)\,dx+g\int x^2\log^3\left(c\left(d+ex^2\right)^p\right)\,dx$$

$$=fx\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log^3\left(c\left(d+ex^2\right)^p\right)-(6efp)\int \frac{x^2\log^2\left(c\right.}{d+}$$

$$=fx\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log^3\left(c\left(d+ex^2\right)^p\right)-(6efp)\int \left(\frac{\log^2\left(c\right(}{}\right.$$

$$=fx\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log^3\left(c\left(d+ex^2\right)^p\right)-(6fp)\int \log^2\left(c\left(d+\right.\right.$$

$$=-6fpx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{2dgpx\log^2\left(c\left(d+ex^2\right)^p\right)}{e}-\frac{2}{3}gpx^3\log^2\left(c\right($$

$$=-6fpx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{2dgpx\log^2\left(c\left(d+ex^2\right)^p\right)}{e}-\frac{2}{3}gpx^3\log^2\left(c\right($$

$$=-6fpx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{2dgpx\log^2\left(c\left(d+ex^2\right)^p\right)}{e}-\frac{2}{3}gpx^3\log^2\left(c\right($$

$$=24fp^2x\log\left(c\left(d+ex^2\right)^p\right)-\frac{32dgp^2x\log\left(c\left(d+ex^2\right)^p\right)}{3e}+\frac{8}{9}gp^2x^3\log\left(c\right.$$

$$=-48fp^3x+\frac{64dgp^3x}{3e}+24fp^2x\log\left(c\left(d+ex^2\right)^p\right)-\frac{32dgp^2x\log\left(c\left(d+\right.\right.}{3e}$$

$$=-48fp^3x+\frac{208dgp^3x}{9e}-\frac{16}{27}gp^3x^3+\frac{48\sqrt{d}\,fp^3\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{64d^{3/}}{}$$

$$=-48fp^3x+\frac{208dgp^3x}{9e}-\frac{16}{27}gp^3x^3+\frac{48\sqrt{d}\,fp^3\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{208d^{}}{}$$

$$=-48fp^3x+\frac{208dgp^3x}{9e}-\frac{16}{27}gp^3x^3+\frac{48\sqrt{d}\,fp^3\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{208d^{}}{}$$

$$=-48fp^3x+\frac{208dgp^3x}{9e}-\frac{16}{27}gp^3x^3+\frac{48\sqrt{d}\,fp^3\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{208d^{}}{}$$

Mathematica [**A**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1460 vs. $2(683) = 1366$.
time = 2.99, size = 1460, normalized size = 2.14

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^2)*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] (g*p^3*x*(-18*(d + e*x^2)*HypergeometricPFQ[{-1/2, 1, 1, 1, 1}, {2, 2, 2, 2
}, (d + e*x^2)/d] + 18*(d + e*x^2)*HypergeometricPFQ[{-1/2, 1, 1, 1}, {2, 2
, 2}, (d + e*x^2)/d]*Log[d + e*x^2] - 9*(d + e*x^2)*HypergeometricPFQ[{-1/2
, 1, 1}, {2, 2}, (d + e*x^2)/d]*Log[d + e*x^2]^2 + 2*d*Log[d + e*x^2]^3 - 2
*d*Sqrt[1 - (d + e*x^2)/d]*Log[d + e*x^2]^3 + 2*(d + e*x^2)*Sqrt[1 - (d + e
*x^2)/d]*Log[d + e*x^2]^3))/(6*e*Sqrt[1 - (d + e*x^2)/d]) + (2*d*g*p*x*(-(p
*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/e + (6*Sqrt[d]*f*p*ArcTan[(Sqrt
[e]*x)/Sqrt[d]]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/Sqrt[e] - (
2*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-(p*Log[d + e*x^2]) + Log[c*(d +
 e*x^2)^p])^2)/e^(3/2) + 3*f*p*x*Log[d + e*x^2]*(-(p*Log[d + e*x^2]) + Log[
c*(d + e*x^2)^p])^2 + g*p*x^3*Log[d + e*x^2]*(-(p*Log[d + e*x^2]) + Log[c*(
d + e*x^2)^p])^2 + f*x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2*(-6*p
 - p*Log[d + e*x^2] + Log[c*(d + e*x^2)^p]) + (g*x^3*(-(p*Log[d + e*x^2]) +
 Log[c*(d + e*x^2)^p])^2*(-2*p - p*Log[d + e*x^2] + Log[c*(d + e*x^2)^p]))/
3 + 3*f*p^2*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])*(x*Log[d + e*x^2]^
2 - (4*((-I)*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2 + Sqrt[e]*x*(-2 + Log[d
+ e*x^2]) - Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-2 + 2*Log[(2*Sqrt[d])/(Sq
rt[d] + I*Sqrt[e]*x)] + Log[d + e*x^2]) - I*Sqrt[d]*PolyLog[2, (I*Sqrt[d] +
 Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)]))/Sqrt[e]) + 3*g*p^2*(-(p*Log[d + e
*x^2]) + Log[c*(d + e*x^2)^p])*((x^3*Log[d + e*x^2]^2)/3 - (4*((9*I)*d^(3/2
)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2 + 3*d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-8
 + 6*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)] + 3*Log[d + e*x^2]) + Sqrt[e]
*x*(24*d - 2*e*x^2 + (-9*d + 3*e*x^2)*Log[d + e*x^2]) + (9*I)*d^(3/2)*PolyL
og[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)]))/(27*e^(3/2))) +
 (f*p^3*(-48*Sqrt[-d^2]*Sqrt[d + e*x^2]*Sqrt[1 - d/(d + e*x^2)]*ArcSin[Sqrt
[d]/Sqrt[d + e*x^2]] - 6*Sqrt[-d^2]*Sqrt[1 - d/(d + e*x^2)]*(8*Sqrt[d]*Hype
rgeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}, d/(d + e*x^2)] + 4*Sqr
t[d]*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, d/(d + e*x^2)]*Log[d +
e*x^2] + Sqrt[d + e*x^2]*ArcSin[Sqrt[d]/Sqrt[d + e*x^2]]*Log[d + e*x^2]^2)
+ Sqrt[-d]*e*x^2*(-48 + 24*Log[d + e*x^2] - 6*Log[d + e*x^2]^2 + Log[d + e*
x^2]^3) + 24*d*Sqrt[e*x^2]*ArcTanh[Sqrt[e*x^2]/Sqrt[-d]]*(Log[d + e*x^2] -
Log[(d + e*x^2)/d]) + 6*(-d)^(3/2)*Sqrt[1 - (d + e*x^2)/d]*(Log[(d + e*x^2)
/d]^2 - 4*Log[(d + e*x^2)/d]*Log[(1 + Sqrt[1 - (d + e*x^2)/d])/2] + 2*Log[(
1 + Sqrt[1 - (d + e*x^2)/d])/2]^2 - 4*PolyLog[2, 1/2 - Sqrt[1 - (d + e*x^2)
/d]/2])))/(Sqrt[-d]*e*x)
```

Maple [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \left(g\,x^{2}+f\right) \ln \left(c \left(e\,x^{2}+d\right)^{p}\right)^{3}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)^3,x)
```

```
[Out] int((g*x^2+f)*ln(c*(e*x^2+d)^p)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(g*p^3*x^3 + 3*f*p^3*x)*log(x^2*e + d)^3 + integrate((g*x^4*e*log(c)^3
+ d*f*log(c)^3 + (d*g*log(c)^3 + f*e*log(c)^3)*x^2 - ((2*g*p^3 - 3*g*p^2*lo
g(c))*x^4*e - 3*d*f*p^2*log(c) - 3*(d*g*p^2*log(c) - (2*f*p^3 - f*p^2*log(c
))*e)*x^2)*log(x^2*e + d)^2 + 3*(g*p*x^4*e*log(c)^2 + d*f*p*log(c)^2 + (d*g
*p*log(c)^2 + f*p*e*log(c)^2)*x^2)*log(x^2*e + d))/(x^2*e + d), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)^3,x, algorithm="fricas")
```

```
[Out] integral((g*x^2 + f)*log((x^2*e + d)^p*c)^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(f + g x^{2}\right) \log {\left(c \left(d + e x^{2}\right)^{p} \right)}^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)**3,x)
```

```
[Out] Integral((f + g*x**2)*log(c*(d + e*x**2)**p)**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)^3,x, algorithm="giac")
[Out] integrate((g*x^2 + f)*log((x^2*e + d)^p*c)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\left(e\,x^2+d\right)^p\right)^3\,\left(g\,x^2+f\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^3*(f + g*x^2),x)
[Out] int(log(c*(d + e*x^2)^p)^3*(f + g*x^2), x)
```

3.278 $\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^2},x\right)$$

```
[Out] Unintegrable(ln(c*(e*x^2+d)^p)^3/(g*x^2+f),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[c*(d + e*x^2)^p]^3/(f + g*x^2),x]
[Out] Defer[Int][Log[c*(d + e*x^2)^p]^3/(f + g*x^2), x]
```

Rubi steps

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx = \int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$$

Mathematica [A]
time = 2.62, size = 0, normalized size = 0.00

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^2),x]
[Out] Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^2), x]
```

Maple [A]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)^3}{g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)^3/(g*x^2+f),x)
```

```
[Out] int(ln(c*(e*x^2+d)^p)^3/(g*x^2+f),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] integrate(log((x^2*e + d)^p*c)^3/(g*x^2 + f), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log((x^2*e + d)^p*c)^3/(g*x^2 + f), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c(d+ex^2)^p\right)^3}{f+gx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)**3/(g*x**2+f),x)
```

```
[Out] Integral(log(c*(d + e*x**2)**p)**3/(f + g*x**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(log((x^2*e + d)^p*c)^3/(g*x^2 + f), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln \left(c \left(e\,x^2+d\right)^p\right)^3}{g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^3/(f + g*x^2),x)
```

```
[Out] int(log(c*(d + e*x^2)^p)^3/(f + g*x^2), x)
```

3.279 $\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2},x\right)$$

```
[Out] Unintegrable(ln(c*(e*x^2+d)^p)^3/(g*x^2+f)^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[Log[c*(d + e*x^2)^p]^3/(f + g*x^2)^2,x]
[Out] Defer[Int][Log[c*(d + e*x^2)^p]^3/(f + g*x^2)^2, x]
```

Rubi steps

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2} dx = \int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2} dx$$

Mathematica [A]
time = 9.79, size = 0, normalized size = 0.00

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^2)^2,x]
[Out] Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^2)^2, x]
```

Maple [A]
time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)^3}{\left(g\,x^2+f\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)^3/(g*x^2+f)^2,x)
```

```
[Out] int(ln(c*(e*x^2+d)^p)^3/(g*x^2+f)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] integrate(log((x^2*e + d)^p*c)^3/(g*x^2 + f)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] integral(log((x^2*e + d)^p*c)^3/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)**3/(g*x**2+f)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((x^2*e + d)^p*c)^3/(g*x^2 + f)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)^3}{\left(g\,x^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^3/(f + g*x^2)^2,x)
[Out] int(log(c*(d + e*x^2)^p)^3/(f + g*x^2)^2, x)
```

3.280 $\int \frac{(f+gx^2)^2}{\log(c(d+ex^2)^p)} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{(f+gx^2)^2}{\log\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable((g*x^2+f)^2/ln(c*(e*x^2+d)^p),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(f+gx^2)^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(f + g*x^2)^2/Log[c*(d + e*x^2)^p],x]
[Out] Defer[Int][(f + g*x^2)^2/Log[c*(d + e*x^2)^p], x]
```

Rubi steps

$$\int \frac{(f+gx^2)^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{(f+gx^2)^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [A]
time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{(f+gx^2)^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f + g*x^2)^2/Log[c*(d + e*x^2)^p],x]
[Out] Integrate[(f + g*x^2)^2/Log[c*(d + e*x^2)^p], x]
```

Maple [A]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\left(g\,x^{2}+f\right)^{2}}{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2/ln(c*(e*x^2+d)^p),x)
[Out] int((g*x^2+f)^2/ln(c*(e*x^2+d)^p),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2/log(c*(e*x^2+d)^p),x, algorithm="maxima")
[Out] integrate((g*x^2 + f)^2/log((x^2*e + d)^p*c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2/log(c*(e*x^2+d)^p),x, algorithm="fricas")
[Out] integral((g^2*x^4 + 2*f*g*x^2 + f^2)/log((x^2*e + d)^p*c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f + gx^{2}\right)^{2}}{\log\left(c\left(d + ex^{2}\right)^{p}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2/ln(c*(e*x**2+d)**p),x)
[Out] Integral((f + g*x**2)**2/log(c*(d + e*x**2)**p), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2/log(c*(e*x^2+d)^p),x, algorithm="giac")
[Out] integrate((g*x^2 + f)^2/log((x^2*e + d)^p*c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(g\,x^2+f\right)^2}{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x^2)^2/log(c*(d + e*x^2)^p),x)
[Out] int((f + g*x^2)^2/log(c*(d + e*x^2)^p), x)
```

3.281 $\int \frac{f+gx^2}{\log(c(d+ex^2)^p)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable((g*x^2+f)/ln(c*(e*x^2+d)^p),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(f + g*x^2)/Log[c*(d + e*x^2)^p],x]
[Out] Defer[Int][(f + g*x^2)/Log[c*(d + e*x^2)^p], x]
```

Rubi steps

$$\int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [A]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f + g*x^2)/Log[c*(d + e*x^2)^p],x]
[Out] Integrate[(f + g*x^2)/Log[c*(d + e*x^2)^p], x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{g\,x^2+f}{\ln\left(c\left(e\,x^2+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)/ln(c*(e*x^2+d)^p),x)
```

```
[Out] int((g*x^2+f)/ln(c*(e*x^2+d)^p),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)/log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] integrate((g*x^2 + f)/log((x^2*e + d)^p*c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)/log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] integral((g*x^2 + f)/log((x^2*e + d)^p*c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f + gx^2}{\log\left(c\left(d + ex^2\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)/ln(c*(e*x**2+d)**p),x)
```

```
[Out] Integral((f + g*x**2)/log(c*(d + e*x**2)**p), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)/log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] integrate((g*x^2 + f)/log((x^2*e + d)^p*c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{g\,x^2+f}{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x^2)/log(c*(d + e*x^2)^p),x)
[Out] int((f + g*x^2)/log(c*(d + e*x^2)^p), x)
```

3.282 $\int \frac{1}{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)} \, dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{1}{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x^2+f)/ln(c*(e*x^2+d)^p),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[1/((f + g*x^2)*Log[c*(d + e*x^2)^p]),x]
[Out] Defer[Int][1/((f + g*x^2)*Log[c*(d + e*x^2)^p]), x]
```

Rubi steps

$$\int \frac{1}{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)} \, dx = \int \frac{1}{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)} \, dx$$

Mathematica [A]
time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((f + g*x^2)*Log[c*(d + e*x^2)^p]),x]
[Out] Integrate[1/((f + g*x^2)*Log[c*(d + e*x^2)^p]), x]
```

Maple [A]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{1}{(g\,x^2+f)\ln\left(c\left(e\,x^2+d\right)^p\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x^2+f)/ln(c*(e*x^2+d)^p),x)
[Out] int(1/(g*x^2+f)/ln(c*(e*x^2+d)^p),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)/log(c*(e*x^2+d)^p),x, algorithm="maxima")
[Out] integrate(1/((g*x^2 + f)*log((x^2*e + d)^p*c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)/log(c*(e*x^2+d)^p),x, algorithm="fricas")
[Out] integral(1/((g*x^2 + f)*log((x^2*e + d)^p*c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x**2+f)/ln(c*(e*x**2+d)**p),x)
[Out] Integral(1/((f + g*x**2)*log(c*(d + e*x**2)**p)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)/log(c*(e*x^2+d)^p),x, algorithm="giac")
[Out] integrate(1/((g*x^2 + f)*log((x^2*e + d)^p*c)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\ln\left(c\left(e\,x^2+d\right)^p\right)\,\left(g\,x^2+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(log(c*(d + e*x^2)^p)*(f + g*x^2)),x)
[Out] int(1/(log(c*(d + e*x^2)^p)*(f + g*x^2)), x)
```

3.283 $\int \frac{1}{(f+gx^2)^2 \log(c(d+ex^2)^p)} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{1}{(f+gx^2)^2 \log(c(d+ex^2)^p)}, x\right)$$

```
[Out] Unintegrable(1/(g*x^2+f)^2/ln(c*(e*x^2+d)^p),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(f+gx^2)^2 \log(c(d+ex^2)^p)} dx$$

Verification is not applicable to the result.

```
[In] Int[1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]),x]
[Out] Defer[Int][1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]), x]
```

Rubi steps

$$\int \frac{1}{(f+gx^2)^2 \log(c(d+ex^2)^p)} dx = \int \frac{1}{(f+gx^2)^2 \log(c(d+ex^2)^p)} dx$$

Mathematica [A]
time = 1.21, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx^2)^2 \log(c(d+ex^2)^p)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]),x]
[Out] Integrate[1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]), x]
```

Maple [A]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{1}{(g\,x^2+f)^2 \ln(c(e\,x^2+d)^p)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x^2+f)^2/ln(c*(e*x^2+d)^p),x)
```

```
[Out] int(1/(g*x^2+f)^2/ln(c*(e*x^2+d)^p),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)^2/log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] integrate(1/((g*x^2 + f)^2*log((x^2*e + d)^p*c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)^2/log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] integral(1/((g^2*x^4 + 2*f*g*x^2 + f^2)*log((x^2*e + d)^p*c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(f + gx^{2}\right)^{2} \log \left(c \left(d + ex^{2}\right)^{p}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x**2+f)**2/ln(c*(e*x**2+d)**p),x)
```

```
[Out] Integral(1/((f + g*x**2)**2*log(c*(d + e*x**2)**p)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)^2/log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] integrate(1/((g*x^2 + f)^2*log((x^2*e + d)^p*c)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\ln\left(c\left(e\,x^2+d\right)^p\right)\,\left(g\,x^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(log(c*(d + e*x^2)^p)*(f + g*x^2)^2),x)
[Out] int(1/(log(c*(d + e*x^2)^p)*(f + g*x^2)^2), x)
```

3.284 $\int \frac{(f+gx^2)^2}{\log^2(c(d+ex^2)^p)}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{(f+gx^2)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable((g*x^2+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(f+gx^2)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(f + g*x^2)^2/Log[c*(d + e*x^2)^p]^2,x]
[Out] Defer[Int][(f + g*x^2)^2/Log[c*(d + e*x^2)^p]^2, x]
```

Rubi steps

$$\int \frac{(f+gx^2)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{(f+gx^2)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [A]
time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{(f+gx^2)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f + g*x^2)^2/Log[c*(d + e*x^2)^p]^2,x]
[Out] Integrate[(f + g*x^2)^2/Log[c*(d + e*x^2)^p]^2, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g\,x^2+f\right)^2}{\ln\left(c\left(e\,x^2+d\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int((g*x^2+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(g^2*x^6*e + (d*g^2 + 2*f*g*e)*x^4 + d*f^2 + (2*d*f*g + f^2*e)*x^2)/(p
^2*x*e*log(x^2*e + d) + p*x*e*log(c)) + integrate(1/2*(5*g^2*x^6*e + 3*(d*g
^2 + 2*f*g*e)*x^4 - d*f^2 + (2*d*f*g + f^2*e)*x^2)/(p^2*x^2*e*log(x^2*e + d
) + p*x^2*e*log(c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral((g^2*x^4 + 2*f*g*x^2 + f^2)/log((x^2*e + d)^p*c)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f + gx^{2}\right)^{2}}{\log\left(c\left(d + ex^{2}\right)^{p}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2/ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Integral((f + g*x**2)**2/log(c*(d + e*x**2)**p)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((g*x^2 + f)^2/log((x^2*e + d)^p*c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(g\,x^2+f\right)^2}{\ln\left(c\,\left(e\,x^2+d\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x^2)^2/log(c*(d + e*x^2)^p)^2,x)
[Out] int((f + g*x^2)^2/log(c*(d + e*x^2)^p)^2, x)
```

3.285 $\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable((g*x^2+f)/ln(c*(e*x^2+d)^p)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(f + g*x^2)/Log[c*(d + e*x^2)^p]^2,x]
[Out] Defer[Int][(f + g*x^2)/Log[c*(d + e*x^2)^p]^2, x]
```

Rubi steps

$$\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [A]
time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f + g*x^2)/Log[c*(d + e*x^2)^p]^2,x]
[Out] Integrate[(f + g*x^2)/Log[c*(d + e*x^2)^p]^2, x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{g\,x^2+f}{\ln\left(c\left(e\,x^2+d\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)/ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int((g*x^2+f)/ln(c*(e*x^2+d)^p)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(g*x^4*e + (d*g + f*e)*x^2 + d*f)/(p^2*x*e*log(x^2*e + d) + p*x*e*log(
c)) + integrate(1/2*(3*g*x^4*e + (d*g + f*e)*x^2 - d*f)/(p^2*x^2*e*log(x^2*
e + d) + p*x^2*e*log(c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral((g*x^2 + f)/log((x^2*e + d)^p*c)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f + gx^2}{\log \left(c \left(d + ex^2\right)^p\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)/ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Integral((f + g*x**2)/log(c*(d + e*x**2)**p)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((g*x^2 + f)/log((x^2*e + d)^p*c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{g\,x^2+f}{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x^2)/log(c*(d + e*x^2)^p)^2,x)
[Out] int((f + g*x^2)/log(c*(d + e*x^2)^p)^2, x)
```

3.286 $\int \frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x^2+f)/ln(c*(e*x^2+d)^p)^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)} dx$$

Verification is not applicable to the result.

```
[In] Int[1/((f + g*x^2)*Log[c*(d + e*x^2)^p]^2),x]
[Out] Defer[Int][1/((f + g*x^2)*Log[c*(d + e*x^2)^p]^2), x]
```

Rubi steps

$$\int \frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)} dx = \int \frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)} dx$$

Mathematica [A]
time = 2.48, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((f + g*x^2)*Log[c*(d + e*x^2)^p]^2),x]
[Out] Integrate[1/((f + g*x^2)*Log[c*(d + e*x^2)^p]^2), x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(g\,x^2+f)\ln\left(c\left(e\,x^2+d\right)^p\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x^2+f)/ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int(1/(g*x^2+f)/ln(c*(e*x^2+d)^p)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(x^2*e + d)/(g*p*x^3*e*log(c) + f*p*x*e*log(c) + (g*p^2*x^3*e + f*p^2*
x*e)*log(x^2*e + d)) - integrate(1/2*(g*x^4*e + (3*d*g - f*e)*x^2 + d*f)/(g
^2*p*x^6*e*log(c) + 2*f*g*p*x^4*e*log(c) + f^2*p*x^2*e*log(c) + (g^2*p^2*x^
6*e + 2*f*g*p^2*x^4*e + f^2*p^2*x^2*e)*log(x^2*e + d)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((g*x^2 + f)*log((x^2*e + d)^p*c)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(f + gx^2) \log{(c (d + ex^2)^p)}^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x**2+f)/ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Integral(1/((f + g*x**2)*log(c*(d + e*x**2)**p)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((g*x^2 + f)*log((x^2*e + d)^p*c)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)^2\,\left(g\,x^2+f\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(log(c*(d + e*x^2)^p)^2*(f + g*x^2)),x)
[Out] int(1/(log(c*(d + e*x^2)^p)^2*(f + g*x^2)), x)
```

3.287 $\int \frac{1}{\left(f+gx^2\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)} \, dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{1}{\left(f+gx^2\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x^2+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\left(f+gx^2\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Int[1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]^2),x]
```

```
[Out] Defer[Int][1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]^2), x]
```

Rubi steps

$$\int \frac{1}{\left(f+gx^2\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)} \, dx = \int \frac{1}{\left(f+gx^2\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)} \, dx$$

Mathematica [A]
time = 5.72, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(f+gx^2\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]^2),x]
```

```
[Out] Integrate[1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]^2), x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(g\,x^2+f\right)^2 \ln\left(c\left(e\,x^2+d\right)^p\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x^2+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int(1/(g*x^2+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(x^2*e + d)/(g^2*p*x^5*e*log(c) + 2*f*g*p*x^3*e*log(c) + f^2*p*x*e*log
(c) + (g^2*p^2*x^5*e + 2*f*g*p^2*x^3*e + f^2*p^2*x*e)*log(x^2*e + d)) - int
egrate(1/2*(3*g*x^4*e + (5*d*g - f*e)*x^2 + d*f)/(g^3*p*x^8*e*log(c) + 3*f*
g^2*p*x^6*e*log(c) + 3*f^2*g*p*x^4*e*log(c) + f^3*p*x^2*e*log(c) + (g^3*p^2
*x^8*e + 3*f*g^2*p^2*x^6*e + 3*f^2*g*p^2*x^4*e + f^3*p^2*x^2*e)*log(x^2*e +
 d)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((g^2*x^4 + 2*f*g*x^2 + f^2)*log((x^2*e + d)^p*c)^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x**2+f)**2/ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^2+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((g*x^2 + f)^2*log((x^2*e + d)^p*c)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)^2\,{\left(g\,x^2+f\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(log(c*(d + e*x^2)^p)^2*(f + g*x^2)^2),x)
```

```
[Out] int(1/(log(c*(d + e*x^2)^p)^2*(f + g*x^2)^2), x)
```

3.288 $\int (f+gx^3)^3 \log\left(c(d+ex^2)^p\right)\,dx$

Optimal. Leaf size=366

$$-2f^3px+\frac{6d^3fg^2px}{7e^3}+\frac{3df^2gpx^2}{4e}-\frac{d^4g^3px^2}{10e^4}-\frac{2d^2fg^2px^3}{7e^2}-\frac{3}{8}f^2gpx^4+\frac{d^3g^3px^4}{20e^3}+\frac{6dfg^2px^5}{35e}-\frac{d^2g^3px^6}{30e^2}-\frac{6}{49}fg^2px^7$$

```
[Out] -2*f^3*p*x+6/7*d^3*f*g^2*p*x/e^3+3/4*d*f^2*g*p*x^2/e-1/10*d^4*g^3*p*x^2/e^4
-2/7*d^2*f*g^2*p*x^3/e^2-3/8*f^2*g*p*x^4+1/20*d^3*g^3*p*x^4/e^3+6/35*d*f*g^
2*p*x^5/e-1/30*d^2*g^3*p*x^6/e^2-6/49*f*g^2*p*x^7+1/40*d*g^3*p*x^8/e-1/50*g
^3*p*x^10-6/7*d^(7/2)*f*g^2*p*arctan(x*e^(1/2)/d^(1/2))/e^(7/2)-3/4*d^2*f^2
*g*p*ln(e*x^2+d)/e^2+1/10*d^5*g^3*p*ln(e*x^2+d)/e^5+f^3*x*ln(c*(e*x^2+d)^p)
+3/4*f^2*g*x^4*ln(c*(e*x^2+d)^p)+3/7*f*g^2*x^7*ln(c*(e*x^2+d)^p)+1/10*g^3*x
^10*ln(c*(e*x^2+d)^p)+2*f^3*p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$,
Rules used = {2521, 2498, 327, 211, 2504, 2442, 45, 2505, 308}

$$-\frac{6d^{7/2}fg^2p\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{7e^{7/2}}+\frac{2\sqrt{d}\,f^3p\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}+f^3x\log\left(c(d+ex^2)^p\right)+\frac{3}{4}f^2gx^4\log\left(c(d+ex^2)^p\right)+\frac{3}{7}fg^2x^7\log\left(c(d+ex^2)^p\right)+\frac{1}{10}g^3x^{10}\log\left(c(d+ex^2)^p\right)+\frac{d^5g^3p\log(d+ex^2)}{10e^5}-\frac{d^4g^3px^2}{10e^4}+\frac{6d^3fg^2px}{7e^3}+\frac{d^3g^3px^4}{20e^3}-\frac{3d^2f^2gp\log(d+ex^2)}{4e^2}-\frac{2d^2fg^2px^3}{7e^2}-\frac{d^2g^3px^6}{30e^2}+\frac{3df^2gpx^2}{4e}+\frac{6dfg^2px^5}{35e}+\frac{dg^3px^8}{40e}-2f^3px-\frac{3}{8}f^2gpx^4-\frac{6}{49}fg^2px^7-\frac{1}{50}g^3px^{10}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^3)^3*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f^3*p*x + (6*d^3*f*g^2*p*x)/(7*e^3) + (3*d*f^2*g*p*x^2)/(4*e) - (d^4*g^3
*p*x^2)/(10*e^4) - (2*d^2*f*g^2*p*x^3)/(7*e^2) - (3*f^2*g*p*x^4)/8 + (d^3*g
^3*p*x^4)/(20*e^3) + (6*d*f*g^2*p*x^5)/(35*e) - (d^2*g^3*p*x^6)/(30*e^2) -
(6*f*g^2*p*x^7)/49 + (d*g^3*p*x^8)/(40*e) - (g^3*p*x^10)/50 + (2*Sqrt[d]*f^
3*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (6*d^(7/2)*f*g^2*p*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/(7*e^(7/2)) - (3*d^2*f^2*g*p*Log[d + e*x^2])/(4*e^2) + (d^5
*g^3*p*Log[d + e*x^2])/(10*e^5) + f^3*x*Log[c*(d + e*x^2)^p] + (3*f^2*g*x^4
*Log[c*(d + e*x^2)^p])/4 + (3*f*g^2*x^7*Log[c*(d + e*x^2)^p])/7 + (g^3*x^10
*Log[c*(d + e*x^2)^p])/10
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
```

```
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$\begin{aligned}\int \left(f+gx^3\right)^3 \log \left(c\left(d+ex^2\right)^p\right)\,dx &= \int \left(f^3 \log \left(c\left(d+ex^2\right)^p\right)+3f^2gx^3 \log \left(c\left(d+ex^2\right)^p\right)+3fg^2x^6 \log \left(c(d\right.\right.\\ &= f^3 \int \log \left(c\left(d+ex^2\right)^p\right)\,dx+\left(3f^2g\right) \int x^3 \log \left(c\left(d+ex^2\right)^p\right)\,dx+\left(3fg\right.\\ &= f^3x \log \left(c\left(d+ex^2\right)^p\right)+\frac{3}{7}fg^2x^7 \log \left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}\left(3f^2g\right) \text{Subst}\Bigg(\int\\ &= -2f^3px+f^3x \log \left(c\left(d+ex^2\right)^p\right)+\frac{3}{4}f^2gx^4 \log \left(c\left(d+ex^2\right)^p\right)+\frac{3}{7}fg^2x^7\\ &= -2f^3px+\frac{6d^3fg^2px}{7e^3}-\frac{2d^2fg^2px^3}{7e^2}+\frac{6dfg^2px^5}{35e}-\frac{6}{49}fg^2px^7+\frac{2\sqrt{d}\,f^3}{}\\ &= -2f^3px+\frac{6d^3fg^2px}{7e^3}+\frac{3df^2gpx^2}{4e}-\frac{d^4g^3px^2}{10e^4}-\frac{2d^2fg^2px^3}{7e^2}-\frac{3}{8}f^2gpx^4\end{aligned}$$

Mathematica [A]
time = 0.18, size = 258, normalized size = 0.70

$$\frac{-epx(2940d^4g^3x+140d^2e^2g^2x^2(60f+7gx^3)-210d^3eg^2(120f+7gx^3)-105de^3gx(210f^2+48fgx^3+7g^2x^6)+3e^4(19600f^3+3675f^2gx^3+1200fg^2x^6+196g^3x^9))-8400\sqrt{d}\,e^{3/2}f(-7e^3f^2+3d^3g^2)\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)+1470d^2g(-15e^3f^2+2d^3g^2)\,p\log\left(d+ex^2\right)+210e^5x(140f^3+105f^2gx^3+60fg^2x^6+14g^3x^9)\log\left(c(d+ex^2)^p\right)}{29400e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^3)^3*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (-(e*p*x*(2940*d^4*g^3*x + 140*d^2*e^2*g^2*x^2*(60*f + 7*g*x^3) - 210*d^3*e
*g^2*(120*f + 7*g*x^3) - 105*d*e^3*g*x*(210*f^2 + 48*f*g*x^3 + 7*g^2*x^6) +
 3*e^4*(19600*f^3 + 3675*f^2*g*x^3 + 1200*f*g^2*x^6 + 196*g^3*x^9))) - 8400
*Sqrt[d]*e^(3/2)*f*(-7*e^3*f^2 + 3*d^3*g^2)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]] +
 1470*d^2*g*(-15*e^3*f^2 + 2*d^3*g^2)*p*Log[d + e*x^2] + 210*e^5*x*(140*f^3
 + 105*f^2*g*x^3 + 60*f*g^2*x^6 + 14*g^3*x^9)*Log[c*(d + e*x^2)^p])/(29400*
e^5)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.99, size = 1311, normalized size = 3.58

method	result	size
risch	Expression too large to display	1311

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)^3*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] -1/7/e^5*p*ln(-3*d^4*e*f*g^2+7*d*e^4*f^3+(-9*d^7*e^3*f^2*g^4+42*d^4*e^6*f^4
*g^2-49*d*e^9*f^6)^(1/2)*x)*(-9*d^7*e^3*f^2*g^4+42*d^4*e^6*f^4*g^2-49*d*e^9
*f^6)^(1/2)+1/7/e^5*p*ln(-3*d^4*e*f*g^2+7*d*e^4*f^3-(-9*d^7*e^3*f^2*g^4+42*
d^4*e^6*f^4*g^2-49*d*e^9*f^6)^(1/2)*x)*(-9*d^7*e^3*f^2*g^4+42*d^4*e^6*f^4*g
^2-49*d*e^9*f^6)^(1/2)-1/10*d^4*g^3*p*x^2/e^4+1/20*d^3*g^3*p*x^4/e^3-1/30*d
^2*g^3*p*x^6/e^2+1/40*d*g^3*p*x^8/e-3/8*I*Pi*f^2*g*x^4*csgn(I*(e*x^2+d)^p)*
csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-3/14*I*Pi*f*g^2*x^7*csgn(I*(e*x^2+d)^p)*csg
n(I*c*(e*x^2+d)^p)*csgn(I*c)+3/4*ln(c)*f^2*g*x^4+3/7*ln(c)*f*g^2*x^7-2*f^3*
p*x-3/4/e^2*p*ln(-3*d^4*e*f*g^2+7*d*e^4*f^3-(-9*d^7*e^3*f^2*g^4+42*d^4*e^6*
f^4*g^2-49*d*e^9*f^6)^(1/2)*x)*d^2*f^2*g-3/4/e^2*p*ln(-3*d^4*e*f*g^2+7*d*e^
4*f^3+(-9*d^7*e^3*f^2*g^4+42*d^4*e^6*f^4*g^2-49*d*e^9*f^6)^(1/2)*x)*d^2*f^2
*g+1/10*ln(c)*g^3*x^10+ln(c)*f^3*x+6/35*d*f*g^2*p*x^5/e+1/10/e^5*p*ln(-3*d^
4*e*f*g^2+7*d*e^4*f^3-(-9*d^7*e^3*f^2*g^4+42*d^4*e^6*f^4*g^2-49*d*e^9*f^6)^
(1/2)*x)*d^5*g^3+1/10/e^5*p*ln(-3*d^4*e*f*g^2+7*d*e^4*f^3+(-9*d^7*e^3*f^2*g
^4+42*d^4*e^6*f^4*g^2-49*d*e^9*f^6)^(1/2)*x)*d^5*g^3-1/2*I*Pi*f^3*csgn(I*c*
(e*x^2+d)^p)^3*x-1/20*I*Pi*g^3*x^10*csgn(I*c*(e*x^2+d)^p)^3+6/7*d^3*f*g^2*p
*x/e^3+3/4*d*f^2*g*p*x^2/e-2/7*d^2*f*g^2*p*x^3/e^2+3/8*I*Pi*f^2*g*x^4*csgn(
I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+3/14*I*Pi*f*g^2*x^7*csgn(I*(e*x^2+d)
^p)*csgn(I*c*(e*x^2+d)^p)^2+3/14*I*Pi*f*g^2*x^7*csgn(I*c*(e*x^2+d)^p)^2*csg
n(I*c)-1/2*I*Pi*f^3*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*x+(
1/10*g^3*x^10+3/7*f*g^2*x^7+3/4*f^2*g*x^4+f^3*x)*ln((e*x^2+d)^p)-1/50*g^3*p
*x^10+3/8*I*Pi*f^2*g*x^4*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/20*I*Pi*g^3*x^
10*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/20*I*Pi*g^3*x^10*c
sgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/2*I*Pi*f^3*csgn(I*(e*x^2+d)^p)
*csgn(I*c*(e*x^2+d)^p)^2*x+1/20*I*Pi*g^3*x^10*csgn(I*c*(e*x^2+d)^p)^2*csgn(
I*c)+1/2*I*Pi*f^3*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*x-3/14*I*Pi*f*g^2*x^7*c
sgn(I*c*(e*x^2+d)^p)^3-3/8*I*Pi*f^2*g*x^4*csgn(I*c*(e*x^2+d)^p)^3-3/8*f^2*g
*p*x^4-6/49*f*g^2*p*x^7
```

Maxima [A]
time = 0.55, size = 266, normalized size = 0.73

$$\frac{1}{29400}\left(1470\left(2d^5g^3 - 15d^2f^2ge^3\right)e^{(-6)}\log\left(x^2e + d\right) - \frac{8400\left(3d^4fg^2 - 7df^3e^3\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{\sqrt{d}} - \left(588g^3x^{10}e^4 - 735dg^3x^8e^3 + 980d^2g^3x^6e^2 + 3600fg^2x^7e^4 - 5040dfg^2x^5e^3 + 8400d^2fg^2x^3e^2 - 735\left(2d^3g^3e - 15f^2ge^4\right)x^4 + 1470\left(2d^4g^3 - 15df^2ge^3\right)x^2 - 8400\left(3d^3fg^2e - 7f^3e^4\right)x\right)e^{(-5)}\right)pe + \frac{1}{140}\left(14g^3x^{10} + 60fg^2x^7 + 105f^2gx^4 + 140f^3x\right)\log\left(\left(x^2e + d\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^3*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] 1/29400*(1470*(2*d^5*g^3 - 15*d^2*f^2*g*e^3)*e^(-6)*log(x^2*e + d) - 8400*(
3*d^4*f*g^2 - 7*d*f^3*e^3)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/sqrt(d) - (58
8*g^3*x^10*e^4 - 735*d*g^3*x^8*e^3 + 980*d^2*g^3*x^6*e^2 + 3600*f*g^2*x^7*e
^4 - 5040*d*f*g^2*x^5*e^3 + 8400*d^2*f*g^2*x^3*e^2 - 735*(2*d^3*g^3*e - 15*
f^2*g*e^4)*x^4 + 1470*(2*d^4*g^3 - 15*d*f^2*g*e^3)*x^2 - 8400*(3*d^3*f*g^2*
e - 7*f^3*e^4)*x)*e^(-5))*p*e + 1/140*(14*g^3*x^10 + 60*f*g^2*x^7 + 105*f^2
*g*x^4 + 140*f^3*x)*log((x^2*e + d)^p*c)
```

Fricas [**A**]
time = 0.38, size = 635, normalized size = 1.73

$$\left[-\frac{1}{29400}\left(2940\,d^4g^3px^2e-210\,(14\,g^3x^{10}+60\,fg^2x^7+105\,f^2gx^4+140\,f^3x)e^5\log(c)-4200\,(3\,d^3fg^2pe^2-7\,f^3pe^5)\sqrt{-de^{(-1)}}\log\left(\frac{x^2e-2\,\sqrt{-de^{(-1)}}\,xe-d}{x^2e+d}\right)+3\,(196\,g^3px^{10}+1200\,fg^2px^7+3675\,f^2gpx^4+19600\,f^3px)e^5-105\,(7\,dg^3px^8+48\,dfg^2px^5+210\,df^2gpx^2)e^4+140\,(7\,d^2g^3px^6+60\,d^2fg^2px^3)e^3-210\,(7\,d^3g^3px^4+120\,d^3fg^2px)e^2-210\,(14\,d^5g^3p-105\,d^2f^2gpe^3+(14\,g^3px^{10}+60\,fg^2px^7+105\,f^2gpx^4+140\,f^3px)e^5)\log(x^2e+d)\right)e^{(-5)},\ -\frac{1}{29400}\left(2940\,d^4g^3px^2e+8400\,(3\,d^3fg^2pe^2-7\,f^3pe^5)\sqrt{d}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{(-\frac{1}{2})}-210\,(14\,g^3x^{10}+60\,fg^2x^7+105\,f^2gx^4+140\,f^3x)e^5\log(c)+3\,(196\,g^3px^{10}+1200\,fg^2px^7+3675\,f^2gpx^4+19600\,f^3px)e^5-105\,(7\,dg^3px^8+48\,dfg^2px^5+210\,df^2gpx^2)e^4+140\,(7\,d^2g^3px^6+60\,d^2fg^2px^3)e^3-210\,(7\,d^3g^3px^4+120\,d^3fg^2px)e^2-210\,(14\,d^5g^3p-105\,d^2f^2gpe^3+(14\,g^3px^{10}+60\,fg^2px^7+105\,f^2gpx^4+140\,f^3px)e^5)\log(x^2e+d)\right)e^{(-5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^3*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [-1/29400*(2940*d^4*g^3*p*x^2*e - 210*(14*g^3*x^10 + 60*f*g^2*x^7 + 105*f^2
*g*x^4 + 140*f^3*x)*e^5*log(c) - 4200*(3*d^3*f*g^2*p*e^2 - 7*f^3*p*e^5)*sqr
t(-d*e^(-1))*log((x^2*e - 2*sqrt(-d*e^(-1))*x*e - d)/(x^2*e + d)) + 3*(196*
g^3*p*x^10 + 1200*f*g^2*p*x^7 + 3675*f^2*g*p*x^4 + 19600*f^3*p*x)*e^5 - 105
*(7*d*g^3*p*x^8 + 48*d*f*g^2*p*x^5 + 210*d*f^2*g*p*x^2)*e^4 + 140*(7*d^2*g^
3*p*x^6 + 60*d^2*f*g^2*p*x^3)*e^3 - 210*(7*d^3*g^3*p*x^4 + 120*d^3*f*g^2*p*
x)*e^2 - 210*(14*d^5*g^3*p - 105*d^2*f^2*g*p*e^3 + (14*g^3*p*x^10 + 60*f*g^
2*p*x^7 + 105*f^2*g*p*x^4 + 140*f^3*p*x)*e^5)*log(x^2*e + d))*e^(-5), -1/29
400*(2940*d^4*g^3*p*x^2*e + 8400*(3*d^3*f*g^2*p*e^2 - 7*f^3*p*e^5)*sqrt(d)*
arctan(x*e^(1/2)/sqrt(d))*e^(-1/2) - 210*(14*g^3*x^10 + 60*f*g^2*x^7 + 105*
f^2*g*x^4 + 140*f^3*x)*e^5*log(c) + 3*(196*g^3*p*x^10 + 1200*f*g^2*p*x^7 +
3675*f^2*g*p*x^4 + 19600*f^3*p*x)*e^5 - 105*(7*d*g^3*p*x^8 + 48*d*f*g^2*p*x
^5 + 210*d*f^2*g*p*x^2)*e^4 + 140*(7*d^2*g^3*p*x^6 + 60*d^2*f*g^2*p*x^3)*e^
3 - 210*(7*d^3*g^3*p*x^4 + 120*d^3*f*g^2*p*x)*e^2 - 210*(14*d^5*g^3*p - 105
*d^2*f^2*g*p*e^3 + (14*g^3*p*x^10 + 60*f*g^2*p*x^7 + 105*f^2*g*p*x^4 + 140*
f^3*p*x)*e^5)*log(x^2*e + d))*e^(-5)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)**3*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.97, size = 354, normalized size = 0.97

$$\frac{1}{20}\,(2\,d^5g^3p-15\,d^2f^2gpe^3)e^{(-6)}\log(x^2e+d)-\frac{2\,(3\,d^4fg^2p-7\,df^3pe^3)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{(-\frac{9}{2})}}{7\,\sqrt{d}}+\frac{1}{29400}\,(2940\,g^3px^{10}e^4\log(x^2e+d)-588\,g^3px^{10}e^4+2940\,g^3x^{10}e^4\log(c)+735\,dg^3px^8e^3-980\,d^2g^3px^6e^2+12600\,fg^2px^7e^4\log(x^2e+d)-3600\,fg^2px^7e^4+1470\,d^3g^3px^4e+12600\,fg^2x^7e^4\log(c)+5040\,dfg^2px^5e^3-2940\,d^4g^3px^2-8400\,d^2fg^2px^3e^2+22050\,f^2gpx^4e^4\log(x^2e+d)-11025\,f^2gpx^4e^4+25200\,d^3fg^2pxe+22050\,f^2gx^4e^4\log(c)+22050\,df^2gpx^2e^3+29400\,f^3pxe^4\log(x^2e+d)-58800\,f^3pxe^4+29400\,f^3xe^4\log(c))e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^3*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 1/20*(2*d^5*g^3*p - 15*d^2*f^2*g*p*e^3)*e^(-5)*log(x^2*e + d) - 2/7*(3*d^4*
f*g^2*p - 7*d*f^3*p*e^3)*arctan(x*e^(1/2)/sqrt(d))*e^(-7/2)/sqrt(d) + 1/294
00*(2940*g^3*p*x^10*e^4*log(x^2*e + d) - 588*g^3*p*x^10*e^4 + 2940*g^3*x^10
*e^4*log(c) + 735*d*g^3*p*x^8*e^3 - 980*d^2*g^3*p*x^6*e^2 + 12600*f*g^2*p*x
^7*e^4*log(x^2*e + d) - 3600*f*g^2*p*x^7*e^4 + 1470*d^3*g^3*p*x^4*e + 12600
*f*g^2*x^7*e^4*log(c) + 5040*d*f*g^2*p*x^5*e^3 - 2940*d^4*g^3*p*x^2 - 8400*
d^2*f*g^2*p*x^3*e^2 + 22050*f^2*g*p*x^4*e^4*log(x^2*e + d) - 11025*f^2*g*p*
x^4*e^4 + 25200*d^3*f*g^2*p*x*e + 22050*f^2*g*x^4*e^4*log(c) + 22050*d*f^2*
g*p*x^2*e^3 + 29400*f^3*p*x*e^4*log(x^2*e + d) - 58800*f^3*p*x*e^4 + 29400*
f^3*x*e^4*log(c))*e^(-4)
```

Mupad [B]
time = 3.33, size = 316, normalized size = 0.86

$$\frac{g^3\,x^{10}\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{10}-2\,f^3\,p\,x-\frac{g^3\,p\,x^{10}}{50}+f^3\,x\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)+\frac{3\,f^2\,g\,x^4\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{4}+\frac{3\,f\,g^2\,x^7\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{7}-\frac{3\,f^2\,g\,p\,x^4}{8}-\frac{6\,f\,g^2\,p\,x^7}{49}+\frac{d\,g^3\,p\,x^8}{40\,e}+\frac{2\,\sqrt{d}\,f^3\,p\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{d^5\,g^3\,p\,\ln\left(e\,x^2+d\right)}{10\,e^5}-\frac{d^2\,g^3\,p\,x^6}{30\,e^2}+\frac{d^3\,g^3\,p\,x^4}{20\,e^3}-\frac{d^4\,g^3\,p\,x^2}{10\,e^4}-\frac{6\,d^{7/2}\,f\,g^2\,p\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{7\,e^{7/2}}-\frac{3\,d^2\,f^2\,g\,p\,\ln\left(e\,x^2+d\right)}{4\,e^2}-\frac{2\,d^2\,f\,g^2\,p\,x^3}{7\,e^2}+\frac{3\,d\,f^2\,g\,p\,x^2}{4\,e}+\frac{6\,d\,f\,g^2\,p\,x^5}{35\,e}+\frac{6\,d^3\,f\,g^2\,p\,x}{7\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)*(f + g*x^3)^3,x)
```

```
[Out] (g^3*x^10*log(c*(d + e*x^2)^p))/10 - 2*f^3*p*x - (g^3*p*x^10)/50 + f^3*x*lo
g(c*(d + e*x^2)^p) + (3*f^2*g*x^4*log(c*(d + e*x^2)^p))/4 + (3*f*g^2*x^7*lo
g(c*(d + e*x^2)^p))/7 - (3*f^2*g*p*x^4)/8 - (6*f*g^2*p*x^7)/49 + (d*g^3*p*x
^8)/(40*e) + (2*d^(1/2)*f^3*p*atan((e^(1/2)*x)/d^(1/2)))/e^(1/2) + (d^5*g^3
*p*log(d + e*x^2))/(10*e^5) - (d^2*g^3*p*x^6)/(30*e^2) + (d^3*g^3*p*x^4)/(2
0*e^3) - (d^4*g^3*p*x^2)/(10*e^4) - (6*d^(7/2)*f*g^2*p*atan((e^(1/2)*x)/d^(
1/2)))/(7*e^(7/2)) - (3*d^2*f^2*g*p*log(d + e*x^2))/(4*e^2) - (2*d^2*f*g^2*
p*x^3)/(7*e^2) + (3*d*f^2*g*p*x^2)/(4*e) + (6*d*f*g^2*p*x^5)/(35*e) + (6*d^
3*f*g^2*p*x)/(7*e^3)
```

3.289 $\int (f+gx^3)^2 \log \left(c(d+ex^2)^p\right) \, dx$

Optimal. Leaf size=231

$$-2f^2px+\frac{2d^3g^2px}{7e^3}+\frac{dfgpx^2}{2e}-\frac{2d^2g^2px^3}{21e^2}-\frac{1}{4}fgpx^4+\frac{2dg^2px^5}{35e}-\frac{2}{49}g^2px^7+\frac{2\sqrt{d}\,f^2p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{2d^{7/2}g^2p\,\mathrm{ta}}{7}$$

```
[Out] -2*f^2*p*x+2/7*d^3*g^2*p*x/e^3+1/2*d*f*g*p*x^2/e-2/21*d^2*g^2*p*x^3/e^2-1/4
*f*g*p*x^4+2/35*d*g^2*p*x^5/e-2/49*g^2*p*x^7-2/7*d^(7/2)*g^2*p*arctan(x*e^(
1/2)/d^(1/2))/e^(7/2)-1/2*d^2*f*g*p*ln(e*x^2+d)/e^2+f^2*x*ln(c*(e*x^2+d)^p)
+1/2*f*g*x^4*ln(c*(e*x^2+d)^p)+1/7*g^2*x^7*ln(c*(e*x^2+d)^p)+2*f^2*p*arctan
(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {2521, 2498, 327, 211, 2504, 2442, 45, 2505, 308}

$$-\frac{2d^{7/2}g^2p\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{7e^{7/2}}+\frac{2\sqrt{d}\,f^2p\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}+f^2x\log\left(c(d+ex^2)^p\right)+\frac{1}{2}fgx^4\log\left(c(d+ex^2)^p\right)+\frac{1}{7}g^2x^7\log\left(c(d+ex^2)^p\right)+\frac{2d^3g^2px}{7e^3}-\frac{d^2fgp\log(d+ex^2)}{2e^2}-\frac{2d^2g^2px^3}{21e^2}+\frac{dfgpx^2}{2e}+\frac{2dg^2px^5}{35e}-2f^2px-\frac{1}{4}fgpx^4-\frac{2}{49}g^2px^7$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^3)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f^2*p*x + (2*d^3*g^2*p*x)/(7*e^3) + (d*f*g*p*x^2)/(2*e) - (2*d^2*g^2*p*x
^3)/(21*e^2) - (f*g*p*x^4)/4 + (2*d*g^2*p*x^5)/(35*e) - (2*g^2*p*x^7)/49 +
(2*Sqrt[d]*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(7/2)*g^2*p*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]])/(7*e^(7/2)) - (d^2*f*g*p*Log[d + e*x^2])/(2*e^2)
 + f^2*x*Log[c*(d + e*x^2)^p] + (f*g*x^4*Log[c*(d + e*x^2)^p])/2 + (g^2*x^7
*Log[c*(d + e*x^2)^p])/7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

Q[m, 2*n - 1]

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
```

```
0] && LtQ[r, 0]))
```

Rubi steps

$$\int \left(f+gx^3\right)^2 \log \left(c\left(d+ex^2\right)^p\right)\,dx = \int \left(f^2 \log \left(c\left(d+ex^2\right)^p\right) + 2fgx^3 \log \left(c\left(d+ex^2\right)^p\right) + g^2x^6 \log \left(c\left(d+e\right.\right.\right.$$

$$= f^2 \int \log \left(c\left(d+ex^2\right)^p\right)\,dx + (2fg)\int x^3 \log \left(c\left(d+ex^2\right)^p\right)\,dx + g^2 \int x$$

$$= f^2 x \log \left(c\left(d+ex^2\right)^p\right) + \frac{1}{7}g^2x^7 \log \left(c\left(d+ex^2\right)^p\right) + (fg)\text{Subst}\left(\int x \log\right.$$

$$= -2f^2px + f^2x \log \left(c\left(d+ex^2\right)^p\right) + \frac{1}{2}fgx^4 \log \left(c\left(d+ex^2\right)^p\right) + \frac{1}{7}g^2x^7 \text{lo}$$

$$= -2f^2px + \frac{2d^3g^2px}{7e^3} - \frac{2d^2g^2px^3}{21e^2} + \frac{2dg^2px^5}{35e} - \frac{2}{49}g^2px^7 + \frac{2\sqrt{d}\,f^2p\tan}{\sqrt{e}}$$

$$= -2f^2px + \frac{2d^3g^2px}{7e^3} + \frac{dfgpx^2}{2e} - \frac{2d^2g^2px^3}{21e^2} - \frac{1}{4}fgpx^4 + \frac{2dg^2px^5}{35e} - \frac{2}{49}g$$

Mathematica [A]
time = 0.15, size = 178, normalized size = 0.77

$$\frac{px(840d^3g^2-280d^2eg^2x^2+42de^2gx(35f+4gx^3)-15e^3(392f^2+49fgx^3+8g^2x^6))}{2940e^3} - \frac{2\sqrt{d}\,(-7e^3f^2+d^3g^2)\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{7e^{7/2}} - \frac{d^2fgp\log(d+ex^2)}{2e^2} + \frac{1}{14}x(14f^2+7fgx^3+2g^2x^6)\log\left(c\left(d+ex^2\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^3)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (p*x*(840*d^3*g^2 - 280*d^2*e*g^2*x^2 + 42*d*e^2*g*x*(35*f + 4*g*x^3) - 15*
e^3*(392*f^2 + 49*f*g*x^3 + 8*g^2*x^6)))/(2940*e^3) - (2*Sqrt[d]*(-7*e^3*f^
2 + d^3*g^2)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(7*e^(7/2)) - (d^2*f*g*p*Log[d
+ e*x^2])/(2*e^2) + (x*(14*f^2 + 7*f*g*x^3 + 2*g^2*x^6)*Log[c*(d + e*x^2)^p
])/14
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.71, size = 869, normalized size = 3.76

method	result
risch	$\frac{\ln(c)fg\,x^4}{2} - \frac{i\pi\,f^2\text{csgn}\left(ic\left(e\,x^2+d\right)^p\right)^3 x}{2} - \frac{2g^2p\,x^7}{49} - \frac{i\pi fg\,x^4\text{csgn}\left(i\left(e\,x^2+d\right)^p\right)\text{csgn}\left(ic\left(e\,x^2+d\right)^p\right)\text{csgn}(ic)}{4} - \frac{i\pi fg\,x^4\text{csgn}\left(ic\left(e\,x\right.\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)^2*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*Pi*f*g*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/14*I*Pi*g^2*
x^7*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/4*I*Pi*f*g*x^4*cs
gn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/2*I*Pi*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(
e*x^2+d)^p)*csgn(I*c)*x+1/2*ln(c)*f*g*x^4-2/49*g^2*p*x^7-1/2*I*Pi*f^2*csgn(
I*c*(e*x^2+d)^p)^3*x-1/14*I*Pi*g^2*x^7*csgn(I*c*(e*x^2+d)^p)^3-2*f^2*p*x+1/
7*ln(c)*g^2*x^7+ln(c)*f^2*x-1/7/e^4*p*ln(-d^4*g^2+7*d*e^3*f^2+(-d^7*e*g^4+1
4*d^4*e^4*f^2*g^2-49*d*e^7*f^4)^(1/2)*x)*(-d^7*e*g^4+14*d^4*e^4*f^2*g^2-49*
d*e^7*f^4)^(1/2)+1/7/e^4*p*ln(-d^4*g^2+7*d*e^3*f^2-(-d^7*e*g^4+14*d^4*e^4*f
^2*g^2-49*d*e^7*f^4)^(1/2)*x)*(-d^7*e*g^4+14*d^4*e^4*f^2*g^2-49*d*e^7*f^4)^
(1/2)+1/2*d*f*g*p*x^2/e+2/7*d^3*g^2*p*x/e^3-2/21*d^2*g^2*p*x^3/e^2+2/35*d*g
^2*p*x^5/e+1/2*I*Pi*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*x+1/2*I
*Pi*f^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*x-1/4*I*Pi*f*g*x^4*csgn(I*(e*x^2+
d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+(1/7*g^2*x^7+1/2*f*g*x^4+f^2*x)*ln((e
*x^2+d)^p)-1/2/e^2*p*ln(-d^4*g^2+7*d*e^3*f^2+(-d^7*e*g^4+14*d^4*e^4*f^2*g^2
-49*d*e^7*f^4)^(1/2)*x)*d^2*f*g-1/2/e^2*p*ln(-d^4*g^2+7*d*e^3*f^2-(-d^7*e*g
^4+14*d^4*e^4*f^2*g^2-49*d*e^7*f^4)^(1/2)*x)*d^2*f*g+1/14*I*Pi*g^2*x^7*csgn
(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/14*I*Pi*g^2*x^7*csgn(I*c*(e*x^2+d
)^p)^2*csgn(I*c)-1/4*I*Pi*f*g*x^4*csgn(I*c*(e*x^2+d)^p)^3-1/4*f*g*p*x^4
```

<u>Maxima</u> [A]
time = 0.51, size = 168, normalized size = 0.73

$$-\frac{1}{2940}\left(1470\,d^2fge^{(-3)}\log\left(x^2e+d\right)+\frac{840\,\left(d^4g^2-7\,df^2e^3\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{\sqrt{d}}+\left(120\,g^2x^7e^3-168\,dg^2x^5e^2+280\,d^2g^2x^3e+735\,fgx^4e^3-1470\,dfgx^2e^2-840\,\left(d^3g^2-7\,f^2e^3\right)x\right)e^{(-4)}\right)pe+\frac{1}{14}\,\left(2\,g^2x^7+7\,fgx^4+14\,f^2x\right)\log\left(\left(x^2e+d\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] -1/2940*(1470*d^2*f*g*e^(-3)*log(x^2*e + d) + 840*(d^4*g^2 - 7*d*f^2*e^3)*a
rctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/sqrt(d) + (120*g^2*x^7*e^3 - 168*d*g^2*x^
5*e^2 + 280*d^2*g^2*x^3*e + 735*f*g*x^4*e^3 - 1470*d*f*g*x^2*e^2 - 840*(d^3
*g^2 - 7*f^2*e^3)*x)*e^(-4))*p*e + 1/14*(2*g^2*x^7 + 7*f*g*x^4 + 14*f^2*x)*
log((x^2*e + d)^p*c)
```

<u>Fricas</u> [A]
time = 0.46, size = 415, normalized size = 1.80

$$\left[-\frac{1}{2940}\left(280\,d^2g^2px^3e-840\,d^3g^2px-210\,\left(2\,g^2x^7+7\,fgx^4+14\,f^2x\right)e^3\log\left(c\right)-420\,\left(d^3g^2p-7\,f^2pe^3\right)\sqrt{-de^{(-1)}}\log\left(\frac{x^2e-2\,\sqrt{-de^{(-1)}}xe-d}{x^2e+d}\right)+15\,\left(8\,g^2px^7+49\,fgpx^4+392\,f^2px\right)e^3-42\,\left(4\,dg^2px^5+35\,dfgpx^2\right)e^2+210\,\left(7\,d^2fgpe-\left(2\,g^2px^7+7\,fgpx^4+14\,f^2px\right)e^3\right)\log\left(x^2e+d\right)\right)e^{(-3)},\ -\frac{1}{2940}\left(280\,d^2g^2px^3e-840\,d^3g^2px+840\,\left(d^3g^2p-7\,f^2pe^3\right)\sqrt{d}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}-210\,\left(2\,g^2x^7+7\,fgx^4+14\,f^2x\right)e^3\log\left(c\right)+15\,\left(8\,g^2px^7+49\,fgpx^4+392\,f^2px\right)e^3-42\,\left(4\,dg^2px^5+35\,dfgpx^2\right)e^2+210\,\left(7\,d^2fgpe-\left(2\,g^2px^7+7\,fgpx^4+14\,f^2px\right)e^3\right)\log\left(x^2e+d\right)\right)e^{(-3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [-1/2940*(280*d^2*g^2*p*x^3*e - 840*d^3*g^2*p*x - 210*(2*g^2*x^7 + 7*f*g*x^
4 + 14*f^2*x)*e^3*log(c) - 420*(d^3*g^2*p - 7*f^2*p*e^3)*sqrt(-d*e^(-1))*lo
g((x^2*e - 2*sqrt(-d*e^(-1))*x*e - d)/(x^2*e + d)) + 15*(8*g^2*p*x^7 + 49*f
*g*p*x^4 + 392*f^2*p*x)*e^3 - 42*(4*d*g^2*p*x^5 + 35*d*f*g*p*x^2)*e^2 + 210
*(7*d^2*f*g*p*e - (2*g^2*p*x^7 + 7*f*g*p*x^4 + 14*f^2*p*x)*e^3)*log(x^2*e +
 d))*e^(-3), -1/2940*(280*d^2*g^2*p*x^3*e - 840*d^3*g^2*p*x + 840*(d^3*g^2*
p - 7*f^2*p*e^3)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2) - 210*(2*g^2*x^
7 + 7*f*g*x^4 + 14*f^2*x)*e^3*log(c) + 15*(8*g^2*p*x^7 + 49*f*g*p*x^4 + 392
*f^2*p*x)*e^3 - 42*(4*d*g^2*p*x^5 + 35*d*f*g*p*x^2)*e^2 + 210*(7*d^2*f*g*p*
e - (2*g^2*p*x^7 + 7*f*g*p*x^4 + 14*f^2*p*x)*e^3)*log(x^2*e + d))*e^(-3)]
```

Sympy [A]
time = 133.12, size = 440, normalized size = 1.90

$$\begin{cases} \left(f^{2} x + \frac{f g x^{4}}{2} + \frac{g^{2} x^{7}}{7}\right) \log\left(0^{p} c\right) & \text{for } d = 0 \wedge e = 0 \\ -2 f^{2} p x + f^{2} x \log\left(c \left(e x^{2}\right)^{p}\right) - \frac{f g p x^{4}}{4} + \frac{f g x^{4} \log\left(c \left(e x^{2}\right)^{p}\right)}{2} - \frac{2 g^{2} p x^{7}}{49} + \frac{g^{2} x^{7} \log\left(c \left(e x^{2}\right)^{p}\right)}{7} & \text{for } d = 0 \\ \left(f^{2} x + \frac{f g x^{4}}{2} + \frac{g^{2} x^{7}}{7}\right) \log\left(c d^{p}\right) & \text{for } e = 0 \\ - \frac{2 d^{4} g^{2} p \log\left(x - \sqrt{- \frac{d}{e}}\right)}{7 e^{4} \sqrt{- \frac{d}{e}}} + \frac{d^{4} g^{2} \log\left(c \left(d + e x^{2}\right)^{p}\right)}{7 e^{4} \sqrt{- \frac{d}{e}}} + \frac{2 d^{3} g^{2} p x}{7 e^{3}} - \frac{d^{2} f g \log\left(c \left(d + e x^{2}\right)^{p}\right)}{2 e^{2}} - \frac{2 d^{2} g^{2} p x^{3}}{21 e^{2}} + \frac{2 d f^{2} p \log\left(x - \sqrt{- \frac{d}{e}}\right)}{e \sqrt{- \frac{d}{e}}} - \frac{d f^{2} \log\left(c \left(d + e x^{2}\right)^{p}\right)}{e \sqrt{- \frac{d}{e}}} + \frac{d f g p x^{2}}{2 e} + \frac{2 d g^{2} p x^{5}}{35 e} - 2 f^{2} p x + f^{2} x \log\left(c \left(d + e x^{2}\right)^{p}\right) - \frac{f g p x^{4}}{4} + \frac{f g x^{4} \log\left(c \left(d + e x^{2}\right)^{p}\right)}{2} - \frac{2 g^{2} p x^{7}}{49} + \frac{g^{2} x^{7} \log\left(c \left(d + e x^{2}\right)^{p}\right)}{7} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)**2*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise((((f**2*x + f*g*x**4/2 + g**2*x**7/7)*log(0**p*c), Eq(d, 0) & Eq(e
, 0)), (-2*f**2*p*x + f**2*x*log(c*(e*x**2)**p) - f*g*p*x**4/4 + f*g*x**4*l
og(c*(e*x**2)**p)/2 - 2*g**2*p*x**7/49 + g**2*x**7*log(c*(e*x**2)**p)/7, Eq
(d, 0)), ((f**2*x + f*g*x**4/2 + g**2*x**7/7)*log(c*d**p), Eq(e, 0)), (-2*d
**4*g**2*p*log(x - sqrt(-d/e))/(7*e**4*sqrt(-d/e)) + d**4*g**2*log(c*(d + e
*x**2)**p)/(7*e**4*sqrt(-d/e)) + 2*d**3*g**2*p*x/(7*e**3) - d**2*f*g*log(c*
(d + e*x**2)**p)/(2*e**2) - 2*d**2*g**2*p*x**3/(21*e**2) + 2*d*f**2*p*log(x
 - sqrt(-d/e))/(e*sqrt(-d/e)) - d*f**2*log(c*(d + e*x**2)**p)/(e*sqrt(-d/e)
) + d*f*g*p*x**2/(2*e) + 2*d*g**2*p*x**5/(35*e) - 2*f**2*p*x + f**2*x*log(c
*(d + e*x**2)**p) - f*g*p*x**4/4 + f*g*x**4*log(c*(d + e*x**2)**p)/2 - 2*g*
*2*p*x**7/49 + g**2*x**7*log(c*(d + e*x**2)**p)/7, True))
```

Giac [A]
time = 5.58, size = 225, normalized size = 0.97

$$-\frac{1}{2} d^{2} f g p e^{(-2)} \log\left(x^{2} e + d\right) - \frac{2 \left(d^{4} g^{2} p - 7 d f^{2} p e^{3}\right) \arctan\left(\frac{x e^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{7}{2}\right)}}{7 \sqrt{d}} + \frac{1}{2940} \left(420 g^{2} p x^{7} e^{3} \log\left(x^{2} e + d\right) - 120 g^{2} p x^{7} e^{3} + 420 g^{2} x^{7} e^{3} \log\left(c\right) + 168 d g^{2} p x^{5} e^{2} - 280 d^{2} g^{2} p x^{3} e + 1470 f g p x^{4} e^{3} \log\left(x^{2} e + d\right) - 735 f g p x^{4} e^{3} + 1470 f g x^{4} e^{3} \log\left(c\right) + 840 d^{3} g^{2} p x + 1470 d f g p x^{2} e^{2} + 2940 f^{2} p x e^{3} \log\left(x^{2} e + d\right) - 5880 f^{2} p x e^{3} + 2940 f^{2} x e^{3} \log\left(c\right)\right) e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] -1/2*d^2*f*g*p*e^(-2)*log(x^2*e + d) - 2/7*(d^4*g^2*p - 7*d*f^2*p*e^3)*arct
an(x*e^(1/2)/sqrt(d))*e^(-7/2)/sqrt(d) + 1/2940*(420*g^2*p*x^7*e^3*log(x^2*
e + d) - 120*g^2*p*x^7*e^3 + 420*g^2*x^7*e^3*log(c) + 168*d*g^2*p*x^5*e^2 -
 280*d^2*g^2*p*x^3*e + 1470*f*g*p*x^4*e^3*log(x^2*e + d) - 735*f*g*p*x^4*e^
```

```
3 + 1470*f*g*x^4*e^3*log(c) + 840*d^3*g^2*p*x + 1470*d*f*g*p*x^2*e^2 + 2940
*f^2*p*x*e^3*log(x^2*e + d) - 5880*f^2*p*x*e^3 + 2940*f^2*x*e^3*log(c))*e^(
-3)
```

Mupad [B]
time = 2.77, size = 317, normalized size = 1.37

$$\frac{g^2\,x^7\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{7}-2\,f^2\,p\,x-\frac{2\,g^2\,p\,x^7}{49}+f^2\,x\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)+\frac{f\,g\,x^4\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{2}-\frac{f\,g\,p\,x^4}{4}+\frac{2\,d\,g^2\,p\,x^5}{35\,e}+\frac{2\,d^3\,g^2\,p\,x}{7\,e^3}-\frac{2\,\sqrt{d}\,f^2\,p\,\mathrm{atan}\left(\frac{7\,\sqrt{d}\,e^{7/2}\,f^2\,p\,x}{d^4\,g^2\,p-7\,d\,e^3\,f^2\,p}-\frac{d^{7/2}\,\sqrt{e}\,g^2\,p\,x}{d^4\,g^2\,p-7\,d\,e^3\,f^2\,p}\right)}{\sqrt{e}}+\frac{2\,d^{7/2}\,g^2\,p\,\mathrm{atan}\left(\frac{7\,\sqrt{d}\,e^{7/2}\,f^2\,p\,x}{d^4\,g^2\,p-7\,d\,e^3\,f^2\,p}-\frac{d^{7/2}\,\sqrt{e}\,g^2\,p\,x}{d^4\,g^2\,p-7\,d\,e^3\,f^2\,p}\right)}{7\,e^{7/2}}-\frac{2\,d^2\,g^2\,p\,x^3}{21\,e^2}+\frac{d\,f\,g\,p\,x^2}{2\,e}-\frac{d^2\,f\,g\,p\,\ln\left(e\,x^2+d\right)}{2\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)*(f + g*x^3)^2,x)
```

```
[Out] (g^2*x^7*log(c*(d + e*x^2)^p))/7 - 2*f^2*p*x - (2*g^2*p*x^7)/49 + f^2*x*log
(c*(d + e*x^2)^p) + (f*g*x^4*log(c*(d + e*x^2)^p))/2 - (f*g*p*x^4)/4 + (2*d
*g^2*p*x^5)/(35*e) + (2*d^3*g^2*p*x)/(7*e^3) - (2*d^(1/2)*f^2*p*atan((7*d^(
1/2)*e^(7/2)*f^2*p*x)/(d^4*g^2*p - 7*d*e^3*f^2*p) - (d^(7/2)*e^(1/2)*g^2*p*
x)/(d^4*g^2*p - 7*d*e^3*f^2*p)))/e^(1/2) + (2*d^(7/2)*g^2*p*atan((7*d^(1/2)
*e^(7/2)*f^2*p*x)/(d^4*g^2*p - 7*d*e^3*f^2*p) - (d^(7/2)*e^(1/2)*g^2*p*x)/(
d^4*g^2*p - 7*d*e^3*f^2*p)))/(7*e^(7/2)) - (2*d^2*g^2*p*x^3)/(21*e^2) + (d*
f*g*p*x^2)/(2*e) - (d^2*f*g*p*log(d + e*x^2))/(2*e^2)
```

3.290 $\int (f+gx^3)\log\left(c(d+ex^2)^p\right)\,dx$

Optimal. Leaf size=110

$$-2fpx+\frac{dgpx^2}{4e}-\frac{1}{8}gpx^4+\frac{2\sqrt{d}\,fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{d^2gp\log\left(d+ex^2\right)}{4e^2}+fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}gx^4\log\left(c\left(d+\right.\right.$$

```
[Out] -2*f*p*x+1/4*d*g*p*x^2/e-1/8*g*p*x^4-1/4*d^2*g*p*ln(e*x^2+d)/e^2+f*x*ln(c*(
e*x^2+d)^p)+1/4*g*x^4*ln(c*(e*x^2+d)^p)+2*f*p*arctan(x*e^(1/2)/d^(1/2))*d^(
1/2)/e^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2521, 2498, 327, 211, 2504, 2442, 45}

$$\frac{2\sqrt{d}\,fp\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}+fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}gx^4\log\left(c\left(d+ex^2\right)^p\right)-\frac{d^2gp\log\left(d+ex^2\right)}{4e^2}+\frac{dgpx^2}{4e}-2fpx-\frac{1}{8}gpx^4$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^3)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f*p*x + (d*g*p*x^2)/(4*e) - (g*p*x^4)/8 + (2*Sqrt[d]*f*p*ArcTan[(Sqrt[e]
*x)/Sqrt[d]])/Sqrt[e] - (d^2*g*p*Log[d + e*x^2])/(4*e^2) + f*x*Log[c*(d + e
*x^2)^p] + (g*x^4*Log[c*(d + e*x^2)^p])/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$\int \left(f+gx^3\right)\log\left(c\left(d+ex^2\right)^p\right)\,dx = \int \left(f\log\left(c\left(d+ex^2\right)^p\right)+gx^3\log\left(c\left(d+ex^2\right)^p\right)\right)\,dx$$
$$= f\int \log\left(c\left(d+ex^2\right)^p\right)\,dx + g\int x^3\log\left(c\left(d+ex^2\right)^p\right)\,dx$$
$$= fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}g\text{Subst}\left(\int x\log\left(c(d+ex)^p\right)\,dx,x,x^2\right)-(2e$$
$$= -2fpx+fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}gx^4\log\left(c\left(d+ex^2\right)^p\right)+(2dfp)\int \frac{}{d-}$$
$$= -2fpx+\frac{2\sqrt{d}\,fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}+fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}gx^4\log\left(c\right.$$
$$= -2fpx+\frac{dgpx^2}{4e}-\frac{1}{8}gpx^4+\frac{2\sqrt{d}\,fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{d^2gp\log\left(d+ex\right.}{4e^2}$$

Mathematica [**A**]
time = 0.04, size = 110, normalized size = 1.00

$$-2fpx+\frac{dgpx^2}{4e}-\frac{1}{8}gpx^4+\frac{2\sqrt{d}\,fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{d^2gp\log\left(d+ex^2\right)}{4e^2}+fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}gx^4\log\left(c\left(d+ex^2\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^3)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f*p*x + (d*g*p*x^2)/(4*e) - (g*p*x^4)/8 + (2*Sqrt[d]*f*p*ArcTan[(Sqrt[e]
*x)/Sqrt[d]])/Sqrt[e] - (d^2*g*p*Log[d + e*x^2])/(4*e^2) + f*x*Log[c*(d + e
*x^2)^p] + (g*x^4*Log[c*(d + e*x^2)^p])/4
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.42, size = 402, normalized size = 3.65

method	result
risch	$\left(\frac{1}{4}g\,x^4+f x\right)\ln\left(\left(e\,x^2+d\right)^p\right)+\frac{i\pi g\,x^4\text{csgn}\left(ic\left(e\,x^2+d\right)^p\right)^2\text{csgn}(ic)}{8}+\frac{i\pi g\,x^4\text{csgn}\left(i\left(e\,x^2+d\right)^p\right)\text{csgn}\left(ic\left(e\,x^2+d\right)^p\right)^2}{8}-\frac{i\pi}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] (1/4*g*x^4+f*x)*ln((e*x^2+d)^p)+1/8*I*Pi*g*x^4*csgn(I*c*(e*x^2+d)^p)^2*csgn
(I*c)+1/8*I*Pi*g*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/2*I*Pi*f
```

```
*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*x-1/2*I*Pi*f*csgn(I*c*
(e*x^2+d)^p)^3*x+1/2*I*Pi*f*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*x-1/8*I*Pi*g*
x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/8*I*Pi*g*x^4*csgn
(I*c*(e*x^2+d)^p)^3+1/2*I*Pi*f*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*
x+1/4*ln(c)*g*x^4-1/8*g*p*x^4+1/4*d*g*p*x^2/e-1/4/e^2*d^2*ln(-(-e*d)^(1/2)*
x+d)*g*p+(-e*d)^(1/2)/e*ln(-(-e*d)^(1/2)*x+d)*f*p+ln(c)*f*x-1/4/e^2*d^2*ln(
(-e*d)^(1/2)*x+d)*g*p-2*f*p*x-(-e*d)^(1/2)/e*ln((-e*d)^(1/2)*x+d)*f*p
```

Maxima [A]
time = 0.50, size = 89, normalized size = 0.81

$$-\frac{1}{8}\left(2\,d^2ge^{(-3)}\log\left(x^2e+d\right)-16\,\sqrt{d}\,f\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{3}{2}\right)}+\left(gx^4e-2\,dgx^2+16\,fxe\right)e^{(-2)}\right)pe+\frac{1}{4}\left(gx^4+4\,fx\right)\log\left(\left(x^2e+d\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] -1/8*(2*d^2*g*e^(-3)*log(x^2*e + d) - 16*sqrt(d)*f*arctan(x*e^(1/2)/sqrt(d)
)*e^(-3/2) + (g*x^4*e - 2*d*g*x^2 + 16*f*x*e)*e^(-2))*p*e + 1/4*(g*x^4 + 4*
f*x)*log((x^2*e + d)^p*c)
```

Fricas [A]
time = 0.40, size = 227, normalized size = 2.06

$$\left[\frac{1}{8}\left(2\,dgpx^2e+8\,\sqrt{-de^{(-1)}}\,fpe^2\log\left(\frac{x^2e+2\,\sqrt{-de^{(-1)}}\,xe-d}{x^2e+d}\right)+2\left(gx^4+4\,fx\right)e^2\log\left(c\right)-\left(gpx^4+16\,fpx\right)e^2-2\left(d^2gp-\left(gpx^4+4\,fpx\right)e^2\right)\log\left(x^2e+d\right)\right)e^{(-2)},\ \frac{1}{8}\left(2\,dgpx^2e+16\,\sqrt{d}\,fp\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{3}{2}}+2\left(gx^4+4\,fx\right)e^2\log\left(c\right)-\left(gpx^4+16\,fpx\right)e^2-2\left(d^2gp-\left(gpx^4+4\,fpx\right)e^2\right)\log\left(x^2e+d\right)\right)e^{(-2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [1/8*(2*d*g*p*x^2*e + 8*sqrt(-d*e^(-1))*f*p*e^2*log((x^2*e + 2*sqrt(-d*e^(-
1))*x*e - d)/(x^2*e + d)) + 2*(g*x^4 + 4*f*x)*e^2*log(c) - (g*p*x^4 + 16*f*
p*x)*e^2 - 2*(d^2*g*p - (g*p*x^4 + 4*f*p*x)*e^2)*log(x^2*e + d))*e^(-2), 1/
8*(2*d*g*p*x^2*e + 16*sqrt(d)*f*p*arctan(x*e^(1/2)/sqrt(d))*e^(3/2) + 2*(g*
x^4 + 4*f*x)*e^2*log(c) - (g*p*x^4 + 16*f*p*x)*e^2 - 2*(d^2*g*p - (g*p*x^4
+ 4*f*p*x)*e^2)*log(x^2*e + d))*e^(-2)]
```

Sympy [A]
time = 17.35, size = 214, normalized size = 1.95

$$\begin{cases}\left(fx+\frac{gx^4}{4}\right)\log\left(0^pc\right) & \text{for}\ d=0\wedge e=0\\ -2fpx+fx\log\left(c\left(ex^2\right)^p\right)-\frac{gpx^4}{8}+\frac{gx^4\log\left(c\left(ex^2\right)^p\right)}{4} & \text{for}\ d=0\\ \left(fx+\frac{gx^4}{4}\right)\log\left(cd^p\right) & \text{for}\ e=0\\ -\frac{d^2g\log\left(c\left(d+ex^2\right)^p\right)}{4e^2}+\frac{2dfp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{e\sqrt{-\frac{d}{e}}}-\frac{df\log\left(c\left(d+ex^2\right)^p\right)}{e\sqrt{-\frac{d}{e}}}+\frac{dgpx^2}{4e}-2fpx+fx\log\left(c\left(d+ex^2\right)^p\right)-\frac{gpx^4}{8}+\frac{gx^4\log\left(c\left(d+ex^2\right)^p\right)}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise(((f*x + g*x**4/4)*log(0**p*c), Eq(d, 0) & Eq(e, 0)), (-2*f*p*x +
f*x*log(c*(e*x**2)**p) - g*p*x**4/8 + g*x**4*log(c*(e*x**2)**p)/4, Eq(d, 0)
), ((f*x + g*x**4/4)*log(c*d**p), Eq(e, 0)), (-d**2*g*log(c*(d + e*x**2)**p
)/(4*e**2) + 2*d*f*p*log(x - sqrt(-d/e))/(e*sqrt(-d/e)) - d*f*log(c*(d + e*
x**2)**p)/(e*sqrt(-d/e)) + d*g*p*x**2/(4*e) - 2*f*p*x + f*x*log(c*(d + e*x*
*2)**p) - g*p*x**4/8 + g*x**4*log(c*(d + e*x**2)**p)/4, True))
```

Giac [A]
time = 3.27, size = 117, normalized size = 1.06

$$-\frac{1}{4}\,d^2gpe^{(-2)}\log\left(x^2e+d\right)+2\,\sqrt{d}\,fp\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}+\frac{1}{8}\,\left(2\,gpx^4e\log\left(x^2e+d\right)-gpx^4e+2\,gx^4e\log\left(c\right)+2\,dgpx^2+8\,fpxe\log\left(x^2e+d\right)-16\,fpxe+8\,fxe\log\left(c\right)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] -1/4*d^2*g*p*e^(-2)*log(x^2*e + d) + 2*sqrt(d)*f*p*arctan(x*e^(1/2)/sqrt(d)
)*e^(-1/2) + 1/8*(2*g*p*x^4*e*log(x^2*e + d) - g*p*x^4*e + 2*g*x^4*e*log(c)
 + 2*d*g*p*x^2 + 8*f*p*x*e*log(x^2*e + d) - 16*f*p*x*e + 8*f*x*e*log(c))*e^
(-1)
```

Mupad [B]
time = 0.92, size = 94, normalized size = 0.85

$$f\,x\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)-\frac{g\,p\,x^4}{8}-2\,f\,p\,x+\frac{g\,x^4\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{4}+\frac{d\,g\,p\,x^2}{4\,e}+\frac{2\,\sqrt{d}\,f\,p\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{d^2\,g\,p\,\ln\left(e\,x^2+d\right)}{4\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)*(f + g*x^3),x)
```

```
[Out] f*x*log(c*(d + e*x^2)^p) - (g*p*x^4)/8 - 2*f*p*x + (g*x^4*log(c*(d + e*x^2)
^p))/4 + (d*g*p*x^2)/(4*e) + (2*d^(1/2)*f*p*atan((e^(1/2)*x)/d^(1/2)))/e^(1
/2) - (d^2*g*p*log(d + e*x^2))/(4*e^2)
```

3.291 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^3}\,dx$

Optimal. Leaf size=1165

$$-\frac{p\log\left(\frac{\sqrt[3]{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\sqrt{e}\,\sqrt[3]{f}+\sqrt{-d}\,\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{f}-\sqrt[3]{g}\,x\right)}{3f^{2/3}\sqrt[3]{g}}-\frac{p\log\left(-\frac{\sqrt[3]{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{\sqrt{e}\,\sqrt[3]{f}-\sqrt{-d}\,\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{f}-\sqrt[3]{g}\,x\right.}{3f^{2/3}\sqrt[3]{g}}$$

```
[Out] 1/3*ln(-f^(1/3)-g^(1/3)*x)*ln(c*(e*x^2+d)^p)/f^(2/3)/g^(1/3)+1/3*(-1)^(2/3)
*ln(-f^(1/3)+(-1)^(1/3)*g^(1/3)*x)*ln(c*(e*x^2+d)^p)/f^(2/3)/g^(1/3)-1/3*(-
1)^(1/3)*ln(-f^(1/3)-(-1)^(2/3)*g^(1/3)*x)*ln(c*(e*x^2+d)^p)/f^(2/3)/g^(1/3
)-1/3*p*ln(-f^(1/3)-g^(1/3)*x)*ln(g^(1/3)*((-d)^(1/2)-x*e^(1/2))/(g^(1/3)*(
-d)^(1/2)+f^(1/3)*e^(1/2)))/f^(2/3)/g^(1/3)-1/3*(-1)^(2/3)*p*ln(-f^(1/3)+(-
1)^(1/3)*g^(1/3)*x)*ln(-(-1)^(1/3)*g^(1/3)*((-d)^(1/2)-x*e^(1/2))/(-(-1)^(1
/3)*g^(1/3)*(-d)^(1/2)+f^(1/3)*e^(1/2)))/f^(2/3)/g^(1/3)+1/3*(-1)^(1/3)*p*l
n(-f^(1/3)-(-1)^(2/3)*g^(1/3)*x)*ln((-1)^(2/3)*g^(1/3)*((-d)^(1/2)-x*e^(1/2
))/((-1)^(2/3)*g^(1/3)*(-d)^(1/2)+f^(1/3)*e^(1/2)))/f^(2/3)/g^(1/3)-1/3*p*l
n(-f^(1/3)-g^(1/3)*x)*ln(-g^(1/3)*((-d)^(1/2)+x*e^(1/2))/(-g^(1/3)*(-d)^(1/
2)+f^(1/3)*e^(1/2)))/f^(2/3)/g^(1/3)-1/3*(-1)^(2/3)*p*ln(-f^(1/3)+(-1)^(1/3
)*g^(1/3)*x)*ln((-1)^(1/3)*g^(1/3)*((-d)^(1/2)+x*e^(1/2))/((-1)^(1/3)*g^(1/
3)*(-d)^(1/2)+f^(1/3)*e^(1/2)))/f^(2/3)/g^(1/3)+1/3*(-1)^(1/3)*p*ln(-f^(1/3
)-(-1)^(2/3)*g^(1/3)*x)*ln(-(-1)^(2/3)*g^(1/3)*((-d)^(1/2)+x*e^(1/2))/(-(-1
)^(2/3)*g^(1/3)*(-d)^(1/2)+f^(1/3)*e^(1/2)))/f^(2/3)/g^(1/3)-1/3*p*polylog(
2,(f^(1/3)+g^(1/3)*x)*e^(1/2)/(-g^(1/3)*(-d)^(1/2)+f^(1/3)*e^(1/2)))/f^(2/3
)/g^(1/3)-1/3*p*polylog(2,(f^(1/3)+g^(1/3)*x)*e^(1/2)/(g^(1/3)*(-d)^(1/2)+f
^(1/3)*e^(1/2)))/f^(2/3)/g^(1/3)-1/3*(-1)^(2/3)*p*polylog(2,(f^(1/3)-(-1)^(
1/3)*g^(1/3)*x)*e^(1/2)/(-(-1)^(1/3)*g^(1/3)*(-d)^(1/2)+f^(1/3)*e^(1/2)))/f
^(2/3)/g^(1/3)-1/3*(-1)^(2/3)*p*polylog(2,(f^(1/3)-(-1)^(1/3)*g^(1/3)*x)*e^
(1/2)/((-1)^(1/3)*g^(1/3)*(-d)^(1/2)+f^(1/3)*e^(1/2)))/f^(2/3)/g^(1/3)+1/3*
(-1)^(1/3)*p*polylog(2,(f^(1/3)+(-1)^(2/3)*g^(1/3)*x)*e^(1/2)/(-(-1)^(2/3)*
g^(1/3)*(-d)^(1/2)+f^(1/3)*e^(1/2)))/f^(2/3)/g^(1/3)+1/3*(-1)^(1/3)*p*polyl
og(2,(f^(1/3)+(-1)^(2/3)*g^(1/3)*x)*e^(1/2)/((-1)^(2/3)*g^(1/3)*(-d)^(1/2)+
f^(1/3)*e^(1/2)))/f^(2/3)/g^(1/3)
```

Rubi [A]

time = 1.15, antiderivative size = 1165, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2521, 2512, 266, 2463, 2441, 2440, 2438}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(f + g*x^3),x]
```

```
[Out] -1/3*(p*Log[(g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/3) + Sqrt[-d]*g^
(1/3))]*Log[-f^(1/3) - g^(1/3)*x])/(f^(2/3)*g^(1/3)) - (p*Log[-((g^(1/3)*(S
qrt[-d] + Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - Sqrt[-d]*g^(1/3)))]*Log[-f^(1/3) -
 g^(1/3)*x])/(3*f^(2/3)*g^(1/3)) - ((-1)^(2/3)*p*Log[-(((-1)^(1/3)*g^(1/3)*
(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - (-1)^(1/3)*Sqrt[-d]*g^(1/3)))]*L
og[-f^(1/3) + (-1)^(1/3)*g^(1/3)*x])/(3*f^(2/3)*g^(1/3)) - ((-1)^(2/3)*p*Lo
g[((-1)^(1/3)*g^(1/3)*(Sqrt[-d] + Sqrt[e]*x))/(Sqrt[e]*f^(1/3) + (-1)^(1/3)
*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) + (-1)^(1/3)*g^(1/3)*x])/(3*f^(2/3)*g^(1/3
)) + ((-1)^(1/3)*p*Log[((-1)^(2/3)*g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]
*f^(1/3) + (-1)^(2/3)*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) - (-1)^(2/3)*g^(1/3)*
x])/(3*f^(2/3)*g^(1/3)) + ((-1)^(1/3)*p*Log[-(((-1)^(2/3)*g^(1/3)*(Sqrt[-d]
 + Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - (-1)^(2/3)*Sqrt[-d]*g^(1/3)))]*Log[-f^(1/
3) - (-1)^(2/3)*g^(1/3)*x])/(3*f^(2/3)*g^(1/3)) + (Log[-f^(1/3) - g^(1/3)*x
]*Log[c*(d + e*x^2)^p])/(3*f^(2/3)*g^(1/3)) + ((-1)^(2/3)*Log[-f^(1/3) + (-
1)^(1/3)*g^(1/3)*x]*Log[c*(d + e*x^2)^p])/(3*f^(2/3)*g^(1/3)) - ((-1)^(1/3)
*Log[-f^(1/3) - (-1)^(2/3)*g^(1/3)*x]*Log[c*(d + e*x^2)^p])/(3*f^(2/3)*g^(1
/3)) - (p*PolyLog[2, (Sqrt[e]*(f^(1/3) + g^(1/3)*x))/(Sqrt[e]*f^(1/3) - Sqr
t[-d]*g^(1/3))])/(3*f^(2/3)*g^(1/3)) - (p*PolyLog[2, (Sqrt[e]*(f^(1/3) + g^
(1/3)*x))/(Sqrt[e]*f^(1/3) + Sqrt[-d]*g^(1/3))])/(3*f^(2/3)*g^(1/3)) - ((-1
)^(2/3)*p*PolyLog[2, (Sqrt[e]*(f^(1/3) - (-1)^(1/3)*g^(1/3)*x))/(Sqrt[e]*f^
(1/3) - (-1)^(1/3)*Sqrt[-d]*g^(1/3))])/(3*f^(2/3)*g^(1/3)) - ((-1)^(2/3)*p*
PolyLog[2, (Sqrt[e]*(f^(1/3) - (-1)^(1/3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) + (-
1)^(1/3)*Sqrt[-d]*g^(1/3))])/(3*f^(2/3)*g^(1/3)) + ((-1)^(1/3)*p*PolyLog[2,
 (Sqrt[e]*(f^(1/3) + (-1)^(2/3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) - (-1)^(2/3)*S
qrt[-d]*g^(1/3))])/(3*f^(2/3)*g^(1/3)) + ((-1)^(1/3)*p*PolyLog[2, (Sqrt[e]*
(f^(1/3) + (-1)^(2/3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) + (-1)^(2/3)*Sqrt[-d]*g^
(1/3))])/(3*f^(2/3)*g^(1/3))
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx = \int \left(-\frac{\log\left(c(d+ex^2)^p\right)}{3f^{2/3}\left(-\sqrt[3]{f}-\sqrt[3]{g}\,x\right)} - \frac{\log\left(c(d+ex^2)^p\right)}{3f^{2/3}\left(-\sqrt[3]{f}+\sqrt[3]{-1}\,\sqrt[3]{g}\,x\right)} - \frac{\log\left(c(d\right.}{3f^{2/3}\left(-\sqrt[3]{f}\,-\right.}$$

$$= -\frac{\int \frac{\log\left(c(d+ex^2)^p\right)}{-\sqrt[3]{f}-\sqrt[3]{g}\,x}dx}{3f^{2/3}} - \frac{\int \frac{\log\left(c(d+ex^2)^p\right)}{-\sqrt[3]{f}+\sqrt[3]{-1}\,\sqrt[3]{g}\,x}dx}{3f^{2/3}} - \frac{\int \frac{\log\left(c(d+ex^2)^p\right)}{-\sqrt[3]{f}-(-1)^{2/3}\sqrt[3]{g}\,x}dx}{3f^{2/3}}$$

$$= \frac{\log\left(-\sqrt[3]{f}-\sqrt[3]{g}\,x\right)\log\left(c(d+ex^2)^p\right)}{3f^{2/3}\sqrt[3]{g}} + \frac{(-1)^{2/3}\log\left(-\sqrt[3]{f}+\sqrt[3]{-1}\,\sqrt[3]{g}\,x\right)\log\left(\right.}{3f^{2/3}\sqrt[3]{g}}$$

$$= \frac{\log\left(-\sqrt[3]{f}-\sqrt[3]{g}\,x\right)\log\left(c(d+ex^2)^p\right)}{3f^{2/3}\sqrt[3]{g}} + \frac{(-1)^{2/3}\log\left(-\sqrt[3]{f}+\sqrt[3]{-1}\,\sqrt[3]{g}\,x\right)\log\left(\right.}{3f^{2/3}\sqrt[3]{g}}$$

$$= \frac{\log\left(-\sqrt[3]{f}-\sqrt[3]{g}\,x\right)\log\left(c(d+ex^2)^p\right)}{3f^{2/3}\sqrt[3]{g}} + \frac{(-1)^{2/3}\log\left(-\sqrt[3]{f}+\sqrt[3]{-1}\,\sqrt[3]{g}\,x\right)\log\left(\right.}{3f^{2/3}\sqrt[3]{g}}$$

$$= -\frac{p\log\left(\frac{\sqrt[3]{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\sqrt{e}\,\sqrt[3]{f}+\sqrt{-d}\,\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{f}-\sqrt[3]{g}\,x\right)}{3f^{2/3}\sqrt[3]{g}} - \frac{p\log\left(-\frac{\sqrt[3]{g}\left(\sqrt{-d}+\sqrt{e}\right.}{\sqrt{e}\,\sqrt[3]{f}-\sqrt{-d}\right.}}{3f^{2/}}$$

$$= -\frac{p\log\left(\frac{\sqrt[3]{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\sqrt{e}\,\sqrt[3]{f}+\sqrt{-d}\,\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{f}-\sqrt[3]{g}\,x\right)}{3f^{2/3}\sqrt[3]{g}} - \frac{p\log\left(-\frac{\sqrt[3]{g}\left(\sqrt{-d}+\sqrt{e}\right.}{\sqrt{e}\,\sqrt[3]{f}-\sqrt{-d}\right.}}{3f^{2/}}$$

$$= -\frac{p\log\left(\frac{\sqrt[3]{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\sqrt{e}\,\sqrt[3]{f}+\sqrt{-d}\,\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{f}-\sqrt[3]{g}\,x\right)}{3f^{2/3}\sqrt[3]{g}} - \frac{p\log\left(-\frac{\sqrt[3]{g}\left(\sqrt{-d}+\sqrt{e}\right.}{\sqrt{e}\,\sqrt[3]{f}-\sqrt{-d}\right.}}{3f^{2/}}$$

Mathematica [A]
time = 0.59, size = 990, normalized size = 0.85

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^3),x]
```

```
[Out] (-(p*Log[(g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/3) + Sqrt[-d]*g^(1/
3))]*Log[-f^(1/3) - g^(1/3)*x]) - p*Log[(g^(1/3)*(Sqrt[-d] + Sqrt[e]*x))/(-
(Sqrt[e]*f^(1/3)) + Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) - g^(1/3)*x] - (-1)^(2/
3)*p*Log[((-1)^(1/3)*g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(-(Sqrt[e]*f^(1/3)) +
(-1)^(1/3)*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) + (-1)^(1/3)*g^(1/3)*x] - (-1)^(
2/3)*p*Log[((-1)^(1/3)*g^(1/3)*(Sqrt[-d] + Sqrt[e]*x))/(Sqrt[e]*f^(1/3) + (
-1)^(1/3)*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) + (-1)^(1/3)*g^(1/3)*x] + (-1)^(1
/3)*p*Log[((-1)^(2/3)*g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/3) + (-
1)^(2/3)*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) - (-1)^(2/3)*g^(1/3)*x] + (-1)^(1/
3)*p*Log[((-1)^(2/3)*g^(1/3)*(Sqrt[-d] + Sqrt[e]*x))/(-(Sqrt[e]*f^(1/3)) +
(-1)^(2/3)*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) - (-1)^(2/3)*g^(1/3)*x] + Log[-f
^(1/3) - g^(1/3)*x]*Log[c*(d + e*x^2)^p] + (-1)^(2/3)*Log[-f^(1/3) + (-1)^(
1/3)*g^(1/3)*x]*Log[c*(d + e*x^2)^p] - (-1)^(1/3)*Log[-f^(1/3) - (-1)^(2/3)
*g^(1/3)*x]*Log[c*(d + e*x^2)^p] - p*PolyLog[2, (Sqrt[e]*(f^(1/3) + g^(1/3)
*x))/(Sqrt[e]*f^(1/3) - Sqrt[-d]*g^(1/3))] - p*PolyLog[2, (Sqrt[e]*(f^(1/3)
 + g^(1/3)*x))/(Sqrt[e]*f^(1/3) + Sqrt[-d]*g^(1/3))] - (-1)^(2/3)*p*PolyLog
[2, (Sqrt[e]*(f^(1/3) - (-1)^(1/3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) - (-1)^(1/3
)*Sqrt[-d]*g^(1/3))] - (-1)^(2/3)*p*PolyLog[2, (Sqrt[e]*(f^(1/3) - (-1)^(1/
3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) + (-1)^(1/3)*Sqrt[-d]*g^(1/3))] + (-1)^(1/3
)*p*PolyLog[2, (Sqrt[e]*(f^(1/3) + (-1)^(2/3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3)
- (-1)^(2/3)*Sqrt[-d]*g^(1/3))] + (-1)^(1/3)*p*PolyLog[2, (Sqrt[e]*(f^(1/3)
 + (-1)^(2/3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) + (-1)^(2/3)*Sqrt[-d]*g^(1/3))])
/(3*f^(2/3)*g^(1/3))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.07, size = 1180, normalized size = 1.01

method	result	size
risch	Expression too large to display	1180

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/(g*x^3+f),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(ln((e*x^2+d)^p)-p*ln(e*x^2+d))/g/(f/g)^(2/3)*ln(x+(f/g)^(1/3))-1/6*(ln
((e*x^2+d)^p)-p*ln(e*x^2+d))/g/(f/g)^(2/3)*ln(x^2-(f/g)^(1/3)*x+(f/g)^(2/3)
)+1/3*(ln((e*x^2+d)^p)-p*ln(e*x^2+d))/g/(f/g)^(2/3)*3^(1/2)*arctan(1/3*3^(1
/2)*(2/(f/g)^(1/3)*x-1))+1/3*p/g*sum(1/_alpha^2*(ln(x-_alpha)*ln(e*x^2+d)-l
n(x-_alpha)*(ln((RootOf(_Z^2*e+2*_Z*_alpha*e+_alpha^2*e+d,index=1)-x+_alpha
)/RootOf(_Z^2*e+2*_Z*_alpha*e+_alpha^2*e+d,index=1))+ln((RootOf(_Z^2*e+2*_Z
*_alpha*e+_alpha^2*e+d,index=2)-x+_alpha)/RootOf(_Z^2*e+2*_Z*_alpha*e+_alph
a^2*e+d,index=2)))-dilog((RootOf(_Z^2*e+2*_Z*_alpha*e+_alpha^2*e+d,index=1)
-x+_alpha)/RootOf(_Z^2*e+2*_Z*_alpha*e+_alpha^2*e+d,index=1))-dilog((RootOf
(_Z^2*e+2*_Z*_alpha*e+_alpha^2*e+d,index=2)-x+_alpha)/RootOf(_Z^2*e+2*_Z*_a
lpha*e+_alpha^2*e+d,index=2))),_alpha=RootOf(_Z^3*g+f))-1/12*I*Pi*csgn(I*c*
(e*x^2+d)^p)^2*csgn(I*c)/g/(f/g)^(2/3)*ln(x^2-(f/g)^(1/3)*x+(f/g)^(2/3))-1/
```

```
6*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/g/(f/g)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/
(f/g)^(1/3)*x-1))+1/6*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/g/(f
/g)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(f/g)^(1/3)*x-1))+1/6*I*Pi*csgn(I*c
*(e*x^2+d)^p)^2*csgn(I*c)/g/(f/g)^(2/3)*ln(x+(f/g)^(1/3))+1/6*I*Pi*csgn(I*c
*(e*x^2+d)^p)^2*csgn(I*c)/g/(f/g)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(f/g)
^(1/3)*x-1))-1/12*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/g/(f/g)^
(2/3)*ln(x^2-(f/g)^(1/3)*x+(f/g)^(2/3))-1/6*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/g/
(f/g)^(2/3)*ln(x+(f/g)^(1/3))-1/6*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+
d)^p)*csgn(I*c)/g/(f/g)^(2/3)*ln(x+(f/g)^(1/3))+1/12*I*Pi*csgn(I*c*(e*x^2+d
)^p)^3/g/(f/g)^(2/3)*ln(x^2-(f/g)^(1/3)*x+(f/g)^(2/3))+1/12*I*Pi*csgn(I*(e*
x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/g/(f/g)^(2/3)*ln(x^2-(f/g)^(1/3)*
x+(f/g)^(2/3))+1/6*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/g/(f/g)
^(2/3)*ln(x+(f/g)^(1/3))-1/6*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)
*csgn(I*c)/g/(f/g)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(f/g)^(1/3)*x-1))+1/
3*ln(c)/g/(f/g)^(2/3)*ln(x+(f/g)^(1/3))-1/6*ln(c)/g/(f/g)^(2/3)*ln(x^2-(f/g
)^(1/3)*x+(f/g)^(2/3))+1/3*ln(c)/g/(f/g)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(
2/(f/g)^(1/3)*x-1))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^3+f),x, algorithm="maxima")
[Out] integrate(log((x^2*e + d)^p*c)/(g*x^3 + f), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^3+f),x, algorithm="fricas")
[Out] integral(log((x^2*e + d)^p*c)/(g*x^3 + f), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/(g*x**3+f),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^3+f),x, algorithm="giac")
```

```
[Out] integrate(log((x^2*e + d)^p*c)/(g*x^3 + f), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{g\,x^3+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)/(f + g*x^3),x)
```

```
[Out] int(log(c*(d + e*x^2)^p)/(f + g*x^3), x)
```

3.292 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}\,dx$

Optimal. Leaf size=1861

result too large to display

```
[Out] -1/9*ln(c*(e*x^2+d)^p)/f^(4/3)/g^(1/3)/(f^(1/3)+g^(1/3)*x)+2/9*ln(f^(1/3)+g
^(1/3)*x)*ln(c*(e*x^2+d)^p)/f^(5/3)/g^(1/3)-2/9*p*polylog(2,(f^(1/3)+g^(1/3
)*x)*e^(1/2)/(-g^(1/3)*(-d)^(1/2)+f^(1/3)*e^(1/2)))/f^(5/3)/g^(1/3)-2/9*p*p
olylog(2,(f^(1/3)+g^(1/3)*x)*e^(1/2)/(g^(1/3)*(-d)^(1/2)+f^(1/3)*e^(1/2)))/
f^(5/3)/g^(1/3)-2/9*e*p*ln(f^(1/3)+g^(1/3)*x)/f/(e*f^(2/3)+d*g^(2/3))/g^(1/
3)+1/9*e*p*ln(e*x^2+d)/f/(e*f^(2/3)+d*g^(2/3))/g^(1/3)+4/9*p*ln(2*f^(1/3)-g
^(1/3)*x*(1-I*3^(1/2)))*ln(-g^(1/3)*(3^(1/2)+I)*((-d)^(1/2)-x*e^(1/2))/(-g^
(1/3)*(3^(1/2)+I)*(-d)^(1/2)+2*I*f^(1/3)*e^(1/2)))/f^(5/3)/g^(1/3)/(1-I*3^(
1/2))+4/9*p*ln(2*f^(1/3)-g^(1/3)*x*(1-I*3^(1/2)))*ln(g^(1/3)*(3^(1/2)+I)*((
-d)^(1/2)+x*e^(1/2))/(g^(1/3)*(3^(1/2)+I)*(-d)^(1/2)+2*I*f^(1/3)*e^(1/2)))/
f^(5/3)/g^(1/3)/(1-I*3^(1/2))+4/9*p*ln(2*f^(1/3)-g^(1/3)*x*(1+I*3^(1/2)))*l
n(-g^(1/3)*(1+I*3^(1/2))*((-d)^(1/2)-x*e^(1/2))/(-g^(1/3)*(1+I*3^(1/2))*(-d
)^(1/2)+2*f^(1/3)*e^(1/2)))/f^(5/3)/g^(1/3)/(1+I*3^(1/2))+4/9*p*ln(2*f^(1/3
)-g^(1/3)*x*(1+I*3^(1/2)))*ln(g^(1/3)*(1+I*3^(1/2))*((-d)^(1/2)+x*e^(1/2))/
(g^(1/3)*(1+I*3^(1/2))*(-d)^(1/2)+2*f^(1/3)*e^(1/2)))/f^(5/3)/g^(1/3)/(1+I*
3^(1/2))+2/9*p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)*e^(1/2)/f^(4/3)/(e*f^(2/3)
+d*g^(2/3))+4/9*p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)*e^(1/2)/f^(4/3)/(2*e*f^
(2/3)-d*g^(2/3)*(1+I*3^(1/2)))+4/9*(-1)^(1/3)*e*p*ln(f^(1/3)+(-1)^(2/3)*g^(
1/3)*x)/f/g^(1/3)/(2*e*f^(2/3)-d*g^(2/3)*(1+I*3^(1/2)))-2/9*(-1)^(1/3)*e*p*
ln(e*x^2+d)/f/g^(1/3)/(2*e*f^(2/3)-d*g^(2/3)*(1+I*3^(1/2)))-ln(c*(e*x^2+d)^
p)/(1+(-1)^(1/3))^4/f^(4/3)/g^(1/3)/((-1)^(2/3)*f^(1/3)+g^(1/3)*x)+1/9*(-1)
^(1/3)*ln(c*(e*x^2+d)^p)/f^(4/3)/g^(1/3)/(f^(1/3)+(-1)^(2/3)*g^(1/3)*x)-2/9
*p*ln(f^(1/3)+g^(1/3)*x)*ln(g^(1/3)*((-d)^(1/2)-x*e^(1/2))/(g^(1/3)*(-d)^(1
/2)+f^(1/3)*e^(1/2)))/f^(5/3)/g^(1/3)-2/9*p*ln(f^(1/3)+g^(1/3)*x)*ln(-g^(1/
3)*((-d)^(1/2)+x*e^(1/2))/(-g^(1/3)*(-d)^(1/2)+f^(1/3)*e^(1/2)))/f^(5/3)/g^
(1/3)-4/9*ln(c*(e*x^2+d)^p)*ln(2*f^(1/3)-g^(1/3)*x*(1-I*3^(1/2)))/f^(5/3)/g
^(1/3)/(1-I*3^(1/2))+4/9*p*polylog(2,(2*f^(1/3)-g^(1/3)*x*(1-I*3^(1/2)))*e^
(1/2)/(g^(1/3)*(1-I*3^(1/2))*(-d)^(1/2)+2*f^(1/3)*e^(1/2)))/f^(5/3)/g^(1/3)
/(1-I*3^(1/2))+4/9*p*polylog(2,(2*f^(1/3)-g^(1/3)*x*(1-I*3^(1/2)))*e^(1/2)/
(I*g^(1/3)*(3^(1/2)+I)*(-d)^(1/2)+2*f^(1/3)*e^(1/2)))/f^(5/3)/g^(1/3)/(1-I*
3^(1/2))-4/9*ln(c*(e*x^2+d)^p)*ln(2*f^(1/3)-g^(1/3)*x*(1+I*3^(1/2)))/f^(5/3
)/g^(1/3)/(1+I*3^(1/2))+4/9*p*polylog(2,(2*f^(1/3)-g^(1/3)*x*(1+I*3^(1/2)))
*e^(1/2)/(-g^(1/3)*(1+I*3^(1/2))*(-d)^(1/2)+2*f^(1/3)*e^(1/2)))/f^(5/3)/g^(
1/3)/(1+I*3^(1/2))+4/9*p*polylog(2,(2*f^(1/3)-g^(1/3)*x*(1+I*3^(1/2)))*e^(1
/2)/(g^(1/3)*(1+I*3^(1/2))*(-d)^(1/2)+2*f^(1/3)*e^(1/2)))/f^(5/3)/g^(1/3)/(
1+I*3^(1/2))+2*(-1)^(1/3)*e*p*ln(f^(1/3)-(-1)^(1/3)*g^(1/3)*x)/(1+(-1)^(1/3
))^4/f/(e*f^(2/3)+(-1)^(2/3)*d*g^(2/3))/g^(1/3)-(-1)^(1/3)*e*p*ln(e*x^2+d)/
(1+(-1)^(1/3))^4/f/(e*f^(2/3)+(-1)^(2/3)*d*g^(2/3))/g^(1/3)+2*(-1)^(2/3)*p*
```

```
arctan(x*e^(1/2)/d^(1/2))*d^(1/2)*e^(1/2)/(1+(-1)^(1/3))^4/f^(4/3)/(e*f^(2/
3)+(-1)^(2/3)*d*g^(2/3))
```

Rubi [A]

time = 2.20, antiderivative size = 1863, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2521, 2513, 815, 649, 211, 266, 2512, 2463, 2441, 2440, 2438}

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(f + g*x^3)^2,x]

[Out] (2*Sqrt[d]*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(9*f^(4/3)*(e*f^(2/3) + d
*g^(2/3))) + (2*(-1)^(2/3)*Sqrt[d]*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(
(1 + (-1)^(1/3))^4*f^(4/3)*(e*f^(2/3) + (-1)^(2/3)*d*g^(2/3))) + (4*Sqrt[d]
*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(9*f^(4/3)*(2*e*f^(2/3) + I*(I - Sq
rt[3])*d*g^(2/3))) - (2*e*p*Log[f^(1/3) + g^(1/3)*x])/(9*f*(e*f^(2/3) + d*g
^(2/3))*g^(1/3)) - (2*p*Log[(g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/
3) + Sqrt[-d]*g^(1/3))]*Log[f^(1/3) + g^(1/3)*x])/(9*f^(5/3)*g^(1/3)) - (2*
p*Log[-((g^(1/3)*(Sqrt[-d] + Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - Sqrt[-d]*g^(1/3
)))]*Log[f^(1/3) + g^(1/3)*x])/(9*f^(5/3)*g^(1/3)) + (2*(-1)^(1/3)*e*p*Log[
f^(1/3) - (-1)^(1/3)*g^(1/3)*x])/((1 + (-1)^(1/3))^4*f*(e*f^(2/3) + (-1)^(2
/3)*d*g^(2/3))*g^(1/3)) + ((2*I)*Sqrt[3]*p*Log[-(((-1)^(1/3)*g^(1/3)*(Sqrt[
-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - (-1)^(1/3)*Sqrt[-d]*g^(1/3)))]*Log[-f^
(1/3) + (-1)^(1/3)*g^(1/3)*x])/((1 + (-1)^(1/3))^5*f^(5/3)*g^(1/3)) + ((2*I
)*Sqrt[3]*p*Log[((-1)^(1/3)*g^(1/3)*(Sqrt[-d] + Sqrt[e]*x))/(Sqrt[e]*f^(1/3
) + (-1)^(1/3)*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) + (-1)^(1/3)*g^(1/3)*x])/((1
 + (-1)^(1/3))^5*f^(5/3)*g^(1/3)) + (4*(-1)^(1/3)*e*p*Log[f^(1/3) + (-1)^(2
/3)*g^(1/3)*x])/(9*f*(2*e*f^(2/3) - (1 + I*Sqrt[3])*d*g^(2/3))*g^(1/3)) - (
2*p*Log[((-1)^(2/3)*g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/3) + (-1)
^(2/3)*Sqrt[-d]*g^(1/3))]*Log[f^(1/3) + (-1)^(2/3)*g^(1/3)*x])/((1 + (-1)^(
1/3))^4*f^(5/3)*g^(1/3)) - (2*p*Log[-(((-1)^(2/3)*g^(1/3)*(Sqrt[-d] + Sqrt[
e]*x))/(Sqrt[e]*f^(1/3) - (-1)^(2/3)*Sqrt[-d]*g^(1/3)))]*Log[f^(1/3) + (-1)
^(2/3)*g^(1/3)*x])/((1 + (-1)^(1/3))^4*f^(5/3)*g^(1/3)) + (e*p*Log[d + e*x^
2])/(9*f*(e*f^(2/3) + d*g^(2/3))*g^(1/3)) - ((-1)^(1/3)*e*p*Log[d + e*x^2])
/((1 + (-1)^(1/3))^4*f*(e*f^(2/3) + (-1)^(2/3)*d*g^(2/3))*g^(1/3)) - (2*(-1
)^(1/3)*e*p*Log[d + e*x^2])/(9*f*(2*e*f^(2/3) - (1 + I*Sqrt[3])*d*g^(2/3))*
g^(1/3)) - Log[c*(d + e*x^2)^p]/(9*f^(4/3)*g^(1/3)*(f^(1/3) + g^(1/3)*x)) -
 Log[c*(d + e*x^2)^p]/((1 + (-1)^(1/3))^4*f^(4/3)*g^(1/3)*((-1)^(2/3)*f^(1/
3) + g^(1/3)*x)) + ((-1)^(1/3)*Log[c*(d + e*x^2)^p])/(9*f^(4/3)*g^(1/3)*(f^
(1/3) + (-1)^(2/3)*g^(1/3)*x)) + (2*Log[f^(1/3) + g^(1/3)*x]*Log[c*(d + e*x
^2)^p])/(9*f^(5/3)*g^(1/3)) - ((2*I)*Sqrt[3]*Log[-f^(1/3) + (-1)^(1/3)*g^(1
/3)*x]*Log[c*(d + e*x^2)^p])/((1 + (-1)^(1/3))^5*f^(5/3)*g^(1/3)) + (2*Log[
f^(1/3) + (-1)^(2/3)*g^(1/3)*x]*Log[c*(d + e*x^2)^p])/((1 + (-1)^(1/3))^4*f
```

```
^(5/3)*g^(1/3)) - (2*p*PolyLog[2, (Sqrt[e]*(f^(1/3) + g^(1/3)*x))/(Sqrt[e]*
f^(1/3) - Sqrt[-d]*g^(1/3))])/(9*f^(5/3)*g^(1/3)) - (2*p*PolyLog[2, (Sqrt[e
]*(f^(1/3) + g^(1/3)*x))/(Sqrt[e]*f^(1/3) + Sqrt[-d]*g^(1/3))])/(9*f^(5/3)*
g^(1/3)) + ((2*I)*Sqrt[3]*p*PolyLog[2, (Sqrt[e]*(f^(1/3) - (-1)^(1/3)*g^(1/
3)*x))/(Sqrt[e]*f^(1/3) - (-1)^(1/3)*Sqrt[-d]*g^(1/3))])/((1 + (-1)^(1/3))^
5*f^(5/3)*g^(1/3)) + ((2*I)*Sqrt[3]*p*PolyLog[2, (Sqrt[e]*(f^(1/3) - (-1)^(
1/3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) + (-1)^(1/3)*Sqrt[-d]*g^(1/3))])/((1 + (-
1)^(1/3))^5*f^(5/3)*g^(1/3)) - (2*p*PolyLog[2, (Sqrt[e]*(f^(1/3) + (-1)^(2/
3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) - (-1)^(2/3)*Sqrt[-d]*g^(1/3))])/((1 + (-1)
^(1/3))^4*f^(5/3)*g^(1/3)) - (2*p*PolyLog[2, (Sqrt[e]*(f^(1/3) + (-1)^(2/3)
*g^(1/3)*x))/(Sqrt[e]*f^(1/3) + (-1)^(2/3)*Sqrt[-d]*g^(1/3))])/((1 + (-1)^(
1/3))^4*f^(5/3)*g^(1/3))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{(f+gx^3)^2}\,dx = \int \left(\frac{\log\left(c(d+ex^2)^p\right)}{9f^{4/3}\left(\sqrt[3]{f}+\sqrt[3]{g}\,x\right)^2} + \frac{2\log\left(c(d+ex^2)^p\right)}{9f^{5/3}\left(\sqrt[3]{f}+\sqrt[3]{g}\,x\right)} + \frac{(-1)^{2/3}\log\left(c(d\right.}{\left(1+\sqrt[3]{-1}\right)^4 f^{4/3}\left(-\sqrt[3]{f}\right.}\right.$$

$$= \frac{2\int \frac{\log(c(d+ex^2)^p)}{\sqrt[3]{f}+\sqrt[3]{g}\,x}\,dx}{9f^{5/3}} + \frac{2\int \frac{\log(c(d+ex^2)^p)}{\sqrt[3]{f}+(-1)^{2/3}\sqrt[3]{g}\,x}\,dx}{9f^{5/3}} - \frac{\left(2(-1)^{5/6}\sqrt{3}\right)\int \frac{\log(c(d+ex^2)^p}{-\sqrt[3]{f}+\sqrt[3]{-1}\,\sqrt[3]{}}}{\left(1+\sqrt[3]{-1}\right)^5 f^{5/3}}$$

$$= -\frac{\log\left(c(d+ex^2)^p\right)}{9f^{4/3}\sqrt[3]{g}\left(\sqrt[3]{f}+\sqrt[3]{g}\,x\right)} + \frac{\sqrt[3]{-1}\,\log\left(c(d+ex^2)^p\right)}{9f^{4/3}\sqrt[3]{g}\left((-1)^{2/3}\sqrt[3]{f}+\sqrt[3]{g}\,x\right)} + \frac{\sqrt[3]{-1}\,\log}{9f^{4/3}\sqrt[3]{g}\left(\sqrt[3]{}\right.}$$

$$= -\frac{\log\left(c(d+ex^2)^p\right)}{9f^{4/3}\sqrt[3]{g}\left(\sqrt[3]{f}+\sqrt[3]{g}\,x\right)} + \frac{\sqrt[3]{-1}\,\log\left(c(d+ex^2)^p\right)}{9f^{4/3}\sqrt[3]{g}\left((-1)^{2/3}\sqrt[3]{f}+\sqrt[3]{g}\,x\right)} + \frac{\sqrt[3]{-1}\,\log}{9f^{4/3}\sqrt[3]{g}\left(\sqrt[3]{}\right.}$$

$$= -\frac{2ep\log\left(\sqrt[3]{f}+\sqrt[3]{g}\,x\right)}{9f\left(ef^{2/3}+dg^{2/3}\right)\sqrt[3]{g}} - \frac{2(-1)^{2/3}ep\log\left(\sqrt[3]{f}-\sqrt[3]{-1}\,\sqrt[3]{g}\,x\right)}{9f\left(ef^{2/3}+(-1)^{2/3}dg^{2/3}\right)\sqrt[3]{g}} + \frac{2\sqrt[3]{-1}\,ep\log}{9f\left(ef^{2/3}\right.}$$

$$= -\frac{2ep\log\left(\sqrt[3]{f}+\sqrt[3]{g}\,x\right)}{9f\left(ef^{2/3}+dg^{2/3}\right)\sqrt[3]{g}} - \frac{2p\log\left(\frac{\sqrt[3]{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\sqrt{e}\,\sqrt[3]{f}+\sqrt{-d}\,\sqrt[3]{g}}\right)\log\left(\sqrt[3]{f}+\sqrt[3]{g}\,x\right)}{9f^{5/3}\sqrt[3]{g}} -$$

$$= \frac{2\sqrt{d}\,\sqrt{e}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{9f^{4/3}\left(ef^{2/3}+dg^{2/3}\right)} + \frac{2\sqrt{d}\,\sqrt{e}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{9f^{4/3}\left(ef^{2/3}-\sqrt[3]{-1}\,dg^{2/3}\right)} + \frac{2\sqrt{d}\,\sqrt{e}\,p\tan^{-1}\left(\right.}{9f^{4/3}\left(ef^{2/3}+(-1)^{2/3}\right.}$$

$$= \frac{2\sqrt{d}\,\sqrt{e}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{9f^{4/3}\left(ef^{2/3}+dg^{2/3}\right)} + \frac{2\sqrt{d}\,\sqrt{e}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{9f^{4/3}\left(ef^{2/3}-\sqrt[3]{-1}\,dg^{2/3}\right)} + \frac{2\sqrt{d}\,\sqrt{e}\,p\tan^{-1}\left(\right.}{9f^{4/3}\left(ef^{2/3}+(-1)^{2/3}\right.}$$

Mathematica [A]
time = 6.84, size = 2168, normalized size = 1.16

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^3)^2,x]
```

```
[Out] (x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]))/(3*f*(f + g*x^3)) + (2*Arc
Tan[(-f^(1/3) + 2*g^(1/3)*x)/(Sqrt[3]*f^(1/3))]*(-(p*Log[d + e*x^2]) + Log[
c*(d + e*x^2)^p]))/(3*Sqrt[3]*f^(5/3)*g^(1/3)) + (2*Log[f^(1/3) + g^(1/3)*x
]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]))/(9*f^(5/3)*g^(1/3)) - ((-(p
*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])*Log[f^(2/3) - f^(1/3)*g^(1/3)*x +
g^(2/3)*x^2])/(9*f^(5/3)*g^(1/3)) + p*(-1/3*((-1 + (-1)^(1/3))*(-(Log[((-I)
*Sqrt[d])/Sqrt[e] + x]/((-1)^(2/3)*f^(1/3) + g^(1/3)*x)) + (Sqrt[e]*(Log[I*
Sqrt[d] - Sqrt[e]*x] - Log[-((-1)^(2/3)*f^(1/3)) - g^(1/3)*x]))/((-1)^(2/3)
*Sqrt[e]*f^(1/3) + I*Sqrt[d]*g^(1/3))))/((1 + (-1)^(1/3))^2*f^(4/3)*g^(1/3)
) - ((-1 + (-1)^(1/3))*(-(Log[(I*Sqrt[d])/Sqrt[e] + x]/((-1)^(2/3)*f^(1/3)
+ g^(1/3)*x)) + (Sqrt[e]*(Log[I*Sqrt[d] + Sqrt[e]*x] - Log[-((-1)^(2/3)*f^(
1/3)) - g^(1/3)*x]))/((-1)^(2/3)*Sqrt[e]*f^(1/3) - I*Sqrt[d]*g^(1/3))))/(3*
(1 + (-1)^(1/3))^2*f^(4/3)*g^(1/3)) + ((-1)^(1/3)*(-(Log[((-I)*Sqrt[d])/Sqr
t[e] + x]/(f^(1/3) + g^(1/3)*x)) + (Sqrt[e]*(Log[I*Sqrt[d] - Sqrt[e]*x] - L
og[f^(1/3) + g^(1/3)*x]))/(Sqrt[e]*f^(1/3) + I*Sqrt[d]*g^(1/3))))/(3*(1 + (
-1)^(1/3))^2*f^(4/3)*g^(1/3)) + ((-1)^(1/3)*(-(Log[(I*Sqrt[d])/Sqrt[e] + x]
/(f^(1/3) + g^(1/3)*x)) + (Sqrt[e]*(Log[I*Sqrt[d] + Sqrt[e]*x] - Log[f^(1/3
) + g^(1/3)*x]))/(Sqrt[e]*f^(1/3) - I*Sqrt[d]*g^(1/3))))/(3*(1 + (-1)^(1/3)
)^2*f^(4/3)*g^(1/3)) - (Log[((-I)*Sqrt[d])/Sqrt[e] + x]/((-1)^(1/3)*f^(1/3)
 - g^(1/3)*x) + (Sqrt[e]*(-Log[I*Sqrt[d] - Sqrt[e]*x] + Log[f^(1/3) + (-1)^
(2/3)*g^(1/3)*x]))/((-1)^(1/3)*Sqrt[e]*f^(1/3) - I*Sqrt[d]*g^(1/3)))/(3*(1
+ (-1)^(1/3))^2*f^(4/3)*g^(1/3)) - (Log[(I*Sqrt[d])/Sqrt[e] + x]/((-1)^(1/3
)*f^(1/3) - g^(1/3)*x) + (Sqrt[e]*(-Log[I*Sqrt[d] + Sqrt[e]*x] + Log[f^(1/3
) + (-1)^(2/3)*g^(1/3)*x]))/((-1)^(1/3)*Sqrt[e]*f^(1/3) + I*Sqrt[d]*g^(1/3)
))/(3*(1 + (-1)^(1/3))^2*f^(4/3)*g^(1/3)) + ((-Log[((-I)*Sqrt[d])/Sqrt[e] +
 x] - Log[(I*Sqrt[d])/Sqrt[e] + x] + Log[d + e*x^2])*((3*f^(2/3)*x)/(f + g*
x^3) - (2*Sqrt[3]*ArcTan[(1 - (2*g^(1/3)*x)/f^(1/3))/Sqrt[3]])/g^(1/3) + (2
*Log[f^(1/3) + g^(1/3)*x])/g^(1/3) - Log[f^(2/3) - f^(1/3)*g^(1/3)*x + g^(2
/3)*x^2]/g^(1/3)))/(9*f^(5/3)) - (2*(Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[((-1)
^(2/3)*f^(1/3) + g^(1/3)*x)/((-1)^(2/3)*f^(1/3) - (I*Sqrt[d]*g^(1/3))/Sqrt[
e])] + PolyLog[2, -((g^(1/3)*(Sqrt[d] - I*Sqrt[e]*x))/((-1)^(1/6)*Sqrt[e]*f
^(1/3) - Sqrt[d]*g^(1/3)))]))/(3*(1 + (-1)^(1/3))^2*f^(5/3)*g^(1/3)) - (2*(
-1 + (-1)^(1/3))*(Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[-((-((-1)^(1/3)*f^(1/3))
 + g^(1/3)*x)/((-1)^(1/3)*f^(1/3) + (I*Sqrt[d]*g^(1/3))/Sqrt[e]))] + PolyLo
g[2, -((g^(1/3)*(Sqrt[d] - I*Sqrt[e]*x))/((-1)^(5/6)*Sqrt[e]*f^(1/3) - Sqrt
[d]*g^(1/3)))]))/(3*(1 + (-1)^(1/3))^2*f^(5/3)*g^(1/3)) + (2*(-1)^(1/3)*(Lo
g[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(f^(1/3) + g^(1/3)*x)/(f^(1/3) + (I*Sqrt[
d]*g^(1/3))/Sqrt[e])] + PolyLog[2, (I*g^(1/3)*(Sqrt[d] + I*Sqrt[e]*x))/(Sqr
t[e]*f^(1/3) + I*Sqrt[d]*g^(1/3))]))/(3*(1 + (-1)^(1/3))^2*f^(5/3)*g^(1/3))
 - (2*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[((-1)^(2/3)*f^(1/3) + g^(1/3)*x)
/((-1)^(2/3)*f^(1/3) + (I*Sqrt[d]*g^(1/3))/Sqrt[e])] + PolyLog[2, (g^(1/3)*
```

```
(Sqrt[d] + I*Sqrt[e]*x))/((-1)^(1/6)*Sqrt[e]*f^(1/3) + Sqrt[d]*g^(1/3))]))/
(3*(1 + (-1)^(1/3))^2*f^(5/3)*g^(1/3)) - (2*(-1 + (-1)^(1/3))*(Log[((-I)*Sq
rt[d])/Sqrt[e] + x]*Log[(-((-1)^(1/3)*f^(1/3)) + g^(1/3)*x)/(-((-1)^(1/3)*f
^(1/3)) + (I*Sqrt[d]*g^(1/3))/Sqrt[e])] + PolyLog[2, (g^(1/3)*(Sqrt[d] + I*
Sqrt[e]*x))/((-1)^(5/6)*Sqrt[e]*f^(1/3) + Sqrt[d]*g^(1/3))]))/(3*(1 + (-1)^
(1/3))^2*f^(5/3)*g^(1/3)) + (2*(-1)^(1/3)*(Log[(I*Sqrt[d])/Sqrt[e] + x]*Log
[(f^(1/3) + g^(1/3)*x)/(f^(1/3) - (I*Sqrt[d]*g^(1/3))/Sqrt[e])] + PolyLog[2
, -((g^(1/3)*(I*Sqrt[d] + Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - I*Sqrt[d]*g^(1/3))
)]))/(3*(1 + (-1)^(1/3))^2*f^(5/3)*g^(1/3)))
```

Maple [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(c(e\,x^2+d)^p\right)}{\left(g\,x^3+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/(g*x^3+f)^2,x)
[Out] int(ln(c*(e*x^2+d)^p)/(g*x^3+f)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^3+f)^2,x, algorithm="maxima")
[Out] integrate(log((x^2*e + d)^p*c)/(g*x^3 + f)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^3+f)^2,x, algorithm="fricas")
[Out] integral(log((x^2*e + d)^p*c)/(g^2*x^6 + 2*f*g*x^3 + f^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/(g*x**3+f)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^3+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((x^2*e + d)^p*c)/(g*x^3 + f)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{{\left(g\,x^3+f\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)/(f + g*x^3)^2,x)
```

```
[Out] int(log(c*(d + e*x^2)^p)/(f + g*x^3)^2, x)
```

3.293 $\int (f + gx^3)^3 \log^2 \left(c(d + ex^2)^p\right) \, dx$

Optimal. Leaf size=1221

$$8f^3p^2x - \frac{1408d^3fg^2p^2x}{245e^3} - \frac{3df^2gp^2x^2}{e} + \frac{d^4g^3p^2x^2}{e^4} + \frac{568d^2fg^2p^2x^3}{735e^2} - \frac{288dfg^2p^2x^5}{1225e} + \frac{24}{343}fg^2p^2x^7 + \frac{3f^2gp^2(d+ex^2}{8e^2}$$

```
[Out] -4*f^3*p*x*ln(c*(e*x^2+d)^p)+3/7*f*g^2*x^7*ln(c*(e*x^2+d)^p)^2+24/343*f*g^2
*p^2*x^7+1/125*g^3*p^2*(e*x^2+d)^5/e^5+f^3*x*ln(c*(e*x^2+d)^p)^2+1/10*g^3*x
^10*ln(c*(e*x^2+d)^p)^2+d^4*g^3*p^2*x^2/e^4+4*I*f^3*p^2*arctan(x*e^(1/2)/d^
(1/2))^2*d^(1/2)/e^(1/2)+4*I*f^3*p^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x*e^(
1/2)))*d^(1/2)/e^(1/2)-d^4*g^3*p*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e^5+d^3*g^3*p*
(e*x^2+d)^2*ln(c*(e*x^2+d)^p)/e^5+1408/245*d^(7/2)*f*g^2*p^2*arctan(x*e^(1/
2)/d^(1/2))/e^(7/2)-3/4*f^2*g*p*(e*x^2+d)^2*ln(c*(e*x^2+d)^p)/e^2-2/3*d^2*g
^3*p*(e*x^2+d)^3*ln(c*(e*x^2+d)^p)/e^5+1/4*d*g^3*p*(e*x^2+d)^4*ln(c*(e*x^2+
d)^p)/e^5+1/5*d^5*g^3*p*ln(e*x^2+d)*ln(c*(e*x^2+d)^p)/e^5-3/2*d*f^2*g*(e*x^
2+d)*ln(c*(e*x^2+d)^p)^2/e^2+4*f^3*p*arctan(x*e^(1/2)/d^(1/2))*ln(c*(e*x^2+
d)^p)*d^(1/2)/e^(1/2)+8*f^3*p^2*arctan(x*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(
1/2)+I*x*e^(1/2)))*d^(1/2)/e^(1/2)+3/8*f^2*g*p^2*(e*x^2+d)^2/e^2-1/2*d^3*g^
3*p^2*(e*x^2+d)^2/e^5+2/9*d^2*g^3*p^2*(e*x^2+d)^3/e^5-1/16*d*g^3*p^2*(e*x^2
+d)^4/e^5+12/7*d^3*f*g^2*p*x*ln(c*(e*x^2+d)^p)/e^3-4/7*d^2*f*g^2*p*x^3*ln(c
*(e*x^2+d)^p)/e^2+12/35*d*f*g^2*p*x^5*ln(c*(e*x^2+d)^p)/e+3*d*f^2*g*p*(e*x^
2+d)*ln(c*(e*x^2+d)^p)/e^2-12/7*d^(7/2)*f*g^2*p*arctan(x*e^(1/2)/d^(1/2))*l
n(c*(e*x^2+d)^p)/e^(7/2)-24/7*d^(7/2)*f*g^2*p^2*arctan(x*e^(1/2)/d^(1/2))*l
n(2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))/e^(7/2)-12/7*I*d^(7/2)*f*g^2*p^2*arctan(
x*e^(1/2)/d^(1/2))^2/e^(7/2)-12/7*I*d^(7/2)*f*g^2*p^2*polylog(2,1-2*d^(1/2)
/(d^(1/2)+I*x*e^(1/2)))/e^(7/2)-1408/245*d^3*f*g^2*p^2*x/e^3-3*d*f^2*g*p^2*
x^2/e+568/735*d^2*f*g^2*p^2*x^3/e^2-288/1225*d*f*g^2*p^2*x^5/e+8*f^3*p^2*x-
1/10*d^5*g^3*p^2*ln(e*x^2+d)^2/e^5-12/49*f*g^2*p*x^7*ln(c*(e*x^2+d)^p)-1/25
*g^3*p*(e*x^2+d)^5*ln(c*(e*x^2+d)^p)/e^5+3/4*f^2*g*(e*x^2+d)^2*ln(c*(e*x^2+
d)^p)^2/e^2-8*f^3*p^2*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)
```

Rubi [A]

time = 1.02, antiderivative size = 1221, normalized size of antiderivative = 1.00, number of steps used = 55, number of rules used = 29, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 1.208$, Rules used = {2521, 2500, 2526, 2498, 327, 211, 2520, 12, 5040, 4964, 2449, 2352, 2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341, 2507, 2505, 308, 2445, 2458, 45, 2372, 14, 2338}

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^3)^3*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] 8*f^3*p^2*x - (1408*d^3*f*g^2*p^2*x)/(245*e^3) - (3*d*f^2*g*p^2*x^2)/e + (d
^4*g^3*p^2*x^2)/e^4 + (568*d^2*f*g^2*p^2*x^3)/(735*e^2) - (288*d*f*g^2*p^2*
```

```
x^5)/(1225*e) + (24*f*g^2*p^2*x^7)/343 + (3*f^2*g*p^2*(d + e*x^2)^2)/(8*e^2
) - (d^3*g^3*p^2*(d + e*x^2)^2)/(2*e^5) + (2*d^2*g^3*p^2*(d + e*x^2)^3)/(9*
e^5) - (d*g^3*p^2*(d + e*x^2)^4)/(16*e^5) + (g^3*p^2*(d + e*x^2)^5)/(125*e^
5) - (8*Sqrt[d]*f^3*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + (1408*d^(7/2
)*f*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(245*e^(7/2)) + ((4*I)*Sqrt[d]*f^3
*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/Sqrt[e] - (((12*I)/7)*d^(7/2)*f*g^2*p^2
*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e^(7/2) + (8*Sqrt[d]*f^3*p^2*ArcTan[(Sqrt[e
]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] - (24*d^(7/
2)*f*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[
e]*x)])/(7*e^(7/2)) - (d^5*g^3*p^2*Log[d + e*x^2]^2)/(10*e^5) - 4*f^3*p*x*L
og[c*(d + e*x^2)^p] + (12*d^3*f*g^2*p*x*Log[c*(d + e*x^2)^p])/(7*e^3) - (4*
d^2*f*g^2*p*x^3*Log[c*(d + e*x^2)^p])/(7*e^2) + (12*d*f*g^2*p*x^5*Log[c*(d
+ e*x^2)^p])/(35*e) - (12*f*g^2*p*x^7*Log[c*(d + e*x^2)^p])/49 + (3*d*f^2*g
*p*(d + e*x^2)*Log[c*(d + e*x^2)^p])/e^2 - (d^4*g^3*p*(d + e*x^2)*Log[c*(d
+ e*x^2)^p])/e^5 - (3*f^2*g*p*(d + e*x^2)^2*Log[c*(d + e*x^2)^p])/(4*e^2) +
 (d^3*g^3*p*(d + e*x^2)^2*Log[c*(d + e*x^2)^p])/e^5 - (2*d^2*g^3*p*(d + e*x
^2)^3*Log[c*(d + e*x^2)^p])/(3*e^5) + (d*g^3*p*(d + e*x^2)^4*Log[c*(d + e*x
^2)^p])/(4*e^5) - (g^3*p*(d + e*x^2)^5*Log[c*(d + e*x^2)^p])/(25*e^5) + (4*
Sqrt[d]*f^3*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/Sqrt[e] - (
12*d^(7/2)*f*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/(7*e^(
7/2)) + (d^5*g^3*p*Log[d + e*x^2]*Log[c*(d + e*x^2)^p])/(5*e^5) + f^3*x*Log
[c*(d + e*x^2)^p]^2 + (3*f*g^2*x^7*Log[c*(d + e*x^2)^p]^2)/7 + (g^3*x^10*Lo
g[c*(d + e*x^2)^p]^2)/10 - (3*d*f^2*g*(d + e*x^2)*Log[c*(d + e*x^2)^p]^2)/(
2*e^2) + (3*f^2*g*(d + e*x^2)^2*Log[c*(d + e*x^2)^p]^2)/(4*e^2) + ((4*I)*Sq
rt[d]*f^3*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e]
- (((12*I)/7)*d^(7/2)*f*g^2*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqr
t[e]*x)])/e^(7/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
```

```
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2500

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[x^n*(
(a + b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
```

```
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(f+gx^3\right)^3 \log^2\left(c\left(d+ex^2\right)^p\right)\,dx = \int \left(f^3\log^2\left(c\left(d+ex^2\right)^p\right)+3f^2gx^3\log^2\left(c\left(d+ex^2\right)^p\right)+3fg^2x^6\log^2\left(\right.\right.$$

$$= f^3\int \log^2\left(c\left(d+ex^2\right)^p\right)\,dx+\left(3f^2g\right)\int x^3\log^2\left(c\left(d+ex^2\right)^p\right)\,dx+\left(3\right.$$

$$= f^3x\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{3}{7}fg^2x^7\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}\left(3f^2g\right)\text{Subst}$$

$$= f^3x\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{3}{7}fg^2x^7\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{10}g^3x^{10}\log^2\left(c\right.$$

$$= f^3x\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{3}{7}fg^2x^7\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{10}g^3x^{10}\log^2\left(c\right.$$

$$= -4f^3px\log\left(c\left(d+ex^2\right)^p\right)+\frac{12d^3fg^2px\log\left(c\left(d+ex^2\right)^p\right)}{7e^3}-\frac{4d^2fg^2px}{}$$

$$= 8f^3p^2x-\frac{24d^3fg^2p^2x}{7e^3}-4f^3px\log\left(c\left(d+ex^2\right)^p\right)+\frac{12d^3fg^2px\log\left(c\left(\right.\right.}{7e^3}$$

$$= 8f^3p^2x-\frac{1408d^3fg^2p^2x}{245e^3}-\frac{3df^2gp^2x^2}{e}+\frac{568d^2fg^2p^2x^3}{735e^2}-\frac{288dfg^2p^2x}{1225e}$$

$$= 8f^3p^2x-\frac{1408d^3fg^2p^2x}{245e^3}-\frac{3df^2gp^2x^2}{e}+\frac{d^4g^3p^2x^2}{e^4}+\frac{568d^2fg^2p^2x^3}{735e^2}-$$

$$= 8f^3p^2x-\frac{1408d^3fg^2p^2x}{245e^3}-\frac{3df^2gp^2x^2}{e}+\frac{d^4g^3p^2x^2}{e^4}+\frac{568d^2fg^2p^2x^3}{735e^2}-$$

$$= 8f^3p^2x-\frac{1408d^3fg^2p^2x}{245e^3}-\frac{3df^2gp^2x^2}{e}+\frac{d^4g^3p^2x^2}{e^4}+\frac{568d^2fg^2p^2x^3}{735e^2}-$$

Mathematica [A]
time = 0.71, size = 1020, normalized size = 0.84

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^3)^3*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] 8*f^3*p^2*x - (1408*d^3*f*g^2*p^2*x)/(245*e^3) - (9*d*f^2*g*p^2*x^2)/(4*e)
+ (137*d^4*g^3*p^2*x^2)/(300*e^4) + (568*d^2*f*g^2*p^2*x^3)/(735*e^2) + (3*
f^2*g*p^2*x^4)/8 - (77*d^3*g^3*p^2*x^4)/(600*e^3) - (288*d*f*g^2*p^2*x^5)/(
1225*e) + (47*d^2*g^3*p^2*x^6)/(900*e^2) + (24*f*g^2*p^2*x^7)/343 - (9*d*g^
3*p^2*x^8)/(400*e) + (g^3*p^2*x^10)/125 - (((4*I)/7)*Sqrt[d]*f*(-7*e^3*f^2
+ 3*d^3*g^2)*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e^(7/2) + (3*d^2*f^2*g*p^2*
Log[d + e*x^2])/(4*e^2) - (77*d^5*g^3*p^2*Log[d + e*x^2])/(300*e^5) + (3*d^
2*f^2*g*p*Log[c*(d + e*x^2)^p])/(2*e^2) - (d^5*g^3*p*Log[c*(d + e*x^2)^p])/
(5*e^5) - 4*f^3*p*x*Log[c*(d + e*x^2)^p] + (12*d^3*f*g^2*p*x*Log[c*(d + e*x
^2)^p])/(7*e^3) + (3*d*f^2*g*p*x^2*Log[c*(d + e*x^2)^p])/(2*e) - (d^4*g^3*p
*x^2*Log[c*(d + e*x^2)^p])/(5*e^4) - (4*d^2*f*g^2*p*x^3*Log[c*(d + e*x^2)^p
])/(7*e^2) - (3*f^2*g*p*x^4*Log[c*(d + e*x^2)^p])/4 + (d^3*g^3*p*x^4*Log[c*
(d + e*x^2)^p])/(10*e^3) + (12*d*f*g^2*p*x^5*Log[c*(d + e*x^2)^p])/(35*e) -
 (d^2*g^3*p*x^6*Log[c*(d + e*x^2)^p])/(15*e^2) - (12*f*g^2*p*x^7*Log[c*(d +
 e*x^2)^p])/49 + (d*g^3*p*x^8*Log[c*(d + e*x^2)^p])/(20*e) - (g^3*p*x^10*Lo
g[c*(d + e*x^2)^p])/25 - (3*d^2*f^2*g*Log[c*(d + e*x^2)^p]^2)/(4*e^2) + (d^
5*g^3*Log[c*(d + e*x^2)^p]^2)/(10*e^5) + f^3*x*Log[c*(d + e*x^2)^p]^2 + (3*
f^2*g*x^4*Log[c*(d + e*x^2)^p]^2)/4 + (3*f*g^2*x^7*Log[c*(d + e*x^2)^p]^2)/
7 + (g^3*x^10*Log[c*(d + e*x^2)^p]^2)/10 - (4*Sqrt[d]*f*p*ArcTan[(Sqrt[e]*x
)/Sqrt[d]]*(490*e^3*f^2*p - 352*d^3*g^2*p - 70*(7*e^3*f^2 - 3*d^3*g^2)*p*Lo
g[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)] - 35*(7*e^3*f^2 - 3*d^3*g^2)*Log[c*(
d + e*x^2)^p]))/(245*e^(7/2)) - (((4*I)/7)*Sqrt[d]*f*(-7*e^3*f^2 + 3*d^3*g^
2)*p^2*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)])/e^(7
/2)
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int \left(g\,x^{3}+f\right)^{3} \ln \left(c\left(e\,x^{2}+d\right)^{p}\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)^3*ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int((g*x^3+f)^3*ln(c*(e*x^2+d)^p)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^3*log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] 1/140*(14*g^3*p^2*x^10 + 60*f*g^2*p^2*x^7 + 105*f^2*g*p^2*x^4 + 140*f^3*p^2
*x)*log(x^2*e + d)^2 + integrate(1/35*(35*g^3*x^11*e*log(c)^2 + 35*d*g^3*x^
9*log(c)^2 + 105*f*g^2*x^8*e*log(c)^2 + 105*d*f*g^2*x^6*log(c)^2 + 105*f^2*
g*x^5*e*log(c)^2 + 105*d*f^2*g*x^3*log(c)^2 + 35*f^3*x^2*e*log(c)^2 + 35*d*
f^3*log(c)^2 + (70*d*g^3*p*x^9*log(c) - 14*(g^3*p^2 - 5*g^3*p*log(c))*x^11*
e + 210*d*f*g^2*p*x^6*log(c) - 30*(2*f*g^2*p^2 - 7*f*g^2*p*log(c))*x^8*e +
210*d*f^2*g*p*x^3*log(c) - 105*(f^2*g*p^2 - 2*f^2*g*p*log(c))*x^5*e + 70*d*
f^3*p*log(c) - 70*(2*f^3*p^2 - f^3*p*log(c))*x^2*e)*log(x^2*e + d))/(x^2*e
+ d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^3*log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral((g^3*x^9 + 3*f*g^2*x^6 + 3*f^2*g*x^3 + f^3)*log((x^2*e + d)^p*c)^2
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(f + gx^{3}\right)^{3} \log \left(c\left(d + ex^{2}\right)^{p}\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)**3*ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Integral((f + g*x**3)**3*log(c*(d + e*x**2)**p)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^3*log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f)^3*log((x^2*e + d)^p*c)^2, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \ln \left(c \left(e x^2+d\right)^p\right)^2 \left(g x^3+f\right)^3 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^2*(f + g*x^3)^3,x)
```

```
[Out] int(log(c*(d + e*x^2)^p)^2*(f + g*x^3)^3, x)
```

3.294 $\int (f+gx^3)^2 \log^2\left(c(d+ex^2)^p\right) dx$

Optimal. Leaf size=835

$$8f^2p^2x-\frac{1408d^3g^2p^2x}{735e^3}-\frac{2dfgp^2x^2}{e}+\frac{568d^2g^2p^2x^3}{2205e^2}-\frac{96dg^2p^2x^5}{1225e}+\frac{8}{343}g^2p^2x^7+\frac{fgp^2(d+ex^2)^2}{4e^2}-\frac{8\sqrt{d}\,f^2p^2\tan^{-1}}{\sqrt{e}}$$

[Out] f^2*x*ln(c*(e*x^2+d)^p)^2-4/49*g^2*p*x^7*ln(c*(e*x^2+d)^p)+1/7*g^2*x^7*ln(c*(e*x^2+d)^p)^2-d*f*g*(e*x^2+d)*ln(c*(e*x^2+d)^p)^2/e^2-1408/735*d^3*g^2*p^2*x/e^3+568/2205*d^2*g^2*p^2*x^3/e^2-96/1225*d*g^2*p^2*x^5/e+1/4*f*g*p^2*(e*x^2+d)^2/e^2+2*d*f*g*p*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e^2+4*f^2*p*arctan(x*e^(1/2)/d^(1/2))*ln(c*(e*x^2+d)^p)*d^(1/2)/e^(1/2)+8*f^2*p^2*arctan(x*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))*d^(1/2)/e^(1/2)+4*I*f^2*p^2*arctan(x*e^(1/2)/d^(1/2))^2*d^(1/2)/e^(1/2)+4*I*f^2*p^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))*d^(1/2)/e^(1/2)-2*d*f*g*p^2*x^2/e+8*f^2*p^2*x+4/7*d^3*g^2*p*x*ln(c*(e*x^2+d)^p)/e^3-4/21*d^2*g^2*p*x^3*ln(c*(e*x^2+d)^p)/e^2+4/35*d*g^2*p*x^5*ln(c*(e*x^2+d)^p)/e-1/2*f*g*p*(e*x^2+d)^2*ln(c*(e*x^2+d)^p)/e^2-4/7*d^(7/2)*g^2*p*arctan(x*e^(1/2)/d^(1/2))*ln(c*(e*x^2+d)^p)/e^(7/2)-8/7*d^(7/2)*g^2*p^2*arctan(x*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))/e^(7/2)-4/7*I*d^(7/2)*g^2*p^2*arctan(x*e^(1/2)/d^(1/2))^2/e^(7/2)-4/7*I*d^(7/2)*g^2*p^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))/e^(7/2)+8/343*g^2*p^2*x^7+1408/735*d^(7/2)*g^2*p^2*arctan(x*e^(1/2)/d^(1/2))/e^(7/2)+1/2*f*g*(e*x^2+d)^2*ln(c*(e*x^2+d)^p)^2/e^2-4*f^2*p*x*ln(c*(e*x^2+d)^p)-8*f^2*p^2*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)

Rubi [A]
time = 0.68, antiderivative size = 835, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 23, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.958$, Rules used = {2521, 2500, 2526, 2498, 327, 211, 2520, 12, 5040, 4964, 2449, 2352, 2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341, 2507, 2505, 308}

$$\frac{8}{343}g^2p^2x^7+\frac{1}{7}g^2\log^2\left(c\left(ex^2+d\right)^p\right)x^7-\frac{4}{49}g^2p\log\left(c\left(ex^2+d\right)^p\right)x^7-\frac{96dg^2p^2x^5}{1225e}+\frac{4dg^2p\log\left(c\left(ex^2+d\right)^p\right)x^5}{35e}+\frac{568d^2g^2p^2x^3}{2205e^2}-\frac{4d^2g^2p\log\left(c\left(ex^2+d\right)^p\right)x^3}{21e^2}-\frac{2dfgp^2x^2}{e}+8f^2p^2x-\frac{1408d^3g^2p^2x}{735e^3}+f^2\log^2\left(c\left(ex^2+d\right)^p\right)x-4f^2p\log\left(c\left(ex^2+d\right)^p\right)x+\frac{4d^3g^2p\log\left(c\left(ex^2+d\right)^p\right)x}{7e^3}+\frac{fgp^2\left(ex^2+d\right)^2}{4e^2}+\frac{4i\sqrt{d}f^2p^2\text{ArcTan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}}-\frac{4id^{7/2}g^2p^2\text{ArcTan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{7e^{7/2}}+\frac{fg\left(ex^2+d\right)^2\log^2\left(c\left(ex^2+d\right)^p\right)}{2e^2}-\frac{dfg\left(ex^2+d\right)\log^2\left(c\left(ex^2+d\right)^p\right)}{e^2}-\frac{8\sqrt{d}f^2p^2\text{ArcTan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{1408d^{7/2}g^2p^2\text{ArcTan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{735e^{7/2}}+\frac{8\sqrt{d}f^2p^2\text{ArcTan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{\sqrt{e}}-\frac{8d^{7/2}g^2p^2\text{ArcTan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{7e^{7/2}}-\frac{fgp\left(ex^2+d\right)^2\log\left(c\left(ex^2+d\right)^p\right)}{2e^2}+\frac{2dfgp\left(ex^2+d\right)\log\left(c\left(ex^2+d\right)^p\right)}{e^2}+\frac{4\sqrt{d}f^2p\text{ArcTan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(ex^2+d\right)^p\right)}{\sqrt{e}}-\frac{4d^{7/2}g^2p\text{ArcTan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(ex^2+d\right)^p\right)}{7e^{7/2}}+\frac{4i\sqrt{d}f^2p^2\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{\sqrt{e}}-\frac{4id^{7/2}g^2p^2\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{7e^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[(f + g*x^3)^2*Log[c*(d + e*x^2)^p]^2,x]

[Out] 8*f^2*p^2*x - (1408*d^3*g^2*p^2*x)/(735*e^3) - (2*d*f*g*p^2*x^2)/e + (568*d^2*g^2*p^2*x^3)/(2205*e^2) - (96*d*g^2*p^2*x^5)/(1225*e) + (8*g^2*p^2*x^7)/343 + (f*g*p^2*(d + e*x^2)^2)/(4*e^2) - (8*Sqrt[d]*f^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + (1408*d^(7/2)*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(735*e^(7/2)) + ((4*I)*Sqrt[d]*f^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/Sqrt[e] - (((4*I)/7)*d^(7/2)*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e^(7/2) + (8*Sqrt[d]*f^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] - (8*d^(7/2)*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*

```
Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/(7*e^(7/2)) - 4*f^2*p*x*Log[c*(d + e*x^2
)^p] + (4*d^3*g^2*p*x*Log[c*(d + e*x^2)^p])/(7*e^3) - (4*d^2*g^2*p*x^3*Log[
c*(d + e*x^2)^p])/(21*e^2) + (4*d*g^2*p*x^5*Log[c*(d + e*x^2)^p])/(35*e) -
(4*g^2*p*x^7*Log[c*(d + e*x^2)^p])/49 + (2*d*f*g*p*(d + e*x^2)*Log[c*(d + e
*x^2)^p])/e^2 - (f*g*p*(d + e*x^2)^2*Log[c*(d + e*x^2)^p])/(2*e^2) + (4*Sqr
t[d]*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/Sqrt[e] - (4*d
^(7/2)*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/(7*e^(7/2))
+ f^2*x*Log[c*(d + e*x^2)^p]^2 + (g^2*x^7*Log[c*(d + e*x^2)^p]^2)/7 - (d*f*
g*(d + e*x^2)*Log[c*(d + e*x^2)^p]^2)/e^2 + (f*g*(d + e*x^2)^2*Log[c*(d + e
*x^2)^p]^2)/(2*e^2) + ((4*I)*Sqrt[d]*f^2*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sq
rt[d] + I*Sqrt[e]*x)])/Sqrt[e] - (((4*I)/7)*d^(7/2)*g^2*p^2*PolyLog[2, 1 -
(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/e^(7/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
```

```
e, n, p}, x]
```

Rule 2500

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[x^n*(
(a + b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
```

```
0] && LtQ[r, 0]))
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(f+gx^3\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)\,dx = \int \left(f^2\log^2\left(c\left(d+ex^2\right)^p\right)+2fgx^3\log^2\left(c\left(d+ex^2\right)^p\right)+g^2x^6\log^2\left(c\right.\right.$$

$$= f^2\int \log^2\left(c\left(d+ex^2\right)^p\right)\,dx+(2fg)\int x^3\log^2\left(c\left(d+ex^2\right)^p\right)\,dx+g^2$$

$$= f^2x\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log^2\left(c\left(d+ex^2\right)^p\right)+(fg)\text{Subst}\left(\int\right.$$

$$= f^2x\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log^2\left(c\left(d+ex^2\right)^p\right)+(fg)\text{Subst}\left(\int\right.$$

$$= f^2x\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{(fg)\text{Subst}\left(\int(d\right.}{}$$

$$= -4f^2px\log\left(c\left(d+ex^2\right)^p\right)+\frac{4d^3g^2px\log\left(c\left(d+ex^2\right)^p\right)}{7e^3}-\frac{4d^2g^2px^3\,\text{lo}}{}$$

$$= 8f^2p^2x-\frac{8d^3g^2p^2x}{7e^3}-4f^2px\log\left(c\left(d+ex^2\right)^p\right)+\frac{4d^3g^2px\log\left(c(d+\right.}{7e^3}$$

$$= 8f^2p^2x-\frac{1408d^3g^2p^2x}{735e^3}-\frac{2dfgp^2x^2}{e}+\frac{568d^2g^2p^2x^3}{2205e^2}-\frac{96dg^2p^2x^5}{1225e}+$$

$$= 8f^2p^2x-\frac{1408d^3g^2p^2x}{735e^3}-\frac{2dfgp^2x^2}{e}+\frac{568d^2g^2p^2x^3}{2205e^2}-\frac{96dg^2p^2x^5}{1225e}+$$

$$= 8f^2p^2x-\frac{1408d^3g^2p^2x}{735e^3}-\frac{2dfgp^2x^2}{e}+\frac{568d^2g^2p^2x^3}{2205e^2}-\frac{96dg^2p^2x^5}{1225e}+$$

$$= 8f^2p^2x-\frac{1408d^3g^2p^2x}{735e^3}-\frac{2dfgp^2x^2}{e}+\frac{568d^2g^2p^2x^3}{2205e^2}-\frac{96dg^2p^2x^5}{1225e}+$$

Mathematica [A]

time = 0.42, size = 475, normalized size = 0.57

$$\frac{-176400i\sqrt{d}\left(-7e^3f^2+d^3g^2\right)p^2\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)^2-1680\sqrt{d}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(2(735e^3f^2-176d^3g^2)p-210(7e^3f^2-d^3g^2)p\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\,x}\right)-105(7e^3f^2-d^3g^2)\log\left(c(d+ex^2)^p\right)\right)+\sqrt{e}\left(p^2x(-591360d^3g^2+79520d^2eg^2x^2-378de^2gx(1225f+64gx^3)+225e^3(10976f^2+343fgx^3+32g^2x^6))+154350d^2efgp^2\log\left(d+ex^2\right)-210p(-840d^3g^2x+70d^2eg(-21f+4gx^3)-42de^2gx^2(35f+4gx^3)+15e^3x(392f^2+49fgx^3+8g^2x^6))\log\left(c(d+ex^2)^p\right)+22050(-7d^2efg+e^3x(14f^2+7fgx^3+2g^2x^6))\log^2\left(c(d+ex^2)^p\right)\right)-176400i\sqrt{d}\left(-7e^3f^2+d^3g^2\right)p^2\mathrm{Li}_2\left(\frac{i\sqrt{d}+\sqrt{e}\,x}{-i\sqrt{d}+\sqrt{e}\,x}\right)}{308700e^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^3)^2*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] ((-176400*I)*Sqrt[d]*(-7*e^3*f^2 + d^3*g^2)*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
^2 - 1680*Sqrt[d]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(2*(735*e^3*f^2 - 176*d^3*g
^2)*p - 210*(7*e^3*f^2 - d^3*g^2)*p*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)
] - 105*(7*e^3*f^2 - d^3*g^2)*Log[c*(d + e*x^2)^p]) + Sqrt[e]*(p^2*x*(-5913
60*d^3*g^2 + 79520*d^2*e*g^2*x^2 - 378*d*e^2*g*x*(1225*f + 64*g*x^3) + 225*
e^3*(10976*f^2 + 343*f*g*x^3 + 32*g^2*x^6)) + 154350*d^2*e*f*g*p^2*Log[d +
e*x^2] - 210*p*(-840*d^3*g^2*x + 70*d^2*e*g*(-21*f + 4*g*x^3) - 42*d*e^2*g*
x^2*(35*f + 4*g*x^3) + 15*e^3*x*(392*f^2 + 49*f*g*x^3 + 8*g^2*x^6))*Log[c*(
d + e*x^2)^p] + 22050*(-7*d^2*e*f*g + e^3*x*(14*f^2 + 7*f*g*x^3 + 2*g^2*x^6
))*Log[c*(d + e*x^2)^p]^2) - (176400*I)*Sqrt[d]*(-7*e^3*f^2 + d^3*g^2)*p^2*
PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)])/(308700*e^(
7/2))
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \left(g\,x^3+f\right)^2 \ln\left(c\left(e\,x^2+d\right)^p\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)^2*ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int((g*x^3+f)^2*ln(c*(e*x^2+d)^p)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] 1/14*(2*g^2*p^2*x^7 + 7*f*g*p^2*x^4 + 14*f^2*p^2*x)*log(x^2*e + d)^2 + inte
grate(1/7*(7*g^2*x^8*e*log(c)^2 + 7*d*g^2*x^6*log(c)^2 + 14*f*g*x^5*e*log(c
)^2 + 14*d*f*g*x^3*log(c)^2 + 7*f^2*x^2*e*log(c)^2 + 7*d*f^2*log(c)^2 + 2*(
7*d*g^2*p*x^6*log(c) - (2*g^2*p^2 - 7*g^2*p*log(c))*x^8*e + 14*d*f*g*p*x^3*
log(c) - 7*(f*g*p^2 - 2*f*g*p*log(c))*x^5*e + 7*d*f^2*p*log(c) - 7*(2*f^2*p
^2 - f^2*p*log(c))*x^2*e)*log(x^2*e + d))/(x^2*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
[Out] integral((g^2*x^6 + 2*f*g*x^3 + f^2)*log((x^2*e + d)^p*c)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(f+gx^3\right)^2 \log \left(c\left(d+ex^2\right)^p\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)**2*ln(c*(e*x**2+d)**p)**2,x)
[Out] Integral((f + g*x**3)**2*log(c*(d + e*x**2)**p)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
[Out] integrate((g*x^3 + f)^2*log((x^2*e + d)^p*c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln \left(c\left(e\,x^2+d\right)^p\right)^2 \left(g\,x^3+f\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^2*(f + g*x^3)^2,x)
[Out] int(log(c*(d + e*x^2)^p)^2*(f + g*x^3)^2, x)
```

3.295 $\int (f + gx^3) \log^2 \left(c(d + ex^2)^p\right) \, dx$

Optimal. Leaf size=395

$$8fp^2x - \frac{dgp^2x^2}{e} + \frac{gp^2(d+ex^2)^2}{8e^2} - \frac{8\sqrt{d}\, fp^2 \tan^{-1}\left(\frac{\sqrt{e}\, x}{\sqrt{d}}\right)}{\sqrt{e}} + \frac{4i\sqrt{d}\, fp^2 \tan^{-1}\left(\frac{\sqrt{e}\, x}{\sqrt{d}}\right)^2}{\sqrt{e}} + \frac{8\sqrt{d}\, fp^2 \tan^{-1}\left(\frac{\sqrt{e}}{\sqrt{}}\right.}{}$$

```
[Out] 8*f*p^2*x-d*g*p^2*x^2/e+1/8*g*p^2*(e*x^2+d)^2/e^2-4*f*p*x*ln(c*(e*x^2+d)^p)
+d*g*p*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e^2-1/4*g*p*(e*x^2+d)^2*ln(c*(e*x^2+d)^p
)/e^2+f*x*ln(c*(e*x^2+d)^p)^2-1/2*d*g*(e*x^2+d)*ln(c*(e*x^2+d)^p)^2/e^2+1/4
*g*(e*x^2+d)^2*ln(c*(e*x^2+d)^p)^2/e^2-8*f*p^2*arctan(x*e^(1/2)/d^(1/2))*d^
(1/2)/e^(1/2)+4*I*f*p^2*arctan(x*e^(1/2)/d^(1/2))^2*d^(1/2)/e^(1/2)+4*f*p*a
rctan(x*e^(1/2)/d^(1/2))*ln(c*(e*x^2+d)^p)*d^(1/2)/e^(1/2)+8*f*p^2*arctan(x
*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))*d^(1/2)/e^(1/2)+4*I*f
*p^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))*d^(1/2)/e^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 20, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.909$, Rules used = {2521, 2500, 2526, 2498, 327, 211, 2520, 12, 5040, 4964, 2449, 2352, 2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

$$\frac{4i\sqrt{d}\, fp^2 \text{PolyLog}\left(2, 1 - \frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\, x}\right)}{\sqrt{e}} + \frac{4\sqrt{d}\, fp \text{ArcTan}\left(\frac{\sqrt{e}\, x}{\sqrt{d}}\right) \log\left(c(d+ex^2)^p\right)}{\sqrt{e}} + \frac{4i\sqrt{d}\, fp^2 \text{ArcTan}\left(\frac{\sqrt{e}\, x}{\sqrt{d}}\right)^2}{\sqrt{e}} - \frac{8\sqrt{d}\, fp^2 \text{ArcTan}\left(\frac{\sqrt{e}\, x}{\sqrt{d}}\right)}{\sqrt{e}} + \frac{8\sqrt{d}\, fp^2 \text{ArcTan}\left(\frac{\sqrt{e}\, x}{\sqrt{d}}\right) \log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\, x}\right)}{\sqrt{e}} + \frac{g(d+ex^2)^2 \log^2\left(c(d+ex^2)^p\right)}{4e^2} - \frac{dg(d+ex^2) \log^2\left(c(d+ex^2)^p\right)}{2e^2} - \frac{gp(d+ex^2)^2 \log\left(c(d+ex^2)^p\right)}{4e^2} + \frac{dgp(d+ex^2) \log\left(c(d+ex^2)^p\right)}{e^2} + fx \log^2\left(c(d+ex^2)^p\right) - 4fpx \log\left(c(d+ex^2)^p\right) + \frac{gp^2(d+ex^2)^2}{8e^2} - \frac{dgp^2x^2}{e} + 8fp^2x$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^3)*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] 8*f*p^2*x - (d*g*p^2*x^2)/e + (g*p^2*(d + e*x^2)^2)/(8*e^2) - (8*Sqrt[d]*f*
p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + ((4*I)*Sqrt[d]*f*p^2*ArcTan[(Sqr
t[e]*x)/Sqrt[d]]^2)/Sqrt[e] + (8*Sqrt[d]*f*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*
Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] - 4*f*p*x*Log[c*(d + e*x^
2)^p] + (d*g*p*(d + e*x^2)*Log[c*(d + e*x^2)^p])/e^2 - (g*p*(d + e*x^2)^2*L
og[c*(d + e*x^2)^p])/(4*e^2) + (4*Sqrt[d]*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*L
og[c*(d + e*x^2)^p])/Sqrt[e] + f*x*Log[c*(d + e*x^2)^p]^2 - (d*g*(d + e*x^2
)*Log[c*(d + e*x^2)^p]^2)/(2*e^2) + (g*(d + e*x^2)^2*Log[c*(d + e*x^2)^p]^2
)/(4*e^2) + ((4*I)*Sqrt[d]*f*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sq
rt[e]*x)])/Sqrt[e]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2500

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[x^n*(
(a + b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
```

```
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \left(f+gx^3\right)\log^2\left(c\left(d+ex^2\right)^p\right)\,dx &= \int \left(f\log^2\left(c\left(d+ex^2\right)^p\right)+gx^3\log^2\left(c\left(d+ex^2\right)^p\right)\right)\,dx \\
&= f\int \log^2\left(c\left(d+ex^2\right)^p\right)\,dx+g\int x^3\log^2\left(c\left(d+ex^2\right)^p\right)\,dx \\
&= fx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}g\text{Subst}\left(\int x\log^2\left(c(d+ex)^p\right)\,dx,x,x^2\right)-(\\
&= fx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}g\text{Subst}\left(\int\left(-\frac{d\log^2\left(c(d+ex)^p\right)}{e}+\frac{(d+ex}{} \right.\right. \\
&= fx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{g\text{Subst}\left(\int(d+ex)\log^2\left(c(d+ex)^p\right)\,dx,x,x^2\right)}{2e} \\
&= -4fpx\log\left(c\left(d+ex^2\right)^p\right)+\frac{4\sqrt{d}\,fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{e}}+ \\
&= 8fp^2x-4fpx\log\left(c\left(d+ex^2\right)^p\right)+\frac{4\sqrt{d}\,fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\log\left(c(d+e\right.}{\sqrt{e}} \\
&= 8fp^2x-\frac{dgp^2x^2}{e}+\frac{gp^2\left(d+ex^2\right)^2}{8e^2}-\frac{8\sqrt{d}\,fp^2\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{4i\sqrt{d}}{} \\
&= 8fp^2x-\frac{dgp^2x^2}{e}+\frac{gp^2\left(d+ex^2\right)^2}{8e^2}-\frac{8\sqrt{d}\,fp^2\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{4i\sqrt{d}}{} \\
&= 8fp^2x-\frac{dgp^2x^2}{e}+\frac{gp^2\left(d+ex^2\right)^2}{8e^2}-\frac{8\sqrt{d}\,fp^2\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{4i\sqrt{d}}{} \\
&= 8fp^2x-\frac{dgp^2x^2}{e}+\frac{gp^2\left(d+ex^2\right)^2}{8e^2}-\frac{8\sqrt{d}\,fp^2\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{4i\sqrt{d}}{}
\end{aligned}
$$

Mathematica [A]

time = 0.12, size = 415, normalized size = 1.05

$$
8fp^2x-\frac{8\sqrt{d}\,fp^2\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{4i\sqrt{d}\,fp^2\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)^2}{\sqrt{e}}+\frac{8\sqrt{d}\,fp^2\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\log\left(\frac{2i\sqrt{d}}{i\sqrt{d}-\sqrt{e}\,x}\right)}{\sqrt{e}}-4fpx\log\left(c\left(d+ex^2\right)^p\right)+\frac{4\sqrt{d}\,fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{e}}+fx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}gx^4\log^2\left(c\left(d+ex^2\right)^p\right)-egp\left(\frac{1}{8}p\left(\frac{2dx^2}{e^2}-\frac{x^4}{e}-\frac{2d^2\log\left(d+ex^2\right)}{e^3}\right)+\frac{x^4\log\left(c\left(d+ex^2\right)^p\right)}{4e}+\frac{d^2\log^2\left(c\left(d+ex^2\right)^p\right)}{4e^3p}+\frac{d\left(px^2-\frac{\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{e}\right)}{2e^2}\right)+\frac{4i\sqrt{d}\,fp^2\text{Li}_2\left(-\frac{i\sqrt{d}+\sqrt{e}\,x}{i\sqrt{d}-\sqrt{e}\,x}\right)}{\sqrt{e}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^3)*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] 8*f*p^2*x - (8*Sqrt[d]*f*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + ((4*I)*
Sqrt[d]*f*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/Sqrt[e] + (8*Sqrt[d]*f*p^2*Arc
Tan[(Sqrt[e]*x)/Sqrt[d]]*Log[((2*I)*Sqrt[d])/(I*Sqrt[d] - Sqrt[e]*x)])/Sqrt
[e] - 4*f*p*x*Log[c*(d + e*x^2)^p] + (4*Sqrt[d]*f*p*ArcTan[(Sqrt[e]*x)/Sqrt
[d]]*Log[c*(d + e*x^2)^p])/Sqrt[e] + f*x*Log[c*(d + e*x^2)^p]^2 + (g*x^4*Lo
g[c*(d + e*x^2)^p]^2)/4 - e*g*p*((p*((2*d*x^2)/e^2 - x^4/e - (2*d^2*Log[d +
 e*x^2])/e^3))/8 + (x^4*Log[c*(d + e*x^2)^p])/(4*e) + (d^2*Log[c*(d + e*x^2
)^p]^2)/(4*e^3*p) + (d*(p*x^2 - ((d + e*x^2)*Log[c*(d + e*x^2)^p])/e))/(2*e
^2)) + ((4*I)*Sqrt[d]*f*p^2*PolyLog[2, -((I*Sqrt[d] + Sqrt[e]*x)/(I*Sqrt[d]
 - Sqrt[e]*x))])/Sqrt[e]
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \left(g\,x^{3}+f\right) \ln \left(c\left(e\,x^{2}+d\right)^{p}\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)*ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int((g*x^3+f)*ln(c*(e*x^2+d)^p)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(g*p^2*x^4 + 4*f*p^2*x)*log(x^2*e + d)^2 + integrate((g*x^5*e*log(c)^2
+ d*g*x^3*log(c)^2 + f*x^2*e*log(c)^2 + d*f*log(c)^2 - ((g*p^2 - 2*g*p*log(
c))*x^5*e - 2*d*g*p*x^3*log(c) + 2*(2*f*p^2 - f*p*log(c))*x^2*e - 2*d*f*p*l
og(c))*log(x^2*e + d))/(x^2*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral((g*x^3 + f)*log((x^2*e + d)^p*c)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (f+gx^3)\log \left(c(d+ex^2)^p\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)*ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Integral((f + g*x**3)*log(c*(d + e*x**2)**p)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f)*log((x^2*e + d)^p*c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln \left(c\left(e\,x^2+d\right)^p\right)^2\,\left(g\,x^3+f\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^2*(f + g*x^3),x)
```

```
[Out] int(log(c*(d + e*x^2)^p)^2*(f + g*x^3), x)
```

3.296 $\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^3} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^3},x\right)$$

```
[Out] Unintegrable(ln(c*(e*x^2+d)^p)^2/(g*x^3+f),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^3} dx$$

Verification is not applicable to the result.

```
[In] Int[Log[c*(d + e*x^2)^p]^2/(f + g*x^3),x]
[Out] Defer[Int][Log[c*(d + e*x^2)^p]^2/(f + g*x^3), x]
```

Rubi steps

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^3} dx = \int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^3} dx$$

Mathematica [A]
time = 10.55, size = 0, normalized size = 0.00

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^3} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^3),x]
[Out] Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^3), x]
```

Maple [A]
time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)^2}{g\,x^3+f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)^2/(g*x^3+f),x)
```

```
[Out] int(ln(c*(e*x^2+d)^p)^2/(g*x^3+f),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^3+f),x, algorithm="maxima")
```

```
[Out] integrate(log((x^2*e + d)^p*c)^2/(g*x^3 + f), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^3+f),x, algorithm="fricas")
```

```
[Out] integral(log((x^2*e + d)^p*c)^2/(g*x^3 + f), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)**2/(g*x**3+f),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^3+f),x, algorithm="giac")
```

```
[Out] integrate(log((x^2*e + d)^p*c)^2/(g*x^3 + f), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)^2}{g\,x^3+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^2/(f + g*x^3),x)
[Out] int(log(c*(d + e*x^2)^p)^2/(f + g*x^3), x)
```

3.297 $\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2},x\right)$$

[Out] Unintegrable(ln(c*(e*x^2+d)^p)^2/(g*x^3+f)^2,x)

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2} dx$$

Verification is not applicable to the result.

[In] Int[Log[c*(d + e*x^2)^p]^2/(f + g*x^3)^2,x]

[Out] Defer[Int][Log[c*(d + e*x^2)^p]^2/(f + g*x^3)^2, x]

Rubi steps

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2} dx = \int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2} dx$$

Mathematica [A]
time = 29.67, size = 0, normalized size = 0.00

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2} dx$$

Verification is not applicable to the result.

[In] Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^3)^2,x]

[Out] Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^3)^2, x]

Maple [A]
time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)^2}{\left(g\,x^3+f\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)^2/(g*x^3+f)^2,x)
```

```
[Out] int(ln(c*(e*x^2+d)^p)^2/(g*x^3+f)^2,x)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^3+f)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^3+f)^2,x, algorithm="fricas")
```

```
[Out] integral(log((x^2*e + d)^p*c)^2/(g^2*x^6 + 2*f*g*x^3 + f^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)**2/(g*x**3+f)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^3+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((x^2*e + d)^p*c)^2/(g*x^3 + f)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)^2}{\left(g\,x^3+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^2/(f + g*x^3)^2,x)
```

```
[Out] int(log(c*(d + e*x^2)^p)^2/(f + g*x^3)^2, x)
```

3.298 $\int (f+gx^3)^2 \log^3\left(c(d+ex^2)^p\right)\,dx$

Optimal. Leaf size=1126

$$-48f^2p^3x+\frac{351136d^3g^2p^3x}{25725e^3}+\frac{6dfgp^3x^2}{e}-\frac{55456d^2g^2p^3x^3}{77175e^2}+\frac{5232dg^2p^3x^5}{42875e}-\frac{48g^2p^3x^7}{2401}-\frac{3fgp^3(d+ex^2)^2}{8e^2}+\frac{48\sqrt{\cdot}}{}$$

```
[Out] 24*f^2*p^2*x*ln(c*(e*x^2+d)^p)+24/343*g^2*p^2*x^7*ln(c*(e*x^2+d)^p)-6*f^2*p
*x*ln(c*(e*x^2+d)^p)^2-6/49*g^2*p*x^7*ln(c*(e*x^2+d)^p)^2+6*d*f^2*p*Uninteg
rable(ln(c*(e*x^2+d)^p)^2/(e*x^2+d),x)-48/2401*g^2*p^3*x^7+1/7*g^2*x^7*ln(c
*(e*x^2+d)^p)^3+f^2*x*ln(c*(e*x^2+d)^p)^3+351136/25725*d^3*g^2*p^3*x/e^3-55
456/77175*d^2*g^2*p^3*x^3/e^2+5232/42875*d*g^2*p^3*x^5/e-3/8*f*g*p^3*(e*x^2
+d)^2/e^2-6*d*f*g*p^2*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e^2+3*d*f*g*p*(e*x^2+d)*l
n(c*(e*x^2+d)^p)^2/e^2+6*d*f*g*p^3*x^2/e-d*f*g*(e*x^2+d)*ln(c*(e*x^2+d)^p)^
3/e^2+1408/245*I*d^(7/2)*g^2*p^3*arctan(x*e^(1/2)/d^(1/2))^2/e^(7/2)+1408/2
45*I*d^(7/2)*g^2*p^3*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))/e^(7/2)-1
408/245*d^3*g^2*p^2*x*ln(c*(e*x^2+d)^p)/e^3+568/735*d^2*g^2*p^2*x^3*ln(c*(e
*x^2+d)^p)/e^2-288/1225*d*g^2*p^2*x^5*ln(c*(e*x^2+d)^p)/e+3/4*f*g*p^2*(e*x^
2+d)^2*ln(c*(e*x^2+d)^p)/e^2+1408/245*d^(7/2)*g^2*p^2*arctan(x*e^(1/2)/d^(1
/2))*ln(c*(e*x^2+d)^p)/e^(7/2)+6/7*d^3*g^2*p*x*ln(c*(e*x^2+d)^p)^2/e^3-2/7*
d^2*g^2*p*x^3*ln(c*(e*x^2+d)^p)^2/e^2+6/35*d*g^2*p*x^5*ln(c*(e*x^2+d)^p)^2/
e-3/4*f*g*p*(e*x^2+d)^2*ln(c*(e*x^2+d)^p)^2/e^2+2816/245*d^(7/2)*g^2*p^3*ar
ctan(x*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))/e^(7/2)-24*f^2*
p^2*arctan(x*e^(1/2)/d^(1/2))*ln(c*(e*x^2+d)^p)*d^(1/2)/e^(1/2)-48*f^2*p^3*
arctan(x*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))*d^(1/2)/e^(1/
2)-24*I*f^2*p^3*arctan(x*e^(1/2)/d^(1/2))^2*d^(1/2)/e^(1/2)-24*I*f^2*p^3*po
lylog(2,1-2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))*d^(1/2)/e^(1/2)-48*f^2*p^3*x-351
136/25725*d^(7/2)*g^2*p^3*arctan(x*e^(1/2)/d^(1/2))/e^(7/2)+1/2*f*g*(e*x^2+
d)^2*ln(c*(e*x^2+d)^p)^3/e^2+48*f^2*p^3*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/e
^(1/2)-6/7*d^4*g^2*p*Unintegrable(ln(c*(e*x^2+d)^p)^2/(e*x^2+d),x)/e^3
```

Rubi [A]
time = 1.69, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (f+gx^3)^2 \log^3\left(c(d+ex^2)^p\right)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(f + g*x^3)^2*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] -48*f^2*p^3*x + (351136*d^3*g^2*p^3*x)/(25725*e^3) + (6*d*f*g*p^3*x^2)/e -
(55456*d^2*g^2*p^3*x^3)/(77175*e^2) + (5232*d*g^2*p^3*x^5)/(42875*e) - (48*
```

g^2*p^3*x^7)/2401 - (3*f*g*p^3*(d + e*x^2)^2)/(8*e^2) + (48*Sqrt[d]*f^2*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (351136*d^(7/2)*g^2*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(25725*e^(7/2)) - ((24*I)*Sqrt[d]*f^2*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/Sqrt[e] + (((1408*I)/245)*d^(7/2)*g^2*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e^(7/2) - (48*Sqrt[d]*f^2*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] + (2816*d^(7/2)*g^2*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/(245*e^(7/2)) + 24*f^2*p^2*x*Log[c*(d + e*x^2)^p] - (1408*d^3*g^2*p^2*x*Log[c*(d + e*x^2)^p])/(245*e^3) + (568*d^2*g^2*p^2*x^3*Log[c*(d + e*x^2)^p])/(735*e^2) - (288*d*g^2*p^2*x^5*Log[c*(d + e*x^2)^p])/(1225*e) + (24*g^2*p^2*x^7*Log[c*(d + e*x^2)^p])/343 - (6*d*f*g*p^2*(d + e*x^2)*Log[c*(d + e*x^2)^p])/e^2 + (3*f*g*p^2*(d + e*x^2)^2*Log[c*(d + e*x^2)^p])/(4*e^2) - (24*Sqrt[d]*f^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/Sqrt[e] + (1408*d^(7/2)*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/(245*e^(7/2)) - 6*f^2*p*x*Log[c*(d + e*x^2)^p]^2 + (6*d^3*g^2*p*x*Log[c*(d + e*x^2)^p]^2)/(7*e^3) - (2*d^2*g^2*p*x^3*Log[c*(d + e*x^2)^p]^2)/(7*e^2) + (6*d*g^2*p*x^5*Log[c*(d + e*x^2)^p]^2)/(35*e) - (6*g^2*p*x^7*Log[c*(d + e*x^2)^p]^2)/49 + (3*d*f*g*p*(d + e*x^2)*Log[c*(d + e*x^2)^p]^2)/e^2 - (3*f*g*p*(d + e*x^2)^2*Log[c*(d + e*x^2)^p]^2)/(4*e^2) + f^2*x*Log[c*(d + e*x^2)^p]^3 + (g^2*x^7*Log[c*(d + e*x^2)^p]^3)/7 - (d*f*g*(d + e*x^2)*Log[c*(d + e*x^2)^p]^3)/e^2 + (f*g*(d + e*x^2)^2*Log[c*(d + e*x^2)^p]^3)/(2*e^2) - ((24*I)*Sqrt[d]*f^2*p^3*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] + (((1408*I)/245)*d^(7/2)*g^2*p^3*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/e^(7/2) + 6*d*f^2*p*Defer[Int][Log[c*(d + e*x^2)^p]^2/(d + e*x^2), x] - (6*d^4*g^2*p*Defer[Int][Log[c*(d + e*x^2)^p]^2/(d + e*x^2), x])/(7*e^3)

Rubi steps

$$\int \left(f+gx^3\right)^2 \log^3\left(c\left(d+ex^2\right)^p\right)\,dx = \int \left(f^2\log^3\left(c\left(d+ex^2\right)^p\right)+2fgx^3\log^3\left(c\left(d+ex^2\right)^p\right)+g^2x^6\log^3\left(c\left(\right.\right.\right.$$

$$= f^2\int \log^3\left(c\left(d+ex^2\right)^p\right)\,dx+(2fg)\int x^3\log^3\left(c\left(d+ex^2\right)^p\right)\,dx+g^2$$

$$= f^2x\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log^3\left(c\left(d+ex^2\right)^p\right)+(fg)\text{Subst}\left(\int\right.$$

$$= f^2x\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log^3\left(c\left(d+ex^2\right)^p\right)+(fg)\text{Subst}\left(\int\right.$$

$$= f^2x\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{(fg)\text{Subst}\left(\int(d\right.}{}$$

$$= -6f^2px\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{6d^3g^2px\log^2\left(c\left(d+ex^2\right)^p\right)}{7e^3}-\frac{2d^2g^2px^3}{}$$

$$= -6f^2px\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{6d^3g^2px\log^2\left(c\left(d+ex^2\right)^p\right)}{7e^3}-\frac{2d^2g^2px^3}{}$$

$$= -6f^2px\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{6d^3g^2px\log^2\left(c\left(d+ex^2\right)^p\right)}{7e^3}-\frac{2d^2g^2px^3}{}$$

$$= \frac{6dfgp^3x^2}{e}-\frac{3fgp^3\left(d+ex^2\right)^2}{8e^2}+24f^2p^2x\log\left(c\left(d+ex^2\right)^p\right)-\frac{1408d^3}{}$$

$$= -48f^2p^3x+\frac{2816d^3g^2p^3x}{245e^3}+\frac{6dfgp^3x^2}{e}-\frac{3fgp^3\left(d+ex^2\right)^2}{8e^2}+24f^2p^2$$

$$= -48f^2p^3x+\frac{351136d^3g^2p^3x}{25725e^3}+\frac{6dfgp^3x^2}{e}-\frac{55456d^2g^2p^3x^3}{77175e^2}+\frac{5232d}{428}$$

$$= -48f^2p^3x+\frac{351136d^3g^2p^3x}{25725e^3}+\frac{6dfgp^3x^2}{e}-\frac{55456d^2g^2p^3x^3}{77175e^2}+\frac{5232d}{428}$$

$$= -48f^2p^3x+\frac{351136d^3g^2p^3x}{25725e^3}+\frac{6dfgp^3x^2}{e}-\frac{55456d^2g^2p^3x^3}{77175e^2}+\frac{5232d}{428}$$

$$= -48f^2p^3x+\frac{351136d^3g^2p^3x}{25725e^3}+\frac{6dfgp^3x^2}{e}-\frac{55456d^2g^2p^3x^3}{77175e^2}+\frac{5232d}{428}$$

Mathematica [A] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2539 vs. 2(1126) = 2252.
time = 6.35, size = 2539, normalized size = 2.25

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^3)^2*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] (f*g*p^3*(d + e*x^2)*(45*d - 3*e*x^2 + (-42*d + 6*e*x^2)*Log[d + e*x^2] + 6
*(3*d - e*x^2)*Log[d + e*x^2]^2 - 4*(d - e*x^2)*Log[d + e*x^2]^3))/(8*e^2)
+ (g^2*p^3*x*(-280*d^3*HypergeometricPFQ[{-3/2, 1, 1, 1}, {2, 2, 2}, 1 + (e
*x^2)/d] - 280*d^2*e*x^2*HypergeometricPFQ[{-3/2, 1, 1, 1}, {2, 2, 2}, 1 +
(e*x^2)/d] - 112*d^3*HypergeometricPFQ[{-5/2, 1, 1, 1, 1}, {2, 2, 2, 2}, 1
+ (e*x^2)/d] - 112*d^2*e*x^2*HypergeometricPFQ[{-5/2, 1, 1, 1, 1}, {2, 2, 2
, 2}, 1 + (e*x^2)/d] + 280*d^3*HypergeometricPFQ[{-3/2, 1, 1, 1, 1}, {2, 2,
 2, 2}, 1 + (e*x^2)/d] + 280*d^2*e*x^2*HypergeometricPFQ[{-3/2, 1, 1, 1, 1}
, {2, 2, 2, 2}, 1 + (e*x^2)/d] - 210*d^3*HypergeometricPFQ[{-1/2, 1, 1, 1,
1}, {2, 2, 2, 2}, 1 + (e*x^2)/d] - 210*d^2*e*x^2*HypergeometricPFQ[{-1/2, 1
, 1, 1, 1}, {2, 2, 2, 2}, 1 + (e*x^2)/d] + 16*d^3*Log[d + e*x^2] + 16*e^3*x
^6*Sqrt[-((e*x^2)/d)]*Log[d + e*x^2] + 280*d^3*HypergeometricPFQ[{-3/2, 1,
1}, {2, 2}, 1 + (e*x^2)/d]*Log[d + e*x^2] + 280*d^2*e*x^2*HypergeometricPFQ
[{-3/2, 1, 1}, {2, 2}, 1 + (e*x^2)/d]*Log[d + e*x^2] - 280*d^3*Hypergeometr
icPFQ[{-3/2, 1, 1, 1}, {2, 2, 2}, 1 + (e*x^2)/d]*Log[d + e*x^2] - 280*d^2*e
*x^2*HypergeometricPFQ[{-3/2, 1, 1, 1}, {2, 2, 2}, 1 + (e*x^2)/d]*Log[d + e
*x^2] + 210*d^3*HypergeometricPFQ[{-1/2, 1, 1, 1}, {2, 2, 2}, 1 + (e*x^2)/d
]*Log[d + e*x^2] + 210*d^2*e*x^2*HypergeometricPFQ[{-1/2, 1, 1, 1}, {2, 2,
2}, 1 + (e*x^2)/d]*Log[d + e*x^2] - 32*d^3*Log[d + e*x^2]^2 + 28*d*e^2*x^4*
Sqrt[-((e*x^2)/d)]*Log[d + e*x^2]^2 - 4*e^3*x^6*Sqrt[-((e*x^2)/d)]*Log[d +
e*x^2]^2 + 140*d^3*HypergeometricPFQ[{-3/2, 1, 1}, {2, 2}, 1 + (e*x^2)/d]*L
og[d + e*x^2]^2 + 140*d^2*e*x^2*HypergeometricPFQ[{-3/2, 1, 1}, {2, 2}, 1 +
 (e*x^2)/d]*Log[d + e*x^2]^2 - 105*d^3*HypergeometricPFQ[{-1/2, 1, 1}, {2,
2}, 1 + (e*x^2)/d]*Log[d + e*x^2]^2 - 105*d^2*e*x^2*HypergeometricPFQ[{-1/2
, 1, 1}, {2, 2}, 1 + (e*x^2)/d]*Log[d + e*x^2]^2 + 10*d^3*Log[d + e*x^2]^3
+ 10*e^3*x^6*Sqrt[-((e*x^2)/d)]*Log[d + e*x^2]^3 + 56*d^2*(d + e*x^2)*Hyper
geometricPFQ[{-5/2, 1, 1, 1}, {2, 2, 2}, 1 + (e*x^2)/d]*(3 + 2*Log[d + e*x^
2]) - 56*d^2*(d + e*x^2)*HypergeometricPFQ[{-5/2, 1, 1}, {2, 2}, 1 + (e*x^2
)/d]*(1 + 3*Log[d + e*x^2] + Log[d + e*x^2]^2)))/(70*e^3*Sqrt[-((e*x^2)/d)]
) - (3*f*g*p^2*(e*x^2*(-6*d + e*x^2) + (6*d^2 + 4*d*e*x^2 - 2*e^2*x^4)*Log[
d + e*x^2] - 2*(d^2 - e^2*x^4)*Log[d + e*x^2]^2)*(p*Log[d + e*x^2] - Log[c*
(d + e*x^2)^p]))/(4*e^2) + (3*d*f*g*p*x^2*(-(p*Log[d + e*x^2]) + Log[c*(d +
 e*x^2)^p])^2)/(2*e) - (2*d^2*g^2*p*x^3*(-(p*Log[d + e*x^2]) + Log[c*(d + e
*x^2)^p])^2)/(7*e^2) + (6*d*g^2*p*x^5*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x
^2)^p])^2)/(35*e) - (6*Sqrt[d]*(-7*e^3*f^2 + d^3*g^2)*p*ArcTan[(Sqrt[e]*x)/
Sqrt[d]]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/(7*e^(7/2)) - (3*d
```

```
^2*f*g*p*Log[d + e*x^2]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/(2*
e^2) + (3*p*x*(14*f^2 + 7*f*g*x^3 + 2*g^2*x^6)*Log[d + e*x^2]*(-(p*Log[d +
e*x^2]) + Log[c*(d + e*x^2)^p])^2)/14 - (g^2*x^7*(6*p + 7*p*Log[d + e*x^2]
- 7*Log[c*(d + e*x^2)^p])*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/4
9 - (f*g*x^4*(3*p + 2*p*Log[d + e*x^2] - 2*Log[c*(d + e*x^2)^p])*(-(p*Log[d
 + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/4 + (x*(-(p*Log[d + e*x^2]) + Log[c*(
d + e*x^2)^p])^2*(-42*e^3*f^2*p + 6*d^3*g^2*p + 7*e^3*f^2*(-(p*Log[d + e*x^
2]) + Log[c*(d + e*x^2)^p])))/(7*e^3) - (3*f^2*p^2*(p*Log[d + e*x^2] - Log[
c*(d + e*x^2)^p])*((4*I)*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2 + 4*Sqrt[d]*
ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-2 + 2*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)
] + Log[d + e*x^2]) + Sqrt[e]*x*(8 - 4*Log[d + e*x^2] + Log[d + e*x^2]^2) +
 (4*I)*Sqrt[d]*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x
)]))/Sqrt[e] + 3*g^2*p^2*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])*((x^7
*Log[d + e*x^2]^2)/7 - (4*((11025*I)*d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2
+ 105*d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-352 + 210*Log[(2*Sqrt[d])/(Sqrt
[d] + I*Sqrt[e]*x)] + 105*Log[d + e*x^2]) + Sqrt[e]*x*(36960*d^3 - 4970*d^2
*e*x^2 + 1512*d*e^2*x^4 - 450*e^3*x^6 - 105*(105*d^3 - 35*d^2*e*x^2 + 21*d*
e^2*x^4 - 15*e^3*x^6)*Log[d + e*x^2]) + (11025*I)*d^(7/2)*PolyLog[2, (I*Sqr
t[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)]))/(77175*e^(7/2))) + (f^2*p^3
*(-48*Sqrt[-d^2]*Sqrt[(e*x^2)/(d + e*x^2)]*Sqrt[d + e*x^2]*ArcSin[Sqrt[d]/S
qrt[d + e*x^2]] + Sqrt[-d]*e*x^2*(-48 + 24*Log[d + e*x^2] - 6*Log[d + e*x^2
]^2 + Log[d + e*x^2]^3) - 6*Sqrt[-d^2]*Sqrt[(e*x^2)/(d + e*x^2)]*(8*Sqrt[d]
*HypergeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}, d/(d + e*x^2)] +
Log[d + e*x^2]*(4*Sqrt[d]*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, d/
(d + e*x^2)] + Sqrt[d + e*x^2]*ArcSin[Sqrt[d]/Sqrt[d + e*x^2]]*Log[d + e*x^
2])) + 24*d*Sqrt[e*x^2]*ArcTanh[Sqrt[e*x^2]/Sqrt[-d]]*(Log[d + e*x^2] - Log
[1 + (e*x^2)/d]) + 6*(-d)^(3/2)*Sqrt[-((e*x^2)/d)]*(Log[1 + (e*x^2)/d]^2 -
4*Log[1 + (e*x^2)/d]*Log[(1 + Sqrt[-((e*x^2)/d)])/2] + 2*Log[(1 + Sqrt[-((e
*x^2)/d)])/2]^2 - 4*PolyLog[2, 1/2 - Sqrt[-((e*x^2)/d)]/2])))/(Sqrt[-d]*e*x
)
```

Maple [A]
time = 0.26, size = 0, normalized size = 0.00

$$\int \left(g\,x^{3}+f\right)^{2} \ln \left(c \left(e\,x^{2}+d\right)^{p}\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)^2*ln(c*(e*x^2+d)^p)^3,x)
[Out] int((g*x^3+f)^2*ln(c*(e*x^2+d)^p)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p)^3,x, algorithm="maxima")
```

```
[Out] 1/14*(2*g^2*p^3*x^7 + 7*f*g*p^3*x^4 + 14*f^2*p^3*x)*log(x^2*e + d)^3 + inte
grate(1/7*(7*g^2*x^8*e*log(c)^3 + 7*d*g^2*x^6*log(c)^3 + 14*f*g*x^5*e*log(c
)^3 + 14*d*f*g*x^3*log(c)^3 + 7*f^2*x^2*e*log(c)^3 + 7*d*f^2*log(c)^3 + 3*(
7*d*g^2*p^2*x^6*log(c) - (2*g^2*p^3 - 7*g^2*p^2*log(c))*x^8*e + 14*d*f*g*p^
2*x^3*log(c) - 7*(f*g*p^3 - 2*f*g*p^2*log(c))*x^5*e + 7*d*f^2*p^2*log(c) -
7*(2*f^2*p^3 - f^2*p^2*log(c))*x^2*e)*log(x^2*e + d)^2 + 21*(g^2*p*x^8*e*lo
g(c)^2 + d*g^2*p*x^6*log(c)^2 + 2*f*g*p*x^5*e*log(c)^2 + 2*d*f*g*p*x^3*log(
c)^2 + f^2*p*x^2*e*log(c)^2 + d*f^2*p*log(c)^2)*log(x^2*e + d))/(x^2*e + d)
, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p)^3,x, algorithm="fricas")
```

```
[Out] integral((g^2*x^6 + 2*f*g*x^3 + f^2)*log((x^2*e + d)^p*c)^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(f + gx^{3}\right)^{2} \log \left(c\left(d + ex^{2}\right)^{p}\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)**2*ln(c*(e*x**2+d)**p)**3,x)
```

```
[Out] Integral((f + g*x**3)**2*log(c*(d + e*x**2)**p)**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p)^3,x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f)^2*log((x^2*e + d)^p*c)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(c\,{\left(e\,x^2+d\right)}^p\right)^3\,{\left(g\,x^3+f\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^3*(f + g*x^3)^2,x)
[Out] int(log(c*(d + e*x^2)^p)^3*(f + g*x^3)^2, x)
```

3.299 $\int (f+gx^3)\log^3\left(c(d+ex^2)^p\right)\,dx$

Optimal. Leaf size=518

$$-48fp^3x+\frac{3dgp^3x^2}{e}-\frac{3gp^3(d+ex^2)^2}{16e^2}+\frac{48\sqrt{d}\,fp^3\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{24i\sqrt{d}\,fp^3\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)^2}{\sqrt{e}}-\frac{48\sqrt{d}\,fp^3\,\mathrm{ta}}{}$$

```
[Out] -48*f*p^3*x+3*d*g*p^3*x^2/e-3/16*g*p^3*(e*x^2+d)^2/e^2+24*f*p^2*x*ln(c*(e*x
^2+d)^p)-3*d*g*p^2*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e^2+3/8*g*p^2*(e*x^2+d)^2*ln
(c*(e*x^2+d)^p)/e^2-6*f*p*x*ln(c*(e*x^2+d)^p)^2+3/2*d*g*p*(e*x^2+d)*ln(c*(e
*x^2+d)^p)^2/e^2-3/8*g*p*(e*x^2+d)^2*ln(c*(e*x^2+d)^p)^2/e^2+f*x*ln(c*(e*x^
2+d)^p)^3-1/2*d*g*(e*x^2+d)*ln(c*(e*x^2+d)^p)^3/e^2+1/4*g*(e*x^2+d)^2*ln(c*
(e*x^2+d)^p)^3/e^2+48*f*p^3*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)-24*I*
f*p^3*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))*d^(1/2)/e^(1/2)-24*f*p^2
*arctan(x*e^(1/2)/d^(1/2))*ln(c*(e*x^2+d)^p)*d^(1/2)/e^(1/2)-48*f*p^3*arcta
n(x*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))*d^(1/2)/e^(1/2)-24
*I*f*p^3*arctan(x*e^(1/2)/d^(1/2))^2*d^(1/2)/e^(1/2)+6*d*f*p*Unintegrable(l
n(c*(e*x^2+d)^p)^2/(e*x^2+d),x)
```

Rubi [A]
time = 0.49, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (f+gx^3)\log^3\left(c(d+ex^2)^p\right)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(f + g*x^3)*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] -48*f*p^3*x + (3*d*g*p^3*x^2)/e - (3*g*p^3*(d + e*x^2)^2)/(16*e^2) + (48*Sq
rt[d]*f*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - ((24*I)*Sqrt[d]*f*p^3*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]]^2)/Sqrt[e] - (48*Sqrt[d]*f*p^3*ArcTan[(Sqrt[e]*x)
/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] + 24*f*p^2*x*Lo
g[c*(d + e*x^2)^p] - (3*d*g*p^2*(d + e*x^2)*Log[c*(d + e*x^2)^p])/e^2 + (3*
g*p^2*(d + e*x^2)^2*Log[c*(d + e*x^2)^p])/(8*e^2) - (24*Sqrt[d]*f*p^2*ArcTa
n[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/Sqrt[e] - 6*f*p*x*Log[c*(d + e
*x^2)^p]^2 + (3*d*g*p*(d + e*x^2)*Log[c*(d + e*x^2)^p]^2)/(2*e^2) - (3*g*p*
(d + e*x^2)^2*Log[c*(d + e*x^2)^p]^2)/(8*e^2) + f*x*Log[c*(d + e*x^2)^p]^3
- (d*g*(d + e*x^2)*Log[c*(d + e*x^2)^p]^3)/(2*e^2) + (g*(d + e*x^2)^2*Log[c
*(d + e*x^2)^p]^3)/(4*e^2) - ((24*I)*Sqrt[d]*f*p^3*PolyLog[2, 1 - (2*Sqrt[d
])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] + 6*d*f*p*Defer[Int][Log[c*(d + e*x^2)
^p]^2/(d + e*x^2), x]
```

Rubi steps

$$\int \left(f+gx^3\right)\log^3\left(c\left(d+ex^2\right)^p\right)\,dx=\int \left(f\log^3\left(c\left(d+ex^2\right)^p\right)+gx^3\log^3\left(c\left(d+ex^2\right)^p\right)\right)\,dx$$

$$=f\int \log^3\left(c\left(d+ex^2\right)^p\right)\,dx+g\int x^3\log^3\left(c\left(d+ex^2\right)^p\right)\,dx$$

$$=fx\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}g\text{Subst}\left(\int x\log^3\left(c(d+ex)^p\right)\,dx,x,x^2\right)-$$

$$=fx\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}g\text{Subst}\left(\int \left(-\frac{d\log^3\left(c(d+ex)^p\right)}{e}+\frac{(d+e}{}\right.\right.$$

$$=fx\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{g\text{Subst}\left(\int (d+ex)\log^3\left(c(d+ex)^p\right)\,dx,x,x^2\right)}{2e}$$

$$=-6fpx\log^2\left(c\left(d+ex^2\right)^p\right)+fx\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{g\text{Subst}\left(\int x\log^3\right.}{}$$

$$=-6fpx\log^2\left(c\left(d+ex^2\right)^p\right)+fx\log^3\left(c\left(d+ex^2\right)^p\right)-\frac{dg\left(d+ex^2\right)\log^3}{2e}$$

$$=-6fpx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{3dgp\left(d+ex^2\right)\log^2\left(c\left(d+ex^2\right)^p\right)}{2e^2}-\frac{3gp(}{}$$

$$=\frac{3dgp^3x^2}{e}-\frac{3gp^3\left(d+ex^2\right)^2}{16e^2}+24fp^2x\log\left(c\left(d+ex^2\right)^p\right)-\frac{3dgp^2(d+e}{}$$

$$=-48fp^3x+\frac{3dgp^3x^2}{e}-\frac{3gp^3\left(d+ex^2\right)^2}{16e^2}+24fp^2x\log\left(c\left(d+ex^2\right)^p\right)-$$

$$=-48fp^3x+\frac{3dgp^3x^2}{e}-\frac{3gp^3\left(d+ex^2\right)^2}{16e^2}+\frac{48\sqrt{d}\,fp^3\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-$$

$$=-48fp^3x+\frac{3dgp^3x^2}{e}-\frac{3gp^3\left(d+ex^2\right)^2}{16e^2}+\frac{48\sqrt{d}\,fp^3\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-$$

$$=-48fp^3x+\frac{3dgp^3x^2}{e}-\frac{3gp^3\left(d+ex^2\right)^2}{16e^2}+\frac{48\sqrt{d}\,fp^3\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-$$

$$=-48fp^3x+\frac{3dgp^3x^2}{e}-\frac{3gp^3\left(d+ex^2\right)^2}{16e^2}+\frac{48\sqrt{d}\,fp^3\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-$$

Mathematica [A] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1146 vs. $2(518) = 1036$.
time = 3.02, size = 1146, normalized size = 2.21

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^3)*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] (g*p^3*(d + e*x^2)*(45*d - 3*e*x^2 + (-42*d + 6*e*x^2)*Log[d + e*x^2] + 6*(
3*d - e*x^2)*Log[d + e*x^2]^2 - 4*(d - e*x^2)*Log[d + e*x^2]^3))/(16*e^2) -
 (3*g*p^2*(e*x^2*(-6*d + e*x^2) + (6*d^2 + 4*d*e*x^2 - 2*e^2*x^4)*Log[d + e
*x^2] - 2*(d^2 - e^2*x^4)*Log[d + e*x^2]^2)*(p*Log[d + e*x^2] - Log[c*(d +
e*x^2)^p]))/(8*e^2) + (3*d*g*p*x^2*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)
^p])^2)/(4*e) + (6*Sqrt[d]*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-(p*Log[d + e*x
^2]) + Log[c*(d + e*x^2)^p])^2)/Sqrt[e] - (3*d^2*g*p*Log[d + e*x^2]*(-(p*Lo
g[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/(4*e^2) + (3*p*x*(4*f + g*x^3)*Log
[d + e*x^2]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/4 - (g*x^4*(3*p
 + 2*p*Log[d + e*x^2] - 2*Log[c*(d + e*x^2)^p])*(-(p*Log[d + e*x^2]) + Log[
c*(d + e*x^2)^p])^2)/8 + f*x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2
*(-6*p - p*Log[d + e*x^2] + Log[c*(d + e*x^2)^p]) - (3*f*p^2*(p*Log[d + e*x
^2] - Log[c*(d + e*x^2)^p])*((4*I)*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2 +
4*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-2 + 2*Log[(2*Sqrt[d])/(Sqrt[d] + I*
Sqrt[e]*x)] + Log[d + e*x^2]) + Sqrt[e]*x*(8 - 4*Log[d + e*x^2] + Log[d + e
*x^2]^2) + (4*I)*Sqrt[d]*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] +
 Sqrt[e]*x)]))/Sqrt[e] + (f*p^3*(-48*Sqrt[-d^2]*Sqrt[(e*x^2)/(d + e*x^2)]*S
qrt[d + e*x^2]*ArcSin[Sqrt[d]/Sqrt[d + e*x^2]] + Sqrt[-d]*e*x^2*(-48 + 24*L
og[d + e*x^2] - 6*Log[d + e*x^2]^2 + Log[d + e*x^2]^3) - 6*Sqrt[-d^2]*Sqrt[
(e*x^2)/(d + e*x^2)]*(8*Sqrt[d]*HypergeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/
2, 3/2, 3/2}, d/(d + e*x^2)] + Log[d + e*x^2]*(4*Sqrt[d]*HypergeometricPFQ[
{1/2, 1/2, 1/2}, {3/2, 3/2}, d/(d + e*x^2)] + Sqrt[d + e*x^2]*ArcSin[Sqrt[d
]/Sqrt[d + e*x^2]]*Log[d + e*x^2])) + 24*d*Sqrt[e*x^2]*ArcTanh[Sqrt[e*x^2]/
Sqrt[-d]]*(Log[d + e*x^2] - Log[1 + (e*x^2)/d]) + 6*(-d)^(3/2)*Sqrt[-((e*x^
2)/d)]*(Log[1 + (e*x^2)/d]^2 - 4*Log[1 + (e*x^2)/d]*Log[(1 + Sqrt[-((e*x^2)
/d)])/2] + 2*Log[(1 + Sqrt[-((e*x^2)/d)])/2]^2 - 4*PolyLog[2, 1/2 - Sqrt[-(
(e*x^2)/d)]/2])))/(Sqrt[-d]*e*x)
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \left(g\,x^{3}+f\right) \ln \left(c{\left(e\,x^{2}+d\right)}^{p}\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)*ln(c*(e*x^2+d)^p)^3,x)
```

```
[Out] int((g*x^3+f)*ln(c*(e*x^2+d)^p)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(g*p^3*x^4 + 4*f*p^3*x)*log(x^2*e + d)^3 + integrate(1/2*(2*g*x^5*e*log
(c)^3 + 2*d*g*x^3*log(c)^3 + 2*f*x^2*e*log(c)^3 + 2*d*f*log(c)^3 + 3*(2*d*g
*p^2*x^3*log(c) - (g*p^3 - 2*g*p^2*log(c))*x^5*e + 2*d*f*p^2*log(c) - 2*(2*
f*p^3 - f*p^2*log(c))*x^2*e)*log(x^2*e + d)^2 + 6*(g*p*x^5*e*log(c)^2 + d*g
*p*x^3*log(c)^2 + f*p*x^2*e*log(c)^2 + d*f*p*log(c)^2)*log(x^2*e + d))/(x^2
*e + d), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p)^3,x, algorithm="fricas")
```

```
[Out] integral((g*x^3 + f)*log((x^2*e + d)^p*c)^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(f + gx^{3}\right) \log \left(c\left(d + ex^{2}\right)^{p}\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)*ln(c*(e*x**2+d)**p)**3,x)
```

```
[Out] Integral((f + g*x**3)*log(c*(d + e*x**2)**p)**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p)^3,x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f)*log((x^2*e + d)^p*c)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln \left(c \left(e x^2+d\right)^p\right)^3 \left(g x^3+f\right) \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^3*(f + g*x^3),x)
[Out] int(log(c*(d + e*x^2)^p)^3*(f + g*x^3), x)
```

3.300 $\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^3}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^3},x\right)$$

```
[Out] Unintegrable(ln(c*(e*x^2+d)^p)^3/(g*x^3+f),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[c*(d + e*x^2)^p]^3/(f + g*x^3),x]
[Out] Defer[Int][Log[c*(d + e*x^2)^p]^3/(f + g*x^3), x]
```

Rubi steps

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^3}\,dx=\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^3}\,dx$$

Mathematica [A]
time = 12.02, size = 0, normalized size = 0.00

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^3),x]
[Out] Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^3), x]
```

Maple [A]
time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)^3}{g\,x^3+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)^3/(g*x^3+f),x)
```

```
[Out] int(ln(c*(e*x^2+d)^p)^3/(g*x^3+f),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^3+f),x, algorithm="maxima")
```

```
[Out] integrate(log((x^2*e + d)^p*c)^3/(g*x^3 + f), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^3+f),x, algorithm="fricas")
```

```
[Out] integral(log((x^2*e + d)^p*c)^3/(g*x^3 + f), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)**3/(g*x**3+f),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^3+f),x, algorithm="giac")
```

```
[Out] integrate(log((x^2*e + d)^p*c)^3/(g*x^3 + f), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln\left(c\,(e\,x^2+d)^p\right)^3}{g\,x^3+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^3/(f + g*x^3),x)
[Out] int(log(c*(d + e*x^2)^p)^3/(f + g*x^3), x)
```

3.301 $\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2},x\right)$$

```
[Out] Unintegrable(ln(c*(e*x^2+d)^p)^3/(g*x^3+f)^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[Log[c*(d + e*x^2)^p]^3/(f + g*x^3)^2,x]
[Out] Defer[Int][Log[c*(d + e*x^2)^p]^3/(f + g*x^3)^2, x]
```

Rubi steps

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2} dx = \int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2} dx$$

Mathematica [A]
time = 34.55, size = 0, normalized size = 0.00

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^3)^2,x]
[Out] Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^3)^2, x]
```

Maple [A]
time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)^3}{\left(g\,x^3+f\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)^3/(g*x^3+f)^2,x)
```

```
[Out] int(ln(c*(e*x^2+d)^p)^3/(g*x^3+f)^2,x)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^3+f)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^3+f)^2,x, algorithm="fricas")
```

```
[Out] integral(log((x^2*e + d)^p*c)^3/(g^2*x^6 + 2*f*g*x^3 + f^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)**3/(g*x**3+f)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^3+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((x^2*e + d)^p*c)^3/(g*x^3 + f)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln \left(c \left(e\,x^2+d\right)^p\right)^3}{\left(g\,x^3+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)^3/(f + g*x^3)^2,x)
```

```
[Out] int(log(c*(d + e*x^2)^p)^3/(f + g*x^3)^2, x)
```

3.302 $\int \frac{\left(f+gx^3\right)^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\left(f+gx^3\right)^2}{\log\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable((g*x^3+f)^2/ln(c*(e*x^2+d)^p),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(f+gx^3\right)^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(f + g*x^3)^2/Log[c*(d + e*x^2)^p],x]
[Out] Defer[Int][(f + g*x^3)^2/Log[c*(d + e*x^2)^p], x]
```

Rubi steps

$$\int \frac{\left(f+gx^3\right)^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{\left(f+gx^3\right)^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [A]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\left(f+gx^3\right)^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f + g*x^3)^2/Log[c*(d + e*x^2)^p],x]
[Out] Integrate[(f + g*x^3)^2/Log[c*(d + e*x^2)^p], x]
```

Maple [A]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\left(g\,x^3+f\right)^2}{\ln\left(c\left(e\,x^2+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)^2/ln(c*(e*x^2+d)^p),x)
```

```
[Out] int((g*x^3+f)^2/ln(c*(e*x^2+d)^p),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2/log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] integrate((g*x^3 + f)^2/log((x^2*e + d)^p*c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2/log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] integral((g^2*x^6 + 2*f*g*x^3 + f^2)/log((x^2*e + d)^p*c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f + gx^{3}\right)^{2}}{\log\left(c\left(d + ex^{2}\right)^{p}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)**2/ln(c*(e*x**2+d)**p),x)
```

```
[Out] Integral((f + g*x**3)**2/log(c*(d + e*x**2)**p), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2/log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f)^2/log((x^2*e + d)^p*c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(g\,x^3+f\right)^2}{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x^3)^2/log(c*(d + e*x^2)^p),x)
[Out] int((f + g*x^3)^2/log(c*(d + e*x^2)^p), x)
```

3.303 $\int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$

Optimal. Leaf size=25

$$\operatorname{Int}\left(\frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable((g*x^3+f)/ln(c*(e*x^2+d)^p),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(f + g*x^3)/Log[c*(d + e*x^2)^p],x]
[Out] Defer[Int][(f + g*x^3)/Log[c*(d + e*x^2)^p], x]
```

Rubi steps

$$\int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [A]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f + g*x^3)/Log[c*(d + e*x^2)^p],x]
[Out] Integrate[(f + g*x^3)/Log[c*(d + e*x^2)^p], x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{g\,x^3+f}{\ln\left(c\left(e\,x^2+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)/ln(c*(e*x^2+d)^p),x)
[Out] int((g*x^3+f)/ln(c*(e*x^2+d)^p),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)/log(c*(e*x^2+d)^p),x, algorithm="maxima")
[Out] integrate((g*x^3 + f)/log((x^2*e + d)^p*c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)/log(c*(e*x^2+d)^p),x, algorithm="fricas")
[Out] integral((g*x^3 + f)/log((x^2*e + d)^p*c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f + gx^3}{\log\left(c\left(d + ex^2\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)/ln(c*(e*x**2+d)**p),x)
[Out] Integral((f + g*x**3)/log(c*(d + e*x**2)**p), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)/log(c*(e*x^2+d)^p),x, algorithm="giac")
[Out] integrate((g*x^3 + f)/log((x^2*e + d)^p*c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{g\,x^3 + f}{\ln\left(c\,\left(e\,x^2 + d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x^3)/log(c*(d + e*x^2)^p),x)
[Out] int((f + g*x^3)/log(c*(d + e*x^2)^p), x)
```

3.304 $\int \frac{1}{(f+gx^3)\log(c(d+ex^2)^p)}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x^3+f)/ln(c*(e*x^2+d)^p),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((f + g*x^3)*Log[c*(d + e*x^2)^p]),x]
[Out] Defer[Int][1/((f + g*x^3)*Log[c*(d + e*x^2)^p]), x]
```

Rubi steps

$$\int \frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [A]
time = 1.46, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((f + g*x^3)*Log[c*(d + e*x^2)^p]),x]
[Out] Integrate[1/((f + g*x^3)*Log[c*(d + e*x^2)^p]), x]
```

Maple [A]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{1}{(g\,x^3+f)\ln\left(c\left(e\,x^2+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x^3+f)/ln(c*(e*x^2+d)^p),x)
[Out] int(1/(g*x^3+f)/ln(c*(e*x^2+d)^p),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)/log(c*(e*x^2+d)^p),x, algorithm="maxima")
[Out] integrate(1/((g*x^3 + f)*log((x^2*e + d)^p*c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)/log(c*(e*x^2+d)^p),x, algorithm="fricas")
[Out] integral(1/((g*x^3 + f)*log((x^2*e + d)^p*c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(f + gx^3)\log\left(c\left(d + ex^2\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x**3+f)/ln(c*(e*x**2+d)**p),x)
[Out] Integral(1/((f + g*x**3)*log(c*(d + e*x**2)**p)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)/log(c*(e*x^2+d)^p),x, algorithm="giac")
[Out] integrate(1/((g*x^3 + f)*log((x^2*e + d)^p*c)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\ln\left(c\left(e\,x^2+d\right)^p\right)\,\left(g\,x^3+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(log(c*(d + e*x^2)^p)*(f + g*x^3)),x)
[Out] int(1/(log(c*(d + e*x^2)^p)*(f + g*x^3)), x)
```

3.305 $\int \frac{1}{\left(f+gx^3\right)^2 \log\left(c\left(d+ex^2\right)^p\right)}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{1}{\left(f+gx^3\right)^2 \log \left(c \left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x^3+f)^2/ln(c*(e*x^2+d)^p),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\left(f+gx^3\right)^2 \log \left(c \left(d+ex^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]),x]
```

```
[Out] Defer[Int][1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]), x]
```

Rubi steps

$$\int \frac{1}{\left(f+gx^3\right)^2 \log \left(c \left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{\left(f+gx^3\right)^2 \log \left(c \left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [A]
time = 5.33, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(f+gx^3\right)^2 \log \left(c \left(d+ex^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]),x]
```

```
[Out] Integrate[1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]), x]
```

Maple [A]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(g\,x^3+f\right)^2 \ln \left(c \left(e\,x^2+d\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x^3+f)^2/ln(c*(e*x^2+d)^p),x)
```

```
[Out] int(1/(g*x^3+f)^2/ln(c*(e*x^2+d)^p),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)^2/log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] integrate(1/((g*x^3 + f)^2*log((x^2*e + d)^p*c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)^2/log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] integral(1/((g^2*x^6 + 2*f*g*x^3 + f^2)*log((x^2*e + d)^p*c)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x**3+f)**2/ln(c*(e*x**2+d)**p),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)^2/log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] integrate(1/((g*x^3 + f)^2*log((x^2*e + d)^p*c)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\ln\left(c\left(e\,x^2+d\right)^p\right)\,\left(g\,x^3+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(log(c*(d + e*x^2)^p)*(f + g*x^3)^2),x)
[Out] int(1/(log(c*(d + e*x^2)^p)*(f + g*x^3)^2), x)
```

3.306 $\int \frac{(f+gx^3)^2}{\log^2(c(d+ex^2)^p)}\,dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(\frac{(f+gx^3)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable((g*x^3+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(f+gx^3)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(f + g*x^3)^2/Log[c*(d + e*x^2)^p]^2,x]
[Out] Defer[Int][(f + g*x^3)^2/Log[c*(d + e*x^2)^p]^2, x]
```

Rubi steps

$$\int \frac{(f+gx^3)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{(f+gx^3)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [A]
time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{(f+gx^3)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f + g*x^3)^2/Log[c*(d + e*x^2)^p]^2,x]
[Out] Integrate[(f + g*x^3)^2/Log[c*(d + e*x^2)^p]^2, x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(g\,x^3+f)^2}{\ln\left(c\left(e\,x^2+d\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int((g*x^3+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(g^2*x^8*e + d*g^2*x^6 + 2*f*g*x^5*e + 2*d*f*g*x^3 + f^2*x^2*e + d*f^2
)/(p^2*x*e*log(x^2*e + d) + p*x*e*log(c)) + integrate(1/2*(7*g^2*x^8*e + 5*
d*g^2*x^6 + 8*f*g*x^5*e + 4*d*f*g*x^3 + f^2*x^2*e - d*f^2)/(p^2*x^2*e*log(x
^2*e + d) + p*x^2*e*log(c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral((g^2*x^6 + 2*f*g*x^3 + f^2)/log((x^2*e + d)^p*c)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f + gx^{3}\right)^{2}}{\log{\left(c\left(d + ex^{2}\right)^{p} \right)}^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)**2/ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Integral((f + g*x**3)**2/log(c*(d + e*x**2)**p)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f)^2/log((x^2*e + d)^p*c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(g\,x^3+f\right)^2}{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x^3)^2/log(c*(d + e*x^2)^p)^2,x)
[Out] int((f + g*x^3)^2/log(c*(d + e*x^2)^p)^2, x)
```

3.307 $\int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable((g*x^3+f)/ln(c*(e*x^2+d)^p)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)} dx$$

Verification is not applicable to the result.

```
[In] Int[(f + g*x^3)/Log[c*(d + e*x^2)^p]^2,x]
[Out] Defer[Int][(f + g*x^3)/Log[c*(d + e*x^2)^p]^2, x]
```

Rubi steps

$$\int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)} dx=\int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)} dx$$

Mathematica [A]
time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f + g*x^3)/Log[c*(d + e*x^2)^p]^2,x]
[Out] Integrate[(f + g*x^3)/Log[c*(d + e*x^2)^p]^2, x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{g\,x^3+f}{\ln\left(c\left(e\,x^2+d\right)^p\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f)/ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int((g*x^3+f)/ln(c*(e*x^2+d)^p)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(g*x^5*e + d*g*x^3 + f*x^2*e + d*f)/(p^2*x*e*log(x^2*e + d) + p*x*e*lo
g(c)) + integrate(1/2*(4*g*x^5*e + 2*d*g*x^3 + f*x^2*e - d*f)/(p^2*x^2*e*lo
g(x^2*e + d) + p*x^2*e*log(c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral((g*x^3 + f)/log((x^2*e + d)^p*c)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f + gx^3}{\log\left(c\left(d + ex^2\right)^p\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f)/ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Integral((f + g*x**3)/log(c*(d + e*x**2)**p)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f)/log((x^2*e + d)^p*c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{g\,x^3+f}{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x^3)/log(c*(d + e*x^2)^p)^2,x)
[Out] int((f + g*x^3)/log(c*(d + e*x^2)^p)^2, x)
```

3.308 $\int \frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$

Optimal. Leaf size=27

$$\operatorname{Int}\left(\frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x^3+f)/ln(c*(e*x^2+d)^p)^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((f + g*x^3)*Log[c*(d + e*x^2)^p]^2),x]
[Out] Defer[Int][1/((f + g*x^3)*Log[c*(d + e*x^2)^p]^2), x]
```

Rubi steps

$$\int \frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [A]
time = 5.26, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((f + g*x^3)*Log[c*(d + e*x^2)^p]^2),x]
[Out] Integrate[1/((f + g*x^3)*Log[c*(d + e*x^2)^p]^2), x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(g\,x^3+f)\ln\left(c\left(e\,x^2+d\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x^3+f)/ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int(1/(g*x^3+f)/ln(c*(e*x^2+d)^p)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(x^2*e + d)/(g*p*x^4*e*log(c) + f*p*x*e*log(c) + (g*p^2*x^4*e + f*p^2*
x*e)*log(x^2*e + d)) - integrate(1/2*(2*g*x^5*e + 4*d*g*x^3 - f*x^2*e + d*f
)/(g^2*p*x^8*e*log(c) + 2*f*g*p*x^5*e*log(c) + f^2*p*x^2*e*log(c) + (g^2*p^
2*x^8*e + 2*f*g*p^2*x^5*e + f^2*p^2*x^2*e)*log(x^2*e + d)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((g*x^3 + f)*log((x^2*e + d)^p*c)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(f + gx^{3}\right) \log{\left(c \left(d + ex^{2}\right)^{p} \right)}^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x**3+f)/ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Integral(1/((f + g*x**3)*log(c*(d + e*x**2)**p)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((g*x^3 + f)*log((x^2*e + d)^p*c)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\ln\left(c\left(e\,x^2+d\right)^p\right)^2\,\left(g\,x^3+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(log(c*(d + e*x^2)^p)^2*(f + g*x^3)),x)
[Out] int(1/(log(c*(d + e*x^2)^p)^2*(f + g*x^3)), x)
```

3.309 $\int \frac{1}{\left(f+gx^3\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{1}{\left(f+gx^3\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x^3+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\left(f+gx^3\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]^2),x]
[Out] Defer[Int][1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]^2), x]
```

Rubi steps

$$\int \frac{1}{\left(f+gx^3\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{\left(f+gx^3\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [A]
time = 7.58, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(f+gx^3\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]^2),x]
[Out] Integrate[1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]^2), x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(g\,x^3+f\right)^2 \ln\left(c\left(e\,x^2+d\right)^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x^3+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int(1/(g*x^3+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(x^2*e + d)/(g^2*p*x^7*e*log(c) + 2*f*g*p*x^4*e*log(c) + f^2*p*x*e*log
(c) + (g^2*p^2*x^7*e + 2*f*g*p^2*x^4*e + f^2*p^2*x*e)*log(x^2*e + d)) - int
egrate(1/2*(5*g*x^5*e + 7*d*g*x^3 - f*x^2*e + d*f)/(g^3*p*x^11*e*log(c) + 3
*f*g^2*p*x^8*e*log(c) + 3*f^2*g*p*x^5*e*log(c) + f^3*p*x^2*e*log(c) + (g^3*
p^2*x^11*e + 3*f*g^2*p^2*x^8*e + 3*f^2*g*p^2*x^5*e + f^3*p^2*x^2*e)*log(x^2
*e + d)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((g^2*x^6 + 2*f*g*x^3 + f^2)*log((x^2*e + d)^p*c)^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x**3+f)**2/ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x^3+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
[Out] integrate(1/((g*x^3 + f)^2*log((x^2*e + d)^p*c)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\ln\left(c\left(e\,x^2+d\right)^p\right)^2\left(g\,x^3+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(log(c*(d + e*x^2)^p)^2*(f + g*x^3)^2),x)
[Out] int(1/(log(c*(d + e*x^2)^p)^2*(f + g*x^3)^2), x)
```

3.310 $\int x^5(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx$

Optimal. Leaf size=142

$$-\frac{d^2(4ef-3dg)px^2}{24e^3}+\frac{d(4ef-3dg)px^4}{48e^2}-\frac{(4ef-3dg)px^6}{72e}-\frac{1}{32}gpx^8+\frac{d^3(4ef-3dg)p\log\left(d+ex^2\right)}{24e^4}+\frac{1}{6}fx^6\log$$

```
[Out] -1/24*d^2*(-3*d*g+4*e*f)*p*x^2/e^3+1/48*d*(-3*d*g+4*e*f)*p*x^4/e^2-1/72*(-3
*d*g+4*e*f)*p*x^6/e-1/32*g*p*x^8+1/24*d^3*(-3*d*g+4*e*f)*p*ln(e*x^2+d)/e^4+
1/6*f*x^6*ln(c*(e*x^2+d)^p)+1/8*g*x^8*ln(c*(e*x^2+d)^p)
```

Rubi [A]

time = 0.15, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.217$, Rules used = {2525, 45, 2461, 12, 78}

$$\frac{1}{6}fx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{8}gx^8\log\left(c\left(d+ex^2\right)^p\right)+\frac{d^3p(4ef-3dg)\log\left(d+ex^2\right)}{24e^4}-\frac{d^2px^2(4ef-3dg)}{24e^3}+\frac{dpx^4(4ef-3dg)}{48e^2}-\frac{px^6(4ef-3dg)}{72e}-\frac{1}{32}gpx^8$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -1/24*(d^2*(4*e*f - 3*d*g)*p*x^2)/e^3 + (d*(4*e*f - 3*d*g)*p*x^4)/(48*e^2)
- ((4*e*f - 3*d*g)*p*x^6)/(72*e) - (g*p*x^8)/32 + (d^3*(4*e*f - 3*d*g)*p*Lo
g[d + e*x^2])/(24*e^4) + (f*x^6*Log[c*(d + e*x^2)^p])/6 + (g*x^8*Log[c*(d +
 e*x^2)^p])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2461

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\int x^5\left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)\,dx &= \frac{1}{2}\text{Subst}\left(\int x^2(f+gx)\log\left(c(d+ex)^p\right)\,dx,x,x^2\right)\\
&= \frac{1}{6}fx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{8}gx^8\log\left(c\left(d+ex^2\right)^p\right)-\frac{1}{2}(ep)\text{Subst}\Bigg(\int \\
&= \frac{1}{6}fx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{8}gx^8\log\left(c\left(d+ex^2\right)^p\right)-\frac{1}{24}(ep)\text{Subst}\Bigg(\int \\
&= \frac{1}{6}fx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{8}gx^8\log\left(c\left(d+ex^2\right)^p\right)-\frac{1}{24}(ep)\text{Subst}\Bigg(\int \\
&= -\frac{d^2(4ef-3dg)px^2}{24e^3}+\frac{d(4ef-3dg)px^4}{48e^2}-\frac{(4ef-3dg)px^6}{72e}-\frac{1}{32}gpx^8
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 170, normalized size = 1.20

$$-\frac{d^2fpx^2}{6e^2}+\frac{d^3gpx^2}{8e^3}+\frac{dfpx^4}{12e}-\frac{d^2gpx^4}{16e^2}-\frac{1}{18}fpx^6+\frac{dgpx^6}{24e}-\frac{1}{32}gpx^8+\frac{d^3fp\log\left(d+ex^2\right)}{6e^3}-\frac{d^4gp\log\left(d+ex^2\right)}{8e^4}+\frac{1}{6}fx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{8}gx^8\log\left(c\left(d+ex^2\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -1/6*(d^2*f*p*x^2)/e^2 + (d^3*g*p*x^2)/(8*e^3) + (d*f*p*x^4)/(12*e) - (d^2*
g*p*x^4)/(16*e^2) - (f*p*x^6)/18 + (d*g*p*x^6)/(24*e) - (g*p*x^8)/32 + (d^3
*f*p*Log[d + e*x^2])/(6*e^3) - (d^4*g*p*Log[d + e*x^2])/(8*e^4) + (f*x^6*Lo
g[c*(d + e*x^2)^p])/6 + (g*x^8*Log[c*(d + e*x^2)^p])/8
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.49, size = 413, normalized size = 2.91

method	result
risch	$\left(\frac{1}{8}g\,x^{8}+\frac{1}{6}f\,x^{6}\right)\ln\left(\left(e\,x^{2}+d\right)^{p}\right)-\frac{i\pi g\,x^{8}\mathrm{csgn}\left(i\left(e\,x^{2}+d\right)^{p}\right)\mathrm{csgn}\left(ic\left(e\,x^{2}+d\right)^{p}\right)\mathrm{csgn}\left(ic\right)}{16}-\frac{i\pi f\,x^{6}\mathrm{csgn}\left(ic\left(e\,x^{2}+d\right)^{p}\right)^{3}}{12}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(g*x^2+f)*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*g*x^8+1/6*f*x^6)*ln((e*x^2+d)^p)-1/16*I*Pi*g*x^8*csgn(I*(e*x^2+d)^p)*c
sgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/12*I*Pi*f*x^6*csgn(I*c*(e*x^2+d)^p)^3+1/12
*I*Pi*f*x^6*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/16*I*Pi*g*x^8*csg
n(I*c*(e*x^2+d)^p)^2*csgn(I*c)+1/16*I*Pi*g*x^8*csgn(I*(e*x^2+d)^p)*csgn(I*c
*(e*x^2+d)^p)^2+1/12*I*Pi*f*x^6*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/16*I*Pi
*g*x^8*csgn(I*c*(e*x^2+d)^p)^3-1/12*I*Pi*f*x^6*csgn(I*(e*x^2+d)^p)*csgn(I*c
*(e*x^2+d)^p)*csgn(I*c)+1/8*ln(c)*g*x^8-1/32*g*p*x^8+1/6*ln(c)*f*x^6+1/24/e
*d*g*p*x^6-1/18*f*p*x^6-1/16/e^2*d^2*g*p*x^4+1/12/e*d*f*p*x^4+1/8/e^3*d^3*g
*p*x^2-1/6/e^2*d^2*f*p*x^2-1/8/e^4*ln(e*x^2+d)*d^4*g*p+1/6/e^3*ln(e*x^2+d)*
d^3*f*p
```

Maxima [A]
time = 0.27, size = 132, normalized size = 0.93

$$-\frac{1}{288}\left(12\left(3\,d^{4}g-4\,d^{3}fe\right)e^{(-5)}\log\left(x^{2}e+d\right)+\left(9\,gx^{8}e^{3}-4\left(3\,dge^{2}-4\,fe^{3}\right)x^{6}+6\left(3\,d^{2}ge-4\,dfe^{2}\right)x^{4}-12\left(3\,d^{3}g-4\,d^{2}fe\right)x^{2}\right)e^{(-4)}\right)pe+\frac{1}{24}\left(3\,gx^{8}+4\,fx^{6}\right)\log\left(\left(x^{2}e+d\right)^{p}c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] -1/288*(12*(3*d^4*g - 4*d^3*f*e)*e^(-5)*log(x^2*e + d) + (9*g*x^8*e^3 - 4*(
3*d*g*e^2 - 4*f*e^3)*x^6 + 6*(3*d^2*g*e - 4*d*f*e^2)*x^4 - 12*(3*d^3*g - 4*
d^2*f*e)*x^2)*e^(-4))*p*e + 1/24*(3*g*x^8 + 4*f*x^6)*log((x^2*e + d)^p*c)
```

Fricas [A]
time = 0.36, size = 147, normalized size = 1.04

$$\frac{1}{288}\left(36\,d^{3}gpx^{2}e+12\left(3\,gx^{8}+4\,fx^{6}\right)e^{4}\log\left(c\right)-\left(9\,gpx^{8}+16\,fpx^{6}\right)e^{4}+12\left(dgpx^{6}+2\,dfpx^{4}\right)e^{3}-6\left(3\,d^{2}gpx^{4}+8\,d^{2}fpx^{2}\right)e^{2}-12\left(3\,d^{4}gp-4\,d^{3}fpe-\left(3\,gpx^{8}+4\,fpx^{6}\right)e^{4}\right)\log\left(x^{2}e+d\right)\right)e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] 1/288*(36*d^3*g*p*x^2*e + 12*(3*g*x^8 + 4*f*x^6)*e^4*log(c) - (9*g*p*x^8 +
16*f*p*x^6)*e^4 + 12*(d*g*p*x^6 + 2*d*f*p*x^4)*e^3 - 6*(3*d^2*g*p*x^4 + 8*d
^2*f*p*x^2)*e^2 - 12*(3*d^4*g*p - 4*d^3*f*p*e - (3*g*p*x^8 + 4*f*p*x^6)*e^4
)*log(x^2*e + d))*e^(-4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(g*x**2+f)*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 413 vs. 2(131) = 262.
time = 3.97, size = 413, normalized size = 2.91

$$\frac{1}{8}\left(x^2e+d\right)^4gpe^{(-4)}\log\left(x^2e+d\right)-\frac{1}{2}\left(x^2e+d\right)^3dgpe^{(-4)}\log\left(x^2e+d\right)+\frac{3}{4}\left(x^2e+d\right)^2d^2gpe^{(-4)}\log\left(x^2e+d\right)-\frac{1}{32}\left(x^2e+d\right)^4gpe^{(-4)}+\frac{1}{6}\left(x^2e+d\right)^3dgpe^{(-4)}-\frac{3}{8}\left(x^2e+d\right)^2d^2gpe^{(-4)}+\frac{1}{6}\left(x^2e+d\right)^3fpe^{(-3)}\log\left(x^2e+d\right)-\frac{1}{2}\left(x^2e+d\right)^2dfpe^{(-3)}\log\left(x^2e+d\right)+\frac{1}{8}\left(x^2e+d\right)^4ge^{(-4)}\log\left(c\right)-\frac{1}{2}\left(x^2e+d\right)^3dge^{(-4)}\log\left(c\right)+\frac{3}{4}\left(x^2e+d\right)^2d^2ge^{(-4)}\log\left(c\right)-\frac{1}{18}\left(x^2e+d\right)^3fpe^{(-3)}+\frac{1}{4}\left(x^2e+d\right)^2dfpe^{(-3)}+\frac{1}{6}\left(x^2e+d\right)^3fe^{(-3)}\log\left(c\right)-\frac{1}{2}\left(x^2e+d\right)^2dfe^{(-3)}\log\left(c\right)+\frac{1}{2}\left(\left(x^2e-\left(x^2e+d\right)\log\left(x^2e+d\right)+d\right)d^3gp-\left(x^2e-\left(x^2e+d\right)\log\left(x^2e+d\right)+d\right)d^2fpe-\left(x^2e+d\right)d^3g\log\left(c\right)+\left(x^2e+d\right)d^2fe\log\left(c\right)\right)e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 1/8*(x^2*e + d)^4*g*p*e^(-4)*log(x^2*e + d) - 1/2*(x^2*e + d)^3*d*g*p*e^(-4
)*log(x^2*e + d) + 3/4*(x^2*e + d)^2*d^2*g*p*e^(-4)*log(x^2*e + d) - 1/32*(
x^2*e + d)^4*g*p*e^(-4) + 1/6*(x^2*e + d)^3*d*g*p*e^(-4) - 3/8*(x^2*e + d)^
2*d^2*g*p*e^(-4) + 1/6*(x^2*e + d)^3*f*p*e^(-3)*log(x^2*e + d) - 1/2*(x^2*e
 + d)^2*d*f*p*e^(-3)*log(x^2*e + d) + 1/8*(x^2*e + d)^4*g*e^(-4)*log(c) - 1
/2*(x^2*e + d)^3*d*g*e^(-4)*log(c) + 3/4*(x^2*e + d)^2*d^2*g*e^(-4)*log(c)
- 1/18*(x^2*e + d)^3*f*p*e^(-3) + 1/4*(x^2*e + d)^2*d*f*p*e^(-3) + 1/6*(x^2
*e + d)^3*f*e^(-3)*log(c) - 1/2*(x^2*e + d)^2*d*f*e^(-3)*log(c) + 1/2*((x^2
*e - (x^2*e + d)*log(x^2*e + d) + d)*d^3*g*p - (x^2*e - (x^2*e + d)*log(x^2
*e + d) + d)*d^2*f*p*e - (x^2*e + d)*d^3*g*log(c) + (x^2*e + d)*d^2*f*e*log
(c))*e^(-4)
```

Mupad [B]
time = 0.32, size = 127, normalized size = 0.89

$$\ln\left(c\left(e\,x^2+d\right)^p\right)\,\left(\frac{g\,x^8}{8}+\frac{f\,x^6}{6}\right)-x^6\,\left(\frac{f\,p}{18}-\frac{d\,g\,p}{24\,e}\right)-\frac{g\,p\,x^8}{32}-\frac{\ln\left(e\,x^2+d\right)\,\left(3\,d^4\,g\,p-4\,d^3\,e\,f\,p\right)}{24\,e^4}+\frac{d\,x^4\,\left(\frac{f\,p}{3}-\frac{d\,g\,p}{4\,e}\right)}{4\,e}-\frac{d^2\,x^2\,\left(\frac{f\,p}{3}-\frac{d\,g\,p}{4\,e}\right)}{2\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*log(c*(d + e*x^2)^p)*(f + g*x^2),x)
```

```
[Out] log(c*(d + e*x^2)^p)*((f*x^6)/6 + (g*x^8)/8) - x^6*((f*p)/18 - (d*g*p)/(24*
e)) - (g*p*x^8)/32 - (log(d + e*x^2)*(3*d^4*g*p - 4*d^3*e*f*p))/(24*e^4) +
(d*x^4*((f*p)/3 - (d*g*p)/(4*e)))/(4*e) - (d^2*x^2*((f*p)/3 - (d*g*p)/(4*e)
))/(2*e^2)
```

3.311 $\int x^3(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx$

Optimal. Leaf size=119

$$\frac{d(3ef-2dg)px^2}{12e^2}-\frac{(3ef-2dg)px^4}{24e}-\frac{1}{18}gpx^6-\frac{d^2(3ef-2dg)p\log\left(d+ex^2\right)}{12e^3}+\frac{1}{4}fx^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{6}gx^6$$

```
[Out] 1/12*d*(-2*d*g+3*e*f)*p*x^2/e^2-1/24*(-2*d*g+3*e*f)*p*x^4/e-1/18*g*p*x^6-1/
12*d^2*(-2*d*g+3*e*f)*p*ln(e*x^2+d)/e^3+1/4*f*x^4*ln(c*(e*x^2+d)^p)+1/6*g*x
^6*ln(c*(e*x^2+d)^p)
```

Rubi [A]
time = 0.12, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2525, 45, 2461, 12, 78}

$$\frac{1}{4}fx^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{6}gx^6\log\left(c\left(d+ex^2\right)^p\right)-\frac{d^2p(3ef-2dg)\log\left(d+ex^2\right)}{12e^3}+\frac{dpx^2(3ef-2dg)}{12e^2}-\frac{px^4(3ef-2dg)}{24e}-\frac{1}{18}gpx^6$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (d*(3*e*f - 2*d*g)*p*x^2)/(12*e^2) - ((3*e*f - 2*d*g)*p*x^4)/(24*e) - (g*p*
x^6)/18 - (d^2*(3*e*f - 2*d*g)*p*Log[d + e*x^2])/(12*e^3) + (f*x^4*Log[c*(d
 + e*x^2)^p])/4 + (g*x^6*Log[c*(d + e*x^2)^p])/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2461

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int x^3\left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)\,dx=\frac{1}{2}\text{Subst}\left(\int x(f+gx)\log\left(c(d+ex)^p\right)\,dx,x,x^2\right)$$

$$=\frac{1}{4}fx^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{6}gx^6\log\left(c\left(d+ex^2\right)^p\right)-\frac{1}{2}(ep)\text{Subst}\Bigg(\int \frac{x}{}$$

$$=\frac{1}{4}fx^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{6}gx^6\log\left(c\left(d+ex^2\right)^p\right)-\frac{1}{12}(ep)\text{Subst}\Bigg(\int$$

$$=\frac{1}{4}fx^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{6}gx^6\log\left(c\left(d+ex^2\right)^p\right)-\frac{1}{12}(ep)\text{Subst}\Bigg(\int$$

$$=\frac{d(3ef-2dg)px^2}{12e^2}-\frac{(3ef-2dg)px^4}{24e}-\frac{1}{18}gpx^6-\frac{d^2(3ef-2dg)p\log}{12e^3}$$

Mathematica [A]
time = 0.02, size = 140, normalized size = 1.18

$$\frac{dfpx^2}{4e}-\frac{d^2gpx^2}{6e^2}-\frac{1}{8}fpx^4+\frac{dgpx^4}{12e}-\frac{1}{18}gpx^6-\frac{d^2fp\log\left(d+ex^2\right)}{4e^2}+\frac{d^3gp\log\left(d+ex^2\right)}{6e^3}+\frac{1}{4}fx^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{6}gx^6\log\left(c\left(d+ex^2\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (d*f*p*x^2)/(4*e) - (d^2*g*p*x^2)/(6*e^2) - (f*p*x^4)/8 + (d*g*p*x^4)/(12*e
) - (g*p*x^6)/18 - (d^2*f*p*Log[d + e*x^2])/(4*e^2) + (d^3*g*p*Log[d + e*x^
2])/(6*e^3) + (f*x^4*Log[c*(d + e*x^2)^p])/4 + (g*x^6*Log[c*(d + e*x^2)^p])
/6
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.25, size = 387, normalized size = 3.25

method	result
risch	$\left(\frac{1}{6}x^6 g+\frac{1}{4}x^4 f\right)\ln\left(\left(e\,x^2+d\right)^p\right)+\frac{i\pi g\,x^6\operatorname{csgn}\left(i\left(e\,x^2+d\right)^p\right)\operatorname{csgn}\left(ic\left(e\,x^2+d\right)^p\right)^2}{12}-\frac{i\pi g\,x^6\operatorname{csgn}\left(i\left(e\,x^2+d\right)^p\right)\operatorname{csgn}\left(ic\left(e\,x^2\right.\right.}{12}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(g*x^2+f)*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] (1/6*x^6*g+1/4*x^4*f)*ln((e*x^2+d)^p)+1/12*I*Pi*g*x^6*csgn(I*(e*x^2+d)^p)*c
sgn(I*c*(e*x^2+d)^p)^2-1/12*I*Pi*g*x^6*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+
d)^p)*csgn(I*c)-1/8*I*Pi*f*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*cs
gn(I*c)-1/8*I*Pi*f*x^4*csgn(I*c*(e*x^2+d)^p)^3+1/8*I*Pi*f*x^4*csgn(I*(e*x^2
+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/8*I*Pi*f*x^4*csgn(I*c*(e*x^2+d)^p)^2*csgn(
I*c)+1/12*I*Pi*g*x^6*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/12*I*Pi*g*x^6*csgn
(I*c*(e*x^2+d)^p)^3+1/6*x^6*ln(c)*g-1/18*g*p*x^6+1/4*x^4*ln(c)*f+1/12/e*x^4
*d*g*p-1/8*x^4*f*p-1/6/e^2*x^2*d^2*g*p+1/4/e*x^2*d*f*p+1/6/e^3*ln(e*x^2+d)*
d^3*g*p-1/4/e^2*ln(e*x^2+d)*d^2*f*p
```

Maxima [A]
time = 0.30, size = 111, normalized size = 0.93

$$\frac{1}{72}\left(6\left(2\,d^3 g-3\,d^2 fe\right)e^{(-4)}\log\left(x^2 e+d\right)-\left(4\,gx^6e^2-3\left(2\,dge-3\,fe^2\right)x^4+6\left(2\,d^2 g-3\,dfe\right)x^2\right)e^{(-3)}\right)pe+\frac{1}{12}\left(2\,gx^6+3\,fx^4\right)\log\left(\left(x^2 e+d\right)^p c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] 1/72*(6*(2*d^3*g - 3*d^2*f*e)*e^(-4)*log(x^2*e + d) - (4*g*x^6*e^2 - 3*(2*d
*g*e - 3*f*e^2)*x^4 + 6*(2*d^2*g - 3*d*f*e)*x^2)*e^(-3))*p*e + 1/12*(2*g*x^
6 + 3*f*x^4)*log((x^2*e + d)^p*c)
```

Fricas [A]
time = 0.34, size = 120, normalized size = 1.01

$$-\frac{1}{72}\left(12\,d^2gpx^2e-6\left(2\,gx^6+3\,fx^4\right)e^3\log\left(c\right)+\left(4\,gpx^6+9\,fpx^4\right)e^3-6\left(dgpx^4+3\,dfpx^2\right)e^2-6\left(2\,d^3gp-3\,d^2fpe+\left(2\,gpx^6+3\,fpx^4\right)e^3\right)\log\left(x^2e+d\right)\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] -1/72*(12*d^2*g*p*x^2*e - 6*(2*g*x^6 + 3*f*x^4)*e^3*log(c) + (4*g*p*x^6 + 9
*f*p*x^4)*e^3 - 6*(d*g*p*x^4 + 3*d*f*p*x^2)*e^2 - 6*(2*d^3*g*p - 3*d^2*f*p*
e + (2*g*p*x^6 + 3*f*p*x^4)*e^3)*log(x^2*e + d))*e^(-3)
```

Sympy [A]
time = 66.92, size = 156, normalized size = 1.31

$$\begin{cases} \frac{d^3 g \log\left(c(d+ex^2)^p\right)}{6e^3} - \frac{d^2 f \log\left(c(d+ex^2)^p\right)}{4e^2} - \frac{d^2 gpx^2}{6e^2} + \frac{dfpx^2}{4e} + \frac{dgpx^4}{12e} - \frac{fpx^4}{8} + \frac{fx^4 \log\left(c(d+ex^2)^p\right)}{4} - \frac{gpx^6}{18} + \frac{gx^6 \log\left(c(d+ex^2)^p\right)}{6} & \text{for } e \neq 0 \\ \left(\frac{fx^4}{4} + \frac{gx^6}{6}\right) \log\left(cd^p\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(g*x**2+f)*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise((d**3*g*log(c*(d + e*x**2)**p)/(6*e**3) - d**2*f*log(c*(d + e*x**
2)**p)/(4*e**2) - d**2*g*p*x**2/(6*e**2) + d*f*p*x**2/(4*e) + d*g*p*x**4/(1
2*e) - f*p*x**4/8 + f*x**4*log(c*(d + e*x**2)**p)/4 - g*p*x**6/18 + g*x**6*
log(c*(d + e*x**2)**p)/6, Ne(e, 0)), ((f*x**4/4 + g*x**6/6)*log(c*d**p), Tr
ue))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 281 vs. 2(110) = 220.
time = 5.46, size = 281, normalized size = 2.36

$$\frac{1}{6} \left(x^2 e + d\right)^3 g p e^{(-3)} \log\left(x^2 e + d\right) - \frac{1}{2} \left(x^2 e + d\right)^2 d g p e^{(-3)} \log\left(x^2 e + d\right) - \frac{1}{18} \left(x^2 e + d\right)^3 g p e^{(-3)} + \frac{1}{4} \left(x^2 e + d\right)^2 d g p e^{(-3)} + \frac{1}{4} \left(x^2 e + d\right)^2 f p e^{(-2)} \log\left(x^2 e + d\right) + \frac{1}{6} \left(x^2 e + d\right)^3 g e^{(-3)} \log\left(c\right) - \frac{1}{2} \left(x^2 e + d\right)^2 d g e^{(-3)} \log\left(c\right) - \frac{1}{8} \left(x^2 e + d\right)^2 f p e^{(-2)} + \frac{1}{4} \left(x^2 e + d\right)^2 f e^{(-2)} \log\left(c\right) - \frac{1}{2} \left(\left(x^2 e - \left(x^2 e + d\right) \log\left(x^2 e + d\right) + d\right) d^2 g p - \left(x^2 e - \left(x^2 e + d\right) \log\left(x^2 e + d\right) + d\right) d f p e - \left(x^2 e + d\right) d^2 g \log\left(c\right) + \left(x^2 e + d\right) d f e \log\left(c\right)\right) e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 1/6*(x^2*e + d)^3*g*p*e^(-3)*log(x^2*e + d) - 1/2*(x^2*e + d)^2*d*g*p*e^(-3
)*log(x^2*e + d) - 1/18*(x^2*e + d)^3*g*p*e^(-3) + 1/4*(x^2*e + d)^2*d*g*p*
e^(-3) + 1/4*(x^2*e + d)^2*f*p*e^(-2)*log(x^2*e + d) + 1/6*(x^2*e + d)^3*g*
e^(-3)*log(c) - 1/2*(x^2*e + d)^2*d*g*e^(-3)*log(c) - 1/8*(x^2*e + d)^2*f*p
*e^(-2) + 1/4*(x^2*e + d)^2*f*e^(-2)*log(c) - 1/2*((x^2*e - (x^2*e + d)*log
(x^2*e + d) + d)*d^2*g*p - (x^2*e - (x^2*e + d)*log(x^2*e + d) + d)*d*f*p*e
 - (x^2*e + d)*d^2*g*log(c) + (x^2*e + d)*d*f*e*log(c))*e^(-3)
```

Mupad [B]
time = 0.31, size = 103, normalized size = 0.87

$$\ln\left(c\left(e\,x^2+d\right)^p\right)\,\left(\frac{g\,x^6}{6}+\frac{f\,x^4}{4}\right)-x^4\,\left(\frac{f\,p}{8}-\frac{d\,g\,p}{12\,e}\right)-\frac{g\,p\,x^6}{18}+\frac{\ln\left(e\,x^2+d\right)\,\left(2\,d^3\,g\,p-3\,d^2\,e\,f\,p\right)}{12\,e^3}+\frac{d\,x^2\,\left(\frac{f\,p}{2}-\frac{d\,g\,p}{3\,e}\right)}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(c*(d + e*x^2)^p)*(f + g*x^2),x)
```

```
[Out] log(c*(d + e*x^2)^p)*((f*x^4)/4 + (g*x^6)/6) - x^4*((f*p)/8 - (d*g*p)/(12*e
)) - (g*p*x^6)/18 + (log(d + e*x^2)*(2*d^3*g*p - 3*d^2*e*f*p))/(12*e^3) + (
d*x^2*((f*p)/2 - (d*g*p)/(3*e)))/(2*e)
```

3.312 $\int x(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx$

Optimal. Leaf size=94

$$-\frac{(ef-dg)px^2}{4e}-\frac{p(f+gx^2)^2}{8g}-\frac{(ef-dg)^2p\log(d+ex^2)}{4e^2g}+\frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{4g}$$

```
[Out] -1/4*(-d*g+e*f)*p*x^2/e-1/8*p*(g*x^2+f)^2/g-1/4*(-d*g+e*f)^2*p*ln(e*x^2+d)/
e^2/g+1/4*(g*x^2+f)^2*ln(c*(e*x^2+d)^p)/g
```

Rubi [A]
time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2525, 2442, 45}

$$\frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{4g}-\frac{p(ef-dg)^2\log(d+ex^2)}{4e^2g}-\frac{px^2(ef-dg)}{4e}-\frac{p(f+gx^2)^2}{8g}$$

Antiderivative was successfully verified.

```
[In] Int[x*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -1/4*((e*f - d*g)*p*x^2)/e - (p*(f + g*x^2)^2)/(8*g) - ((e*f - d*g)^2*p*Log
[d + e*x^2])/(4*e^2*g) + ((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/(4*g)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
```

```
|| IGtQ[q, 0])
```

Rubi steps

$$\int x\left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)\,dx=\frac{1}{2}\text{Subst}\left(\int (f+gx)\log\left(c(d+ex)^p\right)\,dx,x,x^2\right)$$
$$=\frac{\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)}{4g}-\frac{(ep)\text{Subst}\left(\int\frac{(f+gx)^2}{d+ex}\,dx,x,x^2\right)}{4g}$$
$$=\frac{\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)}{4g}-\frac{(ep)\text{Subst}\left(\int\left(\frac{g(ef-dg)}{e^2}+\frac{(ef-dg)^2}{e^2(d+ex)}+\frac{g(f\cdots\right.\right.}{4g}$$
$$=-\frac{(ef-dg)px^2}{4e}-\frac{p\left(f+gx^2\right)^2}{8g}-\frac{(ef-dg)^2p\log\left(d+ex^2\right)}{4e^2g}+\frac{(f+gx\cdots}{}$$

Mathematica [A]
time = 0.04, size = 98, normalized size = 1.04

$$\frac{dgpx^2}{4e}-\frac{1}{8}gpx^4-\frac{d^2gp\log\left(d+ex^2\right)}{4e^2}+\frac{1}{4}gx^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}f\left(-px^2+\frac{\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{e}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (d*g*p*x^2)/(4*e) - (g*p*x^4)/8 - (d^2*g*p*Log[d + e*x^2])/(4*e^2) + (g*x^4
*Log[c*(d + e*x^2)^p])/4 + (f*(-(p*x^2) + ((d + e*x^2)*Log[c*(d + e*x^2)^p]
)/e))/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.44, size = 3275, normalized size = 34.84

method	result	size
risch	Expression too large to display	3275

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(g*x^2+f)*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(g*x^2+f)^2/g*ln((e*x^2+d)^p)+1/8*(-4*I*ln(e*x^2+d)*Pi*d*e*f*g*p*csgn(I
*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+4*e^2*f^2*p^2-Pi^2*e^2*f^2*cs
gn(I*c*(e*x^2+d)^p)^6+4*I*ln(e*x^2+d)*Pi*d*e*f*g*p*csgn(I*c*(e*x^2+d)^p)^2*
csgn(I*c)-2*I*Pi*d*e*g^2*p*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*cs
gn(I*c)+6*I*Pi*e^2*f*g*p*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn
```

(I*c)-2*I*Pi*d*e*f*g*p*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+
2*Pi^2*e^2*f^2*csgn(I*c*(e*x^2+d)^p)^5*csgn(I*c)-Pi^2*e^2*f^2*csgn(I*c*(e*x
^2+d)^p)^4*csgn(I*c)^2-Pi^2*e^2*g^2*x^4*csgn(I*c*(e*x^2+d)^p)^6-Pi^2*e^2*f^
2*csgn(I*(e*x^2+d)^p)^2*csgn(I*c*(e*x^2+d)^p)^4+2*Pi^2*e^2*f^2*csgn(I*(e*x^
2+d)^p)*csgn(I*c*(e*x^2+d)^p)^5+4*I*ln(e*x^2+d)*Pi*d*e*f*g*p*csgn(I*(e*x^2+
d)^p)*csgn(I*c*(e*x^2+d)^p)^2-8*I*Pi*ln(c)*e^2*f*g*x^2*csgn(I*(e*x^2+d)^p)*
csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-Pi^2*e^2*g^2*x^4*csgn(I*(e*x^2+d)^p)^2*csgn
(I*c*(e*x^2+d)^p)^2*csgn(I*c)^2-4*Pi^2*e^2*g^2*x^4*csgn(I*(e*x^2+d)^p)*csgn
(I*c*(e*x^2+d)^p)^4*csgn(I*c)+2*Pi^2*e^2*g^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I
c(e*x^2+d)^p)^3*csgn(I*c)^2-2*Pi^2*e^2*f*g*x^2*csgn(I*(e*x^2+d)^p)^2*csgn
(I*c*(e*x^2+d)^p)^4+4*Pi^2*e^2*f*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+
d)^p)^5-4*I*Pi*e^2*f^2*p*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-4*I*Pi
*e^2*f^2*p*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+2*Pi^2*e^2*g^2*x^4*csgn(I*(e*x
^2+d)^p)^2*csgn(I*c*(e*x^2+d)^p)^3*csgn(I*c)-4*I*Pi*ln(c)*e^2*g^2*x^4*csgn(
I*c*(e*x^2+d)^p)^3+2*I*Pi*e^2*g^2*p*x^4*csgn(I*c*(e*x^2+d)^p)^3+4*I*Pi*ln(c
)*e^2*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+4*I*Pi*ln(c)*e^2*f^2*
csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+4*Pi^2*e^2*f*g*x^2*csgn(I*c*(e*x^2+d)^p)^
5*csgn(I*c)-2*Pi^2*e^2*f*g*x^2*csgn(I*c*(e*x^2+d)^p)^4*csgn(I*c)^2+2*I*ln(e
*x^2+d)*Pi*d^2*g^2*p*csgn(I*c*(e*x^2+d)^p)^3+2*I*ln(e*x^2+d)*Pi*e^2*f^2*p*c
sgn(I*c*(e*x^2+d)^p)^3+4*ln(c)^2*e^2*f^2-2*Pi^2*e^2*f*g*x^2*csgn(I*c*(e*x^2
+d)^p)^6+2*Pi^2*e^2*f^2*csgn(I*(e*x^2+d)^p)^2*csgn(I*c*(e*x^2+d)^p)^3*csgn(
I*c)-Pi^2*e^2*f^2*csgn(I*(e*x^2+d)^p)^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)^2
-Pi^2*e^2*g^2*x^4*csgn(I*(e*x^2+d)^p)^2*csgn(I*c*(e*x^2+d)^p)^4+2*Pi^2*e^2*
g^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^5+2*Pi^2*e^2*g^2*x^4*csgn
(I*c*(e*x^2+d)^p)^5*csgn(I*c)-4*Pi^2*e^2*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(
e*x^2+d)^p)^4*csgn(I*c)+2*Pi^2*e^2*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+
d)^p)^3*csgn(I*c)^2-4*I*Pi*ln(c)*e^2*f^2*csgn(I*c*(e*x^2+d)^p)^3+4*I*Pi*e^2
*f^2*p*csgn(I*c*(e*x^2+d)^p)^3-Pi^2*e^2*g^2*x^4*csgn(I*c*(e*x^2+d)^p)^4*csg
n(I*c)^2-4*ln(e*x^2+d)*d*e*f*g*p^2+4*ln(c)*d*e*g^2*p*x^2-12*ln(c)*e^2*f*g*p
*x^2+4*ln(c)*d*e*f*g*p-8*I*Pi*ln(c)*e^2*f*g*x^2*csgn(I*c*(e*x^2+d)^p)^3-2*I
*Pi*d*e*g^2*p*x^2*csgn(I*c*(e*x^2+d)^p)^3+6*I*Pi*e^2*f*g*p*x^2*csgn(I*c*(e*
x^2+d)^p)^3-2*I*ln(e*x^2+d)*Pi*e^2*f^2*p*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+
4*I*Pi*ln(c)*e^2*g^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+4*I*Pi
*ln(c)*e^2*g^2*x^4*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-2*I*Pi*e^2*g^2*p*x^4*c
sgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-2*I*Pi*e^2*g^2*p*x^4*csgn(I*c*(e
*x^2+d)^p)^2*csgn(I*c)-2*Pi^2*e^2*f*g*x^2*csgn(I*(e*x^2+d)^p)^2*csgn(I*c*(e
*x^2+d)^p)^2*csgn(I*c)^2-8*Pi^2*e^2*f*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e
*x^2+d)^p)^4*csgn(I*c)+4*Pi^2*e^2*f*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x
^2+d)^p)^3*csgn(I*c)^2+4*Pi^2*e^2*f*g*x^2*csgn(I*(e*x^2+d)^p)^2*csgn(I*c*(e
*x^2+d)^p)^3*csgn(I*c)+4*I*Pi*e^2*f^2*p*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2
+d)^p)*csgn(I*c)-2*I*ln(e*x^2+d)*Pi*d^2*g^2*p*csgn(I*(e*x^2+d)^p)*csgn(I*c*
(e*x^2+d)^p)^2-2*I*ln(e*x^2+d)*Pi*d^2*g^2*p*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*
c)-2*I*ln(e*x^2+d)*Pi*e^2*f^2*p*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2
-4*I*Pi*ln(c)*e^2*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-2
*I*Pi*d*e*f*g*p*csgn(I*c*(e*x^2+d)^p)^3+8*I*Pi*ln(c)*e^2*f*g*x^2*csgn(I*c*(

```
e*x^2+d)^p)^2*csgn(I*c)+2*I*Pi*d*e*g^2*p*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(
e*x^2+d)^p)^2-4*I*ln(e*x^2+d)*Pi*d*e*f*g*p*csgn(I*c*(e*x^2+d)^p)^3+2*I*ln(e
*x^2+d)*Pi*e^2*f^2*p*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+2*
I*Pi*d*e*f*g*p*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-4*I*Pi*ln(c)*e^2*g^2*x^4*c
sgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+2*I*Pi*e^2*g^2*p*x^4*csg
n(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+8*I*Pi*ln(c)*e^2*f*g*x^2*c
sgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+2*I*Pi*d*e*g^2*p*x^2*csgn(I*c*(e
*x^2+d)^p)^2*csgn(I*c)-6*I*Pi*e^2*f*g*p*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e
*x^2+d)^p)^2-6*I*Pi*e^2*f*g*p*x^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+2*I*Pi*
d*e*f*g*p*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+2*I*ln(e*x^2+d)*Pi*d^
2*g^2*p*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+8*ln(e*x^2+d)*l
n(c)*d*e*f*g*p-4*ln(e*x^2+d)*ln(c)*d^2*g^2*p-4*ln(e*x^2+d)*ln(c)*e^2*f^2*p-
4*ln(c)*e^2*g^2*p*x^4+8*ln(c)^2*e^2*f*g*x^2+2*ln(e*x^2+d)*d^2*g^2*p^2+2*ln(
e*x^2+d)*e^2*f^2*p^2+4*ln(c)^2*e^2*g^2*x^4-8*ln(c)*e^2*f^2*p+e^2*g^2*p^2*x^
4+d^2*g^2*p^2-2*d*e*g^2*p^2*x^2+4*e^2*f*g*p^2*x^2-4*d*e*f*g*p^2)/e^2/g/(I*P
i*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-I*Pi*csgn(I*(e*x^2+d)^p)*csgn
(I*c*(e*x^2+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^...
```

Maxima [A]
time = 0.27, size = 101, normalized size = 1.07

$$-\frac{\left(2\left(d^2g^2-2\,dfge+f^2e^2\right)e^{(-3)}\log\left(x^2e+d\right)+\left(g^2x^4e-2\left(dg^2-2\,fge\right)x^2\right)e^{(-2)}\right)pe}{8\,g}+\frac{\left(gx^2+f\right)^2\log\left(\left(x^2e+d\right)^pc\right)}{4\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
[Out] -1/8*(2*(d^2*g^2 - 2*d*f*g*e + f^2*e^2)*e^(-3)*log(x^2*e + d) + (g^2*x^4*e
- 2*(d*g^2 - 2*f*g*e)*x^2)*e^(-2))*p*e/g + 1/4*(g*x^2 + f)^2*log((x^2*e + d
)^p*c)/g
```

Fricas [A]
time = 0.38, size = 94, normalized size = 1.00

$$\frac{1}{8}\left(2\,dgpx^2e+2\left(gx^4+2\,fx^2\right)e^2\log\left(c\right)-\left(gpx^4+4\,fpx^2\right)e^2-2\left(d^2gp-2\,dfpe-\left(gpx^4+2\,fpx^2\right)e^2\right)\log\left(x^2e+d\right)\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
[Out] 1/8*(2*d*g*p*x^2*e + 2*(g*x^4 + 2*f*x^2)*e^2*log(c) - (g*p*x^4 + 4*f*p*x^2)
*e^2 - 2*(d^2*g*p - 2*d*f*p*e - (g*p*x^4 + 2*f*p*x^2)*e^2)*log(x^2*e + d))*
e^(-2)
```

Sympy [A]
time = 17.02, size = 126, normalized size = 1.34

$$\begin{cases} -\frac{d^2g\log\left(c\left(d+ex^2\right)^p\right)}{4e^2}+\frac{df\log\left(c\left(d+ex^2\right)^p\right)}{2e}+\frac{dgpx^2}{4e}-\frac{fpx^2}{2}+\frac{fx^2\log\left(c\left(d+ex^2\right)^p\right)}{2}-\frac{gpx^4}{8}+\frac{gx^4\log\left(c\left(d+ex^2\right)^p\right)}{4} & \text{for}\ e\neq 0 \\ \left(\frac{fx^2}{2}+\frac{gx^4}{4}\right)\log\left(cd^p\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x**2+f)*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise((-d**2*g*log(c*(d + e*x**2)**p)/(4*e**2) + d*f*log(c*(d + e*x**2)
**p)/(2*e) + d*g*p*x**2/(4*e) - f*p*x**2/2 + f*x**2*log(c*(d + e*x**2)**p)/
2 - g*p*x**4/8 + g*x**4*log(c*(d + e*x**2)**p)/4, Ne(e, 0)), ((f*x**2/2 + g
*x**4/4)*log(c*d**p), True))
```

Giac [**A**]
time = 4.26, size = 155, normalized size = 1.65

$$\frac{1}{8}\left(2\left(x^2e+d\right)^2gp\log\left(x^2e+d\right)-\left(x^2e+d\right)^2gp+2\left(x^2e+d\right)^2g\log\left(c\right)\right)e^{(-2)}+\frac{1}{2}\left(\left(x^2e-\left(x^2e+d\right)\log\left(x^2e+d\right)+d\right)dgp-\left(x^2e-\left(x^2e+d\right)\log\left(x^2e+d\right)+d\right)fpe-\left(x^2e+d\right)dg\log\left(c\right)+\left(x^2e+d\right)fe\log\left(c\right)\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 1/8*(2*(x^2*e + d)^2*g*p*log(x^2*e + d) - (x^2*e + d)^2*g*p + 2*(x^2*e + d)
^2*g*log(c))*e^(-2) + 1/2*((x^2*e - (x^2*e + d)*log(x^2*e + d) + d)*d*g*p -
 (x^2*e - (x^2*e + d)*log(x^2*e + d) + d)*f*p*e - (x^2*e + d)*d*g*log(c) +
(x^2*e + d)*f*e*log(c))*e^(-2)
```

Mupad [**B**]
time = 0.31, size = 78, normalized size = 0.83

$$\ln\left(c\left(e\,x^2+d\right)^p\right)\,\left(\frac{g\,x^4}{4}+\frac{f\,x^2}{2}\right)-x^2\,\left(\frac{f\,p}{2}-\frac{d\,g\,p}{4\,e}\right)-\frac{g\,p\,x^4}{8}-\frac{\ln\left(e\,x^2+d\right)\,\left(d^2\,g\,p-2\,d\,e\,f\,p\right)}{4\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(c*(d + e*x^2)^p)*(f + g*x^2),x)
```

```
[Out] log(c*(d + e*x^2)^p)*((f*x^2)/2 + (g*x^4)/4) - x^2*((f*p)/2 - (d*g*p)/(4*e)
) - (g*p*x^4)/8 - (log(d + e*x^2)*(d^2*g*p - 2*d*e*f*p))/(4*e^2)
```

3.313 $\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x}\,dx$

Optimal. Leaf size=82

$$-\frac{1}{2}gpx^2+\frac{g(d+ex^2)\log\left(c(d+ex^2)^p\right)}{2e}+\frac{1}{2}f\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)+\frac{1}{2}fp\mathrm{Li}_2\left(1+\frac{ex^2}{d}\right)$$

```
[Out] -1/2*g*p*x^2+1/2*g*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e+1/2*f*ln(-e*x^2/d)*ln(c*(e
*x^2+d)^p)+1/2*f*p*polylog(2,1+e*x^2/d)
```

Rubi [A]
time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2525, 45, 2463, 2436, 2332, 2441, 2352}

$$\frac{1}{2}fp\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)+\frac{1}{2}f\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)+\frac{g(d+ex^2)\log\left(c(d+ex^2)^p\right)}{2e}-\frac{1}{2}gpx^2$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x,x]
```

```
[Out] -1/2*(g*p*x^2) + (g*(d + e*x^2)*Log[c*(d + e*x^2)^p])/(2*e) + (f*Log[-((e*x
^2)/d)]*Log[c*(d + e*x^2)^p])/2 + (f*p*PolyLog[2, 1 + (e*x^2)/d])/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(f+gx)\log\left(c(d+ex)^p\right)}{x}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(g\log\left(c(d+ex)^p\right)+\frac{f\log\left(c(d+ex)^p\right)}{x}\right)\,dx, x, x^2\right)\\
&= \frac{1}{2}f\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x}\,dx, x, x^2\right)+\frac{1}{2}g\text{Subst}\left(\int \log\left(c(d+ex)^p\right.\right.\\
&= \frac{1}{2}f\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{g\text{Subst}\left(\int \log\left(cx^p\right)\,dx, x, d+ex^2\right)}{2e}\\
&= -\frac{1}{2}gpx^2+\frac{g(d+ex^2)\log\left(c\left(d+ex^2\right)^p\right)}{2e}+\frac{1}{2}f\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+e\right.\right.
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 80, normalized size = 0.98

$$\frac{1}{2}g\left(-px^2+\frac{(d+ex^2)\log\left(c\left(d+ex^2\right)^p\right)}{e}\right)+\frac{1}{2}f\left(\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+p\text{Li}_2\left(\frac{d+ex^2}{d}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x,x]
```

```
[Out] (g*(-(p*x^2) + ((d + e*x^2)*Log[c*(d + e*x^2)^p])/e))/2 + (f*(Log[-((e*x^2)
/d)]*Log[c*(d + e*x^2)^p] + p*PolyLog[2, (d + e*x^2)/d]))/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 419, normalized size = 5.11

method	result
risch	$\frac{\ln\left(\left(e\,x^{2}+d\right)^{p}\right) g\,x^{2}}{2}+\ln\left(\left(e\,x^{2}+d\right)^{p}\right) f\ln\left(x\right)-\frac{g p\,x^{2}}{2}+\frac{p g d\ln\left(e\,x^{2}+d\right)}{2 e}-p f\ln\left(x\right)\ln\left(\frac{-e x+\sqrt{-e d}}{\sqrt{-e d}}\right)-p f$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln((e*x^2+d)^p)*g*x^2+ln((e*x^2+d)^p)*f*ln(x)-1/2*g*p*x^2+1/2*p/e*g*d*l
n(e*x^2+d)-p*f*ln(x)*ln((-e*x+(-e*d)^(1/2))/(-e*d)^(1/2))-p*f*ln(x)*ln((e*x
+(-e*d)^(1/2))/(-e*d)^(1/2))-p*f*dilog((-e*x+(-e*d)^(1/2))/(-e*d)^(1/2))-p*
f*dilog((e*x+(-e*d)^(1/2))/(-e*d)^(1/2))+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*c
sgn(I*c)*f*ln(x)+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*f*ln(
x)-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*f*ln(x)-1/4*I*Pi*g*x^2*csgn(I*(e*x^2+d)
^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/4*I*Pi*g*x^2*csgn(I*(e*x^2+d)^p)*csgn
(I*c*(e*x^2+d)^p)^2+1/4*I*Pi*g*x^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/2*I*
Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*f*ln(x)-1/4*I*Pi*g*x
^2*csgn(I*c*(e*x^2+d)^p)^3+1/2*ln(c)*g*x^2+ln(c)*f*ln(x)
```

Maxima [A]
time = 0.59, size = 95, normalized size = 1.16

$$\frac{1}{2}\left(\log\left(x^{2}e+d\right)\log\left(-\frac{x^{2}e+d}{d}+1\right)+\mathrm{Li}_{2}\left(\frac{x^{2}e+d}{d}\right)\right)fp+f\log\left(c\right)\log\left(x\right)-\frac{1}{2}\left(\left(gp-g\log\left(c\right)\right)x^{2}e-\left(gpx^{2}e+dgp\right)\log\left(x^{2}e+d\right)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x,x, algorithm="maxima")
```

```
[Out] 1/2*(log(x^2*e + d)*log(-(x^2*e + d)/d + 1) + dilog((x^2*e + d)/d))*f*p + f
*log(c)*log(x) - 1/2*((g*p - g*log(c))*x^2*e - (g*p*x^2*e + d*g*p)*log(x^2*
e + d))*e^(-1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x,x, algorithm="fricas")
[Out] integral((g*x^2 + f)*log((x^2*e + d)^p*c)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x,x)
[Out] Integral((f + g*x**2)*log(c*(d + e*x**2)**p)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x,x, algorithm="giac")
[Out] integrate((g*x^2 + f)*log((x^2*e + d)^p*c)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,(e\,x^2+d)^p\right)\,\left(g\,x^2+f\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x,x)
[Out] int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x, x)
```

3.314 $\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^3} dx$

Optimal. Leaf size=93

$$\frac{efp\log(x)}{d}-\frac{efp\log\left(d+ex^2\right)}{2d}-\frac{f\log\left(c(d+ex^2)^p\right)}{2x^2}+\frac{1}{2}g\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)+\frac{1}{2}gp\text{Li}_2\left(1+\frac{ex^2}{d}\right)$$

```
[Out] e*f*p*ln(x)/d-1/2*e*f*p*ln(e*x^2+d)/d-1/2*f*ln(c*(e*x^2+d)^p)/x^2+1/2*g*ln(
-e*x^2/d)*ln(c*(e*x^2+d)^p)+1/2*g*p*polylog(2,1+e*x^2/d)
```

Rubi [A]

time = 0.08, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2525, 45, 2463, 2442, 36, 29, 31, 2441, 2352}

$$\frac{1}{2}gp\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)-\frac{f\log\left(c(d+ex^2)^p\right)}{2x^2}+\frac{1}{2}g\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)-\frac{efp\log\left(d+ex^2\right)}{2d}+\frac{efp\log(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^3,x]
```

```
[Out] (e*f*p*Log[x])/d - (e*f*p*Log[d + e*x^2])/(2*d) - (f*Log[c*(d + e*x^2)^p])/
(2*x^2) + (g*Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])/2 + (g*p*PolyLog[2, 1
+ (e*x^2)/d])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(f+gx)\log(c(d+ex)^p)}{x^2}\,dx,x,x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{f\log(c(d+ex)^p)}{x^2}+\frac{g\log(c(d+ex)^p)}{x}\right)\,dx,x,x^2\right)$$
$$= \frac{1}{2}f\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x^2}\,dx,x,x^2\right)+\frac{1}{2}g\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\right.$$
$$= -\frac{f\log\left(c(d+ex^2)^p\right)}{2x^2}+\frac{1}{2}g\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)+\frac{1}{2}(efp)\text{Subs}$$
$$= -\frac{f\log\left(c(d+ex^2)^p\right)}{2x^2}+\frac{1}{2}g\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)+\frac{1}{2}gp\text{Li}_2\left(1+\right.$$
$$= \frac{efp\log(x)}{d}-\frac{efp\log(d+ex^2)}{2d}-\frac{f\log\left(c(d+ex^2)^p\right)}{2x^2}+\frac{1}{2}g\log\left(-\frac{ex^2}{d}\right)$$

Mathematica [A]
time = 0.02, size = 92, normalized size = 0.99

$$\frac{efp\log(x)}{d}-\frac{efp\log(d+ex^2)}{2d}-\frac{f\log\left(c(d+ex^2)^p\right)}{2x^2}+\frac{1}{2}g\left(\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)+p\text{Li}_2\left(\frac{d+ex^2}{d}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^3,x]
```

```
[Out] (e*f*p*Log[x])/d - (e*f*p*Log[d + e*x^2])/(2*d) - (f*Log[c*(d + e*x^2)^p])/
(2*x^2) + (g*(Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p] + p*PolyLog[2, (d + e*
x^2)/d]))/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 421, normalized size = 4.53

method	result
risch	$\ln\left(\left(e\,x^2+d\right)^p\right)g\ln(x)-\frac{\ln\left(\left(e\,x^2+d\right)^p\right)f}{2x^2}+\frac{efp\ln(x)}{d}-\frac{efp\ln(e\,x^2+d)}{2d}-pg\ln(x)\ln\left(\frac{-ex+\sqrt{-ed}}{\sqrt{-ed}}\right)-pg$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] ln((e*x^2+d)^p)*g*ln(x)-1/2*ln((e*x^2+d)^p)*f/x^2+e*f*p*ln(x)/d-1/2*e*f*p*l
n(e*x^2+d)/d-p*g*ln(x)*ln((-e*x+(-e*d)^(1/2))/(-e*d)^(1/2))-p*g*ln(x)*ln((e
*x+(-e*d)^(1/2))/(-e*d)^(1/2))-p*g*dilog((-e*x+(-e*d)^(1/2))/(-e*d)^(1/2))-
```

```
p*g*dilog((e*x+(-e*d)^(1/2))/(-e*d)^(1/2))+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^3
*f/x^2+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*g*ln(x)-1/4*I*P
i*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*f/x^2-1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn
(I*c*(e*x^2+d)^p)^2*f/x^2-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p
)*csgn(I*c)*g*ln(x)+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*g*ln(x)+1/4*
I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*f/x^2-1/2*I*Pi*csg
n(I*c*(e*x^2+d)^p)^3*g*ln(x)+ln(c)*g*ln(x)-1/2*ln(c)*f/x^2
```

Maxima [A]
time = 0.54, size = 99, normalized size = 1.06

$$\frac{1}{2}\left(\log\left(x^2e+d\right)\log\left(-\frac{x^2e+d}{d}+1\right)+\text{Li}_2\left(\frac{x^2e+d}{d}\right)\right)gp+\frac{(fpe+dg\log(c))\log(x)}{d}-\frac{df\log(c)+(fpx^2e+dfp)\log(x^2e+d)}{2\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(log(x^2*e + d)*log(-(x^2*e + d)/d + 1) + dilog((x^2*e + d)/d))*g*p + (
f*p*e + d*g*log(c))*log(x)/d - 1/2*(d*f*log(c) + (f*p*x^2*e + d*f*p)*log(x^
2*e + d))/(d*x^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^3,x, algorithm="fricas")
```

```
[Out] integral((g*x^2 + f)*log((x^2*e + d)^p*c)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**3,x)
```

```
[Out] Integral((f + g*x**2)*log(c*(d + e*x**2)**p)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^3,x, algorithm="giac")
```

```
[Out] integrate((g*x^2 + f)*log((x^2*e + d)^p*c)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(g\,x^2+f\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x^3,x)
```

```
[Out] int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x^3, x)
```

3.315 $\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^5}\,dx$

Optimal. Leaf size=93

$$-\frac{efp}{4dx^2}-\frac{e(ef-2dg)p\log(x)}{2d^2}+\frac{(ef-dg)^2p\log(d+ex^2)}{4d^2f}-\frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{4fx^4}$$

```
[Out] -1/4*e*f*p/d/x^2-1/2*e*(-2*d*g+e*f)*p*ln(x)/d^2+1/4*(-d*g+e*f)^2*p*ln(e*x^2
+d)/d^2/f-1/4*(g*x^2+f)^2*ln(c*(e*x^2+d)^p)/f/x^4
```

Rubi [A]
time = 0.09, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2525, 37, 2461, 12, 90}

$$-\frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{4fx^4}+\frac{p(ef-dg)^2\log(d+ex^2)}{4d^2f}-\frac{ep\log(x)(ef-2dg)}{2d^2}-\frac{efp}{4dx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^5,x]
```

```
[Out] -1/4*(e*f*p)/(d*x^2) - (e*(e*f - 2*d*g)*p*Log[x])/(2*d^2) + ((e*f - d*g)^2*
p*Log[d + e*x^2])/(4*d^2*f) - ((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/(4*f*x^4
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2461

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(f+gx)\log\left(c(d+ex)^p\right)}{x^3}\,dx,x,x^2\right)\\ &= -\frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{4fx^4}-\frac{1}{2}(ep)\text{Subst}\left(\int -\frac{(f+gx)^2}{2fx^2(d+ex)}\,dx,x,x\right.\\ &= -\frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{4fx^4}+\frac{(ep)\text{Subst}\left(\int \frac{(f+gx)^2}{x^2(d+ex)}\,dx,x,x^2\right)}{4f}\\ &= -\frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{4fx^4}+\frac{(ep)\text{Subst}\left(\int \left(\frac{f^2}{dx^2}+\frac{f(-ef+2dg)}{d^2x}+\frac{(-ef+}{d^2(d+}\right.\right.}{4f}\\ &= -\frac{efp}{4dx^2}-\frac{e(ef-2dg)p\log(x)}{2d^2}+\frac{(ef-dg)^2p\log\left(d+ex^2\right)}{4d^2f}-\frac{(f+gx^2)^2}{}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 105, normalized size = 1.13

$$\frac{egp\log(x)}{d}-\frac{egp\log\left(d+ex^2\right)}{2d}+\frac{1}{4}efp\left(-\frac{1}{dx^2}-\frac{2e\log(x)}{d^2}+\frac{e\log\left(d+ex^2\right)}{d^2}\right)-\frac{f\log\left(c(d+ex^2)^p\right)}{4x^4}-\frac{g\log\left(c(d+ex^2)^p\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^5,x]
```

```
[Out] (e*g*p*Log[x])/d - (e*g*p*Log[d + e*x^2])/(2*d) + (e*f*p*(-(1/(d*x^2)) - (2
*e*Log[x])/d^2 + (e*Log[d + e*x^2])/d^2))/4 - (f*Log[c*(d + e*x^2)^p])/(4*x
^4) - (g*Log[c*(d + e*x^2)^p])/(2*x^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.15, size = 392, normalized size = 4.22

method	result
risch	$-\frac{(2g\,x^2+f)\ln((e\,x^2+d)^p)}{4x^4} - \frac{2i\pi\,d^2g\,x^2\mathrm{csgn}(i(e\,x^2+d)^p)\mathrm{csgn}(ic(e\,x^2+d)^p)^2-2i\pi\,d^2g\,x^2\mathrm{csgn}(i(e\,x^2+d)^p)\mathrm{csgn}(ic(e\,x^2+d)^p)\mathrm{cs}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(2*g*x^2+f)/x^4*ln((e*x^2+d)^p)-1/8*(2*I*Pi*d^2*g*x^2*csgn(I*(e*x^2+d)
^p)*csgn(I*c*(e*x^2+d)^p)^2-2*I*Pi*d^2*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(
e*x^2+d)^p)*csgn(I*c)-2*I*Pi*d^2*g*x^2*csgn(I*c*(e*x^2+d)^p)^3+2*I*Pi*d^2*g
*x^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+4*ln(e*x^2+d)*d*e*g*p*x^4-2*ln(e*x^2
+d)*e^2*f*p*x^4-8*ln(x)*d*e*g*p*x^4+4*ln(x)*e^2*f*p*x^4+I*Pi*d^2*f*csgn(I*(
e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-I*Pi*d^2*f*csgn(I*(e*x^2+d)^p)*csgn(I*c
*(e*x^2+d)^p)*csgn(I*c)-I*Pi*d^2*f*csgn(I*c*(e*x^2+d)^p)^3+I*Pi*d^2*f*csgn(
I*c*(e*x^2+d)^p)^2*csgn(I*c)+4*ln(c)*d^2*g*x^2+2*d*e*f*p*x^2+2*ln(c)*d^2*f)
/d^2/x^4
```

Maxima [A]
time = 0.28, size = 83, normalized size = 0.89

$$-\frac{1}{4}\,p\left(\frac{(2\,dg-fe)\log\left(x^2e+d\right)}{d^2}-\frac{(2\,dg-fe)\log\left(x^2\right)}{d^2}+\frac{f}{dx^2}\right)e-\frac{(2\,gx^2+f)\log\left(\left(x^2e+d\right)^pc\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*p*((2*d*g - f*e)*log(x^2*e + d)/d^2 - (2*d*g - f*e)*log(x^2)/d^2 + f/(
d*x^2))*e - 1/4*(2*g*x^2 + f)*log((x^2*e + d)^p*c)/x^4
```

Fricas [A]
time = 0.39, size = 106, normalized size = 1.14

$$-\frac{dfpx^2e+(2\,dgpx^4e-fpx^4e^2+2\,d^2gpx^2+d^2fp)\log\left(x^2e+d\right)+(2\,d^2gx^2+d^2f)\log\left(c\right)-2\,(2\,dgpx^4e-fpx^4e^2)\log\left(x\right)}{4\,d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^5,x, algorithm="fricas")
```

```
[Out] -1/4*(d*f*p*x^2*e + (2*d*g*p*x^4*e - f*p*x^4*e^2 + 2*d^2*g*p*x^2 + d^2*f*p)
*log(x^2*e + d) + (2*d^2*g*x^2 + d^2*f)*log(c) - 2*(2*d*g*p*x^4*e - f*p*x^4
*e^2)*log(x))/(d^2*x^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(83) = 166$.
time = 79.75, size = 167, normalized size = 1.80

$$\begin{cases} -\frac{f\log\left(c(d+ex^2)^p\right)}{4x^4} - \frac{g\log\left(c(d+ex^2)^p\right)}{2x^2} - \frac{efp}{4dx^2} + \frac{egp\log(x)}{d} - \frac{eg\log\left(c(d+ex^2)^p\right)}{2d} - \frac{e^2fp\log(x)}{2d^2} + \frac{e^2f\log\left(c(d+ex^2)^p\right)}{4d^2} & \text{for } d \neq 0 \\ -\frac{fp}{8x^4} - \frac{f\log\left(c(ex^2)^p\right)}{4x^4} - \frac{gp}{2x^2} - \frac{g\log\left(c(ex^2)^p\right)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**5,x)
```

```
[Out] Piecewise((-f*log(c*(d + e*x**2)**p)/(4*x**4) - g*log(c*(d + e*x**2)**p)/(2
*x**2) - e*f*p/(4*d*x**2) + e*g*p*log(x)/d - e*g*log(c*(d + e*x**2)**p)/(2*
d) - e**2*f*p*log(x)/(2*d**2) + e**2*f*log(c*(d + e*x**2)**p)/(4*d**2), Ne(
d, 0)), (-f*p/(8*x**4) - f*log(c*(e*x**2)**p)/(4*x**4) - g*p/(2*x**2) - g*l
og(c*(e*x**2)**p)/(2*x**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(92) = 184$.
time = 3.79, size = 322, normalized size = 3.46

$$-\frac{\left(2\,(x^2e+d)^2dgpe^2\log(x^2e+d) - 2\,(x^2e+d)d^2gpe^2\log(x^2e+d) - 2\,(x^2e+d)^2dgpe^2\log(x^2e) + 4\,(x^2e+d)d^2gpe^2\log(x^2e) - 2\,d^3gpe^2\log(x^2e) - (x^2e+d)^2fpe^3\log(x^2e+d) + 2\,(x^2e+d)dfpe^3\log(x^2e+d) + (x^2e+d)^2fpe^3\log(x^2e) - 2\,(x^2e+d)dfpe^3\log(x^2e) + d^2fpe^3\log(x^2e) + 2\,(x^2e+d)d^2ge^2\log(c) - 2\,d^3ge^2\log(c) + (x^2e+d)dfpe^3 - d^2fpe^3 + d^2fe^3\log(c)\right)e^{(-1)}}{4\left((x^2e+d)^2d^2 - 2\,(x^2e+d)d^3 + d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^5,x, algorithm="giac")
```

```
[Out] -1/4*(2*(x^2*e + d)^2*d*g*p*e^2*log(x^2*e + d) - 2*(x^2*e + d)*d^2*g*p*e^2*
log(x^2*e + d) - 2*(x^2*e + d)^2*d*g*p*e^2*log(x^2*e) + 4*(x^2*e + d)*d^2*g
*p*e^2*log(x^2*e) - 2*d^3*g*p*e^2*log(x^2*e) - (x^2*e + d)^2*f*p*e^3*log(x^
2*e + d) + 2*(x^2*e + d)*d*f*p*e^3*log(x^2*e + d) + (x^2*e + d)^2*f*p*e^3*l
og(x^2*e) - 2*(x^2*e + d)*d*f*p*e^3*log(x^2*e) + d^2*f*p*e^3*log(x^2*e) + 2
*(x^2*e + d)*d^2*g*e^2*log(c) - 2*d^3*g*e^2*log(c) + (x^2*e + d)*d*f*p*e^3
- d^2*f*p*e^3 + d^2*f*e^3*log(c))*e^(-1)/((x^2*e + d)^2*d^2 - 2*(x^2*e + d)
*d^3 + d^4)
```

Mupad [B]
time = 0.37, size = 85, normalized size = 0.91

$$\frac{\ln\left(e\,x^2+d\right)\,\left(e^2\,f\,p-2\,d\,e\,g\,p\right)}{4\,d^2}-\frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(\frac{g\,x^2}{2}+\frac{f}{4}\right)}{x^4}-\frac{\ln\left(x\right)\,\left(e^2\,f\,p-2\,d\,e\,g\,p\right)}{2\,d^2}-\frac{e\,f\,p}{4\,d\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x^5,x)
```

```
[Out] (log(d + e*x^2)*(e^2*f*p - 2*d*e*g*p))/(4*d^2) - (log(c*(d + e*x^2)^p)*(f/4
 + (g*x^2)/2))/x^4 - (log(x)*(e^2*f*p - 2*d*e*g*p))/(2*d^2) - (e*f*p)/(4*d*
x^2)
```

3.316 $\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^7}\,dx$

Optimal. Leaf size=125

$$-\frac{efp}{12dx^4}+\frac{e(2ef-3dg)p}{12d^2x^2}+\frac{e^2(2ef-3dg)p\log(x)}{6d^3}-\frac{e^2(2ef-3dg)p\log(d+ex^2)}{12d^3}-\frac{f\log\left(c(d+ex^2)^p\right)}{6x^6}-g\log$$

```
[Out] -1/12*e*f*p/d/x^4+1/12*e*(-3*d*g+2*e*f)*p/d^2/x^2+1/6*e^2*(-3*d*g+2*e*f)*p*
ln(x)/d^3-1/12*e^2*(-3*d*g+2*e*f)*p*ln(e*x^2+d)/d^3-1/6*f*ln(c*(e*x^2+d)^p)
/x^6-1/4*g*ln(c*(e*x^2+d)^p)/x^4
```

Rubi [A]
time = 0.11, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2525, 45, 2461, 12, 78}

$$-\frac{f\log\left(c(d+ex^2)^p\right)}{6x^6}-\frac{g\log\left(c(d+ex^2)^p\right)}{4x^4}-\frac{e^2p(2ef-3dg)\log(d+ex^2)}{12d^3}+\frac{e^2p\log(x)(2ef-3dg)}{6d^3}+\frac{ep(2ef-3dg)}{12d^2x^2}-\frac{efp}{12dx^4}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^7,x]
```

```
[Out] -1/12*(e*f*p)/(d*x^4) + (e*(2*e*f - 3*d*g)*p)/(12*d^2*x^2) + (e^2*(2*e*f -
3*d*g)*p*Log[x])/(6*d^3) - (e^2*(2*e*f - 3*d*g)*p*Log[d + e*x^2])/(12*d^3)
- (f*Log[c*(d + e*x^2)^p])/(6*x^6) - (g*Log[c*(d + e*x^2)^p])/(4*x^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2461

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^7}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(f+gx)\log\left(c(d+ex)^p\right)}{x^4}\,dx, x, x^2\right)$$
$$= -\frac{f\log\left(c(d+ex^2)^p\right)}{6x^6} - \frac{g\log\left(c(d+ex^2)^p\right)}{4x^4} - \frac{1}{2}(ep)\text{Subst}\left(\int \frac{-2f-3g}{6x^3(d+e}\right.$$
$$= -\frac{f\log\left(c(d+ex^2)^p\right)}{6x^6} - \frac{g\log\left(c(d+ex^2)^p\right)}{4x^4} - \frac{1}{12}(ep)\text{Subst}\left(\int \frac{-2f-3}{x^3(d+e}\right.$$
$$= -\frac{f\log\left(c(d+ex^2)^p\right)}{6x^6} - \frac{g\log\left(c(d+ex^2)^p\right)}{4x^4} - \frac{1}{12}(ep)\text{Subst}\left(\int \left(-\frac{2f}{dx^3}\right.\right.$$
$$= -\frac{efp}{12dx^4} + \frac{e(2ef-3dg)p}{12d^2x^2} + \frac{e^2(2ef-3dg)p\log(x)}{6d^3} - \frac{e^2(2ef-3dg)p\log}{12d^3}$$

Mathematica [A]
time = 0.05, size = 130, normalized size = 1.04

$$\frac{1}{4}egp\left(-\frac{1}{dx^2}-\frac{2e\log(x)}{d^2}+\frac{e\log\left(d+ex^2\right)}{d^2}\right)+\frac{1}{6}efp\left(-\frac{1}{2dx^4}+\frac{e}{d^2x^2}+\frac{2e^2\log(x)}{d^3}-\frac{e^2\log\left(d+ex^2\right)}{d^3}\right)-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{6x^6}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^7,x]
```

```
[Out] (e*g*p*(-(1/(d*x^2)) - (2*e*Log[x])/d^2 + (e*Log[d + e*x^2])/d^2))/4 + (e*f
*p*(-1/2*1/(d*x^4) + e/(d^2*x^2) + (2*e^2*Log[x])/d^3 - (e^2*Log[d + e*x^2]
)/d^3))/6 - (f*Log[c*(d + e*x^2)^p])/(6*x^6) - (g*Log[c*(d + e*x^2)^p])/(4*
x^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.12, size = 428, normalized size = 3.42

method	result
risch	$-\frac{(3g\,x^2+2f)\ln\left((e\,x^2+d)^p\right)}{12x^6} - 12\ln(x)d\,e^2gp\,x^6-8\ln(x)e^3fp\,x^6-6\ln(-e\,x^2-d)\,d\,e^2gp\,x^6+4\ln(-e\,x^2-d)\,e^3fp\,x^6-2i\pi\,d^3f\mathrm{csgn}($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(3*g*x^2+2*f)/x^6*ln((e*x^2+d)^p)-1/24*(12*ln(x)*d*e^2*g*p*x^6-8*ln(x
)*e^3*f*p*x^6-6*ln(-e*x^2-d)*d*e^2*g*p*x^6+4*ln(-e*x^2-d)*e^3*f*p*x^6-2*I*P
i*d^3*f*csgn(I*c*(e*x^2+d)^p)^3-3*I*Pi*d^3*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I
*c*(e*x^2+d)^p)*csgn(I*c)+2*I*Pi*d^3*f*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-2*
I*Pi*d^3*f*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+3*I*Pi*d^3*g
*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+2*I*Pi*d^3*f*csgn(I*(e*x^2
+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+3*I*Pi*d^3*g*x^2*csgn(I*c*(e*x^2+d)^p)^2*csg
n(I*c)-3*I*Pi*d^3*g*x^2*csgn(I*c*(e*x^2+d)^p)^3+6*d^2*e*g*p*x^4-4*d*e^2*f*p
*x^4+6*ln(c)*d^3*g*x^2+2*d^2*e*f*p*x^2+4*ln(c)*d^3*f)/d^3/x^6
```

Maxima [A]
time = 0.32, size = 107, normalized size = 0.86

$$\frac{1}{12}p\left(\frac{(3\,dge-2\,fe^2)\log\left(x^2e+d\right)}{d^3}-\frac{(3\,dge-2\,fe^2)\log\left(x^2\right)}{d^3}-\frac{(3\,dg-2\,fe)x^2+df}{d^2x^4}\right)e-\frac{(3\,gx^2+2\,f)\log\left((x^2e+d)^pc\right)}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^7,x, algorithm="maxima")
```

```
[Out] 1/12*p*((3*d*g*e - 2*f*e^2)*log(x^2*e + d)/d^3 - (3*d*g*e - 2*f*e^2)*log(x^
2)/d^3 - ((3*d*g - 2*f*e)*x^2 + d*f)/(d^2*x^4))*e - 1/12*(3*g*x^2 + 2*f)*lo
g((x^2*e + d)^p*c)/x^6
```

Fricas [A]
time = 0.39, size = 134, normalized size = 1.07

$$\frac{2\,dfpx^4e^2-(3\,d^2gpx^4+d^2fpx^2)e+(3\,dgpx^6e^2-2\,fpx^6e^3-3\,d^3gpx^2-2\,d^3fp)\log\left(x^2e+d\right)-(3\,d^3gx^2+2\,d^3f)\log\left(c\right)-2\,(3\,dgpx^6e^2-2\,fpx^6e^3)\log\left(x\right)}{12\,d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(2*d*f*p*x^4*e^2 - (3*d^2*g*p*x^4 + d^2*f*p*x^2)*e + (3*d*g*p*x^6*e^2
- 2*f*p*x^6*e^3 - 3*d^3*g*p*x^2 - 2*d^3*f*p)*log(x^2*e + d) - (3*d^3*g*x^2
+ 2*d^3*f)*log(c) - 2*(3*d*g*p*x^6*e^2 - 2*f*p*x^6*e^3)*log(x))/(d^3*x^6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**7,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 515 vs. 2(119) = 238.
time = 4.84, size = 515, normalized size = 4.12

$$\frac{\left(3\,(x^2e+d)^3dgpe^3\log\left(x^2e+d\right)-9\,(x^2e+d)^2d^2gpe^3\log\left(x^2e+d\right)+6\,(x^2e+d)d^3gpe^3\log\left(x^2e+d\right)-3\,(x^2e+d)^3dgpe^3\log\left(x^2e\right)+9\,(x^2e+d)^2d^2gpe^3\log\left(x^2e\right)-9\,(x^2e+d)d^3gpe^3\log\left(x^2e\right)+3\,d^4gpe^3\log\left(x^2e\right)-3\,(x^2e+d)^2d^2gpe^3+6\,(x^2e+d)d^3gpe^3-3\,d^4gpe^3-2\,(x^2e+d)^3fpe^4\log\left(x^2e+d\right)+6\,(x^2e+d)^2dfpe^4\log\left(x^2e+d\right)-6\,(x^2e+d)d^2fpe^4\log\left(x^2e+d\right)+2\,(x^2e+d)^3fpe^4\log\left(x^2e\right)-6\,(x^2e+d)^2dfpe^4\log\left(x^2e\right)+6\,(x^2e+d)d^2fpe^4\log\left(x^2e\right)-2\,d^3fpe^4\log\left(x^2e\right)-3\,(x^2e+d)d^3ge^3\log\left(c\right)+3\,d^4ge^3\log\left(c\right)+2\,(x^2e+d)^2dfpe^4-5\,(x^2e+d)d^2fpe^4+3\,d^3fpe^4-2\,d^3fe^4\log\left(c\right)\right)e^{(-1)}}{12\left((x^2e+d)^3d^3-3\,(x^2e+d)^2d^4+3\,(x^2e+d)d^5-d^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^7,x, algorithm="giac")
```

```
[Out] 1/12*(3*(x^2*e + d)^3*d*g*p*e^3*log(x^2*e + d) - 9*(x^2*e + d)^2*d^2*g*p*e^
3*log(x^2*e + d) + 6*(x^2*e + d)*d^3*g*p*e^3*log(x^2*e + d) - 3*(x^2*e + d)
^3*d*g*p*e^3*log(x^2*e) + 9*(x^2*e + d)^2*d^2*g*p*e^3*log(x^2*e) - 9*(x^2*e
 + d)*d^3*g*p*e^3*log(x^2*e) + 3*d^4*g*p*e^3*log(x^2*e) - 3*(x^2*e + d)^2*d
^2*g*p*e^3 + 6*(x^2*e + d)*d^3*g*p*e^3 - 3*d^4*g*p*e^3 - 2*(x^2*e + d)^3*f*
p*e^4*log(x^2*e + d) + 6*(x^2*e + d)^2*d*f*p*e^4*log(x^2*e + d) - 6*(x^2*e
+ d)*d^2*f*p*e^4*log(x^2*e + d) + 2*(x^2*e + d)^3*f*p*e^4*log(x^2*e) - 6*(x
^2*e + d)^2*d*f*p*e^4*log(x^2*e) + 6*(x^2*e + d)*d^2*f*p*e^4*log(x^2*e) - 2
*d^3*f*p*e^4*log(x^2*e) - 3*(x^2*e + d)*d^3*g*e^3*log(c) + 3*d^4*g*e^3*log(
c) + 2*(x^2*e + d)^2*d*f*p*e^4 - 5*(x^2*e + d)*d^2*f*p*e^4 + 3*d^3*f*p*e^4
- 2*d^3*f*e^4*log(c))*e^(-1)/((x^2*e + d)^3*d^3 - 3*(x^2*e + d)^2*d^4 + 3*(
x^2*e + d)*d^5 - d^6)
```

Mupad [B]
time = 0.37, size = 113, normalized size = 0.90

$$\frac{\ln\left(x\right)\,\left(2\,e^3\,f\,p-3\,d\,e^2\,g\,p\right)}{6\,d^3}-\frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(\frac{g\,x^2}{4}+\frac{f}{6}\right)}{x^6}-\frac{\ln\left(e\,x^2+d\right)\,\left(2\,e^3\,f\,p-3\,d\,e^2\,g\,p\right)}{12\,d^3}-\frac{\frac{e\,f\,p}{2\,d}+\frac{e\,p\,x^2\,\left(3\,d\,g-2\,e\,f\right)}{2\,d^2}}{6\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x^7,x)
```

```
[Out] (log(x)*(2*e^3*f*p - 3*d*e^2*g*p))/(6*d^3) - (log(c*(d + e*x^2)^p)*(f/6 + (
g*x^2)/4))/x^6 - (log(d + e*x^2)*(2*e^3*f*p - 3*d*e^2*g*p))/(12*d^3) - ((e*
f*p)/(2*d) + (e*p*x^2*(3*d*g - 2*e*f))/(2*d^2))/(6*x^4)
```

3.317 $\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^9}\,dx$

Optimal. Leaf size=148

$$-\frac{efp}{24dx^6}+\frac{e(3ef-4dg)p}{48d^2x^4}-\frac{e^2(3ef-4dg)p}{24d^3x^2}-\frac{e^3(3ef-4dg)p\log(x)}{12d^4}+\frac{e^3(3ef-4dg)p\log(d+ex^2)}{24d^4}-\frac{f\log(c$$

```
[Out] -1/24*e*f*p/d/x^6+1/48*e*(-4*d*g+3*e*f)*p/d^2/x^4-1/24*e^2*(-4*d*g+3*e*f)*p
/d^3/x^2-1/12*e^3*(-4*d*g+3*e*f)*p*ln(x)/d^4+1/24*e^3*(-4*d*g+3*e*f)*p*ln(e
*x^2+d)/d^4-1/8*f*ln(c*(e*x^2+d)^p)/x^8-1/6*g*ln(c*(e*x^2+d)^p)/x^6
```

Rubi [A]
time = 0.13, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2525, 45, 2461, 12, 78}

$$-\frac{f\log\left(c(d+ex^2)^p\right)}{8x^8}-\frac{g\log\left(c(d+ex^2)^p\right)}{6x^6}+\frac{e^3p(3ef-4dg)\log(d+ex^2)}{24d^4}-\frac{e^3p\log(x)(3ef-4dg)}{12d^4}-\frac{e^2p(3ef-4dg)}{24d^3x^2}+\frac{ep(3ef-4dg)}{48d^2x^4}-\frac{efp}{24dx^6}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^9,x]
```

```
[Out] -1/24*(e*f*p)/(d*x^6) + (e*(3*e*f - 4*d*g)*p)/(48*d^2*x^4) - (e^2*(3*e*f -
4*d*g)*p)/(24*d^3*x^2) - (e^3*(3*e*f - 4*d*g)*p*Log[x])/(12*d^4) + (e^3*(3*
e*f - 4*d*g)*p*Log[d + e*x^2])/(24*d^4) - (f*Log[c*(d + e*x^2)^p])/(8*x^8)
- (g*Log[c*(d + e*x^2)^p])/(6*x^6)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
 c, d, e, f]))))
```

Rule 2461

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^9}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(f+gx)\log\left(c(d+ex)^p\right)}{x^5}\,dx, x, x^2\right)$$

$$= -\frac{f\log\left(c(d+ex^2)^p\right)}{8x^8} - \frac{g\log\left(c(d+ex^2)^p\right)}{6x^6} - \frac{1}{2}(ep)\text{Subst}\left(\int \frac{-3f-4\ldots}{12x^4(d+\ldots}\right.$$

$$= -\frac{f\log\left(c(d+ex^2)^p\right)}{8x^8} - \frac{g\log\left(c(d+ex^2)^p\right)}{6x^6} - \frac{1}{24}(ep)\text{Subst}\left(\int \frac{-3f-4\ldots}{x^4(d+e\ldots}\right.$$

$$= -\frac{f\log\left(c(d+ex^2)^p\right)}{8x^8} - \frac{g\log\left(c(d+ex^2)^p\right)}{6x^6} - \frac{1}{24}(ep)\text{Subst}\left(\int \left(-\frac{3f}{dx^4} \ldots\right.\right.$$

$$= -\frac{efp}{24dx^6} + \frac{e(3ef-4dg)p}{48d^2x^4} - \frac{e^2(3ef-4dg)p}{24d^3x^2} - \frac{e^3(3ef-4dg)p\log(x)}{12d^4} + \ldots$$

Mathematica [A]
time = 0.08, size = 158, normalized size = 1.07

$$\frac{1}{6}egp\left(-\frac{1}{2dx^4}+\frac{e}{d^2x^2}+\frac{2e^2\log(x)}{d^3}-\frac{e^2\log\left(d+ex^2\right)}{d^3}\right)+\frac{1}{8}efp\left(-\frac{1}{3dx^6}+\frac{e}{2d^2x^4}-\frac{e^2}{d^3x^2}-\frac{2e^3\log(x)}{d^4}+\frac{e^3\log\left(d+ex^2\right)}{d^4}\right)-\frac{f\log\left(c(d+ex^2)^p\right)}{8x^8}-\frac{g\log\left(c(d+ex^2)^p\right)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^9,x]
```

```
[Out] (e*g*p*(-1/2*1/(d*x^4) + e/(d^2*x^2) + (2*e^2*Log[x])/d^3 - (e^2*Log[d + e*
x^2])/d^3))/6 + (e*f*p*(-1/3*1/(d*x^6) + e/(2*d^2*x^4) - e^2/(d^3*x^2) - (2
```

```
*e^3*Log[x])/d^4 + (e^3*Log[d + e*x^2])/d^4))/8 - (f*Log[c*(d + e*x^2)^p])/
(8*x^8) - (g*Log[c*(d + e*x^2)^p])/(6*x^6)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.19, size = 448, normalized size = 3.03

method	result
risch	$-\frac{(4g\,x^2+3f)\ln\left(\left(e\,x^2+d\right)^p\right)}{24x^8}-\frac{8\ln\left(e\,x^2+d\right)d\,e^3gp\,x^8-6\ln\left(e\,x^2+d\right)e^4fp\,x^8-16\ln(x)d\,e^3gp\,x^8+12\ln(x)e^4fp\,x^8-4i\pi\,d^4g\,x^2\mathrm{csgn}(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(4*g*x^2+3*f)/x^8*ln((e*x^2+d)^p)-1/48*(8*ln(e*x^2+d)*d*e^3*g*p*x^8-6
*ln(e*x^2+d)*e^4*f*p*x^8-16*ln(x)*d*e^3*g*p*x^8+12*ln(x)*e^4*f*p*x^8-4*I*Pi
*d^4*g*x^2*csgn(I*c*(e*x^2+d)^p)^3+4*I*Pi*d^4*g*x^2*csgn(I*c*(e*x^2+d)^p)^2
*csgn(I*c)+3*I*Pi*d^4*f*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+4*I*Pi*d^4*g*x^2*
csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-8*d^2*e^2*g*p*x^6+6*d*e^3*f*p*x
^6-4*I*Pi*d^4*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+3*I
*Pi*d^4*f*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-3*I*Pi*d^4*f*csgn(I*c
*(e*x^2+d)^p)^3-3*I*Pi*d^4*f*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn
(I*c)+4*d^3*e*g*p*x^4-3*d^2*e^2*f*p*x^4+8*ln(c)*d^4*g*x^2+2*d^3*e*f*p*x^2+6
*ln(c)*d^4*f)/d^4/x^8
```

Maxima [A]
time = 0.33, size = 132, normalized size = 0.89

$$-\frac{1}{48}\,p\left(\frac{2\,(4\,dge^2-3\,fe^3)\log\left(x^2e+d\right)}{d^4}-\frac{2\,(4\,dge^2-3\,fe^3)\log\left(x^2\right)}{d^4}-\frac{2\,(4\,dge-3\,fe^2)x^4-2\,d^2f-(4\,d^2g-3\,dfe)x^2}{d^3x^6}\right)e-\frac{(4\,gx^2+3\,f)\log\left(\left(x^2e+d\right)^pc\right)}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^9,x, algorithm="maxima")
```

```
[Out] -1/48*p*(2*(4*d*g*e^2 - 3*f*e^3)*log(x^2*e + d)/d^4 - 2*(4*d*g*e^2 - 3*f*e^
3)*log(x^2)/d^4 - (2*(4*d*g*e - 3*f*e^2)*x^4 - 2*d^2*f - (4*d^2*g - 3*d*f*e
)*x^2)/(d^3*x^6))*e - 1/24*(4*g*x^2 + 3*f)*log((x^2*e + d)^p*c)/x^8
```

Fricas [A]
time = 0.34, size = 160, normalized size = 1.08

$$-\frac{6\,dfpx^6e^3-(8\,d^2gpx^6+3\,d^2fpx^4)e^2+2\,(2\,d^3gpx^4+d^3fpx^2)e+2\,(4\,dgpx^8e^3-3\,fpx^8e^4+4\,d^4gpx^2+3\,d^4fp)\log\left(x^2e+d\right)+2\,(4\,d^4gx^2+3\,d^4f)\log\left(c\right)-4\,(4\,dgpx^8e^3-3\,fpx^8e^4)\log\left(x\right)}{48\,d^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^9,x, algorithm="fricas")
```

```
[Out] -1/48*(6*d*f*p*x^6*e^3 - (8*d^2*g*p*x^6 + 3*d^2*f*p*x^4)*e^2 + 2*(2*d^3*g*p
*x^4 + d^3*f*p*x^2)*e + 2*(4*d*g*p*x^8*e^3 - 3*f*p*x^8*e^4 + 4*d^4*g*p*x^2
```

```
+ 3*d^4*f*p)*log(x^2*e + d) + 2*(4*d^4*g*x^2 + 3*d^4*f)*log(c) - 4*(4*d*g*p
*x^8*e^3 - 3*f*p*x^8*e^4)*log(x))/(d^4*x^8)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**9,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 674 vs. 2(140) = 280.
time = 3.57, size = 674, normalized size = 4.55

$$-\frac{\left(8\,(x^2e+d)^4dgpe^4\log\left(x^2e+d\right)-32\,(x^2e+d)^3d^2gpe^4\log\left(x^2e+d\right)+48\,(x^2e+d)^2d^3gpe^4\log\left(x^2e+d\right)-24\,(x^2e+d)d^4gpe^4\log\left(x^2e+d\right)-8\,(x^2e+d)^4dgpe^4\log\left(x^2e\right)+32\,(x^2e+d)^3d^2gpe^4\log\left(x^2e\right)-48\,(x^2e+d)^2d^3gpe^4\log\left(x^2e\right)+32\,(x^2e+d)d^4gpe^4\log\left(x^2e\right)-8\,d^5gpe^4\log\left(x^2e\right)-8\,(x^2e+d)^3d^2gpe^4+28\,(x^2e+d)^2d^3gpe^4-32\,(x^2e+d)d^4gpe^4+12\,d^5gpe^4-6\,(x^2e+d)^4fpe^5\log\left(x^2e+d\right)+24\,(x^2e+d)^3dfpe^5\log\left(x^2e+d\right)-36\,(x^2e+d)^2d^2fpe^5\log\left(x^2e+d\right)+24\,(x^2e+d)d^3fpe^5\log\left(x^2e+d\right)+6\,(x^2e+d)^4fpe^5\log\left(x^2e\right)-24\,(x^2e+d)^3dfpe^5\log\left(x^2e\right)+36\,(x^2e+d)^2d^2fpe^5\log\left(x^2e\right)-24\,(x^2e+d)d^3fpe^5\log\left(x^2e\right)+6\,d^4fpe^5\log\left(x^2e\right)+8\,(x^2e+d)d^4ge^4\log\left(c\right)-8\,d^5ge^4\log\left(c\right)+6\,(x^2e+d)^3dfpe^5-21\,(x^2e+d)^2d^2fpe^5+26\,(x^2e+d)d^3fpe^5-11\,d^4fpe^5+6\,d^4fe^5\log\left(c\right)\right)e^{(-1)}}{48\,\left((x^2e+d)^4d^4-4\,(x^2e+d)^3d^5+6\,(x^2e+d)^2d^6-4\,(x^2e+d)d^7+d^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^9,x, algorithm="giac")
```

```
[Out] -1/48*(8*(x^2*e + d)^4*d*g*p*e^4*log(x^2*e + d) - 32*(x^2*e + d)^3*d^2*g*p*
e^4*log(x^2*e + d) + 48*(x^2*e + d)^2*d^3*g*p*e^4*log(x^2*e + d) - 24*(x^2*
e + d)*d^4*g*p*e^4*log(x^2*e + d) - 8*(x^2*e + d)^4*d*g*p*e^4*log(x^2*e) +
32*(x^2*e + d)^3*d^2*g*p*e^4*log(x^2*e) - 48*(x^2*e + d)^2*d^3*g*p*e^4*log(
x^2*e) + 32*(x^2*e + d)*d^4*g*p*e^4*log(x^2*e) - 8*d^5*g*p*e^4*log(x^2*e) -
 8*(x^2*e + d)^3*d^2*g*p*e^4 + 28*(x^2*e + d)^2*d^3*g*p*e^4 - 32*(x^2*e + d
)*d^4*g*p*e^4 + 12*d^5*g*p*e^4 - 6*(x^2*e + d)^4*f*p*e^5*log(x^2*e + d) + 2
4*(x^2*e + d)^3*d*f*p*e^5*log(x^2*e + d) - 36*(x^2*e + d)^2*d^2*f*p*e^5*log
(x^2*e + d) + 24*(x^2*e + d)*d^3*f*p*e^5*log(x^2*e + d) + 6*(x^2*e + d)^4*f
*p*e^5*log(x^2*e) - 24*(x^2*e + d)^3*d*f*p*e^5*log(x^2*e) + 36*(x^2*e + d)^
2*d^2*f*p*e^5*log(x^2*e) - 24*(x^2*e + d)*d^3*f*p*e^5*log(x^2*e) + 6*d^4*f*
p*e^5*log(x^2*e) + 8*(x^2*e + d)*d^4*g*e^4*log(c) - 8*d^5*g*e^4*log(c) + 6*
(x^2*e + d)^3*d*f*p*e^5 - 21*(x^2*e + d)^2*d^2*f*p*e^5 + 26*(x^2*e + d)*d^3
*f*p*e^5 - 11*d^4*f*p*e^5 + 6*d^4*f*e^5*log(c))*e^(-1)/((x^2*e + d)^4*d^4 -
 4*(x^2*e + d)^3*d^5 + 6*(x^2*e + d)^2*d^6 - 4*(x^2*e + d)*d^7 + d^8)
```

Mupad [B]
time = 0.40, size = 134, normalized size = 0.91

$$\frac{\ln\left(e\,x^2+d\right)\,\left(3\,e^4\,f\,p-4\,d\,e^3\,g\,p\right)}{24\,d^4}-\frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(\frac{g\,x^2}{6}+\frac{f}{8}\right)}{x^8}-\frac{\frac{e\,f\,p}{2\,d}+\frac{e\,p\,x^2\,\left(4\,d\,g-3\,e\,f\right)}{4\,d^2}-\frac{e^2\,p\,x^4\,\left(4\,d\,g-3\,e\,f\right)}{2\,d^3}}{12\,x^6}-\frac{\ln\left(x\right)\,\left(3\,e^4\,f\,p-4\,d\,e^3\,g\,p\right)}{12\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x^9,x)
```

```
[Out] (log(d + e*x^2)*(3*e^4*f*p - 4*d*e^3*g*p))/(24*d^4) - (log(c*(d + e*x^2)^p)
*(f/8 + (g*x^2)/6))/x^8 - ((e*f*p)/(2*d) + (e*p*x^2*(4*d*g - 3*e*f))/(4*d^2
) - (e^2*p*x^4*(4*d*g - 3*e*f))/(2*d^3))/(12*x^6) - (log(x)*(3*e^4*f*p - 4*
d*e^3*g*p))/(12*d^4)
```

3.318 $\int x^2(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx$

Optimal. Leaf size=154

$$\frac{2dfpx}{3e}-\frac{2d^2gpx}{5e^2}-\frac{2}{9}fpx^3+\frac{2dgpx^3}{15e}-\frac{2}{25}gpx^5-\frac{2d^{3/2}fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2d^{5/2}gp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{5e^{5/2}}+\frac{1}{3}fx^3\log\left(c(c$$

```
[Out] 2/3*d*f*p*x/e-2/5*d^2*g*p*x/e^2-2/9*f*p*x^3+2/15*d*g*p*x^3/e-2/25*g*p*x^5-2
/3*d^(3/2)*f*p*arctan(x*e^(1/2)/d^(1/2))/e^(3/2)+2/5*d^(5/2)*g*p*arctan(x*e
^(1/2)/d^(1/2))/e^(5/2)+1/3*f*x^3*ln(c*(e*x^2+d)^p)+1/5*g*x^5*ln(c*(e*x^2+d
)^p)
```

Rubi [A]

time = 0.09, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2526, 2505, 308, 211}

$$-\frac{2d^{3/2}fp\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2d^{5/2}gp\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{5e^{5/2}}+\frac{1}{3}fx^3\log\left(c(d+ex^2)^p\right)+\frac{1}{5}gx^5\log\left(c(d+ex^2)^p\right)-\frac{2d^2gpx}{5e^2}+\frac{2dfpx}{3e}+\frac{2dgpx^3}{15e}-\frac{2}{9}fpx^3-\frac{2}{25}gpx^5$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (2*d*f*p*x)/(3*e) - (2*d^2*g*p*x)/(5*e^2) - (2*f*p*x^3)/9 + (2*d*g*p*x^3)/(
15*e) - (2*g*p*x^5)/25 - (2*d^(3/2)*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^(
3/2)) + (2*d^(5/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*e^(5/2)) + (f*x^3*Lo
g[c*(d + e*x^2)^p])/3 + (g*x^5*Log[c*(d + e*x^2)^p])/5
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int x^2(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = \int \left(fx^2\log\left(c(d+ex^2)^p\right)+gx^4\log\left(c(d+ex^2)^p\right)\right)\,dx$$

$$= f\int x^2\log\left(c(d+ex^2)^p\right)\,dx + g\int x^4\log\left(c(d+ex^2)^p\right)\,dx$$

$$= \frac{1}{3}fx^3\log\left(c(d+ex^2)^p\right)+\frac{1}{5}gx^5\log\left(c(d+ex^2)^p\right)-\frac{1}{3}(2efp)\int \frac{x}{d+}$$

$$= \frac{1}{3}fx^3\log\left(c(d+ex^2)^p\right)+\frac{1}{5}gx^5\log\left(c(d+ex^2)^p\right)-\frac{1}{3}(2efp)\int \Big(-$$

$$= \frac{2dfpx}{3e}-\frac{2d^2gpx}{5e^2}-\frac{2}{9}fpx^3+\frac{2dgpx^3}{15e}-\frac{2}{25}gpx^5+\frac{1}{3}fx^3\log\big(c(d+e$$

$$= \frac{2dfpx}{3e}-\frac{2d^2gpx}{5e^2}-\frac{2}{9}fpx^3+\frac{2dgpx^3}{15e}-\frac{2}{25}gpx^5-\frac{2d^{3/2}fp\tan^{-1}\Big(}{3e^{3/2}}$$

Mathematica [A]
time = 0.04, size = 118, normalized size = 0.77

$$\frac{30d^{3/2}(-5ef+3dg)p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)+\sqrt{e}\,x\left(-2p(45d^2g-15de(5f+gx^2)+e^2x^2(25f+9gx^2))+15e^2x^2(5f+3gx^2)\log\left(c(d+ex^2)^p\right)\right)}{225e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (30*d^(3/2)*(-5*e*f + 3*d*g)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[e]*x*(-2*
p*(45*d^2*g - 15*d*e*(5*f + g*x^2) + e^2*x^2*(25*f + 9*g*x^2)) + 15*e^2*x^2
*(5*f + 3*g*x^2)*Log[c*(d + e*x^2)^p]))/(225*e^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.34, size = 453, normalized size = 2.94

method	result

risch	$\left(\frac{1}{5} g\, x^{5}+\frac{1}{3} f\, x^{3}\right) \ln\left(\left(e\, x^{2}+d\right)^{p}\right)-\frac{i \pi g\, x^{5} \operatorname{csgn}\left(i \left(e\, x^{2}+d\right)^{p}\right) \operatorname{csgn}\left(i c \left(e\, x^{2}+d\right)^{p}\right) \operatorname{csgn}\left(i c\right)}{10}-\frac{i \pi f\, x^{3} \operatorname{csgn}\left(i \left(e\, x^{2}+d\right)^{p}\right) \operatorname{csgn}\left(i\right.}{6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(g*x^2+f)*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] (1/5*g*x^5+1/3*f*x^3)*ln((e*x^2+d)^p)-1/10*I*Pi*g*x^5*csgn(I*(e*x^2+d)^p)*c
sgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/6*I*Pi*f*x^3*csgn(I*(e*x^2+d)^p)*csgn(I*c*
(e*x^2+d)^p)*csgn(I*c)+1/6*I*Pi*f*x^3*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d
)^p)^2+1/10*I*Pi*g*x^5*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/10*I*P
i*g*x^5*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/10*I*Pi*g*x^5*csgn(I*c*(e*x^2+d
)^p)^3+1/6*I*Pi*f*x^3*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/6*I*Pi*f*x^3*csgn
(I*c*(e*x^2+d)^p)^3+1/5*ln(c)*g*x^5-2/25*g*p*x^5+1/3*ln(c)*f*x^3+2/15*d*g*p
*x^3/e-2/9*f*p*x^3+1/5/e^3*ln(-(-e*d)^(1/2)*x+d)*(-e*d)^(1/2)*d^2*g*p-1/3/e
^2*(-e*d)^(1/2)*p*d*ln(-(-e*d)^(1/2)*x+d)*f-1/5/e^3*ln((-e*d)^(1/2)*x+d)*(-
e*d)^(1/2)*d^2*g*p+1/3/e^2*(-e*d)^(1/2)*p*d*ln((-e*d)^(1/2)*x+d)*f-2/5*d^2*
g*p*x/e^2+2/3*d*f*p*x/e
```

Maxima [A]
time = 0.52, size = 111, normalized size = 0.72

$$\frac{2}{225} \left(\frac{15 \left(3 d^{3} g-5 d^{2} f e\right) \arctan\left(\frac{x e^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{7}{2}\right)}}{\sqrt{d}}-\left(9 g x^{5} e^{2}-5 \left(3 d g e-5 f e^{2}\right) x^{3}+15 \left(3 d^{2} g-5 d f e\right) x\right) e^{\left(-3\right)}\right) p e+\frac{1}{15} \left(3 g x^{5}+5 f x^{3}\right) \log\left(\left(x^{2} e+d\right)^{p} c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] 2/225*(15*(3*d^3*g - 5*d^2*f*e)*arctan(x*e^(1/2)/sqrt(d))*e^(-7/2)/sqrt(d)
- (9*g*x^5*e^2 - 5*(3*d*g*e - 5*f*e^2)*x^3 + 15*(3*d^2*g - 5*d*f*e)*x)*e^(-
3))*p*e + 1/15*(3*g*x^5 + 5*f*x^3)*log((x^2*e + d)^p*c)
```

Fricas [A]
time = 0.39, size = 283, normalized size = 1.84

$$\left[-\frac{1}{225} \left(90 d^{2} g p x-15 \left(3 g p x^{5}+5 f p x^{3}\right) e^{2} \log\left(x^{2} e+d\right)-15 \left(3 g x^{5}+5 f x^{3}\right) e^{2} \log\left(c\right)+15 \left(3 d^{2} g p-5 d f p e\right) \sqrt{-d e^{\left(-1\right)}} \log\left(\frac{x^{2} e-2 \sqrt{-d e^{\left(-1\right)}} x e-d}{x^{2} e+d}\right)+2 \left(9 g p x^{5}+25 f p x^{3}\right) e^{2}-30 \left(d g p x^{3}+5 d f p x\right) e\right) e^{\left(-2\right)}, -\frac{1}{225} \left(90 d^{2} g p x-30 \left(3 d^{2} g p-5 d f p e\right) \sqrt{d} \arctan\left(\frac{x e^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{1}{2}\right)}-15 \left(3 g p x^{5}+5 f p x^{3}\right) e^{2} \log\left(x^{2} e+d\right)-15 \left(3 g x^{5}+5 f x^{3}\right) e^{2} \log\left(c\right)+2 \left(9 g p x^{5}+25 f p x^{3}\right) e^{2}-30 \left(d g p x^{3}+5 d f p x\right) e\right) e^{\left(-2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [-1/225*(90*d^2*g*p*x - 15*(3*g*p*x^5 + 5*f*p*x^3)*e^2*log(x^2*e + d) - 15*
(3*g*x^5 + 5*f*x^3)*e^2*log(c) + 15*(3*d^2*g*p - 5*d*f*p*e)*sqrt(-d*e^(-1))
*log((x^2*e - 2*sqrt(-d*e^(-1))*x*e - d)/(x^2*e + d)) + 2*(9*g*p*x^5 + 25*f
*p*x^3)*e^2 - 30*(d*g*p*x^3 + 5*d*f*p*x)*e)*e^(-2), -1/225*(90*d^2*g*p*x -
30*(3*d^2*g*p - 5*d*f*p*e)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2) - 15*
```

```
(3*g*p*x^5 + 5*f*p*x^3)*e^2*log(x^2*e + d) - 15*(3*g*x^5 + 5*f*x^3)*e^2*log
(c) + 2*(9*g*p*x^5 + 25*f*p*x^3)*e^2 - 30*(d*g*p*x^3 + 5*d*f*p*x)*e)*e^(-2)
]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(158) = 316$.
time = 34.48, size = 320, normalized size = 2.08

$$\begin{cases} \left(\frac{fx^3}{3} + \frac{gx^5}{5}\right)\log\left(0^p c\right) & \text{for } d = 0 \wedge e = 0 \\ \left(\frac{fx^3}{3} + \frac{gx^5}{5}\right)\log\left(cd^p\right) & \text{for } e = 0 \\ -\frac{2fpx^3}{9} + \frac{fx^3\log\left(c(ex^2)^p\right)}{3} - \frac{2gpx^5}{25} + \frac{gx^5\log\left(c(ex^2)^p\right)}{5} & \text{for } d = 0 \\ \frac{2d^3gp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{5e^3\sqrt{-\frac{d}{e}}} - \frac{d^3g\log\left(c(d+ex^2)^p\right)}{5e^3\sqrt{-\frac{d}{e}}} - \frac{2d^2fp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{3e^2\sqrt{-\frac{d}{e}}} + \frac{d^2f\log\left(c(d+ex^2)^p\right)}{3e^2\sqrt{-\frac{d}{e}}} - \frac{2d^2gpx}{5e^2} + \frac{2dfpx}{3e} + \frac{2dgpx^3}{15e} - \frac{2fpx^3}{9} + \frac{fx^3\log\left(c(d+ex^2)^p\right)}{3} - \frac{2gpx^5}{25} + \frac{gx^5\log\left(c(d+ex^2)^p\right)}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(g*x**2+f)*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise(((f*x**3/3 + g*x**5/5)*log(0**p*c), Eq(d, 0) & Eq(e, 0)), ((f*x**
3/3 + g*x**5/5)*log(c*d**p), Eq(e, 0)), (-2*f*p*x**3/9 + f*x**3*log(c*(e*x*
*2)**p)/3 - 2*g*p*x**5/25 + g*x**5*log(c*(e*x**2)**p)/5, Eq(d, 0)), (2*d**3
*g*p*log(x - sqrt(-d/e))/(5*e**3*sqrt(-d/e)) - d**3*g*log(c*(d + e*x**2)**p
)/(5*e**3*sqrt(-d/e)) - 2*d**2*f*p*log(x - sqrt(-d/e))/(3*e**2*sqrt(-d/e))
+ d**2*f*log(c*(d + e*x**2)**p)/(3*e**2*sqrt(-d/e)) - 2*d**2*g*p*x/(5*e**2)
 + 2*d*f*p*x/(3*e) + 2*d*g*p*x**3/(15*e) - 2*f*p*x**3/9 + f*x**3*log(c*(d +
 e*x**2)**p)/3 - 2*g*p*x**5/25 + g*x**5*log(c*(d + e*x**2)**p)/5, True))
```

Giac [A]
time = 5.43, size = 138, normalized size = 0.90

$$\frac{2\left(3\,d^3gp - 5\,d^2fpe\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{15\,\sqrt{d}} + \frac{1}{225}\left(45\,gpx^5e^2\log\left(x^2e+d\right) - 18\,gpx^5e^2 + 45\,gx^5e^2\log\left(c\right) + 30\,dgpx^3e + 75\,fpx^3e^2\log\left(x^2e+d\right) - 50\,fpx^3e^2 + 75\,fx^3e^2\log\left(c\right) - 90\,d^2gpx + 150\,dfpxe\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 2/15*(3*d^3*g*p - 5*d^2*f*p*e)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/sqrt(d) +
 1/225*(45*g*p*x^5*e^2*log(x^2*e + d) - 18*g*p*x^5*e^2 + 45*g*x^5*e^2*log(c
) + 30*d*g*p*x^3*e + 75*f*p*x^3*e^2*log(x^2*e + d) - 50*f*p*x^3*e^2 + 75*f*
x^3*e^2*log(c) - 90*d^2*g*p*x + 150*d*f*p*x*e)*e^(-2)
```

Mupad [B]
time = 0.32, size = 126, normalized size = 0.82

$$\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(\frac{g\,x^5}{5}+\frac{f\,x^3}{3}\right)-x^3\,\left(\frac{2\,f\,p}{9}-\frac{2\,d\,g\,p}{15\,e}\right)-\frac{2\,g\,p\,x^5}{25}+\frac{d\,x\,\left(\frac{2\,f\,p}{3}-\frac{2\,d\,g\,p}{5\,e}\right)}{e}+\frac{2\,d^{3/2}\,p\,\mathrm{atan}\left(\frac{d^{3/2}\,\sqrt{e}\,p\,x\,\left(3\,d\,g-5\,e\,f\right)}{3\,d^3\,g\,p-5\,d^2\,e\,f\,p}\right)\,\left(3\,d\,g-5\,e\,f\right)}{15\,e^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(c*(d + e*x^2)^p)*(f + g*x^2),x)
```

```
[Out] log(c*(d + e*x^2)^p)*((f*x^3)/3 + (g*x^5)/5) - x^3*((2*f*p)/9 - (2*d*g*p)/(
15*e)) - (2*g*p*x^5)/25 + (d*x*((2*f*p)/3 - (2*d*g*p)/(5*e)))/e + (2*d^(3/2
)*p*atan((d^(3/2)*e^(1/2)*p*x*(3*d*g - 5*e*f))/(3*d^3*g*p - 5*d^2*e*f*p))*(
3*d*g - 5*e*f))/(15*e^(5/2))
```

3.319 $\int (f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx$

Optimal. Leaf size=117

$$-2fpx+\frac{2dgpx}{3e}-\frac{2}{9}gpx^3+\frac{2\sqrt{d}\,fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{2d^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3e^{3/2}}+fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log$$

```
[Out] -2*f*p*x+2/3*d*g*p*x/e-2/9*g*p*x^3-2/3*d^(3/2)*g*p*arctan(x*e^(1/2)/d^(1/2)
)/e^(3/2)+f*x*ln(c*(e*x^2+d)^p)+1/3*g*x^3*ln(c*(e*x^2+d)^p)+2*f*p*arctan(x*
e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2521, 2498, 327, 211, 2505, 308}

$$-\frac{2d^{3/2}gp\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2\sqrt{d}\,fp\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}+fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{2dgpx}{3e}-2fpx-\frac{2}{9}gpx^3$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f*p*x + (2*d*g*p*x)/(3*e) - (2*g*p*x^3)/9 + (2*Sqrt[d]*f*p*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^
(3/2)) + f*x*Log[c*(d + e*x^2)^p] + (g*x^3*Log[c*(d + e*x^2)^p])/3
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$\begin{aligned}
\int \left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)\,dx &= \int \left(f\log\left(c\left(d+ex^2\right)^p\right)+gx^2\log\left(c\left(d+ex^2\right)^p\right)\right)\,dx \\
&= f\int \log\left(c\left(d+ex^2\right)^p\right)\,dx+g\int x^2\log\left(c\left(d+ex^2\right)^p\right)\,dx \\
&= fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log\left(c\left(d+ex^2\right)^p\right)-(2efp)\int \frac{x^2}{d+ex^2}\,dx - \\
&= -2fpx+fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log\left(c\left(d+ex^2\right)^p\right)+(2dfp)\int \frac{}{d-} \\
&= -2fpx+\frac{2dgpx}{3e}-\frac{2}{9}gpx^3+\frac{2\sqrt{d}\,fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}+fx\log\left(c\left(d+ex\right.\right. \\
&= -2fpx+\frac{2dgpx}{3e}-\frac{2}{9}gpx^3+\frac{2\sqrt{d}\,fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{2d^{3/2}gp\tan^{-1}\left(\right.}{3e^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 117, normalized size = 1.00

$$-2fpx+\frac{2dgpx}{3e}-\frac{2}{9}gpx^3+\frac{2\sqrt{d}\,fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{2d^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3e^{3/2}}+fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log\left(c\left(d+ex^2\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f*p*x + (2*d*g*p*x)/(3*e) - (2*g*p*x^3)/9 + (2*Sqrt[d]*f*p*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^
(3/2)) + f*x*Log[c*(d + e*x^2)^p] + (g*x^3*Log[c*(d + e*x^2)^p])/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.04, size = 416, normalized size = 3.56

method	result
risch	$\left(\frac{1}{3} g\,x^{3}+f x \right) \ln \left(\left(e\,x^{2}+d \right)^{p}\right)+\frac{i \pi g\,x^{3} \mathrm{csgn}\left(i \left(e\,x^{2}+d \right)^{p}\right) \mathrm{csgn}\left(i c \left(e\,x^{2}+d \right)^{p}\right)^{2}}{6}+\frac{i \pi g\,x^{3} \mathrm{csgn}\left(i c \left(e\,x^{2}+d \right)^{p}\right)^{2} \mathrm{csgn}\left(i c \right)}{6}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] (1/3*g*x^3+f*x)*ln((e*x^2+d)^p)+1/6*I*Pi*g*x^3*csgn(I*(e*x^2+d)^p)*csgn(I*c
*(e*x^2+d)^p)^2+1/6*I*Pi*g*x^3*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/2*I*Pi*f
*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*x-1/6*I*Pi*g*x^3*csgn(
I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/2*I*Pi*f*csgn(I*c*(e*x^2+d
)^p)^3*x-1/6*I*Pi*g*x^3*csgn(I*c*(e*x^2+d)^p)^3+1/2*I*Pi*f*csgn(I*c*(e*x^2+
d)^p)^2*csgn(I*c)*x+1/2*I*Pi*f*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*
x+1/3*ln(c)*g*x^3-2/9*g*p*x^3+1/3/e^2*(-e*d)^(1/2)*p*ln(-(-e*d)^(1/2)*x-d)*
d*g-1/e*(-e*d)^(1/2)*p*ln(-(-e*d)^(1/2)*x-d)*f-1/3/e^2*(-e*d)^(1/2)*p*ln((-
e*d)^(1/2)*x-d)*d*g+1/e*(-e*d)^(1/2)*p*ln((-e*d)^(1/2)*x-d)*f+ln(c)*f*x+2/3
*d*g*p*x/e-2*f*p*x
```

Maxima [A]
time = 0.53, size = 82, normalized size = 0.70

$$-\frac{2}{9} \, {\left(\frac{3 \, {\left(d^{2} g - 3 \, d f e\right)} \arctan\left(\frac{x e^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{5}{2}\right)}}{\sqrt{d}} + {\left(g x^{3} e - 3 \, {\left(d g - 3 \, f e\right)} x\right)} e^{\left(-2\right)}\right)} p e + \frac{1}{3} \, {\left(g x^{3} + 3 \, f x\right)} \log\left({\left(x^{2} e + d\right)}^{p} c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] -2/9*(3*(d^2*g - 3*d*f*e)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/sqrt(d) + (g*x
^3*e - 3*(d*g - 3*f*e)*x)*e^(-2))*p*e + 1/3*(g*x^3 + 3*f*x)*log((x^2*e + d)
^p*c)
```

Fricas [A]
time = 0.36, size = 217, normalized size = 1.85

$$\left[\frac{1}{9} \, {\left(6 \, d g p x + 3 \, {\left(g p x^{3} + 3 \, f p x\right)} e \log\left(x^{2} e + d\right) + 3 \, {\left(g x^{3} + 3 \, f x\right)} e \log\left(c\right) - 3 \, {\left(d g p - 3 \, f p e\right)} \sqrt{-d e^{\left(-1\right)}} \log\left(\frac{x^{2} e + 2 \, \sqrt{-d e^{\left(-1\right)}} x e - d}{x^{2} e + d}\right) - 2 \, {\left(g p x^{3} + 9 \, f p x\right)} e\right)} e^{\left(-1\right)}, \frac{1}{9} \, {\left(6 \, d g p x - 6 \, {\left(d g p - 3 \, f p e\right)} \sqrt{d} \arctan\left(\frac{x e^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{1}{2}\right)} + 3 \, {\left(g p x^{3} + 3 \, f p x\right)} e \log\left(x^{2} e + d\right) + 3 \, {\left(g x^{3} + 3 \, f x\right)} e \log\left(c\right) - 2 \, {\left(g p x^{3} + 9 \, f p x\right)} e\right)} e^{\left(-1\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [1/9*(6*d*g*p*x + 3*(g*p*x^3 + 3*f*p*x)*e*log(x^2*e + d) + 3*(g*x^3 + 3*f*x
)*e*log(c) - 3*(d*g*p - 3*f*p*e)*sqrt(-d*e^(-1))*log((x^2*e + 2*sqrt(-d*e^(
-1))*x*e - d)/(x^2*e + d)) - 2*(g*p*x^3 + 9*f*p*x)*e)*e^(-1), 1/9*(6*d*g*p*
x - 6*(d*g*p - 3*f*p*e)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2) + 3*(g*p
*x^3 + 3*f*p*x)*e*log(x^2*e + d) + 3*(g*x^3 + 3*f*x)*e*log(c) - 2*(g*p*x^3
+ 9*f*p*x)*e)*e^(-1)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(121) = 242$.
time = 8.83, size = 260, normalized size = 2.22

$$\begin{cases} \left(fx + \frac{gx^3}{3}\right)\log\left(0^p c\right) & \text{for}\ d = 0 \wedge e = 0 \\ -2fpx + fx\log\left(c(ex^2)^p\right) - \frac{2gpx^3}{9} + \frac{gx^3\log\left(c(ex^2)^p\right)}{3} & \text{for}\ d = 0 \\ \left(fx + \frac{gx^3}{3}\right)\log\left(cd^p\right) & \text{for}\ e = 0 \\ -\frac{2d^2gp\log\left(x - \sqrt{-\frac{d}{e}}\right)}{3e^2\sqrt{-\frac{d}{e}}} + \frac{d^2g\log\left(c(d+ex^2)^p\right)}{3e^2\sqrt{-\frac{d}{e}}} + \frac{2dfp\log\left(x - \sqrt{-\frac{d}{e}}\right)}{e\sqrt{-\frac{d}{e}}} - \frac{df\log\left(c(d+ex^2)^p\right)}{e\sqrt{-\frac{d}{e}}} + \frac{2dgpx}{3e} - 2fpx + fx\log\left(c(d+ex^2)^p\right) - \frac{2gpx^3}{9} + \frac{gx^3\log\left(c(d+ex^2)^p\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise(((f*x + g*x**3/3)*log(0**p*c), Eq(d, 0) & Eq(e, 0)), (-2*f*p*x +
f*x*log(c*(e*x**2)**p) - 2*g*p*x**3/9 + g*x**3*log(c*(e*x**2)**p)/3, Eq(d,
0)), ((f*x + g*x**3/3)*log(c*d**p), Eq(e, 0)), (-2*d**2*g*p*log(x - sqrt(-d
/e))/(3*e**2*sqrt(-d/e)) + d**2*g*log(c*(d + e*x**2)**p)/(3*e**2*sqrt(-d/e)
) + 2*d*f*p*log(x - sqrt(-d/e))/(e*sqrt(-d/e)) - d*f*log(c*(d + e*x**2)**p)
/(e*sqrt(-d/e)) + 2*d*g*p*x/(3*e) - 2*f*p*x + f*x*log(c*(d + e*x**2)**p) -
2*g*p*x**3/9 + g*x**3*log(c*(d + e*x**2)**p)/3, True))
```

Giac [A]
time = 5.91, size = 109, normalized size = 0.93

$$-\frac{2\,(d^2gp - 3\,dfpe)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{3}{2}\right)}}{3\,\sqrt{d}} + \frac{1}{9}\,(3\,gpx^3e\log\left(x^2e + d\right) - 2\,gpx^3e + 3\,gx^3e\log\left(c\right) + 9\,fpxe\log\left(x^2e + d\right) + 6\,dgpx - 18\,fpxe + 9\,fxe\log\left(c\right))e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] -2/3*(d^2*g*p - 3*d*f*p*e)*arctan(x*e^(1/2)/sqrt(d))*e^(-3/2)/sqrt(d) + 1/9
*(3*g*p*x^3*e*log(x^2*e + d) - 2*g*p*x^3*e + 3*g*x^3*e*log(c) + 9*f*p*x*e*l
og(x^2*e + d) + 6*d*g*p*x - 18*f*p*x*e + 9*f*x*e*log(c))*e^(-1)
```

Mupad [B]
time = 0.00, size = 97, normalized size = 0.83

$$\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(\frac{g\,x^3}{3}+f\,x\right)-x\,\left(2\,f\,p-\frac{2\,d\,g\,p}{3\,e}\right)-\frac{2\,g\,p\,x^3}{9}-\frac{2\,\sqrt{d}\,\,p\,\mathrm{atan}\left(\frac{\sqrt{d}\,\,\sqrt{e}\,\,p\,x\,\left(d\,g-3\,e\,f\right)}{d^2\,g\,p-3\,d\,e\,f\,p}\right)\,\left(d\,g-3\,e\,f\right)}{3\,e^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)*(f + g*x^2),x)
```

```
[Out] log(c*(d + e*x^2)^p)*(f*x + (g*x^3)/3) - x*(2*f*p - (2*d*g*p)/(3*e)) - (2*g
*p*x^3)/9 - (2*d^(1/2)*p*atan((d^(1/2)*e^(1/2)*p*x*(d*g - 3*e*f))/(d^2*g*p
- 3*d*e*f*p))*(d*g - 3*e*f))/(3*e^(3/2))
```

3.320 $\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^2}\,dx$

Optimal. Leaf size=72

$$-2gpx+\frac{2(ef+dg)p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,\sqrt{e}}-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{x}+gx\log\left(c\left(d+ex^2\right)^p\right)$$

```
[Out] -2*g*p*x-f*ln(c*(e*x^2+d)^p)/x+g*x*ln(c*(e*x^2+d)^p)+2*(d*g+e*f)*p*arctan(x
*e^(1/2)/d^(1/2))/d^(1/2)/e^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 93, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2526, 2498, 327, 211, 2505}

$$\frac{2\sqrt{e}\,fp\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{d}\,gp\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{x}+gx\log\left(c\left(d+ex^2\right)^p\right)-2gpx$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^2,x]
```

```
[Out] -2*g*p*x + (2*Sqrt[e]*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] + (2*Sqrt[d]
*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (f*Log[c*(d + e*x^2)^p])/x + g*
x*Log[c*(d + e*x^2)^p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^2}\,dx = \int \left(g\log\left(c(d+ex^2)^p\right)+\frac{f\log\left(c(d+ex^2)^p\right)}{x^2}\right)dx$$
$$= f\int \frac{\log\left(c(d+ex^2)^p\right)}{x^2}\,dx + g\int \log\left(c(d+ex^2)^p\right)\,dx$$
$$= -\frac{f\log\left(c(d+ex^2)^p\right)}{x} + gx\log\left(c(d+ex^2)^p\right) + (2efp)\int \frac{1}{d+ex^2}\,dx -$$
$$= -2gpx + \frac{2\sqrt{e}\,fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}} - \frac{f\log\left(c(d+ex^2)^p\right)}{x} + gx\log\left(c(d-\right.$$
$$= -2gpx + \frac{2\sqrt{e}\,fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{d}\,gp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{f\log\left(c\right.}{}$$

Mathematica [A]
time = 0.04, size = 62, normalized size = 0.86

$$-2gpx + \frac{2(ef+dg)p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,\sqrt{e}} + \left(-\frac{f}{x}+gx\right)\log\left(c(d+ex^2)^p\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^2,x]
[Out] -2*g*p*x + (2*(e*f + d*g)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqrt[e])
+ (-(f/x) + g*x)*Log[c*(d + e*x^2)^p]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.17, size = 427, normalized size = 5.93

method	result
risch	$-\frac{(-g\,x^2+f)\ln\left((e\,x^2+d)^p\right)}{x}+\frac{i\pi g\,x^2\mathrm{csgn}\left(i(e\,x^2+d)^p\right)\mathrm{csgn}\left(ic(e\,x^2+d)^p\right)^2de-i\pi g\,x^2\mathrm{csgn}\left(i(e\,x^2+d)^p\right)\mathrm{csgn}\left(ic(e\,x^2+d)^p\right)\mathrm{csgn}(ic)d}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(-g*x^2+f)/x*ln((e*x^2+d)^p)+1/2*(I*Pi*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*
(e*x^2+d)^p)^2*d*e-I*Pi*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csg
n(I*c)*d*e-I*Pi*g*x^2*csgn(I*c*(e*x^2+d)^p)^3*d*e+I*Pi*g*x^2*csgn(I*c*(e*x^
2+d)^p)^2*csgn(I*c)*d*e-I*Pi*d*f*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^
2*e+I*Pi*d*f*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*e+I*Pi*d*f
*csgn(I*c*(e*x^2+d)^p)^3*e-I*Pi*d*f*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*e+2*l
n(c)*g*x^2*d*e+2*(-e*d)^(1/2)*p*ln(-(-e*d)^(1/2)*x+d)*g*d*x+2*(-e*d)^(1/2)*
p*ln(-(-e*d)^(1/2)*x+d)*f*e*x-2*(-e*d)^(1/2)*p*ln(-(-e*d)^(1/2)*x-d)*g*d*x-
2*(-e*d)^(1/2)*p*ln(-(-e*d)^(1/2)*x-d)*f*e*x-4*x^2*d*e*g*p-2*ln(c)*d*e*f)/d
/e/x
```

Maxima [A]
time = 0.55, size = 59, normalized size = 0.82

$$-2\left(gxe^{(-1)}-\frac{(dg+fe)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{3}{2}\right)}}{\sqrt{d}}\right)pe+\left(gx-\frac{f}{x}\right)\log\left(\left(x^2e+d\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^2,x, algorithm="maxima")
```

```
[Out] -2*(g*x*e^(-1) - (d*g + f*e)*arctan(x*e^(1/2)/sqrt(d))*e^(-3/2)/sqrt(d))*p*
e + (g*x - f/x)*log((x^2*e + d)^p*c)
```

Fricas [A]
time = 0.39, size = 208, normalized size = 2.89

$$\left[-\frac{\left(2\,dgpx^2e-(dgpx^2-dfp)e\log\left(x^2e+d\right)-(dgx^2-df)e\log\left(c\right)+(dgpx+fpxe)\sqrt{-de}\,\log\left(\frac{x^2e-2\,\sqrt{-de}\,x-d}{x^2e+d}\right)\right)e^{(-1)}}{dx},-\frac{\left(2\,dgpx^2e-2\,(dgpx+fpxe)\sqrt{d}\,\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}-(dgpx^2-dfp)e\log\left(x^2e+d\right)-(dgx^2-df)e\log\left(c\right)\right)e^{(-1)}}{dx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^2,x, algorithm="fricas")
```

```
[Out] [-(2*d*g*p*x^2*e - (d*g*p*x^2 - d*f*p)*e*log(x^2*e + d) - (d*g*x^2 - d*f)*e
*log(c) + (d*g*p*x + f*p*x*e)*sqrt(-d*e)*log((x^2*e - 2*sqrt(-d*e)*x - d)/(
```

```
x^2*e + d)))*e^(-1)/(d*x), -(2*d*g*p*x^2*e - 2*(d*g*p*x + f*p*x*e)*sqrt(d)*
arctan(x*e^(1/2)/sqrt(d))*e^(1/2) - (d*g*p*x^2 - d*f*p)*e*log(x^2*e + d) -
(d*g*x^2 - d*f)*e*log(c))*e^(-1)/(d*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(71) = 142$.

time = 16.96, size = 204, normalized size = 2.83

$$\begin{cases} \left(-\frac{f}{x} + gx\right)\log\left(0^{p}c\right) & \text{for}\ d = 0 \wedge e = 0 \\ \left(-\frac{f}{x} + gx\right)\log\left(cd^{p}\right) & \text{for}\ e = 0 \\ -\frac{2fp}{x} - \frac{f\log\left(c\left(ex^{2}\right)^{p}\right)}{x} - 2gpx + gx\log\left(c\left(ex^{2}\right)^{p}\right) & \text{for}\ d = 0 \\ \frac{2dgp\log\left(x - \sqrt{-\frac{d}{e}}\right)}{e\sqrt{-\frac{d}{e}}} - \frac{dg\log\left(c\left(d + ex^{2}\right)^{p}\right)}{e\sqrt{-\frac{d}{e}}} + \frac{2fp\log\left(x - \sqrt{-\frac{d}{e}}\right)}{\sqrt{-\frac{d}{e}}} - \frac{f\log\left(c\left(d + ex^{2}\right)^{p}\right)}{\sqrt{-\frac{d}{e}}} - \frac{f\log\left(c\left(d + ex^{2}\right)^{p}\right)}{x} - 2gpx + gx\log\left(c\left(d + ex^{2}\right)^{p}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**2,x)
```

```
[Out] Piecewise(((-f/x + g*x)*log(0**p*c), Eq(d, 0) & Eq(e, 0)), ((-f/x + g*x)*lo
g(c*d**p), Eq(e, 0)), (-2*f*p/x - f*log(c*(e*x**2)**p)/x - 2*g*p*x + g*x*lo
g(c*(e*x**2)**p), Eq(d, 0)), (2*d*g*p*log(x - sqrt(-d/e))/(e*sqrt(-d/e)) -
d*g*log(c*(d + e*x**2)**p)/(e*sqrt(-d/e)) + 2*f*p*log(x - sqrt(-d/e))/sqrt(
-d/e) - f*log(c*(d + e*x**2)**p)/sqrt(-d/e) - f*log(c*(d + e*x**2)**p)/x -
2*g*p*x + g*x*log(c*(d + e*x**2)**p), True))
```

Giac [A]

time = 3.13, size = 78, normalized size = 1.08

$$\frac{2\,(dgp + fpe)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{\sqrt{d}} + \frac{gpx^{2}\log\left(x^{2}e + d\right) - 2\,gpx^{2} + gx^{2}\log\left(c\right) - fp\log\left(x^{2}e + d\right) - f\log\left(c\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^2,x, algorithm="giac")
```

```
[Out] 2*(d*g*p + f*p*e)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/sqrt(d) + (g*p*x^2*log
(x^2*e + d) - 2*g*p*x^2 + g*x^2*log(c) - f*p*log(x^2*e + d) - f*log(c))/x
```

Mupad [B]

time = 0.33, size = 83, normalized size = 1.15

$$\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)\,\left(2\,g\,x - \frac{g\,x^{2}+f}{x}\right) - 2\,g\,p\,x + \frac{2\,p\,\mathrm{atan}\left(\frac{2\,\sqrt{e}\,\,p\,x\,\left(d\,g+e\,f\right)}{\sqrt{d}\,\,\left(2\,d\,g\,p+2\,e\,f\,p\right)}\right)\,\left(d\,g+e\,f\right)}{\sqrt{d}\,\,\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x^2,x)
```

```
[Out] log(c*(d + e*x^2)^p)*(2*g*x - (f + g*x^2)/x) - 2*g*p*x + (2*p*atan((2*e^(1/
2)*p*x*(d*g + e*f))/(d^(1/2)*(2*d*g*p + 2*e*f*p)))*(d*g + e*f))/(d^(1/2)*e^
(1/2))
```

3.321 $\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^4}\,dx$

Optimal. Leaf size=108

$$-\frac{2efp}{3dx}-\frac{2e^{3/2}fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3d^{3/2}}+\frac{2\sqrt{e}\,gp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{f\log\left(c(d+ex^2)^p\right)}{3x^3}-\frac{g\log\left(c(d+ex^2)^p\right)}{x}$$

```
[Out] -2/3*e*f*p/d/x-2/3*e^(3/2)*f*p*arctan(x*e^(1/2)/d^(1/2))/d^(3/2)-1/3*f*ln(c
*(e*x^2+d)^p)/x^3-g*ln(c*(e*x^2+d)^p)/x+2*g*p*arctan(x*e^(1/2)/d^(1/2))*e^(
1/2)/d^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2526, 2505, 331, 211}

$$-\frac{2e^{3/2}fp\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3d^{3/2}}+\frac{2\sqrt{e}\,gp\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{f\log\left(c(d+ex^2)^p\right)}{3x^3}-\frac{g\log\left(c(d+ex^2)^p\right)}{x}-\frac{2efp}{3dx}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^4,x]
```

```
[Out] (-2*e*f*p)/(3*d*x) - (2*e^(3/2)*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*d^(3/2)
) + (2*Sqrt[e]*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] - (f*Log[c*(d + e*x
^2)^p])/(3*x^3) - (g*Log[c*(d + e*x^2)^p])/x
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
```

```
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\begin{aligned}\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^4}\,dx &= \int \left(\frac{f\log\left(c(d+ex^2)^p\right)}{x^4}+\frac{g\log\left(c(d+ex^2)^p\right)}{x^2}\right)dx\\ &= f\int \frac{\log\left(c(d+ex^2)^p\right)}{x^4}\,dx+g\int \frac{\log\left(c(d+ex^2)^p\right)}{x^2}\,dx\\ &= -\frac{f\log\left(c(d+ex^2)^p\right)}{3x^3}-\frac{g\log\left(c(d+ex^2)^p\right)}{x}+\frac{1}{3}(2efp)\int \frac{1}{x^2\left(d+ex^2\right)}\,d\\ &= -\frac{2efp}{3dx}+\frac{2\sqrt{e}\,gp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{f\log\left(c(d+ex^2)^p\right)}{3x^3}-\frac{g\log\left(c(d+\right.}{x}\\ &= -\frac{2efp}{3dx}-\frac{2e^{3/2}fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3d^{3/2}}+\frac{2\sqrt{e}\,gp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{f\log\left(c\right.}{}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 96, normalized size = 0.89

$$\frac{2\sqrt{e}\,gp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{2efp\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{ex^2}{d}\right)}{3dx}-\frac{f\log\left(c(d+ex^2)^p\right)}{3x^3}-\frac{g\log\left(c(d+ex^2)^p\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^4,x]
```

```
[Out] (2*Sqrt[e]*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] - (2*e*f*p*Hypergeometr
ic2F1[-1/2, 1, 1/2, -((e*x^2)/d)])/(3*d*x) - (f*Log[c*(d + e*x^2)^p])/(3*x^
3) - (g*Log[c*(d + e*x^2)^p])/x
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.18, size = 442, normalized size = 4.09

method	result
risch	$-\frac{(3g\,x^2+f)\ln\left((e\,x^2+d)^p\right)}{3x^3}-\frac{3i\pi\,d^2g\,x^2\mathrm{csgn}\left(i(e\,x^2+d)^p\right)\mathrm{csgn}\left(ic(e\,x^2+d)^p\right)^2-3i\pi\,d^2g\,x^2\mathrm{csgn}\left(i(e\,x^2+d)^p\right)\mathrm{csgn}\left(ic(e\,x^2+d)^p\right)\mathrm{csgn}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(3*g*x^2+f)/x^3*ln((e*x^2+d)^p)-1/6*(3*I*Pi*d^2*g*x^2*csgn(I*(e*x^2+d)
^p)*csgn(I*c*(e*x^2+d)^p)^2-3*I*Pi*d^2*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(
e*x^2+d)^p)*csgn(I*c)-3*I*Pi*d^2*g*x^2*csgn(I*c*(e*x^2+d)^p)^3+3*I*Pi*d^2*g
*x^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+I*Pi*d^2*f*csgn(I*(e*x^2+d)^p)*csgn(
I*c*(e*x^2+d)^p)^2-I*Pi*d^2*f*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csg
n(I*c)-I*Pi*d^2*f*csgn(I*c*(e*x^2+d)^p)^3+I*Pi*d^2*f*csgn(I*c*(e*x^2+d)^p)^
2*csgn(I*c)-6*(-e*d)^(1/2)*p*ln(-e*x-(-e*d)^(1/2))*g*d*x^3+2*(-e*d)^(1/2)*p
*ln(-e*x-(-e*d)^(1/2))*e*f*x^3+6*(-e*d)^(1/2)*p*ln(-e*x+(-e*d)^(1/2))*g*d*x
^3-2*(-e*d)^(1/2)*p*ln(-e*x+(-e*d)^(1/2))*e*f*x^3+6*ln(c)*d^2*g*x^2+4*d*e*f
*p*x^2+2*ln(c)*d^2*f)/d^2/x^3
```

Maxima [A]
time = 0.49, size = 66, normalized size = 0.61

$$\frac{2}{3}\left(\frac{(3\,dg-fe)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{d^{\frac{3}{2}}}-\frac{f}{dx}\right)pe-\frac{(3\,gx^2+f)\log\left((x^2e+d)^pc\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^4,x, algorithm="maxima")
```

```
[Out] 2/3*((3*d*g - f*e)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d^(3/2) - f/(d*x))*p*
e - 1/3*(3*g*x^2 + f)*log((x^2*e + d)^p*c)/x^3
```

Fricas [A]
time = 0.39, size = 207, normalized size = 1.92

$$\left[-\frac{2\,fpx^2e+(3\,dgpx^3-fpx^3e)\sqrt{-\frac{e}{d}}\log\left(\frac{x^2e-2\,dx\sqrt{-\frac{e}{d}}-d}{x^2e+d}\right)+(3\,dgpx^2+dfp)\log\left(x^2e+d\right)+(3\,dgx^2+df)\log\left(c\right)}{3\,dx^3},-\frac{2\,fpx^2e-\frac{2\,(3\,dgpx^3-fpx^3e)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}}{\sqrt{d}}+(3\,dgpx^2+dfp)\log\left(x^2e+d\right)+(3\,dgx^2+df)\log\left(c\right)}{3\,dx^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/3*(2*f*p*x^2*e + (3*d*g*p*x^3 - f*p*x^3*e)*sqrt(-e/d)*log((x^2*e - 2*d*
x*sqrt(-e/d) - d)/(x^2*e + d)) + (3*d*g*p*x^2 + d*f*p)*log(x^2*e + d) + (3*
```

```
d*g*x^2 + d*f)*log(c))/(d*x^3), -1/3*(2*f*p*x^2*e - 2*(3*d*g*p*x^3 - f*p*x^
3*e)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2)/sqrt(d) + (3*d*g*p*x^2 + d*f*p)*log(
x^2*e + d) + (3*d*g*x^2 + d*f)*log(c))/(d*x^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 879 vs. $2(104) = 208$.
time = 40.83, size = 879, normalized size = 8.14

$$\begin{cases} \left(-\frac{f}{3x^3} - \frac{g}{x}\right)\log\left(0^p c\right) & \text{for } d = 0 \wedge e = 0 \\ \left(-\frac{f}{3x^3} - \frac{g}{x}\right)\log\left(cd^p\right) & \text{for } e = 0 \\ -\frac{2fp}{9x^3} - \frac{f\log\left(c\left(ex^2\right)^p\right)}{3x^3} - \frac{2gp}{x} - \frac{g\log\left(c\left(ex^2\right)^p\right)}{x} & \text{for } d = 0 \\ -\frac{d^2f\sqrt{-\frac{d}{e}}\log\left(c\left(d+ex^2\right)^p\right)}{3d^2x^3\sqrt{-\frac{d}{e}}+3dex^5\sqrt{-\frac{d}{e}}} + \frac{6d^2gpx^3\log\left(x-\sqrt{-\frac{d}{e}}\right)}{3d^2x^3\sqrt{-\frac{d}{e}}+3dex^5\sqrt{-\frac{d}{e}}} - \frac{3d^2gx^3\log\left(c\left(d+ex^2\right)^p\right)}{3d^2x^3\sqrt{-\frac{d}{e}}+3dex^5\sqrt{-\frac{d}{e}}} - \frac{3d^2gx^2\sqrt{-\frac{d}{e}}\log\left(c\left(d+ex^2\right)^p\right)}{3d^2x^3\sqrt{-\frac{d}{e}}+3dex^5\sqrt{-\frac{d}{e}}} - \frac{2dfpx^3\log\left(x-\sqrt{-\frac{d}{e}}\right)}{\frac{3d^2x^3\sqrt{-\frac{d}{e}}}{e}+3dx^5\sqrt{-\frac{d}{e}}} - \frac{2dfpx^2\sqrt{-\frac{d}{e}}}{\frac{3d^2x^3\sqrt{-\frac{d}{e}}}{e}+3dx^5\sqrt{-\frac{d}{e}}} + \frac{dfx^3\log\left(c\left(d+ex^2\right)^p\right)}{\frac{3d^2x^3\sqrt{-\frac{d}{e}}}{e}+3dx^5\sqrt{-\frac{d}{e}}} - \frac{dfx^2\sqrt{-\frac{d}{e}}\log\left(c\left(d+ex^2\right)^p\right)}{\frac{3d^2x^3\sqrt{-\frac{d}{e}}}{e}+3dx^5\sqrt{-\frac{d}{e}}} + \frac{6dgpx^5\log\left(x-\sqrt{-\frac{d}{e}}\right)}{\frac{3d^2x^3\sqrt{-\frac{d}{e}}}{e}+3dx^5\sqrt{-\frac{d}{e}}} - \frac{3dgx^5\log\left(c\left(d+ex^2\right)^p\right)}{\frac{3d^2x^3\sqrt{-\frac{d}{e}}}{e}+3dx^5\sqrt{-\frac{d}{e}}} - \frac{3dgx^4\sqrt{-\frac{d}{e}}\log\left(c\left(d+ex^2\right)^p\right)}{\frac{3d^2x^3\sqrt{-\frac{d}{e}}}{e}+3dx^5\sqrt{-\frac{d}{e}}} - \frac{2efpx^5\log\left(x-\sqrt{-\frac{d}{e}}\right)}{\frac{3d^2x^3\sqrt{-\frac{d}{e}}}{e}+3dx^5\sqrt{-\frac{d}{e}}} - \frac{2efpx^4\sqrt{-\frac{d}{e}}}{\frac{3d^2x^3\sqrt{-\frac{d}{e}}}{e}+3dx^5\sqrt{-\frac{d}{e}}} + \frac{efx^5\log\left(c\left(d+ex^2\right)^p\right)}{\frac{3d^2x^3\sqrt{-\frac{d}{e}}}{e}+3dx^5\sqrt{-\frac{d}{e}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**4,x)
```

```
[Out] Piecewise(((-f/(3*x**3) - g/x)*log(0**p*c), Eq(d, 0) & Eq(e, 0)), ((-f/(3*x
**3) - g/x)*log(c*d**p), Eq(e, 0)), (-2*f*p/(9*x**3) - f*log(c*(e*x**2)**p)
/(3*x**3) - 2*g*p/x - g*log(c*(e*x**2)**p)/x, Eq(d, 0)), (-d**2*f*sqrt(-d/e
)*log(c*(d + e*x**2)**p)/(3*d**2*x**3*sqrt(-d/e) + 3*d*e*x**5*sqrt(-d/e)) +
 6*d**2*g*p*x**3*log(x - sqrt(-d/e))/(3*d**2*x**3*sqrt(-d/e) + 3*d*e*x**5*s
qrt(-d/e)) - 3*d**2*g*x**3*log(c*(d + e*x**2)**p)/(3*d**2*x**3*sqrt(-d/e) +
 3*d*e*x**5*sqrt(-d/e)) - 3*d**2*g*x**2*sqrt(-d/e)*log(c*(d + e*x**2)**p)/(
3*d**2*x**3*sqrt(-d/e) + 3*d*e*x**5*sqrt(-d/e)) - 2*d*f*p*x**3*log(x - sqrt
(-d/e))/(3*d**2*x**3*sqrt(-d/e)/e + 3*d*x**5*sqrt(-d/e)) - 2*d*f*p*x**2*sqr
t(-d/e)/(3*d**2*x**3*sqrt(-d/e)/e + 3*d*x**5*sqrt(-d/e)) + d*f*x**3*log(c*(
d + e*x**2)**p)/(3*d**2*x**3*sqrt(-d/e)/e + 3*d*x**5*sqrt(-d/e)) - d*f*x**2
*sqrt(-d/e)*log(c*(d + e*x**2)**p)/(3*d**2*x**3*sqrt(-d/e)/e + 3*d*x**5*sqr
t(-d/e)) + 6*d*g*p*x**5*log(x - sqrt(-d/e))/(3*d**2*x**3*sqrt(-d/e)/e + 3*d
*x**5*sqrt(-d/e)) - 3*d*g*x**5*log(c*(d + e*x**2)**p)/(3*d**2*x**3*sqrt(-d/
e)/e + 3*d*x**5*sqrt(-d/e)) - 3*d*g*x**4*sqrt(-d/e)*log(c*(d + e*x**2)**p)/
(3*d**2*x**3*sqrt(-d/e)/e + 3*d*x**5*sqrt(-d/e)) - 2*e*f*p*x**5*log(x - sqr
t(-d/e))/(3*d**2*x**3*sqrt(-d/e)/e + 3*d*x**5*sqrt(-d/e)) - 2*e*f*p*x**4*sq
rt(-d/e)/(3*d**2*x**3*sqrt(-d/e)/e + 3*d*x**5*sqrt(-d/e)) + e*f*x**5*log(c*
(d + e*x**2)**p)/(3*d**2*x**3*sqrt(-d/e)/e + 3*d*x**5*sqrt(-d/e)), True))
```

Giac [A]
time = 5.40, size = 92, normalized size = 0.85

$$\frac{2\left(3\,dgpe - fpe^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{3\,d^{\frac{3}{2}}} - \frac{3\,dgpx^2\log\left(x^2e+d\right) + 2\,fpx^2e + 3\,dgx^2\log\left(c\right) + dfp\log\left(x^2e+d\right) + df\log\left(c\right)}{3\,dx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^4,x, algorithm="giac")
```

```
[Out] 2/3*(3*d*g*p*e - f*p*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d^(3/2) - 1/3*
(3*d*g*p*x^2*log(x^2*e + d) + 2*f*p*x^2*e + 3*d*g*x^2*log(c) + d*f*p*log(x^
2*e + d) + d*f*log(c))/(d*x^3)
```

Mupad [B]
time = 0.37, size = 65, normalized size = 0.60

$$\frac{2\sqrt{e}\,p\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(3\,d\,g-e\,f)}{3\,d^{3/2}}-\frac{2\,e\,f\,p}{3\,d\,x}-\frac{\ln\left(c\,(e\,x^2+d)^p\right)\,\left(g\,x^2+\frac{f}{3}\right)}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x^4,x)
```

```
[Out] (2*e^(1/2)*p*atan((e^(1/2)*x)/d^(1/2))*(3*d*g - e*f))/(3*d^(3/2)) - (2*e*f*
p)/(3*d*x) - (log(c*(d + e*x^2)^p)*(f/3 + g*x^2))/x^3
```

3.322 $\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^6}\,dx$

Optimal. Leaf size=140

$$-\frac{2efp}{15dx^3}+\frac{2e^2fp}{5d^2x}-\frac{2egp}{3dx}+\frac{2e^{5/2}fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{5d^{5/2}}-\frac{2e^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3d^{3/2}}-\frac{f\log\left(c(d+ex^2)^p\right)}{5x^5}-\frac{g\log\left(c(d+ex^2)^p\right)}{3x^3}$$

```
[Out] -2/15*e*f*p/d/x^3+2/5*e^2*f*p/d^2/x-2/3*e*g*p/d/x+2/5*e^(5/2)*f*p*arctan(x*
e^(1/2)/d^(1/2))/d^(5/2)-2/3*e^(3/2)*g*p*arctan(x*e^(1/2)/d^(1/2))/d^(3/2)-
1/5*f*ln(c*(e*x^2+d)^p)/x^5-1/3*g*ln(c*(e*x^2+d)^p)/x^3
```

Rubi [A]
time = 0.08, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2526, 2505, 331, 211}

$$\frac{2e^{5/2}fp\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{5d^{5/2}}-\frac{2e^{3/2}gp\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3d^{3/2}}-\frac{f\log\left(c(d+ex^2)^p\right)}{5x^5}-\frac{g\log\left(c(d+ex^2)^p\right)}{3x^3}+\frac{2e^2fp}{5d^2x}-\frac{2efp}{15dx^3}-\frac{2egp}{3dx}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^6,x]
```

```
[Out] (-2*e*f*p)/(15*d*x^3) + (2*e^2*f*p)/(5*d^2*x) - (2*e*g*p)/(3*d*x) + (2*e^(5
/2)*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*d^(5/2)) - (2*e^(3/2)*g*p*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(3*d^(3/2)) - (f*Log[c*(d + e*x^2)^p])/(5*x^5) - (g*Log
[c*(d + e*x^2)^p])/(3*x^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
```

```
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^6}\,dx &= \int \left(\frac{f\log\left(c(d+ex^2)^p\right)}{x^6}+\frac{g\log\left(c(d+ex^2)^p\right)}{x^4}\right)dx \\
&= f\int \frac{\log\left(c(d+ex^2)^p\right)}{x^6}\,dx+g\int \frac{\log\left(c(d+ex^2)^p\right)}{x^4}\,dx \\
&= -\frac{f\log\left(c(d+ex^2)^p\right)}{5x^5}-\frac{g\log\left(c(d+ex^2)^p\right)}{3x^3}+\frac{1}{5}(2efp)\int \frac{1}{x^4\left(d+ex^2\right)}\,dx \\
&= -\frac{2efp}{15dx^3}-\frac{2egp}{3dx}-\frac{f\log\left(c(d+ex^2)^p\right)}{5x^5}-\frac{g\log\left(c(d+ex^2)^p\right)}{3x^3}-\frac{(2e^2fp)}{} \\
&= -\frac{2efp}{15dx^3}+\frac{2e^2fp}{5d^2x}-\frac{2egp}{3dx}-\frac{2e^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3d^{3/2}}-\frac{f\log\left(c(d+ex^2)^p\right)}{5x^5} \\
&= -\frac{2efp}{15dx^3}+\frac{2e^2fp}{5d^2x}-\frac{2egp}{3dx}+\frac{2e^{5/2}fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{5d^{5/2}}-\frac{2e^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3d^{3/2}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.01, size = 101, normalized size = 0.72

$$
-\frac{2efp\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\frac{ex^2}{d}\right)}{15dx^3}-\frac{2egp\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{ex^2}{d}\right)}{3dx}-\frac{f\log\left(c(d+ex^2)^p\right)}{5x^5}-\frac{g\log\left(c(d+ex^2)^p\right)}{3x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^6,x]
```

```
[Out] (-2*e*f*p*Hypergeometric2F1[-3/2, 1, -1/2, -((e*x^2)/d)])/(15*d*x^3) - (2*e
*g*p*Hypergeometric2F1[-1/2, 1, 1/2, -((e*x^2)/d)])/(3*d*x) - (f*Log[c*(d +
 e*x^2)^p])/(5*x^5) - (g*Log[c*(d + e*x^2)^p])/(3*x^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.10, size = 474, normalized size = 3.39

method	result
risch	$-\frac{(5g\,x^2+3f)\ln((e\,x^2+d)^p)}{15x^5} - \frac{-3i\pi\,d^3 f\,\mathrm{csgn}(ic(e\,x^2+d)^p)^3 - 5i\pi\,d^3 g\,x^2\mathrm{csgn}(i(e\,x^2+d)^p)\mathrm{csgn}(ic(e\,x^2+d)^p)\mathrm{csgn}(ic)+3i\pi\,d^3 f\,\mathrm{csg}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(5*g*x^2+3*f)/x^5*ln((e*x^2+d)^p)-1/30*(-3*I*Pi*d^3*f*csgn(I*c*(e*x^2
+d)^p)^3-5*I*Pi*d^3*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*
c)+3*I*Pi*d^3*f*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-3*I*Pi*d^3*f*csgn(I*(e*x^
2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+10*(-e*d)^(1/2)*p*e*ln(-e*x-(-e*d)^
(1/2))*g*d*x^5-6*(-e*d)^(1/2)*p*e^2*ln(-e*x-(-e*d)^(1/2))*f*x^5-10*(-e*d)^(
1/2)*p*e*ln(-e*x+(-e*d)^(1/2))*g*d*x^5+6*(-e*d)^(1/2)*p*e^2*ln(-e*x+(-e*d)^
(1/2))*f*x^5+5*I*Pi*d^3*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+3
*I*Pi*d^3*f*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+5*I*Pi*d^3*g*x^2*cs
gn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-5*I*Pi*d^3*g*x^2*csgn(I*c*(e*x^2+d)^p)^3+20
*d^2*e*g*p*x^4-12*d*e^2*f*p*x^4+10*ln(c)*d^3*g*x^2+4*d^2*e*f*p*x^2+6*ln(c)*
d^3*f)/d^3/x^5
```

Maxima [A]
time = 0.49, size = 86, normalized size = 0.61

$$-\frac{2}{15}\,p\left(\frac{(5\,dge-3\,fe^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{d^{\frac{5}{2}}}+\frac{(5\,dg-3\,fe)x^2+df}{d^2x^3}\right)e-\frac{(5\,gx^2+3\,f)\log\left((x^2e+d)^pc\right)}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^6,x, algorithm="maxima")
```

```
[Out] -2/15*p*((5*d*g*e - 3*f*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d^(5/2) + (
(5*d*g - 3*f*e)*x^2 + d*f)/(d^2*x^3))*e - 1/15*(5*g*x^2 + 3*f)*log((x^2*e +
 d)^p*c)/x^5
```

Fricas [A]
time = 0.36, size = 276, normalized size = 1.97

$$\left[\frac{6\,fpx^4e^2-(5\,dgpx^5e-3\,fpx^5e^2)\sqrt{-\frac{e}{d}}\log\left(\frac{x^2e+2\,dx\sqrt{-\frac{e}{d}}-d}{x^2e+d}\right)-2\,(5\,dgpx^4+dfpx^2)e-(5\,d^2gpx^2+3\,d^2fp)\log\left(x^2e+d\right)-(5\,d^2gx^2+3\,d^2f)\log\left(c\right)}{15\,d^2x^5},\ \frac{6\,fpx^4e^2-\frac{2\,(5\,dgpx^5e-3\,fpx^5e^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}}{\sqrt{d}}-2\,(5\,dgpx^4+dfpx^2)e-(5\,d^2gpx^2+3\,d^2fp)\log\left(x^2e+d\right)-(5\,d^2gx^2+3\,d^2f)\log\left(c\right)}{15\,d^2x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^6,x, algorithm="fricas")
```

[Out] [1/15*(6*f*p*x^4*e^2 - (5*d*g*p*x^5*e - 3*f*p*x^5*e^2)*sqrt(-e/d)*log((x^2*e + 2*d*x*sqrt(-e/d) - d)/(x^2*e + d)) - 2*(5*d*g*p*x^4 + d*f*p*x^2)*e - (5*d^2*g*p*x^2 + 3*d^2*f*p)*log(x^2*e + d) - (5*d^2*g*x^2 + 3*d^2*f)*log(c))/(d^2*x^5), 1/15*(6*f*p*x^4*e^2 - 2*(5*d*g*p*x^5*e - 3*f*p*x^5*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2)/sqrt(d) - 2*(5*d*g*p*x^4 + d*f*p*x^2)*e - (5*d^2*g*p*x^2 + 3*d^2*f*p)*log(x^2*e + d) - (5*d^2*g*x^2 + 3*d^2*f)*log(c))/(d^2*x^5)]

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1108 vs. $2(138) = 276$.

time = 164.84, size = 1108, normalized size = 7.91

$$\begin{cases} \left(-\frac{f}{5x^5} - \frac{g}{3x^3}\right)\log(0^p c) & \text{for } d = 0 \wedge e = 0 \\ \left(-\frac{f}{5x^5} - \frac{g}{3x^3}\right)\log(cd^p) & \text{for } e = 0 \\ -\frac{2fp}{25x^5} - \frac{f\log\left(c(ex^2)^p\right)}{5x^5} - \frac{2gp}{9x^3} - \frac{g\log\left(c(ex^2)^p\right)}{3x^3} & \text{for } d = 0 \\ -\frac{3d^3f\sqrt{-\frac{d}{e}}\log\left(c(d+ex^2)^p\right)}{15d^3x^5\sqrt{-\frac{d}{e}} + 15d^2ex^7\sqrt{-\frac{d}{e}}} - \frac{5d^3gx^2\sqrt{-\frac{d}{e}}\log\left(c(d+ex^2)^p\right)}{15d^3x^5\sqrt{-\frac{d}{e}} + 15d^2ex^7\sqrt{-\frac{d}{e}}} - \frac{2d^2fpx^2\sqrt{-\frac{d}{e}}}{\frac{15d^3x^5\sqrt{-\frac{d}{e}}}{e} + 15d^2x^7\sqrt{-\frac{d}{e}}} - \frac{3d^2fx^2\sqrt{-\frac{d}{e}}\log\left(c(d+ex^2)^p\right)}{\frac{15d^3x^5\sqrt{-\frac{d}{e}}}{e} + 15d^2x^7\sqrt{-\frac{d}{e}}} - \frac{10d^2gpx^5\log\left(x - \sqrt{-\frac{d}{e}}\right)}{\frac{15d^3x^5\sqrt{-\frac{d}{e}}}{e} + 15d^2x^7\sqrt{-\frac{d}{e}}} - \frac{10d^2gpx^4\sqrt{-\frac{d}{e}}}{\frac{15d^3x^5\sqrt{-\frac{d}{e}}}{e} + 15d^2x^7\sqrt{-\frac{d}{e}}} + \frac{5d^2gx^5\log\left(c(d+ex^2)^p\right)}{\frac{15d^3x^5\sqrt{-\frac{d}{e}}}{e} + 15d^2x^7\sqrt{-\frac{d}{e}}} - \frac{5d^2gx^4\sqrt{-\frac{d}{e}}\log\left(c(d+ex^2)^p\right)}{\frac{15d^3x^5\sqrt{-\frac{d}{e}}}{e} + 15d^2x^7\sqrt{-\frac{d}{e}}} + \frac{6defpx^5\log\left(x - \sqrt{-\frac{d}{e}}\right)}{\frac{15d^3x^5\sqrt{-\frac{d}{e}}}{e} + 15d^2x^7\sqrt{-\frac{d}{e}}} + \frac{4defpx^4\sqrt{-\frac{d}{e}}}{\frac{15d^3x^5\sqrt{-\frac{d}{e}}}{e} + 15d^2x^7\sqrt{-\frac{d}{e}}} - \frac{3defx^5\log\left(c(d+ex^2)^p\right)}{\frac{15d^3x^5\sqrt{-\frac{d}{e}}}{e} + 15d^2x^7\sqrt{-\frac{d}{e}}} - \frac{10degpx^7\log\left(x - \sqrt{-\frac{d}{e}}\right)}{\frac{15d^3x^5\sqrt{-\frac{d}{e}}}{e} + 15d^2x^7\sqrt{-\frac{d}{e}}} - \frac{10degpx^6\sqrt{-\frac{d}{e}}}{\frac{15d^3x^5\sqrt{-\frac{d}{e}}}{e} + 15d^2x^7\sqrt{-\frac{d}{e}}} + \frac{5degx^7\log\left(c(d+ex^2)^p\right)}{\frac{15d^3x^5\sqrt{-\frac{d}{e}}}{e} + 15d^2x^7\sqrt{-\frac{d}{e}}} + \frac{6e^2fpx^7\log\left(x - \sqrt{-\frac{d}{e}}\right)}{\frac{15d^3x^5\sqrt{-\frac{d}{e}}}{e} + 15d^2x^7\sqrt{-\frac{d}{e}}} + \frac{6e^2fpx^6\sqrt{-\frac{d}{e}}}{\frac{15d^3x^5\sqrt{-\frac{d}{e}}}{e} + 15d^2x^7\sqrt{-\frac{d}{e}}} - \frac{3e^2fx^7\log\left(c(d+ex^2)^p\right)}{\frac{15d^3x^5\sqrt{-\frac{d}{e}}}{e} + 15d^2x^7\sqrt{-\frac{d}{e}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**6,x)

[Out] Piecewise(((-f/(5*x**5) - g/(3*x**3))*log(0**p*c), Eq(d, 0) & Eq(e, 0)), ((-f/(5*x**5) - g/(3*x**3))*log(c*d**p), Eq(e, 0)), (-2*f*p/(25*x**5) - f*log(c*(e*x**2)**p)/(5*x**5) - 2*g*p/(9*x**3) - g*log(c*(e*x**2)**p)/(3*x**3), Eq(d, 0)), (-3*d**3*f*sqrt(-d/e)*log(c*(d + e*x**2)**p)/(15*d**3*x**5*sqrt(-d/e) + 15*d**2*e*x**7*sqrt(-d/e)) - 5*d**3*g*x**2*sqrt(-d/e)*log(c*(d + e*x**2)**p)/(15*d**3*x**5*sqrt(-d/e) + 15*d**2*e*x**7*sqrt(-d/e)) - 2*d**2*f*p*x**2*sqrt(-d/e)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) - 3*d**2*f*x**2*sqrt(-d/e)*log(c*(d + e*x**2)**p)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) - 10*d**2*g*p*x**5*log(x - sqrt(-d/e))/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) - 10*d**2*g*p*x**4*sqrt(-d/e)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) + 5*d**2*g*x**5*log(c*(d + e*x**2)**p)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) - 5*d**2*g*x**4*sqrt(-d/e)*log(c*(d + e*x**2)**p)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) + 6*d*e*f*p*x**5*log(x - sqrt(-d/e))/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) + 4*d*e*f*p*x**4*sqrt(-d/e)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) - 3*d*e*f*x**5*log(c*(d + e*x**2)**p)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) - 10*d*e*g*p*x**7*log(x - sqrt(-d/e))/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) - 10*d*e*g*p*x**6*sqrt(-d/e)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) + 5*d*e*g*x**7*log(c*(d + e*x**2)**p)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) + 6*e**2*f*p*x**7*log(x - sqrt(-d/e))/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) + 6*e**2*f*p*x**6*sqrt(-d/e)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) - 3*e**2*f*x**7*log(c*(d + e*x**2)**p)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)), True))

Giac [A]

time = 3.91, size = 122, normalized size = 0.87

$$-\frac{2\,(5\,dgpe^2-3\,fpe^3)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{15\,d^{\frac{5}{2}}}-\frac{10\,dgpx^4e-6\,fpx^4e^2+5\,d^2gpx^2\log\left(x^2e+d\right)+2\,dfpx^2e+5\,d^2gx^2\log\left(c\right)+3\,d^2fp\log\left(x^2e+d\right)+3\,d^2f\log\left(c\right)}{15\,d^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^6,x, algorithm="giac")
```

```
[Out] -2/15*(5*d*g*p*e^2 - 3*f*p*e^3)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d^(5/2)
- 1/15*(10*d*g*p*x^4*e - 6*f*p*x^4*e^2 + 5*d^2*g*p*x^2*log(x^2*e + d) + 2*d
*f*p*x^2*e + 5*d^2*g*x^2*log(c) + 3*d^2*f*p*log(x^2*e + d) + 3*d^2*f*log(c)
)/(d^2*x^5)
```

Mupad [B]
time = 0.38, size = 88, normalized size = 0.63

$$-\frac{\frac{2\,e\,f\,p}{d}+\frac{2\,e\,p\,x^2\,(5\,d\,g-3\,e\,f)}{d^2}}{15\,x^3}-\frac{\ln\left(c\,\left(e\,x^2+d\right)^p\right)\,\left(\frac{g\,x^2}{3}+\frac{f}{5}\right)}{x^5}-\frac{2\,e^{3/2}\,p\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(5\,d\,g-3\,e\,f)}{15\,d^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x^6,x)
```

```
[Out] - ((2*e*f*p)/d + (2*e*p*x^2*(5*d*g - 3*e*f))/d^2)/(15*x^3) - (log(c*(d + e*
x^2)^p)*(f/5 + (g*x^2)/3))/x^5 - (2*e^(3/2)*p*atan((e^(1/2)*x)/d^(1/2))*(5*
d*g - 3*e*f))/(15*d^(5/2))
```

3.323 $\int x^5 (f+gx^2)^2 \log \left(c(d+ex^2)^p\right) \, dx$

Optimal. Leaf size=251

$$-\frac{d^2(ef-dg)^2px^2}{2e^4}+\frac{d(ef-2dg)(ef-dg)p(d+ex^2)^2}{4e^5}-\frac{(e^2f^2-6defg+6d^2g^2)\,p(d+ex^2)^3}{18e^5}-\frac{g(ef-2dg)p(}{16e^5}$$

```
[Out] -1/2*d^2*(-d*g+e*f)^2*p*x^2/e^4+1/4*d*(-2*d*g+e*f)*(-d*g+e*f)*p*(e*x^2+d)^2
/e^5-1/18*(6*d^2*g^2-6*d*e*f*g+e^2*f^2)*p*(e*x^2+d)^3/e^5-1/16*g*(-2*d*g+e*
f)*p*(e*x^2+d)^4/e^5-1/50*g^2*p*(e*x^2+d)^5/e^5+1/60*d^3*(6*d^2*g^2-15*d*e*
f*g+10*e^2*f^2)*p*ln(e*x^2+d)/e^5+1/6*f^2*x^6*ln(c*(e*x^2+d)^p)+1/4*f*g*x^8
*ln(c*(e*x^2+d)^p)+1/10*g^2*x^10*ln(c*(e*x^2+d)^p)
```

Rubi [A]
time = 0.32, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2525, 45, 2461, 12, 907}

$$\frac{1}{6}f^2x^6\log\left(c(d+ex^2)^p\right)+\frac{1}{4}fgx^8\log\left(c(d+ex^2)^p\right)+\frac{1}{10}g^2x^{10}\log\left(c(d+ex^2)^p\right)-\frac{d^2px^2(ef-dg)^2}{2e^4}-\frac{p(d+ex^2)^3(6d^2g^2-6defg+e^2f^2)}{18e^5}+\frac{d^3p(6d^2g^2-15defg+10e^2f^2)\log(d+ex^2)}{60e^5}-\frac{gp(d+ex^2)^4(ef-2dg)}{16e^5}+\frac{dp(d+ex^2)^2(ef-2dg)(ef-dg)}{4e^5}-\frac{g^2p(d+ex^2)^5}{50e^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -1/2*(d^2*(e*f - d*g)^2*p*x^2)/e^4 + (d*(e*f - 2*d*g)*(e*f - d*g)*p*(d + e*
x^2)^2)/(4*e^5) - ((e^2*f^2 - 6*d*e*f*g + 6*d^2*g^2)*p*(d + e*x^2)^3)/(18*e
^5) - (g*(e*f - 2*d*g)*p*(d + e*x^2)^4)/(16*e^5) - (g^2*p*(d + e*x^2)^5)/(5
0*e^5) + (d^3*(10*e^2*f^2 - 15*d*e*f*g + 6*d^2*g^2)*p*Log[d + e*x^2])/(60*e
^5) + (f^2*x^6*Log[c*(d + e*x^2)^p])/6 + (f*g*x^8*Log[c*(d + e*x^2)^p])/4 +
 (g^2*x^10*Log[c*(d + e*x^2)^p])/10
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
```

```
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2461

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}\int x^5\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)\,dx &= \frac{1}{2}\text{Subst}\left(\int x^2(f+gx)^2\log\left(c(d+ex)^p\right)\,dx,x,x^2\right)\\ &= \frac{1}{6}f^2x^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}fgx^8\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{10}g^2x^{10}\log\left(\right.\\ &= \frac{1}{6}f^2x^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}fgx^8\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{10}g^2x^{10}\log\left(\right.\\ &= \frac{1}{6}f^2x^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}fgx^8\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{10}g^2x^{10}\log\left(\right.\\ &= -\frac{d^2(ef-dg)^2px^2}{2e^4}+\frac{d(ef-2dg)(ef-dg)p\left(d+ex^2\right)^2}{4e^5}-\frac{(e^2f^2-}{}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 205, normalized size = 0.82

$$\frac{-epx^2\left(360d^4g^2-180d^3eg\left(5f+gx^2\right)-30de^3x^2\left(10f^2+10fgx^2+3g^2x^4\right)+30d^2e^2\left(20f^2+15fgx^2+4g^2x^4\right)+e^4x^4\left(200f^2+225fgx^2+72g^2x^4\right)\right)+60d^3\left(10e^2f^2-15defg+6d^2g^2\right)p\log\left(d+ex^2\right)+60e^5x^6\left(10f^2+15fgx^2+6g^2x^4\right)\log\left(c\left(d+ex^2\right)^p\right)}{3600e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (-(e*p*x^2*(360*d^4*g^2 - 180*d^3*e*g*(5*f + g*x^2) - 30*d*e^3*x^2*(10*f^2
+ 10*f*g*x^2 + 3*g^2*x^4) + 30*d^2*e^2*(20*f^2 + 15*f*g*x^2 + 4*g^2*x^4) +
e^4*x^4*(200*f^2 + 225*f*g*x^2 + 72*g^2*x^4))) + 60*d^3*(10*e^2*f^2 - 15*d*
e*f*g + 6*d^2*g^2)*p*Log[d + e*x^2] + 60*e^5*x^6*(10*f^2 + 15*f*g*x^2 + 6*g
^2*x^4)*Log[c*(d + e*x^2)^p])/(3600*e^5)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.54, size = 687, normalized size = 2.74

method	result
risch	$-\frac{i\pi f g\,x^8 \mathrm{csgn}\left(i\left(e\,x^2+d\right)^p\right) \mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right) \mathrm{csgn}(ic)}{8}+\frac{\ln(c) g^2 x^{10}}{10}+\frac{\ln(c) f^2 x^6}{6}+\frac{d\,g^2 p\,x^8}{40e}-\frac{d^2 g^2 p\,x^6}{30e^2}+\frac{d^3 g^2 p\,x^4}{20e^3}+\frac{d\,f^2 p}{12}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(g*x^2+f)^2*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*ln(c)*g^2*x^10+1/6*ln(c)*f^2*x^6+1/40/e*d*g^2*p*x^8-1/30/e^2*d^2*g^2*p
*x^6+1/20/e^3*d^3*g^2*p*x^4+1/12/e*d*f^2*p*x^4-1/10/e^4*d^4*g^2*p*x^2-1/6/e
^2*d^2*f^2*p*x^2+1/10/e^5*ln(e*x^2+d)*d^5*g^2*p+1/6/e^3*ln(e*x^2+d)*d^3*f^2
*p-1/20*I*Pi*g^2*x^10*csgn(I*c*(e*x^2+d)^p)^3-1/12*I*Pi*f^2*x^6*csgn(I*c*(e
*x^2+d)^p)^3-1/20*I*Pi*g^2*x^10*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*c
sgn(I*c)+1/8*I*Pi*f*g*x^8*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+1/8*I*Pi*f*g*x^
8*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/12*I*Pi*f^2*x^6*csgn(I*(e*x
^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/4/e^3*d^3*f*g*p*x^2-1/4/e^4*ln(e
*x^2+d)*d^4*f*g*p+1/20*I*Pi*g^2*x^10*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+1/20
*I*Pi*g^2*x^10*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/8*I*Pi*f*g*x^8
*csgn(I*c*(e*x^2+d)^p)^3+1/12*I*Pi*f^2*x^6*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c
)+1/12*I*Pi*f^2*x^6*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/12/e*d*f*
g*p*x^6-1/8/e^2*d^2*f*g*p*x^4+(1/10*g^2*x^10+1/4*f*g*x^8+1/6*f^2*x^6)*ln((e
*x^2+d)^p)-1/16*f*g*p*x^8+1/4*ln(c)*f*g*x^8-1/50*g^2*p*x^10-1/18*f^2*p*x^6-
1/8*I*Pi*f*g*x^8*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)
```

Maxima [A]
time = 0.28, size = 218, normalized size = 0.87

$$\frac{1}{3600}\left(60\left(6\,d^5g^2-15\,d^4fge+10\,d^3f^2e^2\right)e^{(-6)}\log\left(x^2e+d\right)-\left(72\,g^2x^{10}e^4-45\left(2\,dg^2e^3-5\,fge^4\right)x^8+20\left(6\,d^2g^2e^2-15\,dfge^3+10\,f^2e^4\right)x^6-30\left(6\,d^3g^2e-15\,d^2fge^2+10\,df^2e^3\right)x^4+60\left(6\,d^4g^2-15\,d^3fge+10\,d^2f^2e^2\right)x^2\right)e^{(-5)}\right)pe+\frac{1}{60}\left(6\,g^2x^{10}+15\,fgx^8+10\,f^2x^6\right)\log\left(\left(x^2e+d\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] 1/3600*(60*(6*d^5*g^2 - 15*d^4*f*g*e + 10*d^3*f^2*e^2)*e^(-6)*log(x^2*e + d
) - (72*g^2*x^10*e^4 - 45*(2*d*g^2*e^3 - 5*f*g*e^4)*x^8 + 20*(6*d^2*g^2*e^2
 - 15*d*f*g*e^3 + 10*f^2*e^4)*x^6 - 30*(6*d^3*g^2*e - 15*d^2*f*g*e^2 + 10*d
*f^2*e^3)*x^4 + 60*(6*d^4*g^2 - 15*d^3*f*g*e + 10*d^2*f^2*e^2)*x^2)*e^(-5))
*p*e + 1/60*(6*g^2*x^10 + 15*f*g*x^8 + 10*f^2*x^6)*log((x^2*e + d)^p*c)
```

Fricas [A]
time = 0.36, size = 252, normalized size = 1.00

$$-\frac{1}{3600}\left(360\,d^4g^2px^2e-60\left(6\,g^2x^{10}+15\,fgx^8+10\,f^2x^6\right)e^5\log\left(c\right)+\left(72\,g^2px^{10}+225\,fgpx^8+200\,f^2px^6\right)e^5-30\left(3\,dg^2px^8+10\,dfgpx^6+10\,df^2px^4\right)e^4+30\left(4\,d^2g^2px^6+15\,d^2fgpx^4+20\,d^2f^2px^2\right)e^3-180\left(d^3g^2px^4+5\,d^3fgpx^2\right)e^2-60\left(6\,d^5g^2p-15\,d^4fgpe+10\,d^3f^2pe^2+\left(6\,g^2px^{10}+15\,fgpx^8+10\,f^2px^6\right)e^5\right)\log\left(x^2e+d\right)\right)e^{(-5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] -1/3600*(360*d^4*g^2*p*x^2*e - 60*(6*g^2*x^10 + 15*f*g*x^8 + 10*f^2*x^6)*e^
5*log(c) + (72*g^2*p*x^10 + 225*f*g*p*x^8 + 200*f^2*p*x^6)*e^5 - 30*(3*d*g^
2*p*x^8 + 10*d*f*g*p*x^6 + 10*d*f^2*p*x^4)*e^4 + 30*(4*d^2*g^2*p*x^6 + 15*d
^2*f*g*p*x^4 + 20*d^2*f^2*p*x^2)*e^3 - 180*(d^3*g^2*p*x^4 + 5*d^3*f*g*p*x^2
)*e^2 - 60*(6*d^5*g^2*p - 15*d^4*f*g*p*e + 10*d^3*f^2*p*e^2 + (6*g^2*p*x^10
 + 15*f*g*p*x^8 + 10*f^2*p*x^6)*e^5)*log(x^2*e + d))*e^(-5)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(g*x**2+f)**2*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 773 vs. 2(241) = 482.
time = 6.93, size = 773, normalized size = 3.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 1/10*(x^2*e + d)^5*g^2*p*e^(-5)*log(x^2*e + d) - 1/2*(x^2*e + d)^4*d*g^2*p*
e^(-5)*log(x^2*e + d) + (x^2*e + d)^3*d^2*g^2*p*e^(-5)*log(x^2*e + d) - (x^
2*e + d)^2*d^3*g^2*p*e^(-5)*log(x^2*e + d) - 1/50*(x^2*e + d)^5*g^2*p*e^(-5
) + 1/8*(x^2*e + d)^4*d*g^2*p*e^(-5) - 1/3*(x^2*e + d)^3*d^2*g^2*p*e^(-5) +
 1/2*(x^2*e + d)^2*d^3*g^2*p*e^(-5) + 1/4*(x^2*e + d)^4*f*g*p*e^(-4)*log(x^
2*e + d) - (x^2*e + d)^3*d*f*g*p*e^(-4)*log(x^2*e + d) + 3/2*(x^2*e + d)^2*
d^2*f*g*p*e^(-4)*log(x^2*e + d) + 1/10*(x^2*e + d)^5*g^2*e^(-5)*log(c) - 1/
2*(x^2*e + d)^4*d*g^2*e^(-5)*log(c) + (x^2*e + d)^3*d^2*g^2*e^(-5)*log(c) -
 (x^2*e + d)^2*d^3*g^2*e^(-5)*log(c) - 1/16*(x^2*e + d)^4*f*g*p*e^(-4) + 1/
3*(x^2*e + d)^3*d*f*g*p*e^(-4) - 3/4*(x^2*e + d)^2*d^2*f*g*p*e^(-4) + 1/6*(
x^2*e + d)^3*f^2*p*e^(-3)*log(x^2*e + d) - 1/2*(x^2*e + d)^2*d*f^2*p*e^(-3)
```

```
*log(x^2*e + d) + 1/4*(x^2*e + d)^4*f*g*e^(-4)*log(c) - (x^2*e + d)^3*d*f*g
*e^(-4)*log(c) + 3/2*(x^2*e + d)^2*d^2*f*g*e^(-4)*log(c) - 1/18*(x^2*e + d)
^3*f^2*p*e^(-3) + 1/4*(x^2*e + d)^2*d*f^2*p*e^(-3) + 1/6*(x^2*e + d)^3*f^2*
e^(-3)*log(c) - 1/2*(x^2*e + d)^2*d*f^2*e^(-3)*log(c) - 1/2*((x^2*e - (x^2*
e + d)*log(x^2*e + d) + d)*d^4*g^2*p - 2*(x^2*e - (x^2*e + d)*log(x^2*e + d
) + d)*d^3*f*g*p*e - (x^2*e + d)*d^4*g^2*log(c) + 2*(x^2*e + d)*d^3*f*g*e*l
og(c) + (x^2*e - (x^2*e + d)*log(x^2*e + d) + d)*d^2*f^2*p*e^2 - (x^2*e + d
)*d^2*f^2*e^2*log(c))*e^(-5)
```

Mupad [B]
time = 0.36, size = 224, normalized size = 0.89

$$\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(\frac{f^2\,x^6}{6}+\frac{f\,g\,x^8}{4}+\frac{g^2\,x^{10}}{10}\right)-x^6\,\left(\frac{f^2\,p}{18}-\frac{d\,\left(\frac{f\,g\,p}{2}-\frac{d\,g^2\,p}{5\,e}\right)}{6\,e}\right)-x^8\,\left(\frac{f\,g\,p}{16}-\frac{d\,g^2\,p}{40\,e}\right)-\frac{g^2\,p\,x^{10}}{50}+\frac{\ln\left(e\,x^2+d\right)\,\left(6\,p\,d^5\,g^2-15\,p\,d^4\,e\,f\,g+10\,p\,d^3\,e^2\,f^2\right)}{60\,e^5}+\frac{d\,x^4\,\left(\frac{f^2\,p}{3}-\frac{d\,\left(\frac{f\,g\,p}{2}-\frac{d\,g^2\,p}{5\,e}\right)}{e}\right)}{4\,e}-\frac{d^2\,x^2\,\left(\frac{f^2\,p}{3}-\frac{d\,\left(\frac{f\,g\,p}{2}-\frac{d\,g^2\,p}{5\,e}\right)}{e}\right)}{2\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*log(c*(d + e*x^2)^p)*(f + g*x^2)^2,x)
```

```
[Out] log(c*(d + e*x^2)^p)*((f^2*x^6)/6 + (g^2*x^10)/10 + (f*g*x^8)/4) - x^6*((f^
2*p)/18 - (d*((f*g*p)/2 - (d*g^2*p)/(5*e)))/(6*e)) - x^8*((f*g*p)/16 - (d*g
^2*p)/(40*e)) - (g^2*p*x^10)/50 + (log(d + e*x^2)*(6*d^5*g^2*p + 10*d^3*e^2
*f^2*p - 15*d^4*e*f*g*p))/(60*e^5) + (d*x^4*((f^2*p)/3 - (d*((f*g*p)/2 - (d
*g^2*p)/(5*e)))/e))/(4*e) - (d^2*x^2*((f^2*p)/3 - (d*((f*g*p)/2 - (d*g^2*p)
/(5*e)))/e))/(2*e^2)
```

3.324 $\int x^3 (f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx$

Optimal. Leaf size=210

$$\frac{d(ef-dg)^2px^2}{2e^3}-\frac{(ef-3dg)(ef-dg)p(d+ex^2)^2}{8e^4}-\frac{g(2ef-3dg)p(d+ex^2)^3}{18e^4}-\frac{g^2p(d+ex^2)^4}{32e^4}-\frac{d^2(6e^2f^2-$$

```
[Out] 1/2*d*(-d*g+e*f)^2*p*x^2/e^3-1/8*(-3*d*g+e*f)*(-d*g+e*f)*p*(e*x^2+d)^2/e^4-
1/18*g*(-3*d*g+2*e*f)*p*(e*x^2+d)^3/e^4-1/32*g^2*p*(e*x^2+d)^4/e^4-1/24*d^2
*(3*d^2*g^2-8*d*e*f*g+6*e^2*f^2)*p*ln(e*x^2+d)/e^4+1/4*f^2*x^4*ln(c*(e*x^2+
d)^p)+1/3*f*g*x^6*ln(c*(e*x^2+d)^p)+1/8*g^2*x^8*ln(c*(e*x^2+d)^p)
```

Rubi [A]
time = 0.25, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2525, 45, 2461, 12, 907}

$$\frac{1}{4}f^2x^4\log\left(c(d+ex^2)^p\right)+\frac{1}{3}fgx^6\log\left(c(d+ex^2)^p\right)+\frac{1}{8}g^2x^8\log\left(c(d+ex^2)^p\right)-\frac{d^2p(3d^2g^2-8defg+6e^2f^2)\log(d+ex^2)}{24e^4}-\frac{gp(d+ex^2)^3(2ef-3dg)}{18e^4}-\frac{p(d+ex^2)^2(ef-3dg)(ef-dg)}{8e^4}-\frac{g^2p(d+ex^2)^4}{32e^4}+\frac{dpx^2(ef-dg)^2}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (d*(e*f - d*g)^2*p*x^2)/(2*e^3) - ((e*f - 3*d*g)*(e*f - d*g)*p*(d + e*x^2)^
2)/(8*e^4) - (g*(2*e*f - 3*d*g)*p*(d + e*x^2)^3)/(18*e^4) - (g^2*p*(d + e*x
^2)^4)/(32*e^4) - (d^2*(6*e^2*f^2 - 8*d*e*f*g + 3*d^2*g^2)*p*Log[d + e*x^2]
)/(24*e^4) + (f^2*x^4*Log[c*(d + e*x^2)^p])/4 + (f*g*x^6*Log[c*(d + e*x^2)^
p])/3 + (g^2*x^8*Log[c*(d + e*x^2)^p])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
```

```
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2461

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int x^3\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)\,dx=\frac{1}{2}\text{Subst}\left(\int x(f+gx)^2\log\left(c(d+ex)^p\right)\,dx,x,x^2\right)$$
$$=\frac{1}{4}f^2x^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}fgx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{8}g^2x^8\log\left(c(d$$
$$=\frac{1}{4}f^2x^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}fgx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{8}g^2x^8\log\left(c(d$$
$$=\frac{1}{4}f^2x^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}fgx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{8}g^2x^8\log\left(c(d$$
$$=\frac{d(ef-dg)^2px^2}{2e^3}-\frac{(ef-3dg)(ef-dg)p\left(d+ex^2\right)^2}{8e^4}-\frac{g(2ef-3dg)p}{18e}$$

Mathematica [A]
time = 0.10, size = 173, normalized size = 0.82

$$\frac{epx^2\left(36d^3g^2-6d^2eg\left(16f+3gx^2\right)+12de^2\left(6f^2+4fgx^2+g^2x^4\right)-e^3x^2\left(36f^2+32fgx^2+9g^2x^4\right)\right)-12d^2\left(6e^2f^2-8defg+3d^2g^2\right)p\log\left(d+ex^2\right)+12e^4x^4\left(6f^2+8fgx^2+3g^2x^4\right)\log\left(c\left(d+ex^2\right)^p\right)}{288e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (e*p*x^2*(36*d^3*g^2 - 6*d^2*e*g*(16*f + 3*g*x^2) + 12*d*e^2*(6*f^2 + 4*f*g
*x^2 + g^2*x^4) - e^3*x^2*(36*f^2 + 32*f*g*x^2 + 9*g^2*x^4)) - 12*d^2*(6*e^
2*f^2 - 8*d*e*f*g + 3*d^2*g^2)*p*Log[d + e*x^2] + 12*e^4*x^4*(6*f^2 + 8*f*g
*x^2 + 3*g^2*x^4)*Log[c*(d + e*x^2)^p])/(288*e^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.58, size = 643, normalized size = 3.06

method	result
risch	$-\frac{p\,x^8 g^2}{32}-\frac{x^6 f g p}{9}-\frac{x^4 f^2 p}{8}+\frac{\ln(c) g^2 x^8}{8}+\frac{x^4\ln(c) f^2}{4}-\frac{i\pi f g\, x^6\operatorname{csgn}(i(e\,x^2+d)^p)\operatorname{csgn}(ic(e\,x^2+d)^p)\operatorname{csgn}(ic)}{6}-\frac{i\pi\, f^2 x^4 \mathrm{cs}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(g*x^2+f)^2*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*p*x^8*g^2-1/9*x^6*f*g*p-1/8*x^4*f^2*p+1/8*ln(c)*g^2*x^8+1/4*x^4*ln(c)
*f^2-1/6*I*Pi*f*g*x^6*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1
/16*I*Pi*g^2*x^8*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/6*I*
Pi*f*g*x^6*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+1/6*I*Pi*f*g*x^6*csgn(I*(e*x^2
+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/8*I*Pi*f^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*
c*(e*x^2+d)^p)*csgn(I*c)+(1/8*g^2*x^8+1/3*x^6*f*g+1/4*x^4*f^2)*ln((e*x^2+d)
^p)+1/3/e^3*ln(e*x^2+d)*d^3*f*g*p+1/16*I*Pi*g^2*x^8*csgn(I*c*(e*x^2+d)^p)^2
*csgn(I*c)+1/16*I*Pi*g^2*x^8*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/
6*I*Pi*f*g*x^6*csgn(I*c*(e*x^2+d)^p)^3+1/8*I*Pi*f^2*x^4*csgn(I*c*(e*x^2+d)^
p)^2*csgn(I*c)+1/8*I*Pi*f^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2
+1/6/e*x^4*d*f*g*p-1/3/e^2*x^2*d^2*f*g*p+1/3*x^6*ln(c)*f*g+1/24/e*x^6*d*g^2
*p-1/16/e^2*x^4*d^2*g^2*p+1/8/e^3*x^2*d^3*g^2*p+1/4/e*x^2*d*f^2*p-1/8/e^4*l
n(e*x^2+d)*d^4*g^2*p-1/4/e^2*ln(e*x^2+d)*d^2*f^2*p-1/16*I*Pi*g^2*x^8*csgn(I
*c*(e*x^2+d)^p)^3-1/8*I*Pi*f^2*x^4*csgn(I*c*(e*x^2+d)^p)^3
```

Maxima [A]
time = 0.28, size = 182, normalized size = 0.87

$$-\frac{1}{288}\left(12\left(3\,d^4g^2-8\,d^3fge+6\,d^2f^2e^2\right)e^{(-5)}\log\left(x^2e+d\right)+\left(9\,g^2x^8e^3-4\left(3\,dg^2e^2-8\,fge^3\right)x^6+6\left(3\,d^2g^2e-8\,dfge^2+6\,f^2e^3\right)x^4-12\left(3\,d^3g^2-8\,d^2fge+6\,df^2e^2\right)x^2\right)e^{(-4)}\right)pe+\frac{1}{24}\left(3\,g^2x^8+8\,fgx^6+6\,f^2x^4\right)\log\left(\left(x^2e+d\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] -1/288*(12*(3*d^4*g^2 - 8*d^3*f*g*e + 6*d^2*f^2*e^2)*e^(-5)*log(x^2*e + d)
+ (9*g^2*x^8*e^3 - 4*(3*d*g^2*e^2 - 8*f*g*e^3)*x^6 + 6*(3*d^2*g^2*e - 8*d*f
*g*e^2 + 6*f^2*e^3)*x^4 - 12*(3*d^3*g^2 - 8*d^2*f*g*e + 6*d*f^2*e^2)*x^2)*e
^(-4))*p*e + 1/24*(3*g^2*x^8 + 8*f*g*x^6 + 6*f^2*x^4)*log((x^2*e + d)^p*c)
```

Fricas [A]
time = 0.38, size = 214, normalized size = 1.02

$$\frac{1}{288}\left(36\,d^3g^2px^2e+12\left(3\,g^2x^8+8\,fgx^6+6\,f^2x^4\right)e^4\log\left(c\right)-\left(9\,g^2px^8+32\,fgpx^6+36\,f^2px^4\right)e^4+12\left(dg^2px^6+4\,dfgpx^4+6\,df^2px^2\right)e^3-6\left(3\,d^2g^2px^4+16\,d^2fgpx^2\right)e^2-12\left(3\,d^4g^2p-8\,d^3fgpe+6\,d^2f^2pe^2-\left(3\,g^2px^8+8\,fgpx^6+6\,f^2px^4\right)e^4\right)\log\left(x^2e+d\right)\right)e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] 1/288*(36*d^3*g^2*p*x^2*e + 12*(3*g^2*x^8 + 8*f*g*x^6 + 6*f^2*x^4)*e^4*log(
c) - (9*g^2*p*x^8 + 32*f*g*p*x^6 + 36*f^2*p*x^4)*e^4 + 12*(d*g^2*p*x^6 + 4*
d*f*g*p*x^4 + 6*d*f^2*p*x^2)*e^3 - 6*(3*d^2*g^2*p*x^4 + 16*d^2*f*g*p*x^2)*e
^2 - 12*(3*d^4*g^2*p - 8*d^3*f*g*p*e + 6*d^2*f^2*p*e^2 - (3*g^2*p*x^8 + 8*f
*g*p*x^6 + 6*f^2*p*x^4)*e^4)*log(x^2*e + d))*e^(-4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(g*x**2+f)**2*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 561 vs. 2(201) = 402.
time = 3.86, size = 561, normalized size = 2.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 1/8*(x^2*e + d)^4*g^2*p*e^(-4)*log(x^2*e + d) - 1/2*(x^2*e + d)^3*d*g^2*p*e
^(-4)*log(x^2*e + d) + 3/4*(x^2*e + d)^2*d^2*g^2*p*e^(-4)*log(x^2*e + d) -
1/32*(x^2*e + d)^4*g^2*p*e^(-4) + 1/6*(x^2*e + d)^3*d*g^2*p*e^(-4) - 3/8*(x
^2*e + d)^2*d^2*g^2*p*e^(-4) + 1/3*(x^2*e + d)^3*f*g*p*e^(-3)*log(x^2*e + d
) - (x^2*e + d)^2*d*f*g*p*e^(-3)*log(x^2*e + d) + 1/8*(x^2*e + d)^4*g^2*e^(
-4)*log(c) - 1/2*(x^2*e + d)^3*d*g^2*e^(-4)*log(c) + 3/4*(x^2*e + d)^2*d^2*
g^2*e^(-4)*log(c) - 1/9*(x^2*e + d)^3*f*g*p*e^(-3) + 1/2*(x^2*e + d)^2*d*f*
g*p*e^(-3) + 1/4*(x^2*e + d)^2*f^2*p*e^(-2)*log(x^2*e + d) + 1/3*(x^2*e + d
)^3*f*g*e^(-3)*log(c) - (x^2*e + d)^2*d*f*g*e^(-3)*log(c) - 1/8*(x^2*e + d)
^2*f^2*p*e^(-2) + 1/4*(x^2*e + d)^2*f^2*e^(-2)*log(c) + 1/2*((x^2*e - (x^2*
e + d)*log(x^2*e + d) + d)*d^3*g^2*p - 2*(x^2*e - (x^2*e + d)*log(x^2*e + d
) + d)*d^2*f*g*p*e - (x^2*e + d)*d^3*g^2*log(c) + 2*(x^2*e + d)*d^2*f*g*e*l
og(c) + (x^2*e - (x^2*e + d)*log(x^2*e + d) + d)*d*f^2*p*e^2 - (x^2*e + d)*
d*f^2*e^2*log(c))*e^(-4)
```

Mupad [**B**]
time = 0.34, size = 184, normalized size = 0.88

$$\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(\frac{f^2\,x^4}{4}+\frac{f\,g\,x^6}{3}+\frac{g^2\,x^8}{8}\right)-x^4\,\left(\frac{f^2\,p}{8}-\frac{d\,\left(\frac{2\,f\,g\,p}{3}-\frac{d\,g^2\,p}{4\,e}\right)}{4\,e}\right)-x^6\,\left(\frac{f\,g\,p}{9}-\frac{d\,g^2\,p}{24\,e}\right)-\frac{g^2\,p\,x^8}{32}-\frac{\ln\left(e\,x^2+d\right)\,\left(3\,p\,d^4\,g^2-8\,p\,d^3\,e\,f\,g+6\,p\,d^2\,e^2\,f^2\right)}{24\,e^4}+\frac{d\,x^2\,\left(\frac{f^2\,p}{2}-\frac{d\,\left(\frac{2\,f\,g\,p}{3}-\frac{d\,g^2\,p}{4\,e}\right)}{e}\right)}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(c*(d + e*x^2)^p)*(f + g*x^2)^2,x)
```

```
[Out] log(c*(d + e*x^2)^p)*((f^2*x^4)/4 + (g^2*x^8)/8 + (f*g*x^6)/3) - x^4*((f^2*
p)/8 - (d*((2*f*g*p)/3 - (d*g^2*p)/(4*e)))/(4*e)) - x^6*((f*g*p)/9 - (d*g^2
*p)/(24*e)) - (g^2*p*x^8)/32 - (log(d + e*x^2)*(3*d^4*g^2*p + 6*d^2*e^2*f^2
*p - 8*d^3*e*f*g*p))/(24*e^4) + (d*x^2*((f^2*p)/2 - (d*((2*f*g*p)/3 - (d*g^
2*p)/(4*e)))/e))/(2*e)
```

3.325 $\int x(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx$

Optimal. Leaf size=124

$$-\frac{(ef-dg)^2px^2}{6e^2}-\frac{(ef-dg)p(f+gx^2)^2}{12eg}-\frac{p(f+gx^2)^3}{18g}-\frac{(ef-dg)^3p\log(d+ex^2)}{6e^3g}+\frac{(f+gx^2)^3\log\left(c(d+ex^2\right.}{6g}$$

```
[Out] -1/6*(-d*g+e*f)^2*p*x^2/e^2-1/12*(-d*g+e*f)*p*(g*x^2+f)^2/e/g-1/18*p*(g*x^2
+f)^3/g-1/6*(-d*g+e*f)^3*p*ln(e*x^2+d)/e^3/g+1/6*(g*x^2+f)^3*ln(c*(e*x^2+d)
^p)/g
```

Rubi [A]
time = 0.09, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2525, 2442, 45}

$$\frac{(f+gx^2)^3\log\left(c(d+ex^2)^p\right)}{6g}-\frac{p(ef-dg)^3\log(d+ex^2)}{6e^3g}-\frac{px^2(ef-dg)^2}{6e^2}-\frac{p(f+gx^2)^2(ef-dg)}{12eg}-\frac{p(f+gx^2)^3}{18g}$$

Antiderivative was successfully verified.

```
[In] Int[x*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -1/6*((e*f - d*g)^2*p*x^2)/e^2 - ((e*f - d*g)*p*(f + g*x^2)^2)/(12*e*g) - (
p*(f + g*x^2)^3)/(18*g) - ((e*f - d*g)^3*p*Log[d + e*x^2])/(6*e^3*g) + ((f
+ g*x^2)^3*Log[c*(d + e*x^2)^p])/(6*g)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
```

```
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int x\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)\,dx=\frac{1}{2}\text{Subst}\left(\int(f+gx)^2\log\left(c(d+ex)^p\right)\,dx,x,x^2\right)$$
$$=\frac{\left(f+gx^2\right)^3\log\left(c\left(d+ex^2\right)^p\right)}{6g}-\frac{(ep)\text{Subst}\left(\int\frac{(f+gx)^3}{d+ex}\,dx,x,x^2\right)}{6g}$$
$$=\frac{\left(f+gx^2\right)^3\log\left(c\left(d+ex^2\right)^p\right)}{6g}-\frac{(ep)\text{Subst}\left(\int\left(\frac{g(ef-dg)^2}{e^3}+\frac{(ef-dg)^3}{e^3(d+ex)}+\right.\right.}{6g}$$
$$=-\frac{(ef-dg)^2px^2}{6e^2}-\frac{(ef-dg)p\left(f+gx^2\right)^2}{12eg}-\frac{p\left(f+gx^2\right)^3}{18g}-\frac{(ef-dg}{}$$

Mathematica [A]
time = 0.08, size = 135, normalized size = 1.09

$$\frac{6d^2g(-3ef+dg)p\log\left(d+ex^2\right)+e\left(-px^2\left(6d^2g^2-3deg\left(6f+gx^2\right)+e^2\left(18f^2+9fgx^2+2g^2x^4\right)\right)+6e\left(3df^2+ex^2\left(3f^2+3fgx^2+g^2x^4\right)\right)\log\left(c\left(d+ex^2\right)^p\right)\right)}{36e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (6*d^2*g*(-3*e*f + d*g)*p*Log[d + e*x^2] + e*(-(p*x^2*(6*d^2*g^2 - 3*d*e*g*
(6*f + g*x^2) + e^2*(18*f^2 + 9*f*g*x^2 + 2*g^2*x^4))) + 6*e*(3*d*f^2 + e*x
^2*(3*f^2 + 3*f*g*x^2 + g^2*x^4))*Log[c*(d + e*x^2)^p]))/(36*e^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.59, size = 605, normalized size = 4.88

method	result
risch	$\frac{\ln(c)fg\,x^4}{2}-\frac{i\pi fg\,x^4\operatorname{csgn}\left(i\left(e\,x^2+d\right)^p\right)\operatorname{csgn}\left(ic\left(e\,x^2+d\right)^p\right)\operatorname{csgn}(ic)}{4}-\frac{i\pi fg\,x^4\operatorname{csgn}\left(ic\left(e\,x^2+d\right)^p\right)^3}{4}-\frac{d^2fgp\ln\left(e\,x^2+d\right)}{2e^2}+\frac{dfgp}{2e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(g*x^2+f)^2*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(c)*f*g*x^4-1/2*d^2*f*g*p*ln(e*x^2+d)/e^2+1/2*d*f*g*p*x^2/e-1/4*f*g*p
*x^4-1/18*g^2*p*x^6-1/2*p*x^2*f^2-1/12*I*g^2*Pi*x^6*csgn(I*(e*x^2+d)^p)*csg
n(I*c*(e*x^2+d)^p)*csgn(I*c)+1/4*I*g*Pi*f*x^4*csgn(I*c*(e*x^2+d)^p)^2*csgn(
I*c)+1/4*I*g*Pi*f*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/4*I*Pi*
```

```
f^2*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/4*I*g*Pi*f*x^
4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/12/e*g^2*d*p*x^4-1/
6/e^2*g^2*d^2*p*x^2+1/6/e^3*g^2*ln(e*x^2+d)*d^3*p+1/2/e*ln(e*x^2+d)*d*f^2*p
-1/4*I*Pi*f^2*x^2*csgn(I*c*(e*x^2+d)^p)^3-1/12*I*g^2*Pi*x^6*csgn(I*c*(e*x^2
+d)^p)^3+1/12*I*g^2*Pi*x^6*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+1/12*I*g^2*Pi*
x^6*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/4*I*g*Pi*f*x^4*csgn(I*c*(
e*x^2+d)^p)^3+1/4*I*Pi*f^2*x^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+1/4*I*Pi*f
^2*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/6*ln(c)*g^2*x^6+1/2*ln
(c)*f^2*x^2+1/6*(g*x^2+f)^3/g*ln((e*x^2+d)^p)-1/6/g*ln(e*x^2+d)*f^3*p
```

Maxima [A]
time = 0.29, size = 151, normalized size = 1.22

$$\frac{(gx^2+f)^3\log\left((x^2e+d)^pc\right)}{6\,g}+\frac{\left(6\left(d^3g^3-3\,d^2fg^2e+3\,df^2ge^2-f^3e^3\right)e^{(-4)}\log\left(x^2e+d\right)-\left(2\,g^3x^6e^2-3\left(dg^3e-3\,fg^2e^2\right)x^4+6\left(d^2g^3-3\,dfg^2e+3\,f^2ge^2\right)x^2\right)e^{(-3)}\right)pe}{36\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] 1/6*(g*x^2 + f)^3*log((x^2*e + d)^p*c)/g + 1/36*(6*(d^3*g^3 - 3*d^2*f*g^2*e
 + 3*d*f^2*g*e^2 - f^3*e^3)*e^(-4)*log(x^2*e + d) - (2*g^3*x^6*e^2 - 3*(d*g
^3*e - 3*f*g^2*e^2)*x^4 + 6*(d^2*g^3 - 3*d*f*g^2*e + 3*f^2*g*e^2)*x^2)*e^(-
3))*p*e/g
```

Fricas [A]
time = 0.35, size = 169, normalized size = 1.36

$$-\frac{1}{36}\left(6\,d^2g^2px^2e-6\left(g^2x^6+3\,fgx^4+3\,f^2x^2\right)e^3\log\left(c\right)+\left(2\,g^2px^6+9\,fgpx^4+18\,f^2px^2\right)e^3-3\left(dg^2px^4+6\,dfgpx^2\right)e^2-6\left(d^3g^2p-3\,d^2fgpe+3\,df^2pe^2+\left(g^2px^6+3\,fgpx^4+3\,f^2px^2\right)e^3\right)\log\left(x^2e+d\right)\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] -1/36*(6*d^2*g^2*p*x^2*e - 6*(g^2*x^6 + 3*f*g*x^4 + 3*f^2*x^2)*e^3*log(c) +
 (2*g^2*p*x^6 + 9*f*g*p*x^4 + 18*f^2*p*x^2)*e^3 - 3*(d*g^2*p*x^4 + 6*d*f*g*
p*x^2)*e^2 - 6*(d^3*g^2*p - 3*d^2*f*g*p*e + 3*d*f^2*p*e^2 + (g^2*p*x^6 + 3*
f*g*p*x^4 + 3*f^2*p*x^2)*e^3)*log(x^2*e + d))*e^(-3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(104) = 208$.
time = 66.76, size = 235, normalized size = 1.90

$$\begin{cases}\frac{d^3g^2\log\left(c(d+ex^2)^p\right)}{6e^3}-\frac{d^2fg\log\left(c(d+ex^2)^p\right)}{2e^2}-\frac{d^2g^2px^2}{6e^2}+\frac{df^2\log\left(c(d+ex^2)^p\right)}{2e}+\frac{dfgpx^2}{2e}+\frac{dg^2px^4}{12e}-\frac{f^2px^2}{2}+\frac{f^2x^2\log\left(c(d+ex^2)^p\right)}{2}-\frac{fgpx^4}{4}+\frac{fgx^4\log\left(c(d+ex^2)^p\right)}{2}-\frac{g^2px^6}{18}+\frac{g^2x^6\log\left(c(d+ex^2)^p\right)}{6} & \text{for } e\neq 0\\ \left(\frac{f^2x^2}{2}+\frac{fgx^4}{2}+\frac{g^2x^6}{6}\right)\log\left(cd^p\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x**2+f)**2*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise((d**3*g**2*log(c*(d + e*x**2)**p)/(6*e**3) - d**2*f*g*log(c*(d +
e*x**2)**p)/(2*e**2) - d**2*g**2*p*x**2/(6*e**2) + d*f**2*log(c*(d + e*x**2
)**p)/(2*e) + d*f*g*p*x**2/(2*e) + d*g**2*p*x**4/(12*e) - f**2*p*x**2/2 + f
**2*x**2*log(c*(d + e*x**2)**p)/2 - f*g*p*x**4/4 + f*g*x**4*log(c*(d + e*x*
*2)**p)/2 - g**2*p*x**6/18 + g**2*x**6*log(c*(d + e*x**2)**p)/6, Ne(e, 0)),
 ((f**2*x**2/2 + f*g*x**4/2 + g**2*x**6/6)*log(c*d**p), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(116) = 232.
time = 6.60, size = 354, normalized size = 2.85

$$\frac{1}{6}\left(x^2e+d\right)^3g^2pe^{(-3)}\log\left(x^2e+d\right)-\frac{1}{2}\left(x^2e+d\right)^2dg^2pe^{(-3)}\log\left(x^2e+d\right)-\frac{1}{18}\left(x^2e+d\right)^3g^2pe^{(-3)}+\frac{1}{4}\left(x^2e+d\right)^2dg^2pe^{(-3)}+\frac{1}{2}\left(x^2e+d\right)^2fgpe^{(-2)}\log\left(x^2e+d\right)+\frac{1}{6}\left(x^2e+d\right)^3g^2e^{(-3)}\log\left(c\right)-\frac{1}{2}\left(x^2e+d\right)^2dg^2e^{(-3)}\log\left(c\right)-\frac{1}{4}\left(x^2e+d\right)^2fgpe^{(-2)}+\frac{1}{2}\left(x^2e+d\right)^2fge^{(-2)}\log\left(c\right)-\frac{1}{2}\left(\left(x^2e-\left(x^2e+d\right)\log\left(x^2e+d\right)+d\right)d^2g^2p-2\left(x^2e-\left(x^2e+d\right)\log\left(x^2e+d\right)+d\right)dfgpe-\left(x^2e+d\right)d^2g^2\log\left(c\right)+2\left(x^2e+d\right)dfge\log\left(c\right)+\left(x^2e-\left(x^2e+d\right)\log\left(x^2e+d\right)+d\right)f^2pe^2-\left(x^2e+d\right)f^2e^2\log\left(c\right)\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 1/6*(x^2*e + d)^3*g^2*p*e^(-3)*log(x^2*e + d) - 1/2*(x^2*e + d)^2*d*g^2*p*e
^(-3)*log(x^2*e + d) - 1/18*(x^2*e + d)^3*g^2*p*e^(-3) + 1/4*(x^2*e + d)^2*
d*g^2*p*e^(-3) + 1/2*(x^2*e + d)^2*f*g*p*e^(-2)*log(x^2*e + d) + 1/6*(x^2*e
 + d)^3*g^2*e^(-3)*log(c) - 1/2*(x^2*e + d)^2*d*g^2*e^(-3)*log(c) - 1/4*(x^
2*e + d)^2*f*g*p*e^(-2) + 1/2*(x^2*e + d)^2*f*g*e^(-2)*log(c) - 1/2*((x^2*e
 - (x^2*e + d)*log(x^2*e + d) + d)*d^2*g^2*p - 2*(x^2*e - (x^2*e + d)*log(x
^2*e + d) + d)*d*f*g*p*e - (x^2*e + d)*d^2*g^2*log(c) + 2*(x^2*e + d)*d*f*g
*e*log(c) + (x^2*e - (x^2*e + d)*log(x^2*e + d) + d)*f^2*p*e^2 - (x^2*e + d
)*f^2*e^2*log(c))*e^(-3)
```

Mupad [**B**]
time = 0.33, size = 142, normalized size = 1.15

$$\ln\left(c\left(e\,x^2+d\right)^p\right)\,\left(\frac{f^2\,x^2}{2}+\frac{f\,g\,x^4}{2}+\frac{g^2\,x^6}{6}\right)-x^2\,\left(\frac{f^2\,p}{2}-\frac{d\,\left(f\,g\,p-\frac{d\,g^2\,p}{3\,e}\right)}{2\,e}\right)-x^4\,\left(\frac{f\,g\,p}{4}-\frac{d\,g^2\,p}{12\,e}\right)+\frac{\ln\left(e\,x^2+d\right)\,\left(p\,d^3\,g^2-3\,p\,d^2\,e\,f\,g+3\,p\,d\,e^2\,f^2\right)}{6\,e^3}-\frac{g^2\,p\,x^6}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(c*(d + e*x^2)^p)*(f + g*x^2)^2,x)
```

```
[Out] log(c*(d + e*x^2)^p)*((f^2*x^2)/2 + (g^2*x^6)/6 + (f*g*x^4)/2) - x^2*((f^2*
p)/2 - (d*(f*g*p - (d*g^2*p)/(3*e)))/(2*e)) - x^4*((f*g*p)/4 - (d*g^2*p)/(1
2*e)) + (log(d + e*x^2)*(d^3*g^2*p + 3*d*e^2*f^2*p - 3*d^2*e*f*g*p))/(6*e^3
) - (g^2*p*x^6)/18
```

3.326 $\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x} \, dx$

Optimal. Leaf size=153

$$-fgpx^2+\frac{dg^2px^2}{4e}-\frac{1}{8}g^2px^4-\frac{d^2g^2p\log(d+ex^2)}{4e^2}+\frac{1}{4}g^2x^4\log\left(c(d+ex^2)^p\right)+\frac{fg(d+ex^2)\log\left(c(d+ex^2)^p\right)}{e}+\frac{1}{2}\cdot$$

```
[Out] -f*g*p*x^2+1/4*d*g^2*p*x^2/e-1/8*g^2*p*x^4-1/4*d^2*g^2*p*ln(e*x^2+d)/e^2+1/
4*g^2*x^4*ln(c*(e*x^2+d)^p)+f*g*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e+1/2*f^2*ln(-e
*x^2/d)*ln(c*(e*x^2+d)^p)+1/2*f^2*p*polylog(2,1+e*x^2/d)
```

Rubi [A]
time = 0.13, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2525, 45, 2463, 2436, 2332, 2441, 2352, 2442}

$$\frac{1}{2}f^2p\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)+\frac{1}{2}f^2\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)+\frac{fg(d+ex^2)\log\left(c(d+ex^2)^p\right)}{e}+\frac{1}{4}g^2x^4\log\left(c(d+ex^2)^p\right)-\frac{d^2g^2p\log(d+ex^2)}{4e^2}+\frac{dg^2px^2}{4e}-fgpx^2-\frac{1}{8}g^2px^4$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x,x]
```

```
[Out] -(f*g*p*x^2) + (d*g^2*p*x^2)/(4*e) - (g^2*p*x^4)/8 - (d^2*g^2*p*Log[d + e*x
^2])/(4*e^2) + (g^2*x^4*Log[c*(d + e*x^2)^p])/4 + (f*g*(d + e*x^2)*Log[c*(d
 + e*x^2)^p])/e + (f^2*Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])/2 + (f^2*p*P
olyLog[2, 1 + (e*x^2)/d])/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{(f+gx^2)^2 \log(c(d+ex^2)^p)}{x}\,dx = \frac{1}{2}\mathrm{Subst}\left(\int \frac{(f+gx)^2\log(c(d+ex)^p)}{x}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\mathrm{Subst}\left(\int \left(2fg\log(c(d+ex)^p) + \frac{f^2\log(c(d+ex)^p)}{x} + g^2x\log(c(d+\right.\right.$$

$$= \frac{1}{2}f^2\mathrm{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^2\right) + (fg)\mathrm{Subst}\left(\int \log(c(d+ex\right.$$

$$= \frac{1}{4}g^2x^4\log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{2}f^2\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right) + \frac{(fg)\mathrm{Su}}{}$$

$$= -fgpx^2 + \frac{1}{4}g^2x^4\log\left(c\left(d+ex^2\right)^p\right) + \frac{fg(d+ex^2)\log\left(c\left(d+ex^2\right)^p\right)}{e} + \frac{1}{2}$$

$$= -fgpx^2 + \frac{dg^2px^2}{4e} - \frac{1}{8}g^2px^4 - \frac{d^2g^2p\log(d+ex^2)}{4e^2} + \frac{1}{4}g^2x^4\log\left(c\left(d+e\right.\right.$$

Mathematica [A]
time = 0.06, size = 121, normalized size = 0.79

$$\frac{-egpx^2(8ef-2dg+egx^2) - 2d^2g^2p\log(d+ex^2) + 2e\left(g(4df+4efx^2+egx^4) + 2ef^2\log\left(-\frac{ex^2}{d}\right)\right)\log\left(c\left(d+ex^2\right)^p\right) + 4e^2f^2p\mathrm{Li}_2\left(1+\frac{ex^2}{d}\right)}{8e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x,x]
```

```
[Out] (-(e*g*p*x^2*(8*e*f - 2*d*g + e*g*x^2)) - 2*d^2*g^2*p*Log[d + e*x^2] + 2*e*
(g*(4*d*f + 4*e*f*x^2 + e*g*x^4) + 2*e*f^2*Log[-((e*x^2)/d)])*Log[c*(d + e*
x^2)^p] + 4*e^2*f^2*p*PolyLog[2, 1 + (e*x^2)/d])/(8*e^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.53, size = 652, normalized size = 4.26

method	result
risch	$-\frac{d^2g^2p\ln(ex^2+d)}{4e^2} - \frac{g^2px^4}{8} - fgp\,x^2 + \frac{dg^2px^2}{4e} - p\,f^2\,\mathrm{dilog}\left(\frac{-ex+\sqrt{-ed}}{\sqrt{-ed}}\right) - p\,f^2\,\mathrm{dilog}\left(\frac{ex+\sqrt{-ed}}{\sqrt{-ed}}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*d^2*g^2*p*ln(e*x^2+d)/e^2-1/8*g^2*p*x^4-f*g*p*x^2+1/4*d*g^2*p*x^2/e+1/
4*ln((e*x^2+d)^p)*g^2*x^4+ln((e*x^2+d)^p)*f^2*ln(x)-p*f^2*dilog((-e*x+(-e*d
)^(1/2))/(-e*d)^(1/2))-p*f^2*dilog((e*x+(-e*d)^(1/2))/(-e*d)^(1/2))+ln(c)*f
^2*ln(x)+ln((e*x^2+d)^p)*f*g*x^2-p*f^2*ln(x)*ln((-e*x+(-e*d)^(1/2))/(-e*d)^
```

```
(1/2))-p*f^2*ln(x)*ln((e*x+(-e*d)^(1/2))/(-e*d)^(1/2))+ln(c)*f*g*x^2-1/8*I*
Pi*g^2*x^4*csgn(I*c*(e*x^2+d)^p)^3-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*f^2*ln(
x)-1/2*I*Pi*f*g*x^2*csgn(I*c*(e*x^2+d)^p)^3+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*cs
gn(I*c*(e*x^2+d)^p)^2*f^2*ln(x)+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*
f^2*ln(x)+1/8*I*Pi*g^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/8*
I*Pi*g^2*x^4*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+p/e*g*d*ln(e*x^2+d)*f-1/2*I*
Pi*f*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/4*ln(c)*g^
2*x^4+1/2*I*Pi*f*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/2*I*Pi
*f*g*x^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csg
n(I*c*(e*x^2+d)^p)*csgn(I*c)*f^2*ln(x)-1/8*I*Pi*g^2*x^4*csgn(I*(e*x^2+d)^p)
*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)
```

Maxima [A]
time = 0.55, size = 157, normalized size = 1.03

$$\frac{1}{2}\left(\log\left(x^2e+d\right)\log\left(-\frac{x^2e+d}{d}+1\right)+\mathrm{Li}_2\left(\frac{x^2e+d}{d}\right)\right)f^2p+f^2\log\left(c\right)\log\left(x\right)-\frac{1}{8}\left(\left(g^2p-2\,g^2\log\left(c\right)\right)x^4e^2-2\left(dg^2pe-4\left(fgp-fg\log\left(c\right)\right)e^2\right)x^2-2\left(g^2px^4e^2+4\,fgpx^2e^2-d^2g^2p+4\,dfgpe\right)\log\left(x^2e+d\right)\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x,x, algorithm="maxima")
```

```
[Out] 1/2*(log(x^2*e + d)*log(-(x^2*e + d)/d + 1) + dilog((x^2*e + d)/d))*f^2*p +
 f^2*log(c)*log(x) - 1/8*((g^2*p - 2*g^2*log(c))*x^4*e^2 - 2*(d*g^2*p*e - 4
*(f*g*p - f*g*log(c))*e^2)*x^2 - 2*(g^2*p*x^4*e^2 + 4*f*g*p*x^2*e^2 - d^2*g
^2*p + 4*d*f*g*p*e)*log(x^2*e + d))*e^(-2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x,x, algorithm="fricas")
```

```
[Out] integral((g^2*x^4 + 2*f*g*x^2 + f^2)*log((x^2*e + d)^p*c)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f + gx^{2}\right)^{2}\log\left(c\left(d + ex^{2}\right)^{p}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x,x)
```

```
[Out] Integral((f + g*x**2)**2*log(c*(d + e*x**2)**p)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x,x, algorithm="giac")
[Out] integrate((g*x^2 + f)^2*log((x^2*e + d)^p*c)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,{\left(g\,x^2+f\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x,x)
[Out] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x, x)
```

3.327 $\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^3} dx$

Optimal. Leaf size=135

$$-\frac{1}{2}g^2px^2+\frac{ef^2p\log(x)}{d}-\frac{ef^2p\log(d+ex^2)}{2d}-\frac{f^2\log\left(c(d+ex^2)^p\right)}{2x^2}+\frac{g^2(d+ex^2)\log\left(c(d+ex^2)^p\right)}{2e}+fg\log\left(-\right.$$

```
[Out] -1/2*g^2*p*x^2+e*f^2*p*ln(x)/d-1/2*e*f^2*p*ln(e*x^2+d)/d-1/2*f^2*ln(c*(e*x^
2+d)^p)/x^2+1/2*g^2*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e+f*g*ln(-e*x^2/d)*ln(c*(e*
x^2+d)^p)+f*g*p*polylog(2,1+e*x^2/d)
```

Rubi [A]
time = 0.13, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {2525, 45, 2463, 2436, 2332, 2442, 36, 29, 31, 2441, 2352}

$$fgp\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)-\frac{f^2\log\left(c(d+ex^2)^p\right)}{2x^2}+fg\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)+\frac{g^2(d+ex^2)\log\left(c(d+ex^2)^p\right)}{2e}-\frac{ef^2p\log(d+ex^2)}{2d}+\frac{ef^2p\log(x)}{d}-\frac{1}{2}g^2px^2$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^3,x]
```

```
[Out] -1/2*(g^2*p*x^2) + (e*f^2*p*Log[x])/d - (e*f^2*p*Log[d + e*x^2])/(2*d) - (f
^2*Log[c*(d + e*x^2)^p])/(2*x^2) + (g^2*(d + e*x^2)*Log[c*(d + e*x^2)^p])/(
2*e) + f*g*Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p] + f*g*p*PolyLog[2, 1 + (e
*x^2)/d]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
```

```
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(f+gx)^2\log\left(c(d+ex)^p\right)}{x^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(g^2\log\left(c(d+ex)^p\right) + \frac{f^2\log\left(c(d+ex)^p\right)}{x^2} + \frac{2fg\log\left(c(d\right.}{x}\right.\right.$$

$$= \frac{1}{2}f^2\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x^2}\,dx, x, x^2\right) + (fg)\text{Subst}\left(\int \frac{\log\left(c(d+e\right.}{x}\right.$$

$$= -\frac{f^2\log\left(c(d+ex^2)^p\right)}{2x^2} + fg\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right) + \frac{g^2\text{Subst}}{}$$

$$= -\frac{1}{2}g^2px^2 - \frac{f^2\log\left(c(d+ex^2)^p\right)}{2x^2} + \frac{g^2(d+ex^2)\log\left(c(d+ex^2)^p\right)}{2e} + fg$$

$$= -\frac{1}{2}g^2px^2 + \frac{ef^2p\log(x)}{d} - \frac{ef^2p\log\left(d+ex^2\right)}{2d} - \frac{f^2\log\left(c(d+ex^2)^p\right)}{2x^2} +$$

Mathematica [A]
time = 0.06, size = 126, normalized size = 0.93

$$\frac{1}{2}\left(-g^2px^2 + \frac{ef^2p(2\log(x)-\log\left(d+ex^2\right))}{d} - \frac{f^2\log\left(c(d+ex^2)^p\right)}{x^2} + \frac{g^2(d+ex^2)\log\left(c(d+ex^2)^p\right)}{e} + 2fg\left(\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right) + p\text{Li}_2\left(1+\frac{ex^2}{d}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^3,x]
```

```
[Out] (-(g^2*p*x^2) + (e*f^2*p*(2*Log[x] - Log[d + e*x^2]))/d - (f^2*Log[c*(d + e
*x^2)^p])/x^2 + (g^2*(d + e*x^2)*Log[c*(d + e*x^2)^p])/e + 2*f*g*(Log[-((e*
x^2)/d)]*Log[c*(d + e*x^2)^p] + p*PolyLog[2, 1 + (e*x^2)/d]))/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.41, size = 642, normalized size = 4.76

method	result
risch	$\frac{e\,f^2p\ln(x)}{d} - \frac{e\,f^2p\ln(e\,x^2+d)}{2d} - \frac{g^2p\,x^2}{2} - i\pi\,\text{csgn}\left(i(e\,x^2+d)^p\right)\text{csgn}\left(ic(e\,x^2+d)^p\right)\text{csgn}(ic)\,fg\ln(x) -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] e*f^2*p*ln(x)/d-1/2*e*f^2*p*ln(e*x^2+d)/d-1/2*g^2*p*x^2-I*Pi*csgn(I*(e*x^2+
d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*f*g*ln(x)-1/4*I*Pi*g^2*x^2*csgn(I*c*(
e*x^2+d)^p)^3+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*f^2/x^2-2*p*f*g*ln(x)*ln((-e
*x+(-e*d)^(1/2))/(-e*d)^(1/2))-2*p*f*g*ln(x)*ln((e*x+(-e*d)^(1/2))/(-e*d)^(
1/2))+1/2*p/e*d*ln(e*x^2+d)*g^2+1/2*ln((e*x^2+d)^p)*x^2*g^2-1/2*ln((e*x^2+d
)^p)*f^2/x^2-1/2*ln(c)*f^2/x^2+2*ln((e*x^2+d)^p)*f*g*ln(x)-2*p*f*g*dilog((-
e*x+(-e*d)^(1/2))/(-e*d)^(1/2))-2*p*f*g*dilog((e*x+(-e*d)^(1/2))/(-e*d)^(1/
2))+2*ln(c)*f*g*ln(x)+1/2*ln(c)*g^2*x^2-1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I
*c*(e*x^2+d)^p)^2*f^2/x^2-I*Pi*csgn(I*c*(e*x^2+d)^p)^3*f*g*ln(x)-1/4*I*Pi*c
sgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*f^2/x^2+1/4*I*Pi*g^2*x^2*csgn(I*(e*x^2+d)^
p)*csgn(I*c*(e*x^2+d)^p)^2+1/4*I*Pi*g^2*x^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*
c)+I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*f*g*ln(x)-1/4*I*Pi*g^2*x^2*csgn(I
*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*
csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*f^2/x^2+I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(
e*x^2+d)^p)^2*f*g*ln(x)
```

Maxima [A]
time = 0.33, size = 160, normalized size = 1.19

$$\left(\log\left(x^2e+d\right)\log\left(-\frac{x^2e+d}{d}+1\right)+\mathrm{Li}_2\left(\frac{x^2e+d}{d}\right)\right)fgp+\frac{\left(f^2pe+2\,dfg\log\left(c\right)\right)\log\left(x\right)}{d}-\frac{\left(\left(dg^2p-dg^2\log\left(c\right)\right)x^4e+df^2e\log\left(c\right)-\left(dg^2px^4e-df^2pe+\left(d^2g^2p-f^2pe^2\right)x^2\right)\log\left(x^2e+d\right)\right)e^{(-1)}}{2\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^3,x, algorithm="maxima")
```

```
[Out] (log(x^2*e + d)*log(-(x^2*e + d)/d + 1) + dilog((x^2*e + d)/d))*f*g*p + (f^
2*p*e + 2*d*f*g*log(c))*log(x)/d - 1/2*((d*g^2*p - d*g^2*log(c))*x^4*e + d*
f^2*e*log(c) - (d*g^2*p*x^4*e - d*f^2*p*e + (d^2*g^2*p - f^2*p*e^2)*x^2)*lo
g(x^2*e + d))*e^(-1)/(d*x^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^3,x, algorithm="fricas")
```

```
[Out] integral((g^2*x^4 + 2*f*g*x^2 + f^2)*log((x^2*e + d)^p*c)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**3,x)
```

```
[Out] Integral((f + g*x**2)**2*log(c*(d + e*x**2)**p)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^3,x, algorithm="giac")
```

```
[Out] integrate((g*x^2 + f)^2*log((x^2*e + d)^p*c)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,{\left(g\,x^2+f\right)}^2}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^3,x)
```

```
[Out] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^3, x)
```

3.328 $\int \frac{(f+gx^2)^2 \log\left(c\left(d+ex^2\right)^p\right)}{x^5} \, dx$

Optimal. Leaf size=172

$$-\frac{ef^2p}{4dx^2}-\frac{e^2f^2p\log(x)}{2d^2}+\frac{2efgp\log(x)}{d}+\frac{e^2f^2p\log\left(d+ex^2\right)}{4d^2}-\frac{efgp\log\left(d+ex^2\right)}{d}-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{4x^4}-fg\log$$

```
[Out] -1/4*e*f^2*p/d/x^2-1/2*e^2*f^2*p*ln(x)/d^2+2*e*f*g*p*ln(x)/d+1/4*e^2*f^2*p*
ln(e*x^2+d)/d^2-e*f*g*p*ln(e*x^2+d)/d-1/4*f^2*ln(c*(e*x^2+d)^p)/x^4-f*g*ln(
c*(e*x^2+d)^p)/x^2+1/2*g^2*ln(-e*x^2/d)*ln(c*(e*x^2+d)^p)+1/2*g^2*p*polylog
(2,1+e*x^2/d)
```

Rubi [A]

time = 0.15, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2525, 45, 2463, 2442, 46, 36, 29, 31, 2441, 2352}

$$\frac{1}{2}g^2p\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{4x^4}-\frac{fg\log\left(c\left(d+ex^2\right)^p\right)}{x^2}+\frac{1}{2}g^2\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{e^2f^2p\log\left(d+ex^2\right)}{4d^2}-\frac{e^2f^2p\log(x)}{2d^2}-\frac{ef^2p}{4dx^2}-\frac{efgp\log\left(d+ex^2\right)}{d}+\frac{2efgp\log(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^5,x]
```

```
[Out] -1/4*(e*f^2*p)/(d*x^2) - (e^2*f^2*p*Log[x])/(2*d^2) + (2*e*f*g*p*Log[x])/d
+ (e^2*f^2*p*Log[d + e*x^2])/(4*d^2) - (e*f*g*p*Log[d + e*x^2])/d - (f^2*Lo
g[c*(d + e*x^2)^p])/(4*x^4) - (f*g*Log[c*(d + e*x^2)^p])/x^2 + (g^2*Log[-((
e*x^2)/d)]*Log[c*(d + e*x^2)^p])/2 + (g^2*p*PolyLog[2, 1 + (e*x^2)/d])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{(f+gx^2)^2 \log(c(d+ex^2)^p)}{x^5}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(f+gx)^2 \log(c(d+ex)^p)}{x^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{f^2 \log(c(d+ex)^p)}{x^3} + \frac{2fg \log(c(d+ex)^p)}{x^2} + \frac{g^2 \log(c(d+}{x}\right.\right.$$
$$= \frac{1}{2}f^2\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x^3}\,dx, x, x^2\right) + (fg)\text{Subst}\left(\int \frac{\log(c(d+ex}{x^2}\right.$$
$$= -\frac{f^2 \log(c(d+ex^2)^p)}{4x^4} - \frac{fg \log(c(d+ex^2)^p)}{x^2} + \frac{1}{2}g^2 \log\left(-\frac{ex^2}{d}\right) \log(c$$
$$= -\frac{f^2 \log(c(d+ex^2)^p)}{4x^4} - \frac{fg \log(c(d+ex^2)^p)}{x^2} + \frac{1}{2}g^2 \log\left(-\frac{ex^2}{d}\right) \log(c$$
$$= -\frac{ef^2p}{4dx^2} - \frac{e^2f^2p\log(x)}{2d^2} + \frac{2efgp\log(x)}{d} + \frac{e^2f^2p\log(d+ex^2)}{4d^2} - \frac{efgp\,\text{lo}}{}$$

Mathematica [A]
time = 0.09, size = 148, normalized size = 0.86

$$\frac{1}{4}\left(\frac{4efgp(2\log(x)-\log(d+ex^2))}{d} - \frac{ef^2p(d+2ex^2\log(x)-ex^2\log(d+ex^2))}{d^2x^2} - \frac{f^2\log(c(d+ex^2)^p)}{x^4} - \frac{4fg\log(c(d+ex^2)^p)}{x^2} + 2g^2\left(\log\left(-\frac{ex^2}{d}\right)\log(c(d+ex^2)^p) + p\text{Li}_2\left(1+\frac{ex^2}{d}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^5,x]
```

```
[Out] ((4*e*f*g*p*(2*Log[x] - Log[d + e*x^2]))/d - (e*f^2*p*(d + 2*e*x^2*Log[x] -
 e*x^2*Log[d + e*x^2]))/(d^2*x^2) - (f^2*Log[c*(d + e*x^2)^p])/x^4 - (4*f*g
*Log[c*(d + e*x^2)^p])/x^2 + 2*g^2*(Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p]
+ p*PolyLog[2, 1 + (e*x^2)/d]))/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.45, size = 663, normalized size = 3.85

method	result
risch	$\frac{2efgp\ln(x)}{d} - \frac{efgp\ln(ex^2+d)}{d} + \frac{e^2f^2p\ln(ex^2+d)}{4d^2} - \frac{e^2f^2p\ln(x)}{2d^2} - \frac{ef^2p}{4dx^2} + \frac{i\pi\,\text{csgn}(i(ex^2+d)^p)\text{csgn}(ic(ex^2+d)^p)\text{csgn}(ic)f}{2x^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 2*e*f*g*p*ln(x)/d-e*f*g*p*ln(e*x^2+d)/d+1/4*e^2*f^2*p*ln(e*x^2+d)/d^2-1/2*e
^2*f^2*p*ln(x)/d^2-1/4*e*f^2*p/d/x^2+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*
```

```
(e*x^2+d)^p)*csgn(I*c)*f*g/x^2+1/8*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*f^2/x^4-1/2
*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*g^2*ln(x)-ln((e*x^2+d)^p)*f*g/x^2-p*g^2*ln(x)
*ln((-e*x+(-e*d)^(1/2))/(-e*d)^(1/2))-p*g^2*ln(x)*ln((e*x+(-e*d)^(1/2))/(-e
*d)^(1/2))-ln(c)*f*g/x^2+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*f*g/x^2-1/8*I*Pi*
csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*f^2/x^4+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*
csgn(I*c)*g^2*ln(x)-1/8*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*f^
2/x^4+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*g^2*ln(x)-1/4*ln
((e*x^2+d)^p)*f^2/x^4+ln((e*x^2+d)^p)*g^2*ln(x)+ln(c)*g^2*ln(x)-p*g^2*dilog
((-e*x+(-e*d)^(1/2))/(-e*d)^(1/2))-p*g^2*dilog((e*x+(-e*d)^(1/2))/(-e*d)^(1
/2))-1/4*ln(c)*f^2/x^4-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2
*f*g/x^2+1/8*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*f^2/x
^4-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*g^2*ln(x)-1
/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*f*g/x^2
```

Maxima [A]
time = 1.24, size = 172, normalized size = 1.00

$$\frac{1}{2}\left(\log\left(x^2e+d\right)\log\left(-\frac{x^2e+d}{d}+1\right)+\mathrm{Li}_2\left(\frac{x^2e+d}{d}\right)\right)g^2p+\frac{\left(4\,dfgpe+2\,d^2g^2\log\left(c\right)-f^2pe^2\right)\log\left(x\right)}{2\,d^2}-\frac{d^2f^2\log\left(c\right)+\left(df^2pe+4\,d^2fg\log\left(c\right)\right)x^2+\left(4\,d^2fgpx^2+d^2f^2p+\left(4\,dfgpe-f^2pe^2\right)x^4\right)\log\left(x^2e+d\right)}{4\,d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*(log(x^2*e + d)*log(-(x^2*e + d)/d + 1) + dilog((x^2*e + d)/d))*g^2*p +
 1/2*(4*d*f*g*p*e + 2*d^2*g^2*log(c) - f^2*p*e^2)*log(x)/d^2 - 1/4*(d^2*f^2
*log(c) + (d*f^2*p*e + 4*d^2*f*g*log(c))*x^2 + (4*d^2*f*g*p*x^2 + d^2*f^2*p
 + (4*d*f*g*p*e - f^2*p*e^2)*x^4)*log(x^2*e + d))/(d^2*x^4)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^5,x, algorithm="fricas")
```

```
[Out] integral((g^2*x^4 + 2*f*g*x^2 + f^2)*log((x^2*e + d)^p*c)/x^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**5,x)
```

```
[Out] Integral((f + g*x**2)**2*log(c*(d + e*x**2)**p)/x**5, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^5,x, algorithm="giac")
[Out] integrate((g*x^2 + f)^2*log((x^2*e + d)^p*c)/x^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,{\left(g\,x^2+f\right)}^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^5,x)
[Out] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^5, x)
```

3.329 $\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^7} dx$

Optimal. Leaf size=130

$$-\frac{ef^2p}{12dx^4}+\frac{ef(ef-3dg)p}{6d^2x^2}+\frac{e(e^2f^2-3defg+3d^2g^2)\,p\log(x)}{3d^3}-\frac{(ef-dg)^3p\log\left(d+ex^2\right)}{6d^3f}-\frac{(f+gx^2)^3\log\left(c\left(\right.\right.}{6fx^6}$$

```
[Out] -1/12*e*f^2*p/d/x^4+1/6*e*f*(-3*d*g+e*f)*p/d^2/x^2+1/3*e*(3*d^2*g^2-3*d*e*f
*g+e^2*f^2)*p*ln(x)/d^3-1/6*(-d*g+e*f)^3*p*ln(e*x^2+d)/d^3/f-1/6*(g*x^2+f)^
3*ln(c*(e*x^2+d)^p)/f/x^6
```

Rubi [A]
time = 0.14, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2525, 37, 2461, 12, 90}

$$-\frac{(f+gx^2)^3\log\left(c(d+ex^2)^p\right)}{6fx^6}-\frac{p(ef-dg)^3\log\left(d+ex^2\right)}{6d^3f}+\frac{efp(ef-3dg)}{6d^2x^2}+\frac{ep\log(x)\left(3d^2g^2-3defg+e^2f^2\right)}{3d^3}-\frac{ef^2p}{12dx^4}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^7,x]
```

```
[Out] -1/12*(e*f^2*p)/(d*x^4) + (e*f*(e*f - 3*d*g)*p)/(6*d^2*x^2) + (e*(e^2*f^2 -
 3*d*e*f*g + 3*d^2*g^2)*p*Log[x])/(3*d^3) - ((e*f - d*g)^3*p*Log[d + e*x^2]
)/(6*d^3*f) - ((f + g*x^2)^3*Log[c*(d + e*x^2)^p])/(6*f*x^6)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2461

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^7}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(f+gx)^2\log\left(c(d+ex)^p\right)}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{(f+gx^2)^3\log\left(c(d+ex^2)^p\right)}{6fx^6} - \frac{1}{2}(ep)\text{Subst}\left(\int -\frac{(f+gx)^3}{3fx^3(d+ex)}\,dx, x,\right.$$

$$= -\frac{(f+gx^2)^3\log\left(c(d+ex^2)^p\right)}{6fx^6} + \frac{(ep)\text{Subst}\left(\int \frac{(f+gx)^3}{x^3(d+ex)}\,dx, x, x^2\right)}{6f}$$

$$= -\frac{(f+gx^2)^3\log\left(c(d+ex^2)^p\right)}{6fx^6} + \frac{(ep)\text{Subst}\left(\int \left(\frac{f^3}{dx^3} + \frac{f^2(-ef+3dg)}{d^2x^2} + \frac{f(e^2\ldots}{}\right.\right.}{6\ldots}$$

$$= -\frac{ef^2p}{12dx^4} + \frac{ef(ef-3dg)p}{6d^2x^2} + \frac{e(e^2f^2-3defg+3d^2g^2)\,p\log(x)}{3d^3} - \frac{(ef-\ldots}{}$$

Mathematica [A]
time = 0.09, size = 141, normalized size = 1.08

$$-\frac{defpx^2(-2efx^2+d(f+6gx^2)) - 4e(e^2f^2-3defg+3d^2g^2)\,px^6\log(x) + 2e(e^2f^2-3defg+3d^2g^2)\,px^6\log\left(d+ex^2\right) + 2d^3(f^2+3fgx^2+3g^2x^4)\log\left(c(d+ex^2)^p\right)}{12d^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^7,x]
```

```
[Out] -1/12*(d*e*f*p*x^2*(-2*e*f*x^2 + d*(f + 6*g*x^2)) - 4*e*(e^2*f^2 - 3*d*e*f*
g + 3*d^2*g^2)*p*x^6*Log[x] + 2*e*(e^2*f^2 - 3*d*e*f*g + 3*d^2*g^2)*p*x^6*L
og[d + e*x^2] + 2*d^3*(f^2 + 3*f*g*x^2 + 3*g^2*x^4)*Log[c*(d + e*x^2)^p])/(
d^3*x^6)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.45, size = 656, normalized size = 5.05

method	result
risch	$-\frac{(3g^2x^4+3fg\,x^2+f^2)\ln((e\,x^2+d)^p)}{6x^6}+\frac{3i\pi\,d^3fg\,x^2\mathrm{csgn}(ic(e\,x^2+d)^p)^3+3i\pi\,d^3g^2x^4\mathrm{csgn}(i(e\,x^2+d)^p)\mathrm{csgn}(ic(e\,x^2+d)^p)\mathrm{csgn}(ic)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(3*g^2*x^4+3*f*g*x^2+f^2)/x^6*ln((e*x^2+d)^p)+1/12*(3*I*Pi*d^3*f*g*x^2
*csgn(I*c*(e*x^2+d)^p)^3+3*I*Pi*d^3*g^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e
*x^2+d)^p)*csgn(I*c)-3*I*Pi*d^3*g^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2
+d)^p)^2-3*I*Pi*d^3*g^2*x^4*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-6*ln(e*x^2+d)
*d^2*e*g^2*p*x^6+6*ln(e*x^2+d)*d*e^2*f*g*p*x^6-2*ln(e*x^2+d)*e^3*f^2*p*x^6+
12*ln(x)*d^2*e*g^2*p*x^6-12*ln(x)*d*e^2*f*g*p*x^6+4*ln(x)*e^3*f^2*p*x^6+3*I
*Pi*d^3*g^2*x^4*csgn(I*c*(e*x^2+d)^p)^3+I*Pi*d^3*f^2*csgn(I*c*(e*x^2+d)^p)^
3-3*I*Pi*d^3*f*g*x^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-3*I*Pi*d^3*f*g*x^2*c
sgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+3*I*Pi*d^3*f*g*x^2*csgn(I*(e*x^2
+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+I*Pi*d^3*f^2*csgn(I*(e*x^2+d)^p)*csg
n(I*c*(e*x^2+d)^p)*csgn(I*c)-I*Pi*d^3*f^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)
-I*Pi*d^3*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-6*ln(c)*d^3*g^2*x
^4-6*d^2*e*f*g*p*x^4+2*d*e^2*f^2*p*x^4-6*ln(c)*d^3*f*g*x^2-d^2*e*f^2*p*x^2-
2*ln(c)*d^3*f^2)/d^3/x^6
```

Maxima [A]
time = 0.27, size = 142, normalized size = 1.09

$$-\frac{1}{12}\,p\left(\frac{2\,(3\,d^2g^2-3\,dfge+f^2e^2)\log\left(x^2e+d\right)}{d^3}-\frac{2\,(3\,d^2g^2-3\,dfge+f^2e^2)\log\left(x^2\right)}{d^3}+\frac{df^2+2\,(3\,dfg-f^2e)x^2}{d^2x^4}\right)e-\frac{(3\,g^2x^4+3\,fgx^2+f^2)\log\left((x^2e+d)^pc\right)}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^7,x, algorithm="maxima")
```

```
[Out] -1/12*p*(2*(3*d^2*g^2 - 3*d*f*g*e + f^2*e^2)*log(x^2*e + d)/d^3 - 2*(3*d^2*
g^2 - 3*d*f*g*e + f^2*e^2)*log(x^2)/d^3 + (d*f^2 + 2*(3*d*f*g - f^2*e)*x^2)
/(d^2*x^4))*e - 1/6*(3*g^2*x^4 + 3*f*g*x^2 + f^2)*log((x^2*e + d)^p*c)/x^6
```

Fricas [A]
time = 0.37, size = 199, normalized size = 1.53

$$\frac{2\,df^2px^4e^2-(6\,d^2fgpx^4+d^2f^2px^2)e-2\,(3\,d^2g^2px^6e-3\,dfgpx^6e^2+3\,d^3g^2px^4+f^2px^6e^3+3\,d^3fgpx^2+d^3f^2p)\log\left(x^2e+d\right)-2\,(3\,d^3g^2x^4+3\,d^3fgx^2+d^3f^2)\log\left(c\right)+4\,(3\,d^2g^2px^6e-3\,dfgpx^6e^2+f^2px^6e^3)\log\left(x\right)}{12\,d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(2*d*f^2*p*x^4*e^2 - (6*d^2*f*g*p*x^4 + d^2*f^2*p*x^2)*e - 2*(3*d^2*g^
2*p*x^6*e - 3*d*f*g*p*x^6*e^2 + 3*d^3*g^2*p*x^4 + f^2*p*x^6*e^3 + 3*d^3*f*g
*p*x^2 + d^3*f^2*p)*log(x^2*e + d) - 2*(3*d^3*g^2*x^4 + 3*d^3*f*g*x^2 + d^3
*f^2)*log(c) + 4*(3*d^2*g^2*p*x^6*e - 3*d*f*g*p*x^6*e^2 + f^2*p*x^6*e^3)*lo
g(x))/(d^3*x^6)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**7,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 791 vs. 2(128) = 256.
time = 5.80, size = 791, normalized size = 6.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^7,x, algorithm="giac")
```

```
[Out] -1/12*(6*(x^2*e + d)^3*d^2*g^2*p*e^2*log(x^2*e + d) - 12*(x^2*e + d)^2*d^3*
g^2*p*e^2*log(x^2*e + d) + 6*(x^2*e + d)*d^4*g^2*p*e^2*log(x^2*e + d) - 6*(
x^2*e + d)^3*d^2*g^2*p*e^2*log(x^2*e) + 18*(x^2*e + d)^2*d^3*g^2*p*e^2*log(
x^2*e) - 18*(x^2*e + d)*d^4*g^2*p*e^2*log(x^2*e) + 6*d^5*g^2*p*e^2*log(x^2*
e) - 6*(x^2*e + d)^3*d*f*g*p*e^3*log(x^2*e + d) + 18*(x^2*e + d)^2*d^2*f*g*
p*e^3*log(x^2*e + d) - 12*(x^2*e + d)*d^3*f*g*p*e^3*log(x^2*e + d) + 6*(x^2
*e + d)^3*d*f*g*p*e^3*log(x^2*e) - 18*(x^2*e + d)^2*d^2*f*g*p*e^3*log(x^2*e
) + 18*(x^2*e + d)*d^3*f*g*p*e^3*log(x^2*e) - 6*d^4*f*g*p*e^3*log(x^2*e) +
6*(x^2*e + d)^2*d^3*g^2*e^2*log(c) - 12*(x^2*e + d)*d^4*g^2*e^2*log(c) + 6*
d^5*g^2*e^2*log(c) + 6*(x^2*e + d)^2*d^2*f*g*p*e^3 - 12*(x^2*e + d)*d^3*f*g
*p*e^3 + 6*d^4*f*g*p*e^3 + 2*(x^2*e + d)^3*f^2*p*e^4*log(x^2*e + d) - 6*(x^
2*e + d)^2*d*f^2*p*e^4*log(x^2*e + d) + 6*(x^2*e + d)*d^2*f^2*p*e^4*log(x^2
*e + d) - 2*(x^2*e + d)^3*f^2*p*e^4*log(x^2*e) + 6*(x^2*e + d)^2*d*f^2*p*e^
4*log(x^2*e) - 6*(x^2*e + d)*d^2*f^2*p*e^4*log(x^2*e) + 2*d^3*f^2*p*e^4*log
(x^2*e) + 6*(x^2*e + d)*d^3*f*g*e^3*log(c) - 6*d^4*f*g*e^3*log(c) - 2*(x^2*
e + d)^2*d*f^2*p*e^4 + 5*(x^2*e + d)*d^2*f^2*p*e^4 - 3*d^3*f^2*p*e^4 + 2*d^
3*f^2*e^4*log(c))*e^(-1)/((x^2*e + d)^3*d^3 - 3*(x^2*e + d)^2*d^4 + 3*(x^2*
e + d)*d^5 - d^6)
```

Mupad [**B**]
time = 0.41, size = 151, normalized size = 1.16

$$\frac{\ln(x)\,\left(3\,p\,d^2\,e\,g^2-3\,p\,d\,e^2\,f\,g+p\,e^3\,f^2\right)}{3\,d^3}-\frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(\frac{f^2}{6}+\frac{f\,g\,x^2}{2}+\frac{g^2\,x^4}{2}\right)}{x^6}-\frac{\ln\left(e\,x^2+d\right)\,\left(3\,p\,d^2\,e\,g^2-3\,p\,d\,e^2\,f\,g+p\,e^3\,f^2\right)}{6\,d^3}-\frac{\frac{e\,f^2\,p}{4\,d}+\frac{e\,f\,p\,x^2\,\left(3\,d\,g-e\,f\right)}{2\,d^2}}{3\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^7,x)

[Out] (log(x)*(e^3*f^2*p + 3*d^2*e*g^2*p - 3*d*e^2*f*g*p))/(3*d^3) - (log(c*(d + e*x^2)^p)*(f^2/6 + (g^2*x^4)/2 + (f*g*x^2)/2))/x^6 - (log(d + e*x^2)*(e^3*f^2*p + 3*d^2*e*g^2*p - 3*d*e^2*f*g*p))/(6*d^3) - ((e*f^2*p)/(4*d) + (e*f*p*x^2*(3*d*g - e*f))/(2*d^2))/(3*x^4)

3.330 $\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^9} \, dx$

Optimal. Leaf size=216

$$-\frac{ef^2p}{24dx^6}+\frac{ef(3ef-8dg)p}{48d^2x^4}-\frac{e(3e^2f^2-8defg+6d^2g^2)\,p}{24d^3x^2}-\frac{e^2(3e^2f^2-8defg+6d^2g^2)\,p\log(x)}{12d^4}+\frac{e^2(3e^2f^2-8\cdots}{}$$

```
[Out] -1/24*e*f^2*p/d/x^6+1/48*e*f*(-8*d*g+3*e*f)*p/d^2/x^4-1/24*e*(6*d^2*g^2-8*d
*e*f*g+3*e^2*f^2)*p/d^3/x^2-1/12*e^2*(6*d^2*g^2-8*d*e*f*g+3*e^2*f^2)*p*ln(x
)/d^4+1/24*e^2*(6*d^2*g^2-8*d*e*f*g+3*e^2*f^2)*p*ln(e*x^2+d)/d^4-1/8*f^2*ln
(c*(e*x^2+d)^p)/x^8-1/3*f*g*ln(c*(e*x^2+d)^p)/x^6-1/4*g^2*ln(c*(e*x^2+d)^p)
/x^4
```

Rubi [A]
time = 0.20, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2525, 45, 2461, 12, 907}

$$-\frac{f^2\log\left(c(d+ex^2)^p\right)}{8x^8}-\frac{fg\log\left(c(d+ex^2)^p\right)}{3x^6}-\frac{g^2\log\left(c(d+ex^2)^p\right)}{4x^4}+\frac{efp(3ef-8dg)}{48d^2x^4}+\frac{e^2p(6d^2g^2-8defg+3e^2f^2)\log\left(d+ex^2\right)}{24d^4}-\frac{e^2p\log(x)\left(6d^2g^2-8defg+3e^2f^2\right)}{12d^4}-\frac{ep(6d^2g^2-8defg+3e^2f^2)}{24d^3x^2}-\frac{ef^2p}{24dx^6}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^9,x]
```

```
[Out] -1/24*(e*f^2*p)/(d*x^6) + (e*f*(3*e*f - 8*d*g)*p)/(48*d^2*x^4) - (e*(3*e^2*
f^2 - 8*d*e*f*g + 6*d^2*g^2)*p)/(24*d^3*x^2) - (e^2*(3*e^2*f^2 - 8*d*e*f*g
+ 6*d^2*g^2)*p*Log[x])/(12*d^4) + (e^2*(3*e^2*f^2 - 8*d*e*f*g + 6*d^2*g^2)*
p*Log[d + e*x^2])/(24*d^4) - (f^2*Log[c*(d + e*x^2)^p])/(8*x^8) - (f*g*Log[
c*(d + e*x^2)^p])/(3*x^6) - (g^2*Log[c*(d + e*x^2)^p])/(4*x^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
```

```
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2461

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right)}{x^9}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(f+gx)^2 \log\left(c(d+ex)^p\right)}{x^5}\,dx, x, x^2\right)\\
&= -\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{8x^8}-\frac{fg\log\left(c\left(d+ex^2\right)^p\right)}{3x^6}-\frac{g^2\log\left(c\left(d+ex^2\right)^p\right)}{4x^4}-\\
&= -\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{8x^8}-\frac{fg\log\left(c\left(d+ex^2\right)^p\right)}{3x^6}-\frac{g^2\log\left(c\left(d+ex^2\right)^p\right)}{4x^4}-\\
&= -\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{8x^8}-\frac{fg\log\left(c\left(d+ex^2\right)^p\right)}{3x^6}-\frac{g^2\log\left(c\left(d+ex^2\right)^p\right)}{4x^4}-\\
&= -\frac{ef^2p}{24dx^6}+\frac{ef(3ef-8dg)p}{48d^2x^4}-\frac{e\left(3e^2f^2-8defg+6d^2g^2\right)p}{24d^3x^2}-\frac{e^2(3e^2f^2
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 184, normalized size = 0.85

$$-\frac{depx^2\left(6e^2f^2x^4-defx^2\left(3f+16gx^2\right)+2d^2\left(f^2+4fgx^2+6g^2x^4\right)\right)+4e^2\left(3e^2f^2-8defg+6d^2g^2\right)px^8\log(x)-2e^2\left(3e^2f^2-8defg+6d^2g^2\right)px^8\log\left(d+ex^2\right)+2d^4\left(3f^2+8fgx^2+6g^2x^4\right)\log\left(c\left(d+ex^2\right)^p\right)}{48d^4x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^9,x]
```

```
[Out] -1/48*(d*e*p*x^2*(6*e^2*f^2*x^4 - d*e*f*x^2*(3*f + 16*g*x^2) + 2*d^2*(f^2 +
 4*f*g*x^2 + 6*g^2*x^4)) + 4*e^2*(3*e^2*f^2 - 8*d*e*f*g + 6*d^2*g^2)*p*x^8*
Log[x] - 2*e^2*(3*e^2*f^2 - 8*d*e*f*g + 6*d^2*g^2)*p*x^8*Log[d + e*x^2] + 2
*d^4*(3*f^2 + 8*f*g*x^2 + 6*g^2*x^4)*Log[c*(d + e*x^2)^p])/(d^4*x^8)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.50, size = 713, normalized size = 3.30

method	result
risch	$-\frac{(6g^2x^4+8fg\,x^2+3f^2)\ln((e\,x^2+d)^p)}{24x^8} - \frac{-6i\pi\,d^4g^2x^4\mathrm{csgn}(i(e\,x^2+d)^p)\mathrm{csgn}(ic(e\,x^2+d)^p)\mathrm{csgn}(ic)+8i\pi\,d^4fg\,x^2\mathrm{csgn}(i(e\,x^2+d)^p)\mathrm{c}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(6*g^2*x^4+8*f*g*x^2+3*f^2)/x^8*ln((e*x^2+d)^p)-1/48*(-6*I*Pi*d^4*g^2
*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+8*I*Pi*d^4*f*g*x^2
*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+8*I*Pi*d^4*f*g*x^2*csgn(I*c*(e
*x^2+d)^p)^2*csgn(I*c)+12*ln(c)*d^4*g^2*x^4+6*ln(c)*d^4*f^2-3*I*Pi*d^4*f^2*
csgn(I*c*(e*x^2+d)^p)^3+16*ln(c)*d^4*f*g*x^2-6*ln(-e*x^2-d)*e^4*f^2*p*x^8+1
2*ln(x)*e^4*f^2*p*x^8+12*d^3*e*g^2*p*x^6+6*d*e^3*f^2*p*x^6-3*d^2*e^2*f^2*p*
x^4+2*d^3*e*f^2*p*x^2+6*I*Pi*d^4*g^2*x^4*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-
8*I*Pi*d^4*f*g*x^2*csgn(I*c*(e*x^2+d)^p)^3-3*I*Pi*d^4*f^2*csgn(I*(e*x^2+d)^
p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+6*I*Pi*d^4*g^2*x^4*csgn(I*(e*x^2+d)^p)*c
sgn(I*c*(e*x^2+d)^p)^2+16*ln(-e*x^2-d)*d*e^3*f*g*p*x^8-32*ln(x)*d*e^3*f*g*p
*x^8-16*d^2*e^2*f*g*p*x^6+8*d^3*e*f*g*p*x^4-8*I*Pi*d^4*f*g*x^2*csgn(I*(e*x^
2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-6*I*Pi*d^4*g^2*x^4*csgn(I*c*(e*x^2+
d)^p)^3+3*I*Pi*d^4*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+3*I*Pi*d
^4*f^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-12*ln(-e*x^2-d)*d^2*e^2*g^2*p*x^8+
24*ln(x)*d^2*e^2*g^2*p*x^8)/d^4/x^8
```

Maxima [A]
time = 0.28, size = 184, normalized size = 0.85

$$\frac{1}{48}\,p\left(\frac{2\,(6\,d^2g^2e-8\,dfge^2+3\,f^2e^3)\log\left(x^2e+d\right)}{d^4}-\frac{2\,(6\,d^2g^2e-8\,dfge^2+3\,f^2e^3)\log\left(x^2\right)}{d^4}-\frac{2\,(6\,d^2g^2-8\,dfge+3\,f^2e^2)x^4+2\,d^2f^2+(8\,d^2fg-3\,df^2e)x^2}{d^3x^6}\right)e-\frac{(6\,g^2x^4+8\,fgx^2+3\,f^2)\log\left((x^2e+d)^pc\right)}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^9,x, algorithm="maxima")
```

```
[Out] 1/48*p*(2*(6*d^2*g^2*e - 8*d*f*g*e^2 + 3*f^2*e^3)*log(x^2*e + d)/d^4 - 2*(6
*d^2*g^2*e - 8*d*f*g*e^2 + 3*f^2*e^3)*log(x^2)/d^4 - (2*(6*d^2*g^2 - 8*d*f*
g*e + 3*f^2*e^2)*x^4 + 2*d^2*f^2 + (8*d^2*f*g - 3*d*f^2*e)*x^2)/(d^3*x^6))*
e - 1/24*(6*g^2*x^4 + 8*f*g*x^2 + 3*f^2)*log((x^2*e + d)^p*c)/x^8
```

Fricas [A]
time = 0.37, size = 243, normalized size = 1.12

$$-\frac{6\,df^2px^6e^3-(16\,d^2fgpx^6+3\,d^2f^2px^4)e^2+2\,(6\,d^3g^2px^6+4\,d^3fgpx^4+d^3f^2px^2)e-2\,(6\,d^2g^2px^8e^2-8\,dfgpx^8e^3+3\,f^2px^8e^4-6\,d^4g^2px^4-8\,d^4fgpx^2-3\,d^4f^2p)\log\left(x^2e+d\right)+2\,(6\,d^4g^2x^4+8\,d^4fgx^2+3\,d^4f^2)\log\left(c\right)+4\,(6\,d^2g^2px^8e^2-8\,dfgpx^8e^3+3\,f^2px^8e^4)\log\left(x\right)}{48\,d^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^9,x, algorithm="fricas")

[Out] -1/48*(6*d*f^2*p*x^6*e^3 - (16*d^2*f*g*p*x^6 + 3*d^2*f^2*p*x^4)*e^2 + 2*(6*d^3*g^2*p*x^6 + 4*d^3*f*g*p*x^4 + d^3*f^2*p*x^2)*e - 2*(6*d^2*g^2*p*x^8*e^2 - 8*d*f*g*p*x^8*e^3 + 3*f^2*p*x^8*e^4 - 6*d^4*g^2*p*x^4 - 8*d^4*f*g*p*x^2 - 3*d^4*f^2*p)*log(x^2*e + d) + 2*(6*d^4*g^2*x^4 + 8*d^4*f*g*x^2 + 3*d^4*f^2)*log(c) + 4*(6*d^2*g^2*p*x^8*e^2 - 8*d*f*g*p*x^8*e^3 + 3*f^2*p*x^8*e^4)*log(x))/(d^4*x^8)

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**9,x)

[Out] Timed out

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1089 vs. $2(206) = 412$.
time = 3.47, size = 1089, normalized size = 5.04

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^9,x, algorithm="giac")

[Out] 1/48*(12*(x^2*e + d)^4*d^2*g^2*p*e^3*log(x^2*e + d) - 48*(x^2*e + d)^3*d^3*g^2*p*e^3*log(x^2*e + d) + 60*(x^2*e + d)^2*d^4*g^2*p*e^3*log(x^2*e + d) - 24*(x^2*e + d)*d^5*g^2*p*e^3*log(x^2*e + d) - 12*(x^2*e + d)^4*d^2*g^2*p*e^3*log(x^2*e) + 48*(x^2*e + d)^3*d^3*g^2*p*e^3*log(x^2*e) - 72*(x^2*e + d)^2*d^4*g^2*p*e^3*log(x^2*e) + 48*(x^2*e + d)*d^5*g^2*p*e^3*log(x^2*e) - 12*d^6*g^2*p*e^3*log(x^2*e) - 12*(x^2*e + d)^3*d^3*g^2*p*e^3 + 36*(x^2*e + d)^2*d^4*g^2*p*e^3 - 36*(x^2*e + d)*d^5*g^2*p*e^3 + 12*d^6*g^2*p*e^3 - 16*(x^2*e + d)^4*d*f*g*p*e^4*log(x^2*e + d) + 64*(x^2*e + d)^3*d^2*f*g*p*e^4*log(x^2*e + d) - 96*(x^2*e + d)^2*d^3*f*g*p*e^4*log(x^2*e + d) + 48*(x^2*e + d)*d^4*f*g*p*e^4*log(x^2*e + d) + 16*(x^2*e + d)^4*d*f*g*p*e^4*log(x^2*e) - 64*(x^2*e + d)^3*d^2*f*g*p*e^4*log(x^2*e) + 96*(x^2*e + d)^2*d^3*f*g*p*e^4*log(x^2*e) - 64*(x^2*e + d)*d^4*f*g*p*e^4*log(x^2*e) + 16*d^5*f*g*p*e^4*log(x^2*e) - 12*(x^2*e + d)^2*d^4*g^2*e^3*log(c) + 24*(x^2*e + d)*d^5*g^2*e^3*log(c) - 12*d^6*g^2*e^3*log(c) + 16*(x^2*e + d)^3*d^2*f*g*p*e^4 - 56*(x^2*e + d)^2*d^3*f*g*p*e^4 + 64*(x^2*e + d)*d^4*f*g*p*e^4 - 24*d^5*f*g*p*e^4 + 6*(x^2*e + d)^4*f^2*p*e^5*log(x^2*e + d) - 24*(x^2*e + d)^3*d*f^2*p*e^5*log(x^2*

```
e + d) + 36*(x^2*e + d)^2*d^2*f^2*p*e^5*log(x^2*e + d) - 24*(x^2*e + d)*d^3
*f^2*p*e^5*log(x^2*e + d) - 6*(x^2*e + d)^4*f^2*p*e^5*log(x^2*e) + 24*(x^2*
e + d)^3*d*f^2*p*e^5*log(x^2*e) - 36*(x^2*e + d)^2*d^2*f^2*p*e^5*log(x^2*e)
 + 24*(x^2*e + d)*d^3*f^2*p*e^5*log(x^2*e) - 6*d^4*f^2*p*e^5*log(x^2*e) - 1
6*(x^2*e + d)*d^4*f*g*e^4*log(c) + 16*d^5*f*g*e^4*log(c) - 6*(x^2*e + d)^3*
d*f^2*p*e^5 + 21*(x^2*e + d)^2*d^2*f^2*p*e^5 - 26*(x^2*e + d)*d^3*f^2*p*e^5
 + 11*d^4*f^2*p*e^5 - 6*d^4*f^2*e^5*log(c))*e^(-1)/((x^2*e + d)^4*d^4 - 4*(
x^2*e + d)^3*d^5 + 6*(x^2*e + d)^2*d^6 - 4*(x^2*e + d)*d^7 + d^8)
```

Mupad [B]

time = 0.43, size = 190, normalized size = 0.88

$$\frac{\ln\left(e\,x^2+d\right)\,\left(6\,p\,d^2\,e^2\,g^2-8\,p\,d\,e^3\,f\,g+3\,p\,e^4\,f^2\right)}{24\,d^4}-\frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(\frac{f^2}{8}+\frac{f\,g\,x^2}{3}+\frac{g^2\,x^4}{4}\right)}{x^8}-\frac{\frac{e\,f^2\,p}{2\,d}+\frac{e\,p\,x^4\,\left(6\,d^2\,g^2-8\,d\,e\,f\,g+3\,e^2\,f^2\right)}{2\,d^3}+\frac{e\,f\,p\,x^2\,\left(8\,d\,g-3\,e\,f\right)}{4\,d^2}}{12\,x^6}-\frac{\ln\left(x\right)\,\left(6\,p\,d^2\,e^2\,g^2-8\,p\,d\,e^3\,f\,g+3\,p\,e^4\,f^2\right)}{12\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^9,x)
```

```
[Out] (log(d + e*x^2)*(3*e^4*f^2*p + 6*d^2*e^2*g^2*p - 8*d*e^3*f*g*p))/(24*d^4) -
 (log(c*(d + e*x^2)^p)*(f^2/8 + (g^2*x^4)/4 + (f*g*x^2)/3))/x^8 - ((e*f^2*p
)/(2*d) + (e*p*x^4*(6*d^2*g^2 + 3*e^2*f^2 - 8*d*e*f*g))/(2*d^3) + (e*f*p*x^
2*(8*d*g - 3*e*f))/(4*d^2))/(12*x^6) - (log(x)*(3*e^4*f^2*p + 6*d^2*e^2*g^2
*p - 8*d*e^3*f*g*p))/(12*d^4)
```

3.331 $\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^{11}} dx$

Optimal. Leaf size=253

$$-\frac{ef^2p}{40dx^8}+\frac{ef(2ef-5dg)p}{60d^2x^6}-\frac{e(6e^2f^2-15defg+10d^2g^2)\,p}{120d^3x^4}+\frac{e^2(6e^2f^2-15defg+10d^2g^2)\,p}{60d^4x^2}+\frac{e^3(6e^2f^2-15$$

```
[Out] -1/40*e*f^2*p/d/x^8+1/60*e*f*(-5*d*g+2*e*f)*p/d^2/x^6-1/120*e*(10*d^2*g^2-1
5*d*e*f*g+6*e^2*f^2)*p/d^3/x^4+1/60*e^2*(10*d^2*g^2-15*d*e*f*g+6*e^2*f^2)*p
/d^4/x^2+1/30*e^3*(10*d^2*g^2-15*d*e*f*g+6*e^2*f^2)*p*ln(x)/d^5-1/60*e^3*(1
0*d^2*g^2-15*d*e*f*g+6*e^2*f^2)*p*ln(e*x^2+d)/d^5-1/10*f^2*ln(c*(e*x^2+d)^p
)/x^10-1/4*f*g*ln(c*(e*x^2+d)^p)/x^8-1/6*g^2*ln(c*(e*x^2+d)^p)/x^6
```

Rubi [A]
time = 0.23, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2525, 45, 2461, 12, 907}

$$-\frac{f^2\log\left(c(d+ex^2)^p\right)}{10x^{10}}-\frac{fg\log\left(c(d+ex^2)^p\right)}{4x^8}-\frac{g^2\log\left(c(d+ex^2)^p\right)}{6x^6}+\frac{efp(2ef-5dg)}{60d^2x^6}-\frac{e^3p(10d^2g^2-15defg+6e^2f^2)\log\left(d+ex^2\right)}{60d^5}+\frac{e^3p\log(x)\left(10d^2g^2-15defg+6e^2f^2\right)}{30d^5}+\frac{e^2p(10d^2g^2-15defg+6e^2f^2)}{60d^4x^2}-\frac{ep(10d^2g^2-15defg+6e^2f^2)}{120d^3x^4}-\frac{ef^2p}{40dx^8}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^11,x]
```

```
[Out] -1/40*(e*f^2*p)/(d*x^8) + (e*f*(2*e*f - 5*d*g)*p)/(60*d^2*x^6) - (e*(6*e^2*
f^2 - 15*d*e*f*g + 10*d^2*g^2)*p)/(120*d^3*x^4) + (e^2*(6*e^2*f^2 - 15*d*e*
f*g + 10*d^2*g^2)*p)/(60*d^4*x^2) + (e^3*(6*e^2*f^2 - 15*d*e*f*g + 10*d^2*g
^2)*p*Log[x])/(30*d^5) - (e^3*(6*e^2*f^2 - 15*d*e*f*g + 10*d^2*g^2)*p*Log[d
 + e*x^2])/(60*d^5) - (f^2*Log[c*(d + e*x^2)^p])/(10*x^10) - (f*g*Log[c*(d
+ e*x^2)^p])/(4*x^8) - (g^2*Log[c*(d + e*x^2)^p])/(6*x^6)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
```

```
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2461

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^{11}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(f+gx)^2 \log\left(c(d+ex)^p\right)}{x^6}\,dx, x, x^2\right)\\
&= -\frac{f^2\log\left(c(d+ex^2)^p\right)}{10x^{10}} - \frac{fg\log\left(c(d+ex^2)^p\right)}{4x^8} - \frac{g^2\log\left(c(d+ex^2)^p\right)}{6x^6} -\\
&= -\frac{f^2\log\left(c(d+ex^2)^p\right)}{10x^{10}} - \frac{fg\log\left(c(d+ex^2)^p\right)}{4x^8} - \frac{g^2\log\left(c(d+ex^2)^p\right)}{6x^6} -\\
&= -\frac{f^2\log\left(c(d+ex^2)^p\right)}{10x^{10}} - \frac{fg\log\left(c(d+ex^2)^p\right)}{4x^8} - \frac{g^2\log\left(c(d+ex^2)^p\right)}{6x^6} -\\
&= -\frac{ef^2p}{40dx^8} + \frac{ef(2ef-5dg)p}{60d^2x^6} - \frac{e(6e^2f^2-15defg+10d^2g^2)\,p}{120d^3x^4} + \frac{e^2(6e^2f
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 215, normalized size = 0.85

$$-\frac{depx^2(-12e^3f^2x^6+6de^2fx^4(f+5gx^2)+d^3(3f^2+10fgx^2+10g^2x^4)-d^2ex^2(4f^2+15fgx^2+20g^2x^4))-4e^3(6e^2f^2-15defg+10d^2g^2)\,px^{10}\log(x)+2e^3(6e^2f^2-15defg+10d^2g^2)\,px^{10}\log\left(d+ex^2\right)+2d^5(6f^2+15fgx^2+10g^2x^4)\log\left(c(d+ex^2)^p\right)}{120d^5x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^11,x]
```

```
[Out] -1/120*(d*e*p*x^2*(-12*e^3*f^2*x^6 + 6*d*e^2*f*x^4*(f + 5*g*x^2) + d^3*(3*f
^2 + 10*f*g*x^2 + 10*g^2*x^4) - d^2*e*x^2*(4*f^2 + 15*f*g*x^2 + 20*g^2*x^4)
) - 4*e^3*(6*e^2*f^2 - 15*d*e*f*g + 10*d^2*g^2)*p*x^10*Log[x] + 2*e^3*(6*e^
2*f^2 - 15*d*e*f*g + 10*d^2*g^2)*p*x^10*Log[d + e*x^2] + 2*d^5*(6*f^2 + 15*
f*g*x^2 + 10*g^2*x^4)*Log[c*(d + e*x^2)^p])/(d^5*x^10)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.51, size = 748, normalized size = 2.96

method	result
risch	$-\frac{(10g^2x^4+15fg\,x^2+6f^2)\ln((e\,x^2+d)^p)}{60x^{10}} - \frac{-15i\pi\,d^5fg\,x^2\mathrm{csgn}(i(e\,x^2+d)^p)\mathrm{csgn}(ic(e\,x^2+d)^p)\mathrm{csgn}(ic)+10i\pi\,d^5g^2x^4\mathrm{csgn}(ic(e\,x^2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -1/60*(10*g^2*x^4+15*f*g*x^2+6*f^2)/x^10*ln((e*x^2+d)^p)-1/120*(-15*I*Pi*d^
5*f*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-10*I*Pi*d^5*g
^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+15*I*Pi*d^5*f*g*
x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+15*I*Pi*d^5*f*g*x^2*csgn(I*
c*(e*x^2+d)^p)^2*csgn(I*c)+6*I*Pi*d^5*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x
^2+d)^p)^2+6*I*Pi*d^5*f^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-10*I*Pi*d^5*g^2
*x^4*csgn(I*c*(e*x^2+d)^p)^3+20*ln(e*x^2+d)*d^2*e^3*g^2*p*x^10-40*ln(x)*d^2
*e^3*g^2*p*x^10+30*d^2*e^3*f*g*p*x^8-15*d^3*e^2*f*g*p*x^6+10*d^4*e*f*g*p*x^
4+10*I*Pi*d^5*g^2*x^4*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-15*I*Pi*d^5*f*g*x^2
*csgn(I*c*(e*x^2+d)^p)^3-6*I*Pi*d^5*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2
+d)^p)*csgn(I*c)+10*I*Pi*d^5*g^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)
^p)^2-30*ln(e*x^2+d)*d*e^4*f*g*p*x^10+60*ln(x)*d*e^4*f*g*p*x^10-6*I*Pi*d^5*
f^2*csgn(I*c*(e*x^2+d)^p)^3+12*ln(e*x^2+d)*e^5*f^2*p*x^10-24*ln(x)*e^5*f^2*
p*x^10+30*ln(c)*d^5*f*g*x^2-20*d^3*e^2*g^2*p*x^8-12*d*e^4*f^2*p*x^8+10*d^4*
e*g^2*p*x^6+6*d^2*e^3*f^2*p*x^6-4*d^3*e^2*f^2*p*x^4+3*d^4*e*f^2*p*x^2+20*ln
(c)*d^5*g^2*x^4+12*ln(c)*d^5*f^2)/d^5/x^10
```

Maxima [A]
time = 0.27, size = 220, normalized size = 0.87

$$-\frac{1}{120}\,p\left(\frac{2\,(10\,d^2g^2e^2-15\,dfge^3+6\,f^2e^4)\log(x^2e+d)}{d^5}-\frac{2\,(10\,d^2g^2e^2-15\,dfge^3+6\,f^2e^4)\log(x^2)}{d^5}-\frac{2\,(10\,d^2g^2e-15\,dfge^2+6\,f^2e^3)x^6-3\,d^3f^2-(10\,d^3g^2-15\,d^2fge+6\,df^2e^2)x^4-2\,(5\,d^3fg-2\,d^2f^2e)x^2}{d^4x^8}\right)e-\frac{(10\,g^2x^4+15\,fgx^2+6\,f^2)\log((x^2e+d)^pc)}{60\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^11,x, algorithm="maxima")
```

```
[Out] -1/120*p*(2*(10*d^2*g^2*e^2 - 15*d*f*g*e^3 + 6*f^2*e^4)*log(x^2*e + d)/d^5
- 2*(10*d^2*g^2*e^2 - 15*d*f*g*e^3 + 6*f^2*e^4)*log(x^2)/d^5 - (2*(10*d^2*g
^2*e - 15*d*f*g*e^2 + 6*f^2*e^3)*x^6 - 3*d^3*f^2 - (10*d^3*g^2 - 15*d^2*f*g
```

```
*e + 6*d*f^2*e^2)*x^4 - 2*(5*d^3*f*g - 2*d^2*f^2*e)*x^2)/(d^4*x^8))*e - 1/6
0*(10*g^2*x^4 + 15*f*g*x^2 + 6*f^2)*log((x^2*e + d)^p*c)/x^10
```

Fricas [A]
time = 0.37, size = 282, normalized size = 1.11

$$\frac{12\,df^2px^8e^4 - 6\,(5\,d^2fgpx^8 + d^2f^2px^6)e^3 + (20\,d^3g^2px^8 + 15\,d^3fgpx^6 + 4\,d^3f^2px^4)e^2 - (10\,d^4g^2px^6 + 10\,d^4fgpx^4 + 3\,d^4f^2px^2)e - 2\,(10\,d^2g^2px^{10}e^3 - 15\,dfgpx^{10}e^4 + 6\,f^2px^{10}e^5 + 10\,d^5g^2px^4 + 15\,d^5fgpx^2 + 6\,d^5f^2p)\log\left(x^2e + d\right) - 2\,(10\,d^5g^2x^4 + 15\,d^5fgx^2 + 6\,d^5f^2)\log\left(c\right) + 4\,(10\,d^2g^2px^{10}e^3 - 15\,dfgpx^{10}e^4 + 6\,f^2px^{10}e^5)\log\left(x\right)}{120\,d^5x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^11,x, algorithm="fricas")
```

```
[Out] 1/120*(12*d*f^2*p*x^8*e^4 - 6*(5*d^2*f*g*p*x^8 + d^2*f^2*p*x^6)*e^3 + (20*d
^3*g^2*p*x^8 + 15*d^3*f*g*p*x^6 + 4*d^3*f^2*p*x^4)*e^2 - (10*d^4*g^2*p*x^6
+ 10*d^4*f*g*p*x^4 + 3*d^4*f^2*p*x^2)*e - 2*(10*d^2*g^2*p*x^10*e^3 - 15*d*f
*g*p*x^10*e^4 + 6*f^2*p*x^10*e^5 + 10*d^5*g^2*p*x^4 + 15*d^5*f*g*p*x^2 + 6*
d^5*f^2*p)*log(x^2*e + d) - 2*(10*d^5*g^2*x^4 + 15*d^5*f*g*x^2 + 6*d^5*f^2)
*log(c) + 4*(10*d^2*g^2*p*x^10*e^3 - 15*d*f*g*p*x^10*e^4 + 6*f^2*p*x^10*e^5
)*log(x))/(d^5*x^10)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**11,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1340 vs. $2(240) = 480$.
time = 5.51, size = 1340, normalized size = 5.30

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^11,x, algorithm="giac")
```

```
[Out] -1/120*(20*(x^2*e + d)^5*d^2*g^2*p*e^4*log(x^2*e + d) - 100*(x^2*e + d)^4*d
^3*g^2*p*e^4*log(x^2*e + d) + 200*(x^2*e + d)^3*d^4*g^2*p*e^4*log(x^2*e + d
) - 180*(x^2*e + d)^2*d^5*g^2*p*e^4*log(x^2*e + d) + 60*(x^2*e + d)*d^6*g^2
*p*e^4*log(x^2*e + d) - 20*(x^2*e + d)^5*d^2*g^2*p*e^4*log(x^2*e) + 100*(x^
2*e + d)^4*d^3*g^2*p*e^4*log(x^2*e) - 200*(x^2*e + d)^3*d^4*g^2*p*e^4*log(x
^2*e) + 200*(x^2*e + d)^2*d^5*g^2*p*e^4*log(x^2*e) - 100*(x^2*e + d)*d^6*g^
2*p*e^4*log(x^2*e) + 20*d^7*g^2*p*e^4*log(x^2*e) - 20*(x^2*e + d)^4*d^3*g^2
*p*e^4 + 90*(x^2*e + d)^3*d^4*g^2*p*e^4 - 150*(x^2*e + d)^2*d^5*g^2*p*e^4 +
```

```
110*(x^2*e + d)*d^6*g^2*p*e^4 - 30*d^7*g^2*p*e^4 - 30*(x^2*e + d)^5*d*f*g*
p*e^5*log(x^2*e + d) + 150*(x^2*e + d)^4*d^2*f*g*p*e^5*log(x^2*e + d) - 300
*(x^2*e + d)^3*d^3*f*g*p*e^5*log(x^2*e + d) + 300*(x^2*e + d)^2*d^4*f*g*p*e
^5*log(x^2*e + d) - 120*(x^2*e + d)*d^5*f*g*p*e^5*log(x^2*e + d) + 30*(x^2*
e + d)^5*d*f*g*p*e^5*log(x^2*e) - 150*(x^2*e + d)^4*d^2*f*g*p*e^5*log(x^2*e
) + 300*(x^2*e + d)^3*d^3*f*g*p*e^5*log(x^2*e) - 300*(x^2*e + d)^2*d^4*f*g*
p*e^5*log(x^2*e) + 150*(x^2*e + d)*d^5*f*g*p*e^5*log(x^2*e) - 30*d^6*f*g*p*
e^5*log(x^2*e) + 20*(x^2*e + d)^2*d^5*g^2*e^4*log(c) - 40*(x^2*e + d)*d^6*g
^2*e^4*log(c) + 20*d^7*g^2*e^4*log(c) + 30*(x^2*e + d)^4*d^2*f*g*p*e^5 - 13
5*(x^2*e + d)^3*d^3*f*g*p*e^5 + 235*(x^2*e + d)^2*d^4*f*g*p*e^5 - 185*(x^2*
e + d)*d^5*f*g*p*e^5 + 55*d^6*f*g*p*e^5 + 12*(x^2*e + d)^5*f^2*p*e^6*log(x^
2*e + d) - 60*(x^2*e + d)^4*d*f^2*p*e^6*log(x^2*e + d) + 120*(x^2*e + d)^3*
d^2*f^2*p*e^6*log(x^2*e + d) - 120*(x^2*e + d)^2*d^3*f^2*p*e^6*log(x^2*e +
d) + 60*(x^2*e + d)*d^4*f^2*p*e^6*log(x^2*e + d) - 12*(x^2*e + d)^5*f^2*p*e
^6*log(x^2*e) + 60*(x^2*e + d)^4*d*f^2*p*e^6*log(x^2*e) - 120*(x^2*e + d)^3
*d^2*f^2*p*e^6*log(x^2*e) + 120*(x^2*e + d)^2*d^3*f^2*p*e^6*log(x^2*e) - 60
*(x^2*e + d)*d^4*f^2*p*e^6*log(x^2*e) + 12*d^5*f^2*p*e^6*log(x^2*e) + 30*(x
^2*e + d)*d^5*f*g*e^5*log(c) - 30*d^6*f*g*e^5*log(c) - 12*(x^2*e + d)^4*d*f
^2*p*e^6 + 54*(x^2*e + d)^3*d^2*f^2*p*e^6 - 94*(x^2*e + d)^2*d^3*f^2*p*e^6
+ 77*(x^2*e + d)*d^4*f^2*p*e^6 - 25*d^5*f^2*p*e^6 + 12*d^5*f^2*e^6*log(c))*
e^(-1)/((x^2*e + d)^5*d^5 - 5*(x^2*e + d)^4*d^6 + 10*(x^2*e + d)^3*d^7 - 10
*(x^2*e + d)^2*d^8 + 5*(x^2*e + d)*d^9 - d^10)
```

Mupad [B]
time = 0.46, size = 225, normalized size = 0.89

$$\frac{\ln(x)\,\left(10\,p\,d^2\,e^3\,g^2-15\,p\,d\,e^4\,f\,g+6\,p\,e^5\,f^2\right)}{30\,d^5}-\frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(\frac{f^2}{10}+\frac{f\,g\,x^2}{4}+\frac{g^2\,x^4}{6}\right)}{x^{10}}-\frac{\ln\left(e\,x^2+d\right)\,\left(10\,p\,d^2\,e^3\,g^2-15\,p\,d\,e^4\,f\,g+6\,p\,e^5\,f^2\right)}{60\,d^5}-\frac{\frac{3\,e\,f^2\,p}{4\,d}-\frac{e^2\,p\,x^6\,\left(10\,d^2\,g^2-15\,d\,e\,f\,g+6\,e^2\,f^2\right)}{2\,d^4}+\frac{e\,p\,x^4\,\left(10\,d^2\,g^2-15\,d\,e\,f\,g+6\,e^2\,f^2\right)}{4\,d^3}+\frac{e\,f\,p\,x^2\,\left(5\,d\,g-2\,e\,f\right)}{2\,d^2}}{30\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^11,x)
```

```
[Out] (log(x)*(6*e^5*f^2*p + 10*d^2*e^3*g^2*p - 15*d*e^4*f*g*p))/(30*d^5) - (log(
c*(d + e*x^2)^p)*(f^2/10 + (g^2*x^4)/6 + (f*g*x^2)/4))/x^10 - (log(d + e*x^
2)*(6*e^5*f^2*p + 10*d^2*e^3*g^2*p - 15*d*e^4*f*g*p))/(60*d^5) - ((3*e*f^2*
p)/(4*d) - (e^2*p*x^6*(10*d^2*g^2 + 6*e^2*f^2 - 15*d*e*f*g))/(2*d^4) + (e*p
*x^4*(10*d^2*g^2 + 6*e^2*f^2 - 15*d*e*f*g))/(4*d^3) + (e*f*p*x^2*(5*d*g - 2
*e*f))/(2*d^2))/(30*x^8)
```

3.332 $\int x^2(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx$

Optimal. Leaf size=278

$$\frac{2df^2px}{3e}-\frac{4d^2fgpx}{5e^2}+\frac{2d^3g^2px}{7e^3}-\frac{2}{9}f^2px^3+\frac{4dfgpx^3}{15e}-\frac{2d^2g^2px^3}{21e^2}-\frac{4}{25}fgpx^5+\frac{2dg^2px^5}{35e}-\frac{2}{49}g^2px^7-\frac{2d^{3/2}f^2p\tan^{-1}}{3e^{3/2}}$$

```
[Out] 2/3*d*f^2*p*x/e-4/5*d^2*f*g*p*x/e^2+2/7*d^3*g^2*p*x/e^3-2/9*f^2*p*x^3+4/15*
d*f*g*p*x^3/e-2/21*d^2*g^2*p*x^3/e^2-4/25*f*g*p*x^5+2/35*d*g^2*p*x^5/e-2/49
*g^2*p*x^7-2/3*d^(3/2)*f^2*p*arctan(x*e^(1/2)/d^(1/2))/e^(3/2)+4/5*d^(5/2)*
f*g*p*arctan(x*e^(1/2)/d^(1/2))/e^(5/2)-2/7*d^(7/2)*g^2*p*arctan(x*e^(1/2)/
d^(1/2))/e^(7/2)+1/3*f^2*x^3*ln(c*(e*x^2+d)^p)+2/5*f*g*x^5*ln(c*(e*x^2+d)^p
)+1/7*g^2*x^7*ln(c*(e*x^2+d)^p)
```

Rubi [A]
time = 0.15, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2526, 2505, 308, 211}

$$-\frac{2d^{3/2}f^2p\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{4d^{5/2}fgp\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{5e^{5/2}}-\frac{2d^{7/2}g^2p\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{7e^{7/2}}+\frac{1}{3}f^2x^3\log\left(c(d+ex^2)^p\right)+\frac{2}{5}fgx^5\log\left(c(d+ex^2)^p\right)+\frac{1}{7}g^2x^7\log\left(c(d+ex^2)^p\right)+\frac{2d^3g^2px}{7e^3}-\frac{4d^2fgpx}{5e^2}-\frac{2d^2g^2px^3}{21e^2}+\frac{2df^2px}{3e}+\frac{4dfgpx^3}{15e}+\frac{2dg^2px^5}{35e}-\frac{2}{9}f^2px^3-\frac{4}{25}fgpx^5-\frac{2}{49}g^2px^7$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (2*d*f^2*p*x)/(3*e) - (4*d^2*f*g*p*x)/(5*e^2) + (2*d^3*g^2*p*x)/(7*e^3) - (
2*f^2*p*x^3)/9 + (4*d*f*g*p*x^3)/(15*e) - (2*d^2*g^2*p*x^3)/(21*e^2) - (4*f
*g*p*x^5)/25 + (2*d*g^2*p*x^5)/(35*e) - (2*g^2*p*x^7)/49 - (2*d^(3/2)*f^2*p
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^(3/2)) + (4*d^(5/2)*f*g*p*ArcTan[(Sqrt[e
]*x)/Sqrt[d]])/(5*e^(5/2)) - (2*d^(7/2)*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/
(7*e^(7/2)) + (f^2*x^3*Log[c*(d + e*x^2)^p])/3 + (2*f*g*x^5*Log[c*(d + e*x^
2)^p])/5 + (g^2*x^7*Log[c*(d + e*x^2)^p])/7
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\begin{aligned}
\int x^2\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)\,dx &= \int \left(f^2x^2\log\left(c\left(d+ex^2\right)^p\right)+2fgx^4\log\left(c\left(d+ex^2\right)^p\right)+g^2x^6\log\left(c\right.\right.\\
&= f^2\int x^2\log\left(c\left(d+ex^2\right)^p\right)\,dx+(2fg)\int x^4\log\left(c\left(d+ex^2\right)^p\right)\,dx+\\
&= \frac{1}{3}f^2x^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{5}fgx^5\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log\left(c\right.\\
&= \frac{1}{3}f^2x^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{5}fgx^5\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log\left(c\right.\\
&= \frac{2df^2px}{3e}-\frac{4d^2fgpx}{5e^2}+\frac{2d^3g^2px}{7e^3}-\frac{2}{9}f^2px^3+\frac{4dfgpx^3}{15e}-\frac{2d^2g^2px^3}{21e^2}-\\
&= \frac{2df^2px}{3e}-\frac{4d^2fgpx}{5e^2}+\frac{2d^3g^2px}{7e^3}-\frac{2}{9}f^2px^3+\frac{4dfgpx^3}{15e}-\frac{2d^2g^2px^3}{21e^2}-
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 188, normalized size = 0.68

$$\frac{-210d^{3/2}(35e^2f^2-42defg+15d^2g^2)\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)+\sqrt{e}\,x\left(2p(1575d^3g^2-105d^2eg(42f+5gx^2)+105de^2(35f^2+14fgx^2+3g^2x^4)-e^3x^2(1225f^2+882fgx^2+225g^2x^4))+105e^3x^2(35f^2+42fgx^2+15g^2x^4)\log\left(c(d+ex^2)^p\right)\right)}{11025e^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (-210*d^(3/2)*(35*e^2*f^2 - 42*d*e*f*g + 15*d^2*g^2)*p*ArcTan[(Sqrt[e]*x)/S
qrt[d]] + Sqrt[e]*x*(2*p*(1575*d^3*g^2 - 105*d^2*e*g*(42*f + 5*g*x^2) + 105
*d*e^2*(35*f^2 + 14*f*g*x^2 + 3*g^2*x^4) - e^3*x^2*(1225*f^2 + 882*f*g*x^2
+ 225*g^2*x^4)) + 105*e^3*x^2*(35*f^2 + 42*f*g*x^2 + 15*g^2*x^4)*Log[c*(d +
 e*x^2)^p]))/(11025*e^(7/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.68, size = 761, normalized size = 2.74

method	result
risch	$-\frac{2g^2p\,x^7}{49}-\frac{4d^2fgpx}{5e^2}+\frac{4dfgp\,x^3}{15e}+\frac{\ln(c)g^2x^7}{7}-\frac{i\pi\,g^2x^7\,\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)^3}{14}+\frac{i\pi\,g^2x^7\,\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)^2\mathrm{csgn}(ic)}{14}-\frac{2f^2p}{9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(g*x^2+f)^2*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] -2/49*g^2*p*x^7-4/5*d^2*f*g*p*x/e^2+4/15*d*f*g*p*x^3/e+1/7*ln(c)*g^2*x^7-2/
9*f^2*p*x^3+2/7*d^3*g^2*p*x/e^3-2/21*d^2*g^2*p*x^3/e^2+2/35*d*g^2*p*x^5/e-4
/25*f*g*p*x^5+2/3*d*f^2*p*x/e+1/3*ln(c)*f^2*x^3-1/5*I*Pi*f*g*x^5*csgn(I*(e*
x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/14*I*Pi*g^2*x^7*csgn(I*c*(e*x^2
+d)^p)^3-1/6*I*Pi*f^2*x^3*csgn(I*c*(e*x^2+d)^p)^3-1/7/e^4*(-e*d)^(1/2)*p*d^
3*ln((-e*d)^(1/2)*x-d)*g^2+1/7/e^4*(-e*d)^(1/2)*p*d^3*ln(-(-e*d)^(1/2)*x-d)
*g^2+1/3/e^2*(-e*d)^(1/2)*p*d*ln(-(-e*d)^(1/2)*x-d)*f^2-1/3/e^2*(-e*d)^(1/2
)*p*d*ln((-e*d)^(1/2)*x-d)*f^2-1/5*I*Pi*f*g*x^5*csgn(I*c*(e*x^2+d)^p)^3+1/1
4*I*Pi*g^2*x^7*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+1/14*I*Pi*g^2*x^
7*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+1/6*I*Pi*f^2*x^3*csgn(I*(e*x^2+d)^p)*cs
gn(I*c*(e*x^2+d)^p)^2+1/6*I*Pi*f^2*x^3*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-2/
5/e^3*(-e*d)^(1/2)*p*d^2*ln(-(-e*d)^(1/2)*x-d)*f*g+2/5/e^3*(-e*d)^(1/2)*p*d
^2*ln((-e*d)^(1/2)*x-d)*f*g-1/14*I*Pi*g^2*x^7*csgn(I*(e*x^2+d)^p)*csgn(I*c*
(e*x^2+d)^p)*csgn(I*c)-1/6*I*Pi*f^2*x^3*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2
+d)^p)*csgn(I*c)+1/5*I*Pi*f*g*x^5*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)
^2+1/5*I*Pi*f*g*x^5*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+(1/7*g^2*x^7+2/5*f*g*
x^5+1/3*f^2*x^3)*ln((e*x^2+d)^p)+2/5*ln(c)*f*g*x^5
```

Maxima [A]
time = 0.49, size = 182, normalized size = 0.65

$$-\frac{2}{11025}\left(\frac{105\left(15\,d^4g^2-42\,d^3fge+35\,d^2f^2e^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{\sqrt{d}}+\left(225\,g^2x^7e^3-63\left(5\,dg^2e^2-14\,fge^3\right)x^5+35\left(15\,d^2g^2e-42\,dfge^2+35\,f^2e^3\right)x^3-105\left(15\,d^3g^2-42\,d^2fge+35\,df^2e^2\right)x\right)e^{(-4)}\right)pe+\frac{1}{105}\left(15\,g^2x^7+42\,fgx^5+35\,f^2x^3\right)\log\left(\left(x^2e+d\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] -2/11025*(105*(15*d^4*g^2 - 42*d^3*f*g*e + 35*d^2*f^2*e^2)*arctan(x*e^(1/2)
/sqrt(d))*e^(-9/2)/sqrt(d) + (225*g^2*x^7*e^3 - 63*(5*d*g^2*e^2 - 14*f*g*e^
3)*x^5 + 35*(15*d^2*g^2*e - 42*d*f*g*e^2 + 35*f^2*e^3)*x^3 - 105*(15*d^3*g^
2 - 42*d^2*f*g*e + 35*d*f^2*e^2)*x)*e^(-4))*p*e + 1/105*(15*g^2*x^7 + 42*f*
g*x^5 + 35*f^2*x^3)*log((x^2*e + d)^p*c)
```

Fricas [A]
time = 0.40, size = 465, normalized size = 1.67

$$\left[\frac{1}{11025}\left(3150\,d^3g^2px+105\left(15\,g^2px^7+42\,fgpx^5+35\,f^2px^3\right)e^3\log\left(x^2e+d\right)+105\left(15\,g^2x^7+42\,fgx^5+35\,f^2x^3\right)e^3\log\left(c\right)+105\left(15\,d^3g^2p-42\,d^2fgpe+35\,df^2pe^2\right)\sqrt{-de^{(-1)}}\log\left(\frac{x^2e-2\sqrt{-de^{(-1)}}\,xe-d}{x^2e+d}\right)-2\left(225\,g^2px^7+882\,fgpx^5+1225\,f^2px^3\right)e^3+210\left(3\,dg^2px^5+14\,dfgpx^3+35\,df^2px\right)e^2-210\left(5\,d^2g^2px^3+42\,d^2fgpx\right)e\right)e^{(-3)},\ \frac{1}{11025}\left(3150\,d^3g^2px-210\left(15\,d^3g^2p-42\,d^2fgpe+35\,df^2pe^2\right)\sqrt{d}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}+105\left(15\,g^2px^7+42\,fgpx^5+35\,f^2px^3\right)e^3\log\left(x^2e+d\right)+105\left(15\,g^2x^7+42\,fgx^5+35\,f^2x^3\right)e^3\log\left(c\right)-2\left(225\,g^2px^7+882\,fgpx^5+1225\,f^2px^3\right)e^3+210\left(3\,dg^2px^5+14\,dfgpx^3+35\,df^2px\right)e^2-210\left(5\,d^2g^2px^3+42\,d^2fgpx\right)e\right)e^{(-3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")

[Out] [1/11025*(3150*d^3*g^2*p*x + 105*(15*g^2*p*x^7 + 42*f*g*p*x^5 + 35*f^2*p*x^3)*e^3*log(x^2*e + d) + 105*(15*g^2*x^7 + 42*f*g*x^5 + 35*f^2*x^3)*e^3*log(c) + 105*(15*d^3*g^2*p - 42*d^2*f*g*p*e + 35*d*f^2*p*e^2)*sqrt(-d*e^(-1))*log((x^2*e - 2*sqrt(-d*e^(-1))*x*e - d)/(x^2*e + d)) - 2*(225*g^2*p*x^7 + 882*f*g*p*x^5 + 1225*f^2*p*x^3)*e^3 + 210*(3*d*g^2*p*x^5 + 14*d*f*g*p*x^3 + 35*d*f^2*p*x)*e^2 - 210*(5*d^2*g^2*p*x^3 + 42*d^2*f*g*p*x)*e)*e^(-3), 1/11025*(3150*d^3*g^2*p*x - 210*(15*d^3*g^2*p - 42*d^2*f*g*p*e + 35*d*f^2*p*e^2)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2) + 105*(15*g^2*p*x^7 + 42*f*g*p*x^5 + 35*f^2*p*x^3)*e^3*log(x^2*e + d) + 105*(15*g^2*x^7 + 42*f*g*x^5 + 35*f^2*x^3)*e^3*log(c) - 2*(225*g^2*p*x^7 + 882*f*g*p*x^5 + 1225*f^2*p*x^3)*e^3 + 210*(3*d*g^2*p*x^5 + 14*d*f*g*p*x^3 + 35*d*f^2*p*x)*e^2 - 210*(5*d^2*g^2*p*x^3 + 42*d^2*f*g*p*x)*e)*e^(-3)]

Sympy [A]
time = 132.53, size = 559, normalized size = 2.01

$$\begin{cases} \left(\frac{f^{2} x^{3}}{3} + \frac{2 f g x^{5}}{5} + \frac{g^{2} x^{7}}{7}\right) \log{\left(0^{p} c \right)} & \text{for}\: d = 0 \wedge e = 0 \\ \left(\frac{f^{2} x^{3}}{3} + \frac{2 f g x^{5}}{5} + \frac{g^{2} x^{7}}{7}\right) \log{\left(c d^{p} \right)} & \text{for}\: e = 0 \\ - \frac{2 f^{2} p x^{3}}{9} + \frac{f^{2} x^{3} \log{\left(c \left(e x^{2}\right)^{p} \right)}}{3} - \frac{4 f g p x^{5}}{25} + \frac{2 f g x^{5} \log{\left(c \left(e x^{2}\right)^{p} \right)}}{5} - \frac{2 g^{2} p x^{7}}{49} + \frac{g^{2} x^{7} \log{\left(c \left(e x^{2}\right)^{p} \right)}}{7} & \text{for}\: d = 0 \\ - \frac{2 d^{4} g^{2} p \log{\left(x - \sqrt{- \frac{d}{e}} \right)}}{7 e^{4} \sqrt{- \frac{d}{e}}} + \frac{d^{4} g^{2} \log{\left(c \left(d + e x^{2}\right)^{p} \right)}}{7 e^{4} \sqrt{- \frac{d}{e}}} + \frac{4 d^{3} f g p \log{\left(x - \sqrt{- \frac{d}{e}} \right)}}{5 e^{3} \sqrt{- \frac{d}{e}}} - \frac{2 d^{3} f g \log{\left(c \left(d + e x^{2}\right)^{p} \right)}}{5 e^{3} \sqrt{- \frac{d}{e}}} + \frac{2 d^{3} g^{2} p x}{7 e^{3}} - \frac{2 d^{2} f^{2} p \log{\left(x - \sqrt{- \frac{d}{e}} \right)}}{3 e^{2} \sqrt{- \frac{d}{e}}} + \frac{d^{2} f^{2} \log{\left(c \left(d + e x^{2}\right)^{p} \right)}}{3 e^{2} \sqrt{- \frac{d}{e}}} - \frac{4 d^{2} f g p x}{5 e^{2}} - \frac{2 d^{2} g^{2} p x^{3}}{21 e^{2}} + \frac{2 d f^{2} p x}{3 e} + \frac{4 d f g p x^{3}}{15 e} + \frac{2 d g^{2} p x^{5}}{35 e} - \frac{2 f^{2} p x^{3}}{9} + \frac{f^{2} x^{3} \log{\left(c \left(d + e x^{2}\right)^{p} \right)}}{3} - \frac{4 f g p x^{5}}{25} + \frac{2 f g x^{5} \log{\left(c \left(d + e x^{2}\right)^{p} \right)}}{5} - \frac{2 g^{2} p x^{7}}{49} + \frac{g^{2} x^{7} \log{\left(c \left(d + e x^{2}\right)^{p} \right)}}{7} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(g*x**2+f)**2*ln(c*(e*x**2+d)**p),x)

[Out] Piecewise(((f**2*x**3/3 + 2*f*g*x**5/5 + g**2*x**7/7)*log(0**p*c), Eq(d, 0) & Eq(e, 0)), ((f**2*x**3/3 + 2*f*g*x**5/5 + g**2*x**7/7)*log(c*d**p), Eq(e, 0)), (-2*f**2*p*x**3/9 + f**2*x**3*log(c*(e*x**2)**p)/3 - 4*f*g*p*x**5/25 + 2*f*g*x**5*log(c*(e*x**2)**p)/5 - 2*g**2*p*x**7/49 + g**2*x**7*log(c*(e*x**2)**p)/7, Eq(d, 0)), (-2*d**4*g**2*p*log(x - sqrt(-d/e))/(7*e**4*sqrt(-d/e)) + d**4*g**2*log(c*(d + e*x**2)**p)/(7*e**4*sqrt(-d/e)) + 4*d**3*f*g*p*log(x - sqrt(-d/e))/(5*e**3*sqrt(-d/e)) - 2*d**3*f*g*log(c*(d + e*x**2)**p)/(5*e**3*sqrt(-d/e)) + 2*d**3*g**2*p*x/(7*e**3) - 2*d**2*f**2*p*log(x - sqrt(-d/e))/(3*e**2*sqrt(-d/e)) + d**2*f**2*log(c*(d + e*x**2)**p)/(3*e**2*sqrt(-d/e)) - 4*d**2*f*g*p*x/(5*e**2) - 2*d**2*g**2*p*x**3/(21*e**2) + 2*d*f**2*p*x/(3*e) + 4*d*f*g*p*x**3/(15*e) + 2*d*g**2*p*x**5/(35*e) - 2*f**2*p*x**3/9 + f**2*x**3*log(c*(d + e*x**2)**p)/3 - 4*f*g*p*x**5/25 + 2*f*g*x**5*log(c*(d + e*x**2)**p)/5 - 2*g**2*p*x**7/49 + g**2*x**7*log(c*(d + e*x**2)**p)/7, True))

Giac [A]
time = 4.70, size = 246, normalized size = 0.88

$$-\frac{2 \, {\left(15 \, d^{4} g^{2} p - 42 \, d^{3} f g p e + 35 \, d^{2} f^{2} p e^{2}\right)} \arctan\left(\frac{x e^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{7}{2}\right)}}{105 \, \sqrt{d}} + \frac{1}{11025} \, {\left(1575 \, g^{2} p x^{7} e^{3} \log\left(x^{2} e + d\right) - 450 \, g^{2} p x^{7} e^{3} + 1575 \, g^{2} x^{7} e^{3} \log\left(c\right) + 630 \, d g^{2} p x^{5} e^{2} + 4410 \, f g p x^{5} e^{3} \log\left(x^{2} e + d\right) - 1764 \, f g p x^{5} e^{3} - 1050 \, d^{2} g^{2} p x^{3} e + 4410 \, f g x^{5} e^{3} \log\left(c\right) + 2940 \, d f g p x^{3} e^{2} + 3675 \, f^{2} p x^{3} e^{3} \log\left(x^{2} e + d\right) + 3150 \, d^{3} g^{2} p x - 2450 \, f^{2} p x^{3} e^{3} - 8820 \, d^{2} f g p x e + 3675 \, f^{2} x^{3} e^{3} \log\left(c\right) + 7350 \, d f^{2} p x e^{2}\right)} e^{\left(-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] -2/105*(15*d^4*g^2*p - 42*d^3*f*g*p*e + 35*d^2*f^2*p*e^2)*arctan(x*e^(1/2)/
sqrt(d))*e^(-7/2)/sqrt(d) + 1/11025*(1575*g^2*p*x^7*e^3*log(x^2*e + d) - 45
0*g^2*p*x^7*e^3 + 1575*g^2*x^7*e^3*log(c) + 630*d*g^2*p*x^5*e^2 + 4410*f*g*
p*x^5*e^3*log(x^2*e + d) - 1764*f*g*p*x^5*e^3 - 1050*d^2*g^2*p*x^3*e + 4410
*f*g*x^5*e^3*log(c) + 2940*d*f*g*p*x^3*e^2 + 3675*f^2*p*x^3*e^3*log(x^2*e +
 d) + 3150*d^3*g^2*p*x - 2450*f^2*p*x^3*e^3 - 8820*d^2*f*g*p*x*e + 3675*f^2
*x^3*e^3*log(c) + 7350*d*f^2*p*x*e^2)*e^(-3)
```

Mupad [B]
time = 0.36, size = 235, normalized size = 0.85

$$\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(\frac{f^2\,x^3}{3}+\frac{2\,f\,g\,x^5}{5}+\frac{g^2\,x^7}{7}\right)-x^3\,\left(\frac{2\,f^2\,p}{9}-\frac{d\,\left(\frac{4\,f\,g\,p}{5}-\frac{2\,d\,g^2\,p}{7\,e}\right)}{3\,e}\right)-x^5\,\left(\frac{4\,f\,g\,p}{25}-\frac{2\,d\,g^2\,p}{35\,e}\right)-\frac{2\,g^2\,p\,x^7}{49}+\frac{d\,x\,\left(\frac{2\,f^2\,p}{3}-\frac{d\,\left(\frac{4\,f\,g\,p}{5}-\frac{2\,d\,g^2\,p}{7\,e}\right)}{e}\right)}{e}-\frac{2\,d^{3/2}\,p\,\mathrm{atan}\left(\frac{d^{3/2}\,\sqrt{e}\,p\,x\,\left(15\,d^2\,g^2-42\,d\,e\,f\,g+35\,e^2\,f^2\right)}{15\,p\,d^4\,g^2-42\,p\,d^3\,e\,f\,g+35\,p\,d^2\,e^2\,f^2}\right)\,\left(15\,d^2\,g^2-42\,d\,e\,f\,g+35\,e^2\,f^2\right)}{105\,e^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(c*(d + e*x^2)^p)*(f + g*x^2)^2,x)
```

```
[Out] log(c*(d + e*x^2)^p)*((f^2*x^3)/3 + (g^2*x^7)/7 + (2*f*g*x^5)/5) - x^3*((2*
f^2*p)/9 - (d*((4*f*g*p)/5 - (2*d*g^2*p)/(7*e)))/(3*e)) - x^5*((4*f*g*p)/25
 - (2*d*g^2*p)/(35*e)) - (2*g^2*p*x^7)/49 + (d*x*((2*f^2*p)/3 - (d*((4*f*g*
p)/5 - (2*d*g^2*p)/(7*e)))/e))/e - (2*d^(3/2)*p*atan((d^(3/2)*e^(1/2)*p*x*(
15*d^2*g^2 + 35*e^2*f^2 - 42*d*e*f*g))/(15*d^4*g^2*p + 35*d^2*e^2*f^2*p - 4
2*d^3*e*f*g*p))*(15*d^2*g^2 + 35*e^2*f^2 - 42*d*e*f*g))/(105*e^(7/2))
```

3.333 $\int (f+gx^2)^2 \log \left(c(d+ex^2)^p\right) \, dx$

Optimal. Leaf size=221

$$-2f^2px+\frac{4dfgpx}{3e}-\frac{2d^2g^2px}{5e^2}-\frac{4}{9}fgpx^3+\frac{2dg^2px^3}{15e}-\frac{2}{25}g^2px^5+\frac{2\sqrt{d}\,f^2p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{4d^{3/2}fgp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3e^{3/2}}$$

```
[Out] -2*f^2*p*x+4/3*d*f*g*p*x/e-2/5*d^2*g^2*p*x/e^2-4/9*f*g*p*x^3+2/15*d*g^2*p*x
^3/e-2/25*g^2*p*x^5-4/3*d^(3/2)*f*g*p*arctan(x*e^(1/2)/d^(1/2))/e^(3/2)+2/5
*d^(5/2)*g^2*p*arctan(x*e^(1/2)/d^(1/2))/e^(5/2)+f^2*x*ln(c*(e*x^2+d)^p)+2/
3*f*g*x^3*ln(c*(e*x^2+d)^p)+1/5*g^2*x^5*ln(c*(e*x^2+d)^p)+2*f^2*p*arctan(x*
e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2521, 2498, 327, 211, 2505, 308}

$$-\frac{4d^{3/2}fgp\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2d^{5/2}g^2p\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{5e^{5/2}}+\frac{2\sqrt{d}\,f^2p\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}+f^2x\log\left(c(d+ex^2)^p\right)+\frac{2}{3}fgx^3\log\left(c(d+ex^2)^p\right)+\frac{1}{5}g^2x^5\log\left(c(d+ex^2)^p\right)-\frac{2d^2g^2px}{5e^2}+\frac{4dfgpx}{3e}+\frac{2dg^2px^3}{15e}-2f^2px-\frac{4}{9}fgpx^3-\frac{2}{25}g^2px^5$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f^2*p*x + (4*d*f*g*p*x)/(3*e) - (2*d^2*g^2*p*x)/(5*e^2) - (4*f*g*p*x^3)/
9 + (2*d*g^2*p*x^3)/(15*e) - (2*g^2*p*x^5)/25 + (2*Sqrt[d]*f^2*p*ArcTan[(Sq
rt[e]*x)/Sqrt[d]])/Sqrt[e] - (4*d^(3/2)*f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/
(3*e^(3/2)) + (2*d^(5/2)*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*e^(5/2)) + f
^2*x*Log[c*(d + e*x^2)^p] + (2*f*g*x^3*Log[c*(d + e*x^2)^p])/3 + (g^2*x^5*L
og[c*(d + e*x^2)^p])/5
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$\int \left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right)\,dx = \int \left(f^2\log\left(c\left(d+ex^2\right)^p\right)+2fgx^2\log\left(c\left(d+ex^2\right)^p\right)+g^2x^4\log\left(c\left(d+e\right.\right.\right.$$

$$= f^2\int \log\left(c\left(d+ex^2\right)^p\right)\,dx+(2fg)\int x^2\log\left(c\left(d+ex^2\right)^p\right)\,dx+g^2\int x$$

$$= f^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}g^2x^5\log\left(c\left(d+e\right.\right.$$

$$= -2f^2px+f^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}g^2x^5\mathrm{lo}$$

$$= -2f^2px+\frac{4dfgpx}{3e}-\frac{2d^2g^2px}{5e^2}-\frac{4}{9}fgpx^3+\frac{2dg^2px^3}{15e}-\frac{2}{25}g^2px^5+\frac{2\sqrt{d}}{}$$

$$= -2f^2px+\frac{4dfgpx}{3e}-\frac{2d^2g^2px}{5e^2}-\frac{4}{9}fgpx^3+\frac{2dg^2px^3}{15e}-\frac{2}{25}g^2px^5+\frac{2\sqrt{d}}{}$$

Mathematica [A]
time = 0.09, size = 151, normalized size = 0.68

$$\frac{30\sqrt{d}\left(15e^2f^2-10defg+3d^2g^2\right)p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)+\sqrt{e}\,x\left(-2p(45d^2g^2-15deg(10f+gx^2)+e^2(225f^2+50fgx^2+9g^2x^4))+15e^2(15f^2+10fgx^2+3g^2x^4)\log\left(c(d+ex^2)^p\right)\right)}{225e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (30*Sqrt[d]*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*p*ArcTan[(Sqrt[e]*x)/Sqrt
[d]] + Sqrt[e]*x*(-2*p*(45*d^2*g^2 - 15*d*e*g*(10*f + g*x^2) + e^2*(225*f^2
 + 50*f*g*x^2 + 9*g^2*x^4)) + 15*e^2*(15*f^2 + 10*f*g*x^2 + 3*g^2*x^4)*Log[
c*(d + e*x^2)^p]))/(225*e^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.37, size = 686, normalized size = 3.10

method	result
risch	$-\frac{i\pi\, g^2x^5\mathrm{csgn}\left(i\left(e\,x^2+d\right)^p\right)\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)\mathrm{csgn}(ic)}{10}-\frac{i\pi\, f^2\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)^3x}{2}+\frac{2\sqrt{-ed}\,p\ln\left(\sqrt{-ed}\,x+d\right)dfg}{3e^2}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/e^2*(-e*d)^(1/2)*p*ln((-e*d)^(1/2)*x+d)*d*f*g-1/e*(-e*d)^(1/2)*p*ln((-e
*d)^(1/2)*x+d)*f^2-4/9*f*g*p*x^3-2*f^2*p*x-2/25*g^2*p*x^5-2/5*d^2*g^2*p*x/e
^2+2/15*d*g^2*p*x^3/e+1/5/e^3*(-e*d)^(1/2)*p*ln(-(-e*d)^(1/2)*x+d)*d^2*g^2-
1/5/e^3*(-e*d)^(1/2)*p*ln((-e*d)^(1/2)*x+d)*d^2*g^2+4/3*d*f*g*p*x/e+2/3*ln(
c)*f*g*x^3+1/10*I*Pi*g^2*x^5*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+1/10*I*Pi*g^
2*x^5*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/3*I*Pi*f*g*x^3*csgn(I*c
*(e*x^2+d)^p)^3+1/2*I*Pi*f^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*x+1/2*I*Pi*f
^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*x+1/5*ln(c)*g^2*x^5+ln(c)*f^
2*x+1/e*(-e*d)^(1/2)*p*ln(-(-e*d)^(1/2)*x+d)*f^2-1/10*I*Pi*g^2*x^5*csgn(I*c
*(e*x^2+d)^p)^3-1/2*I*Pi*f^2*csgn(I*c*(e*x^2+d)^p)^3*x-1/3*I*Pi*f*g*x^3*csg
n(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+(1/5*g^2*x^5+2/3*f*g*x^3+f
^2*x)*ln((e*x^2+d)^p)-2/3/e^2*(-e*d)^(1/2)*p*ln(-(-e*d)^(1/2)*x+d)*d*f*g-1/
10*I*Pi*g^2*x^5*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+1/3*I*P
i*f*g*x^3*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+1/3*I*Pi*f*g*x^3*csgn(I*(e*x^2+
d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/2*I*Pi*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*
x^2+d)^p)*csgn(I*c)*x
```

Maxima [A]
time = 0.57, size = 146, normalized size = 0.66

$$\frac{2}{225}\left(\frac{15\left(3\,d^3g^2-10\,d^2fge+15\,df^2e^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{7}{2}\right)}}{\sqrt{d}}-\left(9\,g^2x^5e^2-5\left(3\,dg^2e-10\,fge^2\right)x^3+15\left(3\,d^2g^2-10\,dfge+15\,f^2e^2\right)x\right)e^{(-3)}\right)pe+\frac{1}{15}\left(3\,g^2x^5+10\,fgx^3+15\,f^2x\right)\log\left(\left(x^2e+d\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")

[Out] 2/225*(15*(3*d^3*g^2 - 10*d^2*f*g*e + 15*d*f^2*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-7/2)/sqrt(d) - (9*g^2*x^5*e^2 - 5*(3*d*g^2*e - 10*f*g*e^2)*x^3 + 15*(3*d^2*g^2 - 10*d*f*g*e + 15*f^2*e^2)*x)*e^(-3))*p*e + 1/15*(3*g^2*x^5 + 10*f*g*x^3 + 15*f^2*x)*log((x^2*e + d)^p*c)

Fricas [A]
time = 0.41, size = 373, normalized size = 1.69

$$\left[-\frac{1}{225}\left(90 d^2 g^2 p x - 15\left(3 g^2 p x^5 + 10 f g p x^3 + 15 f^2 p x\right) e^2 \log\left(x^2 e + d\right) - 15\left(3 g^2 x^5 + 10 f g x^3 + 15 f^2 x\right) e^2 \log\left(c\right) - 15\left(3 d^2 g^2 p - 10 d f g p e + 15 f^2 p e^2\right) \sqrt{-d e^{(-1)}} \log\left(\frac{x^2 e + 2\sqrt{-d e^{(-1)}} x e - d}{x^2 e + d}\right) + 2\left(9 g^2 p x^5 + 50 f g p x^3 + 225 f^2 p x\right) e^2 - 30\left(d g^2 p x^3 + 10 d f g p x\right) e\right) e^{(-2)}, -\frac{1}{225}\left(90 d^2 g^2 p x - 30\left(3 d^2 g^2 p - 10 d f g p e + 15 f^2 p e^2\right) \sqrt{d} \arctan\left(\frac{x e^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{1}{2}\right)} - 15\left(3 g^2 p x^5 + 10 f g p x^3 + 15 f^2 p x\right) e^2 \log\left(x^2 e + d\right) - 15\left(3 g^2 x^5 + 10 f g x^3 + 15 f^2 x\right) e^2 \log\left(c\right) + 2\left(9 g^2 p x^5 + 50 f g p x^3 + 225 f^2 p x\right) e^2 - 30\left(d g^2 p x^3 + 10 d f g p x\right) e\right) e^{(-2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")

[Out] [-1/225*(90*d^2*g^2*p*x - 15*(3*g^2*p*x^5 + 10*f*g*p*x^3 + 15*f^2*p*x)*e^2*log(x^2*e + d) - 15*(3*g^2*x^5 + 10*f*g*x^3 + 15*f^2*x)*e^2*log(c) - 15*(3*d^2*g^2*p - 10*d*f*g*p*e + 15*f^2*p*e^2)*sqrt(-d*e^(-1))*log((x^2*e + 2*sqrt(-d*e^(-1))*x*e - d)/(x^2*e + d)) + 2*(9*g^2*p*x^5 + 50*f*g*p*x^3 + 225*f^2*p*x)*e^2 - 30*(d*g^2*p*x^3 + 10*d*f*g*p*x)*e)*e^(-2), -1/225*(90*d^2*g^2*p*x - 30*(3*d^2*g^2*p - 10*d*f*g*p*e + 15*f^2*p*e^2)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2) - 15*(3*g^2*p*x^5 + 10*f*g*p*x^3 + 15*f^2*p*x)*e^2*log(x^2*e + d) - 15*(3*g^2*x^5 + 10*f*g*x^3 + 15*f^2*x)*e^2*log(c) + 2*(9*g^2*p*x^5 + 50*f*g*p*x^3 + 225*f^2*p*x)*e^2 - 30*(d*g^2*p*x^3 + 10*d*f*g*p*x)*e)*e^(-2)]

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 478 vs. 2(231) = 462.
time = 35.28, size = 478, normalized size = 2.16

$$\begin{cases} \left(f^2 x + \frac{2 f g x^3}{3} + \frac{g^2 x^5}{5}\right) \log\left(0^p c\right) & \text{for}\: d = 0 \wedge e = 0 \\ -2 f^2 p x + f^2 x \log\left(c \left(e x^2\right)^p\right) - \frac{4 f g p x^3}{9} + \frac{2 f g x^3 \log\left(c \left(e x^2\right)^p\right)}{3} - \frac{2 g^2 p x^5}{25} + \frac{g^2 x^5 \log\left(c \left(e x^2\right)^p\right)}{5} & \text{for}\: d = 0 \\ \left(f^2 x + \frac{2 f g x^3}{3} + \frac{g^2 x^5}{5}\right) \log\left(c d^p\right) & \text{for}\: e = 0 \\ \frac{2 d^3 g^2 p \log\left(x - \sqrt{-\frac{d}{e}}\right)}{5 e^3 \sqrt{-\frac{d}{e}}} - \frac{d^3 g^2 \log\left(c \left(d + e x^2\right)^p\right)}{5 e^3 \sqrt{-\frac{d}{e}}} - \frac{4 d^2 f g p \log\left(x - \sqrt{-\frac{d}{e}}\right)}{3 e^2 \sqrt{-\frac{d}{e}}} + \frac{2 d^2 f g \log\left(c \left(d + e x^2\right)^p\right)}{3 e^2 \sqrt{-\frac{d}{e}}} - \frac{2 d^2 g^2 p x}{5 e^2} + \frac{2 d f^2 p \log\left(x - \sqrt{-\frac{d}{e}}\right)}{e \sqrt{-\frac{d}{e}}} - \frac{d f^2 \log\left(c \left(d + e x^2\right)^p\right)}{e \sqrt{-\frac{d}{e}}} + \frac{4 d f g p x}{3 e} + \frac{2 d g^2 p x^3}{15 e} - 2 f^2 p x + f^2 x \log\left(c \left(d + e x^2\right)^p\right) - \frac{4 f g p x^3}{9} + \frac{2 f g x^3 \log\left(c \left(d + e x^2\right)^p\right)}{3} - \frac{2 g^2 p x^5}{25} + \frac{g^2 x^5 \log\left(c \left(d + e x^2\right)^p\right)}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p),x)

[Out] Piecewise(((f**2*x + 2*f*g*x**3/3 + g**2*x**5/5)*log(0**p*c), Eq(d, 0) & Eq(e, 0)), (-2*f**2*p*x + f**2*x*log(c*(e*x**2)**p) - 4*f*g*p*x**3/9 + 2*f*g*x**3*log(c*(e*x**2)**p)/3 - 2*g**2*p*x**5/25 + g**2*x**5*log(c*(e*x**2)**p)/5, Eq(d, 0)), ((f**2*x + 2*f*g*x**3/3 + g**2*x**5/5)*log(c*d**p), Eq(e, 0)), (2*d**3*g**2*p*log(x - sqrt(-d/e))/(5*e**3*sqrt(-d/e)) - d**3*g**2*log(c*(d + e*x**2)**p)/(5*e**3*sqrt(-d/e)) - 4*d**2*f*g*p*log(x - sqrt(-d/e))/(3*e**2*sqrt(-d/e)) + 2*d**2*f*g*log(c*(d + e*x**2)**p)/(3*e**2*sqrt(-d/e)) -

```
 2*d**2*g**2*p*x/(5*e**2) + 2*d*f**2*p*log(x - sqrt(-d/e))/(e*sqrt(-d/e)) -
 d*f**2*log(c*(d + e*x**2)**p)/(e*sqrt(-d/e)) + 4*d*f*g*p*x/(3*e) + 2*d*g**
2*p*x**3/(15*e) - 2*f**2*p*x + f**2*x*log(c*(d + e*x**2)**p) - 4*f*g*p*x**3
/9 + 2*f*g*x**3*log(c*(d + e*x**2)**p)/3 - 2*g**2*p*x**5/25 + g**2*x**5*log
(c*(d + e*x**2)**p)/5, True))
```

Giac [A]
time = 5.34, size = 201, normalized size = 0.91

$$\frac{2\,(3\,d^3g^2p - 10\,d^2fgpe + 15\,df^2pe^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{15\,\sqrt{d}} + \frac{1}{225}\,(45\,g^2px^5e^2\log\left(x^2e+d\right) - 18\,g^2px^5e^2 + 45\,g^2x^5e^2\log\left(c\right) + 30\,dg^2px^3e + 150\,fgpx^3e^2\log\left(x^2e+d\right) - 100\,fgpx^3e^2 + 150\,fgx^3e^2\log\left(c\right) - 90\,d^2g^2px + 300\,dfgpxe + 225\,f^2pxe^2\log\left(x^2e+d\right) - 450\,f^2pxe^2 + 225\,f^2xe^2\log\left(c\right))e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 2/15*(3*d^3*g^2*p - 10*d^2*f*g*p*e + 15*d*f^2*p*e^2)*arctan(x*e^(1/2)/sqrt(
d))*e^(-5/2)/sqrt(d) + 1/225*(45*g^2*p*x^5*e^2*log(x^2*e + d) - 18*g^2*p*x^
5*e^2 + 45*g^2*x^5*e^2*log(c) + 30*d*g^2*p*x^3*e + 150*f*g*p*x^3*e^2*log(x^
2*e + d) - 100*f*g*p*x^3*e^2 + 150*f*g*x^3*e^2*log(c) - 90*d^2*g^2*p*x + 30
0*d*f*g*p*x*e + 225*f^2*p*x*e^2*log(x^2*e + d) - 450*f^2*p*x*e^2 + 225*f^2*
x*e^2*log(c))*e^(-2)
```

Mupad [B]
time = 0.00, size = 193, normalized size = 0.87

$$\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(f^2\,x+\frac{2\,f\,g\,x^3}{3}+\frac{g^2\,x^5}{5}\right)-x\,\left(2\,f^2\,p-\frac{d\,\left(\frac{4\,f\,g\,p}{3}-\frac{2\,d\,g^2\,p}{5\,e}\right)}{e}\right)-x^3\,\left(\frac{4\,f\,g\,p}{9}-\frac{2\,d\,g^2\,p}{15\,e}\right)-\frac{2\,g^2\,p\,x^5}{25}+\frac{2\,\sqrt{d}\,\,p\,\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e}\,\,p\,x\,\left(3\,d^2\,g^2-10\,d\,e\,f\,g+15\,e^2\,f^2\right)}{3\,p\,d^3\,g^2-10\,p\,d^2\,e\,f\,g+15\,p\,d\,e^2\,f^2}\right)\,\left(3\,d^2\,g^2-10\,d\,e\,f\,g+15\,e^2\,f^2\right)}{15\,e^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)*(f + g*x^2)^2,x)
```

```
[Out] log(c*(d + e*x^2)^p)*(f^2*x + (g^2*x^5)/5 + (2*f*g*x^3)/3) - x*(2*f^2*p - (
d*((4*f*g*p)/3 - (2*d*g^2*p)/(5*e)))/e) - x^3*((4*f*g*p)/9 - (2*d*g^2*p)/(1
5*e)) - (2*g^2*p*x^5)/25 + (2*d^(1/2)*p*atan((d^(1/2)*e^(1/2)*p*x*(3*d^2*g^
2 + 15*e^2*f^2 - 10*d*e*f*g))/(3*d^3*g^2*p + 15*d*e^2*f^2*p - 10*d^2*e*f*g*
p))*(3*d^2*g^2 + 15*e^2*f^2 - 10*d*e*f*g))/(15*e^(5/2))
```

3.334 $\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^2} dx$

Optimal. Leaf size=178

$$-4fgpx+\frac{2dg^2px}{3e}-\frac{2}{9}g^2px^3+\frac{2\sqrt{e}\,f^2p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}}+\frac{4\sqrt{d}\,fgp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{2d^{3/2}g^2p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3e^{3/2}}-$$

```
[Out] -4*f*g*p*x+2/3*d*g^2*p*x/e-2/9*g^2*p*x^3-2/3*d^(3/2)*g^2*p*arctan(x*e^(1/2)
/d^(1/2))/e^(3/2)-f^2*ln(c*(e*x^2+d)^p)/x+2*f*g*x*ln(c*(e*x^2+d)^p)+1/3*g^2
*x^3*ln(c*(e*x^2+d)^p)+4*f*g*p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)+2*
f^2*p*arctan(x*e^(1/2)/d^(1/2))*e^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2526, 2498, 327, 211, 2505, 308}

$$-\frac{2d^{3/2}g^2p\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2\sqrt{e}\,f^2p\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}}+\frac{4\sqrt{d}\,fgp\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{x}+2fgx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}g^2x^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{2dg^2px}{3e}-4fgpx-\frac{2}{9}g^2px^3$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^2,x]
```

```
[Out] -4*f*g*p*x + (2*d*g^2*p*x)/(3*e) - (2*g^2*p*x^3)/9 + (2*Sqrt[e]*f^2*p*ArcTa
n[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] + (4*Sqrt[d]*f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[
d]])/Sqrt[e] - (2*d^(3/2)*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^(3/2)) -
(f^2*Log[c*(d + e*x^2)^p])/x + 2*f*g*x*Log[c*(d + e*x^2)^p] + (g^2*x^3*Log[
c*(d + e*x^2)^p])/3
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int \frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{x^2}\,dx = \int \left(2fg\log\left(c(d+ex^2)^p\right)+\frac{f^2\log\left(c(d+ex^2)^p\right)}{x^2}+g^2x^2\log\left(c(d+ex\right.\right.$$

$$= f^2\int \frac{\log\left(c(d+ex^2)^p\right)}{x^2}\,dx+(2fg)\int \log\left(c(d+ex^2)^p\right)\,dx+g^2\int x^2 \mathrm{l}$$

$$= -\frac{f^2\log\left(c(d+ex^2)^p\right)}{x}+2fgx\log\left(c(d+ex^2)^p\right)+\frac{1}{3}g^2x^3\log\left(c(d+ex\right.$$

$$= -4fgpx+\frac{2\sqrt{e}\,f^2p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{f^2\log\left(c(d+ex^2)^p\right)}{x}+2fgx\log$$

$$= -4fgpx+\frac{2dg^2px}{3e}-\frac{2}{9}g^2px^3+\frac{2\sqrt{e}\,f^2p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}}+\frac{4\sqrt{d}\,fgp\,\mathrm{t}}{}$$

$$= -4fgpx+\frac{2dg^2px}{3e}-\frac{2}{9}g^2px^3+\frac{2\sqrt{e}\,f^2p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}}+\frac{4\sqrt{d}\,fgp\,\mathrm{t}}{}$$

Mathematica [A]
time = 0.10, size = 112, normalized size = 0.63

$$\frac{1}{9}\left(-\frac{2gpx(18ef-3dg+egx^2)}{e}+\frac{6(3e^2f^2+6defg-d^2g^2)\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,e^{3/2}}+\left(-\frac{9f^2}{x}+18fgx+3g^2x^3\right)\log\left(c\left(d+ex^2\right)^p\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^2,x]
```

```
[Out] ((-2*g*p*x*(18*e*f - 3*d*g + e*g*x^2))/e + (6*(3*e^2*f^2 + 6*d*e*f*g - d^2*
g^2)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(3/2)) + ((-9*f^2)/x + 18*f*
g*x + 3*g^2*x^3)*Log[c*(d + e*x^2)^p])/9
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.72, size = 702, normalized size = 3.94

method	result
risch	$-\frac{(-g^2x^4-6fg\,x^2+3f^2)\ln((e\,x^2+d)^p)}{3x}+\frac{3i\pi e\,g^2x^4\text{csgn}(ic(e\,x^2+d)^p)^2\text{csgn}(ic)-3i\pi e\,g^2x^4\text{csgn}(ic(e\,x^2+d)^p)^3+3i\pi e\,g^2x^4\text{csgn}(i(e}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-g^2*x^4-6*f*g*x^2+3*f^2)/x*ln((e*x^2+d)^p)+1/18*(3*I*Pi*e*g^2*x^4*cs
gn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-3*I*Pi*e*g^2*x^4*csgn(I*c*(e*x^2+d)^p)^3+3*
I*Pi*e*g^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-3*I*Pi*e*g^2*x^4
*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-18*I*Pi*e*f*g*x^2*csgn
(I*c*(e*x^2+d)^p)^3+18*I*Pi*e*f*g*x^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-9*I
*Pi*e*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+9*I*Pi*e*f^2*csgn(I*c
*(e*x^2+d)^p)^3-18*I*Pi*e*f*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)
*csgn(I*c)-9*I*Pi*e*f^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+18*I*Pi*e*f*g*x^2
*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+9*I*Pi*e*f^2*csgn(I*(e*x^2+d)^
p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+6*ln(c)*e*g^2*x^4-4*e*g^2*p*x^4+36*ln(c)
*e*f*g*x^2+12*d*g^2*p*x^2-72*e*f*g*p*x^2-18*ln(c)*e*f^2+6*sum(_R*ln((2*d^4*
g^4*p^2-24*d^3*e*f*g^3*p^2+60*d^2*e^2*f^2*g^2*p^2+72*d*e^3*f^3*g*p^2+18*e^4
*f^4*p^2+3*_R^2*d*e)*x+(d^3*g^2*p-6*d^2*e*f*g*p-3*d*e^2*f^2*p)*_R),_R=RootO
f(d^4*g^4*p^2-12*d^3*e*f*g^3*p^2+30*d^2*e^2*f^2*g^2*p^2+36*d*e^3*f^3*g*p^2+
9*e^4*f^4*p^2+_Z^2*d*e))*x)/e/x
```

Maxima [A]
time = 0.51, size = 108, normalized size = 0.61

$$-\frac{2}{9}\left(\frac{3\left(d^2g^2-6\,dfge-3\,f^2e^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{\sqrt{d}}+\left(g^2x^3e-3\left(dg^2-6\,fge\right)x\right)e^{(-2)}\right)pe+\frac{1}{3}\left(g^2x^3+6\,fgx-\frac{3\,f^2}{x}\right)\log\left(\left(x^2e+d\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^2,x, algorithm="maxima")
```

```
[Out] -2/9*(3*(d^2*g^2 - 6*d*f*g*e - 3*f^2*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2
)/sqrt(d) + (g^2*x^3*e - 3*(d*g^2 - 6*f*g*e)*x)*e^(-2))*p*e + 1/3*(g^2*x^3
+ 6*f*g*x - 3*f^2/x)*log((x^2*e + d)^p*c)
```

Fricas [A]
time = 0.36, size = 345, normalized size = 1.94

$$\left[\frac{\left(6\,d^{2}g^{2}px^{2}e+3\,(dg^{2}px^{4}+6\,dfgpx^{2}-3\,df^{2}p)e^{2}\log\left(x^{2}e+d\right)+3\,(dg^{2}x^{4}+6\,dfgx^{2}-3\,df^{2})e^{2}\log\left(c\right)+3\,(d^{2}g^{2}px-6\,dfgpxe-3\,f^{2}pxe^{2})\sqrt{-de}\,\log\left(\frac{x^{2}e-2\,\sqrt{-de}\,x-d}{x^{2}e+d}\right)-2\,(dg^{2}px^{4}+18\,dfgpx^{2})e^{2}\right)e^{(-2)}}{9\,dx},\ \frac{\left(6\,d^{2}g^{2}px^{2}e-6\,(d^{2}g^{2}px-6\,dfgpxe-3\,f^{2}pxe^{2})\sqrt{d}\,\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}+3\,(dg^{2}px^{4}+6\,dfgpx^{2}-3\,df^{2}p)e^{2}\log\left(x^{2}e+d\right)+3\,(dg^{2}x^{4}+6\,dfgx^{2}-3\,df^{2})e^{2}\log\left(c\right)-2\,(dg^{2}px^{4}+18\,dfgpx^{2})e^{2}\right)e^{(-2)}}{9\,dx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^2,x, algorithm="fricas")
```

```
[Out] [1/9*(6*d^2*g^2*p*x^2*e + 3*(d*g^2*p*x^4 + 6*d*f*g*p*x^2 - 3*d*f^2*p)*e^2*l
og(x^2*e + d) + 3*(d*g^2*x^4 + 6*d*f*g*x^2 - 3*d*f^2)*e^2*log(c) + 3*(d^2*g
^2*p*x - 6*d*f*g*p*x*e - 3*f^2*p*x*e^2)*sqrt(-d*e)*log((x^2*e - 2*sqrt(-d*e
)*x - d)/(x^2*e + d)) - 2*(d*g^2*p*x^4 + 18*d*f*g*p*x^2)*e^2)*e^(-2)/(d*x),
 1/9*(6*d^2*g^2*p*x^2*e - 6*(d^2*g^2*p*x - 6*d*f*g*p*x*e - 3*f^2*p*x*e^2)*s
qrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2) + 3*(d*g^2*p*x^4 + 6*d*f*g*p*x^2 -
 3*d*f^2*p)*e^2*log(x^2*e + d) + 3*(d*g^2*x^4 + 6*d*f*g*x^2 - 3*d*f^2)*e^2*
log(c) - 2*(d*g^2*p*x^4 + 18*d*f*g*p*x^2)*e^2)*e^(-2)/(d*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(182) = 364$.
time = 65.89, size = 400, normalized size = 2.25

$$\begin{cases} \left(-\frac{f^{2}}{x}+2fgx+\frac{g^{2}x^{3}}{3}\right)\log\left(0^{p}c\right) & \text{for}\: d=0\wedge e=0 \\ -\frac{2f^{2}p}{x}-\frac{f^{2}\log\left(c\left(ex^{2}\right)^{p}\right)}{x}-4fgpx+2fgx\log\left(c\left(ex^{2}\right)^{p}\right)-\frac{2g^{2}px^{3}}{9}+\frac{g^{2}x^{3}\log\left(c\left(ex^{2}\right)^{p}\right)}{3} & \text{for}\: d=0 \\ \left(-\frac{f^{2}}{x}+2fgx+\frac{g^{2}x^{3}}{3}\right)\log\left(cd^{p}\right) & \text{for}\: e=0 \\ -\frac{2d^{2}g^{2}p\log\left(x-\sqrt{-\frac{d}{e}}\right)}{3e^{2}\sqrt{-\frac{d}{e}}}+\frac{d^{2}g^{2}\log\left(c\left(d+ex^{2}\right)^{p}\right)}{3e^{2}\sqrt{-\frac{d}{e}}}+\frac{4dfgp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{e\sqrt{-\frac{d}{e}}}-\frac{2dfg\log\left(c\left(d+ex^{2}\right)^{p}\right)}{e\sqrt{-\frac{d}{e}}}+\frac{2dg^{2}px}{3e}+\frac{2f^{2}p\log\left(x-\sqrt{-\frac{d}{e}}\right)}{\sqrt{-\frac{d}{e}}}-\frac{f^{2}\log\left(c\left(d+ex^{2}\right)^{p}\right)}{\sqrt{-\frac{d}{e}}}-\frac{f^{2}\log\left(c\left(d+ex^{2}\right)^{p}\right)}{x}-4fgpx+2fgx\log\left(c\left(d+ex^{2}\right)^{p}\right)-\frac{2g^{2}px^{3}}{9}+\frac{g^{2}x^{3}\log\left(c\left(d+ex^{2}\right)^{p}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**2,x)
```

```
[Out] Piecewise(((-f**2/x + 2*f*g*x + g**2*x**3/3)*log(0**p*c), Eq(d, 0) & Eq(e,
0)), (-2*f**2*p/x - f**2*log(c*(e*x**2)**p)/x - 4*f*g*p*x + 2*f*g*x*log(c*(
e*x**2)**p) - 2*g**2*p*x**3/9 + g**2*x**3*log(c*(e*x**2)**p)/3, Eq(d, 0)),
((-f**2/x + 2*f*g*x + g**2*x**3/3)*log(c*d**p), Eq(e, 0)), (-2*d**2*g**2*p*
log(x - sqrt(-d/e))/(3*e**2*sqrt(-d/e)) + d**2*g**2*log(c*(d + e*x**2)**p)/
(3*e**2*sqrt(-d/e)) + 4*d*f*g*p*log(x - sqrt(-d/e))/(e*sqrt(-d/e)) - 2*d*f*
g*log(c*(d + e*x**2)**p)/(e*sqrt(-d/e)) + 2*d*g**2*p*x/(3*e) + 2*f**2*p*log
(x - sqrt(-d/e))/sqrt(-d/e) - f**2*log(c*(d + e*x**2)**p)/sqrt(-d/e) - f**2
*log(c*(d + e*x**2)**p)/x - 4*f*g*p*x + 2*f*g*x*log(c*(d + e*x**2)**p) - 2*
g**2*p*x**3/9 + g**2*x**3*log(c*(d + e*x**2)**p)/3, True))
```

Giac [A]
time = 5.25, size = 168, normalized size = 0.94

$$-\frac{2\left(d^2g^2p - 6\,dfgpe - 3\,f^2pe^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{3}{2}\right)}}{3\,\sqrt{d}} + \frac{\left(3\,g^2px^4e\log\left(x^2e + d\right) - 2\,g^2px^4e + 3\,g^2x^4e\log\left(c\right) + 18\,fgpx^2e\log\left(x^2e + d\right) + 6\,dg^2px^2 - 36\,fgpx^2e + 18\,fgx^2e\log\left(c\right) - 9\,f^2pe\log\left(x^2e + d\right) - 9\,f^2e\log\left(c\right)\right)e^{(-1)}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^2,x, algorithm="giac")
```

```
[Out] -2/3*(d^2*g^2*p - 6*d*f*g*p*e - 3*f^2*p*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-
3/2)/sqrt(d) + 1/9*(3*g^2*p*x^4*e*log(x^2*e + d) - 2*g^2*p*x^4*e + 3*g^2*x^
4*e*log(c) + 18*f*g*p*x^2*e*log(x^2*e + d) + 6*d*g^2*p*x^2 - 36*f*g*p*x^2*e
 + 18*f*g*x^2*e*log(c) - 9*f^2*p*e*log(x^2*e + d) - 9*f^2*e*log(c))*e^(-1)/
x
```

Mupad [B]
time = 0.37, size = 180, normalized size = 1.01

$$\frac{2\,p\,\mathrm{atan}\left(\frac{\sqrt{e}\,\,p\,x\,\left(-d^2\,g^2+6\,d\,e\,f\,g+3\,e^2\,f^2\right)}{\sqrt{d}\,\,\left(-p\,d^2\,g^2+6\,p\,d\,e\,f\,g+3\,p\,e^2\,f^2\right)}\right)\,\left(-d^2\,g^2+6\,d\,e\,f\,g+3\,e^2\,f^2\right)}{3\,\sqrt{d}\,\,e^{3/2}} - x\,\left(4\,f\,g\,p-\frac{2\,d\,g^2\,p}{3\,e}\right) - \frac{2\,g^2\,p\,x^3}{9} - \ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(\frac{f^2+2\,f\,g\,x^2+g^2\,x^4}{x}-\frac{\frac{4\,g^2\,x^4}{3}+4\,f\,g\,x^2}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^2,x)
```

```
[Out] (2*p*atan((e^(1/2)*p*x*(3*e^2*f^2 - d^2*g^2 + 6*d*e*f*g))/(d^(1/2)*(3*e^2*f
^2*p - d^2*g^2*p + 6*d*e*f*g*p)))*(3*e^2*f^2 - d^2*g^2 + 6*d*e*f*g))/(3*d^(
1/2)*e^(3/2)) - x*(4*f*g*p - (2*d*g^2*p)/(3*e)) - (2*g^2*p*x^3)/9 - log(c*(
d + e*x^2)^p)*((f^2 + g^2*x^4 + 2*f*g*x^2)/x - ((4*g^2*x^4)/3 + 4*f*g*x^2)/
x)
```

3.335 $\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^4}\,dx$

Optimal. Leaf size=169

$$-\frac{2ef^2p}{3dx}-2g^2px-\frac{2e^{3/2}f^2p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3d^{3/2}}+\frac{4\sqrt{e}\,fgp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{d}\,g^2p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{f^2\log\left(c(\right.}{3}$$

```
[Out] -2/3*e*f^2*p/d/x-2*g^2*p*x-2/3*e^(3/2)*f^2*p*arctan(x*e^(1/2)/d^(1/2))/d^(3
/2)-1/3*f^2*ln(c*(e*x^2+d)^p)/x^3-2*f*g*ln(c*(e*x^2+d)^p)/x+g^2*x*ln(c*(e*x
^2+d)^p)+2*g^2*p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)+4*f*g*p*arctan(x
*e^(1/2)/d^(1/2))*e^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2526, 2498, 327, 211, 2505, 331}

$$-\frac{2e^{3/2}f^2p\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3d^{3/2}}+\frac{4\sqrt{e}\,fgp\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{d}\,g^2p\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{3x^3}-\frac{2fg\log\left(c\left(d+ex^2\right)^p\right)}{x}+g^2x\log\left(c\left(d+ex^2\right)^p\right)-\frac{2ef^2p}{3dx}-2g^2px$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^4,x]
```

```
[Out] (-2*e*f^2*p)/(3*d*x) - 2*g^2*p*x - (2*e^(3/2)*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt
[d]])/(3*d^(3/2)) + (4*Sqrt[e]*f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] +
 (2*Sqrt[d]*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (f^2*Log[c*(d + e*
x^2)^p])/(3*x^3) - (2*f*g*Log[c*(d + e*x^2)^p])/x + g^2*x*Log[c*(d + e*x^2)
^p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\begin{aligned}
\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x^4} \, dx &= \int \left(g^2 \log \left(c(d+ex^2)^p\right) + \frac{f^2 \log \left(c(d+ex^2)^p\right)}{x^4} + \frac{2fg \log \left(c(d+ex^2)^p\right)}{x^2}\right. \\
&= f^2 \int \frac{\log \left(c(d+ex^2)^p\right)}{x^4} \, dx + (2fg) \int \frac{\log \left(c(d+ex^2)^p\right)}{x^2} \, dx + g^2 \int \log \left(c\right. \\
&= -\frac{f^2 \log \left(c(d+ex^2)^p\right)}{3x^3} - \frac{2fg \log \left(c(d+ex^2)^p\right)}{x} + g^2 x \log \left(c(d+ex^2)^p\right) \\
&= -\frac{2ef^2p}{3dx} - 2g^2px + \frac{4\sqrt{e}\, fgp \tan^{-1}\left(\frac{\sqrt{e}\, x}{\sqrt{d}}\right)}{\sqrt{d}} - \frac{f^2 \log \left(c(d+ex^2)^p\right)}{3x^3} - \\
&= -\frac{2ef^2p}{3dx} - 2g^2px - \frac{2e^{3/2} f^2 p \tan^{-1}\left(\frac{\sqrt{e}\, x}{\sqrt{d}}\right)}{3d^{3/2}} + \frac{4\sqrt{e}\, fgp \tan^{-1}\left(\frac{\sqrt{e}\, x}{\sqrt{d}}\right.}{\sqrt{d}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.10, size = 113, normalized size = 0.67

$$-2g^2px+\frac{2g(2ef+dg)p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,\sqrt{e}}-\frac{2ef^2p\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{ex^2}{d}\right)}{3dx}-\frac{(f^2+6fgx^2-3g^2x^4)\log\left(c(d+ex^2)^p\right)}{3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^4,x]

[Out] -2*g^2*p*x + (2*g*(2*e*f + d*g)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqrt[e]) - (2*e*f^2*p*Hypergeometric2F1[-1/2, 1, 1/2, -((e*x^2)/d)])/(3*d*x) - ((f^2 + 6*f*g*x^2 - 3*g^2*x^4)*Log[c*(d + e*x^2)^p])/(3*x^3)

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.45, size = 700, normalized size = 4.14

method	result
risch	$-\frac{(-3g^2x^4+6fg\,x^2+f^2)\ln((e\,x^2+d)^p)}{3x^3}+\frac{-6i\pi dfg\,x^2\mathrm{csgn}(ic(e\,x^2+d)^p)^2\mathrm{csgn}(ic)+3i\pi d\,g^2x^4\mathrm{csgn}(i(e\,x^2+d)^p)\mathrm{csgn}(ic(e\,x^2+d)^p}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^4,x,method=_RETURNVERBOSE)

[Out] -1/3*(-3*g^2*x^4+6*f*g*x^2+f^2)/x^3*ln((e*x^2+d)^p)+1/6*(-6*I*Pi*d*f*g*x^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+3*I*Pi*d*g^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+6*I*Pi*d*f*g*x^2*csgn(I*c*(e*x^2+d)^p)^3+6*I*Pi*d*f*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+3*I*Pi*d*g^2*x^4*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-I*Pi*d*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+I*Pi*d*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-3*I*Pi*d*g^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-3*I*Pi*d*g^2*x^4*csgn(I*c*(e*x^2+d)^p)^3+I*Pi*d*f^2*csgn(I*c*(e*x^2+d)^p)^3-I*Pi*d*f^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-6*I*Pi*d*f*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+6*ln(c)*d*g^2*x^4-12*d*g^2*p*x^4-12*ln(c)*d*f*g*x^2-4*p*e*x^2*f^2+2*sum(_R*ln((18*d^4*g^4*p^2+72*d^3*e*f*g^3*p^2+60*d^2*e^2*f^2*g^2*p^2-24*d*e^3*f^3*g*p^2+2*e^4*f^4*p^2+3*_R^2*d^3*e)*x+(-3*d^4*g^2*p-6*d^3*e*f*g*p+d^2*e^2*f^2*p)*_R),_R=RootOf(9*d^4*g^4*p^2+36*d^3*e*f*g^3*p^2+30*d^2*e^2*f^2*g^2*p^2-12*d*e^3*f^3*g*p^2+e^4*f^4*p^2+_Z^2*d^3*e))*d*x^3-2*ln(c)*d*f^2)/d/x^3

Maxima [A]
time = 0.57, size = 101, normalized size = 0.60

$$-\frac{2}{3}\left(3\,g^2xe^{(-1)}-\frac{(3\,d^2g^2+6\,dfge-f^2e^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{3}{2}\right)}}{d^{\frac{3}{2}}}+\frac{f^2}{dx}\right)pe+\frac{1}{3}\left(3\,g^2x-\frac{6\,fgx^2+f^2}{x^3}\right)\log\left((x^2e+d)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^4,x, algorithm="maxima")
```

```
[Out] -2/3*(3*g^2*x*e^(-1) - (3*d^2*g^2 + 6*d*f*g*e - f^2*e^2)*arctan(x*e^(1/2)/s
qrt(d))*e^(-3/2)/d^(3/2) + f^2/(d*x))*p*e + 1/3*(3*g^2*x - (6*f*g*x^2 + f^2
)/x^3)*log((x^2*e + d)^p*c)
```

Fricas [A]
time = 0.39, size = 365, normalized size = 2.16

$$\left[-\frac{\left(6\,d^2g^2px^4e+2\,df^2px^2e^2-(3\,d^2g^2px^4-6\,d^2fgpx^2-d^2f^2p)e\log\left(x^2e+d\right)-(3\,d^2g^2x^4-6\,d^2fgx^2-d^2f^2)e\log\left(c\right)-(3\,d^2g^2px^3+6\,dfgpx^3e-f^2px^3e^2)\sqrt{-de}\,\log\left(\frac{x^2e+2\,\sqrt{-de}\,x-d}{x^2e+d}\right)\right)e^{(-1)}}{3\,d^2x^3}, -\frac{\left(6\,d^2g^2px^4e+2\,df^2px^2e^2-2\,(3\,d^2g^2px^3+6\,dfgpx^3e-f^2px^3e^2)\sqrt{d}\,\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}-(3\,d^2g^2px^4-6\,d^2fgpx^2-d^2f^2p)e\log\left(x^2e+d\right)-(3\,d^2g^2x^4-6\,d^2fgx^2-d^2f^2)e\log\left(c\right)\right)e^{(-1)}}{3\,d^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/3*(6*d^2*g^2*p*x^4*e + 2*d*f^2*p*x^2*e^2 - (3*d^2*g^2*p*x^4 - 6*d^2*f*g
*p*x^2 - d^2*f^2*p)*e*log(x^2*e + d) - (3*d^2*g^2*x^4 - 6*d^2*f*g*x^2 - d^2
*f^2)*e*log(c) - (3*d^2*g^2*p*x^3 + 6*d*f*g*p*x^3*e - f^2*p*x^3*e^2)*sqrt(-
d*e)*log((x^2*e + 2*sqrt(-d*e)*x - d)/(x^2*e + d)))*e^(-1)/(d^2*x^3), -1/3*
(6*d^2*g^2*p*x^4*e + 2*d*f^2*p*x^2*e^2 - 2*(3*d^2*g^2*p*x^3 + 6*d*f*g*p*x^3
*e - f^2*p*x^3*e^2)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2) - (3*d^2*g^2*
p*x^4 - 6*d^2*f*g*p*x^2 - d^2*f^2*p)*e*log(x^2*e + d) - (3*d^2*g^2*x^4 - 6*
d^2*f*g*x^2 - d^2*f^2)*e*log(c))*e^(-1)/(d^2*x^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(170) = 340$.
time = 80.89, size = 381, normalized size = 2.25

$$\begin{cases} \left(-\frac{f^2}{3x^3}-\frac{2fg}{x}+g^2x\right)\log\left(0^pc\right) & \text{for}\: d=0\wedge e=0 \\ -\frac{2f^2p}{9x^3}-\frac{f^2\log\left(c\left(ex^2\right)^p\right)}{3x^3}-\frac{4fgp}{x}-\frac{2fg\log\left(c\left(ex^2\right)^p\right)}{x}-2g^2px+g^2x\log\left(c\left(ex^2\right)^p\right) & \text{for}\: d=0 \\ \left(-\frac{f^2}{3x^3}-\frac{2fg}{x}+g^2x\right)\log\left(cd^p\right) & \text{for}\: e=0 \\ \frac{2dg^2p\log\left(x-\sqrt{-\frac{d}{e}}\right)}{e\sqrt{-\frac{d}{e}}}-\frac{dg^2\log\left(c\left(d+ex^2\right)^p\right)}{e\sqrt{-\frac{d}{e}}}-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{3x^3}+\frac{4fgp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{\sqrt{-\frac{d}{e}}}-\frac{2fg\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{-\frac{d}{e}}}-\frac{2fg\log\left(c\left(d+ex^2\right)^p\right)}{x}-2g^2px+g^2x\log\left(c\left(d+ex^2\right)^p\right)-\frac{2ef^2p\log\left(x-\sqrt{-\frac{d}{e}}\right)}{3d\sqrt{-\frac{d}{e}}}-\frac{2ef^2p}{3dx}+\frac{ef^2\log\left(c\left(d+ex^2\right)^p\right)}{3d\sqrt{-\frac{d}{e}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**4,x)
```

```
[Out] Piecewise(((-f**2/(3*x**3) - 2*f*g/x + g**2*x)*log(0**p*c), Eq(d, 0) & Eq(e
, 0)), (-2*f**2*p/(9*x**3) - f**2*log(c*(e*x**2)**p)/(3*x**3) - 4*f*g*p/x -
 2*f*g*log(c*(e*x**2)**p)/x - 2*g**2*p*x + g**2*x*log(c*(e*x**2)**p), Eq(d,
 0)), ((-f**2/(3*x**3) - 2*f*g/x + g**2*x)*log(c*d**p), Eq(e, 0)), (2*d*g**
2*p*log(x - sqrt(-d/e))/(e*sqrt(-d/e)) - d*g**2*log(c*(d + e*x**2)**p)/(e*s
qrt(-d/e)) - f**2*log(c*(d + e*x**2)**p)/(3*x**3) + 4*f*g*p*log(x - sqrt(-d
/e))/sqrt(-d/e) - 2*f*g*log(c*(d + e*x**2)**p)/sqrt(-d/e) - 2*f*g*log(c*(d
+ e*x**2)**p)/x - 2*g**2*p*x + g**2*x*log(c*(d + e*x**2)**p) - 2*e*f**2*p*l
og(x - sqrt(-d/e))/(3*d*sqrt(-d/e)) - 2*e*f**2*p/(3*d*x) + e*f**2*log(c*(d
+ e*x**2)**p)/(3*d*sqrt(-d/e)), True))
```

Giac [A]
time = 5.45, size = 154, normalized size = 0.91

$$\frac{2\,(3\,d^2g^2p+6\,dfgpe-f^2pe^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{3\,d^{\frac{3}{2}}}+\frac{3\,dg^2px^4\log\left(x^2e+d\right)-6\,dg^2px^4+3\,dg^2x^4\log\left(c\right)-6\,dfgpx^2\log\left(x^2e+d\right)-2\,f^2px^2e-6\,dfgx^2\log\left(c\right)-df^2p\log\left(x^2e+d\right)-df^2\log\left(c\right)}{3\,dx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^4,x, algorithm="giac")
```

```
[Out] 2/3*(3*d^2*g^2*p + 6*d*f*g*p*e - f^2*p*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1
/2)/d^(3/2) + 1/3*(3*d*g^2*p*x^4*log(x^2*e + d) - 6*d*g^2*p*x^4 + 3*d*g^2*x
^4*log(c) - 6*d*f*g*p*x^2*log(x^2*e + d) - 2*f^2*p*x^2*e - 6*d*f*g*x^2*log(
c) - d*f^2*p*log(x^2*e + d) - d*f^2*log(c))/(d*x^3)
```

Mupad [B]
time = 0.39, size = 108, normalized size = 0.64

$$\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(\frac{8\,g^2\,x}{3}-\frac{\frac{f^2}{3}+2\,f\,g\,x^2+\frac{5\,g^2\,x^4}{3}}{x^3}\right)-2\,g^2\,p\,x+\frac{2\,p\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(3\,d^2\,g^2+6\,d\,e\,f\,g-e^2\,f^2\right)}{3\,d^{3/2}\,\sqrt{e}}-\frac{2\,e\,f^2\,p}{3\,d\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^4,x)
```

```
[Out] log(c*(d + e*x^2)^p)*((8*g^2*x)/3 - (f^2/3 + (5*g^2*x^4)/3 + 2*f*g*x^2)/x^3
) - 2*g^2*p*x + (2*p*atan((e^(1/2)*x)/d^(1/2))*(3*d^2*g^2 - e^2*f^2 + 6*d*e
*f*g))/(3*d^(3/2)*e^(1/2)) - (2*e*f^2*p)/(3*d*x)
```

3.336 $\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^6} \, dx$

Optimal. Leaf size=200

$$-\frac{2ef^2p}{15dx^3}+\frac{2e^2f^2p}{5d^2x}-\frac{4efgp}{3dx}+\frac{2e^{5/2}f^2p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{5d^{5/2}}-\frac{4e^{3/2}fgp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3d^{3/2}}+\frac{2\sqrt{e}\,g^2p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}}-f$$

```
[Out] -2/15*e*f^2*p/d/x^3+2/5*e^2*f^2*p/d^2/x-4/3*e*f*g*p/d/x+2/5*e^(5/2)*f^2*p*a
rctan(x*e^(1/2)/d^(1/2))/d^(5/2)-4/3*e^(3/2)*f*g*p*arctan(x*e^(1/2)/d^(1/2)
)/d^(3/2)-1/5*f^2*ln(c*(e*x^2+d)^p)/x^5-2/3*f*g*ln(c*(e*x^2+d)^p)/x^3-g^2*l
n(c*(e*x^2+d)^p)/x+2*g^2*p*arctan(x*e^(1/2)/d^(1/2))*e^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2526, 2505, 331, 211}

$$\frac{2e^{5/2}f^2p\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{5d^{5/2}}-\frac{4e^{3/2}fgp\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3d^{3/2}}+\frac{2\sqrt{e}\,g^2p\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{f^2\log\left(c(d+ex^2)^p\right)}{5x^5}-\frac{2fg\log\left(c(d+ex^2)^p\right)}{3x^3}-\frac{g^2\log\left(c(d+ex^2)^p\right)}{x}+\frac{2e^2f^2p}{5d^2x}-\frac{2ef^2p}{15dx^3}-\frac{4efgp}{3dx}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^6,x]
```

```
[Out] (-2*e*f^2*p)/(15*d*x^3) + (2*e^2*f^2*p)/(5*d^2*x) - (4*e*f*g*p)/(3*d*x) + (
2*e^(5/2)*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*d^(5/2)) - (4*e^(3/2)*f*g*p
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*d^(3/2)) + (2*Sqrt[e]*g^2*p*ArcTan[(Sqrt[e
]*x)/Sqrt[d]])/Sqrt[d] - (f^2*Log[c*(d + e*x^2)^p])/(5*x^5) - (2*f*g*Log[c*
(d + e*x^2)^p])/(3*x^3) - (g^2*Log[c*(d + e*x^2)^p])/x
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
```

```
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^6}\,dx = \int \left(\frac{f^2 \log\left(c(d+ex^2)^p\right)}{x^6} + \frac{2fg \log\left(c(d+ex^2)^p\right)}{x^4} + \frac{g^2 \log\left(c(d+ex^2)\right.}{x^2}\right.$$

$$= f^2 \int \frac{\log\left(c(d+ex^2)^p\right)}{x^6}\,dx + (2fg)\int \frac{\log\left(c(d+ex^2)^p\right)}{x^4}\,dx + g^2 \int \frac{\log}{}$$

$$= -\frac{f^2 \log\left(c(d+ex^2)^p\right)}{5x^5} - \frac{2fg \log\left(c(d+ex^2)^p\right)}{3x^3} - \frac{g^2 \log\left(c(d+ex^2)^p\right)}{x}$$

$$= -\frac{2ef^2p}{15dx^3} - \frac{4efgp}{3dx} + \frac{2\sqrt{e}\,g^2p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}} - \frac{f^2 \log\left(c(d+ex^2)^p\right)}{5x^5} -$$

$$= -\frac{2ef^2p}{15dx^3} + \frac{2e^2f^2p}{5d^2x} - \frac{4efgp}{3dx} - \frac{4e^{3/2}fgp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3d^{3/2}} + \frac{2\sqrt{e}\,g^2p\,\mathrm{ta}}{\sqrt{}}$$

$$= -\frac{2ef^2p}{15dx^3} + \frac{2e^2f^2p}{5d^2x} - \frac{4efgp}{3dx} + \frac{2e^{5/2}f^2p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{5d^{5/2}} - \frac{4e^{3/2}fgp\,\mathrm{ta}}{3d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 156, normalized size = 0.78

$$\frac{2\sqrt{e}\,g^2p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}} - \frac{2ef^2p\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\frac{ex^2}{d}\right)}{15dx^3} - \frac{4efgp\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{ex^2}{d}\right)}{3dx} - \frac{f^2\log\left(c(d+ex^2)^p\right)}{5x^5} - \frac{2fg\log\left(c(d+ex^2)^p\right)}{3x^3} - \frac{g^2\log\left(c(d+ex^2)^p\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^6,x]
[Out] (2*Sqrt[e]*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] - (2*e*f^2*p*Hypergeo
metric2F1[-3/2, 1, -1/2, -((e*x^2)/d)])/(15*d*x^3) - (4*e*f*g*p*Hypergeomet
```

ric2F1[-1/2, 1, 1/2, -((e*x^2)/d)])/(3*d*x) - (f^2*Log[c*(d + e*x^2)^p])/(5*x^5) - (2*f*g*Log[c*(d + e*x^2)^p])/(3*x^3) - (g^2*Log[c*(d + e*x^2)^p])/x

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.44, size = 736, normalized size = 3.68

method	result
risch	$-\frac{(15g^2x^4+10fg\,x^2+3f^2)\ln((e\,x^2+d)^p)}{15x^5} - \frac{-10i\pi\,d^3fg\,x^2\mathrm{csgn}(ic(e\,x^2+d)^p)^3-15i\pi\,d^3g^2x^4\mathrm{csgn}(i(e\,x^2+d)^p)\mathrm{csgn}(ic(e\,x^2+d)^p)\mathrm{cs}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^6,x,method=_RETURNVERBOSE)

[Out] -1/15*(15*g^2*x^4+10*f*g*x^2+3*f^2)/x^5*ln((e*x^2+d)^p)-1/30*(-10*I*Pi*d^3*f*g*x^2*csgn(I*c*(e*x^2+d)^p)^3-15*I*Pi*d^3*g^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+15*I*Pi*d^3*g^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+15*I*Pi*d^3*g^2*x^4*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-15*I*Pi*d^3*g^2*x^4*csgn(I*c*(e*x^2+d)^p)^3-3*I*Pi*d^3*f^2*csgn(I*c*(e*x^2+d)^p)^3+10*I*Pi*d^3*f*g*x^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+10*I*Pi*d^3*f*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-30*(-e*d)^(1/2)*p*ln(-e*x-(-e*d)^(1/2))*g^2*d^2*x^5+20*(-e*d)^(1/2)*p*ln(-e*x-(-e*d)^(1/2))*e*f*g*d*x^5-6*(-e*d)^(1/2)*p*ln(-e*x-(-e*d)^(1/2))*e^2*f^2*x^5+30*(-e*d)^(1/2)*p*ln(-e*x+(-e*d)^(1/2))*g^2*d^2*x^5-20*(-e*d)^(1/2)*p*ln(-e*x+(-e*d)^(1/2))*e*f*g*d*x^5+6*(-e*d)^(1/2)*p*ln(-e*x+(-e*d)^(1/2))*e^2*f^2*x^5-10*I*Pi*d^3*f*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-3*I*Pi*d^3*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+3*I*Pi*d^3*f^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+3*I*Pi*d^3*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+30*ln(c)*d^3*g^2*x^4+40*d^2*e*f*g*p*x^4-12*d*e^2*f^2*p*x^4+20*ln(c)*d^3*f*g*x^2+4*d^2*e*f^2*p*x^2+6*ln(c)*d^3*f^2)/d^3/x^5

Maxima [A]
time = 0.54, size = 114, normalized size = 0.57

$$\frac{2}{15}\,p\left(\frac{(15\,d^2g^2-10\,dfge+3\,f^2e^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{d^{\frac{5}{2}}}-\frac{df^2+(10\,dfg-3\,f^2e)x^2}{d^2x^3}\right)e-\frac{(15\,g^2x^4+10\,fgx^2+3\,f^2)\log\left((x^2e+d)^pc\right)}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^6,x, algorithm="maxima")

[Out] 2/15*p*((15*d^2*g^2 - 10*d*f*g*e + 3*f^2*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d^(5/2) - (d*f^2 + (10*d*f*g - 3*f^2*e)*x^2)/(d^2*x^3))*e - 1/15*(15*g^2*x^4 + 10*f*g*x^2 + 3*f^2)*log((x^2*e + d)^p*c)/x^5

Fricas [A]
time = 0.36, size = 373, normalized size = 1.86

$$\left[\frac{6\,f^2px^4e^2+(15\,d^2g^2px^5-10\,dfgpx^5e+3\,f^2px^5e^2)\sqrt{-\frac{e}{d}}\log\left(\frac{x^2e+2\,dx\sqrt{-\frac{e}{d}}-d}{x^2e+d}\right)-2\,(10\,dfgpx^4+df^2px^2)e-(15\,d^2g^2px^4+10\,d^2fgpx^2+3\,d^2f^2p)\log(x^2e+d)-(15\,d^2g^2x^4+10\,d^2fgx^2+3\,d^2f^2)\log(c)}{15\,d^2x^5},\ \frac{6\,f^2px^4e^2+\frac{2\,(15\,d^2g^2px^5-10\,dfgpx^5e+3\,f^2px^5e^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}}{\sqrt{d}}-2\,(10\,dfgpx^4+df^2px^2)e-(15\,d^2g^2px^4+10\,d^2fgpx^2+3\,d^2f^2p)\log(x^2e+d)-(15\,d^2g^2x^4+10\,d^2fgx^2+3\,d^2f^2)\log(c)}{15\,d^2x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^6,x, algorithm="fricas")
```

```
[Out] [1/15*(6*f^2*p*x^4*e^2 + (15*d^2*g^2*p*x^5 - 10*d*f*g*p*x^5*e + 3*f^2*p*x^5
*e^2)*sqrt(-e/d)*log((x^2*e + 2*d*x*sqrt(-e/d) - d)/(x^2*e + d)) - 2*(10*d*
f*g*p*x^4 + d*f^2*p*x^2)*e - (15*d^2*g^2*p*x^4 + 10*d^2*f*g*p*x^2 + 3*d^2*f
^2*p)*log(x^2*e + d) - (15*d^2*g^2*x^4 + 10*d^2*f*g*x^2 + 3*d^2*f^2)*log(c)
)/(d^2*x^5), 1/15*(6*f^2*p*x^4*e^2 + 2*(15*d^2*g^2*p*x^5 - 10*d*f*g*p*x^5*e
 + 3*f^2*p*x^5*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2)/sqrt(d) - 2*(10*d*f*g
*p*x^4 + d*f^2*p*x^2)*e - (15*d^2*g^2*p*x^4 + 10*d^2*f*g*p*x^2 + 3*d^2*f^2*
p)*log(x^2*e + d) - (15*d^2*g^2*x^4 + 10*d^2*f*g*x^2 + 3*d^2*f^2)*log(c))/(
d^2*x^5)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1567 vs. $2(199) = 398$.
time = 169.60, size = 1567, normalized size = 7.84

$$\begin{cases} \left(-\frac{f^2}{5x^5} - \frac{2fg}{3x^3} - \frac{g^2}{x}\right)\log\left(0^p c\right) & \text{for } d = 0 \wedge e = 0 \\ \left(-\frac{f^2}{5x^5} - \frac{2fg}{3x^3} - \frac{g^2}{x}\right)\log\left(cd^p\right) & \text{for } e = 0 \\ -\frac{2f^2p}{25x^5} - \frac{f^2\log\left(c\left(ex^2\right)^p\right)}{5x^5} - \frac{4fgp}{9x^3} - \frac{2fg\log\left(c\left(ex^2\right)^p\right)}{3x^3} - \frac{2g^2p}{x} - \frac{g^2\log\left(c\left(ex^2\right)^p\right)}{x} & \text{for } d = 0 \\ \text{[illegible]} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**6,x)
```

```
[Out] Piecewise(((-f**2/(5*x**5) - 2*f*g/(3*x**3) - g**2/x)*log(0**p*c), Eq(d, 0)
 & Eq(e, 0)), ((-f**2/(5*x**5) - 2*f*g/(3*x**3) - g**2/x)*log(c*d**p), Eq(e
, 0)), (-2*f**2*p/(25*x**5) - f**2*log(c*(e*x**2)**p)/(5*x**5) - 4*f*g*p/(9
*x**3) - 2*f*g*log(c*(e*x**2)**p)/(3*x**3) - 2*g**2*p/x - g**2*log(c*(e*x**
2)**p)/x, Eq(d, 0)), (-3*d**3*f**2*sqrt(-d/e)*log(c*(d + e*x**2)**p)/(15*d*
*3*x**5*sqrt(-d/e) + 15*d**2*e*x**7*sqrt(-d/e)) - 10*d**3*f*g*x**2*sqrt(-d/
e)*log(c*(d + e*x**2)**p)/(15*d**3*x**5*sqrt(-d/e) + 15*d**2*e*x**7*sqrt(-d
/e)) + 30*d**3*g**2*p*x**5*log(x - sqrt(-d/e))/(15*d**3*x**5*sqrt(-d/e) + 1
5*d**2*e*x**7*sqrt(-d/e)) - 15*d**3*g**2*x**5*log(c*(d + e*x**2)**p)/(15*d*
*3*x**5*sqrt(-d/e) + 15*d**2*e*x**7*sqrt(-d/e)) - 15*d**3*g**2*x**4*sqrt(-d
/e)*log(c*(d + e*x**2)**p)/(15*d**3*x**5*sqrt(-d/e) + 15*d**2*e*x**7*sqrt(-
d/e)) - 2*d**2*f**2*p*x**2*sqrt(-d/e)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*
x**7*sqrt(-d/e)) - 3*d**2*f**2*x**2*sqrt(-d/e)*log(c*(d + e*x**2)**p)/(15*d
**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) - 20*d**2*f*g*p*x**5*log(x
 - sqrt(-d/e))/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) - 20*d
**2*f*g*p*x**4*sqrt(-d/e)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d
/e)) + 10*d**2*f*g*x**5*log(c*(d + e*x**2)**p)/(15*d**3*x**5*sqrt(-d/e)/e +
 15*d**2*x**7*sqrt(-d/e)) - 10*d**2*f*g*x**4*sqrt(-d/e)*log(c*(d + e*x**2)*
*p)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) + 30*d**2*g**2*p*
x**7*log(x - sqrt(-d/e))/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/
e)) - 15*d**2*g**2*x**7*log(c*(d + e*x**2)**p)/(15*d**3*x**5*sqrt(-d/e)/e +
 15*d**2*x**7*sqrt(-d/e)) - 15*d**2*g**2*x**6*sqrt(-d/e)*log(c*(d + e*x**2)
**p)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) + 6*d*e*f**2*p*x
```

```
**5*log(x - sqrt(-d/e))/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e
)) + 4*d*e*f**2*p*x**4*sqrt(-d/e)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7
*sqrt(-d/e)) - 3*d*e*f**2*x**5*log(c*(d + e*x**2)**p)/(15*d**3*x**5*sqrt(-d
/e)/e + 15*d**2*x**7*sqrt(-d/e)) - 20*d*e*f*g*p*x**7*log(x - sqrt(-d/e))/(1
5*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) - 20*d*e*f*g*p*x**6*sqr
t(-d/e)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) + 10*d*e*f*g*
x**7*log(c*(d + e*x**2)**p)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(
-d/e)) + 6*e**2*f**2*p*x**7*log(x - sqrt(-d/e))/(15*d**3*x**5*sqrt(-d/e)/e
+ 15*d**2*x**7*sqrt(-d/e)) + 6*e**2*f**2*p*x**6*sqrt(-d/e)/(15*d**3*x**5*sq
rt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) - 3*e**2*f**2*x**7*log(c*(d + e*x**2)
**p)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)), True))
```

Giac [A]
time = 4.79, size = 181, normalized size = 0.90

$$\frac{2\,(15\,d^2g^2pe - 10\,dfgpe^2 + 3\,f^2pe^3)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{15\,d^{\frac{5}{2}}} - \frac{15\,d^2g^2px^4\log\left(x^2e+d\right) + 20\,dfgpx^4e + 15\,d^2g^2x^4\log\left(c\right) - 6\,f^2px^4e^2 + 10\,d^2fgpx^2\log\left(x^2e+d\right) + 2\,df^2px^2e + 10\,d^2fgx^2\log\left(c\right) + 3\,d^2f^2p\log\left(x^2e+d\right) + 3\,d^2f^2\log\left(c\right)}{15\,d^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^6,x, algorithm="giac")
```

```
[Out] 2/15*(15*d^2*g^2*p*e - 10*d*f*g*p*e^2 + 3*f^2*p*e^3)*arctan(x*e^(1/2)/sqrt(
d))*e^(-1/2)/d^(5/2) - 1/15*(15*d^2*g^2*p*x^4*log(x^2*e + d) + 20*d*f*g*p*x
^4*e + 15*d^2*g^2*x^4*log(c) - 6*f^2*p*x^4*e^2 + 10*d^2*f*g*p*x^2*log(x^2*e
 + d) + 2*d*f^2*p*x^2*e + 10*d^2*f*g*x^2*log(c) + 3*d^2*f^2*p*log(x^2*e + d
) + 3*d^2*f^2*log(c))/(d^2*x^5)
```

Mupad [B]
time = 0.39, size = 115, normalized size = 0.58

$$\frac{2\,\sqrt{e}\,\,p\,\mathrm{atan}\left(\frac{\sqrt{e}\,\,x}{\sqrt{d}}\right)\,\left(15\,d^2\,g^2-10\,d\,e\,f\,g+3\,e^2\,f^2\right)}{15\,d^{5/2}}-\frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(\frac{f^2}{5}+\frac{2\,f\,g\,x^2}{3}+g^2\,x^4\right)}{x^5}-\frac{\frac{2\,e\,f^2\,p}{d}+\frac{2\,e\,f\,p\,x^2\,\left(10\,d\,g-3\,e\,f\right)}{d^2}}{15\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^6,x)
```

```
[Out] (2*e^(1/2)*p*atan((e^(1/2)*x)/d^(1/2))*(15*d^2*g^2 + 3*e^2*f^2 - 10*d*e*f*g
))/(15*d^(5/2)) - (log(c*(d + e*x^2)^p)*(f^2/5 + g^2*x^4 + (2*f*g*x^2)/3))/
x^5 - ((2*e*f^2*p)/d + (2*e*f*p*x^2*(10*d*g - 3*e*f))/d^2)/(15*x^3)
```

3.337 $\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^8} \, dx$

Optimal. Leaf size=252

$$-\frac{2ef^2p}{35dx^5}+\frac{2e^2f^2p}{21d^2x^3}-\frac{4efgp}{15dx^3}-\frac{2e^3f^2p}{7d^3x}+\frac{4e^2fgp}{5d^2x}-\frac{2eg^2p}{3dx}-\frac{2e^{7/2}f^2p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{7d^{7/2}}+\frac{4e^{5/2}fgp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{5d^{5/2}}-$$

```
[Out] -2/35*e*f^2*p/d/x^5+2/21*e^2*f^2*p/d^2/x^3-4/15*e*f*g*p/d/x^3-2/7*e^3*f^2*p
/d^3/x+4/5*e^2*f*g*p/d^2/x-2/3*e*g^2*p/d/x-2/7*e^(7/2)*f^2*p*arctan(x*e^(1/
2)/d^(1/2))/d^(7/2)+4/5*e^(5/2)*f*g*p*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)-2/3
*e^(3/2)*g^2*p*arctan(x*e^(1/2)/d^(1/2))/d^(3/2)-1/7*f^2*ln(c*(e*x^2+d)^p)/
x^7-2/5*f*g*ln(c*(e*x^2+d)^p)/x^5-1/3*g^2*ln(c*(e*x^2+d)^p)/x^3
```

Rubi [A]
time = 0.14, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2526, 2505, 331, 211}

$$-\frac{2e^{7/2}f^2p\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{7d^{7/2}}+\frac{4e^{5/2}fgp\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{5d^{5/2}}-\frac{2e^{3/2}g^2p\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3d^{3/2}}-\frac{f^2\log\left(c(d+ex^2)^p\right)}{7x^7}-\frac{2fg\log\left(c(d+ex^2)^p\right)}{5x^5}-\frac{g^2\log\left(c(d+ex^2)^p\right)}{3x^3}-\frac{2e^3f^2p}{7d^3x}+\frac{2e^2f^2p}{21d^2x^3}+\frac{4e^2fgp}{5d^2x}-\frac{2ef^2p}{35dx^5}-\frac{4efgp}{15dx^3}-\frac{2eg^2p}{3dx}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^8,x]
```

```
[Out] (-2*e*f^2*p)/(35*d*x^5) + (2*e^2*f^2*p)/(21*d^2*x^3) - (4*e*f*g*p)/(15*d*x^
3) - (2*e^3*f^2*p)/(7*d^3*x) + (4*e^2*f*g*p)/(5*d^2*x) - (2*e*g^2*p)/(3*d*x
) - (2*e^(7/2)*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(7*d^(7/2)) + (4*e^(5/2)*
f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*d^(5/2)) - (2*e^(3/2)*g^2*p*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(3*d^(3/2)) - (f^2*Log[c*(d + e*x^2)^p])/(7*x^7) - (2*f
*g*Log[c*(d + e*x^2)^p])/(5*x^5) - (g^2*Log[c*(d + e*x^2)^p])/(3*x^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^8}\,dx = \int \left(\frac{f^2 \log\left(c(d+ex^2)^p\right)}{x^8} + \frac{2fg \log\left(c(d+ex^2)^p\right)}{x^6} + \frac{g^2 \log\left(c(d+ex^2)^p\right)}{x^4}\right.$$

$$= f^2 \int \frac{\log\left(c(d+ex^2)^p\right)}{x^8}\,dx + (2fg)\int \frac{\log\left(c(d+ex^2)^p\right)}{x^6}\,dx + g^2 \int \frac{\log\left(\right.}{}$$

$$= -\frac{f^2 \log\left(c(d+ex^2)^p\right)}{7x^7} - \frac{2fg \log\left(c(d+ex^2)^p\right)}{5x^5} - \frac{g^2 \log\left(c(d+ex^2)^p\right)}{3x^3} +$$

$$= -\frac{2ef^2p}{35dx^5} - \frac{4efgp}{15dx^3} - \frac{2eg^2p}{3dx} - \frac{f^2 \log\left(c(d+ex^2)^p\right)}{7x^7} - \frac{2fg \log\left(c(d+ex^2\right.}{5x^5}$$

$$= -\frac{2ef^2p}{35dx^5} + \frac{2e^2f^2p}{21d^2x^3} - \frac{4efgp}{15dx^3} + \frac{4e^2fgp}{5d^2x} - \frac{2eg^2p}{3dx} - \frac{2e^{3/2}g^2p\tan^{-1}\left(\frac{\sqrt{e}}{\sqrt{d}}\right.}{3d^{3/2}}$$

$$= -\frac{2ef^2p}{35dx^5} + \frac{2e^2f^2p}{21d^2x^3} - \frac{4efgp}{15dx^3} - \frac{2e^3f^2p}{7d^3x} + \frac{4e^2fgp}{5d^2x} - \frac{2eg^2p}{3dx} + \frac{4e^{5/2}fgp\,\mathrm{ta}}{5}$$

$$= -\frac{2ef^2p}{35dx^5} + \frac{2e^2f^2p}{21d^2x^3} - \frac{4efgp}{15dx^3} - \frac{2e^3f^2p}{7d^3x} + \frac{4e^2fgp}{5d^2x} - \frac{2eg^2p}{3dx} - \frac{2e^{7/2}f^2p\,\mathrm{ta}}{7}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.02, size = 161, normalized size = 0.64

$$-\frac{2ef^2p\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\frac{ex^2}{d}\right)}{35dx^5} - \frac{4efgp\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\frac{ex^2}{d}\right)}{15dx^3} - \frac{2eg^2p\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{ex^2}{d}\right)}{3dx} - \frac{f^2\log\left(c(d+ex^2)^p\right)}{7x^7} - \frac{2fg\log\left(c(d+ex^2)^p\right)}{5x^5} - \frac{g^2\log\left(c(d+ex^2)^p\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^8,x]
```

```
[Out] (-2*e*f^2*p*Hypergeometric2F1[-5/2, 1, -3/2, -((e*x^2)/d)])/(35*d*x^5) - (4
*e*f*g*p*Hypergeometric2F1[-3/2, 1, -1/2, -((e*x^2)/d)])/(15*d*x^3) - (2*e*
g^2*p*Hypergeometric2F1[-1/2, 1, 1/2, -((e*x^2)/d)])/(3*d*x) - (f^2*Log[c*(
d + e*x^2)^p])/(7*x^7) - (2*f*g*Log[c*(d + e*x^2)^p])/(5*x^5) - (g^2*Log[c*
(d + e*x^2)^p])/(3*x^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.43, size = 784, normalized size = 3.11

method	result
risch	$-\frac{\left(35 g^{2} x^{4}+42 f g\,x^{2}+15 f^{2}\right) \ln \left(\left(e\,x^{2}+d\right)^{p}\right)}{105 x^{7}}-\frac{-35 i \pi \,d^{4} g^{2} x^{4} \mathrm{csgn}\left(i \left(e\,x^{2}+d\right)^{p}\right) \mathrm{csgn}\left(i c \left(e\,x^{2}+d\right)^{p}\right) \mathrm{csgn}\left(i c\right)+42 i \pi \,d^{4} f g\,x^{2} \mathrm{csgn}\left(i (e\,x^{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/105*(35*g^2*x^4+42*f*g*x^2+15*f^2)/x^7*ln((e*x^2+d)^p)-1/210*(70*ln(c)*d
^4*g^2*x^4+30*ln(c)*d^4*f^2+84*ln(c)*d^4*f*g*x^2+140*d^3*e*g^2*p*x^6+60*d*e
^3*f^2*p*x^6-20*d^2*e^2*f^2*p*x^4+12*d^3*e*f^2*p*x^2-168*d^2*e^2*f*g*p*x^6+
56*d^3*e*f*g*p*x^4+42*I*Pi*d^4*f*g*x^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-84
*(-e*d)^(1/2)*p*e^2*ln(-e*x-(-e*d)^(1/2))*f*g*d*x^7+84*(-e*d)^(1/2)*p*e^2*l
n(-e*x+(-e*d)^(1/2))*f*g*d*x^7-35*I*Pi*d^4*g^2*x^4*csgn(I*(e*x^2+d)^p)*csgn
(I*c*(e*x^2+d)^p)*csgn(I*c)+42*I*Pi*d^4*f*g*x^2*csgn(I*(e*x^2+d)^p)*csgn(I*
c*(e*x^2+d)^p)^2+30*(-e*d)^(1/2)*p*e^3*ln(-e*x-(-e*d)^(1/2))*f^2*x^7-30*(-e
*d)^(1/2)*p*e^3*ln(-e*x+(-e*d)^(1/2))*f^2*x^7-35*I*Pi*d^4*g^2*x^4*csgn(I*c*
(e*x^2+d)^p)^3+15*I*Pi*d^4*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+
15*I*Pi*d^4*f^2*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-42*I*Pi*d^4*f*g*x^2*csgn(
I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-15*I*Pi*d^4*f^2*csgn(I*c*(e*
x^2+d)^p)^3+35*I*Pi*d^4*g^2*x^4*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2
+35*I*Pi*d^4*g^2*x^4*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)-42*I*Pi*d^4*f*g*x^2*
csgn(I*c*(e*x^2+d)^p)^3-15*I*Pi*d^4*f^2*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2
+d)^p)*csgn(I*c)+70*(-e*d)^(1/2)*p*e*ln(-e*x-(-e*d)^(1/2))*g^2*d^2*x^7-70*(
-e*d)^(1/2)*p*e*ln(-e*x+(-e*d)^(1/2))*g^2*d^2*x^7)/d^4/x^7
```

Maxima [A]
time = 0.56, size = 148, normalized size = 0.59

$$-\frac{2}{105}\,p\left(\frac{(35\,d^2g^2e-42\,dfge^2+15\,f^2e^3)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{d^{\frac{7}{2}}}+\frac{(35\,d^2g^2-42\,dfge+15\,f^2e^2)x^4+3\,d^2f^2+(14\,d^2fg-5\,df^2e)x^2}{d^3x^5}\right)e-\frac{(35\,g^2x^4+42\,fgx^2+15\,f^2)\log\left((x^2e+d)^pc\right)}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^8,x, algorithm="maxima")
```

```
[Out] -2/105*p*((35*d^2*g^2*e - 42*d*f*g*e^2 + 15*f^2*e^3)*arctan(x*e^(1/2)/sqrt(
d))*e^(-1/2)/d^(7/2) + ((35*d^2*g^2 - 42*d*f*g*e + 15*f^2*e^2)*x^4 + 3*d^2*
```

```
f^2 + (14*d^2*f*g - 5*d*f^2*e)*x^2)/(d^3*x^5))*e - 1/105*(35*g^2*x^4 + 42*f
*g*x^2 + 15*f^2)*log((x^2*e + d)^p*c)/x^7
```

Fricas [A]
time = 0.37, size = 456, normalized size = 1.81

$$\left[-\frac{30 f^2 p x^6 e^3 - (35 d^2 g^2 p x^7 e - 42 d f g p x^7 e^2 + 15 f^2 p x^7 e^3)\sqrt{-\frac{e}{d}} \log\left(\frac{x^2 e - 2 d x \sqrt{-\frac{e}{d}} - d}{x^2 e + d}\right) - 2(42 d f g p x^6 + 5 d f^2 p x^4)e^2 + 2(35 d^2 g^2 p x^6 + 14 d^2 f g p x^4 + 3 d^2 f^2 p x^2)e + (35 d^3 g^2 p x^4 + 42 d^3 f g p x^2 + 15 d^3 f^2 p)\log(x^2 e + d) + (35 d^3 g^2 x^4 + 42 d^3 f g x^2 + 15 d^3 f^2)\log(c)}{105 d^3 x^7}, -\frac{30 f^2 p x^6 e^3 + \frac{2(35 d^2 g^2 p x^7 e - 42 d f g p x^7 e^2 + 15 f^2 p x^7 e^3)\arctan\left(\frac{x e^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\frac{1}{2}}}{\sqrt{d}} - 2(42 d f g p x^6 + 5 d f^2 p x^4)e^2 + 2(35 d^2 g^2 p x^6 + 14 d^2 f g p x^4 + 3 d^2 f^2 p x^2)e + (35 d^3 g^2 p x^4 + 42 d^3 f g p x^2 + 15 d^3 f^2 p)\log(x^2 e + d) + (35 d^3 g^2 x^4 + 42 d^3 f g x^2 + 15 d^3 f^2)\log(c)}{105 d^3 x^7}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^8,x, algorithm="fricas")
```

```
[Out] [-1/105*(30*f^2*p*x^6*e^3 - (35*d^2*g^2*p*x^7*e - 42*d*f*g*p*x^7*e^2 + 15*f
^2*p*x^7*e^3)*sqrt(-e/d)*log((x^2*e - 2*d*x*sqrt(-e/d) - d)/(x^2*e + d)) -
2*(42*d*f*g*p*x^6 + 5*d*f^2*p*x^4)*e^2 + 2*(35*d^2*g^2*p*x^6 + 14*d^2*f*g*p
*x^4 + 3*d^2*f^2*p*x^2)*e + (35*d^3*g^2*p*x^4 + 42*d^3*f*g*p*x^2 + 15*d^3*f
^2*p)*log(x^2*e + d) + (35*d^3*g^2*x^4 + 42*d^3*f*g*x^2 + 15*d^3*f^2)*log(c
))/(d^3*x^7), -1/105*(30*f^2*p*x^6*e^3 + 2*(35*d^2*g^2*p*x^7*e - 42*d*f*g*p
*x^7*e^2 + 15*f^2*p*x^7*e^3)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2)/sqrt(d) - 2*
(42*d*f*g*p*x^6 + 5*d*f^2*p*x^4)*e^2 + 2*(35*d^2*g^2*p*x^6 + 14*d^2*f*g*p*x
^4 + 3*d^2*f^2*p*x^2)*e + (35*d^3*g^2*p*x^4 + 42*d^3*f*g*p*x^2 + 15*d^3*f^2
*p)*log(x^2*e + d) + (35*d^3*g^2*x^4 + 42*d^3*f*g*x^2 + 15*d^3*f^2)*log(c))
/(d^3*x^7)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**8,x)
```

```
[Out] Timed out
```

Giac [A]
time = 6.53, size = 222, normalized size = 0.88

$$-\frac{2(35 d^2 g^2 p e^2 - 42 d f g p e^3 + 15 f^2 p e^4)\arctan\left(\frac{x e^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{1}{2}\right)}}{105 d^{\frac{7}{2}}} - \frac{70 d^2 g^2 p x^6 e - 84 d f g p x^6 e^2 + 35 d^3 g^2 p x^4 \log(x^2 e + d) + 30 f^2 p x^6 e^3 + 28 d^2 f g p x^4 e + 35 d^3 g^2 x^4 \log(c) - 10 d f^2 p x^4 e^2 + 42 d^3 f g p x^2 \log(x^2 e + d) + 6 d^2 f^2 p x^2 e + 42 d^3 f g x^2 \log(c) + 15 d^3 f^2 p \log(x^2 e + d) + 15 d^3 f^2 \log(c)}{105 d^3 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^8,x, algorithm="giac")
```

```
[Out] -2/105*(35*d^2*g^2*p*e^2 - 42*d*f*g*p*e^3 + 15*f^2*p*e^4)*arctan(x*e^(1/2)/
sqrt(d))*e^(-1/2)/d^(7/2) - 1/105*(70*d^2*g^2*p*x^6*e - 84*d*f*g*p*x^6*e^2
+ 35*d^3*g^2*p*x^4*log(x^2*e + d) + 30*f^2*p*x^6*e^3 + 28*d^2*f*g*p*x^4*e +
 35*d^3*g^2*x^4*log(c) - 10*d*f^2*p*x^4*e^2 + 42*d^3*f*g*p*x^2*log(x^2*e +
```

```
d) + 6*d^2*f^2*p*x^2*e + 42*d^3*f*g*x^2*log(c) + 15*d^3*f^2*p*log(x^2*e + d
) + 15*d^3*f^2*log(c))/(d^3*x^7)
```

Mupad [B]
time = 0.43, size = 149, normalized size = 0.59

$$-\frac{\frac{6\,e\,f^2\,p}{d}+\frac{2\,e\,p\,x^4\,\left(35\,d^2\,g^2-42\,d\,e\,f\,g+15\,e^2\,f^2\right)}{d^3}+\frac{2\,e\,f\,p\,x^2\,\left(14\,d\,g-5\,e\,f\right)}{d^2}}{105\,x^5}-\frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(\frac{f^2}{7}+\frac{2\,f\,g\,x^2}{5}+\frac{g^2\,x^4}{3}\right)}{x^7}-\frac{2\,e^{3/2}\,p\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(35\,d^2\,g^2-42\,d\,e\,f\,g+15\,e^2\,f^2\right)}{105\,d^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^8,x)
```

```
[Out] - ((6*e*f^2*p)/d + (2*e*p*x^4*(35*d^2*g^2 + 15*e^2*f^2 - 42*d*e*f*g))/d^3 +
 (2*e*f*p*x^2*(14*d*g - 5*e*f))/d^2)/(105*x^5) - (log(c*(d + e*x^2)^p)*(f^2
/7 + (g^2*x^4)/3 + (2*f*g*x^2)/5))/x^7 - (2*e^(3/2)*p*atan((e^(1/2)*x)/d^(1
/2))*(35*d^2*g^2 + 15*e^2*f^2 - 42*d*e*f*g))/(105*d^(7/2))
```

3.338 $\int \frac{x^5 \log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\, dx$

Optimal. Leaf size=188

$$\frac{fpx^2}{2g^2}+\frac{dpx^2}{4eg}-\frac{px^4}{8g}-\frac{d^2p\log\left(d+ex^2\right)}{4e^2g}+\frac{x^4\log\left(c\left(d+ex^2\right)^p\right)}{4g}-\frac{f\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{2eg^2}+\frac{f^2\log\left(c\left(d+ex^2\right)\right.}{2g}$$

```
[Out] 1/2*f*p*x^2/g^2+1/4*d*p*x^2/e/g-1/8*p*x^4/g-1/4*d^2*p*ln(e*x^2+d)/e^2/g+1/4
*x^4*ln(c*(e*x^2+d)^p)/g-1/2*f*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e/g^2+1/2*f^2*ln
(c*(e*x^2+d)^p)*ln(e*(g*x^2+f)/(-d*g+e*f))/g^3+1/2*f^2*p*polylog(2,-g*(e*x^
2+d)/(-d*g+e*f))/g^3
```

Rubi [A]
time = 0.19, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {2525, 45, 2463, 2436, 2332, 2442, 2441, 2440, 2438}

$$\frac{f^2 p \text{PolyLog}\left(2,-\frac{g(d+ex^2)}{ef-dg}\right)}{2g^3}+\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^3}-\frac{f\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{2eg^2}+\frac{x^4\log\left(c\left(d+ex^2\right)^p\right)}{4g}-\frac{d^2p\log\left(d+ex^2\right)}{4e^2g}+\frac{dpx^2}{4eg}+\frac{fpx^2}{2g^2}-\frac{px^4}{8g}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] (f*p*x^2)/(2*g^2) + (d*p*x^2)/(4*e*g) - (p*x^4)/(8*g) - (d^2*p*Log[d + e*x^
2])/(4*e^2*g) + (x^4*Log[c*(d + e*x^2)^p])/(4*g) - (f*(d + e*x^2)*Log[c*(d
+ e*x^2)^p])/(2*e*g^2) + (f^2*Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f
 - d*g)])/(2*g^3) + (f^2*p*PolyLog[2, -((g*(d + e*x^2))/(e*f - d*g))])/(2*g
^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{x^5 \log \left(c(d+ex^2)^p\right)}{f+gx^2} dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2 \log \left(c(d+ex)^p\right)}{f+gx} dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{f \log \left(c(d+ex)^p\right)}{g^2} + \frac{x \log \left(c(d+ex)^p\right)}{g} + \frac{f^2 \log \left(c(d+ex)^p\right)}{g^2(f+gx)}\right) d$$
$$= -\frac{f\text{Subst}\left(\int \log \left(c(d+ex)^p\right) dx, x, x^2\right)}{2g^2} + \frac{f^2\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{f+gx} dx, x, x^2\right)}{2g^2} + \frac{\text{Su}}{}$$
$$= \frac{x^4 \log \left(c(d+ex^2)^p\right)}{4g} + \frac{f^2 \log \left(c(d+ex^2)^p\right) \log \left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^3} - \frac{f\text{Subst}\left(\int \log (cx^p)\right.}{2eg}$$
$$= \frac{fpx^2}{2g^2} + \frac{x^4 \log \left(c(d+ex^2)^p\right)}{4g} - \frac{f(d+ex^2) \log \left(c(d+ex^2)^p\right)}{2eg^2} + \frac{f^2 \log \left(c(d+ex^2\right.}{2}$$
$$= \frac{fpx^2}{2g^2} + \frac{dpx^2}{4eg} - \frac{px^4}{8g} - \frac{d^2p \log (d+ex^2)}{4e^2g} + \frac{x^4 \log \left(c(d+ex^2)^p\right)}{4g} - \frac{f(d+ex^2) \log}{2}$$

Mathematica [A]
time = 0.09, size = 143, normalized size = 0.76

$$\frac{egpx^2(4ef+2dg-egx^2) - 2d^2g^2p \log (d+ex^2) + e \log \left(c(d+ex^2)^p\right) \left(2g(-2df-2efx^2+egx^4) + 4ef^2 \log \left(\frac{e(f+gx^2)}{ef-dg}\right)\right) + 4e^2f^2p\text{Li}_2\left(\frac{g(d+ex^2)}{-ef+dg}\right)}{8e^2g^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] (e*g*p*x^2*(4*e*f + 2*d*g - e*g*x^2) - 2*d^2*g^2*p*Log[d + e*x^2] + e*Log[c
*(d + e*x^2)^p]*(2*g*(-2*d*f - 2*e*f*x^2 + e*g*x^4) + 4*e*f^2*Log[(e*(f + g
*x^2))/(e*f - d*g)]) + 4*e^2*f^2*p*PolyLog[2, (g*(d + e*x^2))/(-(e*f) + d*g
)])/(8*e^2*g^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.80, size = 902, normalized size = 4.80

method	result	size
risch	Expression too large to display	902

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*ln(c*(e*x^2+d)^p)/(g*x^2+f),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*ln((e*x^2+d)^p)/g*x^4-1/2*ln((e*x^2+d)^p)/g^2*f*x^2+1/2*ln((e*x^2+d)^p)
*f^2/g^3*ln(g*x^2+f)-1/2*p*f^2/g^3*sum(ln(x-_alpha)*ln(g*x^2+f)-ln(x-_alpha
```

```
)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(_Z
^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*
g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=
2)))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/Root
Of(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z*_
alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*
f,index=2)),_alpha=RootOf(_Z^2*e+d))-1/8*p*x^4/g+1/4*d*p*x^2/e/g+1/2*f*p*x^
2/g^2-1/4*d^2*p*ln(e*x^2+d)/e^2/g-1/2*p/e/g^2*d*ln(e*x^2+d)*f-1/8*I*Pi*csgn
(I*c*(e*x^2+d)^p)^3/g*x^4-1/8*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p
)*csgn(I*c)/g*x^4+1/8*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/g*x^4+1/4*I*Pi
*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*f^2/g^3*ln(g*x^2+f)+1/4*I*Pi*csgn(I*c*(e
*x^2+d)^p)^3/g^2*f*x^2+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2
*f^2/g^3*ln(g*x^2+f)-1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csg
n(I*c)*f^2/g^3*ln(g*x^2+f)+1/8*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^
p)^2/g*x^4-1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/g^2*f*x^2-1/4*I*Pi*cs
gn(I*c*(e*x^2+d)^p)^3*f^2/g^3*ln(g*x^2+f)-1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn
(I*c*(e*x^2+d)^p)^2/g^2*f*x^2+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+
d)^p)*csgn(I*c)/g^2*f*x^2+1/4*ln(c)/g*x^4-1/2*ln(c)/g^2*f*x^2+1/2*ln(c)*f^2
/g^3*ln(g*x^2+f)
```

Maxima [A]
time = 0.58, size = 181, normalized size = 0.96

$$\frac{\left(\log\left(x^2e+d\right)\log\left(-\frac{gx^2e+dg}{dg-fe}+1\right)+\mathrm{Li}_2\left(\frac{gx^2e+dg}{dg-fe}\right)\right)f^2p}{2\,g^3}+\frac{f^2\log\left(gx^2+f\right)\log\left(c\right)}{2\,g^3}-\frac{\left(\left(gp-2\,g\log\left(c\right)\right)x^4e^2-2\left(dgpe+2\left(fp-f\log\left(c\right)\right)e^2\right)x^2-2\left(gpx^4e^2-2\,fpx^2e^2-d^2gp-2\,dfpe\right)\log\left(x^2e+d\right)\right)e^{(-2)}}{8\,g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] 1/2*(log(x^2*e + d)*log(-(g*x^2*e + d*g)/(d*g - f*e) + 1) + dilog((g*x^2*e
+ d*g)/(d*g - f*e)))*f^2*p/g^3 + 1/2*f^2*log(g*x^2 + f)*log(c)/g^3 - 1/8*((
g*p - 2*g*log(c))*x^4*e^2 - 2*(d*g*p*e + 2*(f*p - f*log(c))*e^2)*x^2 - 2*(g
*p*x^4*e^2 - 2*f*p*x^2*e^2 - d^2*g*p - 2*d*f*p*e)*log(x^2*e + d))*e^(-2)/g^
2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(x^5*log((x^2*e + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
```

```
[Out] Integral(x**5*log(c*(d + e*x**2)**p)/(f + g*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(x^5*log((x^2*e + d)^p*c)/(g*x^2 + f), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*log(c*(d + e*x^2)^p))/(f + g*x^2),x)
```

```
[Out] int((x^5*log(c*(d + e*x^2)^p))/(f + g*x^2), x)
```

3.339 $\int \frac{x^3 \log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$

Optimal. Leaf size=112

$$-\frac{px^2}{2g}+\frac{(d+ex^2)\log\left(c(d+ex^2)^p\right)}{2eg}-\frac{f\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^2}-\frac{fp\text{Li}_2\left(-\frac{g(d+ex^2)}{ef-dg}\right)}{2g^2}$$

```
[Out] -1/2*p*x^2/g+1/2*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e/g-1/2*f*ln(c*(e*x^2+d)^p)*ln
(e*(g*x^2+f)/(-d*g+e*f))/g^2-1/2*f*p*polylog(2,-g*(e*x^2+d)/(-d*g+e*f))/g^2
```

Rubi [A]
time = 0.13, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2525, 45, 2463, 2436, 2332, 2441, 2440, 2438}

$$-\frac{fp\text{PolyLog}\left(2,-\frac{g(d+ex^2)}{ef-dg}\right)}{2g^2}-\frac{f\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^2}+\frac{(d+ex^2)\log\left(c(d+ex^2)^p\right)}{2eg}-\frac{px^2}{2g}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] -1/2*(p*x^2)/g + ((d + e*x^2)*Log[c*(d + e*x^2)^p])/(2*e*g) - (f*Log[c*(d +
 e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)])/(2*g^2) - (f*p*PolyLog[2, -((g
*(d + e*x^2))/(e*f - d*g))])/(2*g^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{x^3 \log(c(d+ex^2)^p)}{f+gx^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x\log(c(d+ex)^p)}{f+gx}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{\log(c(d+ex)^p)}{g} - \frac{f\log(c(d+ex)^p)}{g(f+gx)}\right)\,dx, x, x^2\right)$$
$$= \frac{\text{Subst}\left(\int \log(c(d+ex)^p)\,dx, x, x^2\right)}{2g} - \frac{f\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{f+gx}\,dx, x, x^2\right)}{2g}$$
$$= -\frac{f\log(c(d+ex^2)^p)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^2} + \frac{\text{Subst}\left(\int \log(cx^p)\,dx, x, d+ex^2\right)}{2eg} + \frac{(ef}{}$$
$$= -\frac{px^2}{2g} + \frac{(d+ex^2)\log(c(d+ex^2)^p)}{2eg} - \frac{f\log(c(d+ex^2)^p)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^2} + \frac{(f}{}$$
$$= -\frac{px^2}{2g} + \frac{(d+ex^2)\log(c(d+ex^2)^p)}{2eg} - \frac{f\log(c(d+ex^2)^p)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^2} - \frac{fp}{}$$

Mathematica [A]
time = 0.03, size = 91, normalized size = 0.81

$$-\frac{egpx^2 - \log(c(d+ex^2)^p)\left(dg+egx^2 - ef\log\left(\frac{e(f+gx^2)}{ef-dg}\right)\right) + efp\text{Li}_2\left(\frac{g(d+ex^2)}{-ef+dg}\right)}{2eg^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] -1/2*(e*g*p*x^2 - Log[c*(d + e*x^2)^p]*(d*g + e*g*x^2 - e*f*Log[(e*(f + g*x
^2))/(e*f - d*g)]) + e*f*p*PolyLog[2, (g*(d + e*x^2))/(-(e*f) + d*g)])/(e*g
^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.42, size = 672, normalized size = 6.00

method	result
risch	$\frac{\ln((e\,x^2+d)^p)x^2}{2g} - \frac{\ln((e\,x^2+d)^p)f\ln(g\,x^2+f)}{2g^2} - \frac{p\,x^2}{2g} + \frac{pd\ln(e\,x^2+d)}{2eg} + \frac{pf\left(\sum_{\_\alpha=\text{RootOf}(e\_Z^2+d)}\left(\ln(x-\_\alpha)\ln(g\,x^2+f)\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(e*x^2+d)^p)/(g*x^2+f),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln((e*x^2+d)^p)/g*x^2-1/2*ln((e*x^2+d)^p)*f/g^2*ln(g*x^2+f)-1/2*p*x^2/g
+1/2*p/e/g*d*ln(e*x^2+d)+1/2*p*f/g^2*sum(ln(x-_alpha)*ln(g*x^2+f)-ln(x-_alp
ha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(
_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*
e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,inde
x=2)))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/Ro
otOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z
*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+
e*f,index=2)),_alpha=RootOf(_Z^2*e+d))+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*
c*(e*x^2+d)^p)^2/g*x^2-1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2
*f/g^2*ln(g*x^2+f)-1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(
I*c)/g*x^2+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*f/g
^2*ln(g*x^2+f)-1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/g*x^2+1/4*I*Pi*csgn(I*c*(e*
x^2+d)^p)^3*f/g^2*ln(g*x^2+f)+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/g*
x^2-1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*f/g^2*ln(g*x^2+f)+1/2*ln(c)/
g*x^2-1/2*ln(c)*f/g^2*ln(g*x^2+f)
```

Maxima [A]
time = 0.56, size = 128, normalized size = 1.14

$$-\frac{\left(\log\left(x^2e+d\right)\log\left(-\frac{gx^2e+dg}{dg-fe}+1\right)+\mathrm{Li}_2\left(\frac{gx^2e+dg}{dg-fe}\right)\right)fp}{2\,g^2}-\frac{\left((p-\log\left(c\right))x^2e-\left(px^2e+dp\right)\log\left(x^2e+d\right)\right)e^{(-1)}}{2\,g}-\frac{f\log\left(gx^2+f\right)\log\left(c\right)}{2\,g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] -1/2*(log(x^2*e + d)*log(-(g*x^2*e + d*g)/(d*g - f*e) + 1) + dilog((g*x^2*e
 + d*g)/(d*g - f*e)))*f*p/g^2 - 1/2*((p - log(c))*x^2*e - (p*x^2*e + d*p)*l
og(x^2*e + d))*e^(-1)/g - 1/2*f*log(g*x^2 + f)*log(c)/g^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(x^3*log((x^2*e + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
```

```
[Out] Integral(x**3*log(c*(d + e*x**2)**p)/(f + g*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(x^3*log((x^2*e + d)^p*c)/(g*x^2 + f), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*log(c*(d + e*x^2)^p))/(f + g*x^2),x)
```

```
[Out] int((x^3*log(c*(d + e*x^2)^p))/(f + g*x^2), x)
```

3.340 $\int \frac{x \log \left(c\left(d+ex^2\right)^p\right)}{f+gx^2} \, dx$

Optimal. Leaf size=70

$$\frac{\log \left(c(d+ex^2)^p\right) \log \left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g}+\frac{p\text{Li}_2\left(-\frac{g(d+ex^2)}{ef-dg}\right)}{2g}$$

```
[Out] 1/2*ln(c*(e*x^2+d)^p)*ln(e*(g*x^2+f)/(-d*g+e*f))/g+1/2*p*polylog(2,-g*(e*x^
2+d)/(-d*g+e*f))/g
```

Rubi [A]
time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2525, 2441, 2440, 2438}

$$\frac{p\text{PolyLog}\left(2,-\frac{g(d+ex^2)}{ef-dg}\right)}{2g}+\frac{\log \left(c(d+ex^2)^p\right) \log \left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] (Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)])/(2*g) + (p*PolyLog[
2, -((g*(d + e*x^2))/(e*f - d*g))])/(2*g)
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{f+gx}\,dx, x, x^2\right)\\
&= \frac{\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g} - \frac{(ep)\text{Subst}\left(\int \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx, x, x^2\right)}{2g}\\
&= \frac{\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g} - \frac{p\text{Subst}\left(\int \frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\,dx, x, d+ex^2\right)}{2g}\\
&= \frac{\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g} + \frac{p\text{Li}_2\left(-\frac{g(d+ex^2)}{ef-dg}\right)}{2g}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 64, normalized size = 0.91

$$\frac{\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)+p\text{Li}_2\left(\frac{g(d+ex^2)}{-ef+dg}\right)}{2g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
[Out] (Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)] + p*PolyLog[2, (g*(d
 + e*x^2))/(-(e*f) + d*g)])/(2*g)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.37, size = 472, normalized size = 6.74

method	result
risch	$\frac{\ln(g\,x^2+f)\ln\left((e\,x^2+d)^p\right)}{2g} - \frac{p\left(\sum_{\_\alpha=\text{RootOf}\left(e\_Z^2+d\right)}\left(\ln(x-\_\alpha)\ln(g\,x^2+f)-\ln(x-\_\alpha)\left(\ln\left(\frac{\text{RootOf}\left(e\_Z^2 g+2\_Z\_\alpha ge-dg+\right.}{\text{RootOf}\left(e\_Z^2 g+2\_Z\_\alpha ge-\right.}\right.\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(e*x^2+d)^p)/(g*x^2+f),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/g*ln(g*x^2+f)*ln((e*x^2+d)^p)-1/2/g*p*sum(ln(x-_alpha)*ln(g*x^2+f)-ln(x
-_alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/Ro
otOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_a
lpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f
,index=2)))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alph
a)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((RootOf(_Z^2*e*g
+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g
-d*g+e*f,index=2)),_alpha=RootOf(_Z^2*e+d))+1/4*I/g*ln(g*x^2+f)*Pi*csgn(I*(
e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/4*I/g*ln(g*x^2+f)*Pi*csgn(I*(e*x^2+d)
^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/4*I/g*ln(g*x^2+f)*Pi*csgn(I*c*(e*x^2+
d)^p)^3+1/4*I/g*ln(g*x^2+f)*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+1/2/g*ln(g
*x^2+f)*ln(c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(70) = 140$.
time = 0.32, size = 146, normalized size = 2.09

$$\frac{\left(e^{(-1)}\log\left(gx^2+f\right)\log\left(x^2e+d\right)-\left(\log\left(gx^2+f\right)\log\left(\frac{gx^2e+fe}{dg-fe}+1\right)+\mathrm{Li}_2\left(-\frac{gx^2e+fe}{dg-fe}\right)\right)e^{(-1)}\right)pe}{2\,g}-\frac{p\log\left(gx^2+f\right)\log\left(x^2e+d\right)}{2\,g}+\frac{\log\left(gx^2+f\right)\log\left(\left(x^2e+d\right)^pc\right)}{2\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(-1)*log(g*x^2 + f)*log(x^2*e + d) - (log(g*x^2 + f)*log((g*x^2*e +
f*e)/(d*g - f*e) + 1) + dilog(-(g*x^2*e + f*e)/(d*g - f*e)))*e^(-1))*p*e/g
- 1/2*p*log(g*x^2 + f)*log(x^2*e + d)/g + 1/2*log(g*x^2 + f)*log((x^2*e + d
)^p*c)/g
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(x*log((x^2*e + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
```

```
[Out] Integral(x*log(c*(d + e*x**2)**p)/(f + g*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(x*log((x^2*e + d)^p*c)/(g*x^2 + f), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(c*(d + e*x^2)^p))/(f + g*x^2),x)
```

```
[Out] int((x*log(c*(d + e*x^2)^p))/(f + g*x^2), x)
```

3.341 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x\left(f+gx^2\right)}\,dx$

Optimal. Leaf size=119

$$\frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f}-\frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2f}-\frac{p\text{Li}_2\left(-\frac{g\left(d+ex^2\right)}{ef-dg}\right)}{2f}+\frac{p\text{Li}_2\left(1+\frac{ex^2}{d}\right)}{2f}$$

```
[Out] 1/2*ln(-e*x^2/d)*ln(c*(e*x^2+d)^p)/f-1/2*ln(c*(e*x^2+d)^p)*ln(e*(g*x^2+f)/(
-d*g+e*f))/f-1/2*p*polylog(2,-g*(e*x^2+d)/(-d*g+e*f))/f+1/2*p*polylog(2,1+e
*x^2/d)/f
```

Rubi [A]
time = 0.14, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {2525, 36, 29, 31, 2463, 2441, 2352, 2440, 2438}

$$-\frac{p\text{PolyLog}\left(2,-\frac{g\left(d+ex^2\right)}{ef-dg}\right)}{2f}+\frac{p\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)}{2f}-\frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2f}+\frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(x*(f + g*x^2)),x]
```

```
[Out] (Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])/(2*f) - (Log[c*(d + e*x^2)^p]*Log[
(e*(f + g*x^2))/(e*f - d*g)])/(2*f) - (p*PolyLog[2, -((g*(d + e*x^2))/(e*f
- d*g))])/(2*f) + (p*PolyLog[2, 1 + (e*x^2)/d])/(2*f)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\log \left(c \left(d+e x^2\right)^p\right)}{x \left(f+g x^2\right)} \, dx = \frac{1}{2}\text{Subst}\left(\int \frac{\log \left(c (d+e x)^p\right)}{x (f+g x)} \, dx,x,x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{\log \left(c (d+e x)^p\right)}{f x}-\frac{g \log \left(c (d+e x)^p\right)}{f (f+g x)}\right) \, dx,x,x^2\right)$$
$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x} \, dx,x,x^2\right)}{2 f}-\frac{g\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{f+gx} \, dx,x,x^2\right)}{2 f}$$
$$= \frac{\log \left(-\frac{e x^2}{d}\right) \log \left(c \left(d+e x^2\right)^p\right)}{2 f}-\frac{\log \left(c \left(d+e x^2\right)^p\right) \log \left(\frac{e \left(f+g x^2\right)}{e f-d g}\right)}{2 f}-\frac{(e p)\text{Subst}\left(\int \text{lo}\right.}{2}$$
$$= \frac{\log \left(-\frac{e x^2}{d}\right) \log \left(c \left(d+e x^2\right)^p\right)}{2 f}-\frac{\log \left(c \left(d+e x^2\right)^p\right) \log \left(\frac{e \left(f+g x^2\right)}{e f-d g}\right)}{2 f}+\frac{p\text{Li}_2\left(1+\frac{e x^2}{d}\right)}{2 f}$$
$$= \frac{\log \left(-\frac{e x^2}{d}\right) \log \left(c \left(d+e x^2\right)^p\right)}{2 f}-\frac{\log \left(c \left(d+e x^2\right)^p\right) \log \left(\frac{e \left(f+g x^2\right)}{e f-d g}\right)}{2 f}-\frac{p\text{Li}_2\left(-\frac{g \left(d+e x^2\right)}{e f-d g}\right.}{2 f}$$

Mathematica [A]
time = 0.03, size = 92, normalized size = 0.77

$$\frac{\log \left(c \left(d+e x^2\right)^p\right) \left(\log \left(-\frac{e x^2}{d}\right)-\log \left(\frac{e \left(f+g x^2\right)}{e f-d g}\right)\right)-p\text{Li}_2\left(\frac{g \left(d+e x^2\right)}{-e f+d g}\right)+p\text{Li}_2\left(1+\frac{e x^2}{d}\right)}{2 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(x*(f + g*x^2)),x]
```

```
[Out] (Log[c*(d + e*x^2)^p]*(Log[-((e*x^2)/d)] - Log[(e*(f + g*x^2))/(e*f - d*g)]
) - p*PolyLog[2, (g*(d + e*x^2))/(-(e*f) + d*g)] + p*PolyLog[2, 1 + (e*x^2)
/d])/(2*f)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.50, size = 732, normalized size = 6.15

method	result	size
risch	Expression too large to display	732

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/x/(g*x^2+f),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln((e*x^2+d)^p)/f*ln(g*x^2+f)+ln((e*x^2+d)^p)/f*ln(x)-p/f*ln(x)*ln((-e
*x+(-e*d)^(1/2))/(-e*d)^(1/2))-p/f*ln(x)*ln((e*x+(-e*d)^(1/2))/(-e*d)^(1/2)
```

```
)-p/f*dilog((-e*x+(-e*d)^(1/2))/(-e*d)^(1/2))-p/f*dilog((e*x+(-e*d)^(1/2))/
(-e*d)^(1/2))+1/2*p/f*sum(ln(x-_alpha)*ln(g*x^2+f)-ln(x-_alpha)*(ln((RootOf
(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_
alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,ind
ex=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)))-dilog((R
ootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2
*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g
+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)),_
alpha=RootOf(_Z^2*e+d))+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/f*ln(x)-
1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/f*ln(g*x^2+f)+1/2*I*Pi
*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/f*ln(x)-1/4*I*Pi*csgn(I*c*(e*x
^2+d)^p)^2*csgn(I*c)/f*ln(g*x^2+f)-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e
*x^2+d)^p)*csgn(I*c)/f*ln(x)+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/f*ln(g*x^2+f)
+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/f*ln(g*x^2+f)
-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/f*ln(x)-1/2*ln(c)/f*ln(g*x^2+f)+1/f*ln(c)
*ln(x)
```

Maxima [A]
time = 0.56, size = 148, normalized size = 1.24

$$-\frac{1}{2}p\left(\frac{\left(2\log\left(\frac{x^2e}{d}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{x^2e}{d}\right)\right)e^{(-1)}}{f}-\frac{\left(\log\left(gx^2+f\right)\log\left(\frac{gx^2e+fe}{dg-fe}+1\right)+\mathrm{Li}_2\left(-\frac{gx^2e+fe}{dg-fe}\right)\right)e^{(-1)}}{f}\right)e-\frac{1}{2}\left(\frac{\log\left(gx^2+f\right)}{f}-\frac{\log\left(x^2\right)}{f}\right)\log\left(\left(x^2e+d\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] -1/2*p*((2*log(x^2*e/d + 1)*log(x) + dilog(-x^2*e/d))*e^(-1)/f - (log(g*x^2
 + f)*log((g*x^2*e + f*e)/(d*g - f*e) + 1) + dilog(-(g*x^2*e + f*e)/(d*g -
f*e)))*e^(-1)/f)*e - 1/2*(log(g*x^2 + f)/f - log(x^2)/f)*log((x^2*e + d)^p*
c)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log((x^2*e + d)^p*c)/(g*x^3 + f*x), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/x/(g*x**2+f),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x/(g*x^2+f),x, algorithm="giac")
[Out] integrate(log((x^2*e + d)^p*c)/((g*x^2 + f)*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)}{x\,\left(g\,x^2+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)/(x*(f + g*x^2)),x)
[Out] int(log(c*(d + e*x^2)^p)/(x*(f + g*x^2)), x)
```

3.342 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^3\left(f+gx^2\right)}\,dx$

Optimal. Leaf size=176

$$\frac{ep\log(x)}{df}-\frac{ep\log\left(d+ex^2\right)}{2df}-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{2fx^2}-\frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^2}+\frac{g\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e(\ldots}{e\ldots}\right.}{2f^2}$$

```
[Out] e*p*ln(x)/d/f-1/2*e*p*ln(e*x^2+d)/d/f-1/2*ln(c*(e*x^2+d)^p)/f/x^2-1/2*g*ln(
-e*x^2/d)*ln(c*(e*x^2+d)^p)/f^2+1/2*g*ln(c*(e*x^2+d)^p)*ln(e*(g*x^2+f)/(-d*
g+e*f))/f^2+1/2*g*p*polylog(2,-g*(e*x^2+d)/(-d*g+e*f))/f^2-1/2*g*p*polylog(
2,1+e*x^2/d)/f^2
```

Rubi [A]

time = 0.17, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {2525, 46, 2463, 2442, 36, 29, 31, 2441, 2352, 2440, 2438}

$$\frac{gp\,\text{PolyLog}\left(2,-\frac{g\left(d+ex^2\right)}{ef-dg}\right)}{2f^2}-\frac{gp\,\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)}{2f^2}-\frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^2}+\frac{g\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{2f^2}-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{2fx^2}-\frac{ep\log\left(d+ex^2\right)}{2df}+\frac{ep\log(x)}{df}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(x^3*(f + g*x^2)),x]
```

```
[Out] (e*p*Log[x])/(d*f) - (e*p*Log[d + e*x^2])/(2*d*f) - Log[c*(d + e*x^2)^p]/(2
*f*x^2) - (g*Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])/(2*f^2) + (g*Log[c*(d
+ e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)])/(2*f^2) + (g*p*PolyLog[2, -((
g*(d + e*x^2))/(e*f - d*g))])/(2*f^2) - (g*p*PolyLog[2, 1 + (e*x^2)/d])/(2*
f^2)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^3\left(f+gx^2\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x^2(f+gx)}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{\log\left(c(d+ex)^p\right)}{fx^2} - \frac{g\log\left(c(d+ex)^p\right)}{f^2x} + \frac{g^2\log\left(c(d+ex)^p\right)}{f^2(f+gx)}\right)\,dx, x,$$

$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x^2}\,dx, x, x^2\right)}{2f} - \frac{g\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^2\right)}{2f^2} + \frac{g^2\text{Subst}\left(\int \right.}{}$$

$$= -\frac{\log\left(c(d+ex^2)^p\right)}{2fx^2} - \frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)}{2f^2} + \frac{g\log\left(c(d+ex^2)^p\right)\log\left(\right.}{2f^2}$$

$$= -\frac{\log\left(c(d+ex^2)^p\right)}{2fx^2} - \frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)}{2f^2} + \frac{g\log\left(c(d+ex^2)^p\right)\log\left(\right.}{2f^2}$$

$$= \frac{ep\log(x)}{df} - \frac{ep\log\left(d+ex^2\right)}{2df} - \frac{\log\left(c(d+ex^2)^p\right)}{2fx^2} - \frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right.}{2f^2}$$

Mathematica [A]
time = 0.05, size = 147, normalized size = 0.84

$$\frac{\frac{efp(2\log(x)-\log(d+ex^2))}{d} - \frac{f\log(c(d+ex^2)^p)}{x^2} + g\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right) + gp\text{Li}_2\left(\frac{g(d+ex^2)}{-ef+dg}\right) - g\left(\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right) + p\text{Li}_2\left(1+\frac{ex^2}{d}\right)\right)}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(x^3*(f + g*x^2)),x]
```

```
[Out] ((e*f*p*(2*Log[x] - Log[d + e*x^2]))/d - (f*Log[c*(d + e*x^2)^p])/x^2 + g*L
og[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)] + g*p*PolyLog[2, (g*(d
 + e*x^2))/(-(e*f) + d*g)] - g*(Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p] + p*
PolyLog[2, 1 + (e*x^2)/d]))/(2*f^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.45, size = 942, normalized size = 5.35

method	result	size
risch	Expression too large to display	942

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/x^3/(g*x^2+f),x,method=_RETURNVERBOSE)
```

```
[Out] e*p*ln(x)/d/f-1/2*e*p*ln(e*x^2+d)/d/f-1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c
*(e*x^2+d)^p)^2/f/x^2-1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*g/f^2*ln(g*x^2+f)+1/
2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/f^2*g*ln(x)-1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2
*csgn(I*c)/f/x^2+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/f/x^2+p/f^2*g*ln(x)*ln((-
e*x+(-e*d)^(1/2))/(-e*d)^(1/2))+p/f^2*g*ln(x)*ln((e*x+(-e*d)^(1/2))/(-e*d)^
(1/2))-1/2*ln((e*x^2+d)^p)/f/x^2+1/2*ln((e*x^2+d)^p)*g/f^2*ln(g*x^2+f)-ln((
e*x^2+d)^p)/f^2*g*ln(x)+p/f^2*g*dilog((-e*x+(-e*d)^(1/2))/(-e*d)^(1/2))+p/f
^2*g*dilog((e*x+(-e*d)^(1/2))/(-e*d)^(1/2))+1/2*ln(c)*g/f^2*ln(g*x^2+f)-ln(
c)/f^2*g*ln(x)-1/2*p/f^2*g*sum(ln(x-_alpha)*ln(g*x^2+f)-ln(x-_alpha)*(ln((R
ootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2
*_Z*_alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*
f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)))-dil
og((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(_Z^2*
e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*
g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=
2)),_alpha=RootOf(_Z^2*e+d))-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/f^2
*g*ln(x)+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*g/f^2*ln(g*x^
2+f)-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/f^2*g*ln(x)+1/4*I
*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/f/x^2+1/4*I*Pi*csgn
(I*c*(e*x^2+d)^p)^2*csgn(I*c)*g/f^2*ln(g*x^2+f)-1/2*ln(c)/f/x^2+1/2*I*Pi*cs
gn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/f^2*g*ln(x)-1/4*I*Pi*csgn
(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*g/f^2*ln(g*x^2+f)
```

Maxima [A]
time = 0.58, size = 187, normalized size = 1.06

$$\frac{1}{2}p\left(\frac{\left(2\log\left(\frac{x^2e}{d}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{x^2e}{d}\right)\right)ge^{(-1)}}{f^2}-\frac{\left(\log(gx^2+f)\log\left(\frac{gx^2e+fe}{dg-fe}+1\right)+\mathrm{Li}_2\left(-\frac{gx^2e+fe}{dg-fe}\right)\right)ge^{(-1)}}{f^2}-\frac{\log(x^2e+d)}{df}+\frac{2\log(x)}{df}\right)e+\frac{1}{2}\left(\frac{g\log(gx^2+f)}{f^2}-\frac{g\log(x^2)}{f^2}-\frac{1}{fx^2}\right)\log\left((x^2e+d)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^3/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] 1/2*p*((2*log(x^2*e/d + 1)*log(x) + dilog(-x^2*e/d))*g*e^(-1)/f^2 - (log(g*
x^2 + f)*log((g*x^2*e + f*e)/(d*g - f*e) + 1) + dilog(-(g*x^2*e + f*e)/(d*g
 - f*e)))*g*e^(-1)/f^2 - log(x^2*e + d)/(d*f) + 2*log(x)/(d*f))*e + 1/2*(g*
log(g*x^2 + f)/f^2 - g*log(x^2)/f^2 - 1/(f*x^2))*log((x^2*e + d)^p*c)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^3/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log((x^2*e + d)^p*c)/(g*x^5 + f*x^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/x**3/(g*x**2+f),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^3/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(log((x^2*e + d)^p*c)/((g*x^2 + f)*x^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{x^3\,\left(g\,x^2+f\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)/(x^3*(f + g*x^2)),x)
```

```
[Out] int(log(c*(d + e*x^2)^p)/(x^3*(f + g*x^2)), x)
```

3.343 $\int \frac{x^4 \log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$

Optimal. Leaf size=667

$$\frac{2fpx}{g^2}+\frac{2dpx}{3eg}-\frac{2px^3}{9g}-\frac{2\sqrt{d}\,fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,g^2}-\frac{2d^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3e^{3/2}g}+\frac{2f^{3/2}p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{g^{5/2}}$$

```
[Out] 2*f*p*x/g^2+2/3*d*p*x/e/g-2/9*p*x^3/g-2/3*d^(3/2)*p*arctan(x*e^(1/2)/d^(1/2
))/e^(3/2)/g-f*x*ln(c*(e*x^2+d)^p)/g^2+1/3*x^3*ln(c*(e*x^2+d)^p)/g+f^(3/2)*
arctan(x*g^(1/2)/f^(1/2))*ln(c*(e*x^2+d)^p)/g^(5/2)+2*f^(3/2)*p*arctan(x*g^
(1/2)/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/g^(5/2)-f^(3/2)*p*arctan
(x*g^(1/2)/f^(1/2))*ln(-2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I
*x*g^(1/2))/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2)))/g^(5/2)-f^(3/2)*p*arcta
n(x*g^(1/2)/f^(1/2))*ln(2*((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I
*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))/g^(5/2)-I*f^(3/2)*p*pol
ylog(2,1-2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/g^(5/2)+1/2*I*f^(3/2)*p*polylog(2
,1+2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2
)*f^(1/2)-(-d)^(1/2)*g^(1/2)))/g^(5/2)+1/2*I*f^(3/2)*p*polylog(2,1-2*((-d)^
(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(
-d)^(1/2)*g^(1/2)))/g^(5/2)-2*f*p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/g^2/e^(
1/2)
```

Rubi [A]

time = 0.49, antiderivative size = 667, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {2526, 2498, 327, 211, 2505, 308, 2520, 12, 5048, 4966, 2449, 2352, 2497}

$$\frac{if^{3/2}p\,\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2g^{5/2}}+\frac{if^{3/2}p\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2g^{5/2}}-\frac{if^{3/2}p\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{g^{5/2}}+\frac{f^{3/2}\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{g^{5/2}}-\frac{2d^{3/2}p\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3e^{3/2}g}-\frac{f^{3/2}p\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{g^{5/2}}-\frac{f^{3/2}p\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{g^{5/2}}-\frac{2\sqrt{d}\,fp\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,g^2}+\frac{2f^{3/2}p\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{g^{5/2}}-\frac{fx\log\left(c(d+ex^2)^p\right)}{g^2}+\frac{x^3\log\left(c(d+ex^2)^p\right)}{3g}+\frac{2dpx}{3eg}+\frac{2fpx}{g^2}-\frac{2px^3}{9g}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] (2*f*p*x)/g^2 + (2*d*p*x)/(3*e*g) - (2*p*x^3)/(9*g) - (2*Sqrt[d]*f*p*ArcTan
[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*g^2) - (2*d^(3/2)*p*ArcTan[(Sqrt[e]*x)/Sqrt
[d]])/(3*e^(3/2)*g) + (2*f^(3/2)*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[
f])/(Sqrt[f] - I*Sqrt[g]*x)])/g^(5/2) - (f^(3/2)*p*ArcTan[(Sqrt[g]*x)/Sqrt[
f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - S
qrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2) - (f^(3/2)*p*ArcTan[(Sq
rt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e
]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2) - (f*x*Log
[c*(d + e*x^2)^p])/g^2 + (x^3*Log[c*(d + e*x^2)^p])/(3*g) + (f^(3/2)*ArcTan
[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/g^(5/2) - (I*f^(3/2)*p*PolyLog[
```

```
2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/g^(5/2) + ((I/2)*f^(3/2)*p*Pol
yLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f]
- Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2) + ((I/2)*f^(3/2)*p*P
olyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f
] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\int \frac{x^4 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \left(-\frac{f \log \left(c(d+ex^2)^p\right)}{g^2} + \frac{x^2 \log \left(c(d+ex^2)^p\right)}{g} + \frac{f^2 \log \left(c(d+ex^2)^p\right)}{g^2\,(f+gx^2)}\right)\,dx$$

$$= -\frac{f \int \log \left(c(d+ex^2)^p\right)\,dx}{g^2} + \frac{f^2 \int \frac{\log(c(d+ex^2)^p)}{f+gx^2}\,dx}{g^2} + \frac{\int x^2 \log \left(c(d+ex^2)^p\right)\,dx}{g}$$

$$= -\frac{fx \log \left(c(d+ex^2)^p\right)}{g^2} + \frac{x^3 \log \left(c(d+ex^2)^p\right)}{3g} + \frac{f^{3/2} \tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right) \log \left(c(d+\right.}{g^{5/2}}$$

$$= \frac{2fpx}{g^2} - \frac{fx \log \left(c(d+ex^2)^p\right)}{g^2} + \frac{x^3 \log \left(c(d+ex^2)^p\right)}{3g} + \frac{f^{3/2} \tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right) \log}{g^{5/2}}$$

$$= \frac{2fpx}{g^2} + \frac{2dpx}{3eg} - \frac{2px^3}{9g} - \frac{2\sqrt{d}\,fp \tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,g^2} - \frac{fx \log \left(c(d+ex^2)^p\right)}{g^2} + \frac{x^3}{}$$

$$= \frac{2fpx}{g^2} + \frac{2dpx}{3eg} - \frac{2px^3}{9g} - \frac{2\sqrt{d}\,fp \tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,g^2} - \frac{2d^{3/2}p \tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3e^{3/2}g} - \frac{f}{}$$

$$= \frac{2fpx}{g^2} + \frac{2dpx}{3eg} - \frac{2px^3}{9g} - \frac{2\sqrt{d}\,fp \tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,g^2} - \frac{2d^{3/2}p \tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3e^{3/2}g} + \frac{2}{}$$

$$= \frac{2fpx}{g^2} + \frac{2dpx}{3eg} - \frac{2px^3}{9g} - \frac{2\sqrt{d}\,fp \tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,g^2} - \frac{2d^{3/2}p \tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3e^{3/2}g} + \frac{2}{}$$

$$= \frac{2fpx}{g^2} + \frac{2dpx}{3eg} - \frac{2px^3}{9g} - \frac{2\sqrt{d}\,fp \tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,g^2} - \frac{2d^{3/2}p \tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3e^{3/2}g} + \frac{2}{}$$

Mathematica [A]

time = 0.39, size = 691, normalized size = 1.04

$$-\frac{2px^3}{9g}+\frac{2dp\left(\sqrt{e}\,x-\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\right)}{3e^{3/2}g}+\frac{2fp\left(x-\frac{\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}\right)}{g^2}-\frac{fx\log\left(c(d+ex^2)^p\right)}{g^2}+\frac{x^3\log\left(c(d+ex^2)^p\right)}{3g}+\frac{f^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{g^{5/2}}-\frac{if^{3/2}p\left(\log\left(\frac{\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{i\sqrt{e}\,\sqrt{f}+\sqrt{-d}\,\sqrt{g}}\right)\log\left(1-\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)+\log\left(\frac{\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{-i\sqrt{e}\,\sqrt{f}+\sqrt{-d}\,\sqrt{g}}\right)\log\left(1-\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)-\log\left(\frac{\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{-i\sqrt{e}\,\sqrt{f}+\sqrt{-d}\,\sqrt{g}}\right)\log\left(1+\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)-\log\left(\frac{\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{i\sqrt{e}\,\sqrt{f}+\sqrt{-d}\,\sqrt{g}}\right)\log\left(1+\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)+\mathrm{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}\,x\right)}{\sqrt{e}\,\sqrt{f}-i\sqrt{-d}\,\sqrt{g}}\right)+\mathrm{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}\,x\right)}{\sqrt{e}\,\sqrt{f}+i\sqrt{-d}\,\sqrt{g}}\right)-\mathrm{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}+i\sqrt{g}\,x\right)}{\sqrt{e}\,\sqrt{f}-i\sqrt{-d}\,\sqrt{g}}\right)-\mathrm{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}+i\sqrt{g}\,x\right)}{\sqrt{e}\,\sqrt{f}+i\sqrt{-d}\,\sqrt{g}}\right)\right)}{2g^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] (-2*p*x^3)/(9*g) + (2*d*p*(Sqrt[e]*x - Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])
)/(3*e^(3/2)*g) + (2*f*p*(x - (Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
))/g^2 - (f*x*Log[c*(d + e*x^2)^p])/g^2 + (x^3*Log[c*(d + e*x^2)^p])/(3*g)
+ (f^(3/2)*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/g^(5/2) - ((I/
2)*f^(3/2)*p*(Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqr
t[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + Log[(Sqrt[g]*(Sqrt[-d] + S
qrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)
/Sqrt[f]] - Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sq
rt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] - Log[(Sqrt[g]*(Sqrt[-d] +
Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/S
qrt[f]] + PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I
*Sqrt[-d]*Sqrt[g])] + PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]
*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])] - PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*
x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] - PolyLog[2, (Sqrt[e]*(Sqrt[f]
+ I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])]))/g^(5/2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.55, size = 1011, normalized size = 1.52

method	result	size
risch	Expression too large to display	1011

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*ln(c*(e*x^2+d)^p)/(g*x^2+f),x,method=_RETURNVERBOSE)
```

```
[Out] -2/9*p*x^3/g+2/3*d*p*x/e/g+2*f*p*x/g^2+1/6*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*
c*(e*x^2+d)^p)^2/g*x^3-2/3*p/g*d^2/e/(e*d)^(1/2)*arctan(x*e/(e*d)^(1/2))-2*
p*f/g^2*d/(e*d)^(1/2)*arctan(x*e/(e*d)^(1/2))-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*
csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*f^2/g^2/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))
-1/6*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/g*x^3+(ln((e*x^2+d)^p)-p*ln(e*x^2+d))*f^2
/g^2/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))-p*f/g^2*x*ln(e*x^2+d)+ln(c)*f^2/g^
2/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))+1/3*ln(c)/g*x^3+1/3*(ln((e*x^2+d)^p)-
p*ln(e*x^2+d))/g*x^3+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/g^2*x*f+1/6*I*Pi*csgn
(I*c*(e*x^2+d)^p)^2*csgn(I*c)/g*x^3-(ln((e*x^2+d)^p)-p*ln(e*x^2+d))/g^2*x*f
+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*f^2/g^2/(f*g)^(1/2)*arctan(x*g/
(f*g)^(1/2))+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*f^2/g^2/(
f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e
*x^2+d)^p)*csgn(I*c)/g^2*x*f-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d
```

```
)^p)^2/g^2*x*f-1/6*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)
/g*x^3-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*f^2/g^2/(f*g)^(1/2)*arctan(x*g/(f*g
)^(1/2))-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/g^2*x*f+p*Sum(1/2*(ln(x
-_alpha)*ln(e*x^2+d)-2*e*(1/2*ln(x-_alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha
*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,ind
ex=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x+_alpha)/RootO
f(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)))/e+1/2*(dilog((RootOf(_Z^2*e*g
+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g
+d*g-e*f,index=1))+dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-
x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)))/e))*f^2/g^3/_a
lpha,_alpha=RootOf(_Z^2*g+f))-ln(c)/g^2*x*f+1/3*p/g*x^3*ln(e*x^2+d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
[Out] integrate(x^4*log((x^2*e + d)^p*c)/(g*x^2 + f), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
[Out] integral(x^4*log((x^2*e + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
[Out] Integral(x**4*log(c*(d + e*x**2)**p)/(f + g*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(x^4*log((x^2*e + d)^p*c)/(g*x^2 + f), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{g\,x^2+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*log(c*(d + e*x^2)^p))/(f + g*x^2),x)
```

```
[Out] int((x^4*log(c*(d + e*x^2)^p))/(f + g*x^2), x)
```

3.344 $\int \frac{x^2 \log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$

Optimal. Leaf size=585

$$-\frac{2px}{g}+\frac{2\sqrt{d}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,g}-\frac{2\sqrt{f}\,p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{g^{3/2}}+\frac{\sqrt{f}\,p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\cdots\right.}{}$$

```
[Out] -2*p*x/g+x*ln(c*(e*x^2+d)^p)/g+2*p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/g/e^(1
/2)-arctan(x*g^(1/2)/f^(1/2))*ln(c*(e*x^2+d)^p)*f^(1/2)/g^(3/2)-2*p*arctan(
x*g^(1/2)/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))*f^(1/2)/g^(3/2)+p*ar
ctan(x*g^(1/2)/f^(1/2))*ln(-2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/
2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2)))*f^(1/2)/g^(3/2)+p*a
rctan(x*g^(1/2)/f^(1/2))*ln(2*((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/
2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))*f^(1/2)/g^(3/2)+I*p
*polylog(2,1-2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))*f^(1/2)/g^(3/2)-1/2*I*p*polyl
og(2,1+2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^
(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2)))*f^(1/2)/g^(3/2)-1/2*I*p*polylog(2,1-2*((
-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/
2)+(-d)^(1/2)*g^(1/2)))*f^(1/2)/g^(3/2)
```

Rubi [A]

time = 0.39, antiderivative size = 585, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {2526, 2498, 327, 211, 2520, 12, 5048, 4966, 2449, 2352, 2497}

$$-\frac{i\sqrt{f}\,p\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2g^{3/2}}-\frac{i\sqrt{f}\,p\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2g^{3/2}}+\frac{i\sqrt{f}\,p\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{g^{3/2}}-\frac{\sqrt{f}\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{g^{3/2}}+\frac{\sqrt{f}\,p\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{g^{3/2}}+\frac{\sqrt{f}\,p\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{g^{3/2}}+\frac{2\sqrt{d}\,p\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,g}-\frac{2\sqrt{f}\,p\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{g^{3/2}}+\frac{x\log\left(c\left(d+ex^2\right)^p\right)}{g}-\frac{2px}{g}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] (-2*p*x)/g + (2*Sqrt[d]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*g) - (2*Sqr
t[f]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)]
)/g^(3/2) + (Sqrt[f]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*
(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
I*Sqrt[g]*x))])/g^(3/2) + (Sqrt[f]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqr
t[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g]
)*(Sqrt[f] - I*Sqrt[g]*x))])/g^(3/2) + (x*Log[c*(d + e*x^2)^p])/g - (Sqrt[f
]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/g^(3/2) + (I*Sqrt[f]*p*
PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/g^(3/2) - ((I/2)*Sqrt[
f]*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*
Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(3/2) - ((I/2)*Sqr
```

```
t[f]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e
]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\int \frac{x^2 \log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \left(\frac{\log\left(c(d+ex^2)^p\right)}{g} - \frac{f\log\left(c(d+ex^2)^p\right)}{g\left(f+gx^2\right)}\right)dx$$

$$= \frac{\int \log\left(c(d+ex^2)^p\right)\,dx}{g} - \frac{f\int \frac{\log(c(d+ex^2)^p)}{f+gx^2}\,dx}{g}$$

$$= \frac{x\log\left(c(d+ex^2)^p\right)}{g} - \frac{\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{g^{3/2}} - \frac{(2ep)\int \frac{x^2}{d+ex^2}\,dx}{g}$$

$$= -\frac{2px}{g} + \frac{x\log\left(c(d+ex^2)^p\right)}{g} - \frac{\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{g^{3/2}} + \frac{\left(2e\sqrt{f}\right.}{}$$

$$= -\frac{2px}{g} + \frac{2\sqrt{d}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,g} + \frac{x\log\left(c(d+ex^2)^p\right)}{g} - \frac{\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log}{g^{3/2}}$$

$$= -\frac{2px}{g} + \frac{2\sqrt{d}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,g} + \frac{x\log\left(c(d+ex^2)^p\right)}{g} - \frac{\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log}{g^{3/2}}$$

$$= -\frac{2px}{g} + \frac{2\sqrt{d}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,g} - \frac{2\sqrt{f}\,p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{g^{3/2}} +$$

$$= -\frac{2px}{g} + \frac{2\sqrt{d}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,g} - \frac{2\sqrt{f}\,p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{g^{3/2}} +$$

$$= -\frac{2px}{g} + \frac{2\sqrt{d}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,g} - \frac{2\sqrt{f}\,p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{g^{3/2}} +$$

Mathematica [A]

time = 0.22, size = 680, normalized size = 1.16

$$\frac{-4\sqrt{g}\,px+\frac{4\sqrt{d}\,\sqrt{g}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}+i\sqrt{f}\,p\log\left(\frac{\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{i\sqrt{e}\,\sqrt{f}+\sqrt{-d}\,\sqrt{g}}\right)\log\left(1-\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)+i\sqrt{f}\,p\log\left(\frac{\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{-i\sqrt{e}\,\sqrt{f}+\sqrt{-d}\,\sqrt{g}}\right)\log\left(1-\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)-i\sqrt{f}\,p\log\left(\frac{\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{-i\sqrt{e}\,\sqrt{f}+\sqrt{-d}\,\sqrt{g}}\right)\log\left(1+\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)-i\sqrt{f}\,p\log\left(\frac{\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{i\sqrt{e}\,\sqrt{f}+\sqrt{-d}\,\sqrt{g}}\right)\log\left(1+\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)+2\sqrt{g}\,x\log\left(c(d+ex^2)^p\right)-2\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)+i\sqrt{f}\,p\mathrm{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}\,x\right)}{\sqrt{e}\,\sqrt{f}-i\sqrt{-d}\,\sqrt{g}}\right)+i\sqrt{f}\,p\mathrm{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}\,x\right)}{\sqrt{e}\,\sqrt{f}+i\sqrt{-d}\,\sqrt{g}}\right)-i\sqrt{f}\,p\mathrm{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}+i\sqrt{g}\,x\right)}{\sqrt{e}\,\sqrt{f}-i\sqrt{-d}\,\sqrt{g}}\right)-i\sqrt{f}\,p\mathrm{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}+i\sqrt{g}\,x\right)}{\sqrt{e}\,\sqrt{f}+i\sqrt{-d}\,\sqrt{g}}\right)}{2g^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] (-4*Sqrt[g]*p*x + (4*Sqrt[d]*Sqrt[g]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
 + I*Sqrt[f]*p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sq
rt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + I*Sqrt[f]*p*Log[(Sqrt[g]*
(Sqrt[-d] + Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 -
(I*Sqrt[g]*x)/Sqrt[f]] - I*Sqrt[f]*p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(
(-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] -
I*Sqrt[f]*p*Log[(Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[
-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] + 2*Sqrt[g]*x*Log[c*(d + e*x^2
)^p] - 2*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p] + I*Sqrt[
f]*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt
[-d]*Sqrt[g])] + I*Sqrt[f]*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(
Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])] - I*Sqrt[f]*p*PolyLog[2, (Sqrt[e]*(S
qrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] - I*Sqrt[f]*
p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d
]*Sqrt[g])])/(2*g^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.50, size = 746, normalized size = 1.28

method	result	size
risch	Expression too large to display	746

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(e*x^2+d)^p)/(g*x^2+f),x,method=_RETURNVERBOSE)
```

```
[Out] (ln((e*x^2+d)^p)-p*ln(e*x^2+d))/g*x-(ln((e*x^2+d)^p)-p*ln(e*x^2+d))*f/g/(f*
g)^(1/2)*arctan(x*g/(f*g)^(1/2))+p/g*x*ln(e*x^2+d)-2*p*x/g+2*p/g*d/(e*d)^(1
/2)*arctan(x*e/(e*d)^(1/2))+p*Sum(-1/2*(ln(x-_alpha)*ln(e*x^2+d)-2*e*(1/2*l
n(x-_alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)
/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z
*_alpha*e*g+d*g-e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-
e*f,index=2)))/e+1/2*(dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=
1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1))+dilog((RootO
f(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*
_alpha*e*g+d*g-e*f,index=2)))/e))*f/g^2/_alpha,_alpha=RootOf(_Z^2*g+f))+1/2
*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*f/g/(f*g)^(1/2)*a
rctan(x*g/(f*g)^(1/2))-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*c
sgn(I*c)/g*x-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/g*x+1/2*I*Pi*csgn(I*(e*x^2+d)
```

```
^p)*csgn(I*c*(e*x^2+d)^p)^2/g*x-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*
f/g/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*cs
gn(I*c)/g*x+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*f/g/(f*g)^(1/2)*arctan(x*g/(f*
g)^(1/2))-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*f/g/(f*g)^(1
/2)*arctan(x*g/(f*g)^(1/2))+ln(c)/g*x-ln(c)*f/g/(f*g)^(1/2)*arctan(x*g/(f*g
)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] integrate(x^2*log((x^2*e + d)^p*c)/(g*x^2 + f), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(x^2*log((x^2*e + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \log \left(c(d + ex^2)^p\right)}{f + gx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
```

```
[Out] Integral(x**2*log(c*(d + e*x**2)**p)/(f + g*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(x^2*log((x^2*e + d)^p*c)/(g*x^2 + f), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{g\,x^2+f} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*log(c*(d + e*x^2)^p))/(f + g*x^2),x)
[Out] int((x^2*log(c*(d + e*x^2)^p))/(f + g*x^2), x)
```

3.345 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$

Optimal. Leaf size=533

$$\frac{2p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}}-\frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(i\sqrt{e}\,\sqrt{f}-\sqrt{-d}\,\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}\right)}{\sqrt{f}\,\sqrt{g}}-\frac{p\cdots}{}$$

```
[Out] arctan(x*g^(1/2)/f^(1/2))*ln(c*(e*x^2+d)^p)/f^(1/2)/g^(1/2)+2*p*arctan(x*g^
(1/2)/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)-p*arctan
(x*g^(1/2)/f^(1/2))*ln(-2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I
*x*g^(1/2))/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)-p*arcta
n(x*g^(1/2)/f^(1/2))*ln(2*((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I
*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)-I*p*pol
ylog(2,1-2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)+1/2*I*p*polylog(2
,1+2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2
)*f^(1/2)-(-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)+1/2*I*p*polylog(2,1-2*((-d)^
(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(
-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 533, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {211, 2520, 12, 5048, 4966, 2449, 2352, 2497}

$$\frac{ip\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(-\sqrt{-d}\,\sqrt{g}+i\sqrt{e}\,\sqrt{f}\right)}\right)}{2\sqrt{f}\,\sqrt{g}}+\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(\sqrt{-d}\,\sqrt{g}+i\sqrt{e}\,\sqrt{f}\right)}\right)}{2\sqrt{f}\,\sqrt{g}}-\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}}+\frac{\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\,\sqrt{g}}-\frac{p\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(-\sqrt{-d}\,\sqrt{g}+i\sqrt{e}\,\sqrt{f}\right)}\right)}{\sqrt{f}\,\sqrt{g}}-\frac{p\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(\sqrt{-d}\,\sqrt{g}+i\sqrt{e}\,\sqrt{f}\right)}\right)}{\sqrt{f}\,\sqrt{g}}+\frac{2p\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(f + g*x^2),x]
```

```
[Out] (2*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/
(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*
(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*S
qrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[
g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + (ArcTan[(Sqrt[g]*x)/Sqrt
[f]]*Log[c*(d + e*x^2)^p])/(Sqrt[f]*Sqrt[g]) - (I*p*PolyLog[2, 1 - (2*Sqrt[
f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 + (
2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sq
rt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1
 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d
]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
```

```
/; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\,\sqrt{g}} - (2ep)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}\,\sqrt{g}\,(d+ex^2)}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\,\sqrt{g}} - \frac{(2ep)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{d+ex^2}\,dx}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\,\sqrt{g}} - \frac{(2ep)\int \left(-\frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}\,x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{2\sqrt{e}\left(\sqrt{-d}\right.}\right.}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\,\sqrt{g}} + \frac{\left(\sqrt{e}\,p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{-d}-\sqrt{e}\,x}\,dx}{\sqrt{f}\,\sqrt{g}} - \frac{\left(\sqrt{e}\,p\right)\int \frac{\tan^{-1}\left(\right.}{\sqrt{-d}}}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\,\sqrt{g}}{\left(i\sqrt{e}\,\sqrt{f}-\sqrt{-d}\right.}\right.}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\,\sqrt{g}}{\left(i\sqrt{e}\,\sqrt{f}-\sqrt{-d}\right.}\right.}{\sqrt{f}\,\sqrt{g}}$$

$$= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\,\sqrt{g}}{\left(i\sqrt{e}\,\sqrt{f}-\sqrt{-d}\right.}\right.}{\sqrt{f}\,\sqrt{g}}$$

Mathematica [A]
time = 0.12, size = 564, normalized size = 1.06

$$-\frac{i\left(p\log\left(\frac{\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{i\sqrt{e}\,\sqrt{f}+\sqrt{-d}\,\sqrt{g}}\right)\log\left(1-\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)+p\log\left(\frac{\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{-i\sqrt{e}\,\sqrt{f}+\sqrt{-d}\,\sqrt{g}}\right)\log\left(1-\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)-p\log\left(\frac{\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{-i\sqrt{e}\,\sqrt{f}+\sqrt{-d}\,\sqrt{g}}\right)\log\left(1+\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)-p\log\left(\frac{\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{i\sqrt{e}\,\sqrt{f}+\sqrt{-d}\,\sqrt{g}}\right)\log\left(1+\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)+2i\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)+p\mathrm{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}\,x\right)}{\sqrt{e}\,\sqrt{f}-i\sqrt{-d}\,\sqrt{g}}\right)+p\mathrm{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}\,x\right)}{\sqrt{e}\,\sqrt{f}+i\sqrt{-d}\,\sqrt{g}}\right)-p\mathrm{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}+i\sqrt{g}\,x\right)}{\sqrt{e}\,\sqrt{f}-i\sqrt{-d}\,\sqrt{g}}\right)-p\mathrm{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}+i\sqrt{g}\,x\right)}{\sqrt{e}\,\sqrt{f}+i\sqrt{-d}\,\sqrt{g}}\right)\right)}{2\sqrt{f}\,\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^2),x]
```

```
[Out] ((-1/2*I)*(p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt
[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + p*Log[(Sqrt[g]*(Sqrt[-d] +
Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x
)/Sqrt[f]] - p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] +
 Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] - p*Log[(Sqrt[g]*(Sqrt[-
d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]
*x)/Sqrt[f]] + (2*I)*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p] + p*P
olyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*S
qrt[g])] + p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f]
+ I*Sqrt[-d]*Sqrt[g])] - p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sq
rt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] - p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*S
qrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])]))/(Sqrt[f]*Sqrt[g])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.33, size = 504, normalized size = 0.95

method	result
risch	$\frac{\left(\ln\left(\left(e\,x^2+d\right)^p\right)-p\ln\left(e\,x^2+d\right)\right)\arctan\left(\frac{xg}{\sqrt{fg}}\right)}{\sqrt{fg}}+\frac{p\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^2 g+f\right)}\frac{\ln(x-\_\alpha)\ln\left(e\,x^2+d\right)-\ln(x-\_\alpha)\left(\ln\left(\frac{\mathrm{RootOf}\left(e\_Z^2 \ldots\right.}{\mathrm{RootOf}\left(e\_ \ldots\right.}\right.\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/(g*x^2+f),x,method=_RETURNVERBOSE)
```

```
[Out] (ln((e*x^2+d)^p)-p*ln(e*x^2+d))/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))+1/2*p/g
*sum(1/_alpha*(ln(x-_alpha)*ln(e*x^2+d)-ln(x-_alpha)*(ln((RootOf(_Z^2*e*g+2
*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d
*g-e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x+_al
pha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)))-dilog((RootOf(_Z^2*
e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*
e*g+d*g-e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=
2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2))),_alpha=Root
Of(_Z^2*g+f))+1/2*I/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))*Pi*csgn(I*(e*x^2+d)
^p)*csgn(I*c*(e*x^2+d)^p)^2-1/2*I/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))*Pi*cs
gn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/2*I/(f*g)^(1/2)*arctan(
x*g/(f*g)^(1/2))*Pi*csgn(I*c*(e*x^2+d)^p)^3+1/2*I/(f*g)^(1/2)*arctan(x*g/(f
*g)^(1/2))*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+1/(f*g)^(1/2)*arctan(x*g/(f
*g)^(1/2))*ln(c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
[Out] integrate(log((x^2*e + d)^p*c)/(g*x^2 + f), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
[Out] integral(log((x^2*e + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{f+gx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
[Out] Integral(log(c*(d + e*x**2)**p)/(f + g*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
[Out] integrate(log((x^2*e + d)^p*c)/(g*x^2 + f), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln \left(c\,{\left(e\,x^2+d\right)}^p\right)}{g\,x^2+f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)/(f + g*x^2),x)
[Out] int(log(c*(d + e*x^2)^p)/(f + g*x^2), x)
```

3.346 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^2\left(f+gx^2\right)}\,dx$

Optimal. Leaf size=581

$$\frac{2\sqrt{e}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,f}-\frac{2\sqrt{g}\,p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{f^{3/2}}+\frac{\sqrt{g}\,p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{}}{\left(i\sqrt{e}\,\sqrt{f}\right.}\right.}{f^{3/2}}$$

```
[Out] -ln(c*(e*x^2+d)^p)/f/x+2*p*arctan(x*e^(1/2)/d^(1/2))*e^(1/2)/f/d^(1/2)-arct
an(x*g^(1/2)/f^(1/2))*ln(c*(e*x^2+d)^p)*g^(1/2)/f^(3/2)-2*p*arctan(x*g^(1/2
)/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))*g^(1/2)/f^(3/2)+p*arctan(x*g
^(1/2)/f^(1/2))*ln(-2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g
^(1/2))/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2)))*g^(1/2)/f^(3/2)+p*arctan(x*
g^(1/2)/f^(1/2))*ln(2*((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g
^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))*g^(1/2)/f^(3/2)+I*p*polylog
(2,1-2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))*g^(1/2)/f^(3/2)-1/2*I*p*polylog(2,1+2
*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^
(1/2)-(-d)^(1/2)*g^(1/2)))*g^(1/2)/f^(3/2)-1/2*I*p*polylog(2,1-2*((-d)^(1/2
)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^
(1/2)*g^(1/2)))*g^(1/2)/f^(3/2)
```

Rubi [A]

time = 0.38, antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2526, 2505, 211, 2520, 12, 5048, 4966, 2449, 2352, 2497}

$$-\frac{i\sqrt{g}\,p\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(-\sqrt{-d}\,\sqrt{g}+i\sqrt{e}\,\sqrt{f}\right)}\right)}{2f^{3/2}}-\frac{i\sqrt{g}\,p\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(\sqrt{-d}\,\sqrt{g}+i\sqrt{e}\,\sqrt{f}\right)}\right)}{2f^{3/2}}+\frac{i\sqrt{g}\,p\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{f^{3/2}}-\frac{\sqrt{g}\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^{3/2}}+\frac{\sqrt{g}\,p\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(-\sqrt{-d}\,\sqrt{g}+i\sqrt{e}\,\sqrt{f}\right)}\right)}{f^{3/2}}+\frac{\sqrt{g}\,p\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(\sqrt{-d}\,\sqrt{g}+i\sqrt{e}\,\sqrt{f}\right)}\right)}{f^{3/2}}+\frac{2\sqrt{e}\,p\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,f}-\frac{2\sqrt{g}\,p\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{f^{3/2}}-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{fx}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(x^2*(f + g*x^2)),x]
```

```
[Out] (2*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*f) - (2*Sqrt[g]*p*ArcTan
[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/f^(3/2) + (
Sqrt[g]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - S
qrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))
])/f^(3/2) + (Sqrt[g]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*
(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
I*Sqrt[g]*x))])/f^(3/2) - Log[c*(d + e*x^2)^p]/(f*x) - (Sqrt[g]*ArcTan[(Sqr
t[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/f^(3/2) + (I*Sqrt[g]*p*PolyLog[2, 1
- (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/f^(3/2) - ((I/2)*Sqrt[g]*p*PolyLog[
2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqr
t[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/f^(3/2) - ((I/2)*Sqrt[g]*p*PolyLo
```

```
g[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + S
qrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/f^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^2\left(f+gx^2\right)}\,dx = \int \left(\frac{\log\left(c(d+ex^2)^p\right)}{fx^2} - \frac{g\log\left(c(d+ex^2)^p\right)}{f\left(f+gx^2\right)}\right)\,dx$$

$$= \frac{\int \frac{\log\left(c(d+ex^2)^p\right)}{x^2}\,dx}{f} - \frac{g\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx}{f}$$

$$= -\frac{\log\left(c(d+ex^2)^p\right)}{fx} - \frac{\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{f^{3/2}} + \frac{(2ep)\int \frac{1}{d+ex^2}\,dx}{f} +$$

$$= \frac{2\sqrt{e}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,f} - \frac{\log\left(c(d+ex^2)^p\right)}{fx} - \frac{\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{f^{3/2}}$$

$$= \frac{2\sqrt{e}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,f} - \frac{\log\left(c(d+ex^2)^p\right)}{fx} - \frac{\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{f^{3/2}}$$

$$= \frac{2\sqrt{e}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,f} - \frac{\log\left(c(d+ex^2)^p\right)}{fx} - \frac{\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{f^{3/2}}$$

$$= \frac{2\sqrt{e}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,f} - \frac{2\sqrt{g}\,p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{f^{3/2}} + \frac{\sqrt{g}\,p\tan^{-1}}{}$$

$$= \frac{2\sqrt{e}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,f} - \frac{2\sqrt{g}\,p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{f^{3/2}} + \frac{\sqrt{g}\,p\tan^{-1}}{}$$

$$= \frac{2\sqrt{e}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,f} - \frac{2\sqrt{g}\,p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{f^{3/2}} + \frac{\sqrt{g}\,p\tan^{-1}}{}$$

Mathematica [A]

time = 0.22, size = 673, normalized size = 1.16

$$\frac{\frac{4\sqrt{e}\sqrt{f}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}}+i\sqrt{g}\,p\log\left(\frac{\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}}\right)\log\left(1-\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)+i\sqrt{g}\,p\log\left(\frac{\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{-i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}}\right)\log\left(1-\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)-i\sqrt{g}\,p\log\left(\frac{\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{-i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}}\right)\log\left(1+\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)-i\sqrt{g}\,p\log\left(\frac{\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}}\right)\log\left(1+\frac{i\sqrt{g}\,x}{\sqrt{f}}\right)-\frac{2\sqrt{f}\log\left(c(d+ex^2)^p\right)}{x}-2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)+i\sqrt{g}\,p\operatorname{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}\,x\right)}{\sqrt{e}\sqrt{f}-i\sqrt{-d}\sqrt{g}}\right)+i\sqrt{g}\,p\operatorname{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}\,x\right)}{\sqrt{e}\sqrt{f}+i\sqrt{-d}\sqrt{g}}\right)-i\sqrt{g}\,p\operatorname{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}+i\sqrt{g}\,x\right)}{\sqrt{e}\sqrt{f}-i\sqrt{-d}\sqrt{g}}\right)-i\sqrt{g}\,p\operatorname{Li}_2\left(\frac{\sqrt{e}\left(\sqrt{f}+i\sqrt{g}\,x\right)}{\sqrt{e}\sqrt{f}+i\sqrt{-d}\sqrt{g}}\right)}{2f^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(x^2*(f + g*x^2)),x]
```

```
[Out] ((4*Sqrt[e]*Sqrt[f]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] + I*Sqrt[g]*p*Lo
g[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*
Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + I*Sqrt[g]*p*Log[(Sqrt[g]*(Sqrt[-d] + Sqrt[
e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqr
t[f]] - I*Sqrt[g]*p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt
[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] - I*Sqrt[g]*p*Log[(
Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log
[1 + (I*Sqrt[g]*x)/Sqrt[f]] - (2*Sqrt[f]*Log[c*(d + e*x^2)^p])/x - 2*Sqrt[g
]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p] + I*Sqrt[g]*p*PolyLog[2,
 (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])]
+ I*Sqrt[g]*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f]
 + I*Sqrt[-d]*Sqrt[g])] - I*Sqrt[g]*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt
[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] - I*Sqrt[g]*p*PolyLog[2, (S
qrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])])/(2
*f^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.49, size = 755, normalized size = 1.30

method	result	size
risch	Expression too large to display	755

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/x^2/(g*x^2+f),x,method=_RETURNVERBOSE)
```

```
[Out] -(ln((e*x^2+d)^p)-p*ln(e*x^2+d))/f*g/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))-(l
n((e*x^2+d)^p)-p*ln(e*x^2+d))/f/x+p*Sum(-1/2*(ln(x-_alpha)*ln(e*x^2+d)-2*e*
(1/2*ln(x-_alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_
alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1))+ln((RootOf(_Z^2*e*
g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*
g+d*g-e*f,index=2)))/e+1/2*(dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,
index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1))+dilog(
(RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g
+2*_Z*_alpha*e*g+d*g-e*f,index=2)))/e))/f/_alpha,_alpha=RootOf(_Z^2*g+f))-p
/f/x*ln(e*x^2+d)+2*p/f*e/(e*d)^(1/2)*arctan(x*e/(e*d)^(1/2))-1/2*I*Pi*csgn(
I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/f*g/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/
2))-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/f/x+1/2*I*Pi*csgn(
I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/f*g/(f*g)^(1/2)*arctan(x*g/(
```

```
f*g)^(1/2))+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/f/
x+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/f*g/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))+
1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/f/x-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(
I*c)/f*g/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)
^2*csgn(I*c)/f/x-ln(c)/f*g/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))-ln(c)/f/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^2/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] integrate(log((x^2*e + d)^p*c)/((g*x^2 + f)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^2/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log((x^2*e + d)^p*c)/(g*x^4 + f*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^2\left(f+gx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/x**2/(g*x**2+f),x)
```

```
[Out] Integral(log(c*(d + e*x**2)**p)/(x**2*(f + g*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^2/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(log((x^2*e + d)^p*c)/((g*x^2 + f)*x^2), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)}{x^2\,\left(g\,x^2+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)/(x^2*(f + g*x^2)),x)
[Out] int(log(c*(d + e*x^2)^p)/(x^2*(f + g*x^2)), x)
```

3.347 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^4\left(f+gx^2\right)}\,dx$

Optimal. Leaf size=651

$$-\frac{2ep}{3dfx}-\frac{2e^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3d^{3/2}f}-\frac{2\sqrt{e}\,gp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,f^2}+\frac{2g^{3/2}p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{f^{5/2}}-\frac{g^{3/2}p\,\mathrm{t}}{}$$

```
[Out] -2/3*e*p/d/f/x-2/3*e^(3/2)*p*arctan(x*e^(1/2)/d^(1/2))/d^(3/2)/f-1/3*ln(c*(
e*x^2+d)^p)/f/x^3+g*ln(c*(e*x^2+d)^p)/f^2/x+g^(3/2)*arctan(x*g^(1/2)/f^(1/2
))*ln(c*(e*x^2+d)^p)/f^(5/2)+2*g^(3/2)*p*arctan(x*g^(1/2)/f^(1/2))*ln(2*f^(
1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(5/2)-g^(3/2)*p*arctan(x*g^(1/2)/f^(1/2))*ln(
-2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*
f^(1/2)-(-d)^(1/2)*g^(1/2)))/f^(5/2)-g^(3/2)*p*arctan(x*g^(1/2)/f^(1/2))*ln
(2*((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*
f^(1/2)+(-d)^(1/2)*g^(1/2)))/f^(5/2)-I*g^(3/2)*p*polylog(2,1-2*f^(1/2)/(f^(
1/2)-I*x*g^(1/2)))/f^(5/2)+1/2*I*g^(3/2)*p*polylog(2,1+2*((-d)^(1/2)-x*e^(1
/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^
(1/2)))/f^(5/2)+1/2*I*g^(3/2)*p*polylog(2,1-2*((-d)^(1/2)+x*e^(1/2))*f^(1/2
)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))/f^(
5/2)-2*g*p*arctan(x*e^(1/2)/d^(1/2))*e^(1/2)/f^2/d^(1/2)
```

Rubi [A]

time = 0.43, antiderivative size = 651, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {2526, 2505, 331, 211, 2520, 12, 5048, 4966, 2449, 2352, 2497}

$$\frac{ig^{3/2}p\,\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2f^{5/2}}+\frac{ig^{3/2}p\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2f^{5/2}}-\frac{ig^{3/2}p\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{f^{5/2}}+\frac{g^{3/2}\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^{5/2}}-\frac{2e^{3/2}p\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3d^{3/2}f}-\frac{g^{3/2}p\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{f^{5/2}}-\frac{g^{3/2}p\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{f^{5/2}}-\frac{2\sqrt{e}\,gp\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,f^2}+\frac{2g^{3/2}p\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{f^{5/2}}+\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{f^2x}-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{3fx^3}-\frac{2ep}{3dfx}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(x^4*(f + g*x^2)),x]
```

```
[Out] (-2*e*p)/(3*d*f*x) - (2*e^(3/2)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*d^(3/2)*f
) - (2*Sqrt[e]*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*f^2) + (2*g^(3/2)*
p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/f^(
5/2) - (g^(3/2)*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt
[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqr
t[g]*x))])/f^(5/2) - (g^(3/2)*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*
Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sq
rt[f] - I*Sqrt[g]*x))])/f^(5/2) - Log[c*(d + e*x^2)^p]/(3*f*x^3) + (g*Log[c
*(d + e*x^2)^p])/(f^2*x) + (g^(3/2)*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d +
e*x^2)^p])/f^(5/2) - (I*g^(3/2)*p*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*S
```

```
qrt[g]*x)])/f^(5/2) + ((I/2)*g^(3/2)*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(S
qrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*
Sqrt[g]*x))])/f^(5/2) + ((I/2)*g^(3/2)*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*
(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
I*Sqrt[g]*x))])/f^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
```

```
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^4(f+gx^2)}\,dx = \int \left(\frac{\log\left(c(d+ex^2)^p\right)}{fx^4} - \frac{g\log\left(c(d+ex^2)^p\right)}{f^2x^2} + \frac{g^2\log\left(c(d+ex^2)^p\right)}{f^2(f+gx^2)}\right)dx$$

$$= \frac{\int \frac{\log\left(c(d+ex^2)^p\right)}{x^4}dx}{f} - \frac{g\int \frac{\log\left(c(d+ex^2)^p\right)}{x^2}dx}{f^2} + \frac{g^2\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}dx}{f^2}$$

$$= -\frac{\log\left(c(d+ex^2)^p\right)}{3fx^3} + \frac{g\log\left(c(d+ex^2)^p\right)}{f^2x} + \frac{g^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{f^{5/2}}$$

$$= -\frac{2ep}{3dfx} - \frac{2\sqrt{e}\,gp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,f^2} - \frac{\log\left(c(d+ex^2)^p\right)}{3fx^3} + \frac{g\log\left(c(d+ex^2)^p\right)}{f^2x} + \cdots$$

$$= -\frac{2ep}{3dfx} - \frac{2e^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3d^{3/2}f} - \frac{2\sqrt{e}\,gp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,f^2} - \frac{\log\left(c(d+ex^2)^p\right)}{3fx^3} + \cdots$$

$$= -\frac{2ep}{3dfx} - \frac{2e^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3d^{3/2}f} - \frac{2\sqrt{e}\,gp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,f^2} - \frac{\log\left(c(d+ex^2)^p\right)}{3fx^3} + \cdots$$

$$= -\frac{2ep}{3dfx} - \frac{2e^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3d^{3/2}f} - \frac{2\sqrt{e}\,gp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,f^2} + \frac{2g^{3/2}p\tan^{-1}\left(\frac{\sqrt{g}\,\cdots}{\sqrt{f}}\right)\cdots}{\cdots}$$

$$= -\frac{2ep}{3dfx} - \frac{2e^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3d^{3/2}f} - \frac{2\sqrt{e}\,gp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,f^2} + \frac{2g^{3/2}p\tan^{-1}\left(\frac{\sqrt{g}\,\cdots}{\sqrt{f}}\right)\cdots}{\cdots}$$

$$= -\frac{2ep}{3dfx} - \frac{2e^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3d^{3/2}f} - \frac{2\sqrt{e}\,gp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,f^2} + \frac{2g^{3/2}p\tan^{-1}\left(\frac{\sqrt{g}\,\cdots}{\sqrt{f}}\right)\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order

4 in optimal.
time = 0.19, size = 754, normalized size = 1.16

$$-\frac{2\sqrt{e}\,gp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,f^2}-\frac{2ep\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{ex^2}{d}\right)}{3dfx}-\frac{\log\left(c(d+ex^2)^p\right)}{3fx^3}+\frac{g\log\left(c(d+ex^2)^p\right)}{f^2x}+\frac{g^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{f^{5/2}}-\frac{2eg^{3/2}p\left(\frac{i\left(\frac{\log\left(\frac{\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}}\right)\log\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{e}}+\frac{\text{Li}_2\left(\frac{\sqrt{e}(\sqrt{f}-i\sqrt{g}x)}{\sqrt{e}\sqrt{f}-i\sqrt{-d}\sqrt{g}}\right)}{\sqrt{e}}\right)}{4\sqrt{e}}+\frac{i\left(\frac{\log\left(-\frac{\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g}}\right)\log\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{e}}+\frac{\text{Li}_2\left(\frac{\sqrt{e}(\sqrt{f}-i\sqrt{g}x)}{\sqrt{e}\sqrt{f}+i\sqrt{-d}\sqrt{g}}\right)}{\sqrt{e}}\right)}{4\sqrt{e}}-\frac{i\left(\frac{\log\left(\frac{\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}}\right)\log\left(1+\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{e}}+\frac{\text{Li}_2\left(\frac{\sqrt{e}(\sqrt{f}+i\sqrt{g}x)}{\sqrt{e}\sqrt{f}-i\sqrt{-d}\sqrt{g}}\right)}{\sqrt{e}}\right)}{4\sqrt{e}}-\frac{i\left(\frac{\log\left(-\frac{\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g}}\right)\log\left(1+\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{e}}+\frac{\text{Li}_2\left(\frac{\sqrt{e}(\sqrt{f}+i\sqrt{g}x)}{\sqrt{e}\sqrt{f}+i\sqrt{-d}\sqrt{g}}\right)}{\sqrt{e}}\right)}{4\sqrt{e}}\right)}{f^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(x^4*(f + g*x^2)),x]
```

```
[Out] (-2*Sqrt[e]*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*f^2) - (2*e*p*Hyperge
ometric2F1[-1/2, 1, 1/2, -((e*x^2)/d)])/(3*d*f*x) - Log[c*(d + e*x^2)^p]/(3
*f*x^3) + (g*Log[c*(d + e*x^2)^p])/(f^2*x) + (g^(3/2)*ArcTan[(Sqrt[g]*x)/Sq
rt[f]]*Log[c*(d + e*x^2)^p])/f^(5/2) - (2*e*g^(3/2)*p*(((I/4)*((Log[(Sqrt[g
]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (
I*Sqrt[g]*x)/Sqrt[f]])/Sqrt[e] + PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x
))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])]/Sqrt[e]))/Sqrt[e] + ((I/4)*((Log
[-((Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g]))
]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]])/Sqrt[e] + PolyLog[2, (Sqrt[e]*(Sqrt[f] -
I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])]/Sqrt[e]))/Sqrt[e] - (
(I/4)*((Log[(Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*
Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]])/Sqrt[e] + PolyLog[2, (Sqrt[e]*(Sq
rt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])]/Sqrt[e]))/Sqr
t[e] - ((I/4)*((Log[-((Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] -
 Sqrt[-d]*Sqrt[g]))]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]])/Sqrt[e] + PolyLog[2, (
Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])]/Sq
rt[e]))/Sqrt[e]))/f^(5/2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.46, size = 1005, normalized size = 1.54

method	result	size
risch	Expression too large to display	1005

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/x^4/(g*x^2+f),x,method=_RETURNVERBOSE)
```

```
[Out] (ln((e*x^2+d)^p)-p*ln(e*x^2+d))*g^2/f^2/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))
-1/3*(ln((e*x^2+d)^p)-p*ln(e*x^2+d))/f/x^3+(ln((e*x^2+d)^p)-p*ln(e*x^2+d))/
f^2*g/x+p*Sum(1/2*(ln(x-_alpha)*ln(e*x^2+d)-2*e*(1/2*ln(x-_alpha)*(ln((Root
Of(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z
*_alpha*e*g+d*g-e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,i
ndex=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)))/e+1/2*
(dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_
Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1))+dilog((RootOf(_Z^2*e*g+2*_Z*_alph
```

```
a*e*g+d*g-e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,in
dex=2)))/e))/f^2*g/_alpha,_alpha=RootOf(_Z^2*g+f))+p/f^2*g/x*ln(e*x^2+d)-2*
p/f^2*g*e/(e*d)^(1/2)*arctan(x*e/(e*d)^(1/2))-1/3*p/f/x^3*ln(e*x^2+d)-2/3*p
/f*e^2/d/(e*d)^(1/2)*arctan(x*e/(e*d)^(1/2))-2/3*e*p/d/f/x-1/2*I*Pi*csgn(I*
c*(e*x^2+d)^p)^3*g^2/f^2/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))+1/2*I*Pi*csgn(
I*c*(e*x^2+d)^p)^2*csgn(I*c)*g^2/f^2/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))-1/
6*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/f/x^3+1/2*I*Pi*csgn(I*(e*x^2+d)^p)
*csgn(I*c*(e*x^2+d)^p)^2/f^2*g/x-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/f^2*g/x+1
/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*g^2/f^2/(f*g)^(1/2)*arc
tan(x*g/(f*g)^(1/2))+1/6*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csg
n(I*c)/f/x^3+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/f^2*g/x-1/2*I*Pi*cs
gn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/f^2*g/x+1/6*I*Pi*csgn(I*c
*(e*x^2+d)^p)^3/f/x^3-1/6*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/
f/x^3-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*g^2/f^2/
(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))+ln(c)*g^2/f^2/(f*g)^(1/2)*arctan(x*g/(f
*g)^(1/2))-1/3*ln(c)/f/x^3+ln(c)/f^2*g/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^4/(g*x^2+f),x, algorithm="maxima")
[Out] integrate(log((x^2*e + d)^p*c)/((g*x^2 + f)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^4/(g*x^2+f),x, algorithm="fricas")
[Out] integral(log((x^2*e + d)^p*c)/(g*x^6 + f*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^4\left(f+gx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/x**4/(g*x**2+f),x)
```

```
[Out] Integral(log(c*(d + e*x**2)**p)/(x**4*(f + g*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^4/(g*x^2+f),x, algorithm="giac")
[Out] integrate(log((x^2*e + d)^p*c)/((g*x^2 + f)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{x^4\,\left(g\,x^2+f\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)/(x^4*(f + g*x^2)),x)
[Out] int(log(c*(d + e*x^2)^p)/(x^4*(f + g*x^2)), x)
```

3.348 $\int \frac{x^5 \log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx$

Optimal. Leaf size=199

$$-\frac{px^2}{2g^2}+\frac{ef^2p\log\left(d+ex^2\right)}{2g^3(ef-dg)}+\frac{\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{2eg^2}-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{2g^3\left(f+gx^2\right)}-\frac{ef^2p\log\left(f+gx^2\right)}{2g^3(ef-dg)}-\frac{f\log\left(c\right.}{}$$

```
[Out] -1/2*p*x^2/g^2+1/2*e*f^2*p*ln(e*x^2+d)/g^3/(-d*g+e*f)+1/2*(e*x^2+d)*ln(c*(e
*x^2+d)^p)/e/g^2-1/2*f^2*ln(c*(e*x^2+d)^p)/g^3/(g*x^2+f)-1/2*e*f^2*p*ln(g*x
^2+f)/g^3/(-d*g+e*f)-f*ln(c*(e*x^2+d)^p)*ln(e*(g*x^2+f)/(-d*g+e*f))/g^3-f*p
*polylog(2,-g*(e*x^2+d)/(-d*g+e*f))/g^3
```

Rubi [A]
time = 0.20, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {2525, 45, 2463, 2436, 2332, 2442, 36, 31, 2441, 2440, 2438}

$$-\frac{fp\text{PolyLog}\left(2,-\frac{g\left(d+ex^2\right)}{ef-dg}\right)}{g^3}-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{2g^3\left(f+gx^2\right)}-\frac{f\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{g^3}+\frac{\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{2eg^2}+\frac{ef^2p\log\left(d+ex^2\right)}{2g^3(ef-dg)}-\frac{ef^2p\log\left(f+gx^2\right)}{2g^3(ef-dg)}-\frac{px^2}{2g^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] -1/2*(p*x^2)/g^2 + (e*f^2*p*Log[d + e*x^2])/(2*g^3*(e*f - d*g)) + ((d + e*x
^2)*Log[c*(d + e*x^2)^p])/(2*e*g^2) - (f^2*Log[c*(d + e*x^2)^p])/(2*g^3*(f
+ g*x^2)) - (e*f^2*p*Log[f + g*x^2])/(2*g^3*(e*f - d*g)) - (f*Log[c*(d + e*
x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)])/g^3 - (f*p*PolyLog[2, -((g*(d + e
*x^2))/(e*f - d*g))])/g^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{x^5 \log \left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2 \log \left(c(d+ex)^p\right)}{(f+gx)^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{\log \left(c(d+ex)^p\right)}{g^2} + \frac{f^2 \log \left(c(d+ex)^p\right)}{g^2(f+gx)^2} - \frac{2f \log \left(c(d+ex)^p\right)}{g^2(f+gx)}\right)\,dx$$

$$= \frac{\text{Subst}\left(\int \log \left(c(d+ex)^p\right)\,dx, x, x^2\right)}{2g^2} - \frac{f\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{f+gx}\,dx, x, x^2\right)}{g^2} + \frac{f^2\text{Su}}{}$$

$$= -\frac{f^2 \log \left(c\left(d+ex^2\right)^p\right)}{2g^3\left(f+gx^2\right)} - \frac{f \log \left(c\left(d+ex^2\right)^p\right) \log \left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{g^3} + \frac{\text{Subst}\left(\int \log \left(cx^p\right.\right.}{2e}$$

$$= -\frac{px^2}{2g^2} + \frac{\left(d+ex^2\right) \log \left(c\left(d+ex^2\right)^p\right)}{2eg^2} - \frac{f^2 \log \left(c\left(d+ex^2\right)^p\right)}{2g^3\left(f+gx^2\right)} - \frac{f \log \left(c\left(d+ex^2\right.\right.}{g}$$

$$= -\frac{px^2}{2g^2} + \frac{ef^2p \log \left(d+ex^2\right)}{2g^3(ef-dg)} + \frac{\left(d+ex^2\right) \log \left(c\left(d+ex^2\right)^p\right)}{2eg^2} - \frac{f^2 \log \left(c\left(d+ex^2\right)\right.}{2g^3\left(f+gx^2\right)}$$

Mathematica [A]
time = 0.16, size = 166, normalized size = 0.83

$$-\frac{px^2 - \frac{\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{e} + \frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{g\left(f+gx^2\right)} + \frac{ef^2p\left(\log\left(d+ex^2\right)-\log\left(f+gx^2\right)\right)}{g(-ef+dg)} + \frac{2f\left(\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)+p\text{Li}_2\left(\frac{g\left(d+ex^2\right)}{-ef+dg}\right)\right)}{g}}{2g^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] -1/2*(p*x^2 - ((d + e*x^2)*Log[c*(d + e*x^2)^p])/e + (f^2*Log[c*(d + e*x^2)
^p])/(g*(f + g*x^2)) + (e*f^2*p*(Log[d + e*x^2] - Log[f + g*x^2]))/(g*(-(e*
f) + d*g)) + (2*f*(Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)] +
p*PolyLog[2, (g*(d + e*x^2))/(-(e*f) + d*g)]))/g)/g^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.39, size = 985, normalized size = 4.95

method	result	size
risch	Expression too large to display	985

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln((e*x^2+d)^p)/g^2*x^2-ln((e*x^2+d)^p)*f/g^3*ln(g*x^2+f)-1/2*ln((e*x^2
+d)^p)*f^2/g^3/(g*x^2+f)-1/2*p*x^2/g^2+1/2*p*e/g^3*f^2/(d*g-e*f)*ln(g*x^2+f
)+1/2*p/e/g/(d*g-e*f)*ln(e*x^2+d)*d^2-1/2*p/g^2/(d*g-e*f)*ln(e*x^2+d)*d*f-1
/2*p*e/g^3/(d*g-e*f)*ln(e*x^2+d)*f^2+p*f/g^3*sum(ln(x-_alpha)*ln(g*x^2+f)-l
n(x-_alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)
/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z
*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+
e*f,index=2)))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_a
lpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((RootOf(_Z^2*
e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*
e*g-d*g+e*f,index=2)),_alpha=RootOf(_Z^2*e+d))+1/2*I*Pi*csgn(I*c*(e*x^2+d)^
p)^3*f/g^3*ln(g*x^2+f)-1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2
*f^2/g^3/(g*x^2+f)+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(
I*c)*f/g^3*ln(g*x^2+f)+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/g^2*x^2-1
/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/g^2*x^2-1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I
*c*(e*x^2+d)^p)*csgn(I*c)/g^2*x^2-1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c
)*f^2/g^3/(g*x^2+f)-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*f/g^3*ln(g*x
^2+f)+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*f^2/g^3/(g*x^2+f)+1/4*I*Pi*csgn(I*(e
*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*f^2/g^3/(g*x^2+f)+1/4*I*Pi*csgn(
I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/g^2*x^2-1/2*I*Pi*csgn(I*(e*x^2+d)^p)
*csgn(I*c*(e*x^2+d)^p)^2*f/g^3*ln(g*x^2+f)+1/2*ln(c)/g^2*x^2-ln(c)*f/g^3*ln
(g*x^2+f)-1/2*ln(c)*f^2/g^3/(g*x^2+f)
```

Maxima [A]
time = 0.35, size = 347, normalized size = 1.74

$$-\frac{(2\,dfg\log\left(c\right)-(f^2p+2\,f^2\log\left(c\right))e)\log\left(gx^2+f\right)}{2\,(dg^4-fg^3e)}-\frac{df^2ge\log\left(c\right)-((fg^2p-fg^2\log\left(c\right))e^2-(dg^3p-dg^3\log\left(c\right))e)x^4-f^3e^2\log\left(c\right)-((f^2gp-f^2g\log\left(c\right))e^2-(dfg^2p-dfg^2\log\left(c\right))e)x^2-(d^2fg^2p-2\,df^2gpe+(dg^3pe-fg^2pe^2)x^4+(d^2g^3p-2\,f^2gpe^2)x^2)\log\left(x^2e+d\right)}{2\,(dfg^4e-f^2g^3e^2+(dg^5e-fg^4e^2)x^2)}-\frac{\left(\log\left(x^2e+d\right)\log\left(-\frac{gx^2e+dg}{dg-fe}+1\right)+\mathrm{Li}_2\left(\frac{gx^2e+dg}{dg-fe}\right)\right)fp}{g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*d*f*g*log(c) - (f^2*p + 2*f^2*log(c))*e)*log(g*x^2 + f)/(d*g^4 - f*
g^3*e) - 1/2*(d*f^2*g*e*log(c) - ((f*g^2*p - f*g^2*log(c))*e^2 - (d*g^3*p -
 d*g^3*log(c))*e)*x^4 - f^3*e^2*log(c) - ((f^2*g*p - f^2*g*log(c))*e^2 - (d
*f*g^2*p - d*f*g^2*log(c))*e)*x^2 - (d^2*f*g^2*p - 2*d*f^2*g*p*e + (d*g^3*p
*e - f*g^2*p*e^2)*x^4 + (d^2*g^3*p - 2*f^2*g*p*e^2)*x^2)*log(x^2*e + d))/(d
```

```
*f*g^4*e - f^2*g^3*e^2 + (d*g^5*e - f*g^4*e^2)*x^2) - (log(x^2*e + d)*log(-
(g*x^2*e + d*g)/(d*g - f*e) + 1) + dilog((g*x^2*e + d*g)/(d*g - f*e)))*f*p/
g^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
[Out] integral(x^5*log((x^2*e + d)^p*c)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
[Out] integrate(x^5*log((x^2*e + d)^p*c)/(g*x^2 + f)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{{\left(g\,x^2+f\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*log(c*(d + e*x^2)^p))/(f + g*x^2)^2,x)
[Out] int((x^5*log(c*(d + e*x^2)^p))/(f + g*x^2)^2, x)
```

3.349 $\int \frac{x^3 \log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx$

Optimal. Leaf size=155

$$-\frac{efp\log(d+ex^2)}{2g^2(ef-dg)}+\frac{f\log\left(c(d+ex^2)^p\right)}{2g^2(f+gx^2)}+\frac{efp\log(f+gx^2)}{2g^2(ef-dg)}+\frac{\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^2}+\frac{p\text{Li}_2\left(-\frac{g(d+ex^2)}{ef-dg}\right)}{2g^2}$$

```
[Out] -1/2*e*f*p*ln(e*x^2+d)/g^2/(-d*g+e*f)+1/2*f*ln(c*(e*x^2+d)^p)/g^2/(g*x^2+f)
+1/2*e*f*p*ln(g*x^2+f)/g^2/(-d*g+e*f)+1/2*ln(c*(e*x^2+d)^p)*ln(e*(g*x^2+f)/
(-d*g+e*f))/g^2+1/2*p*polylog(2,-g*(e*x^2+d)/(-d*g+e*f))/g^2
```

Rubi [A]
time = 0.16, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {2525, 45, 2463, 2442, 36, 31, 2441, 2440, 2438}

$$\frac{p\text{PolyLog}\left(2,-\frac{g(d+ex^2)}{ef-dg}\right)}{2g^2}+\frac{f\log\left(c(d+ex^2)^p\right)}{2g^2(f+gx^2)}+\frac{\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^2}-\frac{efp\log(d+ex^2)}{2g^2(ef-dg)}+\frac{efp\log(f+gx^2)}{2g^2(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] -1/2*(e*f*p*Log[d + e*x^2])/(g^2*(e*f - d*g)) + (f*Log[c*(d + e*x^2)^p])/(2
*g^2*(f + g*x^2)) + (e*f*p*Log[f + g*x^2])/(2*g^2*(e*f - d*g)) + (Log[c*(d
+ e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)])/(2*g^2) + (p*PolyLog[2, -((g*
(d + e*x^2))/(e*f - d*g))])/(2*g^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{x^3 \log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x\log\left(c(d+ex)^p\right)}{(f+gx)^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{f\log\left(c(d+ex)^p\right)}{g(f+gx)^2} + \frac{\log\left(c(d+ex)^p\right)}{g(f+gx)}\right)\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{f+gx}\,dx, x, x^2\right)}{2g} - \frac{f\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{(f+gx)^2}\,dx, x, x^2\right)}{2g}$$

$$= \frac{f\log\left(c(d+ex^2)^p\right)}{2g^2\left(f+gx^2\right)} + \frac{\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^2} - \frac{(ep)\text{Subst}\left(\int \frac{\log\left(\frac{e(f+gx}{ef-dg}\right.}{d+ex}\right.}{2g^2}$$

$$= \frac{f\log\left(c(d+ex^2)^p\right)}{2g^2\left(f+gx^2\right)} + \frac{\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^2} - \frac{p\text{Subst}\left(\int \frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\right.}{2g^2}$$

$$= -\frac{efp\log\left(d+ex^2\right)}{2g^2(ef-dg)} + \frac{f\log\left(c(d+ex^2)^p\right)}{2g^2\left(f+gx^2\right)} + \frac{efp\log\left(f+gx^2\right)}{2g^2(ef-dg)} + \frac{\log\left(c(d+ex^2\right)}{}$$

Mathematica [A]
time = 0.07, size = 131, normalized size = 0.85

$$\frac{\frac{efp\log(d+ex^2)}{-ef+dg} + \frac{f\log(c(d+ex^2)^p)}{f+gx^2} + \frac{efp\log(f+gx^2)}{ef-dg} + \log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right) + p\text{Li}_2\left(\frac{g(d+ex^2)}{-ef+dg}\right)}{2g^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] ((e*f*p*Log[d + e*x^2])/(-(e*f) + d*g) + (f*Log[c*(d + e*x^2)^p])/(f + g*x^
2) + (e*f*p*Log[f + g*x^2])/(e*f - d*g) + Log[c*(d + e*x^2)^p]*Log[(e*(f +
g*x^2))/(e*f - d*g)] + p*PolyLog[2, (g*(d + e*x^2))/(-(e*f) + d*g)])/(2*g^2
)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.46, size = 732, normalized size = 4.72

method	result	size
risch	Expression too large to display	732

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln((e*x^2+d)^p)/g^2*ln(g*x^2+f)+1/2*ln((e*x^2+d)^p)*f/g^2/(g*x^2+f)-1/2
*p/g^2*sum(ln(x-_alpha)*ln(g*x^2+f)-ln(x-_alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*
_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e
*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)
/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)))-dilog((RootOf(_Z^2*e*g+
2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-
d*g+e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x
+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)),_alpha=RootOf(_Z
^2*e+d))-1/2*p*e*f/g^2/(d*g-e*f)*ln(g*x^2+f)+1/2*p*e*f/g^2/(d*g-e*f)*ln(e*x
^2+d)+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/g^2*ln(g*x^2+f)+
1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2*f/g^2/(g*x^2+f)-1/4*I*
Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/g^2*ln(g*x^2+f)-1/4*
I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*f/g^2/(g*x^2+f)-1/
4*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/g^2*ln(g*x^2+f)-1/4*I*Pi*csgn(I*c*(e*x^2+d)^
p)^3*f/g^2/(g*x^2+f)+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/g^2*ln(g*x^
2+f)+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*f/g^2/(g*x^2+f)+1/2*ln(c)/g
^2*ln(g*x^2+f)+1/2*ln(c)*f/g^2/(g*x^2+f)
```

Maxima [A]
time = 0.54, size = 193, normalized size = 1.25

$$\frac{(dg\log(c) - (fp + f\log(c))e)\log(gx^2 + f)}{2(dg^3 - fg^2e)} + \frac{dfg\log(c) - f^2e\log(c) + (fgpx^2e + dfgp)\log(x^2e + d)}{2(dfg^3 - f^2g^2e + (dg^4 - fg^3e)x^2)} + \frac{\left(\log(x^2e + d)\log\left(-\frac{gx^2e+dg}{dg-fe} + 1\right) + \mathrm{Li}_2\left(\frac{gx^2e+dg}{dg-fe}\right)\right)p}{2g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(d*g*log(c) - (f*p + f*log(c))*e)*log(g*x^2 + f)/(d*g^3 - f*g^2*e) + 1/
2*(d*f*g*log(c) - f^2*e*log(c) + (f*g*p*x^2*e + d*f*g*p)*log(x^2*e + d))/(d
*f*g^3 - f^2*g^2*e + (d*g^4 - f*g^3*e)*x^2) + 1/2*(log(x^2*e + d)*log(-(g*x
^2*e + d*g)/(d*g - f*e) + 1) + dilog((g*x^2*e + d*g)/(d*g - f*e)))*p/g^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*log((x^2*e + d)^p*c)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*log((x^2*e + d)^p*c)/(g*x^2 + f)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{{\left(g\,x^2+f\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*log(c*(d + e*x^2)^p))/(f + g*x^2)^2,x)
```

```
[Out] int((x^3*log(c*(d + e*x^2)^p))/(f + g*x^2)^2, x)
```

3.350 $\int \frac{x \log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2} dx$

Optimal. Leaf size=83

$$\frac{ep\log\left(d+ex^2\right)}{2g(ef-dg)}-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{2g\left(f+gx^2\right)}-\frac{ep\log\left(f+gx^2\right)}{2g(ef-dg)}$$

```
[Out] 1/2*e*p*ln(e*x^2+d)/g/(-d*g+e*f)-1/2*ln(c*(e*x^2+d)^p)/g/(g*x^2+f)-1/2*e*p*
ln(g*x^2+f)/g/(-d*g+e*f)
```

Rubi [A]
time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2525, 2442, 36, 31}

$$-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{2g\left(f+gx^2\right)}+\frac{ep\log\left(d+ex^2\right)}{2g(ef-dg)}-\frac{ep\log\left(f+gx^2\right)}{2g(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] (e*p*Log[d + e*x^2])/(2*g*(e*f - d*g)) - Log[c*(d + e*x^2)^p]/(2*g*(f + g*x
^2)) - (e*p*Log[f + g*x^2])/(2*g*(e*f - d*g))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x\log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{(f+gx)^2}\,dx,x,x^2\right)\\ &= -\frac{\log\left(c(d+ex^2)^p\right)}{2g\left(f+gx^2\right)}+\frac{(ep)\text{Subst}\left(\int \frac{1}{(d+ex)(f+gx)}\,dx,x,x^2\right)}{2g}\\ &= -\frac{\log\left(c(d+ex^2)^p\right)}{2g\left(f+gx^2\right)}-\frac{(ep)\text{Subst}\left(\int \frac{1}{f+gx}\,dx,x,x^2\right)}{2(ef-dg)}+\frac{(e^2p)\,\text{Subst}\left(\int \frac{1}{d+ex}\,dx,x,x\right.}{2g(ef-dg)}\\ &= \frac{ep\log\left(d+ex^2\right)}{2g(ef-dg)}-\frac{\log\left(c(d+ex^2)^p\right)}{2g\left(f+gx^2\right)}-\frac{ep\log\left(f+gx^2\right)}{2g(ef-dg)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 63, normalized size = 0.76

$$\frac{-\frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}+\frac{ep\left(\log\left(d+ex^2\right)-\log\left(f+gx^2\right)\right)}{ef-dg}}{2g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
[Out] (-(Log[c*(d + e*x^2)^p]/(f + g*x^2)) + (e*p*(Log[d + e*x^2] - Log[f + g*x^2
]))/(e*f - d*g))/(2*g)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.51, size = 371, normalized size = 4.47

method	result
risch	$-\frac{\ln\left((e\,x^2+d)^p\right)}{2g(g\,x^2+f)}-\frac{i\pi g\,\text{csgn}\left(i(e\,x^2+d)^p\right)\text{csgn}\left(ic(e\,x^2+d)^p\right)^2 d-i\pi g\,\text{csgn}\left(i(e\,x^2+d)^p\right)\text{csgn}\left(ic(e\,x^2+d)^p\right)\text{csgn}(ic)d-i\pi g\text{csgn}\left(ic(e\,x^2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/g/(g*x^2+f)*ln((e*x^2+d)^p)-1/4*(I*Pi*g*csgn(I*(e*x^2+d)^p)*csgn(I*c*(
e*x^2+d)^p)^2*d-I*Pi*g*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)*
d-I*Pi*g*csgn(I*c*(e*x^2+d)^p)^3*d+I*Pi*g*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)
*d-I*Pi*e*f*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2+I*Pi*e*f*csgn(I*(e*
x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)+I*Pi*e*f*csgn(I*c*(e*x^2+d)^p)^3-
I*Pi*e*f*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+2*ln(-e*x^2-d)*e*g*p*x^2-2*ln(g*
x^2+f)*e*g*p*x^2+2*ln(-e*x^2-d)*e*f*p-2*p*e*f*ln(g*x^2+f)+2*ln(c)*g*d-2*ln(
c)*e*f)/g/(g*x^2+f)/(d*g-e*f)
```

Maxima [A]
time = 0.28, size = 79, normalized size = 0.95

$$\frac{p\left(\frac{\log\left(gx^2+f\right)}{dg-fe}-\frac{\log\left(x^2e+d\right)}{dg-fe}\right)e}{2\,g}-\frac{\log\left(\left(x^2e+d\right)^pc\right)}{2\,\left(gx^2+f\right)g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] 1/2*p*(log(g*x^2 + f)/(d*g - f*e) - log(x^2*e + d)/(d*g - f*e))*e/g - 1/2*l
og((x^2*e + d)^p*c)/((g*x^2 + f)*g)
```

Fricas [A]
time = 0.35, size = 96, normalized size = 1.16

$$\frac{\left(gpx^2+fp\right)e\log\left(gx^2+f\right)-\left(gpx^2e+dgp\right)\log\left(x^2e+d\right)-\left(dg-fe\right)\log\left(c\right)}{2\,\left(dg^3x^2+dfg^2-\left(fg^2x^2+f^2g\right)e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((g*p*x^2 + f*p)*e*log(g*x^2 + f) - (g*p*x^2*e + d*g*p)*log(x^2*e + d)
- (d*g - f*e)*log(c))/(d*g^3*x^2 + d*f*g^2 - (f*g^2*x^2 + f^2*g)*e)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(83) = 166.
time = 4.44, size = 182, normalized size = 2.19

$$-\frac{\left(x^2e+d\right)gpe\log\left(x^2e+d\right)-\left(x^2e+d\right)gpe\log\left(\left(x^2e+d\right)g-dg+fe\right)+dgpe\log\left(\left(x^2e+d\right)g-dg+fe\right)-fpe^2\log\left(\left(x^2e+d\right)g-dg+fe\right)+dge\log\left(c\right)-fe^2\log\left(c\right)}{2\,\left(\left(x^2e+d\right)dg^3-d^2g^3-\left(x^2e+d\right)fg^2e+2\,dfg^2e-f^2ge^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] -1/2*((x^2*e + d)*g*p*e*log(x^2*e + d) - (x^2*e + d)*g*p*e*log((x^2*e + d)*
g - d*g + f*e) + d*g*p*e*log((x^2*e + d)*g - d*g + f*e) - f*p*e^2*log((x^2*
e + d)*g - d*g + f*e) + d*g*e*log(c) - f*e^2*log(c))/((x^2*e + d)*d*g^3 - d
^2*g^3 - (x^2*e + d)*f*g^2*e + 2*d*f*g^2*e - f^2*g*e^2)
```

Mupad [B]
time = 1.46, size = 80, normalized size = 0.96

$$-\frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{2\,g\,\left(g\,x^2+f\right)}-\frac{e\,p\,\mathrm{atan}\left(\frac{x^2\,\left(d\,g\,1{}\mathrm{i}-e\,f\,1{}\mathrm{i}\right)}{2\,d\,f+d\,g\,x^2+e\,f\,x^2}\right)\,1{}\mathrm{i}}{d\,g^2-e\,f\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(c*(d + e*x^2)^p))/(f + g*x^2)^2,x)
```

```
[Out] - log(c*(d + e*x^2)^p)/(2*g*(f + g*x^2)) - (e*p*atan((x^2*(d*g*1i - e*f*1i)
)/(2*d*f + d*g*x^2 + e*f*x^2))*1i)/(d*g^2 - e*f*g)
```

3.351 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x\left(f+gx^2\right)^2}\,dx$

Optimal. Leaf size=201

$$-\frac{ep\log\left(d+ex^2\right)}{2f(ef-dg)}+\frac{\log\left(c\left(d+ex^2\right)^p\right)}{2f\left(f+gx^2\right)}+\frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^2}+\frac{ep\log\left(f+gx^2\right)}{2f(ef-dg)}-\frac{\log\left(c\left(d+ex^2\right)^p\right)\mathrm{lo}}{2f^2}$$

```
[Out] -1/2*e*p*ln(e*x^2+d)/f/(-d*g+e*f)+1/2*ln(c*(e*x^2+d)^p)/f/(g*x^2+f)+1/2*ln(
-e*x^2/d)*ln(c*(e*x^2+d)^p)/f^2+1/2*e*p*ln(g*x^2+f)/f/(-d*g+e*f)-1/2*ln(c*(
e*x^2+d)^p)*ln(e*(g*x^2+f)/(-d*g+e*f))/f^2-1/2*p*polylog(2,-g*(e*x^2+d)/(-d
*g+e*f))/f^2+1/2*p*polylog(2,1+e*x^2/d)/f^2
```

Rubi [A]
time = 0.19, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$,
Rules used = {2525, 46, 2463, 2441, 2352, 2442, 36, 31, 2440, 2438}

$$-\frac{p\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex^2)}{ef-dg}\right)}{2f^2}+\frac{p\,\mathrm{PolyLog}\left(2,\frac{ex^2}{d}+1\right)}{2f^2}-\frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2f^2}+\frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^2}+\frac{\log\left(c\left(d+ex^2\right)^p\right)}{2f\left(f+gx^2\right)}-\frac{ep\log\left(d+ex^2\right)}{2f(ef-dg)}+\frac{ep\log\left(f+gx^2\right)}{2f(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(x*(f + g*x^2)^2),x]
```

```
[Out] -1/2*(e*p*Log[d + e*x^2])/(f*(e*f - d*g)) + Log[c*(d + e*x^2)^p]/(2*f*(f +
g*x^2)) + (Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])/(2*f^2) + (e*p*Log[f + g
*x^2])/(2*f*(e*f - d*g)) - (Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f -
 d*g)])/(2*f^2) - (p*PolyLog[2, -((g*(d + e*x^2))/(e*f - d*g))])/(2*f^2) +
(p*PolyLog[2, 1 + (e*x^2)/d])/(2*f^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
```

```
n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
```

```
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x\left(f+gx^2\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x(f+gx)^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{\log\left(c(d+ex)^p\right)}{f^2x} - \frac{g\log\left(c(d+ex)^p\right)}{f(f+gx)^2} - \frac{g\log\left(c(d+ex)^p\right)}{f^2(f+gx)}\right)\,dx, x, x$$

$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^2\right)}{2f^2} - \frac{g\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{f+gx}\,dx, x, x^2\right)}{2f^2} - \frac{g\text{Subst}\left(\int \text{lo}\right.}{}$$

$$= \frac{\log\left(c(d+ex^2)^p\right)}{2f\left(f+gx^2\right)} + \frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)}{2f^2} - \frac{\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx}{ef-dg}\right.}{2f^2}$$

$$= \frac{\log\left(c(d+ex^2)^p\right)}{2f\left(f+gx^2\right)} + \frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)}{2f^2} - \frac{\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx}{ef-dg}\right.}{2f^2}$$

$$= -\frac{ep\log\left(d+ex^2\right)}{2f(ef-dg)} + \frac{\log\left(c(d+ex^2)^p\right)}{2f\left(f+gx^2\right)} + \frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)}{2f^2} + \frac{ep\log(}{2f(ef}$$

Mathematica [A]
time = 0.07, size = 170, normalized size = 0.85

$$\frac{\frac{efp\log(d+ex^2)}{-ef+dg} + \frac{f\log(c(d+ex^2)^p)}{f+gx^2} + \log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right) + \frac{efp\log(f+gx^2)}{ef-dg} - \log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right) - p\text{Li}_2\left(\frac{g(d+ex^2)}{-ef+dg}\right) + p\text{Li}_2\left(1+\frac{ex^2}{d}\right)}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(x*(f + g*x^2)^2),x]
```

```
[Out] ((e*f*p*Log[d + e*x^2])/(-(e*f) + d*g) + (f*Log[c*(d + e*x^2)^p])/(f + g*x^
2) + Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p] + (e*f*p*Log[f + g*x^2])/(e*f -
 d*g) - Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)] - p*PolyLog[2
, (g*(d + e*x^2))/(-(e*f) + d*g)] + p*PolyLog[2, 1 + (e*x^2)/d])/(2*f^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.46, size = 984, normalized size = 4.90

method	result	size

risch	Expression too large to display	984

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/x/(g*x^2+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/f^2*ln(x)-1/4*I*Pi*csg
n(I*c*(e*x^2+d)^p)^2*csgn(I*c)/f^2*ln(g*x^2+f)+1/4*I*Pi*csgn(I*c*(e*x^2+d)^
p)^2*csgn(I*c)/f/(g*x^2+f)+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/f^2*l
n(x)-1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/f^2*ln(g*x^2+f)+1
/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2/f/(g*x^2+f)+1/4*I*Pi*cs
gn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/f^2*ln(g*x^2+f)-1/4*I*Pi*
csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)/f/(g*x^2+f)-1/4*I*Pi*cs
gn(I*c*(e*x^2+d)^p)^3/f/(g*x^2+f)-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/f^2*ln(x
)+1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^3/f^2*ln(g*x^2+f)-1/2*ln((e*x^2+d)^p)/f^2*
ln(g*x^2+f)+1/2*ln((e*x^2+d)^p)/f/(g*x^2+f)+ln((e*x^2+d)^p)/f^2*ln(x)+ln(c)
/f^2*ln(x)-1/2*ln(c)/f^2*ln(g*x^2+f)-p/f^2*dilog((-e*x+(-e*d)^(1/2))/(-e*d)
^(1/2))-p/f^2*dilog((e*x+(-e*d)^(1/2))/(-e*d)^(1/2))+1/2*p/f^2*sum(ln(x-_al
pha)*ln(g*x^2+f)-ln(x-_alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,
index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))+ln((Ro
otOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*
_Z*_alpha*e*g-d*g+e*f,index=2)))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g
+e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))-d
ilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^
2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)),_alpha=RootOf(_Z^2*e+d))+1/2*ln(c)/
f/(g*x^2+f)-1/2*p*e/f/(d*g-e*f)*ln(g*x^2+f)+1/2*p*e/f/(d*g-e*f)*ln(e*x^2+d)
-p/f^2*ln(x)*ln((-e*x+(-e*d)^(1/2))/(-e*d)^(1/2))-p/f^2*ln(x)*ln((e*x+(-e*d
)^(1/2))/(-e*d)^(1/2))-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*c
sgn(I*c)/f^2*ln(x)
```

Maxima [A]
time = 0.54, size = 208, normalized size = 1.03

$$-\frac{1}{2}p\left(\frac{\left(2\log\left(\frac{x^2e}{d}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{x^2e}{d}\right)\right)e^{(-1)}}{f^2}-\frac{\left(\log\left(gx^2+f\right)\log\left(\frac{gx^2e+fe}{dg-fe}+1\right)+\mathrm{Li}_2\left(-\frac{gx^2e+fe}{dg-fe}\right)\right)e^{(-1)}}{f^2}+\frac{\log\left(gx^2+f\right)}{dfg-f^2e}-\frac{\log\left(x^2e+d\right)}{dfg-f^2e}\right)e+\frac{1}{2}\left(\frac{1}{fgx^2+f^2}-\frac{\log\left(gx^2+f\right)}{f^2}+\frac{\log\left(x^2\right)}{f^2}\right)\log\left(\left(x^2e+d\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] -1/2*p*((2*log(x^2*e/d + 1)*log(x) + dilog(-x^2*e/d))*e^(-1)/f^2 - (log(g*x
^2 + f)*log((g*x^2*e + f*e)/(d*g - f*e) + 1) + dilog(-(g*x^2*e + f*e)/(d*g
- f*e)))*e^(-1)/f^2 + log(g*x^2 + f)/(d*f*g - f^2*e) - log(x^2*e + d)/(d*f*
g - f^2*e))*e + 1/2*(1/(f*g*x^2 + f^2) - log(g*x^2 + f)/f^2 + log(x^2)/f^2)
*log((x^2*e + d)^p*c)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x/(g*x^2+f)^2,x, algorithm="fricas")
[Out] integral(log((x^2*e + d)^p*c)/(g^2*x^5 + 2*f*g*x^3 + f^2*x), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/x/(g*x**2+f)**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x/(g*x^2+f)^2,x, algorithm="giac")
[Out] integrate(log((x^2*e + d)^p*c)/((g*x^2 + f)^2*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{x\,{\left(g\,x^2+f\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)/(x*(f + g*x^2)^2),x)
[Out] int(log(c*(d + e*x^2)^p)/(x*(f + g*x^2)^2), x)
```

3.352 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^3\left(f+gx^2\right)^2}\,dx$

Optimal. Leaf size=251

$$\frac{ep\log(x)}{df^2}-\frac{ep\log\left(d+ex^2\right)}{2df^2}+\frac{egp\log\left(d+ex^2\right)}{2f^2(ef-dg)}-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{2f^2x^2}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{2f^2\left(f+gx^2\right)}-\frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c(\right.}{f^3}$$

```
[Out] e*p*ln(x)/d/f^2-1/2*e*p*ln(e*x^2+d)/d/f^2+1/2*e*g*p*ln(e*x^2+d)/f^2/(-d*g+e
*f)-1/2*ln(c*(e*x^2+d)^p)/f^2/x^2-1/2*g*ln(c*(e*x^2+d)^p)/f^2/(g*x^2+f)-g*l
n(-e*x^2/d)*ln(c*(e*x^2+d)^p)/f^3-1/2*e*g*p*ln(g*x^2+f)/f^2/(-d*g+e*f)+g*ln
(c*(e*x^2+d)^p)*ln(e*(g*x^2+f)/(-d*g+e*f))/f^3+g*p*polylog(2,-g*(e*x^2+d)/(
-d*g+e*f))/f^3-g*p*polylog(2,1+e*x^2/d)/f^3
```

Rubi [A]
time = 0.23, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {2525, 46, 2463, 2442, 36, 29, 31, 2441, 2352, 2440, 2438}

$$\frac{gp\,\text{PolyLog}\left(2,-\frac{g\left(d+ex^2\right)}{ef-dg}\right)}{f^3}-\frac{gp\,\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)}{f^3}-\frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^3}+\frac{g\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{f^3}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{2f^2\left(f+gx^2\right)}-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{2f^2x^2}+\frac{egp\log\left(d+ex^2\right)}{2f^2(ef-dg)}-\frac{egp\log\left(f+gx^2\right)}{2f^2(ef-dg)}-\frac{ep\log\left(d+ex^2\right)}{2df^2}+\frac{ep\log(x)}{df^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(x^3*(f + g*x^2)^2),x]
```

```
[Out] (e*p*Log[x])/(d*f^2) - (e*p*Log[d + e*x^2])/(2*d*f^2) + (e*g*p*Log[d + e*x^
2])/(2*f^2*(e*f - d*g)) - Log[c*(d + e*x^2)^p]/(2*f^2*x^2) - (g*Log[c*(d +
e*x^2)^p])/(2*f^2*(f + g*x^2)) - (g*Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])
/f^3 - (e*g*p*Log[f + g*x^2])/(2*f^2*(e*f - d*g)) + (g*Log[c*(d + e*x^2)^p]
*Log[(e*(f + g*x^2))/(e*f - d*g)])/f^3 + (g*p*PolyLog[2, -((g*(d + e*x^2))/
(e*f - d*g))])/f^3 - (g*p*PolyLog[2, 1 + (e*x^2)/d])/f^3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^3\left(f+gx^2\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x^2(f+gx)^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{\log\left(c(d+ex)^p\right)}{f^2x^2} - \frac{2g\log\left(c(d+ex)^p\right)}{f^3x} + \frac{g^2\log\left(c(d+ex)^p\right)}{f^2(f+gx)^2} + \frac{2g^2\log}{f^3}\right.\right.$$

$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x^2}\,dx, x, x^2\right)}{2f^2} - \frac{g\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^2\right)}{f^3} + \frac{g^2\text{Subst}\left(\int \frac{\log}{}\right.}{}$$

$$= -\frac{\log\left(c(d+ex^2)^p\right)}{2f^2x^2} - \frac{g\log\left(c(d+ex^2)^p\right)}{2f^2\left(f+gx^2\right)} - \frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)}{f^3} + \frac{g\log}{}$$

$$= -\frac{\log\left(c(d+ex^2)^p\right)}{2f^2x^2} - \frac{g\log\left(c(d+ex^2)^p\right)}{2f^2\left(f+gx^2\right)} - \frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)}{f^3} + \frac{g\log}{}$$

$$= \frac{ep\log(x)}{df^2} - \frac{ep\log\left(d+ex^2\right)}{2df^2} + \frac{egp\log\left(d+ex^2\right)}{2f^2(ef-dg)} - \frac{\log\left(c(d+ex^2)^p\right)}{2f^2x^2} - \frac{g\log\left(c(d+\right.}{2f^2\left(f+\right.}$$

Mathematica [A]
time = 0.12, size = 208, normalized size = 0.83

$$\frac{\frac{efp(2\log(x)-\log(d+ex^2))}{d} - \frac{f\log(c(d+ex^2)^p)}{x^2} - \frac{fg\log(c(d+ex^2)^p)}{f+gx^2} + \frac{efgp(\log(d+ex^2)-\log(f+gx^2))}{ef-dg} + 2g\left(\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right) + p\text{Li}_2\left(\frac{g(d+ex^2)}{-ef+dg}\right)\right) - 2g\left(\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right) + p\text{Li}_2\left(1+\frac{ex^2}{d}\right)\right)}{2f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(x^3*(f + g*x^2)^2),x]
```

```
[Out] ((e*f*p*(2*Log[x] - Log[d + e*x^2]))/d - (f*Log[c*(d + e*x^2)^p])/x^2 - (f*
g*Log[c*(d + e*x^2)^p])/(f + g*x^2) + (e*f*g*p*(Log[d + e*x^2] - Log[f + g*
x^2]))/(e*f - d*g) + 2*g*(Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d
*g)] + p*PolyLog[2, (g*(d + e*x^2))/(-(e*f) + d*g)]) - 2*g*(Log[-((e*x^2)/d
)]*Log[c*(d + e*x^2)^p] + p*PolyLog[2, 1 + (e*x^2)/d]))/(2*f^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.45, size = 1216, normalized size = 4.84

method	result	size
risch	Expression too large to display	1216

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/x^3/(g*x^2+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] e*p*ln(x)/d/f^2+1/2*p*e/f^2*g/(d*g-e*f)*ln(g*x^2+f)-p*e/f^2/(d*g-e*f)*ln(e*
x^2+d)*g+1/2*p*e^2/f/d/(d*g-e*f)*ln(e*x^2+d)+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)
^2*csgn(I*c)*g/f^3*ln(g*x^2+f)-1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)*g
/f^2/(g*x^2+f)-I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)/f^3*g*ln(x)+I*Pi*csgn
(I*c*(e*x^2+d)^p)^3/f^3*g*ln(x)-1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^
2+d)^p)^2/f^2/x^2-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3*g/f^3*ln(g*x^2+f)+1/4*I*
Pi*csgn(I*c*(e*x^2+d)^p)^3*g/f^2/(g*x^2+f)-1/4*I*Pi*csgn(I*c*(e*x^2+d)^p)^2
*csgn(I*c)/f^2/x^2-1/2*ln((e*x^2+d)^p)/f^2/x^2+1/4*I*Pi*csgn(I*c*(e*x^2+d)^
p)^3/f^2/x^2+2*p/f^3*g*ln(x)*ln((-e*x+(-e*d)^(1/2))/(-e*d)^(1/2))+2*p/f^3*g
*ln(x)*ln((e*x+(-e*d)^(1/2))/(-e*d)^(1/2))+ln((e*x^2+d)^p)*g/f^3*ln(g*x^2+f
)-1/2*ln((e*x^2+d)^p)*g/f^2/(g*x^2+f)-2*ln((e*x^2+d)^p)/f^3*g*ln(x)-2*ln(c)
/f^3*g*ln(x)+2*p/f^3*g*dilog((-e*x+(-e*d)^(1/2))/(-e*d)^(1/2))+2*p/f^3*g*di
log((e*x+(-e*d)^(1/2))/(-e*d)^(1/2))-p/f^3*g*sum(ln(x-_alpha)*ln(g*x^2+f)-l
n(x-_alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)
/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z
*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+
e*f,index=2)))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_a
lpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((RootOf(_Z^2*
e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*
e*g-d*g+e*f,index=2)),_alpha=RootOf(_Z^2*e+d))+ln(c)*g/f^3*ln(g*x^2+f)-1/2*
ln(c)*g/f^2/(g*x^2+f)-1/2*ln(c)/f^2/x^2-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I
*c*(e*x^2+d)^p)*csgn(I*c)*g/f^3*ln(g*x^2+f)+I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I
*c*(e*x^2+d)^p)*csgn(I*c)/f^3*g*ln(x)+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c
*(e*x^2+d)^p)*csgn(I*c)*g/f^2/(g*x^2+f)+1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I
*c*(e*x^2+d)^p)^2*g/f^3*ln(g*x^2+f)-1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(
e*x^2+d)^p)^2*g/f^2/(g*x^2+f)-I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p
)^2/f^3*g*ln(x)+1/4*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c
)/f^2/x^2
```

Maxima [A]
time = 0.64, size = 310, normalized size = 1.24

$$-\frac{1}{2}\left(f\left(\frac{g\log\left(gx^2+f\right)}{df^3g-f^4e}-\frac{e\log\left(x^2e+d\right)}{d^2f^2g-df^3e}-\frac{\log\left(x^2\right)}{df^3}\right)-2\,g\left(\frac{\log\left(gx^2+f\right)}{df^2g-f^3e}-\frac{\log\left(x^2e+d\right)}{df^2g-f^3e}\right)-\frac{2\left(2\,\log\left(\frac{x^2e}{d}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{x^2e}{d}\right)\right)ge^{(-1)}}{f^3}+\frac{2\left(\log\left(gx^2+f\right)\log\left(\frac{gx^2e+fe}{dg-fe}+1\right)+\mathrm{Li}_2\left(-\frac{gx^2e+fe}{dg-fe}\right)\right)ge^{(-1)}}{f^3}\right)pe-\frac{1}{2}\left(\frac{2\,gx^2+f}{f^2gx^4+f^3x^2}-\frac{2\,g\log\left(gx^2+f\right)}{f^3}+\frac{2\,g\log\left(x^2\right)}{f^3}\right)\log\left(\left(x^2e+d\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^3/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(f*(g*log(g*x^2 + f)/(d*f^3*g - f^4*e) - e*log(x^2*e + d)/(d^2*f^2*g -
 d*f^3*e) - log(x^2)/(d*f^3)) - 2*g*(log(g*x^2 + f)/(d*f^2*g - f^3*e) - log
(x^2*e + d)/(d*f^2*g - f^3*e)) - 2*(2*log(x^2*e/d + 1)*log(x) + dilog(-x^2*
e/d))*g*e^(-1)/f^3 + 2*(log(g*x^2 + f)*log((g*x^2*e + f*e)/(d*g - f*e) + 1)
 + dilog(-(g*x^2*e + f*e)/(d*g - f*e)))*g*e^(-1)/f^3)*p*e - 1/2*((2*g*x^2 +
 f)/(f^2*g*x^4 + f^3*x^2) - 2*g*log(g*x^2 + f)/f^3 + 2*g*log(x^2)/f^3)*log(
(x^2*e + d)^p*c)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^3/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] integral(log((x^2*e + d)^p*c)/(g^2*x^7 + 2*f*g*x^5 + f^2*x^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/x**3/(g*x**2+f)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^3/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((x^2*e + d)^p*c)/((g*x^2 + f)^2*x^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)}{x^3\,{\left(g\,x^2+f\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)/(x^3*(f + g*x^2)^2),x)
```

```
[Out] int(log(c*(d + e*x^2)^p)/(x^3*(f + g*x^2)^2), x)
```

3.353 $\int \frac{x^4 \log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx$

Optimal. Leaf size=802

$$-\frac{2px}{g^2}+\frac{2\sqrt{d}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,g^2}+\frac{\sqrt{d}\,\sqrt{e}\,fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{g^2(ef-dg)}-\frac{e(-f)^{3/2}p\log\left(\sqrt{-f}-\sqrt{g}\,x\right)}{2g^{5/2}(ef-dg)}-\frac{3\sqrt{f}\,p\tan^{-}}{}$$

```
[Out] -2*p*x/g^2+x*ln(c*(e*x^2+d)^p)/g^2-1/2*e*(-f)^(3/2)*p*ln((-f)^(1/2)-x*g^(1/
2))/g^(5/2)/(-d*g+e*f)+1/2*e*(-f)^(3/2)*p*ln((-f)^(1/2)+x*g^(1/2))/g^(5/2)/
(-d*g+e*f)+2*p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/g^2/e^(1/2)+f*p*arctan(x*e
^(1/2)/d^(1/2))*d^(1/2)*e^(1/2)/g^2/(-d*g+e*f)-3/2*arctan(x*g^(1/2)/f^(1/2)
)*ln(c*(e*x^2+d)^p)*f^(1/2)/g^(5/2)-3*p*arctan(x*g^(1/2)/f^(1/2))*ln(2*f^(1
/2)/(f^(1/2)-I*x*g^(1/2)))*f^(1/2)/g^(5/2)+3/2*p*arctan(x*g^(1/2)/f^(1/2))*
ln(-2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/
2)*f^(1/2)-(-d)^(1/2)*g^(1/2)))*f^(1/2)/g^(5/2)+3/2*p*arctan(x*g^(1/2)/f^(1
/2))*ln(2*((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e
^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))*f^(1/2)/g^(5/2)+3/2*I*p*polylog(2,1-2*f
^(1/2)/(f^(1/2)-I*x*g^(1/2)))*f^(1/2)/g^(5/2)-3/4*I*p*polylog(2,1+2*((-d)^(
1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)-(-
d)^(1/2)*g^(1/2)))*f^(1/2)/g^(5/2)-3/4*I*p*polylog(2,1-2*((-d)^(1/2)+x*e^(1
/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^
(1/2)))*f^(1/2)/g^(5/2)-1/4*f*ln(c*(e*x^2+d)^p)/g^(5/2)/((-f)^(1/2)-x*g^(1/
2))+1/4*f*ln(c*(e*x^2+d)^p)/g^(5/2)/((-f)^(1/2)+x*g^(1/2))
```

Rubi [A]
time = 1.19, antiderivative size = 802, normalized size of antiderivative = 1.00, number of steps used = 43, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {2526, 2498, 327, 211, 2521, 2513, 815, 649, 266, 2520, 12, 5048, 4966, 2449, 2352, 2497}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(x^4*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] (-2*p*x)/g^2 + (2*Sqrt[d]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*g^2) + (S
qrt[d]*Sqrt[e]*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(g^2*(e*f - d*g)) - (e*(-f)
^(3/2)*p*Log[Sqrt[-f] - Sqrt[g]*x])/(2*g^(5/2)*(e*f - d*g)) - (3*Sqrt[f]*p*
ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/g^(5/
2) + (3*Sqrt[f]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt
[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqr
```

```
t[g]*x))])/(2*g^(5/2)) + (3*Sqrt[f]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sq
rt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g
])*(Sqrt[f] - I*Sqrt[g]*x))])/(2*g^(5/2)) + (e*(-f)^(3/2)*p*Log[Sqrt[-f] +
Sqrt[g]*x])/(2*g^(5/2)*(e*f - d*g)) + (x*Log[c*(d + e*x^2)^p])/g^2 - (f*Log
[c*(d + e*x^2)^p])/(4*g^(5/2)*(Sqrt[-f] - Sqrt[g]*x)) + (f*Log[c*(d + e*x^2
)^p])/(4*g^(5/2)*(Sqrt[-f] + Sqrt[g]*x)) - (3*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sq
rt[f]]*Log[c*(d + e*x^2)^p])/(2*g^(5/2)) + (((3*I)/2)*Sqrt[f]*p*PolyLog[2,
1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/g^(5/2) - (((3*I)/4)*Sqrt[f]*p*Po
lyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f]
 - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2) - (((3*I)/4)*Sqrt[f
]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*S
qrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
```

```
0] && LtQ[r, 0]))
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\int \frac{x^4 \log\left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \int \left(\frac{\log\left(c(d+ex^2)^p\right)}{g^2} + \frac{f^2 \log\left(c(d+ex^2)^p\right)}{g^2\left(f+gx^2\right)^2} - \frac{2f\log\left(c(d+ex^2)^p\right)}{g^2\left(f+gx^2\right)}\right)dx$$

$$= \frac{\int \log\left(c(d+ex^2)^p\right)\,dx}{g^2} - \frac{(2f)\int \frac{\log(c(d+ex^2)^p)}{f+gx^2}\,dx}{g^2} + \frac{f^2\int \frac{\log(c(d+ex^2)^p)}{(f+gx^2)^2}\,dx}{g^2}$$

$$= \frac{x\log\left(c(d+ex^2)^p\right)}{g^2} - \frac{2\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{g^{5/2}} + \frac{f^2\int\left(-\frac{g\ldots}{4f\left(\sqrt{\ldots}\right.}\right.}{\ldots}$$

$$= -\frac{2px}{g^2} + \frac{x\log\left(c(d+ex^2)^p\right)}{g^2} - \frac{2\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{g^{5/2}} - \frac{f\int \ldots}{\ldots}$$

$$= -\frac{2px}{g^2} + \frac{2\sqrt{d}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,g^2} + \frac{x\log\left(c(d+ex^2)^p\right)}{g^2} - \frac{f\log\left(c(d+ex^2)^p\right)}{4g^{5/2}\left(\sqrt{-f}-\sqrt{g}\,x\ldots\right.}$$

$$= -\frac{2px}{g^2} + \frac{2\sqrt{d}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,g^2} + \frac{x\log\left(c(d+ex^2)^p\right)}{g^2} - \frac{f\log\left(c(d+ex^2)^p\right)}{4g^{5/2}\left(\sqrt{-f}-\sqrt{g}\,x\ldots\right.}$$

$$= -\frac{2px}{g^2} + \frac{2\sqrt{d}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,g^2} - \frac{e(-f)^{3/2}p\log\left(\sqrt{-f}-\sqrt{g}\,x\right)}{2g^{5/2}(ef-dg)} - \frac{4\sqrt{f}\,p\,t\ldots}{\ldots}$$

$$= -\frac{2px}{g^2} + \frac{2\sqrt{d}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,g^2} - \frac{e(-f)^{3/2}p\log\left(\sqrt{-f}-\sqrt{g}\,x\right)}{2g^{5/2}(ef-dg)} - \frac{4\sqrt{f}\,p\,t\ldots}{\ldots}$$

$$= -\frac{2px}{g^2} + \frac{2\sqrt{d}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,g^2} + \frac{\sqrt{d}\,\sqrt{e}\,fp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{g^2(ef-dg)} - \frac{e(-f)^{3/2}p\log\ldots}{2g^{5/2}\ldots}$$

Mathematica [**A**]
time = 2.93, size = 1349, normalized size = 1.68

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] ((6*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*(p*Log[d + e*x^2] - Log[c*(d + e*x^
2)^p]))/g^(5/2) + (4*x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]))/g^2 +
(2*f*x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]))/(g^2*(f + g*x^2)) + p*
((4*(((-I)*Sqrt[d])/Sqrt[e] + x)*(-1 + Log[((-I)*Sqrt[d])/Sqrt[e] + x]))/g^
2 + (4*((I*Sqrt[d])/Sqrt[e] + x)*(-1 + Log[(I*Sqrt[d])/Sqrt[e] + x]))/g^2 +
 (I*f*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]/(Sqrt[f] + I*Sqrt[g]*x) + (Sqrt[e]*(
-Log[I*Sqrt[d] - Sqrt[e]*x] + Log[I*Sqrt[f] - Sqrt[g]*x]))/(Sqrt[e]*Sqrt[f]
 - Sqrt[d]*Sqrt[g])))/g^(5/2) + (I*f*(Log[(I*Sqrt[d])/Sqrt[e] + x]/(Sqrt[f]
 + I*Sqrt[g]*x) + (Sqrt[e]*(-Log[I*Sqrt[d] + Sqrt[e]*x] + Log[I*Sqrt[f] - S
qrt[g]*x]))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])))/g^(5/2) + (f*((-I)*(Sqrt[
e]*Sqrt[f] + Sqrt[d]*Sqrt[g])*Log[((-I)*Sqrt[d])/Sqrt[e] + x] + Sqrt[e]*(I*
Sqrt[f] + Sqrt[g]*x)*(Log[I*Sqrt[d] - Sqrt[e]*x] - Log[I*Sqrt[f] + Sqrt[g]*
x])))/((Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])*g^(5/2)*(Sqrt[f] - I*Sqrt[g]*x))
 - (f*(-(Log[(I*Sqrt[d])/Sqrt[e] + x]/(I*Sqrt[f] + Sqrt[g]*x)) - (I*Sqrt[e]
*(Log[I*Sqrt[d] + Sqrt[e]*x] - Log[I*Sqrt[f] + Sqrt[g]*x]))/(Sqrt[e]*Sqrt[f
] - Sqrt[d]*Sqrt[g])))/g^(5/2) + 4*((x*(2 + f/(f + g*x^2)))/(2*g^2) - (3*Sq
rt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/(2*g^(5/2)))*(-Log[((-I)*Sqrt[d])/Sqrt[e
] + x] - Log[(I*Sqrt[d])/Sqrt[e] + x] + Log[d + e*x^2]) - ((3*I)*Sqrt[f]*(L
og[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*
Sqrt[f] - Sqrt[d]*Sqrt[g])] + PolyLog[2, -((Sqrt[g]*(Sqrt[d] - I*Sqrt[e]*x)
)/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g]))]))/g^(5/2) + ((3*I)*Sqrt[f]*(Log[(I*
Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f
] + Sqrt[d]*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[
e]*Sqrt[f] + Sqrt[d]*Sqrt[g])]))/g^(5/2) + ((3*I)*Sqrt[f]*(Log[((-I)*Sqrt[d
])/Sqrt[e] + x]*Log[(Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - Sq
rt[d]*Sqrt[g])] + PolyLog[2, -((Sqrt[g]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[e]*S
qrt[f] - Sqrt[d]*Sqrt[g]))]))/g^(5/2) - ((3*I)*Sqrt[f]*(Log[((-I)*Sqrt[d])/
Sqrt[e] + x]*Log[(Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[
d]*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f
] + Sqrt[d]*Sqrt[g])]))/g^(5/2)))/4
```

Maple [**F**]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{x^{4} \ln \left(c (e\,x^{2}+d)^{p}\right)}{\left(g\,x^{2}+f\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
[Out] int(x^4*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
[Out] integrate(x^4*log((x^2*e + d)^p*c)/(g*x^2 + f)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
[Out] integral(x^4*log((x^2*e + d)^p*c)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
[Out] integrate(x^4*log((x^2*e + d)^p*c)/(g*x^2 + f)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{{\left(g\,x^2+f\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*log(c*(d + e*x^2)^p))/(f + g*x^2)^2,x)
[Out] int((x^4*log(c*(d + e*x^2)^p))/(f + g*x^2)^2, x)
```

3.354 $\int \frac{x^2 \log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2} \, dx$

Optimal. Leaf size=746

$$-\frac{\sqrt{d}\,\sqrt{e}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{g(ef-dg)}-\frac{e\sqrt{-f}\,p\log\left(\sqrt{-f}-\sqrt{g}\,x\right)}{2g^{3/2}(ef-dg)}+\frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,g^{3/2}}-\frac{p\tan^{-1}}{}$$

```
[Out] -p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)*e^(1/2)/g/(-d*g+e*f)-1/2*e*p*ln((-f)^(
1/2)-x*g^(1/2))*(-f)^(1/2)/g^(3/2)/(-d*g+e*f)+1/2*e*p*ln((-f)^(1/2)+x*g^(1/
2))*(-f)^(1/2)/g^(3/2)/(-d*g+e*f)+1/2*arctan(x*g^(1/2)/f^(1/2))*ln(c*(e*x^2
+d)^p)/g^(3/2)/f^(1/2)+p*arctan(x*g^(1/2)/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*
x*g^(1/2)))/g^(3/2)/f^(1/2)-1/2*p*arctan(x*g^(1/2)/f^(1/2))*ln(-2*((-d)^(1/
2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)-(-d)
^(1/2)*g^(1/2)))/g^(3/2)/f^(1/2)-1/2*p*arctan(x*g^(1/2)/f^(1/2))*ln(2*((-d)
^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+
(-d)^(1/2)*g^(1/2)))/g^(3/2)/f^(1/2)-1/2*I*p*polylog(2,1-2*f^(1/2)/(f^(1/2)
-I*x*g^(1/2)))/g^(3/2)/f^(1/2)+1/4*I*p*polylog(2,1+2*((-d)^(1/2)-x*e^(1/2))
*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2
)))/g^(3/2)/f^(1/2)+1/4*I*p*polylog(2,1-2*((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^
(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))/g^(3/2)
/f^(1/2)+1/4*ln(c*(e*x^2+d)^p)/g^(3/2)/((-f)^(1/2)-x*g^(1/2))-1/4*ln(c*(e*x
^2+d)^p)/g^(3/2)/((-f)^(1/2)+x*g^(1/2))
```

Rubi [A]

time = 1.03, antiderivative size = 746, normalized size of antiderivative = 1.00, number of steps used = 40, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {2526, 2521, 2513, 815, 649, 211, 266, 2520, 12, 5048, 4966, 2449, 2352, 2497}

$$\frac{ip\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{4\sqrt{f}\,g^{3/2}}+\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{4\sqrt{f}\,g^{3/2}}-\frac{ip\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{2\sqrt{f}\,g^{3/2}}+\frac{\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2\sqrt{f}\,g^{3/2}}-\frac{p\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2\sqrt{f}\,g^{3/2}}-\frac{p\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2\sqrt{f}\,g^{3/2}}-\frac{\sqrt{d}\sqrt{e}\,p\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{g(ef-dg)}+\frac{p\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,g^{3/2}}+\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4g^{3/2}\left(\sqrt{-f}-\sqrt{g}\,x\right)}-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4g^{3/2}\left(\sqrt{-f}+\sqrt{g}\,x\right)}-\frac{e\sqrt{-f}\,p\log\left(\sqrt{-f}-\sqrt{g}\,x\right)}{2g^{3/2}(ef-dg)}+\frac{e\sqrt{-f}\,p\log\left(\sqrt{-f}+\sqrt{g}\,x\right)}{2g^{3/2}(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] -((Sqrt[d]*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(g*(e*f - d*g))) - (e*Sqr
t[-f]*p*Log[Sqrt[-f] - Sqrt[g]*x])/(2*g^(3/2)*(e*f - d*g)) + (p*ArcTan[(Sqr
t[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*g^(3/2)
) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt
[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/
(2*Sqrt[f]*g^(3/2)) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]
*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
 I*Sqrt[g]*x))])/(2*Sqrt[f]*g^(3/2)) + (e*Sqrt[-f]*p*Log[Sqrt[-f] + Sqrt[g]
```

```
*x])/(2*g^(3/2)*(e*f - d*g)) + Log[c*(d + e*x^2)^p]/(4*g^(3/2)*(Sqrt[-f] -
Sqrt[g]*x)) - Log[c*(d + e*x^2)^p]/(4*g^(3/2)*(Sqrt[-f] + Sqrt[g]*x)) + (Ar
cTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/(2*Sqrt[f]*g^(3/2)) - ((I/2
)*p*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*g^(3/2))
+ ((I/4)*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sq
rt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*g^(3/
2)) + ((I/4)*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((
I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*g
^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
```

```
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\int \frac{x^2 \log\left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \int \left(-\frac{f\log\left(c(d+ex^2)^p\right)}{g\left(f+gx^2\right)^2}+\frac{\log\left(c(d+ex^2)^p\right)}{g\left(f+gx^2\right)}\right)dx$$

$$= \frac{\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx}{g} - \frac{f\int \frac{\log\left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx}{g}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\,g^{3/2}} - \frac{f\int \left(-\frac{g\log\left(c(d+ex^2)^p\right)}{4f\left(\sqrt{-f}\,\sqrt{g}-gx\right)^2} - \frac{g\log\left(c(d+ex^2\right.}{4f\left(\sqrt{-f}\,\sqrt{g}\right.}\right.}{g}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\,g^{3/2}} + \frac{1}{4}\int \frac{\log\left(c(d+ex^2)^p\right)}{\left(\sqrt{-f}\,\sqrt{g}-gx\right)^2}\,dx + \frac{1}{4}\int \frac{\log\left(c\right.}{\left(\sqrt{-f}\right.}$$

$$= \frac{\log\left(c(d+ex^2)^p\right)}{4g^{3/2}\left(\sqrt{-f}-\sqrt{g}\,x\right)} - \frac{\log\left(c(d+ex^2)^p\right)}{4g^{3/2}\left(\sqrt{-f}+\sqrt{g}\,x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+\right.}{2\sqrt{f}\,g^{3/2}}$$

$$= \frac{\log\left(c(d+ex^2)^p\right)}{4g^{3/2}\left(\sqrt{-f}-\sqrt{g}\,x\right)} - \frac{\log\left(c(d+ex^2)^p\right)}{4g^{3/2}\left(\sqrt{-f}+\sqrt{g}\,x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+\right.}{2\sqrt{f}\,g^{3/2}}$$

$$= -\frac{e\sqrt{-f}\,p\log\left(\sqrt{-f}-\sqrt{g}\,x\right)}{2g^{3/2}(ef-dg)} + \frac{2p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,g^{3/2}} - \frac{p\,\mathrm{t}}{}$$

$$= -\frac{e\sqrt{-f}\,p\log\left(\sqrt{-f}-\sqrt{g}\,x\right)}{2g^{3/2}(ef-dg)} + \frac{2p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{\sqrt{f}\,g^{3/2}} - \frac{p\,\mathrm{t}}{}$$

$$= -\frac{\sqrt{d}\,\sqrt{e}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{g(ef-dg)} - \frac{e\sqrt{-f}\,p\log\left(\sqrt{-f}-\sqrt{g}\,x\right)}{2g^{3/2}(ef-dg)} + \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right.}{\sqrt{}}$$

Mathematica [A]
time = 2.40, size = 1231, normalized size = 1.65

$$\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(-p\log\left(d+ex^2\right)+\log\left(c\left(d+ex^2\right)^p\right)\right)}{2\sqrt{f}g^{3/2}}+\frac{px\log\left(d+ex^2\right)-x\log\left(c\left(d+ex^2\right)^p\right)}{2fg+2g^2x^2}+\frac{1}{4}p\left(-\frac{i\left(\frac{\log\left(-\frac{i\sqrt{d}}{\sqrt{e}}+x\right)}{\sqrt{f}+i\sqrt{g}x}+\frac{\sqrt{e}\left(-\log\left(i\sqrt{d}-\sqrt{e}x\right)+\log\left(i\sqrt{f}-\sqrt{g}x\right)\right)}{\sqrt{e}\sqrt{f}-\sqrt{d}\sqrt{g}}\right)}{g^{3/2}}-\frac{i\left(\frac{\log\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)}{\sqrt{f}+i\sqrt{g}x}+\frac{\sqrt{e}\left(-\log\left(i\sqrt{d}+\sqrt{e}x\right)+\log\left(i\sqrt{f}-\sqrt{g}x\right)\right)}{\sqrt{e}\sqrt{f}+\sqrt{d}\sqrt{g}}\right)}{g^{3/2}}+\frac{-\left(\left(\sqrt{e}\sqrt{f}+\sqrt{d}\sqrt{g}\right)\log\left(-\frac{i\sqrt{d}}{\sqrt{e}}+x\right)\right)+\sqrt{e}\left(\sqrt{f}-i\sqrt{g}x\right)\left(\log\left(i\sqrt{d}-\sqrt{e}x\right)-\log\left(i\sqrt{f}+\sqrt{g}x\right)\right)}{\left(\sqrt{e}\sqrt{f}+\sqrt{d}\sqrt{g}\right)g^{3/2}\left(i\sqrt{f}+\sqrt{g}x\right)}+\frac{-\frac{\log\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)}{i\sqrt{f}+\sqrt{g}x}-\frac{i\sqrt{e}\left(\log\left(i\sqrt{d}+\sqrt{e}x\right)-\log\left(i\sqrt{f}+\sqrt{g}x\right)\right)}{\sqrt{e}\sqrt{f}-\sqrt{d}\sqrt{g}}}{g^{3/2}}+4\left(-\frac{x}{2g\left(f+gx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{f}g^{3/2}}\right)\left(-\log\left(-\frac{i\sqrt{d}}{\sqrt{e}}+x\right)-\log\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)+\log\left(d+ex^2\right)\right)+\frac{i\left(\log\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)\log\left(\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}x\right)}{\sqrt{e}\sqrt{f}-\sqrt{d}\sqrt{g}}\right)+\text{Li}_2\left(-\frac{\sqrt{g}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{e}\sqrt{f}-\sqrt{d}\sqrt{g}}\right)\right)}{\sqrt{f}g^{3/2}}-\frac{i\left(\log\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)\log\left(\frac{\sqrt{e}\left(\sqrt{f}+i\sqrt{g}x\right)}{\sqrt{e}\sqrt{f}+\sqrt{d}\sqrt{g}}\right)+\text{Li}_2\left(\frac{\sqrt{g}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{e}\sqrt{f}+\sqrt{d}\sqrt{g}}\right)\right)}{\sqrt{f}g^{3/2}}-\frac{i\left(\log\left(-\frac{i\sqrt{d}}{\sqrt{e}}+x\right)\log\left(\frac{\sqrt{e}\left(\sqrt{f}+i\sqrt{g}x\right)}{\sqrt{e}\sqrt{f}-\sqrt{d}\sqrt{g}}\right)+\text{Li}_2\left(-\frac{\sqrt{g}\left(\sqrt{d}+i\sqrt{e}x\right)}{\sqrt{e}\sqrt{f}-\sqrt{d}\sqrt{g}}\right)\right)}{\sqrt{f}g^{3/2}}+\frac{i\left(\log\left(-\frac{i\sqrt{d}}{\sqrt{e}}+x\right)\log\left(\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}x\right)}{\sqrt{e}\sqrt{f}+\sqrt{d}\sqrt{g}}\right)+\text{Li}_2\left(\frac{\sqrt{g}\left(\sqrt{d}+i\sqrt{e}x\right)}{\sqrt{e}\sqrt{f}+\sqrt{d}\sqrt{g}}\right)\right)}{\sqrt{f}g^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]))/
(2*Sqrt[f]*g^(3/2)) + (p*x*Log[d + e*x^2] - x*Log[c*(d + e*x^2)^p])/(2*f*g
+ 2*g^2*x^2) + (p*(((-I)*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]/(Sqrt[f] + I*Sqrt
[g]*x) + (Sqrt[e]*(-Log[I*Sqrt[d] - Sqrt[e]*x] + Log[I*Sqrt[f] - Sqrt[g]*x]
))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g])))/g^(3/2) - (I*(Log[(I*Sqrt[d])/Sqrt
[e] + x]/(Sqrt[f] + I*Sqrt[g]*x) + (Sqrt[e]*(-Log[I*Sqrt[d] + Sqrt[e]*x] +
Log[I*Sqrt[f] - Sqrt[g]*x]))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])))/g^(3/2)
+ (-((Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])*Log[((-I)*Sqrt[d])/Sqrt[e] + x]) +
 Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x)*(Log[I*Sqrt[d] - Sqrt[e]*x] - Log[I*Sqrt[f
] + Sqrt[g]*x]))/((Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])*g^(3/2)*(I*Sqrt[f] +
Sqrt[g]*x)) + (-(Log[(I*Sqrt[d])/Sqrt[e] + x]/(I*Sqrt[f] + Sqrt[g]*x)) - (I
*Sqrt[e]*(Log[I*Sqrt[d] + Sqrt[e]*x] - Log[I*Sqrt[f] + Sqrt[g]*x]))/(Sqrt[e
]*Sqrt[f] - Sqrt[d]*Sqrt[g]))/g^(3/2) + 4*(-1/2*x/(g*(f + g*x^2)) + ArcTan[
(Sqrt[g]*x)/Sqrt[f]]/(2*Sqrt[f]*g^(3/2)))*(-Log[((-I)*Sqrt[d])/Sqrt[e] + x]
 - Log[(I*Sqrt[d])/Sqrt[e] + x] + Log[d + e*x^2]) + (I*(Log[(I*Sqrt[d])/Sqr
t[e] + x]*Log[(Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*
Sqrt[g])] + PolyLog[2, -((Sqrt[g]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f]
 - Sqrt[d]*Sqrt[g]))]))/(Sqrt[f]*g^(3/2)) - (I*(Log[(I*Sqrt[d])/Sqrt[e] + x
]*Log[(Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])
] + PolyLog[2, (Sqrt[g]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]
*Sqrt[g])]))/(Sqrt[f]*g^(3/2)) - (I*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(S
qrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g])] + Poly
Log[2, -((Sqrt[g]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[
g]))]))/(Sqrt[f]*g^(3/2)) + (I*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e
]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])] + PolyLog[2
, (Sqrt[g]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])]))/
(Sqrt[f]*g^(3/2))))/4
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{x^2 \ln\left(c\left(e\,x^2+d\right)^p\right)}{\left(g\,x^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

```
[Out] int(x^2*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
[Out] integrate(x^2*log((x^2*e + d)^p*c)/(g*x^2 + f)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
[Out] integral(x^2*log((x^2*e + d)^p*c)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
[Out] integrate(x^2*log((x^2*e + d)^p*c)/(g*x^2 + f)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{{\left(g\,x^2+f\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*log(c*(d + e*x^2)^p))/(f + g*x^2)^2,x)
[Out] int((x^2*log(c*(d + e*x^2)^p))/(f + g*x^2)^2, x)
```

3.355 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx$

Optimal. Leaf size=751

$$\frac{\sqrt{d}\,\sqrt{e}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{f(ef-dg)}-\frac{ep\log\left(\sqrt{-f}-\sqrt{g}\,x\right)}{2\sqrt{-f}\,\sqrt{g}\,(ef-dg)}+\frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{f^{3/2}\sqrt{g}}-\frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right.}{}$$

```
[Out] p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)*e^(1/2)/f/(-d*g+e*f)+1/2*arctan(x*g^(1/
2)/f^(1/2))*ln(c*(e*x^2+d)^p)/f^(3/2)/g^(1/2)+p*arctan(x*g^(1/2)/f^(1/2))*l
n(2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(3/2)/g^(1/2)-1/2*p*arctan(x*g^(1/2)/f
^(1/2))*ln(-2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/
(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2)))/f^(3/2)/g^(1/2)-1/2*p*arctan(x*g^(1
/2)/f^(1/2))*ln(2*((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/
2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))/f^(3/2)/g^(1/2)-1/2*I*p*polylog
(2,1-2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(3/2)/g^(1/2)+1/4*I*p*polylog(2,1+2
*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^
(1/2)-(-d)^(1/2)*g^(1/2)))/f^(3/2)/g^(1/2)+1/4*I*p*polylog(2,1-2*((-d)^(1/2
)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^
(1/2)*g^(1/2)))/f^(3/2)/g^(1/2)-1/2*e*p*ln((-f)^(1/2)-x*g^(1/2))/(-d*g+e*f)
/(-f)^(1/2)/g^(1/2)+1/2*e*p*ln((-f)^(1/2)+x*g^(1/2))/(-d*g+e*f)/(-f)^(1/2)/
g^(1/2)-1/4*ln(c*(e*x^2+d)^p)/f/g^(1/2)/((-f)^(1/2)-x*g^(1/2))+1/4*ln(c*(e*
x^2+d)^p)/f/g^(1/2)/((-f)^(1/2)+x*g^(1/2))
```

Rubi [A]
time = 0.59, antiderivative size = 751, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {2521, 2513, 815, 649, 211, 266, 2520, 12, 5048, 4966, 2449, 2352, 2497}

$$\frac{ip\,\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{4f^{3/2}\sqrt{g}}+\frac{ip\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{4f^{3/2}\sqrt{g}}-\frac{ip\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{2f^{3/2}\sqrt{g}}+\frac{\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^{3/2}\sqrt{g}}-\frac{p\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2f^{3/2}\sqrt{g}}-\frac{p\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}\,x\right)}{\left(\sqrt{f}-i\sqrt{g}\,x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2f^{3/2}\sqrt{g}}+\frac{\sqrt{d}\sqrt{e}\,p\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{f(ef-dg)}+\frac{p\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{f^{3/2}\sqrt{g}}-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}\,x\right)}+\frac{\log\left(c\left(d+ex^2\right)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}\,x\right)}-\frac{ep\log\left(\sqrt{-f}-\sqrt{g}\,x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}+\frac{ep\log\left(\sqrt{-f}+\sqrt{g}\,x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(f + g*x^2)^2,x]
```

```
[Out] (Sqrt[d]*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(f*(e*f - d*g)) - (e*p*Log[
Sqrt[-f] - Sqrt[g]*x])/(2*Sqrt[-f]*Sqrt[g]*(e*f - d*g)) + (p*ArcTan[(Sqrt[g
]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(f^(3/2)*Sqrt[g]) -
 (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]
*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(2*
f^(3/2)*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(S
qrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*
Sqrt[g]*x))])/(2*f^(3/2)*Sqrt[g]) + (e*p*Log[Sqrt[-f] + Sqrt[g]*x])/(2*Sqrt
```

```
[-f]*Sqrt[g]*(e*f - d*g)) - Log[c*(d + e*x^2)^p]/(4*f*Sqrt[g]*(Sqrt[-f] - S
qrt[g]*x)) + Log[c*(d + e*x^2)^p]/(4*f*Sqrt[g]*(Sqrt[-f] + Sqrt[g]*x)) + (A
rcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/(2*f^(3/2)*Sqrt[g]) - ((I/
2)*p*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(f^(3/2)*Sqrt[g])
 + ((I/4)*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*S
qrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(f^(3/2)*Sqrt
[g]) + ((I/4)*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(
(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(f^(3/2)*
Sqrt[g])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \int \left(-\frac{g\log\left(c(d+ex^2)^p\right)}{4f\left(\sqrt{-f}\,\sqrt{g}-gx\right)^2} - \frac{g\log\left(c(d+ex^2)^p\right)}{4f\left(\sqrt{-f}\,\sqrt{g}+gx\right)^2} - \frac{g\log\left(c(d+ex^2)^p\right)}{2f\left(-fg-g^2x^2\right)}\right)$$

$$= -\frac{g\int \frac{\log(c(d+ex^2)^p)}{\left(\sqrt{-f}\,\sqrt{g}-gx\right)^2}\,dx}{4f} - \frac{g\int \frac{\log(c(d+ex^2)^p)}{\left(\sqrt{-f}\,\sqrt{g}+gx\right)^2}\,dx}{4f} - \frac{g\int \frac{\log(c(d+ex^2)^p)}{-fg-g^2x^2}\,dx}{2f}$$

$$= -\frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}\,x\right)} + \frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}\,x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\right.}{2f^{3/2}\sqrt{g}}$$

$$= -\frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}\,x\right)} + \frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}\,x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\right.}{2f^{3/2}\sqrt{g}}$$

$$= -\frac{ep\log\left(\sqrt{-f}-\sqrt{g}\,x\right)}{2\sqrt{-f}\,\sqrt{g}\,(ef-dg)} + \frac{ep\log\left(\sqrt{-f}+\sqrt{g}\,x\right)}{2\sqrt{-f}\,\sqrt{g}\,(ef-dg)} - \frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}\,x\right.}$$

$$= -\frac{ep\log\left(\sqrt{-f}-\sqrt{g}\,x\right)}{2\sqrt{-f}\,\sqrt{g}\,(ef-dg)} + \frac{ep\log\left(\sqrt{-f}+\sqrt{g}\,x\right)}{2\sqrt{-f}\,\sqrt{g}\,(ef-dg)} - \frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}\,x\right.}$$

$$= \frac{\sqrt{d}\,\sqrt{e}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{f(ef-dg)} - \frac{ep\log\left(\sqrt{-f}-\sqrt{g}\,x\right)}{2\sqrt{-f}\,\sqrt{g}\,(ef-dg)} + \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\right.}{f^{3/2}\sqrt{g}}$$

$$= \frac{\sqrt{d}\,\sqrt{e}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{f(ef-dg)} - \frac{ep\log\left(\sqrt{-f}-\sqrt{g}\,x\right)}{2\sqrt{-f}\,\sqrt{g}\,(ef-dg)} + \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\right.}{f^{3/2}\sqrt{g}}$$

$$= \frac{\sqrt{d}\,\sqrt{e}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{f(ef-dg)} - \frac{ep\log\left(\sqrt{-f}-\sqrt{g}\,x\right)}{2\sqrt{-f}\,\sqrt{g}\,(ef-dg)} + \frac{p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\right.}{f^{3/2}\sqrt{g}}$$

Mathematica [A]
time = 2.33, size = 1236, normalized size = 1.65

$$\frac{1}{2}\left(\frac{x\left(-p\log\left(d+ex^2\right)+\log\left(c\left(d+ex^2\right)^p\right)\right)}{f\left(f+gx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(-p\log\left(d+ex^2\right)+\log\left(c\left(d+ex^2\right)^p\right)\right)}{f^{3/2}\sqrt{g}}+\frac{1}{2}p\left(\frac{i\left(\frac{\log\left(-\frac{i\sqrt{d}}{\sqrt{e}}+x\right)}{\sqrt{f}+i\sqrt{g}x}+\frac{\sqrt{e}\left(-\log\left(i\sqrt{d}-\sqrt{e}x\right)+\log\left(i\sqrt{f}-\sqrt{g}x\right)\right)}{\sqrt{e}\sqrt{f}-\sqrt{d}\sqrt{g}}\right)}{f\sqrt{g}}+\frac{i\left(\frac{\log\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)}{\sqrt{f}+i\sqrt{g}x}+\frac{\sqrt{e}\left(-\log\left(i\sqrt{d}+\sqrt{e}x\right)+\log\left(i\sqrt{f}-\sqrt{g}x\right)\right)}{\sqrt{e}\sqrt{f}+\sqrt{d}\sqrt{g}}\right)}{f\sqrt{g}}+\frac{-i\left(\sqrt{e}\sqrt{f}+\sqrt{d}\sqrt{g}\right)\log\left(-\frac{i\sqrt{d}}{\sqrt{e}}+x\right)+\sqrt{e}\left(i\sqrt{f}+\sqrt{g}x\right)\left(\log\left(i\sqrt{d}-\sqrt{e}x\right)-\log\left(i\sqrt{f}+\sqrt{g}x\right)\right)}{f\left(\sqrt{e}\sqrt{f}+\sqrt{d}\sqrt{g}\right)\sqrt{g}\left(\sqrt{f}-i\sqrt{g}x\right)}-\frac{-\frac{\log\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)}{i\sqrt{f}+\sqrt{g}x}-\frac{i\sqrt{e}\left(\log\left(i\sqrt{d}+\sqrt{e}x\right)-\log\left(i\sqrt{f}+\sqrt{g}x\right)\right)}{\sqrt{e}\sqrt{f}-\sqrt{d}\sqrt{g}}}{f\sqrt{g}}+2\left(\frac{x}{f^2+fgx^2}+\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{f^{3/2}\sqrt{g}}\right)\left(-\log\left(-\frac{i\sqrt{d}}{\sqrt{e}}+x\right)-\log\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)+\log\left(d+ex^2\right)\right)+\frac{i\left(\log\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)\log\left(\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}x\right)}{\sqrt{e}\sqrt{f}-\sqrt{d}\sqrt{g}}\right)+\mathrm{Li}_2\left(-\frac{\sqrt{g}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{e}\sqrt{f}-\sqrt{d}\sqrt{g}}\right)\right)}{f^{3/2}\sqrt{g}}-\frac{i\left(\log\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right)\log\left(\frac{\sqrt{e}\left(\sqrt{f}+i\sqrt{g}x\right)}{\sqrt{e}\sqrt{f}+\sqrt{d}\sqrt{g}}\right)+\mathrm{Li}_2\left(\frac{\sqrt{g}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{e}\sqrt{f}+\sqrt{d}\sqrt{g}}\right)\right)}{f^{3/2}\sqrt{g}}-\frac{i\left(\log\left(-\frac{i\sqrt{d}}{\sqrt{e}}+x\right)\log\left(\frac{\sqrt{e}\left(\sqrt{f}+i\sqrt{g}x\right)}{\sqrt{e}\sqrt{f}-\sqrt{d}\sqrt{g}}\right)+\mathrm{Li}_2\left(-\frac{\sqrt{g}\left(\sqrt{d}+i\sqrt{e}x\right)}{\sqrt{e}\sqrt{f}-\sqrt{d}\sqrt{g}}\right)\right)}{f^{3/2}\sqrt{g}}+\frac{i\left(\log\left(-\frac{i\sqrt{d}}{\sqrt{e}}+x\right)\log\left(\frac{\sqrt{e}\left(\sqrt{f}-i\sqrt{g}x\right)}{\sqrt{e}\sqrt{f}+\sqrt{d}\sqrt{g}}\right)+\mathrm{Li}_2\left(\frac{\sqrt{g}\left(\sqrt{d}+i\sqrt{e}x\right)}{\sqrt{e}\sqrt{f}+\sqrt{d}\sqrt{g}}\right)\right)}{f^{3/2}\sqrt{g}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^2)^2,x]
```

```
[Out] ((x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]))/(f*(f + g*x^2)) + (ArcTan
[(Sqrt[g]*x)/Sqrt[f]]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]))/(f^(3/2
)*Sqrt[g]) + (p*((I*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]/(Sqrt[f] + I*Sqrt[g]*x
) + (Sqrt[e]*(-Log[I*Sqrt[d] - Sqrt[e]*x] + Log[I*Sqrt[f] - Sqrt[g]*x]))/(S
qrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g])))/(f*Sqrt[g]) + (I*(Log[(I*Sqrt[d])/Sqrt[
e] + x]/(Sqrt[f] + I*Sqrt[g]*x) + (Sqrt[e]*(-Log[I*Sqrt[d] + Sqrt[e]*x] + L
og[I*Sqrt[f] - Sqrt[g]*x]))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])))/(f*Sqrt[g
]) + ((-I)*(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])*Log[((-I)*Sqrt[d])/Sqrt[e] +
 x] + Sqrt[e]*(I*Sqrt[f] + Sqrt[g]*x)*(Log[I*Sqrt[d] - Sqrt[e]*x] - Log[I*S
qrt[f] + Sqrt[g]*x]))/(f*(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])*Sqrt[g]*(Sqrt[
f] - I*Sqrt[g]*x)) - (-(Log[(I*Sqrt[d])/Sqrt[e] + x]/(I*Sqrt[f] + Sqrt[g]*x
)) - (I*Sqrt[e]*(Log[I*Sqrt[d] + Sqrt[e]*x] - Log[I*Sqrt[f] + Sqrt[g]*x]))/
(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g]))/(f*Sqrt[g]) + 2*(x/(f^2 + f*g*x^2) + A
rcTan[(Sqrt[g]*x)/Sqrt[f]]/(f^(3/2)*Sqrt[g]))*(-Log[((-I)*Sqrt[d])/Sqrt[e]
+ x] - Log[(I*Sqrt[d])/Sqrt[e] + x] + Log[d + e*x^2]) + (I*(Log[(I*Sqrt[d])
/Sqrt[e] + x]*Log[(Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - Sqrt
[d]*Sqrt[g])] + PolyLog[2, -((Sqrt[g]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[e]*Sqr
t[f] - Sqrt[d]*Sqrt[g]))]))/(f^(3/2)*Sqrt[g]) - (I*(Log[(I*Sqrt[d])/Sqrt[e]
 + x]*Log[(Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt
[g])] + PolyLog[2, (Sqrt[g]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] + Sqr
t[d]*Sqrt[g])]))/(f^(3/2)*Sqrt[g]) - (I*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Lo
g[(Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g])] +
PolyLog[2, -((Sqrt[g]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*S
qrt[g]))]))/(f^(3/2)*Sqrt[g]) + (I*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sq
rt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])] + PolyL
og[2, (Sqrt[g]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])
]))/(f^(3/2)*Sqrt[g])))/2)/2
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(c \left(e\,x^{2}+d\right)^{p}\right)}{\left(g\,x^{2}+f\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

```
[Out] int(ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
[Out] integrate(log((x^2*e + d)^p*c)/(g*x^2 + f)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
[Out] integral(log((x^2*e + d)^p*c)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
[Out] integrate(log((x^2*e + d)^p*c)/(g*x^2 + f)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{{\left(g\,x^2+f\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)/(f + g*x^2)^2,x)
[Out] int(log(c*(d + e*x^2)^p)/(f + g*x^2)^2, x)
```

3.356 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^2\left(f+gx^2\right)^2}\,dx$

Optimal. Leaf size=803

$$\frac{2\sqrt{e}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,f^2}-\frac{\sqrt{d}\,\sqrt{e}\,gp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{f^2(ef-dg)}-\frac{e\sqrt{g}\,p\log\left(\sqrt{-f}-\sqrt{g}\,x\right)}{2(-f)^{3/2}(ef-dg)}-\frac{3\sqrt{g}\,p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log}{f^{5/2}}$$

```
[Out] -ln(c*(e*x^2+d)^p)/f^2/x+2*p*arctan(x*e^(1/2)/d^(1/2))*e^(1/2)/f^2/d^(1/2)-
g*p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)*e^(1/2)/f^2/(-d*g+e*f)-3/2*arctan(x*g
^(1/2)/f^(1/2))*ln(c*(e*x^2+d)^p)*g^(1/2)/f^(5/2)-1/2*e*p*ln((-f)^(1/2)-x*g
^(1/2))*g^(1/2)/(-f)^(3/2)/(-d*g+e*f)-3*p*arctan(x*g^(1/2)/f^(1/2))*ln(2*f^
(1/2)/(f^(1/2)-I*x*g^(1/2)))*g^(1/2)/f^(5/2)+1/2*e*p*ln((-f)^(1/2)+x*g^(1/2
))*g^(1/2)/(-f)^(3/2)/(-d*g+e*f)+3/2*p*arctan(x*g^(1/2)/f^(1/2))*ln(-2*((-d
)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)
-(-d)^(1/2)*g^(1/2)))*g^(1/2)/f^(5/2)+3/2*p*arctan(x*g^(1/2)/f^(1/2))*ln(2*
((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(
1/2)+(-d)^(1/2)*g^(1/2)))*g^(1/2)/f^(5/2)+3/2*I*p*polylog(2,1-2*f^(1/2)/(f^
(1/2)-I*x*g^(1/2)))*g^(1/2)/f^(5/2)-3/4*I*p*polylog(2,1+2*((-d)^(1/2)-x*e^(
1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g
^(1/2)))*g^(1/2)/f^(5/2)-3/4*I*p*polylog(2,1-2*((-d)^(1/2)+x*e^(1/2))*f^(1/
2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))*g^
(1/2)/f^(5/2)+1/4*ln(c*(e*x^2+d)^p)*g^(1/2)/f^2/((-f)^(1/2)-x*g^(1/2))-1/4*
ln(c*(e*x^2+d)^p)*g^(1/2)/f^2/((-f)^(1/2)+x*g^(1/2))
```

Rubi [A]

time = 1.10, antiderivative size = 803, normalized size of antiderivative = 1.00, number of steps used = 42, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {2526, 2505, 211, 2521, 2513, 815, 649, 266, 2520, 12, 5048, 4966, 2449, 2352, 2497}

$$-\frac{\sqrt{d}\,\sqrt{e}\,gp\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{f^2(ef-dg)}+\frac{2\sqrt{e}\,p\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,f^2}-\frac{e\sqrt{g}\,p\log\left(\sqrt{-f}-\sqrt{g}\,x\right)}{2(-f)^{3/2}(ef-dg)}-\frac{3\sqrt{g}\,p\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{f^{5/2}}+\frac{3\sqrt{g}\,p\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(i\sqrt{e}\,\sqrt{f}-\sqrt{-d}\,\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}\right)}{2f^{5/2}}+\frac{3\sqrt{g}\,p\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{e}\,x+\sqrt{-d}\right)}{\left(i\sqrt{e}\,\sqrt{f}+\sqrt{-d}\,\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}\right)}{2f^{5/2}}+\frac{e\sqrt{g}\,p\log\left(\sqrt{g}\,x+\sqrt{-f}\right)}{2(-f)^{3/2}(ef-dg)}-\frac{3\sqrt{g}\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c\left(ex^2+d\right)^p\right)}{2f^{5/2}}-\frac{\log\left(c\left(ex^2+d\right)^p\right)}{f^2x}+\frac{\sqrt{g}\,\log\left(c\left(ex^2+d\right)^p\right)}{4f^2\left(\sqrt{-f}-\sqrt{g}\,x\right)}-\frac{\sqrt{g}\,\log\left(c\left(ex^2+d\right)^p\right)}{4f^2\left(\sqrt{g}\,x+\sqrt{-f}\right)}+\frac{3i\sqrt{g}\,p\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}\,x}\right)}{2f^{5/2}}-\frac{3i\sqrt{g}\,p\text{PolyLog}\left(2,\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(i\sqrt{e}\,\sqrt{f}-\sqrt{-d}\,\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}+1\right)}{4f^{5/2}}-\frac{3i\sqrt{g}\,p\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\,\sqrt{g}\left(\sqrt{e}\,x+\sqrt{-d}\right)}{\left(i\sqrt{e}\,\sqrt{f}+\sqrt{-d}\,\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}\right)}{4f^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^2)^p]/(x^2*(f + g*x^2)^2),x]
```

```
[Out] (2*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*f^2) - (Sqrt[d]*Sqrt[e]*
g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(f^2*(e*f - d*g)) - (e*Sqrt[g]*p*Log[Sqrt[
-f] - Sqrt[g]*x])/(2*(-f)^(3/2)*(e*f - d*g)) - (3*Sqrt[g]*p*ArcTan[(Sqrt[g]
*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/f^(5/2) + (3*Sqrt[g]
*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*
x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(2*f
```

```
^(5/2)) + (3*Sqrt[g]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(
Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I
*Sqrt[g]*x))])/(2*f^(5/2)) + (e*Sqrt[g]*p*Log[Sqrt[-f] + Sqrt[g]*x])/(2*(-f
)^(3/2)*(e*f - d*g)) - Log[c*(d + e*x^2)^p]/(f^2*x) + (Sqrt[g]*Log[c*(d + e
*x^2)^p])/(4*f^2*(Sqrt[-f] - Sqrt[g]*x)) - (Sqrt[g]*Log[c*(d + e*x^2)^p])/(
4*f^2*(Sqrt[-f] + Sqrt[g]*x)) - (3*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[
c*(d + e*x^2)^p])/(2*f^(5/2)) + (((3*I)/2)*Sqrt[g]*p*PolyLog[2, 1 - (2*Sqrt
[f])/(Sqrt[f] - I*Sqrt[g]*x)])/f^(5/2) - (((3*I)/4)*Sqrt[g]*p*PolyLog[2, 1
+ (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]
*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/f^(5/2) - (((3*I)/4)*Sqrt[g]*p*PolyLog
[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sq
rt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/f^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
```

```
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^2\left(f+gx^2\right)^2}\,dx = \int \left(\frac{\log\left(c(d+ex^2)^p\right)}{f^2x^2} - \frac{g\log\left(c(d+ex^2)^p\right)}{f\left(f+gx^2\right)^2} - \frac{g\log\left(c(d+ex^2)^p\right)}{f^2\left(f+gx^2\right)}\right)dx$$

$$= \frac{\int \frac{\log\left(c(d+ex^2)^p\right)}{x^2}\,dx}{f^2} - \frac{g\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx}{f^2} - \frac{g\int \frac{\log\left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx}{f}$$

$$= -\frac{\log\left(c(d+ex^2)^p\right)}{f^2x} - \frac{\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{f^{5/2}} - \frac{g\int\left(-\frac{g\log(c(d+e}{4f\left(\sqrt{-f}\,\sqrt{\right.}\right.}$$

$$= \frac{2\sqrt{e}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,f^2} - \frac{\log\left(c(d+ex^2)^p\right)}{f^2x} - \frac{\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{f^{5/2}}$$

$$= \frac{2\sqrt{e}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,f^2} - \frac{\log\left(c(d+ex^2)^p\right)}{f^2x} + \frac{\sqrt{g}\,\log\left(c(d+ex^2)^p\right)}{4f^2\left(\sqrt{-f}-\sqrt{g}\,x\right)} - \frac{\sqrt{g}\,\log\left(c(\right.}{4f^2\left(\sqrt{-f}\right.}$$

$$= \frac{2\sqrt{e}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,f^2} - \frac{\log\left(c(d+ex^2)^p\right)}{f^2x} + \frac{\sqrt{g}\,\log\left(c(d+ex^2)^p\right)}{4f^2\left(\sqrt{-f}-\sqrt{g}\,x\right)} - \frac{\sqrt{g}\,\log\left(c(\right.}{4f^2\left(\sqrt{-f}\right.}$$

$$= \frac{2\sqrt{e}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,f^2} - \frac{e\sqrt{g}\,p\log\left(\sqrt{-f}-\sqrt{g}\,x\right)}{2(-f)^{3/2}(ef-dg)} - \frac{2\sqrt{g}\,p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log}{f^{5/2}}$$

$$= \frac{2\sqrt{e}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,f^2} - \frac{e\sqrt{g}\,p\log\left(\sqrt{-f}-\sqrt{g}\,x\right)}{2(-f)^{3/2}(ef-dg)} - \frac{2\sqrt{g}\,p\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log}{f^{5/2}}$$

$$= \frac{2\sqrt{e}\,p\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,f^2} - \frac{\sqrt{d}\,\sqrt{e}\,gp\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{f^2(ef-dg)} - \frac{e\sqrt{g}\,p\log\left(\sqrt{-f}-\sqrt{g}\,x\right.}{2(-f)^{3/2}(ef-dg)}$$

Mathematica [A]
time = 3.21, size = 1438, normalized size = 1.79

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(x^2*(f + g*x^2)^2),x]
```

```
[Out] ((4*p*Log[d + e*x^2] - 4*Log[c*(d + e*x^2)^p])/(f^2*x) + (2*g*x*(p*Log[d +
e*x^2] - Log[c*(d + e*x^2)^p]))/(f^2*(f + g*x^2)) + (6*Sqrt[g]*ArcTan[(Sqrt
[g]*x)/Sqrt[f]]*(p*Log[d + e*x^2] - Log[c*(d + e*x^2)^p]))/f^(5/2) + p*(((4
*I)*(Sqrt[e]*x*Log[x] + I*Sqrt[d]*Log[((-I)*Sqrt[d])/Sqrt[e] + x] - Sqrt[e]
*x*Log[I*Sqrt[d] - Sqrt[e]*x]))/(Sqrt[d]*f^2*x) - (4*(I*Sqrt[e]*x*Log[x] +
Sqrt[d]*Log[(I*Sqrt[d])/Sqrt[e] + x] - I*Sqrt[e]*x*Log[I*Sqrt[d] + Sqrt[e]*
x]))/(Sqrt[d]*f^2*x) - (I*Sqrt[g]*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]/(Sqrt[f]
 + I*Sqrt[g]*x) + (Sqrt[e]*(-Log[I*Sqrt[d] - Sqrt[e]*x] + Log[I*Sqrt[f] - S
qrt[g]*x]))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g])))/f^2 - (I*Sqrt[g]*(Log[(I*
Sqrt[d])/Sqrt[e] + x]/(Sqrt[f] + I*Sqrt[g]*x) + (Sqrt[e]*(-Log[I*Sqrt[d] +
Sqrt[e]*x] + Log[I*Sqrt[f] - Sqrt[g]*x]))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g
])))/f^2 + (Sqrt[g]*(-((Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])*Log[((-I)*Sqrt[d
])/Sqrt[e] + x]) + Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x)*(Log[I*Sqrt[d] - Sqrt[e]
*x] - Log[I*Sqrt[f] + Sqrt[g]*x])))/(f^2*(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g]
)*(I*Sqrt[f] + Sqrt[g]*x)) + (Sqrt[g]*(-(Log[(I*Sqrt[d])/Sqrt[e] + x]/(I*Sq
rt[f] + Sqrt[g]*x)) - (I*Sqrt[e]*(Log[I*Sqrt[d] + Sqrt[e]*x] - Log[I*Sqrt[f
] + Sqrt[g]*x]))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g])))/f^2 + 4*(-1/2*(2 + (
g*x^2)/(f + g*x^2))/(f^2*x) - (3*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/(2*f^
(5/2)))*(-Log[((-I)*Sqrt[d])/Sqrt[e] + x] - Log[(I*Sqrt[d])/Sqrt[e] + x] +
Log[d + e*x^2]) - ((3*I)*Sqrt[g]*(Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]
*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g])] + PolyLog[2,
 -((Sqrt[g]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g]))])
)/f^(5/2) + ((3*I)*Sqrt[g]*(Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(Sqrt
[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])] + PolyLog[2, (Sqrt
[g]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])]))/f^(5/2)
 + ((3*I)*Sqrt[g]*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(Sqrt[f] +
I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g])] + PolyLog[2, -((Sqrt[g]*
(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] - Sqrt[d]*Sqrt[g]))]))/f^(5/2) -
((3*I)*Sqrt[g]*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(Sqrt[f] - I*S
qrt[g]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(Sqrt
[d] + I*Sqrt[e]*x))/(Sqrt[e]*Sqrt[f] + Sqrt[d]*Sqrt[g])]))/f^(5/2)))/4
```

Maple [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(c(e\,x^2+d)^p\right)}{x^2\left(g\,x^2+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(e*x^2+d)^p)/x^2/(g*x^2+f)^2,x)
```

```
[Out] int(ln(c*(e*x^2+d)^p)/x^2/(g*x^2+f)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^2/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] integrate(log((x^2*e + d)^p*c)/((g*x^2 + f)^2*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^2/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] integral(log((x^2*e + d)^p*c)/(g^2*x^6 + 2*f*g*x^4 + f^2*x^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(e*x**2+d)**p)/x**2/(g*x**2+f)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(e*x^2+d)^p)/x^2/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((x^2*e + d)^p*c)/((g*x^2 + f)^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln \left(c \left(e\,x^2+d\right)^p\right)}{x^2\,{\left(g\,x^2+f\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^2)^p)/(x^2*(f + g*x^2)^2),x)
[Out] int(log(c*(d + e*x^2)^p)/(x^2*(f + g*x^2)^2), x)
```

3.357 $\int \frac{\log\left(c\left(a+bx^2\right)^n\right)}{a+bx^2} dx$

Optimal. Leaf size=163

$$\frac{in\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}\,\sqrt{b}}+\frac{2n\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^n\right)}{\sqrt{a}\,\sqrt{b}}+\frac{in\text{Li}_2\left(1-\frac{\ }{\sqrt{\ }}\right.}{\sqrt{a}\,\sqrt{\ }}$$

```
[Out] I*n*arctan(x*b^(1/2)/a^(1/2))^2/a^(1/2)/b^(1/2)+arctan(x*b^(1/2)/a^(1/2))*l
n(c*(b*x^2+a)^n)/a^(1/2)/b^(1/2)+2*n*arctan(x*b^(1/2)/a^(1/2))*ln(2*a^(1/2)
/(a^(1/2)+I*x*b^(1/2)))/a^(1/2)/b^(1/2)+I*n*polylog(2,1-2*a^(1/2)/(a^(1/2)+
I*x*b^(1/2)))/a^(1/2)/b^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {211, 2520, 12, 5040, 4964, 2449, 2352}

$$\frac{in\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^n\right)}{\sqrt{a}\,\sqrt{b}}+\frac{in\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}\,\sqrt{b}}+\frac{2n\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(a + b*x^2)^n]/(a + b*x^2),x]
```

```
[Out] (I*n*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/(Sqrt[a]*Sqrt[b]) + (2*n*ArcTan[(Sqrt[b
]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/(Sqrt[a]*Sqrt[b]) +
 (ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[c*(a + b*x^2)^n])/(Sqrt[a]*Sqrt[b]) + (I*
n*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/(Sqrt[a]*Sqrt[b])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\log\left(c(a+bx^2)^n\right)}{a+bx^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^n\right)}{\sqrt{a}\,\sqrt{b}} - (2bn)\int \frac{x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,(a+bx^2)}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^n\right)}{\sqrt{a}\,\sqrt{b}} - \frac{\left(2\sqrt{b}\,n\right)\int \frac{x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a+bx^2}\,dx}{\sqrt{a}}$$

$$= \frac{in\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}\,\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^n\right)}{\sqrt{a}\,\sqrt{b}} + \frac{(2n)\int \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{i-\frac{\sqrt{b}\,x}{\sqrt{a}}}\,dx}{a}$$

$$= \frac{in\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}\,\sqrt{b}} + \frac{2n\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{\sqrt{a}\,\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log(}{\sqrt{a}\,\sqrt{b}}$$

$$= \frac{in\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}\,\sqrt{b}} + \frac{2n\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{\sqrt{a}\,\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log(}{\sqrt{a}\,\sqrt{b}}$$

$$= \frac{in\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}\,\sqrt{b}} + \frac{2n\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\,x}\right)}{\sqrt{a}\,\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log(}{\sqrt{a}\,\sqrt{b}}$$

Mathematica [A]
time = 0.03, size = 128, normalized size = 0.79

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(in\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)+2n\log\left(\frac{2i}{i-\frac{\sqrt{b}\,x}{\sqrt{a}}}\right)+\log\left(c(a+bx^2)^n\right)\right)+in\text{Li}_2\left(\frac{i\sqrt{a}+\sqrt{b}\,x}{-i\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(a + b*x^2)^n]/(a + b*x^2),x]
```

```
[Out] (ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(I*n*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 2*n*Log[(2*
I)/(I - (Sqrt[b]*x)/Sqrt[a])] + Log[c*(a + b*x^2)^n]) + I*n*PolyLog[2, (I*S
qrt[a] + Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)])/(Sqrt[a]*Sqrt[b])
```

Maple [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c(b\,x^{2}+a)^{n}\right)}{b\,x^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(b*x^2+a)^n)/(b*x^2+a),x)
[Out] int(ln(c*(b*x^2+a)^n)/(b*x^2+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^n)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate(log((b*x^2 + a)^n*c)/(b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^n)/(b*x^2+a),x, algorithm="fricas")
[Out] integral(log((b*x^2 + a)^n*c)/(b*x^2 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c(a+bx^{2})^{n}\right)}{a+bx^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(b*x**2+a)**n)/(b*x**2+a),x)
[Out] Integral(log(c*(a + b*x**2)**n)/(a + b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(b*x^2+a)^n)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^n*c)/(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(b\,x^2+a\right)}^n\right)}{b\,x^2+a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(a + b*x^2)^n)/(a + b*x^2),x)
```

```
[Out] int(log(c*(a + b*x^2)^n)/(a + b*x^2), x)
```

3.358 $\int \frac{\log(1-x^2)}{2-x^2}\,dx$

Optimal. Leaf size=239

$$\sqrt{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{2\sqrt{2}}{\sqrt{2}+x}\right)-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(-\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(\sqrt{2}+x\right)}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{}{}\right.}{\sqrt{2}}$$

[Out] 1/2*arctanh(1/2*x*2^(1/2))*ln(-x^2+1)*2^(1/2)-1/2*arctanh(1/2*x*2^(1/2))*ln(-4*(1-x)/(2-2^(1/2))/(x+2^(1/2)))*2^(1/2)-1/2*arctanh(1/2*x*2^(1/2))*ln(4*(1+x)/(2+2^(1/2))/(x+2^(1/2)))*2^(1/2)+1/4*polylog(2,1+4*(1-x)/(2-2^(1/2))/(x+2^(1/2)))*2^(1/2)-1/2*polylog(2,1-2*2^(1/2)/(x+2^(1/2)))*2^(1/2)+1/4*polylog(2,1-4*(1+x)/(2+2^(1/2))/(x+2^(1/2)))*2^(1/2)+arctanh(1/2*x*2^(1/2))*ln(2*2^(1/2)/(x+2^(1/2)))*2^(1/2)

Rubi [A]
time = 0.19, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {212, 2520, 12, 6139, 6057, 2449, 2352, 2497}

$$-\frac{\text{PolyLog}\left(2,1-\frac{2\sqrt{2}}{x+\sqrt{2}}\right)}{\sqrt{2}}+\frac{\text{PolyLog}\left(2,\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(x+\sqrt{2}\right)}+1\right)}{2\sqrt{2}}+\frac{\text{PolyLog}\left(2,1-\frac{4(x+1)}{\left(2+\sqrt{2}\right)\left(x+\sqrt{2}\right)}\right)}{2\sqrt{2}}+\frac{\log\left(1-x^2\right)\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}+\sqrt{2}\log\left(\frac{2\sqrt{2}}{x+\sqrt{2}}\right)\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)-\frac{\log\left(-\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(x+\sqrt{2}\right)}\right)\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\log\left(\frac{4(x+1)}{\left(2+\sqrt{2}\right)\left(x+\sqrt{2}\right)}\right)\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Log[1 - x^2]/(2 - x^2),x]

[Out] Sqrt[2]*ArcTanh[x/Sqrt[2]]*Log[(2*Sqrt[2])/(Sqrt[2] + x)] - (ArcTanh[x/Sqrt[2]]*Log[(-4*(1 - x))/((2 - Sqrt[2])*(Sqrt[2] + x))])/Sqrt[2] - (ArcTanh[x/Sqrt[2]]*Log[(4*(1 + x))/((2 + Sqrt[2])*(Sqrt[2] + x))])/Sqrt[2] + (ArcTanh[x/Sqrt[2]]*Log[1 - x^2])/Sqrt[2] - PolyLog[2, 1 - (2*Sqrt[2])/(Sqrt[2] + x)]/Sqrt[2] + PolyLog[2, 1 + (4*(1 - x))/((2 - Sqrt[2])*(Sqrt[2] + x))]/(2*Sqrt[2]) + PolyLog[2, 1 - (4*(1 + x))/((2 + Sqrt[2])*(Sqrt[2] + x))]/(2*Sqrt[2])

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rubi steps

$$\int \frac{\log(1-x^2)}{2-x^2}\,dx = \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log(1-x^2)}{\sqrt{2}} + 2\int \frac{x\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}\,(1-x^2)}\,dx$$

$$= \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log(1-x^2)}{\sqrt{2}} + \sqrt{2}\int \frac{x\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{1-x^2}\,dx$$

$$= \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log(1-x^2)}{\sqrt{2}} + \sqrt{2}\int \left(-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2(-1+x)} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2(1+x)}\right)dx$$

$$= \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log(1-x^2)}{\sqrt{2}} - \frac{\int \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{-1+x}\,dx}{\sqrt{2}} - \frac{\int \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{1+x}\,dx}{\sqrt{2}}$$

$$= \sqrt{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{2\sqrt{2}}{\sqrt{2}+x}\right) - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(-\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(\sqrt{2}+x\right)}\right)}{\sqrt{2}} -$$

$$= \sqrt{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{2\sqrt{2}}{\sqrt{2}+x}\right) - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(-\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(\sqrt{2}+x\right)}\right)}{\sqrt{2}} -$$

$$= \sqrt{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{2\sqrt{2}}{\sqrt{2}+x}\right) - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(-\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(\sqrt{2}+x\right)}\right)}{\sqrt{2}} -$$

Mathematica [A]
time = 0.10, size = 248, normalized size = 1.04

$$-\frac{\left(\log\left(\sqrt{2}-x\right)-\log\left(\sqrt{2}+x\right)\right)\left(-\log(-1+x)-\log(1+x)+\log\left(1-x^2\right)\right)}{2\sqrt{2}}+\frac{\log\left(1-\frac{-1+x}{-1-\sqrt{2}}\right)\log(-1+x)+\text{Li}_2\left(\frac{-1+x}{-1-\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\log\left(1-\frac{-1+x}{-1+\sqrt{2}}\right)\log(-1+x)+\text{Li}_2\left(\frac{-1+x}{-1+\sqrt{2}}\right)}{2\sqrt{2}}+\frac{\log(1+x)\log\left(1-\frac{1+x}{1-\sqrt{2}}\right)+\text{Li}_2\left(\frac{1+x}{1-\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\log(1+x)\log\left(1-\frac{1+x}{1+\sqrt{2}}\right)+\text{Li}_2\left(\frac{1+x}{1+\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 - x^2]/(2 - x^2),x]
```

```
[Out] -1/2*((Log[Sqrt[2] - x] - Log[Sqrt[2] + x])*(-Log[-1 + x] - Log[1 + x] + Lo
g[1 - x^2]))/Sqrt[2] + (Log[1 - (-1 + x)/(-1 - Sqrt[2])]*Log[-1 + x] + Poly
Log[2, (-1 + x)/(-1 - Sqrt[2])])/(2*Sqrt[2]) - (Log[1 - (-1 + x)/(-1 + Sqrt
```

```
[2])]*Log[-1 + x] + PolyLog[2, (-1 + x)/(-1 + Sqrt[2])])/(2*Sqrt[2]) + (Log
[1 + x]*Log[1 - (1 + x)/(1 - Sqrt[2])] + PolyLog[2, (1 + x)/(1 - Sqrt[2])])
/(2*Sqrt[2]) - (Log[1 + x]*Log[1 - (1 + x)/(1 + Sqrt[2])] + PolyLog[2, (1 +
 x)/(1 + Sqrt[2])])/(2*Sqrt[2])
```

Maple [A]
time = 0.52, size = 194, normalized size = 0.81

method	result
default	$-\frac{\left(\ln\left(x-\sqrt{2}\right)\ln(-x^{2}+1)-\mathrm{dilog}\left(\frac{x+1}{1+\sqrt{2}}\right)-\ln\left(x-\sqrt{2}\right)\ln\left(\frac{x+1}{1+\sqrt{2}}\right)-\mathrm{dilog}\left(\frac{-1+x}{\sqrt{2}-1}\right)-\ln\left(x-\sqrt{2}\right)\ln\left(\frac{-1+x}{\sqrt{2}-1}\right)\right)}{4}$
risch	$-\frac{\sqrt{2}\,\ln(-x^{2}+1)\ln\left(x-\sqrt{2}\right)}{4}+\frac{\sqrt{2}\,\ln\left(x-\sqrt{2}\right)\ln\left(\frac{x+1}{1+\sqrt{2}}\right)}{4}+\frac{\sqrt{2}\,\ln\left(x-\sqrt{2}\right)\ln\left(\frac{-1+x}{\sqrt{2}-1}\right)}{4}+\frac{\sqrt{2}\,\mathrm{dilog}\left(\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-x^2+1)/(-x^2+2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(ln(x-2^(1/2))*ln(-x^2+1)-dilog((x+1)/(1+2^(1/2)))-ln(x-2^(1/2))*ln((x
+1)/(1+2^(1/2)))-dilog((-1+x)/(2^(1/2)-1))-ln(x-2^(1/2))*ln((-1+x)/(2^(1/2)
-1)))*2^(1/2)+1/4*(ln(x+2^(1/2))*ln(-x^2+1)-dilog((x+1)/(-2^(1/2)+1))-ln(x+
2^(1/2))*ln((x+1)/(-2^(1/2)+1))-dilog((-1+x)/(-1-2^(1/2)))-ln(x+2^(1/2))*ln
((-1+x)/(-1-2^(1/2))))*2^(1/2)
```

Maxima [A]
time = 0.51, size = 208, normalized size = 0.87

$$\frac{1}{4}\sqrt{2}\left(\left(\log\left(2x+2\sqrt{2}\right)-\log\left(2x-2\sqrt{2}\right)\right)\log\left(-x^{2}+1\right)-\log\left(x+\sqrt{2}\right)\log\left(-\frac{x+\sqrt{2}}{\sqrt{2}+1}+1\right)+\log\left(x-\sqrt{2}\right)\log\left(\frac{x-\sqrt{2}}{\sqrt{2}+1}+1\right)-\log\left(x+\sqrt{2}\right)\log\left(-\frac{x+\sqrt{2}}{\sqrt{2}-1}+1\right)+\log\left(x-\sqrt{2}\right)\log\left(\frac{x-\sqrt{2}}{\sqrt{2}-1}+1\right)-\mathrm{Li}_2\left(\frac{x+\sqrt{2}}{\sqrt{2}+1}\right)+\mathrm{Li}_2\left(-\frac{x-\sqrt{2}}{\sqrt{2}+1}\right)-\mathrm{Li}_2\left(\frac{x+\sqrt{2}}{\sqrt{2}-1}\right)+\mathrm{Li}_2\left(-\frac{x-\sqrt{2}}{\sqrt{2}-1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-x^2+1)/(-x^2+2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*((log(2*x + 2*sqrt(2)) - log(2*x - 2*sqrt(2)))*log(-x^2 + 1) -
log(x + sqrt(2))*log(-(x + sqrt(2))/(sqrt(2) + 1) + 1) + log(x - sqrt(2))*l
og((x - sqrt(2))/(sqrt(2) + 1) + 1) - log(x + sqrt(2))*log(-(x + sqrt(2))/(
sqrt(2) - 1) + 1) + log(x - sqrt(2))*log((x - sqrt(2))/(sqrt(2) - 1) + 1) -
 dilog((x + sqrt(2))/(sqrt(2) + 1)) + dilog(-(x - sqrt(2))/(sqrt(2) + 1)) -
 dilog((x + sqrt(2))/(sqrt(2) - 1)) + dilog(-(x - sqrt(2))/(sqrt(2) - 1)))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-x^2+1)/(-x^2+2),x, algorithm="fricas")
```

```
[Out] integral(-log(-x^2 + 1)/(x^2 - 2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\log\left(1-x^2\right)}{x^2-2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-x**2+1)/(-x**2+2),x)
```

```
[Out] -Integral(log(1 - x**2)/(x**2 - 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-x^2+1)/(-x^2+2),x, algorithm="giac")
```

```
[Out] integrate(-log(-x^2 + 1)/(x^2 - 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{\ln\left(1-x^2\right)}{x^2-2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-log(1 - x^2)/(x^2 - 2),x)
```

```
[Out] -int(log(1 - x^2)/(x^2 - 2), x)
```

3.359 $\int \frac{\log(d+ex^2)}{1-x^2}\,dx$

Optimal. Leaf size=217

$$2\tanh^{-1}(x)\log\left(\frac{2}{1+x}\right)-\tanh^{-1}(x)\log\left(\frac{2\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(\sqrt{-d}-\sqrt{e}\right)(1+x)}\right)-\tanh^{-1}(x)\log\left(\frac{2\left(\sqrt{-d}+\sqrt{e}\,x\right)}{\left(\sqrt{-d}+\sqrt{e}\right)(1+\right.$$

```
[Out] 2*arctanh(x)*ln(2/(1+x))+arctanh(x)*ln(e*x^2+d)-arctanh(x)*ln(2*((-d)^(1/2)
-x*e^(1/2))/(1+x)/((-d)^(1/2)-e^(1/2)))-arctanh(x)*ln(2*((-d)^(1/2)+x*e^(1/
2))/(1+x)/((-d)^(1/2)+e^(1/2)))-polylog(2,1-2/(1+x))+1/2*polylog(2,1-2*((-d
)^(1/2)-x*e^(1/2))/(1+x)/((-d)^(1/2)-e^(1/2)))+1/2*polylog(2,1-2*((-d)^(1/2
)+x*e^(1/2))/(1+x)/((-d)^(1/2)+e^(1/2)))
```

Rubi [A]
time = 0.17, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {212, 2520, 6139, 6057, 2449, 2352, 2497}

$$\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2\left(\sqrt{-d}-\sqrt{e}\,x\right)}{(x+1)\left(\sqrt{-d}-\sqrt{e}\right)}\right)+\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2\left(\sqrt{-d}+\sqrt{e}\,x\right)}{(x+1)\left(\sqrt{-d}+\sqrt{e}\right)}\right)-\text{PolyLog}\left(2,1-\frac{2}{x+1}\right)+\tanh^{-1}(x)\log\left(d+ex^2\right)-\tanh^{-1}(x)\log\left(\frac{2\left(\sqrt{-d}-\sqrt{e}\,x\right)}{(x+1)\left(\sqrt{-d}-\sqrt{e}\right)}\right)-\tanh^{-1}(x)\log\left(\frac{2\left(\sqrt{-d}+\sqrt{e}\,x\right)}{(x+1)\left(\sqrt{-d}+\sqrt{e}\right)}\right)+2\log\left(\frac{2}{x+1}\right)\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[d + e*x^2]/(1 - x^2),x]
```

```
[Out] 2*ArcTanh[x]*Log[2/(1 + x)] - ArcTanh[x]*Log[(2*(Sqrt[-d] - Sqrt[e]*x))/((S
qrt[-d] - Sqrt[e])*(1 + x))] - ArcTanh[x]*Log[(2*(Sqrt[-d] + Sqrt[e]*x))/((
Sqrt[-d] + Sqrt[e])*(1 + x))] + ArcTanh[x]*Log[d + e*x^2] - PolyLog[2, 1 -
2/(1 + x)] + PolyLog[2, 1 - (2*(Sqrt[-d] - Sqrt[e]*x))/((Sqrt[-d] - Sqrt[e]
)*(1 + x))]/2 + PolyLog[2, 1 - (2*(Sqrt[-d] + Sqrt[e]*x))/((Sqrt[-d] + Sqrt
[e])*(1 + x))]/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rubi steps

$$\int \frac{\log(d+ex^2)}{1-x^2}\,dx = \tanh^{-1}(x)\log\left(d+ex^2\right) - (2e)\int \frac{x\tanh^{-1}(x)}{d+ex^2}\,dx$$

$$= \tanh^{-1}(x)\log\left(d+ex^2\right) - (2e)\int \left(-\frac{\tanh^{-1}(x)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}\,x\right)} + \frac{\tanh^{-1}(x)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}\,x\right.}\right.$$

$$= \tanh^{-1}(x)\log\left(d+ex^2\right) + \sqrt{e}\int \frac{\tanh^{-1}(x)}{\sqrt{-d}-\sqrt{e}\,x}\,dx - \sqrt{e}\int \frac{\tanh^{-1}(x)}{\sqrt{-d}+\sqrt{e}\,x}\,dx$$

$$= 2\tanh^{-1}(x)\log\left(\frac{2}{1+x}\right) - \tanh^{-1}(x)\log\left(\frac{2\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(\sqrt{-d}-\sqrt{e}\right)(1+x)}\right) - \tanh^{-1}(x)\log$$

$$= 2\tanh^{-1}(x)\log\left(\frac{2}{1+x}\right) - \tanh^{-1}(x)\log\left(\frac{2\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(\sqrt{-d}-\sqrt{e}\right)(1+x)}\right) - \tanh^{-1}(x)\log$$

$$= 2\tanh^{-1}(x)\log\left(\frac{2}{1+x}\right) - \tanh^{-1}(x)\log\left(\frac{2\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\left(\sqrt{-d}-\sqrt{e}\right)(1+x)}\right) - \tanh^{-1}(x)\log$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.09, size = 468, normalized size = 2.16

$$\frac{1}{2}\left(\log(1-x)\log\left(-\frac{i\sqrt{d}}{\sqrt{e}}+x\right) - \log\left(\frac{\sqrt{e}(-1+x)}{i\sqrt{d}-\sqrt{e}}\right)\log\left(-\frac{i\sqrt{d}}{\sqrt{e}}+x\right) - \log(1+x)\log\left(-\frac{i\sqrt{d}}{\sqrt{e}}+x\right) + \log\left(-\frac{i\sqrt{e}(1+x)}{\sqrt{d}-i\sqrt{e}}\right)\log\left(-\frac{i\sqrt{d}}{\sqrt{e}}+x\right) + \log(1-x)\log\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right) - \log\left(\frac{\sqrt{e}(-1+x)}{-i\sqrt{d}-\sqrt{e}}\right)\log\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right) - \log(1+x)\log\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right) + \log\left(\frac{i\sqrt{e}(1+x)}{\sqrt{d}+i\sqrt{e}}\right)\log\left(\frac{i\sqrt{d}}{\sqrt{e}}+x\right) - \log(1-x)\log\left(d+ex^2\right) + \log(1+x)\log\left(d+ex^2\right) - \text{Li}_2\left(\frac{\sqrt{d}-i\sqrt{e}\,x}{\sqrt{d}-i\sqrt{e}}\right) + \text{Li}_2\left(\frac{\sqrt{d}-i\sqrt{e}\,x}{\sqrt{d}+i\sqrt{e}}\right) + \text{Li}_2\left(\frac{\sqrt{d}+i\sqrt{e}\,x}{\sqrt{d}-i\sqrt{e}}\right) - \text{Li}_2\left(\frac{\sqrt{d}+i\sqrt{e}\,x}{\sqrt{d}+i\sqrt{e}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d + e*x^2]/(1 - x^2),x]
```

```
[Out] (Log[1 - x]*Log[((-I)*Sqrt[d])/Sqrt[e] + x] - Log[(Sqrt[e]*(-1 + x))/(I*Sqr
t[d] - Sqrt[e])]*Log[((-I)*Sqrt[d])/Sqrt[e] + x] - Log[1 + x]*Log[((-I)*Sqr
t[d])/Sqrt[e] + x] + Log[((-I)*Sqrt[e]*(1 + x))/(Sqrt[d] - I*Sqrt[e])]*Log[
((-I)*Sqrt[d])/Sqrt[e] + x] + Log[1 - x]*Log[(I*Sqrt[d])/Sqrt[e] + x] - Log
[(Sqrt[e]*(-1 + x))/((-I)*Sqrt[d] - Sqrt[e])]*Log[(I*Sqrt[d])/Sqrt[e] + x]
- Log[1 + x]*Log[(I*Sqrt[d])/Sqrt[e] + x] + Log[(I*Sqrt[e]*(1 + x))/(Sqrt[d
] + I*Sqrt[e])]*Log[(I*Sqrt[d])/Sqrt[e] + x] - Log[1 - x]*Log[d + e*x^2] +
Log[1 + x]*Log[d + e*x^2] - PolyLog[2, (Sqrt[d] - I*Sqrt[e]*x)/(Sqrt[d] - I
*Sqrt[e])] + PolyLog[2, (Sqrt[d] - I*Sqrt[e]*x)/(Sqrt[d] + I*Sqrt[e])] + Po
lyLog[2, (Sqrt[d] + I*Sqrt[e]*x)/(Sqrt[d] - I*Sqrt[e])] - PolyLog[2, (Sqrt[
d] + I*Sqrt[e]*x)/(Sqrt[d] + I*Sqrt[e])])/2
```

Maple [A]
time = 0.46, size = 289, normalized size = 1.33

method	result
risch	$\frac{\ln(x+1)\ln(e\,x^2+d)}{2}-\frac{\ln(x+1)\ln\left(\frac{-e(x+1)+\sqrt{-ed}+e}{e+\sqrt{-ed}}\right)}{2}-\frac{\ln(x+1)\ln\left(\frac{e(x+1)+\sqrt{-ed}-e}{-e+\sqrt{-ed}}\right)}{2}-\frac{\mathrm{dilog}\left(\frac{-e(x+1)+\sqrt{-ed}}{e+\sqrt{-ed}}\right.}{2}$
default	$\frac{\ln(x+1)\ln(e\,x^2+d)}{2}-e\left(\frac{\ln(x+1)\left(\ln\left(\frac{-e(x+1)+\sqrt{-ed}+e}{e+\sqrt{-ed}}\right)+\ln\left(\frac{e(x+1)+\sqrt{-ed}-e}{-e+\sqrt{-ed}}\right)\right)}{2e}+\frac{\mathrm{dilog}\left(\frac{-e(x+1)+\sqrt{-ed}+e}{e+\sqrt{-ed}}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*x^2+d)/(-x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(x+1)*ln(e*x^2+d)-e*(1/2*ln(x+1)*(ln((-e*(x+1)+(-e*d)^(1/2)+e)/(e+(-e
*d)^(1/2)))+ln((e*(x+1)+(-e*d)^(1/2)-e)/(-e+(-e*d)^(1/2))))/e+1/2*(dilog((-
e*(x+1)+(-e*d)^(1/2)+e)/(e+(-e*d)^(1/2)))+dilog((e*(x+1)+(-e*d)^(1/2)-e)/(-
e+(-e*d)^(1/2))))/e)-1/2*ln(-1+x)*ln(e*x^2+d)+e*(1/2*ln(-1+x)*(ln((-e*(-1+x
)+(-e*d)^(1/2)-e)/(-e+(-e*d)^(1/2)))+ln((e*(-1+x)+(-e*d)^(1/2)+e)/(e+(-e*d)
^(1/2))))/e+1/2*(dilog((-e*(-1+x)+(-e*d)^(1/2)-e)/(-e+(-e*d)^(1/2)))+dilog(
(e*(-1+x)+(-e*d)^(1/2)+e)/(e+(-e*d)^(1/2))))/e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*x^2+d)/(-x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate(log(x^2*e + d)/(x^2 - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*x^2+d)/(-x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-log(x^2*e + d)/(x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\log\left(d+ex^2\right)}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*x**2+d)/(-x**2+1),x)
```

```
[Out] -Integral(log(d + e*x**2)/(x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*x^2+d)/(-x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-log(x^2*e + d)/(x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{\ln\left(e\,x^2+d\right)}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-log(d + e*x^2)/(x^2 - 1),x)
```

```
[Out] -int(log(d + e*x^2)/(x^2 - 1), x)
```

3.360 $\int \frac{(f+gx^{3n})\log(c(d+ex^n)^p)}{x}\,dx$

Optimal. Leaf size=144

$$-\frac{d^2gpx^n}{3e^2n}+\frac{dgpx^{2n}}{6en}-\frac{gpx^{3n}}{9n}+\frac{d^3gp\log(d+ex^n)}{3e^3n}+\frac{gx^{3n}\log(c(d+ex^n)^p)}{3n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n}+\frac{fp}{}$$

```
[Out] -1/3*d^2*g*p*x^n/e^2/n+1/6*d*g*p*x^(2*n)/e/n-1/9*g*p*x^(3*n)/n+1/3*d^3*g*p*
ln(d+e*x^n)/e^3/n+1/3*g*x^(3*n)*ln(c*(d+e*x^n)^p)/n+f*ln(-e*x^n/d)*ln(c*(d+
e*x^n)^p)/n+f*p*polylog(2,1+e*x^n/d)/n
```

Rubi [A]
time = 0.11, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2525, 14, 2463, 2441, 2352, 2442, 45}

$$\frac{fp\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n}+\frac{gx^{3n}\log(c(d+ex^n)^p)}{3n}+\frac{d^3gp\log(d+ex^n)}{3e^3n}-\frac{d^2gpx^n}{3e^2n}+\frac{dgpx^{2n}}{6en}-\frac{gpx^{3n}}{9n}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^(3*n))*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] -1/3*(d^2*g*p*x^n)/(e^2*n) + (d*g*p*x^(2*n))/(6*e*n) - (g*p*x^(3*n))/(9*n)
+ (d^3*g*p*Log[d + e*x^n])/(3*e^3*n) + (g*x^(3*n)*Log[c*(d + e*x^n)^p])/(3*
n) + (f*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n + (f*p*PolyLog[2, 1 + (e*
x^n)/d])/n
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(f+gx^{3n})\log(c(d+ex^n)^p)}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{(f+gx^3)\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n} \\
&= \frac{\text{Subst}\left(\int \left(\frac{f\log(c(d+ex)^p)}{x} + gx^2\log(c(d+ex)^p)\right)\,dx, x, x^n\right)}{n} \\
&= \frac{f\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n} + \frac{g\text{Subst}\left(\int x^2\log(c(d+ex)^p)\,dx, x,\right.}{n} \\
&= \frac{gx^{3n}\log(c(d+ex^n)^p)}{3n} + \frac{f\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} - \frac{(efp)\text{Subst}}{} \\
&= \frac{gx^{3n}\log(c(d+ex^n)^p)}{3n} + \frac{f\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} + \frac{fp\text{Li}_2(1+}{n} \\
&= -\frac{d^2gpx^n}{3e^2n} + \frac{dgpx^{2n}}{6en} - \frac{gpx^{3n}}{9n} + \frac{d^3gp\log(d+ex^n)}{3e^3n} + \frac{gx^{3n}\log(c(d+e}{3n}
\end{aligned}
$$

Mathematica [A]
time = 0.14, size = 188, normalized size = 1.31

$$
\frac{-6d^2egpx^n + 3de^2gpx^{2n} - 2e^3gpx^{3n} - 6d^3gp\log(x^n) + 18e^3fp\log\left(-\frac{ex^n}{d}\right)\log(d+ex^n) + 6d^3gp\log(dn(d+ex^n)) + 6e^3gx^{3n}\log(c(d+ex^n)^p) + 6n\log(x)\left(d^3gp - 3e^3fp\log(d+ex^n) + 3e^3f\log(c(d+ex^n)^p)\right) + 18e^3fp\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{18e^3n}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^(3*n))*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (-6*d^2*e*g*p*x^n + 3*d*e^2*g*p*x^(2*n) - 2*e^3*g*p*x^(3*n) - 6*d^3*g*p*Log
[x^n] + 18*e^3*f*p*Log[-((e*x^n)/d)]*Log[d + e*x^n] + 6*d^3*g*p*Log[d*n*(d
+ e*x^n)] + 6*e^3*g*x^(3*n)*Log[c*(d + e*x^n)^p] + 6*n*Log[x]*(d^3*g*p - 3*
e^3*f*p*Log[d + e*x^n] + 3*e^3*f*Log[c*(d + e*x^n)^p]) + 18*e^3*f*p*PolyLog
[2, 1 + (e*x^n)/d])/(18*e^3*n)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.40, size = 428, normalized size = 2.97

method	result
risch	$\frac{(g\,x^{3n}+3f\ln(x)n)\ln((d+e\,x^n)^p)}{3n} + \frac{i\pi\,\text{csgn}(i(d+e\,x^n)^p)\text{csgn}(ic(d+e\,x^n)^p)^2 f\ln(x)}{2} + \frac{i\pi\,\text{csgn}(i(d+e\,x^n)^p)\text{csgn}(ic(d+e\,x^n)^p)^2 g\,x}{6n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g*x^(3*n))*ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(g*(x^n)^3+3*f*ln(x)*n)/n*ln((d+e*x^n)^p)+1/2*I*Pi*csgn(I*(d+e*x^n)^p)*
csgn(I*c*(d+e*x^n)^p)^2*f*ln(x)-1/6*I*Pi*csgn(I*c*(d+e*x^n)^p)^3*g*(x^n)^3/
```

```
n-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*f*ln(x)-1/2*
I*Pi*csgn(I*c*(d+e*x^n)^p)^3*f*ln(x)+1/6*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(
I*c)*g*(x^n)^3/n+1/6*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2*g*(x^
n)^3/n-1/6*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*g*(x^n)
^3/n+1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)*f*ln(x)+ln(c)*f*ln(x)+1/3*l
n(c)*g*(x^n)^3/n-1/9*p/n*g*(x^n)^3+1/6*p/e/n*g*d*(x^n)^2-1/3*d^2*g*p*x^n/e^
2/n+1/3*d^3*g*p*ln(d+e*x^n)/e^3/n-p/n*f*dilog((d+e*x^n)/d)-p*f*ln(x)*ln((d+
e*x^n)/d)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(3*n))*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/18*(9*f*n^2*p*e^3*log(x)^2 + 6*d^2*g*p*e^(n*log(x) + 1) - 3*d*g*p*e^(2*n
*log(x) + 2) + 2*(g*p - 3*g*log(c))*e^(3*n*log(x) + 3) - 6*(3*f*n*e^3*log(x
) + g*e^(3*n*log(x) + 3))*log((d + e^(n*log(x) + 1))^p) - 6*(d^3*g*n*p + 3*
f*n*e^3*log(c))*log(x))*e^(-3)/n + integrate(-1/3*(d^4*g*p - 3*d*f*n*p*e^3*
log(x))/(d*x*e^3 + x*e^(n*log(x) + 4)), x)
```

Fricas [**A**]
time = 0.37, size = 143, normalized size = 0.99

$$-\frac{\left(6\,d^{2}gpx^{n}e+18\,fnpe^{3}\log\left(x\right)\log\left(\frac{x^{n}e+d}{d}\right)-3\,dgpx^{2\,n}e^{2}-18\,fne^{3}\log\left(c\right)\log\left(x\right)+18\,fp\mathrm{Li}_2\left(-\frac{x^{n}e+d}{d}+1\right)e^{3}+2\left(gpe^{3}-3\,ge^{3}\log\left(c\right)\right)x^{3\,n}-6\left(d^{3}gp+3\,fnpe^{3}\log\left(x\right)+gpx^{3\,n}e^{3}\right)\log\left(x^{n}e+d\right)\right)e^{(-3)}}{18\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(3*n))*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -1/18*(6*d^2*g*p*x^n*e + 18*f*n*p*e^3*log(x)*log((x^n*e + d)/d) - 3*d*g*p*x
^(2*n)*e^2 - 18*f*n*e^3*log(c)*log(x) + 18*f*p*dilog(-(x^n*e + d)/d + 1)*e^
3 + 2*(g*p*e^3 - 3*g*e^3*log(c))*x^(3*n) - 6*(d^3*g*p + 3*f*n*p*e^3*log(x)
+ g*p*x^(3*n)*e^3)*log(x^n*e + d))*e^(-3)/n
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f + g x^{3 n}\right) \log \left(c \left(d + e x^{n}\right)^{p}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x**(3*n))*ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Integral((f + g*x**(3*n))*log(c*(d + e*x**n)**p)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(3*n))*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
[Out] integrate((g*x^(3*n) + f)*log((x^n*e + d)^p*c)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln \left(c\,{\left(d+e\,x^n\right)}^p\right)\,\left(f+g\,x^{3\,n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^n)^p)*(f + g*x^(3*n)))/x,x)
[Out] int((log(c*(d + e*x^n)^p)*(f + g*x^(3*n)))/x, x)
```

3.361 $\int \frac{(f+gx^{2n})\log(c(d+ex^n)^p)}{x}\,dx$

Optimal. Leaf size=124

$$\frac{dgpx^n}{2en}-\frac{gpx^{2n}}{4n}-\frac{d^2gp\log\left(d+ex^n\right)}{2e^2n}+\frac{gx^{2n}\log\left(c(d+ex^n)^p\right)}{2n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n}+\frac{fp\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}$$

```
[Out] 1/2*d*g*p*x^n/e/n-1/4*g*p*x^(2*n)/n-1/2*d^2*g*p*ln(d+e*x^n)/e^2/n+1/2*g*x^(
2*n)*ln(c*(d+e*x^n)^p)/n+f*ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/n+f*p*polylog(2,1
+e*x^n/d)/n
```

Rubi [A]
time = 0.10, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2525, 14, 2463, 2441, 2352, 2442, 45}

$$\frac{fp\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n}+\frac{gx^{2n}\log\left(c(d+ex^n)^p\right)}{2n}-\frac{d^2gp\log\left(d+ex^n\right)}{2e^2n}+\frac{dgpx^n}{2en}-\frac{gpx^{2n}}{4n}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^(2*n))*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (d*g*p*x^n)/(2*e*n) - (g*p*x^(2*n))/(4*n) - (d^2*g*p*Log[d + e*x^n])/(2*e^2
*n) + (g*x^(2*n)*Log[c*(d + e*x^n)^p])/(2*n) + (f*Log[-((e*x^n)/d)]*Log[c*(
d + e*x^n)^p])/n + (f*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
```

```
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{(f+gx^{2n})\log(c(d+ex^n)^p)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{(f+gx^2)\log(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{f\log(c(d+ex)^p)}{x}+gx\log(c(d+ex)^p)\right)\,dx,x,x^n\right)}{n}$$

$$= \frac{f\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{n}+\frac{g\text{Subst}\left(\int x\log(c(d+ex)^p)\,dx,x,\right.}{n}$$

$$= \frac{gx^{2n}\log(c(d+ex^n)^p)}{2n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n}-\frac{(efp)\text{Subst}}{}$$

$$= \frac{gx^{2n}\log(c(d+ex^n)^p)}{2n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n}+\frac{fp\text{Li}_2(1+}{n}$$

$$= \frac{dgpx^n}{2en}-\frac{gpx^{2n}}{4n}-\frac{d^2gp\log(d+ex^n)}{2e^2n}+\frac{gx^{2n}\log(c(d+ex^n)^p)}{2n}+\frac{f\log}{}$$

Mathematica [A]
time = 0.14, size = 173, normalized size = 1.40

$$\frac{2degpx^n - e^2gpx^{2n} + 2d^2gp\log(x^n) + 4e^2fp\log\left(-\frac{ex^n}{d}\right)\log(d+ex^n) - 2d^2gp\log(dn(d+ex^n)) + 2e^2gx^{2n}\log(c(d+ex^n)^p) - 2n\log(x)\left(d^2gp + 2e^2fp\log(d+ex^n) - 2e^2f\log(c(d+ex^n)^p)\right) + 4e^2fp\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{4e^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^(2*n))*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (2*d*e*g*p*x^n - e^2*g*p*x^(2*n) + 2*d^2*g*p*Log[x^n] + 4*e^2*f*p*Log[-((e*
x^n)/d)]*Log[d + e*x^n] - 2*d^2*g*p*Log[d*n*(d + e*x^n)] + 2*e^2*g*x^(2*n)*
Log[c*(d + e*x^n)^p] - 2*n*Log[x]*(d^2*g*p + 2*e^2*f*p*Log[d + e*x^n] - 2*e
^2*f*Log[c*(d + e*x^n)^p]) + 4*e^2*f*p*PolyLog[2, 1 + (e*x^n)/d])/(4*e^2*n)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 410, normalized size = 3.31

method	result
risch	$\frac{(2f\ln(x)n+g\,x^{2n})\ln((d+e\,x^n)^p)}{2n} - \frac{i\pi\,\text{csgn}(i(d+e\,x^n)^p)\text{csgn}(ic(d+e\,x^n)^p)\text{csgn}(ic)g\,x^{2n}}{4n} + \frac{i\pi\,\text{csgn}(i(d+e\,x^n)^p)\text{csgn}(ic(d+e\,x^n)^p)^2}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g*x^(2*n))*ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*f*ln(x)*n+g*(x^n)^2)/n*ln((d+e*x^n)^p)+1/4*I*Pi*csgn(I*(d+e*x^n)^p)*
csgn(I*c*(d+e*x^n)^p)^2*g*(x^n)^2/n+1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(
d+e*x^n)^p)^2*f*ln(x)-1/4*I*Pi*csgn(I*c*(d+e*x^n)^p)^3*g*(x^n)^2/n+1/4*I*Pi
*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)*g*(x^n)^2/n-1/4*I*Pi*csgn(I*(d+e*x^n)^p)
*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*g*(x^n)^2/n-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*c
sgn(I*c*(d+e*x^n)^p)*csgn(I*c)*f*ln(x)+1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csg
n(I*c)*f*ln(x)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^3*f*ln(x)+ln(c)*f*ln(x)+1/2*l
n(c)*g*(x^n)^2/n-1/4*p/n*g*(x^n)^2+1/2*d*g*p*x^n/e/n-1/2*d^2*g*p*ln(d+e*x^n
)/e^2/n-p/n*f*dilog((d+e*x^n)/d)-p*f*ln(x)*ln((d+e*x^n)/d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(2*n))*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/4*(2*f*n^2*p*e^2*log(x)^2 - 2*d*g*p*e^(n*log(x) + 1) + (g*p - 2*g*log(c)
)*e^(2*n*log(x) + 2) - 2*(2*f*n*e^2*log(x) + g*e^(2*n*log(x) + 2))*log((d +
 e^(n*log(x) + 1))^p) + 2*(d^2*g*n*p - 2*f*n*e^2*log(c))*log(x))*e^(-2)/n +
 integrate(1/2*(2*d*f*n*p*e^2*log(x) + d^3*g*p)/(d*x*e^2 + x*e^(n*log(x) +
3)), x)
```

Fricas [A]
time = 0.36, size = 129, normalized size = 1.04

$$-\frac{\left(4\,fnpe^2\log\left(x\right)\log\left(\frac{x^ne+d}{d}\right)-2\,dgpx^ne-4\,fne^2\log\left(c\right)\log\left(x\right)+4\,fp\mathrm{Li}_2\left(-\frac{x^ne+d}{d}+1\right)e^2+\left(gpe^2-2\,ge^2\log\left(c\right)\right)x^{2\,n}-2\left(2\,fnpe^2\log\left(x\right)-d^2gp+gpx^{2\,n}e^2\right)\log\left(x^ne+d\right)\right)e^{(-2)}}{4\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(2*n))*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -1/4*(4*f*n*p*e^2*log(x)*log((x^n*e + d)/d) - 2*d*g*p*x^n*e - 4*f*n*e^2*log
(c)*log(x) + 4*f*p*dilog(-(x^n*e + d)/d + 1)*e^2 + (g*p*e^2 - 2*g*e^2*log(c
))*x^(2*n) - 2*(2*f*n*p*e^2*log(x) - d^2*g*p + g*p*x^(2*n)*e^2)*log(x^n*e +
 d))*e^(-2)/n
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(f+gx^{2n}\right)\log\left(c(d+ex^n)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x**(2*n))*ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Integral((f + g*x**(2*n))*log(c*(d + e*x**n)**p)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(2*n))*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] integrate((g*x^(2*n) + f)*log((x^n*e + d)^p*c)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)\,\left(f+g\,x^{2\,n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^n)^p)*(f + g*x^(2*n)))/x,x)
```

```
[Out] int((log(c*(d + e*x^n)^p)*(f + g*x^(2*n)))/x, x)
```

3.362 $\int \frac{(f+gx^n)\log(c(d+ex^n)^p)}{x}\,dx$

Optimal. Leaf size=83

$$-\frac{gpx^n}{n}+\frac{g(d+ex^n)\log\left(c(d+ex^n)^p\right)}{en}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n}+\frac{fp\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}$$

```
[Out] -g*p*x^n/n+g*(d+e*x^n)*ln(c*(d+e*x^n)^p)/e/n+f*ln(-e*x^n/d)*ln(c*(d+e*x^n)^
p)/n+f*p*polylog(2,1+e*x^n/d)/n
```

Rubi [A]
time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2525, 45, 2463, 2436, 2332, 2441, 2352}

$$\frac{fp\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n}+\frac{g(d+ex^n)\log\left(c(d+ex^n)^p\right)}{en}-\frac{gpx^n}{n}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^n)*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] -((g*p*x^n)/n) + (g*(d + e*x^n)*Log[c*(d + e*x^n)^p])/(e*n) + (f*Log[-((e*x
^n)/d)]*Log[c*(d + e*x^n)^p])/n + (f*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(f+gx^n)\log\left(c(d+ex^n)^p\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{(f+gx)\log(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{n} \\
&= \frac{\text{Subst}\left(\int \left(g\log\left(c(d+ex)^p\right)+\frac{f\log(c(d+ex)^p)}{x}\right)\,dx,x,x^n\right)}{n} \\
&= \frac{f\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{n}+\frac{g\text{Subst}\left(\int \log\left(c(d+ex)^p\right)\,dx,x,x^n\right.}{n} \\
&= \frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n}+\frac{g\text{Subst}\left(\int \log\left(cx^p\right)\,dx,x,d+ex^n\right)}{en}- \\
&= -\frac{gpx^n}{n}+\frac{g(d+ex^n)\log\left(c(d+ex^n)^p\right)}{en}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right.}{n}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 68, normalized size = 0.82

$$\frac{-egpx^n+\left(dg+egx^n+ef\log\left(-\frac{ex^n}{d}\right)\right)\log\left(c(d+ex^n)^p\right)+efp\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{en}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^n)*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (-(e*g*p*x^n) + (d*g + e*g*x^n + e*f*Log[-((e*x^n)/d)])*Log[c*(d + e*x^n)^p
] + e*f*p*PolyLog[2, 1 + (e*x^n)/d])/(e*n)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 376, normalized size = 4.53

method	result
risch	$\frac{(f\ln(x)n+g\,x^n)\ln((d+e\,x^n)^p)}{n}+\frac{i\pi\,\mathrm{csgn}(i(d+e\,x^n)^p)\mathrm{csgn}(ic(d+e\,x^n)^p)^2 f\ln(x)}{2}+\frac{i\pi\,\mathrm{csgn}(i(d+e\,x^n)^p)\mathrm{csgn}(ic(d+e\,x^n)^p)^2 g\,x^n}{2n}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g*x^n)*ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

```
[Out] (f*ln(x)*n+g*x^n)/n*ln((d+e*x^n)^p)+1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(
d+e*x^n)^p)^2*f*ln(x)+1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2*
g*x^n/n-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*f*ln(x
)-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*g*x^n/n-1/2*
I*Pi*csgn(I*c*(d+e*x^n)^p)^3*f*ln(x)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^3*g*x^n
/n+1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)*f*ln(x)+1/2*I*Pi*csgn(I*c*(d+
e*x^n)^p)^2*csgn(I*c)*g*x^n/n+ln(c)*f*ln(x)+ln(c)*g*x^n/n-g*p*x^n/n+p/e/n*g
*d*ln(d+e*x^n)-p/n*f*dilog((d+e*x^n)/d)-p*f*ln(x)*ln((d+e*x^n)/d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^n)*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/2*(f*n^2*p*e*log(x)^2 + 2*(g*p - g*log(c))*e^(n*log(x) + 1) - 2*(f*n*e*l
og(x) + g*e^(n*log(x) + 1))*log((d + e^(n*log(x) + 1))^p) - 2*(d*g*n*p + f*
n*e*log(c))*log(x))*e^(-1)/n + integrate((d*f*n*p*e*log(x) - d^2*g*p)/(d*x*
e + x*e^(n*log(x) + 2)), x)
```

Fricas [A]
time = 0.36, size = 109, normalized size = 1.31

$$-\frac{\left(fnpe\log\left(x\right)\log\left(\frac{x^ne+d}{d}\right)-fne\log\left(c\right)\log\left(x\right)+fp\mathrm{Li}_2\left(-\frac{x^ne+d}{d}+1\right)e+(gpe-ge\log\left(c\right))x^n-(fnpe\log\left(x\right)+gpx^ne+dgp)\log\left(x^ne+d\right)\right)e^{(-1)}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^n)*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -(f*n*p*e*log(x)*log((x^n*e + d)/d) - f*n*e*log(c)*log(x) + f*p*dilog(-(x^n
*e + d)/d + 1)*e + (g*p*e - g*e*log(c))*x^n - (f*n*p*e*log(x) + g*p*x^n*e +
 d*g*p)*log(x^n*e + d))*e^(-1)/n
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx^n)\log\left(c(d+ex^n)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x**n)*ln(c*(d+e*x**n)**p)/x,x)
[Out] Integral((f + g*x**n)*log(c*(d + e*x**n)**p)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^n)*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
[Out] integrate((g*x^n + f)*log((x^n*e + d)^p*c)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)\,\left(f+g\,x^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^n)^p)*(f + g*x^n))/x,x)
[Out] int((log(c*(d + e*x^n)^p)*(f + g*x^n))/x, x)
```

3.363 $\int \frac{(f+gx^{-n})\log(c(d+ex^n)^p)}{x}\,dx$

Optimal. Leaf size=97

$$\frac{egp\log(x)}{d}-\frac{egp\log\left(d+ex^n\right)}{dn}-\frac{gx^{-n}\log\left(c(d+ex^n)^p\right)}{n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n}+\frac{fp\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}$$

```
[Out] e*g*p*ln(x)/d-e*g*p*ln(d+e*x^n)/d/n-g*ln(c*(d+e*x^n)^p)/n/(x^n)+f*ln(-e*x^n
/d)*ln(c*(d+e*x^n)^p)/n+f*p*polylog(2,1+e*x^n/d)/n
```

Rubi [A]
time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {2525, 14, 2463, 2442, 36, 29, 31, 2441, 2352}

$$\frac{fp\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n}-\frac{gx^{-n}\log\left(c(d+ex^n)^p\right)}{n}-\frac{egp\log\left(d+ex^n\right)}{dn}+\frac{egp\log(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g/x^n)*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (e*g*p*Log[x])/d - (e*g*p*Log[d + e*x^n])/(d*n) - (g*Log[c*(d + e*x^n)^p])/
(n*x^n) + (f*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n + (f*p*PolyLog[2, 1
+ (e*x^n)/d])/n
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{(f+gx^{-n})\log(c(d+ex^n)^p)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\left(f+\frac{g}{x}\right)\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{g\log(c(d+ex)^p)}{x^2}+\frac{f\log(c(d+ex)^p)}{x}\right)\,dx, x, x^n\right)}{n}$$
$$= \frac{f\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n} + \frac{g\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x^2}\,dx, x, x^n\right)}{n}$$
$$= -\frac{gx^{-n}\log(c(d+ex^n)^p)}{n} + \frac{f\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} - \frac{(efp)\text{Subs}}{}$$
$$= -\frac{gx^{-n}\log(c(d+ex^n)^p)}{n} + \frac{f\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} + \frac{fp\text{Li}_2(1+}{n}$$
$$= \frac{egp\log(x)}{d} - \frac{egp\log(d+ex^n)}{dn} - \frac{gx^{-n}\log(c(d+ex^n)^p)}{n} + \frac{f\log\left(-\frac{ex^n}{d}\right.}{}$$

Mathematica [A]
time = 0.21, size = 100, normalized size = 1.03

$$-\frac{egp\log(dn(d+ex^n)) + dgx^{-n}\log(c(d+ex^n)^p) - n\log(x)\left(egp + df\log(c(d+ex^n)^p) - dfp\log\left(1+\frac{ex^n}{d}\right)\right) + dfp\text{Li}_2\left(-\frac{ex^n}{d}\right)}{dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g/x^n)*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] -((e*g*p*Log[d*n*(d + e*x^n)] + (d*g*Log[c*(d + e*x^n)^p])/x^n - n*Log[x]*(
e*g*p + d*f*Log[c*(d + e*x^n)^p] - d*f*p*Log[1 + (e*x^n)/d]) + d*f*p*PolyLo
g[2, -((e*x^n)/d)])/(d*n))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 423, normalized size = 4.36

method	result
risch	$\frac{(f\ln(x)n\,x^n-g)x^{-n}\ln((d+e\,x^n)^p)}{n} + \frac{i\pi\,\text{csgn}(i(d+e\,x^n)^p)\text{csgn}(ic(d+e\,x^n)^p)^2 f\ln(x^n)}{2n} - \frac{i\pi\,\text{csgn}(i(d+e\,x^n)^p)\text{csgn}(ic(d+e\,x^n)^p)^2 g}{2n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g/(x^n))*ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

```
[Out] (f*ln(x)*n*x^n-g)/n/(x^n)*ln((d+e*x^n)^p)+1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*cs
gn(I*c*(d+e*x^n)^p)^2*f*ln(x^n)-1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+
e*x^n)^p)^2*g/(x^n)-1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*cs
gn(I*c)*f*ln(x^n)+1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn
```

```
(I*c)*g/(x^n)-1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3*f*ln(x^n)+1/2*I/n*Pi*csgn(
I*c*(d+e*x^n)^p)^3*g/(x^n)+1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)*f*l
n(x^n)-1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)*g/(x^n)+1/n*ln(c)*f*ln(
x^n)-1/n*ln(c)*g/(x^n)-p/n*f*dilog((d+e*x^n)/d)-p*f*ln(x)*ln((d+e*x^n)/d)-e
*g*p*ln(d+e*x^n)/d/n+p*e/n*g/d*ln(x^n)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^n))*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/2*((f*n^2*p*log(x)^2 - 2*f*n*log(c)*log(x))*x^n + 2*g*log(c) - 2*(f*n*x^
n*log(x) - g)*log((d + e^(n*log(x) + 1))^p))/(n*x^n) + integrate((d*f*n*p*l
og(x) + g*p*e)/(d*x + x*e^(n*log(x) + 1)), x)
```

Fricas [A]
time = 0.36, size = 119, normalized size = 1.23

$$-\frac{dfnpx^n\log(x)\log\left(\frac{x^ne+d}{d}\right)+dfpx^n\mathrm{Li}_2\left(-\frac{x^ne+d}{d}+1\right)+dg\log(c)-(gnpe+dfn\log(c))x^n\log(x)+(dgp-(dfnp\log(x)-gpe)x^n)\log(x^ne+d)}{dnx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^n))*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -(d*f*n*p*x^n*log(x)*log((x^n*e + d)/d) + d*f*p*x^n*dilog(-(x^n*e + d)/d +
1) + d*g*log(c) - (g*n*p*e + d*f*n*log(c))*x^n*log(x) + (d*g*p - (d*f*n*p*l
og(x) - g*p*e)*x^n)*log(x^n*e + d))/(d*n*x^n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{-n}(fx^n+g)\log\left(c(d+ex^n)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x**n))*ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Integral((f*x**n + g)*log(c*(d + e*x**n)**p)/(x*x**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^n))*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] integrate((f + g/x^n)*log((x^n*e + d)^p*c)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\left(d+e\,x^n\right)^p\right)\,\left(f+\frac{g}{x^n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^n)^p)*(f + g/x^n))/x,x)
```

```
[Out] int((log(c*(d + e*x^n)^p)*(f + g/x^n))/x, x)
```

3.364 $\int \frac{(f+gx^{-2n})\log(c(d+ex^n)^p)}{x}\,dx$

Optimal. Leaf size=126

$$-\frac{egpx^{-n}}{2dn}-\frac{e^2gp\log(x)}{2d^2}+\frac{e^2gp\log(d+ex^n)}{2d^2n}-\frac{gx^{-2n}\log(c(d+ex^n)^p)}{2n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n}+\frac{fp\text{Li}$$

[Out] -1/2*e*g*p/d/n/(x^n)-1/2*e^2*g*p*ln(x)/d^2+1/2*e^2*g*p*ln(d+e*x^n)/d^2/n-1/2*g*ln(c*(d+e*x^n)^p)/n/(x^(2*n))+f*ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/n+f*p*polylog(2,1+e*x^n/d)/n

Rubi [A]
time = 0.11, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2525, 14, 2463, 2442, 46, 2441, 2352}

$$\frac{fp\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n}-\frac{gx^{-2n}\log(c(d+ex^n)^p)}{2n}+\frac{e^2gp\log(d+ex^n)}{2d^2n}-\frac{e^2gp\log(x)}{2d^2}-\frac{egpx^{-n}}{2dn}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g/x^(2*n))*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] -1/2*(e*g*p)/(d*n*x^n) - (e^2*g*p*Log[x])/(2*d^2) + (e^2*g*p*Log[d + e*x^n]
)/(2*d^2*n) - (g*Log[c*(d + e*x^n)^p])/(2*n*x^(2*n)) + (f*Log[-((e*x^n)/d)]
*Log[c*(d + e*x^n)^p])/n + (f*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
```

```
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{(f+gx^{-2n})\log(c(d+ex^n)^p)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\left(f+\frac{g}{x^2}\right)\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{g\log(c(d+ex)^p)}{x^3}+\frac{f\log(c(d+ex)^p)}{x}\right)dx, x, x^n\right)}{n}$$
$$= \frac{f\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}+\frac{g\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x^3}\,dx, x, x^n\right)}{n}$$
$$= -\frac{gx^{-2n}\log(c(d+ex^n)^p)}{2n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n}-\frac{(efp)\text{S}}{}$$
$$= -\frac{gx^{-2n}\log(c(d+ex^n)^p)}{2n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n}+\frac{fp\text{Li}_2(}{}$$
$$= -\frac{egpx^{-n}}{2dn}-\frac{e^2gp\log(x)}{2d^2}+\frac{e^2gp\log(d+ex^n)}{2d^2n}-\frac{gx^{-2n}\log(c(d+ex^n}{2n}$$

Mathematica [A]
time = 0.30, size = 148, normalized size = 1.17

$$-\frac{degpx^{-n}-2d^2fp\log\left(-\frac{ex^n}{d}\right)\log(d+ex^n)-e^2gp\log(n(d+ex^n))+d^2gx^{-2n}\log(c(d+ex^n)^p)+n\log(x)\left(e^2gp+2d^2fp\log(d+ex^n)-2d^2f\log(c(d+ex^n)^p)\right)-2d^2fp\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{2d^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g/x^(2*n))*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] -1/2*((d*e*g*p)/x^n - 2*d^2*f*p*Log[-((e*x^n)/d)]*Log[d + e*x^n] - e^2*g*p*
Log[n*(d + e*x^n)] + (d^2*g*Log[c*(d + e*x^n)^p])/x^(2*n) + n*Log[x]*(e^2*g
*p + 2*d^2*f*p*Log[d + e*x^n] - 2*d^2*f*Log[c*(d + e*x^n)^p]) - 2*d^2*f*p*P
olyLog[2, 1 + (e*x^n)/d])/(d^2*n)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 448, normalized size = 3.56

method	result
risch	$\frac{(2f\ln(x)n\,x^{2n}-g)x^{-2n}\ln((d+e\,x^n)^p)}{2n}+\frac{i\pi\,\text{csgn}(i(d+e\,x^n)^p)\text{csgn}(ic(d+e\,x^n)^p)\text{csgn}(ic)g\,x^{-2n}}{4n}-\frac{i\pi\text{csgn}(ic(d+e\,x^n)^p)^3f\ln(x^n)}{2n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g/(x^(2*n)))*ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*f*ln(x)*n*(x^n)^2-g)/n/(x^n)^2*ln((d+e*x^n)^p)+1/4*I/n*Pi*csgn(I*c*(
d+e*x^n)^p)^3*g/(x^n)^2-1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3*f*ln(x^n)-1/4*I/
```

```
n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2*g/(x^n)^2+1/2*I/n*Pi*csgn(
I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2*f*ln(x^n)-1/2*I/n*Pi*csgn(I*(d+e*x^n
)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*f*ln(x^n)+1/4*I/n*Pi*csgn(I*(d+e*x^n)^
p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*g/(x^n)^2+1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^
p)^2*csgn(I*c)*f*ln(x^n)-1/4*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)*g/(x^
n)^2+1/n*ln(c)*f*ln(x^n)-1/2/n*ln(c)*g/(x^n)^2+1/2*e^2*g*p*ln(d+e*x^n)/d^2/
n-1/2*e*g*p/d/n/(x^n)-1/2*p*e^2/n*g/d^2*ln(x^n)-p/n*f*dilog((d+e*x^n)/d)-p*
f*ln(x)*ln((d+e*x^n)/d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^(2*n)))*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/2*(g*p*e^(n*log(x) + 1) + d*g*log(c) + (d*f*n^2*p*log(x)^2 - 2*d*f*n*log
(c)*log(x))*x^(2*n) - (2*d*f*n*x^(2*n)*log(x) - d*g)*log((d + e^(n*log(x) +
 1))^p))/(d*n*x^(2*n)) + integrate(1/2*(2*d^2*f*n*p*log(x) - g*p*e^2)/(d^2*
x + d*x*e^(n*log(x) + 1)), x)
```

Fricas [A]
time = 0.37, size = 154, normalized size = 1.22

$$-\frac{2\,d^2fnpx^{2\,n}\log\left(x\right)\log\left(\frac{x^ne+d}{d}\right)+2\,d^2fpx^{2\,n}\mathrm{Li}_2\left(-\frac{x^ne+d}{d}+1\right)+dgpx^ne+d^2g\log\left(c\right)-\left(2\,d^2fn\log\left(c\right)-gnpe^2\right)x^{2\,n}\log\left(x\right)+\left(d^2gp-\left(2\,d^2fnp\log\left(x\right)+gpe^2\right)x^{2\,n}\right)\log\left(x^ne+d\right)}{2\,d^2nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^(2*n)))*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(2*d^2*f*n*p*x^(2*n)*log(x)*log((x^n*e + d)/d) + 2*d^2*f*p*x^(2*n)*dil
og(-(x^n*e + d)/d + 1) + d*g*p*x^n*e + d^2*g*log(c) - (2*d^2*f*n*log(c) - g
*n*p*e^2)*x^(2*n)*log(x) + (d^2*g*p - (2*d^2*f*n*p*log(x) + g*p*e^2)*x^(2*n
))*log(x^n*e + d))/(d^2*n*x^(2*n))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{-2n}\left(fx^{2n}+g\right)\log\left(c\left(d+ex^n\right)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x**(2*n)))*ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Integral((f*x**(2*n) + g)*log(c*(d + e*x**n)**p)/(x*x**(2*n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^(2*n)))*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
[Out] integrate((f + g/x^(2*n))*log((x^n*e + d)^p*c)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)\,\left(f+\frac{g}{x^{2\,n}}\right)}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^n)^p)*(f + g/x^(2*n)))/x,x)
[Out] int((log(c*(d + e*x^n)^p)*(f + g/x^(2*n)))/x, x)
```

3.365 $\int \frac{(f+gx^{3n})^2 \log(c(d+ex^n)^p)}{x} dx$

Optimal. Leaf size=327

$$-\frac{2d^2fgpx^n}{3e^2n}+\frac{d^5g^2px^n}{6e^5n}+\frac{dfgpx^{2n}}{3en}-\frac{d^4g^2px^{2n}}{12e^4n}-\frac{2fgpx^{3n}}{9n}+\frac{d^3g^2px^{3n}}{18e^3n}-\frac{d^2g^2px^{4n}}{24e^2n}+\frac{dg^2px^{5n}}{30en}-\frac{g^2px^{6n}}{36n}+\frac{2d^3fgp\log}{3e^3}$$

```
[Out] -2/3*d^2*f*g*p*x^n/e^2/n+1/6*d^5*g^2*p*x^n/e^5/n+1/3*d*f*g*p*x^(2*n)/e/n-1/
12*d^4*g^2*p*x^(2*n)/e^4/n-2/9*f*g*p*x^(3*n)/n+1/18*d^3*g^2*p*x^(3*n)/e^3/n
-1/24*d^2*g^2*p*x^(4*n)/e^2/n+1/30*d*g^2*p*x^(5*n)/e/n-1/36*g^2*p*x^(6*n)/n
+2/3*d^3*f*g*p*ln(d+e*x^n)/e^3/n-1/6*d^6*g^2*p*ln(d+e*x^n)/e^6/n+2/3*f*g*x^
(3*n)*ln(c*(d+e*x^n)^p)/n+1/6*g^2*x^(6*n)*ln(c*(d+e*x^n)^p)/n+f^2*ln(-e*x^n
/d)*ln(c*(d+e*x^n)^p)/n+f^2*p*polylog(2,1+e*x^n/d)/n
```

Rubi [A]
time = 0.21, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2525, 272, 45, 2463, 2441, 2352, 2442}

$$\frac{f^2p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{f^2\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n}+\frac{2fgx^{3n}\log\left(c(d+ex^n)^p\right)}{3n}+\frac{g^2x^{6n}\log\left(c(d+ex^n)^p\right)}{6n}-\frac{d^6g^2p\log\left(d+ex^n\right)}{6e^6n}+\frac{d^5g^2px^n}{6e^5n}-\frac{d^4g^2px^{2n}}{12e^4n}+\frac{2d^3fgp\log\left(d+ex^n\right)}{3e^3n}+\frac{d^3g^2px^{3n}}{18e^3n}-\frac{2d^2fgpx^n}{3e^2n}-\frac{d^2g^2px^{4n}}{24e^2n}+\frac{dfgpx^{2n}}{3en}+\frac{dg^2px^{5n}}{30en}-\frac{2fgpx^{3n}}{9n}-\frac{g^2px^{6n}}{36n}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^(3*n))^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (-2*d^2*f*g*p*x^n)/(3*e^2*n) + (d^5*g^2*p*x^n)/(6*e^5*n) + (d*f*g*p*x^(2*n)
)/(3*e*n) - (d^4*g^2*p*x^(2*n))/(12*e^4*n) - (2*f*g*p*x^(3*n))/(9*n) + (d^3
*g^2*p*x^(3*n))/(18*e^3*n) - (d^2*g^2*p*x^(4*n))/(24*e^2*n) + (d*g^2*p*x^(5
*n))/(30*e*n) - (g^2*p*x^(6*n))/(36*n) + (2*d^3*f*g*p*Log[d + e*x^n])/(3*e^
3*n) - (d^6*g^2*p*Log[d + e*x^n])/(6*e^6*n) + (2*f*g*x^(3*n)*Log[c*(d + e*x
^n)^p])/(3*n) + (g^2*x^(6*n)*Log[c*(d + e*x^n)^p])/(6*n) + (f^2*Log[-((e*x^
n)/d)]*Log[c*(d + e*x^n)^p])/n + (f^2*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{(f+gx^{3n})^2 \log(c(d+ex^n)^p)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{(f+gx^3)^2\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{f^2\log(c(d+ex)^p)}{x} + 2fgx^2\log(c(d+ex)^p) + g^2x^5\log(c(d+ex)\right.\right.}{n}$$
$$= \frac{f^2\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n} + \frac{(2fg)\text{Subst}\left(\int x^2\log(c(d+ex)^p)\right.}{n}$$
$$= \frac{2fgx^{3n}\log(c(d+ex^n)^p)}{3n} + \frac{g^2x^{6n}\log(c(d+ex^n)^p)}{6n} + \frac{f^2\log\left(-\frac{ex^n}{d}\right)\log}{n}$$
$$= \frac{2fgx^{3n}\log(c(d+ex^n)^p)}{3n} + \frac{g^2x^{6n}\log(c(d+ex^n)^p)}{6n} + \frac{f^2\log\left(-\frac{ex^n}{d}\right)\log}{n}$$
$$= -\frac{2d^2fgpx^n}{3e^2n} + \frac{d^5g^2px^n}{6e^5n} + \frac{dfgpx^{2n}}{3en} - \frac{d^4g^2px^{2n}}{12e^4n} - \frac{2fgpx^{3n}}{9n} + \frac{d^3g^2px^{3n}}{18e^3n}$$

Mathematica [A]
time = 0.15, size = 341, normalized size = 1.04

$$\frac{-240d^2e^4fgpx^n + 60d^5eg^2px^n + 120de^5fgpx^{2n} - 30d^4e^2g^2px^{2n} - 80e^6fgpx^{3n} + 20d^3e^3g^2px^{3n} - 15d^2e^4g^2px^{4n} + 12de^5g^2px^{5n} - 10e^6g^2px^{6n} + 240d^3e^3fgp\log(d-dx^n) - 60d^6g^2p\log(d-dx^n) + 360e^6f^2p\log\left(-\frac{ex^n}{d}\right)\log(d+ex^n) + 240e^6fgx^{3n}\log(c(d+ex^n)^p) + 60e^6g^2x^{6n}\log(c(d+ex^n)^p) - 60n\log(x)\left(d^3g(-4e^3f+d^3g)p + 6e^6f^2p\log(d+ex^n) - 6e^6f^2\log(c(d+ex^n)^p)\right) + 360e^6f^2p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{360e^6n}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f + g*x^(3*n))^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (-240*d^2*e^4*f*g*p*x^n + 60*d^5*e*g^2*p*x^n + 120*d*e^5*f*g*p*x^(2*n) - 30
*d^4*e^2*g^2*p*x^(2*n) - 80*e^6*f*g*p*x^(3*n) + 20*d^3*e^3*g^2*p*x^(3*n) -
15*d^2*e^4*g^2*p*x^(4*n) + 12*d*e^5*g^2*p*x^(5*n) - 10*e^6*g^2*p*x^(6*n) +
240*d^3*e^3*f*g*p*Log[d - d*x^n] - 60*d^6*g^2*p*Log[d - d*x^n] + 360*e^6*f^
2*p*Log[-((e*x^n)/d)]*Log[d + e*x^n] + 240*e^6*f*g*x^(3*n)*Log[c*(d + e*x^n
)^p] + 60*e^6*g^2*x^(6*n)*Log[c*(d + e*x^n)^p] - 60*n*Log[x]*(d^3*g*(-4*e^3
*f + d^3*g)*p + 6*e^6*f^2*p*Log[d + e*x^n] - 6*e^6*f^2*Log[c*(d + e*x^n)^p]
) + 360*e^6*f^2*p*PolyLog[2, 1 + (e*x^n)/d])/(360*e^6*n)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.34, size = 795, normalized size = 2.43

method	result
risch	$\frac{d^5g^2p\,x^n}{6e^5n} - \frac{d^4g^2p\,x^{2n}}{12e^4n} - \frac{g^2p\,x^{6n}}{36n} + \frac{2d^3fgp\ln(d+e\,x^n)}{3e^3n} - \frac{d^6g^2p\ln(d+e\,x^n)}{6e^6n} + \frac{d^3g^2p\,x^{3n}}{18e^3n} - \frac{d^2g^2p\,x^{4n}}{24e^2n} + \frac{d\,g^2p\,x^{5n}}{30en} - \frac{2d^2}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g*x^(3*n))^2*ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*d^5*g^2*p*x^n/e^5/n+2/3*d^3*f*g*p*ln(d+e*x^n)/e^3/n-1/6*d^6*g^2*p*ln(d+
e*x^n)/e^6/n-2/3*d^2*f*g*p*x^n/e^2/n+1/6*(g^2*(x^n)^6+4*f*g*(x^n)^3+6*f^2*l
n(x)*n)/n*ln((d+e*x^n)^p)-1/36*p/n*g^2*(x^n)^6-p/n*f^2*dilog((d+e*x^n)/d)-p
*f^2*ln(x)*ln((d+e*x^n)/d)+1/6/n*ln(c)*g^2*(x^n)^6+1/n*ln(c)*f^2*ln(x^n)+2/
3/n*ln(c)*f*g*(x^n)^3-2/9*p/n*f*g*(x^n)^3+1/12*I/n*Pi*csgn(I*c*(d+e*x^n)^p)
^2*csgn(I*c)*g^2*(x^n)^6+1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)*f^2*l
n(x^n)+1/3*p/e/n*f*g*d*(x^n)^2+1/12*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+
e*x^n)^p)^2*g^2*(x^n)^6+1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p
)^2*f^2*ln(x^n)-1/3*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3*f*g*(x^n)^3-1/3*I/n*Pi*c
sgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*f*g*(x^n)^3-1/2*I/n*Pi*c
sgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*f^2*ln(x^n)+1/3*I/n*Pi*c
sgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)*f*g*(x^n)^3+1/3*I/n*Pi*csgn(I*(d+e*x^n)^p)
*csgn(I*c*(d+e*x^n)^p)^2*f*g*(x^n)^3-1/12*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I
*c*(d+e*x^n)^p)*csgn(I*c)*g^2*(x^n)^6-1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3*f^
2*ln(x^n)+1/30*p/e/n*g^2*(x^n)^5*d-1/24*p/e^2/n*g^2*(x^n)^4*d^2+1/18*p/e^3/
n*g^2*d^3*(x^n)^3-1/12*p/e^4/n*g^2*(x^n)^2*d^4-1/12*I/n*Pi*csgn(I*c*(d+e*x^
n)^p)^3*g^2*(x^n)^6
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(3*n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/360*(180*f^2*n^2*p*e^6*log(x)^2 + 15*d^2*g^2*p*e^(4*n*log(x) + 4) - 12*d
*g^2*p*e^(5*n*log(x) + 5) - 20*(d^3*g^2*p*e^3 - 4*(f*g*p - 3*f*g*log(c))*e^
6)*x^(3*n) + 30*(d^4*g^2*p*e^2 - 4*d*f*g*p*e^5)*x^(2*n) - 60*(d^5*g^2*p*e -
 4*d^2*f*g*p*e^4)*x^n + 10*(g^2*p - 6*g^2*log(c))*e^(6*n*log(x) + 6) - 60*(
6*f^2*n*e^6*log(x) + g^2*e^(6*n*log(x) + 6) + 4*f*g*e^(3*n*log(x) + 6))*log
((d + e^(n*log(x) + 1))^p) + 60*(d^6*g^2*n*p - 4*d^3*f*g*n*p*e^3 - 6*f^2*n*
e^6*log(c))*log(x))*e^(-6)/n + integrate(1/6*(d^7*g^2*p - 4*d^4*f*g*p*e^3 +
 6*d*f^2*n*p*e^6*log(x))/(d*x*e^6 + x*e^(n*log(x) + 7)), x)
```

Fricas [A]
time = 0.38, size = 273, normalized size = 0.83

$$-\frac{\left(15\,d^2g^2px^{4\,n}e^4+360\,f^2npe^6\log\left(x\right)\log\left(\frac{x^ne+d}{d}\right)-12\,dg^2px^{5\,n}e^5-360\,f^2ne^6\log\left(c\right)\log\left(x\right)+360\,f^2p\mathrm{Li}_2\left(-\frac{x^ne+d}{d}+1\right)e^6+10\,\left(g^2pe^6-6\,g^2e^6\log\left(c\right)\right)x^{6\,n}-20\,\left(d^3g^2pe^3-4\,fgpe^6+12\,fge^6\log\left(c\right)\right)x^{3\,n}+30\,\left(d^4g^2pe^2-4\,dfgpe^5\right)x^{2\,n}-60\,\left(d^5g^2pe-4\,d^2fgpe^4\right)x^n+60\,\left(d^6g^2p-4\,d^3fgpe^3-6\,f^2npe^6\log\left(x\right)-g^2px^{6\,n}e^6-4\,fgpx^{3\,n}e^6\right)\log\left(x^ne+d\right)\right)e^{(-6)}}{360\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(3*n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -1/360*(15*d^2*g^2*p*x^(4*n)*e^4 + 360*f^2*n*p*e^6*log(x)*log((x^n*e + d)/d
) - 12*d*g^2*p*x^(5*n)*e^5 - 360*f^2*n*e^6*log(c)*log(x) + 360*f^2*p*dilog(
```

```
-(x^n*e + d)/d + 1)*e^6 + 10*(g^2*p*e^6 - 6*g^2*e^6*log(c))*x^(6*n) - 20*(d
^3*g^2*p*e^3 - 4*f*g*p*e^6 + 12*f*g*e^6*log(c))*x^(3*n) + 30*(d^4*g^2*p*e^2
 - 4*d*f*g*p*e^5)*x^(2*n) - 60*(d^5*g^2*p*e - 4*d^2*f*g*p*e^4)*x^n + 60*(d^
6*g^2*p - 4*d^3*f*g*p*e^3 - 6*f^2*n*p*e^6*log(x) - g^2*p*x^(6*n)*e^6 - 4*f*
g*p*x^(3*n)*e^6)*log(x^n*e + d))*e^(-6)/n
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x**(3*n))**2*ln(c*(d+e*x**n)**p)/x,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(3*n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
[Out] integrate((g*x^(3*n) + f)^2*log((x^n*e + d)^p*c)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)\,{\left(f+g\,x^{3\,n}\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^n)^p)*(f + g*x^(3*n))^2)/x,x)
[Out] int((log(c*(d + e*x^n)^p)*(f + g*x^(3*n))^2)/x, x)
```

3.366 $\int \frac{(f+gx^{2n})^2 \log(c(d+ex^n)^p)}{x} \, dx$

Optimal. Leaf size=254

$$\frac{dfgpx^n}{en}+\frac{d^3g^2px^n}{4e^3n}-\frac{fgpx^{2n}}{2n}-\frac{d^2g^2px^{2n}}{8e^2n}+\frac{dg^2px^{3n}}{12en}-\frac{g^2px^{4n}}{16n}-\frac{d^2fgp\log(d+ex^n)}{e^2n}-\frac{d^4g^2p\log(d+ex^n)}{4e^4n}+\frac{fgx^{2}}{}$$

```
[Out] d*f*g*p*x^n/e/n+1/4*d^3*g^2*p*x^n/e^3/n-1/2*f*g*p*x^(2*n)/n-1/8*d^2*g^2*p*x
^(2*n)/e^2/n+1/12*d*g^2*p*x^(3*n)/e/n-1/16*g^2*p*x^(4*n)/n-d^2*f*g*p*ln(d+e
*x^n)/e^2/n-1/4*d^4*g^2*p*ln(d+e*x^n)/e^4/n+f*g*x^(2*n)*ln(c*(d+e*x^n)^p)/n
+1/4*g^2*x^(4*n)*ln(c*(d+e*x^n)^p)/n+f^2*ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/n+f
^2*p*polylog(2,1+e*x^n/d)/n
```

Rubi [A]

time = 0.17, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2525, 272, 45, 2463, 2441, 2352, 2442}

$$\frac{f^2p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{f^2\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n}+\frac{fgx^{2n}\log\left(c(d+ex^n)^p\right)}{n}+\frac{g^2x^{4n}\log\left(c(d+ex^n)^p\right)}{4n}-\frac{d^4g^2p\log(d+ex^n)}{4e^4n}+\frac{d^3g^2px^n}{4e^3n}-\frac{d^2fgp\log(d+ex^n)}{e^2n}-\frac{d^2g^2px^{2n}}{8e^2n}+\frac{dfgpx^n}{en}+\frac{dg^2px^{3n}}{12en}-\frac{fgpx^{2n}}{2n}-\frac{g^2px^{4n}}{16n}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^(2*n))^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (d*f*g*p*x^n)/(e*n) + (d^3*g^2*p*x^n)/(4*e^3*n) - (f*g*p*x^(2*n))/(2*n) - (
d^2*g^2*p*x^(2*n))/(8*e^2*n) + (d*g^2*p*x^(3*n))/(12*e*n) - (g^2*p*x^(4*n))
/(16*n) - (d^2*f*g*p*Log[d + e*x^n])/(e^2*n) - (d^4*g^2*p*Log[d + e*x^n])/(
4*e^4*n) + (f*g*x^(2*n)*Log[c*(d + e*x^n)^p])/n + (g^2*x^(4*n)*Log[c*(d + e
*x^n)^p])/(4*n) + (f^2*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n + (f^2*p*P
olyLog[2, 1 + (e*x^n)/d])/n
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{(f+gx^{2n})^2 \log(c(d+ex^n)^p)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{(f+gx^2)^2\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{f^2\log(c(d+ex)^p)}{x} + 2fgx\log(c(d+ex)^p) + g^2x^3\log(c(d+ex$$

$$= \frac{f^2\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n} + \frac{(2fg)\text{Subst}\left(\int x\log(c(d+ex)^p)\right.}{n}$$

$$= \frac{fgx^{2n}\log(c(d+ex^n)^p)}{n} + \frac{g^2x^{4n}\log(c(d+ex^n)^p)}{4n} + \frac{f^2\log\left(-\frac{ex^n}{d}\right)\log}{n}$$

$$= \frac{fgx^{2n}\log(c(d+ex^n)^p)}{n} + \frac{g^2x^{4n}\log(c(d+ex^n)^p)}{4n} + \frac{f^2\log\left(-\frac{ex^n}{d}\right)\log}{n}$$

$$= \frac{dfgpx^n}{en} + \frac{d^3g^2px^n}{4e^3n} - \frac{fgpx^{2n}}{2n} - \frac{d^2g^2px^{2n}}{8e^2n} + \frac{dg^2px^{3n}}{12en} - \frac{g^2px^{4n}}{16n} - \frac{d^2}{}$$

Mathematica [A]
time = 0.15, size = 291, normalized size = 1.15

$$\frac{48de^3fgpx^n + 12d^3eg^2px^n - 24e^4fgpx^{2n} - 6d^2e^2g^2px^{2n} + 4de^3g^2px^{3n} - 3e^4g^2px^{4n} - 48d^2e^2fgp\log(d-dx^n) - 12d^4g^2p\log(d-dx^n) + 48e^4f^2p\log\left(-\frac{ex^n}{d}\right)\log(d+ex^n) + 48e^4fgx^{2n}\log(c(d+ex^n)^p) + 12e^4g^2x^{4n}\log(c(d+ex^n)^p) - 12n\log(x)\left(d^2g(4e^2f+d^2g)p + 4e^4f^2p\log(d+ex^n) - 4e^4f^2\log(c(d+ex^n)^p)\right) + 48e^4f^2p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{48e^4n}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f + g*x^(2*n))^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (48*d*e^3*f*g*p*x^n + 12*d^3*e*g^2*p*x^n - 24*e^4*f*g*p*x^(2*n) - 6*d^2*e^2
*g^2*p*x^(2*n) + 4*d*e^3*g^2*p*x^(3*n) - 3*e^4*g^2*p*x^(4*n) - 48*d^2*e^2*f
*g*p*Log[d - d*x^n] - 12*d^4*g^2*p*Log[d - d*x^n] + 48*e^4*f^2*p*Log[-((e*x
^n)/d)]*Log[d + e*x^n] + 48*e^4*f*g*x^(2*n)*Log[c*(d + e*x^n)^p] + 12*e^4*g
^2*x^(4*n)*Log[c*(d + e*x^n)^p] - 12*n*Log[x]*(d^2*g*(4*e^2*f + d^2*g)*p +
4*e^4*f^2*p*Log[d + e*x^n] - 4*e^4*f^2*Log[c*(d + e*x^n)^p]) + 48*e^4*f^2*p
*PolyLog[2, 1 + (e*x^n)/d])/(48*e^4*n)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.77, size = 734, normalized size = 2.89

method	result
risch	$-\frac{g^2px^{4n}}{16n} - \frac{d^2fgp\ln(d+ex^n)}{e^2n} - \frac{d^4g^2p\ln(d+ex^n)}{4e^4n} + \frac{d^3g^2px^n}{4e^3n} - \frac{d^2g^2px^{2n}}{8e^2n} + \frac{dg^2px^{3n}}{12en} + \frac{dfgpx^n}{en} - \frac{fgpx^{2n}}{2n} - \frac{pf^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g*x^(2*n))^2*ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -d^2*f*g*p*ln(d+e*x^n)/e^2/n-1/4*d^4*g^2*p*ln(d+e*x^n)/e^4/n+1/4*d^3*g^2*p*
x^n/e^3/n+d*f*g*p*x^n/e/n-p/n*f^2*dilog((d+e*x^n)/d)-p*f^2*ln(x)*ln((d+e*x^
n)/d)+1/n*ln(c)*f^2*ln(x^n)+1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)*f^
2*ln(x^n)+1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2*f^2*ln(x^n
)-1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*f^2*ln(x^n
)-1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3*f^2*ln(x^n)+1/8*I/n*Pi*csgn(I*c*(d+e*x
^n)^p)^2*csgn(I*c)*g^2*(x^n)^4+1/8*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e
*x^n)^p)^2*g^2*(x^n)^4-1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3*f*g*(x^n)^2+1/2*I
/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2*f*g*(x^n)^2-1/8*I/n*Pi*cs
gn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*g^2*(x^n)^4+1/2*I/n*Pi*cs
gn(I*c*(d+e*x^n)^p)^2*csgn(I*c)*f*g*(x^n)^2+1/12*p/e/n*g^2*d*(x^n)^3-1/8*p/
e^2/n*g^2*d^2*(x^n)^2-1/8*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3*g^2*(x^n)^4-1/2*I/
n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*f*g*(x^n)^2+1/4*(g
^2*(x^n)^4+4*f^2*ln(x)*n+4*f*g*(x^n)^2)/n*ln((d+e*x^n)^p)-1/16*p/n*g^2*(x^n
)^4+1/4/n*ln(c)*g^2*(x^n)^4+1/n*ln(c)*f*g*(x^n)^2-1/2*p/n*f*g*(x^n)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(2*n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/48*(24*f^2*n^2*p*e^4*log(x)^2 - 4*d*g^2*p*e^(3*n*log(x) + 3) + 6*(d^2*g^
2*p*e^2 + 4*(f*g*p - 2*f*g*log(c))*e^4)*x^(2*n) - 12*(d^3*g^2*p*e + 4*d*f*g
*p*e^3)*x^n + 3*(g^2*p - 4*g^2*log(c))*e^(4*n*log(x) + 4) - 12*(4*f^2*n*e^4
*log(x) + g^2*e^(4*n*log(x) + 4) + 4*f*g*e^(2*n*log(x) + 4))*log((d + e^(n*
log(x) + 1))^p) + 12*(d^4*g^2*n*p + 4*d^2*f*g*n*p*e^2 - 4*f^2*n*e^4*log(c))
*log(x))*e^(-4)/n + integrate(1/4*(d^5*g^2*p + 4*d^3*f*g*p*e^2 + 4*d*f^2*n*
p*e^4*log(x))/(d*x*e^4 + x*e^(n*log(x) + 5)), x)
```

Fricas [A]
time = 0.38, size = 229, normalized size = 0.90

$$-\frac{\left(48 f^{2} n p e^{4} \log\left(x\right) \log\left(\frac{x^{n} e+d}{d}\right) - 4 d g^{2} p x^{3 n} e^{3} - 48 f^{2} n e^{4} \log\left(c\right) \log\left(x\right) + 48 f^{2} p \mathrm{Li}_2\left(-\frac{x^{n} e+d}{d} + 1\right) e^{4} + 3 \left(g^{2} p e^{4} - 4 g^{2} e^{4} \log\left(c\right)\right) x^{4 n} + 6 \left(d^{2} g^{2} p e^{2} + 4 f g p e^{4} - 8 f g e^{4} \log\left(c\right)\right) x^{2 n} - 12 \left(d^{3} g^{2} p e + 4 d f g p e^{3}\right) x^{n} + 12 \left(d^{4} g^{2} p + 4 d^{2} f g p e^{2} - 4 f^{2} n p e^{4} \log\left(x\right) - g^{2} p x^{4 n} e^{4} - 4 f g p x^{2 n} e^{4}\right) \log\left(x^{n} e + d\right)\right) e^{\left(-4\right)}}{48 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(2*n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -1/48*(48*f^2*n*p*e^4*log(x)*log((x^n*e + d)/d) - 4*d*g^2*p*x^(3*n)*e^3 - 4
8*f^2*n*e^4*log(c)*log(x) + 48*f^2*p*dilog(-(x^n*e + d)/d + 1)*e^4 + 3*(g^2
*p*e^4 - 4*g^2*e^4*log(c))*x^(4*n) + 6*(d^2*g^2*p*e^2 + 4*f*g*p*e^4 - 8*f*g
*e^4*log(c))*x^(2*n) - 12*(d^3*g^2*p*e + 4*d*f*g*p*e^3)*x^n + 12*(d^4*g^2*p
 + 4*d^2*f*g*p*e^2 - 4*f^2*n*p*e^4*log(x) - g^2*p*x^(4*n)*e^4 - 4*f*g*p*x^(
2*n)*e^4)*log(x^n*e + d))*e^(-4)/n
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f + gx^{2n}\right)^2 \log\left(c(d + ex^n)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x**(2*n))**2*ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Integral((f + g*x**(2*n))**2*log(c*(d + e*x**n)**p)/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(2*n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] integrate((g*x^(2*n) + f)^2*log((x^n*e + d)^p*c)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(d + e\,x^n\right)^p\right)\,\left(f + g\,x^{2\,n}\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^n)^p)*(f + g*x^(2*n))^2)/x,x)
```

```
[Out] int((log(c*(d + e*x^n)^p)*(f + g*x^(2*n))^2)/x, x)
```

3.367 $\int \frac{(f+gx^n)^2\log(c(d+ex^n)^p)}{x}\,dx$

Optimal. Leaf size=176

$$-\frac{2fgpx^n}{n}+\frac{dg^2px^n}{2en}-\frac{g^2px^{2n}}{4n}-\frac{d^2g^2p\log(d+ex^n)}{2e^2n}+\frac{g^2x^{2n}\log(c(d+ex^n)^p)}{2n}+\frac{2fg(d+ex^n)\log(c(d+ex^n)^p)}{en}+$$

```
[Out] -2*f*g*p*x^n/n+1/2*d*g^2*p*x^n/e/n-1/4*g^2*p*x^(2*n)/n-1/2*d^2*g^2*p*ln(d+e
*x^n)/e^2/n+1/2*g^2*x^(2*n)*ln(c*(d+e*x^n)^p)/n+2*f*g*(d+e*x^n)*ln(c*(d+e*x
^n)^p)/e/n+f^2*ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/n+f^2*p*polylog(2,1+e*x^n/d)/
n
```

Rubi [A]
time = 0.12, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2525, 45, 2463, 2436, 2332, 2441, 2352, 2442}

$$\frac{f^2p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{f^2\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n}+\frac{2fg(d+ex^n)\log(c(d+ex^n)^p)}{en}+\frac{g^2x^{2n}\log(c(d+ex^n)^p)}{2n}-\frac{d^2g^2p\log(d+ex^n)}{2e^2n}+\frac{dg^2px^n}{2en}-\frac{2fgpx^n}{n}-\frac{g^2px^{2n}}{4n}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x^n)^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (-2*f*g*p*x^n)/n + (d*g^2*p*x^n)/(2*e*n) - (g^2*p*x^(2*n))/(4*n) - (d^2*g^2
*p*Log[d + e*x^n])/(2*e^2*n) + (g^2*x^(2*n)*Log[c*(d + e*x^n)^p])/(2*n) + (
2*f*g*(d + e*x^n)*Log[c*(d + e*x^n)^p])/(e*n) + (f^2*Log[-((e*x^n)/d)]*Log[
c*(d + e*x^n)^p])/n + (f^2*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{(f+gx^n)^2\log(c(d+ex^n)^p)}{x}dx = \frac{\text{Subst}\left(\int \frac{(f+gx)^2\log(c(d+ex)^p)}{x}dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(2fg\log(c(d+ex)^p) + \frac{f^2\log(c(d+ex)^p)}{x} + g^2x\log(c(d+ex)^p)\right)\right.}{n}$$

$$= \frac{f^2\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}dx, x, x^n\right)}{n} + \frac{(2fg)\text{Subst}\left(\int \log(c(d+ex)^p)\,dx,\right.}{n}$$

$$= \frac{g^2x^{2n}\log(c(d+ex^n)^p)}{2n} + \frac{f^2\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} + \frac{(2fg)\text{Subst}}{}$$

$$= -\frac{2fgpx^n}{n} + \frac{g^2x^{2n}\log(c(d+ex^n)^p)}{2n} + \frac{2fg(d+ex^n)\log(c(d+ex^n)^p)}{en} +$$

$$= -\frac{2fgpx^n}{n} + \frac{dg^2px^n}{2en} - \frac{g^2px^{2n}}{4n} - \frac{d^2g^2p\log(d+ex^n)}{2e^2n} + \frac{g^2x^{2n}\log(c(d+}{2n}$$

Mathematica [A]
time = 0.14, size = 124, normalized size = 0.70

$$\frac{-egpx^n(8ef-2dg+egx^n)-2d^2g^2p\log(d+ex^n)+2e\left(4dfg+egx^n(4f+gx^n)+2ef^2\log\left(-\frac{ex^n}{d}\right)\right)\log(c(d+ex^n)^p)+4e^2f^2p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{4e^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x^n)^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (-(e*g*p*x^n*(8*e*f - 2*d*g + e*g*x^n)) - 2*d^2*g^2*p*Log[d + e*x^n] + 2*e*
(4*d*f*g + e*g*x^n*(4*f + g*x^n) + 2*e*f^2*Log[-((e*x^n)/d)])*Log[c*(d + e*
x^n)^p] + 4*e^2*f^2*p*PolyLog[2, 1 + (e*x^n)/d])/(4*e^2*n)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.43, size = 665, normalized size = 3.78

method	result
risch	$\frac{(2f^2\ln(x)n+g^2x^{2n}+4fg\,x^n)\ln((d+e\,x^n)^p)}{2n} + \frac{i\pi\,\text{csgn}(i(d+e\,x^n)^p)\text{csgn}(ic(d+e\,x^n)^p)^2x^nfg}{n} - \frac{i\pi\,\text{csgn}(i(d+e\,x^n)^p)\text{csgn}(ic(d+e\,x^n}{4n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g*x^n)^2*ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*f^2*ln(x)*n+g^2*(x^n)^2+4*f*g*x^n)/n*ln((d+e*x^n)^p)+I/n*Pi*csgn(I*(
d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2*x^n*f*g+I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*
csgn(I*c)*x^n*f*g+1/4*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2*(x
```

```
^n)^2*g^2-I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*x^n*f*
g+1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2*f^2*ln(x^n)-1/2*I/
n*Pi*csgn(I*c*(d+e*x^n)^p)^3*f^2*ln(x^n)-I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3*x^n
*f*g+1/4*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)*(x^n)^2*g^2-1/4*I/n*Pi*cs
gn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*(x^n)^2*g^2-1/2*I/n*Pi*cs
gn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*f^2*ln(x^n)+1/2*I/n*Pi*cs
gn(I*c*(d+e*x^n)^p)^2*csgn(I*c)*f^2*ln(x^n)-1/4*I/n*Pi*csgn(I*c*(d+e*x^n)^p
)^3*(x^n)^2*g^2+1/2/n*ln(c)*(x^n)^2*g^2+2/n*ln(c)*x^n*f*g+1/n*ln(c)*f^2*ln(
x^n)-1/4*p/n*g^2*(x^n)^2+1/2*d*g^2*p*x^n/e/n-1/2*d^2*g^2*p*ln(d+e*x^n)/e^2/
n-p/n*f^2*dilog((d+e*x^n)/d)-p*f^2*ln(x)*ln((d+e*x^n)/d)-2*f*g*p*x^n/n+2*p/
e/n*f*g*d*ln(d+e*x^n)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^n)^2*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/4*(2*f^2*n^2*p*e^2*log(x)^2 - 2*(d*g^2*p*e - 4*(f*g*p - f*g*log(c))*e^2)
*x^n + (g^2*p - 2*g^2*log(c))*e^(2*n*log(x) + 2) - 2*(2*f^2*n*e^2*log(x) +
g^2*e^(2*n*log(x) + 2) + 4*f*g*e^(n*log(x) + 2))*log((d + e^(n*log(x) + 1))
^p) + 2*(d^2*g^2*n*p - 4*d*f*g*n*p*e - 2*f^2*n*e^2*log(c))*log(x))*e^(-2)/n
 + integrate(1/2*(2*d*f^2*n*p*e^2*log(x) + d^3*g^2*p - 4*d^2*f*g*p*e)/(d*x*
e^2 + x*e^(n*log(x) + 3)), x)
```

Fricas [A]
time = 0.38, size = 182, normalized size = 1.03

$$-\frac{\left(4\,f^2npe^2\log\left(x\right)\log\left(\frac{x^ne+d}{d}\right)-4\,f^2ne^2\log\left(c\right)\log\left(x\right)+4\,f^2p\mathrm{Li}_2\left(-\frac{x^ne+d}{d}+1\right)e^2+\left(g^2pe^2-2\,g^2e^2\log\left(c\right)\right)x^{2\,n}-2\left(dg^2pe-4\,fgpe^2+4\,fge^2\log\left(c\right)\right)x^n-2\left(2\,f^2npe^2\log\left(x\right)-d^2g^2p+g^2px^{2\,n}e^2+4\,fgpx^ne^2+4\,dfgpe\right)\log\left(x^ne+d\right)\right)e^{(-2)}}{4\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^n)^2*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -1/4*(4*f^2*n*p*e^2*log(x)*log((x^n*e + d)/d) - 4*f^2*n*e^2*log(c)*log(x) +
 4*f^2*p*dilog(-(x^n*e + d)/d + 1)*e^2 + (g^2*p*e^2 - 2*g^2*e^2*log(c))*x^(
2*n) - 2*(d*g^2*p*e - 4*f*g*p*e^2 + 4*f*g*e^2*log(c))*x^n - 2*(2*f^2*n*p*e^
2*log(x) - d^2*g^2*p + g^2*p*x^(2*n)*e^2 + 4*f*g*p*x^n*e^2 + 4*d*f*g*p*e)*l
og(x^n*e + d))*e^(-2)/n
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(f + gx^{n}\right)^{2}\log\left(c\left(d + ex^{n}\right)^{p}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x**n)**2*ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Integral((f + g*x**n)**2*log(c*(d + e*x**n)**p)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^n)^2*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] integrate((g*x^n + f)^2*log((x^n*e + d)^p*c)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)\,{\left(f+g\,x^n\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^n)^p)*(f + g*x^n)^2)/x,x)
```

```
[Out] int((log(c*(d + e*x^n)^p)*(f + g*x^n)^2)/x, x)
```

3.368 $\int \frac{(f+gx^{-n})^2 \log(c(d+ex^n)^p)}{x} \, dx$

Optimal. Leaf size=193

$$-\frac{eg^2px^{-n}}{2dn}+\frac{2efgp\log(x)}{d}-\frac{e^2g^2p\log(x)}{2d^2}-\frac{2efgp\log\left(d+ex^n\right)}{dn}+\frac{e^2g^2p\log\left(d+ex^n\right)}{2d^2n}-\frac{g^2x^{-2n}\log\left(c(d+ex^n\right)^{p}}{2n}$$

```
[Out] -1/2*e*g^2*p/d/n/(x^n)+2*e*f*g*p*ln(x)/d-1/2*e^2*g^2*p*ln(x)/d^2-2*e*f*g*p*
ln(d+e*x^n)/d/n+1/2*e^2*g^2*p*ln(d+e*x^n)/d^2/n-1/2*g^2*ln(c*(d+e*x^n)^p)/n
/(x^(2*n))-2*f*g*ln(c*(d+e*x^n)^p)/n/(x^n)+f^2*ln(-e*x^n/d)*ln(c*(d+e*x^n)^
p)/n+f^2*p*polylog(2,1+e*x^n/d)/n
```

Rubi [A]
time = 0.16, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {2525, 269, 45, 2463, 2442, 46, 36, 29, 31, 2441, 2352}

$$\frac{f^2p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{f^2\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{2fgx^{-n}\log\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{g^2x^{-2n}\log\left(c\left(d+ex^n\right)^p\right)}{2n}+\frac{e^2g^2p\log\left(d+ex^n\right)}{2d^2n}-\frac{e^2g^2p\log(x)}{2d^2}-\frac{2efgp\log\left(d+ex^n\right)}{dn}+\frac{2efgp\log(x)}{d}-\frac{eg^2px^{-n}}{2dn}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g/x^n)^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] -1/2*(e*g^2*p)/(d*n*x^n) + (2*e*f*g*p*Log[x])/d - (e^2*g^2*p*Log[x])/(2*d^2
) - (2*e*f*g*p*Log[d + e*x^n])/(d*n) + (e^2*g^2*p*Log[d + e*x^n])/(2*d^2*n)
 - (g^2*Log[c*(d + e*x^n)^p])/(2*n*x^(2*n)) - (2*f*g*Log[c*(d + e*x^n)^p])/
(n*x^n) + (f^2*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n + (f^2*p*PolyLog[2
, 1 + (e*x^n)/d])/n
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
```

```
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(f+gx^{-n})^2 \log(c(d+ex^n)^p)}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(f+\frac{g}{x}\right)^2 \log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n} \\
&= \frac{\text{Subst}\left(\int \left(\frac{g^2\log(c(d+ex)^p)}{x^3}+\frac{2fg\log(c(d+ex)^p)}{x^2}+\frac{f^2\log(c(d+ex)^p)}{x}\right)dx, x, x^n\right)}{n} \\
&= \frac{f^2\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}+\frac{(2fg)\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x^2}\,dx, x,\right.}{n} \\
&= -\frac{g^2x^{-2n}\log(c(d+ex^n)^p)}{2n}-\frac{2fgx^{-n}\log(c(d+ex^n)^p)}{n}+\frac{f^2\log\left(-\frac{ex}{d}\right.}{} \\
&= -\frac{g^2x^{-2n}\log(c(d+ex^n)^p)}{2n}-\frac{2fgx^{-n}\log(c(d+ex^n)^p)}{n}+\frac{f^2\log\left(-\frac{ex}{d}\right.}{} \\
&= -\frac{eg^2px^{-n}}{2dn}+\frac{2efgp\log(x)}{d}-\frac{e^2g^2p\log(x)}{2d^2}-\frac{2efgp\log(d+ex^n)}{dn}+
\end{aligned}$$

Mathematica [A]
time = 0.45, size = 152, normalized size = 0.79

$$-\frac{eg(4df-eg)p\log(e-ex^{-n})-2d^2f^2n\log(x)\log(c(d+ex^n)^p)+dgx^{-2n}(egpx^n+d(g+4fx^n)\log(c(d+ex^n)^p))+np\log(x)\left(eg(-4df+eg)+2d^2f^2\log\left(1+\frac{ex^n}{d}\right)\right)+2d^2f^2p\text{Li}_2\left(-\frac{ex^n}{d}\right)}{2d^2n}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f + g/x^n)^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] -1/2*(e*g*(4*d*f - e*g)*p*Log[e - e/x^n] - 2*d^2*f^2*n*Log[x]*Log[c*(d + e*
x^n)^p] + (d*g*(e*g*p*x^n + d*(g + 4*f*x^n)*Log[c*(d + e*x^n)^p]))/x^(2*n)
+ n*p*Log[x]*(e*g*(-4*d*f + e*g) + 2*d^2*f^2*Log[1 + (e*x^n)/d]) + 2*d^2*f^
2*p*PolyLog[2, -((e*x^n)/d)])/(d^2*n)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.46, size = 693, normalized size = 3.59

method	result

risch	$\frac{(2f^2\ln(x)n\,x^{2n}-4fg\,x^n-g^2)x^{-2n}\ln((d+e\,x^n)^p)}{2n} - \frac{i\pi\,\mathrm{csgn}(i(d+e\,x^n)^p)\mathrm{csgn}(ic(d+e\,x^n)^p)^2g^2x^{-2n}}{4n} + \frac{i\pi\,\mathrm{csgn}(i(d+e\,x^n)^p)\mathrm{csgn}(ic}{4n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g/(x^n))^2*ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*f^2*ln(x)*n*(x^n)^2-4*f*g*x^n-g^2)/n/(x^n)^2*ln((d+e*x^n)^p)+I/n*Pi*
csgn(I*c*(d+e*x^n)^p)^3*f*g/(x^n)+1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(
d+e*x^n)^p)^2*f^2*ln(x^n)-1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3*f^2*ln(x^n)-I/
n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)*f*g/(x^n)+I/n*Pi*csgn(I*(d+e*x^n)^p)
*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*f*g/(x^n)-1/4*I/n*Pi*csgn(I*(d+e*x^n)^p)*c
sgn(I*c*(d+e*x^n)^p)^2*g^2/(x^n)^2-1/4*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(
I*c)*g^2/(x^n)^2-I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2*f*g/(x^
n)-1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*f^2*ln(x^
n)+1/4*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*g^2/(x^n)
^2+1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)*f^2*ln(x^n)+1/4*I/n*Pi*csgn
(I*c*(d+e*x^n)^p)^3*g^2/(x^n)^2+1/n*ln(c)*f^2*ln(x^n)-1/2/n*ln(c)*g^2/(x^n)
^2-2/n*ln(c)*f*g/(x^n)-2*e*f*g*p*ln(d+e*x^n)/d/n+2*p*e/n*f*g/d*ln(x^n)+1/2*
e^2*g^2*p*ln(d+e*x^n)/d^2/n-1/2*e*g^2*p/d/n/(x^n)-1/2*p*e^2/n*g^2/d^2*ln(x^
n)-p/n*f^2*dilog((d+e*x^n)/d)-p*f^2*ln(x)*ln((d+e*x^n)/d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/2*(d*g^2*log(c) + (d*f^2*n^2*p*log(x)^2 - 2*d*f^2*n*log(c)*log(x))*x^(2*
n) + (g^2*p*e + 4*d*f*g*log(c))*x^n - (2*d*f^2*n*x^(2*n)*log(x) - 4*d*f*g*x
^n - d*g^2)*log((d + e^(n*log(x) + 1))^p))/(d*n*x^(2*n)) + integrate(1/2*(2
*d^2*f^2*n*p*log(x) + 4*d*f*g*p*e - g^2*p*e^2)/(d^2*x + d*x*e^(n*log(x) + 1
)), x)
```

Fricas [A]
time = 0.35, size = 211, normalized size = 1.09

$$-\frac{2\,d^2f^2npx^{2\,n}\log\left(x\right)\log\left(\frac{x^ne+d}{d}\right)+2\,d^2f^2px^{2\,n}\mathrm{Li}_2\left(-\frac{x^ne+d}{d}+1\right)+d^2g^2\log\left(c\right)-\left(4\,dfgnpe+2\,d^2f^2n\log\left(c\right)-g^2npe^2\right)x^{2\,n}\log\left(x\right)+\left(dg^2pe+4\,d^2fg\log\left(c\right)\right)x^n+\left(4\,d^2fgpx^n+d^2g^2p-\left(2\,d^2f^2np\log\left(x\right)-4\,dfgpe+g^2pe^2\right)x^{2\,n}\right)\log\left(x^ne+d\right)}{2\,d^2nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(2*d^2*f^2*n*p*x^(2*n)*log(x)*log((x^n*e + d)/d) + 2*d^2*f^2*p*x^(2*n)
*dilog(-(x^n*e + d)/d + 1) + d^2*g^2*log(c) - (4*d*f*g*n*p*e + 2*d^2*f^2*n*
```

```
log(c) - g^2*n*p*e^2)*x^(2*n)*log(x) + (d*g^2*p*e + 4*d^2*f*g*log(c))*x^n +
 (4*d^2*f*g*p*x^n + d^2*g^2*p - (2*d^2*f^2*n*p*log(x) - 4*d*f*g*p*e + g^2*p
*e^2)*x^(2*n))*log(x^n*e + d))/(d^2*n*x^(2*n))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-2n}(fx^n+g)^2\log\left(c(d+ex^n)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x**n))**2*ln(c*(d+e*x**n)**p)/x,x)
[Out] Integral((f*x**n + g)**2*log(c*(d + e*x**n)**p)/(x*x**(2*n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
[Out] integrate((f + g/x^n)^2*log((x^n*e + d)^p*c)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)\,{\left(f+\frac{g}{x^n}\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^n)^p)*(f + g/x^n)^2)/x,x)
[Out] int((log(c*(d + e*x^n)^p)*(f + g/x^n)^2)/x, x)
```

3.369 $\int \frac{(f+gx^{-2n})^2 \log(c(d+ex^n)^p)}{x} \, dx$

Optimal. Leaf size=257

$$-\frac{eg^2px^{-3n}}{12dn}+\frac{e^2g^2px^{-2n}}{8d^2n}-\frac{efgpx^{-n}}{dn}-\frac{e^3g^2px^{-n}}{4d^3n}-\frac{e^2fgp\log(x)}{d^2}-\frac{e^4g^2p\log(x)}{4d^4}+\frac{e^2fgp\log(d+ex^n)}{d^2n}+\frac{e^4g^2p\log(}{4d}$$

```
[Out] -1/12*e*g^2*p/d/n/(x^(3*n))+1/8*e^2*g^2*p/d^2/n/(x^(2*n))-e*f*g*p/d/n/(x^n)
-1/4*e^3*g^2*p/d^3/n/(x^n)-e^2*f*g*p*ln(x)/d^2-1/4*e^4*g^2*p*ln(x)/d^4+e^2*
f*g*p*ln(d+e*x^n)/d^2/n+1/4*e^4*g^2*p*ln(d+e*x^n)/d^4/n-1/4*g^2*ln(c*(d+e*x
^n)^p)/n/(x^(4*n))-f*g*ln(c*(d+e*x^n)^p)/n/(x^(2*n))+f^2*ln(-e*x^n/d)*ln(c*
(d+e*x^n)^p)/n+f^2*p*polylog(2,1+e*x^n/d)/n
```

Rubi [A]

time = 0.20, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2525, 269, 272, 45, 2463, 2442, 46, 2441, 2352}

$$\frac{f^2p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{f^2\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n}-\frac{fgx^{-2n}\log\left(c(d+ex^n)^p\right)}{n}-\frac{g^2x^{-4n}\log\left(c(d+ex^n)^p\right)}{4n}+\frac{e^4g^2p\log\left(d+ex^n\right)}{4d^4n}-\frac{e^4g^2p\log(x)}{4d^4}-\frac{e^3g^2px^{-n}}{4d^3n}+\frac{e^2fgp\log\left(d+ex^n\right)}{d^2n}-\frac{e^2fgp\log(x)}{d^2}+\frac{e^2g^2px^{-2n}}{8d^2n}-\frac{efgpx^{-n}}{dn}-\frac{eg^2px^{-3n}}{12dn}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g/x^(2*n))^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] -1/12*(e*g^2*p)/(d*n*x^(3*n)) + (e^2*g^2*p)/(8*d^2*n*x^(2*n)) - (e*f*g*p)/(
d*n*x^n) - (e^3*g^2*p)/(4*d^3*n*x^n) - (e^2*f*g*p*Log[x])/d^2 - (e^4*g^2*p*
Log[x])/(4*d^4) + (e^2*f*g*p*Log[d + e*x^n])/(d^2*n) + (e^4*g^2*p*Log[d + e
*x^n])/(4*d^4*n) - (g^2*Log[c*(d + e*x^n)^p])/(4*n*x^(4*n)) - (f*g*Log[c*(d
 + e*x^n)^p])/(n*x^(2*n)) + (f^2*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n
+ (f^2*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{(f+gx^{-2n})^2 \log(c(d+ex^n)^p)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\left(f+\frac{g}{x^2}\right)^2 \log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{g^2\log(c(d+ex)^p)}{x^5} + \frac{2fg\log(c(d+ex)^p)}{x^3} + \frac{f^2\log(c(d+ex)^p)}{x}\right)\,dx, x, x^n\right)}{n}$$

$$= \frac{f^2\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n} + \frac{(2fg)\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x^3}\,dx, x, x\right.}{n}$$

$$= -\frac{g^2x^{-4n}\log(c(d+ex^n)^p)}{4n} - \frac{fgx^{-2n}\log(c(d+ex^n)^p)}{n} + \frac{f^2\log\left(-\frac{ex^n}{d}\right.}{}$$

$$= -\frac{g^2x^{-4n}\log(c(d+ex^n)^p)}{4n} - \frac{fgx^{-2n}\log(c(d+ex^n)^p)}{n} + \frac{f^2\log\left(-\frac{ex^n}{d}\right.}{}$$

$$= -\frac{eg^2px^{-3n}}{12dn} + \frac{e^2g^2px^{-2n}}{8d^2n} - \frac{efgpx^{-n}}{dn} - \frac{e^3g^2px^{-n}}{4d^3n} - \frac{e^2fgp\log(x)}{d^2} -$$

Mathematica [A]
time = 0.41, size = 204, normalized size = 0.79

$$\frac{g(6e^2(4d^2f+e^2g)p\log(e-ex^{-n}) - dx^{-4n}(epx^n(-3degx^n+6e^2gx^{2n}+2d^2(g+12fx^{2n}))+6d^3(g+4fx^{2n})\log(c(d+ex^n)^p))) - 6n\log(x)\left(e^2g(4d^2f+e^2g)p - 4d^4f^2\log(c(d+ex^n)^p) + 4d^4f^2p\log\left(1+\frac{ex^n}{d}\right)\right) - 24d^4f^2p\text{Li}_2\left(-\frac{ex^n}{d}\right)}{24d^4n}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f + g/x^(2*n))^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (g*(6*e^2*(4*d^2*f + e^2*g)*p*Log[e - e/x^n] - (d*(e*p*x^n*(-3*d*e*g*x^n +
6*e^2*g*x^(2*n) + 2*d^2*(g + 12*f*x^(2*n))) + 6*d^3*(g + 4*f*x^(2*n))*Log[c
*(d + e*x^n)^p]))/x^(4*n)) - 6*n*Log[x]*(e^2*g*(4*d^2*f + e^2*g)*p - 4*d^4*
f^2*Log[c*(d + e*x^n)^p] + 4*d^4*f^2*p*Log[1 + (e*x^n)/d]) - 24*d^4*f^2*p*P
olyLog[2, -((e*x^n)/d)])/(24*d^4*n)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.46, size = 755, normalized size = 2.94

method	result
risch	$\frac{e^2fgp\ln(d+e\,x^n)}{d^2n} + \frac{e^4g^2p\ln(d+e\,x^n)}{4d^4n} - \frac{p\,f^2\,\text{dilog}\left(\frac{d+e\,x^n}{d}\right)}{n} - p\,f^2\ln(x)\ln\left(\frac{d+e\,x^n}{d}\right) + \frac{\ln(c)f^2\ln(x^n)}{n} + \frac{i\pi\text{csgn}(ic(d+e}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g/(x^(2*n)))^2*ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

```
[Out] e^2*f*g*p*ln(d+e*x^n)/d^2/n-1/4*e^3*g^2*p/d^3/n/(x^n)+1/4*e^4*g^2*p*ln(d+e*
x^n)/d^4/n-e*f*g*p/d/n/(x^n)-p/n*f^2*dilog((d+e*x^n)/d)-p*f^2*ln(x)*ln((d+e
*x^n)/d)+1/n*ln(c)*f^2*ln(x^n)+1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)
*f^2*ln(x^n)+1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2*f^2*ln(
x^n)-1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)*f^2*ln(
x^n)-1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3*f^2*ln(x^n)-p*e^2/n*f*g/d^2*ln(x^n)
+1/4*(4*f^2*ln(x)*n*(x^n)^4-4*f*g*(x^n)^2-g^2)/n/(x^n)^4*ln((d+e*x^n)^p)-1/
4/n*ln(c)*g^2/(x^n)^4-1/n*ln(c)*f*g/(x^n)^2-1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p
)^2*csgn(I*c)*f*g/(x^n)^2-1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)
^p)^2*f*g/(x^n)^2+1/8*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn
(I*c)*g^2/(x^n)^4-1/12*p*e/n*g^2/d/(x^n)^3+1/8*p*e^2/n*g^2/d^2/(x^n)^2-1/4*
p*e^4/n*g^2/d^4*ln(x^n)+1/8*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3*g^2/(x^n)^4-1/8*
I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2*g^2/(x^n)^4+1/2*I/n*Pi*c
sgn(I*c*(d+e*x^n)^p)^3*f*g/(x^n)^2-1/8*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(
I*c)*g^2/(x^n)^4+1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(
I*c)*f*g/(x^n)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^(2*n)))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/24*(2*d^2*g^2*p*e^(n*log(x) + 1) + 6*d^3*g^2*log(c) + 12*(d^3*f^2*n^2*p*
log(x)^2 - 2*d^3*f^2*n*log(c)*log(x))*x^(4*n) + 6*(4*d^2*f*g*p*e + g^2*p*e^
3)*x^(3*n) + 3*(8*d^3*f*g*log(c) - d*g^2*p*e^2)*x^(2*n) - 6*(4*d^3*f^2*n*x^
(4*n)*log(x) - 4*d^3*f*g*x^(2*n) - d^3*g^2)*log((d + e^(n*log(x) + 1))^p))/
(d^3*n*x^(4*n)) + integrate(1/4*(4*d^4*f^2*n*p*log(x) - 4*d^2*f*g*p*e^2 - g
^2*p*e^4)/(d^4*x + d^3*x*e^(n*log(x) + 1)), x)
```

Fricas [A]
time = 0.40, size = 265, normalized size = 1.03

$$-\frac{24\,d^4f^2npx^{4\,n}\log\left(x\right)\log\left(\frac{x^ne+d}{d}\right)+24\,d^4f^2px^{4\,n}\mathrm{Li}_2\left(-\frac{x^ne+d}{d}+1\right)+2\,d^3g^2px^ne+6\,d^4g^2\log\left(c\right)-6\,(4\,d^4f^2n\log\left(c\right)-4\,d^2fgnpe^2-g^2npe^4)x^{4\,n}\log\left(x\right)+6\,(4\,d^3fgpe+dg^2pe^3)x^{3\,n}+3\,(8\,d^4fg\log\left(c\right)-d^2g^2pe^2)x^{2\,n}+6\,(4\,d^4fgpx^{2\,n}+d^4g^2p-(4\,d^4f^2np\log\left(x\right)+4\,d^2fgpe^2+g^2pe^4)x^{4\,n})\log\left(x^ne+d\right)}{24\,d^4nx^{4\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^(2*n)))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -1/24*(24*d^4*f^2*n*p*x^(4*n)*log(x)*log((x^n*e + d)/d) + 24*d^4*f^2*p*x^(4
*n)*dilog(-(x^n*e + d)/d + 1) + 2*d^3*g^2*p*x^n*e + 6*d^4*g^2*log(c) - 6*(4
*d^4*f^2*n*log(c) - 4*d^2*f*g*n*p*e^2 - g^2*n*p*e^4)*x^(4*n)*log(x) + 6*(4*
d^3*f*g*p*e + d*g^2*p*e^3)*x^(3*n) + 3*(8*d^4*f*g*log(c) - d^2*g^2*p*e^2)*x
^(2*n) + 6*(4*d^4*f*g*p*x^(2*n) + d^4*g^2*p - (4*d^4*f^2*n*p*log(x) + 4*d^2
*f*g*p*e^2 + g^2*p*e^4)*x^(4*n))*log(x^n*e + d))/(d^4*n*x^(4*n))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x**(2*n)))**2*ln(c*(d+e*x**n)**p)/x,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^(2*n)))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
[Out] integrate((f + g/x^(2*n))^2*log((x^n*e + d)^p*c)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(d+e\,x^n\right)^p\right)\,\left(f+\frac{g}{x^{2\,n}}\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^n)^p)*(f + g/x^(2*n))^2)/x,x)
[Out] int((log(c*(d + e*x^n)^p)*(f + g/x^(2*n))^2)/x, x)
```

3.370 $\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{2n})}\,dx$

Optimal. Leaf size=266

$$\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{fn}-\frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}\,x^n\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2fn}-\frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e\left(\sqrt{-f}\right.}{e\sqrt{-f}}\right.}{2fn}$$

[Out] ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/f/n-1/2*ln(c*(d+e*x^n)^p)*ln(e*((-f)^(1/2)-x^n*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))/f/n-1/2*ln(c*(d+e*x^n)^p)*ln(e*((-f)^(1/2)+x^n*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/f/n+p*polylog(2,1+e*x^n/d)/f/n-1/2*p*polylog(2,-(d+e*x^n)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/f/n-1/2*p*polylog(2,(d+e*x^n)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/f/n

Rubi [A]
time = 0.30, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {2525, 272, 36, 29, 31, 2463, 2441, 2352, 266, 2440, 2438}

$$-\frac{p\text{PolyLog}\left(2,-\frac{\sqrt{g}\,(d+ex^n)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2fn}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt{g}\,(d+ex^n)}{d\sqrt{g}+e\sqrt{-f}}\right)}{2fn}+\frac{p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{fn}-\frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}\,x^n\right)}{d\sqrt{g}+e\sqrt{-f}}\right)}{2fn}-\frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}\,x^n\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2fn}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{fn}$$

Antiderivative was successfully verified.

[In] Int[Log[c*(d + e*x^n)^p]/(x*(f + g*x^(2*n))),x]

[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/(f*n) - (Log[c*(d + e*x^n)^p]*Log[(e*(Sqrt[-f] - Sqrt[g]*x^n))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*f*n) - (Log[c*(d + e*x^n)^p]*Log[(e*(Sqrt[-f] + Sqrt[g]*x^n))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*f*n) - (p*PolyLog[2, -((Sqrt[g]*(d + e*x^n))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*f*n) - (p*PolyLog[2, (Sqrt[g]*(d + e*x^n))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*f*n) + (p*PolyLog[2, 1 + (e*x^n)/d])/(f*n)

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}\,dx = \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x(f+gx^2)}\,dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{\log(c(d+ex)^p)}{fx} - \frac{gx\log(c(d+ex)^p)}{f(f+gx^2)}\right)\,dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{fn} - \frac{g\text{Subst}\left(\int \frac{x\log(c(d+ex)^p)}{f+gx^2}\,dx, x, x^n\right)}{fn}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{fn} - \frac{g\text{Subst}\left(\int \left(-\frac{\log(c(d+ex)^p)}{2\sqrt{g}\left(\sqrt{-f}-\sqrt{g}\,x\right)} + \frac{\log(c(d\cdots}{2\sqrt{g}\left(\sqrt{-\cdots}\right.}\right.\right.}{fn}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{fn} + \frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{fn} + \frac{\sqrt{g}\,\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt{-f}-\sqrt{g}\,x}\,dx, x\cdots\right.}{2fn}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{fn} - \frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}\,x^n\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2fn} - \frac{\log\cdots}{\cdots}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{fn} - \frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}\,x^n\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2fn} - \frac{\log\cdots}{\cdots}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{fn} - \frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}\,x^n\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2fn} - \frac{\log\cdots}{\cdots}$$

Mathematica [F]
time = 4.15, size = 0, normalized size = 0.00

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g*x^(2*n))),x]
```

```
[Out] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g*x^(2*n))), x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.90, size = 695, normalized size = 2.61

method	result
risch	$-\frac{\ln((d+e\,x^n)^p)\ln(f+g\,x^{2n})}{2nf}+\frac{\ln((d+e\,x^n)^p)\ln(x^n)}{nf}-\frac{p\,\mathrm{dilog}\left(\frac{d+e\,x^n}{d}\right)}{fn}-\frac{p\ln(x^n)\ln\left(\frac{d+e\,x^n}{d}\right)}{nf}+\frac{p\ln(d+e\,x^n)\ln(f+g\,x^{2n})}{2nf}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)/x/(f+g*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/n*ln((d+e*x^n)^p)/f*ln(f+g*(x^n)^2)+1/n*ln((d+e*x^n)^p)/f*ln(x^n)-1/f*
p/n*dilog((d+e*x^n)/d)-1/n*p/f*ln(x^n)*ln((d+e*x^n)/d)+1/2/n*p/f*ln(d+e*x^n
)*ln(f+g*(x^n)^2)-1/2/n*p/f*ln(d+e*x^n)*ln((e*(-f*g)^(1/2)-g*(d+e*x^n)+d*g)
/(e*(-f*g)^(1/2)+d*g))-1/2/n*p/f*ln(d+e*x^n)*ln((e*(-f*g)^(1/2)+g*(d+e*x^n)
-d*g)/(e*(-f*g)^(1/2)-d*g))-1/2/n*p/f*dilog((e*(-f*g)^(1/2)-g*(d+e*x^n)+d*g
)/(e*(-f*g)^(1/2)+d*g))-1/2/n*p/f*dilog((e*(-f*g)^(1/2)+g*(d+e*x^n)-d*g)/(e
*(-f*g)^(1/2)-d*g))-1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3/f*ln(x^n)+1/4*I/n*Pi
*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)/f*ln(f+g*(x^n)^2)+1/4*
I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3/f*ln(f+g*(x^n)^2)-1/4*I/n*Pi*csgn(I*(d+e*x^n
)^p)*csgn(I*c*(d+e*x^n)^p)^2/f*ln(f+g*(x^n)^2)-1/2*I/n*Pi*csgn(I*(d+e*x^n)^
p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)/f*ln(x^n)+1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)
*csgn(I*c*(d+e*x^n)^p)^2/f*ln(x^n)+1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(
I*c)/f*ln(x^n)-1/4*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)/f*ln(f+g*(x^n)^
2)-1/2/n*ln(c)/f*ln(f+g*(x^n)^2)+1/n*ln(c)/f*ln(x^n)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(log((x^n*e + d)^p*c)/((g*x^(2*n) + f)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^(2*n)),x, algorithm="fricas")
[Out] integral(log((x^n*e + d)^p*c)/(g*x*x^(2*n) + f*x), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x/(f+g*x**(2*n)),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^(2*n)),x, algorithm="giac")
[Out] integrate(log((x^n*e + d)^p*c)/((g*x^(2*n) + f)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\left(d+e\,x^n\right)^p\right)}{x\,\left(f+g\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/(x*(f + g*x^(2*n))),x)
[Out] int(log(c*(d + e*x^n)^p)/(x*(f + g*x^(2*n))), x)
```

3.371 $\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^n)}\,dx$

Optimal. Leaf size=121

$$\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{fn}-\frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e(f+gx^n)}{ef-dg}\right)}{fn}-\frac{p\text{Li}_2\left(-\frac{g(d+ex^n)}{ef-dg}\right)}{fn}+\frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{fn}$$

```
[Out] ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/f/n-ln(c*(d+e*x^n)^p)*ln(e*(f+g*x^n)/(-d*g+e
*f))/f/n-p*polylog(2,-g*(d+e*x^n)/(-d*g+e*f))/f/n+p*polylog(2,1+e*x^n/d)/f/
n
```

Rubi [A]
time = 0.14, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {2525, 36, 29, 31, 2463, 2441, 2352, 2440, 2438}

$$-\frac{p\text{PolyLog}\left(2,-\frac{g(d+ex^n)}{ef-dg}\right)}{fn}+\frac{p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{fn}-\frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e(f+gx^n)}{ef-dg}\right)}{fn}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{fn}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/(x*(f + g*x^n)),x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/(f*n) - (Log[c*(d + e*x^n)^p]*Log[
(e*(f + g*x^n))/(e*f - d*g)])/(f*n) - (p*PolyLog[2, -((g*(d + e*x^n))/(e*f
- d*g))])/(f*n) + (p*PolyLog[2, 1 + (e*x^n)/d])/(f*n)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^n)}\,dx = \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x(f+gx)}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{\log(c(d+ex)^p)}{fx} - \frac{g\log(c(d+ex)^p)}{f(f+gx)}\right)\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{fn} - \frac{g\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{f+gx}\,dx, x, x^n\right)}{fn}$$
$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{fn} - \frac{\log(c(d+ex^n)^p)\log\left(\frac{e(f+gx^n)}{ef-dg}\right)}{fn} - \frac{(ep)\text{Subst}\left(\int \frac{\log(}{d}\right.}{f}$$
$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{fn} - \frac{\log(c(d+ex^n)^p)\log\left(\frac{e(f+gx^n)}{ef-dg}\right)}{fn} + \frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{fn} +$$
$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{fn} - \frac{\log(c(d+ex^n)^p)\log\left(\frac{e(f+gx^n)}{ef-dg}\right)}{fn} - \frac{p\text{Li}_2\left(-\frac{g(d+ex^n)}{ef-dg}\right)}{fn}$$

Mathematica [A]
time = 0.06, size = 92, normalized size = 0.76

$$\frac{\log(c(d+ex^n)^p)\left(\log\left(-\frac{ex^n}{d}\right) - \log\left(\frac{e(f+gx^n)}{ef-dg}\right)\right) - p\text{Li}_2\left(\frac{g(d+ex^n)}{-ef+dg}\right) + p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g*x^n)),x]
```

```
[Out] (Log[c*(d + e*x^n)^p]*(Log[-((e*x^n)/d)] - Log[(e*(f + g*x^n))/(e*f - d*g)]
) - p*PolyLog[2, (g*(d + e*x^n))/(-(e*f) + d*g)] + p*PolyLog[2, 1 + (e*x^n)
/d])/(f*n)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.78, size = 532, normalized size = 4.40

method	result
risch	$-\frac{\ln((d+e\,x^n)^p)\ln(f+g\,x^n)}{nf} + \frac{\ln((d+e\,x^n)^p)\ln(x^n)}{nf} - \frac{p\,\text{dilog}\left(\frac{d+e\,x^n}{d}\right)}{fn} - \frac{p\ln(x^n)\ln\left(\frac{d+e\,x^n}{d}\right)}{nf} + \frac{p\,\text{dilog}\left(\frac{(f+g\,x^n)e+dg-ef}{dg-ef}\right)}{nf}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)/x/(f+g*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] -1/n*ln((d+e*x^n)^p)/f*ln(f+g*x^n)+1/n*ln((d+e*x^n)^p)/f*ln(x^n)-1/f*p/n*di
log((d+e*x^n)/d)-1/n*p/f*ln(x^n)*ln((d+e*x^n)/d)+1/n*p/f*dilog(((f+g*x^n)*e
+d*g-e*f)/(d*g-e*f))+1/n*p/f*ln(f+g*x^n)*ln(((f+g*x^n)*e+d*g-e*f)/(d*g-e*f)
)+1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3/f*ln(f+g*x^n)-1/2*I/n*Pi*csgn(I*c*(d+e
*x^n)^p)^3/f*ln(x^n)-1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*c
sgn(I*c)/f*ln(x^n)-1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2/f
*ln(f+g*x^n)-1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)/f*ln(f+g*x^n)+1/2
*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2/f*ln(x^n)+1/2*I/n*Pi*cs
gn(I*c*(d+e*x^n)^p)^2*csgn(I*c)/f*ln(x^n)+1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*cs
gn(I*c*(d+e*x^n)^p)*csgn(I*c)/f*ln(f+g*x^n)-1/n*ln(c)/f*ln(f+g*x^n)+1/n*ln(
c)/f*ln(x^n)
```

Maxima [A]
time = 0.79, size = 169, normalized size = 1.40

$$np\left(\frac{\left(\log\left(gx^n+f\right)\log\left(\frac{fe+ge^{(n\log(x)+1)}}{dg-fe}+1\right)+\operatorname{Li}_2\left(-\frac{fe+ge^{(n\log(x)+1)}}{dg-fe}\right)\right)e^{(-1)}}{fn^2}-\frac{\left(\log\left(x^n\right)\log\left(\frac{e^{(n\log(x)+1)}}{d}+1\right)+\operatorname{Li}_2\left(-\frac{e^{(n\log(x)+1)}}{d}\right)\right)e^{(-1)}}{fn^2}\right)e-\left(\frac{\log\left(gx^n+f\right)}{fn}-\frac{\log\left(x^n\right)}{fn}\right)\log\left(\left(x^ne+d\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^n),x, algorithm="maxima")
```

```
[Out] n*p*((log(g*x^n + f)*log((f*e + g*e^(n*log(x) + 1))/(d*g - f*e) + 1) + dilo
g(-(f*e + g*e^(n*log(x) + 1))/(d*g - f*e)))*e^(-1)/(f*n^2) - (log(x^n)*log(
e^(n*log(x) + 1)/d + 1) + dilog(-e^(n*log(x) + 1)/d))*e^(-1)/(f*n^2))*e - (
log(g*x^n + f)/(f*n) - log(x^n)/(f*n))*log((x^n*e + d)^p*c)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^n),x, algorithm="fricas")
```

```
[Out] integral(log((x^n*e + d)^p*c)/(g*x*x^n + f*x), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x/(f+g*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^n),x, algorithm="giac")
```

```
[Out] integrate(log((x^n*e + d)^p*c)/((g*x^n + f)*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)}{x\,\left(f+g\,x^n\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/(x*(f + g*x^n)),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)/(x*(f + g*x^n)), x)
```

3.372 $\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{-n})}\,dx$

Optimal. Leaf size=70

$$\frac{\log\left(c(d+ex^n)^p\right)\log\left(-\frac{e(g+fx^n)}{df-eg}\right)}{fn}+\frac{p\text{Li}_2\left(\frac{f(d+ex^n)}{df-eg}\right)}{fn}$$

```
[Out] ln(c*(d+e*x^n)^p)*ln(-e*(g+f*x^n)/(d*f-e*g))/f/n+p*polylog(2,f*(d+e*x^n)/(d
*f-e*g))/f/n
```

Rubi [A]
time = 0.11, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2525, 2459, 2441, 2440, 2438}

$$\frac{p\text{PolyLog}\left(2,\frac{f(d+ex^n)}{df-eg}\right)}{fn}+\frac{\log\left(c(d+ex^n)^p\right)\log\left(-\frac{e(fx^n+g)}{df-eg}\right)}{fn}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/(x*(f + g/x^n)),x]
```

```
[Out] (Log[c*(d + e*x^n)^p]*Log[-((e*(g + f*x^n))/(d*f - e*g))])/(f*n) + (p*PolyL
og[2, (f*(d + e*x^n))/(d*f - e*g)])/(f*n)
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2459

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)/(x_))^(q_.)*(x_)^(m_.), x_Symbol] :> Int[(g + f*x)^q*(a + b*Log[c*(d + e*
x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q}, x] && EqQ[m, q] &&
 IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{-n})}\,dx &= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\left(f+\frac{g}{x}\right)x}\,dx, x, x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{g+fx}\,dx, x, x^n\right)}{n}\\
&= \frac{\log(c(d+ex^n)^p)\log\left(-\frac{e(g+fx^n)}{df-eg}\right)}{fn} - \frac{(ep)\text{Subst}\left(\int \frac{\log\left(\frac{e(g+fx)}{-df+eg}\right)}{d+ex}\,dx, x, x^n\right)}{fn}\\
&= \frac{\log(c(d+ex^n)^p)\log\left(-\frac{e(g+fx^n)}{df-eg}\right)}{fn} - \frac{p\text{Subst}\left(\int \frac{\log\left(1+\frac{fx}{-df+eg}\right)}{x}\,dx, x, d+ex^n\right)}{fn}\\
&= \frac{\log(c(d+ex^n)^p)\log\left(-\frac{e(g+fx^n)}{df-eg}\right)}{fn} + \frac{p\text{Li}_2\left(\frac{f(d+ex^n)}{df-eg}\right)}{fn}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 64, normalized size = 0.91

$$\frac{\log(c(d+ex^n)^p)\log\left(\frac{e(g+fx^n)}{-df+eg}\right)+p\text{Li}_2\left(\frac{f(d+ex^n)}{df-eg}\right)}{fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g/x^n)),x]
[Out] (Log[c*(d + e*x^n)^p]*Log[(e*(g + f*x^n))/(-(d*f) + e*g)] + p*PolyLog[2, (f
*(d + e*x^n))/(d*f - e*g)])/(f*n)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.92, size = 298, normalized size = 4.26

method	result
risch	$\frac{\ln(g+f\,x^{n})\ln((d+e\,x^{n})^{p})}{nf}-\frac{p\,\mathrm{dilog}\left(\frac{(g+f\,x^{n})e+df-eg}{df-eg}\right)}{nf}-\frac{p\ln(g+f\,x^{n})\ln\left(\frac{(g+f\,x^{n})e+df-eg}{df-eg}\right)}{nf}+\frac{i\ln(g+f\,x^{n})\pi\,\mathrm{csgn}(i(d+e\,x^{n})}{2nf}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)/x/(f+g/(x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*ln(g+f*x^n)/f*ln((d+e*x^n)^p)-1/n/f*p*dilog(((g+f*x^n)*e+d*f-e*g)/(d*f-
e*g))-1/n/f*p*ln(g+f*x^n)*ln(((g+f*x^n)*e+d*f-e*g)/(d*f-e*g))+1/2*I/n*ln(g+
f*x^n)/f*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-1/2*I/n*ln(g+f*x^n)
/f*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/2*I/n*ln(g+f*x^
n)/f*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I/n*ln(g+f*x^n)/f*Pi*csgn(I*c*(d+e*x^n)
^p)^2*csgn(I*c)+1/n*ln(g+f*x^n)/f*ln(c)
```

Maxima [A]
time = 0.65, size = 123, normalized size = 1.76

$$\left(\frac{\log\left(f+\frac{g}{x^{n}}\right)}{fn}-\frac{\log\left(\frac{1}{x^{n}}\right)}{fn}\right)\log\left((x^{n}e+d)^{p}c\right)-\frac{\left(\log\left(fx^{n}+g\right)\log\left(\frac{ge+fe^{(n\log(x)+1)}}{df-ge}+1\right)+\mathrm{Li}_{2}\left(-\frac{ge+fe^{(n\log(x)+1)}}{df-ge}\right)\right)p}{fn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^n)),x, algorithm="maxima")
```

```
[Out] (log(f + g/x^n)/(f*n) - log(1/(x^n))/(f*n))*log((x^n*e + d)^p*c) - (log(f*x
^n + g)*log((g*e + f*e^(n*log(x) + 1))/(d*f - g*e) + 1) + dilog(-(g*e + f*e
^(n*log(x) + 1))/(d*f - g*e)))*p/(f*n)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^n)),x, algorithm="fricas")
```

```
[Out] integral(x^n*log((x^n*e + d)^p*c)/(f*x*x^n + g*x), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x/(f+g/(x**n)),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^n)),x, algorithm="giac")
```

```
[Out] integrate(log((x^n*e + d)^p*c)/((f + g/x^n)*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)}{x\,\left(f+\frac{g}{x^n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/(x*(f + g/x^n)),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)/(x*(f + g/x^n)), x)
```

3.373 $\int \frac{\log(c(d+ex^n)^p)}{x\left(f+gx^{-2n}\right)}\,dx$

Optimal. Leaf size=221

$$\frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}\,x^n\right)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2fn}+\frac{\log\left(c(d+ex^n)^p\right)\log\left(-\frac{e\left(\sqrt{g}+\sqrt{-f}\,x^n\right)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2fn}+\frac{p\mathrm{Li}_2\left(\frac{\sqrt{-f}\,(d\ldots}{d\sqrt{-f}-\ldots}\right.}{2fn}$$

```
[Out] 1/2*ln(c*(d+e*x^n)^p)*ln(-e*(x^n*(-f)^(1/2)+g^(1/2))/(d*(-f)^(1/2)-e*g^(1/2
)))/f/n+1/2*ln(c*(d+e*x^n)^p)*ln(e*(-x^n*(-f)^(1/2)+g^(1/2))/(d*(-f)^(1/2)+
e*g^(1/2)))/f/n+1/2*p*polylog(2,(d+e*x^n)*(-f)^(1/2)/(d*(-f)^(1/2)-e*g^(1/2
)))/f/n+1/2*p*polylog(2,(d+e*x^n)*(-f)^(1/2)/(d*(-f)^(1/2)+e*g^(1/2)))/f/n
```

Rubi [A]
time = 0.31, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2525, 269, 266, 2463, 2441, 2440, 2438}

$$\frac{p\text{PolyLog}\left(2,\frac{\sqrt{-f}\,(d+ex^n)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2fn}+\frac{p\text{PolyLog}\left(2,\frac{\sqrt{-f}\,(d+ex^n)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2fn}+\frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}\,x^n\right)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2fn}+\frac{\log\left(c(d+ex^n)^p\right)\log\left(-\frac{e\left(\sqrt{-f}\,x^n+\sqrt{g}\right)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2fn}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/(x*(f + g/x^(2*n))),x]
```

```
[Out] (Log[c*(d + e*x^n)^p]*Log[(e*(Sqrt[g] - Sqrt[-f]*x^n))/(d*Sqrt[-f] + e*Sqrt
[g])])/(2*f*n) + (Log[c*(d + e*x^n)^p]*Log[-((e*(Sqrt[g] + Sqrt[-f]*x^n))/(
d*Sqrt[-f] - e*Sqrt[g]))])/(2*f*n) + (p*PolyLog[2, (Sqrt[-f]*(d + e*x^n))/(
d*Sqrt[-f] - e*Sqrt[g])])/(2*f*n) + (p*PolyLog[2, (Sqrt[-f]*(d + e*x^n))/(d
*Sqrt[-f] + e*Sqrt[g])])/(2*f*n)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx = \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\left(f+\frac{g}{x^2}\right)x}\,dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{\sqrt{-f}\,\log(c(d+ex)^p)}{2f\left(\sqrt{g}-\sqrt{-f}\,x\right)}+\frac{\sqrt{-f}\,\log(c(d+ex)^p)}{2f\left(\sqrt{g}+\sqrt{-f}\,x\right)}\right)dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt{g}-\sqrt{-f}\,x}\,dx, x, x^n\right)}{2\sqrt{-f}\,n} - \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt{g}+\sqrt{-f}\,x}\,dx, x, x^n\right)}{2\sqrt{-f}\,n}$$

$$= \frac{\log \left(c(d+ex^n)^p\right)\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}\,x^n\right)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2fn} + \frac{\log \left(c(d+ex^n)^p\right)\log\left(-\frac{e\left(\sqrt{g}+\sqrt{\right.}}{d\sqrt{-f}\,-}\right.}{2fn}$$

$$= \frac{\log \left(c(d+ex^n)^p\right)\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}\,x^n\right)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2fn} + \frac{\log \left(c(d+ex^n)^p\right)\log\left(-\frac{e\left(\sqrt{g}+\sqrt{\right.}}{d\sqrt{-f}\,-}\right.}{2fn}$$

$$= \frac{\log \left(c(d+ex^n)^p\right)\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}\,x^n\right)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2fn} + \frac{\log \left(c(d+ex^n)^p\right)\log\left(-\frac{e\left(\sqrt{g}+\sqrt{\right.}}{d\sqrt{-f}\,-}\right.}{2fn}$$

Mathematica [F]
time = 1.32, size = 0, normalized size = 0.00

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g/x^(2*n))),x]
```

```
[Out] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g/x^(2*n))), x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.85, size = 461, normalized size = 2.09

method	result
risch	$\frac{\ln(g+f\,x^{2n})\ln((d+e\,x^{n})^{p})}{2nf}-\frac{p\ln(d+e\,x^{n})\ln(g+f\,x^{2n})}{2nf}+\frac{p\ln(d+e\,x^{n})\ln\left(\frac{e\sqrt{-fg}-f(d+e\,x^{n})+df}{e\sqrt{-fg}+df}\right)}{2nf}+\frac{p\ln(d+e\,x^{n})\ln\left(\frac{e\sqrt{-}}{}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n))),x,method=_RETURNVERBOSE)

[Out] 1/2/n/f*ln(g+f*(x^n)^2)*ln((d+e*x^n)^p)-1/2/n/f*p*ln(d+e*x^n)*ln(g+f*(x^n)^2)+1/2/n/f*p*ln(d+e*x^n)*ln((e*(-f*g)^(1/2)-f*(d+e*x^n)+d*f)/(e*(-f*g)^(1/2)+d*f))+1/2/n/f*p*ln(d+e*x^n)*ln((e*(-f*g)^(1/2)+f*(d+e*x^n)-d*f)/(e*(-f*g)^(1/2)-d*f))+1/2/n/f*p*dilog((e*(-f*g)^(1/2)-f*(d+e*x^n)+d*f)/(e*(-f*g)^(1/2)+d*f))+1/2/n/f*p*dilog((e*(-f*g)^(1/2)+f*(d+e*x^n)-d*f)/(e*(-f*g)^(1/2)-d*f))+1/4*I/n/f*ln(g+f*(x^n)^2)*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-1/4*I/n/f*ln(g+f*(x^n)^2)*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/4*I/n/f*ln(g+f*(x^n)^2)*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/4*I/n/f*ln(g+f*(x^n)^2)*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+1/2/n/f*ln(g+f*(x^n)^2)*ln(c)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n))),x, algorithm="maxima")

[Out] integrate(log((x^n*e + d)^p*c)/((f + g/x^(2*n))*x), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n))),x, algorithm="fricas")

[Out] integral(x^(2*n)*log((x^n*e + d)^p*c)/(f*x*x^(2*n) + g*x), x)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x/(f+g/(x**(2*n))),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n))),x, algorithm="giac")
[Out] integrate(log((x^n*e + d)^p*c)/((f + g/x^(2*n))*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)}{x\,\left(f+\frac{g}{x^{2\,n}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/(x*(f + g/x^(2*n))),x)
[Out] int(log(c*(d + e*x^n)^p)/(x*(f + g/x^(2*n))), x)
```

3.374 $\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{2n})^2}\,dx$

Optimal. Leaf size=419

$$-\frac{de\sqrt{g}\,p\tan^{-1}\left(\frac{\sqrt{g}\,x^n}{\sqrt{f}}\right)}{2f^{3/2}\left(e^2f+d^2g\right)n}-\frac{e^2p\log\left(d+ex^n\right)}{2f\left(e^2f+d^2g\right)n}+\frac{\log\left(c(d+ex^n)^p\right)}{2fn\left(f+gx^{2n}\right)}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{f^2n}-\frac{\log\left(c(d+e\right.}{}$$

```
[Out] -1/2*e^2*p*ln(d+e*x^n)/f/(d^2*g+e^2*f)/n+1/2*ln(c*(d+e*x^n)^p)/f/n/(f+g*x^(
2*n))+ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/f^2/n+1/4*e^2*p*ln(f+g*x^(2*n))/f/(d^2
*g+e^2*f)/n-1/2*ln(c*(d+e*x^n)^p)*ln(e*((-f)^(1/2)-x^n*g^(1/2))/(e*(-f)^(1/
2)+d*g^(1/2)))/f^2/n-1/2*ln(c*(d+e*x^n)^p)*ln(e*((-f)^(1/2)+x^n*g^(1/2))/(e
*(-f)^(1/2)-d*g^(1/2)))/f^2/n+p*polylog(2,1+e*x^n/d)/f^2/n-1/2*p*polylog(2,
-(d+e*x^n)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/f^2/n-1/2*p*polylog(2,(d+e*x^n
)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/f^2/n-1/2*d*e*p*arctan(x^n*g^(1/2)/f^(1
/2))*g^(1/2)/f^(3/2)/(d^2*g+e^2*f)/n
```

Rubi [A]
time = 0.40, antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.518$, Rules used = {2525, 272, 46, 2463, 2441, 2352, 2460, 720, 31, 649, 211, 266, 2440, 2438}

$$-\frac{p\text{PolyLog}\left(2,-\frac{\sqrt{g}\,(d+ex^n)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2n}-\frac{p\text{PolyLog}\left(2,\frac{\sqrt{g}\,(d+ex^n)}{d\sqrt{g}+e\sqrt{-f}}\right)}{2f^2n}+\frac{p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{f^2n}-\frac{de\sqrt{g}\,p\text{ArcTan}\left(\frac{\sqrt{g}\,x^n}{\sqrt{f}}\right)}{2f^{3/2}n\left(d^2g+e^2f\right)}-\frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}\,x^n\right)}{d\sqrt{g}+e\sqrt{-f}}\right)}{2f^2n}-\frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}\,x^n\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2n}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{f^2n}+\frac{\log\left(c(d+ex^n)^p\right)}{2fn\left(f+gx^{2n}\right)}+\frac{e^2p\log\left(f+gx^{2n}\right)}{4fn\left(d^2g+e^2f\right)}-\frac{e^2p\log\left(d+ex^n\right)}{2fn\left(d^2g+e^2f\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/(x*(f + g*x^(2*n))^2),x]
```

```
[Out] -1/2*(d*e*Sqrt[g]*p*ArcTan[(Sqrt[g]*x^n)/Sqrt[f]])/(f^(3/2)*(e^2*f + d^2*g)
*n) - (e^2*p*Log[d + e*x^n])/(2*f*(e^2*f + d^2*g)*n) + Log[c*(d + e*x^n)^p]
/(2*f*n*(f + g*x^(2*n))) + (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/(f^2*n)
 - (Log[c*(d + e*x^n)^p]*Log[(e*(Sqrt[-f] - Sqrt[g]*x^n))/(e*Sqrt[-f] + d*S
qrt[g])])/(2*f^2*n) - (Log[c*(d + e*x^n)^p]*Log[(e*(Sqrt[-f] + Sqrt[g]*x^n)
)/(e*Sqrt[-f] - d*Sqrt[g])])/(2*f^2*n) + (e^2*p*Log[f + g*x^(2*n)])/(4*f*(e
^2*f + d^2*g)*n) - (p*PolyLog[2, -((Sqrt[g]*(d + e*x^n))/(e*Sqrt[-f] - d*Sq
rt[g]))])/(2*f^2*n) - (p*PolyLog[2, (Sqrt[g]*(d + e*x^n))/(e*Sqrt[-f] + d*S
qrt[g])])/(2*f^2*n) + (p*PolyLog[2, 1 + (e*x^n)/d])/(f^2*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2460

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(x_)^(m_.)*(
(f_.) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Simp[(f + g*x^r)^(q + 1)*((a
+ b*Log[c*(d + e*x)^n])^p/(g*r*(q + 1))), x] - Dist[b*e*n*(p/(g*r*(q + 1)))
, Int[(f + g*x^r)^(q + 1)*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x
], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q, r}, x] && EqQ[m, r - 1] && Ne
Q[q, -1] && IGtQ[p, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x(f+gx^2)^2}\,dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{\log(c(d+ex)^p)}{f^2x} - \frac{gx\log(c(d+ex)^p)}{f(f+gx^2)^2} - \frac{gx\log(c(d+ex)^p)}{f^2(f+gx^2)}\right)dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{f^2n} - \frac{g\text{Subst}\left(\int \frac{x\log(c(d+ex)^p)}{f+gx^2}\,dx, x, x^n\right)}{f^2n} - \frac{g\text{Subst}\left(\int\right.}{}$$

$$= \frac{\log\left(c(d+ex^n)^p\right)}{2fn\left(f+gx^{2n}\right)} + \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{f^2n} - \frac{g\text{Subst}\left(\int \left(-\frac{\log(c(d+ex)}{2\sqrt{g}\left(\sqrt{-f}-\right.}\right.\right.}{}$$

$$= \frac{\log\left(c(d+ex^n)^p\right)}{2fn\left(f+gx^{2n}\right)} + \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{f^2n} + \frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{f^2n} + \frac{\sqrt{g}\,\text{Subst}\left(\right.}{}$$

$$= -\frac{e^2p\log\left(d+ex^n\right)}{2f\left(e^2f+d^2g\right)n} + \frac{\log\left(c(d+ex^n)^p\right)}{2fn\left(f+gx^{2n}\right)} + \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{f^2n} - \frac{\log\left(c(d\right.}{}$$

$$= -\frac{de\sqrt{g}\,p\tan^{-1}\left(\frac{\sqrt{g}\,x^n}{\sqrt{f}}\right)}{2f^{3/2}\left(e^2f+d^2g\right)n} - \frac{e^2p\log\left(d+ex^n\right)}{2f\left(e^2f+d^2g\right)n} + \frac{\log\left(c(d+ex^n)^p\right)}{2fn\left(f+gx^{2n}\right)} + \frac{\log\left(-\frac{ex^n}{d}\right)}{}$$

$$= -\frac{de\sqrt{g}\,p\tan^{-1}\left(\frac{\sqrt{g}\,x^n}{\sqrt{f}}\right)}{2f^{3/2}\left(e^2f+d^2g\right)n} - \frac{e^2p\log\left(d+ex^n\right)}{2f\left(e^2f+d^2g\right)n} + \frac{\log\left(c(d+ex^n)^p\right)}{2fn\left(f+gx^{2n}\right)} + \frac{\log\left(-\frac{ex^n}{d}\right)}{}$$

Mathematica [F]
time = 5.92, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g*x^(2*n))^2),x]
```

[Out] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g*x^(2*n))^2), x]

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.75, size = 1036, normalized size = 2.47

method	result
risch	$-\frac{e^{2} p \ln\left(d+e\,x^{n}\right)}{2 f \left(d^{2} g+f\,e^{2}\right) n}+\frac{e^{2} p \ln\left(f+g\,x^{2 n}\right)}{4 f \left(d^{2} g+f\,e^{2}\right) n}-\frac{i \pi \operatorname{csgn}\left(i c \left(d+e\,x^{n}\right)^{p}\right)^{2} \operatorname{csgn}\left(i c\right) \ln\left(f+g\,x^{2 n}\right)}{4 n\,f^{2}}+\frac{i \pi \operatorname{csgn}\left(i c \left(d+e\,x^{n}\right)^{p}\right)^{2} \operatorname{csgn}\left(i c\right)}{4 n f \left(f+g\,x^{2 n}\right)}+ i \pi \operatorname{csgn}($

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(c*(d+e*x^n)^p)/x/(f+g*x^(2*n))^2,x,method=_RETURNVERBOSE)

[Out] -1/2*e^2*p*ln(d+e*x^n)/f/(d^2*g+e^2*f)/n-1/4*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2/f^2*ln(f+g*(x^n)^2)+1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2/f^2*ln(x^n)-1/4*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)/f^2*ln(f+g*(x^n)^2)+1/4*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)/f/(f+g*(x^n)^2)+1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)/f^2*ln(x^n)+1/4*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2/f/(f+g*(x^n)^2)+1/4/n*p*e^2/f/(d^2*g+e^2*f)*ln(f+g*(x^n)^2)+1/4*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3/f^2*ln(f+g*(x^n)^2)-1/4*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3/f/(f+g*(x^n)^2)-1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3/f^2*ln(x^n)-1/n*p/f^2*ln(x^n)*ln((d+e*x^n)/d)+1/2/n*p/f^2*ln(d+e*x^n)*ln(f+g*(x^n)^2)-1/2/n*p/f^2*ln(d+e*x^n)*ln((e*(-f*g)^(1/2)-g*(d+e*x^n)+d*g)/(e*(-f*g)^(1/2)+d*g))-1/2/n*p/f^2*ln(d+e*x^n)*ln((e*(-f*g)^(1/2)+g*(d+e*x^n)-d*g)/(e*(-f*g)^(1/2)-d*g))-1/n*p/f^2*dilog((d+e*x^n)/d)-1/2/n*p/f^2*dilog((e*(-f*g)^(1/2)-g*(d+e*x^n)+d*g)/(e*(-f*g)^(1/2)+d*g))-1/2/n*p/f^2*dilog((e*(-f*g)^(1/2)+g*(d+e*x^n)-d*g)/(e*(-f*g)^(1/2)-d*g))-1/2/n*ln(c)/f^2*ln(f+g*(x^n)^2)+1/2/n*ln(c)/f/(f+g*(x^n)^2)+1/n*ln(c)/f^2*ln(x^n)-1/2/n*ln((d+e*x^n)^p)/f^2*ln(f+g*(x^n)^2)+1/2/n*ln((d+e*x^n)^p)/f/(f+g*(x^n)^2)+1/n*ln((d+e*x^n)^p)/f^2*ln(x^n)-1/4*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)/f/(f+g*(x^n)^2)-1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)/f^2*ln(x^n)+1/4*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)/f^2*ln(f+g*(x^n)^2)-1/2/n*p*e/f*g/(d^2*g+e^2*f)*d/(f*g)^(1/2)*arctan(x^n*g/(f*g)^(1/2))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^(2*n))^2,x, algorithm="maxima")

[Out] integrate(log((x^n*e + d)^p*c)/((g*x^(2*n) + f)^2*x), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^(2*n))^2,x, algorithm="fricas")
[Out] integral(log((x^n*e + d)^p*c)/(g^2*x*x^(4*n) + 2*f*g*x*x^(2*n) + f^2*x), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x/(f+g*x**(2*n))**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^(2*n))^2,x, algorithm="giac")
[Out] integrate(log((x^n*e + d)^p*c)/((g*x^(2*n) + f)^2*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)}{x\,{\left(f+g\,x^{2\,n}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/(x*(f + g*x^(2*n))^2),x)
[Out] int(log(c*(d + e*x^n)^p)/(x*(f + g*x^(2*n))^2), x)
```

3.375 $\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^n)^2}\,dx$

Optimal. Leaf size=204

$$-\frac{ep\log\left(d+ex^n\right)}{f(ef-dg)n}+\frac{\log\left(c(d+ex^n)^p\right)}{fn\left(f+gx^n\right)}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{f^2n}+\frac{ep\log\left(f+gx^n\right)}{f(ef-dg)n}-\frac{\log\left(c(d+ex^n)^p\right)\log}{f^2n}$$

```
[Out] -e*p*ln(d+e*x^n)/f/(-d*g+e*f)/n+ln(c*(d+e*x^n)^p)/f/n/(f+g*x^n)+ln(-e*x^n/d
)*ln(c*(d+e*x^n)^p)/f^2/n+e*p*ln(f+g*x^n)/f/(-d*g+e*f)/n-ln(c*(d+e*x^n)^p)*
ln(e*(f+g*x^n)/(-d*g+e*f))/f^2/n-p*polylog(2,-g*(d+e*x^n)/(-d*g+e*f))/f^2/n
+p*polylog(2,1+e*x^n/d)/f^2/n
```

Rubi [A]
time = 0.18, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2525, 46, 2463, 2441, 2352, 2442, 36, 31, 2440, 2438}

$$-\frac{p\text{PolyLog}\left(2,-\frac{g(d+ex^n)}{ef-dg}\right)}{f^2n}+\frac{p\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{f^2n}-\frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e(f+gx^n)}{ef-dg}\right)}{f^2n}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{f^2n}+\frac{\log\left(c(d+ex^n)^p\right)}{fn\left(f+gx^n\right)}-\frac{ep\log\left(d+ex^n\right)}{fn(ef-dg)}+\frac{ep\log\left(f+gx^n\right)}{fn(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/(x*(f + g*x^n)^2),x]
```

```
[Out] -((e*p*Log[d + e*x^n])/(f*(e*f - d*g)*n)) + Log[c*(d + e*x^n)^p]/(f*n*(f +
g*x^n)) + (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/(f^2*n) + (e*p*Log[f + g
*x^n])/(f*(e*f - d*g)*n) - (Log[c*(d + e*x^n)^p]*Log[(e*(f + g*x^n))/(e*f -
 d*g)])/(f^2*n) - (p*PolyLog[2, -((g*(d + e*x^n))/(e*f - d*g))])/(f^2*n) +
(p*PolyLog[2, 1 + (e*x^n)/d])/(f^2*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
```

```
|| IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x(f+gx)^2}\,dx,x,x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{\log(c(d+ex)^p)}{f^2x}-\frac{g\log(c(d+ex)^p)}{f(f+gx)^2}-\frac{g\log(c(d+ex)^p)}{f^2(f+gx)}\right)\,dx,x,x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{f^2n}-\frac{g\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{f+gx}\,dx,x,x^n\right)}{f^2n}-\frac{g\text{Subst}\left(\int \frac{\log(}{(}\right.}{}$$
$$= \frac{\log\left(c(d+ex^n)^p\right)}{fn\left(f+gx^n\right)}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{f^2n}-\frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e(f+gx^n)}{ef-dg}\right)}{f^2n}$$
$$= \frac{\log\left(c(d+ex^n)^p\right)}{fn\left(f+gx^n\right)}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{f^2n}-\frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e(f+gx^n)}{ef-dg}\right)}{f^2n}$$
$$= -\frac{ep\log\left(d+ex^n\right)}{f(ef-dg)n}+\frac{\log\left(c(d+ex^n)^p\right)}{fn\left(f+gx^n\right)}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{f^2n}+\frac{ep\log\left(f+\right.}{f(ef-}$$

Mathematica [A]
time = 0.12, size = 171, normalized size = 0.84

$$\frac{-\frac{efp\log(d+ex^n)}{ef-dg}+\frac{f\log(c(d+ex^n)^p)}{f+gx^n}+\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)+\frac{efp\log(f+gx^n)}{ef-dg}-\log\left(c(d+ex^n)^p\right)\log\left(\frac{e(f+gx^n)}{ef-dg}\right)-p\text{Li}_2\left(\frac{g(d+ex^n)}{-ef+dg}\right)+p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{f^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g*x^n)^2),x]
```

```
[Out] (-((e*f*p*Log[d + e*x^n])/(e*f - d*g)) + (f*Log[c*(d + e*x^n)^p])/(f + g*x^
n) + Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + (e*f*p*Log[f + g*x^n])/(e*f -
 d*g) - Log[c*(d + e*x^n)^p]*Log[(e*(f + g*x^n))/(e*f - d*g)] - p*PolyLog[2
, (g*(d + e*x^n))/(-(e*f) + d*g)] + p*PolyLog[2, 1 + (e*x^n)/d])/(f^2*n)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.84, size = 805, normalized size = 3.95

method	result

risch	$\frac{\ln((d+e\,x^n)^p)}{nf(f+g\,x^n)}-\frac{\ln((d+e\,x^n)^p)\ln(f+g\,x^n)}{n\,f^2}+\frac{\ln((d+e\,x^n)^p)\ln(x^n)}{n\,f^2}-\frac{pe\ln(f+g\,x^n)}{nf(dg-ef)}+\frac{pe\ln(d+e\,x^n)}{nf(dg-ef)}-\frac{p\,\mathrm{dilog}\left(\frac{d+e\,x^n}{d}\right)}{n\,f^2}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)/x/(f+g*x^n)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*ln((d+e*x^n)^p)/f/(f+g*x^n)-1/n*ln((d+e*x^n)^p)/f^2*ln(f+g*x^n)+1/n*ln(
(d+e*x^n)^p)/f^2*ln(x^n)-1/n*p*e/f/(d*g-e*f)*ln(f+g*x^n)+1/n*p*e/f/(d*g-e*f
)*ln(d+e*x^n)-1/n*p/f^2*dilog((d+e*x^n)/d)-1/n*p/f^2*ln(x^n)*ln((d+e*x^n)/d
)+1/n*p/f^2*dilog(((f+g*x^n)*e+d*g-e*f)/(d*g-e*f))+1/n*p/f^2*ln(f+g*x^n)*ln
(((f+g*x^n)*e+d*g-e*f)/(d*g-e*f))-1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(
d+e*x^n)^p)^2/f^2*ln(f+g*x^n)+1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)/
f^2*ln(x^n)+1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)/f/(f+g*x^n)-1/2*I/
n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)/f/(f+g*x^n)-1/2*I/
n*Pi*csgn(I*c*(d+e*x^n)^p)^3/f/(f+g*x^n)+1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*csg
n(I*c*(d+e*x^n)^p)^2/f/(f+g*x^n)+1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3/f^2*ln(
f+g*x^n)-1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3/f^2*ln(x^n)-1/2*I/n*Pi*csgn(I*c
*(d+e*x^n)^p)^2*csgn(I*c)/f^2*ln(f+g*x^n)+1/2*I/n*Pi*csgn(I*(d+e*x^n)^p)*cs
gn(I*c*(d+e*x^n)^p)*csgn(I*c)/f^2*ln(f+g*x^n)-1/2*I/n*Pi*csgn(I*(d+e*x^n)^p
)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)/f^2*ln(x^n)+1/2*I/n*Pi*csgn(I*(d+e*x^n)^p
)*csgn(I*c*(d+e*x^n)^p)^2/f^2*ln(x^n)+1/n*ln(c)/f/(f+g*x^n)-1/n*ln(c)/f^2*l
n(f+g*x^n)+1/n*ln(c)/f^2*ln(x^n)
```

Maxima [A]
time = 0.35, size = 251, normalized size = 1.23

$$np\left(\frac{\log\left(\left(d+e^{(n\log(x)+1)}\right)e^{(-1)}\right)}{dfgn^2-f^2n^2e}-\frac{\log\left(\frac{gx^n+f}{g}\right)}{dfgn^2-f^2n^2e}+\frac{\left(\log\left(gx^n+f\right)\log\left(\frac{fe+ge^{(n\log(x)+1)}}{dg-fe}+1\right)+\mathrm{Li}_2\left(-\frac{fe+ge^{(n\log(x)+1)}}{dg-fe}\right)\right)e^{(-1)}}{f^2n^2}-\frac{\left(\log\left(x^n\right)\log\left(\frac{e^{(n\log(x)+1)}}{d}+1\right)+\mathrm{Li}_2\left(-\frac{e^{(n\log(x)+1)}}{d}\right)\right)e^{(-1)}}{f^2n^2}\right)e+\left(\frac{1}{fgnx^n+f^2n}-\frac{\log\left(gx^n+f\right)}{f^2n}+\frac{\log\left(x^n\right)}{f^2n}\right)\log\left(\left(x^ne+d\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^n)^2,x, algorithm="maxima")
```

```
[Out] n*p*(log((d + e^(n*log(x) + 1))*e^(-1))/(d*f*g*n^2 - f^2*n^2*e) - log((g*x^
n + f)/g)/(d*f*g*n^2 - f^2*n^2*e) + (log(g*x^n + f)*log((f*e + g*e^(n*log(x
) + 1))/(d*g - f*e) + 1) + dilog(-(f*e + g*e^(n*log(x) + 1))/(d*g - f*e)))*
e^(-1)/(f^2*n^2) - (log(x^n)*log(e^(n*log(x) + 1)/d + 1) + dilog(-e^(n*log(
x) + 1)/d))*e^(-1)/(f^2*n^2))*e + (1/(f*g*n*x^n + f^2*n) - log(g*x^n + f)/(
f^2*n) + log(x^n)/(f^2*n))*log((x^n*e + d)^p*c)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral(log((x^n*e + d)^p*c)/(g^2*x*x^(2*n) + 2*f*g*x*x^n + f^2*x), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x/(f+g*x**n)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(log((x^n*e + d)^p*c)/((g*x^n + f)^2*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln \left(c\,{\left(d+e\,x^n\right)}^p\right)}{x\,{\left(f+g\,x^n\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/(x*(f + g*x^n)^2),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)/(x*(f + g*x^n)^2), x)
```

3.376 $\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{-n})^2}\,dx$

Optimal. Leaf size=156

$$\frac{egp\log(d+ex^n)}{f^2(df-eg)n}+\frac{g\log(c(d+ex^n)^p)}{f^2n(g+fx^n)}-\frac{egp\log(g+fx^n)}{f^2(df-eg)n}+\frac{\log(c(d+ex^n)^p)\log\left(-\frac{e(g+fx^n)}{df-eg}\right)}{f^2n}+\frac{p\text{Li}_2\left(\frac{f(d+ex^n)}{df-eg}\right)}{f^2n}$$

```
[Out] e*g*p*ln(d+e*x^n)/f^2/(d*f-e*g)/n+g*ln(c*(d+e*x^n)^p)/f^2/n/(g+f*x^n)-e*g*p
*ln(g+f*x^n)/f^2/(d*f-e*g)/n+ln(c*(d+e*x^n)^p)*ln(-e*(g+f*x^n)/(d*f-e*g))/f
^2/n+p*polylog(2,f*(d+e*x^n)/(d*f-e*g))/f^2/n
```

Rubi [A]

time = 0.19, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {2525, 269, 45, 2463, 2442, 36, 31, 2441, 2440, 2438}

$$\frac{p\text{PolyLog}\left(2,\frac{f(d+ex^n)}{df-eg}\right)}{f^2n}+\frac{g\log(c(d+ex^n)^p)}{f^2n(fx^n+g)}+\frac{\log(c(d+ex^n)^p)\log\left(-\frac{e(fx^n+g)}{df-eg}\right)}{f^2n}+\frac{egp\log(d+ex^n)}{f^2n(df-eg)}-\frac{egp\log(fx^n+g)}{f^2n(df-eg)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/(x*(f + g/x^n)^2),x]
```

```
[Out] (e*g*p*Log[d + e*x^n])/(f^2*(d*f - e*g)*n) + (g*Log[c*(d + e*x^n)^p])/(f^2*
n*(g + f*x^n)) - (e*g*p*Log[g + f*x^n])/(f^2*(d*f - e*g)*n) + (Log[c*(d + e
*x^n)^p]*Log[-((e*(g + f*x^n))/(d*f - e*g))])/(f^2*n) + (p*PolyLog[2, (f*(d
 + e*x^n))/(d*f - e*g)])/(f^2*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
```

```
|| IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x \left(f+gx^{-n}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\left(f+\frac{g}{x}\right)^2 x}\,dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{g\log(c(d+ex)^p)}{f(g+fx)^2}+\frac{\log(c(d+ex)^p)}{f(g+fx)}\right)dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{g+fx}\,dx, x, x^n\right)}{fn} - \frac{g\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{(g+fx)^2}\,dx, x, x^n\right)}{fn}$$

$$= \frac{g\log\left(c(d+ex^n)^p\right)}{f^2n\left(g+fx^n\right)} + \frac{\log\left(c(d+ex^n)^p\right)\log\left(-\frac{e(g+fx^n)}{df-eg}\right)}{f^2n} - \frac{(ep)\text{Subst}\left(\int \frac{\log\left(\frac{e(g+fx)}{-df+eg}\right.}{d+ex}\right.}{f^2n}$$

$$= \frac{g\log\left(c(d+ex^n)^p\right)}{f^2n\left(g+fx^n\right)} + \frac{\log\left(c(d+ex^n)^p\right)\log\left(-\frac{e(g+fx^n)}{df-eg}\right)}{f^2n} - \frac{p\text{Subst}\left(\int \frac{\log\left(1+\frac{fx}{-df+eg}\right.}{x}\right.}{f^2n}$$

$$= \frac{egp\log\left(d+ex^n\right)}{f^2(df-eg)n} + \frac{g\log\left(c(d+ex^n)^p\right)}{f^2n\left(g+fx^n\right)} - \frac{egp\log\left(g+fx^n\right)}{f^2(df-eg)n} + \frac{\log\left(c(d+ex^n)^p\right)\ldots}{f^2n}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 433 vs. 2(156) = 312.
time = 1.13, size = 433, normalized size = 2.78

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g/x^n)^2),x]
```

```
[Out] (g*p*Log[f - f/x^n] + f*p*x^n*Log[f - f/x^n] - g*n*p*Log[x]*Log[f - f/x^n]
- f*n*p*x^n*Log[x]*Log[f - f/x^n] - p*Log[e + d/x^n]*(-(f*x^n) + (g + f*x^n
)*Log[f - f/x^n]) - f*x^n*Log[c*(d + e*x^n)^p] + g*Log[f - f/x^n]*Log[c*(d
+ e*x^n)^p] + f*x^n*Log[f - f/x^n]*Log[c*(d + e*x^n)^p] + g*n*p*Log[x]*Log[
1 + (f*x^n)/g] + f*n*p*x^n*Log[x]*Log[1 + (f*x^n)/g] + p*(g + f*x^n)*PolyLo
g[2, -((f*x^n)/g)])/(f^2*n*(g + f*x^n)) - (p*(-((d*f*Log[e + d/x^n])/(d*f -
 e*g)) + (f*x^n*Log[e + d/x^n])/(g + f*x^n) + Log[-(d/(e*x^n))]*Log[e + d/x
^n] + (d*f*Log[f + g/x^n])/(d*f - e*g) - Log[e + d/x^n]*Log[(d*(f + g/x^n))
/(d*f - e*g)] - PolyLog[2, -((g*(e + d/x^n))/(d*f - e*g))] + PolyLog[2, 1 +
 d/(e*x^n)]))/(f^2*n)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.09, size = 589, normalized size = 3.78

method	result
risch	$\frac{\ln((d+e\,x^n)^p)g}{n\,f^2(g+f\,x^n)}+\frac{\ln((d+e\,x^n)^p)\ln(g+f\,x^n)}{n\,f^2}-\frac{p\,\mathrm{dilog}\left(\frac{(g+f\,x^n)e+df-eg}{df-eg}\right)}{n\,f^2}-\frac{p\ln(g+f\,x^n)\ln\left(\frac{(g+f\,x^n)e+df-eg}{df-eg}\right)}{n\,f^2}-\frac{egp\ln(g+f}{f^2(df-eg)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)/x/(f+g/(x^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*ln((d+e*x^n)^p)/f^2*g/(g+f*x^n)+1/n*ln((d+e*x^n)^p)/f^2*ln(g+f*x^n)-1/n
/f^2*p*dilog(((g+f*x^n)*e+d*f-e*g)/(d*f-e*g))-1/n/f^2*p*ln(g+f*x^n)*ln(((g+
f*x^n)*e+d*f-e*g)/(d*f-e*g))-e*g*p*ln(g+f*x^n)/f^2/(d*f-e*g)/n+1/n/f^2*p*e*
g/(d*f-e*g)*ln((g+f*x^n)*e+d*f-e*g)-1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3/f^2*
ln(g+f*x^n)-1/2*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3/f^2*g/(g+f*x^n)-1/2*I/n*Pi*c
sgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)/f^2*g/(g+f*x^n)+1/2*I/n*
Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2/f^2*g/(g+f*x^n)-1/2*I/n*Pi*c
sgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)/f^2*ln(g+f*x^n)+1/2*I/n*
Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)/f^2*g/(g+f*x^n)+1/2*I/n*Pi*csgn(I*(d+e
*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2/f^2*ln(g+f*x^n)+1/2*I/n*Pi*csgn(I*c*(d+e*x
^n)^p)^2*csgn(I*c)/f^2*ln(g+f*x^n)+1/n*ln(c)/f^2*g/(g+f*x^n)+1/n*ln(c)/f^2*
ln(g+f*x^n)
```

Maxima [A]
time = 0.41, size = 222, normalized size = 1.42

$$np\left(\frac{d\log\left(\left(d+e^{(n\log(x)+1)}\right)e^{(-1)}\right)}{df^2n^2e-fgn^2e^2}-\frac{g\log\left(\frac{fx^n+g}{f}\right)}{df^3n^2-f^2gn^2e}-\frac{\left(\log\left(fx^n+g\right)\log\left(\frac{ge+fe^{(n\log(x)+1)}}{df-ge}+1\right)+\mathrm{Li}_2\left(-\frac{ge+fe^{(n\log(x)+1)}}{df-ge}\right)\right)e^{(-1)}}{f^2n^2}\right)e-\left(\frac{1}{f^2n+\frac{fgn}{x^n}}-\frac{\log\left(f+\frac{g}{x^n}\right)}{f^2n}+\frac{\log\left(\frac{1}{x^n}\right)}{f^2n}\right)\log\left(\left(x^ne+d\right)^pc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^n))^2,x, algorithm="maxima")
```

```
[Out] n*p*(d*log((d + e^(n*log(x) + 1))*e^(-1))/(d*f^2*n^2*e - f*g*n^2*e^2) - g*l
og((f*x^n + g)/f)/(d*f^3*n^2 - f^2*g*n^2*e) - (log(f*x^n + g)*log((g*e + f*
e^(n*log(x) + 1))/(d*f - g*e) + 1) + dilog(-(g*e + f*e^(n*log(x) + 1))/(d*f
 - g*e)))*e^(-1)/(f^2*n^2))*e - (1/(f^2*n + f*g*n/x^n) - log(f + g/x^n)/(f^
2*n) + log(1/(x^n))/(f^2*n))*log((x^n*e + d)^p*c)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^n))^2,x, algorithm="fricas")
```

```
[Out] integral(log((x^n*e + d)^p*c)/(f^2*x + 2*f*g*x*x^n/x^(2*n) + g^2*x/x^(2*n))
, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x/(f+g/(x**n))**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^n))^2,x, algorithm="giac")
[Out] integrate(log((x^n*e + d)^p*c)/((f + g/x^n)^2*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\left(d+e\,x^n\right)^p\right)}{x\,\left(f+\frac{g}{x^n}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/(x*(f + g/x^n)^2),x)
[Out] int(log(c*(d + e*x^n)^p)/(x*(f + g/x^n)^2), x)
```

3.377 $\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{-2n})^2} \, dx$

Optimal. Leaf size=377

$$-\frac{de\sqrt{g}\,p\tan^{-1}\left(\frac{\sqrt{f}\,x^n}{\sqrt{g}}\right)}{2f^{3/2}\left(d^2f+e^2g\right)n}-\frac{e^2gp\log\left(d+ex^n\right)}{2f^2\left(d^2f+e^2g\right)n}+\frac{g\log\left(c(d+ex^n)^p\right)}{2f^2n\left(g+fx^{2n}\right)}+\frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}\,x\right)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2f^2n}$$

```
[Out] -1/2*e^2*g*p*ln(d+e*x^n)/f^2/(d^2*f+e^2*g)/n+1/2*g*ln(c*(d+e*x^n)^p)/f^2/n/
(g+f*x^(2*n))+1/4*e^2*g*p*ln(g+f*x^(2*n))/f^2/(d^2*f+e^2*g)/n+1/2*ln(c*(d+e
*x^n)^p)*ln(-e*(x^n*(-f)^(1/2)+g^(1/2))/(d*(-f)^(1/2)-e*g^(1/2)))/f^2/n+1/2
*ln(c*(d+e*x^n)^p)*ln(e*(-x^n*(-f)^(1/2)+g^(1/2))/(d*(-f)^(1/2)+e*g^(1/2)))
/f^2/n+1/2*p*polylog(2,(d+e*x^n)*(-f)^(1/2)/(d*(-f)^(1/2)-e*g^(1/2)))/f^2/n
+1/2*p*polylog(2,(d+e*x^n)*(-f)^(1/2)/(d*(-f)^(1/2)+e*g^(1/2)))/f^2/n-1/2*d
*e*p*arctan(x^n*f^(1/2)/g^(1/2))*g^(1/2)/f^(3/2)/(d^2*f+e^2*g)/n
```

Rubi [A]
time = 0.45, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.518$, Rules used = {2525, 269, 272, 45, 2463, 2460, 720, 31, 649, 211, 266, 2441, 2440, 2438}

$$\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{-f}\,(d+ex^n)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2f^2n}+\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{-f}\,(d+ex^n)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2f^2n}-\frac{de\sqrt{g}\,p\,\text{ArcTan}\left(\frac{\sqrt{f}\,x^n}{\sqrt{g}}\right)}{2f^{3/2}n\left(d^2f+e^2g\right)}+\frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}\,x^n\right)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2f^2n}+\frac{\log\left(c(d+ex^n)^p\right)\log\left(-\frac{e\left(\sqrt{-f}\,x^n+\sqrt{g}\right)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2f^2n}+\frac{g\log\left(c(d+ex^n)^p\right)}{2f^2n\left(fx^{2n}+g\right)}+\frac{e^2gp\log\left(fx^{2n}+g\right)}{4f^2n\left(d^2f+e^2g\right)}-\frac{e^2gp\log\left(d+ex^n\right)}{2f^2n\left(d^2f+e^2g\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*x^n)^p]/(x*(f + g/x^(2*n))^2),x]
```

```
[Out] -1/2*(d*e*Sqrt[g]*p*ArcTan[(Sqrt[f]*x^n)/Sqrt[g]])/(f^(3/2)*(d^2*f + e^2*g)
*n) - (e^2*g*p*Log[d + e*x^n])/(2*f^2*(d^2*f + e^2*g)*n) + (g*Log[c*(d + e*
x^n)^p])/(2*f^2*n*(g + f*x^(2*n))) + (Log[c*(d + e*x^n)^p]*Log[(e*(Sqrt[g]
- Sqrt[-f]*x^n))/(d*Sqrt[-f] + e*Sqrt[g])])/(2*f^2*n) + (Log[c*(d + e*x^n)^
p]*Log[-((e*(Sqrt[g] + Sqrt[-f]*x^n))/(d*Sqrt[-f] - e*Sqrt[g]))])/(2*f^2*n)
 + (e^2*g*p*Log[g + f*x^(2*n)])/(4*f^2*(d^2*f + e^2*g)*n) + (p*PolyLog[2, (
Sqrt[-f]*(d + e*x^n))/(d*Sqrt[-f] - e*Sqrt[g])])/(2*f^2*n) + (p*PolyLog[2,
(Sqrt[-f]*(d + e*x^n))/(d*Sqrt[-f] + e*Sqrt[g])])/(2*f^2*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2460

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(x_)^(m_.)*(
(f_.) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Simp[(f + g*x^r)^(q + 1)*((a
+ b*Log[c*(d + e*x)^n])^p/(g*r*(q + 1))), x] - Dist[b*e*n*(p/(g*r*(q + 1)))
, Int[(f + g*x^r)^(q + 1)*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x
], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q, r}, x] && EqQ[m, r - 1] && Ne
Q[q, -1] && IGtQ[p, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{-2n})^2}\,dx = \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\left(f+\frac{g}{x^2}\right)^2 x}\,dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{gx\log(c(d+ex)^p)}{f(g+fx^2)^2} + \frac{x\log(c(d+ex)^p)}{f(g+fx^2)}\right)dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \frac{x\log(c(d+ex)^p)}{g+fx^2}\,dx, x, x^n\right)}{fn} - \frac{g\text{Subst}\left(\int \frac{x\log(c(d+ex)^p)}{(g+fx^2)^2}\,dx, x, x^n\right)}{fn}$$

$$= \frac{g\log(c(d+ex^n)^p)}{2f^2n(g+fx^{2n})} + \frac{\text{Subst}\left(\int \left(-\frac{\sqrt{-f}\log(c(d+ex)^p)}{2f\left(\sqrt{g}-\sqrt{-f}\,x\right)} + \frac{\sqrt{-f}\log(c(d+ex)^p)}{2f\left(\sqrt{g}+\sqrt{-f}\,x\right)}\right)dx,\right.}{fn}$$

$$= \frac{g\log(c(d+ex^n)^p)}{2f^2n(g+fx^{2n})} - \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt{g}-\sqrt{-f}\,x}\,dx, x, x^n\right)}{2(-f)^{3/2}n} + \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt{g}+\sqrt{-f}\right.}{2(-f)^{3/2}n}$$

$$= -\frac{e^2gp\log(d+ex^n)}{2f^2(d^2f+e^2g)n} + \frac{g\log(c(d+ex^n)^p)}{2f^2n(g+fx^{2n})} + \frac{\log(c(d+ex^n)^p)\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}\right.}{d\sqrt{-f}+e\sqrt{}}\right.}{2f^2n}$$

$$= -\frac{de\sqrt{g}\,p\tan^{-1}\left(\frac{\sqrt{f}\,x^n}{\sqrt{g}}\right)}{2f^{3/2}(d^2f+e^2g)n} - \frac{e^2gp\log(d+ex^n)}{2f^2(d^2f+e^2g)n} + \frac{g\log(c(d+ex^n)^p)}{2f^2n(g+fx^{2n})} + \frac{\log(c(d}{}$$

$$= -\frac{de\sqrt{g}\,p\tan^{-1}\left(\frac{\sqrt{f}\,x^n}{\sqrt{g}}\right)}{2f^{3/2}(d^2f+e^2g)n} - \frac{e^2gp\log(d+ex^n)}{2f^2(d^2f+e^2g)n} + \frac{g\log(c(d+ex^n)^p)}{2f^2n(g+fx^{2n})} + \frac{\log(c(d}{}$$

Mathematica [F]
time = 0.87, size = 0, normalized size = 0.00

$$\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{-2n})^2}\,dx$$

Verification is not applicable to the result.

[In] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g/x^(2*n))^2),x]

[Out] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g/x^(2*n))^2), x]

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.94, size = 810, normalized size = 2.15

method	result
risch	$\frac{\ln((d+e\,x^n)^p)g}{2n\,f^2(g+f\,x^{2n})}+\frac{\ln((d+e\,x^n)^p)\ln(g+f\,x^{2n})}{2n\,f^2}-\frac{p\ln(d+e\,x^n)\ln(g+f\,x^{2n})}{2n\,f^2}+\frac{p\ln(d+e\,x^n)\ln\left(\frac{e\sqrt{-fg}-f(d+e\,x^n)+df}{e\sqrt{-fg}+df}\right)}{2n\,f^2}+$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n)))^2,x,method=_RETURNVERBOSE)

[Out] 1/2/n*ln((d+e*x^n)^p)/f^2*g/(g+f*(x^n)^2)+1/2/n*ln((d+e*x^n)^p)/f^2*ln(g+f*(x^n)^2)-1/2/n*p/f^2*ln(d+e*x^n)*ln(g+f*(x^n)^2)+1/2/n*p/f^2*ln(d+e*x^n)*ln((e*(-f*g)^(1/2)-f*(d+e*x^n)+d*f)/(e*(-f*g)^(1/2)+d*f))+1/2/n*p/f^2*ln(d+e*x^n)*ln((e*(-f*g)^(1/2)+f*(d+e*x^n)-d*f)/(e*(-f*g)^(1/2)-d*f))+1/2/n*p/f^2*dilog((e*(-f*g)^(1/2)-f*(d+e*x^n)+d*f)/(e*(-f*g)^(1/2)+d*f))+1/2/n*p/f^2*dilog((e*(-f*g)^(1/2)+f*(d+e*x^n)-d*f)/(e*(-f*g)^(1/2)-d*f))-1/2*e^2*g*p*ln(d+e*x^n)/f^2/(d^2*f+e^2*g)/n+1/4/n*p*e^2/f^2*g/(d^2*f+e^2*g)*ln(g+f*(x^n)^2)-1/2/n*p*e/f*g/(d^2*f+e^2*g)*d/(f*g)^(1/2)*arctan(x^n*f/(f*g)^(1/2))+1/4*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)/f^2*ln(g+f*(x^n)^2)+1/4*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)/f^2*g/(g+f*(x^n)^2)-1/4*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)/f^2*ln(g+f*(x^n)^2)+1/4*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2/f^2*ln(g+f*(x^n)^2)-1/4*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3/f^2*g/(g+f*(x^n)^2)+1/4*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2/f^2*g/(g+f*(x^n)^2)-1/4*I/n*Pi*csgn(I*c*(d+e*x^n)^p)^3/f^2*ln(g+f*(x^n)^2)-1/4*I/n*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)/f^2*g/(g+f*(x^n)^2)+1/2/n*ln(c)/f^2*g/(g+f*(x^n)^2)+1/2/n*ln(c)/f^2*ln(g+f*(x^n)^2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n)))^2,x, algorithm="maxima")

[Out] integrate(log((x^n*e + d)^p*c)/((f + g/x^(2*n))^2*x), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n)))^2,x, algorithm="fricas")
```

```
[Out] integral(log((x^n*e + d)^p*c)/(f^2*x + 2*f*g*x*x^(2*n)/x^(4*n) + g^2*x/x^(4
*n)), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)/x/(f+g/(x**(2*n)))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3008 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n)))^2,x, algorithm="giac")
```

```
[Out] integrate(log((x^n*e + d)^p*c)/((f + g/x^(2*n))^2*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)}{x\,{\left(f+\frac{g}{x^{2\,n}}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)/(x*(f + g/x^(2*n))^2),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)/(x*(f + g/x^(2*n))^2), x)
```

3.378 $\int \frac{\log(c(d+ex^n))}{x(ce-(1-cd)x^{-n})}\,dx$

Optimal. Leaf size=25

$$-\frac{\text{Li}_2(1-c(d+ex^n))}{cen}$$

[Out] -polylog(2,1-c*(d+e*x^n))/c/e/n

Rubi [A]
time = 0.10, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2525, 2459, 2440, 2438}

$$-\frac{\text{PolyLog}(2,1-c(d+ex^n))}{cen}$$

Antiderivative was successfully verified.

[In] Int[Log[c*(d + e*x^n)]/(x*(c*e - (1 - c*d)/x^n)),x]

[Out] -(PolyLog[2, 1 - c*(d + e*x^n)]/(c*e*n))

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2459

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)/(x_))^(q_.)*(x_)^(m_.), x_Symbol] :> Int[(g + f*x)^q*(a + b*Log[c*(d + e*
x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q}, x] && EqQ[m, q] &&
 IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
```

```
|| IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\log (c(d+ex^n))}{x\,(ce-(1-cd)x^{-n})}\,dx &= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex))}{\left(ce+\frac{-1+cd}{x}\right)x}\,dx, x, x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex))}{-1+cd+cex}\,dx, x, x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, -1+cd+cex^n\right)}{cen}\\
&= -\frac{\text{Li}_2(1-c(d+ex^n))}{cen}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 25, normalized size = 1.00

$$-\frac{\text{Li}_2(1-c(d+ex^n))}{cen}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*x^n)]/(x*(c*e - (1 - c*d)/x^n)),x]
[Out] -(PolyLog[2, 1 - c*(d + e*x^n)]/(c*e*n))
```

Maple [A]
time = 1.18, size = 23, normalized size = 0.92

method	result
derivativedivides	$-\frac{\text{dilog}(ce\,x^n+cd)}{nce}$
default	$-\frac{\text{dilog}(ce\,x^n+cd)}{nce}$
risch	$\frac{\ln(1-c(d+e\,x^n))\ln(d+e\,x^n)}{enc}-\frac{\ln(1-c(d+e\,x^n))\ln(c(d+e\,x^n))}{enc}-\frac{\text{dilog}(c(d+e\,x^n))}{enc}+\frac{i\ln(-1+cd+ce\,x^n)\pi\,\text{csgn}(i($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n))/x/(c*e+(c*d-1)/(x^n)),x,method=_RETURNVERBOSE)
[Out] -1/n*dilog(c*e*x^n+c*d)/c/e
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(24) = 48$.
time = 0.37, size = 114, normalized size = 4.56

$$\left(\frac{e^{(-1)}\log\left(ce+\frac{cd-1}{x^n}\right)}{cn}-\frac{e^{(-1)}\log\left(\frac{1}{x^n}\right)}{cn}\right)\log\left((x^ne+d)c\right)-\frac{\left(\log\left(cd+ce^{(n\log(x)+1)}\right)\log\left(cd+ce^{(n\log(x)+1)}-1\right)+\text{Li}_2\left(-cd-ce^{(n\log(x)+1)}+1\right)\right)e^{(-1)}}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n))/x/(c*e+(c*d-1)/(x^n)),x, algorithm="maxima")
```

```
[Out] (e^(-1)*log(c*e + (c*d - 1)/x^n)/(c*n) - e^(-1)*log(1/(x^n))/(c*n))*log((x^
n*e + d)*c) - (log(c*d + c*e^(n*log(x) + 1))*log(c*d + c*e^(n*log(x) + 1) -
 1) + dilog(-c*d - c*e^(n*log(x) + 1) + 1))*e^(-1)/(c*n)
```

Fricas [A]
time = 0.38, size = 25, normalized size = 1.00

$$-\frac{\text{Li}_2(-cx^ne-cd+1)\,e^{(-1)}}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n))/x/(c*e+(c*d-1)/(x^n)),x, algorithm="fricas")
```

```
[Out] -dilog(-c*x^n*e - c*d + 1)*e^(-1)/(c*n)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n))/x/(c*e+(c*d-1)/(x**n)),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n))/x/(c*e+(c*d-1)/(x^n)),x, algorithm="giac")
```

```
[Out] integrate(log((x^n*e + d)*c)/((c*e + (c*d - 1)/x^n)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln\left(c\left(d+e\,x^n\right)\right)}{x\,\left(c\,e+\frac{c\,d-1}{x^n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n))/(x*(c*e + (c*d - 1)/x^n)),x)
[Out] int(log(c*(d + e*x^n))/(x*(c*e + (c*d - 1)/x^n)), x)
```

3.379 $\int \frac{x^{-1+n}\log(c(d+ex^n))}{-1+cd+cex^n}\,dx$

Optimal. Leaf size=25

$$-\frac{\text{Li}_2(1-c(d+ex^n))}{cen}$$

[Out] -polylog(2,1-c*(d+e*x^n))/c/e/n

Rubi [A]
time = 0.07, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2525, 2440, 2438}

$$-\frac{\text{PolyLog}(2,1-c(d+ex^n))}{cen}$$

Antiderivative was successfully verified.

[In] Int[(x^(-1 + n)*Log[c*(d + e*x^n)])/(-1 + c*d + c*e*x^n),x]

[Out] -(PolyLog[2, 1 - c*(d + e*x^n)]/(c*e*n))

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{x^{-1+n} \log (c(d+ex^n))}{-1+cd+cex^n} \, dx = \frac{\text{Subst}\left(\int \frac{\log(c(d+ex))}{-1+cd+cex} \, dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \frac{\log(1+x)}{x} \, dx, x, -1+cd+cex^n\right)}{cen}$$
$$= -\frac{\text{Li}_2(1-c(d+ex^n))}{cen}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 1.00

$$-\frac{\text{Li}_2(1-c(d+ex^n))}{cen}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + n)*Log[c*(d + e*x^n)])/(-1 + c*d + c*e*x^n),x]
[Out] -(PolyLog[2, 1 - c*(d + e*x^n)]/(c*e*n))
```

Maple [A]
time = 0.51, size = 23, normalized size = 0.92

method	result
default	$-\frac{\text{dilog}(ce\,x^n+cd)}{nce}$
risch	$\frac{\ln(1-c(d+e\,x^n))\ln(d+e\,x^n)}{enc} - \frac{\ln(1-c(d+e\,x^n))\ln(c(d+e\,x^n))}{enc} - \frac{\text{dilog}(c(d+e\,x^n))}{enc} + \frac{i\ln(-1+cd+ce\,x^n)\pi\,\text{csgn}(i(d+e\,x^n))\text{csgn}}{2nce}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*ln(c*(d+e*x^n))/(-1+c*d+c*e*x^n),x,method=_RETURNVERBOSE)
[Out] -1/n*dilog(c*e*x^n+c*d)/c/e
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(24) = 48.
time = 0.29, size = 118, normalized size = 4.72

$$\frac{e^{(-1)}\log\left(cx^ne+cd-1\right)\log\left((x^ne+d)c\right)}{cn} - \frac{e^{(-1)}\log\left(cx^ne+cd-1\right)\log\left(x^ne+d\right)}{cn} + \frac{\left(\log\left(-cd-ce^{(n\log(x)+1)}+1\right)\log\left(d+e^{(n\log(x)+1)}\right)+\text{Li}_2\left(cd+ce^{(n\log(x)+1)}\right)\right)e^{(-1)}}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*log(c*(d+e*x^n))/(-1+c*d+c*e*x^n),x, algorithm="maxima")
```

```
[Out] e^(-1)*log(c*x^n*e + c*d - 1)*log((x^n*e + d)*c)/(c*n) - e^(-1)*log(c*x^n*e
 + c*d - 1)*log(x^n*e + d)/(c*n) + (log(-c*d - c*e^(n*log(x) + 1) + 1)*log(
d + e^(n*log(x) + 1)) + dilog(c*d + c*e^(n*log(x) + 1)))*e^(-1)/(c*n)
```

Fricas [A]
time = 0.39, size = 25, normalized size = 1.00

$$-\frac{\text{Li}_2(-cx^ne-cd+1)\,e^{(-1)}}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*log(c*(d+e*x^n))/(-1+c*d+c*e*x^n),x, algorithm="fricas")
[Out] -dilog(-c*x^n*e - c*d + 1)*e^(-1)/(c*n)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*ln(c*(d+e*x**n))/(-1+c*d+c*e*x**n),x)
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*log(c*(d+e*x^n))/(-1+c*d+c*e*x^n),x, algorithm="giac")
[Out] integrate(x^(n - 1)*log((x^n*e + d)*c)/(c*x^n*e + c*d - 1), x)
```

Mupad [B]
time = 0.65, size = 21, normalized size = 0.84

$$-\frac{\text{Li}_2(c\,(d+e\,x^n))}{c\,e\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(n - 1)*log(c*(d + e*x^n)))/(c*d + c*e*x^n - 1),x)
[Out] -dilog(c*(d + e*x^n))/(c*e*n)
```

3.380 $\int \frac{\log\left(c\left(d+ex^{-n}\right)\right)}{x(ce-(1-cd)x^n)} \, dx$

Optimal. Leaf size=26

$$\frac{\text{Li}_2(1-c(d+ex^{-n}))}{cen}$$

[Out] polylog(2,1-c*(d+e/(x^n)))/c/e/n

Rubi [A]

time = 0.10, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2525, 2459, 2440, 2438}

$$\frac{\text{PolyLog}(2, 1-c(d+ex^{-n}))}{cen}$$

Antiderivative was successfully verified.

[In] Int[Log[c*(d + e/x^n)]/(x*(c*e - (1 - c*d)*x^n)),x]

[Out] PolyLog[2, 1 - c*(d + e/x^n)]/(c*e*n)

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2459

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)/(x_))^(q_.)*(x_)^(m_.), x_Symbol] :> Int[(g + f*x)^q*(a + b*Log[c*(d + e*
x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q}, x] && EqQ[m, q] &&
 IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
```

```
|| IGtQ[q, 0])
```

Rubi steps

$$\int \frac{\log \left(c(d+ex^{-n})\right)}{x\left(ce-(1-cd)x^n\right)}\,dx = -\frac{\text{Subst}\left(\int \frac{\log(c(d+ex))}{\left(ce+\frac{-1+cd}{x}\right)x}\,dx, x, x^{-n}\right)}{n}$$
$$= -\frac{\text{Subst}\left(\int \frac{\log(c(d+ex))}{-1+cd+cex}\,dx, x, x^{-n}\right)}{n}$$
$$= -\frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, -1+cd+cex^{-n}\right)}{cen}$$
$$= \frac{\text{Li}_2(1-cd-cex^{-n})}{cen}$$

Mathematica [A]
time = 0.05, size = 34, normalized size = 1.31

$$\frac{\text{Li}_2(-x^{-n}(ce-x^n+cdx^n))}{cen}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e/x^n)]/(x*(c*e - (1 - c*d)*x^n)),x]
[Out] PolyLog[2, -((c*e - x^n + c*d*x^n)/x^n)]/(c*e*n)
```

Maple [A]
time = 2.57, size = 24, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\text{dilog}\left(cd+ce\,x^{-n}\right)}{nce}$	24
default	$\frac{\text{dilog}\left(cd+ce\,x^{-n}\right)}{nce}$	24
risch	Expression too large to display	1900

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e/(x^n)))/x/(c*e-(-c*d+1)*x^n),x,method=_RETURNVERBOSE)
[Out] 1/n*dilog(c*d+c*e/(x^n))/c/e
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(x^n)))/x/(c*e-(-c*d+1)*x^n),x, algorithm="maxima")
```

```
[Out] n*integrate(log(x)/(c*d*x*x^n + c*x*e), x) + (log(d*x^n + e)*log(x) + log(c
)*log(x) - log(x)*log(x^n))*e^(-1)/c - e^(-1)*log(c)*log(((c*d - 1)*x^n + c
*e)/(c*d - 1))/(c*n) - (log(d*x^n + e)*log(((c*d^2 - d)*x^n + (c*d - 1)*e)*
e^(-1) + 1) + dilog(-((c*d^2 - d)*x^n + (c*d - 1)*e)*e^(-1)))*e^(-1)/(c*n)
+ (log(x^n)*log((c*d - 1)*e^(n*log(x) - 1)/c + 1) + dilog(-(c*d - 1)*e^(n*l
og(x) - 1)/c))*e^(-1)/(c*n)
```

Fricas [A]
time = 0.45, size = 30, normalized size = 1.15

$$\frac{\mathrm{Li}_2\left(-\frac{cdx^n+ce}{x^n}+1\right)e^{(-1)}}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(x^n)))/x/(c*e-(-c*d+1)*x^n),x, algorithm="fricas")
```

```
[Out] dilog(-(c*d*x^n + c*e)/x^n + 1)*e^(-1)/(c*n)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e/(x**n)))/x/(c*e-(-c*d+1)*x**n),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(x^n)))/x/(c*e-(-c*d+1)*x^n),x, algorithm="giac")
```

```
[Out] integrate(log(c*(d + e/x^n))/(((c*d - 1)*x^n + c*e)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln\left(c\left(d+\frac{e}{x^n}\right)\right)}{x\,\left(c\,e+x^n\,\left(c\,d-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e/x^n))/(x*(c*e + x^n*(c*d - 1))),x)
```

```
[Out] int(log(c*(d + e/x^n))/(x*(c*e + x^n*(c*d - 1))), x)
```

3.381 $\int \frac{(f+gx^{2n})^2 \log^q(c(d+ex^n)^p)}{x} dx$

Optimal. Leaf size=608

$$\frac{4^{-1-q}g^2(d+ex^n)^4\left(c(d+ex^n)^p\right)^{-4/p}\Gamma\left(1+q,-\frac{4\log(c(d+ex^n)^p)}{p}\right)\log^q\left(c(d+ex^n)^p\right)\left(-\frac{\log(c(d+ex^n)^p)}{p}\right)^{-q}}{e^4n}-\frac{3^{-q}dg^2}{}$$

```
[Out] 4^(-1-q)*g^2*(d+e*x^n)^4*GAMMA(1+q,-4*ln(c*(d+e*x^n)^p)/p)*ln(c*(d+e*x^n)^p
)^q/e^4/n/((c*(d+e*x^n)^p)^(4/p))/((-ln(c*(d+e*x^n)^p)/p)^q)-d*g^2*(d+e*x^n
)^3*GAMMA(1+q,-3*ln(c*(d+e*x^n)^p)/p)*ln(c*(d+e*x^n)^p)^q/(3^q)/e^4/n/((c*(
d+e*x^n)^p)^(3/p))/((-ln(c*(d+e*x^n)^p)/p)^q)+f*g*(d+e*x^n)^2*GAMMA(1+q,-2*
ln(c*(d+e*x^n)^p)/p)*ln(c*(d+e*x^n)^p)^q/(2^q)/e^2/n/((c*(d+e*x^n)^p)^(2/p)
)/((-ln(c*(d+e*x^n)^p)/p)^q)+3*2^(-1-q)*d^2*g^2*(d+e*x^n)^2*GAMMA(1+q,-2*ln
(c*(d+e*x^n)^p)/p)*ln(c*(d+e*x^n)^p)^q/e^4/n/((c*(d+e*x^n)^p)^(2/p))/((-ln(
c*(d+e*x^n)^p)/p)^q)-2*d*f*g*(d+e*x^n)*GAMMA(1+q,-ln(c*(d+e*x^n)^p)/p)*ln(c
*(d+e*x^n)^p)^q/e^2/n/((c*(d+e*x^n)^p)^(1/p))/((-ln(c*(d+e*x^n)^p)/p)^q)-d^
3*g^2*(d+e*x^n)*GAMMA(1+q,-ln(c*(d+e*x^n)^p)/p)*ln(c*(d+e*x^n)^p)^q/e^4/n/(
(c*(d+e*x^n)^p)^(1/p))/((-ln(c*(d+e*x^n)^p)/p)^q)+f^2*Unintegrable(ln(c*(d+
e*x^n)^p)^q/x,x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(f+gx^{2n})^2 \log^q\left(c(d+ex^n)^p\right)}{x} dx$$

Verification is not applicable to the result.

```
[In] Int[((f + g*x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x,x]
```

```
[Out] Defer[Int][((f + g*x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Rubi steps

$$\int \frac{(f+gx^{2n})^2 \log^q\left(c(d+ex^n)^p\right)}{x} dx = \int \frac{(f+gx^{2n})^2 \log^q\left(c(d+ex^n)^p\right)}{x} dx$$

Mathematica [A]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{(f+gx^{2n})^2 \log^q\left(c(d+ex^n)^p\right)}{x} dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f + g*x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x,x]
```

```
[Out] Integrate[((f + g*x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Maple [A]
time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(f+g\,x^{2n}\right)^{2} \ln\left(c(d+e\,x^{n})^{p}\right)^{q}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g*x^(2*n))^2*ln(c*(d+e*x^n)^p)^q/x,x)
```

```
[Out] int((f+g*x^(2*n))^2*ln(c*(d+e*x^n)^p)^q/x,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(2*n))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(2*n))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="fricas")
```

```
[Out] integral((g^2*x^(4*n) + 2*f*g*x^(2*n) + f^2)*log((x^n*e + d)^p*c)^q/x, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x**(2*n))**2*ln(c*(d+e*x**n)**p)**q/x,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^(2*n))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="giac")
[Out] integrate((g*x^(2*n) + f)^2*log((x^n*e + d)^p*c)^q/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)^q\,{\left(f+g\,x^{2\,n}\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^n)^p)^q*(f + g*x^(2*n))^2)/x,x)
[Out] int((log(c*(d + e*x^n)^p)^q*(f + g*x^(2*n))^2)/x, x)
```

3.382 $\int \frac{(f+gx^n)^2 \log^q(c(d+ex^n)^p)}{x}\,dx$

Optimal. Leaf size=307

$$\frac{2^{-1-q}g^2(d+ex^n)^2\left(c(d+ex^n)^p\right)^{-2/p}\Gamma\left(1+q,-\frac{2\log(c(d+ex^n)^p)}{p}\right)\log^q\left(c(d+ex^n)^p\right)\left(-\frac{\log(c(d+ex^n)^p)}{p}\right)^{-q}}{e^2n}+\frac{2fg(c$$

```
[Out] 2^(-1-q)*g^2*(d+e*x^n)^2*GAMMA(1+q,-2*ln(c*(d+e*x^n)^p)/p)*ln(c*(d+e*x^n)^p
)^q/e^2/n/((c*(d+e*x^n)^p)^(2/p))/((-ln(c*(d+e*x^n)^p)/p)^q)+2*f*g*(d+e*x^n
)*GAMMA(1+q,-ln(c*(d+e*x^n)^p)/p)*ln(c*(d+e*x^n)^p)^q/e/n/((c*(d+e*x^n)^p)^
(1/p))/((-ln(c*(d+e*x^n)^p)/p)^q)-d*g^2*(d+e*x^n)*GAMMA(1+q,-ln(c*(d+e*x^n)
^p)/p)*ln(c*(d+e*x^n)^p)^q/e^2/n/((c*(d+e*x^n)^p)^(1/p))/((-ln(c*(d+e*x^n)^
p)/p)^q)+f^2*Unintegrable(ln(c*(d+e*x^n)^p)^q/x,x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(f+gx^n)^2 \log^q\left(c(d+ex^n)^p\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((f + g*x^n)^2*Log[c*(d + e*x^n)^p]^q)/x,x]
```

```
[Out] Defer[Int][((f + g*x^n)^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Rubi steps

$$\int \frac{(f+gx^n)^2 \log^q\left(c(d+ex^n)^p\right)}{x}\,dx=\int \frac{(f+gx^n)^2 \log^q\left(c(d+ex^n)^p\right)}{x}\,dx$$

Mathematica [A]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{(f+gx^n)^2 \log^q\left(c(d+ex^n)^p\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f + g*x^n)^2*Log[c*(d + e*x^n)^p]^q)/x,x]
```

```
[Out] Integrate[((f + g*x^n)^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Maple [A]
time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\left(f+g\,x^{n}\right)^{2} \ln \left(c \left(d+e\,x^{n}\right)^{p}\right)^{q}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g*x^n)^2*ln(c*(d+e*x^n)^p)^q/x,x)
```

```
[Out] int((f+g*x^n)^2*ln(c*(d+e*x^n)^p)^q/x,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^n)^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^n)^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="fricas")
```

```
[Out] integral((g^2*x^(2*n) + 2*f*g*x^n + f^2)*log((x^n*e + d)^p*c)^q/x, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x**n)**2*ln(c*(d+e*x**n)**p)**q/x,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g*x^n)^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="giac")
```

```
[Out] integrate((g*x^n + f)^2*log((x^n*e + d)^p*c)^q/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)}^q\,{\left(f+g\,x^n\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^n)^p)^q*(f + g*x^n)^2)/x,x)
```

```
[Out] int((log(c*(d + e*x^n)^p)^q*(f + g*x^n)^2)/x, x)
```

3.383 $\int \frac{(f+gx^{-n})^2 \log^q(c(d+ex^n)^p)}{x} \, dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{(f+gx^{-n})^2 \log^q\left(c(d+ex^n)^p\right)}{x},x\right)$$

```
[Out] Unintegrable((f+g/(x^n))^2*ln(c*(d+e*x^n)^p)^q/x,x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(f+gx^{-n})^2 \log^q\left(c(d+ex^n)^p\right)}{x} \, dx$$

Verification is not applicable to the result.

```
[In] Int[((f + g/x^n)^2*Log[c*(d + e*x^n)^p]^q)/x,x]
[Out] Defer[Int][((f + g/x^n)^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Rubi steps

$$\int \frac{(f+gx^{-n})^2 \log^q\left(c(d+ex^n)^p\right)}{x} \, dx = \int \frac{(f+gx^{-n})^2 \log^q\left(c(d+ex^n)^p\right)}{x} \, dx$$

Mathematica [A]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{(f+gx^{-n})^2 \log^q\left(c(d+ex^n)^p\right)}{x} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f + g/x^n)^2*Log[c*(d + e*x^n)^p]^q)/x,x]
[Out] Integrate[((f + g/x^n)^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Maple [A]
time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{(f+g\,x^{-n})^2 \ln\left(c(d+e\,x^n)^p\right)^q}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g/(x^n))^2*ln(c*(d+e*x^n)^p)^q/x,x)
[Out] int((f+g/(x^n))^2*ln(c*(d+e*x^n)^p)^q/x,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^n))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^n))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="fricas")
[Out] integral((f^2*x^(2*n) + 2*f*g*x^n + g^2)*log((x^n*e + d)^p*c)^q/(x*x^(2*n))
, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x**n))**2*ln(c*(d+e*x**n)**p)**q/x,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^n))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="giac")
[Out] integrate((f + g/x^n)^2*log((x^n*e + d)^p*c)^q/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln \left(c \left(d+e\,x^n\right)^p\right)^q\,\left(f+\frac{g}{x^n}\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^n)^p)^q*(f + g/x^n)^2)/x,x)
[Out] int((log(c*(d + e*x^n)^p)^q*(f + g/x^n)^2)/x, x)
```

3.384 $\int \frac{(f+gx^{-2n})^2 \log^q(c(d+ex^n)^p)}{x} dx$

Optimal. Leaf size=32

$$\operatorname{Int}\left(\frac{(f+gx^{-2n})^2 \log^q(c(d+ex^n)^p)}{x},x\right)$$

```
[Out] Unintegrable((f+g/(x^(2*n)))^2*ln(c*(d+e*x^n)^p)^q/x,x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(f+gx^{-2n})^2 \log^q(c(d+ex^n)^p)}{x} dx$$

Verification is not applicable to the result.

```
[In] Int[((f + g/x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x,x]
[Out] Defer[Int][((f + g/x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Rubi steps

$$\int \frac{(f+gx^{-2n})^2 \log^q(c(d+ex^n)^p)}{x} dx = \int \frac{(f+gx^{-2n})^2 \log^q(c(d+ex^n)^p)}{x} dx$$

Mathematica [A]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(f+gx^{-2n})^2 \log^q(c(d+ex^n)^p)}{x} dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f + g/x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x,x]
[Out] Integrate[((f + g/x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Maple [A]
time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{(f+g\,x^{-2n})^2 \ln(c(d+e\,x^n)^p)^q}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g/(x^(2*n)))^2*ln(c*(d+e*x^n)^p)^q/x,x)
```

```
[Out] int((f+g/(x^(2*n)))^2*ln(c*(d+e*x^n)^p)^q/x,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^(2*n)))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^(2*n)))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="fricas")
```

```
[Out] integral((f^2*x^(4*n) + 2*f*g*x^(2*n) + g^2)*log((x^n*e + d)^p*c)^q/(x*x^(4
*n)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x**(2*n)))**2*ln(c*(d+e*x**n)**p)**q/x,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/(x^(2*n)))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="giac")
```

```
[Out] integrate((f + g/x^(2*n))^2*log((x^n*e + d)^p*c)^q/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln \left(c \left(d+e\,x^n\right)^p\right)^q\,\left(f+\frac{g}{x^{2\,n}}\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*(d + e*x^n)^p)^q*(f + g/x^(2*n))^2)/x,x)
[Out] int((log(c*(d + e*x^n)^p)^q*(f + g/x^(2*n))^2)/x, x)
```

3.385 $\int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^{2n})} dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)},x\right)$$

```
[Out] Unintegrable(ln(c*(d+e*x^n)^p)^q/x/(f+g*x^(2*n)),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^(2*n))),x]
[Out] Defer[Int][Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^(2*n))), x]
```

Rubi steps

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}\,dx = \int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}\,dx$$

Mathematica [A]
time = 1.59, size = 0, normalized size = 0.00

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^(2*n))),x]
[Out] Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^(2*n))), x]
```

Maple [A]
time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c(d+e\,x^n)^p\right)^q}{x\left(f+g\,x^{2n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)^q/x/(f+g*x^(2*n)),x)
[Out] int(ln(c*(d+e*x^n)^p)^q/x/(f+g*x^(2*n)),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g*x^(2*n)),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g*x^(2*n)),x, algorithm="fricas")
[Out] integral(log((x^n*e + d)^p*c)^q/(g*x*x^(2*n) + f*x), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)**q/x/(f+g*x**(2*n)),x)
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g*x^(2*n)),x, algorithm="giac")
[Out] integrate(log((x^n*e + d)^p*c)^q/((g*x^(2*n) + f)*x), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)^q}{x\,\left(f+g\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)^q/(x*(f + g*x^(2*n))),x)
[Out] int(log(c*(d + e*x^n)^p)^q/(x*(f + g*x^(2*n))), x)
```

3.386 $\int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^n)} dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)},x\right)$$

```
[Out] Unintegrable(ln(c*(d+e*x^n)^p)^q/x/(f+g*x^n),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^n)),x]
[Out] Defer[Int][Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^n)), x]
```

Rubi steps

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)}\,dx = \int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)}\,dx$$

Mathematica [A]
time = 1.16, size = 0, normalized size = 0.00

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^n)),x]
[Out] Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^n)), x]
```

Maple [A]
time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c(d+e\,x^n)^p\right)^q}{x\left(f+g\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)^q/x/(f+g*x^n),x)
```

```
[Out] int(ln(c*(d+e*x^n)^p)^q/x/(f+g*x^n),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g*x^n),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g*x^n),x, algorithm="fricas")
```

```
[Out] integral(log((x^n*e + d)^p*c)^q/(g*x*x^n + f*x), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c(d+ex^n)^p\right)^q}{x\left(f+gx^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)**q/x/(f+g*x**n),x)
```

```
[Out] Integral(log(c*(d + e*x**n)**p)**q/(x*(f + g*x**n)), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g*x^n),x, algorithm="giac")
```

```
[Out] integrate(log((x^n*e + d)^p*c)^q/((g*x^n + f)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(c\left(d+e\,x^n\right)^p\right)^q}{x\,\left(f+g\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)^q/(x*(f + g*x^n)),x)
[Out] int(log(c*(d + e*x^n)^p)^q/(x*(f + g*x^n)), x)
```

3.387 $\int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^{-n})} dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)},x\right)$$

```
[Out] Unintegrable(ln(c*(d+e*x^n)^p)^q/x/(f+g/(x^n)),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^n)),x]
[Out] Defer[Int][Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^n)), x]
```

Rubi steps

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)}\,dx = \int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)}\,dx$$

Mathematica [A]
time = 1.20, size = 0, normalized size = 0.00

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^n)),x]
[Out] Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^n)), x]
```

Maple [A]
time = 0.77, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c(d+e\,x^n)^p\right)^q}{x\left(f+g\,x^{-n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)^q/x/(f+g/(x^n)),x)
```

```
[Out] int(ln(c*(d+e*x^n)^p)^q/x/(f+g/(x^n)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g/(x^n)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g/(x^n)),x, algorithm="fricas")
```

```
[Out] integral(x^n*log((x^n*e + d)^p*c)^q/(f*x*x^n + g*x), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)**q/x/(f+g/(x**n)),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g/(x^n)),x, algorithm="giac")
```

```
[Out] integrate(log((x^n*e + d)^p*c)^q/((f + g/x^n)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)^q}{x\,\left(f+\frac{g}{x^n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)^q/(x*(f + g/x^n)),x)
[Out] int(log(c*(d + e*x^n)^p)^q/(x*(f + g/x^n)), x)
```

3.388 $\int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^{-2n})} dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)},x\right)$$

```
[Out] Unintegrable(ln(c*(d+e*x^n)^p)^q/x/(f+g/(x^(2*n))),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^(2*n))),x]
[Out] Defer[Int][Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^(2*n))), x]
```

Rubi steps

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx = \int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx$$

Mathematica [A]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^(2*n))),x]
[Out] Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^(2*n))), x]
```

Maple [A]
time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c(d+e\,x^n)^p\right)^q}{x\left(f+g\,x^{-2n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*x^n)^p)^q/x/(f+g/(x^(2*n))),x)
[Out] int(ln(c*(d+e*x^n)^p)^q/x/(f+g/(x^(2*n))),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g/(x^(2*n))),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g/(x^(2*n))),x, algorithm="fricas")
[Out] integral(x^(2*n)*log((x^n*e + d)^p*c)^q/(f*x*x^(2*n) + g*x), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*x**n)**p)**q/x/(f+g/(x**(2*n))),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5011 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g/(x^(2*n))),x, algorithm="giac")
[Out] integrate(log((x^n*e + d)^p*c)^q/((f + g/x^(2*n))*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)^q}{x\,\left(f+\frac{g}{x^{2\,n}}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*x^n)^p)^q/(x*(f + g/x^(2*n))),x)
[Out] int(log(c*(d + e*x^n)^p)^q/(x*(f + g/x^(2*n))), x)
```

3.389 $\int \frac{\log(x)\log(d+ex^m)}{x}\,dx$

Optimal. Leaf size=69

$$\frac{1}{2}\log^2(x)\log\left(d+ex^m\right)-\frac{1}{2}\log^2(x)\log\left(1+\frac{ex^m}{d}\right)-\frac{\log(x)\text{Li}_2\left(-\frac{ex^m}{d}\right)}{m}+\frac{\text{Li}_3\left(-\frac{ex^m}{d}\right)}{m^2}$$

```
[Out] 1/2*ln(x)^2*ln(d+e*x^m)-1/2*ln(x)^2*ln(1+e*x^m/d)-ln(x)*polylog(2,-e*x^m/d)
/m+polylog(3,-e*x^m/d)/m^2
```

Rubi [A]
time = 0.08, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = $\{2422, 2375, 2421, 6724\}$

$$\frac{\text{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m^2}-\frac{\log(x)\text{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m}+\frac{1}{2}\log^2(x)\log\left(d+ex^m\right)-\frac{1}{2}\log^2(x)\log\left(\frac{ex^m}{d}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Log[x]*Log[d + e*x^m])/x,x]
```

```
[Out] (Log[x]^2*Log[d + e*x^m])/2 - (Log[x]^2*Log[1 + (e*x^m)/d])/2 - (Log[x]*Pol
yLog[2, -((e*x^m)/d)])/m + PolyLog[3, -((e*x^m)/d)]/m^2
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
```

```
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log(x)\log(d+ex^m)}{x}\,dx &= \frac{1}{2}\log^2(x)\log(d+ex^m) - \frac{1}{2}(em)\int \frac{x^{-1+m}\log^2(x)}{d+ex^m}\,dx \\
&= \frac{1}{2}\log^2(x)\log(d+ex^m) - \frac{1}{2}\log^2(x)\log\left(1+\frac{ex^m}{d}\right) + \int \frac{\log(x)\log\left(1+\frac{ex^m}{d}\right)}{x} \\
&= \frac{1}{2}\log^2(x)\log(d+ex^m) - \frac{1}{2}\log^2(x)\log\left(1+\frac{ex^m}{d}\right) - \frac{\log(x)\text{Li}_2\left(-\frac{ex^m}{d}\right)}{m} + \int \\
&= \frac{1}{2}\log^2(x)\log(d+ex^m) - \frac{1}{2}\log^2(x)\log\left(1+\frac{ex^m}{d}\right) - \frac{\log(x)\text{Li}_2\left(-\frac{ex^m}{d}\right)}{m} + \text{Li}_3
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 75, normalized size = 1.09

$$
-\frac{1}{6}\log^2(x)\left(m\log(x)+3\log\left(1+\frac{dx^{-m}}{e}\right)-3\log(d+ex^m)\right)+\frac{\log(x)\text{Li}_2\left(-\frac{dx^{-m}}{e}\right)}{m}+\frac{\text{Li}_3\left(-\frac{dx^{-m}}{e}\right)}{m^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[x]*Log[d + e*x^m])/x,x]
```

```
[Out] -1/6*(Log[x]^2*(m*Log[x] + 3*Log[1 + d/(e*x^m)] - 3*Log[d + e*x^m])) + (Log
[x]*PolyLog[2, -(d/(e*x^m))])/m + PolyLog[3, -(d/(e*x^m))]/m^2
```

Maple [A]
time = 0.69, size = 66, normalized size = 0.96

method	result	size
risch	$\frac{\ln(x)^2\ln(d+e\,x^m)}{2}-\frac{\ln(x)^2\ln\left(1+\frac{e\,x^m}{d}\right)}{2}-\frac{\ln(x)\,\text{polylog}\left(2,-\frac{e\,x^m}{d}\right)}{m}+\frac{\text{polylog}\left(3,-\frac{e\,x^m}{d}\right)}{m^2}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*ln(d+e*x^m)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(x)^2*ln(d+e*x^m)-1/2*ln(x)^2*ln(1+e*x^m/d)-ln(x)*polylog(2,-e*x^m/d)
/m+polylog(3,-e*x^m/d)/m^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log(d+e*x^m)/x,x, algorithm="maxima")
```

```
[Out] -1/6*m*log(x)^3 + d*m*integrate(1/2*log(x)^2/(d*x + x*e^(m*log(x) + 1)), x)
 + 1/2*log(d + e^(m*log(x) + 1))*log(x)^2
```

Fricas [A]
time = 0.36, size = 80, normalized size = 1.16

$$\frac{m^2\log\left(x^me+d\right)\log\left(x\right)^2-m^2\log\left(x\right)^2\log\left(\frac{x^me+d}{d}\right)-2\,m\mathrm{Li}_2\left(-\frac{x^me+d}{d}+1\right)\log\left(x\right)+2\,\mathrm{polylog}\left(3,-\frac{x^me}{d}\right)}{2\,m^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log(d+e*x^m)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(m^2*log(x^m*e + d)*log(x)^2 - m^2*log(x)^2*log((x^m*e + d)/d) - 2*m*di
log(-(x^m*e + d)/d + 1)*log(x) + 2*polylog(3, -x^m*e/d))/m^2
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*ln(d+e*x**m)/x,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log(d+e*x^m)/x,x, algorithm="giac")
```

```
[Out] integrate(log(x^m*e + d)*log(x)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln (d + e\, x^m)\, \ln (x)}{x} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(d + e*x^m)*log(x))/x,x)
```

```
[Out] int((log(d + e*x^m)*log(x))/x, x)
```

3.390 $\int \frac{\log\left(\frac{a+x}{x}\right)}{x} dx$

Optimal. Leaf size=8

$$\text{Li}_2\left(-\frac{a}{x}\right)$$

[Out] polylog(2,-a/x)

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.50, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{2497\}$

$$\text{PolyLog}\left(2, 1 - \frac{a+x}{x}\right)$$

Antiderivative was successfully verified.

[In] Int[Log[(a + x)/x]/x,x]

[Out] PolyLog[2, 1 - (a + x)/x]

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(\frac{a+x}{x}\right)}{x} dx = \text{Li}_2\left(1 - \frac{a+x}{x}\right)$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 34 vs. 2(8) = 16.
time = 0.00, size = 34, normalized size = 4.25

$$-\log\left(-\frac{a}{x}\right)\log\left(\frac{a+x}{x}\right) - \text{Li}_2\left(-\frac{-a-x}{x}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Log[(a + x)/x]/x,x]

[Out] -(Log[-(a/x)]*Log[(a + x)/x]) - PolyLog[2, -((-a - x)/x)]

Maple [A]
time = 0.44, size = 9, normalized size = 1.12

method	result	size
derivativedivides	$\mathrm{dilog}\left(1+\frac{a}{x}\right)$	9
default	$\mathrm{dilog}\left(1+\frac{a}{x}\right)$	9
risch	$\mathrm{dilog}\left(1+\frac{a}{x}\right)$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((a+x)/x)/x,x,method=_RETURNVERBOSE)
```

```
[Out] dilog(1+a/x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(7) = 14$.
time = 0.37, size = 59, normalized size = 7.38

$$-(\log(a+x)-\log(x))\log(x)+\log(a+x)\log(x)-\frac{1}{2}\log(x)^2+\log(x)\log\left(\frac{a+x}{x}\right)-\log(x)\log\left(\frac{x}{a}+1\right)-\mathrm{Li}_2\left(-\frac{x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a+x)/x)/x,x, algorithm="maxima")
```

```
[Out] -(log(a + x) - log(x))*log(x) + log(a + x)*log(x) - 1/2*log(x)^2 + log(x)*l
og((a + x)/x) - log(x)*log(x/a + 1) - dilog(-x/a)
```

Fricas [A]
time = 0.36, size = 11, normalized size = 1.38

$$\mathrm{Li}_2\left(-\frac{a+x}{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a+x)/x)/x,x, algorithm="fricas")
```

```
[Out] dilog(-(a + x)/x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left(\frac{a}{x}+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((a+x)/x)/x,x)
```

[Out] Integral(log(a/x + 1)/x, x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(7) = 14$.
time = 4.21, size = 68, normalized size = 8.50

$$-\frac{a^3\left(\frac{1}{\frac{a+x}{x}-1} - \log\left(\frac{|a+x|}{|x|}\right) + \log\left(\left|\frac{a+x}{x} - 1\right|\right)\right) + \frac{a^3\log\left(\frac{a+x}{x}\right)}{\left(\frac{a+x}{x}-1\right)^2}}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log((a+x)/x)/x,x, algorithm="giac")

[Out] -1/2*(a^3*(1/((a + x)/x - 1) - log(abs(a + x)/abs(x)) + log(abs((a + x)/x - 1))) + a^3*log((a + x)/x)/((a + x)/x - 1)^2)/a^2

Mupad [B]
time = 0.31, size = 8, normalized size = 1.00

$$\mathrm{polylog}\left(2,-\frac{a}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log((a + x)/x)/x,x)

[Out] polylog(2, -a/x)

3.391 $\int \frac{\log\left(\frac{a+x^2}{x^2}\right)}{x}\,dx$

Optimal. Leaf size=12

$$\frac{1}{2}\text{Li}_2\left(-\frac{a}{x^2}\right)$$

```
[Out] 1/2*polylog(2,-a/x^2)
```

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2511, 2438}

$$\frac{1}{2}\text{PolyLog}\left(2,-\frac{a}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[(a + x^2)/x^2]/x,x]
```

```
[Out] PolyLog[2, -(a/x^2)]/2
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2511

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*((f_.)*(x_))^(m_.), x_Symbo
l] :> Int[(f*x)^m*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b,
c, f, m, p, q}, x] && BinomialQ[v, x] &&  !BinomialMatchQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(\frac{a+x^2}{x^2}\right)}{x}\,dx &= \int \frac{\log\left(1+\frac{a}{x^2}\right)}{x}\,dx\\ &= \frac{1}{2}\text{Li}_2\left(-\frac{a}{x^2}\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$\frac{1}{2}\text{Li}_2\left(-\frac{a}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(a + x^2)/x^2]/x,x]
```

```
[Out] PolyLog[2, -(a/x^2)]/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(10) = 20$.

time = 0.09, size = 86, normalized size = 7.17

method	result
risch	$-\ln\left(\frac{1}{x}\right)\ln\left(1+\frac{a}{x^2}\right)+\ln\left(\frac{1}{x}\right)\ln\left(1+\frac{\sqrt{-a}}{x}\right)+\ln\left(\frac{1}{x}\right)\ln\left(1-\frac{\sqrt{-a}}{x}\right)+\operatorname{dilog}\left(1+\sqrt{\right.$
derivativedivides	$-\ln\left(\frac{1}{x}\right)\ln\left(1+\frac{a}{x^2}\right)+2a\left(\frac{\ln\left(\frac{1}{x}\right)\left(\ln\left(1+\frac{\sqrt{-a}}{x}\right)+\ln\left(1-\frac{\sqrt{-a}}{x}\right)\right)}{2a}+\frac{\operatorname{dilog}\left(1+\frac{\sqrt{-a}}{x}\right)+\operatorname{dilog}\left(1-\right.}{2a}\right.$
default	$-\ln\left(\frac{1}{x}\right)\ln\left(1+\frac{a}{x^2}\right)+2a\left(\frac{\ln\left(\frac{1}{x}\right)\left(\ln\left(1+\frac{\sqrt{-a}}{x}\right)+\ln\left(1-\frac{\sqrt{-a}}{x}\right)\right)}{2a}+\frac{\operatorname{dilog}\left(1+\frac{\sqrt{-a}}{x}\right)+\operatorname{dilog}\left(1-\right.}{2a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((x^2+a)/x^2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -ln(1/x)*ln(1+a/x^2)+2*a*(1/2*ln(1/x)*(ln(1+1/x*(-a)^(1/2))+ln(1-1/x*(-a)^(
1/2)))/a+1/2*(dilog(1+1/x*(-a)^(1/2))+dilog(1-1/x*(-a)^(1/2)))/a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(9) = 18$.

time = 0.37, size = 69, normalized size = 5.75

$$-\left(\log\left(x^2+a\right)-2\log\left(x\right)\right)\log\left(x\right)+\log\left(x^2+a\right)\log\left(x\right)-\log\left(x\right)^2-\log\left(x\right)\log\left(\frac{x^2}{a}+1\right)+\log\left(x\right)\log\left(\frac{x^2+a}{x^2}\right)-\frac{1}{2}\,\mathrm{Li}_2\left(-\frac{x^2}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((x^2+a)/x^2)/x,x, algorithm="maxima")
```

```
[Out] -(log(x^2 + a) - 2*log(x))*log(x) + log(x^2 + a)*log(x) - log(x)^2 - log(x)
*log(x^2/a + 1) + log(x)*log((x^2 + a)/x^2) - 1/2*dilog(-x^2/a)
```

Fricas [A]

time = 0.34, size = 15, normalized size = 1.25

$$\frac{1}{2}\,\mathrm{Li}_2\left(-\frac{x^2+a}{x^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((x^2+a)/x^2)/x,x, algorithm="fricas")
```

```
[Out] 1/2*dilog(-(x^2 + a)/x^2 + 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{a}{x^2}+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((x**2+a)/x**2)/x,x)
```

```
[Out] Integral(log(a/x**2 + 1)/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((x^2+a)/x^2)/x,x, algorithm="giac")
```

```
[Out] integrate(log((x^2 + a)/x^2)/x, x)
```

Mupad [**B**]
time = 0.29, size = 10, normalized size = 0.83

$$\frac{\mathrm{polylog}\left(2,-\frac{a}{x^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((a + x^2)/x^2)/x,x)
```

```
[Out] polylog(2, -a/x^2)/2
```

3.392 $\int \frac{\log(x^{-n}(a+x^n))}{x}\,dx$

Optimal. Leaf size=14

$$\frac{\text{Li}_2(-ax^{-n})}{n}$$

```
[Out] polylog(2,-a/(x^n))/n
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2511, 2438}

$$\frac{\text{PolyLog}(2, -ax^{-n})}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(a + x^n)/x^n]/x,x]
```

```
[Out] PolyLog[2, -(a/x^n)]/n
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2511

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*((f_.)*(x_))^(m_.), x_Symbo
l] :> Int[(f*x)^m*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b,
c, f, m, p, q}, x] && BinomialQ[v, x] &&  !BinomialMatchQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(x^{-n}(a+x^n))}{x}\,dx &= \int \frac{\log(1+ax^{-n})}{x}\,dx\\ &= \frac{\text{Li}_2(-ax^{-n})}{n}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{\text{Li}_2(-ax^{-n})}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(a + x^n)/x^n]/x,x]
```

```
[Out] PolyLog[2, -(a/x^n)]/n
```

Maple [A]
time = 1.83, size = 15, normalized size = 1.07

method	result
derivativedivides	$\frac{\operatorname{dilog}(1+a\,x^{-n})}{n}$
default	$\frac{\operatorname{dilog}(1+a\,x^{-n})}{n}$
risch	$-\ln(x)\ln(x^{n})+\frac{n\ln(x)^{2}}{2}+\frac{i\pi\ln(x)\operatorname{csgn}(ix^{-n})\operatorname{csgn}(ix^{-n}(a+x^{n}))^{2}}{2}+\frac{i\pi\ln(x)\operatorname{csgn}(i(a+x^{n}))\operatorname{csgn}(ix^{-n}(a+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((a+x^n)/(x^n))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*dilog(1+a/(x^n))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a+x^n)/(x^n))/x,x, algorithm="maxima")
```

```
[Out] a*n*integrate(log(x)/(a*x + x*x^n), x) + log(a + x^n)*log(x) - log(x)*log(x
^n)
```

Fricas [A]
time = 0.42, size = 19, normalized size = 1.36

$$\frac{\operatorname{Li}_2\left(-\frac{a+x^n}{x^n}+1\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a+x^n)/(x^n))/x,x, algorithm="fricas")
```

```
[Out] dilog(-(a + x^n)/x^n + 1)/n
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(ax^{-n}+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((a+x**n)/(x**n))/x,x)
```

```
[Out] Integral(log(a/x**n + 1)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a+x^n)/(x^n))/x,x, algorithm="giac")
```

```
[Out] integrate(log((a + x^n)/x^n)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\ln\left(\frac{a+x^n}{x^n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((a + x^n)/x^n)/x,x)
```

```
[Out] int(log((a + x^n)/x^n)/x, x)
```

3.393 $\int \frac{\log\left(\frac{a+bx}{x}\right)}{x} dx$

Optimal. Leaf size=35

$$-\log\left(b+\frac{a}{x}\right)\log\left(-\frac{a}{bx}\right)-\text{Li}_2\left(1+\frac{a}{bx}\right)$$

```
[Out] -ln(b+a/x)*ln(-a/b/x)-polylog(2,1+a/b/x)
```

Rubi [A]
time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2511, 2504, 2441, 2352}

$$-\text{PolyLog}\left(2,\frac{a}{bx}+1\right)-\log\left(\frac{a}{x}+b\right)\log\left(-\frac{a}{bx}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[(a + b*x)/x]/x,x]
```

```
[Out] -(Log[b + a/x]*Log[-(a/(b*x))]) - PolyLog[2, 1 + a/(b*x)]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2511

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*((f_.)*(x_))^(m_.), x_Symbo
l] :> Int[(f*x)^m*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b,
```

```
c, f, m, p, q}, x] && BinomialQ[v, x] &&  !BinomialMatchQ[v, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log\left(\frac{a+bx}{x}\right)}{x}\,dx &= \int \frac{\log\left(b+\frac{a}{x}\right)}{x}\,dx \\
&= -\mathrm{Subst}\left(\int \frac{\log(b+ax)}{x}\,dx, x, \frac{1}{x}\right) \\
&= -\log\left(b+\frac{a}{x}\right)\log\left(-\frac{a}{bx}\right) + a\mathrm{Subst}\left(\int \frac{\log\left(-\frac{ax}{b}\right)}{b+ax}\,dx, x, \frac{1}{x}\right) \\
&= -\log\left(b+\frac{a}{x}\right)\log\left(-\frac{a}{bx}\right) - \mathrm{Li}_2\left(1+\frac{a}{bx}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 36, normalized size = 1.03

$$
-\log\left(b+\frac{a}{x}\right)\log\left(-\frac{a}{bx}\right) - \mathrm{Li}_2\left(\frac{b+\frac{a}{x}}{b}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(a + b*x)/x]/x,x]
[Out] -(Log[b + a/x]*Log[-(a/(b*x))]) - PolyLog[2, (b + a/x)/b]
```

Maple [A]
time = 0.72, size = 34, normalized size = 0.97

method	result	size
derivativedivides	$-\operatorname{dilog}\left(-\frac{a}{bx}\right)-\ln\left(b+\frac{a}{x}\right)\ln\left(-\frac{a}{bx}\right)$	34
default	$-\operatorname{dilog}\left(-\frac{a}{bx}\right)-\ln\left(b+\frac{a}{x}\right)\ln\left(-\frac{a}{bx}\right)$	34
risch	$-\operatorname{dilog}\left(-\frac{a}{bx}\right)-\ln\left(b+\frac{a}{x}\right)\ln\left(-\frac{a}{bx}\right)$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((b*x+a)/x)/x,x,method=_RETURNVERBOSE)
[Out] -dilog(-a/b/x)-ln(b+a/x)*ln(-a/b/x)
```

Maxima [A]
time = 0.27, size = 67, normalized size = 1.91

$$
-(\log(bx+a)-\log(x))\log(x)+\log(bx+a)\log(x)-\log\left(\frac{bx}{a}+1\right)\log(x)-\frac{1}{2}\log(x)^2+\log(x)\log\left(\frac{bx+a}{x}\right)-\mathrm{Li}_2\left(-\frac{bx}{a}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x+a)/x)/x,x, algorithm="maxima")
```

```
[Out] -(log(b*x + a) - log(x))*log(x) + log(b*x + a)*log(x) - log(b*x/a + 1)*log(
x) - 1/2*log(x)^2 + log(x)*log((b*x + a)/x) - dilog(-b*x/a)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x+a)/x)/x,x, algorithm="fricas")
```

```
[Out] integral(log((b*x + a)/x)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{a}{x} + b\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((b*x+a)/x)/x,x)
```

```
[Out] Integral(log(a/x + b)/x, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(34) = 68.
time = 4.34, size = 204, normalized size = 5.83

$$\frac{a^3\left(\frac{\log\left(\frac{|bx+a|}{|x|}\right)}{b^2} - \frac{\log\left(\left|-b+\frac{bx+a}{x}\right|\right)}{b^2} + \frac{1}{\left(b-\frac{bx+a}{x}\right)b}\right) - \frac{a^3\log\left(-\left(a-\frac{b}{\frac{\left(a-\frac{b}{\frac{b}{a}-\frac{bx+a}{ax}}\right)\left(\frac{b}{a}-\frac{bx+a}{ax}\right)}{a}+\frac{b}{a}}\right)\left(\frac{\left(a-\frac{b}{\frac{b}{a}-\frac{bx+a}{ax}}\right)\left(\frac{b}{a}-\frac{bx+a}{ax}\right)}{a}+\frac{b}{a}\right)\right)}{\left(b-\frac{bx+a}{x}\right)^2}}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x+a)/x)/x,x, algorithm="giac")
```

```
[Out] 1/2*(a^3*(log(abs(b*x + a)/abs(x))/b^2 - log(abs(-b + (b*x + a)/x))/b^2 + 1
/((b - (b*x + a)/x)*b)) - a^3*log(-(a - b/((a - b/(b/a - (b*x + a)/(a*x)))*
(b/a - (b*x + a)/(a*x))/a + b/a))*((a - b/(b/a - (b*x + a)/(a*x)))*(b/a - (
b*x + a)/(a*x))/a + b/a))/(b - (b*x + a)/x)^2)/a^2
```

Mupad [B]
time = 0.41, size = 37, normalized size = 1.06

$$-\mathrm{polylog}\left(2,\frac{a}{b\,x}+1\right)-\ln\left(\frac{a+b\,x}{x}\right)\,\ln\left(-\frac{a}{b\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((a + b*x)/x)/x,x)
[Out] - polylog(2, a/(b*x) + 1) - log((a + b*x)/x)*log(-a/(b*x))
```

3.394 $\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{x}\,dx$

Optimal. Leaf size=39

$$-\frac{1}{2}\log\left(b+\frac{a}{x^2}\right)\log\left(-\frac{a}{bx^2}\right)-\frac{1}{2}\text{Li}_2\left(1+\frac{a}{bx^2}\right)$$

```
[Out] -1/2*ln(b+a/x^2)*ln(-a/b/x^2)-1/2*polylog(2,1+a/b/x^2)
```

Rubi [A]
time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2511, 2504, 2441, 2352}

$$-\frac{1}{2}\text{PolyLog}\left(2,\frac{a}{bx^2}+1\right)-\frac{1}{2}\log\left(\frac{a}{x^2}+b\right)\log\left(-\frac{a}{bx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[(a + b*x^2)/x^2]/x,x]
```

```
[Out] -1/2*(Log[b + a/x^2]*Log[-(a/(b*x^2))]) - PolyLog[2, 1 + a/(b*x^2)]/2
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2511

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*((f_.)*(x_))^(m_.), x_Symbo
l] :> Int[(f*x)^m*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b,
```

```
c, f, m, p, q}, x] && BinomialQ[v, x] &&  !BinomialMatchQ[v, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{x}\,dx &= \int \frac{\log\left(b+\frac{a}{x^2}\right)}{x}\,dx\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{\log(b+ax)}{x}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= -\frac{1}{2}\log\left(b+\frac{a}{x^2}\right)\log\left(-\frac{a}{bx^2}\right)+\frac{1}{2}a\text{Subst}\left(\int \frac{\log\left(-\frac{ax}{b}\right)}{b+ax}\,dx, x, \frac{1}{x^2}\right)\\
&= -\frac{1}{2}\log\left(b+\frac{a}{x^2}\right)\log\left(-\frac{a}{bx^2}\right)-\frac{1}{2}\text{Li}_2\left(1+\frac{a}{bx^2}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 40, normalized size = 1.03

$$-\frac{1}{2}\log\left(b+\frac{a}{x^2}\right)\log\left(-\frac{a}{bx^2}\right)-\frac{1}{2}\text{Li}_2\left(\frac{b+\frac{a}{x^2}}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(a + b*x^2)/x^2]/x,x]
[Out] -1/2*(Log[b + a/x^2]*Log[-(a/(b*x^2))]) - PolyLog[2, (b + a/x^2)/b]/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(35) = 70$.
time = 0.03, size = 118, normalized size = 3.03

method	result
risch	$-\ln\left(\frac{1}{x}\right)\ln\left(b+\frac{a}{x^2}\right)+\ln\left(\frac{1}{x}\right)\ln\left(\frac{\frac{a}{x}+\sqrt{-ba}}{\sqrt{-ba}}\right)+\ln\left(\frac{1}{x}\right)\ln\left(\frac{-\frac{a}{x}+\sqrt{-ba}}{\sqrt{-ba}}\right)+\operatorname{dilog}\left(\frac{-\frac{a}{x}+\sqrt{}}{\sqrt{-}}\right.$
derivativedivides	$-\ln\left(\frac{1}{x}\right)\ln\left(b+\frac{a}{x^2}\right)+2a\left(\frac{\ln\left(\frac{1}{x}\right)\left(\ln\left(\frac{-\frac{a}{x}+\sqrt{-ba}}{\sqrt{-ba}}\right)+\ln\left(\frac{\frac{a}{x}+\sqrt{-ba}}{\sqrt{-ba}}\right)\right)}{2a}+\frac{\operatorname{dilog}\left(\frac{-\frac{a}{x}+\sqrt{-ba}}{\sqrt{-ba}}\right)+}{2a}\right.$
default	$-\ln\left(\frac{1}{x}\right)\ln\left(b+\frac{a}{x^2}\right)+2a\left(\frac{\ln\left(\frac{1}{x}\right)\left(\ln\left(\frac{-\frac{a}{x}+\sqrt{-ba}}{\sqrt{-ba}}\right)+\ln\left(\frac{\frac{a}{x}+\sqrt{-ba}}{\sqrt{-ba}}\right)\right)}{2a}+\frac{\operatorname{dilog}\left(\frac{-\frac{a}{x}+\sqrt{-ba}}{\sqrt{-ba}}\right)+}{2a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((b*x^2+a)/x^2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -ln(1/x)*ln(b+a/x^2)+2*a*(1/2*ln(1/x)*(ln((-a/x+(-b*a)^(1/2))/(-b*a)^(1/2))
+ln((a/x+(-b*a)^(1/2))/(-b*a)^(1/2)))/a+1/2*(dilog((-a/x+(-b*a)^(1/2))/(-b*
a)^(1/2))+dilog((a/x+(-b*a)^(1/2))/(-b*a)^(1/2)))/a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(34) = 68$.
time = 0.28, size = 77, normalized size = 1.97

$$-\left(\log\left(bx^2+a\right)-2\log\left(x\right)\right)\log\left(x\right)+\log\left(bx^2+a\right)\log\left(x\right)-\log\left(\frac{bx^2}{a}+1\right)\log\left(x\right)-\log\left(x\right)^2+\log\left(x\right)\log\left(\frac{bx^2+a}{x^2}\right)-\frac{1}{2}\,\mathrm{Li}_2\left(-\frac{bx^2}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x^2+a)/x^2)/x,x, algorithm="maxima")
```

```
[Out] -(log(b*x^2 + a) - 2*log(x))*log(x) + log(b*x^2 + a)*log(x) - log(b*x^2/a +
 1)*log(x) - log(x)^2 + log(x)*log((b*x^2 + a)/x^2) - 1/2*dilog(-b*x^2/a)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x^2+a)/x^2)/x,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)/x^2)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{a}{x^2}+b\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((b*x**2+a)/x**2)/x,x)
```

```
[Out] Integral(log(a/x**2 + b)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x^2+a)/x^2)/x,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)/x^2)/x, x)
```

Mupad [B]
time = 0.44, size = 33, normalized size = 0.85

$$-\frac{\mathrm{Li}_2\left(-\frac{a}{b\,x^2}\right)}{2}-\frac{\ln\left(b+\frac{a}{x^2}\right)\,\ln\left(-\frac{a}{b\,x^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((a + b*x^2)/x^2)/x,x)
```

```
[Out] - dilog(-a/(b*x^2))/2 - (log(b + a/x^2)*log(-a/(b*x^2)))/2
```

3.395 $\int \frac{\log(x^{-n}(a+bx^n))}{x}\,dx$

Optimal. Leaf size=47

$$-\frac{\log\left(-\frac{ax^{-n}}{b}\right)\log\left(b+ax^{-n}\right)}{n}-\frac{\text{Li}_2\left(1+\frac{ax^{-n}}{b}\right)}{n}$$

```
[Out] -ln(-a/b/(x^n))*ln(b+a/(x^n))/n-polylog(2,1+a/b/(x^n))/n
```

Rubi [A]
time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2511, 2504, 2441, 2352}

$$-\frac{\text{PolyLog}\left(2,\frac{ax^{-n}}{b}+1\right)}{n}-\frac{\log\left(-\frac{ax^{-n}}{b}\right)\log\left(ax^{-n}+b\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(a + b*x^n)/x^n]/x,x]
```

```
[Out] -((Log[-(a/(b*x^n))]*Log[b + a/x^n])/n) - PolyLog[2, 1 + a/(b*x^n)]/n
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2511

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*((f_.)*(x_))^(m_.), x_Symbo
l] :> Int[(f*x)^m*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b,
```

`c, f, m, p, q}, x] && BinomialQ[v, x] && !BinomialMatchQ[v, x]`

Rubi steps

$$\int \frac{\log (x^{-n}(a+bx^n))}{x}\,dx = \int \frac{\log (b+ax^{-n})}{x}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{\log(b+ax)}{x}\,dx, x, x^{-n}\right)}{n}$$
$$= -\frac{\log\left(-\frac{ax^{-n}}{b}\right)\log (b+ax^{-n})}{n} + \frac{a\text{Subst}\left(\int \frac{\log\left(-\frac{ax}{b}\right)}{b+ax}\,dx, x, x^{-n}\right)}{n}$$
$$= -\frac{\log\left(-\frac{ax^{-n}}{b}\right)\log (b+ax^{-n})}{n} - \frac{\text{Li}_2\left(1+\frac{ax^{-n}}{b}\right)}{n}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 0.94

$$-\frac{\log\left(-\frac{ax^{-n}}{b}\right)\log (b+ax^{-n}) + \text{Li}_2\left(\frac{b+ax^{-n}}{b}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(a + b*x^n)/x^n]/x,x]
[Out] -((Log[-(a/(b*x^n))]*Log[b + a/x^n] + PolyLog[2, (b + a/x^n)/b])/n)
```

Maple [A]
time = 1.96, size = 44, normalized size = 0.94

method	result
derivativedivides	$\frac{-\operatorname{dilog}\left(-\frac{a\,x^{-n}}{b}\right)-\ln\left(b+a\,x^{-n}\right)\ln\left(-\frac{a\,x^{-n}}{b}\right)}{n}$
default	$\frac{-\operatorname{dilog}\left(-\frac{a\,x^{-n}}{b}\right)-\ln\left(b+a\,x^{-n}\right)\ln\left(-\frac{a\,x^{-n}}{b}\right)}{n}$
risch	$-\ln(x)\ln(x^n)+\frac{n\ln(x)^2}{2}+\frac{i\pi\ln(x)\operatorname{csgn}(ix^{-n})\operatorname{csgn}(ix^{-n}(a+b\,x^n))^2}{2}+\frac{i\pi\ln(x)\operatorname{csgn}(i(a+b\,x^n))\operatorname{csgn}(ix^{-n}(a-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((a+b*x^n)/(x^n))/x,x,method=_RETURNVERBOSE)
[Out] 1/n*(-dilog(-a/b/(x^n))-ln(b+a/(x^n))*ln(-a/b/(x^n)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a+b*x^n)/(x^n))/x,x, algorithm="maxima")
```

```
[Out] a*n*integrate(log(x)/(b*x*x^n + a*x), x) + log(b*x^n + a)*log(x) - log(x)*l
og(x^n)
```

Fricas [A]
time = 0.35, size = 67, normalized size = 1.43

$$\frac{n^2\log(x)^2 - 2\,n\log(x)\log\left(\frac{bx^n+a}{a}\right) + 2\,n\log(x)\log\left(\frac{bx^n+a}{x^n}\right) - 2\,\mathrm{Li}_2\left(-\frac{bx^n+a}{a}+1\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a+b*x^n)/(x^n))/x,x, algorithm="fricas")
```

```
[Out] 1/2*(n^2*log(x)^2 - 2*n*log(x)*log((b*x^n + a)/a) + 2*n*log(x)*log((b*x^n +
 a)/x^n) - 2*dilog(-(b*x^n + a)/a + 1))/n
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(ax^{-n}+b)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((a+b*x**n)/(x**n))/x,x)
```

```
[Out] Integral(log(a/x**n + b)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a+b*x^n)/(x^n))/x,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^n + a)/x^n)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(\frac{a+b\,x^n}{x^n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((a + b*x^n)/x^n)/x,x)
[Out] int(log((a + b*x^n)/x^n)/x, x)
```

3.396 $\int \frac{\log\left(\frac{a+bx}{x}\right)}{c+dx}\,dx$

Optimal. Leaf size=105

$$\frac{\log\left(b+\frac{a}{x}\right)\log(c+dx)}{d}+\frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d}-\frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{d}-\frac{\mathrm{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{d}+\frac{\mathrm{Li}_2\left(1+\frac{dx}{c}\right)}{d}$$

```
[Out] ln(b+a/x)*ln(d*x+c)/d+ln(-d*x/c)*ln(d*x+c)/d-ln(-d*(b*x+a)/(-a*d+b*c))*ln(d
*x+c)/d-polylog(2,b*(d*x+c)/(-a*d+b*c))/d+polylog(2,1+d*x/c)/d
```

Rubi [A]
time = 0.12, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2515, 2512, 266, 2463, 2441, 2352, 2440, 2438}

$$-\frac{\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{d}+\frac{\text{PolyLog}\left(2,\frac{dx}{c}+1\right)}{d}+\frac{\log\left(\frac{a}{x}+b\right)\log(c+dx)}{d}-\frac{\log(c+dx)\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{d}+\frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(a + b*x)/x]/(c + d*x),x]
```

```
[Out] (Log[b + a/x]*Log[c + d*x])/d + (Log[-((d*x)/c)]*Log[c + d*x])/d - (Log[-((
d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/d - PolyLog[2, (b*(c + d*x))/(b*c
- a*d)]/d + PolyLog[2, 1 + (d*x)/c]/d
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2515

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*(u_)^(r_.), x_Symbol] :> In
t[ExpandToSum[u, x]^r*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a,
 b, c, p, q, r}, x] && LinearQ[u, x] && BinomialQ[v, x] &&  !(LinearMatchQ[
u, x] && BinomialMatchQ[v, x])
```

Rubi steps

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{c+dx}\,dx = \int \frac{\log\left(b+\frac{a}{x}\right)}{c+dx}\,dx$$
$$= \frac{\log\left(b+\frac{a}{x}\right)\log(c+dx)}{d} + \frac{a\int \frac{\log(c+dx)}{\left(b+\frac{a}{x}\right)x^2}\,dx}{d}$$
$$= \frac{\log\left(b+\frac{a}{x}\right)\log(c+dx)}{d} + \frac{a\int \left(\frac{\log(c+dx)}{ax} - \frac{b\log(c+dx)}{a(a+bx)}\right)dx}{d}$$
$$= \frac{\log\left(b+\frac{a}{x}\right)\log(c+dx)}{d} + \frac{\int \frac{\log(c+dx)}{x}\,dx}{d} - \frac{b\int \frac{\log(c+dx)}{a+bx}\,dx}{d}$$
$$= \frac{\log\left(b+\frac{a}{x}\right)\log(c+dx)}{d} + \frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} - \frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{d} - \int \frac{\log}{c+}$$
$$= \frac{\log\left(b+\frac{a}{x}\right)\log(c+dx)}{d} + \frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} - \frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{d} + \frac{\mathrm{Li}_2(1-}{d}$$
$$= \frac{\log\left(b+\frac{a}{x}\right)\log(c+dx)}{d} + \frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} - \frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{d} - \frac{\mathrm{Li}_2\left(\frac{b(}{b}\right.}{d}$$

Mathematica [A]
time = 0.02, size = 80, normalized size = 0.76

$$\frac{\left(\log\left(b+\frac{a}{x}\right)+\log\left(-\frac{dx}{c}\right)-\log\left(\frac{d(a+bx)}{-bc+ad}\right)\right)\log(c+dx)-\mathrm{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)+\mathrm{Li}_2\left(1+\frac{dx}{c}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(a + b*x)/x]/(c + d*x),x]
```

```
[Out] ((Log[b + a/x] + Log[-((d*x)/c)] - Log[(d*(a + b*x))/(-(b*c) + a*d)])*Log[c
 + d*x] - PolyLog[2, (b*(c + d*x))/(b*c - a*d)] + PolyLog[2, 1 + (d*x)/c])/
d
```

Maple [A]
time = 2.33, size = 126, normalized size = 1.20

method	result	size
risch	$\frac{\mathrm{dilog}\left(\frac{ad-cb+c\left(b+\frac{a}{x}\right)}{ad-cb}\right)}{d}+\frac{\ln\left(b+\frac{a}{x}\right)\ln\left(\frac{ad-cb+c\left(b+\frac{a}{x}\right)}{ad-cb}\right)}{d}-\frac{\ln\left(b+\frac{a}{x}\right)\ln\left(-\frac{a}{bx}\right)}{d}-\frac{\mathrm{dilog}\left(-\frac{a}{bx}\right)}{d}$	114

derivativedivides	$-a\left(-\frac{\left(\frac{\operatorname{dilog}\left(\frac{ad-cb+c\left(b+\frac{a}{x}\right)}{ad-cb}\right)}{c}+\frac{\ln\left(b+\frac{a}{x}\right)\ln\left(\frac{ad-cb+c\left(b+\frac{a}{x}\right)}{ad-cb}\right)}{c}\right)c}{da}+\frac{\operatorname{dilog}\left(-\frac{a}{bx}\right)+\ln\left(b+\frac{a}{x}\right)\ln\left(-\frac{a}{bx}\right)}{da}\right)$	126
default	$-a\left(-\frac{\left(\frac{\operatorname{dilog}\left(\frac{ad-cb+c\left(b+\frac{a}{x}\right)}{ad-cb}\right)}{c}+\frac{\ln\left(b+\frac{a}{x}\right)\ln\left(\frac{ad-cb+c\left(b+\frac{a}{x}\right)}{ad-cb}\right)}{c}\right)c}{da}+\frac{\operatorname{dilog}\left(-\frac{a}{bx}\right)+\ln\left(b+\frac{a}{x}\right)\ln\left(-\frac{a}{bx}\right)}{da}\right)$	126

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((b*x+a)/x)/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -a*(-(dilog((a*d-c*b+c*(b+a/x))/(a*d-b*c))/c+ln(b+a/x)*ln((a*d-c*b+c*(b+a/x
))/(a*d-b*c))/c)*c/d/a+(dilog(-a/b/x)+ln(b+a/x)*ln(-a/b/x))/d/a)
```

Maxima [A]
time = 0.34, size = 124, normalized size = 1.18

$$-\frac{(\log(bx+a)-\log(x))\log(dx+c)}{d}+\frac{\log(dx+c)\log\left(\frac{bx+a}{x}\right)}{d}-\frac{\log\left(\frac{dx}{c}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{dx}{c}\right)}{d}+\frac{\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x+a)/x)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -(log(b*x + a) - log(x))*log(d*x + c)/d + log(d*x + c)*log((b*x + a)/x)/d -
 (log(d*x/c + 1)*log(x) + dilog(-d*x/c))/d + (log(b*x + a)*log((b*d*x + a*d
)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x+a)/x)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(log((b*x + a)/x)/(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left(\frac{a}{x}+b\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((b*x+a)/x)/(d*x+c),x)
```

```
[Out] Integral(log(a/x + b)/(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x+a)/x)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(log((b*x + a)/x)/(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(\frac{a+b\,x}{x}\right)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((a + b*x)/x)/(c + d*x),x)
```

```
[Out] int(log((a + b*x)/x)/(c + d*x), x)
```

3.397 $\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{c+dx}\,dx$

Optimal. Leaf size=227

$$\frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d}+\frac{2\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d}-\frac{\log\left(\frac{d\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,c+\sqrt{-a}\,d}\right)\log(c+dx)}{d}-\frac{\log\left(-\frac{d\left(\sqrt{-a}+\sqrt{\cdots}\right.}{\sqrt{b}\,c-\sqrt{-\cdots}}\right.}{d}$$

```
[Out] ln(b+a/x^2)*ln(d*x+c)/d+2*ln(-d*x/c)*ln(d*x+c)/d-ln(d*x+c)*ln(d*((-a)^(1/2)
-x*b^(1/2))/(d*(-a)^(1/2)+c*b^(1/2)))/d-ln(d*x+c)*ln(-d*((-a)^(1/2)+x*b^(1/
2))/(-d*(-a)^(1/2)+c*b^(1/2)))/d+2*polylog(2,1+d*x/c)/d-polylog(2,(d*x+c)*b
^(1/2)/(-d*(-a)^(1/2)+c*b^(1/2)))/d-polylog(2,(d*x+c)*b^(1/2)/(d*(-a)^(1/2)
+c*b^(1/2)))/d
```

Rubi [A]
time = 0.27, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2515, 2512, 266, 2463, 2441, 2352, 2440, 2438}

$$-\frac{\text{PolyLog}\left(2,\frac{\sqrt{b}(c+dx)}{\sqrt{b}c-\sqrt{-a}d}\right)}{d}-\frac{\text{PolyLog}\left(2,\frac{\sqrt{b}(c+dx)}{\sqrt{-a}d+\sqrt{b}c}\right)}{d}+\frac{2\text{PolyLog}\left(2,\frac{dx}{c}+1\right)}{d}+\frac{\log\left(\frac{a}{x^2}+b\right)\log(c+dx)}{d}-\frac{\log(c+dx)\log\left(\frac{d\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{-a}d+\sqrt{b}c}\right)}{d}-\frac{\log(c+dx)\log\left(-\frac{d\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}c-\sqrt{-a}d}\right)}{d}+\frac{2\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(a + b*x^2)/x^2]/(c + d*x),x]
```

```
[Out] (Log[b + a/x^2]*Log[c + d*x])/d + (2*Log[-((d*x)/c)]*Log[c + d*x])/d - (Log
[(d*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*c + Sqrt[-a]*d)]*Log[c + d*x])/d - (Lo
g[-((d*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*c - Sqrt[-a]*d))]*Log[c + d*x])/d -
 PolyLog[2, (Sqrt[b]*(c + d*x))/(Sqrt[b]*c - Sqrt[-a]*d)]/d - PolyLog[2, (S
qrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[-a]*d)]/d + (2*PolyLog[2, 1 + (d*x)/c])
/d
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2515

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*(u_)^(r_.), x_Symbol] :> In
t[ExpandToSum[u, x]^r*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a,
 b, c, p, q, r}, x] && LinearQ[u, x] && BinomialQ[v, x] &&  !(LinearMatchQ[
u, x] && BinomialMatchQ[v, x])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{c+dx}\,dx &= \int \frac{\log\left(b+\frac{a}{x^2}\right)}{c+dx}\,dx \\
&= \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d} + \frac{(2a)\int \frac{\log(c+dx)}{\left(b+\frac{a}{x^2}\right)x^3}\,dx}{d} \\
&= \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d} + \frac{(2a)\int \left(\frac{\log(c+dx)}{ax} - \frac{bx\log(c+dx)}{a(a+bx^2)}\right)dx}{d} \\
&= \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d} + \frac{2\int \frac{\log(c+dx)}{x}\,dx}{d} - \frac{(2b)\int \frac{x\log(c+dx)}{a+bx^2}\,dx}{d} \\
&= \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d} + \frac{2\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} - 2\int \frac{\log\left(-\frac{dx}{c}\right)}{c+dx}\,dx - \frac{(2b)\int \left(-\frac{\cdots}{2\sqrt{\cdots}}\cdots\right.}{\cdots} \\
&= \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d} + \frac{2\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} + \frac{2\mathrm{Li}_2\left(1+\frac{dx}{c}\right)}{d} + \frac{\sqrt{b}\int \frac{\log(c+dx)}{\sqrt{-a}-\sqrt{b}\cdots}\cdots}{d} \\
&= \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d} + \frac{2\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} - \frac{\log\left(\frac{d\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,c+\sqrt{-a}\,d}\right)\log(c+dx\cdots}{d} \\
&= \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d} + \frac{2\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} - \frac{\log\left(\frac{d\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,c+\sqrt{-a}\,d}\right)\log(c+dx\cdots}{d} \\
&= \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d} + \frac{2\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} - \frac{\log\left(\frac{d\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,c+\sqrt{-a}\,d}\right)\log(c+dx\cdots}{d}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 228, normalized size = 1.00

$$
\frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d} + \frac{2\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} - \frac{\log\left(\frac{d\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,c+\sqrt{-a}\,d}\right)\log(c+dx)}{d} - \frac{\log\left(-\frac{d\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,c-\sqrt{-a}\,d}\right)\log(c+dx)}{d} + \frac{2\mathrm{Li}_2\left(\frac{c+dx}{c}\right)}{d} - \frac{\mathrm{Li}_2\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt{b}\,c-\sqrt{-a}\,d}\right)}{d} - \frac{\mathrm{Li}_2\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt{b}\,c+\sqrt{-a}\,d}\right)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(a + b*x^2)/x^2]/(c + d*x),x]
```

```
[Out] (Log[b + a/x^2]*Log[c + d*x])/d + (2*Log[-((d*x)/c)]*Log[c + d*x])/d - (Log
[(d*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*c + Sqrt[-a]*d)]*Log[c + d*x])/d - (Lo
```

```
g[-((d*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*c - Sqrt[-a]*d))]*Log[c + d*x])/d +
 (2*PolyLog[2, (c + d*x)/c])/d - PolyLog[2, (Sqrt[b]*(c + d*x))/(Sqrt[b]*c
- Sqrt[-a]*d)]/d - PolyLog[2, (Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[-a]*d)]
/d
```

Maple [A]
time = 0.88, size = 330, normalized size = 1.45

method	result
derivativedivides	$\frac{\frac{\ln\left(\frac{c}{x}+d\right)\ln\left(b+\frac{a}{x^2}\right)}{c}-\frac{2a\left(\frac{\ln\left(\frac{c}{x}+d\right)\left(\ln\left(\frac{c\sqrt{-ba}+ad-\left(\frac{c}{x}+d\right)a}{c\sqrt{-ba}+ad}\right)+\ln\left(\frac{c\sqrt{-ba}-ad+\left(\frac{c}{x}+d\right)a}{c\sqrt{-ba}-ad}\right)\right)}{2a}+\frac{\operatorname{dilog}\left(\frac{c\sqrt{-ba}}{c\sqrt{-}}\right.}{}\right.}{c}}{d}$
default	$\frac{\frac{\ln\left(\frac{c}{x}+d\right)\ln\left(b+\frac{a}{x^2}\right)}{c}-\frac{2a\left(\frac{\ln\left(\frac{c}{x}+d\right)\left(\ln\left(\frac{c\sqrt{-ba}+ad-\left(\frac{c}{x}+d\right)a}{c\sqrt{-ba}+ad}\right)+\ln\left(\frac{c\sqrt{-ba}-ad+\left(\frac{c}{x}+d\right)a}{c\sqrt{-ba}-ad}\right)\right)}{2a}+\frac{\operatorname{dilog}\left(\frac{c\sqrt{-ba}}{c\sqrt{-}}\right.}{}\right.}{c}}{d}$
risch	$\frac{\ln\left(\frac{c}{x}+d\right)\ln\left(b+\frac{a}{x^2}\right)}{d}-\frac{\ln\left(\frac{c}{x}+d\right)\ln\left(\frac{c\sqrt{-ba}+ad-\left(\frac{c}{x}+d\right)a}{c\sqrt{-ba}+ad}\right)}{d}-\frac{\ln\left(\frac{c}{x}+d\right)\ln\left(\frac{c\sqrt{-ba}-ad+\left(\frac{c}{x}+d\right)a}{c\sqrt{-ba}-ad}\right)}{d}-\frac{\operatorname{dilog}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((b*x^2+a)/x^2)/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] (ln(c/x+d)/c*ln(b+a/x^2)-2/c*a*(1/2*ln(c/x+d)*(ln((c*(-b*a)^(1/2)+a*d-(c/x+
d)*a)/(c*(-b*a)^(1/2)+a*d))+ln((c*(-b*a)^(1/2)-a*d+(c/x+d)*a)/(c*(-b*a)^(1/
2)-a*d)))/a+1/2*(dilog((c*(-b*a)^(1/2)+a*d-(c/x+d)*a)/(c*(-b*a)^(1/2)+a*d))
+dilog((c*(-b*a)^(1/2)-a*d+(c/x+d)*a)/(c*(-b*a)^(1/2)-a*d)))/a))/d*c-(ln(1/
x)*ln(b+a/x^2)-2*a*(1/2*ln(1/x)*(ln((-a/x+(-b*a)^(1/2))/(-b*a)^(1/2))+ln((a
/x+(-b*a)^(1/2))/(-b*a)^(1/2)))/a+1/2*(dilog((-a/x+(-b*a)^(1/2))/(-b*a)^(1/
2))+dilog((a/x+(-b*a)^(1/2))/(-b*a)^(1/2)))/a))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x^2+a)/x^2)/(d*x+c),x, algorithm="maxima")
[Out] integrate(log((b*x^2 + a)/x^2)/(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x^2+a)/x^2)/(d*x+c),x, algorithm="fricas")
[Out] integral(log((b*x^2 + a)/x^2)/(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{a}{x^2} + b\right)}{c + dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((b*x**2+a)/x**2)/(d*x+c),x)
[Out] Integral(log(a/x**2 + b)/(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((b*x^2+a)/x^2)/(d*x+c),x, algorithm="giac")
[Out] integrate(log((b*x^2 + a)/x^2)/(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(\frac{b\,x^2+a}{x^2}\right)}{c+d\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((a + b*x^2)/x^2)/(c + d*x),x)
[Out] int(log((a + b*x^2)/x^2)/(c + d*x), x)
```

3.398 $\int \frac{\log(x^{-n}(a+bx^n))}{c+dx} dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{\log(b+ax^{-n})}{c+dx},x\right)$$

```
[Out] Unintegrable(ln(b+a/(x^n))/(d*x+c),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log(x^{-n}(a+bx^n))}{c+dx} dx$$

Verification is not applicable to the result.

```
[In] Int[Log[(a + b*x^n)/x^n]/(c + d*x),x]
[Out] Defer[Int][Log[b + a/x^n]/(c + d*x), x]
```

Rubi steps

$$\int \frac{\log(x^{-n}(a+bx^n))}{c+dx} dx = \int \frac{\log(b+ax^{-n})}{c+dx} dx$$

Mathematica [A]
time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\log(x^{-n}(a+bx^n))}{c+dx} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[(a + b*x^n)/x^n]/(c + d*x),x]
[Out] Integrate[Log[(a + b*x^n)/x^n]/(c + d*x), x]
```

Maple [A]
time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\ln((a+b\,x^n)\,x^{-n})}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((a+b*x^n)/(x^n))/(d*x+c),x)
[Out] int(ln((a+b*x^n)/(x^n))/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a+b*x^n)/(x^n))/(d*x+c),x, algorithm="maxima")
[Out] integrate(log((b*x^n + a)/x^n)/(d*x + c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a+b*x^n)/(x^n))/(d*x+c),x, algorithm="fricas")
[Out] integral(log((b*x^n + a)/x^n)/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(ax^{-n}+b\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((a+b*x**n)/(x**n))/(d*x+c),x)
[Out] Integral(log(a/x**n + b)/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a+b*x^n)/(x^n))/(d*x+c),x, algorithm="giac")
[Out] integrate(log((b*x^n + a)/x^n)/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\ln\left(\frac{a+b\,x^n}{x^n}\right)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((a + b*x^n)/x^n)/(c + d*x),x)
[Out] int(log((a + b*x^n)/x^n)/(c + d*x), x)
```

3.399 $\int (fx)^q (a+b\log (c(d+ex^m)^n))\, dx$

Optimal. Leaf size=92

$$-\frac{bemnx^{1+m}(fx)^q\,{}_2F_1\left(1,\frac{1+m+q}{m};\frac{1+2m+q}{m};-\frac{ex^m}{d}\right)}{d(1+q)(1+m+q)}+\frac{(fx)^{1+q}\,(a+b\log (c(d+ex^m)^n))}{f(1+q)}$$

```
[Out] -b*e*m*n*x^(1+m)*(f*x)^q*hypergeom([1, (1+m+q)/m],[(1+2*m+q)/m],-e*x^m/d)/d
/(1+q)/(1+m+q)+(f*x)^(1+q)*(a+b*ln(c*(d+e*x^m)^n))/f/(1+q)
```

Rubi [A]
time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2505, 20, 371}

$$\frac{(fx)^{q+1}\,(a+b\log (c(d+ex^m)^n))}{f(q+1)}-\frac{bemnx^{m+1}(fx)^q\,{}_2F_1\left(1,\frac{m+q+1}{m};\frac{2m+q+1}{m};-\frac{ex^m}{d}\right)}{d(q+1)(m+q+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^q*(a + b*Log[c*(d + e*x^m)^n]),x]
```

```
[Out] -((b*e*m*n*x^(1 + m)*(f*x)^q*Hypergeometric2F1[1, (1 + m + q)/m, (1 + 2*m +
 q)/m, -((e*x^m)/d)])/(d*(1 + q)*(1 + m + q))) + ((f*x)^(1 + q)*(a + b*Log[
c*(d + e*x^m)^n]))/(f*(1 + q))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (fx)^q (a+b\log(c(d+ex^m)^n))\,dx = \frac{(fx)^{1+q}(a+b\log(c(d+ex^m)^n))}{f(1+q)} - \frac{(bemn)\int \frac{x^{-1+m}(fx)^{1+q}}{d+ex^m}\,dx}{f(1+q)}$$
$$= \frac{(fx)^{1+q}(a+b\log(c(d+ex^m)^n))}{f(1+q)} - \frac{(bemnx^{-q}(fx)^q)\int \frac{x^{m+q}}{d+ex^m}\,dx}{1+q}$$
$$= -\frac{bemnx^{1+m}(fx)^q\,{}_2F_1\left(1,\frac{1+m+q}{m};\frac{1+2m+q}{m};-\frac{ex^m}{d}\right)}{d(1+q)(1+m+q)} + \frac{(fx)^{1+q}(a+b\log}{f(1}$$

Mathematica [A]
time = 0.04, size = 82, normalized size = 0.89

$$\frac{x(fx)^q\left(-bemnx^m\,{}_2F_1\left(1,\frac{1+m+q}{m};\frac{1+2m+q}{m};-\frac{ex^m}{d}\right)+d(1+m+q)(a+b\log(c(d+ex^m)^n))\right)}{d(1+q)(1+m+q)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^q*(a + b*Log[c*(d + e*x^m)^n]),x]
```

```
[Out] (x*(f*x)^q*(-(b*e*m*n*x^m*Hypergeometric2F1[1, (1 + m + q)/m, (1 + 2*m + q)
/m, -((e*x^m)/d)]) + d*(1 + m + q)*(a + b*Log[c*(d + e*x^m)^n])))/(d*(1 + q
)*(1 + m + q))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (fx)^q (a+b\ln(c(d+e\,x^m)^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^q*(a+b*ln(c*(d+e*x^m)^n)),x)
```

```
[Out] int((f*x)^q*(a+b*ln(c*(d+e*x^m)^n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^q*(a+b*log(c*(d+e*x^m)^n)),x, algorithm="maxima")
```

```
[Out] (d^2*f^q*m^2*n*integrate(x^q/((m*(q + 1) - q^2 - 2*q - 1)*d^2 + 2*(m*(q + 1
) - q^2 - 2*q - 1)*d*e^(m*log(x) + 1) + (m*(q + 1) - q^2 - 2*q - 1)*e^(2*m*
```

```
log(x) + 2)), x) - (((m*(q + 1) - q^2 - 2*q - 1)*d*f^q*x + (m*(q + 1) - q^2
 - 2*q - 1)*f^q*x*e^(m*log(x) + 1))*x^q*log((d + e^(m*log(x) + 1))^n) + (((
m*(q + 1) - q^2 - 2*q - 1)*f^q*log(c) - (m^2*n - m*n*(q + 1))*f^q)*x*e^(m*l
og(x) + 1) - (d*f^q*m^2*n - (m*(q + 1) - q^2 - 2*q - 1)*d*f^q*log(c))*x)*x^
q)/((q^3 - (q^2 + 2*q + 1)*m + 3*q^2 + 3*q + 1)*d + (q^3 - (q^2 + 2*q + 1)*
m + 3*q^2 + 3*q + 1)*e^(m*log(x) + 1)))*b + (f*x)^(q + 1)*a/(f*(q + 1))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^q*(a+b*log(c*(d+e*x^m)^n)),x, algorithm="fricas")
[Out] integral((f*x)^q*b*log((x^m*e + d)^n*c) + (f*x)^q*a, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^q (a + b\log (c(d + ex^m)^n))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**q*(a+b*ln(c*(d+e*x**m)**n)),x)
[Out] Integral((f*x)**q*(a + b*log(c*(d + e*x**m)**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^q*(a+b*log(c*(d+e*x^m)^n)),x, algorithm="giac")
[Out] integrate((b*log((x^m*e + d)^n*c) + a)*(f*x)^q, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (f\,x)^q\,(a + b\,\ln (c\,(d + e\,x^m)^n))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^q*(a + b*log(c*(d + e*x^m)^n)),x)
[Out] int((f*x)^q*(a + b*log(c*(d + e*x^m)^n)), x)
```

3.400 $\int x^3 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx$

Optimal. Leaf size=166

$$\frac{bd^7n\sqrt{x}}{4e^7}-\frac{bd^6nx}{8e^6}+\frac{bd^5nx^{3/2}}{12e^5}-\frac{bd^4nx^2}{16e^4}+\frac{bd^3nx^{5/2}}{20e^3}-\frac{bd^2nx^3}{24e^2}+\frac{bdnx^{7/2}}{28e}-\frac{1}{32}bnx^4-\frac{bd^8n\log\left(d+e\sqrt{x}\right)}{4e^8}+\frac{1}{4}x^4\left(a\right.$$

```
[Out] -1/8*b*d^6*n*x/e^6+1/12*b*d^5*n*x^(3/2)/e^5-1/16*b*d^4*n*x^2/e^4+1/20*b*d^3
*n*x^(5/2)/e^3-1/24*b*d^2*n*x^3/e^2+1/28*b*d*n*x^(7/2)/e-1/32*b*n*x^4-1/4*b
*d^8*n*ln(d+e*x^(1/2))/e^8+1/4*x^4*(a+b*ln(c*(d+e*x^(1/2))^n))+1/4*b*d^7*n*
x^(1/2)/e^7
```

Rubi [A]
time = 0.09, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2504, 2442, 45}

$$\frac{1}{4}x^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)-\frac{bd^8n\log\left(d+e\sqrt{x}\right)}{4e^8}+\frac{bd^7n\sqrt{x}}{4e^7}-\frac{bd^6nx}{8e^6}+\frac{bd^5nx^{3/2}}{12e^5}-\frac{bd^4nx^2}{16e^4}+\frac{bd^3nx^{5/2}}{20e^3}-\frac{bd^2nx^3}{24e^2}+\frac{bdnx^{7/2}}{28e}-\frac{1}{32}bnx^4$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*Sqrt[x])^n]),x]
```

```
[Out] (b*d^7*n*Sqrt[x])/(4*e^7) - (b*d^6*n*x)/(8*e^6) + (b*d^5*n*x^(3/2))/(12*e^5
) - (b*d^4*n*x^2)/(16*e^4) + (b*d^3*n*x^(5/2))/(20*e^3) - (b*d^2*n*x^3)/(24
*e^2) + (b*d*n*x^(7/2))/(28*e) - (b*n*x^4)/32 - (b*d^8*n*Log[d + e*Sqrt[x]]
)/(4*e^8) + (x^4*(a + b*Log[c*(d + e*Sqrt[x])^n]))/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx = 2\text{Subst}\left(\int x^7(a+b\log\left(c(d+ex)^n\right))\,dx, x, \sqrt{x}\right)$$

$$= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right) - \frac{1}{4}(ben)\text{Subst}\left(\int \frac{x^8}{d+ex}\,dx, x, \sqrt{x}\right)$$

$$= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right) - \frac{1}{4}(ben)\text{Subst}\left(\int \left(-\frac{d^7}{e^8}+\frac{d^6x}{e^7}-\frac{d^5}{e}\right.\right.$$

$$= \frac{bd^7n\sqrt{x}}{4e^7} - \frac{bd^6nx}{8e^6} + \frac{bd^5nx^{3/2}}{12e^5} - \frac{bd^4nx^2}{16e^4} + \frac{bd^3nx^{5/2}}{20e^3} - \frac{bd^2nx^3}{24e^2} + \frac{bdn}{2}$$

Mathematica [A]
time = 0.10, size = 159, normalized size = 0.96

$$\frac{ax^4}{4} - \frac{1}{4}ben\left(-\frac{d^7\sqrt{x}}{e^8} + \frac{d^6x}{2e^7} - \frac{d^5x^{3/2}}{3e^6} + \frac{d^4x^2}{4e^5} - \frac{d^3x^{5/2}}{5e^4} + \frac{d^2x^3}{6e^3} - \frac{dx^{7/2}}{7e^2} + \frac{x^4}{8e} + \frac{d^8\log\left(d+e\sqrt{x}\right)}{e^9}\right) + \frac{1}{4}bx^4\log\left(c\left(d+e\sqrt{x}\right)^n\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*Sqrt[x])^n]),x]
[Out] (a*x^4)/4 - (b*e*n*(-((d^7*Sqrt[x])/e^8) + (d^6*x)/(2*e^7) - (d^5*x^(3/2))/
(3*e^6) + (d^4*x^2)/(4*e^5) - (d^3*x^(5/2))/(5*e^4) + (d^2*x^3)/(6*e^3) - (
d*x^(7/2))/(7*e^2) + x^4/(8*e) + (d^8*Log[d + e*Sqrt[x]])/e^9))/4 + (b*x^4*
Log[c*(d + e*Sqrt[x])^n])/4
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int x^3\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(1/2))^n)),x)
[Out] int(x^3*(a+b*ln(c*(d+e*x^(1/2))^n)),x)
```

Maxima [A]
time = 0.27, size = 124, normalized size = 0.75

$$\frac{1}{4}bx^4\log\left(\left(\sqrt{x}e+d\right)^nc\right) + \frac{1}{4}ax^4 - \frac{1}{3360}\left(840\,d^8e^{(-9)}\log\left(\sqrt{x}e+d\right) + \left(420\,d^6xe - 840\,d^7\sqrt{x} - 280\,d^5x^{\frac{3}{2}}e^2 + 210\,d^4x^2e^3 - 168\,d^3x^{\frac{5}{2}}e^4 + 140\,d^2x^3e^5 - 120\,dx^{\frac{7}{2}}e^6 + 105\,x^4e^7\right)e^{(-8)}\right)bne$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4*log((sqrt(x)*e + d)^n*c) + 1/4*a*x^4 - 1/3360*(840*d^8*e^(-9)*log
(sqrt(x)*e + d) + (420*d^6*x*e - 840*d^7*sqrt(x) - 280*d^5*x^(3/2)*e^2 + 21
0*d^4*x^2*e^3 - 168*d^3*x^(5/2)*e^4 + 140*d^2*x^3*e^5 - 120*d*x^(7/2)*e^6 +
 105*x^4*e^7)*e^(-8))*b*n*e
```

Fricas [A]
time = 0.46, size = 137, normalized size = 0.83

$$-\frac{1}{3360}\left(420\,bd^6nxe^2+210\,bd^4nx^2e^4+140\,bd^2nx^3e^6-840\,bx^4e^8\log\left(c\right)+105\,(bn-8\,a)x^4e^8+840\,\left(bd^8n-bnx^4e^8\right)\log\left(\sqrt{x}\,e+d\right)-8\,\left(105\,bd^7ne+35\,bd^5nxe^3+21\,bd^3nx^2e^5+15\,bdnx^3e^7\right)\sqrt{x}\right)e^{(-8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="fricas")
```

```
[Out] -1/3360*(420*b*d^6*n*x*e^2 + 210*b*d^4*n*x^2*e^4 + 140*b*d^2*n*x^3*e^6 - 84
0*b*x^4*e^8*log(c) + 105*(b*n - 8*a)*x^4*e^8 + 840*(b*d^8*n - b*n*x^4*e^8)*
log(sqrt(x)*e + d) - 8*(105*b*d^7*n*e + 35*b*d^5*n*x*e^3 + 21*b*d^3*n*x^2*e
^5 + 15*b*d*n*x^3*e^7)*sqrt(x))*e^(-8)
```

Sympy [A]
time = 9.72, size = 155, normalized size = 0.93

$$\frac{ax^4}{4}+b\left(-\frac{en\left(\frac{2d^8\left(\begin{cases}\frac{\sqrt{x}}{d} & \text{for}\ e=0\\ \frac{\log\left(d+e\sqrt{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^8}-\frac{2d^7\sqrt{x}}{e^8}+\frac{d^6x}{e^7}-\frac{2d^5x^{\frac{3}{2}}}{3e^6}+\frac{d^4x^2}{2e^5}-\frac{2d^3x^{\frac{5}{2}}}{5e^4}+\frac{d^2x^3}{3e^3}-\frac{2dx^{\frac{7}{2}}}{7e^2}+\frac{x^4}{4e}\right)}{8}+\frac{x^4\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(1/2))**n)),x)
```

```
[Out] a*x**4/4 + b*(-e*n*(2*d**8*Piecewise((sqrt(x)/d, Eq(e, 0)), (log(d + e*sqrt
(x))/e, True))/e**8 - 2*d**7*sqrt(x)/e**8 + d**6*x/e**7 - 2*d**5*x**(3/2)/(
3*e**6) + d**4*x**2/(2*e**5) - 2*d**3*x**(5/2)/(5*e**4) + d**2*x**3/(3*e**3
) - 2*d*x**(7/2)/(7*e**2) + x**4/(4*e))/8 + x**4*log(c*(d + e*sqrt(x))**n)/
4)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 357 vs. 2(128) = 256.

time = 2.71, size = 357, normalized size = 2.15

$$\frac{1}{3360}\left(840\,bx^4e\log\left(c\right)+840\,ax^4e+\left(840\left(\sqrt{x}e+d\right)^8e^{(-7)}\log\left(\sqrt{x}e+d\right)-6720\left(\sqrt{x}e+d\right)^7de^{(-7)}\log\left(\sqrt{x}e+d\right)+23520\left(\sqrt{x}e+d\right)^6d^2e^{(-7)}\log\left(\sqrt{x}e+d\right)-47040\left(\sqrt{x}e+d\right)^5d^3e^{(-7)}\log\left(\sqrt{x}e+d\right)+58800\left(\sqrt{x}e+d\right)^4d^4e^{(-7)}\log\left(\sqrt{x}e+d\right)-47040\left(\sqrt{x}e+d\right)^3d^5e^{(-7)}\log\left(\sqrt{x}e+d\right)+23520\left(\sqrt{x}e+d\right)^2d^6e^{(-7)}\log\left(\sqrt{x}e+d\right)-6720\left(\sqrt{x}e+d\right)d^7e^{(-7)}\log\left(\sqrt{x}e+d\right)-105\left(\sqrt{x}e+d\right)^8e^{(-7)}+960\left(\sqrt{x}e+d\right)^7de^{(-7)}-3920\left(\sqrt{x}e+d\right)^6d^2e^{(-7)}+9408\left(\sqrt{x}e+d\right)^5d^3e^{(-7)}-14700\left(\sqrt{x}e+d\right)^4d^4e^{(-7)}+15680\left(\sqrt{x}e+d\right)^3d^5e^{(-7)}-11760\left(\sqrt{x}e+d\right)^2d^6e^{(-7)}+6720\left(\sqrt{x}e+d\right)d^7e^{(-7)}\right)bn\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="giac")
```

```
[Out] 1/3360*(840*b*x^4*e*log(c) + 840*a*x^4*e + (840*(sqrt(x)*e + d)^8*e^(-7)*lo
g(sqrt(x)*e + d) - 6720*(sqrt(x)*e + d)^7*d*e^(-7)*log(sqrt(x)*e + d) + 235
20*(sqrt(x)*e + d)^6*d^2*e^(-7)*log(sqrt(x)*e + d) - 47040*(sqrt(x)*e + d)^
5*d^3*e^(-7)*log(sqrt(x)*e + d) + 58800*(sqrt(x)*e + d)^4*d^4*e^(-7)*log(sq
rt(x)*e + d) - 47040*(sqrt(x)*e + d)^3*d^5*e^(-7)*log(sqrt(x)*e + d) + 2352
0*(sqrt(x)*e + d)^2*d^6*e^(-7)*log(sqrt(x)*e + d) - 6720*(sqrt(x)*e + d)*d^
7*e^(-7)*log(sqrt(x)*e + d) - 105*(sqrt(x)*e + d)^8*e^(-7) + 960*(sqrt(x)*e
 + d)^7*d*e^(-7) - 3920*(sqrt(x)*e + d)^6*d^2*e^(-7) + 9408*(sqrt(x)*e + d)
^5*d^3*e^(-7) - 14700*(sqrt(x)*e + d)^4*d^4*e^(-7) + 15680*(sqrt(x)*e + d)^
3*d^5*e^(-7) - 11760*(sqrt(x)*e + d)^2*d^6*e^(-7) + 6720*(sqrt(x)*e + d)*d^
7*e^(-7))*b*n)*e^(-1)
```

Mupad [B]
time = 0.51, size = 137, normalized size = 0.83

$$\frac{a\,x^4}{4}-\frac{b\,n\,x^4}{32}+\frac{b\,x^4\,\ln\left(c\,{\left(d+e\,\sqrt{x}\,\right)}^n\right)}{4}+\frac{b\,d\,n\,x^{7/2}}{28\,e}-\frac{b\,d^6\,n\,x}{8\,e^6}-\frac{b\,d^8\,n\,\ln\left(d+e\,\sqrt{x}\,\right)}{4\,e^8}-\frac{b\,d^2\,n\,x^3}{24\,e^2}-\frac{b\,d^4\,n\,x^2}{16\,e^4}+\frac{b\,d^3\,n\,x^{5/2}}{20\,e^3}+\frac{b\,d^5\,n\,x^{3/2}}{12\,e^5}+\frac{b\,d^7\,n\,\sqrt{x}}{4\,e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e*x^(1/2))^n)),x)
```

```
[Out] (a*x^4)/4 - (b*n*x^4)/32 + (b*x^4*log(c*(d + e*x^(1/2))^n))/4 + (b*d*n*x^(7
/2))/(28*e) - (b*d^6*n*x)/(8*e^6) - (b*d^8*n*log(d + e*x^(1/2)))/(4*e^8) -
(b*d^2*n*x^3)/(24*e^2) - (b*d^4*n*x^2)/(16*e^4) + (b*d^3*n*x^(5/2))/(20*e^3
) + (b*d^5*n*x^(3/2))/(12*e^5) + (b*d^7*n*x^(1/2))/(4*e^7)
```

3.401 $\int x^2 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx$

Optimal. Leaf size=134

$$\frac{bd^5n\sqrt{x}}{3e^5}-\frac{bd^4nx}{6e^4}+\frac{bd^3nx^{3/2}}{9e^3}-\frac{bd^2nx^2}{12e^2}+\frac{bdnx^{5/2}}{15e}-\frac{1}{18}bnx^3-\frac{bd^6n\log\left(d+e\sqrt{x}\right)}{3e^6}+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right.\right.\right.$$

```
[Out] -1/6*b*d^4*n*x/e^4+1/9*b*d^3*n*x^(3/2)/e^3-1/12*b*d^2*n*x^2/e^2+1/15*b*d*n*
x^(5/2)/e-1/18*b*n*x^3-1/3*b*d^6*n*ln(d+e*x^(1/2))/e^6+1/3*x^3*(a+b*ln(c*(d
+e*x^(1/2))^n))+1/3*b*d^5*n*x^(1/2)/e^5
```

Rubi [A]
time = 0.07, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2504, 2442, 45}

$$\frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)-\frac{bd^6n\log\left(d+e\sqrt{x}\right)}{3e^6}+\frac{bd^5n\sqrt{x}}{3e^5}-\frac{bd^4nx}{6e^4}+\frac{bd^3nx^{3/2}}{9e^3}-\frac{bd^2nx^2}{12e^2}+\frac{bdnx^{5/2}}{15e}-\frac{1}{18}bnx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*Sqrt[x])^n]),x]
```

```
[Out] (b*d^5*n*Sqrt[x])/(3*e^5) - (b*d^4*n*x)/(6*e^4) + (b*d^3*n*x^(3/2))/(9*e^3)
 - (b*d^2*n*x^2)/(12*e^2) + (b*d*n*x^(5/2))/(15*e) - (b*n*x^3)/18 - (b*d^6*
n*Log[d + e*Sqrt[x]])/(3*e^6) + (x^3*(a + b*Log[c*(d + e*Sqrt[x])^n]))/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
```

```
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx = 2\text{Subst}\left(\int x^5(a+b\log(c(d+ex)^n))\,dx, x, \sqrt{x}\right)$$
$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right) - \frac{1}{3}(ben)\text{Subst}\left(\int \frac{x^6}{d+ex}\,dx, x, \sqrt{x}\right.$$
$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right) - \frac{1}{3}(ben)\text{Subst}\left(\int\left(-\frac{d^5}{e^6}+\frac{d^4x}{e^5}-\frac{d^3}{e}\right.\right.$$
$$= \frac{bd^5n\sqrt{x}}{3e^5} - \frac{bd^4nx}{6e^4} + \frac{bd^3nx^{3/2}}{9e^3} - \frac{bd^2nx^2}{12e^2} + \frac{bdnx^{5/2}}{15e} - \frac{1}{18}bnx^3 - \frac{bd^6n}{}$$

Mathematica [A]
time = 0.07, size = 131, normalized size = 0.98

$$\frac{ax^3}{3} - \frac{1}{3}ben\left(-\frac{d^5\sqrt{x}}{e^6}+\frac{d^4x}{2e^5}-\frac{d^3x^{3/2}}{3e^4}+\frac{d^2x^2}{4e^3}-\frac{dx^{5/2}}{5e^2}+\frac{x^3}{6e}+\frac{d^6\log\left(d+e\sqrt{x}\right)}{e^7}\right)+\frac{1}{3}bx^3\log\left(c\left(d+e\sqrt{x}\right)^n\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*Sqrt[x])^n]),x]
[Out] (a*x^3)/3 - (b*e*n*(-((d^5*Sqrt[x])/e^6) + (d^4*x)/(2*e^5) - (d^3*x^(3/2))/
(3*e^4) + (d^2*x^2)/(4*e^3) - (d*x^(5/2))/(5*e^2) + x^3/(6*e) + (d^6*Log[d
+ e*Sqrt[x]])/e^7))/3 + (b*x^3*Log[c*(d + e*Sqrt[x])^n])/3
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(1/2))^n)),x)
[Out] int(x^2*(a+b*ln(c*(d+e*x^(1/2))^n)),x)
```

Maxima [A]
time = 0.28, size = 104, normalized size = 0.78

$$\frac{1}{3}bx^3\log\left(\left(\sqrt{x}\,e+d\right)^nc\right)+\frac{1}{3}ax^3-\frac{1}{180}\left(60\,d^6e^{(-7)}\log\left(\sqrt{x}\,e+d\right)+\left(30\,d^4xe-60\,d^5\sqrt{x}-20\,d^3x^{\frac{3}{2}}e^2+15\,d^2x^2e^3-12\,dx^{\frac{5}{2}}e^4+10\,x^3e^5\right)e^{(-6)}\right)bne$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="maxima")

[Out] 1/3*b*x^3*log((sqrt(x)*e + d)^n*c) + 1/3*a*x^3 - 1/180*(60*d^6*e^(-7)*log(sqrt(x)*e + d) + (30*d^4*x*e - 60*d^5*sqrt(x) - 20*d^3*x^(3/2)*e^2 + 15*d^2*x^2*e^3 - 12*d*x^(5/2)*e^4 + 10*x^3*e^5)*e^(-6))*b*n*e

Fricas [A]
time = 0.39, size = 113, normalized size = 0.84

$$-\frac{1}{180}\left(30\,bd^4nxe^2+15\,bd^2nx^2e^4-60\,bx^3e^6\log\left(c\right)+10\,(bn-6\,a)x^3e^6+60\,\left(bd^6n-bnx^3e^6\right)\log\left(\sqrt{x}\,e+d\right)-4\,\left(15\,bd^5ne+5\,bd^3nxe^3+3\,bdnx^2e^5\right)\sqrt{x}\,\right)e^{(-6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="fricas")

[Out] -1/180*(30*b*d^4*n*x*e^2 + 15*b*d^2*n*x^2*e^4 - 60*b*x^3*e^6*log(c) + 10*(b*n - 6*a)*x^3*e^6 + 60*(b*d^6*n - b*n*x^3*e^6)*log(sqrt(x)*e + d) - 4*(15*b*d^5*n*e + 5*b*d^3*n*x*e^3 + 3*b*d*n*x^2*e^5)*sqrt(x))*e^(-6)

Sympy [A]
time = 3.83, size = 128, normalized size = 0.96

$$\frac{ax^3}{3}+b\left(-\frac{en\left(\frac{2d^6\left(\begin{cases}\frac{\sqrt{x}}{d} & \text{for}\ e=0\\ \frac{\log\left(d+e\sqrt{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^6}-\frac{2d^5\sqrt{x}}{e^6}+\frac{d^4x}{e^5}-\frac{2d^3x^{\frac{3}{2}}}{3e^4}+\frac{d^2x^2}{2e^3}-\frac{2dx^{\frac{5}{2}}}{5e^2}+\frac{x^3}{3e}\right)}{6}+\frac{x^3\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/2))**n)),x)

[Out] a*x**3/3 + b*(-e*n*(2*d**6*Piecewise((sqrt(x)/d, Eq(e, 0)), (log(d + e*sqrt(x))/e, True))/e**6 - 2*d**5*sqrt(x)/e**6 + d**4*x/e**5 - 2*d**3*x**(3/2)/(3*e**4) + d**2*x**2/(2*e**3) - 2*d*x**(5/2)/(5*e**2) + x**3/(3*e))/6 + x**3*log(c*(d + e*sqrt(x))**n)/3)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(104) = 208.
time = 5.63, size = 271, normalized size = 2.02

$$\frac{1}{180}\left(60\,bx^3e\log\left(c\right)+60\,ax^3e+\left(60\left(\sqrt{x}\,e+d\right)^6e^{(-5)}\log\left(\sqrt{x}\,e+d\right)-360\left(\sqrt{x}\,e+d\right)^5de^{(-5)}\log\left(\sqrt{x}\,e+d\right)+900\left(\sqrt{x}\,e+d\right)^4d^2e^{(-5)}\log\left(\sqrt{x}\,e+d\right)-1200\left(\sqrt{x}\,e+d\right)^3d^3e^{(-5)}\log\left(\sqrt{x}\,e+d\right)+900\left(\sqrt{x}\,e+d\right)^2d^4e^{(-5)}\log\left(\sqrt{x}\,e+d\right)-360\left(\sqrt{x}\,e+d\right)d^5e^{(-5)}\log\left(\sqrt{x}\,e+d\right)-10\left(\sqrt{x}\,e+d\right)^6e^{(-5)}+72\left(\sqrt{x}\,e+d\right)^5de^{(-5)}-225\left(\sqrt{x}\,e+d\right)^4d^2e^{(-5)}+400\left(\sqrt{x}\,e+d\right)^3d^3e^{(-5)}-450\left(\sqrt{x}\,e+d\right)^2d^4e^{(-5)}+360\left(\sqrt{x}\,e+d\right)d^5e^{(-5)}\right)bn\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="giac")
```

```
[Out] 1/180*(60*b*x^3*e*log(c) + 60*a*x^3*e + (60*(sqrt(x)*e + d)^6*e^(-5)*log(sq
rt(x)*e + d) - 360*(sqrt(x)*e + d)^5*d*e^(-5)*log(sqrt(x)*e + d) + 900*(sqr
t(x)*e + d)^4*d^2*e^(-5)*log(sqrt(x)*e + d) - 1200*(sqrt(x)*e + d)^3*d^3*e^
(-5)*log(sqrt(x)*e + d) + 900*(sqrt(x)*e + d)^2*d^4*e^(-5)*log(sqrt(x)*e +
d) - 360*(sqrt(x)*e + d)*d^5*e^(-5)*log(sqrt(x)*e + d) - 10*(sqrt(x)*e + d)
^6*e^(-5) + 72*(sqrt(x)*e + d)^5*d*e^(-5) - 225*(sqrt(x)*e + d)^4*d^2*e^(-5
) + 400*(sqrt(x)*e + d)^3*d^3*e^(-5) - 450*(sqrt(x)*e + d)^2*d^4*e^(-5) + 3
60*(sqrt(x)*e + d)*d^5*e^(-5))*b*n)*e^(-1)
```

Mupad [B]
time = 0.42, size = 111, normalized size = 0.83

$$\frac{a\,x^3}{3}-\frac{b\,n\,x^3}{18}+\frac{b\,x^3\,\ln\left(c\,{\left(d+e\,\sqrt{x}\,\right)}^n\right)}{3}+\frac{b\,d\,n\,x^{5/2}}{15\,e}-\frac{b\,d^4\,n\,x}{6\,e^4}-\frac{b\,d^6\,n\,\ln\left(d+e\,\sqrt{x}\,\right)}{3\,e^6}-\frac{b\,d^2\,n\,x^2}{12\,e^2}+\frac{b\,d^3\,n\,x^{3/2}}{9\,e^3}+\frac{b\,d^5\,n\,\sqrt{x}}{3\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e*x^(1/2))^n)),x)
```

```
[Out] (a*x^3)/3 - (b*n*x^3)/18 + (b*x^3*log(c*(d + e*x^(1/2))^n))/3 + (b*d*n*x^(5
/2))/(15*e) - (b*d^4*n*x)/(6*e^4) - (b*d^6*n*log(d + e*x^(1/2)))/(3*e^6) -
(b*d^2*n*x^2)/(12*e^2) + (b*d^3*n*x^(3/2))/(9*e^3) + (b*d^5*n*x^(1/2))/(3*e
^5)
```

3.402 $\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx$

Optimal. Leaf size=102

$$\frac{bd^3n\sqrt{x}}{2e^3}-\frac{bd^2nx}{4e^2}+\frac{bdnx^{3/2}}{6e}-\frac{1}{8}bnx^2-\frac{bd^4n\log\left(d+e\sqrt{x}\right)}{2e^4}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)$$

```
[Out] -1/4*b*d^2*n*x/e^2+1/6*b*d*n*x^(3/2)/e-1/8*b*n*x^2-1/2*b*d^4*n*ln(d+e*x^(1/
2))/e^4+1/2*x^2*(a+b*ln(c*(d+e*x^(1/2))^n))+1/2*b*d^3*n*x^(1/2)/e^3
```

Rubi [A]

time = 0.05, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2504, 2442, 45}

$$\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)-\frac{bd^4n\log\left(d+e\sqrt{x}\right)}{2e^4}+\frac{bd^3n\sqrt{x}}{2e^3}-\frac{bd^2nx}{4e^2}+\frac{bdnx^{3/2}}{6e}-\frac{1}{8}bnx^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*Sqrt[x])^n]),x]
```

```
[Out] (b*d^3*n*Sqrt[x])/(2*e^3) - (b*d^2*n*x)/(4*e^2) + (b*d*n*x^(3/2))/(6*e) - (
b*n*x^2)/8 - (b*d^4*n*Log[d + e*Sqrt[x]])/(2*e^4) + (x^2*(a + b*Log[c*(d +
e*Sqrt[x])^n]))/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
```

```
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx = 2\text{Subst}\left(\int x^3(a+b\log(c(d+ex)^n))\,dx, x, \sqrt{x}\right)$$
$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right) - \frac{1}{2}(ben)\text{Subst}\left(\int \frac{x^4}{d+ex}\,dx, x, \sqrt{x}\right)$$
$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right) - \frac{1}{2}(ben)\text{Subst}\left(\int\left(-\frac{d^3}{e^4}+\frac{d^2x}{e^3}-\frac{dx}{e^2}\right.\right.$$
$$= \frac{bd^3n\sqrt{x}}{2e^3} - \frac{bd^2nx}{4e^2} + \frac{bdnx^{3/2}}{6e} - \frac{1}{8}bnx^2 - \frac{bd^4n\log\left(d+e\sqrt{x}\right)}{2e^4} + \frac{1}{2}x^2($$

Mathematica [A]
time = 0.03, size = 107, normalized size = 1.05

$$\frac{bd^3n\sqrt{x}}{2e^3} - \frac{bd^2nx}{4e^2} + \frac{bdnx^{3/2}}{6e} + \frac{ax^2}{2} - \frac{1}{8}bnx^2 - \frac{bd^4n\log\left(d+e\sqrt{x}\right)}{2e^4} + \frac{1}{2}bx^2\log\left(c\left(d+e\sqrt{x}\right)^n\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*Sqrt[x])^n]),x]
[Out] (b*d^3*n*Sqrt[x])/(2*e^3) - (b*d^2*n*x)/(4*e^2) + (b*d*n*x^(3/2))/(6*e) + (
a*x^2)/2 - (b*n*x^2)/8 - (b*d^4*n*Log[d + e*Sqrt[x]])/(2*e^4) + (b*x^2*Log[
c*(d + e*Sqrt[x])^n])/2
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e*x^(1/2))^n)),x)
[Out] int(x*(a+b*ln(c*(d+e*x^(1/2))^n)),x)
```

Maxima [A]
time = 0.28, size = 84, normalized size = 0.82

$$-\frac{1}{24}\left(12\,d^4e^{(-5)}\log\left(\sqrt{x}\,e+d\right)+\left(6\,d^2xe-12\,d^3\sqrt{x}-4\,dx^{\frac{3}{2}}e^2+3\,x^2e^3\right)e^{(-4)}\right)bne+\frac{1}{2}\,bx^2\log\left(\left(\sqrt{x}\,e+d\right)^nc\right)+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="maxima")
```

```
[Out] -1/24*(12*d^4*e^(-5)*log(sqrt(x)*e + d) + (6*d^2*x*e - 12*d^3*sqrt(x) - 4*d
*x^(3/2)*e^2 + 3*x^2*e^3)*e^(-4))*b*n*e + 1/2*b*x^2*log((sqrt(x)*e + d)^n*c
) + 1/2*a*x^2
```

Fricas [A]
time = 0.38, size = 88, normalized size = 0.86

$$-\frac{1}{24}\left(6\,bd^2nxe^2-12\,bx^2e^4\log\left(c\right)+3\,(bn-4\,a)x^2e^4+12\,\left(bd^4n-bnx^2e^4\right)\log\left(\sqrt{x}\,e+d\right)-4\,\left(3\,bd^3ne+bdnxe^3\right)\sqrt{x}\,\right)e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="fricas")
```

```
[Out] -1/24*(6*b*d^2*n*x*e^2 - 12*b*x^2*e^4*log(c) + 3*(b*n - 4*a)*x^2*e^4 + 12*(
b*d^4*n - b*n*x^2*e^4)*log(sqrt(x)*e + d) - 4*(3*b*d^3*n*e + b*d*n*x*e^3)*s
qrt(x))*e^(-4)
```

Sympy [A]
time = 1.90, size = 100, normalized size = 0.98

$$\frac{ax^2}{2}+b\left(-\frac{en\left(\frac{2d^4\left(\begin{cases}\frac{\sqrt{x}}{d} & \text{for}\ e=0\\ \frac{\log\left(d+e\sqrt{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^4}-\frac{2d^3\sqrt{x}}{e^4}+\frac{d^2x}{e^3}-\frac{2dx^{\frac{3}{2}}}{3e^2}+\frac{x^2}{2e}\right)}{4}+\frac{x^2\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/2))**n)),x)
```

```
[Out] a*x**2/2 + b*(-e*n*(2*d**4*Piecewise((sqrt(x)/d, Eq(e, 0)), (log(d + e*sqrt
(x))/e, True))/e**4 - 2*d**3*sqrt(x)/e**4 + d**2*x/e**3 - 2*d*x**(3/2)/(3*e
**2) + x**2/(2*e))/4 + x**2*log(c*(d + e*sqrt(x))**n)/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(80) = 160.
time = 4.16, size = 185, normalized size = 1.81

$$\frac{1}{24}\left(12\,bx^2e\log\left(c\right)+12\,ax^2e+\left(12\,\left(\sqrt{x}\,e+d\right)^4e^{(-3)}\log\left(\sqrt{x}\,e+d\right)-48\,\left(\sqrt{x}\,e+d\right)^3de^{(-3)}\log\left(\sqrt{x}\,e+d\right)+72\,\left(\sqrt{x}\,e+d\right)^2d^2e^{(-3)}\log\left(\sqrt{x}\,e+d\right)-48\,\left(\sqrt{x}\,e+d\right)d^3e^{(-3)}\log\left(\sqrt{x}\,e+d\right)-3\,\left(\sqrt{x}\,e+d\right)^4e^{(-3)}+16\,\left(\sqrt{x}\,e+d\right)^3de^{(-3)}-36\,\left(\sqrt{x}\,e+d\right)^2d^2e^{(-3)}+48\,\left(\sqrt{x}\,e+d\right)d^3e^{(-3)}\right)bn\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="giac")
```

```
[Out] 1/24*(12*b*x^2*e*log(c) + 12*a*x^2*e + (12*(sqrt(x)*e + d)^4*e^(-3)*log(sqr
t(x)*e + d) - 48*(sqrt(x)*e + d)^3*d*e^(-3)*log(sqrt(x)*e + d) + 72*(sqrt(x
)*e + d)^2*d^2*e^(-3)*log(sqrt(x)*e + d) - 48*(sqrt(x)*e + d)*d^3*e^(-3)*lo
g(sqrt(x)*e + d) - 3*(sqrt(x)*e + d)^4*e^(-3) + 16*(sqrt(x)*e + d)^3*d*e^(-
3) - 36*(sqrt(x)*e + d)^2*d^2*e^(-3) + 48*(sqrt(x)*e + d)*d^3*e^(-3))*b*n)*
e^(-1)
```

Mupad [B]
time = 0.41, size = 85, normalized size = 0.83

$$\frac{a\,x^2}{2}-\frac{b\,n\,x^2}{8}+\frac{b\,x^2\,\ln\left(c\,{\left(d+e\,\sqrt{x}\,\right)}^n\right)}{2}-\frac{b\,d^2\,n\,x}{4\,e^2}+\frac{b\,d\,n\,x^{3/2}}{6\,e}-\frac{b\,d^4\,n\,\ln\left(d+e\,\sqrt{x}\,\right)}{2\,e^4}+\frac{b\,d^3\,n\,\sqrt{x}}{2\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e*x^(1/2))^n)),x)
```

```
[Out] (a*x^2)/2 - (b*n*x^2)/8 + (b*x^2*log(c*(d + e*x^(1/2))^n))/2 - (b*d^2*n*x)/
(4*e^2) + (b*d*n*x^(3/2))/(6*e) - (b*d^4*n*log(d + e*x^(1/2)))/(2*e^4) + (b
*d^3*n*x^(1/2))/(2*e^3)
```

3.403 $\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx$

Optimal. Leaf size=60

$$\frac{bdn\sqrt{x}}{e}+ax-\frac{bnx}{2}-\frac{bd^2n\log\left(d+e\sqrt{x}\right)}{e^2}+bx\log\left(c\left(d+e\sqrt{x}\right)^n\right)$$

```
[Out] a*x-1/2*b*n*x-b*d^2*n*ln(d+e*x^(1/2))/e^2+b*x*ln(c*(d+e*x^(1/2))^n)+b*d*n*x
^(1/2)/e
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2498, 272, 45}

$$ax+bx\log\left(c\left(d+e\sqrt{x}\right)^n\right)-\frac{bd^2n\log\left(d+e\sqrt{x}\right)}{e^2}+\frac{bdn\sqrt{x}}{e}-\frac{bnx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Log[c*(d + e*Sqrt[x])^n],x]
```

```
[Out] (b*d*n*Sqrt[x])/e + a*x - (b*n*x)/2 - (b*d^2*n*Log[d + e*Sqrt[x]])/e^2 + b*
x*Log[c*(d + e*Sqrt[x])^n]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\int (a+b\log (c(d+e\sqrt{x})^n))\,dx = ax+b\int \log (c(d+e\sqrt{x})^n)\,dx$$
$$= ax+bx\log (c(d+e\sqrt{x})^n)-\frac{1}{2}(ben)\int \frac{\sqrt{x}}{d+e\sqrt{x}}\,dx$$
$$= ax+bx\log (c(d+e\sqrt{x})^n)-(ben)\text{Subst}\left(\int \frac{x^2}{d+ex}\,dx,x,\sqrt{x}\right)$$
$$= ax+bx\log (c(d+e\sqrt{x})^n)-(ben)\text{Subst}\left(\int \left(-\frac{d}{e^2}+\frac{x}{e}+\frac{d^2}{e^2(d+ex)}\right.\right.$$
$$= \frac{bdn\sqrt{x}}{e}+ax-\frac{bnx}{2}-\frac{bd^2n\log (d+e\sqrt{x})}{e^2}+bx\log (c(d+e\sqrt{x})^n)$$

Mathematica [A]
time = 0.02, size = 60, normalized size = 1.00

$$\frac{bdn\sqrt{x}}{e}+ax-\frac{bnx}{2}-\frac{bd^2n\log (d+e\sqrt{x})}{e^2}+bx\log (c(d+e\sqrt{x})^n)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Log[c*(d + e*Sqrt[x])^n],x]
```

```
[Out] (b*d*n*Sqrt[x])/e + a*x - (b*n*x)/2 - (b*d^2*n*Log[d + e*Sqrt[x]])/e^2 + b*
x*Log[c*(d + e*Sqrt[x])^n]
```

Maple [A]
time = 0.04, size = 53, normalized size = 0.88

method	result	size
default	$ax-\frac{bnx}{2}-\frac{b\,d^2n\ln\left(d+e\sqrt{x}\right)}{e^2}+bx\ln\left(c\left(d+e\sqrt{x}\right)^n\right)+\frac{bdn\sqrt{x}}{e}$	53
derivativedivides	$ax-\frac{bnx}{2}+bx\ln\left(c\,\mathrm{e}^{n\ln(d+e\sqrt{x})}\right)-\frac{b\,d^2n\ln\left(d+e\sqrt{x}\right)}{e^2}+\frac{bdn\sqrt{x}}{e}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*ln(c*(d+e*x^(1/2))^n),x,method=_RETURNVERBOSE)
```

```
[Out] a*x-1/2*b*n*x-b*d^2*n*ln(d+e*x^(1/2))/e^2+b*x*ln(c*(d+e*x^(1/2))^n)+b*d*n*x
^(1/2)/e
```

Maxima [A]
time = 0.33, size = 59, normalized size = 0.98

$$-\frac{1}{2}\left(\left(2\,d^2e^{(-3)}\log\left(\sqrt{x}\,e+d\right)+\left(xe-2\,d\sqrt{x}\right)e^{(-2)}\right)ne-2\,x\log\left(\left(\sqrt{x}\,e+d\right)^nc\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e*x^(1/2))^n),x, algorithm="maxima")
```

```
[Out] -1/2*((2*d^2*e^(-3)*log(sqrt(x)*e + d) + (x*e - 2*d*sqrt(x))*e^(-2))*n*e -
2*x*log((sqrt(x)*e + d)^n*c))*b + a*x
```

Fricas [A]
time = 0.40, size = 60, normalized size = 1.00

$$\frac{1}{2}\left(2\,bdn\sqrt{x}\,e+2\,bxe^{2}\log\left(c\right)-\left(bn-2\,a\right)xe^{2}-2\left(bd^{2}n-bnxe^{2}\right)\log\left(\sqrt{x}\,e+d\right)\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e*x^(1/2))^n),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*n*sqrt(x)*e + 2*b*x*e^2*log(c) - (b*n - 2*a)*x*e^2 - 2*(b*d^2*n
- b*n*x*e^2)*log(sqrt(x)*e + d))*e^(-2)
```

Sympy [A]
time = 0.68, size = 66, normalized size = 1.10

$$ax+b\left(-\frac{en\left(\frac{2d^{2}\left(\begin{cases}\frac{\sqrt{x}}{d} & \text{for } e=0\\ \frac{\log\left(d+e\sqrt{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^{2}}-\frac{2d\sqrt{x}}{e^{2}}+\frac{x}{e}\right)}{2}+x\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*ln(c*(d+e*x**(1/2))**n),x)
```

```
[Out] a*x + b*(-e*n*(2*d**2*Piecewise((sqrt(x)/d, Eq(e, 0)), (log(d + e*sqrt(x))/
e, True))/e**2 - 2*d*sqrt(x)/e**2 + x/e)/2 + x*log(c*(d + e*sqrt(x))**n))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(52) = 104.
time = 4.83, size = 107, normalized size = 1.78

$$\frac{1}{2}\left(\left(2\left(\sqrt{x}\,e+d\right)^{2}\log\left(\sqrt{x}\,e+d\right)-4\left(\sqrt{x}\,e+d\right)d\log\left(\sqrt{x}\,e+d\right)-\left(\sqrt{x}\,e+d\right)^{2}+4\left(\sqrt{x}\,e+d\right)d\right)ne^{(-1)}+2\left(\left(\sqrt{x}\,e+d\right)^{2}-2\left(\sqrt{x}\,e+d\right)d\right)e^{(-1)}\log\left(c\right)\right)be^{(-1)}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e*x^(1/2))^n),x, algorithm="giac")
```

```
[Out] 1/2*((2*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d) - 4*(sqrt(x)*e + d)*d*log(sqrt
(x)*e + d) - (sqrt(x)*e + d)^2 + 4*(sqrt(x)*e + d)*d)*n*e^(-1) + 2*((sqrt(x
)*e + d)^2 - 2*(sqrt(x)*e + d)*d)*e^(-1)*log(c))*b*e^(-1) + a*x
```

Mupad [B]
time = 0.43, size = 52, normalized size = 0.87

$$a\,x+b\,x\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)-\frac{b\,n\,\left(e^2\,x+2\,d^2\,\ln\left(d+e\,\sqrt{x}\right)-2\,d\,e\,\sqrt{x}\right)}{2\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*log(c*(d + e*x^(1/2))^n),x)
```

```
[Out] a*x + b*x*log(c*(d + e*x^(1/2))^n) - (b*n*(e^2*x + 2*d^2*log(d + e*x^(1/2))
 - 2*d*e*x^(1/2)))/(2*e^2)
```

3.404 $\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x}\,dx$

Optimal. Leaf size=51

$$2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt{x}}{d}\right)+2bn\text{Li}_2\left(1+\frac{e\sqrt{x}}{d}\right)$$

```
[Out] 2*ln(-e*x^(1/2)/d)*(a+b*ln(c*(d+e*x^(1/2))^n))+2*b*n*polylog(2,1+e*x^(1/2)/
d)
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2504, 2441, 2352}

$$2bn\text{PolyLog}\left(2,\frac{e\sqrt{x}}{d}+1\right)+2\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])/x,x]
```

```
[Out] 2*(a + b*Log[c*(d + e*Sqrt[x])^n])*Log[-((e*Sqrt[x])/d)] + 2*b*n*PolyLog[2,
 1 + (e*Sqrt[x])/d]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x}\,dx = 2\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x}\,dx, x, \sqrt{x}\right)$$
$$= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt{x}}{d}\right) - (2ben)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}$$
$$= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt{x}}{d}\right) + 2bn\text{Li}_2\left(1+\frac{e\sqrt{x}}{d}\right)$$

Mathematica [A]
time = 0.00, size = 53, normalized size = 1.04

$$2b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\log\left(-\frac{e\sqrt{x}}{d}\right) + a\log(x) + 2bn\text{Li}_2\left(\frac{d+e\sqrt{x}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])/x,x]
```

```
[Out] 2*b*Log[c*(d + e*Sqrt[x])^n]*Log[-((e*Sqrt[x])/d)] + a*Log[x] + 2*b*n*PolyL
og[2, (d + e*Sqrt[x])/d]
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))/x,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))/x,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(47) = 94.
time = 0.41, size = 122, normalized size = 2.39

$$-2\left(\log\left(\sqrt{x}\right)\log\left(\frac{e^{\left(\frac{1}{2}\log(x)+1\right)}}{d}+1\right)+\text{Li}_2\left(-\frac{e^{\left(\frac{1}{2}\log(x)+1\right)}}{d}\right)\right)bn + \frac{bdn\log\left(\sqrt{x}e+d\right)\log(x) + (bd\log(c)+ad)\log(x) - \frac{bnxe\log(x)-2bnxe}{\sqrt{x}}}{d} + \frac{2\left(bne^{\left(\frac{1}{2}\log(x)+1\right)}\log\left(\sqrt{x}\right) - bne^{\left(\frac{1}{2}\log(x)+1\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x,x, algorithm="maxima")
```

```
[Out] -2*(log(sqrt(x))*log(e^(1/2*log(x) + 1)/d + 1) + dilog(-e^(1/2*log(x) + 1)/
d))*b*n + (b*d*n*log(sqrt(x)*e + d)*log(x) + (b*d*log(c) + a*d)*log(x) - (b
```

```
*n*x*e*log(x) - 2*b*n*x*e)/sqrt(x))/d + 2*(b*n*e^(1/2*log(x) + 1)*log(sqrt(
x)) - b*n*e^(1/2*log(x) + 1))/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x,x, algorithm="fricas")
[Out] integral((b*log((sqrt(x)*e + d)^n*c) + a)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\log \left(c\left(d + e\sqrt{x}\right)^{n}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))/x,x)
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x,x, algorithm="giac")
[Out] integrate((b*log((sqrt(x)*e + d)^n*c) + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + b\,\ln \left(c\,{\left(d + e\,\sqrt{x}\right)}^n\right)}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))/x,x)
[Out] int((a + b*log(c*(d + e*x^(1/2))^n))/x, x)
```

3.405 $\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^2}\,dx$

Optimal. Leaf size=70

$$-\frac{ben}{d\sqrt{x}}+\frac{be^2n\log\left(d+e\sqrt{x}\right)}{d^2}-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x}-\frac{be^2n\log(x)}{2d^2}$$

```
[Out] -1/2*b*e^2*n*ln(x)/d^2+b*e^2*n*ln(d+e*x^(1/2))/d^2+(-a-b*ln(c*(d+e*x^(1/2))
^n))/x-b*e*n/d/x^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2504, 2442, 46}

$$-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x}+\frac{be^2n\log\left(d+e\sqrt{x}\right)}{d^2}-\frac{be^2n\log(x)}{2d^2}-\frac{ben}{d\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])/x^2,x]
```

```
[Out] -((b*e*n)/(d*Sqrt[x])) + (b*e^2*n*Log[d + e*Sqrt[x]])/d^2 - (a + b*Log[c*(d
 + e*Sqrt[x])^n])/x - (b*e^2*n*Log[x])/(2*d^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
```

```
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^2}\,dx = 2\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x} + (ben)\text{Subst}\left(\int \frac{1}{x^2(d+ex)}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x} + (ben)\text{Subst}\left(\int \left(\frac{1}{dx^2} - \frac{e}{d^2x} + \frac{e^2}{d^2(d+ex)}\right.\right.$$
$$= -\frac{ben}{d\sqrt{x}} + \frac{be^2n\log\left(d+e\sqrt{x}\right)}{d^2} - \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x} - \frac{be^2n\log(x}{2d^2}$$

Mathematica [A]
time = 0.03, size = 67, normalized size = 0.96

$$-\frac{a}{x} - \frac{b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x} + ben\left(-\frac{1}{d\sqrt{x}} + \frac{e\log\left(d+e\sqrt{x}\right)}{d^2} - \frac{e\log(x)}{2d^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])/x^2,x]
[Out] -(a/x) - (b*Log[c*(d + e*Sqrt[x])^n])/x + b*e*n*(-(1/(d*Sqrt[x])) + (e*Log[
d + e*Sqrt[x]])/d^2 - (e*Log[x])/(2*d^2))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))/x^2,x)
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))/x^2,x)
```

Maxima [A]
time = 0.27, size = 66, normalized size = 0.94

$$\frac{1}{2}bn\left(\frac{2e\log\left(\sqrt{x}\,e+d\right)}{d^2} - \frac{e\log(x)}{d^2} - \frac{2}{d\sqrt{x}}\right)e - \frac{b\log\left(\left(\sqrt{x}\,e+d\right)^nc\right)}{x} - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*n*(2*e*log(sqrt(x)*e + d)/d^2 - e*log(x)/d^2 - 2/(d*sqrt(x)))*e - b*l
og((sqrt(x)*e + d)^n*c)/x - a/x
```

Fricas [A]
time = 0.39, size = 64, normalized size = 0.91

$$-\frac{bnxe^2\log\left(\sqrt{x}\right) + bdn\sqrt{x}\,e + bd^2\log(c) + ad^2 + (bd^2n - bnxe^2)\log\left(\sqrt{x}\,e + d\right)}{d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^2,x, algorithm="fricas")
```

```
[Out] -(b*n*x*e^2*log(sqrt(x)) + b*d*n*sqrt(x)*e + b*d^2*log(c) + a*d^2 + (b*d^2*
n - b*n*x*e^2)*log(sqrt(x)*e + d))/(d^2*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 410 vs. 2(66) = 132.
time = 14.69, size = 410, normalized size = 5.86

$$\begin{cases} -\frac{2ad^3\sqrt{x}}{2d^3x^{\frac{3}{2}}+2d^2ex^2} - \frac{2ad^2ex}{2d^3x^{\frac{3}{2}}+2d^2ex^2} - \frac{2bd^3\sqrt{x}\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2d^3x^{\frac{3}{2}}+2d^2ex^2} - \frac{2bd^2enx}{2d^3x^{\frac{3}{2}}+2d^2ex^2} - \frac{2bd^2ex\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2d^3x^{\frac{3}{2}}+2d^2ex^2} - \frac{bde^2nx^{\frac{3}{2}}\log(x)}{2d^3x^{\frac{3}{2}}+2d^2ex^2} - \frac{2bde^2nx^{\frac{3}{2}}}{2d^3x^{\frac{3}{2}}+2d^2ex^2} + \frac{2bde^2x^{\frac{3}{2}}\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2d^3x^{\frac{3}{2}}+2d^2ex^2} - \frac{be^3nx^2\log(x)}{2d^3x^{\frac{3}{2}}+2d^2ex^2} + \frac{2be^3x^2\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2d^3x^{\frac{3}{2}}+2d^2ex^2} & \text{for}\ d \neq 0 \\ -\frac{a}{x} - \frac{bn}{2x} - \frac{b\log\left(c\left(e\sqrt{x}\right)^n\right)}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))/x**2,x)
```

```
[Out] Piecewise((-2*a*d**3*sqrt(x)/(2*d**3*x**(3/2) + 2*d**2*e*x**2) - 2*a*d**2*e
*x/(2*d**3*x**(3/2) + 2*d**2*e*x**2) - 2*b*d**3*sqrt(x)*log(c*(d + e*sqrt(x
))**n)/(2*d**3*x**(3/2) + 2*d**2*e*x**2) - 2*b*d**2*e*n*x/(2*d**3*x**(3/2)
+ 2*d**2*e*x**2) - 2*b*d**2*e*x*log(c*(d + e*sqrt(x))**n)/(2*d**3*x**(3/2)
+ 2*d**2*e*x**2) - b*d*e**2*n*x**(3/2)*log(x)/(2*d**3*x**(3/2) + 2*d**2*e*x
**2) - 2*b*d*e**2*n*x**(3/2)/(2*d**3*x**(3/2) + 2*d**2*e*x**2) + 2*b*d*e**2
*x**(3/2)*log(c*(d + e*sqrt(x))**n)/(2*d**3*x**(3/2) + 2*d**2*e*x**2) - b*e
**3*n*x**2*log(x)/(2*d**3*x**(3/2) + 2*d**2*e*x**2) + 2*b*e**3*x**2*log(c*(
d + e*sqrt(x))**n)/(2*d**3*x**(3/2) + 2*d**2*e*x**2), Ne(d, 0)), (-a/x - b*
n/(2*x) - b*log(c*(e*sqrt(x))**n)/x, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(63) = 126.
time = 3.94, size = 187, normalized size = 2.67

$$\frac{\left(\left(\sqrt{x}\,e+d\right)^2bne^3\log\left(\sqrt{x}\,e+d\right) - 2\left(\sqrt{x}\,e+d\right)bdne^3\log\left(\sqrt{x}\,e+d\right) - \left(\sqrt{x}\,e+d\right)^2bne^3\log\left(\sqrt{x}\,e\right) + 2\left(\sqrt{x}\,e+d\right)bdne^3\log\left(\sqrt{x}\,e\right) - bd^2ne^3\log\left(\sqrt{x}\,e\right) - \left(\sqrt{x}\,e+d\right)bdne^3 + bd^2ne^3 - bd^2e^3\log(c) - ad^2e^3\right)e^{(-1)}}{\left(\sqrt{x}\,e+d\right)^2d^2 - 2\left(\sqrt{x}\,e+d\right)d^3 + d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^2,x, algorithm="giac")
```

```
[Out] ((sqrt(x)*e + d)^2*b*n*e^3*log(sqrt(x)*e + d) - 2*(sqrt(x)*e + d)*b*d*n*e^3
*log(sqrt(x)*e + d) - (sqrt(x)*e + d)^2*b*n*e^3*log(sqrt(x)*e) + 2*(sqrt(x)
*e + d)*b*d*n*e^3*log(sqrt(x)*e) - b*d^2*n*e^3*log(sqrt(x)*e) - (sqrt(x)*e
+ d)*b*d*n*e^3 + b*d^2*n*e^3 - b*d^2*e^3*log(c) - a*d^2*e^3)*e^(-1)/((sqrt(
x)*e + d)^2*d^2 - 2*(sqrt(x)*e + d)*d^3 + d^4)
```

Mupad [B]
time = 0.80, size = 58, normalized size = 0.83

$$\frac{2\,b\,e^2\,n\,\mathrm{atanh}\left(\frac{2\,e\,\sqrt{x}}{d}+1\right)}{d^2}-\frac{b\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)}{x}-\frac{b\,e\,n}{d\,\sqrt{x}}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))/x^2,x)
```

```
[Out] (2*b*e^2*n*atanh((2*e*x^(1/2))/d + 1))/d^2 - (b*log(c*(d + e*x^(1/2))^n))/x
 - (b*e*n)/(d*x^(1/2)) - a/x
```

3.406 $\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^3}\,dx$

Optimal. Leaf size=109

$$-\frac{ben}{6dx^{3/2}}+\frac{be^2n}{4d^2x}-\frac{be^3n}{2d^3\sqrt{x}}+\frac{be^4n\log\left(d+e\sqrt{x}\right)}{2d^4}-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2x^2}-\frac{be^4n\log(x)}{4d^4}$$

```
[Out] -1/6*b*e*n/d/x^(3/2)+1/4*b*e^2*n/d^2/x-1/4*b*e^4*n*ln(x)/d^4+1/2*b*e^4*n*ln
(d+e*x^(1/2))/d^4+1/2*(-a-b*ln(c*(d+e*x^(1/2))^n))/x^2-1/2*b*e^3*n/d^3/x^(1
/2)
```

Rubi [A]
time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2504, 2442, 46}

$$-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2x^2}+\frac{be^4n\log\left(d+e\sqrt{x}\right)}{2d^4}-\frac{be^4n\log(x)}{4d^4}-\frac{be^3n}{2d^3\sqrt{x}}+\frac{be^2n}{4d^2x}-\frac{ben}{6dx^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])/x^3,x]
```

```
[Out] -1/6*(b*e*n)/(d*x^(3/2)) + (b*e^2*n)/(4*d^2*x) - (b*e^3*n)/(2*d^3*Sqrt[x])
+ (b*e^4*n*Log[d + e*Sqrt[x]])/(2*d^4) - (a + b*Log[c*(d + e*Sqrt[x])^n])/(
2*x^2) - (b*e^4*n*Log[x])/(4*d^4)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^3}\,dx = 2\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^5}\,dx,x,\sqrt{x}\right)$$
$$= -\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2x^2} + \frac{1}{2}(ben)\text{Subst}\left(\int \frac{1}{x^4(d+ex)}\,dx,x,\sqrt{x}\right)$$
$$= -\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2x^2} + \frac{1}{2}(ben)\text{Subst}\left(\int \left(\frac{1}{dx^4} - \frac{e}{d^2x^3} + \frac{e^2}{d^3x^2} - \right.\right.$$
$$= -\frac{ben}{6dx^{3/2}} + \frac{be^2n}{4d^2x} - \frac{be^3n}{2d^3\sqrt{x}} + \frac{be^4n\log\left(d+e\sqrt{x}\right)}{2d^4} - \frac{a+b\log\left(c\left(d+e\right.\right.}{2x^2}$$

Mathematica [A]
time = 0.03, size = 104, normalized size = 0.95

$$-\frac{a}{2x^2} - \frac{b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2x^2} + \frac{1}{2}ben\left(-\frac{1}{3dx^{3/2}} + \frac{e}{2d^2x} - \frac{e^2}{d^3\sqrt{x}} + \frac{e^3\log\left(d+e\sqrt{x}\right)}{d^4} - \frac{e^3\log(x)}{2d^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])/x^3,x]
[Out] -1/2*a/x^2 - (b*Log[c*(d + e*Sqrt[x])^n])/(2*x^2) + (b*e*n*(-1/3*1/(d*x^(3/
2)) + e/(2*d^2*x) - e^2/(d^3*Sqrt[x]) + (e^3*Log[d + e*Sqrt[x]])/d^4 - (e^3
*Log[x])/(2*d^4)))/2
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))/x^3,x)
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))/x^3,x)
```

Maxima [A]
time = 0.29, size = 84, normalized size = 0.77

$$\frac{1}{12}bn\left(\frac{6e^3\log\left(\sqrt{x}e+d\right)}{d^4} - \frac{3e^3\log(x)}{d^4} + \frac{3d\sqrt{x}e - 2d^2 - 6xe^2}{d^3x^{\frac{3}{2}}}\right)e - \frac{b\log\left(\left(\sqrt{x}e+d\right)^nc\right)}{2x^2} - \frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^3,x, algorithm="maxima")
```

```
[Out] 1/12*b*n*(6*e^3*log(sqrt(x)*e + d)/d^4 - 3*e^3*log(x)/d^4 + (3*d*sqrt(x)*e
- 2*d^2 - 6*x*e^2)/(d^3*x^(3/2)))*e - 1/2*b*log((sqrt(x)*e + d)^n*c)/x^2 -
1/2*a/x^2
```

Fricas [A]
time = 0.37, size = 95, normalized size = 0.87

$$\frac{3\,bd^2nxe^2 - 6\,bd^4\log\left(c\right) - 6\,bnx^2e^4\log\left(\sqrt{x}\right) - 6\,ad^4 - 6\,\left(bd^4n - bnx^2e^4\right)\log\left(\sqrt{x}\,e + d\right) - 2\,\left(bd^3ne + 3\,bdnxe^3\right)\sqrt{x}}{12\,d^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*d^2*n*x*e^2 - 6*b*d^4*log(c) - 6*b*n*x^2*e^4*log(sqrt(x)) - 6*a*d
^4 - 6*(b*d^4*n - b*n*x^2*e^4)*log(sqrt(x)*e + d) - 2*(b*d^3*n*e + 3*b*d*n*
x*e^3)*sqrt(x))/(d^4*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 493 vs. 2(105) = 210.
time = 107.56, size = 493, normalized size = 4.52

$$\begin{cases} - \frac{6ad^5\sqrt{x}}{12d^5x^{\frac{5}{2}}+12d^4ex^3} - \frac{6ad^4ex}{12d^5x^{\frac{5}{2}}+12d^4ex^3} - \frac{6bd^5\sqrt{x}\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{12d^5x^{\frac{5}{2}}+12d^4ex^3} - \frac{2bd^4enx}{12d^5x^{\frac{5}{2}}+12d^4ex^3} - \frac{6bd^4ex\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{12d^5x^{\frac{5}{2}}+12d^4ex^3} + \frac{bd^3e^2nx^{\frac{3}{2}}}{12d^5x^{\frac{5}{2}}+12d^4ex^3} - \frac{3bd^2e^3nx^2}{12d^5x^{\frac{5}{2}}+12d^4ex^3} - \frac{3bde^4nx^{\frac{5}{2}}\log\left(x\right)}{12d^5x^{\frac{5}{2}}+12d^4ex^3} - \frac{6bde^4nx^{\frac{5}{2}}}{12d^5x^{\frac{5}{2}}+12d^4ex^3} + \frac{6bde^4x^{\frac{5}{2}}\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{12d^5x^{\frac{5}{2}}+12d^4ex^3} - \frac{3be^5nx^3\log\left(x\right)}{12d^5x^{\frac{5}{2}}+12d^4ex^3} + \frac{6be^5x^3\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{12d^5x^{\frac{5}{2}}+12d^4ex^3} & \text{for}\: d \neq 0 \\ - \frac{a}{2x^2} - \frac{bn}{8x^2} - \frac{b\log\left(c\left(e\sqrt{x}\right)^n\right)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))/x**3,x)
```

```
[Out] Piecewise((-6*a*d**5*sqrt(x)/(12*d**5*x**(5/2) + 12*d**4*e*x**3) - 6*a*d**4
*e*x/(12*d**5*x**(5/2) + 12*d**4*e*x**3) - 6*b*d**5*sqrt(x)*log(c*(d + e*sq
rt(x))**n)/(12*d**5*x**(5/2) + 12*d**4*e*x**3) - 2*b*d**4*e*n*x/(12*d**5*x*
*(5/2) + 12*d**4*e*x**3) - 6*b*d**4*e*x*log(c*(d + e*sqrt(x))**n)/(12*d**5*
x**(5/2) + 12*d**4*e*x**3) + b*d**3*e**2*n*x**(3/2)/(12*d**5*x**(5/2) + 12*
d**4*e*x**3) - 3*b*d**2*e**3*n*x**2/(12*d**5*x**(5/2) + 12*d**4*e*x**3) - 3
*b*d*e**4*n*x**(5/2)*log(x)/(12*d**5*x**(5/2) + 12*d**4*e*x**3) - 6*b*d*e**
4*n*x**(5/2)/(12*d**5*x**(5/2) + 12*d**4*e*x**3) + 6*b*d*e**4*x**(5/2)*log(
c*(d + e*sqrt(x))**n)/(12*d**5*x**(5/2) + 12*d**4*e*x**3) - 3*b*e**5*n*x**3
*log(x)/(12*d**5*x**(5/2) + 12*d**4*e*x**3) + 6*b*e**5*x**3*log(c*(d + e*sq
rt(x))**n)/(12*d**5*x**(5/2) + 12*d**4*e*x**3), Ne(d, 0)), (-a/(2*x**2) - b
*n/(8*x**2) - b*log(c*(e*sqrt(x))**n)/(2*x**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 366 vs. 2(88) = 176.
time = 4.84, size = 366, normalized size = 3.36

$$\frac{\left(6\left(\sqrt{x}e+d\right)^4bne^5\log\left(\sqrt{x}e+d\right) - 24\left(\sqrt{x}e+d\right)^3bdne^5\log\left(\sqrt{x}e+d\right) + 36\left(\sqrt{x}e+d\right)^2bd^2ne^5\log\left(\sqrt{x}e+d\right) - 24\left(\sqrt{x}e+d\right)bd^3ne^5\log\left(\sqrt{x}e+d\right) - 6\left(\sqrt{x}e+d\right)^4bne^5\log\left(\sqrt{x}e\right) + 24\left(\sqrt{x}e+d\right)^3bdne^5\log\left(\sqrt{x}e\right) - 36\left(\sqrt{x}e+d\right)^2bd^2ne^5\log\left(\sqrt{x}e\right) + 24\left(\sqrt{x}e+d\right)bd^3ne^5\log\left(\sqrt{x}e\right) - 6\,bd^4ne^5\log\left(\sqrt{x}e\right) - 6\left(\sqrt{x}e+d\right)^3bdne^5 + 21\left(\sqrt{x}e+d\right)^2bd^2ne^5 - 26\left(\sqrt{x}e+d\right)bd^3ne^5 + 11\,bd^4ne^5 - 6\,bd^4e^5\log\left(c\right) - 6\,ad^4e^5\right)e^{(-1)}}{12\left(\left(\sqrt{x}e+d\right)^4d^4 - 4\left(\sqrt{x}e+d\right)^3d^5 + 6\left(\sqrt{x}e+d\right)^2d^6 - 4\left(\sqrt{x}e+d\right)d^7 + d^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^3,x, algorithm="giac")
```

```
[Out] 1/12*(6*(sqrt(x)*e + d)^4*b*n*e^5*log(sqrt(x)*e + d) - 24*(sqrt(x)*e + d)^3
*b*d*n*e^5*log(sqrt(x)*e + d) + 36*(sqrt(x)*e + d)^2*b*d^2*n*e^5*log(sqrt(x
)*e + d) - 24*(sqrt(x)*e + d)*b*d^3*n*e^5*log(sqrt(x)*e + d) - 6*(sqrt(x)*e
 + d)^4*b*n*e^5*log(sqrt(x)*e) + 24*(sqrt(x)*e + d)^3*b*d*n*e^5*log(sqrt(x)
*e) - 36*(sqrt(x)*e + d)^2*b*d^2*n*e^5*log(sqrt(x)*e) + 24*(sqrt(x)*e + d)*
b*d^3*n*e^5*log(sqrt(x)*e) - 6*b*d^4*n*e^5*log(sqrt(x)*e) - 6*(sqrt(x)*e +
d)^3*b*d*n*e^5 + 21*(sqrt(x)*e + d)^2*b*d^2*n*e^5 - 26*(sqrt(x)*e + d)*b*d^
3*n*e^5 + 11*b*d^4*n*e^5 - 6*b*d^4*e^5*log(c) - 6*a*d^4*e^5)*e^(-1)/((sqrt(
x)*e + d)^4*d^4 - 4*(sqrt(x)*e + d)^3*d^5 + 6*(sqrt(x)*e + d)^2*d^6 - 4*(sq
rt(x)*e + d)*d^7 + d^8)
```

Mupad [B]
time = 0.63, size = 83, normalized size = 0.76

$$\frac{b\,e^4\,n\,\mathrm{atanh}\left(\frac{2\,e\,\sqrt{x}}{d}+1\right)}{d^4}-\frac{\frac{b\,e\,n}{3\,d}+\frac{b\,e^3\,n\,x}{d^3}-\frac{b\,e^2\,n\,\sqrt{x}}{2\,d^2}}{2\,x^{3/2}}-\frac{b\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)}{2\,x^2}-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))/x^3,x)
```

```
[Out] (b*e^4*n*atanh((2*e*x^(1/2))/d + 1))/d^4 - ((b*e*n)/(3*d) + (b*e^3*n*x)/d^3
 - (b*e^2*n*x^(1/2))/(2*d^2))/(2*x^(3/2)) - (b*log(c*(d + e*x^(1/2))^n))/(2
*x^2) - a/(2*x^2)
```

3.407 $\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^4}\,dx$

Optimal. Leaf size=141

$$-\frac{ben}{15dx^{5/2}}+\frac{be^2n}{12d^2x^2}-\frac{be^3n}{9d^3x^{3/2}}+\frac{be^4n}{6d^4x}-\frac{be^5n}{3d^5\sqrt{x}}+\frac{be^6n\log\left(d+e\sqrt{x}\right)}{3d^6}-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{3x^3}-\frac{be^6n\log(x)}{6d^6}$$

```
[Out] -1/15*b*e*n/d/x^(5/2)+1/12*b*e^2*n/d^2/x^2-1/9*b*e^3*n/d^3/x^(3/2)+1/6*b*e^
4*n/d^4/x-1/6*b*e^6*n*ln(x)/d^6+1/3*b*e^6*n*ln(d+e*x^(1/2))/d^6+1/3*(-a-b*l
n(c*(d+e*x^(1/2))^n))/x^3-1/3*b*e^5*n/d^5/x^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2504, 2442, 46}

$$-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{3x^3}+\frac{be^6n\log\left(d+e\sqrt{x}\right)}{3d^6}-\frac{be^6n\log(x)}{6d^6}-\frac{be^5n}{3d^5\sqrt{x}}+\frac{be^4n}{6d^4x}-\frac{be^3n}{9d^3x^{3/2}}+\frac{be^2n}{12d^2x^2}-\frac{ben}{15dx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])/x^4,x]
```

```
[Out] -1/15*(b*e*n)/(d*x^(5/2)) + (b*e^2*n)/(12*d^2*x^2) - (b*e^3*n)/(9*d^3*x^(3/
2)) + (b*e^4*n)/(6*d^4*x) - (b*e^5*n)/(3*d^5*Sqrt[x]) + (b*e^6*n*Log[d + e*
Sqrt[x]])/(3*d^6) - (a + b*Log[c*(d + e*Sqrt[x])^n])/(3*x^3) - (b*e^6*n*Log
[x])/(6*d^6)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^4}\,dx = 2\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^7}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{3x^3} + \frac{1}{3}(ben)\text{Subst}\left(\int \frac{1}{x^6(d+ex)}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{3x^3} + \frac{1}{3}(ben)\text{Subst}\left(\int \left(\frac{1}{dx^6} - \frac{e}{d^2x^5} + \frac{e^2}{d^3x^4} - \right.\right.$$
$$= -\frac{ben}{15dx^{5/2}} + \frac{be^2n}{12d^2x^2} - \frac{be^3n}{9d^3x^{3/2}} + \frac{be^4n}{6d^4x} - \frac{be^5n}{3d^5\sqrt{x}} + \frac{be^6n\log\left(d+e\sqrt{x}\right.}{3d^6}$$

Mathematica [A]
time = 0.09, size = 132, normalized size = 0.94

$$-\frac{a}{3x^3} - \frac{b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{3x^3} + \frac{1}{3}ben\left(-\frac{1}{5dx^{5/2}} + \frac{e}{4d^2x^2} - \frac{e^2}{3d^3x^{3/2}} + \frac{e^3}{2d^4x} - \frac{e^4}{d^5\sqrt{x}} + \frac{e^5\log\left(d+e\sqrt{x}\right)}{d^6} - \frac{e^5\log(x)}{2d^6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (b*Log[c*(d + e*Sqrt[x])^n])/(3*x^3) + (b*e*n*(-1/5*1/(d*x^(5/
2)) + e/(4*d^2*x^2) - e^2/(3*d^3*x^(3/2)) + e^3/(2*d^4*x) - e^4/(d^5*Sqrt[x
]) + (e^5*Log[d + e*Sqrt[x]])/d^6 - (e^5*Log[x])/(2*d^6)))/3
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))/x^4,x)
```

Maxima [A]
time = 0.27, size = 104, normalized size = 0.74

$$\frac{1}{180}bn\left(\frac{60\,e^5\log\left(\sqrt{x}\,e+d\right)}{d^6} - \frac{30\,e^5\log(x)}{d^6} + \frac{15\,d^3\sqrt{x}\,e - 12\,d^4 - 20\,d^2xe^2 + 30\,dx^{\frac{3}{2}}e^3 - 60\,x^2e^4}{d^5x^{\frac{5}{2}}}\right)e - \frac{b\log\left(\left(\sqrt{x}\,e+d\right)^nc\right)}{3\,x^3} - \frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^4,x, algorithm="maxima")
```

```
[Out] 1/180*b*n*(60*e^5*log(sqrt(x)*e + d)/d^6 - 30*e^5*log(x)/d^6 + (15*d^3*sqrt
(x)*e - 12*d^4 - 20*d^2*x*e^2 + 30*d*x^(3/2)*e^3 - 60*x^2*e^4)/(d^5*x^(5/2)
))*e - 1/3*b*log((sqrt(x)*e + d)^n*c)/x^3 - 1/3*a/x^3
```

Fricas [**A**]
time = 0.37, size = 120, normalized size = 0.85

$$\frac{15\,bd^4nxe^2-60\,bd^6\log\left(c\right)-60\,ad^6+30\,bd^2nx^2e^4-60\,bnx^3e^6\log\left(\sqrt{x}\right)-60\left(bd^6n-bnx^3e^6\right)\log\left(\sqrt{x}\,e+d\right)-4\left(3\,bd^5ne+5\,bd^3nxe^3+15\,bdnx^2e^5\right)\sqrt{x}}{180\,d^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^4,x, algorithm="fricas")
```

```
[Out] 1/180*(15*b*d^4*n*x*e^2 - 60*b*d^6*log(c) - 60*a*d^6 + 30*b*d^2*n*x^2*e^4 -
 60*b*n*x^3*e^6*log(sqrt(x)) - 60*(b*d^6*n - b*n*x^3*e^6)*log(sqrt(x)*e + d
) - 4*(3*b*d^5*n*e + 5*b*d^3*n*x*e^3 + 15*b*d*n*x^2*e^5)*sqrt(x))/(d^6*x^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))/x**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 542 vs. 2(112) = 224.
time = 4.91, size = 542, normalized size = 3.84

$$\frac{\left(60\left(\sqrt{x}\,e+d\right)^6bne^7\log\left(\sqrt{x}\,e+d\right)-360\left(\sqrt{x}\,e+d\right)^5bdne^7\log\left(\sqrt{x}\,e+d\right)+900\left(\sqrt{x}\,e+d\right)^4bd^2ne^7\log\left(\sqrt{x}\,e+d\right)-1200\left(\sqrt{x}\,e+d\right)^3bd^3ne^7\log\left(\sqrt{x}\,e+d\right)+900\left(\sqrt{x}\,e+d\right)^2bd^4ne^7\log\left(\sqrt{x}\,e+d\right)-360\left(\sqrt{x}\,e+d\right)bd^5ne^7\log\left(\sqrt{x}\,e+d\right)-60\left(\sqrt{x}\,e+d\right)^6bne^7\log\left(\sqrt{x}\,e\right)+360\left(\sqrt{x}\,e+d\right)^5bdne^7\log\left(\sqrt{x}\,e\right)-900\left(\sqrt{x}\,e+d\right)^4bd^2ne^7\log\left(\sqrt{x}\,e\right)+1200\left(\sqrt{x}\,e+d\right)^3bd^3ne^7\log\left(\sqrt{x}\,e\right)-900\left(\sqrt{x}\,e+d\right)^2bd^4ne^7\log\left(\sqrt{x}\,e\right)+360\left(\sqrt{x}\,e+d\right)bd^5ne^7\log\left(\sqrt{x}\,e\right)-60\,bd^6ne^7\log\left(\sqrt{x}\,e\right)-60\left(\sqrt{x}\,e+d\right)^5bdne^7+330\left(\sqrt{x}\,e+d\right)^4bd^2ne^7-740\left(\sqrt{x}\,e+d\right)^3bd^3ne^7+855\left(\sqrt{x}\,e+d\right)^2bd^4ne^7-522\left(\sqrt{x}\,e+d\right)bd^5ne^7+137\,bd^6ne^7-60\,bd^6e^7\log\left(c\right)-60\,ad^6e^7\right)e^{(-1)}}{180\left(\left(\sqrt{x}\,e+d\right)^6d^6-6\left(\sqrt{x}\,e+d\right)^5d^7+15\left(\sqrt{x}\,e+d\right)^4d^8-20\left(\sqrt{x}\,e+d\right)^3d^9+15\left(\sqrt{x}\,e+d\right)^2d^{10}-6\left(\sqrt{x}\,e+d\right)d^{11}+d^{12}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^4,x, algorithm="giac")
```

```
[Out] 1/180*(60*(sqrt(x)*e + d)^6*b*n*e^7*log(sqrt(x)*e + d) - 360*(sqrt(x)*e + d
)^5*b*d*n*e^7*log(sqrt(x)*e + d) + 900*(sqrt(x)*e + d)^4*b*d^2*n*e^7*log(sq
rt(x)*e + d) - 1200*(sqrt(x)*e + d)^3*b*d^3*n*e^7*log(sqrt(x)*e + d) + 900*
(sqrt(x)*e + d)^2*b*d^4*n*e^7*log(sqrt(x)*e + d) - 360*(sqrt(x)*e + d)*b*d^
5*n*e^7*log(sqrt(x)*e + d) - 60*(sqrt(x)*e + d)^6*b*n*e^7*log(sqrt(x)*e) +
360*(sqrt(x)*e + d)^5*b*d*n*e^7*log(sqrt(x)*e) - 900*(sqrt(x)*e + d)^4*b*d^
2*n*e^7*log(sqrt(x)*e) + 1200*(sqrt(x)*e + d)^3*b*d^3*n*e^7*log(sqrt(x)*e)
- 900*(sqrt(x)*e + d)^2*b*d^4*n*e^7*log(sqrt(x)*e) + 360*(sqrt(x)*e + d)*b*
```

```
d^5*n*e^7*log(sqrt(x)*e) - 60*b*d^6*n*e^7*log(sqrt(x)*e) - 60*(sqrt(x)*e +
d)^5*b*d*n*e^7 + 330*(sqrt(x)*e + d)^4*b*d^2*n*e^7 - 740*(sqrt(x)*e + d)^3*
b*d^3*n*e^7 + 855*(sqrt(x)*e + d)^2*b*d^4*n*e^7 - 522*(sqrt(x)*e + d)*b*d^5
*n*e^7 + 137*b*d^6*n*e^7 - 60*b*d^6*e^7*log(c) - 60*a*d^6*e^7)*e^(-1)/((sqr
t(x)*e + d)^6*d^6 - 6*(sqrt(x)*e + d)^5*d^7 + 15*(sqrt(x)*e + d)^4*d^8 - 20
*(sqrt(x)*e + d)^3*d^9 + 15*(sqrt(x)*e + d)^2*d^10 - 6*(sqrt(x)*e + d)*d^11
 + d^12)
```

Mupad [B]
time = 0.76, size = 110, normalized size = 0.78

$$\frac{2\,b\,e^6\,n\,\mathrm{atanh}\left(\frac{2\,e\,\sqrt{x}}{d}+1\right)}{3\,d^6}-\frac{\frac{b\,e\,n}{5\,d}+\frac{b\,e^3\,n\,x}{3\,d^3}-\frac{b\,e^2\,n\,\sqrt{x}}{4\,d^2}+\frac{b\,e^5\,n\,x^2}{d^5}-\frac{b\,e^4\,n\,x^{3/2}}{2\,d^4}}{3\,x^{5/2}}-\frac{b\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)}{3\,x^3}-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))/x^4,x)
[Out] (2*b*e^6*n*atanh((2*e*x^(1/2))/d + 1))/(3*d^6) - ((b*e*n)/(5*d) + (b*e^3*n*
x)/(3*d^3) - (b*e^2*n*x^(1/2))/(4*d^2) + (b*e^5*n*x^2)/d^5 - (b*e^4*n*x^(3/
2))/(2*d^4))/(3*x^(5/2)) - (b*log(c*(d + e*x^(1/2))^n))/(3*x^3) - a/(3*x^3)
```

3.408 $\int x^2 \left(a+b \log \left(c \left(d+e \sqrt{x}\right)^n\right)\right)^2 \, dx$

Optimal. Leaf size=480

$$\frac{5b^2d^4n^2\left(d+e\sqrt{x}\right)^2}{2e^6}-\frac{40b^2d^3n^2\left(d+e\sqrt{x}\right)^3}{27e^6}+\frac{5b^2d^2n^2\left(d+e\sqrt{x}\right)^4}{8e^6}-\frac{4b^2dn^2\left(d+e\sqrt{x}\right)^5}{25e^6}+\frac{b^2n^2\left(d+e\sqrt{x}\right)^6}{54e^6}$$

```
[Out] 1/3*b^2*d^6*n^2*ln(d+e*x^(1/2))^2/e^6-2/3*b*d^6*n*ln(d+e*x^(1/2))*(a+b*ln(c
*(d+e*x^(1/2))^n))/e^6+1/3*x^3*(a+b*ln(c*(d+e*x^(1/2))^n))^2-4*b^2*d^5*n^2*
x^(1/2)/e^5+4*b*d^5*n*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))/e^6+5/2*b^2
*d^4*n^2*(d+e*x^(1/2))^2/e^6-5*b*d^4*n*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(
1/2))^2/e^6-40/27*b^2*d^3*n^2*(d+e*x^(1/2))^3/e^6+40/9*b*d^3*n*(a+b*ln(c*(d
+e*x^(1/2))^n))*(d+e*x^(1/2))^3/e^6+5/8*b^2*d^2*n^2*(d+e*x^(1/2))^4/e^6-5/2
*b*d^2*n*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))^4/e^6-4/25*b^2*d*n^2*(d+
e*x^(1/2))^5/e^6+4/5*b*d*n*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))^5/e^6+
1/54*b^2*n^2*(d+e*x^(1/2))^6/e^6-1/9*b*n*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x
^(1/2))^6/e^6
```

Rubi [A]
time = 0.33, antiderivative size = 480, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2504, 2445, 2458, 45, 2372, 12, 14, 2338}

$$-\frac{2bd^6n\log\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3e^6}+\frac{4bd^5n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^6}-\frac{5bd^4n\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^6}+\frac{40bd^3n\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{9e^6}-\frac{5bd^2n\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2e^6}+\frac{4bdn\left(d+e\sqrt{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{5e^6}-\frac{bn\left(d+e\sqrt{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{9e^6}+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2+\frac{b^2d^6n^2\log^2\left(d+e\sqrt{x}\right)}{3e^6}-\frac{4b^2d^5n^2\sqrt{x}}{e^5}+\frac{5b^2d^4n^2\left(d+e\sqrt{x}\right)^2}{2e^6}-\frac{40b^2d^3n^2\left(d+e\sqrt{x}\right)^3}{27e^6}+\frac{5b^2d^2n^2\left(d+e\sqrt{x}\right)^4}{8e^6}-\frac{4b^2dn^2\left(d+e\sqrt{x}\right)^5}{25e^6}+\frac{b^2n^2\left(d+e\sqrt{x}\right)^6}{54e^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2,x]
```

```
[Out] (5*b^2*d^4*n^2*(d + e*Sqrt[x])^2)/(2*e^6) - (40*b^2*d^3*n^2*(d + e*Sqrt[x])
^3)/(27*e^6) + (5*b^2*d^2*n^2*(d + e*Sqrt[x])^4)/(8*e^6) - (4*b^2*d*n^2*(d
+ e*Sqrt[x])^5)/(25*e^6) + (b^2*n^2*(d + e*Sqrt[x])^6)/(54*e^6) - (4*b^2*d^
5*n^2*Sqrt[x])/e^5 + (b^2*d^6*n^2*Log[d + e*Sqrt[x]]^2)/(3*e^6) + (4*b*d^5*
n*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n]))/e^6 - (5*b*d^4*n*(d + e
*Sqrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n]))/e^6 + (40*b*d^3*n*(d + e*Sqrt
[x])^3*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(9*e^6) - (5*b*d^2*n*(d + e*Sqrt[x
])^4*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(2*e^6) + (4*b*d*n*(d + e*Sqrt[x])^5
*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(5*e^6) - (b*n*(d + e*Sqrt[x])^6*(a + b*
Log[c*(d + e*Sqrt[x])^n]))/(9*e^6) - (2*b*d^6*n*Log[d + e*Sqrt[x]]*(a + b*L
og[c*(d + e*Sqrt[x])^n]))/(3*e^6) + (x^3*(a + b*Log[c*(d + e*Sqrt[x])^n])^2
)/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx = 2\text{Subst}\left(\int x^5(a+b\log(c(d+ex)^n))^2\,dx, x, \sqrt{x}\right)$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2 - \frac{1}{3}(2ben)\text{Subst}\left(\int \frac{x^6(a+b\log(c(}{d+e}\right.$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2 - \frac{1}{3}(2bn)\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^6(a+}{x}\right.$$

$$= \frac{1}{90}bn\left(\frac{360d^5\left(d+e\sqrt{x}\right)}{e^6} - \frac{450d^4\left(d+e\sqrt{x}\right)^2}{e^6} + \frac{400d^3\left(d+e\sqrt{x}\right)^3}{e^6}\right.$$

$$= \frac{1}{90}bn\left(\frac{360d^5\left(d+e\sqrt{x}\right)}{e^6} - \frac{450d^4\left(d+e\sqrt{x}\right)^2}{e^6} + \frac{400d^3\left(d+e\sqrt{x}\right)^3}{e^6}\right.$$

$$= \frac{1}{90}bn\left(\frac{360d^5\left(d+e\sqrt{x}\right)}{e^6} - \frac{450d^4\left(d+e\sqrt{x}\right)^2}{e^6} + \frac{400d^3\left(d+e\sqrt{x}\right)^3}{e^6}\right.$$

$$= \frac{5b^2d^4n^2\left(d+e\sqrt{x}\right)^2}{2e^6} - \frac{40b^2d^3n^2\left(d+e\sqrt{x}\right)^3}{27e^6} + \frac{5b^2d^2n^2\left(d+e\sqrt{x}\right)^4}{8e^6}$$

$$= \frac{5b^2d^4n^2\left(d+e\sqrt{x}\right)^2}{2e^6} - \frac{40b^2d^3n^2\left(d+e\sqrt{x}\right)^3}{27e^6} + \frac{5b^2d^2n^2\left(d+e\sqrt{x}\right)^4}{8e^6}$$

Mathematica [A]
time = 0.24, size = 341, normalized size = 0.71

$$\frac{1800b^2d^6n^2\log^2\left(d+e\sqrt{x}\right)+180bd^6n\log\left(d+e\sqrt{x}\right)\left(-20a+49bn-20b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)+e\sqrt{x}\left(1800a^2e^5x^{5/2}+60abn\left(60d^5-30d^4e\sqrt{x}+20d^3e^2x-15d^2e^3x^{3/2}+12de^4x^2-10e^5x^{5/2}\right)+b^2n^2\left(-8820d^5+2610d^4e\sqrt{x}-1140d^3e^2x+555d^2e^3x^{3/2}-264de^4x^2+100e^5x^{5/2}\right)+60b\left(60ae^5x^{5/2}+bn\left(60d^5-30d^4e\sqrt{x}+20d^3e^2x-15d^2e^3x^{3/2}+12de^4x^2-10e^5x^{5/2}\right)\right)\log\left(c\left(d+e\sqrt{x}\right)^n\right)+1800b^2e^5x^{5/2}\log^2\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{5400e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2,x]
```

```
[Out] (1800*b^2*d^6*n^2*Log[d + e*Sqrt[x]]^2 + 180*b*d^6*n*Log[d + e*Sqrt[x]]*(-2
0*a + 49*b*n - 20*b*Log[c*(d + e*Sqrt[x])^n]) + e*Sqrt[x]*(1800*a^2*e^5*x^(
5/2) + 60*a*b*n*(60*d^5 - 30*d^4*e*Sqrt[x] + 20*d^3*e^2*x - 15*d^2*e^3*x^(3
/2) + 12*d*e^4*x^2 - 10*e^5*x^(5/2)) + b^2*n^2*(-8820*d^5 + 2610*d^4*e*Sqrt
[x] - 1140*d^3*e^2*x + 555*d^2*e^3*x^(3/2) - 264*d*e^4*x^2 + 100*e^5*x^(5/2
```

```
)) + 60*b*(60*a*e^5*x^(5/2) + b*n*(60*d^5 - 30*d^4*e*Sqrt[x] + 20*d^3*e^2*x
 - 15*d^2*e^3*x^(3/2) + 12*d*e^4*x^2 - 10*e^5*x^(5/2)))*Log[c*(d + e*Sqrt[x
])^n] + 1800*b^2*e^5*x^(5/2)*Log[c*(d + e*Sqrt[x])^n]^2))/(5400*e^6)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^{2} \left(a +b \ln \left(c \left(d +e \sqrt{x}\right)^{n}\right)\right)^{2}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(1/2))^n))^2,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e*x^(1/2))^n))^2,x)
```

Maxima [A]
time = 0.29, size = 317, normalized size = 0.66

$$\frac{1}{3} b^{2} x^{3} \log\left(\left(\sqrt{x} e + d\right)^{n} c\right)^{2} + \frac{2}{3} a b x^{3} \log\left(\left(\sqrt{x} e + d\right)^{n} c\right) + \frac{1}{3} a^{2} x^{3} - \frac{1}{90} \left(60 d^{6} e^{(-7)} \log\left(\sqrt{x} e + d\right) + \left(30 d^{4} x e - 60 d^{5} \sqrt{x} - 20 d^{3} x^{\frac{3}{2}} e^{2} + 15 d^{2} x^{2} e^{3} - 12 d x^{\frac{5}{2}} e^{4} + 10 x^{3} e^{5}\right) e^{(-6)}\right) a b n e + \frac{1}{5400} \left(\left(1800 d^{6} \log\left(\sqrt{x} e + d\right)^{2} + 8820 d^{6} \log\left(\sqrt{x} e + d\right) - 8820 d^{5} \sqrt{x} e + 2610 d^{4} x e^{2} - 1140 d^{3} x^{\frac{3}{2}} e^{3} + 555 d^{2} x^{2} e^{4} - 264 d x^{\frac{5}{2}} e^{5} + 100 x^{3} e^{6}\right) n^{2} e^{(-6)} - 60 \left(60 d^{6} e^{(-7)} \log\left(\sqrt{x} e + d\right) + \left(30 d^{4} x e - 60 d^{5} \sqrt{x} - 20 d^{3} x^{\frac{3}{2}} e^{2} + 15 d^{2} x^{2} e^{3} - 12 d x^{\frac{5}{2}} e^{4} + 10 x^{3} e^{5}\right) e^{(-6)}\right) n e \log\left(\left(\sqrt{x} e + d\right)^{n} c\right)\right) b^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3*log((sqrt(x)*e + d)^n*c)^2 + 2/3*a*b*x^3*log((sqrt(x)*e + d)^n*
c) + 1/3*a^2*x^3 - 1/90*(60*d^6*e^(-7)*log(sqrt(x)*e + d) + (30*d^4*x*e - 6
0*d^5*sqrt(x) - 20*d^3*x^(3/2)*e^2 + 15*d^2*x^2*e^3 - 12*d*x^(5/2)*e^4 + 10
*x^3*e^5)*e^(-6))*a*b*n*e + 1/5400*((1800*d^6*log(sqrt(x)*e + d)^2 + 8820*d
^6*log(sqrt(x)*e + d) - 8820*d^5*sqrt(x)*e + 2610*d^4*x*e^2 - 1140*d^3*x^(3
/2)*e^3 + 555*d^2*x^2*e^4 - 264*d*x^(5/2)*e^5 + 100*x^3*e^6)*n^2*e^(-6) - 6
0*(60*d^6*e^(-7)*log(sqrt(x)*e + d) + (30*d^4*x*e - 60*d^5*sqrt(x) - 20*d^3
*x^(3/2)*e^2 + 15*d^2*x^2*e^3 - 12*d*x^(5/2)*e^4 + 10*x^3*e^5)*e^(-6))*n*e*
log((sqrt(x)*e + d)^n*c))*b^2
```

Fricas [A]
time = 0.38, size = 451, normalized size = 0.94

$$\frac{1}{5400} \left(1800 b^{2} x^{3} e^{6} \log\left(c\right)^{2} + 100 \left(b^{2} n^{2} - 6 a b n + 18 a^{2}\right) x^{3} e^{6} + 15 \left(37 b^{2} d^{2} n^{2} - 60 a b d^{2} n\right) x^{2} e^{4} + 90 \left(29 b^{2} d^{4} n^{2} - 20 a b d^{4} n\right) x e^{2} - 1800 \left(b^{2} d^{6} n^{2} - b^{2} n^{2} x^{3} e^{6}\right) \log\left(\sqrt{x} e + d\right)^{2} + 60 \left(147 b^{2} d^{6} n^{2} - 30 b^{2} d^{4} n^{2} x e^{2} - 60 a b d^{6} n - 15 b^{2} d^{2} n^{2} x^{2} e^{4} - 10 \left(b^{2} n^{2} - 6 a b n\right) x^{3} e^{6} - 60 \left(b^{2} d^{6} n - b^{2} n x^{3} e^{6}\right) \log\left(c\right) + 4 \left(15 b^{2} d^{5} n^{2} e + 5 b^{2} d^{3} n^{2} x e^{3} + 3 b^{2} d n^{2} x^{2} e^{5}\right) \sqrt{x}\right) \log\left(\sqrt{x} e + d\right) - 300 \left(6 b^{2} d^{4} n x e^{2} + 3 b^{2} d^{2} n x^{2} e^{4} + 2 \left(b^{2} n - 6 a b\right) x^{3} e^{6}\right) \log\left(c\right) - 12 \left(2 \left(11 b^{2} d n^{2} - 30 a b d n\right) x^{2} e^{5} + 5 \left(19 b^{2} d^{3} n^{2} - 20 a b d^{3} n\right) x e^{3} + 15 \left(49 b^{2} d^{5} n^{2} - 20 a b d^{5} n\right) e - 20 \left(15 b^{2} d^{5} n e + 5 b^{2} d^{3} n x e^{3} + 3 b^{2} d n x^{2} e^{5}\right) \log\left(c\right)\right) \sqrt{x}\right) e^{(-6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="fricas")
```

```
[Out] 1/5400*(1800*b^2*x^3*e^6*log(c)^2 + 100*(b^2*n^2 - 6*a*b*n + 18*a^2)*x^3*e^
6 + 15*(37*b^2*d^2*n^2 - 60*a*b*d^2*n)*x^2*e^4 + 90*(29*b^2*d^4*n^2 - 20*a*
b*d^4*n)*x*e^2 - 1800*(b^2*d^6*n^2 - b^2*n^2*x^3*e^6)*log(sqrt(x)*e + d)^2
+ 60*(147*b^2*d^6*n^2 - 30*b^2*d^4*n^2*x*e^2 - 60*a*b*d^6*n - 15*b^2*d^2*n^
2*x^2*e^4 - 10*(b^2*n^2 - 6*a*b*n)*x^3*e^6 - 60*(b^2*d^6*n - b^2*n*x^3*e^6)
```

```
*log(c) + 4*(15*b^2*d^5*n^2*e + 5*b^2*d^3*n^2*x*e^3 + 3*b^2*d*n^2*x^2*e^5)*
sqrt(x))*log(sqrt(x)*e + d) - 300*(6*b^2*d^4*n*x*e^2 + 3*b^2*d^2*n*x^2*e^4
+ 2*(b^2*n - 6*a*b)*x^3*e^6)*log(c) - 12*(2*(11*b^2*d*n^2 - 30*a*b*d*n)*x^2
*e^5 + 5*(19*b^2*d^3*n^2 - 20*a*b*d^3*n)*x*e^3 + 15*(49*b^2*d^5*n^2 - 20*a*
b*d^5*n)*e - 20*(15*b^2*d^5*n*e + 5*b^2*d^3*n*x*e^3 + 3*b^2*d*n*x^2*e^5)*lo
g(c))*sqrt(x))*e^(-6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + b \log \left(c \left(d + e \sqrt{x}\right)^n\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/2))**n))**2,x)
```

```
[Out] Integral(x**2*(a + b*log(c*(d + e*sqrt(x))**n))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 956 vs. 2(419) = 838.
time = 4.05, size = 956, normalized size = 1.99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="giac")
```

```
[Out] 1/5400*(1800*b^2*x^3*e*log(c)^2 + 3600*a*b*x^3*e*log(c) + 1800*a^2*x^3*e +
(1800*(sqrt(x)*e + d)^6*e^(-5)*log(sqrt(x)*e + d)^2 - 10800*(sqrt(x)*e + d)
^5*d*e^(-5)*log(sqrt(x)*e + d)^2 + 27000*(sqrt(x)*e + d)^4*d^2*e^(-5)*log(s
qrt(x)*e + d)^2 - 36000*(sqrt(x)*e + d)^3*d^3*e^(-5)*log(sqrt(x)*e + d)^2 +
 27000*(sqrt(x)*e + d)^2*d^4*e^(-5)*log(sqrt(x)*e + d)^2 - 10800*(sqrt(x)*e
 + d)*d^5*e^(-5)*log(sqrt(x)*e + d)^2 - 600*(sqrt(x)*e + d)^6*e^(-5)*log(sq
rt(x)*e + d) + 4320*(sqrt(x)*e + d)^5*d*e^(-5)*log(sqrt(x)*e + d) - 13500*(
sqrt(x)*e + d)^4*d^2*e^(-5)*log(sqrt(x)*e + d) + 24000*(sqrt(x)*e + d)^3*d^
3*e^(-5)*log(sqrt(x)*e + d) - 27000*(sqrt(x)*e + d)^2*d^4*e^(-5)*log(sqrt(x
)*e + d) + 21600*(sqrt(x)*e + d)*d^5*e^(-5)*log(sqrt(x)*e + d) + 100*(sqrt(
x)*e + d)^6*e^(-5) - 864*(sqrt(x)*e + d)^5*d*e^(-5) + 3375*(sqrt(x)*e + d)^
4*d^2*e^(-5) - 8000*(sqrt(x)*e + d)^3*d^3*e^(-5) + 13500*(sqrt(x)*e + d)^2*
d^4*e^(-5) - 21600*(sqrt(x)*e + d)*d^5*e^(-5))*b^2*n^2 + 60*(60*(sqrt(x)*e
+ d)^6*e^(-5)*log(sqrt(x)*e + d) - 360*(sqrt(x)*e + d)^5*d*e^(-5)*log(sqrt(
x)*e + d) + 900*(sqrt(x)*e + d)^4*d^2*e^(-5)*log(sqrt(x)*e + d) - 1200*(sqr
t(x)*e + d)^3*d^3*e^(-5)*log(sqrt(x)*e + d) + 900*(sqrt(x)*e + d)^2*d^4*e^(
-5)*log(sqrt(x)*e + d) - 360*(sqrt(x)*e + d)*d^5*e^(-5)*log(sqrt(x)*e + d)
- 10*(sqrt(x)*e + d)^6*e^(-5) + 72*(sqrt(x)*e + d)^5*d*e^(-5) - 225*(sqrt(x
)*e + d)^4*d^2*e^(-5) + 400*(sqrt(x)*e + d)^3*d^3*e^(-5) - 450*(sqrt(x)*e +
```

```
d)^2*d^4*e^(-5) + 360*(sqrt(x)*e + d)*d^5*e^(-5))*b^2*n*log(c) + 60*(60*(s
qrt(x)*e + d)^6*e^(-5)*log(sqrt(x)*e + d) - 360*(sqrt(x)*e + d)^5*d*e^(-5)*
log(sqrt(x)*e + d) + 900*(sqrt(x)*e + d)^4*d^2*e^(-5)*log(sqrt(x)*e + d) -
1200*(sqrt(x)*e + d)^3*d^3*e^(-5)*log(sqrt(x)*e + d) + 900*(sqrt(x)*e + d)^
2*d^4*e^(-5)*log(sqrt(x)*e + d) - 360*(sqrt(x)*e + d)*d^5*e^(-5)*log(sqrt(x
)*e + d) - 10*(sqrt(x)*e + d)^6*e^(-5) + 72*(sqrt(x)*e + d)^5*d*e^(-5) - 22
5*(sqrt(x)*e + d)^4*d^2*e^(-5) + 400*(sqrt(x)*e + d)^3*d^3*e^(-5) - 450*(sq
rt(x)*e + d)^2*d^4*e^(-5) + 360*(sqrt(x)*e + d)*d^5*e^(-5))*a*b*n)*e^(-1)
```

Mupad [B]

time = 1.78, size = 434, normalized size = 0.90

$$\frac{a^2\,x^3}{3}+\frac{b^2\,x^3\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)^2}{3}+\frac{b^2\,n^2\,x^3}{54}+\frac{2\,a\,b\,x^3\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)}{3}-\frac{b^2\,d^6\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)^2}{3\,e^6}-\frac{a\,b\,n\,x^3}{9}-\frac{b^2\,n\,x^3\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)}{9}+\frac{49\,b^2\,d^6\,n^2\,\ln\left(d+e\,\sqrt{x}\right)}{30\,e^6}+\frac{37\,b^2\,d^2\,n^2\,x^2}{360\,e^2}-\frac{19\,b^2\,d^3\,n^2\,x^{3/2}}{90\,e^3}-\frac{49\,b^2\,d^5\,n^2\,\sqrt{x}}{30\,e^5}-\frac{11\,b^2\,d\,n^2\,x^{5/2}}{225\,e}+\frac{29\,b^2\,d^4\,n^2\,x}{60\,e^4}-\frac{b^2\,d^2\,n\,x^2\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)}{6\,e^2}+\frac{2\,b^2\,d^3\,n\,x^{3/2}\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)}{9\,e^3}+\frac{2\,b^2\,d^5\,n\,\sqrt{x}\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)}{3\,e^5}+\frac{2\,a\,b\,d\,n\,x^{5/2}}{15\,e}-\frac{a\,b\,d^4\,n\,x}{3\,e^4}-\frac{2\,a\,b\,d^6\,n\,\ln\left(d+e\,\sqrt{x}\right)}{3\,e^6}+\frac{2\,b^2\,d\,n\,x^{5/2}\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)}{15\,e}-\frac{b^2\,d^4\,n\,x\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)}{3\,e^4}-\frac{a\,b\,d^2\,n\,x^2}{6\,e^2}+\frac{2\,a\,b\,d^3\,n\,x^{3/2}}{9\,e^3}+\frac{2\,a\,b\,d^5\,n\,\sqrt{x}}{3\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e*x^(1/2))^n))^2,x)
```

```
[Out] (a^2*x^3)/3 + (b^2*x^3*log(c*(d + e*x^(1/2))^n)^2)/3 + (b^2*n^2*x^3)/54 + (
2*a*b*x^3*log(c*(d + e*x^(1/2))^n))/3 - (b^2*d^6*log(c*(d + e*x^(1/2))^n)^2
)/(3*e^6) - (a*b*n*x^3)/9 - (b^2*n*x^3*log(c*(d + e*x^(1/2))^n))/9 + (49*b^
2*d^6*n^2*log(d + e*x^(1/2)))/(30*e^6) + (37*b^2*d^2*n^2*x^2)/(360*e^2) - (
19*b^2*d^3*n^2*x^(3/2))/(90*e^3) - (49*b^2*d^5*n^2*x^(1/2))/(30*e^5) - (11*
b^2*d*n^2*x^(5/2))/(225*e) + (29*b^2*d^4*n^2*x)/(60*e^4) - (b^2*d^2*n*x^2*l
og(c*(d + e*x^(1/2))^n))/(6*e^2) + (2*b^2*d^3*n*x^(3/2)*log(c*(d + e*x^(1/2
))^n))/(9*e^3) + (2*b^2*d^5*n*x^(1/2)*log(c*(d + e*x^(1/2))^n))/(3*e^5) + (
2*a*b*d*n*x^(5/2))/(15*e) - (a*b*d^4*n*x)/(3*e^4) - (2*a*b*d^6*n*log(d + e*
x^(1/2)))/(3*e^6) + (2*b^2*d*n*x^(5/2)*log(c*(d + e*x^(1/2))^n))/(15*e) - (
b^2*d^4*n*x*log(c*(d + e*x^(1/2))^n))/(3*e^4) - (a*b*d^2*n*x^2)/(6*e^2) + (
2*a*b*d^3*n*x^(3/2))/(9*e^3) + (2*a*b*d^5*n*x^(1/2))/(3*e^5)
```

3.409 $\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$

Optimal. Leaf size=342

$$\frac{3b^2d^2n^2\left(d+e\sqrt{x}\right)^2}{2e^4}-\frac{4b^2dn^2\left(d+e\sqrt{x}\right)^3}{9e^4}+\frac{b^2n^2\left(d+e\sqrt{x}\right)^4}{16e^4}-\frac{4b^2d^3n^2\sqrt{x}}{e^3}+\frac{b^2d^4n^2\log^2\left(d+e\sqrt{x}\right)}{2e^4}+\frac{4bd^3n}{}$$

[Out] 1/2*b^2*d^4*n^2*ln(d+e*x^(1/2))^2/e^4-b*d^4*n*ln(d+e*x^(1/2))*(a+b*ln(c*(d+e*x^(1/2))^n))/e^4+1/2*x^2*(a+b*ln(c*(d+e*x^(1/2))^n))^2-4*b^2*d^3*n^2*x^(1/2)/e^3+4*b*d^3*n*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))/e^4+3/2*b^2*d^2*n^2*(d+e*x^(1/2))^2/e^4-3*b*d^2*n*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))^2/e^4-4/9*b^2*d*n^2*(d+e*x^(1/2))^3/e^4+4/3*b*d*n*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))^3/e^4+1/16*b^2*n^2*(d+e*x^(1/2))^4/e^4-1/4*b*n*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))^4/e^4

Rubi [A]
time = 0.24, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2504, 2445, 2458, 45, 2372, 12, 14, 2338}

$$-\frac{bd^4n\log\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^4}+\frac{4bd^3n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^4}-\frac{3bd^2n\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^4}+\frac{4bdn\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3e^4}-\frac{bn\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{4e^4}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2+\frac{b^2d^4n^2\log^2\left(d+e\sqrt{x}\right)}{2e^4}-\frac{4b^2d^3n^2\sqrt{x}}{e^3}+\frac{3b^2d^2n^2\left(d+e\sqrt{x}\right)^2}{2e^4}-\frac{4b^2dn^2\left(d+e\sqrt{x}\right)^3}{9e^4}+\frac{b^2n^2\left(d+e\sqrt{x}\right)^4}{16e^4}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*Log[c*(d + e*Sqrt[x])^n])^2,x]

[Out] (3*b^2*d^2*n^2*(d + e*Sqrt[x])^2)/(2*e^4) - (4*b^2*d*n^2*(d + e*Sqrt[x])^3)/(9*e^4) + (b^2*n^2*(d + e*Sqrt[x])^4)/(16*e^4) - (4*b^2*d^3*n^2*Sqrt[x])/e^3 + (b^2*d^4*n^2*Log[d + e*Sqrt[x]]^2)/(2*e^4) + (4*b*d^3*n*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n]))/e^4 - (3*b*d^2*n*(d + e*Sqrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n]))/e^4 + (4*b*d*n*(d + e*Sqrt[x])^3*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(3*e^4) - (b*n*(d + e*Sqrt[x])^4*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(4*e^4) - (b*d^4*n*Log[d + e*Sqrt[x]]*(a + b*Log[c*(d + e*Sqrt[x])^n]))/e^4 + (x^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/2

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx = 2\text{Subst}\left(\int x^3(a+b\log\left(c(d+ex)^n\right))^2\,dx,x,\sqrt{x}\right)$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2-(ben)\text{Subst}\left(\int \frac{x^4(a+b\log\left(c(d+\right.}{d+ex}\right.$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2-(bn)\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^4(a+b\log}{x}\right.$$

$$= \frac{1}{12}bn\left(\frac{48d^3\left(d+e\sqrt{x}\right)}{e^4}-\frac{36d^2\left(d+e\sqrt{x}\right)^2}{e^4}+\frac{16d\left(d+e\sqrt{x}\right)^3}{e^4}-\frac{3(}{}\right.$$

$$= \frac{1}{12}bn\left(\frac{48d^3\left(d+e\sqrt{x}\right)}{e^4}-\frac{36d^2\left(d+e\sqrt{x}\right)^2}{e^4}+\frac{16d\left(d+e\sqrt{x}\right)^3}{e^4}-\frac{3(}{}\right.$$

$$= \frac{1}{12}bn\left(\frac{48d^3\left(d+e\sqrt{x}\right)}{e^4}-\frac{36d^2\left(d+e\sqrt{x}\right)^2}{e^4}+\frac{16d\left(d+e\sqrt{x}\right)^3}{e^4}-\frac{3(}{}\right.$$

$$= \frac{3b^2d^2n^2\left(d+e\sqrt{x}\right)^2}{2e^4}-\frac{4b^2dn^2\left(d+e\sqrt{x}\right)^3}{9e^4}+\frac{b^2n^2\left(d+e\sqrt{x}\right)^4}{16e^4}-\frac{4b}{}$$

$$= \frac{3b^2d^2n^2\left(d+e\sqrt{x}\right)^2}{2e^4}-\frac{4b^2dn^2\left(d+e\sqrt{x}\right)^3}{9e^4}+\frac{b^2n^2\left(d+e\sqrt{x}\right)^4}{16e^4}-\frac{4b}{}$$

Mathematica [**A**]
time = 0.17, size = 269, normalized size = 0.79

$$\frac{72b^2d^4n^2\log^2\left(d+e\sqrt{x}\right)+12bd^4n\log\left(d+e\sqrt{x}\right)\left(-12a+25bn-12b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)+e\sqrt{x}\left(72a^2e^3x^{3/2}+12abn\left(12d^3-6d^2e\sqrt{x}+4de^2x-3e^3x^{3/2}\right)+b^2n^2\left(-300d^3+78d^2e\sqrt{x}-28de^2x+9e^3x^{3/2}\right)+12b\left(12ae^3x^{3/2}+bn\left(12d^3-6d^2e\sqrt{x}+4de^2x-3e^3x^{3/2}\right)\right)\log\left(c\left(d+e\sqrt{x}\right)^n\right)+72b^2e^3x^{3/2}\log^2\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{144e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*Sqrt[x])^n])^2,x]
```

```
[Out] (72*b^2*d^4*n^2*Log[d + e*Sqrt[x]]^2 + 12*b*d^4*n*Log[d + e*Sqrt[x]]*(-12*a
 + 25*b*n - 12*b*Log[c*(d + e*Sqrt[x])^n]) + e*Sqrt[x]*(72*a^2*e^3*x^(3/2)
+ 12*a*b*n*(12*d^3 - 6*d^2*e*Sqrt[x] + 4*d*e^2*x - 3*e^3*x^(3/2)) + b^2*n^2
*(-300*d^3 + 78*d^2*e*Sqrt[x] - 28*d*e^2*x + 9*e^3*x^(3/2)) + 12*b*(12*a*e^
3*x^(3/2) + b*n*(12*d^3 - 6*d^2*e*Sqrt[x] + 4*d*e^2*x - 3*e^3*x^(3/2)))*Log
[c*(d + e*Sqrt[x])^n] + 72*b^2*e^3*x^(3/2)*Log[c*(d + e*Sqrt[x])^n]^2))/(14
4*e^4)
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int x\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e*x^(1/2))^n))^2,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e*x^(1/2))^n))^2,x)
```

Maxima [A]
time = 0.29, size = 256, normalized size = 0.75

$$\frac{1}{2}\,b^2x^2\log\left(\left(\sqrt{x}\,e+d\right)^nc\right)^2-\frac{1}{12}\left(12\,d^4e^{(-5)}\log\left(\sqrt{x}\,e+d\right)+\left(6\,d^2xe-12\,d^3\sqrt{x}-4\,dx^{\frac{3}{2}}e^2+3\,x^2e^3\right)e^{(-4)}\right)abne+abx^2\log\left(\left(\sqrt{x}\,e+d\right)^nc\right)+\frac{1}{2}\,a^2x^2+\frac{1}{144}\left(\left(72\,d^4\log\left(\sqrt{x}\,e+d\right)^2+300\,d^4\log\left(\sqrt{x}\,e+d\right)-300\,d^3\sqrt{x}\,e+78\,d^2xe^2-28\,dx^{\frac{3}{2}}e^3+9\,x^2e^4\right)n^2e^{(-4)}-12\left(12\,d^4e^{(-5)}\log\left(\sqrt{x}\,e+d\right)+\left(6\,d^2xe-12\,d^3\sqrt{x}-4\,dx^{\frac{3}{2}}e^2+3\,x^2e^3\right)e^{(-4)}\right)ne\log\left(\left(\sqrt{x}\,e+d\right)^nc\right)\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*log((sqrt(x)*e + d)^n*c)^2 - 1/12*(12*d^4*e^(-5)*log(sqrt(x)*e
+ d) + (6*d^2*x*e - 12*d^3*sqrt(x) - 4*d*x^(3/2)*e^2 + 3*x^2*e^3)*e^(-4))*a
*b*n*e + a*b*x^2*log((sqrt(x)*e + d)^n*c) + 1/2*a^2*x^2 + 1/144*((72*d^4*lo
g(sqrt(x)*e + d)^2 + 300*d^4*log(sqrt(x)*e + d) - 300*d^3*sqrt(x)*e + 78*d^
2*x*e^2 - 28*d*x^(3/2)*e^3 + 9*x^2*e^4)*n^2*e^(-4) - 12*(12*d^4*e^(-5)*log(
sqrt(x)*e + d) + (6*d^2*x*e - 12*d^3*sqrt(x) - 4*d*x^(3/2)*e^2 + 3*x^2*e^3)
*e^(-4))*n*e*log((sqrt(x)*e + d)^n*c))*b^2
```

Fricas [A]
time = 0.39, size = 333, normalized size = 0.97

$$\frac{1}{144}\left(72\,b^2x^2e^4\log\left(c\right)^2+9\left(b^2n^2-4\,abn+8\,a^2\right)x^2e^4+6\left(13\,b^2d^2n^2-12\,abd^2n\right)xe^2-72\left(b^2d^4n^2-b^2n^2x^2e^4\right)\log\left(\sqrt{x}\,e+d\right)^2+12\left(25\,b^2d^4n^2-6\,b^2d^2n^2xe^2-12\,abd^4n-3\left(b^2n^2-4\,abn\right)x^2e^4-12\left(b^2d^4n-b^2nx^2e^4\right)\log\left(c\right)+4\left(3\,b^2d^3n^2e+b^2dn^2xe^3\right)\sqrt{x}\right)\log\left(\sqrt{x}\,e+d\right)-36\left(2\,b^2d^2nxe^2+\left(b^2n-4\,ab\right)x^2e^4\right)\log\left(c\right)-4\left(\left(7\,b^2dn^2-12\,abdn\right)xe^3+3\left(25\,b^2d^3n^2-12\,abd^3n\right)e-12\left(3\,b^2d^3ne+b^2dnxe^3\right)\log\left(c\right)\right)\sqrt{x}\right)e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="fricas")
```

```
[Out] 1/144*(72*b^2*x^2*e^4*log(c)^2 + 9*(b^2*n^2 - 4*a*b*n + 8*a^2)*x^2*e^4 + 6*
(13*b^2*d^2*n^2 - 12*a*b*d^2*n)*x*e^2 - 72*(b^2*d^4*n^2 - b^2*n^2*x^2*e^4)*
log(sqrt(x)*e + d)^2 + 12*(25*b^2*d^4*n^2 - 6*b^2*d^2*n^2*x*e^2 - 12*a*b*d^
4*n - 3*(b^2*n^2 - 4*a*b*n)*x^2*e^4 - 12*(b^2*d^4*n - b^2*n*x^2*e^4)*log(c)
 + 4*(3*b^2*d^3*n^2*e + b^2*d*n^2*x*e^3)*sqrt(x))*log(sqrt(x)*e + d) - 36*(
2*b^2*d^2*n*x*e^2 + (b^2*n - 4*a*b)*x^2*e^4)*log(c) - 4*((7*b^2*d*n^2 - 12*
a*b*d*n)*x*e^3 + 3*(25*b^2*d^3*n^2 - 12*a*b*d^3*n)*e - 12*(3*b^2*d^3*n*e +
b^2*d*n*x*e^3)*log(c))*sqrt(x))*e^(-4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/2))**n))**2,x)
```

[Out] Integral(x*(a + b*log(c*(d + e*sqrt(x))**n))**2, x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 642 vs. $2(301) = 602$.
time = 4.24, size = 642, normalized size = 1.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="giac")
```

```
[Out] 1/144*(72*b^2*x^2*e*log(c)^2 + 144*a*b*x^2*e*log(c) + (72*(sqrt(x)*e + d)^4
*e^(-3)*log(sqrt(x)*e + d)^2 - 288*(sqrt(x)*e + d)^3*d*e^(-3)*log(sqrt(x)*e
 + d)^2 + 432*(sqrt(x)*e + d)^2*d^2*e^(-3)*log(sqrt(x)*e + d)^2 - 288*(sqrt
(x)*e + d)*d^3*e^(-3)*log(sqrt(x)*e + d)^2 - 36*(sqrt(x)*e + d)^4*e^(-3)*lo
g(sqrt(x)*e + d) + 192*(sqrt(x)*e + d)^3*d*e^(-3)*log(sqrt(x)*e + d) - 432*
(sqrt(x)*e + d)^2*d^2*e^(-3)*log(sqrt(x)*e + d) + 576*(sqrt(x)*e + d)*d^3*e
^(-3)*log(sqrt(x)*e + d) + 9*(sqrt(x)*e + d)^4*e^(-3) - 64*(sqrt(x)*e + d)^
3*d*e^(-3) + 216*(sqrt(x)*e + d)^2*d^2*e^(-3) - 576*(sqrt(x)*e + d)*d^3*e^(
-3))*b^2*n^2 + 72*a^2*x^2*e + 12*(12*(sqrt(x)*e + d)^4*e^(-3)*log(sqrt(x)*e
 + d) - 48*(sqrt(x)*e + d)^3*d*e^(-3)*log(sqrt(x)*e + d) + 72*(sqrt(x)*e +
d)^2*d^2*e^(-3)*log(sqrt(x)*e + d) - 48*(sqrt(x)*e + d)*d^3*e^(-3)*log(sqrt
(x)*e + d) - 3*(sqrt(x)*e + d)^4*e^(-3) + 16*(sqrt(x)*e + d)^3*d*e^(-3) - 3
6*(sqrt(x)*e + d)^2*d^2*e^(-3) + 48*(sqrt(x)*e + d)*d^3*e^(-3))*b^2*n*log(c
) + 12*(12*(sqrt(x)*e + d)^4*e^(-3)*log(sqrt(x)*e + d) - 48*(sqrt(x)*e + d)
^3*d*e^(-3)*log(sqrt(x)*e + d) + 72*(sqrt(x)*e + d)^2*d^2*e^(-3)*log(sqrt(x
)*e + d) - 48*(sqrt(x)*e + d)*d^3*e^(-3)*log(sqrt(x)*e + d) - 3*(sqrt(x)*e
+ d)^4*e^(-3) + 16*(sqrt(x)*e + d)^3*d*e^(-3) - 36*(sqrt(x)*e + d)^2*d^2*e^
(-3) + 48*(sqrt(x)*e + d)*d^3*e^(-3))*a*b*n)*e^(-1)
```

Mupad [B]
time = 0.56, size = 420, normalized size = 1.23

$$x\left(\frac{d\left(\frac{d\left(2a^2-a\,b\,n+\frac{b^2n^2}{4}\right)}{e}-\frac{d\left(6a^2-b^2n^2\right)}{3e}\right)}{2e}+\frac{b^2d^2n^2}{4e^2}\right)-x^{3/2}\left(\frac{d\left(2a^2-a\,b\,n+\frac{b^2n^2}{4}\right)}{3e}-\frac{d\left(6a^2-b^2n^2\right)}{9e}\right)+\ln\left(c\left(d+e\sqrt{x}\right)^n\right)^2\left(\frac{b^2x^2}{2}-\frac{b^2d^4}{2e^4}\right)+x^2\left(\frac{a^2}{2}-\frac{a\,b\,n}{4}+\frac{b^2n^2}{16}\right)-\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\left(x^{3/2}\left(\frac{b\,d\,(4a-b\,n)}{3e}-\frac{4a\,b\,d}{3e}\right)-\frac{b\,x^2\,(4a-b\,n)}{4}+\frac{d^2\sqrt{x}\left(\frac{b\,d\,(4a-b\,n)}{e}-\frac{4a\,b\,d}{e}\right)}{e^2}-\frac{d\,x\left(\frac{b\,d\,(4a-b\,n)}{e}-\frac{4a\,b\,d}{e}\right)}{2e}\right)-\sqrt{x}\left(\frac{d\left(\frac{d\left(\frac{d\left(2a^2-a\,b\,n+\frac{b^2n^2}{4}\right)}{e}-\frac{d\left(6a^2-b^2n^2\right)}{3e}\right)}{e}+\frac{b^2d^2n^2}{2e^2}\right)}{e}+\frac{b^2d^3n^2}{e^3}\right)+\frac{\ln\left(d+e\sqrt{x}\right)\left(25\,b^2d^4n^2-12\,a\,b\,d^4n\right)}{12e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e*x^(1/2))^n))^2,x)
```

```
[Out] x*((d*((d*(2*a^2 + (b^2*n^2)/4 - a*b*n))/e - (d*(6*a^2 - b^2*n^2))/(3*e)))/
(2*e) + (b^2*d^2*n^2)/(4*e^2)) - x^(3/2)*((d*(2*a^2 + (b^2*n^2)/4 - a*b*n))
/(3*e) - (d*(6*a^2 - b^2*n^2))/(9*e)) + log(c*(d + e*x^(1/2))^n)^2*((b^2*x^
2)/2 - (b^2*d^4)/(2*e^4)) + x^2*(a^2/2 + (b^2*n^2)/16 - (a*b*n)/4) - log(c*
(d + e*x^(1/2))^n)*(x^(3/2)*((b*d*(4*a - b*n))/(3*e) - (4*a*b*d)/(3*e)) - (
b*x^2*(4*a - b*n))/4 + (d^2*x^(1/2)*((b*d*(4*a - b*n))/e - (4*a*b*d)/e))/e^
2 - (d*x*((b*d*(4*a - b*n))/e - (4*a*b*d)/e))/(2*e)) - x^(1/2)*((d*((d*((d*
```

```
(2*a^2 + (b^2*n^2)/4 - a*b*n))/e - (d*(6*a^2 - b^2*n^2))/(3*e)))/e + (b^2*d
^2*n^2)/(2*e^2)))/e + (b^2*d^3*n^2)/e^3) + (log(d + e*x^(1/2))*(25*b^2*d^4*
n^2 - 12*a*b*d^4*n))/(12*e^4)
```

3.410 $\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$

Optimal. Leaf size=195

$$\frac{b^2n^2\left(d+e\sqrt{x}\right)^2}{2e^2}+\frac{4abdn\sqrt{x}}{e}-\frac{4b^2dn^2\sqrt{x}}{e}+\frac{4b^2dn\left(d+e\sqrt{x}\right)\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{e^2}-\frac{bn\left(d+e\sqrt{x}\right)^2\left(a+b\log\right.}{e^2}$$

```
[Out] 4*a*b*d*n*x^(1/2)/e-4*b^2*d*n^2*x^(1/2)/e+4*b^2*d*n*ln(c*(d+e*x^(1/2))^n)*(
d+e*x^(1/2))/e^2-2*d*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1/2))/e^2+1/2*b^
2*n^2*(d+e*x^(1/2))^2/e^2-b*n*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))^2/e
^2+(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1/2))^2/e^2
```

Rubi [A]

time = 0.13, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2501, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

$$-\frac{bn\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^2}+\frac{\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^2}-\frac{2d\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^2}+\frac{4abdn\sqrt{x}}{e}+\frac{4b^2dn\left(d+e\sqrt{x}\right)\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{e^2}+\frac{b^2n^2\left(d+e\sqrt{x}\right)^2}{2e^2}-\frac{4b^2dn^2\sqrt{x}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^2,x]
```

```
[Out] (b^2*n^2*(d + e*Sqrt[x])^2)/(2*e^2) + (4*a*b*d*n*Sqrt[x])/e - (4*b^2*d*n^2*
Sqrt[x])/e + (4*b^2*d*n*(d + e*Sqrt[x])*Log[c*(d + e*Sqrt[x])^n])/e^2 - (b*
n*(d + e*Sqrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n]))/e^2 - (2*d*(d + e*Sqr
t[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/e^2 + ((d + e*Sqrt[x])^2*(a + b*L
og[c*(d + e*Sqrt[x])^n])^2)/e^2
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2501

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx = 2\text{Subst}\left(\int x(a+b\log\left(c(d+ex)^n\right))^2\,dx, x, \sqrt{x}\right)$$

$$= 2\text{Subst}\left(\int\left(-\frac{d(a+b\log\left(c(d+ex)^n\right))^2}{e}+\frac{(d+ex)\left(a+b\log\left(c(d+e\right.\right.}{e}\right.\right.$$

$$= \frac{2\text{Subst}\left(\int (d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx, x, \sqrt{x}\right)}{e} - \frac{(2d)\text{Subst}\left(\int\right.}{}$$

$$= \frac{2\text{Subst}\left(\int x(a+b\log\left(cx^n\right))^2\,dx, x, d+e\sqrt{x}\right)}{e^2} - \frac{(2d)\text{Subst}\left(\int (a+b\,\text{lo}\right.}{}$$

$$= -\frac{2d\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^2}+\frac{\left(d+e\sqrt{x}\right)^2\left(a+b\log\right.}{e^2}$$

$$= \frac{b^2n^2\left(d+e\sqrt{x}\right)^2}{2e^2}+\frac{4abdn\sqrt{x}}{e}-\frac{bn\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{\right.\right.\right.}}{e^2}$$

$$= \frac{b^2n^2\left(d+e\sqrt{x}\right)^2}{2e^2}+\frac{4abdn\sqrt{x}}{e}-\frac{4b^2dn^2\sqrt{x}}{e}+\frac{4b^2dn\left(d+e\sqrt{x}\right)\log\left(\right.}{e^2}$$

Mathematica [A]
time = 0.06, size = 150, normalized size = 0.77

$$\frac{-2abn\left(d-e\sqrt{x}\right)^2+b^2en^2\left(-6d+e\sqrt{x}\right)\sqrt{x}-2a^2(d^2-e^2x)+2b\left(d+e\sqrt{x}\right)\left(-2ad+3bdn+2ae\sqrt{x}-ben\sqrt{x}\right)\log\left(c\left(d+e\sqrt{x}\right)^n\right)-2b^2(d^2-e^2x)\log^2\left(c\left(d+e\sqrt{x}\right)^n\right)}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^2,x]
```

```
[Out] (-2*a*b*n*(d - e*Sqrt[x])^2 + b^2*e*n^2*(-6*d + e*Sqrt[x])*Sqrt[x] - 2*a^2*
(d^2 - e^2*x) + 2*b*(d + e*Sqrt[x])*(-2*a*d + 3*b*d*n + 2*a*e*Sqrt[x] - b*e
*n*Sqrt[x])*Log[c*(d + e*Sqrt[x])^n] - 2*b^2*(d^2 - e^2*x)*Log[c*(d + e*Sqr
t[x])^n]^2)/(2*e^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))^2,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))^2,x)
```

Maxima [A]
time = 0.28, size = 184, normalized size = 0.94

$$-\left(\left(2\,d^2e^{(-3)}\log\left(\sqrt{x}e+d\right)+\left(xe-2\,d\sqrt{x}\right)e^{(-2)}\right)ne-2\,x\log\left(\left(\sqrt{x}e+d\right)^nc\right)\right)ab+\frac{1}{2}\left(\left(2\,d^2\log\left(\sqrt{x}e+d\right)^2+6\,d^2\log\left(\sqrt{x}e+d\right)-6\,d\sqrt{x}e+xe^2\right)n^2e^{(-2)}-2\left(2\,d^2e^{(-3)}\log\left(\sqrt{x}e+d\right)+\left(xe-2\,d\sqrt{x}\right)e^{(-2)}\right)ne\log\left(\left(\sqrt{x}e+d\right)^nc\right)+2\,x\log\left(\left(\sqrt{x}e+d\right)^nc\right)^2\right)b^2+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="maxima")
```

```
[Out] -((2*d^2*e^(-3)*log(sqrt(x)*e + d) + (x*e - 2*d*sqrt(x))*e^(-2))*n*e - 2*x*
log((sqrt(x)*e + d)^n*c))*a*b + 1/2*((2*d^2*log(sqrt(x)*e + d)^2 + 6*d^2*lo
g(sqrt(x)*e + d) - 6*d*sqrt(x)*e + x*e^2)*n^2*e^(-2) - 2*(2*d^2*e^(-3)*log(
sqrt(x)*e + d) + (x*e - 2*d*sqrt(x))*e^(-2))*n*e*log((sqrt(x)*e + d)^n*c) +
 2*x*log((sqrt(x)*e + d)^n*c)^2)*b^2 + a^2*x
```

Fricas [A]
time = 0.40, size = 213, normalized size = 1.09

$$\frac{1}{2}\left(2\,b^2xe^2\log\left(c\right)^2-2\,\left(b^2n-2\,ab\right)xe^2\log\left(c\right)+\left(b^2n^2-2\,abn+2\,a^2\right)xe^2-2\,\left(b^2d^2n^2-b^2n^2xe^2\right)\log\left(\sqrt{x}\,e+d\right)^2+2\,\left(2\,b^2dn^2\sqrt{x}\,e+3\,b^2d^2n^2-2\,abd^2n-\left(b^2n^2-2\,abn\right)xe^2-2\,\left(b^2d^2n-b^2nxe^2\right)\log\left(c\right)\right)\log\left(\sqrt{x}\,e+d\right)+2\,\left(2\,b^2dne\log\left(c\right)-\left(3\,b^2dn^2-2\,abdn\right)e\right)\sqrt{x}\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*x*e^2*log(c)^2 - 2*(b^2*n - 2*a*b)*x*e^2*log(c) + (b^2*n^2 - 2*a
*b*n + 2*a^2)*x*e^2 - 2*(b^2*d^2*n^2 - b^2*n^2*x*e^2)*log(sqrt(x)*e + d)^2
+ 2*(2*b^2*d*n^2*sqrt(x)*e + 3*b^2*d^2*n^2 - 2*a*b*d^2*n - (b^2*n^2 - 2*a*b
*n)*x*e^2 - 2*(b^2*d^2*n - b^2*n*x*e^2)*log(c))*log(sqrt(x)*e + d) + 2*(2*b
^2*d*n*e*log(c) - (3*b^2*d*n^2 - 2*a*b*d*n)*e)*sqrt(x))*e^(-2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \log \left(c \left(d + e \sqrt{x}\right)^{n}\right)\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 361 vs. 2(173) = 346.
time = 5.50, size = 361, normalized size = 1.85

$$\frac{1}{2}\left(\left(2\left(\sqrt{x}e+d\right)^2\log\left(\sqrt{x}e+d\right)^2-4\left(\sqrt{x}e+d\right)d\log\left(\sqrt{x}e+d\right)^2-2\left(\sqrt{x}e+d\right)^2\log\left(\sqrt{x}e+d\right)+8\left(\sqrt{x}e+d\right)d\log\left(\sqrt{x}e+d\right)+\left(\sqrt{x}e+d\right)^2-8\left(\sqrt{x}e+d\right)d\right)b^2n^2e^{(-1)}+2\left(2\left(\sqrt{x}e+d\right)^2\log\left(\sqrt{x}e+d\right)-4\left(\sqrt{x}e+d\right)d\log\left(\sqrt{x}e+d\right)-\left(\sqrt{x}e+d\right)^2+4\left(\sqrt{x}e+d\right)d\right)b^2ne^{(-1)}\log\left(c\right)+2\left(\left(\sqrt{x}e+d\right)^2-2\left(\sqrt{x}e+d\right)d\right)b^2e^{(-1)}\log\left(c\right)^2+2\left(2\left(\sqrt{x}e+d\right)^2\log\left(\sqrt{x}e+d\right)-4\left(\sqrt{x}e+d\right)d\log\left(\sqrt{x}e+d\right)-\left(\sqrt{x}e+d\right)^2+4\left(\sqrt{x}e+d\right)d\right)abne^{(-1)}+4\left(\left(\sqrt{x}e+d\right)^2-2\left(\sqrt{x}e+d\right)d\right)abe^{(-1)}\log\left(c\right)+2\left(\left(\sqrt{x}e+d\right)^2-2\left(\sqrt{x}e+d\right)d\right)a^2e^{(-1)}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="giac")
```

```
[Out] 1/2*((2*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d)^2 - 4*(sqrt(x)*e + d)*d*log(sq
rt(x)*e + d)^2 - 2*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d) + 8*(sqrt(x)*e + d)
*d*log(sqrt(x)*e + d) + (sqrt(x)*e + d)^2 - 8*(sqrt(x)*e + d)*d)*b^2*n^2*e^
```

```
(-1) + 2*(2*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d) - 4*(sqrt(x)*e + d)*d*log(
sqrt(x)*e + d) - (sqrt(x)*e + d)^2 + 4*(sqrt(x)*e + d)*d)*b^2*n*e^(-1)*log(
c) + 2*((sqrt(x)*e + d)^2 - 2*(sqrt(x)*e + d)*d)*b^2*e^(-1)*log(c)^2 + 2*(2
*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d) - 4*(sqrt(x)*e + d)*d*log(sqrt(x)*e +
 d) - (sqrt(x)*e + d)^2 + 4*(sqrt(x)*e + d)*d)*a*b*n*e^(-1) + 4*((sqrt(x)*e
 + d)^2 - 2*(sqrt(x)*e + d)*d)*a*b*e^(-1)*log(c) + 2*((sqrt(x)*e + d)^2 - 2
*(sqrt(x)*e + d)*d)*a^2*e^(-1))*e^(-1)
```

Mupad [B]
time = 0.47, size = 186, normalized size = 0.95

$$x\left(a^2-a\,b\,n+\frac{b^2\,n^2}{2}\right)-\sqrt{x}\,\left(\frac{d\,\left(2\,a^2-2\,a\,b\,n+b^2\,n^2\right)}{e}-\frac{2\,d\,\left(a^2-b^2\,n^2\right)}{e}\right)+\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)^2\,\left(b^2\,x-\frac{b^2\,d^2}{e^2}\right)-\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)\,\left(\sqrt{x}\,\left(\frac{2\,b\,d\,\left(2\,a-b\,n\right)}{e}-\frac{4\,a\,b\,d}{e}\right)-b\,x\,\left(2\,a-b\,n\right)\right)+\frac{\ln\left(d+e\,\sqrt{x}\right)\,\left(3\,b^2\,d^2\,n^2-2\,a\,b\,d^2\,n\right)}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))^2,x)
```

```
[Out] x*(a^2 + (b^2*n^2)/2 - a*b*n) - x^(1/2)*((d*(2*a^2 + b^2*n^2 - 2*a*b*n))/e
- (2*d*(a^2 - b^2*n^2))/e) + log(c*(d + e*x^(1/2))^n)^2*(b^2*x - (b^2*d^2)/
e^2) - log(c*(d + e*x^(1/2))^n)*(x^(1/2)*((2*b*d*(2*a - b*n))/e - (4*a*b*d)
/e) - b*x*(2*a - b*n)) + (log(d + e*x^(1/2))*(3*b^2*d^2*n^2 - 2*a*b*d^2*n))
/e^2
```

3.411 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x}\,dx$

Optimal. Leaf size=93

$$2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt{x}}{d}\right)+4bn\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\text{Li}_2\left(1+\frac{e\sqrt{x}}{d}\right)-4b^2n^2\text{Li}_3\left(1\right.$$

```
[Out] 2*ln(-e*x^(1/2)/d)*(a+b*ln(c*(d+e*x^(1/2))^n))^2+4*b*n*(a+b*ln(c*(d+e*x^(1/
2))^n))*polylog(2,1+e*x^(1/2)/d)-4*b^2*n^2*polylog(3,1+e*x^(1/2)/d)
```

Rubi [A]
time = 0.09, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2504, 2443, 2481, 2421, 6724}

$$4bn\text{PolyLog}\left(2,\frac{e\sqrt{x}}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)-4b^2n^2\text{PolyLog}\left(3,\frac{e\sqrt{x}}{d}+1\right)+2\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x,x]
```

```
[Out] 2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2*Log[-((e*Sqrt[x])/d)] + 4*b*n*(a + b*L
og[c*(d + e*Sqrt[x])^n])*PolyLog[2, 1 + (e*Sqrt[x])/d] - 4*b^2*n^2*PolyLog[
3, 1 + (e*Sqrt[x])/d]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
```

```
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x}\,dx = 2\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x}\,dx, x, \sqrt{x}\right)$$

$$= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt{x}}{d}\right) - (4ben)\text{Subst}\left(\int \frac{\log\left(-\right.}{}\right.$$

$$= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt{x}}{d}\right) - (4bn)\text{Subst}\left(\int \frac{(a+b\,\mathrm{l}}{}\right.$$

$$= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt{x}}{d}\right) + 4bn\left(a+b\log\left(c\left(d+e\sqrt{\right.\right.\right.$$

$$= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt{x}}{d}\right) + 4bn\left(a+b\log\left(c\left(d+e\sqrt{\right.\right.\right.$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 195 vs. 2(93) = 186.
time = 0.08, size = 195, normalized size = 2.10

$$\left(a-bn\log\left(d+e\sqrt{x}\right)+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\log(x)+2bn\left(a-bn\log\left(d+e\sqrt{x}\right)+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\left(\left(\log\left(d+e\sqrt{x}\right)-\log\left(1+\frac{e\sqrt{x}}{d}\right)\right)\log(x)-2\text{Li}_2\left(-\frac{e\sqrt{x}}{d}\right)\right)+2b^2n^2\left(\log^2\left(d+e\sqrt{x}\right)\log\left(-\frac{e\sqrt{x}}{d}\right)+2\log\left(d+e\sqrt{x}\right)\text{Li}_2\left(1+\frac{e\sqrt{x}}{d}\right)-2\text{Li}_3\left(1+\frac{e\sqrt{x}}{d}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x,x]
```

```
[Out] (a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^2*Log[x] + 2*b*n*
(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])*((Log[d + e*Sqrt[
x]] - Log[1 + (e*Sqrt[x])/d])*Log[x] - 2*PolyLog[2, -((e*Sqrt[x])/d)]) + 2*
b^2*n^2*(Log[d + e*Sqrt[x]]^2*Log[-((e*Sqrt[x])/d)] + 2*Log[d + e*Sqrt[x]]*
PolyLog[2, 1 + (e*Sqrt[x])/d] - 2*PolyLog[3, 1 + (e*Sqrt[x])/d])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\sqrt{x}\,\right)^{n}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))^2/x,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))^2/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x,x, algorithm="maxima")
```

```
[Out] b^2*n^2*log(sqrt(x)*e + d)^2*log(x) + integrate(((b^2*log(c)^2 + 2*a*b*log(
c) + a^2)*x*e - (b^2*n*x*e*log(x) - 2*(b^2*log(c) + a*b)*x*e - 2*(b^2*d*log
(c) + a*b*d)*sqrt(x))*n*log(sqrt(x)*e + d) + (b^2*d*log(c)^2 + 2*a*b*d*log(
c) + a^2*d)*sqrt(x))/(x^2*e + d*x^(3/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((sqrt(x)*e + d)^n*c)^2 + 2*a*b*log((sqrt(x)*e + d)^n*c) +
 a^2)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\,\right)^{n}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**2/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))**2/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((sqrt(x)*e + d)^n*c) + a)^2/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))^2/x,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/2))^n))^2/x, x)
```

3.412 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=155

$$-\frac{2ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2\sqrt{x}}-\frac{2be^2n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2}-\frac{(a-}{}$$

```
[Out] b^2*e^2*n^2*ln(x)/d^2-(a+b*ln(c*(d+e*x^(1/2))^n))^2/x-2*b*e^2*n*(a+b*ln(c*(
d+e*x^(1/2))^n))*ln(1-d/(d+e*x^(1/2)))/d^2+2*b^2*e^2*n^2*polylog(2,d/(d+e*x
^(1/2)))/d^2-2*b*e*n*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))/d^2/x^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2504, 2445, 2458, 2389, 2379, 2438, 2351, 31}

$$\frac{2b^2e^2n^2\text{PolyLog}\left(2,\frac{d}{d+e\sqrt{x}}\right)}{d^2}-\frac{2be^2n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2}-\frac{2ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2\sqrt{x}}-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x}+\frac{b^2e^2n^2\log(x)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^2,x]
```

```
[Out] (-2*b*e*n*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(d^2*Sqrt[x]) -
 (2*b*e^2*n*Log[1 - d/(d + e*Sqrt[x])]*(a + b*Log[c*(d + e*Sqrt[x])^n]))/d^
2 - (a + b*Log[c*(d + e*Sqrt[x])^n])^2/x + (b^2*e^2*n^2*Log[x])/d^2 + (2*b^
2*e^2*n^2*PolyLog[2, d/(d + e*Sqrt[x])])/d^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
```

```
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^2}\,dx = 2\text{Subst}\left(\int \frac{(a+b\log\left(c(d+ex)^n\right))^2}{x^3}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x} + (2ben)\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex\right.}{x^2(d+ex)}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x} + (2bn)\text{Subst}\left(\int \frac{a+b\log\left(cx^n\right)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x} + \frac{(2bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+\right.}{d}$$

$$= -\frac{2ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2\sqrt{x}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{\right.\right.\right.}{x}$$

$$= -\frac{2ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2\sqrt{x}} + \frac{e^2\left(a+b\log\left(c\left(d+\right.\right.\right.}{d^2}$$

$$= -\frac{2ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2\sqrt{x}} + \frac{e^2\left(a+b\log\left(c\left(d+\right.\right.\right.}{d^2}$$

Mathematica [A]
time = 0.12, size = 188, normalized size = 1.21

$$2\left(-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2x}+ben\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{d\sqrt{x}}+\frac{e\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2bd^2n}-\frac{e\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt{x}}{d}\right)}{d^2}+\frac{ben\left(-\frac{\log\left(d+e\sqrt{x}\right)}{d}+\frac{\log(x)}{2d}\right)}{d}-\frac{ben\text{Li}_2\left(\frac{d+e\sqrt{x}}{d}\right)}{d^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^2,x]
```

```
[Out] 2*(-1/2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x + b*e*n*(-((a + b*Log[c*(d + e
*Sqrt[x])^n])/(d*Sqrt[x])) + (e*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(2*b*d^
2*n) - (e*(a + b*Log[c*(d + e*Sqrt[x])^n])*Log[-((e*Sqrt[x])/d)])/d^2 + (b*
e*n*(-(Log[d + e*Sqrt[x]]/d) + Log[x]/(2*d)))/d - (b*e*n*PolyLog[2, (d + e*
Sqrt[x])/d])/d^2))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))^2/x^2,x)
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))^2/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^2,x, algorithm="maxima")
[Out] 2*(log(sqrt(x))*log(e^(1/2*log(x) + 1)/d + 1) + dilog(-e^(1/2*log(x) + 1)/d
))*b^2*n^2*e^2/d^2 - 2*((n^2 - n*log(c))*b^2 - a*b*n)*e^2*log(d + e^(1/2*lo
g(x) + 1))/d^2 - 2*(b^2*n*log(c) + a*b*n)*e^2*log(sqrt(x))/d^2 + integrate(
(b^2*d^2*n^2*e^2 + b^2*n^2*x*e^4)/x, x)/d^4 - 1/3*(6*b^2*d^2*n^2*e^(1/2*log
(x) + 3)*log(sqrt(x)) - 12*b^2*d^2*n^2*e^(1/2*log(x) + 3) + 3*b^2*d*n^2*e^(
log(x) + 4) - 2*b^2*n^2*e^(3/2*log(x) + 5))/d^5 - 1/3*(3*b^2*d^5*n^2*sqrt(x
)*log(sqrt(x)*e + d)^2 + 3*b^2*d^3*n^2*x^(3/2)*e^2*log(sqrt(x)*e + d)^2 - 3
*b^2*d^2*n^2*x^2*e^3*log(x) + 12*b^2*d^2*n^2*x^2*e^3 + 2*b^2*n^2*x^3*e^5 +
6*(b^2*d^4*n*log(c) + a*b*d^4*n)*x*e - 3*(2*b^2*d^3*n*x^(3/2)*e^2*log(sqrt(
x)*e + d) - 2*b^2*d^4*n*x*e - (b^2*d^3*n*x*e^2*log(x) + 2*b^2*d^5*log(c) +
2*a*b*d^5)*sqrt(x))*n*log(sqrt(x)*e + d))/(d^5*x^(3/2))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^2,x, algorithm="fricas")
[Out] integral((b^2*log((sqrt(x)*e + d)^n*c)^2 + 2*a*b*log((sqrt(x)*e + d)^n*c) +
 a^2)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \log \left(c \left(d + e \sqrt{x}\right)^{n}\right)\right)^{2}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**2/x**2,x)
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))**2/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^2,x, algorithm="giac")
[Out] integrate((b*log((sqrt(x)*e + d)^n*c) + a)^2/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,\sqrt{x}\,\right)}^n\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))^2/x^2,x)
[Out] int((a + b*log(c*(d + e*x^(1/2))^n))^2/x^2, x)
```

3.413 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=293

$$-\frac{b^2e^2n^2}{6d^2x}+\frac{5b^2e^3n^2}{6d^3\sqrt{x}}-\frac{5b^2e^4n^2\log\left(d+e\sqrt{x}\right)}{6d^4}-\frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3dx^{3/2}}+\frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right.\right.}{2d^2x}$$

```
[Out] -1/6*b^2*e^2*n^2/d^2/x+11/12*b^2*e^4*n^2*ln(x)/d^4-5/6*b^2*e^4*n^2*ln(d+e*x
^(1/2))/d^4-1/3*b*e*n*(a+b*ln(c*(d+e*x^(1/2))^n))/d/x^(3/2)+1/2*b*e^2*n*(a+
b*ln(c*(d+e*x^(1/2))^n))/d^2/x-1/2*(a+b*ln(c*(d+e*x^(1/2))^n))^2/x^2-b*e^4*
n*(a+b*ln(c*(d+e*x^(1/2))^n))*ln(1-d/(d+e*x^(1/2)))/d^4+b^2*e^4*n^2*polylog
(2,d/(d+e*x^(1/2)))/d^4+5/6*b^2*e^3*n^2/d^3/x^(1/2)-b*e^3*n*(a+b*ln(c*(d+e*
x^(1/2))^n))*(d+e*x^(1/2))/d^4/x^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2504, 2445, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$\frac{b^2e^4n^2\text{PolyLog}\left(2,\frac{d}{d+e\sqrt{x}}\right)}{d^4}-\frac{be^4n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^4}-\frac{be^3n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^4\sqrt{x}}+\frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^2x}-\frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3dx^{3/2}}-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2x^2}-\frac{5b^2e^4n^2\log\left(d+e\sqrt{x}\right)}{6d^4}+\frac{11b^2e^4n^2\log(x)}{12d^4}+\frac{5b^2e^3n^2}{6d^3\sqrt{x}}-\frac{b^2e^2n^2}{6d^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^3,x]
```

```
[Out] -1/6*(b^2*e^2*n^2)/(d^2*x) + (5*b^2*e^3*n^2)/(6*d^3*Sqrt[x]) - (5*b^2*e^4*n
^2*Log[d + e*Sqrt[x]])/(6*d^4) - (b*e*n*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(
3*d*x^(3/2)) + (b*e^2*n*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(2*d^2*x) - (b*e^
3*n*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(d^4*Sqrt[x]) - (b*e^
4*n*Log[1 - d/(d + e*Sqrt[x])]*(a + b*Log[c*(d + e*Sqrt[x])^n]))/d^4 - (a +
 b*Log[c*(d + e*Sqrt[x])^n])^2/(2*x^2) + (11*b^2*e^4*n^2*Log[x])/(12*d^4) +
 (b^2*e^4*n^2*PolyLog[2, d/(d + e*Sqrt[x])])/d^4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^3}\,dx = 2\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x^5}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2x^2} + (ben)\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right.}{x^4(d+ex)}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2x^2} + (bn)\text{Subst}\left(\int \frac{a+b\log\left(cx^n\right)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x,\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2x^2} + \frac{(bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+e\right.}{d}$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3dx^{3/2}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2x^2} - \cdots$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3dx^{3/2}} + \frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^2x} - \cdots$$

$$= -\frac{b^2e^2n^2}{6d^2x} + \frac{b^2e^3n^2}{3d^3\sqrt{x}} - \frac{b^2e^4n^2\log\left(d+e\sqrt{x}\right)}{3d^4} - \frac{ben\left(a+b\log\left(c\left(d+\cdots\right.\right.\right.}{3dx^{3/2}}$$

$$= -\frac{b^2e^2n^2}{6d^2x} + \frac{5b^2e^3n^2}{6d^3\sqrt{x}} - \frac{5b^2e^4n^2\log\left(d+e\sqrt{x}\right)}{6d^4} - \frac{ben\left(a+b\log\left(c\left(d+\cdots\right.\right.\right.}{3dx^{3/2}}$$

$$= -\frac{b^2e^2n^2}{6d^2x} + \frac{5b^2e^3n^2}{6d^3\sqrt{x}} - \frac{5b^2e^4n^2\log\left(d+e\sqrt{x}\right)}{6d^4} - \frac{ben\left(a+b\log\left(c\left(d+\cdots\right.\right.\right.}{3dx^{3/2}}$$

Mathematica [A]
time = 0.23, size = 353, normalized size = 1.20

$$-\frac{6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2+\frac{e\sqrt{x}\left(4bd^3n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)-6bd^2en\sqrt{x}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)+12bde^2nx\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)-6e^3x^{3/2}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2+12be^3nx^{3/2}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt{x}}{d}\right)+6b^2e^3n^2x^{3/2}\left(2\log\left(d+e\sqrt{x}\right)-\log(x)\right)-3b^2e^2n^2x\left(2d-2e\sqrt{x}\log\left(d+e\sqrt{x}\right)+e\sqrt{x}\log(x)\right)+2b^2en^2\sqrt{x}\left(d\left(d-2e\sqrt{x}\right)+2e^2x\log\left(d+e\sqrt{x}\right)-e^2x\log(x)\right)+12b^2e^3n^2x^{3/2}\text{Li}_2\left(1+\frac{e\sqrt{x}}{d}\right)\right)}{d^4}}{12x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^3,x]
```

```
[Out] -1/12*(6*(a + b*Log[c*(d + e*Sqrt[x])^n])^2 + (e*Sqrt[x]*(4*b*d^3*n*(a + b*
Log[c*(d + e*Sqrt[x])^n]) - 6*b*d^2*e*n*Sqrt[x]*(a + b*Log[c*(d + e*Sqrt[x]
)^n]) + 12*b*d*e^2*n*x*(a + b*Log[c*(d + e*Sqrt[x])^n]) - 6*e^3*x^(3/2)*(a
+ b*Log[c*(d + e*Sqrt[x])^n])^2 + 12*b*e^3*n*x^(3/2)*(a + b*Log[c*(d + e*Sq
```

```
rt[x])^n])*Log[-((e*Sqrt[x])/d)] + 6*b^2*e^3*n^2*x^(3/2)*(2*Log[d + e*Sqrt[
x]] - Log[x]) - 3*b^2*e^2*n^2*x*(2*d - 2*e*Sqrt[x]*Log[d + e*Sqrt[x]] + e*S
qrt[x]*Log[x]) + 2*b^2*e*n^2*Sqrt[x]*(d*(d - 2*e*Sqrt[x]) + 2*e^2*x*Log[d +
 e*Sqrt[x]] - e^2*x*Log[x]) + 12*b^2*e^3*n^2*x^(3/2)*PolyLog[2, 1 + (e*Sqrt
[x])/d]))/d^4)/x^2
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\ln\left(c\left(d + e\sqrt{x}\,\right)^{n}\right)\right)^{2}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))^2/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))^2/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b^2*n^2*log(sqrt(x)*e + d)^2/x^2 + integrate(1/2*(2*(b^2*log(c)^2 + 2*
a*b*log(c) + a^2)*x*e + (b^2*n*x*e + 4*(b^2*log(c) + a*b)*x*e + 4*(b^2*d*lo
g(c) + a*b*d)*sqrt(x))*n*log(sqrt(x)*e + d) + 2*(b^2*d*log(c)^2 + 2*a*b*d*l
og(c) + a^2*d)*sqrt(x))/(x^4*e + d*x^(7/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((sqrt(x)*e + d)^n*c)^2 + 2*a*b*log((sqrt(x)*e + d)^n*c) +
 a^2)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\log\left(c\left(d + e\sqrt{x}\,\right)^{n}\right)\right)^{2}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**2/x**3,x)
```

```
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))**2/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((sqrt(x)*e + d)^n*c) + a)^2/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,\sqrt{x}\,\right)}^n\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))^2/x^3,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/2))^n))^2/x^3, x)
```

3.414 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^4}\,dx$

Optimal. Leaf size=408

$$-\frac{b^2e^2n^2}{30d^2x^2}+\frac{b^2e^3n^2}{10d^3x^{3/2}}-\frac{47b^2e^4n^2}{180d^4x}+\frac{77b^2e^5n^2}{90d^5\sqrt{x}}-\frac{77b^2e^6n^2\log\left(d+e\sqrt{x}\right)}{90d^6}-\frac{2ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{15dx^{5/2}}+\frac{be^2\ldots}{}$$

```
[Out] -1/30*b^2*e^2*n^2/d^2/x^2+1/10*b^2*e^3*n^2/d^3/x^(3/2)-47/180*b^2*e^4*n^2/d
^4/x+137/180*b^2*e^6*n^2*ln(x)/d^6-77/90*b^2*e^6*n^2*ln(d+e*x^(1/2))/d^6-2/
15*b*e*n*(a+b*ln(c*(d+e*x^(1/2))^n))/d/x^(5/2)+1/6*b*e^2*n*(a+b*ln(c*(d+e*x
^(1/2))^n))/d^2/x^2-2/9*b*e^3*n*(a+b*ln(c*(d+e*x^(1/2))^n))/d^3/x^(3/2)+1/3
*b*e^4*n*(a+b*ln(c*(d+e*x^(1/2))^n))/d^4/x-1/3*(a+b*ln(c*(d+e*x^(1/2))^n))^
2/x^3-2/3*b*e^6*n*(a+b*ln(c*(d+e*x^(1/2))^n))*ln(1-d/(d+e*x^(1/2)))/d^6+2/3
*b^2*e^6*n^2*polylog(2,d/(d+e*x^(1/2)))/d^6+77/90*b^2*e^5*n^2/d^5/x^(1/2)-2
/3*b*e^5*n*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))/d^6/x^(1/2)
```

Rubi [A]
time = 0.62, antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2504, 2445, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$\frac{2b^2e^6n^2\text{PolyLog}\left(2,\frac{d}{d+e\sqrt{x}}\right)}{3d^6}-\frac{2be^6n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3d^6}-\frac{2be^5n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3d^6\sqrt{x}}+\frac{be^4n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3d^4x}-\frac{2be^3n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{9d^3x^{3/2}}+\frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{6d^2x^2}-\frac{2ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{15dx^{5/2}}-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{3x^3}-\frac{77b^2e^6n^2\log\left(d+e\sqrt{x}\right)}{90d^6}+\frac{137b^2e^6n^2\log(x)}{180d^6}+\frac{77b^2e^5n^2}{90d^5\sqrt{x}}-\frac{47b^2e^4n^2}{180d^4x}+\frac{b^2e^3n^2}{10d^3x^{3/2}}-\frac{b^2e^2n^2}{30d^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^4,x]
```

```
[Out] -1/30*(b^2*e^2*n^2)/(d^2*x^2) + (b^2*e^3*n^2)/(10*d^3*x^(3/2)) - (47*b^2*e^
4*n^2)/(180*d^4*x) + (77*b^2*e^5*n^2)/(90*d^5*Sqrt[x]) - (77*b^2*e^6*n^2*Lo
g[d + e*Sqrt[x]])/(90*d^6) - (2*b*e*n*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(15
*d*x^(5/2)) + (b*e^2*n*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(6*d^2*x^2) - (2*b
*e^3*n*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(9*d^3*x^(3/2)) + (b*e^4*n*(a + b*
Log[c*(d + e*Sqrt[x])^n]))/(3*d^4*x) - (2*b*e^5*n*(d + e*Sqrt[x])*(a + b*Lo
g[c*(d + e*Sqrt[x])^n]))/(3*d^6*Sqrt[x]) - (2*b*e^6*n*Log[1 - d/(d + e*Sqrt
[x])]*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(3*d^6) - (a + b*Log[c*(d + e*Sqrt[
x])^n])^2/(3*x^3) + (137*b^2*e^6*n^2*Log[x])/(180*d^6) + (2*b^2*e^6*n^2*Pol
yLog[2, d/(d + e*Sqrt[x])])/(3*d^6)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^4}\,dx = 2\text{Subst}\left(\int \frac{(a+b\log\left(c(d+ex)^n\right))^2}{x^7}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{3x^3} + \frac{1}{3}(2ben)\text{Subst}\left(\int \frac{a+b\log\left(c(d+e\right.}{x^6(d+ex)}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{3x^3} + \frac{1}{3}(2bn)\text{Subst}\left(\int \frac{a+b\log\left(cx^n\right)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx,\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{3x^3} + \frac{(2bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+\right.}{3d}$$

$$= -\frac{2ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{15dx^{5/2}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{3x^3} -$$

$$= -\frac{2ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{15dx^{5/2}} + \frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{6d^2x^2}$$

$$= -\frac{b^2e^2n^2}{30d^2x^2} + \frac{2b^2e^3n^2}{45d^3x^{3/2}} - \frac{b^2e^4n^2}{15d^4x} + \frac{2b^2e^5n^2}{15d^5\sqrt{x}} - \frac{2b^2e^6n^2\log\left(d+e\sqrt{x}\right)}{15d^6}$$

$$= -\frac{b^2e^2n^2}{30d^2x^2} + \frac{b^2e^3n^2}{10d^3x^{3/2}} - \frac{3b^2e^4n^2}{20d^4x} + \frac{3b^2e^5n^2}{10d^5\sqrt{x}} - \frac{3b^2e^6n^2\log\left(d+e\sqrt{x}\right.}{10d^6}$$

$$= -\frac{b^2e^2n^2}{30d^2x^2} + \frac{b^2e^3n^2}{10d^3x^{3/2}} - \frac{47b^2e^4n^2}{180d^4x} + \frac{47b^2e^5n^2}{90d^5\sqrt{x}} - \frac{47b^2e^6n^2\log\left(d+e\sqrt{\right.}}{90d^6}$$

$$= -\frac{b^2e^2n^2}{30d^2x^2} + \frac{b^2e^3n^2}{10d^3x^{3/2}} - \frac{47b^2e^4n^2}{180d^4x} + \frac{77b^2e^5n^2}{90d^5\sqrt{x}} - \frac{77b^2e^6n^2\log\left(d+e\sqrt{\right.}}{90d^6}$$

$$= -\frac{b^2e^2n^2}{30d^2x^2} + \frac{b^2e^3n^2}{10d^3x^{3/2}} - \frac{47b^2e^4n^2}{180d^4x} + \frac{77b^2e^5n^2}{90d^5\sqrt{x}} - \frac{77b^2e^6n^2\log\left(d+e\sqrt{\right.}}{90d^6}$$

Mathematica [A]
time = 0.20, size = 538, normalized size = 1.32

$$-\frac{60a^2d^6+24abd^5en\sqrt{x}-30abd^4e^2nx+6b^2d^4e^2n^2x+40abd^3e^3nx^{3/2}-18b^2d^3e^3n^2x^{3/2}-60abd^2e^4nx^2+47b^2d^2e^4n^2x^2+120abde^5nx^{5/2}-154b^2de^5n^2x^{5/2}-60a^2e^6x^3+274b^2e^6n^2x^3\log\left(d+e\sqrt{x}\right)+120abd^6\log\left(c\left(d+e\sqrt{x}\right)^n\right)+24b^2d^5en\sqrt{x}\log\left(c\left(d+e\sqrt{x}\right)^n\right)-30b^2d^4e^2nx\log\left(c\left(d+e\sqrt{x}\right)^n\right)+40b^2d^3e^3nx^{3/2}\log\left(c\left(d+e\sqrt{x}\right)^n\right)-60b^2d^2e^4nx^2\log\left(c\left(d+e\sqrt{x}\right)^n\right)+120b^2de^5nx^{5/2}\log\left(c\left(d+e\sqrt{x}\right)^n\right)-120abe^6x^3\log\left(c\left(d+e\sqrt{x}\right)^n\right)+60b^2d^6\log^2\left(c\left(d+e\sqrt{x}\right)^n\right)-60b^2e^6x^3\log^2\left(c\left(d+e\sqrt{x}\right)^n\right)+120abe^6nx^3\log\left(-\frac{e\sqrt{x}}{d}\right)+120b^2e^6nx^3\log\left(c\left(d+e\sqrt{x}\right)^n\right)\log\left(-\frac{e\sqrt{x}}{d}\right)-137b^2e^6n^2x^3\log(x)+120b^2e^6n^2x^3\text{Li}_2\left(1+\frac{e\sqrt{x}}{d}\right)}{180d^6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^4,x]
```

```
[Out] -1/180*(60*a^2*d^6 + 24*a*b*d^5*e*n*Sqrt[x] - 30*a*b*d^4*e^2*n*x + 6*b^2*d^
4*e^2*n^2*x + 40*a*b*d^3*e^3*n*x^(3/2) - 18*b^2*d^3*e^3*n^2*x^(3/2) - 60*a*
b*d^2*e^4*n*x^2 + 47*b^2*d^2*e^4*n^2*x^2 + 120*a*b*d*e^5*n*x^(5/2) - 154*b^
2*d*e^5*n^2*x^(5/2) - 60*a^2*e^6*x^3 + 274*b^2*e^6*n^2*x^3*Log[d + e*Sqrt[x
]] + 120*a*b*d^6*Log[c*(d + e*Sqrt[x])^n] + 24*b^2*d^5*e*n*Sqrt[x]*Log[c*(d
 + e*Sqrt[x])^n] - 30*b^2*d^4*e^2*n*x*Log[c*(d + e*Sqrt[x])^n] + 40*b^2*d^3
*e^3*n*x^(3/2)*Log[c*(d + e*Sqrt[x])^n] - 60*b^2*d^2*e^4*n*x^2*Log[c*(d + e
*Sqrt[x])^n] + 120*b^2*d*e^5*n*x^(5/2)*Log[c*(d + e*Sqrt[x])^n] - 120*a*b*e
^6*x^3*Log[c*(d + e*Sqrt[x])^n] + 60*b^2*d^6*Log[c*(d + e*Sqrt[x])^n]^2 - 6
0*b^2*e^6*x^3*Log[c*(d + e*Sqrt[x])^n]^2 + 120*a*b*e^6*n*x^3*Log[-((e*Sqrt[
x])/d)] + 120*b^2*e^6*n*x^3*Log[c*(d + e*Sqrt[x])^n]*Log[-((e*Sqrt[x])/d)]
- 137*b^2*e^6*n^2*x^3*Log[x] + 120*b^2*e^6*n^2*x^3*PolyLog[2, 1 + (e*Sqrt[x
])/d])/(d^6*x^3)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\ln\left(c\left(d + e\sqrt{x}\,\right)^{n}\right)\right)^{2}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))^2/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))^2/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^2*n^2*log(sqrt(x)*e + d)^2/x^3 + integrate(1/3*(3*(b^2*log(c)^2 + 2*
a*b*log(c) + a^2)*x*e + (b^2*n*x*e + 6*(b^2*log(c) + a*b)*x*e + 6*(b^2*d*lo
g(c) + a*b*d)*sqrt(x))*n*log(sqrt(x)*e + d) + 3*(b^2*d*log(c)^2 + 2*a*b*d*l
og(c) + a^2*d)*sqrt(x))/(x^5*e + d*x^(9/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((sqrt(x)*e + d)^n*c)^2 + 2*a*b*log((sqrt(x)*e + d)^n*c) +
 a^2)/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \log \left(c \left(d + e \sqrt{x}\right)^{n}\right)\right)^{2}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**2/x**4,x)
```

```
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))**2/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log((sqrt(x)*e + d)^n*c) + a)^2/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)\right)}^2}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))^2/x^4,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/2))^n))^2/x^4, x)
```

3.415 $\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=907

$$-\frac{15b^3d^4n^3\left(d+e\sqrt{x}\right)^2}{4e^6}+\frac{40b^3d^3n^3\left(d+e\sqrt{x}\right)^3}{27e^6}-\frac{15b^3d^2n^3\left(d+e\sqrt{x}\right)^4}{32e^6}+\frac{12b^3dn^3\left(d+e\sqrt{x}\right)^5}{125e^6}-\frac{b^3n^3\left(d+e\sqrt{x}\right.}{108e^6}$$

```
[Out] 1/3*(a+b*ln(c*(d+e*x^(1/2))^n))^3*(d+e*x^(1/2))^6/e^6-2*d^5*(a+b*ln(c*(d+e*
x^(1/2))^n))^3*(d+e*x^(1/2))/e^6+5*d^4*(a+b*ln(c*(d+e*x^(1/2))^n))^3*(d+e*x
^(1/2))^2/e^6-20/3*d^3*(a+b*ln(c*(d+e*x^(1/2))^n))^3*(d+e*x^(1/2))^3/e^6+5*
d^2*(a+b*ln(c*(d+e*x^(1/2))^n))^3*(d+e*x^(1/2))^4/e^6-2*d*(a+b*ln(c*(d+e*x^
(1/2))^n))^3*(d+e*x^(1/2))^5/e^6-1/108*b^3*n^3*(d+e*x^(1/2))^6/e^6-15/4*b*d
^2*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1/2))^4/e^6-12/25*b^2*d*n^2*(a+b
*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))^5/e^6+6/5*b*d*n*(a+b*ln(c*(d+e*x^(1/2
))^n))^2*(d+e*x^(1/2))^5/e^6-12*a*b^2*d^5*n^2*x^(1/2)/e^5-12*b^3*d^5*n^2*ln
(c*(d+e*x^(1/2))^n)*(d+e*x^(1/2))/e^6+6*b*d^5*n*(a+b*ln(c*(d+e*x^(1/2))^n))
^2*(d+e*x^(1/2))/e^6+15/2*b^2*d^4*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1
/2))^2/e^6-15/2*b*d^4*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1/2))^2/e^6-4
0/9*b^2*d^3*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))^3/e^6+20/3*b*d^3*
n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1/2))^3/e^6+15/8*b^2*d^2*n^2*(a+b*l
n(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))^4/e^6+12*b^3*d^5*n^3*x^(1/2)/e^5-15/4*b
^3*d^4*n^3*(d+e*x^(1/2))^2/e^6+40/27*b^3*d^3*n^3*(d+e*x^(1/2))^3/e^6-15/32*
b^3*d^2*n^3*(d+e*x^(1/2))^4/e^6+12/125*b^3*d*n^3*(d+e*x^(1/2))^5/e^6+1/18*b
^2*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))^6/e^6-1/6*b*n*(a+b*ln(c*(d
+e*x^(1/2))^n))^2*(d+e*x^(1/2))^6/e^6
```

Rubi [A]
time = 0.67, antiderivative size = 907, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3,x]
```

```
[Out] (-15*b^3*d^4*n^3*(d + e*Sqrt[x])^2)/(4*e^6) + (40*b^3*d^3*n^3*(d + e*Sqrt[x
])^3)/(27*e^6) - (15*b^3*d^2*n^3*(d + e*Sqrt[x])^4)/(32*e^6) + (12*b^3*d*n^
3*(d + e*Sqrt[x])^5)/(125*e^6) - (b^3*n^3*(d + e*Sqrt[x])^6)/(108*e^6) - (1
2*a*b^2*d^5*n^2*Sqrt[x])/e^5 + (12*b^3*d^5*n^3*Sqrt[x])/e^5 - (12*b^3*d^5*n
^2*(d + e*Sqrt[x])*Log[c*(d + e*Sqrt[x])^n])/e^6 + (15*b^2*d^4*n^2*(d + e*S
qrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(2*e^6) - (40*b^2*d^3*n^2*(d +
e*Sqrt[x])^3*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(9*e^6) + (15*b^2*d^2*n^2*(d
 + e*Sqrt[x])^4*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(8*e^6) - (12*b^2*d*n^2*(
d + e*Sqrt[x])^5*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(25*e^6) + (b^2*n^2*(d +
```

```
 e*Sqrt[x])^6*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(18*e^6) + (6*b*d^5*n*(d +
e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/e^6 - (15*b*d^4*n*(d + e*Sqr
t[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(2*e^6) + (20*b*d^3*n*(d + e*Sq
rt[x])^3*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(3*e^6) - (15*b*d^2*n*(d + e*S
qrt[x])^4*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(4*e^6) + (6*b*d*n*(d + e*Sqr
t[x])^5*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(5*e^6) - (b*n*(d + e*Sqrt[x])^
6*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(6*e^6) - (2*d^5*(d + e*Sqrt[x])*(a +
 b*Log[c*(d + e*Sqrt[x])^n])^3)/e^6 + (5*d^4*(d + e*Sqrt[x])^2*(a + b*Log[c
*(d + e*Sqrt[x])^n])^3)/e^6 - (20*d^3*(d + e*Sqrt[x])^3*(a + b*Log[c*(d + e
*Sqrt[x])^n])^3)/(3*e^6) + (5*d^2*(d + e*Sqrt[x])^4*(a + b*Log[c*(d + e*Sqr
t[x])^n])^3)/e^6 - (2*d*(d + e*Sqrt[x])^5*(a + b*Log[c*(d + e*Sqrt[x])^n])^
3)/e^6 + ((d + e*Sqrt[x])^6*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/(3*e^6)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
```

```
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx = 2\text{Subst}\left(\int x^5(a+b\log\left(c(d+ex)^n\right))^3\,dx, x, \sqrt{x}\right)$$
$$= 2\text{Subst}\left(\int\left(-\frac{d^5(a+b\log\left(c(d+ex)^n\right))^3}{e^5}+\frac{5d^4(d+ex)\left(a+b\log\left(c\right.\right.}{e^5}\right.\right.$$
$$= \frac{2\text{Subst}\left(\int(d+ex)^5\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx, x, \sqrt{x}\right)}{e^5} - \frac{(10d)\text{Sub}}{}$$
$$= \frac{2\text{Subst}\left(\int x^5(a+b\log\left(cx^n\right))^3\,dx, x, d+e\sqrt{x}\right)}{e^6} - \frac{(10d)\text{Subst}\left(\int x^4(c\right.}{}$$
$$= -\frac{2d^5\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^6}+\frac{5d^4\left(d+e\sqrt{x}\right)^2(a}{}$$
$$= \frac{6bd^5n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^6} - \frac{15bd^4n\left(d+e\sqrt{x}\right)^2}{}$$
$$= -\frac{15b^3d^4n^3\left(d+e\sqrt{x}\right)^2}{4e^6}+\frac{40b^3d^3n^3\left(d+e\sqrt{x}\right)^3}{27e^6}-\frac{15b^3d^2n^3\left(d+e\sqrt{\right.}{32e^6}$$
$$= -\frac{15b^3d^4n^3\left(d+e\sqrt{x}\right)^2}{4e^6}+\frac{40b^3d^3n^3\left(d+e\sqrt{x}\right)^3}{27e^6}-\frac{15b^3d^2n^3\left(d+e\sqrt{\right.}{32e^6}$$

Mathematica [A]
time = 0.45, size = 661, normalized size = 0.73

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3,x]

[Out] (-36000*b^3*d^6*n^3*Log[d + e*Sqrt[x]]^3 + 5400*b^2*d^6*n^2*Log[d + e*Sqrt[x]]^2*(20*a - 49*b*n + 20*b*Log[c*(d + e*Sqrt[x])^n]) - 60*b*d^6*n*Log[d + e*Sqrt[x]]*(1800*a^2 - 8820*a*b*n + 13489*b^2*n^2 + 180*b*(20*a - 49*b*n)*Log[c*(d + e*Sqrt[x])^n] + 1800*b^2*Log[c*(d + e*Sqrt[x])^n]^2) + e*Sqrt[x]*(36000*a^3*e^5*x^(5/2) + b^3*n^3*(809340*d^5 - 140070*d^4*e*Sqrt[x] + 41180*d^3*e^2*x - 13785*d^2*e^3*x^(3/2) + 4368*d*e^4*x^2 - 1000*e^5*x^(5/2)) - 60*a*b^2*n^2*(8820*d^5 - 2610*d^4*e*Sqrt[x] + 1140*d^3*e^2*x - 555*d^2*e^3*x^(3/2) + 264*d*e^4*x^2 - 100*e^5*x^(5/2)) + 1800*a^2*b*n*(60*d^5 - 30*d^4*e*Sqrt[x] + 20*d^3*e^2*x - 15*d^2*e^3*x^(3/2) + 12*d*e^4*x^2 - 10*e^5*x^(5/2)) + 60*b*(1800*a^2*e^5*x^(5/2) + 60*a*b*n*(60*d^5 - 30*d^4*e*Sqrt[x] + 20*d^3*e^2*x - 15*d^2*e^3*x^(3/2) + 12*d*e^4*x^2 - 10*e^5*x^(5/2)) + b^2*n^2*(-8820*d^5 + 2610*d^4*e*Sqrt[x] - 1140*d^3*e^2*x + 555*d^2*e^3*x^(3/2) - 264*d*e^4*x^2 + 100*e^5*x^(5/2)))*Log[c*(d + e*Sqrt[x])^n] + 1800*b^2*(60*a*e^5*x^(5/2) + b*n*(60*d^5 - 30*d^4*e*Sqrt[x] + 20*d^3*e^2*x - 15*d^2*e^3*x^(3/2) + 12*d*e^4*x^2 - 10*e^5*x^(5/2)))*Log[c*(d + e*Sqrt[x])^n]^2 + 36000*b^3*e^5*x^(5/2)*Log[c*(d + e*Sqrt[x])^n]^3))/(108000*e^6)

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\ln\left(c\left(d+e\sqrt{x}\,\right)^{n}\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a+b*ln(c*(d+e*x^(1/2))^n))^3,x)

[Out] int(x^2*(a+b*ln(c*(d+e*x^(1/2))^n))^3,x)

Maxima [A]
time = 0.32, size = 655, normalized size = 0.72

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="maxima")

[Out] 1/3*b^3*x^3*log((sqrt(x)*e + d)^n*c)^3 + a*b^2*x^3*log((sqrt(x)*e + d)^n*c)^2 + a^2*b*x^3*log((sqrt(x)*e + d)^n*c) + 1/3*a^3*x^3 - 1/60*(60*d^6*e^(-7)*log(sqrt(x)*e + d) + (30*d^4*x*e - 60*d^5*sqrt(x) - 20*d^3*x^(3/2)*e^2 + 15*d^2*x^2*e^3 - 12*d*x^(5/2)*e^4 + 10*x^3*e^5)*e^(-6))*a^2*b*n*e + 1/1800*((1800*d^6*log(sqrt(x)*e + d)^2 + 8820*d^6*log(sqrt(x)*e + d) - 8820*d^5*sqrt(x)*e + 2610*d^4*x*e^2 - 1140*d^3*x^(3/2)*e^3 + 555*d^2*x^2*e^4 - 264*d*x^(5/2)*e^5 + 100*x^3*e^6)*n^2*e^(-6) - 60*(60*d^6*e^(-7)*log(sqrt(x)*e + d)

```
+ (30*d^4*x*e - 60*d^5*sqrt(x) - 20*d^3*x^(3/2)*e^2 + 15*d^2*x^2*e^3 - 12*d
*x^(5/2)*e^4 + 10*x^3*e^5)*e^(-6))*n*e*log((sqrt(x)*e + d)^n*c))*a*b^2 - 1/
108000*(1800*(60*d^6*e^(-7)*log(sqrt(x)*e + d) + (30*d^4*x*e - 60*d^5*sqrt(
x) - 20*d^3*x^(3/2)*e^2 + 15*d^2*x^2*e^3 - 12*d*x^(5/2)*e^4 + 10*x^3*e^5)*e
^(-6))*n*e*log((sqrt(x)*e + d)^n*c)^2 + ((36000*d^6*log(sqrt(x)*e + d)^3 +
264600*d^6*log(sqrt(x)*e + d)^2 + 809340*d^6*log(sqrt(x)*e + d) - 809340*d^
5*sqrt(x)*e + 140070*d^4*x*e^2 - 41180*d^3*x^(3/2)*e^3 + 13785*d^2*x^2*e^4
- 4368*d*x^(5/2)*e^5 + 1000*x^3*e^6)*n^2*e^(-7) - 60*(1800*d^6*log(sqrt(x)*
e + d)^2 + 8820*d^6*log(sqrt(x)*e + d) - 8820*d^5*sqrt(x)*e + 2610*d^4*x*e^
2 - 1140*d^3*x^(3/2)*e^3 + 555*d^2*x^2*e^4 - 264*d*x^(5/2)*e^5 + 100*x^3*e^
6)*n*e^(-7)*log((sqrt(x)*e + d)^n*c))*n*e)*b^3
```

Fricas [A]
time = 0.44, size = 1098, normalized size = 1.21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="fricas")
```

```
[Out] 1/108000*(36000*b^3*x^3*e^6*log(c)^3 - 1000*(b^3*n^3 - 6*a*b^2*n^2 + 18*a^2
*b*n - 36*a^3)*x^3*e^6 - 15*(919*b^3*d^2*n^3 - 2220*a*b^2*d^2*n^2 + 1800*a^
2*b*d^2*n)*x^2*e^4 - 36000*(b^3*d^6*n^3 - b^3*n^3*x^3*e^6)*log(sqrt(x)*e +
d)^3 - 30*(4669*b^3*d^4*n^3 - 5220*a*b^2*d^4*n^2 + 1800*a^2*b*d^4*n)*x*e^2
+ 1800*(147*b^3*d^6*n^3 - 30*b^3*d^4*n^3*x*e^2 - 60*a*b^2*d^6*n^2 - 15*b^3*
d^2*n^3*x^2*e^4 - 10*(b^3*n^3 - 6*a*b^2*n^2)*x^3*e^6 - 60*(b^3*d^6*n^2 - b^
3*n^2*x^3*e^6)*log(c) + 4*(15*b^3*d^5*n^3*e + 5*b^3*d^3*n^3*x*e^3 + 3*b^3*d
*n^3*x^2*e^5)*sqrt(x))*log(sqrt(x)*e + d)^2 - 9000*(6*b^3*d^4*n*x*e^2 + 3*b
^3*d^2*n*x^2*e^4 + 2*(b^3*n - 6*a*b^2)*x^3*e^6)*log(c)^2 - 60*(13489*b^3*d^
6*n^3 - 8820*a*b^2*d^6*n^2 + 1800*a^2*b*d^6*n - 100*(b^3*n^3 - 6*a*b^2*n^2
+ 18*a^2*b*n)*x^3*e^6 - 15*(37*b^3*d^2*n^3 - 60*a*b^2*d^2*n^2)*x^2*e^4 - 90
*(29*b^3*d^4*n^3 - 20*a*b^2*d^4*n^2)*x*e^2 + 1800*(b^3*d^6*n - b^3*n*x^3*e^
6)*log(c)^2 - 60*(147*b^3*d^6*n^2 - 30*b^3*d^4*n^2*x*e^2 - 60*a*b^2*d^6*n -
 15*b^3*d^2*n^2*x^2*e^4 - 10*(b^3*n^2 - 6*a*b^2*n)*x^3*e^6)*log(c) + 12*(2*
(11*b^3*d*n^3 - 30*a*b^2*d*n^2)*x^2*e^5 + 5*(19*b^3*d^3*n^3 - 20*a*b^2*d^3*
n^2)*x*e^3 + 15*(49*b^3*d^5*n^3 - 20*a*b^2*d^5*n^2)*e - 20*(15*b^3*d^5*n^2*
e + 5*b^3*d^3*n^2*x*e^3 + 3*b^3*d*n^2*x^2*e^5)*log(c))*sqrt(x))*log(sqrt(x)
*e + d) + 300*(20*(b^3*n^2 - 6*a*b^2*n + 18*a^2*b)*x^3*e^6 + 3*(37*b^3*d^2*
n^2 - 60*a*b^2*d^2*n)*x^2*e^4 + 18*(29*b^3*d^4*n^2 - 20*a*b^2*d^4*n)*x*e^2)
*log(c) + 4*(12*(91*b^3*d*n^3 - 330*a*b^2*d*n^2 + 450*a^2*b*d*n)*x^2*e^5 +
5*(2059*b^3*d^3*n^3 - 3420*a*b^2*d^3*n^2 + 1800*a^2*b*d^3*n)*x*e^3 + 1800*(
15*b^3*d^5*n*e + 5*b^3*d^3*n*x*e^3 + 3*b^3*d*n*x^2*e^5)*log(c)^2 + 15*(1348
9*b^3*d^5*n^3 - 8820*a*b^2*d^5*n^2 + 1800*a^2*b*d^5*n)*e - 180*(2*(11*b^3*d
*n^2 - 30*a*b^2*d*n)*x^2*e^5 + 5*(19*b^3*d^3*n^2 - 20*a*b^2*d^3*n)*x*e^3 +
15*(49*b^3*d^5*n^2 - 20*a*b^2*d^5*n)*e)*log(c))*sqrt(x))*e^(-6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/2))**n))**3,x)
```

```
[Out] Integral(x**2*(a + b*log(c*(d + e*sqrt(x))**n))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2223 vs. $2(803) = 1606$.
time = 4.97, size = 2223, normalized size = 2.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="giac")
```

```
[Out] 1/108000*(36000*b^3*x^3*e*log(c)^3 + 108000*a*b^2*x^3*e*log(c)^2 + 108000*a
^2*b*x^3*e*log(c) + (36000*(sqrt(x)*e + d)^6*e^(-5)*log(sqrt(x)*e + d)^3 -
216000*(sqrt(x)*e + d)^5*d*e^(-5)*log(sqrt(x)*e + d)^3 + 540000*(sqrt(x)*e
+ d)^4*d^2*e^(-5)*log(sqrt(x)*e + d)^3 - 720000*(sqrt(x)*e + d)^3*d^3*e^(-5
)*log(sqrt(x)*e + d)^3 + 540000*(sqrt(x)*e + d)^2*d^4*e^(-5)*log(sqrt(x)*e
+ d)^3 - 216000*(sqrt(x)*e + d)*d^5*e^(-5)*log(sqrt(x)*e + d)^3 - 18000*(sq
rt(x)*e + d)^6*e^(-5)*log(sqrt(x)*e + d)^2 + 129600*(sqrt(x)*e + d)^5*d*e^(
-5)*log(sqrt(x)*e + d)^2 - 405000*(sqrt(x)*e + d)^4*d^2*e^(-5)*log(sqrt(x)*
e + d)^2 + 720000*(sqrt(x)*e + d)^3*d^3*e^(-5)*log(sqrt(x)*e + d)^2 - 81000
0*(sqrt(x)*e + d)^2*d^4*e^(-5)*log(sqrt(x)*e + d)^2 + 648000*(sqrt(x)*e + d
)*d^5*e^(-5)*log(sqrt(x)*e + d)^2 + 6000*(sqrt(x)*e + d)^6*e^(-5)*log(sqrt(
x)*e + d) - 51840*(sqrt(x)*e + d)^5*d*e^(-5)*log(sqrt(x)*e + d) + 202500*(s
qrt(x)*e + d)^4*d^2*e^(-5)*log(sqrt(x)*e + d) - 480000*(sqrt(x)*e + d)^3*d^
3*e^(-5)*log(sqrt(x)*e + d) + 810000*(sqrt(x)*e + d)^2*d^4*e^(-5)*log(sqrt(
x)*e + d) - 1296000*(sqrt(x)*e + d)*d^5*e^(-5)*log(sqrt(x)*e + d) - 1000*(s
qrt(x)*e + d)^6*e^(-5) + 10368*(sqrt(x)*e + d)^5*d*e^(-5) - 50625*(sqrt(x)*
e + d)^4*d^2*e^(-5) + 160000*(sqrt(x)*e + d)^3*d^3*e^(-5) - 405000*(sqrt(x)
*e + d)^2*d^4*e^(-5) + 1296000*(sqrt(x)*e + d)*d^5*e^(-5))*b^3*n^3 + 36000*
a^3*x^3*e + 60*(1800*(sqrt(x)*e + d)^6*e^(-5)*log(sqrt(x)*e + d)^2 - 10800*
(sqrt(x)*e + d)^5*d*e^(-5)*log(sqrt(x)*e + d)^2 + 27000*(sqrt(x)*e + d)^4*d
^2*e^(-5)*log(sqrt(x)*e + d)^2 - 36000*(sqrt(x)*e + d)^3*d^3*e^(-5)*log(sqr
t(x)*e + d)^2 + 27000*(sqrt(x)*e + d)^2*d^4*e^(-5)*log(sqrt(x)*e + d)^2 - 1
0800*(sqrt(x)*e + d)*d^5*e^(-5)*log(sqrt(x)*e + d)^2 - 600*(sqrt(x)*e + d)^
6*e^(-5)*log(sqrt(x)*e + d) + 4320*(sqrt(x)*e + d)^5*d*e^(-5)*log(sqrt(x)*e
 + d) - 13500*(sqrt(x)*e + d)^4*d^2*e^(-5)*log(sqrt(x)*e + d) + 24000*(sqrt
```

```
(x)*e + d)^3*d^3*e^(-5)*log(sqrt(x)*e + d) - 27000*(sqrt(x)*e + d)^2*d^4*e^
(-5)*log(sqrt(x)*e + d) + 21600*(sqrt(x)*e + d)*d^5*e^(-5)*log(sqrt(x)*e +
d) + 100*(sqrt(x)*e + d)^6*e^(-5) - 864*(sqrt(x)*e + d)^5*d*e^(-5) + 3375*(
sqrt(x)*e + d)^4*d^2*e^(-5) - 8000*(sqrt(x)*e + d)^3*d^3*e^(-5) + 13500*(sq
rt(x)*e + d)^2*d^4*e^(-5) - 21600*(sqrt(x)*e + d)*d^5*e^(-5))*b^3*n^2*log(c
) + 1800*(60*(sqrt(x)*e + d)^6*e^(-5)*log(sqrt(x)*e + d) - 360*(sqrt(x)*e +
 d)^5*d*e^(-5)*log(sqrt(x)*e + d) + 900*(sqrt(x)*e + d)^4*d^2*e^(-5)*log(sq
rt(x)*e + d) - 1200*(sqrt(x)*e + d)^3*d^3*e^(-5)*log(sqrt(x)*e + d) + 900*(
sqrt(x)*e + d)^2*d^4*e^(-5)*log(sqrt(x)*e + d) - 360*(sqrt(x)*e + d)*d^5*e^
(-5)*log(sqrt(x)*e + d) - 10*(sqrt(x)*e + d)^6*e^(-5) + 72*(sqrt(x)*e + d)^
5*d*e^(-5) - 225*(sqrt(x)*e + d)^4*d^2*e^(-5) + 400*(sqrt(x)*e + d)^3*d^3*e
^(-5) - 450*(sqrt(x)*e + d)^2*d^4*e^(-5) + 360*(sqrt(x)*e + d)*d^5*e^(-5))*
b^3*n*log(c)^2 + 60*(1800*(sqrt(x)*e + d)^6*e^(-5)*log(sqrt(x)*e + d)^2 - 1
0800*(sqrt(x)*e + d)^5*d*e^(-5)*log(sqrt(x)*e + d)^2 + 27000*(sqrt(x)*e + d
)^4*d^2*e^(-5)*log(sqrt(x)*e + d)^2 - 36000*(sqrt(x)*e + d)^3*d^3*e^(-5)*lo
g(sqrt(x)*e + d)^2 + 27000*(sqrt(x)*e + d)^2*d^4*e^(-5)*log(sqrt(x)*e + d)^
2 - 10800*(sqrt(x)*e + d)*d^5*e^(-5)*log(sqrt(x)*e + d)^2 - 600*(sqrt(x)*e
+ d)^6*e^(-5)*log(sqrt(x)*e + d) + 4320*(sqrt(x)*e + d)^5*d*e^(-5)*log(sqrt
(x)*e + d) - 13500*(sqrt(x)*e + d)^4*d^2*e^(-5)*log(sqrt(x)*e + d) + 24000*
(sqrt(x)*e + d)^3*d^3*e^(-5)*log(sqrt(x)*e + d) - 27000*(sqrt(x)*e + d)^2*d
^4*e^(-5)*log(sqrt(x)*e + d) + 21600*(sqrt(x)*e + d)*d^5*e^(-5)*log(sqrt(x)
*e + d) + 100*(sqrt(x)*e + d)^6*e^(-5) - 864*(sqrt(x)*e + d)^5*d*e^(-5) + 3
375*(sqrt(x)*e + d)^4*d^2*e^(-5) - 8000*(sqrt(x)*e + d)^3*d^3*e^(-5) + 1350
0*(sqrt(x)*e + d)^2*d^4*e^(-5) - 21600*(sqrt(x)*e + d)*d^5*e^(-5))*a*b^2*n^
2 + 3600*(60*(sqrt(x)*e + d)^6*e^(-5)*log(sqrt(x)*e + d) - 360*(sqrt(x)*e +
 d)^5*d*e^(-5)*log(sqrt(x)*e + d) + 900*(sqrt(x)*e + d)^4*d^2*e^(-5)*log(sq
rt(x)*e + d) - 1200*(sqrt(x)*e + d)^3*d^3*e^(-5)*log(sqrt(x)*e + d) + 900*(
sqrt(x)*e + d)^2*d^4*e^(-5)*log(sqrt(x)*e + d) - 360*(sqrt(x)*e + d)*d^5*e^
(-5)*log(sqrt(x)*e + d) - 10*(sqrt(x)*e + d)^6*e^(-5) + 72*(sqrt(x)*e + d)^
5*d*e^(-5) - 225*(sqrt(x)*e + d)^4*d^2*e^(-5) + 400*(sqrt(x)*e + d)^3*d^3*e
^(-5) - 450*(sqrt(x)*e + d)^2*d^4*e^(-5) + 360*(sqrt(x)*e + d)*d^5*e^(-5))*
a*b^2*n*log(c) + 1800*(60*(sqrt(x)*e + d)^6*e^(-5)*log(sqrt(x)*e + d) - 360
*(sqrt(x)*e + d)^5*d*e^(-5)*log(sqrt(x)*e + d) + 900*(sqrt(x)*e + d)^4*d^2*
e^(-5)*log(sqrt(x)*e + d) - 1200*(sqrt(x)*e + d)^3*d^3*e^(-5)*log(sqrt(x)*e
 + d) + 900*(sqrt(x)*e + d)^2*d^4*e^(-5)*log(sqrt(x)*e + d) - 360*(sqrt(x)*
e + d)*d^5*e^(-5)*log(sqrt(x)*e + d) - 10*(sqrt(x)*e + d)^6*e^(-5) + 72*(sq
rt(x)*e + d)^5*d*e^(-5) - 225*(sqrt(x)*e + d)^4*d^2*e^(-5) + 400*(sqrt(x)*e
 + d)^3*d^3*e^(-5) - 450*(sqrt(x)*e + d)^2*d^4*e^(-5) + 360*(sqrt(x)*e + d)
*d^5*e^(-5))*a^2*b*n)*e^(-1)
```

Mupad [B]
time = 8.18, size = 976, normalized size = 1.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e*x^(1/2))^n))^3,x)
```

```
[Out] (a^3*x^3)/3 + (b^3*x^3*log(c*(d + e*x^(1/2))^n)^3)/3 - (b^3*n^3*x^3)/108 +
a*b^2*x^3*log(c*(d + e*x^(1/2))^n)^2 - (b^3*n*x^3*log(c*(d + e*x^(1/2))^n)^
2)/6 + (b^3*n^2*x^3*log(c*(d + e*x^(1/2))^n))/18 + (a*b^2*n^2*x^3)/18 - (b^
3*d^6*log(c*(d + e*x^(1/2))^n)^3)/(3*e^6) + a^2*b*x^3*log(c*(d + e*x^(1/2))
^n) - (a^2*b*n*x^3)/6 - (a*b^2*n*x^3*log(c*(d + e*x^(1/2))^n))/3 - (13489*b
^3*d^6*n^3*log(d + e*x^(1/2)))/(1800*e^6) - (919*b^3*d^2*n^3*x^2)/(7200*e^2
) + (2059*b^3*d^3*n^3*x^(3/2))/(5400*e^3) + (13489*b^3*d^5*n^3*x^(1/2))/(18
00*e^5) - (a*b^2*d^6*log(c*(d + e*x^(1/2))^n)^2)/e^6 + (49*b^3*d^6*n*log(c*
(d + e*x^(1/2))^n)^2)/(20*e^6) + (91*b^3*d*n^3*x^(5/2))/(2250*e) - (4669*b^
3*d^4*n^3*x)/(3600*e^4) - (a^2*b*d^6*n*log(d + e*x^(1/2)))/e^6 + (b^3*d*n*x
^(5/2)*log(c*(d + e*x^(1/2))^n)^2)/(5*e) - (11*b^3*d*n^2*x^(5/2)*log(c*(d +
 e*x^(1/2))^n))/(75*e) - (b^3*d^4*n*x*log(c*(d + e*x^(1/2))^n)^2)/(2*e^4) +
 (29*b^3*d^4*n^2*x*log(c*(d + e*x^(1/2))^n))/(20*e^4) - (a^2*b*d^2*n*x^2)/(
4*e^2) - (11*a*b^2*d*n^2*x^(5/2))/(75*e) + (29*a*b^2*d^4*n^2*x)/(20*e^4) +
(a^2*b*d^3*n*x^(3/2))/(3*e^3) + (a^2*b*d^5*n*x^(1/2))/e^5 + (49*a*b^2*d^6*n
^2*log(d + e*x^(1/2)))/(10*e^6) - (b^3*d^2*n*x^2*log(c*(d + e*x^(1/2))^n)^2
)/(4*e^2) + (37*b^3*d^2*n^2*x^2*log(c*(d + e*x^(1/2))^n))/(120*e^2) + (b^3*
d^3*n*x^(3/2)*log(c*(d + e*x^(1/2))^n)^2)/(3*e^3) - (19*b^3*d^3*n^2*x^(3/2)
*log(c*(d + e*x^(1/2))^n))/(30*e^3) + (b^3*d^5*n*x^(1/2)*log(c*(d + e*x^(1/
2))^n)^2)/e^5 - (49*b^3*d^5*n^2*x^(1/2)*log(c*(d + e*x^(1/2))^n))/(10*e^5)
+ (37*a*b^2*d^2*n^2*x^2)/(120*e^2) - (19*a*b^2*d^3*n^2*x^(3/2))/(30*e^3) -
(49*a*b^2*d^5*n^2*x^(1/2))/(10*e^5) + (a^2*b*d*n*x^(5/2))/(5*e) - (a^2*b*d^
4*n*x)/(2*e^4) + (2*a*b^2*d*n*x^(5/2)*log(c*(d + e*x^(1/2))^n))/(5*e) - (a*
b^2*d^4*n*x*log(c*(d + e*x^(1/2))^n))/e^4 - (a*b^2*d^2*n*x^2*log(c*(d + e*x
^(1/2))^n))/(2*e^2) + (2*a*b^2*d^3*n*x^(3/2)*log(c*(d + e*x^(1/2))^n))/(3*e
^3) + (2*a*b^2*d^5*n*x^(1/2)*log(c*(d + e*x^(1/2))^n))/e^5
```

3.416 $\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=595

$$-\frac{9b^3d^2n^3\left(d+e\sqrt{x}\right)^2}{4e^4}+\frac{4b^3dn^3\left(d+e\sqrt{x}\right)^3}{9e^4}-\frac{3b^3n^3\left(d+e\sqrt{x}\right)^4}{64e^4}-\frac{12ab^2d^3n^2\sqrt{x}}{e^3}+\frac{12b^3d^3n^3\sqrt{x}}{e^3}-\frac{12b^3d^3n^2}{}$$

```
[Out] -12*a*b^2*d^3*n^2*x^(1/2)/e^3+12*b^3*d^3*n^3*x^(1/2)/e^3-12*b^3*d^3*n^2*ln(
c*(d+e*x^(1/2))^n)*(d+e*x^(1/2))/e^4+6*b*d^3*n*(a+b*ln(c*(d+e*x^(1/2))^n))^
2*(d+e*x^(1/2))/e^4-2*d^3*(a+b*ln(c*(d+e*x^(1/2))^n))^3*(d+e*x^(1/2))/e^4-9
/4*b^3*d^2*n^3*(d+e*x^(1/2))^2/e^4+9/2*b^2*d^2*n^2*(a+b*ln(c*(d+e*x^(1/2))^
n))*(d+e*x^(1/2))^2/e^4-9/2*b*d^2*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1
/2))^2/e^4+3*d^2*(a+b*ln(c*(d+e*x^(1/2))^n))^3*(d+e*x^(1/2))^2/e^4+4/9*b^3*
d*n^3*(d+e*x^(1/2))^3/e^4-4/3*b^2*d*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^
(1/2))^3/e^4+2*b*d*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1/2))^3/e^4-2*d*
(a+b*ln(c*(d+e*x^(1/2))^n))^3*(d+e*x^(1/2))^3/e^4-3/64*b^3*n^3*(d+e*x^(1/2)
)^4/e^4+3/16*b^2*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))^4/e^4-3/8*b*
n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1/2))^4/e^4+1/2*(a+b*ln(c*(d+e*x^(1
/2))^n))^3*(d+e*x^(1/2))^4/e^4
```

Rubi [A]

time = 0.40, antiderivative size = 595, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*Sqrt[x])^n])^3,x]
```

```
[Out] (-9*b^3*d^2*n^3*(d + e*Sqrt[x])^2)/(4*e^4) + (4*b^3*d*n^3*(d + e*Sqrt[x])^3
)/(9*e^4) - (3*b^3*n^3*(d + e*Sqrt[x])^4)/(64*e^4) - (12*a*b^2*d^3*n^2*Sqrt
[x])/e^3 + (12*b^3*d^3*n^3*Sqrt[x])/e^3 - (12*b^3*d^3*n^2*(d + e*Sqrt[x])*L
og[c*(d + e*Sqrt[x])^n])/e^4 + (9*b^2*d^2*n^2*(d + e*Sqrt[x])^2*(a + b*Log[
c*(d + e*Sqrt[x])^n]))/(2*e^4) - (4*b^2*d*n^2*(d + e*Sqrt[x])^3*(a + b*Log[
c*(d + e*Sqrt[x])^n]))/(3*e^4) + (3*b^2*n^2*(d + e*Sqrt[x])^4*(a + b*Log[c*
(d + e*Sqrt[x])^n]))/(16*e^4) + (6*b*d^3*n*(d + e*Sqrt[x])*(a + b*Log[c*(d
+ e*Sqrt[x])^n])^2)/e^4 - (9*b*d^2*n*(d + e*Sqrt[x])^2*(a + b*Log[c*(d + e*
Sqrt[x])^n])^2)/(2*e^4) + (2*b*d*n*(d + e*Sqrt[x])^3*(a + b*Log[c*(d + e*Sq
rt[x])^n])^2)/e^4 - (3*b*n*(d + e*Sqrt[x])^4*(a + b*Log[c*(d + e*Sqrt[x])^n
])^2)/(8*e^4) - (2*d^3*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/
e^4 + (3*d^2*(d + e*Sqrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/e^4 - (2
*d*(d + e*Sqrt[x])^3*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/e^4 + ((d + e*Sqrt
[x])^4*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/(2*e^4)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx &= 2\text{Subst}\left(\int x^3(a+b\log(c(d+ex)^n))^3\,dx,x,\sqrt{x}\right)\\
&= 2\text{Subst}\left(\int\left(-\frac{d^3(a+b\log(c(d+ex)^n))^3}{e^3}+\frac{3d^2(d+ex)\,(a+b\log(c\right.\right.\\
&= \frac{2\text{Subst}\left(\int(d+ex)^3\,(a+b\log(c(d+ex)^n))^3\,dx,x,\sqrt{x}\right)}{e^3}-\frac{(6d)\text{Subst}}{}\\
&= \frac{2\text{Subst}\left(\int x^3(a+b\log(cx^n))^3\,dx,x,d+e\sqrt{x}\right)}{e^4}-\frac{(6d)\text{Subst}\left(\int x^2(a+\right.}{}\\
&= -\frac{2d^3\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^4}+\frac{3d^2\left(d+e\sqrt{x}\right)^2(a+}{}\\
&= \frac{6bd^3n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^4}-\frac{9bd^2n\left(d+e\sqrt{x}\right)^2(}{}\\
&= -\frac{9b^3d^2n^3\left(d+e\sqrt{x}\right)^2}{4e^4}+\frac{4b^3dn^3\left(d+e\sqrt{x}\right)^3}{9e^4}-\frac{3b^3n^3\left(d+e\sqrt{x}\right)^4}{64e^4}-\\
&= -\frac{9b^3d^2n^3\left(d+e\sqrt{x}\right)^2}{4e^4}+\frac{4b^3dn^3\left(d+e\sqrt{x}\right)^3}{9e^4}-\frac{3b^3n^3\left(d+e\sqrt{x}\right)^4}{64e^4}-
\end{aligned}$$

Mathematica [A]
time = 0.32, size = 517, normalized size = 0.87

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*Sqrt[x])^n])^3,x]
```

```
[Out] (-288*b^3*d^4*n^3*Log[d + e*Sqrt[x]]^3 + 72*b^2*d^4*n^2*Log[d + e*Sqrt[x]]^
2*(12*a - 25*b*n + 12*b*Log[c*(d + e*Sqrt[x])^n]) - 12*b*d^4*n*Log[d + e*Sq
rt[x]]*(72*a^2 - 300*a*b*n + 415*b^2*n^2 + 12*b*(12*a - 25*b*n)*Log[c*(d +
e*Sqrt[x])^n] + 72*b^2*Log[c*(d + e*Sqrt[x])^n]^2) + e*Sqrt[x]*(288*a^3*e^3
*x^(3/2) + b^3*n^3*(4980*d^3 - 690*d^2*e*Sqrt[x] + 148*d*e^2*x - 27*e^3*x^(
3/2)) - 12*a*b^2*n^2*(300*d^3 - 78*d^2*e*Sqrt[x] + 28*d*e^2*x - 9*e^3*x^(3/
2)) + 72*a^2*b*n*(12*d^3 - 6*d^2*e*Sqrt[x] + 4*d*e^2*x - 3*e^3*x^(3/2)) + 1
2*b*(72*a^2*e^3*x^(3/2) + 12*a*b*n*(12*d^3 - 6*d^2*e*Sqrt[x] + 4*d*e^2*x -
```

```
3*e^3*x^(3/2)) + b^2*n^2*(-300*d^3 + 78*d^2*e*Sqrt[x] - 28*d*e^2*x + 9*e^3*
x^(3/2)))*Log[c*(d + e*Sqrt[x])^n] + 72*b^2*(12*a*e^3*x^(3/2) + b*n*(12*d^3
 - 6*d^2*e*Sqrt[x] + 4*d*e^2*x - 3*e^3*x^(3/2)))*Log[c*(d + e*Sqrt[x])^n]^2
 + 288*b^3*e^3*x^(3/2)*Log[c*(d + e*Sqrt[x])^n]^3))/(576*e^4)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int x\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e*x^(1/2))^n))^3,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e*x^(1/2))^n))^3,x)
```

Maxima [A]
time = 0.31, size = 537, normalized size = 0.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*x^2*log((sqrt(x)*e + d)^n*c)^3 + 3/2*a*b^2*x^2*log((sqrt(x)*e + d)^
n*c)^2 - 1/8*(12*d^4*e^(-5)*log(sqrt(x)*e + d) + (6*d^2*x*e - 12*d^3*sqrt(x
) - 4*d*x^(3/2)*e^2 + 3*x^2*e^3)*e^(-4))*a^2*b*n*e + 3/2*a^2*b*x^2*log((sqr
t(x)*e + d)^n*c) + 1/2*a^3*x^2 + 1/48*((72*d^4*log(sqrt(x)*e + d)^2 + 300*d
^4*log(sqrt(x)*e + d) - 300*d^3*sqrt(x)*e + 78*d^2*x*e^2 - 28*d*x^(3/2)*e^3
 + 9*x^2*e^4)*n^2*e^(-4) - 12*(12*d^4*e^(-5)*log(sqrt(x)*e + d) + (6*d^2*x*
e - 12*d^3*sqrt(x) - 4*d*x^(3/2)*e^2 + 3*x^2*e^3)*e^(-4))*n*e*log((sqrt(x)*
e + d)^n*c))*a*b^2 - 1/576*(72*(12*d^4*e^(-5)*log(sqrt(x)*e + d) + (6*d^2*x
*e - 12*d^3*sqrt(x) - 4*d*x^(3/2)*e^2 + 3*x^2*e^3)*e^(-4))*n*e*log((sqrt(x)
*e + d)^n*c)^2 + ((288*d^4*log(sqrt(x)*e + d)^3 + 1800*d^4*log(sqrt(x)*e +
d)^2 + 4980*d^4*log(sqrt(x)*e + d) - 4980*d^3*sqrt(x)*e + 690*d^2*x*e^2 - 1
48*d*x^(3/2)*e^3 + 27*x^2*e^4)*n^2*e^(-5) - 12*(72*d^4*log(sqrt(x)*e + d)^2
 + 300*d^4*log(sqrt(x)*e + d) - 300*d^3*sqrt(x)*e + 78*d^2*x*e^2 - 28*d*x^(
3/2)*e^3 + 9*x^2*e^4)*n*e^(-5)*log((sqrt(x)*e + d)^n*c))*n*e)*b^3
```

Fricas [A]
time = 0.45, size = 798, normalized size = 1.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="fricas")
```

```
[Out] 1/576*(288*b^3*x^2*e^4*log(c)^3 - 9*(3*b^3*n^3 - 12*a*b^2*n^2 + 24*a^2*b*n
- 32*a^3)*x^2*e^4 - 288*(b^3*d^4*n^3 - b^3*n^3*x^2*e^4)*log(sqrt(x)*e + d)^
3 - 6*(115*b^3*d^2*n^3 - 156*a*b^2*d^2*n^2 + 72*a^2*b*d^2*n)*x*e^2 + 72*(25
*b^3*d^4*n^3 - 6*b^3*d^2*n^3*x*e^2 - 12*a*b^2*d^4*n^2 - 3*(b^3*n^3 - 4*a*b^
2*n^2)*x^2*e^4 - 12*(b^3*d^4*n^2 - b^3*n^2*x^2*e^4)*log(c) + 4*(3*b^3*d^3*n
^3*e + b^3*d*n^3*x*e^3)*sqrt(x))*log(sqrt(x)*e + d)^2 - 216*(2*b^3*d^2*n*x*
e^2 + (b^3*n - 4*a*b^2)*x^2*e^4)*log(c)^2 - 12*(415*b^3*d^4*n^3 - 300*a*b^2
*d^4*n^2 + 72*a^2*b*d^4*n - 9*(b^3*n^3 - 4*a*b^2*n^2 + 8*a^2*b*n)*x^2*e^4 -
 6*(13*b^3*d^2*n^3 - 12*a*b^2*d^2*n^2)*x*e^2 + 72*(b^3*d^4*n - b^3*n*x^2*e^
4)*log(c)^2 - 12*(25*b^3*d^4*n^2 - 6*b^3*d^2*n^2*x*e^2 - 12*a*b^2*d^4*n - 3
*(b^3*n^2 - 4*a*b^2*n)*x^2*e^4)*log(c) + 4*((7*b^3*d*n^3 - 12*a*b^2*d*n^2)*
x*e^3 + 3*(25*b^3*d^3*n^3 - 12*a*b^2*d^3*n^2)*e - 12*(3*b^3*d^3*n^2*e + b^3
*d*n^2*x*e^3)*log(c))*sqrt(x))*log(sqrt(x)*e + d) + 36*(3*(b^3*n^2 - 4*a*b^
2*n + 8*a^2*b)*x^2*e^4 + 2*(13*b^3*d^2*n^2 - 12*a*b^2*d^2*n)*x*e^2)*log(c)
+ 4*((37*b^3*d*n^3 - 84*a*b^2*d*n^2 + 72*a^2*b*d*n)*x*e^3 + 72*(3*b^3*d^3*n
*e + b^3*d*n*x*e^3)*log(c)^2 + 3*(415*b^3*d^3*n^3 - 300*a*b^2*d^3*n^2 + 72*
a^2*b*d^3*n)*e - 12*((7*b^3*d*n^2 - 12*a*b^2*d*n)*x*e^3 + 3*(25*b^3*d^3*n^2
 - 12*a*b^2*d^3*n)*e)*log(c))*sqrt(x))*e^(-4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/2))**n))**3,x)
```

```
[Out] Integral(x*(a + b*log(c*(d + e*sqrt(x))**n))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1483 vs. $2(529) = 1058$.
time = 4.57, size = 1483, normalized size = 2.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="giac")
```

```
[Out] 1/576*(288*b^3*x^2*e*log(c)^3 + 864*a*b^2*x^2*e*log(c)^2 + (288*(sqrt(x)*e
+ d)^4*e^(-3)*log(sqrt(x)*e + d)^3 - 1152*(sqrt(x)*e + d)^3*d*e^(-3)*log(sq
rt(x)*e + d)^3 + 1728*(sqrt(x)*e + d)^2*d^2*e^(-3)*log(sqrt(x)*e + d)^3 - 1
152*(sqrt(x)*e + d)*d^3*e^(-3)*log(sqrt(x)*e + d)^3 - 216*(sqrt(x)*e + d)^4
*e^(-3)*log(sqrt(x)*e + d)^2 + 1152*(sqrt(x)*e + d)^3*d*e^(-3)*log(sqrt(x)*
e + d)^2 - 2592*(sqrt(x)*e + d)^2*d^2*e^(-3)*log(sqrt(x)*e + d)^2 + 3456*(s
qrt(x)*e + d)*d^3*e^(-3)*log(sqrt(x)*e + d)^2 + 108*(sqrt(x)*e + d)^4*e^(-3
```

```
)*log(sqrt(x)*e + d) - 768*(sqrt(x)*e + d)^3*d*e^(-3)*log(sqrt(x)*e + d) +
2592*(sqrt(x)*e + d)^2*d^2*e^(-3)*log(sqrt(x)*e + d) - 6912*(sqrt(x)*e + d)
*d^3*e^(-3)*log(sqrt(x)*e + d) - 27*(sqrt(x)*e + d)^4*e^(-3) + 256*(sqrt(x)
*e + d)^3*d*e^(-3) - 1296*(sqrt(x)*e + d)^2*d^2*e^(-3) + 6912*(sqrt(x)*e +
d)*d^3*e^(-3))*b^3*n^3 + 12*(72*(sqrt(x)*e + d)^4*e^(-3)*log(sqrt(x)*e + d)
^2 - 288*(sqrt(x)*e + d)^3*d*e^(-3)*log(sqrt(x)*e + d)^2 + 432*(sqrt(x)*e +
 d)^2*d^2*e^(-3)*log(sqrt(x)*e + d)^2 - 288*(sqrt(x)*e + d)*d^3*e^(-3)*log(
sqrt(x)*e + d)^2 - 36*(sqrt(x)*e + d)^4*e^(-3)*log(sqrt(x)*e + d) + 192*(sq
rt(x)*e + d)^3*d*e^(-3)*log(sqrt(x)*e + d) - 432*(sqrt(x)*e + d)^2*d^2*e^(-
3)*log(sqrt(x)*e + d) + 576*(sqrt(x)*e + d)*d^3*e^(-3)*log(sqrt(x)*e + d) +
 9*(sqrt(x)*e + d)^4*e^(-3) - 64*(sqrt(x)*e + d)^3*d*e^(-3) + 216*(sqrt(x)*
e + d)^2*d^2*e^(-3) - 576*(sqrt(x)*e + d)*d^3*e^(-3))*b^3*n^2*log(c) + 864*
a^2*b*x^2*e*log(c) + 72*(12*(sqrt(x)*e + d)^4*e^(-3)*log(sqrt(x)*e + d) - 4
8*(sqrt(x)*e + d)^3*d*e^(-3)*log(sqrt(x)*e + d) + 72*(sqrt(x)*e + d)^2*d^2*
e^(-3)*log(sqrt(x)*e + d) - 48*(sqrt(x)*e + d)*d^3*e^(-3)*log(sqrt(x)*e + d
) - 3*(sqrt(x)*e + d)^4*e^(-3) + 16*(sqrt(x)*e + d)^3*d*e^(-3) - 36*(sqrt(x
)*e + d)^2*d^2*e^(-3) + 48*(sqrt(x)*e + d)*d^3*e^(-3))*b^3*n*log(c)^2 + 12*
(72*(sqrt(x)*e + d)^4*e^(-3)*log(sqrt(x)*e + d)^2 - 288*(sqrt(x)*e + d)^3*d
*e^(-3)*log(sqrt(x)*e + d)^2 + 432*(sqrt(x)*e + d)^2*d^2*e^(-3)*log(sqrt(x)
*e + d)^2 - 288*(sqrt(x)*e + d)*d^3*e^(-3)*log(sqrt(x)*e + d)^2 - 36*(sqrt(
x)*e + d)^4*e^(-3)*log(sqrt(x)*e + d) + 192*(sqrt(x)*e + d)^3*d*e^(-3)*log(
sqrt(x)*e + d) - 432*(sqrt(x)*e + d)^2*d^2*e^(-3)*log(sqrt(x)*e + d) + 576*
(sqrt(x)*e + d)*d^3*e^(-3)*log(sqrt(x)*e + d) + 9*(sqrt(x)*e + d)^4*e^(-3)
- 64*(sqrt(x)*e + d)^3*d*e^(-3) + 216*(sqrt(x)*e + d)^2*d^2*e^(-3) - 576*(s
qrt(x)*e + d)*d^3*e^(-3))*a*b^2*n^2 + 288*a^3*x^2*e + 144*(12*(sqrt(x)*e +
d)^4*e^(-3)*log(sqrt(x)*e + d) - 48*(sqrt(x)*e + d)^3*d*e^(-3)*log(sqrt(x)*
e + d) + 72*(sqrt(x)*e + d)^2*d^2*e^(-3)*log(sqrt(x)*e + d) - 48*(sqrt(x)*e
 + d)*d^3*e^(-3)*log(sqrt(x)*e + d) - 3*(sqrt(x)*e + d)^4*e^(-3) + 16*(sqrt
(x)*e + d)^3*d*e^(-3) - 36*(sqrt(x)*e + d)^2*d^2*e^(-3) + 48*(sqrt(x)*e + d
)*d^3*e^(-3))*a*b^2*n*log(c) + 72*(12*(sqrt(x)*e + d)^4*e^(-3)*log(sqrt(x)*
e + d) - 48*(sqrt(x)*e + d)^3*d*e^(-3)*log(sqrt(x)*e + d) + 72*(sqrt(x)*e +
 d)^2*d^2*e^(-3)*log(sqrt(x)*e + d) - 48*(sqrt(x)*e + d)*d^3*e^(-3)*log(sqr
t(x)*e + d) - 3*(sqrt(x)*e + d)^4*e^(-3) + 16*(sqrt(x)*e + d)^3*d*e^(-3) -
36*(sqrt(x)*e + d)^2*d^2*e^(-3) + 48*(sqrt(x)*e + d)*d^3*e^(-3))*a^2*b*n)*e
^(-1)
```

Mupad [B]
time = 0.77, size = 840, normalized size = 1.41

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e*x^(1/2))^n))^3,x)
```

```
[Out] log(c*(d + e*x^(1/2))^n)^3*((b^3*x^2)/2 - (b^3*d^4)/(2*e^4)) - x^(3/2)*((d*
(2*a^3 - (3*b^3*n^3)/16 + (3*a*b^2*n^2)/4 - (3*a^2*b*n)/2))/(3*e) - (d*(24*
```

```
a^3 + 7*b^3*n^3 - 12*a*b^2*n^2))/(36*e)) - log(c*(d + e*x^(1/2))^n)^2*((x^(
3/2)*((b^2*d*(4*a - b*n))/e - (4*a*b^2*d)/e))/2 - (3*b^2*x^2*(4*a - b*n))/8
 + (d*(12*a*b^2*d^3 - 25*b^3*d^3*n))/(8*e^4) + (d^2*x^(1/2)*((6*b^2*d*(4*a
- b*n))/e - (24*a*b^2*d)/e))/(4*e^2) - (d*x*((6*b^2*d*(4*a - b*n))/e - (24*
a*b^2*d)/e))/(8*e)) + x*((d*((d*(2*a^3 - (3*b^3*n^3)/16 + (3*a*b^2*n^2)/4 -
 (3*a^2*b*n)/2))/e - (d*(24*a^3 + 7*b^3*n^3 - 12*a*b^2*n^2))/(12*e)))/(2*e)
 + (b^2*d^2*n^2*(12*a - 13*b*n))/(16*e^2)) - x^(1/2)*((d*((d*((d*(2*a^3 - (
3*b^3*n^3)/16 + (3*a*b^2*n^2)/4 - (3*a^2*b*n)/2))/e - (d*(24*a^3 + 7*b^3*n^
3 - 12*a*b^2*n^2))/(12*e)))/e + (b^2*d^2*n^2*(12*a - 13*b*n))/(8*e^2)))/e +
 (b^2*d^3*n^2*(12*a - 25*b*n))/(4*e^3)) + x^2*(a^3/2 - (3*b^3*n^3)/64 + (3*
a*b^2*n^2)/16 - (3*a^2*b*n)/8) + (log(c*(d + e*x^(1/2))^n)*((x^(3/2)*(16*b*
d*e^3*(6*a^2 - b^2*n^2) - 12*b*d*e^3*(8*a^2 + b^2*n^2 - 4*a*b*n)))/(12*e^2)
 - (x*((d*(16*b*d*e^3*(6*a^2 - b^2*n^2) - 12*b*d*e^3*(8*a^2 + b^2*n^2 - 4*a
*b*n)))/e - 24*b^3*d^2*e^2*n^2))/(8*e^2) + (x^(1/2)*((d*((d*(16*b*d*e^3*(6*
a^2 - b^2*n^2) - 12*b*d*e^3*(8*a^2 + b^2*n^2 - 4*a*b*n)))/e - 24*b^3*d^2*e^
2*n^2))/e - 48*b^3*d^3*e*n^2))/(4*e^2) + (3*b*e^2*x^2*(8*a^2 + b^2*n^2 - 4*
a*b*n))/4))/(4*e^2) - (log(d + e*x^(1/2))*(415*b^3*d^4*n^3 - 300*a*b^2*d^4*
n^2 + 72*a^2*b*d^4*n))/(48*e^4)
```

3.417 $\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=284

$$-\frac{3b^3n^3\left(d+e\sqrt{x}\right)^2}{4e^2}-\frac{12ab^2dn^2\sqrt{x}}{e}+\frac{12b^3dn^3\sqrt{x}}{e}-\frac{12b^3dn^2\left(d+e\sqrt{x}\right)\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{e^2}+\frac{3b^2n^2\left(d+e\right.}{}$$

```
[Out] -12*a*b^2*d*n^2*x^(1/2)/e+12*b^3*d*n^3*x^(1/2)/e-12*b^3*d*n^2*ln(c*(d+e*x^(
1/2))^n)*(d+e*x^(1/2))/e^2+6*b*d*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1/
2))/e^2-2*d*(a+b*ln(c*(d+e*x^(1/2))^n))^3*(d+e*x^(1/2))/e^2-3/4*b^3*n^3*(d+
e*x^(1/2))^2/e^2+3/2*b^2*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))^2/e^
2-3/2*b*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1/2))^2/e^2+(a+b*ln(c*(d+e*
x^(1/2))^n))^3*(d+e*x^(1/2))^2/e^2
```

Rubi [A]
time = 0.17, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2501, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

$$\frac{3b^2n^2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2e^2}-\frac{12ab^2dn^2\sqrt{x}}{e}-\frac{3bn\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2e^2}+\frac{6bdn\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^2}+\frac{\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^2}-\frac{2d\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^2}-\frac{12b^3dn^2\left(d+e\sqrt{x}\right)\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{e^2}-\frac{3b^3n^3\left(d+e\sqrt{x}\right)^2}{4e^2}+\frac{12b^3dn^3\sqrt{x}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^3,x]
```

```
[Out] (-3*b^3*n^3*(d + e*Sqrt[x])^2)/(4*e^2) - (12*a*b^2*d*n^2*Sqrt[x])/e + (12*b
^3*d*n^3*Sqrt[x])/e - (12*b^3*d*n^2*(d + e*Sqrt[x])*Log[c*(d + e*Sqrt[x])^n
])/e^2 + (3*b^2*n^2*(d + e*Sqrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(2*
e^2) + (6*b*d*n*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/e^2 - (
3*b*n*(d + e*Sqrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(2*e^2) - (2*d*
(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/e^2 + ((d + e*Sqrt[x])^
2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/e^2
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
```

```
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2501

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx = 2\text{Subst}\left(\int x(a+b\log(c(d+ex)^n))^3\,dx, x, \sqrt{x}\right)$$

$$= 2\text{Subst}\left(\int \left(-\frac{d(a+b\log(c(d+ex)^n))^3}{e} + \frac{(d+ex)\,(a+b\log(c(d+$$

$$= \frac{2\text{Subst}\left(\int (d+ex)\,(a+b\log(c(d+ex)^n))^3\,dx, x, \sqrt{x}\right)}{e} - \frac{(2d)\text{Subst}(}{}$$

$$= \frac{2\text{Subst}\left(\int x(a+b\log(cx^n))^3\,dx, x, d+e\sqrt{x}\right)}{e^2} - \frac{(2d)\text{Subst}\left(\int (a+b\right.}{}$$

$$= -\frac{2d\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^2} + \frac{\left(d+e\sqrt{x}\right)^2\left(a+b\log\right.}{e^2}$$

$$= \frac{6bdn\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^2} - \frac{3bn\left(d+e\sqrt{x}\right)^2\left(a+\right.}{}$$

$$= -\frac{3b^3n^3\left(d+e\sqrt{x}\right)^2}{4e^2} - \frac{12ab^2dn^2\sqrt{x}}{e} + \frac{3b^2n^2\left(d+e\sqrt{x}\right)^2\left(a+b\log\right.}{2e^2}$$

$$= -\frac{3b^3n^3\left(d+e\sqrt{x}\right)^2}{4e^2} - \frac{12ab^2dn^2\sqrt{x}}{e} + \frac{12b^3dn^3\sqrt{x}}{e} - \frac{12b^3dn^2(d+}{}$$

Mathematica [A]
time = 0.14, size = 241, normalized size = 0.85

$$\frac{-8d\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3+4\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3+24bdn\left(\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2-2bn\left(e(a-bn)\sqrt{x}+b\left(d+e\sqrt{x}\right)\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\right)-3bn\left(2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2+bn\left(ben\left(2d\sqrt{x}+ex\right)-2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\right)\right)}{4e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^3,x]
```

```
[Out] (-8*d*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^3 + 4*(d + e*Sqrt[x]
)^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3 + 24*b*d*n*((d + e*Sqrt[x])*(a + b*L
og[c*(d + e*Sqrt[x])^n])^2 - 2*b*n*(e*(a - b*n)*Sqrt[x] + b*(d + e*Sqrt[x])
*Log[c*(d + e*Sqrt[x])^n])) - 3*b*n*(2*(d + e*Sqrt[x])^2*(a + b*Log[c*(d +
e*Sqrt[x])^n])^2 + b*n*(b*e*n*(2*d*Sqrt[x] + e*x) - 2*(d + e*Sqrt[x])^2*(a
+ b*Log[c*(d + e*Sqrt[x])^n]))))/(4*e^2)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))^3,x)
```

Maxima [A]
time = 0.30, size = 394, normalized size = 1.39

$$-\frac{3}{2}\left(\left(2\,d^2e^{(-3)}\log\left(\sqrt{x}e+d\right)+\left(xe-2\,d\sqrt{x}\right)e^{(-2)}\right)ne-2\,x\log\left(\left(\sqrt{x}e+d\right)^nc\right)\right)a^2b+\frac{3}{2}\left(\left(2\,d^2\log\left(\sqrt{x}e+d\right)^2+6\,d^2\log\left(\sqrt{x}e+d\right)-6\,d\sqrt{x}e+xe^2\right)n^2e^{(-2)}-2\left(2\,d^2e^{(-3)}\log\left(\sqrt{x}e+d\right)+\left(xe-2\,d\sqrt{x}\right)e^{(-2)}\right)ne\log\left(\left(\sqrt{x}e+d\right)^nc\right)+2\,x\log\left(\left(\sqrt{x}e+d\right)^nc\right)^2\right)ab^2-\frac{1}{4}\left(6\left(2\,d^2e^{(-3)}\log\left(\sqrt{x}e+d\right)+\left(xe-2\,d\sqrt{x}\right)e^{(-2)}\right)ne\log\left(\left(\sqrt{x}e+d\right)^nc\right)^2-4\,x\log\left(\left(\sqrt{x}e+d\right)^nc\right)^3+\left(\left(4\,d^2\log\left(\sqrt{x}e+d\right)^3+18\,d^2\log\left(\sqrt{x}e+d\right)^2+42\,d^2\log\left(\sqrt{x}e+d\right)-42\,d\sqrt{x}e+3\,xe^2\right)n^2e^{(-3)}-6\left(2\,d^2\log\left(\sqrt{x}e+d\right)^2+6\,d^2\log\left(\sqrt{x}e+d\right)-6\,d\sqrt{x}e+xe^2\right)ne^{(-3)}\log\left(\left(\sqrt{x}e+d\right)^nc\right)\right)ne\right)b^3+a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="maxima")
```

```
[Out] -3/2*((2*d^2*e^(-3)*log(sqrt(x)*e + d) + (x*e - 2*d*sqrt(x))*e^(-2))*n*e -
2*x*log((sqrt(x)*e + d)^n*c))*a^2*b + 3/2*((2*d^2*log(sqrt(x)*e + d)^2 + 6*
d^2*log(sqrt(x)*e + d) - 6*d*sqrt(x)*e + x*e^2)*n^2*e^(-2) - 2*(2*d^2*e^(-3
)*log(sqrt(x)*e + d) + (x*e - 2*d*sqrt(x))*e^(-2))*n*e*log((sqrt(x)*e + d)^
n*c) + 2*x*log((sqrt(x)*e + d)^n*c)^2)*a*b^2 - 1/4*(6*(2*d^2*e^(-3)*log(sqr
t(x)*e + d) + (x*e - 2*d*sqrt(x))*e^(-2))*n*e*log((sqrt(x)*e + d)^n*c)^2 -
4*x*log((sqrt(x)*e + d)^n*c)^3 + ((4*d^2*log(sqrt(x)*e + d)^3 + 18*d^2*log(
sqrt(x)*e + d)^2 + 42*d^2*log(sqrt(x)*e + d) - 42*d*sqrt(x)*e + 3*x*e^2)*n^
2*e^(-3) - 6*(2*d^2*log(sqrt(x)*e + d)^2 + 6*d^2*log(sqrt(x)*e + d) - 6*d*s
qrt(x)*e + x*e^2)*n*e^(-3)*log((sqrt(x)*e + d)^n*c))*n*e)*b^3 + a^3*x
```

Fricas [A]
time = 0.42, size = 496, normalized size = 1.75

$$\frac{1}{4}\left(4\,b^3xe^2\log\left(c\right)^3-6\left(b^3n-2\,ab^2\right)xe^2\log\left(c\right)^2-4\left(b^3d^2n^3-b^3n^3xe^2\right)\log\left(\sqrt{x}e+d\right)^3+6\left(b^3n^2-2\,ab^2n+2\,a^2b\right)xe^2\log\left(c\right)-\left(3\,b^3n^3-6\,ab^2n^2+6\,a^2bn-4\,a^3\right)xe^2+6\left(2\,b^3dn^3\sqrt{x}e+3\,b^3d^2n^3-2\,ab^2d^2n^2-\left(b^3n^3-2\,ab^2n^2\right)xe^2-2\left(b^3d^2n^2-b^3n^2xe^2\right)\log\left(c\right)\right)\log\left(\sqrt{x}e+d\right)^2-6\left(7\,b^3d^2n^3-6\,ab^2d^2n^2+2\,a^2bd^2n-\left(b^3n^3-2\,ab^2n^2+2\,a^2bn\right)xe^2+2\left(b^3d^2n-b^3nxe^2\right)\log\left(c\right)^2-2\left(3\,b^3d^2n^2-2\,ab^2d^2n-\left(b^3n^2-2\,ab^2n\right)xe^2\right)\log\left(c\right)-2\left(2\,b^3dn^2e\log\left(c\right)-\left(3\,b^3dn^3-2\,ab^2dn^2\right)e\right)\sqrt{x}\right)\log\left(\sqrt{x}e+d\right)+6\left(2\,b^3dne\log\left(c\right)^2-2\left(3\,b^3dn^2-2\,ab^2dn\right)e\log\left(c\right)+\left(7\,b^3dn^3-6\,ab^2dn^2+2\,a^2bdn\right)e\right)\sqrt{x}\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^3*x*e^2*log(c)^3 - 6*(b^3*n - 2*a*b^2)*x*e^2*log(c)^2 - 4*(b^3*d^2
*n^3 - b^3*n^3*x*e^2)*log(sqrt(x)*e + d)^3 + 6*(b^3*n^2 - 2*a*b^2*n + 2*a^2
*b)*x*e^2*log(c) - (3*b^3*n^3 - 6*a*b^2*n^2 + 6*a^2*b*n - 4*a^3)*x*e^2 + 6*
(2*b^3*d*n^3*sqrt(x)*e + 3*b^3*d^2*n^3 - 2*a*b^2*d^2*n^2 - (b^3*n^3 - 2*a*b
^2*n^2)*x*e^2 - 2*(b^3*d^2*n^2 - b^3*n^2*x*e^2)*log(c))*log(sqrt(x)*e + d)^
2 - 6*(7*b^3*d^2*n^3 - 6*a*b^2*d^2*n^2 + 2*a^2*b*d^2*n - (b^3*n^3 - 2*a*b^2
*n^2 + 2*a^2*b*n)*x*e^2 + 2*(b^3*d^2*n - b^3*n*x*e^2)*log(c)^2 - 2*(3*b^3*d
^2*n^2 - 2*a*b^2*d^2*n - (b^3*n^2 - 2*a*b^2*n)*x*e^2)*log(c) - 2*(2*b^3*d*n
^2*e*log(c) - (3*b^3*d*n^3 - 2*a*b^2*d*n^2)*e)*sqrt(x))*log(sqrt(x)*e + d)
+ 6*(2*b^3*d*n*e*log(c)^2 - 2*(3*b^3*d*n^2 - 2*a*b^2*d*n)*e*log(c) + (7*b^3
*d*n^3 - 6*a*b^2*d*n^2 + 2*a^2*b*d*n)*e)*sqrt(x))*e^(-2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**3,x)
```

```
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 763 vs. $2(252) = 504$.
time = 2.59, size = 763, normalized size = 2.69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="giac")
```

```
[Out] 1/4*((4*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d)^3 - 8*(sqrt(x)*e + d)*d*log(sq
rt(x)*e + d)^3 - 6*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d)^2 + 24*(sqrt(x)*e +
 d)*d*log(sqrt(x)*e + d)^2 + 6*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d) - 48*(s
qrt(x)*e + d)*d*log(sqrt(x)*e + d) - 3*(sqrt(x)*e + d)^2 + 48*(sqrt(x)*e +
d)*d)*b^3*n^3*e^(-1) + 6*(2*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d)^2 - 4*(sqr
t(x)*e + d)*d*log(sqrt(x)*e + d)^2 - 2*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d)
 + 8*(sqrt(x)*e + d)*d*log(sqrt(x)*e + d) + (sqrt(x)*e + d)^2 - 8*(sqrt(x)*
e + d)*d)*b^3*n^2*e^(-1)*log(c) + 6*(2*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d)
 - 4*(sqrt(x)*e + d)*d*log(sqrt(x)*e + d) - (sqrt(x)*e + d)^2 + 4*(sqrt(x)*
e + d)*d)*b^3*n*e^(-1)*log(c)^2 + 4*((sqrt(x)*e + d)^2 - 2*(sqrt(x)*e + d)*
d)*b^3*e^(-1)*log(c)^3 + 6*(2*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d)^2 - 4*(s
qrt(x)*e + d)*d*log(sqrt(x)*e + d)^2 - 2*(sqrt(x)*e + d)^2*log(sqrt(x)*e +
d) + 8*(sqrt(x)*e + d)*d*log(sqrt(x)*e + d) + (sqrt(x)*e + d)^2 - 8*(sqrt(x
)*e + d)*d)*a*b^2*n^2*e^(-1) + 12*(2*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d) -
 4*(sqrt(x)*e + d)*d*log(sqrt(x)*e + d) - (sqrt(x)*e + d)^2 + 4*(sqrt(x)*e
+ d)*d)*a*b^2*n*e^(-1)*log(c) + 12*((sqrt(x)*e + d)^2 - 2*(sqrt(x)*e + d)*d
)*a*b^2*e^(-1)*log(c)^2 + 6*(2*(sqrt(x)*e + d)^2*log(sqrt(x)*e + d) - 4*(sq
rt(x)*e + d)*d*log(sqrt(x)*e + d) - (sqrt(x)*e + d)^2 + 4*(sqrt(x)*e + d)*d
)*a^2*b*n*e^(-1) + 12*((sqrt(x)*e + d)^2 - 2*(sqrt(x)*e + d)*d)*a^2*b*e^(-1
)*log(c) + 4*((sqrt(x)*e + d)^2 - 2*(sqrt(x)*e + d)*d)*a^3*e^(-1))*e^(-1)
```

Mupad [B]
time = 0.60, size = 350, normalized size = 1.23

$$x\left(a^3-\frac{3a^2bn}{2}+\frac{3ab^2n^2}{2}-\frac{3b^3n^3}{4}\right)-\sqrt{x}\left(\frac{d\left(2a^3-3a^2bn+3ab^2n^2-\frac{3b^3n^3}{2}\right)}{e}-\frac{d\left(2a^3-6ab^2n^2+9b^3n^3\right)}{e}\right)+\ln\left(c\left(d+e\sqrt{x}\right)^n\right)^3\left(b^3x-\frac{b^3d^2}{e^2}\right)-\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\left(\sqrt{x}\left(\frac{3bd\left(2a^2-2abn+b^2n^2\right)}{e}-\frac{6bd\left(a^2-b^2n^2\right)}{e}\right)-\frac{3bx\left(2a^2-2abn+b^2n^2\right)}{2}\right)-\ln\left(c\left(d+e\sqrt{x}\right)^n\right)^2\left(\sqrt{x}\left(\frac{3b^2d\left(2a-bn\right)}{e}-\frac{6ab^2d}{e}\right)+\frac{3d\left(2ab^2d-3b^3dn\right)}{2e^2}-\frac{3b^2x\left(2a-bn\right)}{2}\right)-\frac{\ln\left(d+e\sqrt{x}\right)\left(6a^2bd^2n-18ab^2d^2n^2+21b^3d^2n^3\right)}{2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))^3,x)
```

```
[Out] x*(a^3 - (3*b^3*n^3)/4 + (3*a*b^2*n^2)/2 - (3*a^2*b*n)/2) - x^(1/2)*((d*(2*
a^3 - (3*b^3*n^3)/2 + 3*a*b^2*n^2 - 3*a^2*b*n))/e - (d*(2*a^3 + 9*b^3*n^3 -
```

```
6*a*b^2*n^2))/e) + log(c*(d + e*x^(1/2))^n)^3*(b^3*x - (b^3*d^2)/e^2) - lo
g(c*(d + e*x^(1/2))^n)*(x^(1/2)*((3*b*d*(2*a^2 + b^2*n^2 - 2*a*b*n))/e - (6
*b*d*(a^2 - b^2*n^2))/e) - (3*b*x*(2*a^2 + b^2*n^2 - 2*a*b*n))/2) - log(c*(
d + e*x^(1/2))^n)^2*(x^(1/2)*((3*b^2*d*(2*a - b*n))/e - (6*a*b^2*d)/e) + (3
*d*(2*a*b^2*d - 3*b^3*d*n))/(2*e^2) - (3*b^2*x*(2*a - b*n))/2) - (log(d + e
*x^(1/2))*(21*b^3*d^2*n^3 - 18*a*b^2*d^2*n^2 + 6*a^2*b*d^2*n))/(2*e^2)
```

3.418 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x}\,dx$

Optimal. Leaf size=135

$$2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt{x}}{d}\right)+6bn\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\text{Li}_2\left(1+\frac{e\sqrt{x}}{d}\right)-12b^2n^2\left(a+\right.$$

```
[Out] 2*ln(-e*x^(1/2)/d)*(a+b*ln(c*(d+e*x^(1/2))^n))^3+6*b*n*(a+b*ln(c*(d+e*x^(1/
2))^n))^2*polylog(2,1+e*x^(1/2)/d)-12*b^2*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*p
olylog(3,1+e*x^(1/2)/d)+12*b^3*n^3*polylog(4,1+e*x^(1/2)/d)
```

Rubi [A]
time = 0.14, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2504, 2443, 2481, 2421, 2430, 6724}

$$-12b^2n^2\text{PolyLog}\left(3,\frac{e\sqrt{x}}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)+6bn\text{PolyLog}\left(2,\frac{e\sqrt{x}}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2+12b^3n^3\text{PolyLog}\left(4,\frac{e\sqrt{x}}{d}+1\right)+2\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^3/x,x]
```

```
[Out] 2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3*Log[-((e*Sqrt[x])/d)] + 6*b*n*(a + b*L
og[c*(d + e*Sqrt[x])^n])^2*PolyLog[2, 1 + (e*Sqrt[x])/d] - 12*b^2*n^2*(a +
b*Log[c*(d + e*Sqrt[x])^n])*PolyLog[3, 1 + (e*Sqrt[x])/d] + 12*b^3*n^3*Poly
Log[4, 1 + (e*Sqrt[x])/d]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
```

```
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x}\,dx = 2\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{x}\,dx, x, \sqrt{x}\right)$$

$$= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt{x}}{d}\right) - (6ben)\text{Subst}\left(\int \frac{\log\left(\right.}{}\right.$$

$$= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt{x}}{d}\right) - (6bn)\text{Subst}\left(\int \frac{(a+b}{}\right.$$

$$= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt{x}}{d}\right) + 6bn\left(a+b\log\left(c\left(d+e\right.\right.\right.$$

$$= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt{x}}{d}\right) + 6bn\left(a+b\log\left(c\left(d+e\right.\right.\right.$$

$$= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt{x}}{d}\right) + 6bn\left(a+b\log\left(c\left(d+e\right.\right.\right.$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 333 vs. 2(135) = 270.
time = 0.09, size = 333, normalized size = 2.47

$$\left(a-bn\log\left(d+e\sqrt{x}\right)+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\log(x)+3bn\left(a-bn\log\left(d+e\sqrt{x}\right)+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\left(\left(\log\left(d+e\sqrt{x}\right)-\log\left(1+\frac{e\sqrt{x}}{d}\right)\right)\log(x)-2\text{Li}_2\left(-\frac{e\sqrt{x}}{d}\right)\right)+6b^2n^2\left(a-bn\log\left(d+e\sqrt{x}\right)+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\left(\log^2\left(d+e\sqrt{x}\right)\log\left(-\frac{e\sqrt{x}}{d}\right)+2\log\left(d+e\sqrt{x}\right)\text{Li}_2\left(1+\frac{e\sqrt{x}}{d}\right)-2\text{Li}_3\left(1+\frac{e\sqrt{x}}{d}\right)\right)+2b^3n^3\left(\log^3\left(d+e\sqrt{x}\right)\log\left(-\frac{e\sqrt{x}}{d}\right)+3\log^2\left(d+e\sqrt{x}\right)\text{Li}_2\left(1+\frac{e\sqrt{x}}{d}\right)-6\log\left(d+e\sqrt{x}\right)\text{Li}_3\left(1+\frac{e\sqrt{x}}{d}\right)+6\text{Li}_4\left(1+\frac{e\sqrt{x}}{d}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^3/x,x]
```

```
[Out] (a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^3*Log[x] + 3*b*n*
(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^2*((Log[d + e*Sqr
t[x]] - Log[1 + (e*Sqrt[x])/d])*Log[x] - 2*PolyLog[2, -((e*Sqrt[x])/d)]) +
6*b^2*n^2*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])*(Log[d
+ e*Sqrt[x]]^2*Log[-((e*Sqrt[x])/d)] + 2*Log[d + e*Sqrt[x]]*PolyLog[2, 1 +
(e*Sqrt[x])/d] - 2*PolyLog[3, 1 + (e*Sqrt[x])/d]) + 2*b^3*n^3*(Log[d + e*Sq
rt[x]]^3*Log[-((e*Sqrt[x])/d)] + 3*Log[d + e*Sqrt[x]]^2*PolyLog[2, 1 + (e*S
qrt[x])/d] - 6*Log[d + e*Sqrt[x]]*PolyLog[3, 1 + (e*Sqrt[x])/d] + 6*PolyLog
[4, 1 + (e*Sqrt[x])/d])
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))^3/x,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))^3/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x,x, algorithm="maxima")
```

```
[Out] b^3*n^3*log(sqrt(x)*e + d)^3*log(x) + integrate(-1/2*(3*(b^3*n*x*e*log(x) -
 2*(b^3*log(c) + a*b^2)*x*e - 2*(b^3*d*log(c) + a*b^2*d)*sqrt(x))*n^2*log(s
qrt(x)*e + d)^2 - 2*(b^3*log(c)^3 + 3*a*b^2*log(c)^2 + 3*a^2*b*log(c) + a^3
)*x*e - 6*((b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*b)*x*e + (b^3*d*log(c)^2 +
2*a*b^2*d*log(c) + a^2*b*d)*sqrt(x))*n*log(sqrt(x)*e + d) - 2*(b^3*d*log(c)
^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*sqrt(x))/(x^2*e + d*x^(
3/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((sqrt(x)*e + d)^n*c)^3 + 3*a*b^2*log((sqrt(x)*e + d)^n*c)
^2 + 3*a^2*b*log((sqrt(x)*e + d)^n*c) + a^3)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \log \left(c \left(d + e \sqrt{x}\right)^{n}\right)\right)^{3}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**3/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))**3/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((sqrt(x)*e + d)^n*c) + a)^3/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,\sqrt{x}\,\right)}^n\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))^3/x,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/2))^n))^3/x, x)
```

3.419 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^2}\,dx$

Optimal. Leaf size=263

$$-\frac{3ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{d^2\sqrt{x}}-\frac{3be^2n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{d^2}-\frac{(a+$$

```
[Out] 6*b^2*e^2*n^2*ln(-e*x^(1/2)/d)*(a+b*ln(c*(d+e*x^(1/2))^n))/d^2-(a+b*ln(c*(d
+e*x^(1/2))^n))^3/x-3*b*e^2*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*ln(1-d/(d+e*x^(
1/2)))/d^2+6*b^2*e^2*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*polylog(2,d/(d+e*x^(1/
2)))/d^2+6*b^3*e^2*n^3*polylog(2,1+e*x^(1/2)/d)/d^2+6*b^3*e^2*n^3*polylog(3
,d/(d+e*x^(1/2)))/d^2-3*b*e*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1/2))/d
^2/x^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2504, 2445, 2458, 2389, 2379, 2421, 6724, 2355, 2354, 2438}

$$\frac{6b^2e^2n^2\text{PolyLog}\left(2,\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2}+\frac{6b^3e^2n^3\text{PolyLog}\left(2,\frac{e\sqrt{x}}{d}+1\right)}{d^2}+\frac{6b^3e^2n^3\text{PolyLog}\left(3,\frac{d}{d+e\sqrt{x}}\right)}{d^2}+\frac{6b^2e^2n^2\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2}-\frac{3be^2n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{d^2}-\frac{3ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{d^2\sqrt{x}}-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^3/x^2,x]
```

```
[Out] (-3*b*e*n*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(d^2*Sqrt[x])
 - (3*b*e^2*n*Log[1 - d/(d + e*Sqrt[x])]*(a + b*Log[c*(d + e*Sqrt[x])^n])^2
)/d^2 - (a + b*Log[c*(d + e*Sqrt[x])^n])^3/x + (6*b^2*e^2*n^2*(a + b*Log[c*
(d + e*Sqrt[x])^n])*Log[-((e*Sqrt[x])/d)])/d^2 + (6*b^2*e^2*n^2*(a + b*Log[
c*(d + e*Sqrt[x])^n])*PolyLog[2, d/(d + e*Sqrt[x])])/d^2 + (6*b^3*e^2*n^3*P
olyLog[2, 1 + (e*Sqrt[x])/d])/d^2 + (6*b^3*e^2*n^3*PolyLog[3, d/(d + e*Sqrt
[x])])/d^2
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
```

```
, p}, x] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^2}\,dx = 2\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{x^3}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x} + (3ben)\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)\right.\right.}{x^2(d+ex)}\right.$$
$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x} + (3bn)\text{Subst}\left(\int \frac{\left(a+b\log\left(cx^n\right)\right)^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx,\right.$$
$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x} + \frac{(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+\right.}{d}$$
$$= -\frac{3ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{d^2\sqrt{x}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{\right.\right.\right.}{x}$$
$$= -\frac{3ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{d^2\sqrt{x}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{\right.\right.\right.}{x}$$
$$= -\frac{3ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{d^2\sqrt{x}} + \frac{e^2\left(a+b\log\left(c\left(d+e\right.\right.\right.}{d^2}$$
$$= -\frac{3ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{d^2\sqrt{x}} + \frac{e^2\left(a+b\log\left(c\left(d+e\right.\right.\right.}{d^2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 536 vs. 2(263) = 526.

time = 0.54, size = 536, normalized size = 2.04

$$\frac{-3bden\sqrt{x}\left(a-bn\log\left(d+e\sqrt{x}\right)+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2-3bd^2n\log\left(d+e\sqrt{x}\right)\left(a-bn\log\left(d+e\sqrt{x}\right)+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2+3be^2nx\log\left(d+e\sqrt{x}\right)\left(a-bn\log\left(d+e\sqrt{x}\right)+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2-d^2\left(a-bn\log\left(d+e\sqrt{x}\right)+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3-\frac{3}{2}be^2nx\left(a-bn\log\left(d+e\sqrt{x}\right)+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\log(x)+3b^2n^2\left(a-bn\log\left(d+e\sqrt{x}\right)+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\left(\left(d+e\sqrt{x}\right)\log\left(d+e\sqrt{x}\right)\left(-2e\sqrt{x}+\left(-d+e\sqrt{x}\right)\log\left(d+e\sqrt{x}\right)\right)-2e^2x\left(-1+\log\left(d+e\sqrt{x}\right)\right)\log\left(-\frac{e\sqrt{x}}{d}\right)-2e^2x\operatorname{Li}_2\left(1+\frac{e\sqrt{x}}{d}\right)\right)+b^3n^3\left(\left(d+e\sqrt{x}\right)\log^2\left(d+e\sqrt{x}\right)\left(-3e\sqrt{x}+\left(-d+e\sqrt{x}\right)\log\left(d+e\sqrt{x}\right)\right)-3e^2x\left(-2+\log\left(d+e\sqrt{x}\right)\right)\log\left(d+e\sqrt{x}\right)\log\left(-\frac{e\sqrt{x}}{d}\right)-6e^2x\left(-1+\log\left(d+e\sqrt{x}\right)\right)\operatorname{Li}_2\left(1+\frac{e\sqrt{x}}{d}\right)+6e^2x\operatorname{Li}_3\left(1+\frac{e\sqrt{x}}{d}\right)\right)}{d^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^3/x^2,x]
```

```
[Out] (-3*b*d*e*n*Sqrt[x]*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n
])^2 - 3*b*d^2*n*Log[d + e*Sqrt[x]]*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(
d + e*Sqrt[x])^n])^2 + 3*b*e^2*n*x*Log[d + e*Sqrt[x]]*(a - b*n*Log[d + e*Sq
rt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^2 - d^2*(a - b*n*Log[d + e*Sqrt[x]] +
b*Log[c*(d + e*Sqrt[x])^n])^3 - (3*b*e^2*n*x*(a - b*n*Log[d + e*Sqrt[x]] +
b*Log[c*(d + e*Sqrt[x])^n])^2*Log[x])/2 + 3*b^2*n^2*(a - b*n*Log[d + e*Sqrt
[x]] + b*Log[c*(d + e*Sqrt[x])^n])*((d + e*Sqrt[x])*Log[d + e*Sqrt[x]]*(-2*
e*Sqrt[x] + (-d + e*Sqrt[x])*Log[d + e*Sqrt[x]]) - 2*e^2*x*(-1 + Log[d + e*
Sqrt[x]])*Log[-((e*Sqrt[x])/d)] - 2*e^2*x*PolyLog[2, 1 + (e*Sqrt[x])/d]) +
b^3*n^3*((d + e*Sqrt[x])*Log[d + e*Sqrt[x]]^2*(-3*e*Sqrt[x] + (-d + e*Sqrt[
x])*Log[d + e*Sqrt[x]]) - 3*e^2*x*(-2 + Log[d + e*Sqrt[x]])*Log[d + e*Sqrt[
x]]*Log[-((e*Sqrt[x])/d)] - 6*e^2*x*(-1 + Log[d + e*Sqrt[x]])*PolyLog[2, 1
+ (e*Sqrt[x])/d] + 6*e^2*x*PolyLog[3, 1 + (e*Sqrt[x])/d]))/(d^2*x)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{3}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))^3/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))^3/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^3*d^2*n^3*sqrt(x)*log(sqrt(x)*e + d)^3 - 3*(2*b^3*n*x^(3/2)*e^2*l
og(sqrt(x)*e + d) - 2*b^3*d*n*x*e - (b^3*n*x*e^2*log(x) + 2*b^3*d^2*log(c)
+ 2*a*b^2*d^2)*sqrt(x))*n^2*log(sqrt(x)*e + d)^2)/(d^2*x^(3/2)) - integrate
(-1/2*(2*(b^3*d^2*log(c)^3 + 3*a*b^2*d^2*log(c)^2 + 3*a^2*b*d^2*log(c) + a^
3*d^2)*x^(3/2)*e - 3*(2*b^3*n^2*x^(5/2)*e^3*log(sqrt(x)*e + d) - 2*b^3*d*n^
```

```
2*x^2*e^2 - 2*(b^3*d^2*log(c)^2 + 2*a*b^2*d^2*log(c) + a^2*b*d^2)*x^(3/2)*e
 - 2*(b^3*d^3*log(c)^2 + 2*a*b^2*d^3*log(c) + a^2*b*d^3)*x - (b^3*n^2*x^2*e
^3*log(x) + 2*(b^3*d^2*n*log(c) + a*b^2*d^2*n)*x*e)*sqrt(x))*n*log(sqrt(x)*
e + d) + 2*(b^3*d^3*log(c)^3 + 3*a*b^2*d^3*log(c)^2 + 3*a^2*b*d^3*log(c) +
a^3*d^3)*x)/(d^2*x^(7/2)*e + d^3*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x^2,x, algorithm="fricas")
[Out] integral((b^3*log((sqrt(x)*e + d)^n*c)^3 + 3*a*b^2*log((sqrt(x)*e + d)^n*c)
^2 + 3*a^2*b*log((sqrt(x)*e + d)^n*c) + a^3)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \log{\left(c \left(d + e \sqrt{x}\right)^{n} \right)}\right)^{3}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**3/x**2,x)
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))**3/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x^2,x, algorithm="giac")
[Out] integrate((b*log((sqrt(x)*e + d)^n*c) + a)^3/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)\right)}^3}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))^3/x^2,x)
[Out] int((a + b*log(c*(d + e*x^(1/2))^n))^3/x^2, x)
```

3.420 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^3}\,dx$

Optimal. Leaf size=573

$$-\frac{b^3e^3n^3}{2d^3\sqrt{x}}+\frac{b^3e^4n^3\log\left(d+e\sqrt{x}\right)}{2d^4}-\frac{b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^2x}+\frac{5b^2e^3n^2\left(d+e\sqrt{x}\right)\left(a+b\log\left(c(d\right.\right.}{2d^4\sqrt{x}}$$

```
[Out] -3/2*b^3*e^4*n^3*ln(x)/d^4+1/2*b^3*e^4*n^3*ln(d+e*x^(1/2))/d^4-1/2*b^2*e^2*
n^2*(a+b*ln(c*(d+e*x^(1/2))^n))/d^2/x+3*b^2*e^4*n^2*ln(-e*x^(1/2)/d)*(a+b*l
n(c*(d+e*x^(1/2))^n))/d^4-1/2*b*e*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2/d/x^(3/2)
+3/4*b*e^2*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2/d^2/x-1/2*(a+b*ln(c*(d+e*x^(1/2)
)^n))^3/x^2+5/2*b^2*e^4*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*ln(1-d/(d+e*x^(1/2)
))/d^4-3/2*b*e^4*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*ln(1-d/(d+e*x^(1/2)))/d^4-
5/2*b^3*e^4*n^3*polylog(2,d/(d+e*x^(1/2)))/d^4+3*b^2*e^4*n^2*(a+b*ln(c*(d+e
*x^(1/2))^n))*polylog(2,d/(d+e*x^(1/2)))/d^4+3*b^3*e^4*n^3*polylog(2,1+e*x^
(1/2)/d)/d^4+3*b^3*e^4*n^3*polylog(3,d/(d+e*x^(1/2)))/d^4-1/2*b^3*e^3*n^3/d
^3/x^(1/2)+5/2*b^2*e^3*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))/d^4/x^
(1/2)-3/2*b*e^3*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1/2))/d^4/x^(1/2)
```

Rubi [A]
time = 0.85, antiderivative size = 573, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 14, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {2504, 2445, 2458, 2389, 2379, 2421, 6724, 2355, 2354, 2438, 2356, 2351, 31, 46}

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^3/x^3,x]
```

```
[Out] -1/2*(b^3*e^3*n^3)/(d^3*Sqrt[x]) + (b^3*e^4*n^3*Log[d + e*Sqrt[x]])/(2*d^4)
 - (b^2*e^2*n^2*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(2*d^2*x) + (5*b^2*e^3*n^
2*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(2*d^4*Sqrt[x]) + (5*b^
2*e^4*n^2*Log[1 - d/(d + e*Sqrt[x])]*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(2*d
^4) - (b*e*n*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(2*d*x^(3/2)) + (3*b*e^2*n
*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(4*d^2*x) - (3*b*e^3*n*(d + e*Sqrt[x])
*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(2*d^4*Sqrt[x]) - (3*b*e^4*n*Log[1 - d
/(d + e*Sqrt[x])]*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(2*d^4) - (a + b*Log[
c*(d + e*Sqrt[x])^n])^3/(2*x^2) + (3*b^2*e^4*n^2*(a + b*Log[c*(d + e*Sqrt[x
])^n])*Log[-((e*Sqrt[x])/d)])/d^4 - (3*b^3*e^4*n^3*Log[x])/(2*d^4) - (5*b^3
*e^4*n^3*PolyLog[2, d/(d + e*Sqrt[x])])/(2*d^4) + (3*b^2*e^4*n^2*(a + b*Log
[c*(d + e*Sqrt[x])^n])*PolyLog[2, d/(d + e*Sqrt[x])])/d^4 + (3*b^3*e^4*n^3*
```

```
PolyLog[2, 1 + (e*Sqrt[x])/d])/d^4 + (3*b^3*e^4*n^3*PolyLog[3, d/(d + e*Sqr
t[x])])/d^4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^3}\,dx = 2\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x^5}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2x^2} + \frac{1}{2}(3ben)\text{Subst}\left(\int \frac{(a+b\log(c(d+\ldots}{x^4(d+ex)\ldots}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2x^2} + \frac{1}{2}(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\ldots\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2x^2} + \frac{(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d\ldots\right.}{2d}$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2dx^{3/2}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2x^2} - \ldots$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2dx^{3/2}} + \frac{3be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\ldots}{4d^2x}$$

$$= -\frac{b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^2x} - \frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\ldots}{2dx^{3/2}}$$

$$= -\frac{b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^2x} + \frac{5b^2e^3n^2\left(d+e\sqrt{x}\right)\left(a+b\log\ldots\right.}{2d^4\sqrt{x}}$$

$$= -\frac{b^3e^3n^3}{2d^3\sqrt{x}} + \frac{b^3e^4n^3\log\left(d+e\sqrt{x}\right)}{2d^4} - \frac{b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\ldots\right.\right.}{2d^2x}$$

$$= -\frac{b^3e^3n^3}{2d^3\sqrt{x}} + \frac{b^3e^4n^3\log\left(d+e\sqrt{x}\right)}{2d^4} - \frac{b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\ldots\right.\right.}{2d^2x}$$

Mathematica [A]
time = 0.77, size = 841, normalized size = 1.47

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^3/x^3,x]
```

```
[Out] -1/4*(2*b*d^3*e*n*Sqrt[x]*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt
[x])^n])^2 - 3*b*d^2*e^2*n*x*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*S
qrt[x])^n])^2 + 6*b*d*e^3*n*x^(3/2)*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(
d + e*Sqrt[x])^n])^2 + 6*b*d^4*n*Log[d + e*Sqrt[x]]*(a - b*n*Log[d + e*Sqrt
[x]] + b*Log[c*(d + e*Sqrt[x])^n])^2 - 6*b*e^4*n*x^2*Log[d + e*Sqrt[x]]*(a
- b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^2 + 2*d^4*(a - b*n*L
og[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^3 + 3*b*e^4*n*x^2*(a - b*n*
Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^2*Log[x] - 2*b^2*n^2*(a -
b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])*(-3*(d^4 - e^4*x^2)*Lo
g[d + e*Sqrt[x]]^2 + e^2*x*(-d^2 + 5*d*e*Sqrt[x] + 11*e^2*x*Log[-((e*Sqrt[x
])/d)]) - Log[d + e*Sqrt[x]]*(2*d^3*e*Sqrt[x] - 3*d^2*e^2*x + 6*d*e^3*x^(3/
2) + 11*e^4*x^2 + 6*e^4*x^2*Log[-((e*Sqrt[x])/d)]) - 6*e^4*x^2*PolyLog[2, 1
 + (e*Sqrt[x])/d]) + b^3*n^3*(d^2*e^2*x*(2 - 3*Log[d + e*Sqrt[x]])*Log[d +
e*Sqrt[x]] + 2*d^3*e*Sqrt[x]*Log[d + e*Sqrt[x]]^2 + 2*d^4*Log[d + e*Sqrt[x]
]^3 + 2*d*e^3*x^(3/2)*(1 - 5*Log[d + e*Sqrt[x]] + 3*Log[d + e*Sqrt[x]]^2) +
 12*e^4*x^2*(-Log[d + e*Sqrt[x]] + Log[-((e*Sqrt[x])/d)]) + 11*e^4*x^2*(Log
[d + e*Sqrt[x]]*(Log[d + e*Sqrt[x]] - 2*Log[-((e*Sqrt[x])/d)]) - 2*PolyLog[
2, 1 + (e*Sqrt[x])/d]) - 2*e^4*x^2*(Log[d + e*Sqrt[x]]^2*(Log[d + e*Sqrt[x]
] - 3*Log[-((e*Sqrt[x])/d)]) - 6*Log[d + e*Sqrt[x]]*PolyLog[2, 1 + (e*Sqrt[
x])/d] + 6*PolyLog[3, 1 + (e*Sqrt[x])/d])))/(d^4*x^2)
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\sqrt{x}\,\right)^{n}\right)\right)^{3}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))^3/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))^3/x^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b^3*n^3*log(sqrt(x)*e + d)^3/x^2 + integrate(1/4*(3*(b^3*n*x*e + 4*(b^
3*log(c) + a*b^2)*x*e + 4*(b^3*d*log(c) + a*b^2*d)*sqrt(x))*n^2*log(sqrt(x)
*e + d)^2 + 4*(b^3*log(c)^3 + 3*a*b^2*log(c)^2 + 3*a^2*b*log(c) + a^3)*x*e
+ 12*((b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*b)*x*e + (b^3*d*log(c)^2 + 2*a*b
^2*d*log(c) + a^2*b*d)*sqrt(x))*n*log(sqrt(x)*e + d) + 4*(b^3*d*log(c)^3 +
```

```
3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*sqrt(x))/(x^4*e + d*x^(7/2))
, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x^3,x, algorithm="fricas")
[Out] integral((b^3*log((sqrt(x)*e + d)^n*c)^3 + 3*a*b^2*log((sqrt(x)*e + d)^n*c)
^2 + 3*a^2*b*log((sqrt(x)*e + d)^n*c) + a^3)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\log\left(c\left(d + e\sqrt{x}\right)^{n}\right)\right)^{3}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**3/x**3,x)
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))**3/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x^3,x, algorithm="giac")
[Out] integrate((b*log((sqrt(x)*e + d)^n*c) + a)^3/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)\right)}^3}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))^3/x^3,x)
[Out] int((a + b*log(c*(d + e*x^(1/2))^n))^3/x^3, x)
```

3.421 $\int x^3 \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right) dx$

Optimal. Leaf size=171

$$\frac{be^7n\sqrt{x}}{4d^7}-\frac{be^6nx}{8d^6}+\frac{be^5nx^{3/2}}{12d^5}-\frac{be^4nx^2}{16d^4}+\frac{be^3nx^{5/2}}{20d^3}-\frac{be^2nx^3}{24d^2}+\frac{benx^{7/2}}{28d}-\frac{be^8n\log\left(d+\frac{e}{\sqrt{x}}\right)}{4d^8}+\frac{1}{4}x^4\left(a+b\log\left(c\right.\right.$$

```
[Out] -1/8*b*e^6*n*x/d^6+1/12*b*e^5*n*x^(3/2)/d^5-1/16*b*e^4*n*x^2/d^4+1/20*b*e^3
*n*x^(5/2)/d^3-1/24*b*e^2*n*x^3/d^2+1/28*b*e*n*x^(7/2)/d-1/8*b*e^8*n*ln(x)/
d^8-1/4*b*e^8*n*ln(d+e/x^(1/2))/d^8+1/4*x^4*(a+b*ln(c*(d+e/x^(1/2))^n))+1/4
*b*e^7*n*x^(1/2)/d^7
```

Rubi [A]
time = 0.08, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2504, 2442, 46}

$$\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{be^8n\log\left(d+\frac{e}{\sqrt{x}}\right)}{4d^8}-\frac{be^8n\log(x)}{8d^8}+\frac{be^7n\sqrt{x}}{4d^7}-\frac{be^6nx}{8d^6}+\frac{be^5nx^{3/2}}{12d^5}-\frac{be^4nx^2}{16d^4}+\frac{be^3nx^{5/2}}{20d^3}-\frac{be^2nx^3}{24d^2}+\frac{benx^{7/2}}{28d}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e/Sqrt[x])^n]),x]
```

```
[Out] (b*e^7*n*Sqrt[x])/(4*d^7) - (b*e^6*n*x)/(8*d^6) + (b*e^5*n*x^(3/2))/(12*d^5
) - (b*e^4*n*x^2)/(16*d^4) + (b*e^3*n*x^(5/2))/(20*d^3) - (b*e^2*n*x^3)/(24
*d^2) + (b*e*n*x^(7/2))/(28*d) - (b*e^8*n*Log[d + e/Sqrt[x]])/(4*d^8) + (x^
4*(a + b*Log[c*(d + e/Sqrt[x])^n]))/4 - (b*e^8*n*Log[x])/(8*d^8)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)dx=-\left(2\text{Subst}\left(\int\frac{a+b\log\left(c(d+ex)^n\right)}{x^9}\,dx,x,\frac{1}{\sqrt{x}}\right)\right)$$
$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{1}{4}(ben)\text{Subst}\left(\int\frac{1}{x^8(d+ex}\right.$$
$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{1}{4}(ben)\text{Subst}\left(\int\left(\frac{1}{dx^8}-\right.\right.$$
$$=\frac{be^7n\sqrt{x}}{4d^7}-\frac{be^6nx}{8d^6}+\frac{be^5nx^{3/2}}{12d^5}-\frac{be^4nx^2}{16d^4}+\frac{be^3nx^{5/2}}{20d^3}-\frac{be^2nx^3}{24d^2}+$$

Mathematica [A]
time = 0.10, size = 158, normalized size = 0.92

$$\frac{ax^4}{4}+\frac{1}{4}bx^4\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-\frac{1}{4}ben\left(-\frac{e^6\sqrt{x}}{d^7}+\frac{e^5x}{2d^6}-\frac{e^4x^{3/2}}{3d^5}+\frac{e^3x^2}{4d^4}-\frac{e^2x^{5/2}}{5d^3}+\frac{ex^3}{6d^2}-\frac{x^{7/2}}{7d}+\frac{e^7\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^8}+\frac{e^7\log(x)}{2d^8}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e/Sqrt[x])^n]),x]
```

```
[Out] (a*x^4)/4 + (b*x^4*Log[c*(d + e/Sqrt[x])^n])/4 - (b*e*n*(-((e^6*Sqrt[x])/d^
7) + (e^5*x)/(2*d^6) - (e^4*x^(3/2))/(3*d^5) + (e^3*x^2)/(4*d^4) - (e^2*x^(
5/2))/(5*d^3) + (e*x^3)/(6*d^2) - x^(7/2)/(7*d) + (e^7*Log[d + e/Sqrt[x]])/
d^8 + (e^7*Log[x])/(2*d^8)))/4
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^3\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e/x^(1/2))^n)),x)
```

[Out] int(x^3*(a+b*ln(c*(d+e/x^(1/2))^n)),x)

Maxima [A]
time = 0.29, size = 115, normalized size = 0.67

$$\frac{1}{4}\,bx^4\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+\frac{1}{4}\,ax^4+\frac{1}{1680}\,bn\left(\frac{60\,d^6x^{\frac{7}{2}}-70\,d^5x^3e+84\,d^4x^{\frac{5}{2}}e^2-105\,d^3x^2e^3+140\,d^2x^{\frac{3}{2}}e^4-210\,dxe^5+420\,\sqrt{x}\,e^6}{d^7}-\frac{420\,e^7\log\left(d\sqrt{x}+e\right)}{d^8}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4*log(c*(d + e/sqrt(x))^n) + 1/4*a*x^4 + 1/1680*b*n*((60*d^6*x^(7/2
) - 70*d^5*x^3*e + 84*d^4*x^(5/2)*e^2 - 105*d^3*x^2*e^3 + 140*d^2*x^(3/2)*e
^4 - 210*d*x*e^5 + 420*sqrt(x)*e^6)/d^7 - 420*e^7*log(d*sqrt(x) + e)/d^8)*e
```

Fricas [A]
time = 0.38, size = 176, normalized size = 1.03

$$\frac{420\,bd^8x^4\log\left(c\right)+420\,ad^8x^4-70\,bd^6nx^3e^2-420\,bd^8n\log\left(\sqrt{x}\right)-105\,bd^4nx^2e^4-210\,bd^2nxe^6+420\,\left(bd^8n-bne^8\right)\log\left(d\sqrt{x}+e\right)+420\,\left(bd^8nx^4-bd^8n\right)\log\left(\frac{dx+\sqrt{x}\,e}{x}\right)+4\,\left(15\,bd^7nx^3e+21\,bd^5nx^2e^3+35\,bd^3nxe^5+105\,bdne^7\right)\sqrt{x}}{1680\,d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="fricas")
```

```
[Out] 1/1680*(420*b*d^8*x^4*log(c) + 420*a*d^8*x^4 - 70*b*d^6*n*x^3*e^2 - 420*b*d
^8*n*log(sqrt(x)) - 105*b*d^4*n*x^2*e^4 - 210*b*d^2*n*x*e^6 + 420*(b*d^8*n
- b*n*e^8)*log(d*sqrt(x) + e) + 420*(b*d^8*n*x^4 - b*d^8*n)*log((d*x + sqrt
(x)*e)/x) + 4*(15*b*d^7*n*x^3*e + 21*b*d^5*n*x^2*e^3 + 35*b*d^3*n*x*e^5 + 1
05*b*d*n*e^7)*sqrt(x))/d^8
```

Sympy [A]
time = 59.31, size = 162, normalized size = 0.95

$$\frac{ax^4}{4}+b\left(\frac{en\left(\frac{2x^{\frac{7}{2}}}{7d}-\frac{ex^3}{3d^2}+\frac{2e^2x^{\frac{5}{2}}}{5d^3}-\frac{e^3x^2}{2d^4}+\frac{2e^4x^{\frac{3}{2}}}{3d^5}-\frac{e^5x}{d^6}+\frac{2e^6\sqrt{x}}{d^7}-\frac{2e^8\left(\begin{cases}\frac{1}{d\sqrt{x}} & \text{for}\ e=0\\ \frac{\log\left(d+\frac{e}{\sqrt{x}}\right)}{e} & \text{otherwise}\end{cases}\right)}{d^8}+\frac{2e^7\log\left(\frac{1}{\sqrt{x}}\right)}{d^8}\right)}{8}+\frac{x^4\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e/x**(1/2))**n)),x)
```

```
[Out] a*x**4/4 + b*(e*n*(2*x**(7/2)/(7*d) - e*x**3/(3*d**2) + 2*e**2*x**(5/2)/(5*
d**3) - e**3*x**2/(2*d**4) + 2*e**4*x**(3/2)/(3*d**5) - e**5*x/d**6 + 2*e**
6*sqrt(x)/d**7 - 2*e**8*Piecewise((1/(d*sqrt(x)), Eq(e, 0)), (log(d + e/sqr
t(x))/e, True))/d**8 + 2*e**7*log(1/sqrt(x))/d**8)/8 + x**4*log(c*(d + e/sq
rt(x))**n)/4)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(134) = 268.
time = 4.71, size = 272, normalized size = 1.59

$$\frac{1}{4} b x^4 \log(c) + \frac{1}{4} a x^4 - \frac{1}{1680} \left(\left(\frac{420 \log\left(\frac{|d\sqrt{x}+e|}{\sqrt{|x|}}\right)}{d^8} - \frac{420\log\left(\left|-d+\frac{d\sqrt{x}+e}{\sqrt{x}}\right|\right)}{d^8} + \frac{1089 d^7 - \frac{4683 (d\sqrt{x}+e) d^6}{\sqrt{x}} + \frac{9639 (d\sqrt{x}+e)^2 d^5}{x} - \frac{11165 (d\sqrt{x}+e)^3 d^4}{x^{\frac{3}{2}}} + \frac{7490(d\sqrt{x}+e)^4 d^3}{x^2} - \frac{2730 (d\sqrt{x}+e)^5 d^2}{x^{\frac{5}{2}}} + \frac{420 (d\sqrt{x}+e)^6 d}{x^3}}{\left(d - \frac{d\sqrt{x}+e}{\sqrt{x}}\right)^7 d^8} \right) e^9 - \frac{420 e^9 \log\left(\left(d e^{(-1)} - \frac{(d\sqrt{x}+e) e^{(-1)}}{\sqrt{x}}\right) \left(\frac{d}{d e^{(-1)} - \frac{(d\sqrt{x}+e)e^{(-1)}}{\sqrt{x}}} - e\right)\right)}{\left(d-\frac{d\sqrt{x}+e}{\sqrt{x}}\right)^8}\right) b n e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="giac")
```

```
[Out] 1/4*b*x^4*log(c) + 1/4*a*x^4 - 1/1680*((420*log(abs(d*sqrt(x) + e)/sqrt(abs
(x)))/d^8 - 420*log(abs(-d + (d*sqrt(x) + e)/sqrt(x)))/d^8 + (1089*d^7 - 46
83*(d*sqrt(x) + e)*d^6/sqrt(x) + 9639*(d*sqrt(x) + e)^2*d^5/x - 11165*(d*sq
rt(x) + e)^3*d^4/x^(3/2) + 7490*(d*sqrt(x) + e)^4*d^3/x^2 - 2730*(d*sqrt(x)
 + e)^5*d^2/x^(5/2) + 420*(d*sqrt(x) + e)^6*d/x^3)/((d - (d*sqrt(x) + e)/sq
rt(x))^7*d^8))*e^9 - 420*e^9*log((d*e^(-1) - (d*sqrt(x) + e)*e^(-1)/sqrt(x)
)*(d/(d*e^(-1) - (d*sqrt(x) + e)*e^(-1)/sqrt(x)) - e))/(d - (d*sqrt(x) + e)
/sqrt(x))^8)*b*n*e^(-1)
```

Mupad [B]
time = 0.98, size = 140, normalized size = 0.82

$$\frac{\frac{b\,d\,e^7\,n\,\sqrt{x}}{4} - \frac{b\,d^2\,e^6\,n\,x}{8} + \frac{b\,d^7\,e\,n\,x^{7/2}}{28} - \frac{b\,d^4\,e^4\,n\,x^2}{16} - \frac{b\,d^6\,e^2\,n\,x^3}{24} + \frac{b\,d^3\,e^5\,n\,x^{3/2}}{12} + \frac{b\,d^5\,e^3\,n\,x^{5/2}}{20} + \frac{b\,e^8\,n\,\mathrm{atan}\left(\frac{d\,1\mathrm{i} + \frac{e\,2\mathrm{i}}{\sqrt{x}}}{d}\right)\,1\mathrm{i}}{2}}{d^8} + \frac{a\,x^4}{4} + \frac{b\,x^4\,\ln\left(c\,\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e/x^(1/2))^n)),x)
```

```
[Out] ((b*e^8*n*atan((d*1i + (e*2i)/x^(1/2))/d)*1i)/2 - (b*d^2*e^6*n*x)/8 + (b*d*
e^7*n*x^(1/2))/4 + (b*d^7*e*n*x^(7/2))/28 - (b*d^4*e^4*n*x^2)/16 - (b*d^6*e
^2*n*x^3)/24 + (b*d^3*e^5*n*x^(3/2))/12 + (b*d^5*e^3*n*x^(5/2))/20)/d^8 + (
a*x^4)/4 + (b*x^4*log(c*(d + e/x^(1/2))^n))/4
```

3.422 $\int x^2 \left(a+b \log \left(c \left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right) \, dx$

Optimal. Leaf size=139

$$\frac{be^5n\sqrt{x}}{3d^5}-\frac{be^4nx}{6d^4}+\frac{be^3nx^{3/2}}{9d^3}-\frac{be^2nx^2}{12d^2}+\frac{benx^{5/2}}{15d}-\frac{be^6n\log\left(d+\frac{e}{\sqrt{x}}\right)}{3d^6}+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-$$

```
[Out] -1/6*b*e^4*n*x/d^4+1/9*b*e^3*n*x^(3/2)/d^3-1/12*b*e^2*n*x^2/d^2+1/15*b*e*n*
x^(5/2)/d-1/6*b*e^6*n*ln(x)/d^6-1/3*b*e^6*n*ln(d+e/x^(1/2))/d^6+1/3*x^3*(a+
b*ln(c*(d+e/x^(1/2))^n))+1/3*b*e^5*n*x^(1/2)/d^5
```

Rubi [A]
time = 0.07, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2504, 2442, 46}

$$\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{be^6n\log\left(d+\frac{e}{\sqrt{x}}\right)}{3d^6}-\frac{be^6n\log(x)}{6d^6}+\frac{be^5n\sqrt{x}}{3d^5}-\frac{be^4nx}{6d^4}+\frac{be^3nx^{3/2}}{9d^3}-\frac{be^2nx^2}{12d^2}+\frac{benx^{5/2}}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e/Sqrt[x])^n]),x]
```

```
[Out] (b*e^5*n*Sqrt[x])/(3*d^5) - (b*e^4*n*x)/(6*d^4) + (b*e^3*n*x^(3/2))/(9*d^3)
 - (b*e^2*n*x^2)/(12*d^2) + (b*e*n*x^(5/2))/(15*d) - (b*e^6*n*Log[d + e/Sqr
t[x]])/(3*d^6) + (x^3*(a + b*Log[c*(d + e/Sqrt[x])^n]))/3 - (b*e^6*n*Log[x]
)/(6*d^6)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)dx = -\left(2\text{Subst}\left(\int\frac{a+b\log\left(c(d+ex)^n\right)}{x^7}dx,x,\frac{1}{\sqrt{x}}\right)\right)$$
$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{1}{3}(ben)\text{Subst}\left(\int\frac{1}{x^6(d+ex$$
$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{1}{3}(ben)\text{Subst}\left(\int\left(\frac{1}{dx^6}-\frac{}{d$$
$$=\frac{be^5n\sqrt{x}}{3d^5}-\frac{be^4nx}{6d^4}+\frac{be^3nx^{3/2}}{9d^3}-\frac{be^2nx^2}{12d^2}+\frac{benx^{5/2}}{15d}-\frac{be^6n\log\left(\right.}{3d$$

Mathematica [A]
time = 0.06, size = 130, normalized size = 0.94

$$\frac{ax^3}{3}+\frac{1}{3}bx^3\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-\frac{1}{3}ben\left(-\frac{e^4\sqrt{x}}{d^5}+\frac{e^3x}{2d^4}-\frac{e^2x^{3/2}}{3d^3}+\frac{ex^2}{4d^2}-\frac{x^{5/2}}{5d}+\frac{e^5\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^6}+\frac{e^5\log(x)}{2d^6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/Sqrt[x])^n]),x]
[Out] (a*x^3)/3 + (b*x^3*Log[c*(d + e/Sqrt[x])^n])/3 - (b*e*n*(-((e^4*Sqrt[x])/d^
5) + (e^3*x)/(2*d^4) - (e^2*x^(3/2))/(3*d^3) + (e*x^2)/(4*d^2) - x^(5/2)/(5
*d) + (e^5*Log[d + e/Sqrt[x]])/d^6 + (e^5*Log[x])/(2*d^6)))/3
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e/x^(1/2))^n)),x)
[Out] int(x^2*(a+b*ln(c*(d+e/x^(1/2))^n)),x)
```

Maxima [A]
time = 0.30, size = 95, normalized size = 0.68

$$\frac{1}{3}\,bx^3\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+\frac{1}{3}\,ax^3+\frac{1}{180}\,bn\left(\frac{12\,d^4x^{\frac{5}{2}}-15\,d^3x^2e+20\,d^2x^{\frac{3}{2}}e^2-30\,dxe^3+60\,\sqrt{x}\,e^4}{d^5}-\frac{60\,e^5\log\left(d\sqrt{x}+e\right)}{d^6}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3*log(c*(d + e/sqrt(x))^n) + 1/3*a*x^3 + 1/180*b*n*((12*d^4*x^(5/2)
 - 15*d^3*x^2*e + 20*d^2*x^(3/2)*e^2 - 30*d*x*e^3 + 60*sqrt(x)*e^4)/d^5 - 6
0*e^5*log(d*sqrt(x) + e)/d^6)*e
```

Fricas [A]
time = 0.38, size = 152, normalized size = 1.09

$$\frac{60\,bd^6x^3\log\left(c\right)+60\,ad^6x^3-15\,bd^4nx^2e^2-60\,bd^6n\log\left(\sqrt{x}\right)-30\,bd^2nxe^4+60\,(bd^6n-bne^6)\log\left(d\sqrt{x}+e\right)+60\,(bd^6nx^3-bd^6n)\log\left(\frac{dx+\sqrt{x}\,e}{x}\right)+4\,(3\,bd^5nx^2e+5\,bd^3nxe^3+15\,bdne^5)\sqrt{x}}{180\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="fricas")
```

```
[Out] 1/180*(60*b*d^6*x^3*log(c) + 60*a*d^6*x^3 - 15*b*d^4*n*x^2*e^2 - 60*b*d^6*n
*log(sqrt(x)) - 30*b*d^2*n*x*e^4 + 60*(b*d^6*n - b*n*e^6)*log(d*sqrt(x) + e
) + 60*(b*d^6*n*x^3 - b*d^6*n)*log((d*x + sqrt(x)*e)/x) + 4*(3*b*d^5*n*x^2*
e + 5*b*d^3*n*x*e^3 + 15*b*d*n*e^5)*sqrt(x))/d^6
```

Sympy [A]
time = 21.15, size = 134, normalized size = 0.96

$$\frac{ax^3}{3}+b\left(\frac{en\left(\frac{2x^{\frac{5}{2}}}{5d}-\frac{ex^2}{2d^2}+\frac{2e^2x^{\frac{3}{2}}}{3d^3}-\frac{e^3x}{d^4}+\frac{2e^4\sqrt{x}}{d^5}-\frac{2e^6\left(\begin{cases}\frac{1}{d\sqrt{x}} & \text{for}\ e=0\\ \frac{\log\left(d+\frac{e}{\sqrt{x}}\right)}{e} & \text{otherwise}\end{cases}\right)}{d^6}+\frac{2e^5\log\left(\frac{1}{\sqrt{x}}\right)}{d^6}\right)}{6}+\frac{x^3\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(1/2))**n)),x)
```

```
[Out] a*x**3/3 + b*(e*n*(2*x**(5/2)/(5*d) - e*x**2/(2*d**2) + 2*e**2*x**(3/2)/(3*
d**3) - e**3*x/d**4 + 2*e**4*sqrt(x)/d**5 - 2*e**6*Piecewise((1/(d*sqrt(x))
, Eq(e, 0)), (log(d + e/sqrt(x))/e, True))/d**6 + 2*e**5*log(1/sqrt(x))/d**
6)/6 + x**3*log(c*(d + e/sqrt(x))**n)/3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(110) = 220.

time = 3.09, size = 236, normalized size = 1.70

$$\frac{1}{3}bx^3\log(c)+\frac{1}{3}ax^3-\frac{1}{180}\left(\left(\frac{60\log\left(\frac{|d\sqrt{x}+e|}{\sqrt{|x|}}\right)}{d^6}-\frac{60\log\left(\left|-d+\frac{d\sqrt{x}+e}{\sqrt{x}}\right|\right)}{d^6}+\frac{137d^5-\frac{385\left(d\sqrt{x}+e\right)d^4}{\sqrt{x}}+\frac{470\left(d\sqrt{x}+e\right)^2d^3}{x}-\frac{270\left(d\sqrt{x}+e\right)^3d^2}{x^{\frac{3}{2}}}+\frac{60\left(d\sqrt{x}+e\right)^4d}{x^2}}{\left(d-\frac{d\sqrt{x}+e}{\sqrt{x}}\right)^5d^6}\right)e^7-\frac{60e^7\log\left(\left(de^{(-1)}-\frac{\left(d\sqrt{x}+e\right)e^{(-1)}}{\sqrt{x}}\right)\left(\frac{d}{de^{(-1)}-\frac{\left(d\sqrt{x}+e\right)e^{(-1)}}{\sqrt{x}}}-e\right)\right)}{\left(d-\frac{d\sqrt{x}+e}{\sqrt{x}}\right)^6}\right)bne^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="giac")
```

```
[Out] 1/3*b*x^3*log(c) + 1/3*a*x^3 - 1/180*((60*log(abs(d*sqrt(x) + e)/sqrt(abs(x
)))/d^6 - 60*log(abs(-d + (d*sqrt(x) + e)/sqrt(x)))/d^6 + (137*d^5 - 385*(d
*sqrt(x) + e)*d^4/sqrt(x) + 470*(d*sqrt(x) + e)^2*d^3/x - 270*(d*sqrt(x) +
e)^3*d^2/x^(3/2) + 60*(d*sqrt(x) + e)^4*d/x^2)/((d - (d*sqrt(x) + e)/sqrt(x
))^5*d^6))*e^7 - 60*e^7*log((d*e^(-1) - (d*sqrt(x) + e)*e^(-1)/sqrt(x))*(d/
(d*e^(-1) - (d*sqrt(x) + e)*e^(-1)/sqrt(x)) - e))/(d - (d*sqrt(x) + e)/sqrt
(x))^6)*b*n*e^(-1)
```

Mupad [B]

time = 0.69, size = 106, normalized size = 0.76

$$\frac{a\,x^3}{3}+\frac{b\,\left(60\,d^6\,x^3\,\ln\left(c\,{\left(d+\frac{e}{\sqrt{x}}\right)}^n\right)-120\,e^6\,n\,\mathrm{atanh}\left(\frac{2\,e}{d\,\sqrt{x}}+1\right)-15\,d^4\,e^2\,n\,x^2+20\,d^3\,e^3\,n\,x^{3/2}-30\,d^2\,e^4\,n\,x+60\,d\,e^5\,n\,\sqrt{x}+12\,d^5\,e\,n\,x^{5/2}\right)}{180\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e/x^(1/2))^n)),x)
```

```
[Out] (a*x^3)/3 + (b*(60*d^6*x^3*log(c*(d + e/x^(1/2))^n) - 120*e^6*n*atanh((2*e)
/(d*x^(1/2)) + 1) - 15*d^4*e^2*n*x^2 + 20*d^3*e^3*n*x^(3/2) - 30*d^2*e^4*n*
x + 60*d*e^5*n*x^(1/2) + 12*d^5*e*n*x^(5/2)))/(180*d^6)
```

3.423 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx$

Optimal. Leaf size=107

$$\frac{be^3n\sqrt{x}}{2d^3}-\frac{be^2nx}{4d^2}+\frac{benx^{3/2}}{6d}-\frac{be^4n\log\left(d+\frac{e}{\sqrt{x}}\right)}{2d^4}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{be^4n\log(x)}{4d^4}$$

```
[Out] -1/4*b*e^2*n*x/d^2+1/6*b*e*n*x^(3/2)/d-1/4*b*e^4*n*ln(x)/d^4-1/2*b*e^4*n*ln
(d+e/x^(1/2))/d^4+1/2*x^2*(a+b*ln(c*(d+e/x^(1/2))^n))+1/2*b*e^3*n*x^(1/2)/d
^3
```

Rubi [A]
time = 0.05, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2504, 2442, 46}

$$\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{be^4n\log\left(d+\frac{e}{\sqrt{x}}\right)}{2d^4}-\frac{be^4n\log(x)}{4d^4}+\frac{be^3n\sqrt{x}}{2d^3}-\frac{be^2nx}{4d^2}+\frac{benx^{3/2}}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e/Sqrt[x])^n]),x]
```

```
[Out] (b*e^3*n*Sqrt[x])/(2*d^3) - (b*e^2*n*x)/(4*d^2) + (b*e*n*x^(3/2))/(6*d) - (
b*e^4*n*Log[d + e/Sqrt[x]])/(2*d^4) + (x^2*(a + b*Log[c*(d + e/Sqrt[x])^n])
)/2 - (b*e^4*n*Log[x])/(4*d^4)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=-\left(2\text{Subst}\left(\int\frac{a+b\log\left(c(d+ex)^n\right)}{x^5}\,dx,x,\frac{1}{\sqrt{x}}\right)\right)$$
$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{1}{2}(ben)\text{Subst}\left(\int\frac{1}{x^4(d+ex)}\right.$$
$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{1}{2}(ben)\text{Subst}\left(\int\left(\frac{1}{dx^4}-\frac{e}{d^2}\right.\right.$$
$$=\frac{be^3n\sqrt{x}}{2d^3}-\frac{be^2nx}{4d^2}+\frac{benx^{3/2}}{6d}-\frac{be^4n\log\left(d+\frac{e}{\sqrt{x}}\right)}{2d^4}+\frac{1}{2}x^2\left(a+\right.$$

Mathematica [A]
time = 0.02, size = 102, normalized size = 0.95

$$\frac{ax^2}{2}+\frac{1}{2}bx^2\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-\frac{1}{2}ben\left(-\frac{e^2\sqrt{x}}{d^3}+\frac{ex}{2d^2}-\frac{x^{3/2}}{3d}+\frac{e^3\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^4}+\frac{e^3\log(x)}{2d^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])^n]),x]
```

```
[Out] (a*x^2)/2 + (b*x^2*Log[c*(d + e/Sqrt[x])^n])/2 - (b*e*n*(-((e^2*Sqrt[x])/d^
3) + (e*x)/(2*d^2) - x^(3/2)/(3*d) + (e^3*Log[d + e/Sqrt[x]])/d^4 + (e^3*Lo
g[x])/(2*d^4)))/2
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(1/2))^n)),x)
```

```
[Out] int(x*(a+b*ln(c*(d+e/x^(1/2))^n)),x)
```

Maxima [A]
time = 0.31, size = 75, normalized size = 0.70

$$\frac{1}{12}\,bn\left(\frac{2\,d^2x^{\frac{3}{2}}-3\,dxe+6\,\sqrt{x}\,e^2}{d^3}-\frac{6\,e^3\log\left(d\sqrt{x}\,+e\right)}{d^4}\right)e+\frac{1}{2}\,bx^2\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="maxima")
```

```
[Out] 1/12*b*n*((2*d^2*x^(3/2) - 3*d*x*e + 6*sqrt(x)*e^2)/d^3 - 6*e^3*log(d*sqrt(
x) + e)/d^4)*e + 1/2*b*x^2*log(c*(d + e/sqrt(x))^n) + 1/2*a*x^2
```

Fricas [A]
time = 0.39, size = 127, normalized size = 1.19

$$\frac{6\,bd^4x^2\log\left(c\right)+6\,ad^4x^2-6\,bd^4n\log\left(\sqrt{x}\right)-3\,bd^2nxe^2+6\left(bd^4n-bne^4\right)\log\left(d\sqrt{x}\,+e\right)+6\left(bd^4nx^2-bd^4n\right)\log\left(\frac{dx+\sqrt{x}\,e}{x}\right)+2\left(bd^3nxe+3\,bdne^3\right)\sqrt{x}}{12\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="fricas")
```

```
[Out] 1/12*(6*b*d^4*x^2*log(c) + 6*a*d^4*x^2 - 6*b*d^4*n*log(sqrt(x)) - 3*b*d^2*n
*x*e^2 + 6*(b*d^4*n - b*n*e^4)*log(d*sqrt(x) + e) + 6*(b*d^4*n*x^2 - b*d^4*
n)*log((d*x + sqrt(x)*e)/x) + 2*(b*d^3*n*x*e + 3*b*d*n*e^3)*sqrt(x))/d^4
```

Sympy [A]
time = 7.49, size = 88, normalized size = 0.82

$$\frac{ax^2}{2}+b\left(\frac{en\left(\frac{2x^{\frac{3}{2}}}{3d}-\frac{ex}{d^2}-\frac{2e^3\left(\begin{cases}\frac{\sqrt{x}}{e} & \text{for}\ d=0\\ \frac{\log\left(d\sqrt{x}+e\right)}{d} & \text{otherwise}\end{cases}\right)}{d^3}+\frac{2e^2\sqrt{x}}{d^3}\right)}{4}+\frac{x^2\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/2))**n)),x)
```

```
[Out] a*x**2/2 + b*(e*n*(2*x**(3/2)/(3*d) - e*x/d**2 - 2*e**3*Piecewise((sqrt(x)/
e, Eq(d, 0)), (log(d*sqrt(x) + e)/d, True))/d**3 + 2*e**2*sqrt(x)/d**3)/4 +
 x**2*log(c*(d + e/sqrt(x))**n)/2)
```

Giac [A]
time = 4.25, size = 81, normalized size = 0.76

$$\frac{1}{2}\,bx^2\log\left(c\right)+\frac{1}{12}\left(6\,x^2\log\left(d+\frac{e}{\sqrt{x}}\right)+\left(\frac{2\,d^2x^{\frac{3}{2}}-3\,dxe+6\,\sqrt{x}\,e^2}{d^3}-\frac{6\,e^3\log\left(\left|d\sqrt{x}+e\right|\right)}{d^4}\right)e\right)bn+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2*log(c) + 1/12*(6*x^2*log(d + e/sqrt(x)) + ((2*d^2*x^(3/2) - 3*d*x
*e + 6*sqrt(x)*e^2)/d^3 - 6*e^3*log(abs(d*sqrt(x) + e))/d^4)*e)*b*n + 1/2*a
*x^2
```

Mupad [B]
time = 0.83, size = 86, normalized size = 0.80

$$\frac{x^{3/2}\left(\frac{b\,e\,n}{3\,d}-\frac{b\,e^2\,n}{2\,d^2\,\sqrt{x}}+\frac{b\,e^3\,n}{d^3\,x}\right)}{2}+\frac{a\,x^2}{2}+\frac{b\,x^2\,\ln\left(c\,{\left(d+\frac{e}{\sqrt{x}}\right)}^n\right)}{2}-\frac{b\,e^4\,n\,\mathrm{atanh}\left(\frac{2\,e}{d\,\sqrt{x}}+1\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e/x^(1/2))^n)),x)
```

```
[Out] (x^(3/2)*((b*e*n)/(3*d) - (b*e^2*n)/(2*d^2*x^(1/2)) + (b*e^3*n)/(d^3*x)))/2
 + (a*x^2)/2 + (b*x^2*log(c*(d + e/x^(1/2))^n))/2 - (b*e^4*n*atanh((2*e)/(d
*x^(1/2)) + 1))/d^4
```

3.424 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx$

Optimal. Leaf size=53

$$\frac{ben\sqrt{x}}{d}+ax+bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-\frac{be^2n\log\left(e+d\sqrt{x}\right)}{d^2}$$

```
[Out] a*x+b*x*ln(c*(d+e/x^(1/2))^n)-b*e^2*n*ln(e+d*x^(1/2))/d^2+b*e*n*x^(1/2)/d
```

Rubi [A]
time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2498, 269, 196, 45}

$$ax+bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-\frac{be^2n\log\left(d\sqrt{x}+e\right)}{d^2}+\frac{ben\sqrt{x}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Log[c*(d + e/Sqrt[x])^n],x]
```

```
[Out] (b*e*n*Sqrt[x])/d + a*x + b*x*Log[c*(d + e/Sqrt[x])^n] - (b*e^2*n*Log[e + d
*Sqrt[x]])/d^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
```

```
e, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx &= ax+b\int \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\,dx\\
&= ax+bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+\frac{1}{2}(ben)\int \frac{1}{\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}}\,dx\\
&= ax+bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+\frac{1}{2}(ben)\int \frac{1}{e+d\sqrt{x}}\,dx\\
&= ax+bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+(ben)\text{Subst}\left(\int \frac{x}{e+dx}\,dx,x,\sqrt{x}\right)\\
&= ax+bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+(ben)\text{Subst}\left(\int \left(\frac{1}{d}-\frac{e}{d(e+dx)}\right)\right.\\
&= \frac{ben\sqrt{x}}{d}+ax+bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-\frac{be^2n\log\left(e+d\sqrt{x}\right)}{d^2}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 62, normalized size = 1.17

$$
ax+bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-ben\left(-\frac{\sqrt{x}}{d}+\frac{e\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^2}+\frac{e\log(x)}{2d^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Log[c*(d + e/Sqrt[x])^n],x]
[Out] a*x + b*x*Log[c*(d + e/Sqrt[x])^n] - b*e*n*(-(Sqrt[x]/d) + (e*Log[d + e/Sqr
t[x]])/d^2 + (e*Log[x])/(2*d^2))
```

Maple [A]
time = 0.05, size = 94, normalized size = 1.77

method	result	size
default	$ax+xb\ln\left(c\left(\frac{e+d\sqrt{x}}{\sqrt{x}}\right)^n\right)+\frac{ben\sqrt{x}}{d}+\frac{b\,e^2n\ln\left(d\sqrt{x}-e\right)}{2d^2}-\frac{b\,e^2n\ln\left(e+d\sqrt{x}\right)}{2d^2}-\frac{b\,e^2n\ln\left(d^2x-e^2\right)}{2d^2}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*ln(c*(d+e/x^(1/2))^n),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+x*b*ln(c*((e+d*x^(1/2))/x^(1/2))^n)+b*e*n*x^(1/2)/d+1/2*b*e^2*n/d^2*ln(
d*x^(1/2)-e)-1/2*b*e^2*n*ln(e+d*x^(1/2))/d^2-1/2*b*e^2*n*ln(d^2*x-e^2)/d^2
```

Maxima [A]
time = 0.29, size = 52, normalized size = 0.98

$$-\left(n\left(\frac{e\log\left(d\sqrt{x}+e\right)}{d^2}-\frac{\sqrt{x}}{d}\right)e-x\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/x^(1/2))^n),x, algorithm="maxima")
```

```
[Out] -(n*(e*log(d*sqrt(x) + e)/d^2 - sqrt(x)/d)*e - x*log(c*(d + e/sqrt(x))^n))*
b + a*x
```

Fricas [A]
time = 0.39, size = 93, normalized size = 1.75

$$\frac{bd^2x\log\left(c\right)-bd^2n\log\left(\sqrt{x}\right)+bdn\sqrt{x}\,e+ad^2x+\left(bd^2n-bne^2\right)\log\left(d\sqrt{x}+e\right)+\left(bd^2nx-bd^2n\right)\log\left(\frac{dx+\sqrt{x}\,e}{x}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/x^(1/2))^n),x, algorithm="fricas")
```

```
[Out] (b*d^2*x*log(c) - b*d^2*n*log(sqrt(x)) + b*d*n*sqrt(x)*e + a*d^2*x + (b*d^2
*n - b*n*e^2)*log(d*sqrt(x) + e) + (b*d^2*n*x - b*d^2*n)*log((d*x + sqrt(x)
*e)/x))/d^2
```

Sympy [A]

time = 3.46, size = 76, normalized size = 1.43

$$ax+b\left(\frac{en\left(\frac{2\sqrt{x}}{d}-\frac{2e^{2}\left(\begin{cases}\frac{1}{d\sqrt{x}} & \text{for } e=0\\ \frac{\log\left(d+\frac{e}{\sqrt{x}}\right)}{e} & \text{otherwise}\end{cases}\right)}{d^{2}}+\frac{2e\log\left(\frac{1}{\sqrt{x}}\right)}{d^{2}}\right)}{2}+x\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*ln(c*(d+e/x**(1/2))**n),x)
```

```
[Out] a*x + b*(e*n*(2*sqrt(x)/d - 2*e**2*Piecewise((1/(d*sqrt(x)), Eq(e, 0)), (lo
g(d + e/sqrt(x))/e, True))/d**2 + 2*e*log(1/sqrt(x))/d**2)/2 + x*log(c*(d +
 e/sqrt(x))**n))
```

Giac [A]
time = 4.15, size = 56, normalized size = 1.06

$$-\left(\left(\left(\frac{e\log\left(\left|d\sqrt{x}+e\right|\right)}{d^{2}}-\frac{\sqrt{x}}{d}\right)e-x\log\left(d+\frac{e}{\sqrt{x}}\right)\right)n-x\log\left(c\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/x^(1/2))^n),x, algorithm="giac")
```

```
[Out] -(((e*log(abs(d*sqrt(x) + e))/d^2 - sqrt(x)/d)*e - x*log(d + e/sqrt(x)))*n
- x*log(c))*b + a*x
```

Mupad [B]
time = 0.38, size = 44, normalized size = 0.83

$$a\,x+b\,x\,\ln\left(c\,\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)-\frac{b\,e\,n\,\left(e\,\ln\left(e+d\,\sqrt{x}\right)-d\,\sqrt{x}\right)}{d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*log(c*(d + e/x^(1/2))^n),x)
```

```
[Out] a*x + b*x*log(c*(d + e/x^(1/2))^n) - (b*e*n*(e*log(e + d*x^(1/2)) - d*x^(1/
2)))/d^2
```

3.425 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}\,dx$

Optimal. Leaf size=51

$$-2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt{x}}\right)-2bn\text{Li}_2\left(1+\frac{e}{d\sqrt{x}}\right)$$

```
[Out] -2*(a+b*ln(c*(d+e/x^(1/2))^n))*ln(-e/d/x^(1/2))-2*b*n*polylog(2,1+e/d/x^(1/
2))
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2504, 2441, 2352}

$$-2bn\text{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)-2\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])/x,x]
```

```
[Out] -2*(a + b*Log[c*(d + e/Sqrt[x])^n])*Log[-(e/(d*Sqrt[x]))] - 2*b*n*PolyLog[2
, 1 + e/(d*Sqrt[x])]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}\,dx = -\left(2\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt{x}}\right) + (2ben)\text{Subst}\left(\int \frac{\log}{d}\right.$$

$$= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt{x}}\right) - 2bn\text{Li}_2\left(1+\frac{e}{d\sqrt{x}}\right)$$

Mathematica [**A**]
time = 0.00, size = 53, normalized size = 1.04

$$-2b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\log\left(-\frac{e}{d\sqrt{x}}\right) + a\log(x) - 2bn\text{Li}_2\left(\frac{d+\frac{e}{\sqrt{x}}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])/x,x]
```

```
[Out] -2*b*Log[c*(d + e/Sqrt[x])^n]*Log[-(e/(d*Sqrt[x]))] + a*Log[x] - 2*b*n*Poly
Log[2, (d + e/Sqrt[x])/d]
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))/x,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))/x,x)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(47) = 94.
time = 0.69, size = 127, normalized size = 2.49

$$-2\left(\log\left(de^{\left(\frac{1}{2}\log(x)-1\right)}+1\right)\log\left(\sqrt{x}\right)+\text{Li}_2\left(-de^{\left(\frac{1}{2}\log(x)-1\right)}\right)\right)bn+\frac{1}{4}\left(4bne\log\left(d\sqrt{x}+e\right)\log(x)+bne\log(x)^2+4bdn\sqrt{x}\log(x)-4be\log(x)\log\left(x^{\frac{1}{2}n}\right)-8bdn\sqrt{x}+4(b\log(c)+a)e\log(x)-\frac{4\left(bdnx\log(x)-2bdnx\right)}{\sqrt{x}}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x,x, algorithm="maxima")
```

```
[Out] -2*(log(d*e^(1/2*log(x) - 1) + 1)*log(sqrt(x)) + dilog(-d*e^(1/2*log(x) - 1
)))*b*n + 1/4*(4*b*n*e*log(d*sqrt(x) + e)*log(x) + b*n*e*log(x)^2 + 4*b*d*n
*sqrt(x)*log(x) - 4*b*e*log(x)*log(x^(1/2*n)) - 8*b*d*n*sqrt(x) + 4*(b*log(
c) + a)*e*log(x) - 4*(b*d*n*x*log(x) - 2*b*d*n*x)/sqrt(x))*e^(-1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*((d*x + sqrt(x)*e)/x)^n) + a)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\log\left(c\left(d + \frac{e}{\sqrt{x}}\right)^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e/sqrt(x))**n))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + b\,\ln\left(c\,\left(d + \frac{e}{\sqrt{x}}\right)^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))/x,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(1/2))^n))/x, x)
```

3.426 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^2}\,dx$

Optimal. Leaf size=65

$$\frac{bn}{2x}-\frac{bdn}{e\sqrt{x}}+\frac{bd^2n\log\left(d+\frac{e}{\sqrt{x}}\right)}{e^2}-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}$$

```
[Out] 1/2*b*n/x+b*d^2*n*ln(d+e/x^(1/2))/e^2+(-a-b*ln(c*(d+e/x^(1/2))^n))/x-b*d*n/
e/x^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2504, 2442, 45}

$$-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}+\frac{bd^2n\log\left(d+\frac{e}{\sqrt{x}}\right)}{e^2}-\frac{bdn}{e\sqrt{x}}+\frac{bn}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])/x^2,x]
```

```
[Out] (b*n)/(2*x) - (b*d*n)/(e*Sqrt[x]) + (b*d^2*n*Log[d + e/Sqrt[x]])/e^2 - (a +
 b*Log[c*(d + e/Sqrt[x])^n])/x
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^2}\,dx &= -\left(2\text{Subst}\left(\int x(a+b\log\left(c(d+ex)^n\right))\,dx, x, \frac{1}{\sqrt{x}}\right)\right)\\ &= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x} + (ben)\text{Subst}\left(\int \frac{x^2}{d+ex}\,dx, x, \frac{1}{\sqrt{x}}\right)\\ &= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x} + (ben)\text{Subst}\left(\int \left(-\frac{d}{e^2}+\frac{x}{e}+\frac{d^2}{e^2(d+ex)}\right)dx, x, \frac{1}{\sqrt{x}}\right)\\ &= \frac{bn}{2x} - \frac{bdn}{e\sqrt{x}} + \frac{bd^2n\log\left(d+\frac{e}{\sqrt{x}}\right)}{e^2} - \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 68, normalized size = 1.05

$$-\frac{a}{x}+\frac{bn}{2x}-\frac{bdn}{e\sqrt{x}}+\frac{bd^2n\log\left(d+\frac{e}{\sqrt{x}}\right)}{e^2}-\frac{b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])/x^2,x]
```

```
[Out] -(a/x) + (b*n)/(2*x) - (b*d*n)/(e*Sqrt[x]) + (b*d^2*n*Log[d + e/Sqrt[x]])/e
^2 - (b*Log[c*(d + e/Sqrt[x])^n])/x
```

Maple [**A**]
time = 0.03, size = 63, normalized size = 0.97

method	result	size
derivativedivides	$-\frac{a}{x}+\frac{bn}{2x}-\frac{b\ln\left(c\,e^{n\ln\left(d+\frac{e}{\sqrt{x}}\right)}\right)}{x}+\frac{b\,d^2n\ln\left(d+\frac{e}{\sqrt{x}}\right)}{e^2}-\frac{bdn}{e\sqrt{x}}$	63

default	$-\frac{a}{x}+\frac{bn}{2x}-\frac{b\ln\left(c\,\mathrm{e}^{n\ln\left(d+\frac{e}{\sqrt{x}}\right)}\right)}{x}+\frac{b\,d^{2}n\ln\left(d+\frac{e}{\sqrt{x}}\right)}{e^{2}}-\frac{bdn}{e\sqrt{x}}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/x+1/2*b*n/x-b/x*ln(c*exp(n*ln(d+e/x^(1/2))))+b*d^2*n*ln(d+e/x^(1/2))/e^2
-b*d*n/e/x^(1/2)
```

Maxima [A]
time = 0.28, size = 76, normalized size = 1.17

$$\frac{1}{2}\left(2\,d^{2}e^{(-3)}\log\left(d\sqrt{x}+e\right)-d^{2}e^{(-3)}\log\left(x\right)-\frac{\left(2\,d\sqrt{x}-e\right)e^{(-2)}}{x}\right)bne-\frac{b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)}{x}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*d^2*e^(-3)*log(d*sqrt(x) + e) - d^2*e^(-3)*log(x) - (2*d*sqrt(x) - e
)*e^(-2)/x)*b*n*e - b*log(c*(d + e/sqrt(x))^n)/x - a/x
```

Fricas [A]
time = 0.36, size = 67, normalized size = 1.03

$$-\frac{\left(2\,bdn\sqrt{x}\,e+2\,be^{2}\log\left(c\right)-(bn-2\,a)e^{2}-2\left(bd^{2}nx-bne^{2}\right)\log\left(\frac{dx+\sqrt{x}\,e}{x}\right)\right)e^{(-2)}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*n*sqrt(x)*e + 2*b*e^2*log(c) - (b*n - 2*a)*e^2 - 2*(b*d^2*n*x -
 b*n*e^2)*log((d*x + sqrt(x)*e)/x))*e^(-2)/x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(58) = 116$.
time = 51.83, size = 360, normalized size = 5.54

$$\begin{cases}-\frac{2ade^{2}x^{3}}{2de^{2}x^{4}+2e^{3}x^{\frac{7}{2}}}-\frac{2ae^{3}x^{\frac{5}{2}}}{2de^{2}x^{4}+2e^{3}x^{\frac{7}{2}}}+\frac{2bd^{3}x^{4}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)}{2de^{2}x^{4}+2e^{3}x^{\frac{7}{2}}}-\frac{2bd^{2}enx^{\frac{7}{2}}}{2de^{2}x^{4}+2e^{3}x^{\frac{7}{2}}}+\frac{2bd^{2}ex^{\frac{7}{2}}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)}{2de^{2}x^{4}+2e^{3}x^{\frac{7}{2}}}-\frac{bde^{2}nx^{3}}{2de^{2}x^{4}+2e^{3}x^{\frac{7}{2}}}-\frac{2bde^{2}x^{3}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)}{2de^{2}x^{4}+2e^{3}x^{\frac{7}{2}}}+\frac{be^{3}nx^{\frac{5}{2}}}{2de^{2}x^{4}+2e^{3}x^{\frac{7}{2}}}-\frac{2be^{3}x^{\frac{5}{2}}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)}{2de^{2}x^{4}+2e^{3}x^{\frac{7}{2}}} & \text{for}\: e \neq 0 \\ -\frac{a+b\log\left(cd^{n}\right)}{x} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))/x**2,x)
```

```
[Out] Piecewise((-2*a*d*e**2*x**3/(2*d*e**2*x**4 + 2*e**3*x**(7/2)) - 2*a*e**3*x*
*(5/2)/(2*d*e**2*x**4 + 2*e**3*x**(7/2)) + 2*b*d**3*x**4*log(c*(d + e/sqrt(
x))**n)/(2*d*e**2*x**4 + 2*e**3*x**(7/2)) - 2*b*d**2*e*n*x**(7/2)/(2*d*e**2
*x**4 + 2*e**3*x**(7/2)) + 2*b*d**2*e*x**(7/2)*log(c*(d + e/sqrt(x))**n)/(2
*d*e**2*x**4 + 2*e**3*x**(7/2)) - b*d*e**2*n*x**3/(2*d*e**2*x**4 + 2*e**3*x
**(7/2)) - 2*b*d*e**2*x**3*log(c*(d + e/sqrt(x))**n)/(2*d*e**2*x**4 + 2*e**
3*x**(7/2)) + b*e**3*n*x**(5/2)/(2*d*e**2*x**4 + 2*e**3*x**(7/2)) - 2*b*e**
3*x**(5/2)*log(c*(d + e/sqrt(x))**n)/(2*d*e**2*x**4 + 2*e**3*x**(7/2)), Ne(
e, 0)), (-(a + b*log(c*d**n))/x, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(57) = 114$.
time = 3.45, size = 162, normalized size = 2.49

$$\frac{1}{2}\left(\frac{4\,(d\sqrt{x}+e)bdn\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)}{\sqrt{x}}-\frac{2\,(d\sqrt{x}+e)^2bn\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)}{x}-\frac{4\,(d\sqrt{x}+e)bdn}{\sqrt{x}}+\frac{4\,(d\sqrt{x}+e)bd\log(c)}{\sqrt{x}}+\frac{(d\sqrt{x}+e)^2bn}{x}-\frac{2\,(d\sqrt{x}+e)^2b\log(c)}{x}+\frac{4\,(d\sqrt{x}+e)ad}{\sqrt{x}}-\frac{2\,(d\sqrt{x}+e)^2a}{x}\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*(d*sqrt(x) + e)*b*d*n*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x) - 2*(d*sq
rt(x) + e)^2*b*n*log((d*sqrt(x) + e)/sqrt(x))/x - 4*(d*sqrt(x) + e)*b*d*n/s
qrt(x) + 4*(d*sqrt(x) + e)*b*d*log(c)/sqrt(x) + (d*sqrt(x) + e)^2*b*n/x - 2
*(d*sqrt(x) + e)^2*b*log(c)/x + 4*(d*sqrt(x) + e)*a*d/sqrt(x) - 2*(d*sqrt(x
) + e)^2*a/x)*e^(-2)
```

Mupad [B]
time = 0.43, size = 60, normalized size = 0.92

$$\frac{b\,n}{2\,x}-\frac{a}{x}-\frac{b\,\ln\left(c\,{\left(d+\frac{e}{\sqrt{x}}\right)}^n\right)}{x}-\frac{b\,d\,n}{e\,\sqrt{x}}+\frac{b\,d^2\,n\,\ln\left(d+\frac{e}{\sqrt{x}}\right)}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))/x^2,x)
```

```
[Out] (b*n)/(2*x) - a/x - (b*log(c*(d + e/x^(1/2))^n))/x - (b*d*n)/(e*x^(1/2)) +
(b*d^2*n*log(d + e/x^(1/2)))/e^2
```

3.427 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^3}\,dx$

Optimal. Leaf size=104

$$\frac{bn}{8x^2}-\frac{bdn}{6ex^{3/2}}+\frac{bd^2n}{4e^2x}-\frac{bd^3n}{2e^3\sqrt{x}}+\frac{bd^4n\log\left(d+\frac{e}{\sqrt{x}}\right)}{2e^4}-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2x^2}$$

```
[Out] 1/8*b*n/x^2-1/6*b*d*n/e/x^(3/2)+1/4*b*d^2*n/e^2/x+1/2*b*d^4*n*ln(d+e/x^(1/2
))/e^4+1/2*(-a-b*ln(c*(d+e/x^(1/2))^n))/x^2-1/2*b*d^3*n/e^3/x^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2504, 2442, 45}

$$-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2x^2}+\frac{bd^4n\log\left(d+\frac{e}{\sqrt{x}}\right)}{2e^4}-\frac{bd^3n}{2e^3\sqrt{x}}+\frac{bd^2n}{4e^2x}-\frac{bdn}{6ex^{3/2}}+\frac{bn}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])/x^3,x]
```

```
[Out] (b*n)/(8*x^2) - (b*d*n)/(6*e*x^(3/2)) + (b*d^2*n)/(4*e^2*x) - (b*d^3*n)/(2*
e^3*Sqrt[x]) + (b*d^4*n*Log[d + e/Sqrt[x]])/(2*e^4) - (a + b*Log[c*(d + e/S
qrt[x])^n])/(2*x^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^3}\,dx &= -\left(2\text{Subst}\left(\int x^3(a+b\log\left(c(d+ex)^n\right))\,dx,x,\frac{1}{\sqrt{x}}\right)\right)\\ &= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2x^2}+\frac{1}{2}(ben)\text{Subst}\left(\int\frac{x^4}{d+ex}\,dx,x,\frac{1}{\sqrt{x}}\right)\\ &= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2x^2}+\frac{1}{2}(ben)\text{Subst}\left(\int\left(-\frac{d^3}{e^4}+\frac{d^2x}{e^3}-\frac{dx^2}{e^2}\right.\right.\\ &= \frac{bn}{8x^2}-\frac{bdn}{6ex^{3/2}}+\frac{bd^2n}{4e^2x}-\frac{bd^3n}{2e^3\sqrt{x}}+\frac{bd^4n\log\left(d+\frac{e}{\sqrt{x}}\right)}{2e^4}-\frac{a+b\log\left(\right.}{}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 109, normalized size = 1.05

$$-\frac{a}{2x^2}+\frac{bn}{8x^2}-\frac{bdn}{6ex^{3/2}}+\frac{bd^2n}{4e^2x}-\frac{bd^3n}{2e^3\sqrt{x}}+\frac{bd^4n\log\left(d+\frac{e}{\sqrt{x}}\right)}{2e^4}-\frac{b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])/x^3,x]
```

```
[Out] -1/2*a/x^2 + (b*n)/(8*x^2) - (b*d*n)/(6*e*x^(3/2)) + (b*d^2*n)/(4*e^2*x) -
(b*d^3*n)/(2*e^3*Sqrt[x]) + (b*d^4*n*Log[d + e/Sqrt[x]])/(2*e^4) - (b*Log[c
*(d + e/Sqrt[x])^n])/(2*x^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int\frac{a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))/x^3,x)
```

Maxima [A]
time = 0.28, size = 94, normalized size = 0.90

$$\frac{1}{24}\left(12\,d^4e^{(-5)}\log\left(d\sqrt{x}+e\right)-6\,d^4e^{(-5)}\log\left(x\right)-\frac{\left(12\,d^3x^{\frac{3}{2}}-6\,d^2xe+4\,d\sqrt{x}\,e^2-3\,e^3\right)e^{(-4)}}{x^2}\right)bne-\frac{b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2\,x^2}-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^3,x, algorithm="maxima")
```

```
[Out] 1/24*(12*d^4*e^(-5)*log(d*sqrt(x) + e) - 6*d^4*e^(-5)*log(x) - (12*d^3*x^(3
/2) - 6*d^2*x*e + 4*d*sqrt(x)*e^2 - 3*e^3)*e^(-4)/x^2)*b*n*e - 1/2*b*log(c*
(d + e/sqrt(x))^n)/x^2 - 1/2*a/x^2
```

Fricas [A]
time = 0.38, size = 91, normalized size = 0.88

$$\frac{\left(6\,bd^2nxe^2-12\,be^4\log\left(c\right)+3\,(bn-4\,a)e^4+12\,(bd^4nx^2-bne^4)\log\left(\frac{dx+\sqrt{x}\,e}{x}\right)-4\,(3\,bd^3nxe+bdne^3)\sqrt{x}\right)e^{(-4)}}{24\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^3,x, algorithm="fricas")
```

```
[Out] 1/24*(6*b*d^2*n*x*e^2 - 12*b*e^4*log(c) + 3*(b*n - 4*a)*e^4 + 12*(b*d^4*n*x
^2 - b*n*e^4)*log((d*x + sqrt(x)*e)/x) - 4*(3*b*d^3*n*x*e + b*d*n*e^3)*sqrt
(x))*e^(-4)/x^2
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))/x**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(82) = 164.
time = 3.32, size = 349, normalized size = 3.36

$$\frac{1}{24}\left(\frac{48\left(d\sqrt{x}+e\right)bd^3n\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)}{\sqrt{x}}-\frac{72\left(d\sqrt{x}+e\right)^2bd^2n\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)}{x}-\frac{48\left(d\sqrt{x}+e\right)bd^3n}{\sqrt{x}}+\frac{48\left(d\sqrt{x}+e\right)bd^3\log\left(c\right)}{\sqrt{x}}+\frac{48\left(d\sqrt{x}+e\right)^3bdn\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)}{x^{\frac{3}{2}}}+\frac{36\left(d\sqrt{x}+e\right)^2bd^2n}{x}-\frac{72\left(d\sqrt{x}+e\right)^2bd^2\log\left(c\right)}{x}-\frac{12\left(d\sqrt{x}+e\right)^4bn\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)}{x^2}-\frac{16\left(d\sqrt{x}+e\right)^3bdn}{x^{\frac{3}{2}}}+\frac{48\left(d\sqrt{x}+e\right)ad^3}{\sqrt{x}}+\frac{48\left(d\sqrt{x}+e\right)^3bd\log\left(c\right)}{x^{\frac{3}{2}}}+\frac{3\left(d\sqrt{x}+e\right)^4bn}{x^2}-\frac{72\left(d\sqrt{x}+e\right)^2ad^2}{x}-\frac{12\left(d\sqrt{x}+e\right)^4b\log\left(c\right)}{x^2}+\frac{48\left(d\sqrt{x}+e\right)^3ad}{x^{\frac{3}{2}}}-\frac{12\left(d\sqrt{x}+e\right)^4a}{x^2}\right)e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^3,x, algorithm="giac")
```

```
[Out] 1/24*(48*(d*sqrt(x) + e)*b*d^3*n*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x) - 72*
(d*sqrt(x) + e)^2*b*d^2*n*log((d*sqrt(x) + e)/sqrt(x))/x - 48*(d*sqrt(x) +
e)*b*d^3*n/sqrt(x) + 48*(d*sqrt(x) + e)*b*d^3*log(c)/sqrt(x) + 48*(d*sqrt(x
) + e)^3*b*d*n*log((d*sqrt(x) + e)/sqrt(x))/x^(3/2) + 36*(d*sqrt(x) + e)^2*
b*d^2*n/x - 72*(d*sqrt(x) + e)^2*b*d^2*log(c)/x - 12*(d*sqrt(x) + e)^4*b*n*
log((d*sqrt(x) + e)/sqrt(x))/x^2 - 16*(d*sqrt(x) + e)^3*b*d*n/x^(3/2) + 48*
(d*sqrt(x) + e)*a*d^3/sqrt(x) + 48*(d*sqrt(x) + e)^3*b*d*log(c)/x^(3/2) + 3
*(d*sqrt(x) + e)^4*b*n/x^2 - 72*(d*sqrt(x) + e)^2*a*d^2/x - 12*(d*sqrt(x) +
 e)^4*b*log(c)/x^2 + 48*(d*sqrt(x) + e)^3*a*d/x^(3/2) - 12*(d*sqrt(x) + e)^
4*a/x^2)*e^(-4)
```

Mupad [B]
time = 0.42, size = 87, normalized size = 0.84

$$\frac{b\,n}{8\,x^2}-\frac{a}{2\,x^2}-\frac{b\,\ln\left(c\,{\left(d+\frac{e}{\sqrt{x}}\right)}^n\right)}{2\,x^2}-\frac{b\,d\,n}{6\,e\,x^{3/2}}+\frac{b\,d^4\,n\,\ln\left(d+\frac{e}{\sqrt{x}}\right)}{2\,e^4}+\frac{b\,d^2\,n}{4\,e^2\,x}-\frac{b\,d^3\,n}{2\,e^3\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))/x^3,x)
```

```
[Out] (b*n)/(8*x^2) - a/(2*x^2) - (b*log(c*(d + e/x^(1/2))^n))/(2*x^2) - (b*d*n)/
(6*e*x^(3/2)) + (b*d^4*n*log(d + e/x^(1/2)))/(2*e^4) + (b*d^2*n)/(4*e^2*x)
- (b*d^3*n)/(2*e^3*x^(1/2))
```

3.428 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^4}\,dx$

Optimal. Leaf size=136

$$\frac{bn}{18x^3}-\frac{bdn}{15ex^{5/2}}+\frac{bd^2n}{12e^2x^2}-\frac{bd^3n}{9e^3x^{3/2}}+\frac{bd^4n}{6e^4x}-\frac{bd^5n}{3e^5\sqrt{x}}+\frac{bd^6n\log\left(d+\frac{e}{\sqrt{x}}\right)}{3e^6}-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3x^3}$$

```
[Out] 1/18*b*n/x^3-1/15*b*d*n/e/x^(5/2)+1/12*b*d^2*n/e^2/x^2-1/9*b*d^3*n/e^3/x^(3
/2)+1/6*b*d^4*n/e^4/x+1/3*b*d^6*n*ln(d+e/x^(1/2))/e^6+1/3*(-a-b*ln(c*(d+e/x
^(1/2))^n))/x^3-1/3*b*d^5*n/e^5/x^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2504, 2442, 45}

$$-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3x^3}+\frac{bd^6n\log\left(d+\frac{e}{\sqrt{x}}\right)}{3e^6}-\frac{bd^5n}{3e^5\sqrt{x}}+\frac{bd^4n}{6e^4x}-\frac{bd^3n}{9e^3x^{3/2}}+\frac{bd^2n}{12e^2x^2}-\frac{bdn}{15ex^{5/2}}+\frac{bn}{18x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])/x^4,x]
```

```
[Out] (b*n)/(18*x^3) - (b*d*n)/(15*e*x^(5/2)) + (b*d^2*n)/(12*e^2*x^2) - (b*d^3*n
)/(9*e^3*x^(3/2)) + (b*d^4*n)/(6*e^4*x) - (b*d^5*n)/(3*e^5*Sqrt[x]) + (b*d^
6*n*Log[d + e/Sqrt[x]])/(3*e^6) - (a + b*Log[c*(d + e/Sqrt[x])^n])/(3*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^4}\,dx = -\left(2\mathrm{Subst}\left(\int x^5(a+b\log\left(c(d+ex)^n\right))\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3x^3}+\frac{1}{3}(ben)\mathrm{Subst}\left(\int \frac{x^6}{d+ex}\,dx, x, \frac{1}{\sqrt{x}}\right)$$

$$= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3x^3}+\frac{1}{3}(ben)\mathrm{Subst}\left(\int \left(-\frac{d^5}{e^6}+\frac{d^4x}{e^5}-\frac{d^3x^2}{e^4}\right.\right.$$

$$= \frac{bn}{18x^3}-\frac{bdn}{15ex^{5/2}}+\frac{bd^2n}{12e^2x^2}-\frac{bd^3n}{9e^3x^{3/2}}+\frac{bd^4n}{6e^4x}-\frac{bd^5n}{3e^5\sqrt{x}}+\frac{bd^6n\log\left(d\right.}{3e^6}$$

Mathematica [A]
time = 0.06, size = 133, normalized size = 0.98

$$-\frac{a}{3x^3}+\frac{1}{3}ben\left(\frac{1}{6ex^3}-\frac{d}{5e^2x^{5/2}}+\frac{d^2}{4e^3x^2}-\frac{d^3}{3e^4x^{3/2}}+\frac{d^4}{2e^5x}-\frac{d^5}{e^6\sqrt{x}}+\frac{d^6\log\left(d+\frac{e}{\sqrt{x}}\right)}{e^7}\right)-\frac{b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])/x^4,x]
```

```
[Out] -1/3*a/x^3 + (b*e*n*(1/(6*e*x^3) - d/(5*e^2*x^(5/2)) + d^2/(4*e^3*x^2) - d^
3/(3*e^4*x^(3/2)) + d^4/(2*e^5*x) - d^5/(e^6*Sqrt[x]) + (d^6*Log[d + e/Sqrt
[x]])/e^7))/3 - (b*Log[c*(d + e/Sqrt[x])^n])/(3*x^3)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))/x^4,x)
```

Maxima [A]
time = 0.29, size = 114, normalized size = 0.84

$$\frac{1}{180}\left(60\,d^{6}e^{(-7)}\log\left(d\sqrt{x}+e\right)-30\,d^{6}e^{(-7)}\log\left(x\right)-\frac{\left(60\,d^{5}x^{\frac{5}{2}}-30\,d^{4}x^{2}e+20\,d^{3}x^{\frac{3}{2}}e^{2}-15\,d^{2}xe^{3}+12\,d\sqrt{x}\,e^{4}-10\,e^{5}\right)e^{(-6)}}{x^{3}}\right)bne-\frac{b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)}{3\,x^{3}}-\frac{a}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^4,x, algorithm="maxima")
```

```
[Out] 1/180*(60*d^6*e^(-7)*log(d*sqrt(x) + e) - 30*d^6*e^(-7)*log(x) - (60*d^5*x^
(5/2) - 30*d^4*x^2*e + 20*d^3*x^(3/2)*e^2 - 15*d^2*x*e^3 + 12*d*sqrt(x)*e^4
 - 10*e^5)*e^(-6)/x^3)*b*n*e - 1/3*b*log(c*(d + e/sqrt(x))^n)/x^3 - 1/3*a/x
^3
```

Fricas [A]
time = 0.43, size = 116, normalized size = 0.85

$$\frac{\left(30\,bd^{4}nx^{2}e^{2}+15\,bd^{2}nxe^{4}-60\,be^{6}\log\left(c\right)+10\,(bn-6\,a)e^{6}+60\,(bd^{6}nx^{3}-bne^{6})\log\left(\frac{dx+\sqrt{x}\,e}{x}\right)-4\,(15\,bd^{5}nx^{2}e+5\,bd^{3}nxe^{3}+3\,bdne^{5})\sqrt{x}\right)e^{(-6)}}{180\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^4,x, algorithm="fricas")
```

```
[Out] 1/180*(30*b*d^4*n*x^2*e^2 + 15*b*d^2*n*x*e^4 - 60*b*e^6*log(c) + 10*(b*n -
6*a)*e^6 + 60*(b*d^6*n*x^3 - b*n*e^6)*log((d*x + sqrt(x)*e)/x) - 4*(15*b*d^
5*n*x^2*e + 5*b*d^3*n*x*e^3 + 3*b*d*n*e^5)*sqrt(x))*e^(-6)/x^3
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))/x**4,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 535 vs. 2(106) = 212.
time = 4.29, size = 535, normalized size = 3.93

$$\frac{1}{180}\left(\frac{360\,(d\sqrt{x}+e)bd^{5}n\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)}{\sqrt{x}}-\frac{900\,(d\sqrt{x}+e)^{2}bd^{4}n\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)}{x}-\frac{360\,(d\sqrt{x}+e)bd^{5}n}{\sqrt{x}}+\frac{360\,(d\sqrt{x}+e)bd^{5}\log(c)}{\sqrt{x}}+\frac{1200\,(d\sqrt{x}+e)^{3}bd^{3}n\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)}{x^{\frac{3}{2}}}+\frac{450\,(d\sqrt{x}+e)^{2}bd^{4}n}{x}-\frac{900\,(d\sqrt{x}+e)^{2}bd^{4}\log(c)}{x}-\frac{900\,(d\sqrt{x}+e)^{4}bd^{2}n\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)}{x^{2}}-\frac{400\,(d\sqrt{x}+e)^{3}bd^{3}n}{x^{\frac{3}{2}}}+\frac{360\,(d\sqrt{x}+e)ad^{5}}{\sqrt{x}}+\frac{1200\,(d\sqrt{x}+e)^{3}bd^{3}\log(c)}{x^{\frac{3}{2}}}+\frac{360\,(d\sqrt{x}+e)^{5}bdn\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)}{x^{\frac{5}{2}}}+\frac{225\,(d\sqrt{x}+e)^{4}bd^{2}n}{x^{2}}-\frac{900\,(d\sqrt{x}+e)^{2}ad^{4}}{x}-\frac{900\,(d\sqrt{x}+e)^{4}bd^{2}\log(c)}{x^{2}}-\frac{60\,(d\sqrt{x}+e)^{6}bn\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)}{x^{3}}-\frac{72\,(d\sqrt{x}+e)^{5}bdn}{x^{\frac{5}{2}}}+\frac{1200\,(d\sqrt{x}+e)^{3}ad^{3}}{x^{\frac{3}{2}}}+\frac{360\,(d\sqrt{x}+e)^{5}bd\log(c)}{x^{\frac{5}{2}}}+\frac{10\,(d\sqrt{x}+e)^{6}bn}{x^{3}}-\frac{900\,(d\sqrt{x}+e)^{4}ad^{2}}{x^{2}}-\frac{60\,(d\sqrt{x}+e)^{6}b\log(c)}{x^{3}}+\frac{360\,(d\sqrt{x}+e)^{5}ad}{x^{\frac{5}{2}}}-\frac{60\,(d\sqrt{x}+e)^{6}a}{x^{3}}\right)e^{(-6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^4,x, algorithm="giac")
```

```
[Out] 1/180*(360*(d*sqrt(x) + e)*b*d^5*n*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x) - 9
00*(d*sqrt(x) + e)^2*b*d^4*n*log((d*sqrt(x) + e)/sqrt(x))/x - 360*(d*sqrt(x
) + e)*b*d^5*n/sqrt(x) + 360*(d*sqrt(x) + e)*b*d^5*log(c)/sqrt(x) + 1200*(d
*sqrt(x) + e)^3*b*d^3*n*log((d*sqrt(x) + e)/sqrt(x))/x^(3/2) + 450*(d*sqrt(
x) + e)^2*b*d^4*n/x - 900*(d*sqrt(x) + e)^2*b*d^4*log(c)/x - 900*(d*sqrt(x)
 + e)^4*b*d^2*n*log((d*sqrt(x) + e)/sqrt(x))/x^2 - 400*(d*sqrt(x) + e)^3*b*
d^3*n/x^(3/2) + 360*(d*sqrt(x) + e)*a*d^5/sqrt(x) + 1200*(d*sqrt(x) + e)^3*
b*d^3*log(c)/x^(3/2) + 360*(d*sqrt(x) + e)^5*b*d*n*log((d*sqrt(x) + e)/sqrt
(x))/x^(5/2) + 225*(d*sqrt(x) + e)^4*b*d^2*n/x^2 - 900*(d*sqrt(x) + e)^2*a*
d^4/x - 900*(d*sqrt(x) + e)^4*b*d^2*log(c)/x^2 - 60*(d*sqrt(x) + e)^6*b*n*l
og((d*sqrt(x) + e)/sqrt(x))/x^3 - 72*(d*sqrt(x) + e)^5*b*d*n/x^(5/2) + 1200
*(d*sqrt(x) + e)^3*a*d^3/x^(3/2) + 360*(d*sqrt(x) + e)^5*b*d*log(c)/x^(5/2)
 + 10*(d*sqrt(x) + e)^6*b*n/x^3 - 900*(d*sqrt(x) + e)^4*a*d^2/x^2 - 60*(d*s
qrt(x) + e)^6*b*log(c)/x^3 + 360*(d*sqrt(x) + e)^5*a*d/x^(5/2) - 60*(d*sqrt
(x) + e)^6*a/x^3)*e^(-6)
```

Mupad [B]
time = 0.44, size = 113, normalized size = 0.83

$$\frac{b\,n}{18\,x^3}-\frac{a}{3\,x^3}-\frac{b\,\ln\left(c\,{\left(d+\frac{e}{\sqrt{x}}\right)}^n\right)}{3\,x^3}-\frac{b\,d\,n}{15\,e\,x^{5/2}}+\frac{b\,d^6\,n\,\ln\left(d+\frac{e}{\sqrt{x}}\right)}{3\,e^6}+\frac{b\,d^2\,n}{12\,e^2\,x^2}+\frac{b\,d^4\,n}{6\,e^4\,x}-\frac{b\,d^3\,n}{9\,e^3\,x^{3/2}}-\frac{b\,d^5\,n}{3\,e^5\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))/x^4,x)
```

```
[Out] (b*n)/(18*x^3) - a/(3*x^3) - (b*log(c*(d + e/x^(1/2))^n))/(3*x^3) - (b*d*n)
/(15*e*x^(5/2)) + (b*d^6*n*log(d + e/x^(1/2)))/(3*e^6) + (b*d^2*n)/(12*e^2*
x^2) + (b*d^4*n)/(6*e^4*x) - (b*d^3*n)/(9*e^3*x^(3/2)) - (b*d^5*n)/(3*e^5*x
^(1/2))
```

3.429 $\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 \, dx$

Optimal. Leaf size=404

$$-\frac{77b^2e^5n^2\sqrt{x}}{90d^5}+\frac{47b^2e^4n^2x}{180d^4}-\frac{b^2e^3n^2x^{3/2}}{10d^3}+\frac{b^2e^2n^2x^2}{30d^2}+\frac{77b^2e^6n^2\log\left(d+\frac{e}{\sqrt{x}}\right)}{90d^6}+\frac{2be^5n\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+\right.}{3d^6}$$

```
[Out] 47/180*b^2*e^4*n^2*x/d^4-1/10*b^2*e^3*n^2*x^(3/2)/d^3+1/30*b^2*e^2*n^2*x^2/
d^2+137/180*b^2*e^6*n^2*ln(x)/d^6+77/90*b^2*e^6*n^2*ln(d+e/x^(1/2))/d^6-1/3
*b*e^4*n*x*(a+b*ln(c*(d+e/x^(1/2))^n))/d^4+2/9*b*e^3*n*x^(3/2)*(a+b*ln(c*(d
+e/x^(1/2))^n))/d^3-1/6*b*e^2*n*x^2*(a+b*ln(c*(d+e/x^(1/2))^n))/d^2+2/15*b*
e*n*x^(5/2)*(a+b*ln(c*(d+e/x^(1/2))^n))/d+2/3*b*e^6*n*ln(1-d/(d+e/x^(1/2)))
*(a+b*ln(c*(d+e/x^(1/2))^n))/d^6+1/3*x^3*(a+b*ln(c*(d+e/x^(1/2))^n))^2-2/3*
b^2*e^6*n^2*polylog(2,d/(d+e/x^(1/2)))/d^6-77/90*b^2*e^5*n^2*x^(1/2)/d^5+2/
3*b*e^5*n*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))*x^(1/2)/d^6
```

Rubi [A]
time = 0.62, antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2504, 2445, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$-\frac{2b^2e^6n^2\text{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{3d^6}+\frac{2be^6n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d^6}+\frac{2be^5n\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d^6}-\frac{be^4nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d^4}+\frac{2be^3nx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{9d^3}-\frac{be^2nx^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{6d^2}+\frac{2benx^{5/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{15d}+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2+\frac{77b^2e^6n^2\log\left(d+\frac{e}{\sqrt{x}}\right)}{90d^6}+\frac{137b^2e^6n^2\log(x)}{180d^6}-\frac{77b^2e^5n^2\sqrt{x}}{90d^5}+\frac{47b^2e^4n^2x}{180d^4}-\frac{b^2e^3n^2x^{3/2}}{10d^3}+\frac{b^2e^2n^2x^2}{30d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2,x]
```

```
[Out] (-77*b^2*e^5*n^2*Sqrt[x])/(90*d^5) + (47*b^2*e^4*n^2*x)/(180*d^4) - (b^2*e^
3*n^2*x^(3/2))/(10*d^3) + (b^2*e^2*n^2*x^2)/(30*d^2) + (77*b^2*e^6*n^2*Log[
d + e/Sqrt[x]])/(90*d^6) + (2*b*e^5*n*(d + e/Sqrt[x])*Sqrt[x]*(a + b*Log[c*
(d + e/Sqrt[x])^n]))/(3*d^6) - (b*e^4*n*x*(a + b*Log[c*(d + e/Sqrt[x])^n]))
/(3*d^4) + (2*b*e^3*n*x^(3/2)*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(9*d^3) - (
b*e^2*n*x^2*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(6*d^2) + (2*b*e*n*x^(5/2)*(a
 + b*Log[c*(d + e/Sqrt[x])^n]))/(15*d) + (2*b*e^6*n*Log[1 - d/(d + e/Sqrt[x
])]*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(3*d^6) + (x^3*(a + b*Log[c*(d + e/Sq
rt[x])^n])^2)/3 + (137*b^2*e^6*n^2*Log[x])/(180*d^6) - (2*b^2*e^6*n^2*PolyL
og[2, d/(d + e/Sqrt[x])])/(3*d^6)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
```

```
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\,dx=-\left(2\text{Subst}\left(\int\frac{(a+b\log(c(d+ex)^n))^2}{x^7}\,dx,x,\frac{1}{\sqrt{x}}\right)\right)$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2-\frac{1}{3}(2ben)\text{Subst}\left(\int\frac{a+b\,\mathrm{lo}}{x}\right.$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2-\frac{1}{3}(2bn)\text{Subst}\left(\int\frac{a+b\log}{x\left(-\frac{d}{e}\right.}\right.$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2-\frac{(2bn)\text{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\right.}{3d}$$

$$=\frac{2benx^{5/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{15d}+\frac{1}{3}x^3\left(a+b\log\left(c\left(d\right.\right.\right.$$

$$=-\frac{be^2nx^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{6d^2}+\frac{2benx^{5/2}\left(a+b\log\left(\right.\right.}{15d}$$

$$=-\frac{2b^2e^5n^2\sqrt{x}}{15d^5}+\frac{b^2e^4n^2x}{15d^4}-\frac{2b^2e^3n^2x^{3/2}}{45d^3}+\frac{b^2e^2n^2x^2}{30d^2}+\frac{2b^2e^6n^2\,\mathrm{l}}{}$$

$$=-\frac{3b^2e^5n^2\sqrt{x}}{10d^5}+\frac{3b^2e^4n^2x}{20d^4}-\frac{b^2e^3n^2x^{3/2}}{10d^3}+\frac{b^2e^2n^2x^2}{30d^2}+\frac{3b^2e^6n^2\,\mathrm{l}}{}$$

$$=-\frac{47b^2e^5n^2\sqrt{x}}{90d^5}+\frac{47b^2e^4n^2x}{180d^4}-\frac{b^2e^3n^2x^{3/2}}{10d^3}+\frac{b^2e^2n^2x^2}{30d^2}+\frac{47b^2e^6}{}$$

$$=-\frac{77b^2e^5n^2\sqrt{x}}{90d^5}+\frac{47b^2e^4n^2x}{180d^4}-\frac{b^2e^3n^2x^{3/2}}{10d^3}+\frac{b^2e^2n^2x^2}{30d^2}+\frac{77b^2e^6}{}$$

$$=-\frac{77b^2e^5n^2\sqrt{x}}{90d^5}+\frac{47b^2e^4n^2x}{180d^4}-\frac{b^2e^3n^2x^{3/2}}{10d^3}+\frac{b^2e^2n^2x^2}{30d^2}+\frac{77b^2e^6}{}$$

Mathematica [A]
time = 0.72, size = 379, normalized size = 0.94

$$\frac{60d^6x^3\left(a-bn\log\left(d+\frac{e}{\sqrt{x}}\right)+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2+2bn\left(a-bn\log\left(d+\frac{e}{\sqrt{x}}\right)+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\left(de\sqrt{x}\left(60e^4-30de^3\sqrt{x}+20d^2e^2x-15d^3ex^{3/2}+12d^4x^2\right)-60(e^6-d^6x^3)\log\left(d+\frac{e}{\sqrt{x}}\right)+60e^6\log\left(\frac{e}{\sqrt{x}}\right)\right)+b^2n^2\left(-60(e^6-d^6x^3)\log^2\left(d+\frac{e}{\sqrt{x}}\right)+e^2\left(-154de^3\sqrt{x}+47d^2e^2x-18d^3ex^{3/2}+6d^4x^2-274e^4\log\left(-\frac{e}{d\sqrt{x}}\right)\right)+2e\log\left(d+\frac{e}{\sqrt{x}}\right)\left(137e^5+60de^4\sqrt{x}-30d^2e^3x+20d^3e^2x^{3/2}-15d^4ex^2+12d^5x^{5/2}+60e^5\log\left(-\frac{e}{d\sqrt{x}}\right)\right)+120e^6\text{Li}_2\left(1+\frac{e}{d\sqrt{x}}\right)\right)}{180d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2,x]
```

```
[Out] (60*d^6*x^3*(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x])^n])^2 + 2
*b*n*(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x])^n])*(d*e*Sqrt[x]
*(60*e^4 - 30*d*e^3*Sqrt[x] + 20*d^2*e^2*x - 15*d^3*e*x^(3/2) + 12*d^4*x^2)
 - 60*(e^6 - d^6*x^3)*Log[d + e/Sqrt[x]] + 60*e^6*Log[e/Sqrt[x]]) + b^2*n^2
*(-60*(e^6 - d^6*x^3)*Log[d + e/Sqrt[x]]^2 + e^2*(-154*d*e^3*Sqrt[x] + 47*d
^2*e^2*x - 18*d^3*e*x^(3/2) + 6*d^4*x^2 - 274*e^4*Log[-(e/(d*Sqrt[x]))]) +
2*e*Log[d + e/Sqrt[x]]*(137*e^5 + 60*d*e^4*Sqrt[x] - 30*d^2*e^3*x + 20*d^3*
e^2*x^(3/2) - 15*d^4*e*x^2 + 12*d^5*x^(5/2) + 60*e^5*Log[-(e/(d*Sqrt[x]))])
 + 120*e^6*PolyLog[2, 1 + e/(d*Sqrt[x])]))/(180*d^6)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^{2} \left(a+b \ln \left(c \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e/x^(1/2))^n))^2,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e/x^(1/2))^n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*n^2*x^3*log(d*sqrt(x) + e)^2 - integrate(-1/3*(3*(b^2*log(c)^2 + 2*
a*b*log(c) + a^2)*x^(5/2)*e + 3*(b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d)*x
^3 - (b^2*d*n*x^3 - 6*(b^2*log(c) + a*b)*x^(5/2)*e - 6*(b^2*d*log(c) + a*b*
d)*x^3 + 6*(b^2*d*x^3 + b^2*x^(5/2)*e)*log(x^(1/2*n)))*n*log(d*sqrt(x) + e)
 + 3*(b^2*d*x^3 + b^2*x^(5/2)*e)*log(x^(1/2*n))^2 - 6*((b^2*log(c) + a*b)*x
^(5/2)*e + (b^2*d*log(c) + a*b*d)*x^3)*log(x^(1/2*n)))/(d*x + sqrt(x)*e), x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*log(c*((d*x + sqrt(x)*e)/x)^n)^2 + 2*a*b*x^2*log(c*((d*x +
 sqrt(x)*e)/x)^n) + a^2*x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(1/2))**n))**2,x)
```

```
[Out] Integral(x**2*(a + b*log(c*(d + e/sqrt(x))**n))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^2*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{\sqrt{x}}\right)}^n\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e/x^(1/2))^n))^2,x)
```

```
[Out] int(x^2*(a + b*log(c*(d + e/x^(1/2))^n))^2, x)
```

3.430 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\,dx$

Optimal. Leaf size=288

$$-\frac{5b^2e^3n^2\sqrt{x}}{6d^3}+\frac{b^2e^2n^2x}{6d^2}+\frac{5b^2e^4n^2\log\left(d+\frac{e}{\sqrt{x}}\right)}{6d^4}+\frac{be^3n\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^4}-\frac{be}{}$$

```
[Out] 1/6*b^2*e^2*n^2*x/d^2+11/12*b^2*e^4*n^2*ln(x)/d^4+5/6*b^2*e^4*n^2*ln(d+e/x^
(1/2))/d^4-1/2*b*e^2*n*x*(a+b*ln(c*(d+e/x^(1/2))^n))/d^2+1/3*b*e*n*x^(3/2)*
(a+b*ln(c*(d+e/x^(1/2))^n))/d+b*e^4*n*ln(1-d/(d+e/x^(1/2)))*(a+b*ln(c*(d+e/
x^(1/2))^n))/d^4+1/2*x^2*(a+b*ln(c*(d+e/x^(1/2))^n))^2-b^2*e^4*n^2*polylog(
2,d/(d+e/x^(1/2)))/d^4-5/6*b^2*e^3*n^2*x^(1/2)/d^3+b*e^3*n*(a+b*ln(c*(d+e/x
^(1/2))^n))*(d+e/x^(1/2))*x^(1/2)/d^4
```

Rubi [A]
time = 0.37, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2504, 2445, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$-\frac{b^2e^4n^2\text{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{d^4}+\frac{be^4n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^4}+\frac{be^3n\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^4}-\frac{be^2nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^2}+\frac{benx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2+\frac{5b^2e^4n^2\log\left(d+\frac{e}{\sqrt{x}}\right)}{6d^4}+\frac{11b^2e^4n^2\log(x)}{12d^4}-\frac{5b^2e^3n^2\sqrt{x}}{6d^3}+\frac{b^2e^2n^2x}{6d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e/Sqrt[x])^n])^2,x]
```

```
[Out] (-5*b^2*e^3*n^2*Sqrt[x])/(6*d^3) + (b^2*e^2*n^2*x)/(6*d^2) + (5*b^2*e^4*n^2
*Log[d + e/Sqrt[x]])/(6*d^4) + (b*e^3*n*(d + e/Sqrt[x])*Sqrt[x]*(a + b*Log[
c*(d + e/Sqrt[x])^n]))/d^4 - (b*e^2*n*x*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(
2*d^2) + (b*e*n*x^(3/2)*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(3*d) + (b*e^4*n*
Log[1 - d/(d + e/Sqrt[x])]*(a + b*Log[c*(d + e/Sqrt[x])^n]))/d^4 + (x^2*(a
+ b*Log[c*(d + e/Sqrt[x])^n])^2)/2 + (11*b^2*e^4*n^2*Log[x])/(12*d^4) - (b^
2*e^4*n^2*PolyLog[2, d/(d + e/Sqrt[x])])/d^4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
```

```
n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2dx = -\left(2\text{Subst}\left(\int\frac{(a+b\log(c(d+ex)^n))^2}{x^5}dx,x,\frac{1}{\sqrt{x}}\right)\right)$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2-(ben)\text{Subst}\left(\int\frac{a+b\log(c\cdots}{x^4(d\cdots}\right.$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2-(bn)\text{Subst}\left(\int\frac{a+b\log(cx\cdots}{x\left(-\frac{d}{e}+\frac{x}{e}\cdots\right.}\right.$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2-\frac{(bn)\text{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^4}dx\cdots\right.}{d}$$

$$= \frac{benx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\cdots\right.\right.\right.$$

$$= -\frac{be^2nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^2}+\frac{benx^{3/2}\left(a+b\log\left(c\left(\cdots\right.\right.\right.}{3d}$$

$$= -\frac{b^2e^3n^2\sqrt{x}}{3d^3}+\frac{b^2e^2n^2x}{6d^2}+\frac{b^2e^4n^2\log\left(d+\frac{e}{\sqrt{x}}\right)}{3d^4}+\frac{be^3n\left(d+\cdots\right.}{\cdots}$$

$$= -\frac{5b^2e^3n^2\sqrt{x}}{6d^3}+\frac{b^2e^2n^2x}{6d^2}+\frac{5b^2e^4n^2\log\left(d+\frac{e}{\sqrt{x}}\right)}{6d^4}+\frac{be^3n\left(d+\cdots\right.}{\cdots}$$

$$= -\frac{5b^2e^3n^2\sqrt{x}}{6d^3}+\frac{b^2e^2n^2x}{6d^2}+\frac{5b^2e^4n^2\log\left(d+\frac{e}{\sqrt{x}}\right)}{6d^4}+\frac{be^3n\left(d+\cdots\right.}{\cdots}$$

Mathematica [A]
time = 0.37, size = 307, normalized size = 1.07

$$\frac{3d^4x^2\left(a-bn\log\left(d+\frac{e}{\sqrt{x}}\right)+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2+bn\left(a-bn\log\left(d+\frac{e}{\sqrt{x}}\right)+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\left(de\sqrt{x}\left(6e^2-3de\sqrt{x}+2d^2x\right)-6(e^4-d^4x^2)\log\left(d+\frac{e}{\sqrt{x}}\right)+6e^4\log\left(\frac{e}{\sqrt{x}}\right)\right)-b^2n^2\left(3(e^4-d^4x^2)\log^2\left(d+\frac{e}{\sqrt{x}}\right)+e^2\left(5de\sqrt{x}-d^2x+11e^2\log\left(-\frac{e}{d\sqrt{x}}\right)\right)-e\log\left(d+\frac{e}{\sqrt{x}}\right)\left(11e^3+6de^2\sqrt{x}-3d^2ex+2d^3x^{3/2}+6e^3\log\left(-\frac{e}{d\sqrt{x}}\right)\right)-6e^4\text{Li}_2\left(1+\frac{e}{d\sqrt{x}}\right)\right)}{6d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])^n])^2,x]
```

```
[Out] (3*d^4*x^2*(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x])^n])^2 + b*
n*(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x])^n])*(d*e*Sqrt[x]*(6
*e^2 - 3*d*e*Sqrt[x] + 2*d^2*x) - 6*(e^4 - d^4*x^2)*Log[d + e/Sqrt[x]] + 6*
```

```
e^4*Log[e/Sqrt[x]]) - b^2*n^2*(3*(e^4 - d^4*x^2)*Log[d + e/Sqrt[x]]^2 + e^2
*(5*d*e*Sqrt[x] - d^2*x + 11*e^2*Log[-(e/(d*Sqrt[x]))]) - e*Log[d + e/Sqrt[
x]]*(11*e^3 + 6*d*e^2*Sqrt[x] - 3*d^2*e*x + 2*d^3*x^(3/2) + 6*e^3*Log[-(e/(
d*Sqrt[x]))]) - 6*e^4*PolyLog[2, 1 + e/(d*Sqrt[x])]))/(6*d^4)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(1/2))^n))^2,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e/x^(1/2))^n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*n^2*x^2*log(d*sqrt(x) + e)^2 - integrate(-1/2*(2*(b^2*log(c)^2 + 2*
a*b*log(c) + a^2)*x^(3/2)*e + 2*(b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d)*x
^2 - (b^2*d*n*x^2 - 4*(b^2*log(c) + a*b)*x^(3/2)*e - 4*(b^2*d*log(c) + a*b*
d)*x^2 + 4*(b^2*d*x^2 + b^2*x^(3/2)*e)*log(x^(1/2*n)))*n*log(d*sqrt(x) + e)
 + 2*(b^2*d*x^2 + b^2*x^(3/2)*e)*log(x^(1/2*n))^2 - 4*((b^2*log(c) + a*b)*x
^(3/2)*e + (b^2*d*log(c) + a*b*d)*x^2)*log(x^(1/2*n)))/(d*x + sqrt(x)*e), x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x*log(c*((d*x + sqrt(x)*e)/x)^n)^2 + 2*a*b*x*log(c*((d*x + sqr
t(x)*e)/x)^n) + a^2*x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/2))**n))**2,x)
```

```
[Out] Integral(x*(a + b*log(c*(d + e/sqrt(x))**n))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^2*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e/x^(1/2))^n))^2,x)
```

```
[Out] int(x*(a + b*log(c*(d + e/x^(1/2))^n))^2, x)
```

3.431 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 dx$

Optimal. Leaf size=152

$$\frac{2ben\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2}+\frac{2be^2n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right.\right.}{d^2}$$

```
[Out] b^2*e^2*n^2*ln(x)/d^2+2*b*e^2*n*ln(1-d/(d+e/x^(1/2)))*(a+b*ln(c*(d+e/x^(1/2
))^n))/d^2+x*(a+b*ln(c*(d+e/x^(1/2))^n))^2-2*b^2*e^2*n^2*polylog(2,d/(d+e/x
^(1/2)))/d^2+2*b*e*n*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))*x^(1/2)/d^2
```

Rubi [A]
time = 0.19, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {2501, 2504, 2445, 2458, 2389, 2379, 2438, 2351, 31}

$$-\frac{2b^2e^2n^2\text{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{d^2}+\frac{2be^2n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2}+\frac{2ben\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2}+x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2+\frac{b^2e^2n^2\log(x)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^2,x]
```

```
[Out] (2*b*e*n*(d + e/Sqrt[x])*Sqrt[x]*(a + b*Log[c*(d + e/Sqrt[x])^n]))/d^2 + (2
*b*e^2*n*Log[1 - d/(d + e/Sqrt[x])]*(a + b*Log[c*(d + e/Sqrt[x])^n]))/d^2 +
 x*(a + b*Log[c*(d + e/Sqrt[x])^n])^2 + (b^2*e^2*n^2*Log[x])/d^2 - (2*b^2*e
^2*n^2*PolyLog[2, d/(d + e/Sqrt[x])])/d^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
```

```
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2501

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\,dx = 2\mathrm{Subst}\left(\int x\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^n\right)\right)^2\,dx, x, \sqrt{x}\right)$$

$$= -\left(2\mathrm{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 - (2ben)\mathrm{Subst}\left(\int \frac{a+b\log(c(d+}{x^2(d+ex}\right.$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 - (2bn)\mathrm{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,d\right.$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 - \frac{(2bn)\mathrm{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x,\right.}{d}$$

$$= \frac{2ben\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2} + x\left(a+b\log\right.$$

$$= \frac{2ben\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2} - \frac{e^2\left(a+b\log\right.}{}$$

$$= \frac{2ben\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2} - \frac{e^2\left(a+b\log\right.}{}$$

Mathematica [A]
time = 0.09, size = 170, normalized size = 1.12

$$x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2+\frac{ben\left(2ad\sqrt{x}+2bd\sqrt{x}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-2e\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\log\left(e+d\sqrt{x}\right)+ben\left(2\log\left(d+\frac{e}{\sqrt{x}}\right)+\log(x)\right)+ben\left(\log\left(e+d\sqrt{x}\right)\left(\log\left(e+d\sqrt{x}\right)-2\log\left(-\frac{d\sqrt{x}}{e}\right)\right)-2\mathrm{Li}_2\left(1+\frac{d\sqrt{x}}{e}\right)\right)\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^2,x]
```

```
[Out] x*(a + b*Log[c*(d + e/Sqrt[x])^n])^2 + (b*e*n*(2*a*d*Sqrt[x] + 2*b*d*Sqrt[x
]*Log[c*(d + e/Sqrt[x])^n] - 2*e*(a + b*Log[c*(d + e/Sqrt[x])^n])*Log[e + d
*Sqrt[x]] + b*e*n*(2*Log[d + e/Sqrt[x]] + Log[x]) + b*e*n*(Log[e + d*Sqrt[x
]]*(Log[e + d*Sqrt[x]] - 2*Log[-((d*Sqrt[x])/e)]) - 2*PolyLog[2, 1 + (d*Sqr
t[x])/e])))/d^2
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="maxima")
```

```
[Out] -2*(n*(e*log(d*sqrt(x) + e)/d^2 - sqrt(x)/d)*e - x*log(c*(d + e/sqrt(x))^n)
)*a*b + (n^2*x*log(d*sqrt(x) + e)^2 - integrate(-(d*x*log(c)^2 + sqrt(x)*e*
log(c)^2 - (d*n*x - 2*d*x*log(c) - 2*sqrt(x)*e*log(c) + 2*(d*x + sqrt(x)*e)
*log(x^(1/2*n)))*n*log(d*sqrt(x) + e) + (d*x + sqrt(x)*e)*log(x^(1/2*n))^2
- 2*(d*x*log(c) + sqrt(x)*e*log(c))*log(x^(1/2*n)))/(d*x + sqrt(x)*e), x))*
b^2 + a^2*x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*log(c*((d*x + sqrt(x)*e)/x)^n)^2 + 2*a*b*log(c*((d*x + sqrt(x)
*e)/x)^n) + a^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e/sqrt(x))**n))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\ln\left(c\,{\left(d+\frac{e}{\sqrt{x}}\right)}^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))^2,x)
[Out] int((a + b*log(c*(d + e/x^(1/2))^n))^2, x)
```

3.432 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x}\,dx$

Optimal. Leaf size=93

$$-2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt{x}}\right)-4bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\text{Li}_2\left(1+\frac{e}{d\sqrt{x}}\right)+4b$$

```
[Out] -2*(a+b*ln(c*(d+e/x^(1/2))^n))^2*ln(-e/d/x^(1/2))-4*b*n*(a+b*ln(c*(d+e/x^(1
/2))^n))*polylog(2,1+e/d/x^(1/2))+4*b^2*n^2*polylog(3,1+e/d/x^(1/2))
```

Rubi [A]
time = 0.08, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2504, 2443, 2481, 2421, 6724}

$$-4bn\text{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)+4b^2n^2\text{PolyLog}\left(3,\frac{e}{d\sqrt{x}}+1\right)-2\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x,x]
```

```
[Out] -2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2*Log[-(e/(d*Sqrt[x]))] - 4*b*n*(a + b*
Log[c*(d + e/Sqrt[x])^n])*PolyLog[2, 1 + e/(d*Sqrt[x])] + 4*b^2*n^2*PolyLog
[3, 1 + e/(d*Sqrt[x])]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
```

```
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x}\,dx &= -\left(2\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)\\
&= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt{x}}\right)+(4ben)\text{Subst}\Bigg(\\
&= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt{x}}\right)+(4bn)\text{Subst}\Bigg(\int\\
&= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt{x}}\right)-4bn\Bigg(a+b\log\\
&= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt{x}}\right)-4bn\Bigg(a+b\log
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 386 vs. 2(93) = 186.
time = 0.22, size = 386, normalized size = 4.15

$$
\left(a-bn\log\left(d+\frac{e}{\sqrt{x}}\right)+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\log(x)+2bn\left(a-bn\log\left(d+\frac{e}{\sqrt{x}}\right)+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\left(\left(\log\left(d+\frac{e}{\sqrt{x}}\right)-\log\left(1+\frac{e}{d\sqrt{x}}\right)\right)\log(x)+2\text{Li}_2\left(-\frac{e}{d\sqrt{x}}\right)\right)+\frac{1}{12}b^2n^2\left(24\log^2\left(\frac{e}{d}+\sqrt{x}\right)\log\left(-\frac{d\sqrt{x}}{e}\right)+12\log^2\left(d+\frac{e}{\sqrt{x}}\right)\log(x)-12\log^2\left(\frac{e}{d}+\sqrt{x}\right)\log(x)-24\log\left(d+\frac{e}{\sqrt{x}}\right)\log\left(1+\frac{d\sqrt{x}}{e}\right)\log(x)+24\log\left(\frac{e}{d}+\sqrt{x}\right)\log\left(1+\frac{d\sqrt{x}}{e}\right)\log(x)+6\log\left(d+\frac{e}{\sqrt{x}}\right)\log^2(x)-6\log\left(1+\frac{d\sqrt{x}}{e}\right)\log^2(x)+\log^3(x)+48\log\left(\frac{e}{d}+\sqrt{x}\right)\text{Li}_2\left(1+\frac{d\sqrt{x}}{e}\right)-48\left(\log\left(d+\frac{e}{\sqrt{x}}\right)-\log\left(\frac{e}{d}+\sqrt{x}\right)\right)\text{Li}_2\left(-\frac{d\sqrt{x}}{e}\right)-48\text{Li}_3\left(1+\frac{d\sqrt{x}}{e}\right)-48\text{Li}_3\left(-\frac{d\sqrt{x}}{e}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x,x]
```

```
[Out] (a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x])^n])^2*Log[x] + 2*b*n*
(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x])^n])*((Log[d + e/Sqrt[
x]] - Log[1 + e/(d*Sqrt[x])])*Log[x] + 2*PolyLog[2, -(e/(d*Sqrt[x]))]) + (b
^2*n^2*(24*Log[e/d + Sqrt[x]]^2*Log[-((d*Sqrt[x])/e)] + 12*Log[d + e/Sqrt[x
]]^2*Log[x] - 12*Log[e/d + Sqrt[x]]^2*Log[x] - 24*Log[d + e/Sqrt[x]]*Log[1
+ (d*Sqrt[x])/e]*Log[x] + 24*Log[e/d + Sqrt[x]]*Log[1 + (d*Sqrt[x])/e]*Log[
x] + 6*Log[d + e/Sqrt[x]]*Log[x]^2 - 6*Log[1 + (d*Sqrt[x])/e]*Log[x]^2 + Lo
g[x]^3 + 48*Log[e/d + Sqrt[x]]*PolyLog[2, 1 + (d*Sqrt[x])/e] - 48*(Log[d +
e/Sqrt[x]] - Log[e/d + Sqrt[x]])*PolyLog[2, -((d*Sqrt[x])/e)] - 48*PolyLog[
3, 1 + (d*Sqrt[x])/e] - 48*PolyLog[3, -((d*Sqrt[x])/e)]))/12
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))^2/x,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))^2/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x,x, algorithm="maxima")
```

```
[Out] b^2*n^2*log(d*sqrt(x) + e)^2*log(x) - integrate(((b^2*d*n*x*log(x) - 2*(b^2
*log(c) + a*b)*sqrt(x)*e - 2*(b^2*d*log(c) + a*b*d)*x + 2*(b^2*d*x + b^2*sq
rt(x)*e)*log(x^(1/2*n)))*n*log(d*sqrt(x) + e) - (b^2*d*x + b^2*sqrt(x)*e)*l
og(x^(1/2*n))^2 - (b^2*log(c)^2 + 2*a*b*log(c) + a^2)*sqrt(x)*e - (b^2*d*lo
g(c)^2 + 2*a*b*d*log(c) + a^2*d)*x + 2*((b^2*log(c) + a*b)*sqrt(x)*e + (b^2
*d*log(c) + a*b*d)*x)*log(x^(1/2*n)))/(d*x^2 + x^(3/2)*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*((d*x + sqrt(x)*e)/x)^n)^2 + 2*a*b*log(c*((d*x + sqrt(x
)*e)/x)^n) + a^2)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \log\left(c \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**2/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e/sqrt(x))**n))**2/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^2/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b\,\ln\left(c\,\left(d + \frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))^2/x,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(1/2))^n))^2/x, x)
```

3.433 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=195

$$-\frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^2}-\frac{4abdn}{e\sqrt{x}}+\frac{4b^2dn^2}{e\sqrt{x}}-\frac{4b^2dn\left(d+\frac{e}{\sqrt{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{e^2}+\frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\,\mathrm{lo}\right.}{e^2}$$

```
[Out] -4*b^2*d*n*ln(c*(d+e/x^(1/2))^n)*(d+e/x^(1/2))/e^2+2*d*(a+b*ln(c*(d+e/x^(1/
2))^n))^2*(d+e/x^(1/2))/e^2-1/2*b^2*n^2*(d+e/x^(1/2))^2/e^2+b*n*(a+b*ln(c*(
d+e/x^(1/2))^n))*(d+e/x^(1/2))^2/e^2-(a+b*ln(c*(d+e/x^(1/2))^n))^2*(d+e/x^(
1/2))^2/e^2-4*a*b*d*n/e/x^(1/2)+4*b^2*d*n^2/e/x^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

$$\frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^2}-\frac{\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^2}+\frac{2d\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^2}-\frac{4abdn}{e\sqrt{x}}-\frac{4b^2dn\left(d+\frac{e}{\sqrt{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{e^2}-\frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^2}+\frac{4b^2dn^2}{e\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x^2,x]
```

```
[Out] -1/2*(b^2*n^2*(d + e/Sqrt[x])^2)/e^2 - (4*a*b*d*n)/(e*Sqrt[x]) + (4*b^2*d*n
^2)/(e*Sqrt[x]) - (4*b^2*d*n*(d + e/Sqrt[x])*Log[c*(d + e/Sqrt[x])^n])/e^2
+ (b*n*(d + e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n]))/e^2 + (2*d*(d +
e/Sqrt[x])*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/e^2 - ((d + e/Sqrt[x])^2*(a
+ b*Log[c*(d + e/Sqrt[x])^n])^2)/e^2
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
```

```
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^2}\,dx = -\left(2\text{Subst}\left(\int x(a+b\log\left(c(d+ex)^n\right))^2\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\text{Subst}\left(\int\left(-\frac{d(a+b\log\left(c(d+ex)^n\right))^2}{e}+\frac{(d+ex)\left(a+b\log\right.}{e}\right.\right.\right.$$

$$= -\frac{2\text{Subst}\left(\int(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx, x, \frac{1}{\sqrt{x}}\right)}{e}+\frac{(2d)}{}$$

$$= -\frac{2\text{Subst}\left(\int x(a+b\log\left(cx^n\right))^2\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^2}+\frac{(2d)\text{Subst}\left(\int\right.}{}$$

$$= \frac{2d\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^2}-\frac{\left(d+\frac{e}{\sqrt{x}}\right)^2\left(\right.}{}$$

$$= -\frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^2}-\frac{4abdn}{e\sqrt{x}}+\frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d\right.\right.\right.}{e^2}$$

$$= -\frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^2}-\frac{4abdn}{e\sqrt{x}}+\frac{4b^2dn^2}{e\sqrt{x}}-\frac{4b^2dn\left(d+\frac{e}{\sqrt{x}}\right)\log\left(\right.}{e^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.21, size = 298, normalized size = 1.53

$$-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2+\frac{bn\left(4ade\sqrt{x}-4bden\sqrt{x}+bn\left(e\left(e-2d\sqrt{x}\right)+2d^2x\log\left(d+\frac{e}{\sqrt{x}}\right)\right)+4bd\left(e+d\sqrt{x}\right)\sqrt{x}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-2e^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-4d^2x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\log\left(e+d\sqrt{x}\right)-4d^2x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt{x}}\right)-4bd^2nx\text{Li}_2\left(1+\frac{e}{d\sqrt{x}}\right)+2bd^2nx\left(\log\left(e+d\sqrt{x}\right)\left(\log\left(e+d\sqrt{x}\right)-2\log\left(-\frac{d\sqrt{x}}{e}\right)\right)-2\text{Li}_2\left(1+\frac{d\sqrt{x}}{e}\right)\right)\right)}{e^2}}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x^2,x]
```

```
[Out] -1/2*(2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2 + (b*n*(4*a*d*e*Sqrt[x] - 4*b*d*
e*n*Sqrt[x] + b*n*(e*(e - 2*d*Sqrt[x]) + 2*d^2*x*Log[d + e/Sqrt[x]]) + 4*b*
d*(e + d*Sqrt[x])*Sqrt[x]*Log[c*(d + e/Sqrt[x])^n] - 2*e^2*(a + b*Log[c*(d
+ e/Sqrt[x])^n]) - 4*d^2*x*(a + b*Log[c*(d + e/Sqrt[x])^n])*Log[e + d*Sqrt[
x]] - 4*d^2*x*(a + b*Log[c*(d + e/Sqrt[x])^n])*Log[-(e/(d*Sqrt[x]))] - 4*b*
d^2*n*x*PolyLog[2, 1 + e/(d*Sqrt[x])] + 2*b*d^2*n*x*(Log[e + d*Sqrt[x]]*(Lo
g[e + d*Sqrt[x]] - 2*Log[-((d*Sqrt[x])/e)]) - 2*PolyLog[2, 1 + (d*Sqrt[x])/
e])))/e^2)/x
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))^2/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))^2/x^2,x)
```

Maxima [A]
time = 0.28, size = 252, normalized size = 1.29

$$\left(2\,d^{2}e^{(-3)}\log\left(d\sqrt{x}+e\right)-d^{2}e^{(-3)}\log\left(x\right)-\frac{\left(2\,d\sqrt{x}-e\right)e^{(-2)}}{x}\right)abne+\frac{1}{4}\left(4\left(2\,d^{2}e^{(-3)}\log\left(d\sqrt{x}+e\right)-d^{2}e^{(-3)}\log\left(x\right)-\frac{\left(2\,d\sqrt{x}-e\right)e^{(-2)}}{x}\right)ne\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)-\frac{\left(4\,d^{2}x\log\left(d\sqrt{x}+e\right)^{2}+d^{2}x\log\left(x\right)^{2}-6\,d^{2}x\log\left(x\right)-12\,d\sqrt{x}e-4\left(d^{2}x\log\left(x\right)-3\,d^{2}x\right)\log\left(d\sqrt{x}+e\right)+2\,e^{2}\right)n^{2}e^{(-2)}}{x}\right)b^{2}-\frac{b^{2}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)^{2}}{x}-\frac{2\,ab\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)}{x}-\frac{a^{2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^2,x, algorithm="maxima")
```

```
[Out] (2*d^2*e^(-3)*log(d*sqrt(x) + e) - d^2*e^(-3)*log(x) - (2*d*sqrt(x) - e)*e^
(-2)/x)*a*b*n*e + 1/4*(4*(2*d^2*e^(-3)*log(d*sqrt(x) + e) - d^2*e^(-3)*log(
x) - (2*d*sqrt(x) - e)*e^(-2)/x)*n*e*log(c*(d + e/sqrt(x))^n) - (4*d^2*x*lo
g(d*sqrt(x) + e)^2 + d^2*x*log(x)^2 - 6*d^2*x*log(x) - 12*d*sqrt(x)*e - 4*(
d^2*x*log(x) - 3*d^2*x)*log(d*sqrt(x) + e) + 2*e^2)*n^2*e^(-2)/x)*b^2 - b^2
*log(c*(d + e/sqrt(x))^n)^2/x - 2*a*b*log(c*(d + e/sqrt(x))^n)/x - a^2/x
```

Fricas [A]
time = 0.37, size = 227, normalized size = 1.16

$$-\frac{\left(2\,b^{2}e^{2}\log\left(c\right)^{2}-2\left(b^{2}n-2\,ab\right)e^{2}\log\left(c\right)-2\left(b^{2}d^{2}n^{2}x-b^{2}n^{2}e^{2}\right)\log\left(\frac{dx+\sqrt{x}e}{x}\right)^{2}+\left(b^{2}n^{2}-2\,abn+2\,a^{2}\right)e^{2}+2\left(2\,b^{2}dn^{2}\sqrt{x}e+\left(3\,b^{2}d^{2}n^{2}-2\,abd^{2}n\right)x-\left(b^{2}n^{2}-2\,abn\right)e^{2}-2\left(b^{2}d^{2}nx-b^{2}ne^{2}\right)\log\left(c\right)\right)\log\left(\frac{dx+\sqrt{x}e}{x}\right)+2\left(2\,b^{2}dne\log\left(c\right)-\left(3\,b^{2}dn^{2}-2\,abdn\right)e\right)\sqrt{x}\right)e^{(-2)}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*e^2*log(c)^2 - 2*(b^2*n - 2*a*b)*e^2*log(c) - 2*(b^2*d^2*n^2*x
- b^2*n^2*e^2)*log((d*x + sqrt(x)*e)/x)^2 + (b^2*n^2 - 2*a*b*n + 2*a^2)*e^2
 + 2*(2*b^2*d*n^2*sqrt(x)*e + (3*b^2*d^2*n^2 - 2*a*b*d^2*n)*x - (b^2*n^2 -
2*a*b*n)*e^2 - 2*(b^2*d^2*n*x - b^2*n*e^2)*log(c))*log((d*x + sqrt(x)*e)/x)
 + 2*(2*b^2*d*n*e*log(c) - (3*b^2*d*n^2 - 2*a*b*d*n)*e)*sqrt(x))*e^(-2)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**2/x**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e/sqrt(x))**n))**2/x**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 503 vs. 2(173) = 346.
time = 5.20, size = 503, normalized size = 2.58

$$\frac{1}{2}\left(\frac{4\,(d\sqrt{x}+e)b^2dn^2\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)^2}{\sqrt{x}}-\frac{2\,(d\sqrt{x}+e)^2b^2n^2\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)^2}{x}-\frac{8\,(d\sqrt{x}+e)b^2dn^2\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)}{\sqrt{x}}+\frac{8\,(d\sqrt{x}+e)b^2dn\log(c)\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)}{\sqrt{x}}+\frac{2\,(d\sqrt{x}+e)^2b^2n^2\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)}{x}-\frac{4\,(d\sqrt{x}+e)^2b^2n\log(c)\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)}{x}+\frac{8\,(d\sqrt{x}+e)b^2dn^2}{\sqrt{x}}-\frac{8\,(d\sqrt{x}+e)b^2dn\log(c)}{\sqrt{x}}+\frac{4\,(d\sqrt{x}+e)b^2d\log(c)^2}{\sqrt{x}}+\frac{8\,(d\sqrt{x}+e)abdn\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)}{\sqrt{x}}-\frac{(d\sqrt{x}+e)^2b^2n^2}{x}+\frac{2\,(d\sqrt{x}+e)^2b^2n\log(c)}{x}-\frac{2\,(d\sqrt{x}+e)^2b^2\log(c)^2}{x}-\frac{4\,(d\sqrt{x}+e)^2abn\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)}{x}-\frac{8\,(d\sqrt{x}+e)abdn}{\sqrt{x}}+\frac{8\,(d\sqrt{x}+e)abd\log(c)}{\sqrt{x}}+\frac{2\,(d\sqrt{x}+e)^2abn}{x}-\frac{4\,(d\sqrt{x}+e)^2ab\log(c)}{x}+\frac{4\,(d\sqrt{x}+e)a^2d}{\sqrt{x}}-\frac{2\,(d\sqrt{x}+e)^2a^2}{x}\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*(d*sqrt(x) + e)*b^2*d*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/sqrt(x) - 2
*(d*sqrt(x) + e)^2*b^2*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/x - 8*(d*sqrt(x)
+ e)*b^2*d*n^2*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x) + 8*(d*sqrt(x) + e)*b^2
*d*n*log(c)*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x) + 2*(d*sqrt(x) + e)^2*b^2*
n^2*log((d*sqrt(x) + e)/sqrt(x))/x - 4*(d*sqrt(x) + e)^2*b^2*n*log(c)*log((
d*sqrt(x) + e)/sqrt(x))/x + 8*(d*sqrt(x) + e)*b^2*d*n^2/sqrt(x) - 8*(d*sqrt
(x) + e)*b^2*d*n*log(c)/sqrt(x) + 4*(d*sqrt(x) + e)*b^2*d*log(c)^2/sqrt(x)
+ 8*(d*sqrt(x) + e)*a*b*d*n*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x) - (d*sqrt(
x) + e)^2*b^2*n^2/x + 2*(d*sqrt(x) + e)^2*b^2*n*log(c)/x - 2*(d*sqrt(x) + e
)^2*b^2*log(c)^2/x - 4*(d*sqrt(x) + e)^2*a*b*n*log((d*sqrt(x) + e)/sqrt(x))
/x - 8*(d*sqrt(x) + e)*a*b*d*n/sqrt(x) + 8*(d*sqrt(x) + e)*a*b*d*log(c)/sqr
t(x) + 2*(d*sqrt(x) + e)^2*a*b*n/x - 4*(d*sqrt(x) + e)^2*a*b*log(c)/x + 4*(
d*sqrt(x) + e)*a^2*d/sqrt(x) - 2*(d*sqrt(x) + e)^2*a^2/x)*e^(-2)
```

Mupad [B]
time = 0.47, size = 193, normalized size = 0.99

$$\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\left(\frac{\frac{2\,b\,d\,(2\,a-b\,n)}{e}-\frac{4\,a\,b\,d}{e}}{\sqrt{x}}-\frac{b\,(2\,a-b\,n)}{x}\right)-\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)^2\left(\frac{b^2}{x}-\frac{b^2\,d^2}{e^2}\right)+\frac{\frac{d\,(2\,a^2-2\,a\,b\,n+b^2\,n^2)}{e}-\frac{2\,d\,(a^2-b^2\,n^2)}{e}}{\sqrt{x}}-\frac{a^2-a\,b\,n+\frac{b^2\,n^2}{2}}{x}-\frac{\ln\left(d+\frac{e}{\sqrt{x}}\right)\,(3\,b^2\,d^2\,n^2-2\,a\,b\,d^2\,n)}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))^2/x^2,x)
```

```
[Out] log(c*(d + e/x^(1/2))^n)*(((2*b*d*(2*a - b*n))/e - (4*a*b*d)/e)/x^(1/2) - (
b*(2*a - b*n))/x) - log(c*(d + e/x^(1/2))^n)^2*(b^2/x - (b^2*d^2)/e^2) + ((
d*(2*a^2 + b^2*n^2 - 2*a*b*n))/e - (2*d*(a^2 - b^2*n^2))/e)/x^(1/2) - (a^2
+ (b^2*n^2)/2 - a*b*n)/x - (log(d + e/x^(1/2))*(3*b^2*d^2*n^2 - 2*a*b*d^2*n
))/e^2
```

3.434 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=341

$$-\frac{3b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^4}+\frac{4b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)^3}{9e^4}-\frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4}{16e^4}+\frac{4b^2d^3n^2}{e^3\sqrt{x}}-\frac{b^2d^4n^2\log^2\left(d+\frac{e}{\sqrt{x}}\right)}{2e^4}-\frac{4bd^3n}{}$$

```
[Out] -1/2*b^2*d^4*n^2*ln(d+e/x^(1/2))^2/e^4+b*d^4*n*ln(d+e/x^(1/2))*(a+b*ln(c*(d
+e/x^(1/2))^n))/e^4-1/2*(a+b*ln(c*(d+e/x^(1/2))^n))^2/x^2-4*b*d^3*n*(a+b*ln
(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))/e^4-3/2*b^2*d^2*n^2*(d+e/x^(1/2))^2/e^4+
3*b*d^2*n*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))^2/e^4+4/9*b^2*d*n^2*(d+
e/x^(1/2))^3/e^4-4/3*b*d*n*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))^3/e^4-
1/16*b^2*n^2*(d+e/x^(1/2))^4/e^4+1/4*b*n*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x
^(1/2))^4/e^4+4*b^2*d^3*n^2/e^3/x^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2504, 2445, 2458, 45, 2372, 12, 14, 2338}

$$\frac{bd^4n\log\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^4}-\frac{4bd^3n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^4}+\frac{3bd^2n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^4}-\frac{4bdn\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3e^4}+\frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{4e^4}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2x^2}-\frac{b^2d^4n^2\log^2\left(d+\frac{e}{\sqrt{x}}\right)}{2e^4}+\frac{4b^2d^3n^2}{e^3\sqrt{x}}-\frac{3b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^4}+\frac{4b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)^3}{9e^4}-\frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4}{16e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x^3,x]
```

```
[Out] (-3*b^2*d^2*n^2*(d + e/Sqrt[x])^2)/(2*e^4) + (4*b^2*d*n^2*(d + e/Sqrt[x])^3
)/(9*e^4) - (b^2*n^2*(d + e/Sqrt[x])^4)/(16*e^4) + (4*b^2*d^3*n^2)/(e^3*Sqr
t[x]) - (b^2*d^4*n^2*Log[d + e/Sqrt[x]]^2)/(2*e^4) - (4*b*d^3*n*(d + e/Sqrt
[x])*(a + b*Log[c*(d + e/Sqrt[x])^n]))/e^4 + (3*b*d^2*n*(d + e/Sqrt[x])^2*(
a + b*Log[c*(d + e/Sqrt[x])^n]))/e^4 - (4*b*d*n*(d + e/Sqrt[x])^3*(a + b*Lo
g[c*(d + e/Sqrt[x])^n]))/(3*e^4) + (b*n*(d + e/Sqrt[x])^4*(a + b*Log[c*(d +
 e/Sqrt[x])^n]))/(4*e^4) + (b*d^4*n*Log[d + e/Sqrt[x]]*(a + b*Log[c*(d + e/
Sqrt[x])^n]))/e^4 - (a + b*Log[c*(d + e/Sqrt[x])^n])^2/(2*x^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^3}\,dx = -\left(2\text{Subst}\left(\int x^3\left(a+b\log\left(c(d+ex)^n\right)\right)^2\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2x^2} + (ben)\text{Subst}\left(\int \frac{x^4(a+b\log(c(}{d+e}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2x^2} + (bn)\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^4(a+}{x}\right.$$

$$= -\frac{1}{12}bn\left(\frac{48d^3\left(d+\frac{e}{\sqrt{x}}\right)}{e^4} - \frac{36d^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{e^4} + \frac{16d\left(d+\frac{e}{\sqrt{x}}\right)^3}{e^4}\right.$$

$$= -\frac{1}{12}bn\left(\frac{48d^3\left(d+\frac{e}{\sqrt{x}}\right)}{e^4} - \frac{36d^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{e^4} + \frac{16d\left(d+\frac{e}{\sqrt{x}}\right)^3}{e^4}\right.$$

$$= -\frac{1}{12}bn\left(\frac{48d^3\left(d+\frac{e}{\sqrt{x}}\right)}{e^4} - \frac{36d^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{e^4} + \frac{16d\left(d+\frac{e}{\sqrt{x}}\right)^3}{e^4}\right.$$

$$= -\frac{3b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^4} + \frac{4b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)^3}{9e^4} - \frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4}{16e^4}$$

$$= -\frac{3b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^4} + \frac{4b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)^3}{9e^4} - \frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4}{16e^4}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.25, size = 473, normalized size = 1.39

$$-\frac{72e^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2+bn\left(-36ae^4+9be^4n+48ade^3\sqrt{x}-28bde^3n\sqrt{x}-72ad^2e^2x+78bd^2e^2nx+144ad^3ex^{3/2}-300bd^3enx^{3/2}+156bd^4nx^2\log\left(d+\frac{e}{\sqrt{x}}\right)-36be^4\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+48bde^3\sqrt{x}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-72bd^2e^2x\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+144bd^3ex^{3/2}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+144bd^4x^2\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-144ad^4x^2\log\left(e+d\sqrt{x}\right)-144bd^4x^2\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\log\left(e+d\sqrt{x}\right)+72bd^4nx^2\log^2\left(e+d\sqrt{x}\right)-144ad^4x^2\log\left(-\frac{e}{d\sqrt{x}}\right)-144bd^4x^2\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\log\left(-\frac{e}{d\sqrt{x}}\right)-144bd^4nx^2\log\left(e+d\sqrt{x}\right)\log\left(-\frac{d\sqrt{x}}{e}\right)-144bd^4nx^2\text{Li}_2\left(1+\frac{e}{d\sqrt{x}}\right)-144bd^4nx^2\text{Li}_2\left(1+\frac{d\sqrt{x}}{e}\right)\right)}{144e^4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x^3,x]
```

```
[Out] -1/144*(72*e^4*(a + b*Log[c*(d + e/Sqrt[x])^n])^2 + b*n*(-36*a*e^4 + 9*b*e^
4*n + 48*a*d*e^3*Sqrt[x] - 28*b*d*e^3*n*Sqrt[x] - 72*a*d^2*e^2*x + 78*b*d^2
*e^2*n*x + 144*a*d^3*e*x^(3/2) - 300*b*d^3*e*n*x^(3/2) + 156*b*d^4*n*x^2*Lo
g[d + e/Sqrt[x]] - 36*b*e^4*Log[c*(d + e/Sqrt[x])^n] + 48*b*d*e^3*Sqrt[x]*L
og[c*(d + e/Sqrt[x])^n] - 72*b*d^2*e^2*x*Log[c*(d + e/Sqrt[x])^n] + 144*b*d
^3*e*x^(3/2)*Log[c*(d + e/Sqrt[x])^n] + 144*b*d^4*x^2*Log[c*(d + e/Sqrt[x])
^n] - 144*a*d^4*x^2*Log[e + d*Sqrt[x]] - 144*b*d^4*x^2*Log[c*(d + e/Sqrt[x]
)^n]*Log[e + d*Sqrt[x]] + 72*b*d^4*n*x^2*Log[e + d*Sqrt[x]]^2 - 144*a*d^4*x
^2*Log[-(e/(d*Sqrt[x]))] - 144*b*d^4*x^2*Log[c*(d + e/Sqrt[x])^n]*Log[-(e/(
d*Sqrt[x]))] - 144*b*d^4*n*x^2*Log[e + d*Sqrt[x]]*Log[-((d*Sqrt[x])/e)] - 1
44*b*d^4*n*x^2*PolyLog[2, 1 + e/(d*Sqrt[x])] - 144*b*d^4*n*x^2*PolyLog[2, 1
 + (d*Sqrt[x])/e]))/(e^4*x^2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))^2/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))^2/x^3,x)
```

Maxima [A]
time = 0.31, size = 319, normalized size = 0.94

$$\frac{1}{12}\left(12\,d^{4}e^{(-5)}\log\left(d\sqrt{x}+e\right)-6\,d^{4}e^{(-5)}\log\left(x\right)-\frac{\left(12\,d^{3}x^{\frac{3}{2}}-6\,d^{2}xe+4\,d\sqrt{x}\,e^{2}-3\,e^{3}\right)e^{(-4)}}{x^{2}}\right)abne+\frac{1}{144}\left(12\left(12\,d^{4}e^{(-5)}\log\left(d\sqrt{x}+e\right)-6\,d^{4}e^{(-5)}\log\left(x\right)-\frac{\left(12\,d^{3}x^{\frac{3}{2}}-6\,d^{2}xe+4\,d\sqrt{x}\,e^{2}-3\,e^{3}\right)e^{(-4)}}{x^{2}}\right)ne\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)-\frac{\left(72\,d^{4}x^{2}\log\left(d\sqrt{x}+e\right)^{2}+18\,d^{4}x^{2}\log\left(x\right)^{2}-150\,d^{4}x^{2}\log\left(x\right)-300\,d^{3}x^{\frac{3}{2}}e+78\,d^{2}xe^{2}-28\,d\sqrt{x}\,e^{3}-12\left(6\,d^{4}x^{2}\log\left(x\right)-25\,d^{4}x^{2}\right)\log\left(d\sqrt{x}+e\right)+9\,e^{4}\right)n^{2}e^{(-4)}}{x^{2}}\right)b^{2}-\frac{b^{2}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)^{2}}{2\,x^{2}}-\frac{ab\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)}{x^{2}}-\frac{a^{2}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/12*(12*d^4*e^(-5)*log(d*sqrt(x) + e) - 6*d^4*e^(-5)*log(x) - (12*d^3*x^(3
/2) - 6*d^2*x*e + 4*d*sqrt(x)*e^2 - 3*e^3)*e^(-4)/x^2)*a*b*n*e + 1/144*(12*
(12*d^4*e^(-5)*log(d*sqrt(x) + e) - 6*d^4*e^(-5)*log(x) - (12*d^3*x^(3/2) -
 6*d^2*x*e + 4*d*sqrt(x)*e^2 - 3*e^3)*e^(-4)/x^2)*n*e*log(c*(d + e/sqrt(x))
^n) - (72*d^4*x^2*log(d*sqrt(x) + e)^2 + 18*d^4*x^2*log(x)^2 - 150*d^4*x^2*
log(x) - 300*d^3*x^(3/2)*e + 78*d^2*x*e^2 - 28*d*sqrt(x)*e^3 - 12*(6*d^4*x^
2*log(x) - 25*d^4*x^2)*log(d*sqrt(x) + e) + 9*e^4)*n^2*e^(-4)/x^2)*b^2 - 1/
2*b^2*log(c*(d + e/sqrt(x))^n)^2/x^2 - a*b*log(c*(d + e/sqrt(x))^n)/x^2 - 1
/2*a^2/x^2
```

Fricas [**A**]
time = 0.37, size = 342, normalized size = 1.00

$$-\frac{\left(72\,b^{2}e^{4}\log\left(c\right)^{2}+6\,(13\,b^{2}d^{2}n^{2}-12\,abd^{2}n)xe^{2}-72\,(b^{2}d^{4}n^{2}x^{2}-b^{2}n^{2}e^{4})\log\left(\frac{dx+\sqrt{x}\,e}{x}\right)^{2}+9\,(b^{2}n^{2}-4\,abn+8\,a^{2})e^{4}-36\,(2\,b^{2}d^{2}nxe^{2}+(b^{2}n-4\,ab)e^{4})\log\left(c\right)-12\,(6\,b^{2}d^{2}n^{2}xe^{2}-(25\,b^{2}d^{4}n^{2}-12\,abd^{4}n)x^{2}+3\,(b^{2}n^{2}-4\,abn)e^{4}+12\,(b^{2}d^{4}nx^{2}-b^{2}ne^{4})\log\left(c\right)-4\,(3\,b^{2}d^{3}n^{2}xe+b^{2}dn^{2}e^{3})\sqrt{x})\log\left(\frac{dx+\sqrt{x}\,e}{x}\right)-4\,(3\,(25\,b^{2}d^{3}n^{2}-12\,abd^{3}n)xe+(7\,b^{2}dn^{2}-12\,abdn)e^{3}-12\,(3\,b^{2}d^{3}nxe+b^{2}dne^{3})\log\left(c\right))\sqrt{x}\right)e^{(-4)}}{144\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/144*(72*b^2*e^4*log(c)^2 + 6*(13*b^2*d^2*n^2 - 12*a*b*d^2*n)*x*e^2 - 72*
(b^2*d^4*n^2*x^2 - b^2*n^2*e^4)*log((d*x + sqrt(x)*e)/x)^2 + 9*(b^2*n^2 - 4
*a*b*n + 8*a^2)*e^4 - 36*(2*b^2*d^2*n*x*e^2 + (b^2*n - 4*a*b)*e^4)*log(c) -
 12*(6*b^2*d^2*n^2*x*e^2 - (25*b^2*d^4*n^2 - 12*a*b*d^4*n)*x^2 + 3*(b^2*n^2
 - 4*a*b*n)*e^4 + 12*(b^2*d^4*n*x^2 - b^2*n*e^4)*log(c) - 4*(3*b^2*d^3*n^2*
x*e + b^2*d*n^2*e^3)*sqrt(x))*log((d*x + sqrt(x)*e)/x) - 4*(3*(25*b^2*d^3*n
^2 - 12*a*b*d^3*n)*x*e + (7*b^2*d*n^2 - 12*a*b*d*n)*e^3 - 12*(3*b^2*d^3*n*x
*e + b^2*d*n*e^3)*log(c))*sqrt(x))*e^(-4)/x^2
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**2/x**3,x)
```

```
[Out] Integral((a + b*log(c*(d + e/sqrt(x))**n))**2/x**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1071 vs. 2(300) = 600.
time = 4.36, size = 1071, normalized size = 3.14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^3,x, algorithm="giac")
```

```
[Out] 1/144*(288*(d*sqrt(x) + e)*b^2*d^3*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/sqrt(
x) - 432*(d*sqrt(x) + e)^2*b^2*d^2*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/x - 5
76*(d*sqrt(x) + e)*b^2*d^3*n^2*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x) + 576*(
d*sqrt(x) + e)*b^2*d^3*n*log(c)*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x) + 288*
(d*sqrt(x) + e)^3*b^2*d*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/x^(3/2) + 432*(d
*sqrt(x) + e)^2*b^2*d^2*n^2*log((d*sqrt(x) + e)/sqrt(x))/x - 864*(d*sqrt(x)
 + e)^2*b^2*d^2*n*log(c)*log((d*sqrt(x) + e)/sqrt(x))/x - 72*(d*sqrt(x) + e
```

```
)^4*b^2*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/x^2 + 576*(d*sqrt(x) + e)*b^2*d^
3*n^2/sqrt(x) - 576*(d*sqrt(x) + e)*b^2*d^3*n*log(c)/sqrt(x) + 288*(d*sqrt(
x) + e)*b^2*d^3*log(c)^2/sqrt(x) - 192*(d*sqrt(x) + e)^3*b^2*d*n^2*log((d*s
qrt(x) + e)/sqrt(x))/x^(3/2) + 576*(d*sqrt(x) + e)*a*b*d^3*n*log((d*sqrt(x)
 + e)/sqrt(x))/sqrt(x) + 576*(d*sqrt(x) + e)^3*b^2*d*n*log(c)*log((d*sqrt(x
) + e)/sqrt(x))/x^(3/2) - 216*(d*sqrt(x) + e)^2*b^2*d^2*n^2/x + 432*(d*sqrt
(x) + e)^2*b^2*d^2*n*log(c)/x - 432*(d*sqrt(x) + e)^2*b^2*d^2*log(c)^2/x +
36*(d*sqrt(x) + e)^4*b^2*n^2*log((d*sqrt(x) + e)/sqrt(x))/x^2 - 864*(d*sqrt
(x) + e)^2*a*b*d^2*n*log((d*sqrt(x) + e)/sqrt(x))/x - 144*(d*sqrt(x) + e)^4
*b^2*n*log(c)*log((d*sqrt(x) + e)/sqrt(x))/x^2 + 64*(d*sqrt(x) + e)^3*b^2*d
*n^2/x^(3/2) - 576*(d*sqrt(x) + e)*a*b*d^3*n/sqrt(x) - 192*(d*sqrt(x) + e)^
3*b^2*d*n*log(c)/x^(3/2) + 576*(d*sqrt(x) + e)*a*b*d^3*log(c)/sqrt(x) + 288
*(d*sqrt(x) + e)^3*b^2*d*log(c)^2/x^(3/2) + 576*(d*sqrt(x) + e)^3*a*b*d*n*l
og((d*sqrt(x) + e)/sqrt(x))/x^(3/2) - 9*(d*sqrt(x) + e)^4*b^2*n^2/x^2 + 432
*(d*sqrt(x) + e)^2*a*b*d^2*n/x + 36*(d*sqrt(x) + e)^4*b^2*n*log(c)/x^2 - 86
4*(d*sqrt(x) + e)^2*a*b*d^2*log(c)/x - 72*(d*sqrt(x) + e)^4*b^2*log(c)^2/x^
2 - 144*(d*sqrt(x) + e)^4*a*b*n*log((d*sqrt(x) + e)/sqrt(x))/x^2 - 192*(d*s
qrt(x) + e)^3*a*b*d*n/x^(3/2) + 288*(d*sqrt(x) + e)*a^2*d^3/sqrt(x) + 576*(
d*sqrt(x) + e)^3*a*b*d*log(c)/x^(3/2) + 36*(d*sqrt(x) + e)^4*a*b*n/x^2 - 43
2*(d*sqrt(x) + e)^2*a^2*d^2/x - 144*(d*sqrt(x) + e)^4*a*b*log(c)/x^2 + 288*
(d*sqrt(x) + e)^3*a^2*d/x^(3/2) - 72*(d*sqrt(x) + e)^4*a^2/x^2)*e^(-4)
```

Mupad [B]
time = 0.56, size = 424, normalized size = 1.24

$$\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\left(\frac{\frac{b\,d\,(4\,a-b\,n)}{3\,e}-\frac{4\,a\,b\,d}{3\,e}}{x^{3/2}}-\frac{b\,(4\,a-b\,n)}{4\,x^2}-\frac{d\left(\frac{b\,d\,(4\,a-b\,n)}{e}-\frac{4\,a\,b\,d}{e}\right)}{2\,e\,x}+\frac{d^2\left(\frac{b\,d\,(4\,a-b\,n)}{e}-\frac{4\,a\,b\,d}{e}\right)}{e^2\,\sqrt{x}}\right)+\frac{\frac{d\left(2\,a^2-a\,b\,n+\frac{b^2\,n^2}{4}\right)}{3\,e}-\frac{d\,(6\,a^2-b^2\,n^2)}{9\,e}}{x^{3/2}}-\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)^2\left(\frac{b^2}{2\,x^2}-\frac{b^2\,d^4}{2\,e^4}\right)-\frac{\frac{a^2}{2}-\frac{a\,b\,n}{4}+\frac{b^2\,n^2}{16}}{x^2}-\frac{\frac{d\left(\frac{d\left(2\,a^2-a\,b\,n+\frac{b^2\,n^2}{4}\right)}{e}-\frac{d\,(6\,a^2-b^2\,n^2)}{3\,e}\right)}{2\,e}+\frac{b^2\,d^2\,n^2}{4\,e^2}}{x}+\frac{\frac{d\left(\frac{d\left(\frac{d\left(2\,a^2-a\,b\,n+\frac{b^2\,n^2}{4}\right)}{e}-\frac{d\,(6\,a^2-b^2\,n^2)}{3\,e}\right)}{e}+\frac{b^2\,d^2\,n^2}{2\,e^2}\right)}{e}+\frac{b^2\,d^3\,n^2}{e^3}}{\sqrt{x}}-\frac{\ln\left(d+\frac{e}{\sqrt{x}}\right)\,(25\,b^2\,d^4\,n^2-12\,a\,b\,d^4\,n)}{12\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))^2/x^3,x)
```

```
[Out] log(c*(d + e/x^(1/2))^n)*(((b*d*(4*a - b*n))/(3*e) - (4*a*b*d)/(3*e))/x^(3/
2) - (b*(4*a - b*n))/(4*x^2) - (d*((b*d*(4*a - b*n))/e - (4*a*b*d)/e))/(2*e
*x) + (d^2*((b*d*(4*a - b*n))/e - (4*a*b*d)/e))/(e^2*x^(1/2))) + ((d*(2*a^2
 + (b^2*n^2)/4 - a*b*n))/(3*e) - (d*(6*a^2 - b^2*n^2))/(9*e))/x^(3/2) - log
(c*(d + e/x^(1/2))^n)^2*(b^2/(2*x^2) - (b^2*d^4)/(2*e^4)) - (a^2/2 + (b^2*n
^2)/16 - (a*b*n)/4)/x^2 - ((d*((d*(2*a^2 + (b^2*n^2)/4 - a*b*n))/e - (d*(6*
a^2 - b^2*n^2))/(3*e)))/(2*e) + (b^2*d^2*n^2)/(4*e^2))/x + ((d*((d*((d*(2*a
^2 + (b^2*n^2)/4 - a*b*n))/e - (d*(6*a^2 - b^2*n^2))/(3*e)))/e + (b^2*d^2*n
^2)/(2*e^2)))/e + (b^2*d^3*n^2)/e^3)/x^(1/2) - (log(d + e/x^(1/2))*(25*b^2*
d^4*n^2 - 12*a*b*d^4*n))/(12*e^4)
```

3.435 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^4}\,dx$

Optimal. Leaf size=480

$$-\frac{5b^2d^4n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^6}+\frac{40b^2d^3n^2\left(d+\frac{e}{\sqrt{x}}\right)^3}{27e^6}-\frac{5b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4}{8e^6}+\frac{4b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)^5}{25e^6}-\frac{b^2n^2\left(d+\frac{e}{\sqrt{}}\right.}{54e^6}$$

```
[Out] -1/3*b^2*d^6*n^2*ln(d+e/x^(1/2))^2/e^6+2/3*b*d^6*n*ln(d+e/x^(1/2))*(a+b*ln(
c*(d+e/x^(1/2))^n))/e^6-1/3*(a+b*ln(c*(d+e/x^(1/2))^n))^2/x^3-4*b*d^5*n*(a+
b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))/e^6-5/2*b^2*d^4*n^2*(d+e/x^(1/2))^2/
e^6+5*b*d^4*n*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))^2/e^6+40/27*b^2*d^3
*n^2*(d+e/x^(1/2))^3/e^6-40/9*b*d^3*n*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1
/2))^3/e^6-5/8*b^2*d^2*n^2*(d+e/x^(1/2))^4/e^6+5/2*b*d^2*n*(a+b*ln(c*(d+e/x
^(1/2))^n))*(d+e/x^(1/2))^4/e^6+4/25*b^2*d*n^2*(d+e/x^(1/2))^5/e^6-4/5*b*d*
n*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))^5/e^6-1/54*b^2*n^2*(d+e/x^(1/2)
)^6/e^6+1/9*b*n*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))^6/e^6+4*b^2*d^5*n
^2/e^5/x^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 480, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2504, 2445, 2458, 45, 2372, 12, 14, 2338}

$$\frac{2bd^6n\log\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3e^6}-\frac{4bd^5n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^6}+\frac{5bd^4n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^6}-\frac{40bd^3n\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{9e^6}+\frac{5bd^2n\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^6}-\frac{4bdn\left(d+\frac{e}{\sqrt{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{5e^6}+\frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{9e^6}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{3x^3}-\frac{b^2d^6n^2\log^2\left(d+\frac{e}{\sqrt{x}}\right)}{3e^6}+\frac{4b^2d^5n^2}{e^5\sqrt{x}}-\frac{5b^2d^4n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^6}+\frac{40b^2d^3n^2\left(d+\frac{e}{\sqrt{x}}\right)^3}{27e^6}-\frac{5b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4}{8e^6}+\frac{4b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)^5}{25e^6}-\frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^6}{54e^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x^4,x]
```

```
[Out] (-5*b^2*d^4*n^2*(d + e/Sqrt[x])^2)/(2*e^6) + (40*b^2*d^3*n^2*(d + e/Sqrt[x]
)^3)/(27*e^6) - (5*b^2*d^2*n^2*(d + e/Sqrt[x])^4)/(8*e^6) + (4*b^2*d*n^2*(d
 + e/Sqrt[x])^5)/(25*e^6) - (b^2*n^2*(d + e/Sqrt[x])^6)/(54*e^6) + (4*b^2*d
^5*n^2)/(e^5*Sqrt[x]) - (b^2*d^6*n^2*Log[d + e/Sqrt[x]]^2)/(3*e^6) - (4*b*d
^5*n*(d + e/Sqrt[x])*(a + b*Log[c*(d + e/Sqrt[x])^n]))/e^6 + (5*b*d^4*n*(d
+ e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n]))/e^6 - (40*b*d^3*n*(d + e/S
qrt[x])^3*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(9*e^6) + (5*b*d^2*n*(d + e/Sqr
t[x])^4*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(2*e^6) - (4*b*d*n*(d + e/Sqrt[x]
)^5*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(5*e^6) + (b*n*(d + e/Sqrt[x])^6*(a +
 b*Log[c*(d + e/Sqrt[x])^n]))/(9*e^6) + (2*b*d^6*n*Log[d + e/Sqrt[x]]*(a +
b*Log[c*(d + e/Sqrt[x])^n]))/(3*e^6) - (a + b*Log[c*(d + e/Sqrt[x])^n])^2/(
3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^4}\,dx = -\left(2\text{Subst}\left(\int x^5(a+b\log(c(d+ex)^n))^2\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{3x^3} + \frac{1}{3}(2ben)\text{Subst}\left(\int \frac{x^6(a+b\log}{d}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{3x^3} + \frac{1}{3}(2bn)\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}{}\right.$$

$$= -\frac{1}{90}bn\left(\frac{360d^5\left(d+\frac{e}{\sqrt{x}}\right)}{e^6} - \frac{450d^4\left(d+\frac{e}{\sqrt{x}}\right)^2}{e^6} + \frac{400d^3\left(d+\right.}{e^6}\right.$$

$$= -\frac{1}{90}bn\left(\frac{360d^5\left(d+\frac{e}{\sqrt{x}}\right)}{e^6} - \frac{450d^4\left(d+\frac{e}{\sqrt{x}}\right)^2}{e^6} + \frac{400d^3\left(d+\right.}{e^6}\right.$$

$$= -\frac{1}{90}bn\left(\frac{360d^5\left(d+\frac{e}{\sqrt{x}}\right)}{e^6} - \frac{450d^4\left(d+\frac{e}{\sqrt{x}}\right)^2}{e^6} + \frac{400d^3\left(d+\right.}{e^6}\right.$$

$$= -\frac{5b^2d^4n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^6} + \frac{40b^2d^3n^2\left(d+\frac{e}{\sqrt{x}}\right)^3}{27e^6} - \frac{5b^2d^2n^2\left(d+\right.}{8e^6}$$

$$= -\frac{5b^2d^4n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^6} + \frac{40b^2d^3n^2\left(d+\frac{e}{\sqrt{x}}\right)^3}{27e^6} - \frac{5b^2d^2n^2\left(d+\right.}{8e^6}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.23, size = 692, normalized size = 1.44

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x^4,x]
```

```
[Out] (-1800*a^2*e^6 + 600*a*b*e^6*n - 100*b^2*e^6*n^2 - 720*a*b*d*e^5*n*Sqrt[x]
+ 264*b^2*d*e^5*n^2*Sqrt[x] + 900*a*b*d^2*e^4*n*x - 555*b^2*d^2*e^4*n^2*x -
 1200*a*b*d^3*e^3*n*x^(3/2) + 1140*b^2*d^3*e^3*n^2*x^(3/2) + 1800*a*b*d^4*e
^2*n*x^2 - 2610*b^2*d^4*e^2*n^2*x^2 - 3600*a*b*d^5*e*n*x^(5/2) + 8820*b^2*d
^5*e*n^2*x^(5/2) - 5220*b^2*d^6*n^2*x^3*Log[d + e/Sqrt[x]] - 3600*a*b*e^6*L
og[c*(d + e/Sqrt[x])^n] + 600*b^2*e^6*n*Log[c*(d + e/Sqrt[x])^n] - 720*b^2*
d*e^5*n*Sqrt[x]*Log[c*(d + e/Sqrt[x])^n] + 900*b^2*d^2*e^4*n*x*Log[c*(d + e
/Sqrt[x])^n] - 1200*b^2*d^3*e^3*n*x^(3/2)*Log[c*(d + e/Sqrt[x])^n] + 1800*b
^2*d^4*e^2*n*x^2*Log[c*(d + e/Sqrt[x])^n] - 3600*b^2*d^5*e*n*x^(5/2)*Log[c*
(d + e/Sqrt[x])^n] - 3600*b^2*d^6*n*x^3*Log[c*(d + e/Sqrt[x])^n] - 1800*b^2
*e^6*Log[c*(d + e/Sqrt[x])^n]^2 + 3600*a*b*d^6*n*x^3*Log[e + d*Sqrt[x]] + 3
600*b^2*d^6*n*x^3*Log[c*(d + e/Sqrt[x])^n]*Log[e + d*Sqrt[x]] - 1800*b^2*d^
6*n^2*x^3*Log[e + d*Sqrt[x]]^2 + 3600*a*b*d^6*n*x^3*Log[-(e/(d*Sqrt[x]))] +
 3600*b^2*d^6*n*x^3*Log[c*(d + e/Sqrt[x])^n]*Log[-(e/(d*Sqrt[x]))] + 3600*b
^2*d^6*n^2*x^3*Log[e + d*Sqrt[x]]*Log[-((d*Sqrt[x])/e)] + 3600*b^2*d^6*n^2*
x^3*PolyLog[2, 1 + e/(d*Sqrt[x])] + 3600*b^2*d^6*n^2*x^3*PolyLog[2, 1 + (d*
Sqrt[x])/e])/(5400*e^6*x^3)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))^2/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))^2/x^4,x)
```

Maxima [A]
time = 0.33, size = 379, normalized size = 0.79

$$\frac{1}{90}\left(60\,d^{6}e^{(-7)}\log\left(d\sqrt{x}+e\right)-30\,d^{6}e^{(-7)}\log\left(x\right)-\frac{\left(60\,d^{5}x^{\frac{5}{2}}-30\,d^{4}x^{2}e+20\,d^{3}x^{\frac{3}{2}}e^{2}-15\,d^{2}xe^{3}+12\,d\sqrt{x}\,e^{4}-10\,e^{5}\right)e^{(-6)}}{x^{3}}\right)abne+\frac{1}{5400}\left(60\left(60\,d^{6}e^{(-7)}\log\left(d\sqrt{x}+e\right)-30\,d^{6}e^{(-7)}\log\left(x\right)-\frac{\left(60\,d^{5}x^{\frac{5}{2}}-30\,d^{4}x^{2}e+20\,d^{3}x^{\frac{3}{2}}e^{2}-15\,d^{2}xe^{3}+12\,d\sqrt{x}\,e^{4}-10\,e^{5}\right)e^{(-6)}}{x^{3}}\right)ne\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)-\frac{\left(1800\,d^{6}x^{3}\log\left(d\sqrt{x}+e\right)^{2}+450\,d^{6}x^{3}\log\left(x\right)^{2}-4410\,d^{6}x^{3}\log\left(x\right)-8820\,d^{5}x^{\frac{5}{2}}e+2610\,d^{4}x^{2}e^{2}-1140\,d^{3}x^{\frac{3}{2}}e^{3}+555\,d^{2}xe^{4}-264\,d\sqrt{x}\,e^{5}-180\left(10\,d^{6}x^{3}\log\left(x\right)-49\,d^{6}x^{3}\right)\log\left(d\sqrt{x}+e\right)+100\,e^{6}\right)n^{2}e^{(-6)}}{x^{3}}\right)b^{2}-\frac{b^{2}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)^{2}}{3\,x^{3}}-\frac{2\,ab\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)}{3\,x^{3}}-\frac{a^{2}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/90*(60*d^6*e^(-7)*log(d*sqrt(x) + e) - 30*d^6*e^(-7)*log(x) - (60*d^5*x^(
5/2) - 30*d^4*x^2*e + 20*d^3*x^(3/2)*e^2 - 15*d^2*x*e^3 + 12*d*sqrt(x)*e^4
- 10*e^5)*e^(-6)/x^3)*a*b*n*e + 1/5400*(60*(60*d^6*e^(-7)*log(d*sqrt(x) + e
) - 30*d^6*e^(-7)*log(x) - (60*d^5*x^(5/2) - 30*d^4*x^2*e + 20*d^3*x^(3/2)*
e^2 - 15*d^2*x*e^3 + 12*d*sqrt(x)*e^4 - 10*e^5)*e^(-6)/x^3)*n*e*log(c*(d +
e/sqrt(x))^n) - (1800*d^6*x^3*log(d*sqrt(x) + e)^2 + 450*d^6*x^3*log(x)^2 -
```

```
 4410*d^6*x^3*log(x) - 8820*d^5*x^(5/2)*e + 2610*d^4*x^2*e^2 - 1140*d^3*x^(
3/2)*e^3 + 555*d^2*x*e^4 - 264*d*sqrt(x)*e^5 - 180*(10*d^6*x^3*log(x) - 49*
d^6*x^3)*log(d*sqrt(x) + e) + 100*e^6)*n^2*e^(-6)/x^3)*b^2 - 1/3*b^2*log(c*
(d + e/sqrt(x))^n)^2/x^3 - 2/3*a*b*log(c*(d + e/sqrt(x))^n)/x^3 - 1/3*a^2/x
^3
```

Fricas [**A**]
time = 0.38, size = 460, normalized size = 0.96

$$-\frac{\left(1800\,b^2e^6\log\left(c\right)^2+90\,(29\,b^2d^4n^2-20\,abd^4n)x^2e^2+15\,(37\,b^2d^2n^2-60\,abd^2n)xe^4-1800\,(b^2d^6n^2x^3-b^2n^2e^6)\log\left(\frac{dx+\sqrt{x}\,e}{x}\right)^2+100\,(b^2n^2-6\,abn+18\,a^2)e^6-300\,(6\,b^2d^4nx^2e^2+3\,b^2d^2nxe^4+2\,(b^2n-6\,ab)e^6)\log\left(c\right)-60\,(30\,b^2d^4n^2x^2e^2+15\,b^2d^2n^2xe^4-3\,(49\,b^2d^6n^2-20\,abd^6n)x^3+10\,(b^2n^2-6\,abn)e^6+60\,(b^2d^6nx^3-b^2ne^6)\log\left(c\right)-4\,(15\,b^2d^5n^2x^2e+5\,b^2d^3n^2xe^3+3\,b^2dn^2e^5)\sqrt{x})\log\left(\frac{dx+\sqrt{x}\,e}{x}\right)-12\,(15\,(49\,b^2d^5n^2-20\,abd^5n)x^2e+5\,(19\,b^2d^3n^2-20\,abd^3n)xe^3+2\,(11\,b^2dn^2-30\,abdn)e^5-20\,(15\,b^2d^5nx^2e+5\,b^2d^3nxe^3+3\,b^2dne^5)\log\left(c\right))\sqrt{x}\right)e^{(-6)}}{5400\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/5400*(1800*b^2*e^6*log(c)^2 + 90*(29*b^2*d^4*n^2 - 20*a*b*d^4*n)*x^2*e^2
 + 15*(37*b^2*d^2*n^2 - 60*a*b*d^2*n)*x*e^4 - 1800*(b^2*d^6*n^2*x^3 - b^2*n
^2*e^6)*log((d*x + sqrt(x)*e)/x)^2 + 100*(b^2*n^2 - 6*a*b*n + 18*a^2)*e^6 -
 300*(6*b^2*d^4*n*x^2*e^2 + 3*b^2*d^2*n*x*e^4 + 2*(b^2*n - 6*a*b)*e^6)*log(
c) - 60*(30*b^2*d^4*n^2*x^2*e^2 + 15*b^2*d^2*n^2*x*e^4 - 3*(49*b^2*d^6*n^2
- 20*a*b*d^6*n)*x^3 + 10*(b^2*n^2 - 6*a*b*n)*e^6 + 60*(b^2*d^6*n*x^3 - b^2*
n*e^6)*log(c) - 4*(15*b^2*d^5*n^2*x^2*e + 5*b^2*d^3*n^2*x*e^3 + 3*b^2*d*n^2
*e^5)*sqrt(x))*log((d*x + sqrt(x)*e)/x) - 12*(15*(49*b^2*d^5*n^2 - 20*a*b*d
^5*n)*x^2*e + 5*(19*b^2*d^3*n^2 - 20*a*b*d^3*n)*x*e^3 + 2*(11*b^2*d*n^2 - 3
0*a*b*d*n)*e^5 - 20*(15*b^2*d^5*n*x^2*e + 5*b^2*d^3*n*x*e^3 + 3*b^2*d*n*e^5
)*log(c))*sqrt(x))*e^(-6)/x^3
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**2/x**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1639 vs. $2(419) = 838$.
time = 3.76, size = 1639, normalized size = 3.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^4,x, algorithm="giac")
```

```
[Out] 1/5400*(10800*(d*sqrt(x) + e)*b^2*d^5*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/sq
rt(x) - 27000*(d*sqrt(x) + e)^2*b^2*d^4*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/
```

```
x - 21600*(d*sqrt(x) + e)*b^2*d^5*n^2*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x)
+ 21600*(d*sqrt(x) + e)*b^2*d^5*n*log(c)*log((d*sqrt(x) + e)/sqrt(x))/sqrt(
x) + 36000*(d*sqrt(x) + e)^3*b^2*d^3*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/x^(
3/2) + 27000*(d*sqrt(x) + e)^2*b^2*d^4*n^2*log((d*sqrt(x) + e)/sqrt(x))/x -
 54000*(d*sqrt(x) + e)^2*b^2*d^4*n*log(c)*log((d*sqrt(x) + e)/sqrt(x))/x -
27000*(d*sqrt(x) + e)^4*b^2*d^2*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/x^2 + 21
600*(d*sqrt(x) + e)*b^2*d^5*n^2/sqrt(x) - 21600*(d*sqrt(x) + e)*b^2*d^5*n*l
og(c)/sqrt(x) + 10800*(d*sqrt(x) + e)*b^2*d^5*log(c)^2/sqrt(x) - 24000*(d*s
qrt(x) + e)^3*b^2*d^3*n^2*log((d*sqrt(x) + e)/sqrt(x))/x^(3/2) + 21600*(d*s
qrt(x) + e)*a*b*d^5*n*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x) + 72000*(d*sqrt(
x) + e)^3*b^2*d^3*n*log(c)*log((d*sqrt(x) + e)/sqrt(x))/x^(3/2) + 10800*(d*
sqrt(x) + e)^5*b^2*d*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/x^(5/2) - 13500*(d*
sqrt(x) + e)^2*b^2*d^4*n^2/x + 27000*(d*sqrt(x) + e)^2*b^2*d^4*n*log(c)/x -
 27000*(d*sqrt(x) + e)^2*b^2*d^4*log(c)^2/x + 13500*(d*sqrt(x) + e)^4*b^2*d
^2*n^2*log((d*sqrt(x) + e)/sqrt(x))/x^2 - 54000*(d*sqrt(x) + e)^2*a*b*d^4*n
*log((d*sqrt(x) + e)/sqrt(x))/x - 54000*(d*sqrt(x) + e)^4*b^2*d^2*n*log(c)*
log((d*sqrt(x) + e)/sqrt(x))/x^2 - 1800*(d*sqrt(x) + e)^6*b^2*n^2*log((d*sq
rt(x) + e)/sqrt(x))^2/x^3 + 8000*(d*sqrt(x) + e)^3*b^2*d^3*n^2/x^(3/2) - 21
600*(d*sqrt(x) + e)*a*b*d^5*n/sqrt(x) - 24000*(d*sqrt(x) + e)^3*b^2*d^3*n*l
og(c)/x^(3/2) + 21600*(d*sqrt(x) + e)*a*b*d^5*log(c)/sqrt(x) + 36000*(d*sqr
t(x) + e)^3*b^2*d^3*log(c)^2/x^(3/2) - 4320*(d*sqrt(x) + e)^5*b^2*d*n^2*log
((d*sqrt(x) + e)/sqrt(x))/x^(5/2) + 72000*(d*sqrt(x) + e)^3*a*b*d^3*n*log((
d*sqrt(x) + e)/sqrt(x))/x^(3/2) + 21600*(d*sqrt(x) + e)^5*b^2*d*n*log(c)*lo
g((d*sqrt(x) + e)/sqrt(x))/x^(5/2) - 3375*(d*sqrt(x) + e)^4*b^2*d^2*n^2/x^2
 + 27000*(d*sqrt(x) + e)^2*a*b*d^4*n/x + 13500*(d*sqrt(x) + e)^4*b^2*d^2*n*
log(c)/x^2 - 54000*(d*sqrt(x) + e)^2*a*b*d^4*log(c)/x - 27000*(d*sqrt(x) +
e)^4*b^2*d^2*log(c)^2/x^2 + 600*(d*sqrt(x) + e)^6*b^2*n^2*log((d*sqrt(x) +
e)/sqrt(x))/x^3 - 54000*(d*sqrt(x) + e)^4*a*b*d^2*n*log((d*sqrt(x) + e)/sqr
t(x))/x^2 - 3600*(d*sqrt(x) + e)^6*b^2*n*log(c)*log((d*sqrt(x) + e)/sqrt(x)
)/x^3 + 864*(d*sqrt(x) + e)^5*b^2*d*n^2/x^(5/2) - 24000*(d*sqrt(x) + e)^3*a
*b*d^3*n/x^(3/2) + 10800*(d*sqrt(x) + e)*a^2*d^5/sqrt(x) - 4320*(d*sqrt(x)
+ e)^5*b^2*d*n*log(c)/x^(5/2) + 72000*(d*sqrt(x) + e)^3*a*b*d^3*log(c)/x^(3
/2) + 10800*(d*sqrt(x) + e)^5*b^2*d*log(c)^2/x^(5/2) + 21600*(d*sqrt(x) + e
)^5*a*b*d*n*log((d*sqrt(x) + e)/sqrt(x))/x^(5/2) - 100*(d*sqrt(x) + e)^6*b^
2*n^2/x^3 + 13500*(d*sqrt(x) + e)^4*a*b*d^2*n/x^2 - 27000*(d*sqrt(x) + e)^2
*a^2*d^4/x + 600*(d*sqrt(x) + e)^6*b^2*n*log(c)/x^3 - 54000*(d*sqrt(x) + e)
^4*a*b*d^2*log(c)/x^2 - 1800*(d*sqrt(x) + e)^6*b^2*log(c)^2/x^3 - 3600*(d*s
qrt(x) + e)^6*a*b*n*log((d*sqrt(x) + e)/sqrt(x))/x^3 - 4320*(d*sqrt(x) + e)
^5*a*b*d*n/x^(5/2) + 36000*(d*sqrt(x) + e)^3*a^2*d^3/x^(3/2) + 21600*(d*sqr
t(x) + e)^5*a*b*d*log(c)/x^(5/2) + 600*(d*sqrt(x) + e)^6*a*b*n/x^3 - 27000*
(d*sqrt(x) + e)^4*a^2*d^2/x^2 - 3600*(d*sqrt(x) + e)^6*a*b*log(c)/x^3 + 108
00*(d*sqrt(x) + e)^5*a^2*d/x^(5/2) - 1800*(d*sqrt(x) + e)^6*a^2/x^3)*e^(-6)
```

Mupad [B]

time = 1.77, size = 440, normalized size = 0.92

$$\frac{b^2 d^6 \ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)^2}{3 e^6}-\frac{b^2 \ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)^2}{3 x^3}-\frac{b^2 n^2}{54 x^3}-\frac{2 a b \ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3 x^3}-\frac{a^2}{3 x^3}+\frac{a b n}{9 x^3}+\frac{b^2 n \ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{9 x^3}-\frac{49 b^2 d^6 n^2 \ln\left(d+\frac{e}{\sqrt{x}}\right)}{30 e^6}-\frac{37 b^2 d^2 n^2}{360 e^2 x^2}-\frac{29 b^2 d^4 n^2}{60 e^4 x}+\frac{19 b^2 d^3 n^2}{90 e^3 x^{3/2}}+\frac{49 b^2 d^5 n^2}{30 e^5 \sqrt{x}}+\frac{11 b^2 d n^2}{225 e x^{5/2}}+\frac{b^2 d^2 n \ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{6 e^2 x^2}+\frac{b^2 d^4 n \ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3 e^4 x}-\frac{2 b^2 d^3 n \ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{9 e^3 x^{3/2}}-\frac{2 b^2 d^5 n \ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3 e^5 \sqrt{x}}-\frac{2 a b d n}{15 e x^{5/2}}+\frac{2 a b d^6 n \ln\left(d+\frac{e}{\sqrt{x}}\right)}{3 e^6}-\frac{2 b^2 d n \ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{15 e x^{5/2}}+\frac{a b d^2 n}{6 e^2 x^2}+\frac{a b d^4 n}{3 e^4 x}-\frac{2 a b d^3 n}{9 e^3 x^{3/2}}-\frac{2 a b d^5 n}{3 e^5 \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))^2/x^4,x)
```

```
[Out] (b^2*d^6*log(c*(d + e/x^(1/2))^n)^2)/(3*e^6) - (b^2*log(c*(d + e/x^(1/2))^n
)^2)/(3*x^3) - (b^2*n^2)/(54*x^3) - (2*a*b*log(c*(d + e/x^(1/2))^n))/(3*x^3
) - a^2/(3*x^3) + (a*b*n)/(9*x^3) + (b^2*n*log(c*(d + e/x^(1/2))^n))/(9*x^3
) - (49*b^2*d^6*n^2*log(d + e/x^(1/2)))/(30*e^6) - (37*b^2*d^2*n^2)/(360*e^
2*x^2) - (29*b^2*d^4*n^2)/(60*e^4*x) + (19*b^2*d^3*n^2)/(90*e^3*x^(3/2)) +
(49*b^2*d^5*n^2)/(30*e^5*x^(1/2)) + (11*b^2*d*n^2)/(225*e*x^(5/2)) + (b^2*d
^2*n*log(c*(d + e/x^(1/2))^n))/(6*e^2*x^2) + (b^2*d^4*n*log(c*(d + e/x^(1/2
))^n))/(3*e^4*x) - (2*b^2*d^3*n*log(c*(d + e/x^(1/2))^n))/(9*e^3*x^(3/2)) -
 (2*b^2*d^5*n*log(c*(d + e/x^(1/2))^n))/(3*e^5*x^(1/2)) - (2*a*b*d*n)/(15*e
*x^(5/2)) + (2*a*b*d^6*n*log(d + e/x^(1/2)))/(3*e^6) - (2*b^2*d*n*log(c*(d
+ e/x^(1/2))^n))/(15*e*x^(5/2)) + (a*b*d^2*n)/(6*e^2*x^2) + (a*b*d^4*n)/(3*
e^4*x) - (2*a*b*d^3*n)/(9*e^3*x^(3/2)) - (2*a*b*d^5*n)/(3*e^5*x^(1/2))
```

3.436 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=569

$$\frac{b^3e^3n^3\sqrt{x}}{2d^3}-\frac{b^3e^4n^3\log\left(d+\frac{e}{\sqrt{x}}\right)}{2d^4}-\frac{5b^2e^3n^2\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^4}+\frac{b^2e^2n^2x\left(a+\right.}{}$$

```
[Out] -3/2*b^3*e^4*n^3*ln(x)/d^4-1/2*b^3*e^4*n^3*ln(d+e/x^(1/2))/d^4+1/2*b^2*e^2*
n^2*x*(a+b*ln(c*(d+e/x^(1/2))^n))/d^2-5/2*b^2*e^4*n^2*ln(1-d/(d+e/x^(1/2)))
*(a+b*ln(c*(d+e/x^(1/2))^n))/d^4-3/4*b*e^2*n*x*(a+b*ln(c*(d+e/x^(1/2))^n))^
2/d^2+1/2*b*e*n*x^(3/2)*(a+b*ln(c*(d+e/x^(1/2))^n))^2/d+3/2*b*e^4*n*ln(1-d/
(d+e/x^(1/2)))*(a+b*ln(c*(d+e/x^(1/2))^n))^2/d^4+1/2*x^2*(a+b*ln(c*(d+e/x^(
1/2))^n))^3-3*b^2*e^4*n^2*(a+b*ln(c*(d+e/x^(1/2))^n))*ln(-e/d/x^(1/2))/d^4+
5/2*b^3*e^4*n^3*polylog(2,d/(d+e/x^(1/2)))/d^4-3*b^2*e^4*n^2*(a+b*ln(c*(d+e
/x^(1/2))^n))*polylog(2,d/(d+e/x^(1/2)))/d^4-3*b^3*e^4*n^3*polylog(2,1+e/d/
x^(1/2))/d^4-3*b^3*e^4*n^3*polylog(3,d/(d+e/x^(1/2)))/d^4+1/2*b^3*e^3*n^3*x
^(1/2)/d^3-5/2*b^2*e^3*n^2*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))*x^(1/2
)/d^4+3/2*b*e^3*n*(a+b*ln(c*(d+e/x^(1/2))^n))^2*(d+e/x^(1/2))*x^(1/2)/d^4
```

Rubi [A]
time = 0.87, antiderivative size = 569, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {2504, 2445, 2458, 2389, 2379, 2421, 6724, 2355, 2354, 2438, 2356, 2351, 31, 46}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e/Sqrt[x])^n])^3,x]
```

```
[Out] (b^3*e^3*n^3*Sqrt[x])/(2*d^3) - (b^3*e^4*n^3*Log[d + e/Sqrt[x]])/(2*d^4) -
(5*b^2*e^3*n^2*(d + e/Sqrt[x])*Sqrt[x]*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(2
*d^4) + (b^2*e^2*n^2*x*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(2*d^2) - (5*b^2*e
^4*n^2*Log[1 - d/(d + e/Sqrt[x])]*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(2*d^4)
 + (3*b*e^3*n*(d + e/Sqrt[x])*Sqrt[x]*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(
2*d^4) - (3*b*e^2*n*x*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(4*d^2) + (b*e*n*
x^(3/2)*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(2*d) + (3*b*e^4*n*Log[1 - d/(d
 + e/Sqrt[x])]*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(2*d^4) + (x^2*(a + b*Lo
g[c*(d + e/Sqrt[x])^n])^3)/2 - (3*b^2*e^4*n^2*(a + b*Log[c*(d + e/Sqrt[x])^
n])*Log[-(e/(d*Sqrt[x]))])/d^4 - (3*b^3*e^4*n^3*Log[x])/(2*d^4) + (5*b^3*e^
4*n^3*PolyLog[2, d/(d + e/Sqrt[x])])/(2*d^4) - (3*b^2*e^4*n^2*(a + b*Log[c*
```

```
(d + e/Sqrt[x])^n])*PolyLog[2, d/(d + e/Sqrt[x])])/d^4 - (3*b^3*e^4*n^3*Pol
yLog[2, 1 + e/(d*Sqrt[x])])/d^4 - (3*b^3*e^4*n^3*PolyLog[3, d/(d + e/Sqrt[x
])])/d^4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 dx = -\left(2\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x^5}dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 - \frac{1}{2}(3ben)\text{Subst}\left(\int \frac{(a+b\log}{x^4}\right.$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 - \frac{1}{2}(3bn)\text{Subst}\left(\int \frac{(a+b\log}{x\left(-\frac{d}{e}+\right.}\right.$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 - \frac{(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\right.}{2d}$$

$$= \frac{benx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2d} + \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\right.\right.\right.$$

$$= -\frac{3be^2nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{4d^2} + \frac{benx^{3/2}\left(a+b\log\left(c\left(\right.\right.\right.}{2d}$$

$$= \frac{b^2e^2n^2x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^2} + \frac{3be^3n\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(\right.}{}$$

$$= -\frac{5b^2e^3n^2\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^4} + \frac{b^2e^2n}{}$$

$$= \frac{b^3e^3n^3\sqrt{x}}{2d^3} - \frac{b^3e^4n^3\log\left(d+\frac{e}{\sqrt{x}}\right)}{2d^4} - \frac{5b^2e^3n^2\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(\right.}{}$$

$$= \frac{b^3e^3n^3\sqrt{x}}{2d^3} - \frac{b^3e^4n^3\log\left(d+\frac{e}{\sqrt{x}}\right)}{2d^4} - \frac{5b^2e^3n^2\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(\right.}{}$$

Mathematica [A]
time = 0.61, size = 777, normalized size = 1.37

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])^n])^3,x]
```

```
[Out] (6*b*d*e^3*n*Sqrt[x]*(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x])^
n])^2 - 3*b*d^2*e^2*n*x*(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x
])^n])^2 + 2*b*d^3*e*n*x^(3/2)*(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e
/Sqrt[x])^n])^2 + 6*b*d^4*n*x^2*Log[d + e/Sqrt[x]]*(a - b*n*Log[d + e/Sqrt[
x]] + b*Log[c*(d + e/Sqrt[x])^n])^2 + 2*d^4*x^2*(a - b*n*Log[d + e/Sqrt[x]]
 + b*Log[c*(d + e/Sqrt[x])^n])^3 - 6*b*e^4*n*(a - b*n*Log[d + e/Sqrt[x]] +
b*Log[c*(d + e/Sqrt[x])^n])^2*Log[e + d*Sqrt[x]] - 2*b^2*n^2*(a - b*n*Log[d
 + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x])^n])*(3*(e^4 - d^4*x^2)*Log[d + e/Sq
rt[x]]^2 + e^2*(5*d*e*Sqrt[x] - d^2*x + 11*e^2*Log[-(e/(d*Sqrt[x]))]) - e*L
og[d + e/Sqrt[x]]*(11*e^3 + 6*d*e^2*Sqrt[x] - 3*d^2*e*x + 2*d^3*x^(3/2) + 6
*e^3*Log[-(e/(d*Sqrt[x]))]) - 6*e^4*PolyLog[2, 1 + e/(d*Sqrt[x])]) + b^3*n^
3*(d^2*e^2*x*(2 - 3*Log[d + e/Sqrt[x]])*Log[d + e/Sqrt[x]] + 2*d^3*e*x^(3/2
)*Log[d + e/Sqrt[x]]^2 + 2*d^4*x^2*Log[d + e/Sqrt[x]]^3 + 2*d*e^3*Sqrt[x]*(
1 - 5*Log[d + e/Sqrt[x]] + 3*Log[d + e/Sqrt[x]]^2) + 12*e^4*(-Log[d + e/Sqr
t[x]] + Log[-(e/(d*Sqrt[x]))]) + 11*e^4*(Log[d + e/Sqrt[x]]*(Log[d + e/Sqrt
[x]] - 2*Log[-(e/(d*Sqrt[x]))]) - 2*PolyLog[2, 1 + e/(d*Sqrt[x])]) - 2*e^4*
(Log[d + e/Sqrt[x]]^2*(Log[d + e/Sqrt[x]] - 3*Log[-(e/(d*Sqrt[x]))]) - 6*Lo
g[d + e/Sqrt[x]]*PolyLog[2, 1 + e/(d*Sqrt[x])] + 6*PolyLog[3, 1 + e/(d*Sqrt
[x])])))/(4*d^4)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(1/2))^n))^3,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e/x^(1/2))^n))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*n^3*x^2*log(d*sqrt(x) + e)^3 - integrate(1/4*(3*(b^3*d*n*x^2 - 4*(b
^3*log(c) + a*b^2)*x^(3/2)*e - 4*(b^3*d*log(c) + a*b^2*d)*x^2 + 4*(b^3*d*x^
2 + b^3*x^(3/2)*e)*log(x^(1/2*n)))*n^2*log(d*sqrt(x) + e)^2 + 4*(b^3*d*x^2
+ b^3*x^(3/2)*e)*log(x^(1/2*n))^3 - 4*(b^3*log(c)^3 + 3*a*b^2*log(c)^2 + 3*
a^2*b*log(c) + a^3)*x^(3/2)*e - 4*(b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*
a^2*b*d*log(c) + a^3*d)*x^2 - 12*((b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*b)*x
^(3/2)*e + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x^2 + (b^3*d*x^2 +
```

```
 b^3*x^(3/2)*e)*log(x^(1/2*n))^2 - 2*((b^3*log(c) + a*b^2)*x^(3/2)*e + (b^3
*d*log(c) + a*b^2*d)*x^2)*log(x^(1/2*n)))*n*log(d*sqrt(x) + e) - 12*((b^3*l
og(c) + a*b^2)*x^(3/2)*e + (b^3*d*log(c) + a*b^2*d)*x^2)*log(x^(1/2*n))^2 +
 12*((b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*b)*x^(3/2)*e + (b^3*d*log(c)^2 +
2*a*b^2*d*log(c) + a^2*b*d)*x^2)*log(x^(1/2*n)))/(d*x + sqrt(x)*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*log(c*((d*x + sqrt(x)*e)/x)^n)^3 + 3*a*b^2*x*log(c*((d*x + s
qrt(x)*e)/x)^n)^2 + 3*a^2*b*x*log(c*((d*x + sqrt(x)*e)/x)^n) + a^3*x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/2))**n))**3,x)
```

```
[Out] Integral(x*(a + b*log(c*(d + e/sqrt(x))**n))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^3*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,{\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{\sqrt{x}}\right)}^{n}\right)\right)}^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e/x^(1/2))^n))^3,x)
```

```
[Out] int(x*(a + b*log(c*(d + e/x^(1/2))^n))^3, x)
```

3.437 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=260

$$\frac{3ben\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{d^2}+\frac{3be^2n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{d^2}$$

```
[Out] 3*b*e^2*n*ln(1-d/(d+e/x^(1/2)))*(a+b*ln(c*(d+e/x^(1/2))^n))^2/d^2+x*(a+b*ln
(c*(d+e/x^(1/2))^n))^3-6*b^2*e^2*n^2*(a+b*ln(c*(d+e/x^(1/2))^n))*ln(-e/d/x^
(1/2))/d^2-6*b^2*e^2*n^2*(a+b*ln(c*(d+e/x^(1/2))^n))*polylog(2,d/(d+e/x^(1/
2)))/d^2-6*b^3*e^2*n^3*polylog(2,1+e/d/x^(1/2))/d^2-6*b^3*e^2*n^3*polylog(3
,d/(d+e/x^(1/2)))/d^2+3*b*e*n*(a+b*ln(c*(d+e/x^(1/2))^n))^2*(d+e/x^(1/2))*x
^(1/2)/d^2
```

Rubi [A]
time = 0.34, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {2501, 2504, 2445, 2458, 2389, 2379, 2421, 6724, 2355, 2354, 2438}

$$-\frac{6b^2e^2n^2\text{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2}-\frac{6b^3e^2n^3\text{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)}{d^2}-\frac{6b^3e^2n^3\text{PolyLog}\left(3,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{d^2}-\frac{6b^2e^2n^2\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2}+\frac{3be^2n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{d^2}+\frac{3ben\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{d^2}+x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^3,x]
```

```
[Out] (3*b*e*n*(d + e/Sqrt[x])*Sqrt[x]*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/d^2 +
(3*b*e^2*n*Log[1 - d/(d + e/Sqrt[x])]*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/d
^2 + x*(a + b*Log[c*(d + e/Sqrt[x])^n])^3 - (6*b^2*e^2*n^2*(a + b*Log[c*(d
+ e/Sqrt[x])^n])*Log[-(e/(d*Sqrt[x]))])/d^2 - (6*b^2*e^2*n^2*(a + b*Log[c*(
d + e/Sqrt[x])^n])*PolyLog[2, d/(d + e/Sqrt[x])])/d^2 - (6*b^3*e^2*n^3*Poly
Log[2, 1 + e/(d*Sqrt[x])])/d^2 - (6*b^3*e^2*n^3*PolyLog[3, d/(d + e/Sqrt[x]
)])/d^2
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
```

```
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2501

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 dx = 2\text{Subst}\left(\int x\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^n\right)\right)^3 dx, x, \sqrt{x}\right)$$

$$= -\left(2\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x^3} dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 - (3ben)\text{Subst}\left(\int \frac{(a+b\log(c(d}{x^2(d+e}\right.$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 - (3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\right.$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 - \frac{(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2} dx,\right.}{d}$$

$$= \frac{3ben\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{d^2} + x\left(a+b\log\right.$$

$$= \frac{3ben\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{d^2} + x\left(a+b\log\right.$$

$$= \frac{3ben\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{d^2} - \frac{e^2\left(a+b\log\right.}{}$$

$$= \frac{3ben\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{d^2} - \frac{e^2\left(a+b\log\right.}{}$$

Mathematica [A]
time = 0.43, size = 476, normalized size = 1.83

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^3,x]
```

```
[Out] (3*b*d*e*n*Sqrt[x]*(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x])^n]
)^2 + 3*b*d^2*n*x*Log[d + e/Sqrt[x]]*(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*
(d + e/Sqrt[x])^n])^2 + d^2*x*(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/
Sqrt[x])^n])^3 - 3*b*e^2*n*(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqr
```

```
t[x])^n])^2*Log[e + d*Sqrt[x]] + 3*b^2*n^2*(a - b*n*Log[d + e/Sqrt[x]] + b*
Log[c*(d + e/Sqrt[x])^n])*((-e^2 + d^2*x)*Log[d + e/Sqrt[x]]^2 - 2*e^2*Log[
-(e/(d*Sqrt[x]))] + 2*e*Log[d + e/Sqrt[x]]*(e + d*Sqrt[x] + e*Log[-(e/(d*Sq
rt[x]))]) + 2*e^2*PolyLog[2, 1 + e/(d*Sqrt[x])]) + b^3*n^3*(Log[d + e/Sqrt[
x]]*((-e^2 + d^2*x)*Log[d + e/Sqrt[x]]^2 - 6*e^2*Log[-(e/(d*Sqrt[x]))] + 3*
e*Log[d + e/Sqrt[x]]*(e + d*Sqrt[x] + e*Log[-(e/(d*Sqrt[x]))])) + 6*e^2*(-1
 + Log[d + e/Sqrt[x]])*PolyLog[2, 1 + e/(d*Sqrt[x])] - 6*e^2*PolyLog[3, 1 +
 e/(d*Sqrt[x])]))/d^2
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3,x, algorithm="maxima")
```

```
[Out] b^3*n^3*x*log(d*sqrt(x) + e)^3 - 3*(n*(e*log(d*sqrt(x) + e)/d^2 - sqrt(x)/d
)*e - x*log(c*(d + e/sqrt(x))^n))*a^2*b + a^3*x - integrate(1/2*(3*(b^3*d*n
*x - 2*(b^3*log(c) + a*b^2)*sqrt(x)*e - 2*(b^3*d*log(c) + a*b^2*d)*x + 2*(b
^3*d*x + b^3*sqrt(x)*e)*log(x^(1/2*n)))*n^2*log(d*sqrt(x) + e)^2 + 2*(b^3*d
*x + b^3*sqrt(x)*e)*log(x^(1/2*n))^3 - 6*((b^3*d*x + b^3*sqrt(x)*e)*log(x^(
1/2*n))^2 + (b^3*log(c)^2 + 2*a*b^2*log(c))*sqrt(x)*e + (b^3*d*log(c)^2 + 2
*a*b^2*d*log(c))*x - 2*((b^3*log(c) + a*b^2)*sqrt(x)*e + (b^3*d*log(c) + a*
b^2*d)*x)*log(x^(1/2*n)))*n*log(d*sqrt(x) + e) - 6*((b^3*log(c) + a*b^2)*sq
rt(x)*e + (b^3*d*log(c) + a*b^2*d)*x)*log(x^(1/2*n))^2 - 2*(b^3*log(c)^3 +
3*a*b^2*log(c)^2)*sqrt(x)*e - 2*(b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2)*x + 6
*((b^3*log(c)^2 + 2*a*b^2*log(c))*sqrt(x)*e + (b^3*d*log(c)^2 + 2*a*b^2*d*l
og(c))*x)*log(x^(1/2*n)))/(d*x + sqrt(x)*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*log(c*((d*x + sqrt(x)*e)/x)^n)^3 + 3*a*b^2*log(c*((d*x + sqrt(
x)*e)/x)^n)^2 + 3*a^2*b*log(c*((d*x + sqrt(x)*e)/x)^n) + a^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\log\left(c\left(d + \frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**3,x)
```

```
[Out] Integral((a + b*log(c*(d + e/sqrt(x))**n))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{\sqrt{x}}\right)}^n\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))^3,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(1/2))^n))^3, x)
```

3.438 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x}\,dx$

Optimal. Leaf size=135

$$-2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt{x}}\right)-6bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\text{Li}_2\left(1+\frac{e}{d\sqrt{x}}\right)+1$$

```
[Out] -2*(a+b*ln(c*(d+e/x^(1/2))^n))^3*ln(-e/d/x^(1/2))-6*b*n*(a+b*ln(c*(d+e/x^(1
/2))^n))^2*polylog(2,1+e/d/x^(1/2))+12*b^2*n^2*(a+b*ln(c*(d+e/x^(1/2))^n))*
polylog(3,1+e/d/x^(1/2))-12*b^3*n^3*polylog(4,1+e/d/x^(1/2))
```

Rubi [A]
time = 0.13, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2504, 2443, 2481, 2421, 2430, 6724}

$$12b^2n^2\text{PolyLog}\left(3,\frac{e}{d\sqrt{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-6bn\text{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2-12b^3n^3\text{PolyLog}\left(4,\frac{e}{d\sqrt{x}}+1\right)-2\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x,x]
```

```
[Out] -2*(a + b*Log[c*(d + e/Sqrt[x])^n])^3*Log[-(e/(d*Sqrt[x]))] - 6*b*n*(a + b*
Log[c*(d + e/Sqrt[x])^n])^2*PolyLog[2, 1 + e/(d*Sqrt[x])] + 12*b^2*n^2*(a +
 b*Log[c*(d + e/Sqrt[x])^n])*PolyLog[3, 1 + e/(d*Sqrt[x])] - 12*b^3*n^3*Pol
yLog[4, 1 + e/(d*Sqrt[x])]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
```

```
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x}\,dx = -\left(2\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt{x}}\right) + (6ben)\text{Subst}\Bigg($$

$$= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt{x}}\right) + (6bn)\text{Subst}\Bigg($$

$$= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt{x}}\right) - 6bn\Big(a+b\log$$

$$= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt{x}}\right) - 6bn\Big(a+b\log$$

$$= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt{x}}\right) - 6bn\Big(a+b\log$$

<u>**Mathematica**</u> [**B**] Leaf count is larger than twice the leaf count of optimal. 532 vs. 2(135) = 270.
time = 0.21, size = 532, normalized size = 3.94

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x,x]
```

```
[Out] (a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x])^n])^3*Log[x] + 3*b*n*
(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x])^n])^2*((Log[d + e/Sqr
t[x]] - Log[1 + e/(d*Sqrt[x])])*Log[x] + 2*PolyLog[2, -(e/(d*Sqrt[x]))]) +
6*b^2*n^2*(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x])^n])*(Log[e/
d + Sqrt[x]]^2*Log[-((d*Sqrt[x])/e)] + (Log[d + e/Sqrt[x]]^2*Log[x])/2 - (L
og[e/d + Sqrt[x]]^2*Log[x])/2 - Log[d + e/Sqrt[x]]*Log[1 + (d*Sqrt[x])/e]*L
og[x] + Log[e/d + Sqrt[x]]*Log[1 + (d*Sqrt[x])/e]*Log[x] + (Log[d + e/Sqrt[
x]]*Log[x]^2)/4 - (Log[1 + (d*Sqrt[x])/e]*Log[x]^2)/4 + Log[x]^3/24 + 2*Log
[e/d + Sqrt[x]]*PolyLog[2, 1 + (d*Sqrt[x])/e] - 2*(Log[d + e/Sqrt[x]] - Log
[e/d + Sqrt[x]])*PolyLog[2, -((d*Sqrt[x])/e)] - 2*PolyLog[3, 1 + (d*Sqrt[x]
)/e] - 2*PolyLog[3, -((d*Sqrt[x])/e)]) - 2*b^3*n^3*(Log[d + e/Sqrt[x]]^3*Lo
g[-(e/(d*Sqrt[x]))] + 3*Log[d + e/Sqrt[x]]^2*PolyLog[2, 1 + e/(d*Sqrt[x])]
- 6*Log[d + e/Sqrt[x]]*PolyLog[3, 1 + e/(d*Sqrt[x])] + 6*PolyLog[4, 1 + e/(
d*Sqrt[x])])
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))^3/x,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))^3/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x,x, algorithm="maxima")
```

```
[Out] b^3*n^3*log(d*sqrt(x) + e)^3*log(x) - integrate(1/2*(3*(b^3*d*n*x*log(x) -
2*(b^3*log(c) + a*b^2)*sqrt(x)*e - 2*(b^3*d*log(c) + a*b^2*d)*x + 2*(b^3*d*
x + b^3*sqrt(x)*e)*log(x^(1/2*n)))*n^2*log(d*sqrt(x) + e)^2 + 2*(b^3*d*x +
b^3*sqrt(x)*e)*log(x^(1/2*n))^3 - 6*((b^3*d*x + b^3*sqrt(x)*e)*log(x^(1/2*n
))^2 + (b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*b)*sqrt(x)*e + (b^3*d*log(c)^2
+ 2*a*b^2*d*log(c) + a^2*b*d)*x - 2*((b^3*log(c) + a*b^2)*sqrt(x)*e + (b^3*
d*log(c) + a*b^2*d)*x)*log(x^(1/2*n)))*n*log(d*sqrt(x) + e) - 6*((b^3*log(c
) + a*b^2)*sqrt(x)*e + (b^3*d*log(c) + a*b^2*d)*x)*log(x^(1/2*n))^2 - 2*(b^
3*log(c)^3 + 3*a*b^2*log(c)^2 + 3*a^2*b*log(c) + a^3)*sqrt(x)*e - 2*(b^3*d*
log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x + 6*((b^3*log(c
)^2 + 2*a*b^2*log(c) + a^2*b)*sqrt(x)*e + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c
) + a^2*b*d)*x)*log(x^(1/2*n)))/(d*x^2 + x^(3/2)*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*((d*x + sqrt(x)*e)/x)^n)^3 + 3*a*b^2*log(c*((d*x + sqrt
(x)*e)/x)^n)^2 + 3*a^2*b*log(c*((d*x + sqrt(x)*e)/x)^n) + a^3)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \log \left(c \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**3/x,x)
[Out] Integral((a + b*log(c*(d + e/sqrt(x))**n))**3/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^3/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b \ln \left(c \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))^3/x,x)
[Out] int((a + b*log(c*(d + e/x^(1/2))^n))^3/x, x)
```

3.439 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^2}\,dx$

Optimal. Leaf size=285

$$\frac{3b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^2}+\frac{12ab^2dn^2}{e\sqrt{x}}-\frac{12b^3dn^3}{e\sqrt{x}}+\frac{12b^3dn^2\left(d+\frac{e}{\sqrt{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{e^2}-\frac{3b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(\right.}{}$$

```
[Out] 12*b^3*d*n^2*ln(c*(d+e/x^(1/2))^n)*(d+e/x^(1/2))/e^2-6*b*d*n*(a+b*ln(c*(d+e
/x^(1/2))^n))^2*(d+e/x^(1/2))/e^2+2*d*(a+b*ln(c*(d+e/x^(1/2))^n))^3*(d+e/x^
(1/2))/e^2+3/4*b^3*n^3*(d+e/x^(1/2))^2/e^2-3/2*b^2*n^2*(a+b*ln(c*(d+e/x^(1/
2))^n))*(d+e/x^(1/2))^2/e^2+3/2*b*n*(a+b*ln(c*(d+e/x^(1/2))^n))^2*(d+e/x^(1
/2))^2/e^2-(a+b*ln(c*(d+e/x^(1/2))^n))^3*(d+e/x^(1/2))^2/e^2+12*a*b^2*d*n^2
/e/x^(1/2)-12*b^3*d*n^3/e/x^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

$$-\frac{3b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^2}+\frac{12ab^2dn^2}{e\sqrt{x}}+\frac{3bn\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2e^2}-\frac{6bdn\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^2}-\frac{\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^2}+\frac{2d\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^2}+\frac{12b^3dn^2\left(d+\frac{e}{\sqrt{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{e^2}+\frac{3b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^2}-\frac{12b^3dn^3}{e\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x^2,x]
```

```
[Out] (3*b^3*n^3*(d + e/Sqrt[x])^2)/(4*e^2) + (12*a*b^2*d*n^2)/(e*Sqrt[x]) - (12*
b^3*d*n^3)/(e*Sqrt[x]) + (12*b^3*d*n^2*(d + e/Sqrt[x])*Log[c*(d + e/Sqrt[x]
)^n])/e^2 - (3*b^2*n^2*(d + e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n]))/
(2*e^2) - (6*b*d*n*(d + e/Sqrt[x])*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/e^2
+ (3*b*n*(d + e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(2*e^2) + (2
*d*(d + e/Sqrt[x])*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/e^2 - ((d + e/Sqrt[x
])^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/e^2
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^2}\,dx = -\left(2\text{Subst}\left(\int x(a+b\log\left(c(d+ex)^n\right))^3\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\text{Subst}\left(\int \left(-\frac{d(a+b\log\left(c(d+ex)^n\right))^3}{e} + \frac{(d+ex)\left(a+b\log\right(}{e}\right.\right.\right.$$

$$= -\frac{2\text{Subst}\left(\int (d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx, x, \frac{1}{\sqrt{x}}\right)}{e} + \frac{(2d)\text{S}}{}$$

$$= -\frac{2\text{Subst}\left(\int x(a+b\log\left(cx^n\right))^3\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^2} + \frac{(2d)\text{Subst}\left(\int (a\right.}{}$$

$$= \frac{2d\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^2} - \frac{\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a\right.}{}$$

$$= -\frac{6bdn\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^2} + \frac{3bn\left(d+\frac{e}{\sqrt{x}}\right.}{}$$

$$= \frac{3b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^2} + \frac{12ab^2dn^2}{e\sqrt{x}} - \frac{3b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\right.\right.}{2e^2}$$

$$= \frac{3b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^2} + \frac{12ab^2dn^2}{e\sqrt{x}} - \frac{12b^3dn^3}{e\sqrt{x}} + \frac{12b^3dn^2\left(d+\frac{e}{\sqrt{x}}\right)\text{l}}{e^2}$$

Mathematica [A]
time = 0.37, size = 558, normalized size = 1.96

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x^2,x]
```

```
[Out] (-4*a^3*e^2 + 6*a^2*b*e^2*n - 6*a*b^2*e^2*n^2 + 3*b^3*e^2*n^3 - 12*a^2*b*d*
e*n*Sqrt[x] + 36*a*b^2*d*e*n^2*Sqrt[x] - 42*b^3*d*e*n^3*Sqrt[x] - 8*b^3*d^2
*n^3*x*Log[d + e/Sqrt[x]]^3 - 4*b^3*e^2*Log[c*(d + e/Sqrt[x])^n]^3 + 12*a^2
*b*d^2*n*x*Log[e + d*Sqrt[x]] - 36*a*b^2*d^2*n^2*x*Log[e + d*Sqrt[x]] + 42*
b^3*d^2*n^3*x*Log[e + d*Sqrt[x]] + 6*b^2*d^2*n^2*x*Log[d + e/Sqrt[x]]*(-2*a
 + 3*b*n - 2*b*Log[c*(d + e/Sqrt[x])^n])*(2*Log[e + d*Sqrt[x]] - Log[x]) -
6*a^2*b*d^2*n*x*Log[x] + 18*a*b^2*d^2*n^2*x*Log[x] - 21*b^3*d^2*n^3*x*Log[x
```

```
] + 6*b^2*d^2*n^2*x*Log[d + e/Sqrt[x]]^2*(2*a - 3*b*n + 2*b*Log[c*(d + e/Sq
rt[x])^n] + 2*b*n*Log[e + d*Sqrt[x]] - b*n*Log[x]) + 6*b^2*Log[c*(d + e/Sqr
t[x])^n]^2*(e*(-2*a*e + b*n*(e - 2*d*Sqrt[x])) + 2*b*d^2*n*x*Log[e + d*Sqrt
[x]] - b*d^2*n*x*Log[x]) - 6*b*Log[c*(d + e/Sqrt[x])^n]*(e*(2*a^2*e + b^2*n
^2*(e - 6*d*Sqrt[x]) - 2*a*b*n*(e - 2*d*Sqrt[x])) + 2*b*d^2*n*(-2*a + 3*b*n
)*x*Log[e + d*Sqrt[x]] + b*d^2*n*(2*a - 3*b*n)*x*Log[x]))/(4*e^2*x)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))^3/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))^3/x^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 579 vs. 2(253) = 506.
time = 0.33, size = 579, normalized size = 2.03

$$\frac{3}{2}\left(2\,d^{2}e^{(-3)}\log\left(d\sqrt{x}+e\right)-d^{2}e^{(-3)}\log\left(x\right)-\frac{\left(2\,d\sqrt{x}-e\right)e^{(-2)}}{x}\right)a^{2}bne-\frac{b^{3}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)^{3}}{x}+\frac{3}{4}\left(4\left(2\,d^{2}e^{(-3)}\log\left(d\sqrt{x}+e\right)-d^{2}e^{(-3)}\log\left(x\right)-\frac{\left(2\,d\sqrt{x}-e\right)e^{(-2)}}{x}\right)ne\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)-\frac{\left(4\,d^{2}x\log\left(d\sqrt{x}+e\right)^{2}+d^{2}x\log\left(x\right)^{2}-6\,d^{2}x\log\left(x\right)-12\,d\sqrt{x}e-4\left(d^{2}x\log\left(x\right)-3\,d^{2}x\right)\log\left(d\sqrt{x}+e\right)+2\,e^{2}\right)n^{2}e^{(-2)}}{x}\right)ab^{2}+\frac{1}{8}\left(12\left(2\,d^{2}e^{(-3)}\log\left(d\sqrt{x}+e\right)-d^{2}e^{(-3)}\log\left(x\right)-\frac{\left(2\,d\sqrt{x}-e\right)e^{(-2)}}{x}\right)ne\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)^{2}+\left(\frac{\left(8\,d^{2}x\log\left(d\sqrt{x}+e\right)^{3}-d^{2}x\log\left(x\right)^{3}+9\,d^{2}x\log\left(x\right)^{2}-42\,d^{2}x\log\left(x\right)-12\left(d^{2}x\log\left(x\right)-3\,d^{2}x\right)\log\left(d\sqrt{x}+e\right)^{2}-84\,d\sqrt{x}e+6\left(d^{2}x\log\left(x\right)^{2}-6\,d^{2}x\log\left(x\right)+14\,d^{2}x\right)\log\left(d\sqrt{x}+e\right)+6\,e^{2}\right)n^{2}e^{(-3)}}{x}-\frac{6\left(4\,d^{2}x\log\left(d\sqrt{x}+e\right)^{2}+d^{2}x\log\left(x\right)^{2}-6\,d^{2}x\log\left(x\right)-12\,d\sqrt{x}e-4\left(d^{2}x\log\left(x\right)-3\,d^{2}x\right)\log\left(d\sqrt{x}+e\right)+2\,e^{2}\right)ne^{(-3)}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)}{x}\right)ne\right)b^{3}-\frac{3\,ab^{2}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)^{2}}{x}-\frac{3\,a^{2}b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)}{x}-\frac{a^{3}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^2,x, algorithm="maxima")
```

```
[Out] 3/2*(2*d^2*e^(-3)*log(d*sqrt(x) + e) - d^2*e^(-3)*log(x) - (2*d*sqrt(x) - e
)*e^(-2)/x)*a^2*b*n*e - b^3*log(c*(d + e/sqrt(x))^n)^3/x + 3/4*(4*(2*d^2*e^
(-3)*log(d*sqrt(x) + e) - d^2*e^(-3)*log(x) - (2*d*sqrt(x) - e)*e^(-2)/x)*n
*e*log(c*(d + e/sqrt(x))^n) - (4*d^2*x*log(d*sqrt(x) + e)^2 + d^2*x*log(x)^
2 - 6*d^2*x*log(x) - 12*d*sqrt(x)*e - 4*(d^2*x*log(x) - 3*d^2*x)*log(d*sqrt
(x) + e) + 2*e^2)*n^2*e^(-2)/x)*a*b^2 + 1/8*(12*(2*d^2*e^(-3)*log(d*sqrt(x)
 + e) - d^2*e^(-3)*log(x) - (2*d*sqrt(x) - e)*e^(-2)/x)*n*e*log(c*(d + e/sq
rt(x))^n)^2 + ((8*d^2*x*log(d*sqrt(x) + e)^3 - d^2*x*log(x)^3 + 9*d^2*x*log
(x)^2 - 42*d^2*x*log(x) - 12*(d^2*x*log(x) - 3*d^2*x)*log(d*sqrt(x) + e)^2
- 84*d*sqrt(x)*e + 6*(d^2*x*log(x)^2 - 6*d^2*x*log(x) + 14*d^2*x)*log(d*sqr
t(x) + e) + 6*e^2)*n^2*e^(-3)/x - 6*(4*d^2*x*log(d*sqrt(x) + e)^2 + d^2*x*l
og(x)^2 - 6*d^2*x*log(x) - 12*d*sqrt(x)*e - 4*(d^2*x*log(x) - 3*d^2*x)*log(
d*sqrt(x) + e) + 2*e^2)*n*e^(-3)*log(c*(d + e/sqrt(x))^n)/x)*n*e)*b^3 - 3*a
*b^2*log(c*(d + e/sqrt(x))^n)^2/x - 3*a^2*b*log(c*(d + e/sqrt(x))^n)/x - a^
3/x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 519 vs. 2(253) = 506.

time = 0.40, size = 519, normalized size = 1.82

$$-\frac{\left(4 b^3 e^2 \log(c)^3 - 6 (b^3 n - 2 a b^2) e^2 \log(c)^2 - 4 (b^3 d^2 n^3 x - b^3 n^3 e^2) \log\left(\frac{dx+\sqrt{x}\, e}{x}\right)^3 + 6 (b^3 n^2 - 2 a b^2 n + 2 a^2 b) e^2 \log(c) + 6 \left(2 b^3 d n^3 \sqrt{x}\, e + (3 b^3 d^2 n^3 - 2 a b^2 d^2 n^2) x - (b^3 n^3 - 2 a b^2 n^2) e^2 - 2 (b^3 d^2 n^2 x - b^3 n^2 e^2) \log(c)\right) \log\left(\frac{dx+\sqrt{x}\, e}{x}\right)^2 - (3 b^3 n^3 - 6 a b^2 n^2 + 6 a^2 b n - 4 a^3) e^2 - 6 \left(2 (b^3 d^2 n x - b^3 n e^2) \log(c)^2 + (7 b^3 d^2 n^3 - 6 a b^2 d^2 n^2 + 2 a^2 b d^2 n) x - (b^3 n^3 - 2 a b^2 n^2 + 2 a^2 b n) e^2 - 2 \left((3 b^3 d^2 n^2 - 2 a b^2 d^2 n) x - (b^3 n^2 - 2 a b^2 n) e^2\right) \log(c) - 2 \left(2 b^3 d n^2 e \log(c) - (3 b^3 d n^3 - 2 a b^2 d n^2) e\right) \sqrt{x}\right) \log\left(\frac{dx+\sqrt{x}\, e}{x}\right) + 6 \left(2 b^3 d n e \log(c)^2 - 2 (3 b^3 d n^2 - 2 a b^2 d n) e \log(c) + (7 b^3 d n^3 - 6 a b^2 d n^2 + 2 a^2 b d n) e\right) \sqrt{x}\right) e^{(-2)}}{4 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^3*e^2*log(c)^3 - 6*(b^3*n - 2*a*b^2)*e^2*log(c)^2 - 4*(b^3*d^2*n^
3*x - b^3*n^3*e^2)*log((d*x + sqrt(x)*e)/x)^3 + 6*(b^3*n^2 - 2*a*b^2*n + 2*
a^2*b)*e^2*log(c) + 6*(2*b^3*d*n^3*sqrt(x)*e + (3*b^3*d^2*n^3 - 2*a*b^2*d^2
*n^2)*x - (b^3*n^3 - 2*a*b^2*n^2)*e^2 - 2*(b^3*d^2*n^2*x - b^3*n^2*e^2)*log
(c))*log((d*x + sqrt(x)*e)/x)^2 - (3*b^3*n^3 - 6*a*b^2*n^2 + 6*a^2*b*n - 4*
a^3)*e^2 - 6*(2*(b^3*d^2*n*x - b^3*n*e^2)*log(c)^2 + (7*b^3*d^2*n^3 - 6*a*b
^2*d^2*n^2 + 2*a^2*b*d^2*n)*x - (b^3*n^3 - 2*a*b^2*n^2 + 2*a^2*b*n)*e^2 - 2
*((3*b^3*d^2*n^2 - 2*a*b^2*d^2*n)*x - (b^3*n^2 - 2*a*b^2*n)*e^2)*log(c) - 2
*(2*b^3*d*n^2*e*log(c) - (3*b^3*d*n^3 - 2*a*b^2*d*n^2)*e)*sqrt(x))*log((d*x
 + sqrt(x)*e)/x) + 6*(2*b^3*d*n*e*log(c)^2 - 2*(3*b^3*d*n^2 - 2*a*b^2*d*n)*
e*log(c) + (7*b^3*d*n^3 - 6*a*b^2*d*n^2 + 2*a^2*b*d*n)*e)*sqrt(x))*e^(-2)/x
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \log\left(c \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**3/x**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e/sqrt(x))**n))**3/x**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1127 vs. $2(253) = 506$.
time = 4.68, size = 1127, normalized size = 3.95

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^2,x, algorithm="giac")
```

```
[Out] 1/4*(8*(d*sqrt(x) + e)*b^3*d*n^3*log((d*sqrt(x) + e)/sqrt(x))^3/sqrt(x) - 4
*(d*sqrt(x) + e)^2*b^3*n^3*log((d*sqrt(x) + e)/sqrt(x))^3/x - 24*(d*sqrt(x)
 + e)*b^3*d*n^3*log((d*sqrt(x) + e)/sqrt(x))^2/sqrt(x) + 24*(d*sqrt(x) + e)
*b^3*d*n^2*log(c)*log((d*sqrt(x) + e)/sqrt(x))^2/sqrt(x) + 6*(d*sqrt(x) + e
)^2*b^3*n^3*log((d*sqrt(x) + e)/sqrt(x))^2/x - 12*(d*sqrt(x) + e)^2*b^3*n^2
```

```
*log(c)*log((d*sqrt(x) + e)/sqrt(x))^2/x + 48*(d*sqrt(x) + e)*b^3*d*n^3*log
((d*sqrt(x) + e)/sqrt(x))/sqrt(x) - 48*(d*sqrt(x) + e)*b^3*d*n^2*log(c)*log
((d*sqrt(x) + e)/sqrt(x))/sqrt(x) + 24*(d*sqrt(x) + e)*b^3*d*n*log(c)^2*log
((d*sqrt(x) + e)/sqrt(x))/sqrt(x) + 24*(d*sqrt(x) + e)*a*b^2*d*n^2*log((d*s
qrt(x) + e)/sqrt(x))^2/sqrt(x) - 6*(d*sqrt(x) + e)^2*b^3*n^3*log((d*sqrt(x)
 + e)/sqrt(x))/x + 12*(d*sqrt(x) + e)^2*b^3*n^2*log(c)*log((d*sqrt(x) + e)/
sqrt(x))/x - 12*(d*sqrt(x) + e)^2*b^3*n*log(c)^2*log((d*sqrt(x) + e)/sqrt(x
))/x - 12*(d*sqrt(x) + e)^2*a*b^2*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/x - 48
*(d*sqrt(x) + e)*b^3*d*n^3/sqrt(x) + 48*(d*sqrt(x) + e)*b^3*d*n^2*log(c)/sq
rt(x) - 24*(d*sqrt(x) + e)*b^3*d*n*log(c)^2/sqrt(x) + 8*(d*sqrt(x) + e)*b^3
*d*log(c)^3/sqrt(x) - 48*(d*sqrt(x) + e)*a*b^2*d*n^2*log((d*sqrt(x) + e)/sq
rt(x))/sqrt(x) + 48*(d*sqrt(x) + e)*a*b^2*d*n*log(c)*log((d*sqrt(x) + e)/sq
rt(x))/sqrt(x) + 3*(d*sqrt(x) + e)^2*b^3*n^3/x - 6*(d*sqrt(x) + e)^2*b^3*n^
2*log(c)/x + 6*(d*sqrt(x) + e)^2*b^3*n*log(c)^2/x - 4*(d*sqrt(x) + e)^2*b^3
*log(c)^3/x + 12*(d*sqrt(x) + e)^2*a*b^2*n^2*log((d*sqrt(x) + e)/sqrt(x))/x
 - 24*(d*sqrt(x) + e)^2*a*b^2*n*log(c)*log((d*sqrt(x) + e)/sqrt(x))/x + 48*
(d*sqrt(x) + e)*a*b^2*d*n^2/sqrt(x) - 48*(d*sqrt(x) + e)*a*b^2*d*n*log(c)/s
qrt(x) + 24*(d*sqrt(x) + e)*a*b^2*d*log(c)^2/sqrt(x) + 24*(d*sqrt(x) + e)*a
^2*b*d*n*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x) - 6*(d*sqrt(x) + e)^2*a*b^2*n
^2/x + 12*(d*sqrt(x) + e)^2*a*b^2*n*log(c)/x - 12*(d*sqrt(x) + e)^2*a*b^2*l
og(c)^2/x - 12*(d*sqrt(x) + e)^2*a^2*b*n*log((d*sqrt(x) + e)/sqrt(x))/x - 2
4*(d*sqrt(x) + e)*a^2*b*d*n/sqrt(x) + 24*(d*sqrt(x) + e)*a^2*b*d*log(c)/sqr
t(x) + 6*(d*sqrt(x) + e)^2*a^2*b*n/x - 12*(d*sqrt(x) + e)^2*a^2*b*log(c)/x
+ 8*(d*sqrt(x) + e)*a^3*d/sqrt(x) - 4*(d*sqrt(x) + e)^2*a^3/x)*e^(-2)
```

Mupad [B]
time = 0.62, size = 357, normalized size = 1.25

$$\frac{\frac{d\left(2a^3-3a^2bn+3ab^2n^2-\frac{3b^3n^3}{2}\right)}{e}-\frac{d\left(2a^3-6ab^2n^2+9b^3n^3\right)}{e}}{\sqrt{x}}-\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)^3\left(\frac{b^3}{x}-\frac{b^3d^2}{e^2}\right)+\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\left(\frac{\frac{3bd\left(2a^2-2abn+b^2n^2\right)}{e}-\frac{6bd\left(a^2-b^2n^2\right)}{e}}{\sqrt{x}}-\frac{3b\left(2a^2-2abn+b^2n^2\right)}{2x}\right)+\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)^2\left(\frac{\frac{3b^2d\left(2a-bn\right)}{e}-\frac{6ab^2d}{e}}{\sqrt{x}}-\frac{3b^2\left(2a-bn\right)}{2x}+\frac{3d\left(2ab^2d-3b^3dn\right)}{2e^2}\right)-\frac{a^3-\frac{3a^2bn}{2}+\frac{3ab^2n^2}{2}-\frac{3b^3n^3}{4}}{x}+\frac{\ln\left(d+\frac{e}{\sqrt{x}}\right)\left(6a^2bd^2n-18ab^2d^2n^2+21b^3d^2n^3\right)}{2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))^3/x^2,x)
```

```
[Out] ((d*(2*a^3 - (3*b^3*n^3)/2 + 3*a*b^2*n^2 - 3*a^2*b*n))/e - (d*(2*a^3 + 9*b^
3*n^3 - 6*a*b^2*n^2))/e)/x^(1/2) - log(c*(d + e/x^(1/2))^n)^3*(b^3/x - (b^3
*d^2)/e^2) + log(c*(d + e/x^(1/2))^n)*(((3*b*d*(2*a^2 + b^2*n^2 - 2*a*b*n))
/e - (6*b*d*(a^2 - b^2*n^2))/e)/x^(1/2) - (3*b*(2*a^2 + b^2*n^2 - 2*a*b*n))
/(2*x)) + log(c*(d + e/x^(1/2))^n)^2*(((3*b^2*d*(2*a - b*n))/e - (6*a*b^2*d
)/e)/x^(1/2) - (3*b^2*(2*a - b*n))/(2*x) + (3*d*(2*a*b^2*d - 3*b^3*d*n))/(2
*e^2)) - (a^3 - (3*b^3*n^3)/4 + (3*a*b^2*n^2)/2 - (3*a^2*b*n)/2)/x + (log(d
 + e/x^(1/2))*(21*b^3*d^2*n^3 - 18*a*b^2*d^2*n^2 + 6*a^2*b*d^2*n))/(2*e^2)
```

3.440 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^3}\,dx$

Optimal. Leaf size=595

$$\frac{9b^3d^2n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^4}-\frac{4b^3dn^3\left(d+\frac{e}{\sqrt{x}}\right)^3}{9e^4}+\frac{3b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^4}{64e^4}+\frac{12ab^2d^3n^2}{e^3\sqrt{x}}-\frac{12b^3d^3n^3}{e^3\sqrt{x}}+\frac{12b^3d^3n^2\left(d+\frac{e}{\sqrt{x}}\right.}{}$$

```
[Out] 12*b^3*d^3*n^2*ln(c*(d+e/x^(1/2))^n)*(d+e/x^(1/2))/e^4-6*b*d^3*n*(a+b*ln(c*
(d+e/x^(1/2))^n))^2*(d+e/x^(1/2))/e^4+2*d^3*(a+b*ln(c*(d+e/x^(1/2))^n))^3*(
d+e/x^(1/2))/e^4+9/4*b^3*d^2*n^3*(d+e/x^(1/2))^2/e^4-9/2*b^2*d^2*n^2*(a+b*l
n(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))^2/e^4+9/2*b*d^2*n*(a+b*ln(c*(d+e/x^(1/2
))^n))^2*(d+e/x^(1/2))^2/e^4-3*d^2*(a+b*ln(c*(d+e/x^(1/2))^n))^3*(d+e/x^(1/
2))^2/e^4-4/9*b^3*d*n^3*(d+e/x^(1/2))^3/e^4+4/3*b^2*d*n^2*(a+b*ln(c*(d+e/x^
(1/2))^n))*(d+e/x^(1/2))^3/e^4-2*b*d*n*(a+b*ln(c*(d+e/x^(1/2))^n))^2*(d+e/x
^(1/2))^3/e^4+2*d*(a+b*ln(c*(d+e/x^(1/2))^n))^3*(d+e/x^(1/2))^3/e^4+3/64*b^
3*n^3*(d+e/x^(1/2))^4/e^4-3/16*b^2*n^2*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(
1/2))^4/e^4+3/8*b*n*(a+b*ln(c*(d+e/x^(1/2))^n))^2*(d+e/x^(1/2))^4/e^4-1/2*(
a+b*ln(c*(d+e/x^(1/2))^n))^3*(d+e/x^(1/2))^4/e^4+12*a*b^2*d^3*n^2/e^3/x^(1/
2)-12*b^3*d^3*n^3/e^3/x^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 595, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x^3,x]
```

```
[Out] (9*b^3*d^2*n^3*(d + e/Sqrt[x])^2)/(4*e^4) - (4*b^3*d*n^3*(d + e/Sqrt[x])^3)
/(9*e^4) + (3*b^3*n^3*(d + e/Sqrt[x])^4)/(64*e^4) + (12*a*b^2*d^3*n^2)/(e^3
*Sqrt[x]) - (12*b^3*d^3*n^3)/(e^3*Sqrt[x]) + (12*b^3*d^3*n^2*(d + e/Sqrt[x]
)*Log[c*(d + e/Sqrt[x])^n])/e^4 - (9*b^2*d^2*n^2*(d + e/Sqrt[x])^2*(a + b*L
og[c*(d + e/Sqrt[x])^n]))/(2*e^4) + (4*b^2*d*n^2*(d + e/Sqrt[x])^3*(a + b*L
og[c*(d + e/Sqrt[x])^n]))/(3*e^4) - (3*b^2*n^2*(d + e/Sqrt[x])^4*(a + b*Log
[c*(d + e/Sqrt[x])^n]))/(16*e^4) - (6*b*d^3*n*(d + e/Sqrt[x])*(a + b*Log[c*
(d + e/Sqrt[x])^n])^2)/e^4 + (9*b*d^2*n*(d + e/Sqrt[x])^2*(a + b*Log[c*(d +
 e/Sqrt[x])^n])^2)/(2*e^4) - (2*b*d*n*(d + e/Sqrt[x])^3*(a + b*Log[c*(d + e
/Sqrt[x])^n])^2)/e^4 + (3*b*n*(d + e/Sqrt[x])^4*(a + b*Log[c*(d + e/Sqrt[x]
)^n])^2)/(8*e^4) + (2*d^3*(d + e/Sqrt[x])*(a + b*Log[c*(d + e/Sqrt[x])^n])^
```

```
3)/e^4 - (3*d^2*(d + e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/e^4 +
 (2*d*(d + e/Sqrt[x])^3*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/e^4 - ((d + e/S
qrt[x])^4*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/(2*e^4)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^3}\,dx = -\left(2\text{Subst}\left(\int x^3(a+b\log\left(c(d+ex)^n\right))^3\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\text{Subst}\left(\int\left(-\frac{d^3(a+b\log\left(c(d+ex)^n\right))^3}{e^3}+\frac{3d^2(d+ex)\left(a+b\right.}{e^3}\right.\right.\right.$$

$$= -\frac{2\text{Subst}\left(\int (d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx, x, \frac{1}{\sqrt{x}}\right)}{e^3}+\frac{(6d)\text{S}}{}$$

$$= -\frac{2\text{Subst}\left(\int x^3(a+b\log\left(cx^n\right))^3\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^4}+\frac{(6d)\text{Subst}\left(\int x\right.}{}$$

$$= \frac{2d^3\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^4}-\frac{3d^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{}$$

$$= -\frac{6bd^3n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^4}+\frac{9bd^2n\left(d+\right.}{}$$

$$= \frac{9b^3d^2n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^4}-\frac{4b^3dn^3\left(d+\frac{e}{\sqrt{x}}\right)^3}{9e^4}+\frac{3b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^4}{64e^4}$$

$$= \frac{9b^3d^2n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^4}-\frac{4b^3dn^3\left(d+\frac{e}{\sqrt{x}}\right)^3}{9e^4}+\frac{3b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^4}{64e^4}$$

Mathematica [A]
time = 0.64, size = 766, normalized size = 1.29

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x^3,x]
```

```
[Out] (-288*a^3*e^4 + 216*a^2*b*e^4*n - 108*a*b^2*e^4*n^2 + 27*b^3*e^4*n^3 - 288*
a^2*b*d*e^3*n*Sqrt[x] + 336*a*b^2*d*e^3*n^2*Sqrt[x] - 148*b^3*d*e^3*n^3*Sqr
t[x] + 432*a^2*b*d^2*e^2*n*x - 936*a*b^2*d^2*e^2*n^2*x + 690*b^3*d^2*e^2*n^
3*x - 864*a^2*b*d^3*e*n*x^(3/2) + 3600*a*b^2*d^3*e*n^2*x^(3/2) - 4980*b^3*d
^3*e*n^3*x^(3/2) - 576*b^3*d^4*n^3*x^2*Log[d + e/Sqrt[x]]^3 - 288*b^3*e^4*L
og[c*(d + e/Sqrt[x])^n]^3 + 864*a^2*b*d^4*n*x^2*Log[e + d*Sqrt[x]] - 3600*a
*b^2*d^4*n^2*x^2*Log[e + d*Sqrt[x]] + 4980*b^3*d^4*n^3*x^2*Log[e + d*Sqrt[x
]] + 72*b^2*d^4*n^2*x^2*Log[d + e/Sqrt[x]]*(-12*a + 25*b*n - 12*b*Log[c*(d
+ e/Sqrt[x])^n])*(2*Log[e + d*Sqrt[x]] - Log[x]) - 432*a^2*b*d^4*n*x^2*Log[
x] + 1800*a*b^2*d^4*n^2*x^2*Log[x] - 2490*b^3*d^4*n^3*x^2*Log[x] + 72*b^2*d
^4*n^2*x^2*Log[d + e/Sqrt[x]]^2*(12*a - 25*b*n + 12*b*Log[c*(d + e/Sqrt[x])
^n] + 12*b*n*Log[e + d*Sqrt[x]] - 6*b*n*Log[x]) + 72*b^2*Log[c*(d + e/Sqrt[
x])^n]^2*(e*(-12*a*e^3 + 3*b*e^3*n - 4*b*d*e^2*n*Sqrt[x] + 6*b*d^2*e*n*x -
12*b*d^3*n*x^(3/2)) + 12*b*d^4*n*x^2*Log[e + d*Sqrt[x]] - 6*b*d^4*n*x^2*Log
[x]) - 12*b*Log[c*(d + e/Sqrt[x])^n]*(72*a^2*e^4 + b^2*e*n^2*(9*e^3 - 28*d*
e^2*Sqrt[x] + 78*d^2*e*x - 300*d^3*x^(3/2)) - 12*a*b*e*n*(3*e^3 - 4*d*e^2*S
qrt[x] + 6*d^2*e*x - 12*d^3*x^(3/2)) + 12*b*d^4*n*(-12*a + 25*b*n)*x^2*Log[
e + d*Sqrt[x]] + 6*b*d^4*n*(12*a - 25*b*n)*x^2*Log[x]))/(576*e^4*x^2)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))^3/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))^3/x^3,x)
```

Maxima [A]
time = 0.32, size = 731, normalized size = 1.23

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*(12*d^4*e^(-5)*log(d*sqrt(x) + e) - 6*d^4*e^(-5)*log(x) - (12*d^3*x^(3/
2) - 6*d^2*x*e + 4*d*sqrt(x)*e^2 - 3*e^3)*e^(-4)/x^2)*a^2*b*n*e + 1/48*(12*
(12*d^4*e^(-5)*log(d*sqrt(x) + e) - 6*d^4*e^(-5)*log(x) - (12*d^3*x^(3/2) -
 6*d^2*x*e + 4*d*sqrt(x)*e^2 - 3*e^3)*e^(-4)/x^2)*n*e*log(c*(d + e/sqrt(x))
^n) - (72*d^4*x^2*log(d*sqrt(x) + e)^2 + 18*d^4*x^2*log(x)^2 - 150*d^4*x^2*
```

```
log(x) - 300*d^3*x^(3/2)*e + 78*d^2*x*e^2 - 28*d*sqrt(x)*e^3 - 12*(6*d^4*x^
2*log(x) - 25*d^4*x^2)*log(d*sqrt(x) + e) + 9*e^4)*n^2*e^(-4)/x^2)*a*b^2 +
1/576*(72*(12*d^4*e^(-5)*log(d*sqrt(x) + e) - 6*d^4*e^(-5)*log(x) - (12*d^3
*x^(3/2) - 6*d^2*x*e + 4*d*sqrt(x)*e^2 - 3*e^3)*e^(-4)/x^2)*n*e*log(c*(d +
e/sqrt(x))^n)^2 + ((288*d^4*x^2*log(d*sqrt(x) + e)^3 - 36*d^4*x^2*log(x)^3
+ 450*d^4*x^2*log(x)^2 - 2490*d^4*x^2*log(x) - 4980*d^3*x^(3/2)*e + 690*d^2
*x*e^2 - 72*(6*d^4*x^2*log(x) - 25*d^4*x^2)*log(d*sqrt(x) + e)^2 - 148*d*sq
rt(x)*e^3 + 12*(18*d^4*x^2*log(x)^2 - 150*d^4*x^2*log(x) + 415*d^4*x^2)*log
(d*sqrt(x) + e) + 27*e^4)*n^2*e^(-5)/x^2 - 12*(72*d^4*x^2*log(d*sqrt(x) + e
)^2 + 18*d^4*x^2*log(x)^2 - 150*d^4*x^2*log(x) - 300*d^3*x^(3/2)*e + 78*d^2
*x*e^2 - 28*d*sqrt(x)*e^3 - 12*(6*d^4*x^2*log(x) - 25*d^4*x^2)*log(d*sqrt(x
) + e) + 9*e^4)*n*e^(-5)*log(c*(d + e/sqrt(x))^n)/x^2)*n*e)*b^3 - 1/2*b^3*l
og(c*(d + e/sqrt(x))^n)^3/x^2 - 3/2*a*b^2*log(c*(d + e/sqrt(x))^n)^2/x^2 -
3/2*a^2*b*log(c*(d + e/sqrt(x))^n)/x^2 - 1/2*a^3/x^2
```

Fricas [**A**]
time = 0.43, size = 815, normalized size = 1.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^3,x, algorithm="fricas")
```

```
[Out] -1/576*(288*b^3*e^4*log(c)^3 - 288*(b^3*d^4*n^3*x^2 - b^3*n^3*e^4)*log((d*x
 + sqrt(x)*e)/x)^3 - 6*(115*b^3*d^2*n^3 - 156*a*b^2*d^2*n^2 + 72*a^2*b*d^2*
n)*x*e^2 - 216*(2*b^3*d^2*n*x*e^2 + (b^3*n - 4*a*b^2)*e^4)*log(c)^2 - 72*(6
*b^3*d^2*n^3*x*e^2 - (25*b^3*d^4*n^3 - 12*a*b^2*d^4*n^2)*x^2 + 3*(b^3*n^3 -
 4*a*b^2*n^2)*e^4 + 12*(b^3*d^4*n^2*x^2 - b^3*n^2*e^4)*log(c) - 4*(3*b^3*d^
3*n^3*x*e + b^3*d*n^3*e^3)*sqrt(x))*log((d*x + sqrt(x)*e)/x)^2 - 9*(3*b^3*n
^3 - 12*a*b^2*n^2 + 24*a^2*b*n - 32*a^3)*e^4 + 36*(2*(13*b^3*d^2*n^2 - 12*a
*b^2*d^2*n)*x*e^2 + 3*(b^3*n^2 - 4*a*b^2*n + 8*a^2*b)*e^4)*log(c) - 12*((41
5*b^3*d^4*n^3 - 300*a*b^2*d^4*n^2 + 72*a^2*b*d^4*n)*x^2 - 6*(13*b^3*d^2*n^3
 - 12*a*b^2*d^2*n^2)*x*e^2 + 72*(b^3*d^4*n*x^2 - b^3*n*e^4)*log(c)^2 - 9*(b
^3*n^3 - 4*a*b^2*n^2 + 8*a^2*b*n)*e^4 + 12*(6*b^3*d^2*n^2*x*e^2 - (25*b^3*d
^4*n^2 - 12*a*b^2*d^4*n)*x^2 + 3*(b^3*n^2 - 4*a*b^2*n)*e^4)*log(c) + 4*(3*(
25*b^3*d^3*n^3 - 12*a*b^2*d^3*n^2)*x*e + (7*b^3*d*n^3 - 12*a*b^2*d*n^2)*e^3
 - 12*(3*b^3*d^3*n^2*x*e + b^3*d*n^2*e^3)*log(c))*sqrt(x))*log((d*x + sqrt(
x)*e)/x) + 4*(3*(415*b^3*d^3*n^3 - 300*a*b^2*d^3*n^2 + 72*a^2*b*d^3*n)*x*e
+ 72*(3*b^3*d^3*n*x*e + b^3*d*n*e^3)*log(c)^2 + (37*b^3*d*n^3 - 84*a*b^2*d*
n^2 + 72*a^2*b*d*n)*e^3 - 12*(3*(25*b^3*d^3*n^2 - 12*a*b^2*d^3*n)*x*e + (7*
b^3*d*n^2 - 12*a*b^2*d*n)*e^3)*log(c))*sqrt(x))*e^(-4)/x^2
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**3/x**3,x)
```

```
[Out] Integral((a + b*log(c*(d + e/sqrt(x))**n))**3/x**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2389 vs. $2(529) = 1058$.
time = 4.60, size = 2389, normalized size = 4.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^3,x, algorithm="giac")
```

```
[Out] 1/576*(1152*(d*sqrt(x) + e)*b^3*d^3*n^3*log((d*sqrt(x) + e)/sqrt(x))^3/sqrt
(x) - 1728*(d*sqrt(x) + e)^2*b^3*d^2*n^3*log((d*sqrt(x) + e)/sqrt(x))^3/x -
 3456*(d*sqrt(x) + e)*b^3*d^3*n^3*log((d*sqrt(x) + e)/sqrt(x))^2/sqrt(x) +
3456*(d*sqrt(x) + e)*b^3*d^3*n^2*log(c)*log((d*sqrt(x) + e)/sqrt(x))^2/sqrt
(x) + 1152*(d*sqrt(x) + e)^3*b^3*d*n^3*log((d*sqrt(x) + e)/sqrt(x))^3/x^(3/
2) + 2592*(d*sqrt(x) + e)^2*b^3*d^2*n^3*log((d*sqrt(x) + e)/sqrt(x))^2/x -
5184*(d*sqrt(x) + e)^2*b^3*d^2*n^2*log(c)*log((d*sqrt(x) + e)/sqrt(x))^2/x
- 288*(d*sqrt(x) + e)^4*b^3*n^3*log((d*sqrt(x) + e)/sqrt(x))^3/x^2 + 6912*(
d*sqrt(x) + e)*b^3*d^3*n^3*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x) - 6912*(d*s
qrt(x) + e)*b^3*d^3*n^2*log(c)*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x) + 3456*
(d*sqrt(x) + e)*b^3*d^3*n*log(c)^2*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x) - 1
152*(d*sqrt(x) + e)^3*b^3*d*n^3*log((d*sqrt(x) + e)/sqrt(x))^2/x^(3/2) + 34
56*(d*sqrt(x) + e)*a*b^2*d^3*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/sqrt(x) + 3
456*(d*sqrt(x) + e)^3*b^3*d*n^2*log(c)*log((d*sqrt(x) + e)/sqrt(x))^2/x^(3/
2) - 2592*(d*sqrt(x) + e)^2*b^3*d^2*n^3*log((d*sqrt(x) + e)/sqrt(x))/x + 51
84*(d*sqrt(x) + e)^2*b^3*d^2*n^2*log(c)*log((d*sqrt(x) + e)/sqrt(x))/x - 51
84*(d*sqrt(x) + e)^2*b^3*d^2*n*log(c)^2*log((d*sqrt(x) + e)/sqrt(x))/x + 21
6*(d*sqrt(x) + e)^4*b^3*n^3*log((d*sqrt(x) + e)/sqrt(x))^2/x^2 - 5184*(d*sq
rt(x) + e)^2*a*b^2*d^2*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/x - 864*(d*sqrt(x
) + e)^4*b^3*n^2*log(c)*log((d*sqrt(x) + e)/sqrt(x))^2/x^2 - 6912*(d*sqrt(x
) + e)*b^3*d^3*n^3/sqrt(x) + 6912*(d*sqrt(x) + e)*b^3*d^3*n^2*log(c)/sqrt(x
) - 3456*(d*sqrt(x) + e)*b^3*d^3*n*log(c)^2/sqrt(x) + 1152*(d*sqrt(x) + e)*
b^3*d^3*log(c)^3/sqrt(x) + 768*(d*sqrt(x) + e)^3*b^3*d*n^3*log((d*sqrt(x) +
 e)/sqrt(x))/x^(3/2) - 6912*(d*sqrt(x) + e)*a*b^2*d^3*n^2*log((d*sqrt(x) +
e)/sqrt(x))/sqrt(x) - 2304*(d*sqrt(x) + e)^3*b^3*d*n^2*log(c)*log((d*sqrt(x
```

```
) + e)/sqrt(x))/x^(3/2) + 6912*(d*sqrt(x) + e)*a*b^2*d^3*n*log(c)*log((d*sq
rt(x) + e)/sqrt(x))/sqrt(x) + 3456*(d*sqrt(x) + e)^3*b^3*d*n*log(c)^2*log((
d*sqrt(x) + e)/sqrt(x))/x^(3/2) + 3456*(d*sqrt(x) + e)^3*a*b^2*d*n^2*log((d
*sqrt(x) + e)/sqrt(x))^2/x^(3/2) + 1296*(d*sqrt(x) + e)^2*b^3*d^2*n^3/x - 2
592*(d*sqrt(x) + e)^2*b^3*d^2*n^2*log(c)/x + 2592*(d*sqrt(x) + e)^2*b^3*d^2
*n*log(c)^2/x - 1728*(d*sqrt(x) + e)^2*b^3*d^2*log(c)^3/x - 108*(d*sqrt(x)
+ e)^4*b^3*n^3*log((d*sqrt(x) + e)/sqrt(x))/x^2 + 5184*(d*sqrt(x) + e)^2*a*
b^2*d^2*n^2*log((d*sqrt(x) + e)/sqrt(x))/x + 432*(d*sqrt(x) + e)^4*b^3*n^2*
log(c)*log((d*sqrt(x) + e)/sqrt(x))/x^2 - 10368*(d*sqrt(x) + e)^2*a*b^2*d^2
*n*log(c)*log((d*sqrt(x) + e)/sqrt(x))/x - 864*(d*sqrt(x) + e)^4*b^3*n*log(
c)^2*log((d*sqrt(x) + e)/sqrt(x))/x^2 - 864*(d*sqrt(x) + e)^4*a*b^2*n^2*log
((d*sqrt(x) + e)/sqrt(x))^2/x^2 - 256*(d*sqrt(x) + e)^3*b^3*d*n^3/x^(3/2) +
 6912*(d*sqrt(x) + e)*a*b^2*d^3*n^2/sqrt(x) + 768*(d*sqrt(x) + e)^3*b^3*d*n
^2*log(c)/x^(3/2) - 6912*(d*sqrt(x) + e)*a*b^2*d^3*n*log(c)/sqrt(x) - 1152*
(d*sqrt(x) + e)^3*b^3*d*n*log(c)^2/x^(3/2) + 3456*(d*sqrt(x) + e)*a*b^2*d^3
*log(c)^2/sqrt(x) + 1152*(d*sqrt(x) + e)^3*b^3*d*log(c)^3/x^(3/2) - 2304*(d
*sqrt(x) + e)^3*a*b^2*d*n^2*log((d*sqrt(x) + e)/sqrt(x))/x^(3/2) + 3456*(d*
sqrt(x) + e)*a^2*b*d^3*n*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x) + 6912*(d*sqr
t(x) + e)^3*a*b^2*d*n*log(c)*log((d*sqrt(x) + e)/sqrt(x))/x^(3/2) + 27*(d*s
qrt(x) + e)^4*b^3*n^3/x^2 - 2592*(d*sqrt(x) + e)^2*a*b^2*d^2*n^2/x - 108*(d
*sqrt(x) + e)^4*b^3*n^2*log(c)/x^2 + 5184*(d*sqrt(x) + e)^2*a*b^2*d^2*n*log
(c)/x + 216*(d*sqrt(x) + e)^4*b^3*n*log(c)^2/x^2 - 5184*(d*sqrt(x) + e)^2*a
*b^2*d^2*log(c)^2/x - 288*(d*sqrt(x) + e)^4*b^3*log(c)^3/x^2 + 432*(d*sqrt(
x) + e)^4*a*b^2*n^2*log((d*sqrt(x) + e)/sqrt(x))/x^2 - 5184*(d*sqrt(x) + e)
^2*a^2*b*d^2*n*log((d*sqrt(x) + e)/sqrt(x))/x - 1728*(d*sqrt(x) + e)^4*a*b^
2*n*log(c)*log((d*sqrt(x) + e)/sqrt(x))/x^2 + 768*(d*sqrt(x) + e)^3*a*b^2*d
*n^2/x^(3/2) - 3456*(d*sqrt(x) + e)*a^2*b*d^3*n/sqrt(x) - 2304*(d*sqrt(x) +
 e)^3*a*b^2*d*n*log(c)/x^(3/2) + 3456*(d*sqrt(x) + e)*a^2*b*d^3*log(c)/sqrt
(x) + 3456*(d*sqrt(x) + e)^3*a*b^2*d*log(c)^2/x^(3/2) + 3456*(d*sqrt(x) + e
)^3*a^2*b*d*n*log((d*sqrt(x) + e)/sqrt(x))/x^(3/2) - 108*(d*sqrt(x) + e)^4*
a*b^2*n^2/x^2 + 2592*(d*sqrt(x) + e)^2*a^2*b*d^2*n/x + 432*(d*sqrt(x) + e)^
4*a*b^2*n*log(c)/x^2 - 5184*(d*sqrt(x) + e)^2*a^2*b*d^2*log(c)/x - 864*(d*s
qrt(x) + e)^4*a*b^2*log(c)^2/x^2 - 864*(d*sqrt(x) + e)^4*a^2*b*n*log((d*sqr
t(x) + e)/sqrt(x))/x^2 - 1152*(d*sqrt(x) + e)^3*a^2*b*d*n/x^(3/2) + 1152*(d
*sqrt(x) + e)*a^3*d^3/sqrt(x) + 3456*(d*sqrt(x) + e)^3*a^2*b*d*log(c)/x^(3/
2) + 216*(d*sqrt(x) + e)^4*a^2*b*n/x^2 - 1728*(d*sqrt(x) + e)^2*a^3*d^2/x -
 864*(d*sqrt(x) + e)^4*a^2*b*log(c)/x^2 + 1152*(d*sqrt(x) + e)^3*a^3*d/x^(3
/2) - 288*(d*sqrt(x) + e)^4*a^3/x^2)*e^(-4)
```

Mupad [B]
time = 0.80, size = 846, normalized size = 1.42

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))^3/x^3,x)

[Out] ((d*(2*a^3 - (3*b^3*n^3)/16 + (3*a*b^2*n^2)/4 - (3*a^2*b*n)/2))/(3*e) - (d*
(24*a^3 + 7*b^3*n^3 - 12*a*b^2*n^2))/(36*e))/x^(3/2) - log(c*(d + e/x^(1/2)
)^n)^3*(b^3/(2*x^2) - (b^3*d^4)/(2*e^4)) + ((d*((d*((d*(2*a^3 - (3*b^3*n^3)
/16 + (3*a*b^2*n^2)/4 - (3*a^2*b*n)/2))/e - (d*(24*a^3 + 7*b^3*n^3 - 12*a*b
^2*n^2))/(12*e)))/e + (b^2*d^2*n^2*(12*a - 13*b*n))/(8*e^2)))/e + (b^2*d^3*
n^2*(12*a - 25*b*n))/(4*e^3))/x^(1/2) + log(c*(d + e/x^(1/2))^n)^2*(((b^2*d
*(4*a - b*n))/e - (4*a*b^2*d)/e)/(2*x^(3/2)) - (3*b^2*(4*a - b*n))/(8*x^2)
+ (d*(12*a*b^2*d^3 - 25*b^3*d^3*n))/(8*e^4) - (d*((6*b^2*d*(4*a - b*n))/e -
 (24*a*b^2*d)/e))/(8*e*x) + (d^2*((6*b^2*d*(4*a - b*n))/e - (24*a*b^2*d)/e)
)/(4*e^2*x^(1/2))) - ((d*((d*(2*a^3 - (3*b^3*n^3)/16 + (3*a*b^2*n^2)/4 - (3
*a^2*b*n)/2))/e - (d*(24*a^3 + 7*b^3*n^3 - 12*a*b^2*n^2))/(12*e)))/(2*e) +
(b^2*d^2*n^2*(12*a - 13*b*n))/(16*e^2))/x - (a^3/2 - (3*b^3*n^3)/64 + (3*a*
b^2*n^2)/16 - (3*a^2*b*n)/8)/x^2 - (log(c*(d + e/x^(1/2))^n)*((16*b*d*e^3*(
6*a^2 - b^2*n^2) - 12*b*d*e^3*(8*a^2 + b^2*n^2 - 4*a*b*n))/(12*e^2*x^(3/2))
 + ((d*((d*(16*b*d*e^3*(6*a^2 - b^2*n^2) - 12*b*d*e^3*(8*a^2 + b^2*n^2 - 4*
a*b*n)))/e - 24*b^3*d^2*e^2*n^2))/e - 48*b^3*d^3*e*n^2)/(4*e^2*x^(1/2)) - (
(d*(16*b*d*e^3*(6*a^2 - b^2*n^2) - 12*b*d*e^3*(8*a^2 + b^2*n^2 - 4*a*b*n)))
/e - 24*b^3*d^2*e^2*n^2)/(8*e^2*x) + (3*b*e^2*(8*a^2 + b^2*n^2 - 4*a*b*n))/
(4*x^2)))/(4*e^2) + (log(d + e/x^(1/2))*(415*b^3*d^4*n^3 - 300*a*b^2*d^4*n^
2 + 72*a^2*b*d^4*n))/(48*e^4)
```

3.441 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^4}\,dx$

Optimal. Leaf size=907

$$\frac{15b^3d^4n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^6}-\frac{40b^3d^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^3}{27e^6}+\frac{15b^3d^2n^3\left(d+\frac{e}{\sqrt{x}}\right)^4}{32e^6}-\frac{12b^3dn^3\left(d+\frac{e}{\sqrt{x}}\right)^5}{125e^6}+\frac{b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^6}{108e^6}$$

```
[Out] 15/4*b^3*d^4*n^3*(d+e/x^(1/2))^2/e^6-40/27*b^3*d^3*n^3*(d+e/x^(1/2))^3/e^6+
15/32*b^3*d^2*n^3*(d+e/x^(1/2))^4/e^6-12/125*b^3*d*n^3*(d+e/x^(1/2))^5/e^6-
1/18*b^2*n^2*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))^6/e^6+1/6*b*n*(a+b*l
n(c*(d+e/x^(1/2))^n))^2*(d+e/x^(1/2))^6/e^6-12*b^3*d^5*n^3/e^5/x^(1/2)-1/3*
(a+b*ln(c*(d+e/x^(1/2))^n))^3*(d+e/x^(1/2))^6/e^6+20/3*d^3*(a+b*ln(c*(d+e/x
^(1/2))^n))^3*(d+e/x^(1/2))^3/e^6-5*d^2*(a+b*ln(c*(d+e/x^(1/2))^n))^3*(d+e/
x^(1/2))^4/e^6+2*d*(a+b*ln(c*(d+e/x^(1/2))^n))^3*(d+e/x^(1/2))^5/e^6+1/108*
b^3*n^3*(d+e/x^(1/2))^6/e^6+2*d^5*(a+b*ln(c*(d+e/x^(1/2))^n))^3*(d+e/x^(1/2
))/e^6-5*d^4*(a+b*ln(c*(d+e/x^(1/2))^n))^3*(d+e/x^(1/2))^2/e^6+12*b^3*d^5*n
^2*ln(c*(d+e/x^(1/2))^n)*(d+e/x^(1/2))/e^6-6*b*d^5*n*(a+b*ln(c*(d+e/x^(1/2)
)^n))^2*(d+e/x^(1/2))/e^6-15/2*b^2*d^4*n^2*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e
/x^(1/2))^2/e^6+15/2*b*d^4*n*(a+b*ln(c*(d+e/x^(1/2))^n))^2*(d+e/x^(1/2))^2/
e^6+40/9*b^2*d^3*n^2*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))^3/e^6-20/3*b
*d^3*n*(a+b*ln(c*(d+e/x^(1/2))^n))^2*(d+e/x^(1/2))^3/e^6-15/8*b^2*d^2*n^2*(
a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))^4/e^6+15/4*b*d^2*n*(a+b*ln(c*(d+e/
x^(1/2))^n))^2*(d+e/x^(1/2))^4/e^6+12/25*b^2*d*n^2*(a+b*ln(c*(d+e/x^(1/2))^
n))*(d+e/x^(1/2))^5/e^6-6/5*b*d*n*(a+b*ln(c*(d+e/x^(1/2))^n))^2*(d+e/x^(1/2
))^5/e^6+12*a*b^2*d^5*n^2/e^5/x^(1/2)
```

Rubi [A]
time = 0.64, antiderivative size = 907, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x^4,x]
```

```
[Out] (15*b^3*d^4*n^3*(d + e/Sqrt[x])^2)/(4*e^6) - (40*b^3*d^3*n^3*(d + e/Sqrt[x]
)^3)/(27*e^6) + (15*b^3*d^2*n^3*(d + e/Sqrt[x])^4)/(32*e^6) - (12*b^3*d*n^3
*(d + e/Sqrt[x])^5)/(125*e^6) + (b^3*n^3*(d + e/Sqrt[x])^6)/(108*e^6) + (12
*a*b^2*d^5*n^2)/(e^5*Sqrt[x]) - (12*b^3*d^5*n^3)/(e^5*Sqrt[x]) + (12*b^3*d^
5*n^2*(d + e/Sqrt[x])*Log[c*(d + e/Sqrt[x])^n])/e^6 - (15*b^2*d^4*n^2*(d +
```

```
e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(2*e^6) + (40*b^2*d^3*n^2*(d
 + e/Sqrt[x])^3*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(9*e^6) - (15*b^2*d^2*n^2
*(d + e/Sqrt[x])^4*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(8*e^6) + (12*b^2*d*n^
2*(d + e/Sqrt[x])^5*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(25*e^6) - (b^2*n^2*(
d + e/Sqrt[x])^6*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(18*e^6) - (6*b*d^5*n*(d
 + e/Sqrt[x])*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/e^6 + (15*b*d^4*n*(d + e/
Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(2*e^6) - (20*b*d^3*n*(d + e
/Sqrt[x])^3*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(3*e^6) + (15*b*d^2*n*(d +
e/Sqrt[x])^4*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(4*e^6) - (6*b*d*n*(d + e/
Sqrt[x])^5*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(5*e^6) + (b*n*(d + e/Sqrt[x
])^6*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(6*e^6) + (2*d^5*(d + e/Sqrt[x])*(
a + b*Log[c*(d + e/Sqrt[x])^n])^3)/e^6 - (5*d^4*(d + e/Sqrt[x])^2*(a + b*Lo
g[c*(d + e/Sqrt[x])^n])^3)/e^6 + (20*d^3*(d + e/Sqrt[x])^3*(a + b*Log[c*(d
+ e/Sqrt[x])^n])^3)/(3*e^6) - (5*d^2*(d + e/Sqrt[x])^4*(a + b*Log[c*(d + e/
Sqrt[x])^n])^3)/e^6 + (2*d*(d + e/Sqrt[x])^5*(a + b*Log[c*(d + e/Sqrt[x])^n
])^3)/e^6 - ((d + e/Sqrt[x])^6*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/(3*e^6)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^4}\,dx = -\left(2\text{Subst}\left(\int x^5(a+b\log\left(c(d+ex)^n\right))^3\,dx,x,\frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\text{Subst}\left(\int\left(-\frac{d^5(a+b\log\left(c(d+ex)^n\right))^3}{e^5}+\frac{5d^4(d+ex)\,(a+\right.\right.\right.$$

$$= -\frac{2\text{Subst}\left(\int(d+ex)^5\,(a+b\log\left(c(d+ex)^n\right))^3\,dx,x,\frac{1}{\sqrt{x}}\right)}{e^5}+\frac{(10}{}$$

$$= -\frac{2\text{Subst}\left(\int x^5(a+b\log\left(cx^n\right))^3\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^6}+\frac{(10d)\text{Subst}\left(\right.}{}$$

$$= \frac{2d^5\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^6}-\frac{5d^4\left(d+\frac{e}{\sqrt{x}}\right.}{}$$

$$= -\frac{6bd^5n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^6}+\frac{15bd^4n\left(d\right.}{}$$

$$= \frac{15b^3d^4n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^6}-\frac{40b^3d^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^3}{27e^6}+\frac{15b^3d^2n^3\left(d+\right.}{32e^6}$$

$$= \frac{15b^3d^4n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^6}-\frac{40b^3d^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^3}{27e^6}+\frac{15b^3d^2n^3\left(d+\right.}{32e^6}$$

Mathematica [A]
time = 1.03, size = 950, normalized size = 1.05

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x^4,x]
```

```
[Out] (-36000*a^3*e^6 + 18000*a^2*b*e^6*n - 6000*a*b^2*e^6*n^2 + 1000*b^3*e^6*n^3
 - 21600*a^2*b*d*e^5*n*Sqrt[x] + 15840*a*b^2*d*e^5*n^2*Sqrt[x] - 4368*b^3*d
*e^5*n^3*Sqrt[x] + 27000*a^2*b*d^2*e^4*n*x - 33300*a*b^2*d^2*e^4*n^2*x + 13
785*b^3*d^2*e^4*n^3*x - 36000*a^2*b*d^3*e^3*n*x^(3/2) + 68400*a*b^2*d^3*e^3
*n^2*x^(3/2) - 41180*b^3*d^3*e^3*n^3*x^(3/2) + 54000*a^2*b*d^4*e^2*n*x^2 -
156600*a*b^2*d^4*e^2*n^2*x^2 + 140070*b^3*d^4*e^2*n^3*x^2 - 108000*a^2*b*d^
5*e*n*x^(5/2) + 529200*a*b^2*d^5*e*n^2*x^(5/2) - 809340*b^3*d^5*e*n^3*x^(5/
```

2) - 72000*b^3*d^6*n^3*x^3*Log[d + e/Sqrt[x]]^3 - 36000*b^3*e^6*Log[c*(d + e/Sqrt[x])^n]^3 + 108000*a^2*b*d^6*n*x^3*Log[e + d*Sqrt[x]] - 529200*a*b^2*d^6*n^2*x^3*Log[e + d*Sqrt[x]] + 809340*b^3*d^6*n^3*x^3*Log[e + d*Sqrt[x]] + 5400*b^2*d^6*n^2*x^3*Log[d + e/Sqrt[x]]*(-20*a + 49*b*n - 20*b*Log[c*(d + e/Sqrt[x])^n])*(2*Log[e + d*Sqrt[x]] - Log[x]) - 54000*a^2*b*d^6*n*x^3*Log[x] + 264600*a*b^2*d^6*n^2*x^3*Log[x] - 404670*b^3*d^6*n^3*x^3*Log[x] + 5400*b^2*d^6*n^2*x^3*Log[d + e/Sqrt[x]]^2*(20*a - 49*b*n + 20*b*Log[c*(d + e/Sqrt[x])^n] + 20*b*n*Log[e + d*Sqrt[x]] - 10*b*n*Log[x]) + 1800*b^2*Log[c*(d + e/Sqrt[x])^n]^2*(e*(-60*a*e^5 + 10*b*e^5*n - 12*b*d*e^4*n*Sqrt[x] + 15*b*d^2*e^3*n*x - 20*b*d^3*e^2*n*x^(3/2) + 30*b*d^4*e*n*x^2 - 60*b*d^5*n*x^(5/2)) + 60*b*d^6*n*x^3*Log[e + d*Sqrt[x]] - 30*b*d^6*n*x^3*Log[x]) - 60*b*Log[c*(d + e/Sqrt[x])^n]*(1800*a^2*e^6 + b^2*e*n^2*(100*e^5 - 264*d*e^4*Sqrt[x] + 555*d^2*e^3*x - 1140*d^3*e^2*x^(3/2) + 2610*d^4*e*x^2 - 8820*d^5*x^(5/2)) - 60*a*b*e*n*(10*e^5 - 12*d*e^4*Sqrt[x] + 15*d^2*e^3*x - 20*d^3*e^2*x^(3/2) + 30*d^4*e*x^2 - 60*d^5*x^(5/2)) + 180*b*d^6*n*(-20*a + 49*b*n)*x^3*Log[e + d*Sqrt[x]] + 90*b*d^6*n*(20*a - 49*b*n)*x^3*Log[x]))/(108000*e^6*x^3)

Maple [F]
time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*ln(c*(d+e/x^(1/2))^n))^3/x^4,x)

[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))^3/x^4,x)

Maxima [A]
time = 0.33, size = 851, normalized size = 0.94

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^4,x, algorithm="maxima")

[Out] 1/60*(60*d^6*e^(-7)*log(d*sqrt(x) + e) - 30*d^6*e^(-7)*log(x) - (60*d^5*x^(5/2) - 30*d^4*x^2*e + 20*d^3*x^(3/2)*e^2 - 15*d^2*x*e^3 + 12*d*sqrt(x)*e^4 - 10*e^5)*e^(-6)/x^3)*a^2*b*n*e + 1/1800*(60*(60*d^6*e^(-7)*log(d*sqrt(x) + e) - 30*d^6*e^(-7)*log(x) - (60*d^5*x^(5/2) - 30*d^4*x^2*e + 20*d^3*x^(3/2)*e^2 - 15*d^2*x*e^3 + 12*d*sqrt(x)*e^4 - 10*e^5)*e^(-6)/x^3)*n*e*log(c*(d + e/sqrt(x))^n) - (1800*d^6*x^3*log(d*sqrt(x) + e)^2 + 450*d^6*x^3*log(x)^2 - 4410*d^6*x^3*log(x) - 8820*d^5*x^(5/2)*e + 2610*d^4*x^2*e^2 - 1140*d^3*x^(3/2)*e^3 + 555*d^2*x*e^4 - 264*d*sqrt(x)*e^5 - 180*(10*d^6*x^3*log(x) - 49*d^6*x^3)*log(d*sqrt(x) + e) + 100*e^6)*n^2*e^(-6)/x^3)*a*b^2 + 1/108000*(

```
1800*(60*d^6*e^(-7)*log(d*sqrt(x) + e) - 30*d^6*e^(-7)*log(x) - (60*d^5*x^(
5/2) - 30*d^4*x^2*e + 20*d^3*x^(3/2)*e^2 - 15*d^2*x*e^3 + 12*d*sqrt(x)*e^4
- 10*e^5)*e^(-6)/x^3)*n*e*log(c*(d + e/sqrt(x))^n)^2 + n*((36000*d^6*x^3*lo
g(d*sqrt(x) + e)^3 - 4500*d^6*x^3*log(x)^3 + 66150*d^6*x^3*log(x)^2 - 40467
0*d^6*x^3*log(x) - 809340*d^5*x^(5/2)*e + 140070*d^4*x^2*e^2 - 41180*d^3*x^
(3/2)*e^3 + 13785*d^2*x*e^4 - 5400*(10*d^6*x^3*log(x) - 49*d^6*x^3)*log(d*s
qrt(x) + e)^2 - 4368*d*sqrt(x)*e^5 + 60*(450*d^6*x^3*log(x)^2 - 4410*d^6*x^
3*log(x) + 13489*d^6*x^3)*log(d*sqrt(x) + e) + 1000*e^6)*n^2*e^(-7)/x^3 - 6
0*(1800*d^6*x^3*log(d*sqrt(x) + e)^2 + 450*d^6*x^3*log(x)^2 - 4410*d^6*x^3*
log(x) - 8820*d^5*x^(5/2)*e + 2610*d^4*x^2*e^2 - 1140*d^3*x^(3/2)*e^3 + 555
*d^2*x*e^4 - 264*d*sqrt(x)*e^5 - 180*(10*d^6*x^3*log(x) - 49*d^6*x^3)*log(d
*sqrt(x) + e) + 100*e^6)*n*e^(-7)*log(c*(d + e/sqrt(x))^n)/x^3)*e)*b^3 - 1/
3*b^3*log(c*(d + e/sqrt(x))^n)^3/x^3 - a*b^2*log(c*(d + e/sqrt(x))^n)^2/x^3
 - a^2*b*log(c*(d + e/sqrt(x))^n)/x^3 - 1/3*a^3/x^3
```

Fricas [**A**]
time = 0.43, size = 1115, normalized size = 1.23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^4,x, algorithm="fricas")
```

```
[Out] -1/108000*(36000*b^3*e^6*log(c)^3 - 30*(4669*b^3*d^4*n^3 - 5220*a*b^2*d^4*n
^2 + 1800*a^2*b*d^4*n)*x^2*e^2 - 36000*(b^3*d^6*n^3*x^3 - b^3*n^3*e^6)*log(
(d*x + sqrt(x)*e)/x)^3 - 15*(919*b^3*d^2*n^3 - 2220*a*b^2*d^2*n^2 + 1800*a^
2*b*d^2*n)*x*e^4 - 9000*(6*b^3*d^4*n*x^2*e^2 + 3*b^3*d^2*n*x*e^4 + 2*(b^3*n
 - 6*a*b^2)*e^6)*log(c)^2 - 1800*(30*b^3*d^4*n^3*x^2*e^2 + 15*b^3*d^2*n^3*x
*e^4 - 3*(49*b^3*d^6*n^3 - 20*a*b^2*d^6*n^2)*x^3 + 10*(b^3*n^3 - 6*a*b^2*n^
2)*e^6 + 60*(b^3*d^6*n^2*x^3 - b^3*n^2*e^6)*log(c) - 4*(15*b^3*d^5*n^3*x^2*
e + 5*b^3*d^3*n^3*x*e^3 + 3*b^3*d*n^3*e^5)*sqrt(x))*log((d*x + sqrt(x)*e)/x
)^2 - 1000*(b^3*n^3 - 6*a*b^2*n^2 + 18*a^2*b*n - 36*a^3)*e^6 + 300*(18*(29*
b^3*d^4*n^2 - 20*a*b^2*d^4*n)*x^2*e^2 + 3*(37*b^3*d^2*n^2 - 60*a*b^2*d^2*n)
*x*e^4 + 20*(b^3*n^2 - 6*a*b^2*n + 18*a^2*b)*e^6)*log(c) - 60*((13489*b^3*d
^6*n^3 - 8820*a*b^2*d^6*n^2 + 1800*a^2*b*d^6*n)*x^3 - 90*(29*b^3*d^4*n^3 -
20*a*b^2*d^4*n^2)*x^2*e^2 - 15*(37*b^3*d^2*n^3 - 60*a*b^2*d^2*n^2)*x*e^4 +
1800*(b^3*d^6*n*x^3 - b^3*n*e^6)*log(c)^2 - 100*(b^3*n^3 - 6*a*b^2*n^2 + 18
*a^2*b*n)*e^6 + 60*(30*b^3*d^4*n^2*x^2*e^2 + 15*b^3*d^2*n^2*x*e^4 - 3*(49*b
^3*d^6*n^2 - 20*a*b^2*d^6*n)*x^3 + 10*(b^3*n^2 - 6*a*b^2*n)*e^6)*log(c) + 1
2*(15*(49*b^3*d^5*n^3 - 20*a*b^2*d^5*n^2)*x^2*e + 5*(19*b^3*d^3*n^3 - 20*a*
b^2*d^3*n^2)*x*e^3 + 2*(11*b^3*d*n^3 - 30*a*b^2*d*n^2)*e^5 - 20*(15*b^3*d^5
*n^2*x^2*e + 5*b^3*d^3*n^2*x*e^3 + 3*b^3*d*n^2*e^5)*log(c))*sqrt(x))*log((d
*x + sqrt(x)*e)/x) + 4*(15*(13489*b^3*d^5*n^3 - 8820*a*b^2*d^5*n^2 + 1800*a
^2*b*d^5*n)*x^2*e + 5*(2059*b^3*d^3*n^3 - 3420*a*b^2*d^3*n^2 + 1800*a^2*b*d
^3*n)*x*e^3 + 1800*(15*b^3*d^5*n*x^2*e + 5*b^3*d^3*n*x*e^3 + 3*b^3*d*n*e^5)
```

```
*log(c)^2 + 12*(91*b^3*d*n^3 - 330*a*b^2*d*n^2 + 450*a^2*b*d*n)*e^5 - 180*(
15*(49*b^3*d^5*n^2 - 20*a*b^2*d^5*n)*x^2*e + 5*(19*b^3*d^3*n^2 - 20*a*b^2*d
^3*n)*x*e^3 + 2*(11*b^3*d*n^2 - 30*a*b^2*d*n)*e^5)*log(c))*sqrt(x))*e^(-6)/
x^3
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**3/x**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3651 vs. 2(803) = 1606.
time = 6.09, size = 3651, normalized size = 4.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^4,x, algorithm="giac")
```

```
[Out] 1/108000*(216000*(d*sqrt(x) + e)*b^3*d^5*n^3*log((d*sqrt(x) + e)/sqrt(x))^3
/sqrt(x) - 540000*(d*sqrt(x) + e)^2*b^3*d^4*n^3*log((d*sqrt(x) + e)/sqrt(x)
)^3/x - 648000*(d*sqrt(x) + e)*b^3*d^5*n^3*log((d*sqrt(x) + e)/sqrt(x))^2/s
qrt(x) + 648000*(d*sqrt(x) + e)*b^3*d^5*n^2*log(c)*log((d*sqrt(x) + e)/sqrt
(x))^2/sqrt(x) + 720000*(d*sqrt(x) + e)^3*b^3*d^3*n^3*log((d*sqrt(x) + e)/s
qrt(x))^3/x^(3/2) + 810000*(d*sqrt(x) + e)^2*b^3*d^4*n^3*log((d*sqrt(x) + e
)/sqrt(x))^2/x - 1620000*(d*sqrt(x) + e)^2*b^3*d^4*n^2*log(c)*log((d*sqrt(x
) + e)/sqrt(x))^2/x - 540000*(d*sqrt(x) + e)^4*b^3*d^2*n^3*log((d*sqrt(x) +
 e)/sqrt(x))^3/x^2 + 1296000*(d*sqrt(x) + e)*b^3*d^5*n^3*log((d*sqrt(x) + e
)/sqrt(x))/sqrt(x) - 1296000*(d*sqrt(x) + e)*b^3*d^5*n^2*log(c)*log((d*sqrt
(x) + e)/sqrt(x))/sqrt(x) + 648000*(d*sqrt(x) + e)*b^3*d^5*n*log(c)^2*log((
d*sqrt(x) + e)/sqrt(x))/sqrt(x) - 720000*(d*sqrt(x) + e)^3*b^3*d^3*n^3*log(
(d*sqrt(x) + e)/sqrt(x))^2/x^(3/2) + 648000*(d*sqrt(x) + e)*a*b^2*d^5*n^2*l
og((d*sqrt(x) + e)/sqrt(x))^2/sqrt(x) + 2160000*(d*sqrt(x) + e)^3*b^3*d^3*n
^2*log(c)*log((d*sqrt(x) + e)/sqrt(x))^2/x^(3/2) + 216000*(d*sqrt(x) + e)^5
*b^3*d*n^3*log((d*sqrt(x) + e)/sqrt(x))^3/x^(5/2) - 810000*(d*sqrt(x) + e)^
2*b^3*d^4*n^3*log((d*sqrt(x) + e)/sqrt(x))/x + 1620000*(d*sqrt(x) + e)^2*b^
3*d^4*n^2*log(c)*log((d*sqrt(x) + e)/sqrt(x))/x - 1620000*(d*sqrt(x) + e)^2
*b^3*d^4*n*log(c)^2*log((d*sqrt(x) + e)/sqrt(x))/x + 405000*(d*sqrt(x) + e)
^4*b^3*d^2*n^3*log((d*sqrt(x) + e)/sqrt(x))^2/x^2 - 1620000*(d*sqrt(x) + e)
^2*a*b^2*d^4*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/x - 1620000*(d*sqrt(x) + e)
^4*b^3*d^2*n^2*log(c)*log((d*sqrt(x) + e)/sqrt(x))^2/x^2 - 36000*(d*sqrt(x)
```

+ e)^6*b^3*n^3*log((d*sqrt(x) + e)/sqrt(x))^3/x^3 - 1296000*(d*sqrt(x) + e
)*b^3*d^5*n^3/sqrt(x) + 1296000*(d*sqrt(x) + e)*b^3*d^5*n^2*log(c)/sqrt(x)
- 648000*(d*sqrt(x) + e)*b^3*d^5*n*log(c)^2/sqrt(x) + 216000*(d*sqrt(x) + e
)*b^3*d^5*log(c)^3/sqrt(x) + 480000*(d*sqrt(x) + e)^3*b^3*d^3*n^3*log((d*sq
rt(x) + e)/sqrt(x))/x^(3/2) - 1296000*(d*sqrt(x) + e)*a*b^2*d^5*n^2*log((d*
sqrt(x) + e)/sqrt(x))/sqrt(x) - 1440000*(d*sqrt(x) + e)^3*b^3*d^3*n^2*log(c
)*log((d*sqrt(x) + e)/sqrt(x))/x^(3/2) + 1296000*(d*sqrt(x) + e)*a*b^2*d^5*
n*log(c)*log((d*sqrt(x) + e)/sqrt(x))/sqrt(x) + 2160000*(d*sqrt(x) + e)^3*b
^3*d^3*n*log(c)^2*log((d*sqrt(x) + e)/sqrt(x))/x^(3/2) - 129600*(d*sqrt(x)
+ e)^5*b^3*d*n^3*log((d*sqrt(x) + e)/sqrt(x))^2/x^(5/2) + 2160000*(d*sqrt(x
) + e)^3*a*b^2*d^3*n^2*log((d*sqrt(x) + e)/sqrt(x))^2/x^(3/2) + 648000*(d*s
qrt(x) + e)^5*b^3*d*n^2*log(c)*log((d*sqrt(x) + e)/sqrt(x))^2/x^(5/2) + 405
000*(d*sqrt(x) + e)^2*b^3*d^4*n^3/x - 810000*(d*sqrt(x) + e)^2*b^3*d^4*n^2*
log(c)/x + 810000*(d*sqrt(x) + e)^2*b^3*d^4*n*log(c)^2/x - 540000*(d*sqrt(x
) + e)^2*b^3*d^4*log(c)^3/x - 202500*(d*sqrt(x) + e)^4*b^3*d^2*n^3*log((d*s
qrt(x) + e)/sqrt(x))/x^2 + 1620000*(d*sqrt(x) + e)^2*a*b^2*d^4*n^2*log((d*s
qrt(x) + e)/sqrt(x))/x + 810000*(d*sqrt(x) + e)^4*b^3*d^2*n^2*log(c)*log((d
sqrt(x) + e)/sqrt(x))/x^2 - 3240000(d*sqrt(x) + e)^2*a*b^2*d^4*n*log(c)*l
og((d*sqrt(x) + e)/sqrt(x))/x - 1620000*(d*sqrt(x) + e)^4*b^3*d^2*n*log(c)^
2*log((d*sqrt(x) + e)/sqrt(x))/x^2 + 18000*(d*sqrt(x) + e)^6*b^3*n^3*log((d
sqrt(x) + e)/sqrt(x))^2/x^3 - 1620000(d*sqrt(x) + e)^4*a*b^2*d^2*n^2*log(
(d*sqrt(x) + e)/sqrt(x))^2/x^2 - 108000*(d*sqrt(x) + e)^6*b^3*n^2*log(c)*lo
g((d*sqrt(x) + e)/sqrt(x))^2/x^3 - 160000*(d*sqrt(x) + e)^3*b^3*d^3*n^3/x^(
3/2) + 1296000*(d*sqrt(x) + e)*a*b^2*d^5*n^2/sqrt(x) + 480000*(d*sqrt(x) +
e)^3*b^3*d^3*n^2*log(c)/x^(3/2) - 1296000*(d*sqrt(x) + e)*a*b^2*d^5*n*log(c
)/sqrt(x) - 720000*(d*sqrt(x) + e)^3*b^3*d^3*n*log(c)^2/x^(3/2) + 648000*(d
*sqrt(x) + e)*a*b^2*d^5*log(c)^2/sqrt(x) + 720000*(d*sqrt(x) + e)^3*b^3*d^3
log(c)^3/x^(3/2) + 51840(d*sqrt(x) + e)^5*b^3*d*n^3*log((d*sqrt(x) + e)/s
qrt(x))/x^(5/2) - 1440000*(d*sqrt(x) + e)^3*a*b^2*d^3*n^2*log((d*sqrt(x) +
e)/sqrt(x))/x^(3/2) + 648000*(d*sqrt(x) + e)*a^2*b*d^5*n*log((d*sqrt(x) + e
)/sqrt(x))/sqrt(x) - 259200*(d*sqrt(x) + e)^5*b^3*d*n^2*log(c)*log((d*sqrt(
x) + e)/sqrt(x))/x^(5/2) + 4320000*(d*sqrt(x) + e)^3*a*b^2*d^3*n*log(c)*log
((d*sqrt(x) + e)/sqrt(x))/x^(3/2) + 648000*(d*sqrt(x) + e)^5*b^3*d*n*log(c)
^2*log((d*sqrt(x) + e)/sqrt(x))/x^(5/2) + 648000*(d*sqrt(x) + e)^5*a*b^2*d*
n^2*log((d*sqrt(x) + e)/sqrt(x))^2/x^(5/2) + 50625*(d*sqrt(x) + e)^4*b^3*d^
2*n^3/x^2 - 810000*(d*sqrt(x) + e)^2*a*b^2*d^4*n^2/x - 202500*(d*sqrt(x) +
e)^4*b^3*d^2*n^2*log(c)/x^2 + 1620000*(d*sqrt(x) + e)^2*a*b^2*d^4*n*log(c)/
x + 405000*(d*sqrt(x) + e)^4*b^3*d^2*n*log(c)^2/x^2 - 1620000*(d*sqrt(x) +
e)^2*a*b^2*d^4*log(c)^2/x - 540000*(d*sqrt(x) + e)^4*b^3*d^2*log(c)^3/x^2 -
6000*(d*sqrt(x) + e)^6*b^3*n^3*log((d*sqrt(x) + e)/sqrt(x))/x^3 + 810000*(
d*sqrt(x) + e)^4*a*b^2*d^2*n^2*log((d*sqrt(x) + e)/sqrt(x))/x^2 - 1620000*(
d*sqrt(x) + e)^2*a^2*b*d^4*n*log((d*sqrt(x) + e)/sqrt(x))/x + 36000*(d*sqrt
(x) + e)^6*b^3*n^2*log(c)*log((d*sqrt(x) + e)/sqrt(x))/x^3 - 3240000*(d*sqr
t(x) + e)^4*a*b^2*d^2*n*log(c)*log((d*sqrt(x) + e)/sqrt(x))/x^2 - 108000*(d
*sqrt(x) + e)^6*b^3*n*log(c)^2*log((d*sqrt(x) +...

Mupad [B]
time = 8.18, size = 989, normalized size = 1.09

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))^3/x^4,x)
```

```
[Out] (b^3*n^3)/(108*x^3) - (b^3*log(c*(d + e/x^(1/2))^n)^3)/(3*x^3) - a^3/(3*x^3
) - (a*b^2*log(c*(d + e/x^(1/2))^n)^2)/x^3 + (b^3*n*log(c*(d + e/x^(1/2))^n
)^2)/(6*x^3) - (b^3*n^2*log(c*(d + e/x^(1/2))^n))/(18*x^3) - (a*b^2*n^2)/(1
8*x^3) + (b^3*d^6*log(c*(d + e/x^(1/2))^n)^3)/(3*e^6) - (a^2*b*log(c*(d + e
/x^(1/2))^n))/x^3 + (a^2*b*n)/(6*x^3) + (a*b^2*n*log(c*(d + e/x^(1/2))^n))/
(3*x^3) + (13489*b^3*d^6*n^3*log(d + e/x^(1/2)))/(1800*e^6) + (919*b^3*d^2*
n^3)/(7200*e^2*x^2) + (4669*b^3*d^4*n^3)/(3600*e^4*x) - (2059*b^3*d^3*n^3)/
(5400*e^3*x^(3/2)) - (13489*b^3*d^5*n^3)/(1800*e^5*x^(1/2)) + (a*b^2*d^6*lo
g(c*(d + e/x^(1/2))^n)^2)/e^6 - (49*b^3*d^6*n*log(c*(d + e/x^(1/2))^n)^2)/(
20*e^6) - (91*b^3*d*n^3)/(2250*e*x^(5/2)) + (a^2*b*d^6*n*log(d + e/x^(1/2))
)/e^6 - (b^3*d*n*log(c*(d + e/x^(1/2))^n)^2)/(5*e*x^(5/2)) + (11*b^3*d*n^2*
log(c*(d + e/x^(1/2))^n))/(75*e*x^(5/2)) + (a^2*b*d^2*n)/(4*e^2*x^2) + (a^2
*b*d^4*n)/(2*e^4*x) + (11*a*b^2*d*n^2)/(75*e*x^(5/2)) - (a^2*b*d^3*n)/(3*e^
3*x^(3/2)) - (a^2*b*d^5*n)/(e^5*x^(1/2)) - (49*a*b^2*d^6*n^2*log(d + e/x^(1
/2)))/(10*e^6) + (b^3*d^2*n*log(c*(d + e/x^(1/2))^n)^2)/(4*e^2*x^2) - (37*b
^3*d^2*n^2*log(c*(d + e/x^(1/2))^n))/(120*e^2*x^2) + (b^3*d^4*n*log(c*(d +
e/x^(1/2))^n)^2)/(2*e^4*x) - (29*b^3*d^4*n^2*log(c*(d + e/x^(1/2))^n))/(20*
e^4*x) - (b^3*d^3*n*log(c*(d + e/x^(1/2))^n)^2)/(3*e^3*x^(3/2)) + (19*b^3*d
^3*n^2*log(c*(d + e/x^(1/2))^n))/(30*e^3*x^(3/2)) - (b^3*d^5*n*log(c*(d + e
/x^(1/2))^n)^2)/(e^5*x^(1/2)) + (49*b^3*d^5*n^2*log(c*(d + e/x^(1/2))^n))/(
10*e^5*x^(1/2)) - (37*a*b^2*d^2*n^2)/(120*e^2*x^2) - (29*a*b^2*d^4*n^2)/(20
*e^4*x) + (19*a*b^2*d^3*n^2)/(30*e^3*x^(3/2)) + (49*a*b^2*d^5*n^2)/(10*e^5*
x^(1/2)) - (a^2*b*d*n)/(5*e*x^(5/2)) - (2*a*b^2*d*n*log(c*(d + e/x^(1/2))^n
))/(5*e*x^(5/2)) + (a*b^2*d^2*n*log(c*(d + e/x^(1/2))^n))/(2*e^2*x^2) + (a*
b^2*d^4*n*log(c*(d + e/x^(1/2))^n))/(e^4*x) - (2*a*b^2*d^3*n*log(c*(d + e/x
^(1/2))^n))/(3*e^3*x^(3/2)) - (2*a*b^2*d^5*n*log(c*(d + e/x^(1/2))^n))/(e^5
*x^(1/2))
```

3.442 $\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx$

Optimal. Leaf size=234

$$\frac{bd^{11}n\sqrt[3]{x}}{4e^{11}}-\frac{bd^{10}nx^{2/3}}{8e^{10}}+\frac{bd^9nx}{12e^9}-\frac{bd^8nx^{4/3}}{16e^8}+\frac{bd^7nx^{5/3}}{20e^7}-\frac{bd^6nx^2}{24e^6}+\frac{bd^5nx^{7/3}}{28e^5}-\frac{bd^4nx^{8/3}}{32e^4}+\frac{bd^3nx^3}{36e^3}-\frac{bd^2nx^{10/3}}{40e^2}+b$$

```
[Out] 1/4*b*d^11*n*x^(1/3)/e^11-1/8*b*d^10*n*x^(2/3)/e^10+1/12*b*d^9*n*x/e^9-1/16
*b*d^8*n*x^(4/3)/e^8+1/20*b*d^7*n*x^(5/3)/e^7-1/24*b*d^6*n*x^2/e^6+1/28*b*d
^5*n*x^(7/3)/e^5-1/32*b*d^4*n*x^(8/3)/e^4+1/36*b*d^3*n*x^3/e^3-1/40*b*d^2*n
*x^(10/3)/e^2+1/44*b*d*n*x^(11/3)/e-1/48*b*n*x^4-1/4*b*d^12*n*ln(d+e*x^(1/3
))/e^12+1/4*x^4*(a+b*ln(c*(d+e*x^(1/3))^n))
```

Rubi [A]
time = 0.12, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2504, 2442, 45}

$$\frac{1}{4}x^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{bd^{12}n\log\left(d+e\sqrt[3]{x}\right)}{4e^{12}}+\frac{bd^{11}n\sqrt[3]{x}}{4e^{11}}-\frac{bd^{10}nx^{2/3}}{8e^{10}}+\frac{bd^9nx}{12e^9}-\frac{bd^8nx^{4/3}}{16e^8}+\frac{bd^7nx^{5/3}}{20e^7}-\frac{bd^6nx^2}{24e^6}+\frac{bd^5nx^{7/3}}{28e^5}-\frac{bd^4nx^{8/3}}{32e^4}+\frac{bd^3nx^3}{36e^3}-\frac{bd^2nx^{10/3}}{40e^2}+\frac{bdnx^{11/3}}{44e}-\frac{1}{48}bnx^4$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(1/3))^n]),x]
```

```
[Out] (b*d^11*n*x^(1/3))/(4*e^11) - (b*d^10*n*x^(2/3))/(8*e^10) + (b*d^9*n*x)/(12
*e^9) - (b*d^8*n*x^(4/3))/(16*e^8) + (b*d^7*n*x^(5/3))/(20*e^7) - (b*d^6*n*
x^2)/(24*e^6) + (b*d^5*n*x^(7/3))/(28*e^5) - (b*d^4*n*x^(8/3))/(32*e^4) + (
b*d^3*n*x^3)/(36*e^3) - (b*d^2*n*x^(10/3))/(40*e^2) + (b*d*n*x^(11/3))/(44*
e) - (b*n*x^4)/48 - (b*d^12*n*Log[d + e*x^(1/3)])/(4*e^12) + (x^4*(a + b*Lo
g[c*(d + e*x^(1/3))^n]))/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx = 3\text{Subst}\left(\int x^{11}(a+b\log(c(d+ex)^n))\,dx, x, \sqrt[3]{x}\right)$$
$$= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right) - \frac{1}{4}(ben)\text{Subst}\left(\int \frac{x^{12}}{d+ex}\,dx, x, \sqrt[3]{x}\right)$$
$$= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right) - \frac{1}{4}(ben)\text{Subst}\left(\int\left(-\frac{d^{11}}{e^{12}}+\frac{d^{10}x}{e^{11}}-\right.\right.$$
$$= \frac{bd^{11}n\sqrt[3]{x}}{4e^{11}} - \frac{bd^{10}nx^{2/3}}{8e^{10}} + \frac{bd^9nx}{12e^9} - \frac{bd^8nx^{4/3}}{16e^8} + \frac{bd^7nx^{5/3}}{20e^7} - \frac{bd^6nx^2}{24e^6} +$$

Mathematica [A]
time = 0.16, size = 219, normalized size = 0.94

$$\frac{ax^4}{4}-\frac{1}{4}ben\left(-\frac{d^{11}\sqrt[3]{x}}{e^{12}}+\frac{d^{10}x^{2/3}}{2e^{11}}-\frac{d^9x}{3e^{10}}+\frac{d^8x^{4/3}}{4e^9}-\frac{d^7x^{5/3}}{5e^8}+\frac{d^6x^2}{6e^7}-\frac{d^5x^{7/3}}{7e^6}+\frac{d^4x^{8/3}}{8e^5}-\frac{d^3x^3}{9e^4}+\frac{d^2x^{10/3}}{10e^3}-\frac{dx^{11/3}}{11e^2}+\frac{x^4}{12e}+\frac{d^{12}\log\left(d+e\sqrt[3]{x}\right)}{e^{13}}\right)+\frac{1}{4}bx^4\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(1/3))^n]),x]
[Out] (a*x^4)/4 - (b*e*n*(-((d^11*x^(1/3))/e^12) + (d^10*x^(2/3))/(2*e^11) - (d^9
*x)/(3*e^10) + (d^8*x^(4/3))/(4*e^9) - (d^7*x^(5/3))/(5*e^8) + (d^6*x^2)/(6
*e^7) - (d^5*x^(7/3))/(7*e^6) + (d^4*x^(8/3))/(8*e^5) - (d^3*x^3)/(9*e^4) +
 (d^2*x^(10/3))/(10*e^3) - (d*x^(11/3))/(11*e^2) + x^4/(12*e) + (d^12*Log[d
 + e*x^(1/3)])/e^13))/4 + (b*x^4*Log[c*(d + e*x^(1/3))^n])/4
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^3\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(1/3))^n)),x)
[Out] int(x^3*(a+b*ln(c*(d+e*x^(1/3))^n)),x)
```

Maxima [A]
time = 0.28, size = 164, normalized size = 0.70

$$\frac{1}{4}\,bx^4\log\left(\left(x^{\frac{1}{3}}e+d\right)^nc\right)+\frac{1}{4}\,ax^4-\frac{1}{110880}\left(27720\,d^{12}e^{(-13)}\log\left(x^{\frac{1}{3}}e+d\right)+\left(13860\,d^{10}x^{\frac{2}{3}}e-27720\,d^{11}x^{\frac{1}{3}}-9240\,d^9xe^2+6930\,d^8x^{\frac{4}{3}}e^3-5544\,d^7x^{\frac{5}{3}}e^4+4620\,d^6x^2e^5-3960\,d^5x^{\frac{7}{3}}e^6+3465\,d^4x^{\frac{8}{3}}e^7-3080\,d^3x^3e^8+2772\,d^2x^{\frac{10}{3}}e^9-2520\,dx^{\frac{11}{3}}e^{10}+2310\,x^4e^{11}\right)e^{(-12)}\right)bne$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4*log((x^(1/3)*e + d)^n*c) + 1/4*a*x^4 - 1/110880*(27720*d^12*e^(-1
3)*log(x^(1/3)*e + d) + (13860*d^10*x^(2/3)*e - 27720*d^11*x^(1/3) - 9240*d
^9*x*e^2 + 6930*d^8*x^(4/3)*e^3 - 5544*d^7*x^(5/3)*e^4 + 4620*d^6*x^2*e^5 -
 3960*d^5*x^(7/3)*e^6 + 3465*d^4*x^(8/3)*e^7 - 3080*d^3*x^3*e^8 + 2772*d^2*
x^(10/3)*e^9 - 2520*d*x^(11/3)*e^10 + 2310*x^4*e^11)*e^(-12))*b*n*e
```

Fricas [A]
time = 0.45, size = 186, normalized size = 0.79

$$\frac{1}{110880}\left(9240\,bd^9nxe^3-4620\,bd^6nx^2e^6+3080\,bd^3nx^3e^9+27720\,bx^4e^{12}\log\left(c\right)-2310\,(bn-12\,a)x^4e^{12}-27720\,\left(bd^{12}n-bnx^4e^{12}\right)\log\left(x^{\frac{1}{3}}e+d\right)-63\,\left(220\,bd^{10}ne^2-88\,bd^7nxe^5+55\,bd^4nx^2e^8-40\,bdnx^3e^{11}\right)x^{\frac{2}{3}}+198\,\left(140\,bd^{11}ne-35\,bd^8nxe^4+20\,bd^5nx^2e^7-14\,bd^2nx^3e^{10}\right)x^{\frac{1}{3}}\right)e^{(-12)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="fricas")
```

```
[Out] 1/110880*(9240*b*d^9*n*x*e^3 - 4620*b*d^6*n*x^2*e^6 + 3080*b*d^3*n*x^3*e^9
+ 27720*b*x^4*e^12*log(c) - 2310*(b*n - 12*a)*x^4*e^12 - 27720*(b*d^12*n -
b*n*x^4*e^12)*log(x^(1/3)*e + d) - 63*(220*b*d^10*n*e^2 - 88*b*d^7*n*x*e^5
+ 55*b*d^4*n*x^2*e^8 - 40*b*d*n*x^3*e^11)*x^(2/3) + 198*(140*b*d^11*n*e - 3
5*b*d^8*n*x*e^4 + 20*b*d^5*n*x^2*e^7 - 14*b*d^2*n*x^3*e^10)*x^(1/3))*e^(-12
)
```

Sympy [A]
time = 44.14, size = 216, normalized size = 0.92

$$\frac{ax^4}{4}+b\left(-\frac{en\left(\frac{3d^{12}\left(\begin{cases}\frac{\sqrt[3]{x}}{d} & \text{for}\ e=0\\ \frac{\log\left(d+e\sqrt[3]{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^{12}}-\frac{3d^{11}\sqrt[3]{x}}{e^{12}}+\frac{3d^{10}x^{\frac{2}{3}}}{2e^{11}}-\frac{d^9x}{e^{10}}+\frac{3d^8x^{\frac{4}{3}}}{4e^9}-\frac{3d^7x^{\frac{5}{3}}}{5e^8}+\frac{d^6x^2}{2e^7}-\frac{3d^5x^{\frac{7}{3}}}{7e^6}+\frac{3d^4x^{\frac{8}{3}}}{8e^5}-\frac{d^3x^3}{3e^4}+\frac{3d^2x^{\frac{10}{3}}}{10e^3}-\frac{3dx^{\frac{11}{3}}}{11e^2}+\frac{x^4}{4e}\right)}{12}+\frac{x^4\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(1/3))**n)),x)
```

```
[Out] a*x**4/4 + b*(-e*n*(3*d**12*Piecewise((x**(1/3)/d, Eq(e, 0)), (log(d + e*x*
*(1/3))/e, True))/e**12 - 3*d**11*x**(1/3)/e**12 + 3*d**10*x**(2/3)/(2*e**1
```

```
1) - d**9*x/e**10 + 3*d**8*x**(4/3)/(4*e**9) - 3*d**7*x**(5/3)/(5*e**8) + d
**6*x**2/(2*e**7) - 3*d**5*x**(7/3)/(7*e**6) + 3*d**4*x**(8/3)/(8*e**5) - d
**3*x**3/(3*e**4) + 3*d**2*x**(10/3)/(10*e**3) - 3*d*x**(11/3)/(11*e**2) +
x**4/(4*e))/12 + x**4*log(c*(d + e*x**(1/3))**n)/4)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 529 vs. 2(176) = 352.
time = 3.65, size = 529, normalized size = 2.26

$$\frac{1}{110880}\left(27720 b x^4 e \log(c) + 27720 a x^4 e + \left(27720 \left(x^{\frac{1}{3}} e + d\right)^{12} e^{(-11)} \log\left(x^{\frac{1}{3}} e + d\right) - 332640 \left(x^{\frac{1}{3}} e + d\right)^{11} d e^{(-11)} \log\left(x^{\frac{1}{3}} e + d\right) + 1829520 \left(x^{\frac{1}{3}} e + d\right)^{10} d^2 e^{(-11)} \log\left(x^{\frac{1}{3}} e + d\right) - 6098400 \left(x^{\frac{1}{3}} e + d\right)^{9} d^3 e^{(-11)} \log\left(x^{\frac{1}{3}} e + d\right) + 13721400 \left(x^{\frac{1}{3}} e + d\right)^{8} d^4 e^{(-11)} \log\left(x^{\frac{1}{3}} e + d\right) - 21954240 \left(x^{\frac{1}{3}} e + d\right)^{7} d^5 e^{(-11)} \log\left(x^{\frac{1}{3}} e + d\right) + 25613280 \left(x^{\frac{1}{3}} e + d\right)^{6} d^6 e^{(-11)} \log\left(x^{\frac{1}{3}} e + d\right) - 21954240 \left(x^{\frac{1}{3}} e + d\right)^{5} d^7 e^{(-11)} \log\left(x^{\frac{1}{3}} e + d\right) + 13721400 \left(x^{\frac{1}{3}} e + d\right)^{4} d^8 e^{(-11)} \log\left(x^{\frac{1}{3}} e + d\right) - 6098400 \left(x^{\frac{1}{3}} e + d\right)^{3} d^9 e^{(-11)} \log\left(x^{\frac{1}{3}} e + d\right) + 1829520 \left(x^{\frac{1}{3}} e + d\right)^{2} d^{10} e^{(-11)} \log\left(x^{\frac{1}{3}} e + d\right) - 332640 \left(x^{\frac{1}{3}} e + d\right) d^{11} e^{(-11)} \log\left(x^{\frac{1}{3}} e + d\right) - 2310 \left(x^{\frac{1}{3}} e + d\right)^{12} e^{(-11)} + 30240 \left(x^{\frac{1}{3}} e + d\right)^{11} d e^{(-11)} - 182952 \left(x^{\frac{1}{3}} e + d\right)^{10} d^2 e^{(-11)} + 677600 \left(x^{\frac{1}{3}} e + d\right)^{9} d^3 e^{(-11)} - 1715175 \left(x^{\frac{1}{3}} e + d\right)^{8} d^4 e^{(-11)} + 3136320 \left(x^{\frac{1}{3}} e + d\right)^{7} d^5 e^{(-11)} - 4268880 \left(x^{\frac{1}{3}} e + d\right)^{6} d^6 e^{(-11)} + 4390848 \left(x^{\frac{1}{3}} e + d\right)^{5} d^7 e^{(-11)} - 3430350 \left(x^{\frac{1}{3}} e + d\right)^{4} d^8 e^{(-11)} + 2032800 \left(x^{\frac{1}{3}} e + d\right)^{3} d^9 e^{(-11)} - 914760 \left(x^{\frac{1}{3}} e + d\right)^{2} d^{10} e^{(-11)} + 332640 \left(x^{\frac{1}{3}} e + d\right) d^{11} e^{(-11)}\right) b n\right) e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="giac")
```

```
[Out] 1/110880*(27720*b*x^4*e*log(c) + 27720*a*x^4*e + (27720*(x^(1/3)*e + d)^12*
e^(-11)*log(x^(1/3)*e + d) - 332640*(x^(1/3)*e + d)^11*d*e^(-11)*log(x^(1/3
)*e + d) + 1829520*(x^(1/3)*e + d)^10*d^2*e^(-11)*log(x^(1/3)*e + d) - 6098
400*(x^(1/3)*e + d)^9*d^3*e^(-11)*log(x^(1/3)*e + d) + 13721400*(x^(1/3)*e
+ d)^8*d^4*e^(-11)*log(x^(1/3)*e + d) - 21954240*(x^(1/3)*e + d)^7*d^5*e^(-
11)*log(x^(1/3)*e + d) + 25613280*(x^(1/3)*e + d)^6*d^6*e^(-11)*log(x^(1/3)
*e + d) - 21954240*(x^(1/3)*e + d)^5*d^7*e^(-11)*log(x^(1/3)*e + d) + 13721
400*(x^(1/3)*e + d)^4*d^8*e^(-11)*log(x^(1/3)*e + d) - 6098400*(x^(1/3)*e +
 d)^3*d^9*e^(-11)*log(x^(1/3)*e + d) + 1829520*(x^(1/3)*e + d)^2*d^10*e^(-1
1)*log(x^(1/3)*e + d) - 332640*(x^(1/3)*e + d)*d^11*e^(-11)*log(x^(1/3)*e +
 d) - 2310*(x^(1/3)*e + d)^12*e^(-11) + 30240*(x^(1/3)*e + d)^11*d*e^(-11)
- 182952*(x^(1/3)*e + d)^10*d^2*e^(-11) + 677600*(x^(1/3)*e + d)^9*d^3*e^(-
11) - 1715175*(x^(1/3)*e + d)^8*d^4*e^(-11) + 3136320*(x^(1/3)*e + d)^7*d^5
*e^(-11) - 4268880*(x^(1/3)*e + d)^6*d^6*e^(-11) + 4390848*(x^(1/3)*e + d)^
5*d^7*e^(-11) - 3430350*(x^(1/3)*e + d)^4*d^8*e^(-11) + 2032800*(x^(1/3)*e
+ d)^3*d^9*e^(-11) - 914760*(x^(1/3)*e + d)^2*d^10*e^(-11) + 332640*(x^(1/3
)*e + d)*d^11*e^(-11))*b*n)*e^(-1)
```

Mupad [B]
time = 0.65, size = 189, normalized size = 0.81

$$\frac{a\,x^4}{4}-\frac{b\,n\,x^4}{48}+\frac{b\,x^4\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{4}+\frac{b\,d\,n\,x^{11/3}}{44\,e}+\frac{b\,d^9\,n\,x}{12\,e^9}-\frac{b\,d^{12}\,n\,\ln\left(d+e\,x^{1/3}\right)}{4\,e^{12}}+\frac{b\,d^3\,n\,x^3}{36\,e^3}-\frac{b\,d^6\,n\,x^2}{24\,e^6}-\frac{b\,d^2\,n\,x^{10/3}}{40\,e^2}-\frac{b\,d^4\,n\,x^{8/3}}{32\,e^4}+\frac{b\,d^5\,n\,x^{7/3}}{28\,e^5}+\frac{b\,d^7\,n\,x^{5/3}}{20\,e^7}-\frac{b\,d^8\,n\,x^{4/3}}{16\,e^8}-\frac{b\,d^{10}\,n\,x^{2/3}}{8\,e^{10}}+\frac{b\,d^{11}\,n\,x^{1/3}}{4\,e^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e*x^(1/3))^n)),x)
```

```
[Out] (a*x^4)/4 - (b*n*x^4)/48 + (b*x^4*log(c*(d + e*x^(1/3))^n))/4 + (b*d*n*x^(1
1/3))/(44*e) + (b*d^9*n*x)/(12*e^9) - (b*d^12*n*log(d + e*x^(1/3)))/(4*e^12
) + (b*d^3*n*x^3)/(36*e^3) - (b*d^6*n*x^2)/(24*e^6) - (b*d^2*n*x^(10/3))/(4
0*e^2) - (b*d^4*n*x^(8/3))/(32*e^4) + (b*d^5*n*x^(7/3))/(28*e^5) + (b*d^7*n
*x^(5/3))/(20*e^7) - (b*d^8*n*x^(4/3))/(16*e^8) - (b*d^10*n*x^(2/3))/(8*e^1
0) + (b*d^11*n*x^(1/3))/(4*e^11)
```

3.443 $\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx$

Optimal. Leaf size=185

$$-\frac{bd^8n\sqrt[3]{x}}{3e^8}+\frac{bd^7nx^{2/3}}{6e^7}-\frac{bd^6nx}{9e^6}+\frac{bd^5nx^{4/3}}{12e^5}-\frac{bd^4nx^{5/3}}{15e^4}+\frac{bd^3nx^2}{18e^3}-\frac{bd^2nx^{7/3}}{21e^2}+\frac{bdnx^{8/3}}{24e}-\frac{1}{27}bnx^3+\frac{bd^9n\log(d+}{3e^9}$$

```
[Out] -1/3*b*d^8*n*x^(1/3)/e^8+1/6*b*d^7*n*x^(2/3)/e^7-1/9*b*d^6*n*x/e^6+1/12*b*d
^5*n*x^(4/3)/e^5-1/15*b*d^4*n*x^(5/3)/e^4+1/18*b*d^3*n*x^2/e^3-1/21*b*d^2*n
*x^(7/3)/e^2+1/24*b*d*n*x^(8/3)/e-1/27*b*n*x^3+1/3*b*d^9*n*ln(d+e*x^(1/3))/
e^9+1/3*x^3*(a+b*ln(c*(d+e*x^(1/3))^n))
```

Rubi [A]
time = 0.09, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2504, 2442, 45}

$$\frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)+\frac{bd^9n\log\left(d+e\sqrt[3]{x}\right)}{3e^9}-\frac{bd^8n\sqrt[3]{x}}{3e^8}+\frac{bd^7nx^{2/3}}{6e^7}-\frac{bd^6nx}{9e^6}+\frac{bd^5nx^{4/3}}{12e^5}-\frac{bd^4nx^{5/3}}{15e^4}+\frac{bd^3nx^2}{18e^3}-\frac{bd^2nx^{7/3}}{21e^2}+\frac{bdnx^{8/3}}{24e}-\frac{1}{27}bnx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(1/3))^n]),x]
```

```
[Out] -1/3*(b*d^8*n*x^(1/3))/e^8 + (b*d^7*n*x^(2/3))/(6*e^7) - (b*d^6*n*x)/(9*e^6
) + (b*d^5*n*x^(4/3))/(12*e^5) - (b*d^4*n*x^(5/3))/(15*e^4) + (b*d^3*n*x^2)
/(18*e^3) - (b*d^2*n*x^(7/3))/(21*e^2) + (b*d*n*x^(8/3))/(24*e) - (b*n*x^3)
/27 + (b*d^9*n*Log[d + e*x^(1/3)])/(3*e^9) + (x^3*(a + b*Log[c*(d + e*x^(1/
3))^n]))/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)dx &= 3\text{Subst}\left(\int x^8(a+b\log\left(c(d+ex)^n\right))\,dx,x,\sqrt[3]{x}\right)\\ &= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{1}{3}(ben)\text{Subst}\left(\int\frac{x^9}{d+ex}\,dx,x,\sqrt[3]{x}\right.\\ &= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{1}{3}(ben)\text{Subst}\left(\int\left(\frac{d^8}{e^9}-\frac{d^7x}{e^8}+\frac{d^6x^2}{e^7}\right.\right.\\ &= -\frac{bd^8n\sqrt[3]{x}}{3e^8}+\frac{bd^7nx^{2/3}}{6e^7}-\frac{bd^6nx}{9e^6}+\frac{bd^5nx^{4/3}}{12e^5}-\frac{bd^4nx^{5/3}}{15e^4}+\frac{bd^3nx^2}{18e^3}-\end{aligned}$$

Mathematica [A]
time = 0.10, size = 176, normalized size = 0.95

$$\frac{ax^3}{3}-\frac{1}{3}ben\left(\frac{d^8\sqrt[3]{x}}{e^9}-\frac{d^7x^{2/3}}{2e^8}+\frac{d^6x}{3e^7}-\frac{d^5x^{4/3}}{4e^6}+\frac{d^4x^{5/3}}{5e^5}-\frac{d^3x^2}{6e^4}+\frac{d^2x^{7/3}}{7e^3}-\frac{dx^{8/3}}{8e^2}+\frac{x^3}{9e}-\frac{d^9\log\left(d+e\sqrt[3]{x}\right)}{e^{10}}\right)+\frac{1}{3}bx^3\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(1/3))^n]),x]
```

```
[Out] (a*x^3)/3 - (b*e*n*((d^8*x^(1/3))/e^9 - (d^7*x^(2/3))/(2*e^8) + (d^6*x)/(3*
e^7) - (d^5*x^(4/3))/(4*e^6) + (d^4*x^(5/3))/(5*e^5) - (d^3*x^2)/(6*e^4) +
(d^2*x^(7/3))/(7*e^3) - (d*x^(8/3))/(8*e^2) + x^3/(9*e) - (d^9*Log[d + e*x^
(1/3)])/e^10))/3 + (b*x^3*Log[c*(d + e*x^(1/3))^n])/3
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(1/3))^n)),x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e*x^(1/3))^n)),x)
```

Maxima [A]
time = 0.27, size = 134, normalized size = 0.72

$$\frac{1}{3}bx^3\log\left(\left(x^{\frac{1}{3}}e+d\right)^nc\right)+\frac{1}{3}ax^3+\frac{1}{7560}\left(2520\,d^9e^{(-10)}\log\left(x^{\frac{1}{3}}e+d\right)+\left(1260\,d^7x^{\frac{2}{3}}e-2520\,d^8x^{\frac{1}{3}}-840\,d^6xe^2+630\,d^5x^{\frac{4}{3}}e^3-504\,d^4x^{\frac{5}{3}}e^4+420\,d^3x^2e^5-360\,d^2x^{\frac{7}{3}}e^6+315\,dx^{\frac{8}{3}}e^7-280\,x^3e^8\right)e^{(-9)}\right)bne$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3*log((x^(1/3)*e + d)^n*c) + 1/3*a*x^3 + 1/7560*(2520*d^9*e^(-10)*l
og(x^(1/3)*e + d) + (1260*d^7*x^(2/3)*e - 2520*d^8*x^(1/3) - 840*d^6*x*e^2
+ 630*d^5*x^(4/3)*e^3 - 504*d^4*x^(5/3)*e^4 + 420*d^3*x^2*e^5 - 360*d^2*x^(
7/3)*e^6 + 315*d*x^(8/3)*e^7 - 280*x^3*e^8)*e^(-9))*b*n*e
```

Fricas [A]
time = 0.38, size = 149, normalized size = 0.81

$$-\frac{1}{7560}\left(840\,bd^6nxe^3-420\,bd^3nx^2e^6-2520\,bx^3e^9\log\left(c\right)+280\,(bn-9\,a)x^3e^9-2520\,\left(bd^9n+bnx^3e^9\right)\log\left(x^{\frac{1}{3}}e+d\right)-63\,\left(20\,bd^7ne^2-8\,bd^4nxe^5+5\,bdnx^2e^8\right)x^{\frac{2}{3}}+90\,\left(28\,bd^8ne-7\,bd^5nxe^4+4\,bd^2nx^2e^7\right)x^{\frac{1}{3}}\right)e^{(-9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="fricas")
```

```
[Out] -1/7560*(840*b*d^6*n*x*e^3 - 420*b*d^3*n*x^2*e^6 - 2520*b*x^3*e^9*log(c) +
280*(b*n - 9*a)*x^3*e^9 - 2520*(b*d^9*n + b*n*x^3*e^9)*log(x^(1/3)*e + d) -
 63*(20*b*d^7*n*e^2 - 8*b*d^4*n*x*e^5 + 5*b*d*n*x^2*e^8)*x^(2/3) + 90*(28*b
*d^8*n*e - 7*b*d^5*n*x*e^4 + 4*b*d^2*n*x^2*e^7)*x^(1/3))*e^(-9)
```

Sympy [A]
time = 9.84, size = 173, normalized size = 0.94

$$\frac{ax^3}{3}+b\left(-\frac{en\left(-\frac{3d^9\left(\begin{cases}\frac{\sqrt[3]{x}}{d} & \text{for}\ e=0\\ \frac{\log\left(d+e\sqrt[3]{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^9}+\frac{3d^8\sqrt[3]{x}}{e^9}-\frac{3d^7x^{\frac{2}{3}}}{2e^8}+\frac{d^6x}{e^7}-\frac{3d^5x^{\frac{4}{3}}}{4e^6}+\frac{3d^4x^{\frac{5}{3}}}{5e^5}-\frac{d^3x^2}{2e^4}+\frac{3d^2x^{\frac{7}{3}}}{7e^3}-\frac{3dx^{\frac{8}{3}}}{8e^2}+\frac{x^3}{3e}\right)}{9}+\frac{x^3\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/3))**n)),x)
```

```
[Out] a*x**3/3 + b*(-e*n*(-3*d**9*Piecewise((x**(1/3)/d, Eq(e, 0)), (log(d + e*x*
*(1/3))/e, True))/e**9 + 3*d**8*x**(1/3)/e**9 - 3*d**7*x**(2/3)/(2*e**8) +
d**6*x/e**7 - 3*d**5*x**(4/3)/(4*e**6) + 3*d**4*x**(5/3)/(5*e**5) - d**3*x*
*2/(2*e**4) + 3*d**2*x**(7/3)/(7*e**3) - 3*d*x**(8/3)/(8*e**2) + x**3/(3*e)
)/9 + x**3*log(c*(d + e*x**(1/3))**n)/3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 400 vs. 2(140) = 280.
time = 5.90, size = 400, normalized size = 2.16

$$\frac{1}{7560}\left(2520\,bx^3e\log\left(c\right)+2520\,ax^3e+\left(2520\,\left(x^{\frac{1}{3}}e+d\right)^9e^{(-8)}\log\left(x^{\frac{1}{3}}e+d\right)-22680\,\left(x^{\frac{1}{3}}e+d\right)^8de^{(-8)}\log\left(x^{\frac{1}{3}}e+d\right)+90720\,\left(x^{\frac{1}{3}}e+d\right)^7d^2e^{(-8)}\log\left(x^{\frac{1}{3}}e+d\right)-211680\,\left(x^{\frac{1}{3}}e+d\right)^6d^3e^{(-8)}\log\left(x^{\frac{1}{3}}e+d\right)+317520\,\left(x^{\frac{1}{3}}e+d\right)^5d^4e^{(-8)}\log\left(x^{\frac{1}{3}}e+d\right)-317520\,\left(x^{\frac{1}{3}}e+d\right)^4d^5e^{(-8)}\log\left(x^{\frac{1}{3}}e+d\right)+211680\,\left(x^{\frac{1}{3}}e+d\right)^3d^6e^{(-8)}\log\left(x^{\frac{1}{3}}e+d\right)-90720\,\left(x^{\frac{1}{3}}e+d\right)^2d^7e^{(-8)}\log\left(x^{\frac{1}{3}}e+d\right)+22680\,\left(x^{\frac{1}{3}}e+d\right)d^8e^{(-8)}\log\left(x^{\frac{1}{3}}e+d\right)-280\,\left(x^{\frac{1}{3}}e+d\right)^9e^{(-8)}+2835\,\left(x^{\frac{1}{3}}e+d\right)^8de^{(-8)}-12960\,\left(x^{\frac{1}{3}}e+d\right)^7d^2e^{(-8)}+35280\,\left(x^{\frac{1}{3}}e+d\right)^6d^3e^{(-8)}-63504\,\left(x^{\frac{1}{3}}e+d\right)^5d^4e^{(-8)}+79380\,\left(x^{\frac{1}{3}}e+d\right)^4d^5e^{(-8)}-70560\,\left(x^{\frac{1}{3}}e+d\right)^3d^6e^{(-8)}+45360\,\left(x^{\frac{1}{3}}e+d\right)^2d^7e^{(-8)}-22680\,\left(x^{\frac{1}{3}}e+d\right)d^8e^{(-8)}\right)bn\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="giac")
```

```
[Out] 1/7560*(2520*b*x^3*e*log(c) + 2520*a*x^3*e + (2520*(x^(1/3)*e + d)^9*e^(-8)
*log(x^(1/3)*e + d) - 22680*(x^(1/3)*e + d)^8*d*e^(-8)*log(x^(1/3)*e + d) +
 90720*(x^(1/3)*e + d)^7*d^2*e^(-8)*log(x^(1/3)*e + d) - 211680*(x^(1/3)*e
+ d)^6*d^3*e^(-8)*log(x^(1/3)*e + d) + 317520*(x^(1/3)*e + d)^5*d^4*e^(-8)*
log(x^(1/3)*e + d) - 317520*(x^(1/3)*e + d)^4*d^5*e^(-8)*log(x^(1/3)*e + d)
 + 211680*(x^(1/3)*e + d)^3*d^6*e^(-8)*log(x^(1/3)*e + d) - 90720*(x^(1/3)*
e + d)^2*d^7*e^(-8)*log(x^(1/3)*e + d) + 22680*(x^(1/3)*e + d)*d^8*e^(-8)*l
og(x^(1/3)*e + d) - 280*(x^(1/3)*e + d)^9*e^(-8) + 2835*(x^(1/3)*e + d)^8*d
*e^(-8) - 12960*(x^(1/3)*e + d)^7*d^2*e^(-8) + 35280*(x^(1/3)*e + d)^6*d^3*
e^(-8) - 63504*(x^(1/3)*e + d)^5*d^4*e^(-8) + 79380*(x^(1/3)*e + d)^4*d^5*e
^(-8) - 70560*(x^(1/3)*e + d)^3*d^6*e^(-8) + 45360*(x^(1/3)*e + d)^2*d^7*e^
(-8) - 22680*(x^(1/3)*e + d)*d^8*e^(-8))*b*n)*e^(-1)
```

Mupad [B]
time = 0.51, size = 150, normalized size = 0.81

$$\frac{a\,x^3}{3}-\frac{b\,n\,x^3}{27}+\frac{b\,x^3\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{3}+\frac{b\,d\,n\,x^{8/3}}{24\,e}-\frac{b\,d^6\,n\,x}{9\,e^6}+\frac{b\,d^9\,n\,\ln\left(d+e\,x^{1/3}\right)}{3\,e^9}+\frac{b\,d^3\,n\,x^2}{18\,e^3}-\frac{b\,d^2\,n\,x^{7/3}}{21\,e^2}-\frac{b\,d^4\,n\,x^{5/3}}{15\,e^4}+\frac{b\,d^5\,n\,x^{4/3}}{12\,e^5}+\frac{b\,d^7\,n\,x^{2/3}}{6\,e^7}-\frac{b\,d^8\,n\,x^{1/3}}{3\,e^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e*x^(1/3))^n)),x)
```

```
[Out] (a*x^3)/3 - (b*n*x^3)/27 + (b*x^3*log(c*(d + e*x^(1/3))^n))/3 + (b*d*n*x^(8
/3))/(24*e) - (b*d^6*n*x)/(9*e^6) + (b*d^9*n*log(d + e*x^(1/3)))/(3*e^9) +
(b*d^3*n*x^2)/(18*e^3) - (b*d^2*n*x^(7/3))/(21*e^2) - (b*d^4*n*x^(5/3))/(15
*e^4) + (b*d^5*n*x^(4/3))/(12*e^5) + (b*d^7*n*x^(2/3))/(6*e^7) - (b*d^8*n*x
^(1/3))/(3*e^8)
```

3.444 $\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx$

Optimal. Leaf size=136

$$\frac{bd^5n\sqrt[3]{x}}{2e^5}-\frac{bd^4nx^{2/3}}{4e^4}+\frac{bd^3nx}{6e^3}-\frac{bd^2nx^{4/3}}{8e^2}+\frac{bdnx^{5/3}}{10e}-\frac{1}{12}bnx^2-\frac{bd^6n\log\left(d+e\sqrt[3]{x}\right)}{2e^6}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right.\right.\right.$$

```
[Out] 1/2*b*d^5*n*x^(1/3)/e^5-1/4*b*d^4*n*x^(2/3)/e^4+1/6*b*d^3*n*x/e^3-1/8*b*d^2
*n*x^(4/3)/e^2+1/10*b*d*n*x^(5/3)/e-1/12*b*n*x^2-1/2*b*d^6*n*ln(d+e*x^(1/3)
)/e^6+1/2*x^2*(a+b*ln(c*(d+e*x^(1/3))^n))
```

Rubi [A]
time = 0.06, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2504, 2442, 45}

$$\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{bd^6n\log\left(d+e\sqrt[3]{x}\right)}{2e^6}+\frac{bd^5n\sqrt[3]{x}}{2e^5}-\frac{bd^4nx^{2/3}}{4e^4}+\frac{bd^3nx}{6e^3}-\frac{bd^2nx^{4/3}}{8e^2}+\frac{bdnx^{5/3}}{10e}-\frac{1}{12}bnx^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*x^(1/3))^n]),x]
```

```
[Out] (b*d^5*n*x^(1/3))/(2*e^5) - (b*d^4*n*x^(2/3))/(4*e^4) + (b*d^3*n*x)/(6*e^3)
 - (b*d^2*n*x^(4/3))/(8*e^2) + (b*d*n*x^(5/3))/(10*e) - (b*n*x^2)/12 - (b*d
^6*n*Log[d + e*x^(1/3)])/(2*e^6) + (x^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
```

```
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx = 3\text{Subst}\left(\int x^5(a+b\log(c(d+ex)^n))\,dx,x,\sqrt[3]{x}\right)$$
$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{1}{2}(ben)\text{Subst}\left(\int\frac{x^6}{d+ex}\,dx,x,\sqrt[3]{x}\right)$$
$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{1}{2}(ben)\text{Subst}\left(\int\left(-\frac{d^5}{e^6}+\frac{d^4x}{e^5}-\frac{d^3x}{e^4}\right.\right.$$
$$=\frac{bd^5n\sqrt[3]{x}}{2e^5}-\frac{bd^4nx^{2/3}}{4e^4}+\frac{bd^3nx}{6e^3}-\frac{bd^2nx^{4/3}}{8e^2}+\frac{bdnx^{5/3}}{10e}-\frac{1}{12}bnx^2-\frac{bd^6}{}$$

Mathematica [A]
time = 0.07, size = 133, normalized size = 0.98

$$\frac{ax^2}{2}-\frac{1}{2}ben\left(-\frac{d^5\sqrt[3]{x}}{e^6}+\frac{d^4x^{2/3}}{2e^5}-\frac{d^3x}{3e^4}+\frac{d^2x^{4/3}}{4e^3}-\frac{dx^{5/3}}{5e^2}+\frac{x^2}{6e}+\frac{d^6\log\left(d+e\sqrt[3]{x}\right)}{e^7}\right)+\frac{1}{2}bx^2\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(1/3))^n]),x]
[Out] (a*x^2)/2 - (b*e*n*(-((d^5*x^(1/3))/e^6) + (d^4*x^(2/3))/(2*e^5) - (d^3*x)/
(3*e^4) + (d^2*x^(4/3))/(4*e^3) - (d*x^(5/3))/(5*e^2) + x^2/(6*e) + (d^6*Lo
g[d + e*x^(1/3)])/e^7))/2 + (b*x^2*Log[c*(d + e*x^(1/3))^n])/2
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e*x^(1/3))^n)),x)
[Out] int(x*(a+b*ln(c*(d+e*x^(1/3))^n)),x)
```

Maxima [A]
time = 0.29, size = 104, normalized size = 0.76

$$-\frac{1}{120}\left(60\,d^6e^{(-7)}\log\left(x^{\frac{1}{3}}e+d\right)+\left(30\,d^4x^{\frac{2}{3}}e-60\,d^5x^{\frac{1}{3}}-20\,d^3xe^2+15\,d^2x^{\frac{4}{3}}e^3-12\,dx^{\frac{5}{3}}e^4+10\,x^2e^5\right)e^{(-6)}\right)bne+\frac{1}{2}\,bx^2\log\left(\left(x^{\frac{1}{3}}e+d\right)^nc\right)+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="maxima")
```

```
[Out] -1/120*(60*d^6*e^(-7)*log(x^(1/3)*e + d) + (30*d^4*x^(2/3)*e - 60*d^5*x^(1/
3) - 20*d^3*x*e^2 + 15*d^2*x^(4/3)*e^3 - 12*d*x^(5/3)*e^4 + 10*x^2*e^5)*e^(
-6))*b*n*e + 1/2*b*x^2*log((x^(1/3)*e + d)^n*c) + 1/2*a*x^2
```

Fricas [A]
time = 0.40, size = 114, normalized size = 0.84

$$\frac{1}{120}\left(20\,bd^3nxe^3+60\,bx^2e^6\log\left(c\right)-10\,(bn-6\,a)x^2e^6-60\,\left(bd^6n-bnx^2e^6\right)\log\left(x^{\frac{1}{3}}e+d\right)-6\,\left(5\,bd^4ne^2-2\,bdnxe^5\right)x^{\frac{2}{3}}+15\,\left(4\,bd^5ne-bd^2nxe^4\right)x^{\frac{1}{3}}\right)e^{(-6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="fricas")
```

```
[Out] 1/120*(20*b*d^3*n*x*e^3 + 60*b*x^2*e^6*log(c) - 10*(b*n - 6*a)*x^2*e^6 - 60
*(b*d^6*n - b*n*x^2*e^6)*log(x^(1/3)*e + d) - 6*(5*b*d^4*n*e^2 - 2*b*d*n*x*
e^5)*x^(2/3) + 15*(4*b*d^5*n*e - b*d^2*n*x*e^4)*x^(1/3))*e^(-6)
```

Sympy [A]
time = 2.70, size = 131, normalized size = 0.96

$$\frac{ax^2}{2}+b\left(-\frac{en\left(\frac{3d^6\left(\begin{cases}\frac{\sqrt[3]{x}}{d} & \text{for}\ e=0\\ \frac{\log\left(d+e\sqrt[3]{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^6}-\frac{3d^5\sqrt[3]{x}}{e^6}+\frac{3d^4x^{\frac{2}{3}}}{2e^5}-\frac{d^3x}{e^4}+\frac{3d^2x^{\frac{4}{3}}}{4e^3}-\frac{3dx^{\frac{5}{3}}}{5e^2}+\frac{x^2}{2e}\right)}{6}+\frac{x^2\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/3))**n)),x)
```

```
[Out] a*x**2/2 + b*(-e*n*(3*d**6*Piecewise((x**(1/3)/d, Eq(e, 0)), (log(d + e*x**
(1/3))/e, True))/e**6 - 3*d**5*x**(1/3)/e**6 + 3*d**4*x**(2/3)/(2*e**5) - d
**3*x/e**4 + 3*d**2*x**(4/3)/(4*e**3) - 3*d*x**(5/3)/(5*e**2) + x**2/(2*e))
/6 + x**2*log(c*(d + e*x**(1/3))**n)/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(104) = 208.
time = 4.37, size = 271, normalized size = 1.99

$$\frac{1}{120}\left(60\,bx^2e\log\left(c\right)+60\,ax^2e+\left(60\,\left(x^{\frac{1}{3}}e+d\right)^6e^{(-5)}\log\left(x^{\frac{1}{3}}e+d\right)-360\,\left(x^{\frac{1}{3}}e+d\right)^5de^{(-5)}\log\left(x^{\frac{1}{3}}e+d\right)+900\,\left(x^{\frac{1}{3}}e+d\right)^4d^2e^{(-5)}\log\left(x^{\frac{1}{3}}e+d\right)-1200\,\left(x^{\frac{1}{3}}e+d\right)^3d^3e^{(-5)}\log\left(x^{\frac{1}{3}}e+d\right)+900\,\left(x^{\frac{1}{3}}e+d\right)^2d^4e^{(-5)}\log\left(x^{\frac{1}{3}}e+d\right)-360\,\left(x^{\frac{1}{3}}e+d\right)d^5e^{(-5)}\log\left(x^{\frac{1}{3}}e+d\right)-10\,\left(x^{\frac{1}{3}}e+d\right)^6e^{(-5)}+72\,\left(x^{\frac{1}{3}}e+d\right)^5de^{(-5)}-225\,\left(x^{\frac{1}{3}}e+d\right)^4d^2e^{(-5)}+400\,\left(x^{\frac{1}{3}}e+d\right)^3d^3e^{(-5)}-450\,\left(x^{\frac{1}{3}}e+d\right)^2d^4e^{(-5)}+360\,\left(x^{\frac{1}{3}}e+d\right)d^5e^{(-5)}\right)bn\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="giac")

[Out] 1/120*(60*b*x^2*e*log(c) + 60*a*x^2*e + (60*(x^(1/3)*e + d)^6*e^(-5)*log(x^(1/3)*e + d) - 360*(x^(1/3)*e + d)^5*d*e^(-5)*log(x^(1/3)*e + d) + 900*(x^(1/3)*e + d)^4*d^2*e^(-5)*log(x^(1/3)*e + d) - 1200*(x^(1/3)*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e + d) + 900*(x^(1/3)*e + d)^2*d^4*e^(-5)*log(x^(1/3)*e + d) - 360*(x^(1/3)*e + d)*d^5*e^(-5)*log(x^(1/3)*e + d) - 10*(x^(1/3)*e + d)^6*e^(-5) + 72*(x^(1/3)*e + d)^5*d*e^(-5) - 225*(x^(1/3)*e + d)^4*d^2*e^(-5) + 400*(x^(1/3)*e + d)^3*d^3*e^(-5) - 450*(x^(1/3)*e + d)^2*d^4*e^(-5) + 360*(x^(1/3)*e + d)*d^5*e^(-5))*b*n)*e^(-1)

Mupad [B]
time = 0.42, size = 111, normalized size = 0.82

$$\frac{a\,x^2}{2}-\frac{b\,n\,x^2}{12}+\frac{b\,x^2\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{2}+\frac{b\,d^3\,n\,x}{6\,e^3}+\frac{b\,d\,n\,x^{5/3}}{10\,e}-\frac{b\,d^6\,n\,\ln\left(d+e\,x^{1/3}\right)}{2\,e^6}-\frac{b\,d^2\,n\,x^{4/3}}{8\,e^2}-\frac{b\,d^4\,n\,x^{2/3}}{4\,e^4}+\frac{b\,d^5\,n\,x^{1/3}}{2\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a + b*log(c*(d + e*x^(1/3))^n)),x)

[Out] (a*x^2)/2 - (b*n*x^2)/12 + (b*x^2*log(c*(d + e*x^(1/3))^n))/2 + (b*d^3*n*x)/(6*e^3) + (b*d*n*x^(5/3))/(10*e) - (b*d^6*n*log(d + e*x^(1/3)))/(2*e^6) - (b*d^2*n*x^(4/3))/(8*e^2) - (b*d^4*n*x^(2/3))/(4*e^4) + (b*d^5*n*x^(1/3))/(2*e^5)

3.445 $\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx$

Optimal. Leaf size=77

$$-\frac{bd^2n\sqrt[3]{x}}{e^2}+\frac{bdnx^{2/3}}{2e}+ax-\frac{bnx}{3}+\frac{bd^3n\log\left(d+e\sqrt[3]{x}\right)}{e^3}+bx\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)$$

```
[Out] -b*d^2*n*x^(1/3)/e^2+1/2*b*d*n*x^(2/3)/e+a*x-1/3*b*n*x+b*d^3*n*ln(d+e*x^(1/
3))/e^3+b*x*ln(c*(d+e*x^(1/3))^n)
```

Rubi [A]
time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2498, 272, 45}

$$ax+bx\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)+\frac{bd^3n\log\left(d+e\sqrt[3]{x}\right)}{e^3}-\frac{bd^2n\sqrt[3]{x}}{e^2}+\frac{bdnx^{2/3}}{2e}-\frac{bnx}{3}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Log[c*(d + e*x^(1/3))^n],x]
```

```
[Out] -((b*d^2*n*x^(1/3))/e^2) + (b*d*n*x^(2/3))/(2*e) + a*x - (b*n*x)/3 + (b*d^3
*n*Log[d + e*x^(1/3)])/e^3 + b*x*Log[c*(d + e*x^(1/3))^n]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx = ax+b\int \log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\,dx$$
$$= ax+bx\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)-\frac{1}{3}(ben)\int \frac{\sqrt[3]{x}}{d+e\sqrt[3]{x}}\,dx$$
$$= ax+bx\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)-(ben)\text{Subst}\left(\int \frac{x^3}{d+ex}\,dx,x,\sqrt[3]{x}\right)$$
$$= ax+bx\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)-(ben)\text{Subst}\left(\int \left(\frac{d^2}{e^3}-\frac{dx}{e^2}+\frac{x^2}{e}-\frac{d^3}{e^3(d+\right.\right.$$
$$= -\frac{bd^2n\sqrt[3]{x}}{e^2}+\frac{bdnx^{2/3}}{2e}+ax-\frac{bnx}{3}+\frac{bd^3n\log\left(d+e\sqrt[3]{x}\right)}{e^3}+bx\log\left(c\left(d\right.\right.$$

Mathematica [A]
time = 0.03, size = 77, normalized size = 1.00

$$-\frac{bd^2n\sqrt[3]{x}}{e^2}+\frac{bdnx^{2/3}}{2e}+ax-\frac{bnx}{3}+\frac{bd^3n\log\left(d+e\sqrt[3]{x}\right)}{e^3}+bx\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Log[c*(d + e*x^(1/3))^n],x]
```

```
[Out] -((b*d^2*n*x^(1/3))/e^2) + (b*d*n*x^(2/3))/(2*e) + a*x - (b*n*x)/3 + (b*d^3
*n*Log[d + e*x^(1/3)])/e^3 + b*x*Log[c*(d + e*x^(1/3))^n]
```

Maple [A]
time = 0.02, size = 66, normalized size = 0.86

method	result	size
default	$-\frac{b\,d^2 n\,x^{\frac{1}{3}}}{e^2}+\frac{bdn\,x^{\frac{2}{3}}}{2e}+ax-\frac{bnx}{3}+\frac{b\,d^3 n\ln\left(d+e\,x^{\frac{1}{3}}\right)}{e^3}+bx\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^n\right)$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*ln(c*(d+e*x^(1/3))^n),x,method=_RETURNVERBOSE)
```

```
[Out] -b*d^2*n*x^(1/3)/e^2+1/2*b*d*n*x^(2/3)/e+a*x-1/3*b*n*x+b*d^3*n*ln(d+e*x^(1/
3))/e^3+b*x*ln(c*(d+e*x^(1/3))^n)
```

Maxima [A]
time = 0.27, size = 70, normalized size = 0.91

$$\frac{1}{6}\left(\left(6\,d^3e^{(-4)}\log\left(x^{\frac{1}{3}}e+d\right)+\left(3\,dx^{\frac{2}{3}}e-6\,d^2x^{\frac{1}{3}}-2\,xe^2\right)e^{(-3)}\right)ne+6\,x\log\left(\left(x^{\frac{1}{3}}e+d\right)^nc\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e*x^(1/3))^n),x, algorithm="maxima")
```

```
[Out] 1/6*((6*d^3*e^(-4)*log(x^(1/3)*e + d) + (3*d*x^(2/3)*e - 6*d^2*x^(1/3) - 2*
x*e^2)*e^(-3))*n*e + 6*x*log((x^(1/3)*e + d)^n*c))*b + a*x
```

Fricas [A]
time = 0.38, size = 71, normalized size = 0.92

$$-\frac{1}{6}\left(6\,bd^2nx^{\frac{1}{3}}e-3\,bdnx^{\frac{2}{3}}e^2-6\,bxe^3\log\left(c\right)+2\,(bn-3\,a)xe^3-6\,\left(bd^3n+bnxe^3\right)\log\left(x^{\frac{1}{3}}e+d\right)\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e*x^(1/3))^n),x, algorithm="fricas")
```

```
[Out] -1/6*(6*b*d^2*n*x^(1/3)*e - 3*b*d*n*x^(2/3)*e^2 - 6*b*x*e^3*log(c) + 2*(b*n
 - 3*a)*x*e^3 - 6*(b*d^3*n + b*n*x*e^3)*log(x^(1/3)*e + d))*e^(-3)
```

Sympy [A]
time = 0.75, size = 82, normalized size = 1.06

$$ax+b\left(-\frac{en\left(-\frac{3d^3\left(\begin{cases}\frac{\sqrt[3]{x}}{d} & \text{for } e=0\\ \frac{\log\left(d+e\sqrt[3]{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^3}+\frac{3d^2\sqrt[3]{x}}{e^3}-\frac{3dx^{\frac{2}{3}}}{2e^2}+\frac{x}{e}\right)}{3}+x\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*ln(c*(d+e*x**(1/3))**n),x)
```

```
[Out] a*x + b*(-e*n*(-3*d**3*Piecewise((x**(1/3)/d, Eq(e, 0)), (log(d + e*x**(1/3
))/e, True))/e**3 + 3*d**2*x**(1/3)/e**3 - 3*d*x**(2/3)/(2*e**2) + x/e)/3 +
 x*log(c*(d + e*x**(1/3))**n))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(64) = 128.
time = 4.38, size = 135, normalized size = 1.75

$$\frac{1}{6}\left(6\,xe\log\left(c\right)+\left(6\left(x^{\frac{1}{3}}e+d\right)^3e^{(-2)}\log\left(x^{\frac{1}{3}}e+d\right)-18\left(x^{\frac{1}{3}}e+d\right)^2de^{(-2)}\log\left(x^{\frac{1}{3}}e+d\right)+18\left(x^{\frac{1}{3}}e+d\right)d^2e^{(-2)}\log\left(x^{\frac{1}{3}}e+d\right)-2\left(x^{\frac{1}{3}}e+d\right)^3e^{(-2)}+9\left(x^{\frac{1}{3}}e+d\right)^2de^{(-2)}-18\left(x^{\frac{1}{3}}e+d\right)d^2e^{(-2)}\right)n\right)be^{(-1)}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e*x^(1/3))^n),x, algorithm="giac")
```

```
[Out] 1/6*(6*x*e*log(c) + (6*(x^(1/3)*e + d)^3*e^(-2)*log(x^(1/3)*e + d) - 18*(x^
(1/3)*e + d)^2*d*e^(-2)*log(x^(1/3)*e + d) + 18*(x^(1/3)*e + d)*d^2*e^(-2)*
log(x^(1/3)*e + d) - 2*(x^(1/3)*e + d)^3*e^(-2) + 9*(x^(1/3)*e + d)^2*d*e^(
-2) - 18*(x^(1/3)*e + d)*d^2*e^(-2))*n)*b*e^(-1) + a*x
```

Mupad [B]
time = 0.34, size = 65, normalized size = 0.84

$$a\,x+b\,x\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)-\frac{b\,n\,x}{3}+\frac{b\,d\,n\,x^{2/3}}{2\,e}+\frac{b\,d^3\,n\,\ln\left(d+e\,x^{1/3}\right)}{e^3}-\frac{b\,d^2\,n\,x^{1/3}}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*log(c*(d + e*x^(1/3))^n),x)
```

```
[Out] a*x + b*x*log(c*(d + e*x^(1/3))^n) - (b*n*x)/3 + (b*d*n*x^(2/3))/(2*e) + (b
*d^3*n*log(d + e*x^(1/3)))/e^3 - (b*d^2*n*x^(1/3))/e^2
```

3.446 $\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}\,dx$

Optimal. Leaf size=51

$$3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt[3]{x}}{d}\right)+3bn\text{Li}_2\left(1+\frac{e\sqrt[3]{x}}{d}\right)$$

```
[Out] 3*(a+b*ln(c*(d+e*x^(1/3))^n))*ln(-e*x^(1/3)/d)+3*b*n*polylog(2,1+e*x^(1/3)/
d)
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2504, 2441, 2352}

$$3bn\text{PolyLog}\left(2,\frac{e\sqrt[3]{x}}{d}+1\right)+3\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])/x,x]
```

```
[Out] 3*(a + b*Log[c*(d + e*x^(1/3))^n])*Log[-((e*x^(1/3))/d)] + 3*b*n*PolyLog[2,
 1 + (e*x^(1/3))/d]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}\,dx = 3\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x}\,dx, x, \sqrt[3]{x}\right)$$
$$= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt[3]{x}}{d}\right) - (3ben)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\right.$$
$$= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt[3]{x}}{d}\right) + 3bn\text{Li}_2\left(1+\frac{e\sqrt[3]{x}}{d}\right)$$

Mathematica [**A**]
time = 0.00, size = 53, normalized size = 1.04

$$3b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\log\left(-\frac{e\sqrt[3]{x}}{d}\right) + a\log(x) + 3bn\text{Li}_2\left(\frac{d+e\sqrt[3]{x}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])/x,x]
```

```
[Out] 3*b*Log[c*(d + e*x^(1/3))^n]*Log[-((e*x^(1/3))/d)] + a*Log[x] + 3*b*n*PolyL
og[2, (d + e*x^(1/3))/d]
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))/x,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))/x,x)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(47) = 94.
time = 0.44, size = 183, normalized size = 3.59

$$-3\left(\log\left(x^{\frac{1}{3}}\right)\log\left(\frac{e^{\left(\frac{1}{3}\log(x)+1\right)}}{d}+1\right)+\text{Li}_2\left(-\frac{e^{\left(\frac{1}{3}\log(x)+1\right)}}{d}\right)\right)bn+\frac{4\,bd^2\log\left(\left(x^{\frac{1}{3}}e+d\right)^n\right)\log(x)+4\left(bd^2\log(c)+ad^2\right)\log(x)+\frac{2\,bnxe^2\log(x)-3\,bnxe^2}{x^{\frac{1}{3}}}-\frac{4\left(bdnxe\log(x)-3\,bdnxe\right)}{x^{\frac{2}{3}}}}{4\,d^2}-\frac{3\left(4\,bdne^{\left(\frac{1}{3}\log(x)+1\right)}-bne^{\left(\frac{2}{3}\log(x)+2\right)}-2\left(2\,bdne^{\left(\frac{1}{3}\log(x)+1\right)}-bne^{\left(\frac{2}{3}\log(x)+2\right)}\right)\log\left(x^{\frac{1}{3}}\right)\right)}{4\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x,x, algorithm="maxima")
```

```
[Out] -3*(log(x^(1/3))*log(e^(1/3*log(x) + 1)/d + 1) + dilog(-e^(1/3*log(x) + 1)/
d))*b*n + 1/4*(4*b*d^2*log((x^(1/3)*e + d)^n)*log(x) + 4*(b*d^2*log(c) + a*
```

```
d^2)*log(x) + (2*b*n*x*e^2*log(x) - 3*b*n*x*e^2)/x^(1/3) - 4*(b*d*n*x*e*log
(x) - 3*b*d*n*x*e)/x^(2/3))/d^2 - 3/4*(4*b*d*n*e^(1/3*log(x) + 1) - b*n*e^(
2/3*log(x) + 2) - 2*(2*b*d*n*e^(1/3*log(x) + 1) - b*n*e^(2/3*log(x) + 2))*l
og(x^(1/3)))/d^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x,x, algorithm="fricas")
[Out] integral((b*log((x^(1/3)*e + d)^n*c) + a)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\log\left(c\left(d + e\sqrt[3]{x}\right)^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))/x,x)
[Out] Integral((a + b*log(c*(d + e*x**(1/3))**n))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x,x, algorithm="giac")
[Out] integrate((b*log((x^(1/3)*e + d)^n*c) + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + b\,\ln\left(c\,{\left(d + e\,x^{1/3}\right)}^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))/x,x)
[Out] int((a + b*log(c*(d + e*x^(1/3))^n))/x, x)
```

3.447 $\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^2}\,dx$

Optimal. Leaf size=87

$$-\frac{ben}{2dx^{2/3}}+\frac{be^2n}{d^2\sqrt[3]{x}}-\frac{be^3n\log\left(d+e\sqrt[3]{x}\right)}{d^3}-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}+\frac{be^3n\log(x)}{3d^3}$$

```
[Out] -1/2*b*e*n/d/x^(2/3)+b*e^2*n/d^2/x^(1/3)-b*e^3*n*ln(d+e*x^(1/3))/d^3+(-a-b*
ln(c*(d+e*x^(1/3))^n))/x+1/3*b*e^3*n*ln(x)/d^3
```

Rubi [A]
time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2504, 2442, 46}

$$-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}-\frac{be^3n\log\left(d+e\sqrt[3]{x}\right)}{d^3}+\frac{be^3n\log(x)}{3d^3}+\frac{be^2n}{d^2\sqrt[3]{x}}-\frac{ben}{2dx^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])/x^2,x]
```

```
[Out] -1/2*(b*e*n)/(d*x^(2/3)) + (b*e^2*n)/(d^2*x^(1/3)) - (b*e^3*n*Log[d + e*x^(
1/3)])/d^3 - (a + b*Log[c*(d + e*x^(1/3))^n])/x + (b*e^3*n*Log[x])/(3*d^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
```

```
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^2}\,dx &= 3\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^4}\,dx,x,\sqrt[3]{x}\right)\\ &= -\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}+(ben)\text{Subst}\left(\int \frac{1}{x^3(d+ex)}\,dx,x,\sqrt[3]{x}\right)\\ &= -\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}+(ben)\text{Subst}\left(\int \left(\frac{1}{dx^3}-\frac{e}{d^2x^2}+\frac{e^2}{d^3x}-\frac{}{d^3(}\right.\right.\\ &= -\frac{ben}{2dx^{2/3}}+\frac{be^2n}{d^2\sqrt[3]{x}}-\frac{be^3n\log\left(d+e\sqrt[3]{x}\right)}{d^3}-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}+\end{aligned}$$

Mathematica [A]
time = 0.03, size = 84, normalized size = 0.97

$$-\frac{a}{x}-\frac{b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}+ben\left(-\frac{1}{2dx^{2/3}}+\frac{e}{d^2\sqrt[3]{x}}-\frac{e^2\log\left(d+e\sqrt[3]{x}\right)}{d^3}+\frac{e^2\log(x)}{3d^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])/x^2,x]
```

```
[Out] -(a/x) - (b*Log[c*(d + e*x^(1/3))^n])/x + b*e*n*(-1/2*1/(d*x^(2/3)) + e/(d^
2*x^(1/3)) - (e^2*Log[d + e*x^(1/3)])/d^3 + (e^2*Log[x])/(3*d^3))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^n\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))/x^2,x)
```

Maxima [A]
time = 0.27, size = 77, normalized size = 0.89

$$-\frac{1}{6}\,bn\left(\frac{6\,e^2\log\left(x^{\frac{1}{3}}e+d\right)}{d^3}-\frac{2\,e^2\log\left(x\right)}{d^3}-\frac{3\left(2\,x^{\frac{1}{3}}e-d\right)}{d^2x^{\frac{2}{3}}}\right)e-\frac{b\log\left(\left(x^{\frac{1}{3}}e+d\right)^nc\right)}{x}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^2,x, algorithm="maxima")
```

```
[Out] -1/6*b*n*(6*e^2*log(x^(1/3)*e + d)/d^3 - 2*e^2*log(x)/d^3 - 3*(2*x^(1/3)*e
- d)/(d^2*x^(2/3)))*e - b*log((x^(1/3)*e + d)^n*c)/x - a/x
```

Fricas [A]
time = 0.50, size = 79, normalized size = 0.91

$$-\frac{bd^2nx^{\frac{1}{3}}e + 2\,bd^3\log\left(c\right) - 2\,bnxe^3\log\left(x^{\frac{1}{3}}\right) - 2\,bdnx^{\frac{2}{3}}e^2 + 2\,ad^3 + 2\left(bd^3n + bnxe^3\right)\log\left(x^{\frac{1}{3}}e + d\right)}{2\,d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b*d^2*n*x^(1/3)*e + 2*b*d^3*log(c) - 2*b*n*x*e^3*log(x^(1/3)) - 2*b*d
*n*x^(2/3)*e^2 + 2*a*d^3 + 2*(b*d^3*n + b*n*x*e^3)*log(x^(1/3)*e + d))/(d^3
*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 450 vs. 2(83) = 166.
time = 106.56, size = 450, normalized size = 5.17

$$\begin{cases} -\frac{6ad^4x^{\frac{2}{3}}}{6d^4x^{\frac{5}{3}}+6d^3ex^2} - \frac{6ad^3ex}{6d^4x^{\frac{5}{3}}+6d^3ex^2} - \frac{6bd^4x^{\frac{2}{3}}\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{6d^4x^{\frac{5}{3}}+6d^3ex^2} - \frac{3bd^3enx}{6d^4x^{\frac{5}{3}}+6d^3ex^2} - \frac{6bd^3ex\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{6d^4x^{\frac{5}{3}}+6d^3ex^2} + \frac{3bd^2e^2nx^{\frac{4}{3}}}{6d^4x^{\frac{5}{3}}+6d^3ex^2} + \frac{2bde^3nx^{\frac{5}{3}}\log\left(x\right)}{6d^4x^{\frac{5}{3}}+6d^3ex^2} + \frac{6bde^3nx^{\frac{5}{3}}}{6d^4x^{\frac{5}{3}}+6d^3ex^2} - \frac{6bde^3x^{\frac{5}{3}}\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{6d^4x^{\frac{5}{3}}+6d^3ex^2} + \frac{2be^4nx^2\log\left(x\right)}{6d^4x^{\frac{5}{3}}+6d^3ex^2} - \frac{6be^4x^2\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{6d^4x^{\frac{5}{3}}+6d^3ex^2} & \text{for}\: d \neq 0 \\ -\frac{a}{x} - \frac{bn}{3x} - \frac{b\log\left(c\left(e\sqrt[3]{x}\right)^n\right)}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))/x**2,x)
```

```
[Out] Piecewise((-6*a*d**4*x**(2/3)/(6*d**4*x**(5/3) + 6*d**3*e*x**2) - 6*a*d**3*
e*x/(6*d**4*x**(5/3) + 6*d**3*e*x**2) - 6*b*d**4*x**(2/3)*log(c*(d + e*x**(
1/3))**n)/(6*d**4*x**(5/3) + 6*d**3*e*x**2) - 3*b*d**3*e*n*x/(6*d**4*x**(5/
3) + 6*d**3*e*x**2) - 6*b*d**3*e*x*log(c*(d + e*x**(1/3))**n)/(6*d**4*x**(5
/3) + 6*d**3*e*x**2) + 3*b*d**2*e**2*n*x**(4/3)/(6*d**4*x**(5/3) + 6*d**3*e
*x**2) + 2*b*d*e**3*n*x**(5/3)*log(x)/(6*d**4*x**(5/3) + 6*d**3*e*x**2) + 6
*b*d*e**3*n*x**(5/3)/(6*d**4*x**(5/3) + 6*d**3*e*x**2) - 6*b*d*e**3*x**(5/3
)*log(c*(d + e*x**(1/3))**n)/(6*d**4*x**(5/3) + 6*d**3*e*x**2) + 2*b*e**4*n
*x**2*log(x)/(6*d**4*x**(5/3) + 6*d**3*e*x**2) - 6*b*e**4*x**2*log(c*(d + e
*x**(1/3))**n)/(6*d**4*x**(5/3) + 6*d**3*e*x**2), Ne(d, 0)), (-a/x - b*n/(3
*x) - b*log(c*(e*x**(1/3))**n)/x, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(75) = 150.
time = 4.20, size = 280, normalized size = 3.22

$$-\frac{\left(2\left(x^{\frac{1}{3}}e + d\right)^3 bne^4\log\left(x^{\frac{1}{3}}e + d\right) - 6\left(x^{\frac{1}{3}}e + d\right)^2 bdne^4\log\left(x^{\frac{1}{3}}e + d\right) + 6\left(x^{\frac{1}{3}}e + d\right)bd^2ne^4\log\left(x^{\frac{1}{3}}e + d\right) - 2\left(x^{\frac{1}{3}}e + d\right)^3 bne^4\log\left(x^{\frac{1}{3}}e\right) + 6\left(x^{\frac{1}{3}}e + d\right)^2 bdne^4\log\left(x^{\frac{1}{3}}e\right) - 6\left(x^{\frac{1}{3}}e + d\right)bd^2ne^4\log\left(x^{\frac{1}{3}}e\right) + 2\,bd^3ne^4\log\left(x^{\frac{1}{3}}e\right) - 2\left(x^{\frac{1}{3}}e + d\right)^2 bdne^4 + 5\left(x^{\frac{1}{3}}e + d\right)bd^2ne^4 - 3\,bd^3ne^4 + 2\,bd^3e^4\log\left(c\right) + 2\,ad^3e^4\right)e^{(-1)}}{2\left(\left(x^{\frac{1}{3}}e + d\right)^3 d^3 - 3\left(x^{\frac{1}{3}}e + d\right)^2 d^4 + 3\left(x^{\frac{1}{3}}e + d\right)d^5 - d^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(x^(1/3)*e + d)^3*b*n*e^4*log(x^(1/3)*e + d) - 6*(x^(1/3)*e + d)^2*
b*d*n*e^4*log(x^(1/3)*e + d) + 6*(x^(1/3)*e + d)*b*d^2*n*e^4*log(x^(1/3)*e
+ d) - 2*(x^(1/3)*e + d)^3*b*n*e^4*log(x^(1/3)*e) + 6*(x^(1/3)*e + d)^2*b*d
*n*e^4*log(x^(1/3)*e) - 6*(x^(1/3)*e + d)*b*d^2*n*e^4*log(x^(1/3)*e) + 2*b*
d^3*n*e^4*log(x^(1/3)*e) - 2*(x^(1/3)*e + d)^2*b*d*n*e^4 + 5*(x^(1/3)*e + d
)*b*d^2*n*e^4 - 3*b*d^3*n*e^4 + 2*b*d^3*e^4*log(c) + 2*a*d^3*e^4)*e^(-1)/((
x^(1/3)*e + d)^3*d^3 - 3*(x^(1/3)*e + d)^2*d^4 + 3*(x^(1/3)*e + d)*d^5 - d^
6)
```

Mupad [B]
time = 0.59, size = 74, normalized size = 0.85

$$-\frac{\frac{b\,e\,n}{2\,d}-\frac{b\,e^2\,n\,x^{1/3}}{d^2}}{x^{2/3}}-\frac{a}{x}-\frac{b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{x}-\frac{2\,b\,e^3\,n\,\mathrm{atanh}\left(\frac{2\,e\,x^{1/3}}{d}+1\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))/x^2,x)
```

```
[Out] - ((b*e*n)/(2*d) - (b*e^2*n*x^(1/3))/d^2)/x^(2/3) - a/x - (b*log(c*(d + e*x
^(1/3))^n))/x - (2*b*e^3*n*atanh((2*e*x^(1/3))/d + 1))/d^3
```

3.448 $\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^3}\,dx$

Optimal. Leaf size=143

$$-\frac{ben}{10dx^{5/3}}+\frac{be^2n}{8d^2x^{4/3}}-\frac{be^3n}{6d^3x}+\frac{be^4n}{4d^4x^{2/3}}-\frac{be^5n}{2d^5\sqrt[3]{x}}+\frac{be^6n\log\left(d+e\sqrt[3]{x}\right)}{2d^6}-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{2x^2}-\frac{be^6n\log(x)}{6d^6}$$

```
[Out] -1/10*b*e*n/d/x^(5/3)+1/8*b*e^2*n/d^2/x^(4/3)-1/6*b*e^3*n/d^3/x+1/4*b*e^4*n
/d^4/x^(2/3)-1/2*b*e^5*n/d^5/x^(1/3)+1/2*b*e^6*n*ln(d+e*x^(1/3))/d^6+1/2*(-
a-b*ln(c*(d+e*x^(1/3))^n))/x^2-1/6*b*e^6*n*ln(x)/d^6
```

Rubi [A]
time = 0.06, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2504, 2442, 46}

$$-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{2x^2}+\frac{be^6n\log\left(d+e\sqrt[3]{x}\right)}{2d^6}-\frac{be^6n\log(x)}{6d^6}-\frac{be^5n}{2d^5\sqrt[3]{x}}+\frac{be^4n}{4d^4x^{2/3}}-\frac{be^3n}{6d^3x}+\frac{be^2n}{8d^2x^{4/3}}-\frac{ben}{10dx^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])/x^3,x]
```

```
[Out] -1/10*(b*e*n)/(d*x^(5/3)) + (b*e^2*n)/(8*d^2*x^(4/3)) - (b*e^3*n)/(6*d^3*x)
 + (b*e^4*n)/(4*d^4*x^(2/3)) - (b*e^5*n)/(2*d^5*x^(1/3)) + (b*e^6*n*Log[d +
 e*x^(1/3)])/(2*d^6) - (a + b*Log[c*(d + e*x^(1/3))^n])/(2*x^2) - (b*e^6*n*
Log[x])/(6*d^6)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^3}\,dx &= 3\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^7}\,dx, x, \sqrt[3]{x}\right)\\ &= -\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{2x^2}+\frac{1}{2}(ben)\text{Subst}\left(\int \frac{1}{x^6(d+ex)}\,dx, x, \sqrt[3]{x}\right)\\ &= -\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{2x^2}+\frac{1}{2}(ben)\text{Subst}\left(\int \left(\frac{1}{dx^6}-\frac{e}{d^2x^5}+\frac{e^2}{d^3x^4}-\right.\right.\\ &= -\frac{ben}{10dx^{5/3}}+\frac{be^2n}{8d^2x^{4/3}}-\frac{be^3n}{6d^3x}+\frac{be^4n}{4d^4x^{2/3}}-\frac{be^5n}{2d^5\sqrt[3]{x}}+\frac{be^6n\log\left(d+e\sqrt[3]{x}\right.}{2d^6}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 134, normalized size = 0.94

$$-\frac{a}{2x^2}-\frac{b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{2x^2}+\frac{1}{2}ben\left(-\frac{1}{5dx^{5/3}}+\frac{e}{4d^2x^{4/3}}-\frac{e^2}{3d^3x}+\frac{e^3}{2d^4x^{2/3}}-\frac{e^4}{d^5\sqrt[3]{x}}+\frac{e^5\log\left(d+e\sqrt[3]{x}\right)}{d^6}-\frac{e^5\log(x)}{3d^6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])/x^3,x]
```

```
[Out] -1/2*a/x^2 - (b*Log[c*(d + e*x^(1/3))^n])/(2*x^2) + (b*e*n*(-1/5*1/(d*x^(5/
3)) + e/(4*d^2*x^(4/3)) - e^2/(3*d^3*x) + e^3/(2*d^4*x^(2/3)) - e^4/(d^5*x^
(1/3)) + (e^5*Log[d + e*x^(1/3)])/d^6 - (e^5*Log[x])/(3*d^6)))/2
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^n\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))/x^3,x)
```

Maxima [A]
time = 0.28, size = 104, normalized size = 0.73

$$\frac{1}{120}bn\left(\frac{60\,e^5\log\left(x^{\frac{1}{3}}e+d\right)}{d^6}-\frac{20\,e^5\log(x)}{d^6}+\frac{15\,d^3x^{\frac{1}{3}}e-12\,d^4-20\,d^2x^{\frac{2}{3}}e^2+30\,dxe^3-60\,x^{\frac{4}{3}}e^4}{d^5x^{\frac{5}{3}}}\right)e-\frac{b\log\left(\left(x^{\frac{1}{3}}e+d\right)^nc\right)}{2\,x^2}-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^3,x, algorithm="maxima")
```

```
[Out] 1/120*b*n*(60*e^5*log(x^(1/3)*e + d)/d^6 - 20*e^5*log(x)/d^6 + (15*d^3*x^(1
/3)*e - 12*d^4 - 20*d^2*x^(2/3)*e^2 + 30*d*x*e^3 - 60*x^(4/3)*e^4)/(d^5*x^(
5/3)))*e - 1/2*b*log((x^(1/3)*e + d)^n*c)/x^2 - 1/2*a/x^2
```

Fricas [A]
time = 0.37, size = 120, normalized size = 0.84

$$-\frac{60\,bd^{6}\log\left(c\right)+60\,ad^{6}+20\,bd^{3}nxe^{3}+60\,bnx^{2}e^{6}\log\left(x^{\frac{1}{3}}\right)+60\,(bd^{6}n-bnx^{2}e^{6})\log\left(x^{\frac{1}{3}}e+d\right)-15\,(bd^{4}ne^{2}-4\,bdnxe^{5})x^{\frac{2}{3}}+6\,(2\,bd^{5}ne-5\,bd^{2}nxe^{4})x^{\frac{1}{3}}}{120\,d^{6}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^3,x, algorithm="fricas")
```

```
[Out] -1/120*(60*b*d^6*log(c) + 60*a*d^6 + 20*b*d^3*n*x*e^3 + 60*b*n*x^2*e^6*log(
x^(1/3)) + 60*(b*d^6*n - b*n*x^2*e^6)*log(x^(1/3)*e + d) - 15*(b*d^4*n*e^2
- 4*b*d*n*x*e^5)*x^(2/3) + 6*(2*b*d^5*n*e - 5*b*d^2*n*x*e^4)*x^(1/3))/(d^6*
x^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))/x**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 542 vs. 2(112) = 224.
time = 3.93, size = 542, normalized size = 3.79

$$\frac{\left(60\left(x^{\frac{1}{3}}e+d\right)^{6}bne^{7}\log\left(x^{\frac{1}{3}}e+d\right)-360\left(x^{\frac{1}{3}}e+d\right)^{5}bdne^{7}\log\left(x^{\frac{1}{3}}e+d\right)+900\left(x^{\frac{1}{3}}e+d\right)^{4}bd^{2}ne^{7}\log\left(x^{\frac{1}{3}}e+d\right)-1200\left(x^{\frac{1}{3}}e+d\right)^{3}bd^{3}ne^{7}\log\left(x^{\frac{1}{3}}e+d\right)+900\left(x^{\frac{1}{3}}e+d\right)^{2}bd^{4}ne^{7}\log\left(x^{\frac{1}{3}}e+d\right)-360\left(x^{\frac{1}{3}}e+d\right)bd^{5}ne^{7}\log\left(x^{\frac{1}{3}}e+d\right)-60\left(x^{\frac{1}{3}}e+d\right)^{6}bne^{7}\log\left(x^{\frac{1}{3}}e\right)+360\left(x^{\frac{1}{3}}e+d\right)^{5}bdne^{7}\log\left(x^{\frac{1}{3}}e\right)-900\left(x^{\frac{1}{3}}e+d\right)^{4}bd^{2}ne^{7}\log\left(x^{\frac{1}{3}}e\right)+1200\left(x^{\frac{1}{3}}e+d\right)^{3}bd^{3}ne^{7}\log\left(x^{\frac{1}{3}}e\right)-900\left(x^{\frac{1}{3}}e+d\right)^{2}bd^{4}ne^{7}\log\left(x^{\frac{1}{3}}e\right)+360\left(x^{\frac{1}{3}}e+d\right)bd^{5}ne^{7}\log\left(x^{\frac{1}{3}}e\right)-60\,bd^{6}ne^{7}\log\left(x^{\frac{1}{3}}e\right)-60\left(x^{\frac{1}{3}}e+d\right)^{5}bdne^{7}+330\left(x^{\frac{1}{3}}e+d\right)^{4}bd^{2}ne^{7}-740\left(x^{\frac{1}{3}}e+d\right)^{3}bd^{3}ne^{7}+855\left(x^{\frac{1}{3}}e+d\right)^{2}bd^{4}ne^{7}-522\left(x^{\frac{1}{3}}e+d\right)bd^{5}ne^{7}+137\,bd^{6}ne^{7}-60\,bd^{6}e^{7}\log\left(c\right)-60\,ad^{6}e^{7}\right)e^{(-1)}}{120\left(\left(x^{\frac{1}{3}}e+d\right)^{6}d^{6}-6\left(x^{\frac{1}{3}}e+d\right)^{5}d^{7}+15\left(x^{\frac{1}{3}}e+d\right)^{4}d^{8}-20\left(x^{\frac{1}{3}}e+d\right)^{3}d^{9}+15\left(x^{\frac{1}{3}}e+d\right)^{2}d^{10}-6\left(x^{\frac{1}{3}}e+d\right)d^{11}+d^{12}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^3,x, algorithm="giac")
```

```
[Out] 1/120*(60*(x^(1/3)*e + d)^6*b*n*e^7*log(x^(1/3)*e + d) - 360*(x^(1/3)*e + d
)^5*b*d*n*e^7*log(x^(1/3)*e + d) + 900*(x^(1/3)*e + d)^4*b*d^2*n*e^7*log(x^
(1/3)*e + d) - 1200*(x^(1/3)*e + d)^3*b*d^3*n*e^7*log(x^(1/3)*e + d) + 900*
(x^(1/3)*e + d)^2*b*d^4*n*e^7*log(x^(1/3)*e + d) - 360*(x^(1/3)*e + d)*b*d^
5*n*e^7*log(x^(1/3)*e + d) - 60*(x^(1/3)*e + d)^6*b*n*e^7*log(x^(1/3)*e) +
360*(x^(1/3)*e + d)^5*b*d*n*e^7*log(x^(1/3)*e) - 900*(x^(1/3)*e + d)^4*b*d^
```

```
2*n*e^7*log(x^(1/3)*e) + 1200*(x^(1/3)*e + d)^3*b*d^3*n*e^7*log(x^(1/3)*e)
- 900*(x^(1/3)*e + d)^2*b*d^4*n*e^7*log(x^(1/3)*e) + 360*(x^(1/3)*e + d)*b*
d^5*n*e^7*log(x^(1/3)*e) - 60*b*d^6*n*e^7*log(x^(1/3)*e) - 60*(x^(1/3)*e +
d)^5*b*d*n*e^7 + 330*(x^(1/3)*e + d)^4*b*d^2*n*e^7 - 740*(x^(1/3)*e + d)^3*
b*d^3*n*e^7 + 855*(x^(1/3)*e + d)^2*b*d^4*n*e^7 - 522*(x^(1/3)*e + d)*b*d^5
*n*e^7 + 137*b*d^6*n*e^7 - 60*b*d^6*e^7*log(c) - 60*a*d^6*e^7)*e^(-1)/((x^(
1/3)*e + d)^6*d^6 - 6*(x^(1/3)*e + d)^5*d^7 + 15*(x^(1/3)*e + d)^4*d^8 - 20
*(x^(1/3)*e + d)^3*d^9 + 15*(x^(1/3)*e + d)^2*d^10 - 6*(x^(1/3)*e + d)*d^11
 + d^12)
```

Mupad [B]
time = 0.66, size = 109, normalized size = 0.76

$$\frac{b\,e^6\,n\,\mathrm{atanh}\left(\frac{2\,e\,x^{1/3}}{d}+1\right)}{d^6}-\frac{\frac{b\,e\,n}{5\,d}-\frac{b\,e^4\,n\,x}{2\,d^4}-\frac{b\,e^2\,n\,x^{1/3}}{4\,d^2}+\frac{b\,e^3\,n\,x^{2/3}}{3\,d^3}+\frac{b\,e^5\,n\,x^{4/3}}{d^5}}{2\,x^{5/3}}-\frac{b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{2\,x^2}-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))/x^3,x)
```

```
[Out] (b*e^6*n*atanh((2*e*x^(1/3))/d + 1))/d^6 - ((b*e*n)/(5*d) - (b*e^4*n*x)/(2*
d^4) - (b*e^2*n*x^(1/3))/(4*d^2) + (b*e^3*n*x^(2/3))/(3*d^3) + (b*e^5*n*x^(
4/3))/d^5)/(2*x^(5/3)) - (b*log(c*(d + e*x^(1/3))^n))/(2*x^2) - a/(2*x^2)
```

3.449 $\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^4}\,dx$

Optimal. Leaf size=192

$$-\frac{ben}{24dx^{8/3}}+\frac{be^2n}{21d^2x^{7/3}}-\frac{be^3n}{18d^3x^2}+\frac{be^4n}{15d^4x^{5/3}}-\frac{be^5n}{12d^5x^{4/3}}+\frac{be^6n}{9d^6x}-\frac{be^7n}{6d^7x^{2/3}}+\frac{be^8n}{3d^8\sqrt[3]{x}}-\frac{be^9n\log\left(d+e\sqrt[3]{x}\right)}{3d^9}-\frac{a+b\,\mathrm{l}}{}$$

```
[Out] -1/24*b*e*n/d/x^(8/3)+1/21*b*e^2*n/d^2/x^(7/3)-1/18*b*e^3*n/d^3/x^2+1/15*b*
e^4*n/d^4/x^(5/3)-1/12*b*e^5*n/d^5/x^(4/3)+1/9*b*e^6*n/d^6/x-1/6*b*e^7*n/d^
7/x^(2/3)+1/3*b*e^8*n/d^8/x^(1/3)-1/3*b*e^9*n*ln(d+e*x^(1/3))/d^9+1/3*(-a-b
*ln(c*(d+e*x^(1/3))^n))/x^3+1/9*b*e^9*n*ln(x)/d^9
```

Rubi [A]
time = 0.09, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2504, 2442, 46}

$$-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{3x^3}-\frac{be^9n\log\left(d+e\sqrt[3]{x}\right)}{3d^9}+\frac{be^9n\log(x)}{9d^9}+\frac{be^8n}{3d^8\sqrt[3]{x}}-\frac{be^7n}{6d^7x^{2/3}}+\frac{be^6n}{9d^6x}-\frac{be^5n}{12d^5x^{4/3}}+\frac{be^4n}{15d^4x^{5/3}}-\frac{be^3n}{18d^3x^2}+\frac{be^2n}{21d^2x^{7/3}}-\frac{ben}{24dx^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])/x^4,x]
```

```
[Out] -1/24*(b*e*n)/(d*x^(8/3)) + (b*e^2*n)/(21*d^2*x^(7/3)) - (b*e^3*n)/(18*d^3*
x^2) + (b*e^4*n)/(15*d^4*x^(5/3)) - (b*e^5*n)/(12*d^5*x^(4/3)) + (b*e^6*n)/
(9*d^6*x) - (b*e^7*n)/(6*d^7*x^(2/3)) + (b*e^8*n)/(3*d^8*x^(1/3)) - (b*e^9*
n*Log[d + e*x^(1/3)])/(3*d^9) - (a + b*Log[c*(d + e*x^(1/3))^n])/(3*x^3) +
(b*e^9*n*Log[x])/(9*d^9)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^4}\,dx = 3\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^{10}}\,dx, x, \sqrt[3]{x}\right)$$
$$= -\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{3x^3} + \frac{1}{3}(ben)\text{Subst}\left(\int \frac{1}{x^9(d+ex)}\,dx, x, \sqrt[3]{x}\right)$$
$$= -\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{3x^3} + \frac{1}{3}(ben)\text{Subst}\left(\int \left(\frac{1}{dx^9} - \frac{e}{d^2x^8} + \frac{e^2}{d^3x^7} - \right.\right.$$
$$= -\frac{ben}{24dx^{8/3}} + \frac{be^2n}{21d^2x^{7/3}} - \frac{be^3n}{18d^3x^2} + \frac{be^4n}{15d^4x^{5/3}} - \frac{be^5n}{12d^5x^{4/3}} + \frac{be^6n}{9d^6x} - \frac{be^7n}{6d^7x^2}$$

Mathematica [A]
time = 0.14, size = 177, normalized size = 0.92

$$-\frac{a}{3x^3} - \frac{b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{3x^3} + \frac{1}{3}ben\left(-\frac{1}{8dx^{8/3}} + \frac{e}{7d^2x^{7/3}} - \frac{e^2}{6d^3x^2} + \frac{e^3}{5d^4x^{5/3}} - \frac{e^4}{4d^5x^{4/3}} + \frac{e^5}{3d^6x} - \frac{e^6}{2d^7x^{2/3}} + \frac{e^7}{d^8\sqrt[3]{x}} - \frac{e^8\log\left(d+e\sqrt[3]{x}\right)}{d^9} + \frac{e^8\log(x)}{3d^9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (b*Log[c*(d + e*x^(1/3))^n])/(3*x^3) + (b*e*n*(-1/8*1/(d*x^(8/
3)) + e/(7*d^2*x^(7/3)) - e^2/(6*d^3*x^2) + e^3/(5*d^4*x^(5/3)) - e^4/(4*d^
5*x^(4/3)) + e^5/(3*d^6*x) - e^6/(2*d^7*x^(2/3)) + e^7/(d^8*x^(1/3)) - (e^8
*Log[d + e*x^(1/3)])/d^9 + (e^8*Log[x])/(3*d^9)))/3
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^n\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))/x^4,x)
```

Maxima [A]
time = 0.28, size = 135, normalized size = 0.70

$$-\frac{1}{2520}\,bn\left(\frac{840\,e^8\log\left(x^{\frac{1}{3}}e+d\right)}{d^9}-\frac{280\,e^8\log\left(x\right)}{d^9}-\frac{120\,d^6x^{\frac{1}{3}}e-105\,d^7-140\,d^5x^{\frac{2}{3}}e^2+168\,d^4xe^3-210\,d^3x^{\frac{4}{3}}e^4+280\,d^2x^{\frac{5}{3}}e^5-420\,dx^2e^6+840\,x^{\frac{7}{3}}e^7}{d^8x^{\frac{8}{3}}}\right)e-\frac{b\log\left(\left(x^{\frac{1}{3}}e+d\right)^nc\right)}{3\,x^3}-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^4,x, algorithm="maxima")
```

```
[Out] -1/2520*b*n*(840*e^8*log(x^(1/3)*e + d)/d^9 - 280*e^8*log(x)/d^9 - (120*d^6
*x^(1/3)*e - 105*d^7 - 140*d^5*x^(2/3)*e^2 + 168*d^4*x*e^3 - 210*d^3*x^(4/3
)*e^4 + 280*d^2*x^(5/3)*e^5 - 420*d*x^2*e^6 + 840*x^(7/3)*e^7)/(d^8*x^(8/3)
))*e - 1/3*b*log((x^(1/3)*e + d)^n*c)/x^3 - 1/3*a/x^3
```

Fricas [A]
time = 0.39, size = 156, normalized size = 0.81

$$-\frac{840\,bd^9\log\left(c\right)+840\,ad^9+140\,bd^6nxe^3-280\,bd^3nx^2e^6-840\,bnx^3e^9\log\left(x^{\frac{1}{3}}\right)+840\,\left(bd^9n+bnx^3e^9\right)\log\left(x^{\frac{1}{3}}e+d\right)-30\,\left(4\,bd^7ne^2-7\,bd^4nxe^5+28\,bdnx^2e^8\right)x^{\frac{2}{3}}+21\,\left(5\,bd^8ne-8\,bd^5nxe^4+20\,bd^2nx^2e^7\right)x^{\frac{1}{3}}}{2520\,d^9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^4,x, algorithm="fricas")
```

```
[Out] -1/2520*(840*b*d^9*log(c) + 840*a*d^9 + 140*b*d^6*n*x*e^3 - 280*b*d^3*n*x^2
*e^6 - 840*b*n*x^3*e^9*log(x^(1/3)) + 840*(b*d^9*n + b*n*x^3*e^9)*log(x^(1/
3)*e + d) - 30*(4*b*d^7*n*e^2 - 7*b*d^4*n*x*e^5 + 28*b*d*n*x^2*e^8)*x^(2/3)
 + 21*(5*b*d^8*n*e - 8*b*d^5*n*x*e^4 + 20*b*d^2*n*x^2*e^7)*x^(1/3))/(d^9*x^
3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))/x**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 808 vs. 2(148) = 296.
time = 6.25, size = 808, normalized size = 4.21

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^4,x, algorithm="giac")
```

```
[Out] -1/2520*(840*(x^(1/3)*e + d)^9*b*n*e^10*log(x^(1/3)*e + d) - 7560*(x^(1/3)*
e + d)^8*b*d*n*e^10*log(x^(1/3)*e + d) + 30240*(x^(1/3)*e + d)^7*b*d^2*n*e^
10*log(x^(1/3)*e + d) - 70560*(x^(1/3)*e + d)^6*b*d^3*n*e^10*log(x^(1/3)*e
+ d) + 105840*(x^(1/3)*e + d)^5*b*d^4*n*e^10*log(x^(1/3)*e + d) - 105840*(x
^(1/3)*e + d)^4*b*d^5*n*e^10*log(x^(1/3)*e + d) + 70560*(x^(1/3)*e + d)^3*b
*d^6*n*e^10*log(x^(1/3)*e + d) - 30240*(x^(1/3)*e + d)^2*b*d^7*n*e^10*log(x
^(1/3)*e + d) + 7560*(x^(1/3)*e + d)*b*d^8*n*e^10*log(x^(1/3)*e + d) - 840*
(x^(1/3)*e + d)^9*b*n*e^10*log(x^(1/3)*e) + 7560*(x^(1/3)*e + d)^8*b*d*n*e^
10*log(x^(1/3)*e) - 30240*(x^(1/3)*e + d)^7*b*d^2*n*e^10*log(x^(1/3)*e) + 7
0560*(x^(1/3)*e + d)^6*b*d^3*n*e^10*log(x^(1/3)*e) - 105840*(x^(1/3)*e + d)
^5*b*d^4*n*e^10*log(x^(1/3)*e) + 105840*(x^(1/3)*e + d)^4*b*d^5*n*e^10*log(
x^(1/3)*e) - 70560*(x^(1/3)*e + d)^3*b*d^6*n*e^10*log(x^(1/3)*e) + 30240*(x
^(1/3)*e + d)^2*b*d^7*n*e^10*log(x^(1/3)*e) - 7560*(x^(1/3)*e + d)*b*d^8*n*
e^10*log(x^(1/3)*e) + 840*b*d^9*n*e^10*log(x^(1/3)*e) - 840*(x^(1/3)*e + d)
^8*b*d*n*e^10 + 7140*(x^(1/3)*e + d)^7*b*d^2*n*e^10 - 26740*(x^(1/3)*e + d)
^6*b*d^3*n*e^10 + 57750*(x^(1/3)*e + d)^5*b*d^4*n*e^10 - 78918*(x^(1/3)*e +
 d)^4*b*d^5*n*e^10 + 70252*(x^(1/3)*e + d)^3*b*d^6*n*e^10 - 40188*(x^(1/3)*
e + d)^2*b*d^7*n*e^10 + 13827*(x^(1/3)*e + d)*b*d^8*n*e^10 - 2283*b*d^9*n*e
^10 + 840*b*d^9*e^10*log(c) + 840*a*d^9*e^10)*e^(-1)/((x^(1/3)*e + d)^9*d^9
 - 9*(x^(1/3)*e + d)^8*d^10 + 36*(x^(1/3)*e + d)^7*d^11 - 84*(x^(1/3)*e + d
)^6*d^12 + 126*(x^(1/3)*e + d)^5*d^13 - 126*(x^(1/3)*e + d)^4*d^14 + 84*(x^
(1/3)*e + d)^3*d^15 - 36*(x^(1/3)*e + d)^2*d^16 + 9*(x^(1/3)*e + d)*d^17 -
d^18)
```

Mupad [B]
time = 0.61, size = 154, normalized size = 0.80

$$-\frac{\frac{a\,d^9}{3}+\frac{b\,d^9\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{3}+\frac{b\,d^6\,e^3\,n\,x}{18}+\frac{b\,d^8\,e\,n\,x^{1/3}}{24}-\frac{b\,d\,e^8\,n\,x^{8/3}}{3}-\frac{b\,d^3\,e^6\,n\,x^2}{9}-\frac{b\,d^7\,e^2\,n\,x^{2/3}}{21}-\frac{b\,d^5\,e^4\,n\,x^{4/3}}{15}+\frac{b\,d^4\,e^5\,n\,x^{5/3}}{12}+\frac{b\,d^2\,e^7\,n\,x^{7/3}}{6}}{d^9\,x^3}-\frac{2\,b\,e^9\,n\,\mathrm{atanh}\left(\frac{2\,e\,x^{1/3}}{d}+1\right)}{3\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))/x^4,x)
```

```
[Out] - ((a*d^9)/3 + (b*d^9*log(c*(d + e*x^(1/3))^n))/3 + (b*d^6*e^3*n*x)/18 + (b
*d^8*e*n*x^(1/3))/24 - (b*d*e^8*n*x^(8/3))/3 - (b*d^3*e^6*n*x^2)/9 - (b*d^7
*e^2*n*x^(2/3))/21 - (b*d^5*e^4*n*x^(4/3))/15 + (b*d^4*e^5*n*x^(5/3))/12 +
(b*d^2*e^7*n*x^(7/3))/6)/(d^9*x^3) - (2*b*e^9*n*atanh((2*e*x^(1/3))/d + 1))
/(3*d^9)
```

3.450 $\int x^2 \left(a+b \log \left(c \left(d+e \sqrt[3]{x}\right)^n\right)\right)^2 \, dx$

Optimal. Leaf size=680

$$-\frac{6b^2d^7n^2\left(d+e\sqrt[3]{x}\right)^2}{e^9}+\frac{56b^2d^6n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^9}-\frac{21b^2d^5n^2\left(d+e\sqrt[3]{x}\right)^4}{4e^9}+\frac{84b^2d^4n^2\left(d+e\sqrt[3]{x}\right)^5}{25e^9}-\frac{14b^2d^3n^2(d-}{9e^9}$$

[Out] -6*b^2*d^7*n^2*(d+e*x^(1/3))^2/e^9+56/9*b^2*d^6*n^2*(d+e*x^(1/3))^3/e^9-21/
4*b^2*d^5*n^2*(d+e*x^(1/3))^4/e^9+84/25*b^2*d^4*n^2*(d+e*x^(1/3))^5/e^9-14/
9*b^2*d^3*n^2*(d+e*x^(1/3))^6/e^9+24/49*b^2*d^2*n^2*(d+e*x^(1/3))^7/e^9-3/3
2*b^2*d*n^2*(d+e*x^(1/3))^8/e^9+2/243*b^2*n^2*(d+e*x^(1/3))^9/e^9+6*b^2*d^8
*n^2*x^(1/3)/e^8-1/3*b^2*d^9*n^2*ln(d+e*x^(1/3))^2/e^9-6*b*d^8*n*(d+e*x^(1/
3))*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9+12*b*d^7*n*(d+e*x^(1/3))^2*(a+b*ln(c*(d
+e*x^(1/3))^n))/e^9-56/3*b*d^6*n*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n)
)/e^9+21*b*d^5*n*(d+e*x^(1/3))^4*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9-84/5*b*d^4
n(d+e*x^(1/3))^5*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9+28/3*b*d^3*n*(d+e*x^(1/3
))^6*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9-24/7*b*d^2*n*(d+e*x^(1/3))^7*(a+b*ln(c
*(d+e*x^(1/3))^n))/e^9+3/4*b*d*n*(d+e*x^(1/3))^8*(a+b*ln(c*(d+e*x^(1/3))^n)
)/e^9-2/27*b*n*(d+e*x^(1/3))^9*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9+2/3*b*d^9*n*
ln(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9+1/3*x^3*(a+b*ln(c*(d+e*x^(1
/3))^n))^2

Rubi [A]
time = 0.47, antiderivative size = 680, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2504, 2445, 2458, 45, 2372, 12, 14, 2338}

Antiderivative was successfully verified.

[In] Int[x^2*(a + b*Log[c*(d + e*x^(1/3))^n])^2,x]

[Out] (-6*b^2*d^7*n^2*(d + e*x^(1/3))^2)/e^9 + (56*b^2*d^6*n^2*(d + e*x^(1/3))^3)
/(9*e^9) - (21*b^2*d^5*n^2*(d + e*x^(1/3))^4)/(4*e^9) + (84*b^2*d^4*n^2*(d
+ e*x^(1/3))^5)/(25*e^9) - (14*b^2*d^3*n^2*(d + e*x^(1/3))^6)/(9*e^9) + (24
*b^2*d^2*n^2*(d + e*x^(1/3))^7)/(49*e^9) - (3*b^2*d*n^2*(d + e*x^(1/3))^8)/
(32*e^9) + (2*b^2*n^2*(d + e*x^(1/3))^9)/(243*e^9) + (6*b^2*d^8*n^2*x^(1/3)
)/e^8 - (b^2*d^9*n^2*Log[d + e*x^(1/3)]^2)/(3*e^9) - (6*b*d^8*n*(d + e*x^(1
/3))*(a + b*Log[c*(d + e*x^(1/3))^n]))/e^9 + (12*b*d^7*n*(d + e*x^(1/3))^2*
(a + b*Log[c*(d + e*x^(1/3))^n]))/e^9 - (56*b*d^6*n*(d + e*x^(1/3))^3*(a +
b*Log[c*(d + e*x^(1/3))^n]))/(3*e^9) + (21*b*d^5*n*(d + e*x^(1/3))^4*(a + b
Log[c(d + e*x^(1/3))^n]))/e^9 - (84*b*d^4*n*(d + e*x^(1/3))^5*(a + b*Log[
c*(d + e*x^(1/3))^n]))/(5*e^9) + (28*b*d^3*n*(d + e*x^(1/3))^6*(a + b*Log[c
*(d + e*x^(1/3))^n]))/(3*e^9) - (24*b*d^2*n*(d + e*x^(1/3))^7*(a + b*Log[c*
(d + e*x^(1/3))^n]))/(7*e^9) + (3*b*d*n*(d + e*x^(1/3))^8*(a + b*Log[c*(d +

```
 e*x^(1/3))^n]))/(4*e^9) - (2*b*n*(d + e*x^(1/3))^9*(a + b*Log[c*(d + e*x^(
1/3))^n]))/(27*e^9) + (2*b*d^9*n*Log[d + e*x^(1/3)]*(a + b*Log[c*(d + e*x^(
1/3))^n]))/(3*e^9) + (x^3*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
```

```
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx=3\text{Subst}\left(\int x^8(a+b\log\left(c(d+ex)^n\right))^2\,dx,x,\sqrt[3]{x}\right)$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2-\frac{1}{3}(2ben)\text{Subst}\left(\int\frac{x^9(a+b\log(c(}{d+e}\right.$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2-\frac{1}{3}(2bn)\text{Subst}\left(\int\frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^9(a+}{x}\right.$$

$$=-\frac{bn\left(\frac{22680d^8\left(d+e\sqrt[3]{x}\right)}{e^9}-\frac{45360d^7\left(d+e\sqrt[3]{x}\right)^2}{e^9}+\frac{70560d^6\left(d+e\sqrt[3]{x}\right)^3}{e^9}-\frac{7938}{}\right.}{}$$

$$=-\frac{bn\left(\frac{22680d^8\left(d+e\sqrt[3]{x}\right)}{e^9}-\frac{45360d^7\left(d+e\sqrt[3]{x}\right)^2}{e^9}+\frac{70560d^6\left(d+e\sqrt[3]{x}\right)^3}{e^9}-\frac{7938}{}\right.}{}$$

$$=-\frac{bn\left(\frac{22680d^8\left(d+e\sqrt[3]{x}\right)}{e^9}-\frac{45360d^7\left(d+e\sqrt[3]{x}\right)^2}{e^9}+\frac{70560d^6\left(d+e\sqrt[3]{x}\right)^3}{e^9}-\frac{7938}{}\right.}{}$$

$$=-\frac{6b^2d^7n^2\left(d+e\sqrt[3]{x}\right)^2}{e^9}+\frac{56b^2d^6n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^9}-\frac{21b^2d^5n^2\left(d+e\sqrt[3]{x}\right.}{4e^9}$$

$$=-\frac{6b^2d^7n^2\left(d+e\sqrt[3]{x}\right)^2}{e^9}+\frac{56b^2d^6n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^9}-\frac{21b^2d^5n^2\left(d+e\sqrt[3]{x}\right.}{4e^9}$$

Mathematica [A]
time = 0.38, size = 458, normalized size = 0.67

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(1/3))^n])^2,x]
```

```
[Out] (-3175200*b^2*d^9*n^2*Log[d + e*x^(1/3)]^2 + 2520*b*d^9*n*Log[d + e*x^(1/3)
]*(2520*a - 7129*b*n + 2520*b*Log[c*(d + e*x^(1/3))^n]) + e*x^(1/3)*(317520
0*a^2*e^8*x^(8/3) - 2520*a*b*n*(2520*d^8 - 1260*d^7*e*x^(1/3) + 840*d^6*e^2
*x^(2/3) - 630*d^5*e^3*x + 504*d^4*e^4*x^(4/3) - 420*d^3*e^5*x^(5/3) + 360*
d^2*e^6*x^2 - 315*d*e^7*x^(7/3) + 280*e^8*x^(8/3)) + b^2*n^2*(17965080*d^8
- 5807340*d^7*e*x^(1/3) + 2813160*d^6*e^2*x^(2/3) - 1580670*d^5*e^3*x + 947
016*d^4*e^4*x^(4/3) - 577500*d^3*e^5*x^(5/3) + 343800*d^2*e^6*x^2 - 187425*
d*e^7*x^(7/3) + 78400*e^8*x^(8/3)) - 2520*b*(-2520*a*e^8*x^(8/3) + b*n*(252
0*d^8 - 1260*d^7*e*x^(1/3) + 840*d^6*e^2*x^(2/3) - 630*d^5*e^3*x + 504*d^4*
e^4*x^(4/3) - 420*d^3*e^5*x^(5/3) + 360*d^2*e^6*x^2 - 315*d*e^7*x^(7/3) + 2
80*e^8*x^(8/3)))*Log[c*(d + e*x^(1/3))^n] + 3175200*b^2*e^8*x^(8/3)*Log[c*(
d + e*x^(1/3))^n]^2))/(9525600*e^9)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^{2}\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(1/3))^n))^2,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e*x^(1/3))^n))^2,x)
```

Maxima [A]
time = 0.29, size = 407, normalized size = 0.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3*log((x^(1/3)*e + d)^n*c)^2 + 2/3*a*b*x^3*log((x^(1/3)*e + d)^n*
c) + 1/3*a^2*x^3 + 1/3780*(2520*d^9*e^(-10)*log(x^(1/3)*e + d) + (1260*d^7*
x^(2/3)*e - 2520*d^8*x^(1/3) - 840*d^6*x*e^2 + 630*d^5*x^(4/3)*e^3 - 504*d^
4*x^(5/3)*e^4 + 420*d^3*x^2*e^5 - 360*d^2*x^(7/3)*e^6 + 315*d*x^(8/3)*e^7 -
 280*x^3*e^8)*e^(-9))*a*b*n*e - 1/9525600*((3175200*d^9*log(x^(1/3)*e + d)^
2 + 17965080*d^9*log(x^(1/3)*e + d) - 17965080*d^8*x^(1/3)*e + 5807340*d^7*
```

```
x^(2/3)*e^2 - 2813160*d^6*x*e^3 + 1580670*d^5*x^(4/3)*e^4 - 947016*d^4*x^(5
/3)*e^5 + 577500*d^3*x^2*e^6 - 343800*d^2*x^(7/3)*e^7 + 187425*d*x^(8/3)*e^
8 - 78400*x^3*e^9)*n^2*e^(-9) - 2520*(2520*d^9*e^(-10)*log(x^(1/3)*e + d) +
 (1260*d^7*x^(2/3)*e - 2520*d^8*x^(1/3) - 840*d^6*x*e^2 + 630*d^5*x^(4/3)*e
^3 - 504*d^4*x^(5/3)*e^4 + 420*d^3*x^2*e^5 - 360*d^2*x^(7/3)*e^6 + 315*d*x^
(8/3)*e^7 - 280*x^3*e^8)*e^(-9))*n*e*log((x^(1/3)*e + d)^n*c))*b^2
```

Fricas [A]
time = 0.43, size = 623, normalized size = 0.92

$$\frac{1}{9525600}\left(3175200\,b^2x^3e^9\log(c)^2+39200\,(2b^2n^2-18abn+81a^2)x^3e^9-2100\,(275b^2d^3n^2-504abd^3n)x^2e^6+840\,(3349b^2d^6n^2-2520abd^6n)xe^3+3175200\,(b^2d^9n^2+b^2n^2x^3e^9)\log\left(x^{\frac{1}{3}}e+d\right)^2-2520\left(7129b^2d^9n^2-2520abd^9n+840b^2d^6n^2xe^3-420b^2d^3n^2x^2e^6+280\,(b^2n^2-9abn)x^3e^9-2520\,(b^2d^9n+b^2nx^3e^9)\log(c)-63\,(20b^2d^7n^2e^2-8b^2d^4n^2xe^5+5b^2dn^2x^2e^8)x^{\frac{2}{3}}+90\,(28b^2d^8n^2e-7b^2d^5n^2xe^4+4b^2d^2n^2x^2e^7)x^{\frac{1}{3}}\right)\log\left(x^{\frac{1}{3}}e+d\right)-352800\,(6b^2d^6nxe^3-3b^2d^3nx^2e^6+2\,(b^2n-9ab)x^3e^9)\log(c)-63\left(175\,(17b^2dn^2-72abdn)x^2e^8-8\,(1879b^2d^4n^2-2520abd^4n)xe^5+20\,(4609b^2d^7n^2-2520abd^7n)e^2-2520\,(20b^2d^7ne^2-8b^2d^4nxe^5+5b^2dnx^2e^8)\log(c)\right)x^{\frac{2}{3}}+90\left(20\,(191b^2d^2n^2-504abd^2n)x^2e^7-7\,(2509b^2d^5n^2-2520abd^5n)xe^4+28\,(7129b^2d^8n^2-2520abd^8n)e-2520\,(28b^2d^8ne-7b^2d^5nxe^4+4b^2d^2nx^2e^7)\log(c)\right)x^{\frac{1}{3}}\right)e^{-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="fricas")
```

```
[Out] 1/9525600*(3175200*b^2*x^3*e^9*log(c)^2 + 39200*(2*b^2*n^2 - 18*a*b*n + 81*
a^2)*x^3*e^9 - 2100*(275*b^2*d^3*n^2 - 504*a*b*d^3*n)*x^2*e^6 + 840*(3349*b
^2*d^6*n^2 - 2520*a*b*d^6*n)*x*e^3 + 3175200*(b^2*d^9*n^2 + b^2*n^2*x^3*e^9
)*log(x^(1/3)*e + d)^2 - 2520*(7129*b^2*d^9*n^2 - 2520*a*b*d^9*n + 840*b^2*
d^6*n^2*x*e^3 - 420*b^2*d^3*n^2*x^2*e^6 + 280*(b^2*n^2 - 9*a*b*n)*x^3*e^9 -
 2520*(b^2*d^9*n + b^2*n*x^3*e^9)*log(c) - 63*(20*b^2*d^7*n^2*e^2 - 8*b^2*d
^4*n^2*x*e^5 + 5*b^2*d*n^2*x^2*e^8)*x^(2/3) + 90*(28*b^2*d^8*n^2*e - 7*b^2*
d^5*n^2*x*e^4 + 4*b^2*d^2*n^2*x^2*e^7)*x^(1/3))*log(x^(1/3)*e + d) - 352800
*(6*b^2*d^6*n*x*e^3 - 3*b^2*d^3*n*x^2*e^6 + 2*(b^2*n - 9*a*b)*x^3*e^9)*log(
c) - 63*(175*(17*b^2*d*n^2 - 72*a*b*d*n)*x^2*e^8 - 8*(1879*b^2*d^4*n^2 - 25
20*a*b*d^4*n)*x*e^5 + 20*(4609*b^2*d^7*n^2 - 2520*a*b*d^7*n)*e^2 - 2520*(20
*b^2*d^7*n*e^2 - 8*b^2*d^4*n*x*e^5 + 5*b^2*d*n*x^2*e^8)*log(c))*x^(2/3) + 9
0*(20*(191*b^2*d^2*n^2 - 504*a*b*d^2*n)*x^2*e^7 - 7*(2509*b^2*d^5*n^2 - 252
0*a*b*d^5*n)*x*e^4 + 28*(7129*b^2*d^8*n^2 - 2520*a*b*d^8*n)*e - 2520*(28*b^
2*d^8*n*e - 7*b^2*d^5*n*x*e^4 + 4*b^2*d^2*n*x^2*e^7)*log(c))*x^(1/3))*e^(-9
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/3))**n))**2,x)
```

```
[Out] Integral(x**2*(a + b*log(c*(d + e*x**(1/3))**n))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1427 vs. 2(596) = 1192.
time = 3.77, size = 1427, normalized size = 2.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="giac")
```

```
[Out] 1/9525600*(3175200*b^2*x^3*e*log(c)^2 + 6350400*a*b*x^3*e*log(c) + 3175200*
a^2*x^3*e + (3175200*(x^(1/3)*e + d)^9*e^(-8)*log(x^(1/3)*e + d)^2 - 285768
00*(x^(1/3)*e + d)^8*d*e^(-8)*log(x^(1/3)*e + d)^2 + 114307200*(x^(1/3)*e +
 d)^7*d^2*e^(-8)*log(x^(1/3)*e + d)^2 - 266716800*(x^(1/3)*e + d)^6*d^3*e^(
-8)*log(x^(1/3)*e + d)^2 + 400075200*(x^(1/3)*e + d)^5*d^4*e^(-8)*log(x^(1/
3)*e + d)^2 - 400075200*(x^(1/3)*e + d)^4*d^5*e^(-8)*log(x^(1/3)*e + d)^2 +
 266716800*(x^(1/3)*e + d)^3*d^6*e^(-8)*log(x^(1/3)*e + d)^2 - 114307200*(x
^(1/3)*e + d)^2*d^7*e^(-8)*log(x^(1/3)*e + d)^2 + 28576800*(x^(1/3)*e + d)*
d^8*e^(-8)*log(x^(1/3)*e + d)^2 - 705600*(x^(1/3)*e + d)^9*e^(-8)*log(x^(1/
3)*e + d) + 7144200*(x^(1/3)*e + d)^8*d*e^(-8)*log(x^(1/3)*e + d) - 3265920
0*(x^(1/3)*e + d)^7*d^2*e^(-8)*log(x^(1/3)*e + d) + 88905600*(x^(1/3)*e + d
)^6*d^3*e^(-8)*log(x^(1/3)*e + d) - 160030080*(x^(1/3)*e + d)^5*d^4*e^(-8)*
log(x^(1/3)*e + d) + 200037600*(x^(1/3)*e + d)^4*d^5*e^(-8)*log(x^(1/3)*e +
 d) - 177811200*(x^(1/3)*e + d)^3*d^6*e^(-8)*log(x^(1/3)*e + d) + 114307200
*(x^(1/3)*e + d)^2*d^7*e^(-8)*log(x^(1/3)*e + d) - 57153600*(x^(1/3)*e + d)
*d^8*e^(-8)*log(x^(1/3)*e + d) + 78400*(x^(1/3)*e + d)^9*e^(-8) - 893025*(x
^(1/3)*e + d)^8*d*e^(-8) + 4665600*(x^(1/3)*e + d)^7*d^2*e^(-8) - 14817600*
(x^(1/3)*e + d)^6*d^3*e^(-8) + 32006016*(x^(1/3)*e + d)^5*d^4*e^(-8) - 5000
9400*(x^(1/3)*e + d)^4*d^5*e^(-8) + 59270400*(x^(1/3)*e + d)^3*d^6*e^(-8) -
 57153600*(x^(1/3)*e + d)^2*d^7*e^(-8) + 57153600*(x^(1/3)*e + d)*d^8*e^(-8
))*b^2*n^2 + 2520*(2520*(x^(1/3)*e + d)^9*e^(-8)*log(x^(1/3)*e + d) - 22680
*(x^(1/3)*e + d)^8*d*e^(-8)*log(x^(1/3)*e + d) + 90720*(x^(1/3)*e + d)^7*d^
2*e^(-8)*log(x^(1/3)*e + d) - 211680*(x^(1/3)*e + d)^6*d^3*e^(-8)*log(x^(1/
3)*e + d) + 317520*(x^(1/3)*e + d)^5*d^4*e^(-8)*log(x^(1/3)*e + d) - 317520
*(x^(1/3)*e + d)^4*d^5*e^(-8)*log(x^(1/3)*e + d) + 211680*(x^(1/3)*e + d)^3
*d^6*e^(-8)*log(x^(1/3)*e + d) - 90720*(x^(1/3)*e + d)^2*d^7*e^(-8)*log(x^(
1/3)*e + d) + 22680*(x^(1/3)*e + d)*d^8*e^(-8)*log(x^(1/3)*e + d) - 280*(x^
(1/3)*e + d)^9*e^(-8) + 2835*(x^(1/3)*e + d)^8*d*e^(-8) - 12960*(x^(1/3)*e
+ d)^7*d^2*e^(-8) + 35280*(x^(1/3)*e + d)^6*d^3*e^(-8) - 63504*(x^(1/3)*e +
 d)^5*d^4*e^(-8) + 79380*(x^(1/3)*e + d)^4*d^5*e^(-8) - 70560*(x^(1/3)*e +
d)^3*d^6*e^(-8) + 45360*(x^(1/3)*e + d)^2*d^7*e^(-8) - 22680*(x^(1/3)*e + d
)*d^8*e^(-8))*b^2*n*log(c) + 2520*(2520*(x^(1/3)*e + d)^9*e^(-8)*log(x^(1/3
)*e + d) - 22680*(x^(1/3)*e + d)^8*d*e^(-8)*log(x^(1/3)*e + d) + 90720*(x^(
1/3)*e + d)^7*d^2*e^(-8)*log(x^(1/3)*e + d) - 211680*(x^(1/3)*e + d)^6*d^3*
e^(-8)*log(x^(1/3)*e + d) + 317520*(x^(1/3)*e + d)^5*d^4*e^(-8)*log(x^(1/3)
*e + d) - 317520*(x^(1/3)*e + d)^4*d^5*e^(-8)*log(x^(1/3)*e + d) + 211680*(
x^(1/3)*e + d)^3*d^6*e^(-8)*log(x^(1/3)*e + d) - 90720*(x^(1/3)*e + d)^2*d^
7*e^(-8)*log(x^(1/3)*e + d) + 22680*(x^(1/3)*e + d)*d^8*e^(-8)*log(x^(1/3)*
e + d) - 280*(x^(1/3)*e + d)^9*e^(-8) + 2835*(x^(1/3)*e + d)^8*d*e^(-8) - 1
2960*(x^(1/3)*e + d)^7*d^2*e^(-8) + 35280*(x^(1/3)*e + d)^6*d^3*e^(-8) - 63
504*(x^(1/3)*e + d)^5*d^4*e^(-8) + 79380*(x^(1/3)*e + d)^4*d^5*e^(-8) - 705
```

```
60*(x^(1/3)*e + d)^3*d^6*e^(-8) + 45360*(x^(1/3)*e + d)^2*d^7*e^(-8) - 2268
0*(x^(1/3)*e + d)*d^8*e^(-8))*a*b*n)*e^(-1)
```

Mupad [B]
time = 4.70, size = 608, normalized size = 0.89

$$\frac{a^2 x^3}{3}+\frac{b^2 x^3 \ln\left(c\left(d+e x^{1/3}\right)^n\right)^2}{3}+\frac{2 b^2 n^2 x^3}{243}+\frac{2 a b x^3 \ln\left(c\left(d+e x^{1/3}\right)^n\right)}{3}+\frac{b^2 d^9 \ln\left(c\left(d+e x^{1/3}\right)^n\right)^2}{3 e^9}-\frac{2 a b n x^3}{27}-\frac{2 b^2 n x^3 \ln\left(c\left(d+e x^{1/3}\right)^n\right)}{27}-\frac{7129 b^2 d^9 n^2 \ln\left(d+e x^{1/3}\right)}{3780 e^9}-\frac{275 b^2 d^3 n^2 x^2}{4536 e^3}+\frac{191 b^2 d^2 n^2 x^{7/3}}{5292 e^2}+\frac{1879 b^2 d^4 n^2 x^{5/3}}{18900 e^4}-\frac{2509 b^2 d^5 n^2 x^{4/3}}{15120 e^5}-\frac{4609 b^2 d^7 n^2 x^{2/3}}{7560 e^7}+\frac{7129 b^2 d^8 n^2 x^{1/3}}{3780 e^8}-\frac{17 b^2 d n^2 x^{8/3}}{864 e}+\frac{3349 b^2 d^6 n^2 x}{11340 e^6}+\frac{b^2 d^3 n x^2 \ln\left(c\left(d+e x^{1/3}\right)^n\right)}{9 e^3}-\frac{2 b^2 d^2 n x^{7/3} \ln\left(c\left(d+e x^{1/3}\right)^n\right)}{21 e^2}-\frac{2 b^2 d^4 n x^{5/3} \ln\left(c\left(d+e x^{1/3}\right)^n\right)}{15 e^4}+\frac{b^2 d^5 n x^{4/3} \ln\left(c\left(d+e x^{1/3}\right)^n\right)}{6 e^5}+\frac{b^2 d^7 n x^{2/3} \ln\left(c\left(d+e x^{1/3}\right)^n\right)}{3 e^7}-\frac{2 b^2 d^8 n x^{1/3} \ln\left(c\left(d+e x^{1/3}\right)^n\right)}{3 e^8}+\frac{a b d n x^{8/3}}{12 e}-\frac{2 a b d^6 n x}{9 e^6}+\frac{2 a b d^9 n \ln\left(d+e x^{1/3}\right)}{3 e^9}+\frac{b^2 d n x^{8/3} \ln\left(c\left(d+e x^{1/3}\right)^n\right)}{12 e}-\frac{2 b^2 d^6 n x \ln\left(c\left(d+e x^{1/3}\right)^n\right)}{9 e^6}+\frac{a b d^3 n x^2}{9 e^3}-\frac{2 a b d^2 n x^{7/3}}{21 e^2}-\frac{2 a b d^4 n x^{5/3}}{15 e^4}+\frac{a b d^5 n x^{4/3}}{6 e^5}+\frac{a b d^7 n x^{2/3}}{3 e^7}-\frac{2 a b d^8 n x^{1/3}}{3 e^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e*x^(1/3))^n))^2,x)
```

```
[Out] (a^2*x^3)/3 + (b^2*x^3*log(c*(d + e*x^(1/3))^n)^2)/3 + (2*b^2*n^2*x^3)/243
+ (2*a*b*x^3*log(c*(d + e*x^(1/3))^n))/3 + (b^2*d^9*log(c*(d + e*x^(1/3))^n
)^2)/(3*e^9) - (2*a*b*n*x^3)/27 - (2*b^2*n*x^3*log(c*(d + e*x^(1/3))^n))/27
 - (7129*b^2*d^9*n^2*log(d + e*x^(1/3)))/(3780*e^9) - (275*b^2*d^3*n^2*x^2)
/(4536*e^3) + (191*b^2*d^2*n^2*x^(7/3))/(5292*e^2) + (1879*b^2*d^4*n^2*x^(5
/3))/(18900*e^4) - (2509*b^2*d^5*n^2*x^(4/3))/(15120*e^5) - (4609*b^2*d^7*n
^2*x^(2/3))/(7560*e^7) + (7129*b^2*d^8*n^2*x^(1/3))/(3780*e^8) - (17*b^2*d*
n^2*x^(8/3))/(864*e) + (3349*b^2*d^6*n^2*x)/(11340*e^6) + (b^2*d^3*n*x^2*lo
g(c*(d + e*x^(1/3))^n))/(9*e^3) - (2*b^2*d^2*n*x^(7/3)*log(c*(d + e*x^(1/3)
)^n))/(21*e^2) - (2*b^2*d^4*n*x^(5/3)*log(c*(d + e*x^(1/3))^n))/(15*e^4) +
(b^2*d^5*n*x^(4/3)*log(c*(d + e*x^(1/3))^n))/(6*e^5) + (b^2*d^7*n*x^(2/3)*l
og(c*(d + e*x^(1/3))^n))/(3*e^7) - (2*b^2*d^8*n*x^(1/3)*log(c*(d + e*x^(1/3
))^n))/(3*e^8) + (a*b*d*n*x^(8/3))/(12*e) - (2*a*b*d^6*n*x)/(9*e^6) + (2*a*
b*d^9*n*log(d + e*x^(1/3)))/(3*e^9) + (b^2*d*n*x^(8/3)*log(c*(d + e*x^(1/3)
)^n))/(12*e) - (2*b^2*d^6*n*x*log(c*(d + e*x^(1/3))^n))/(9*e^6) + (a*b*d^3*
n*x^2)/(9*e^3) - (2*a*b*d^2*n*x^(7/3))/(21*e^2) - (2*a*b*d^4*n*x^(5/3))/(15
*e^4) + (a*b*d^5*n*x^(4/3))/(6*e^5) + (a*b*d^7*n*x^(2/3))/(3*e^7) - (2*a*b*
d^8*n*x^(1/3))/(3*e^8)
```

3.451 $\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$

Optimal. Leaf size=480

$$\frac{15b^2d^4n^2\left(d+e\sqrt[3]{x}\right)^2}{4e^6}-\frac{20b^2d^3n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^6}+\frac{15b^2d^2n^2\left(d+e\sqrt[3]{x}\right)^4}{16e^6}-\frac{6b^2dn^2\left(d+e\sqrt[3]{x}\right)^5}{25e^6}+\frac{b^2n^2\left(d+e\sqrt[3]{x}\right.}{36e^6}$$

```
[Out] 15/4*b^2*d^4*n^2*(d+e*x^(1/3))^2/e^6-20/9*b^2*d^3*n^2*(d+e*x^(1/3))^3/e^6+1
5/16*b^2*d^2*n^2*(d+e*x^(1/3))^4/e^6-6/25*b^2*d*n^2*(d+e*x^(1/3))^5/e^6+1/3
6*b^2*n^2*(d+e*x^(1/3))^6/e^6-6*b^2*d^5*n^2*x^(1/3)/e^5+1/2*b^2*d^6*n^2*ln(
d+e*x^(1/3))^2/e^6+6*b*d^5*n*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6-
15/2*b*d^4*n*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6+20/3*b*d^3*n*(
d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6-15/4*b*d^2*n*(d+e*x^(1/3))^4
*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6+6/5*b*d*n*(d+e*x^(1/3))^5*(a+b*ln(c*(d+e*x
^(1/3))^n))/e^6-1/6*b*n*(d+e*x^(1/3))^6*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6-b*d
^6*n*ln(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6+1/2*x^2*(a+b*ln(c*(d+e
*x^(1/3))^n))^2
```

Rubi [A]

time = 0.31, antiderivative size = 480, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2504, 2445, 2458, 45, 2372, 12, 14, 2338}

$$-\frac{bd^6n\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^6}+\frac{6bd^5n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^6}-\frac{15bd^4n\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{2e^6}+\frac{20bd^3n\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^6}-\frac{15bd^2n\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4e^6}+\frac{6bdn\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5e^6}-\frac{bn\left(d+e\sqrt[3]{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{6e^6}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2+\frac{b^2d^6n^2\log^2\left(d+e\sqrt[3]{x}\right)}{2e^6}-\frac{6b^2d^5n^2\sqrt[3]{x}}{e^5}+\frac{15b^2d^4n^2\left(d+e\sqrt[3]{x}\right)^2}{4e^6}-\frac{20b^2d^3n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^6}+\frac{15b^2d^2n^2\left(d+e\sqrt[3]{x}\right)^4}{16e^6}-\frac{6b^2dn^2\left(d+e\sqrt[3]{x}\right)^5}{25e^6}+\frac{b^2n^2\left(d+e\sqrt[3]{x}\right)^6}{36e^6}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*x^(1/3))^n])^2,x]
```

```
[Out] (15*b^2*d^4*n^2*(d + e*x^(1/3))^2)/(4*e^6) - (20*b^2*d^3*n^2*(d + e*x^(1/3)
)^3)/(9*e^6) + (15*b^2*d^2*n^2*(d + e*x^(1/3))^4)/(16*e^6) - (6*b^2*d*n^2*(
d + e*x^(1/3))^5)/(25*e^6) + (b^2*n^2*(d + e*x^(1/3))^6)/(36*e^6) - (6*b^2*
d^5*n^2*x^(1/3))/e^5 + (b^2*d^6*n^2*Log[d + e*x^(1/3)]^2)/(2*e^6) + (6*b*d^
5*n*(d + e*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n]))/e^6 - (15*b*d^4*n*(d
+ e*x^(1/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/(2*e^6) + (20*b*d^3*n*(d +
 e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n]))/(3*e^6) - (15*b*d^2*n*(d +
e*x^(1/3))^4*(a + b*Log[c*(d + e*x^(1/3))^n]))/(4*e^6) + (6*b*d*n*(d + e*x^
(1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n]))/(5*e^6) - (b*n*(d + e*x^(1/3))^6
*(a + b*Log[c*(d + e*x^(1/3))^n]))/(6*e^6) - (b*d^6*n*Log[d + e*x^(1/3)]*(a
 + b*Log[c*(d + e*x^(1/3))^n]))/e^6 + (x^2*(a + b*Log[c*(d + e*x^(1/3))^n])
^2)/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx = 3\text{Subst}\left(\int x^5(a+b\log\left(c(d+ex)^n\right))^2\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 - (ben)\text{Subst}\left(\int \frac{x^6(a+b\log(c(d-}{d+ex}\right.$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 - (bn)\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^6(a+b\,l}{x}\right.$$

$$= \frac{1}{60}bn\left(\frac{360d^5\left(d+e\sqrt[3]{x}\right)}{e^6} - \frac{450d^4\left(d+e\sqrt[3]{x}\right)^2}{e^6} + \frac{400d^3\left(d+e\sqrt[3]{x}\right)^3}{e^6}\right.$$

$$= \frac{1}{60}bn\left(\frac{360d^5\left(d+e\sqrt[3]{x}\right)}{e^6} - \frac{450d^4\left(d+e\sqrt[3]{x}\right)^2}{e^6} + \frac{400d^3\left(d+e\sqrt[3]{x}\right)^3}{e^6}\right.$$

$$= \frac{1}{60}bn\left(\frac{360d^5\left(d+e\sqrt[3]{x}\right)}{e^6} - \frac{450d^4\left(d+e\sqrt[3]{x}\right)^2}{e^6} + \frac{400d^3\left(d+e\sqrt[3]{x}\right)^3}{e^6}\right.$$

$$= \frac{15b^2d^4n^2\left(d+e\sqrt[3]{x}\right)^2}{4e^6} - \frac{20b^2d^3n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^6} + \frac{15b^2d^2n^2\left(d+e\sqrt[3]{x}\right.}{16e^6}$$

$$= \frac{15b^2d^4n^2\left(d+e\sqrt[3]{x}\right)^2}{4e^6} - \frac{20b^2d^3n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^6} + \frac{15b^2d^2n^2\left(d+e\sqrt[3]{x}\right.}{16e^6}$$

Mathematica [A]
time = 0.26, size = 347, normalized size = 0.72

$$\frac{1800b^2d^6n^2\log^2\left(d+e\sqrt[3]{x}\right)+180bd^6n\log\left(d+e\sqrt[3]{x}\right)\left(-20a+49bn-20b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)+e\sqrt[3]{x}\left(1800a^2e^5x^{5/3}+60abn\left(60d^5-30d^4e\sqrt[3]{x}+20d^3e^2x^{2/3}-15d^2e^3x+12de^4x^{4/3}-10e^5x^{5/3}\right)+b^2n^2\left(-8820d^5+2610d^4e\sqrt[3]{x}-1140d^3e^2x^{2/3}+555d^2e^3x-264de^4x^{4/3}+100e^5x^{5/3}\right)+60b\left(60ae^5x^{5/3}+bn\left(60d^5-30d^4e\sqrt[3]{x}+20d^3e^2x^{2/3}-15d^2e^3x+12de^4x^{4/3}-10e^5x^{5/3}\right)\right)\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)+1800b^2e^5x^{5/3}\log^2\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3600e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(1/3))^n])^2,x]
```

```
[Out] (1800*b^2*d^6*n^2*Log[d + e*x^(1/3)]^2 + 180*b*d^6*n*Log[d + e*x^(1/3)]*(-2
0*a + 49*b*n - 20*b*Log[c*(d + e*x^(1/3))^n]) + e*x^(1/3)*(1800*a^2*e^5*x^(
5/3) + 60*a*b*n*(60*d^5 - 30*d^4*e*x^(1/3) + 20*d^3*e^2*x^(2/3) - 15*d^2*e^
3*x + 12*d*e^4*x^(4/3) - 10*e^5*x^(5/3)) + b^2*n^2*(-8820*d^5 + 2610*d^4*e*
x^(1/3) - 1140*d^3*e^2*x^(2/3) + 555*d^2*e^3*x - 264*d*e^4*x^(4/3) + 100*e^
```

```
5*x^(5/3)) + 60*b*(60*a*e^5*x^(5/3) + b*n*(60*d^5 - 30*d^4*e*x^(1/3) + 20*d
^3*e^2*x^(2/3) - 15*d^2*e^3*x + 12*d*e^4*x^(4/3) - 10*e^5*x^(5/3)))*Log[c*(
d + e*x^(1/3))^n] + 1800*b^2*e^5*x^(5/3)*Log[c*(d + e*x^(1/3))^n]^2))/(3600
*e^6)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e*x^(1/3))^n))^2,x)
[Out] int(x*(a+b*ln(c*(d+e*x^(1/3))^n))^2,x)
```

Maxima [A]
time = 0.31, size = 316, normalized size = 0.66

$$\frac{1}{2}\,b^{2}x^{2}\log\left(\left(x^{\frac{1}{3}}e+d\right)^{n}c\right)^{2}-\frac{1}{60}\left(60\,d^{6}e^{(-7)}\log\left(x^{\frac{1}{3}}e+d\right)+\left(30\,d^{4}x^{\frac{2}{3}}e-60\,d^{5}x^{\frac{1}{3}}-20\,d^{3}xe^{2}+15\,d^{2}x^{\frac{4}{3}}e^{3}-12\,dx^{\frac{5}{3}}e^{4}+10\,x^{2}e^{5}\right)e^{(-6)}\right)abne+abx^{2}\log\left(\left(x^{\frac{1}{3}}e+d\right)^{n}c\right)+\frac{1}{2}\,a^{2}x^{2}+\frac{1}{3600}\left(\left(1800\,d^{6}\log\left(x^{\frac{1}{3}}e+d\right)^{2}+8820\,d^{6}\log\left(x^{\frac{1}{3}}e+d\right)-8820\,d^{5}x^{\frac{1}{3}}e+2610\,d^{4}x^{\frac{2}{3}}e^{2}-1140\,d^{3}xe^{3}+555\,d^{2}x^{\frac{4}{3}}e^{4}-264\,dx^{\frac{5}{3}}e^{5}+100\,x^{2}e^{6}\right)n^{2}e^{(-6)}-60\left(60\,d^{6}e^{(-7)}\log\left(x^{\frac{1}{3}}e+d\right)+\left(30\,d^{4}x^{\frac{2}{3}}e-60\,d^{5}x^{\frac{1}{3}}-20\,d^{3}xe^{2}+15\,d^{2}x^{\frac{4}{3}}e^{3}-12\,dx^{\frac{5}{3}}e^{4}+10\,x^{2}e^{5}\right)e^{(-6)}\right)ne\log\left(\left(x^{\frac{1}{3}}e+d\right)^{n}c\right)\right)b^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="maxima")
[Out] 1/2*b^2*x^2*log((x^(1/3)*e + d)^n*c)^2 - 1/60*(60*d^6*e^(-7)*log(x^(1/3)*e
+ d) + (30*d^4*x^(2/3)*e - 60*d^5*x^(1/3) - 20*d^3*x*e^2 + 15*d^2*x^(4/3)*e
^3 - 12*d*x^(5/3)*e^4 + 10*x^2*e^5)*e^(-6))*a*b*n*e + a*b*x^2*log((x^(1/3)*
e + d)^n*c) + 1/2*a^2*x^2 + 1/3600*((1800*d^6*log(x^(1/3)*e + d)^2 + 8820*d
^6*log(x^(1/3)*e + d) - 8820*d^5*x^(1/3)*e + 2610*d^4*x^(2/3)*e^2 - 1140*d^
3*x*e^3 + 555*d^2*x^(4/3)*e^4 - 264*d*x^(5/3)*e^5 + 100*x^2*e^6)*n^2*e^(-6)
 - 60*(60*d^6*e^(-7)*log(x^(1/3)*e + d) + (30*d^4*x^(2/3)*e - 60*d^5*x^(1/3
) - 20*d^3*x*e^2 + 15*d^2*x^(4/3)*e^3 - 12*d*x^(5/3)*e^4 + 10*x^2*e^5)*e^(-
6))*n*e*log((x^(1/3)*e + d)^n*c))*b^2
```

Fricas [A]
time = 0.40, size = 452, normalized size = 0.94

$$\frac{1}{3600}\left(1800\,b^{2}x^{2}e^{6}\log\left(c\right)^{2}+100\,(b^{2}n^{2}-6\,abn+18\,a^{2})x^{2}e^{6}-60\,(19\,b^{2}d^{3}n^{2}-20\,abd^{3}n)xe^{3}-1800\,(b^{2}d^{6}n^{2}-b^{2}n^{2}x^{2}e^{6})\log\left(x^{\frac{1}{3}}e+d\right)^{2}+60\left(147\,b^{2}d^{6}n^{2}-60\,abd^{6}n+20\,b^{2}d^{3}n^{2}xe^{3}-10\,(b^{2}n^{2}-6\,abn)x^{2}e^{6}-60\,(b^{2}d^{6}n-b^{2}nx^{2}e^{6})\log\left(c\right)-6\,(5\,b^{2}d^{4}n^{2}e^{2}-2\,b^{2}dn^{2}xe^{5})x^{\frac{2}{3}}+15\,(4\,b^{2}d^{5}n^{2}e-b^{2}d^{2}n^{2}xe^{4})x^{\frac{1}{3}}\right)\log\left(x^{\frac{1}{3}}e+d\right)+600\,(2\,b^{2}d^{3}nxe^{3}-(b^{2}n-6\,ab)x^{2}e^{6})\log\left(c\right)-6\,(4\,(11\,b^{2}dn^{2}-30\,abdn)xe^{5}-15\,(29\,b^{2}d^{4}n^{2}-20\,abd^{4}n)e^{2}+60\,(5\,b^{2}d^{4}ne^{2}-2\,b^{2}dnxe^{5})\log\left(c\right))x^{\frac{2}{3}}+15\,((37\,b^{2}d^{2}n^{2}-60\,abd^{2}n)xe^{4}-12\,(49\,b^{2}d^{5}n^{2}-20\,abd^{5}n)e+60\,(4\,b^{2}d^{5}ne-b^{2}d^{2}nxe^{4})\log\left(c\right))x^{\frac{1}{3}}\right)e^{(-6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="fricas")
[Out] 1/3600*(1800*b^2*x^2*e^6*log(c)^2 + 100*(b^2*n^2 - 6*a*b*n + 18*a^2)*x^2*e^
6 - 60*(19*b^2*d^3*n^2 - 20*a*b*d^3*n)*x*e^3 - 1800*(b^2*d^6*n^2 - b^2*n^2*
x^2*e^6)*log(x^(1/3)*e + d)^2 + 60*(147*b^2*d^6*n^2 - 60*a*b*d^6*n + 20*b^2
*d^3*n^2*x*e^3 - 10*(b^2*n^2 - 6*a*b*n)*x^2*e^6 - 60*(b^2*d^6*n - b^2*n*x^2
```

```
*e^6)*log(c) - 6*(5*b^2*d^4*n^2*e^2 - 2*b^2*d*n^2*x*e^5)*x^(2/3) + 15*(4*b^
2*d^5*n^2*e - b^2*d^2*n^2*x*e^4)*x^(1/3))*log(x^(1/3)*e + d) + 600*(2*b^2*d
^3*n*x*e^3 - (b^2*n - 6*a*b)*x^2*e^6)*log(c) - 6*(4*(11*b^2*d*n^2 - 30*a*b*
d*n)*x*e^5 - 15*(29*b^2*d^4*n^2 - 20*a*b*d^4*n)*e^2 + 60*(5*b^2*d^4*n*e^2 -
 2*b^2*d*n*x*e^5)*log(c))*x^(2/3) + 15*((37*b^2*d^2*n^2 - 60*a*b*d^2*n)*x*e
^4 - 12*(49*b^2*d^5*n^2 - 20*a*b*d^5*n)*e + 60*(4*b^2*d^5*n*e - b^2*d^2*n*x
*e^4)*log(c))*x^(1/3))*e^(-6)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/3))**n))**2,x)
```

```
[Out] Integral(x*(a + b*log(c*(d + e*x**(1/3))**n))**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 956 vs. 2(419) = 838.
time = 3.29, size = 956, normalized size = 1.99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="giac")
```

```
[Out] 1/3600*(1800*b^2*x^2*e*log(c)^2 + 3600*a*b*x^2*e*log(c) + (1800*(x^(1/3)*e
+ d)^6*e^(-5)*log(x^(1/3)*e + d)^2 - 10800*(x^(1/3)*e + d)^5*d*e^(-5)*log(x
^(1/3)*e + d)^2 + 27000*(x^(1/3)*e + d)^4*d^2*e^(-5)*log(x^(1/3)*e + d)^2 -
 36000*(x^(1/3)*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e + d)^2 + 27000*(x^(1/3)*e
 + d)^2*d^4*e^(-5)*log(x^(1/3)*e + d)^2 - 10800*(x^(1/3)*e + d)*d^5*e^(-5)*
log(x^(1/3)*e + d)^2 - 600*(x^(1/3)*e + d)^6*e^(-5)*log(x^(1/3)*e + d) + 43
20*(x^(1/3)*e + d)^5*d*e^(-5)*log(x^(1/3)*e + d) - 13500*(x^(1/3)*e + d)^4*
d^2*e^(-5)*log(x^(1/3)*e + d) + 24000*(x^(1/3)*e + d)^3*d^3*e^(-5)*log(x^(1
/3)*e + d) - 27000*(x^(1/3)*e + d)^2*d^4*e^(-5)*log(x^(1/3)*e + d) + 21600*
(x^(1/3)*e + d)*d^5*e^(-5)*log(x^(1/3)*e + d) + 100*(x^(1/3)*e + d)^6*e^(-5
) - 864*(x^(1/3)*e + d)^5*d*e^(-5) + 3375*(x^(1/3)*e + d)^4*d^2*e^(-5) - 80
00*(x^(1/3)*e + d)^3*d^3*e^(-5) + 13500*(x^(1/3)*e + d)^2*d^4*e^(-5) - 2160
0*(x^(1/3)*e + d)*d^5*e^(-5))*b^2*n^2 + 1800*a^2*x^2*e + 60*(60*(x^(1/3)*e
+ d)^6*e^(-5)*log(x^(1/3)*e + d) - 360*(x^(1/3)*e + d)^5*d*e^(-5)*log(x^(1/
3)*e + d) + 900*(x^(1/3)*e + d)^4*d^2*e^(-5)*log(x^(1/3)*e + d) - 1200*(x^(
1/3)*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e + d) + 900*(x^(1/3)*e + d)^2*d^4*e^(
-5)*log(x^(1/3)*e + d) - 360*(x^(1/3)*e + d)*d^5*e^(-5)*log(x^(1/3)*e + d)
- 10*(x^(1/3)*e + d)^6*e^(-5) + 72*(x^(1/3)*e + d)^5*d*e^(-5) - 225*(x^(1/3
```

```
)*e + d)^4*d^2*e^(-5) + 400*(x^(1/3)*e + d)^3*d^3*e^(-5) - 450*(x^(1/3)*e +
 d)^2*d^4*e^(-5) + 360*(x^(1/3)*e + d)*d^5*e^(-5))*b^2*n*log(c) + 60*(60*(x
^(1/3)*e + d)^6*e^(-5)*log(x^(1/3)*e + d) - 360*(x^(1/3)*e + d)^5*d*e^(-5)*
log(x^(1/3)*e + d) + 900*(x^(1/3)*e + d)^4*d^2*e^(-5)*log(x^(1/3)*e + d) -
1200*(x^(1/3)*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e + d) + 900*(x^(1/3)*e + d)^
2*d^4*e^(-5)*log(x^(1/3)*e + d) - 360*(x^(1/3)*e + d)*d^5*e^(-5)*log(x^(1/3
)*e + d) - 10*(x^(1/3)*e + d)^6*e^(-5) + 72*(x^(1/3)*e + d)^5*d*e^(-5) - 22
5*(x^(1/3)*e + d)^4*d^2*e^(-5) + 400*(x^(1/3)*e + d)^3*d^3*e^(-5) - 450*(x^
(1/3)*e + d)^2*d^4*e^(-5) + 360*(x^(1/3)*e + d)*d^5*e^(-5))*a*b*n)*e^(-1)
```

Mupad [B]
time = 1.72, size = 431, normalized size = 0.90

$$\frac{a^2 x^2}{2}+\frac{b^2 x^2 \ln \left(c \left(d+e x^{1/3}\right)^n\right)^2}{2}+\frac{b^2 n^2 x^2}{36}+a b x^2 \ln \left(c \left(d+e x^{1/3}\right)^n\right)-\frac{b^2 d^6 \ln \left(c \left(d+e x^{1/3}\right)^n\right)^2}{2 e^6}-\frac{a b n x^2}{6}-\frac{b^2 n x^2 \ln \left(c \left(d+e x^{1/3}\right)^n\right)}{6}+\frac{49 b^2 d^6 n^2 \ln \left(d+e x^{1/3}\right)}{20 e^6}+\frac{37 b^2 d^2 n^2 x^{4/3}}{240 e^2}+\frac{29 b^2 d^4 n^2 x^{2/3}}{40 e^4}-\frac{49 b^2 d^5 n^2 x^{1/3}}{20 e^5}-\frac{19 b^2 d^3 n^2 x}{60 e^3}-\frac{11 b^2 d n^2 x^{5/3}}{150 e}-\frac{b^2 d^2 n x^{4/3} \ln \left(c \left(d+e x^{1/3}\right)^n\right)}{4 e^2}-\frac{b^2 d^4 n x^{2/3} \ln \left(c \left(d+e x^{1/3}\right)^n\right)}{2 e^4}+\frac{b^2 d^5 n x^{1/3} \ln \left(c \left(d+e x^{1/3}\right)^n\right)}{e^5}+\frac{a b d^3 n x}{3 e^3}+\frac{a b d n x^{5/3}}{5 e}-\frac{a b d^6 n \ln \left(d+e x^{1/3}\right)}{e^6}+\frac{b^2 d^3 n x \ln \left(c \left(d+e x^{1/3}\right)^n\right)}{3 e^3}+\frac{b^2 d n x^{5/3} \ln \left(c \left(d+e x^{1/3}\right)^n\right)}{5 e}-\frac{a b d^2 n x^{4/3}}{4 e^2}-\frac{a b d^4 n x^{2/3}}{2 e^4}+\frac{a b d^5 n x^{1/3}}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e*x^(1/3))^n))^2,x)
```

```
[Out] (a^2*x^2)/2 + (b^2*x^2*log(c*(d + e*x^(1/3))^n)^2)/2 + (b^2*n^2*x^2)/36 + a
*b*x^2*log(c*(d + e*x^(1/3))^n) - (b^2*d^6*log(c*(d + e*x^(1/3))^n)^2)/(2*e
^6) - (a*b*n*x^2)/6 - (b^2*n*x^2*log(c*(d + e*x^(1/3))^n))/6 + (49*b^2*d^6*
n^2*log(d + e*x^(1/3)))/(20*e^6) + (37*b^2*d^2*n^2*x^(4/3))/(240*e^2) + (29
*b^2*d^4*n^2*x^(2/3))/(40*e^4) - (49*b^2*d^5*n^2*x^(1/3))/(20*e^5) - (19*b^
2*d^3*n^2*x)/(60*e^3) - (11*b^2*d*n^2*x^(5/3))/(150*e) - (b^2*d^2*n*x^(4/3)
*log(c*(d + e*x^(1/3))^n))/(4*e^2) - (b^2*d^4*n*x^(2/3)*log(c*(d + e*x^(1/3
))^n))/(2*e^4) + (b^2*d^5*n*x^(1/3)*log(c*(d + e*x^(1/3))^n))/e^5 + (a*b*d^
3*n*x)/(3*e^3) + (a*b*d*n*x^(5/3))/(5*e) - (a*b*d^6*n*log(d + e*x^(1/3)))/e
^6 + (b^2*d^3*n*x*log(c*(d + e*x^(1/3))^n))/(3*e^3) + (b^2*d*n*x^(5/3)*log(
c*(d + e*x^(1/3))^n))/(5*e) - (a*b*d^2*n*x^(4/3))/(4*e^2) - (a*b*d^4*n*x^(2
/3))/(2*e^4) + (a*b*d^5*n*x^(1/3))/e^5
```

3.452 $\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$

Optimal. Leaf size=267

$$-\frac{3b^2dn^2\left(d+e\sqrt[3]{x}\right)^2}{2e^3}+\frac{2b^2n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^3}+\frac{6b^2d^2n^2\sqrt[3]{x}}{e^2}-\frac{b^2d^3n^2\log^2\left(d+e\sqrt[3]{x}\right)}{e^3}-\frac{6bd^2n\left(d+e\sqrt[3]{x}\right)\left(a+\cdots\right.}{e^3\cdots}$$

```
[Out] -3/2*b^2*d*n^2*(d+e*x^(1/3))^2/e^3+2/9*b^2*n^2*(d+e*x^(1/3))^3/e^3+6*b^2*d^
2*n^2*x^(1/3)/e^2-b^2*d^3*n^2*ln(d+e*x^(1/3))^2/e^3-6*b*d^2*n*(d+e*x^(1/3))
*(a+b*ln(c*(d+e*x^(1/3))^n))/e^3+3*b*d*n*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(
1/3))^n))/e^3-2/3*b*n*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))/e^3+2*b*d
^3*n*ln(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))/e^3+x*(a+b*ln(c*(d+e*x^(1/
3))^n))^2
```

Rubi [A]
time = 0.20, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2501, 2445, 2458, 45, 2372, 12, 14, 2338}

$$\frac{2bd^3n\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^3}-\frac{6bd^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^3}+\frac{3bdn\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^3}-\frac{2bn\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^3}+x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2-\frac{b^2d^3n^2\log^2\left(d+e\sqrt[3]{x}\right)}{e^3}+\frac{6b^2d^2n^2\sqrt[3]{x}}{e^2}-\frac{3b^2dn^2\left(d+e\sqrt[3]{x}\right)^2}{2e^3}+\frac{2b^2n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])^2,x]
```

```
[Out] (-3*b^2*d*n^2*(d + e*x^(1/3))^2)/(2*e^3) + (2*b^2*n^2*(d + e*x^(1/3))^3)/(9
*e^3) + (6*b^2*d^2*n^2*x^(1/3))/e^2 - (b^2*d^3*n^2*Log[d + e*x^(1/3)]^2)/e^
3 - (6*b*d^2*n*(d + e*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n]))/e^3 + (3*b
*d*n*(d + e*x^(1/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/e^3 - (2*b*n*(d +
e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n]))/(3*e^3) + (2*b*d^3*n*Log[d +
 e*x^(1/3)]*(a + b*Log[c*(d + e*x^(1/3))^n]))/e^3 + x*(a + b*Log[c*(d + e*x
^(1/3))^n])^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2501

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx = 3\text{Subst}\left(\int x^2(a+b\log\left(c(d+ex)^n\right))^2\,dx, x, \sqrt[3]{x}\right)$$

$$= x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 - (2ben)\text{Subst}\left(\int \frac{x^3(a+b\log(c(d+e}{d+ex}\right.$$

$$= x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 - (2bn)\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^3(a+b\log}{x}\right.$$

$$= -\frac{1}{3}bn\left(\frac{18d^2\left(d+e\sqrt[3]{x}\right)}{e^3} - \frac{9d\left(d+e\sqrt[3]{x}\right)^2}{e^3} + \frac{2\left(d+e\sqrt[3]{x}\right)^3}{e^3} - \frac{6d^3\log}{}\right.$$

$$= -\frac{1}{3}bn\left(\frac{18d^2\left(d+e\sqrt[3]{x}\right)}{e^3} - \frac{9d\left(d+e\sqrt[3]{x}\right)^2}{e^3} + \frac{2\left(d+e\sqrt[3]{x}\right)^3}{e^3} - \frac{6d^3\log}{}\right.$$

$$= -\frac{1}{3}bn\left(\frac{18d^2\left(d+e\sqrt[3]{x}\right)}{e^3} - \frac{9d\left(d+e\sqrt[3]{x}\right)^2}{e^3} + \frac{2\left(d+e\sqrt[3]{x}\right)^3}{e^3} - \frac{6d^3\log}{}\right.$$

$$= -\frac{3b^2dn^2\left(d+e\sqrt[3]{x}\right)^2}{2e^3} + \frac{2b^2n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^3} + \frac{6b^2d^2n^2\sqrt[3]{x}}{e^2} - \frac{1}{3}bn\left(\frac{18}{}\right.$$

$$= -\frac{3b^2dn^2\left(d+e\sqrt[3]{x}\right)^2}{2e^3} + \frac{2b^2n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^3} + \frac{6b^2d^2n^2\sqrt[3]{x}}{e^2} - \frac{b^2d^3n^2\,\mathrm{l}}{}$$

Mathematica [A]
time = 0.10, size = 249, normalized size = 0.93

$$\frac{-36abd^2en\sqrt[3]{x}+66b^2d^2en^2\sqrt[3]{x}+18abde^2nx^{2/3}-15b^2de^2n^2x^{2/3}+18a^2e^3x-12abe^3nx+4b^2e^3n^2x-18b^2d^3n^2\log^2\left(d+e\sqrt[3]{x}\right)-6b\left(-6ae^3x+b\left(6d^2en\sqrt[3]{x}-3de^2nx^{2/3}+2e^3nx\right)\right)\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)+18b^2e^3x\log^2\left(c\left(d+e\sqrt[3]{x}\right)^n\right)+6bd^3n\log\left(d+e\sqrt[3]{x}\right)\left(6a-11bn+6b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{18e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^2,x]
```

```
[Out] (-36*a*b*d^2*e*n*x^(1/3) + 66*b^2*d^2*e*n^2*x^(1/3) + 18*a*b*d*e^2*n*x^(2/3
) - 15*b^2*d*e^2*n^2*x^(2/3) + 18*a^2*e^3*x - 12*a*b*e^3*n*x + 4*b^2*e^3*n^
2*x - 18*b^2*d^3*n^2*Log[d + e*x^(1/3)]^2 - 6*b*(-6*a*e^3*x + b*(6*d^2*e*n*
x^(1/3) - 3*d*e^2*n*x^(2/3) + 2*e^3*n*x))*Log[c*(d + e*x^(1/3))^n] + 18*b^2
*e^3*x*Log[c*(d + e*x^(1/3))^n]^2 + 6*b*d^3*n*Log[d + e*x^(1/3)]*(6*a - 11*
b*n + 6*b*Log[c*(d + e*x^(1/3))^n]))/(18*e^3)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))^2,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))^2,x)
```

Maxima [A]
time = 0.28, size = 217, normalized size = 0.81

$$\frac{1}{3}\left(\left(6\,d^3e^{(-4)}\log\left(x^{\frac{1}{3}}e+d\right)+\left(3\,dx^{\frac{2}{3}}e-6\,d^2x^{\frac{1}{3}}-2\,xe^2\right)e^{(-3)}\right)ne+6\,x\log\left(\left(x^{\frac{1}{3}}e+d\right)^nc\right)\right)ab-\frac{1}{18}\left(\left(18\,d^3\log\left(x^{\frac{1}{3}}e+d\right)^2+66\,d^3\log\left(x^{\frac{1}{3}}e+d\right)-66\,d^2x^{\frac{1}{3}}e+15\,dx^{\frac{2}{3}}e^2-4\,xe^3\right)n^2e^{(-3)}-6\left(6\,d^3e^{(-4)}\log\left(x^{\frac{1}{3}}e+d\right)+\left(3\,dx^{\frac{2}{3}}e-6\,d^2x^{\frac{1}{3}}-2\,xe^2\right)e^{(-3)}\right)ne\log\left(\left(x^{\frac{1}{3}}e+d\right)^nc\right)-18\,x\log\left(\left(x^{\frac{1}{3}}e+d\right)^nc\right)^2\right)b^2+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/3*((6*d^3*e^(-4)*log(x^(1/3)*e + d) + (3*d*x^(2/3)*e - 6*d^2*x^(1/3) - 2*
x*e^2)*e^(-3))*n*e + 6*x*log((x^(1/3)*e + d)^n*c))*a*b - 1/18*((18*d^3*log(
x^(1/3)*e + d)^2 + 66*d^3*log(x^(1/3)*e + d) - 66*d^2*x^(1/3)*e + 15*d*x^(2
/3)*e^2 - 4*x*e^3)*n^2*e^(-3) - 6*(6*d^3*e^(-4)*log(x^(1/3)*e + d) + (3*d*x
^(2/3)*e - 6*d^2*x^(1/3) - 2*x*e^2)*e^(-3))*n*e*log((x^(1/3)*e + d)^n*c) -
18*x*log((x^(1/3)*e + d)^n*c)^2)*b^2 + a^2*x
```

Fricas [A]
time = 0.37, size = 272, normalized size = 1.02

$$\frac{1}{18}\left(18\,b^2xe^3\log\left(c\right)^2-12\,(b^2n-3\,ab)xe^3\log\left(c\right)+2\,(2\,b^2n^2-6\,abn+9\,a^2)xe^3+18\,(b^2d^3n^2+b^2n^2xe^3)\log\left(x^{\frac{1}{3}}e+d\right)^2-6\left(6\,b^2d^2n^2x^{\frac{1}{3}}e+11\,b^2d^3n^2-3\,b^2dn^2x^{\frac{2}{3}}e^2-6\,abd^3n+2\,(b^2n^2-3\,abn)xe^3-6\,(b^2d^3n+b^2nxe^3)\log\left(c\right)\right)\log\left(x^{\frac{1}{3}}e+d\right)+3\,(6\,b^2dne^2\log\left(c\right)-(5\,b^2dn^2-6\,abdn)e^2)x^{\frac{2}{3}}-6\,(6\,b^2d^2ne\log\left(c\right)-(11\,b^2d^2n^2-6\,abd^2n)e)x^{\frac{1}{3}}\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="fricas")
```

```
[Out] 1/18*(18*b^2*x*e^3*log(c)^2 - 12*(b^2*n - 3*a*b)*x*e^3*log(c) + 2*(2*b^2*n^
2 - 6*a*b*n + 9*a^2)*x*e^3 + 18*(b^2*d^3*n^2 + b^2*n^2*x*e^3)*log(x^(1/3)*e
 + d)^2 - 6*(6*b^2*d^2*n^2*x^(1/3)*e + 11*b^2*d^3*n^2 - 3*b^2*d*n^2*x^(2/3)
*e^2 - 6*a*b*d^3*n + 2*(b^2*n^2 - 3*a*b*n)*x*e^3 - 6*(b^2*d^3*n + b^2*n*x*e
^3)*log(c))*log(x^(1/3)*e + d) + 3*(6*b^2*d*n*e^2*log(c) - (5*b^2*d*n^2 - 6
*a*b*d*n)*e^2)*x^(2/3) - 6*(6*b^2*d^2*n*e*log(c) - (11*b^2*d^2*n^2 - 6*a*b*
d^2*n)*e)*x^(1/3))*e^(-3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**(1/3))**n))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 479 vs. 2(239) = 478.
time = 4.15, size = 479, normalized size = 1.79

$$\frac{1}{18}\left(18 b^{2} x e \log\left(c\right)^{2} + \left(18 \left(x^{\frac{1}{3}} e + d\right)^{3} e^{(-2)} \log\left(x^{\frac{1}{3}} e + d\right)^{2} - 54 \left(x^{\frac{1}{3}} e + d\right)^{2} d e^{(-2)} \log\left(x^{\frac{1}{3}} e + d\right)^{2} + 54 \left(x^{\frac{1}{3}} e + d\right) d^{2} e^{(-2)} \log\left(x^{\frac{1}{3}} e + d\right)^{2} - 12 \left(x^{\frac{1}{3}} e + d\right)^{3} e^{(-2)} \log\left(x^{\frac{1}{3}} e + d\right) + 54 \left(x^{\frac{1}{3}} e + d\right)^{2} d e^{(-2)} \log\left(x^{\frac{1}{3}} e + d\right) - 108 \left(x^{\frac{1}{3}} e + d\right) d^{2} e^{(-2)} \log\left(x^{\frac{1}{3}} e + d\right) + 4 \left(x^{\frac{1}{3}} e + d\right)^{3} e^{(-2)} - 27 \left(x^{\frac{1}{3}} e + d\right)^{2} d e^{(-2)} + 108 \left(x^{\frac{1}{3}} e + d\right) d^{2} e^{(-2)}\right) b^{2} n^{2} + 6 \left(6 \left(x^{\frac{1}{3}} e + d\right)^{3} e^{(-2)} \log\left(x^{\frac{1}{3}} e + d\right) - 18 \left(x^{\frac{1}{3}} e + d\right)^{2} d e^{(-2)} \log\left(x^{\frac{1}{3}} e + d\right) + 18 \left(x^{\frac{1}{3}} e + d\right) d^{2} e^{(-2)} \log\left(x^{\frac{1}{3}} e + d\right) - 2 \left(x^{\frac{1}{3}} e + d\right)^{3} e^{(-2)} + 9 \left(x^{\frac{1}{3}} e + d\right)^{2} d e^{(-2)} - 18 \left(x^{\frac{1}{3}} e + d\right) d^{2} e^{(-2)}\right) b^{2} n \log\left(c\right) + 36 a b x e \log\left(c\right) + 6 \left(6 \left(x^{\frac{1}{3}} e + d\right)^{3} e^{(-2)} \log\left(x^{\frac{1}{3}} e + d\right) - 18 \left(x^{\frac{1}{3}} e + d\right)^{2} d e^{(-2)} \log\left(x^{\frac{1}{3}} e + d\right) + 18 \left(x^{\frac{1}{3}} e + d\right) d^{2} e^{(-2)} \log\left(x^{\frac{1}{3}} e + d\right) - 2 \left(x^{\frac{1}{3}} e + d\right)^{3} e^{(-2)} + 9 \left(x^{\frac{1}{3}} e + d\right)^{2} d e^{(-2)} - 18 \left(x^{\frac{1}{3}} e + d\right) d^{2} e^{(-2)}\right) a b n + 18 a^{2} x e\right) e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="giac")
```

```
[Out] 1/18*(18*b^2*x*e*log(c)^2 + (18*(x^(1/3)*e + d)^3*e^(-2)*log(x^(1/3)*e + d)
^2 - 54*(x^(1/3)*e + d)^2*d*e^(-2)*log(x^(1/3)*e + d)^2 + 54*(x^(1/3)*e + d
)*d^2*e^(-2)*log(x^(1/3)*e + d)^2 - 12*(x^(1/3)*e + d)^3*e^(-2)*log(x^(1/3)
*e + d) + 54*(x^(1/3)*e + d)^2*d*e^(-2)*log(x^(1/3)*e + d) - 108*(x^(1/3)*e
 + d)*d^2*e^(-2)*log(x^(1/3)*e + d) + 4*(x^(1/3)*e + d)^3*e^(-2) - 27*(x^(1
/3)*e + d)^2*d*e^(-2) + 108*(x^(1/3)*e + d)*d^2*e^(-2))*b^2*n^2 + 6*(6*(x^(
1/3)*e + d)^3*e^(-2)*log(x^(1/3)*e + d) - 18*(x^(1/3)*e + d)^2*d*e^(-2)*log
(x^(1/3)*e + d) + 18*(x^(1/3)*e + d)*d^2*e^(-2)*log(x^(1/3)*e + d) - 2*(x^(
1/3)*e + d)^3*e^(-2) + 9*(x^(1/3)*e + d)^2*d*e^(-2) - 18*(x^(1/3)*e + d)*d^
2*e^(-2))*b^2*n*log(c) + 36*a*b*x*e*log(c) + 6*(6*(x^(1/3)*e + d)^3*e^(-2)*
log(x^(1/3)*e + d) - 18*(x^(1/3)*e + d)^2*d*e^(-2)*log(x^(1/3)*e + d) + 18*
(x^(1/3)*e + d)*d^2*e^(-2)*log(x^(1/3)*e + d) - 2*(x^(1/3)*e + d)^3*e^(-2)
+ 9*(x^(1/3)*e + d)^2*d*e^(-2) - 18*(x^(1/3)*e + d)*d^2*e^(-2))*a*b*n + 18*
a^2*x*e)*e^(-1)
```

Mupad [B]
time = 0.51, size = 290, normalized size = 1.09

$$\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)\,\left(\frac{2\,b\,x\,\left(3\,a-b\,n\right)}{3}-x^{2/3}\,\left(\frac{b\,d\,\left(3\,a-b\,n\right)}{e}-\frac{3\,a\,b\,d}{e}\right)+\frac{d\,x^{1/3}\,\left(\frac{2\,b\,d\,\left(3\,a-b\,n\right)}{e}-\frac{6\,a\,b\,d}{e}\right)}{e}\right)-x^{2/3}\,\left(\frac{d\,\left(3\,a^2-2\,a\,b\,n+\frac{2\,b^2\,n^2}{3}\right)}{2\,e}-\frac{d\,\left(3\,a^2-b^2\,n^2\right)}{2\,e}\right)+x^{1/3}\,\left(\frac{d\,\left(\frac{d\,\left(3\,a^2-2\,a\,b\,n+\frac{2\,b^2\,n^2}{3}\right)}{e}-\frac{d\,\left(3\,a^2-b^2\,n^2\right)}{e}\right)}{e}+\frac{2\,b^2\,d^2\,n^2}{e^2}\right)+x\,\left(a^2-\frac{2\,a\,b\,n}{3}+\frac{2\,b^2\,n^2}{9}\right)+\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)^2\,\left(b^2\,x+\frac{b^2\,d^3}{e^3}\right)-\frac{\ln\left(d+e\,x^{1/3}\right)\,\left(11\,b^2\,d^3\,n^2-6\,a\,b\,d^3\,n\right)}{3\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))^2,x)
```

```
[Out] log(c*(d + e*x^(1/3))^n)*((2*b*x*(3*a - b*n))/3 - x^(2/3)*((b*d*(3*a - b*n)
)/e - (3*a*b*d)/e) + (d*x^(1/3)*((2*b*d*(3*a - b*n))/e - (6*a*b*d)/e))/e) -
 x^(2/3)*((d*(3*a^2 + (2*b^2*n^2)/3 - 2*a*b*n))/(2*e) - (d*(3*a^2 - b^2*n^2
))/(2*e)) + x^(1/3)*((d*((d*(3*a^2 + (2*b^2*n^2)/3 - 2*a*b*n))/e - (d*(3*a^
2 - b^2*n^2))/e))/e + (2*b^2*d^2*n^2)/e^2) + x*(a^2 + (2*b^2*n^2)/9 - (2*a*
b*n)/3) + log(c*(d + e*x^(1/3))^n)^2*(b^2*x + (b^2*d^3)/e^3) - (log(d + e*x
^(1/3))*(11*b^2*d^3*n^2 - 6*a*b*d^3*n))/(3*e^3)
```

3.453 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x}\,dx$

Optimal. Leaf size=93

$$3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt[3]{x}}{d}\right)+6bn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\text{Li}_2\left(1+\frac{e\sqrt[3]{x}}{d}\right)-6b^2n^2\text{Li}_3\left(1-\right.$$

```
[Out] 3*(a+b*ln(c*(d+e*x^(1/3))^n))^2*ln(-e*x^(1/3)/d)+6*b*n*(a+b*ln(c*(d+e*x^(1/
3))^n))*polylog(2,1+e*x^(1/3)/d)-6*b^2*n^2*polylog(3,1+e*x^(1/3)/d)
```

Rubi [A]
time = 0.09, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2504, 2443, 2481, 2421, 6724}

$$6bn\text{PolyLog}\left(2,\frac{e\sqrt[3]{x}}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-6b^2n^2\text{PolyLog}\left(3,\frac{e\sqrt[3]{x}}{d}+1\right)+3\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])^2/x,x]
```

```
[Out] 3*(a + b*Log[c*(d + e*x^(1/3))^n])^2*Log[-((e*x^(1/3))/d)] + 6*b*n*(a + b*L
og[c*(d + e*x^(1/3))^n])*PolyLog[2, 1 + (e*x^(1/3))/d] - 6*b^2*n^2*PolyLog[
3, 1 + (e*x^(1/3))/d]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
```

```
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x}\,dx = 3\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x}\,dx, x, \sqrt[3]{x}\right)$$

$$= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt[3]{x}}{d}\right) - (6ben)\text{Subst}\left(\int \frac{\log(\cdots}{\cdots}\right.$$

$$= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt[3]{x}}{d}\right) - (6bn)\text{Subst}\left(\int \frac{(a+b\cdots}{\cdots}\right.$$

$$= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt[3]{x}}{d}\right) + 6bn\left(a+b\log\left(c\left(d+e\cdots\right.\right.\right.$$

$$= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt[3]{x}}{d}\right) + 6bn\left(a+b\log\left(c\left(d+e\cdots\right.\right.\right.$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 195 vs. 2(93) = 186.
time = 0.07, size = 195, normalized size = 2.10

$$\left(a-bn\log\left(d+e\sqrt[3]{x}\right)+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\log(x)+2bn\left(a-bn\log\left(d+e\sqrt[3]{x}\right)+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\left(\left(\log\left(d+e\sqrt[3]{x}\right)-\log\left(1+\frac{e\sqrt[3]{x}}{d}\right)\right)\log(x)-3\text{Li}_2\left(-\frac{e\sqrt[3]{x}}{d}\right)\right)+3b^2n^2\left(\log^2\left(d+e\sqrt[3]{x}\right)\log\left(-\frac{e\sqrt[3]{x}}{d}\right)+2\log\left(d+e\sqrt[3]{x}\right)\text{Li}_2\left(1+\frac{e\sqrt[3]{x}}{d}\right)-2\text{Li}_3\left(1+\frac{e\sqrt[3]{x}}{d}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^2/x,x]
```

```
[Out] (a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^2*Log[x] + 2*b*n*
(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])*((Log[d + e*x^(1/
3)] - Log[1 + (e*x^(1/3))/d])*Log[x] - 3*PolyLog[2, -((e*x^(1/3))/d)]) + 3*
b^2*n^2*(Log[d + e*x^(1/3)]^2*Log[-((e*x^(1/3))/d)] + 2*Log[d + e*x^(1/3)]*
PolyLog[2, 1 + (e*x^(1/3))/d] - 2*PolyLog[3, 1 + (e*x^(1/3))/d])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln \left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))^2/x,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))^2/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x,x, algorithm="maxima")
```

```
[Out] b^2*log((x^(1/3)*e + d)^n)^2*log(x) + integrate(1/3*(3*(b^2*log(c)^2 + 2*a*
b*log(c) + a^2)*x*e - 2*(b^2*n*x*e*log(x) - 3*(b^2*log(c) + a*b)*x*e - 3*(b
^2*d*log(c) + a*b*d)*x^(2/3))*log((x^(1/3)*e + d)^n) + 3*(b^2*d*log(c)^2 +
2*a*b*d*log(c) + a^2*d)*x^(2/3))/(x^2*e + d*x^(5/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((x^(1/3)*e + d)^n*c)^2 + 2*a*b*log((x^(1/3)*e + d)^n*c) +
 a^2)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log \left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))**2/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**(1/3))**n))**2/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^(1/3)*e + d)^n*c) + a)^2/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))^2/x,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/3))^n))^2/x, x)
```

3.454 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=231

$$-\frac{b^2e^2n^2}{d^2\sqrt[3]{x}}+\frac{b^2e^3n^2\log\left(d+e\sqrt[3]{x}\right)}{d^3}-\frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{dx^{2/3}}+\frac{2be^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right.\right.\right.}{d^3\sqrt[3]{x}}$$

```
[Out] -b^2*e^2*n^2/d^2/x^(1/3)+b^2*e^3*n^2*ln(d+e*x^(1/3))/d^3-b*e*n*(a+b*ln(c*(d
+e*x^(1/3))^n))/d/x^(2/3)+2*b*e^2*n*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n
))/d^3/x^(1/3)+2*b*e^3*n*ln(1-d/(d+e*x^(1/3)))*(a+b*ln(c*(d+e*x^(1/3))^n))/
d^3-(a+b*ln(c*(d+e*x^(1/3))^n))^2/x-b^2*e^3*n^2*ln(x)/d^3-2*b^2*e^3*n^2*pol
ylog(2,d/(d+e*x^(1/3)))/d^3
```

Rubi [A]
time = 0.29, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2504, 2445, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$-\frac{2b^2e^3n^2\text{PolyLog}\left(2,\frac{d}{d+e\sqrt[3]{x}}\right)}{d^3}+\frac{2be^3n\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3}+\frac{2be^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3\sqrt[3]{x}}-\frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{dx^{2/3}}-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x}+\frac{b^2e^3n^2\log\left(d+e\sqrt[3]{x}\right)}{d^3}-\frac{b^2e^3n^2\log(x)}{d^3}-\frac{b^2e^2n^2}{d^2\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])^2/x^2,x]
```

```
[Out] -((b^2*e^2*n^2)/(d^2*x^(1/3))) + (b^2*e^3*n^2*Log[d + e*x^(1/3)])/d^3 - (b*
e*n*(a + b*Log[c*(d + e*x^(1/3))^n]))/(d*x^(2/3)) + (2*b*e^2*n*(d + e*x^(1/
3))*(a + b*Log[c*(d + e*x^(1/3))^n]))/(d^3*x^(1/3)) + (2*b*e^3*n*Log[1 - d/
(d + e*x^(1/3))]*(a + b*Log[c*(d + e*x^(1/3))^n]))/d^3 - (a + b*Log[c*(d +
e*x^(1/3))^n])^2/x - (b^2*e^3*n^2*Log[x])/d^3 - (2*b^2*e^3*n^2*PolyLog[2, d
/(d + e*x^(1/3))])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
```

```
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^2}\,dx = 3\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^4}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x} + (2ben)\text{Subst}\left(\int \frac{a+b\log(c(d+ex)^n}{x^3(d+ex)}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x} + (2bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, c\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x} + \frac{(2bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+e\right.}{d}$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{dx^{2/3}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x} - \frac{(2b}{}$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{dx^{2/3}} + \frac{2be^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c(d+\right.\right.}{d^3\sqrt[3]{x}}$$

$$= -\frac{b^2e^2n^2}{d^2\sqrt[3]{x}} + \frac{b^2e^3n^2\log\left(d+e\sqrt[3]{x}\right)}{d^3} - \frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{dx^{2/3}} +$$

$$= -\frac{b^2e^2n^2}{d^2\sqrt[3]{x}} + \frac{b^2e^3n^2\log\left(d+e\sqrt[3]{x}\right)}{d^3} - \frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{dx^{2/3}} +$$

Mathematica [A]

time = 0.16, size = 274, normalized size = 1.19

$$3\left(-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{3x}+\frac{2}{3}ben\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{2dx^{2/3}}+\frac{e\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^2\sqrt[3]{x}}-\frac{e^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2bd^3n}+\frac{e^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt[3]{x}}{d}\right)}{d^3}-\frac{be^2n\left(-\frac{\log\left(d+e\sqrt[3]{x}\right)}{d}+\frac{\log(x)}{3d}\right)}{d^2}-\frac{ben\left(\frac{1}{d\sqrt[3]{x}}-\frac{e\log\left(d+e\sqrt[3]{x}\right)}{d^2}+\frac{e\log(x)}{3d^2}\right)}{2d}+\frac{be^2n\text{Li}_2\left(\frac{d+e\sqrt[3]{x}}{d}\right)}{d^3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^2/x^2,x]
```

```
[Out] 3*(-1/3*(a + b*Log[c*(d + e*x^(1/3))^n])^2/x + (2*b*e*n*(-1/2*(a + b*Log[c*
(d + e*x^(1/3))^n])/(d*x^(2/3)) + (e*(a + b*Log[c*(d + e*x^(1/3))^n]))/(d^2
*x^(1/3)) - (e^2*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(2*b*d^3*n) + (e^2*(a
+ b*Log[c*(d + e*x^(1/3))^n])*Log[-((e*x^(1/3))/d)])/d^3 - (b*e^2*n*(-(Log[
d + e*x^(1/3)]/d) + Log[x]/(3*d)))/d^2 - (b*e*n*(1/(d*x^(1/3)) - (e*Log[d +
 e*x^(1/3)])/d^2 + (e*Log[x])/(3*d^2)))/(2*d) + (b*e^2*n*PolyLog[2, (d + e*
x^(1/3))/d])/d^3))/3)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^{2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))^2/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))^2/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x^2,x, algorithm="maxima")
```

```
[Out] -2*(log(x^(1/3))*log(e^(1/3*log(x) + 1)/d + 1) + dilog(-e^(1/3*log(x) + 1)/
d))*b^2*n^2*e^3/d^3 + ((3*n^2 - 2*n*log(c))*b^2 - 2*a*b*n)*e^3*log(d + e^(1
/3*log(x) + 1))/d^3 + 2*(b^2*n*log(c) + a*b*n)*e^3*log(x^(1/3))/d^3 + integ
rate(-(b^2*d^3*n^2*e^3 - b^2*n^2*x*e^6)/x, x)/d^6 - 1/20*(100*b^2*d^4*n^2*e
^(1/3*log(x) + 4) - 40*b^2*d^3*n^2*e^(2/3*log(x) + 5) + 20*b^2*d^2*n^2*e^(l
og(x) + 6) - 15*b^2*d*n^2*e^(4/3*log(x) + 7) + 12*b^2*n^2*e^(5/3*log(x) + 8
) - 20*(2*b^2*d^4*n^2*e^(1/3*log(x) + 4) - b^2*d^3*n^2*e^(2/3*log(x) + 5))*
log(x^(1/3)))/d^8 + 1/60*(60*b^2*d^5*n^2*x^(5/3)*e^3*log(x^(1/3)*e + d)^2 -
 60*b^2*d^8*x^(2/3)*log((x^(1/3)*e + d)^n)^2 - 40*b^2*d^4*n^2*x^2*e^4*log(x
) + 300*b^2*d^4*n^2*x^2*e^4 - 45*b^2*d*n^2*x^3*e^7 - 60*(b^2*d^7*n*log(c) +
```

```
a*b*d^7*n)*x*e - 20*(3*b^2*d^7*n*x*e + 6*b^2*d^5*n*x^(5/3)*e^3*log(x^(1/3)
*e + d) - 6*b^2*d^6*n*x^(4/3)*e^2 + 2*(3*b^2*d^8*log(c) - b^2*d^5*n*x*e^3*l
og(x) + 3*a*b*d^8)*x^(2/3))*log((x^(1/3)*e + d)^n) - 60*(b^2*d^8*log(c)^2 +
 2*a*b*d^8*log(c) + a^2*d^8)*x^(2/3) + 4*(5*b^2*d^3*n^2*x^2*e^5*log(x) - 15
*b^2*d^3*n^2*x^2*e^5 + 9*b^2*n^2*x^3*e^8 + 30*(b^2*d^6*n*log(c) + a*b*d^6*n
)*x*e^2)*x^(1/3) - 60*(b^2*d^6*n^2*x^2*e^2 + b^2*d^3*n^2*x^3*e^5)/x^(2/3))/
(d^8*x^(5/3))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((x^(1/3)*e + d)^n*c)^2 + 2*a*b*log((x^(1/3)*e + d)^n*c) +
 a^2)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \log \left(c \left(d + e \sqrt[3]{x}\right)^{n}\right)\right)^{2}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))**2/x**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**(1/3))**n))**2/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^(1/3)*e + d)^n*c) + a)^2/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)\right)}^2}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))^2/x^2,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/3))^n))^2/x^2, x)
```

3.455 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=405

$$-\frac{b^2e^2n^2}{20d^2x^{4/3}}+\frac{3b^2e^3n^2}{20d^3x}-\frac{47b^2e^4n^2}{120d^4x^{2/3}}+\frac{77b^2e^5n^2}{60d^5\sqrt[3]{x}}-\frac{77b^2e^6n^2\log\left(d+e\sqrt[3]{x}\right)}{60d^6}-\frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5dx^{5/3}}+\frac{be}{}$$

```
[Out] -1/20*b^2*e^2*n^2/d^2/x^(4/3)+3/20*b^2*e^3*n^2/d^3/x-47/120*b^2*e^4*n^2/d^4
/x^(2/3)+77/60*b^2*e^5*n^2/d^5/x^(1/3)-77/60*b^2*e^6*n^2*ln(d+e*x^(1/3))/d^
6-1/5*b*e*n*(a+b*ln(c*(d+e*x^(1/3))^n))/d/x^(5/3)+1/4*b*e^2*n*(a+b*ln(c*(d+
e*x^(1/3))^n))/d^2/x^(4/3)-1/3*b*e^3*n*(a+b*ln(c*(d+e*x^(1/3))^n))/d^3/x+1/
2*b*e^4*n*(a+b*ln(c*(d+e*x^(1/3))^n))/d^4/x^(2/3)-b*e^5*n*(d+e*x^(1/3))*(a+
b*ln(c*(d+e*x^(1/3))^n))/d^6/x^(1/3)-b*e^6*n*ln(1-d/(d+e*x^(1/3)))*(a+b*ln(
c*(d+e*x^(1/3))^n))/d^6-1/2*(a+b*ln(c*(d+e*x^(1/3))^n))^2/x^2+137/180*b^2*e
^6*n^2*ln(x)/d^6+b^2*e^6*n^2*polylog(2,d/(d+e*x^(1/3)))/d^6
```

Rubi [A]
time = 0.63, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2504, 2445, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$\frac{b^2e^6n^2\text{PolyLog}\left(2,\frac{d}{d+e\sqrt[3]{x}}\right)}{d^6}-\frac{be^6n\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^6}-\frac{be^5n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^6\sqrt[3]{x}}+\frac{be^4n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{2d^4x^{2/3}}-\frac{be^3n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3d^3x}+\frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4d^2x^{4/3}}-\frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5dx^{5/3}}-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2x^2}-\frac{77b^2e^6n^2\log\left(d+e\sqrt[3]{x}\right)}{60d^6}+\frac{137b^2e^6n^2\log(x)}{180d^6}+\frac{77b^2e^5n^2}{60d^5\sqrt[3]{x}}-\frac{47b^2e^4n^2}{120d^4x^{2/3}}+\frac{3b^2e^3n^2}{20d^3x}-\frac{b^2e^2n^2}{20d^2x^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])^2/x^3,x]
```

```
[Out] -1/20*(b^2*e^2*n^2)/(d^2*x^(4/3)) + (3*b^2*e^3*n^2)/(20*d^3*x) - (47*b^2*e^
4*n^2)/(120*d^4*x^(2/3)) + (77*b^2*e^5*n^2)/(60*d^5*x^(1/3)) - (77*b^2*e^6*
n^2*Log[d + e*x^(1/3)])/(60*d^6) - (b*e*n*(a + b*Log[c*(d + e*x^(1/3))^n]))
/(5*d*x^(5/3)) + (b*e^2*n*(a + b*Log[c*(d + e*x^(1/3))^n]))/(4*d^2*x^(4/3))
 - (b*e^3*n*(a + b*Log[c*(d + e*x^(1/3))^n]))/(3*d^3*x) + (b*e^4*n*(a + b*L
og[c*(d + e*x^(1/3))^n]))/(2*d^4*x^(2/3)) - (b*e^5*n*(d + e*x^(1/3))*(a + b
*Log[c*(d + e*x^(1/3))^n]))/(d^6*x^(1/3)) - (b*e^6*n*Log[1 - d/(d + e*x^(1/
3))]*(a + b*Log[c*(d + e*x^(1/3))^n]))/d^6 - (a + b*Log[c*(d + e*x^(1/3))^n
])^2/(2*x^2) + (137*b^2*e^6*n^2*Log[x])/(180*d^6) + (b^2*e^6*n^2*PolyLog[2,
 d/(d + e*x^(1/3))])/d^6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^3}\,dx = 3\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^7}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2x^2} + (ben)\text{Subst}\left(\int \frac{a+b\log(c(d+ex)^n)}{x^6(d+ex)}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2x^2} + (bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2x^2} + \frac{(bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+e\sqrt[3]{}\right.}{d}$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5dx^{5/3}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2x^2} - \frac{(be}{}$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5dx^{5/3}} + \frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4d^2x^{4/3}} -$$

$$= -\frac{b^2e^2n^2}{20d^2x^{4/3}} + \frac{b^2e^3n^2}{15d^3x} - \frac{b^2e^4n^2}{10d^4x^{2/3}} + \frac{b^2e^5n^2}{5d^5\sqrt[3]{x}} - \frac{b^2e^6n^2\log\left(d+e\sqrt[3]{x}\right)}{5d^6} -$$

$$= -\frac{b^2e^2n^2}{20d^2x^{4/3}} + \frac{3b^2e^3n^2}{20d^3x} - \frac{9b^2e^4n^2}{40d^4x^{2/3}} + \frac{9b^2e^5n^2}{20d^5\sqrt[3]{x}} - \frac{9b^2e^6n^2\log\left(d+e\sqrt[3]{x}\right.}{20d^6}$$

$$= -\frac{b^2e^2n^2}{20d^2x^{4/3}} + \frac{3b^2e^3n^2}{20d^3x} - \frac{47b^2e^4n^2}{120d^4x^{2/3}} + \frac{47b^2e^5n^2}{60d^5\sqrt[3]{x}} - \frac{47b^2e^6n^2\log\left(d+e\sqrt[3]{}\right.}{60d^6}$$

$$= -\frac{b^2e^2n^2}{20d^2x^{4/3}} + \frac{3b^2e^3n^2}{20d^3x} - \frac{47b^2e^4n^2}{120d^4x^{2/3}} + \frac{77b^2e^5n^2}{60d^5\sqrt[3]{x}} - \frac{77b^2e^6n^2\log\left(d+e\sqrt[3]{}\right.}{60d^6}$$

$$= -\frac{b^2e^2n^2}{20d^2x^{4/3}} + \frac{3b^2e^3n^2}{20d^3x} - \frac{47b^2e^4n^2}{120d^4x^{2/3}} + \frac{77b^2e^5n^2}{60d^5\sqrt[3]{x}} - \frac{77b^2e^6n^2\log\left(d+e\sqrt[3]{}\right.}{60d^6}$$

Mathematica [A]
time = 0.20, size = 533, normalized size = 1.32

$$-\frac{180a^2d^6 + 72abd^5en\sqrt[3]{x} - 90abd^4e^2nx^{2/3} + 18b^2d^4e^2n^2x^{2/3} + 120abd^3e^3nx - 54b^2d^3e^3n^2x - 180abd^2e^4nx^{4/3} + 141b^2d^2e^4n^2x^{4/3} + 360abde^5nx^{5/3} - 462b^2de^5n^2x^{5/3} + 822b^2e^6n^2x^2\log\left(d+e\sqrt[3]{x}\right) + 360abd^6\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right) + 72b^2d^5en\sqrt[3]{x}\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right) - 90b^2d^4e^2nx^{2/3}\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right) + 120b^2d^3e^3nx\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right) - 180b^2d^2e^4nx^{4/3}\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right) + 360b^2de^5nx^{5/3}\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right) - 360abe^6x^2\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right) + 180b^2d^6\log^2\left(c\left(d+e\sqrt[3]{x}\right)^n\right) - 180b^2e^6x^2\log^2\left(c\left(d+e\sqrt[3]{x}\right)^n\right) + 360abe^6nx^2\log\left(-\frac{e\sqrt[3]{x}}{d}\right) + 360b^2e^6nx^2\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\log\left(-\frac{e\sqrt[3]{x}}{d}\right) - 274b^2e^6n^2x^2\log(x) + 360b^2e^6n^2x^2\text{Li}_2\left(1+\frac{e\sqrt[3]{x}}{d}\right)}{360d^6x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^2/x^3,x]
```

```
[Out] -1/360*(180*a^2*d^6 + 72*a*b*d^5*e*n*x^(1/3) - 90*a*b*d^4*e^2*n*x^(2/3) + 1
8*b^2*d^4*e^2*n^2*x^(2/3) + 120*a*b*d^3*e^3*n*x - 54*b^2*d^3*e^3*n^2*x - 18
0*a*b*d^2*e^4*n*x^(4/3) + 141*b^2*d^2*e^4*n^2*x^(4/3) + 360*a*b*d*e^5*n*x^(
5/3) - 462*b^2*d*e^5*n^2*x^(5/3) + 822*b^2*e^6*n^2*x^2*Log[d + e*x^(1/3)] +
 360*a*b*d^6*Log[c*(d + e*x^(1/3))^n] + 72*b^2*d^5*e*n*x^(1/3)*Log[c*(d + e
*x^(1/3))^n] - 90*b^2*d^4*e^2*n*x^(2/3)*Log[c*(d + e*x^(1/3))^n] + 120*b^2*
d^3*e^3*n*x*Log[c*(d + e*x^(1/3))^n] - 180*b^2*d^2*e^4*n*x^(4/3)*Log[c*(d +
 e*x^(1/3))^n] + 360*b^2*d*e^5*n*x^(5/3)*Log[c*(d + e*x^(1/3))^n] - 360*a*b
*e^6*x^2*Log[c*(d + e*x^(1/3))^n] + 180*b^2*d^6*Log[c*(d + e*x^(1/3))^n]^2
- 180*b^2*e^6*x^2*Log[c*(d + e*x^(1/3))^n]^2 + 360*a*b*e^6*n*x^2*Log[-((e*x
^(1/3))/d)] + 360*b^2*e^6*n*x^2*Log[c*(d + e*x^(1/3))^n]*Log[-((e*x^(1/3))/
d)] - 274*b^2*e^6*n^2*x^2*Log[x] + 360*b^2*e^6*n^2*x^2*PolyLog[2, 1 + (e*x^
(1/3))/d])/(d^6*x^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^{2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))^2/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))^2/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b^2*log((x^(1/3)*e + d)^n)^2/x^2 + integrate(1/3*(3*(b^2*log(c)^2 + 2*
a*b*log(c) + a^2)*x*e + (b^2*n*x*e + 6*(b^2*log(c) + a*b)*x*e + 6*(b^2*d*lo
g(c) + a*b*d)*x^(2/3))*log((x^(1/3)*e + d)^n) + 3*(b^2*d*log(c)^2 + 2*a*b*d
*log(c) + a^2*d)*x^(2/3))/(x^4*e + d*x^(11/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((x^(1/3)*e + d)^n*c)^2 + 2*a*b*log((x^(1/3)*e + d)^n*c) +
 a^2)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \log \left(c\left(d + e\sqrt[3]{x}\right)^{n}\right)\right)^{2}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))**2/x**3,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**(1/3))**n))**2/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^(1/3)*e + d)^n*c) + a)^2/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)\right)}^2}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))^2/x^3,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/3))^n))^2/x^3, x)
```

3.456 $\int x^3 \left(a + b \log \left(c\left(d + e\sqrt[3]{x}\right)^n\right)\right)^3 \, dx$

Optimal. Leaf size=1835

result too large to display

```
[Out] -18*b^3*d^11*n^2*(d+e*x^(1/3))*ln(c*(d+e*x^(1/3))^n)/e^12+99/4*b^2*d^10*n^2
*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n))/e^12-110/3*b^2*d^9*n^2*(d+e*x^
(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))/e^12+1485/32*b^2*d^8*n^2*(d+e*x^(1/3))
^4*(a+b*ln(c*(d+e*x^(1/3))^n))/e^12-1188/25*b^2*d^7*n^2*(d+e*x^(1/3))^5*(a+
b*ln(c*(d+e*x^(1/3))^n))/e^12+77/2*b^2*d^6*n^2*(d+e*x^(1/3))^6*(a+b*ln(c*(d
+e*x^(1/3))^n))/e^12-1188/49*b^2*d^5*n^2*(d+e*x^(1/3))^7*(a+b*ln(c*(d+e*x^(
1/3))^n))/e^12+1485/128*b^2*d^4*n^2*(d+e*x^(1/3))^8*(a+b*ln(c*(d+e*x^(1/3))
^n))/e^12-110/27*b^2*d^3*n^2*(d+e*x^(1/3))^9*(a+b*ln(c*(d+e*x^(1/3))^n))/e^
12+99/100*b^2*d^2*n^2*(d+e*x^(1/3))^10*(a+b*ln(c*(d+e*x^(1/3))^n))/e^12-18/
121*b^2*d*n^2*(d+e*x^(1/3))^11*(a+b*ln(c*(d+e*x^(1/3))^n))/e^12+9*b*d^11*n*
(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^12-99/4*b*d^10*n*(d+e*x^(1/3)
)^2*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^12+55*b*d^9*n*(d+e*x^(1/3))^3*(a+b*ln(c
*(d+e*x^(1/3))^n))^2/e^12+1/4*(d+e*x^(1/3))^12*(a+b*ln(c*(d+e*x^(1/3))^n))^
3/e^12-1485/16*b*d^8*n*(d+e*x^(1/3))^4*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^12+5
94/5*b*d^7*n*(d+e*x^(1/3))^5*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^12-231/2*b*d^6
*n*(d+e*x^(1/3))^6*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^12+594/7*b*d^5*n*(d+e*x^
(1/3))^7*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^12-1485/32*b*d^4*n*(d+e*x^(1/3))^8
*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^12+55/3*b*d^3*n*(d+e*x^(1/3))^9*(a+b*ln(c*
(d+e*x^(1/3))^n))^2/e^12-99/20*b*d^2*n*(d+e*x^(1/3))^10*(a+b*ln(c*(d+e*x^(1
/3))^n))^2/e^12+9/11*b*d*n*(d+e*x^(1/3))^11*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e
^12+1/96*b^2*n^2*(d+e*x^(1/3))^12*(a+b*ln(c*(d+e*x^(1/3))^n))/e^12-1/16*b*n
*(d+e*x^(1/3))^12*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^12-18*a*b^2*d^11*n^2*x^(1
/3)/e^11-3*d^11*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^12+33/2*d^10*
(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^12-55*d^9*(d+e*x^(1/3))^3*(
a+b*ln(c*(d+e*x^(1/3))^n))^3/e^12+495/4*d^8*(d+e*x^(1/3))^4*(a+b*ln(c*(d+e*
x^(1/3))^n))^3/e^12-198*d^7*(d+e*x^(1/3))^5*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e
^12+231*d^6*(d+e*x^(1/3))^6*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^12-198*d^5*(d+e
*x^(1/3))^7*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^12+495/4*d^4*(d+e*x^(1/3))^8*(a
+b*ln(c*(d+e*x^(1/3))^n))^3/e^12-55*d^3*(d+e*x^(1/3))^9*(a+b*ln(c*(d+e*x^(1
/3))^n))^3/e^12+33/2*d^2*(d+e*x^(1/3))^10*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^1
2-3*d*(d+e*x^(1/3))^11*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^12-99/8*b^3*d^10*n^3
*(d+e*x^(1/3))^2/e^12+110/9*b^3*d^9*n^3*(d+e*x^(1/3))^3/e^12-1485/128*b^3*d
^8*n^3*(d+e*x^(1/3))^4/e^12+1188/125*b^3*d^7*n^3*(d+e*x^(1/3))^5/e^12-77/12
*b^3*d^6*n^3*(d+e*x^(1/3))^6/e^12+1188/343*b^3*d^5*n^3*(d+e*x^(1/3))^7/e^12
-1485/1024*b^3*d^4*n^3*(d+e*x^(1/3))^8/e^12+110/243*b^3*d^3*n^3*(d+e*x^(1/3
))^9/e^12-99/1000*b^3*d^2*n^3*(d+e*x^(1/3))^10/e^12+18/1331*b^3*d*n^3*(d+e*
x^(1/3))^11/e^12+18*b^3*d^11*n^3*x^(1/3)/e^11-1/1152*b^3*n^3*(d+e*x^(1/3))^
12/e^12
```

Rubi [A]

time = 1.43, antiderivative size = 1835, normalized size of antiderivative = 1.00, number of steps used = 52, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

Too large to display

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

```
[Out] (-99*b^3*d^10*n^3*(d + e*x^(1/3))^2)/(8*e^12) + (110*b^3*d^9*n^3*(d + e*x^(
1/3))^3)/(9*e^12) - (1485*b^3*d^8*n^3*(d + e*x^(1/3))^4)/(128*e^12) + (1188
*b^3*d^7*n^3*(d + e*x^(1/3))^5)/(125*e^12) - (77*b^3*d^6*n^3*(d + e*x^(1/3)
)^6)/(12*e^12) + (1188*b^3*d^5*n^3*(d + e*x^(1/3))^7)/(343*e^12) - (1485*b^
3*d^4*n^3*(d + e*x^(1/3))^8)/(1024*e^12) + (110*b^3*d^3*n^3*(d + e*x^(1/3))
^9)/(243*e^12) - (99*b^3*d^2*n^3*(d + e*x^(1/3))^10)/(1000*e^12) + (18*b^3*
d*n^3*(d + e*x^(1/3))^11)/(1331*e^12) - (b^3*n^3*(d + e*x^(1/3))^12)/(1152*
e^12) - (18*a*b^2*d^11*n^2*x^(1/3))/e^11 + (18*b^3*d^11*n^3*x^(1/3))/e^11 -
 (18*b^3*d^11*n^2*(d + e*x^(1/3))*Log[c*(d + e*x^(1/3))^n])/e^12 + (99*b^2*
d^10*n^2*(d + e*x^(1/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/(4*e^12) - (11
0*b^2*d^9*n^2*(d + e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n]))/(3*e^12)
+ (1485*b^2*d^8*n^2*(d + e*x^(1/3))^4*(a + b*Log[c*(d + e*x^(1/3))^n]))/(32
*e^12) - (1188*b^2*d^7*n^2*(d + e*x^(1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n
]))/(25*e^12) + (77*b^2*d^6*n^2*(d + e*x^(1/3))^6*(a + b*Log[c*(d + e*x^(1/
3))^n]))/(2*e^12) - (1188*b^2*d^5*n^2*(d + e*x^(1/3))^7*(a + b*Log[c*(d + e
*x^(1/3))^n]))/(49*e^12) + (1485*b^2*d^4*n^2*(d + e*x^(1/3))^8*(a + b*Log[c
*(d + e*x^(1/3))^n]))/(128*e^12) - (110*b^2*d^3*n^2*(d + e*x^(1/3))^9*(a +
b*Log[c*(d + e*x^(1/3))^n]))/(27*e^12) + (99*b^2*d^2*n^2*(d + e*x^(1/3))^10
*(a + b*Log[c*(d + e*x^(1/3))^n]))/(100*e^12) - (18*b^2*d*n^2*(d + e*x^(1/3
))^11*(a + b*Log[c*(d + e*x^(1/3))^n]))/(121*e^12) + (b^2*n^2*(d + e*x^(1/3
))^12*(a + b*Log[c*(d + e*x^(1/3))^n]))/(96*e^12) + (9*b*d^11*n*(d + e*x^(1
/3))*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^12 - (99*b*d^10*n*(d + e*x^(1/3)
)^2*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(4*e^12) + (55*b*d^9*n*(d + e*x^(1/
3))^3*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^12 - (1485*b*d^8*n*(d + e*x^(1/
3))^4*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(16*e^12) + (594*b*d^7*n*(d + e*x
^(1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(5*e^12) - (231*b*d^6*n*(d +
e*x^(1/3))^6*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(2*e^12) + (594*b*d^5*n*(d
 + e*x^(1/3))^7*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(7*e^12) - (1485*b*d^4*
n*(d + e*x^(1/3))^8*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(32*e^12) + (55*b*d
^3*n*(d + e*x^(1/3))^9*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(3*e^12) - (99*b
*d^2*n*(d + e*x^(1/3))^10*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(20*e^12) + (
9*b*d*n*(d + e*x^(1/3))^11*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(11*e^12) -
(b*n*(d + e*x^(1/3))^12*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(16*e^12) - (3*
d^11*(d + e*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^12 + (33*d^10*(d
 + e*x^(1/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/(2*e^12) - (55*d^9*(d +
 e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^12 + (495*d^8*(d + e*x^
```

```
(1/3))^4*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/(4*e^12) - (198*d^7*(d + e*x^(
1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^12 + (231*d^6*(d + e*x^(1/3))
^6*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^12 - (198*d^5*(d + e*x^(1/3))^7*(a
 + b*Log[c*(d + e*x^(1/3))^n])^3)/e^12 + (495*d^4*(d + e*x^(1/3))^8*(a + b*
Log[c*(d + e*x^(1/3))^n])^3)/(4*e^12) - (55*d^3*(d + e*x^(1/3))^9*(a + b*Lo
g[c*(d + e*x^(1/3))^n])^3)/e^12 + (33*d^2*(d + e*x^(1/3))^10*(a + b*Log[c*(
d + e*x^(1/3))^n])^3)/(2*e^12) - (3*d*(d + e*x^(1/3))^11*(a + b*Log[c*(d +
e*x^(1/3))^n])^3)/e^12 + ((d + e*x^(1/3))^12*(a + b*Log[c*(d + e*x^(1/3))^n
])^3)/(4*e^12)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx = 3\text{Subst}\left(\int x^{11}(a+b\log(c(d+ex)^n))^3\,dx,x,\sqrt[3]{x}\right)$$
$$= 3\text{Subst}\left(\int\left(-\frac{d^{11}(a+b\log(c(d+ex)^n))^3}{e^{11}}+\frac{11d^{10}(d+ex)\left(a+b\log\right.}{e^{11}}\right.\right.$$
$$= \frac{3\text{Subst}\left(\int(d+ex)^{11}\left(a+b\log(c(d+ex)^n)\right)^3\,dx,x,\sqrt[3]{x}\right)}{e^{11}}-\frac{(33d)\text{Sub}}{}$$
$$= \frac{3\text{Subst}\left(\int x^{11}(a+b\log(cx^n))^3\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}}-\frac{(33d)\text{Subst}\left(\int x^{10}\right.}{}$$
$$= -\frac{3d^{11}\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^{12}}+\frac{33d^{10}\left(d+e\sqrt[3]{x}\right)^2}{}$$
$$= \frac{9bd^{11}n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^{12}}-\frac{99bd^{10}n\left(d+e\sqrt[3]{x}\right.}{}$$
$$= -\frac{99b^3d^{10}n^3\left(d+e\sqrt[3]{x}\right)^2}{8e^{12}}+\frac{110b^3d^9n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^{12}}-\frac{1485b^3d^8n^3\left(d-\right.}{128e^{12}}$$
$$= -\frac{99b^3d^{10}n^3\left(d+e\sqrt[3]{x}\right)^2}{8e^{12}}+\frac{110b^3d^9n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^{12}}-\frac{1485b^3d^8n^3\left(d-\right.}{128e^{12}}$$

Mathematica [A]
time = 1.05, size = 1117, normalized size = 0.61

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

```
[Out] (-3550000608000*b^3*d^12*n^3*Log[d + e*x^(1/3)]^3 + 384199200*b^2*d^12*n^2*
Log[d + e*x^(1/3)]^2*(27720*a - 86021*b*n + 27720*b*Log[c*(d + e*x^(1/3))^n
]) - 27720*b*d^12*n*Log[d + e*x^(1/3)]*(384199200*a^2 - 2384502120*a*b*n +
4301068993*b^2*n^2 + 27720*b*(27720*a - 86021*b*n)*Log[c*(d + e*x^(1/3))^n]
 + 384199200*b^2*Log[c*(d + e*x^(1/3))^n]^2) + e*x^(1/3)*(3550000608000*a^3
*e^11*x^(11/3) + b^3*n^3*(119225632485960*d^11 - 26563616859780*d^10*e*x^(1
/3) + 10242678720120*d^9*e^2*x^(2/3) - 4836309598890*d^8*e^3*x + 2516628075
192*d^7*e^4*x^(4/3) - 1373077023780*d^6*e^5*x^(5/3) + 761128152840*d^5*e^6*
x^2 - 417533743935*d^4*e^7*x^(7/3) + 220161492320*d^3*e^8*x^(8/3) - 1069449
90768*d^2*e^9*x^3 + 44119404000*d*e^10*x^(10/3) - 12326391000*e^11*x^(11/3)
) - 27720*a*b^2*n^2*(2384502120*d^11 - 808051860*d^10*e*x^(1/3) + 410634840
*d^9*e^2*x^(2/3) - 243942930*d^8*e^3*x + 156734424*d^7*e^4*x^(4/3) - 104998
740*d^6*e^5*x^(5/3) + 71703720*d^5*e^6*x^2 - 49019355*d^4*e^7*x^(7/3) + 329
00560*d^3*e^8*x^(8/3) - 21072744*d^2*e^9*x^3 + 12171600*d*e^10*x^(10/3) - 5
336100*e^11*x^(11/3)) + 384199200*a^2*b*n*(27720*d^11 - 13860*d^10*e*x^(1/3
) + 9240*d^9*e^2*x^(2/3) - 6930*d^8*e^3*x + 5544*d^7*e^4*x^(4/3) - 4620*d^6
*e^5*x^(5/3) + 3960*d^5*e^6*x^2 - 3465*d^4*e^7*x^(7/3) + 3080*d^3*e^8*x^(8/
3) - 2772*d^2*e^9*x^3 + 2520*d*e^10*x^(10/3) - 2310*e^11*x^(11/3)) + 27720*
b*(384199200*a^2*e^11*x^(11/3) + 27720*a*b*n*(27720*d^11 - 13860*d^10*e*x^(
1/3) + 9240*d^9*e^2*x^(2/3) - 6930*d^8*e^3*x + 5544*d^7*e^4*x^(4/3) - 4620*
d^6*e^5*x^(5/3) + 3960*d^5*e^6*x^2 - 3465*d^4*e^7*x^(7/3) + 3080*d^3*e^8*x^
(8/3) - 2772*d^2*e^9*x^3 + 2520*d*e^10*x^(10/3) - 2310*e^11*x^(11/3)) + b^2
*n^2*(-2384502120*d^11 + 808051860*d^10*e*x^(1/3) - 410634840*d^9*e^2*x^(2/
3) + 243942930*d^8*e^3*x - 156734424*d^7*e^4*x^(4/3) + 104998740*d^6*e^5*x^
(5/3) - 71703720*d^5*e^6*x^2 + 49019355*d^4*e^7*x^(7/3) - 32900560*d^3*e^8*
x^(8/3) + 21072744*d^2*e^9*x^3 - 12171600*d*e^10*x^(10/3) + 5336100*e^11*x^
(11/3)))*Log[c*(d + e*x^(1/3))^n] + 384199200*b^2*(27720*a*e^11*x^(11/3) +
b*n*(27720*d^11 - 13860*d^10*e*x^(1/3) + 9240*d^9*e^2*x^(2/3) - 6930*d^8*e^
3*x + 5544*d^7*e^4*x^(4/3) - 4620*d^6*e^5*x^(5/3) + 3960*d^5*e^6*x^2 - 3465
*d^4*e^7*x^(7/3) + 3080*d^3*e^8*x^(8/3) - 2772*d^2*e^9*x^3 + 2520*d*e^10*x^
(10/3) - 2310*e^11*x^(11/3)))*Log[c*(d + e*x^(1/3))^n]^2 + 3550000608000*b^
3*e^11*x^(11/3)*Log[c*(d + e*x^(1/3))^n]^3))/(14200002432000*e^12)
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int x^3\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(1/3))^n))^3,x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e*x^(1/3))^n))^3,x)
```

Maxima [A]
time = 0.32, size = 1017, normalized size = 0.55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*x^4*log((x^(1/3)*e + d)^n*c)^3 + 3/4*a*b^2*x^4*log((x^(1/3)*e + d)^
n*c)^2 + 3/4*a^2*b*x^4*log((x^(1/3)*e + d)^n*c) + 1/4*a^3*x^4 - 1/36960*(27
720*d^12*e^(-13)*log(x^(1/3)*e + d) + (13860*d^10*x^(2/3)*e - 27720*d^11*x^
(1/3) - 9240*d^9*x*e^2 + 6930*d^8*x^(4/3)*e^3 - 5544*d^7*x^(5/3)*e^4 + 4620
*d^6*x^2*e^5 - 3960*d^5*x^(7/3)*e^6 + 3465*d^4*x^(8/3)*e^7 - 3080*d^3*x^3*e
^8 + 2772*d^2*x^(10/3)*e^9 - 2520*d*x^(11/3)*e^10 + 2310*x^4*e^11)*e^(-12))
*a^2*b*n*e + 1/512265600*((384199200*d^12*log(x^(1/3)*e + d)^2 + 2384502120
*d^12*log(x^(1/3)*e + d) - 2384502120*d^11*x^(1/3)*e + 808051860*d^10*x^(2/
3)*e^2 - 410634840*d^9*x*e^3 + 243942930*d^8*x^(4/3)*e^4 - 156734424*d^7*x^
(5/3)*e^5 + 104998740*d^6*x^2*e^6 - 71703720*d^5*x^(7/3)*e^7 + 49019355*d^4
*x^(8/3)*e^8 - 32900560*d^3*x^3*e^9 + 21072744*d^2*x^(10/3)*e^10 - 12171600
*d*x^(11/3)*e^11 + 5336100*x^4*e^12)*n^2*e^(-12) - 27720*(27720*d^12*e^(-13
)*log(x^(1/3)*e + d) + (13860*d^10*x^(2/3)*e - 27720*d^11*x^(1/3) - 9240*d^
9*x*e^2 + 6930*d^8*x^(4/3)*e^3 - 5544*d^7*x^(5/3)*e^4 + 4620*d^6*x^2*e^5 -
3960*d^5*x^(7/3)*e^6 + 3465*d^4*x^(8/3)*e^7 - 3080*d^3*x^3*e^8 + 2772*d^2*x
^(10/3)*e^9 - 2520*d*x^(11/3)*e^10 + 2310*x^4*e^11)*e^(-12))*n*e*log((x^(1/
3)*e + d)^n*c))*a*b^2 - 1/14200002432000*(384199200*(27720*d^12*e^(-13)*log
(x^(1/3)*e + d) + (13860*d^10*x^(2/3)*e - 27720*d^11*x^(1/3) - 9240*d^9*x*e
^2 + 6930*d^8*x^(4/3)*e^3 - 5544*d^7*x^(5/3)*e^4 + 4620*d^6*x^2*e^5 - 3960*
d^5*x^(7/3)*e^6 + 3465*d^4*x^(8/3)*e^7 - 3080*d^3*x^3*e^8 + 2772*d^2*x^(10/
3)*e^9 - 2520*d*x^(11/3)*e^10 + 2310*x^4*e^11)*e^(-12))*n*e*log((x^(1/3)*e
+ d)^n*c)^2 + ((3550000608000*d^12*log(x^(1/3)*e + d)^3 + 33049199383200*d^
12*log(x^(1/3)*e + d)^2 + 119225632485960*d^12*log(x^(1/3)*e + d) - 1192256
32485960*d^11*x^(1/3)*e + 26563616859780*d^10*x^(2/3)*e^2 - 10242678720120*
d^9*x*e^3 + 4836309598890*d^8*x^(4/3)*e^4 - 2516628075192*d^7*x^(5/3)*e^5 +
 1373077023780*d^6*x^2*e^6 - 761128152840*d^5*x^(7/3)*e^7 + 417533743935*d^
4*x^(8/3)*e^8 - 220161492320*d^3*x^3*e^9 + 106944990768*d^2*x^(10/3)*e^10 -
 44119404000*d*x^(11/3)*e^11 + 12326391000*x^4*e^12)*n^2*e^(-13) - 27720*(3
84199200*d^12*log(x^(1/3)*e + d)^2 + 2384502120*d^12*log(x^(1/3)*e + d) - 2
384502120*d^11*x^(1/3)*e + 808051860*d^10*x^(2/3)*e^2 - 410634840*d^9*x*e^3
 + 243942930*d^8*x^(4/3)*e^4 - 156734424*d^7*x^(5/3)*e^5 + 104998740*d^6*x^
2*e^6 - 71703720*d^5*x^(7/3)*e^7 + 49019355*d^4*x^(8/3)*e^8 - 32900560*d^3*
x^3*e^9 + 21072744*d^2*x^(10/3)*e^10 - 12171600*d*x^(11/3)*e^11 + 5336100*x
^4*e^12)*n*e^(-13)*log((x^(1/3)*e + d)^n*c))*n*e)*b^3
```

Fricas [A]
time = 0.49, size = 1985, normalized size = 1.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="fricas")

[Out] 1/14200002432000*(3550000608000*b^3*x^4*e^12*log(c)^3 - 12326391000*(b^3*n^3 - 12*a*b^2*n^2 + 72*a^2*b*n - 288*a^3)*x^4*e^12 + 603680*(364699*b^3*d^3*n^3 - 1510740*a*b^2*d^3*n^2 + 1960200*a^2*b*d^3*n)*x^3*e^9 - 4620*(297202819*b^3*d^6*n^3 - 629992440*a*b^2*d^6*n^2 + 384199200*a^2*b*d^6*n)*x^2*e^6 - 3550000608000*(b^3*d^12*n^3 - b^3*n^3*x^4*e^12)*log(x^(1/3)*e + d)^3 + 9240*(1108515013*b^3*d^9*n^3 - 1231904520*a*b^2*d^9*n^2 + 384199200*a^2*b*d^9*n)*x*e^3 + 384199200*(86021*b^3*d^12*n^3 - 27720*a*b^2*d^12*n^2 + 9240*b^3*d^9*n^3*x*e^3 - 4620*b^3*d^6*n^3*x^2*e^6 + 3080*b^3*d^3*n^3*x^3*e^9 - 2310*(b^3*n^3 - 12*a*b^2*n^2)*x^4*e^12 - 27720*(b^3*d^12*n^2 - b^3*n^2*x^4*e^12)*log(c) - 63*(220*b^3*d^10*n^3*e^2 - 88*b^3*d^7*n^3*x*e^5 + 55*b^3*d^4*n^3*x^2*e^8 - 40*b^3*d*n^3*x^3*e^11)*x^(2/3) + 198*(140*b^3*d^11*n^3*e - 35*b^3*d^8*n^3*x*e^4 + 20*b^3*d^5*n^3*x^2*e^7 - 14*b^3*d^2*n^3*x^3*e^10)*x^(1/3))*log(x^(1/3)*e + d)^2 + 295833384000*(12*b^3*d^9*n*x*e^3 - 6*b^3*d^6*n*x^2*e^6 + 4*b^3*d^3*n*x^3*e^9 - 3*(b^3*n - 12*a*b^2)*x^4*e^12)*log(c)^2 - 27720*(4301068993*b^3*d^12*n^3 - 2384502120*a*b^2*d^12*n^2 + 384199200*a^2*b*d^12*n - 5336100*(b^3*n^3 - 12*a*b^2*n^2 + 72*a^2*b*n)*x^4*e^12 + 43120*(763*b^3*d^3*n^3 - 1980*a*b^2*d^3*n^2)*x^3*e^9 - 4620*(22727*b^3*d^6*n^3 - 27720*a*b^2*d^6*n^2)*x^2*e^6 + 9240*(44441*b^3*d^9*n^3 - 27720*a*b^2*d^9*n^2)*x*e^3 + 384199200*(b^3*d^12*n - b^3*n*x^4*e^12)*log(c)^2 - 27720*(86021*b^3*d^12*n^2 - 27720*a*b^2*d^12*n + 9240*b^3*d^9*n^2*x*e^3 - 4620*b^3*d^6*n^2*x^2*e^6 + 3080*b^3*d^3*n^2*x^3*e^9 - 2310*(b^3*n^2 - 12*a*b^2*n)*x^4*e^12)*log(c) + 63*(8400*(23*b^3*d*n^3 - 132*a*b^2*d*n^2)*x^3*e^11 - 385*(2021*b^3*d^4*n^3 - 3960*a*b^2*d^4*n^2)*x^2*e^8 + 88*(28271*b^3*d^7*n^3 - 27720*a*b^2*d^7*n^2)*x*e^5 - 220*(58301*b^3*d^10*n^3 - 27720*a*b^2*d^10*n^2)*e^2 + 27720*(220*b^3*d^10*n^2*e^2 - 88*b^3*d^7*n^2*x*e^5 + 55*b^3*d^4*n^2*x^2*e^8 - 40*b^3*d*n^2*x^3*e^11)*log(c))*x^(2/3) - 198*(588*(181*b^3*d^2*n^3 - 660*a*b^2*d^2*n^2)*x^3*e^10 - 20*(18107*b^3*d^5*n^3 - 27720*a*b^2*d^5*n^2)*x^2*e^7 + 35*(35201*b^3*d^8*n^3 - 27720*a*b^2*d^8*n^2)*x*e^4 - 140*(86021*b^3*d^11*n^3 - 27720*a*b^2*d^11*n^2)*e + 27720*(140*b^3*d^11*n^2*e - 35*b^3*d^8*n^2*x*e^4 + 20*b^3*d^5*n^2*x^2*e^7 - 14*b^3*d^2*n^2*x^3*e^10)*log(c))*x^(1/3))*log(x^(1/3)*e + d) + 42688800*(3465*(b^3*n^2 - 12*a*b^2*n + 72*a^2*b)*x^4*e^12 - 28*(763*b^3*d^3*n^2 - 1980*a*b^2*d^3*n)*x^3*e^9 + 3*(22727*b^3*d^6*n^2 - 27720*a*b^2*d^6*n)*x^2*e^6 - 6*(44441*b^3*d^9*n^2 - 27720*a*b^2*d^9*n)*x*e^3)*log(c) + 63*(1764000*(397*b^3*d*n^3 - 3036*a*b^2*d*n^2 + 8712*a^2*b*d*n)*x^3*e^11 - 2695*(2459191*b^3*d^4*n^3 - 8003160*a*b^2*d^4*n^2 + 7840800*a^2*b*d^4*n)*x^2*e^8 + 88*(453937243*b^3*d^7*n^3 - 783672120*a*b^2*d^7*n^2 + 384199200*a^2*b*d^7*n)*x*e^5 - 384199200*(220*b^3*d^10*n*e^2 - 88*b^3*d^7*n*x*e^5 + 55*b^3*d^4*n*x^2*e^8 - 40*b^3*d*n*x^3*e^11)*log(c)^2 - 220*(1916566873*b^3*d^10*n^3 - 1616103720*a*b^2*d^10*n^2 + 384199200*a^2*b*d^10*n)*e^2 - 27720*(8400*(23*b^3*d*n^2 - 132*a*b^2*d*n)*x^3*e^11 - 385*(2021*b^3*d^4*n^2 - 3960*a*b^2*d^4*n)*x^2*e^8 + 88*(28271*b^3*d^7*n^2 - 27720*a*b^2*d^7

```
*n)*x*e^5 - 220*(58301*b^3*d^10*n^2 - 27720*a*b^2*d^10*n)*e^2)*log(c))*x^(2
/3) - 198*(24696*(21871*b^3*d^2*n^3 - 119460*a*b^2*d^2*n^2 + 217800*a^2*b*d
^2*n)*x^3*e^10 - 20*(192204079*b^3*d^5*n^3 - 501926040*a*b^2*d^5*n^2 + 3841
99200*a^2*b*d^5*n)*x^2*e^7 + 35*(697880173*b^3*d^8*n^3 - 975771720*a*b^2*d^
8*n^2 + 384199200*a^2*b*d^8*n)*x*e^4 - 384199200*(140*b^3*d^11*n*e - 35*b^3
*d^8*n*x*e^4 + 20*b^3*d^5*n*x^2*e^7 - 14*b^3*d^2*n*x^3*e^10)*log(c)^2 - 140
*(4301068993*b^3*d^11*n^3 - 2384502120*a*b^2*d^11*n^2 + 384199200*a^2*b*d^1
1*n)*e - 27720*(588*(181*b^3*d^2*n^2 - 660*a*b^2*d^2*n)*x^3*e^10 - 20*(1810
7*b^3*d^5*n^2 - 27720*a*b^2*d^5*n)*x^2*e^7 + 35*(35201*b^3*d^8*n^2 - 27720*
a*b^2*d^8*n)*x*e^4 - 140*(86021*b^3*d^11*n^2 - 27720*a*b^2*d^11*n)*e)*log(c
))*x^(1/3))*e^(-12)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(1/3))**n))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 4443 vs. 2(1625) = 3250.
time = 4.08, size = 4443, normalized size = 2.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="giac")
```

```
[Out] 1/14200002432000*(3550000608000*b^3*x^4*e*log(c)^3 + 10650001824000*a*b^2*x
^4*e*log(c)^2 + 10650001824000*a^2*b*x^4*e*log(c) + 3550000608000*a^3*x^4*e
 + (3550000608000*(x^(1/3)*e + d)^12*e^(-11)*log(x^(1/3)*e + d)^3 - 4260000
7296000*(x^(1/3)*e + d)^11*d*e^(-11)*log(x^(1/3)*e + d)^3 + 234300040128000
*(x^(1/3)*e + d)^10*d^2*e^(-11)*log(x^(1/3)*e + d)^3 - 781000133760000*(x^(
1/3)*e + d)^9*d^3*e^(-11)*log(x^(1/3)*e + d)^3 + 1757250300960000*(x^(1/3)*
e + d)^8*d^4*e^(-11)*log(x^(1/3)*e + d)^3 - 2811600481536000*(x^(1/3)*e + d
)^7*d^5*e^(-11)*log(x^(1/3)*e + d)^3 + 3280200561792000*(x^(1/3)*e + d)^6*d
^6*e^(-11)*log(x^(1/3)*e + d)^3 - 2811600481536000*(x^(1/3)*e + d)^5*d^7*e^
(-11)*log(x^(1/3)*e + d)^3 + 1757250300960000*(x^(1/3)*e + d)^4*d^8*e^(-11)
*log(x^(1/3)*e + d)^3 - 781000133760000*(x^(1/3)*e + d)^3*d^9*e^(-11)*log(x
^(1/3)*e + d)^3 + 234300040128000*(x^(1/3)*e + d)^2*d^10*e^(-11)*log(x^(1/3
)*e + d)^3 - 42600007296000*(x^(1/3)*e + d)*d^11*e^(-11)*log(x^(1/3)*e + d)
^3 - 887500152000*(x^(1/3)*e + d)^12*e^(-11)*log(x^(1/3)*e + d)^2 + 1161818
3808000*(x^(1/3)*e + d)^11*d*e^(-11)*log(x^(1/3)*e + d)^2 - 70290012038400*
```

(x^(1/3)*e + d)^10*d^2*e^(-11)*log(x^(1/3)*e + d)^2 + 260333377920000*(x^(1/3)*e + d)^9*d^3*e^(-11)*log(x^(1/3)*e + d)^2 - 658968862860000*(x^(1/3)*e + d)^8*d^4*e^(-11)*log(x^(1/3)*e + d)^2 + 1204971634944000*(x^(1/3)*e + d)^7*d^5*e^(-11)*log(x^(1/3)*e + d)^2 - 1640100280896000*(x^(1/3)*e + d)^6*d^6*e^(-11)*log(x^(1/3)*e + d)^2 + 1686960288921600*(x^(1/3)*e + d)^5*d^7*e^(-11)*log(x^(1/3)*e + d)^2 - 1317937725720000*(x^(1/3)*e + d)^4*d^8*e^(-11)*log(x^(1/3)*e + d)^2 + 781000133760000*(x^(1/3)*e + d)^3*d^9*e^(-11)*log(x^(1/3)*e + d)^2 - 351450060192000*(x^(1/3)*e + d)^2*d^10*e^(-11)*log(x^(1/3)*e + d)^2 + 127800021888000*(x^(1/3)*e + d)*d^11*e^(-11)*log(x^(1/3)*e + d)^2 + 147916692000*(x^(1/3)*e + d)^12*e^(-11)*log(x^(1/3)*e + d) - 2112397056000*(x^(1/3)*e + d)^11*d*e^(-11)*log(x^(1/3)*e + d) + 14058002407680*(x^(1/3)*e + d)^10*d^2*e^(-11)*log(x^(1/3)*e + d) - 57851861760000*(x^(1/3)*e + d)^9*d^3*e^(-11)*log(x^(1/3)*e + d) + 164742215715000*(x^(1/3)*e + d)^8*d^4*e^(-11)*log(x^(1/3)*e + d) - 344277609984000*(x^(1/3)*e + d)^7*d^5*e^(-11)*log(x^(1/3)*e + d) + 546700093632000*(x^(1/3)*e + d)^6*d^6*e^(-11)*log(x^(1/3)*e + d) - 674784115568640*(x^(1/3)*e + d)^5*d^7*e^(-11)*log(x^(1/3)*e + d) + 658968862860000*(x^(1/3)*e + d)^4*d^8*e^(-11)*log(x^(1/3)*e + d) - 520666755840000*(x^(1/3)*e + d)^3*d^9*e^(-11)*log(x^(1/3)*e + d) + 351450060192000*(x^(1/3)*e + d)^2*d^10*e^(-11)*log(x^(1/3)*e + d) - 255600043776000*(x^(1/3)*e + d)*d^11*e^(-11)*log(x^(1/3)*e + d) - 12326391000*(x^(1/3)*e + d)^12*e^(-11) + 192036096000*(x^(1/3)*e + d)^11*d*e^(-11) - 1405800240768*(x^(1/3)*e + d)^10*d^2*e^(-11) + 6427984640000*(x^(1/3)*e + d)^9*d^3*e^(-11) - 20592776964375*(x^(1/3)*e + d)^8*d^4*e^(-11) + 49182515712000*(x^(1/3)*e + d)^7*d^5*e^(-11) - 91116682272000*(x^(1/3)*e + d)^6*d^6*e^(-11) + 134956823113728*(x^(1/3)*e + d)^5*d^7*e^(-11) - 164742215715000*(x^(1/3)*e + d)^4*d^8*e^(-11) + 173555585280000*(x^(1/3)*e + d)^3*d^9*e^(-11) - 175725030096000*(x^(1/3)*e + d)^2*d^10*e^(-11) + 255600043776000*(x^(1/3)*e + d)*d^11*e^(-11))*b^3*n^3 + 27720*(384199200*(x^(1/3)*e + d)^12*e^(-11)*log(x^(1/3)*e + d)^2 - 4610390400*(x^(1/3)*e + d)^11*d*e^(-11)*log(x^(1/3)*e + d)^2 + 25357147200*(x^(1/3)*e + d)^10*d^2*e^(-11)*log(x^(1/3)*e + d)^2 - 84523824000*(x^(1/3)*e + d)^9*d^3*e^(-11)*log(x^(1/3)*e + d)^2 + 190178604000*(x^(1/3)*e + d)^8*d^4*e^(-11)*log(x^(1/3)*e + d)^2 - 304285766400*(x^(1/3)*e + d)^7*d^5*e^(-11)*log(x^(1/3)*e + d)^2 + 355000060800*(x^(1/3)*e + d)^6*d^6*e^(-11)*log(x^(1/3)*e + d)^2 - 304285766400*(x^(1/3)*e + d)^5*d^7*e^(-11)*log(x^(1/3)*e + d)^2 + 190178604000*(x^(1/3)*e + d)^4*d^8*e^(-11)*log(x^(1/3)*e + d)^2 - 84523824000*(x^(1/3)*e + d)^3*d^9*e^(-11)*log(x^(1/3)*e + d)^2 + 25357147200*(x^(1/3)*e + d)^2*d^10*e^(-11)*log(x^(1/3)*e + d)^2 - 4610390400*(x^(1/3)*e + d)*d^11*e^(-11)*log(x^(1/3)*e + d)^2 - 64033200*(x^(1/3)*e + d)^12*e^(-11)*log(x^(1/3)*e + d) + 838252800*(x^(1/3)*e + d)^11*d*e^(-11)*log(x^(1/3)*e + d) - 5071429440*(x^(1/3)*e + d)^10*d^2*e^(-11)*log(x^(1/3)*e + d) + 18783072000*(x^(1/3)*e + d)^9*d^3*e^(-11)*log(x^(1/3)*e + d) - 47544651000*(x^(1/3)*e + d)^8*d^4*e^(-11)*log(x^(1/3)*e + d) + 86938790400*(x^(1/3)*e + d)^7*d^5*e^(-11)*log(x^(1/3)*e + d) - 118333353600*(x^(1/3)*e + d)^6*d^6*e^(-11)*log(x^(1/3)*e + d) + 121714306560*(x^(1/3)*e + d)^5*d^7*e^(-11)*log(x^(1/3)*e + d) - 95089302000*(x^(1/3)*e + d)^4*d^8*e^(-11)*log(x^(1/3

```
)*e + d) + 56349216000*(x^(1/3)*e + d)^3*d^9*e^(-11)*log(x^(1/3)*e + d) - 2
5357147200*(x^(1/3)*e + d)^2*d^10*e^(-11)*log(x^(1/3)*e + d) + 9220780800*(
x^(1/3)*e + d)*d^11*e^(-11)*log(x^(1/3)*e + d) + 5336100*(x^(1/3)*e + d)^12
*e^(-11) - 76204800*(x^(1/3)*e + d)^11*d*e^(-11) + 507142944*(x^(1/3)*e + d
)^10*d^2*e^(-11) - 2087008000*(x^(1/3)*e + d)^9*d^3*e^(-11) + 5943081375*(x
^(1/3)*e + d)^8*d^4*e^(-11) - 12419827200*(x^(1/3)*e + d)^7*d^5*e^(-11) + 1
9722225600*(x^(1/3)*e + d)^6*d^6*e^(-11) - 2434...
```

Mupad [B]
time = 8.47, size = 1802, normalized size = 0.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e*x^(1/3))^n))^3,x)
```

```
[Out] (a^3*x^4)/4 + (b^3*x^4*log(c*(d + e*x^(1/3))^n)^3)/4 - (b^3*n^3*x^4)/1152 +
 (3*a*b^2*x^4*log(c*(d + e*x^(1/3))^n)^2)/4 - (b^3*n*x^4*log(c*(d + e*x^(1/
3))^n)^2)/16 + (b^3*n^2*x^4*log(c*(d + e*x^(1/3))^n))/96 + (a*b^2*n^2*x^4)/
96 - (b^3*d^12*log(c*(d + e*x^(1/3))^n)^3)/(4*e^12) + (3*a^2*b*x^4*log(c*(d
 + e*x^(1/3))^n))/4 - (a^2*b*n*x^4)/16 - (a*b^2*n*x^4*log(c*(d + e*x^(1/3))
^n))/8 - (4301068993*b^3*d^12*n^3*log(d + e*x^(1/3)))/(512265600*e^12) + (3
64699*b^3*d^3*n^3*x^3)/(23522400*e^3) - (297202819*b^3*d^6*n^3*x^2)/(307359
3600*e^6) - (21871*b^3*d^2*n^3*x^(10/3))/(2904000*e^2) - (2459191*b^3*d^4*n
^3*x^(8/3))/(83635200*e^4) + (192204079*b^3*d^5*n^3*x^(7/3))/(3585859200*e^
5) + (453937243*b^3*d^7*n^3*x^(5/3))/(2561328000*e^7) - (697880173*b^3*d^8*
n^3*x^(4/3))/(2049062400*e^8) - (1916566873*b^3*d^10*n^3*x^(2/3))/(10245312
00*e^10) + (4301068993*b^3*d^11*n^3*x^(1/3))/(512265600*e^11) - (3*a*b^2*d^
12*log(c*(d + e*x^(1/3))^n)^2)/(4*e^12) + (86021*b^3*d^12*n*log(c*(d + e*x^
(1/3))^n)^2)/(36960*e^12) + (397*b^3*d*n^3*x^(11/3))/(127776*e) + (11085150
13*b^3*d^9*n^3*x)/(1536796800*e^9) - (3*a^2*b*d^12*n*log(d + e*x^(1/3)))/(4
*e^12) + (3*b^3*d*n*x^(11/3)*log(c*(d + e*x^(1/3))^n)^2)/(44*e) - (23*b^3*d
*n^2*x^(11/3)*log(c*(d + e*x^(1/3))^n))/(968*e) + (b^3*d^9*n*x*log(c*(d + e
*x^(1/3))^n)^2)/(4*e^9) - (44441*b^3*d^9*n^2*x*log(c*(d + e*x^(1/3))^n))/(5
5440*e^9) + (a^2*b*d^3*n*x^3)/(12*e^3) - (a^2*b*d^6*n*x^2)/(8*e^6) - (23*a*
b^2*d*n^2*x^(11/3))/(968*e) - (3*a^2*b*d^2*n*x^(10/3))/(40*e^2) - (3*a^2*b*
d^4*n*x^(8/3))/(32*e^4) - (44441*a*b^2*d^9*n^2*x)/(55440*e^9) + (3*a^2*b*d^
5*n*x^(7/3))/(28*e^5) + (3*a^2*b*d^7*n*x^(5/3))/(20*e^7) - (3*a^2*b*d^8*n*x
^(4/3))/(16*e^8) - (3*a^2*b*d^10*n*x^(2/3))/(8*e^10) + (3*a^2*b*d^11*n*x^(1
/3))/(4*e^11) + (86021*a*b^2*d^12*n^2*log(d + e*x^(1/3)))/(18480*e^12) + (b
^3*d^3*n*x^3*log(c*(d + e*x^(1/3))^n)^2)/(12*e^3) - (763*b^3*d^3*n^2*x^3*lo
g(c*(d + e*x^(1/3))^n))/(11880*e^3) - (b^3*d^6*n*x^2*log(c*(d + e*x^(1/3))^
n)^2)/(8*e^6) + (22727*b^3*d^6*n^2*x^2*log(c*(d + e*x^(1/3))^n))/(110880*e^
6) - (3*b^3*d^2*n*x^(10/3)*log(c*(d + e*x^(1/3))^n)^2)/(40*e^2) + (181*b^3*
d^2*n^2*x^(10/3)*log(c*(d + e*x^(1/3))^n))/(4400*e^2) - (3*b^3*d^4*n*x^(8/3
```

)*log(c*(d + e*x^(1/3))^n)^2)/(32*e^4) + (2021*b^3*d^4*n^2*x^(8/3)*log(c*(d + e*x^(1/3))^n))/(21120*e^4) + (3*b^3*d^5*n*x^(7/3)*log(c*(d + e*x^(1/3))^n)^2)/(28*e^5) - (18107*b^3*d^5*n^2*x^(7/3)*log(c*(d + e*x^(1/3))^n))/(129360*e^5) + (3*b^3*d^7*n*x^(5/3)*log(c*(d + e*x^(1/3))^n)^2)/(20*e^7) - (28271*b^3*d^7*n^2*x^(5/3)*log(c*(d + e*x^(1/3))^n))/(92400*e^7) - (3*b^3*d^8*n*x^(4/3)*log(c*(d + e*x^(1/3))^n)^2)/(16*e^8) + (35201*b^3*d^8*n^2*x^(4/3)*log(c*(d + e*x^(1/3))^n))/(73920*e^8) - (3*b^3*d^10*n*x^(2/3)*log(c*(d + e*x^(1/3))^n)^2)/(8*e^10) + (58301*b^3*d^10*n^2*x^(2/3)*log(c*(d + e*x^(1/3))^n))/(36960*e^10) + (3*b^3*d^11*n*x^(1/3)*log(c*(d + e*x^(1/3))^n)^2)/(4*e^11) - (86021*b^3*d^11*n^2*x^(1/3)*log(c*(d + e*x^(1/3))^n))/(18480*e^11) - (763*a*b^2*d^3*n^2*x^3)/(11880*e^3) + (22727*a*b^2*d^6*n^2*x^2)/(110880*e^6) + (181*a*b^2*d^2*n^2*x^(10/3))/(4400*e^2) + (2021*a*b^2*d^4*n^2*x^(8/3))/(21120*e^4) - (18107*a*b^2*d^5*n^2*x^(7/3))/(129360*e^5) - (28271*a*b^2*d^7*n^2*x^(5/3))/(92400*e^7) + (35201*a*b^2*d^8*n^2*x^(4/3))/(73920*e^8) + (58301*a*b^2*d^10*n^2*x^(2/3))/(36960*e^10) - (86021*a*b^2*d^11*n^2*x^(1/3))/(18480*e^11) + (3*a^2*b*d*n*x^(11/3))/(44*e) + (a^2*b*d^9*n*x)/(4*e^9) + (3*a*b^2*d*n*x^(11/3)*log(c*(d + e*x^(1/3))^n))/(22*e) + (a*b^2*d^9*n*x*log(c*(d + e*x^(1/3))^n))/(2*e^9) + (a*b^2*d^3*n*x^3*log(c*(d + e*x^(1/3))^n))/(6*e^3) - (a*b^2*d^6*n*x^2*log(c*(d + e*x^(1/3))^n))/(4*e^6) - (3*a*b^2*d^2*n*x^(10/3)*log(c*(d + e*x^(1/3))^n))/(20*e^2) - (3*a*b^2*d^4*n*x^(8/3)*log(c*(d + e*x^(1/3))^n))/(16*e^4) + (3*a*b^2*d^5*n*x^(7/3)*log(c*(d + e*x^(1/3))^n))/(14*e^5) + (3*a*b^2*d^7*n*x^(5/3)*log(c*(d + e*x^(1/3))^n))/(10*e^7) - (3*a*b^2*d^8*n*x^(4/3)*log(c*(d + e*x^(1/3))^n))/(8*e^8) - (3*a*b^2*d^10*n*x^(2/3)*log(c*(d + e*x^(1/3))^n))/(4*e^10) + (3*a*b^2*d^11*n*x^(1/3)*log(c*(d + e*x^(1/3))^n))/(2*e^11)

3.457 $\int x^2 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3 dx$

Optimal. Leaf size=1357

$$\frac{9b^3d^7n^3\left(d+e\sqrt[3]{x}\right)^2}{e^9}-\frac{56b^3d^6n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^9}+\frac{63b^3d^5n^3\left(d+e\sqrt[3]{x}\right)^4}{16e^9}-\frac{252b^3d^4n^3\left(d+e\sqrt[3]{x}\right)^5}{125e^9}+\frac{7b^3d^3n^3\left(d+\cdots\right.}{9e^9}$$

```
[Out] 1/3*(d+e*x^(1/3))^9*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^9-9*b*d^8*n*(d+e*x^(1/3
))*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^9+18*b*d^7*n*(d+e*x^(1/3))^2*(a+b*ln(c*(
d+e*x^(1/3))^n))^2/e^9-28*b*d^6*n*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n
))^2/e^9+63/2*b*d^5*n*(d+e*x^(1/3))^4*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^9-126
/5*b*d^4*n*(d+e*x^(1/3))^5*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^9+14*b*d^3*n*(d+
e*x^(1/3))^6*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^9-36/7*b*d^2*n*(d+e*x^(1/3))^7
*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^9+9/8*b*d*n*(d+e*x^(1/3))^8*(a+b*ln(c*(d+e
*x^(1/3))^n))^2/e^9+18*b^3*d^8*n^2*(d+e*x^(1/3))*ln(c*(d+e*x^(1/3))^n)/e^9-
18*b^2*d^7*n^2*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9+56/3*b^2*d^6
*n^2*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9-63/4*b^2*d^5*n^2*(d+e*
x^(1/3))^4*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9+252/25*b^2*d^4*n^2*(d+e*x^(1/3))
^5*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9-14/3*b^2*d^3*n^2*(d+e*x^(1/3))^6*(a+b*ln
(c*(d+e*x^(1/3))^n))/e^9+72/49*b^2*d^2*n^2*(d+e*x^(1/3))^7*(a+b*ln(c*(d+e*x
^(1/3))^n))/e^9-9/32*b^2*d*n^2*(d+e*x^(1/3))^8*(a+b*ln(c*(d+e*x^(1/3))^n))/
e^9+2/81*b^2*n^2*(d+e*x^(1/3))^9*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9-1/9*b*n*(d
+e*x^(1/3))^9*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^9-252/125*b^3*d^4*n^3*(d+e*x^
(1/3))^5/e^9+7/9*b^3*d^3*n^3*(d+e*x^(1/3))^6/e^9-72/343*b^3*d^2*n^3*(d+e*x^
(1/3))^7/e^9+9/256*b^3*d*n^3*(d+e*x^(1/3))^8/e^9-18*b^3*d^8*n^3*x^(1/3)/e^8
+18*a*b^2*d^8*n^2*x^(1/3)/e^8+3*d^8*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n
))^3/e^9-12*d^7*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^9+28*d^6*(d
+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^9-42*d^5*(d+e*x^(1/3))^4*(a+b
*ln(c*(d+e*x^(1/3))^n))^3/e^9+42*d^4*(d+e*x^(1/3))^5*(a+b*ln(c*(d+e*x^(1/3)
)^n))^3/e^9-28*d^3*(d+e*x^(1/3))^6*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^9+12*d^2
*(d+e*x^(1/3))^7*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^9-3*d*(d+e*x^(1/3))^8*(a+b
*ln(c*(d+e*x^(1/3))^n))^3/e^9+9*b^3*d^7*n^3*(d+e*x^(1/3))^2/e^9-56/9*b^3*d^
6*n^3*(d+e*x^(1/3))^3/e^9+63/16*b^3*d^5*n^3*(d+e*x^(1/3))^4/e^9-2/729*b^3*n
^3*(d+e*x^(1/3))^9/e^9
```

Rubi [A]

time = 1.02, antiderivative size = 1357, normalized size of antiderivative = 1.00, number of steps used = 40, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

```
[Out] (9*b^3*d^7*n^3*(d + e*x^(1/3))^2)/e^9 - (56*b^3*d^6*n^3*(d + e*x^(1/3))^3)/
(9*e^9) + (63*b^3*d^5*n^3*(d + e*x^(1/3))^4)/(16*e^9) - (252*b^3*d^4*n^3*(d
 + e*x^(1/3))^5)/(125*e^9) + (7*b^3*d^3*n^3*(d + e*x^(1/3))^6)/(9*e^9) - (7
2*b^3*d^2*n^3*(d + e*x^(1/3))^7)/(343*e^9) + (9*b^3*d*n^3*(d + e*x^(1/3))^8
)/(256*e^9) - (2*b^3*n^3*(d + e*x^(1/3))^9)/(729*e^9) + (18*a*b^2*d^8*n^2*x
^(1/3))/e^8 - (18*b^3*d^8*n^3*x^(1/3))/e^8 + (18*b^3*d^8*n^2*(d + e*x^(1/3)
)*Log[c*(d + e*x^(1/3))^n])/e^9 - (18*b^2*d^7*n^2*(d + e*x^(1/3))^2*(a + b*
Log[c*(d + e*x^(1/3))^n]))/e^9 + (56*b^2*d^6*n^2*(d + e*x^(1/3))^3*(a + b*L
og[c*(d + e*x^(1/3))^n]))/(3*e^9) - (63*b^2*d^5*n^2*(d + e*x^(1/3))^4*(a +
b*Log[c*(d + e*x^(1/3))^n]))/(4*e^9) + (252*b^2*d^4*n^2*(d + e*x^(1/3))^5*(
a + b*Log[c*(d + e*x^(1/3))^n]))/(25*e^9) - (14*b^2*d^3*n^2*(d + e*x^(1/3))
^6*(a + b*Log[c*(d + e*x^(1/3))^n]))/(3*e^9) + (72*b^2*d^2*n^2*(d + e*x^(1/
3))^7*(a + b*Log[c*(d + e*x^(1/3))^n]))/(49*e^9) - (9*b^2*d*n^2*(d + e*x^(1
/3))^8*(a + b*Log[c*(d + e*x^(1/3))^n]))/(32*e^9) + (2*b^2*n^2*(d + e*x^(1/
3))^9*(a + b*Log[c*(d + e*x^(1/3))^n]))/(81*e^9) - (9*b*d^8*n*(d + e*x^(1/3
))*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^9 + (18*b*d^7*n*(d + e*x^(1/3))^2*
(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^9 - (28*b*d^6*n*(d + e*x^(1/3))^3*(a
+ b*Log[c*(d + e*x^(1/3))^n])^2)/e^9 + (63*b*d^5*n*(d + e*x^(1/3))^4*(a + b
*Log[c*(d + e*x^(1/3))^n])^2)/(2*e^9) - (126*b*d^4*n*(d + e*x^(1/3))^5*(a +
 b*Log[c*(d + e*x^(1/3))^n])^2)/(5*e^9) + (14*b*d^3*n*(d + e*x^(1/3))^6*(a
+ b*Log[c*(d + e*x^(1/3))^n])^2)/e^9 - (36*b*d^2*n*(d + e*x^(1/3))^7*(a + b
*Log[c*(d + e*x^(1/3))^n])^2)/(7*e^9) + (9*b*d*n*(d + e*x^(1/3))^8*(a + b*L
og[c*(d + e*x^(1/3))^n])^2)/(8*e^9) - (b*n*(d + e*x^(1/3))^9*(a + b*Log[c*(
d + e*x^(1/3))^n])^2)/(9*e^9) + (3*d^8*(d + e*x^(1/3))*(a + b*Log[c*(d + e*
x^(1/3))^n])^3)/e^9 - (12*d^7*(d + e*x^(1/3))^2*(a + b*Log[c*(d + e*x^(1/3)
)^n])^3)/e^9 + (28*d^6*(d + e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n])^3
)/e^9 - (42*d^5*(d + e*x^(1/3))^4*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^9 +
 (42*d^4*(d + e*x^(1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^9 - (28*d^
3*(d + e*x^(1/3))^6*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^9 + (12*d^2*(d +
e*x^(1/3))^7*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^9 - (3*d*(d + e*x^(1/3))
^8*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^9 + ((d + e*x^(1/3))^9*(a + b*Log[
c*(d + e*x^(1/3))^n])^3)/(3*e^9)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
```

```
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx = 3\text{Subst}\left(\int x^8(a+b\log\left(c(d+ex)^n\right))^3\,dx, x, \sqrt[3]{x}\right)$$

$$= 3\text{Subst}\left(\int\left(\frac{d^8(a+b\log\left(c(d+ex)^n\right))^3}{e^8} - \frac{8d^7(d+ex)\left(a+b\log\left(c\right.\right.}{e^8}\right.\right.$$

$$= \frac{3\text{Subst}\left(\int(d+ex)^8\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx, x, \sqrt[3]{x}\right)}{e^8} - \frac{(24d)\text{Su}}{}$$

$$= \frac{3\text{Subst}\left(\int x^8(a+b\log\left(cx^n\right))^3\,dx, x, d+e\sqrt[3]{x}\right)}{e^9} - \frac{(24d)\text{Subst}\left(\int x^7\right.}{}$$

$$= \frac{3d^8\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^9} - \frac{12d^7\left(d+e\sqrt[3]{x}\right)^2(a}{}$$

$$= -\frac{9bd^8n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^9} + \frac{18bd^7n\left(d+e\sqrt[3]{\cdot}\right.}{}$$

$$= \frac{9b^3d^7n^3\left(d+e\sqrt[3]{x}\right)^2}{e^9} - \frac{56b^3d^6n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^9} + \frac{63b^3d^5n^3\left(d+e\sqrt[3]{x}\right.}{16e^9}$$

$$= \frac{9b^3d^7n^3\left(d+e\sqrt[3]{x}\right)^2}{e^9} - \frac{56b^3d^6n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^9} + \frac{63b^3d^5n^3\left(d+e\sqrt[3]{x}\right.}{16e^9}$$

Mathematica [A]
time = 0.71, size = 895, normalized size = 0.66

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

```
[Out] (2667168000*b^3*d^9*n^3*Log[d + e*x^(1/3)]^3 + 3175200*b^2*d^9*n^2*Log[d +
e*x^(1/3)]^2*(-2520*a + 7129*b*n - 2520*b*Log[c*(d + e*x^(1/3))^n]) + 2520*
b*d^9*n*Log[d + e*x^(1/3)]*(3175200*a^2 - 17965080*a*b*n + 30300391*b^2*n^2
 + 2520*b*(2520*a - 7129*b*n)*Log[c*(d + e*x^(1/3))^n] + 3175200*b^2*Log[c*
(d + e*x^(1/3))^n]^2) + e*x^(1/3)*(2667168000*a^3*e^8*x^(8/3) + b^3*n^3*(-7
6356985320*d^8 + 15542491860*d^7*e*x^(1/3) - 5483495640*d^6*e^2*x^(2/3) + 2
340330930*d^5*e^3*x - 1075607064*d^4*e^4*x^(4/3) + 498592500*d^3*e^5*x^(5/3
) - 219465000*d^2*e^6*x^2 + 83734875*d*e^7*x^(7/3) - 21952000*e^8*x^(8/3))
- 3175200*a^2*b*n*(2520*d^8 - 1260*d^7*e*x^(1/3) + 840*d^6*e^2*x^(2/3) - 63
0*d^5*e^3*x + 504*d^4*e^4*x^(4/3) - 420*d^3*e^5*x^(5/3) + 360*d^2*e^6*x^2 -
 315*d*e^7*x^(7/3) + 280*e^8*x^(8/3)) + 2520*a*b^2*n^2*(17965080*d^8 - 5807
340*d^7*e*x^(1/3) + 2813160*d^6*e^2*x^(2/3) - 1580670*d^5*e^3*x + 947016*d^
4*e^4*x^(4/3) - 577500*d^3*e^5*x^(5/3) + 343800*d^2*e^6*x^2 - 187425*d*e^7*
x^(7/3) + 78400*e^8*x^(8/3)) + 2520*b*(3175200*a^2*e^8*x^(8/3) - 2520*a*b*n
```

```
*(2520*d^8 - 1260*d^7*e*x^(1/3) + 840*d^6*e^2*x^(2/3) - 630*d^5*e^3*x + 504
*d^4*e^4*x^(4/3) - 420*d^3*e^5*x^(5/3) + 360*d^2*e^6*x^2 - 315*d*e^7*x^(7/3
) + 280*e^8*x^(8/3)) + b^2*n^2*(17965080*d^8 - 5807340*d^7*e*x^(1/3) + 2813
160*d^6*e^2*x^(2/3) - 1580670*d^5*e^3*x + 947016*d^4*e^4*x^(4/3) - 577500*d
^3*e^5*x^(5/3) + 343800*d^2*e^6*x^2 - 187425*d*e^7*x^(7/3) + 78400*e^8*x^(8
/3)))*Log[c*(d + e*x^(1/3))^n] - 3175200*b^2*(-2520*a*e^8*x^(8/3) + b*n*(25
20*d^8 - 1260*d^7*e*x^(1/3) + 840*d^6*e^2*x^(2/3) - 630*d^5*e^3*x + 504*d^4
*e^4*x^(4/3) - 420*d^3*e^5*x^(5/3) + 360*d^2*e^6*x^2 - 315*d*e^7*x^(7/3) +
280*e^8*x^(8/3)))*Log[c*(d + e*x^(1/3))^n]^2 + 2667168000*b^3*e^8*x^(8/3)*L
og[c*(d + e*x^(1/3))^n]^3))/(8001504000*e^9)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^2 \left(a+b\ln \left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^{3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(1/3))^n))^3,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e*x^(1/3))^n))^3,x)
```

Maxima [A]
time = 0.31, size = 835, normalized size = 0.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3*log((x^(1/3)*e + d)^n*c)^3 + a*b^2*x^3*log((x^(1/3)*e + d)^n*c)
^2 + a^2*b*x^3*log((x^(1/3)*e + d)^n*c) + 1/3*a^3*x^3 + 1/2520*(2520*d^9*e^
(-10)*log(x^(1/3)*e + d) + (1260*d^7*x^(2/3)*e - 2520*d^8*x^(1/3) - 840*d^6
*x*e^2 + 630*d^5*x^(4/3)*e^3 - 504*d^4*x^(5/3)*e^4 + 420*d^3*x^2*e^5 - 360*
d^2*x^(7/3)*e^6 + 315*d*x^(8/3)*e^7 - 280*x^3*e^8)*e^(-9))*a^2*b*n*e - 1/31
75200*((3175200*d^9*log(x^(1/3)*e + d)^2 + 17965080*d^9*log(x^(1/3)*e + d)
- 17965080*d^8*x^(1/3)*e + 5807340*d^7*x^(2/3)*e^2 - 2813160*d^6*x*e^3 + 15
80670*d^5*x^(4/3)*e^4 - 947016*d^4*x^(5/3)*e^5 + 577500*d^3*x^2*e^6 - 34380
0*d^2*x^(7/3)*e^7 + 187425*d*x^(8/3)*e^8 - 78400*x^3*e^9)*n^2*e^(-9) - 2520
*(2520*d^9*e^(-10)*log(x^(1/3)*e + d) + (1260*d^7*x^(2/3)*e - 2520*d^8*x^(1
/3) - 840*d^6*x*e^2 + 630*d^5*x^(4/3)*e^3 - 504*d^4*x^(5/3)*e^4 + 420*d^3*x
^2*e^5 - 360*d^2*x^(7/3)*e^6 + 315*d*x^(8/3)*e^7 - 280*x^3*e^8)*e^(-9))*n*e
*log((x^(1/3)*e + d)^n*c))*a*b^2 + 1/8001504000*(3175200*(2520*d^9*e^(-10)*
log(x^(1/3)*e + d) + (1260*d^7*x^(2/3)*e - 2520*d^8*x^(1/3) - 840*d^6*x*e^2
 + 630*d^5*x^(4/3)*e^3 - 504*d^4*x^(5/3)*e^4 + 420*d^3*x^2*e^5 - 360*d^2*x^
(7/3)*e^6 + 315*d*x^(8/3)*e^7 - 280*x^3*e^8)*e^(-9))*n*e*log((x^(1/3)*e + d
```

```
)^n*c)^2 + ((2667168000*d^9*log(x^(1/3)*e + d)^3 + 22636000800*d^9*log(x^(1
/3)*e + d)^2 + 76356985320*d^9*log(x^(1/3)*e + d) - 76356985320*d^8*x^(1/3)
*e + 15542491860*d^7*x^(2/3)*e^2 - 5483495640*d^6*x*e^3 + 2340330930*d^5*x^
(4/3)*e^4 - 1075607064*d^4*x^(5/3)*e^5 + 498592500*d^3*x^2*e^6 - 219465000*
d^2*x^(7/3)*e^7 + 83734875*d*x^(8/3)*e^8 - 21952000*x^3*e^9)*n^2*e^(-10) -
2520*(3175200*d^9*log(x^(1/3)*e + d)^2 + 17965080*d^9*log(x^(1/3)*e + d) -
17965080*d^8*x^(1/3)*e + 5807340*d^7*x^(2/3)*e^2 - 2813160*d^6*x*e^3 + 1580
670*d^5*x^(4/3)*e^4 - 947016*d^4*x^(5/3)*e^5 + 577500*d^3*x^2*e^6 - 343800*
d^2*x^(7/3)*e^7 + 187425*d*x^(8/3)*e^8 - 78400*x^3*e^9)*n*e^(-10)*log((x^(1
/3)*e + d)^n*c))*n*e)*b^3
```

Fricas [A]
time = 0.46, size = 1544, normalized size = 1.14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="fricas")
```

```
[Out] 1/8001504000*(2667168000*b^3*x^3*e^9*log(c)^3 - 10976000*(2*b^3*n^3 - 18*a*
b^2*n^2 + 81*a^2*b*n - 243*a^3)*x^3*e^9 + 10500*(47485*b^3*d^3*n^3 - 138600
*a*b^2*d^3*n^2 + 127008*a^2*b*d^3*n)*x^2*e^6 + 2667168000*(b^3*d^9*n^3 + b^
3*n^3*x^3*e^9)*log(x^(1/3)*e + d)^3 - 840*(6527971*b^3*d^6*n^3 - 8439480*a*
b^2*d^6*n^2 + 3175200*a^2*b*d^6*n)*x*e^3 - 3175200*(7129*b^3*d^9*n^3 - 2520
*a*b^2*d^9*n^2 + 840*b^3*d^6*n^3*x*e^3 - 420*b^3*d^3*n^3*x^2*e^6 + 280*(b^3
*n^3 - 9*a*b^2*n^2)*x^3*e^9 - 2520*(b^3*d^9*n^2 + b^3*n^2*x^3*e^9)*log(c) -
 63*(20*b^3*d^7*n^3*e^2 - 8*b^3*d^4*n^3*x*e^5 + 5*b^3*d*n^3*x^2*e^8)*x^(2/3
) + 90*(28*b^3*d^8*n^3*e - 7*b^3*d^5*n^3*x*e^4 + 4*b^3*d^2*n^3*x^2*e^7)*x^(
1/3))*log(x^(1/3)*e + d)^2 - 444528000*(6*b^3*d^6*n*x*e^3 - 3*b^3*d^3*n*x^2
*e^6 + 2*(b^3*n - 9*a*b^2)*x^3*e^9)*log(c)^2 + 2520*(30300391*b^3*d^9*n^3 -
 17965080*a*b^2*d^9*n^2 + 3175200*a^2*b*d^9*n + 39200*(2*b^3*n^3 - 18*a*b^2
*n^2 + 81*a^2*b*n)*x^3*e^9 - 2100*(275*b^3*d^3*n^3 - 504*a*b^2*d^3*n^2)*x^2
*e^6 + 840*(3349*b^3*d^6*n^3 - 2520*a*b^2*d^6*n^2)*x*e^3 + 3175200*(b^3*d^9
*n + b^3*n*x^3*e^9)*log(c)^2 - 2520*(7129*b^3*d^9*n^2 - 2520*a*b^2*d^9*n +
840*b^3*d^6*n^2*x*e^3 - 420*b^3*d^3*n^2*x^2*e^6 + 280*(b^3*n^2 - 9*a*b^2*n)
*x^3*e^9)*log(c) - 63*(175*(17*b^3*d*n^3 - 72*a*b^2*d*n^2)*x^2*e^8 - 8*(187
9*b^3*d^4*n^3 - 2520*a*b^2*d^4*n^2)*x*e^5 + 20*(4609*b^3*d^7*n^3 - 2520*a*b
^2*d^7*n^2)*e^2 - 2520*(20*b^3*d^7*n^2*e^2 - 8*b^3*d^4*n^2*x*e^5 + 5*b^3*d*
n^2*x^2*e^8)*log(c))*x^(2/3) + 90*(20*(191*b^3*d^2*n^3 - 504*a*b^2*d^2*n^2)
*x^2*e^7 - 7*(2509*b^3*d^5*n^3 - 2520*a*b^2*d^5*n^2)*x*e^4 + 28*(7129*b^3*d
^8*n^3 - 2520*a*b^2*d^8*n^2)*e - 2520*(28*b^3*d^8*n^2*e - 7*b^3*d^5*n^2*x*e
^4 + 4*b^3*d^2*n^2*x^2*e^7)*log(c))*x^(1/3))*log(x^(1/3)*e + d) + 352800*(2
80*(2*b^3*n^2 - 18*a*b^2*n + 81*a^2*b)*x^3*e^9 - 15*(275*b^3*d^3*n^2 - 504*
a*b^2*d^3*n)*x^2*e^6 + 6*(3349*b^3*d^6*n^2 - 2520*a*b^2*d^6*n)*x*e^3)*log(c
) + 63*(6125*(217*b^3*d*n^3 - 1224*a*b^2*d*n^2 + 2592*a^2*b*d*n)*x^2*e^8 -
```

```
8*(2134141*b^3*d^4*n^3 - 4735080*a*b^2*d^4*n^2 + 3175200*a^2*b*d^4*n)*x*e^5
 + 3175200*(20*b^3*d^7*n*e^2 - 8*b^3*d^4*n*x*e^5 + 5*b^3*d*n*x^2*e^8)*log(c
)^2 + 20*(12335311*b^3*d^7*n^3 - 11614680*a*b^2*d^7*n^2 + 3175200*a^2*b*d^7
*n)*e^2 - 2520*(175*(17*b^3*d*n^2 - 72*a*b^2*d*n)*x^2*e^8 - 8*(1879*b^3*d^4
*n^2 - 2520*a*b^2*d^4*n)*x*e^5 + 20*(4609*b^3*d^7*n^2 - 2520*a*b^2*d^7*n)*e
^2)*log(c))*x^(2/3) - 90*(100*(24385*b^3*d^2*n^3 - 96264*a*b^2*d^2*n^2 + 12
7008*a^2*b*d^2*n)*x^2*e^7 - 7*(3714811*b^3*d^5*n^3 - 6322680*a*b^2*d^5*n^2
+ 3175200*a^2*b*d^5*n)*x*e^4 + 3175200*(28*b^3*d^8*n*e - 7*b^3*d^5*n*x*e^4
+ 4*b^3*d^2*n*x^2*e^7)*log(c)^2 + 28*(30300391*b^3*d^8*n^3 - 17965080*a*b^2
*d^8*n^2 + 3175200*a^2*b*d^8*n)*e - 2520*(20*(191*b^3*d^2*n^2 - 504*a*b^2*d
^2*n)*x^2*e^7 - 7*(2509*b^3*d^5*n^2 - 2520*a*b^2*d^5*n)*x*e^4 + 28*(7129*b^
3*d^8*n^2 - 2520*a*b^2*d^8*n)*e)*log(c))*x^(1/3))*e^(-9)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/3))**n))**3,x)
```

```
[Out] Integral(x**2*(a + b*log(c*(d + e*x**(1/3))**n))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 3333 vs. 2(1214) = 2428.
time = 3.78, size = 3333, normalized size = 2.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="giac")
```

```
[Out] 1/8001504000*(2667168000*b^3*x^3*e*log(c)^3 + 8001504000*a*b^2*x^3*e*log(c)
^2 + 8001504000*a^2*b*x^3*e*log(c) + (2667168000*(x^(1/3)*e + d)^9*e^(-8)*l
og(x^(1/3)*e + d)^3 - 24004512000*(x^(1/3)*e + d)^8*d*e^(-8)*log(x^(1/3)*e
+ d)^3 + 96018048000*(x^(1/3)*e + d)^7*d^2*e^(-8)*log(x^(1/3)*e + d)^3 - 22
4042112000*(x^(1/3)*e + d)^6*d^3*e^(-8)*log(x^(1/3)*e + d)^3 + 336063168000
*(x^(1/3)*e + d)^5*d^4*e^(-8)*log(x^(1/3)*e + d)^3 - 336063168000*(x^(1/3)*
e + d)^4*d^5*e^(-8)*log(x^(1/3)*e + d)^3 + 224042112000*(x^(1/3)*e + d)^3*d
^6*e^(-8)*log(x^(1/3)*e + d)^3 - 96018048000*(x^(1/3)*e + d)^2*d^7*e^(-8)*l
og(x^(1/3)*e + d)^3 + 24004512000*(x^(1/3)*e + d)*d^8*e^(-8)*log(x^(1/3)*e
+ d)^3 - 889056000*(x^(1/3)*e + d)^9*e^(-8)*log(x^(1/3)*e + d)^2 + 90016920
00*(x^(1/3)*e + d)^8*d*e^(-8)*log(x^(1/3)*e + d)^2 - 41150592000*(x^(1/3)*e
 + d)^7*d^2*e^(-8)*log(x^(1/3)*e + d)^2 + 112021056000*(x^(1/3)*e + d)^6*d^
3*e^(-8)*log(x^(1/3)*e + d)^2 - 201637900800*(x^(1/3)*e + d)^5*d^4*e^(-8)*l
```

og(x^(1/3)*e + d)^2 + 252047376000*(x^(1/3)*e + d)^4*d^5*e^(-8)*log(x^(1/3)
e + d)^2 - 224042112000(x^(1/3)*e + d)^3*d^6*e^(-8)*log(x^(1/3)*e + d)^2
+ 144027072000*(x^(1/3)*e + d)^2*d^7*e^(-8)*log(x^(1/3)*e + d)^2 - 72013536
000*(x^(1/3)*e + d)*d^8*e^(-8)*log(x^(1/3)*e + d)^2 + 197568000*(x^(1/3)*e
+ d)^9*e^(-8)*log(x^(1/3)*e + d) - 2250423000*(x^(1/3)*e + d)^8*d*e^(-8)*lo
g(x^(1/3)*e + d) + 11757312000*(x^(1/3)*e + d)^7*d^2*e^(-8)*log(x^(1/3)*e +
 d) - 37340352000*(x^(1/3)*e + d)^6*d^3*e^(-8)*log(x^(1/3)*e + d) + 8065516
0320*(x^(1/3)*e + d)^5*d^4*e^(-8)*log(x^(1/3)*e + d) - 126023688000*(x^(1/3
)*e + d)^4*d^5*e^(-8)*log(x^(1/3)*e + d) + 149361408000*(x^(1/3)*e + d)^3*d
^6*e^(-8)*log(x^(1/3)*e + d) - 144027072000*(x^(1/3)*e + d)^2*d^7*e^(-8)*lo
g(x^(1/3)*e + d) + 144027072000*(x^(1/3)*e + d)*d^8*e^(-8)*log(x^(1/3)*e +
d) - 21952000*(x^(1/3)*e + d)^9*e^(-8) + 281302875*(x^(1/3)*e + d)^8*d*e^(-
8) - 1679616000*(x^(1/3)*e + d)^7*d^2*e^(-8) + 6223392000*(x^(1/3)*e + d)^6
*d^3*e^(-8) - 16131032064*(x^(1/3)*e + d)^5*d^4*e^(-8) + 31505922000*(x^(1/
3)*e + d)^4*d^5*e^(-8) - 49787136000*(x^(1/3)*e + d)^3*d^6*e^(-8) + 7201353
6000*(x^(1/3)*e + d)^2*d^7*e^(-8) - 144027072000*(x^(1/3)*e + d)*d^8*e^(-8)
)*b^3*n^3 + 2667168000*a^3*x^3*e + 2520*(3175200*(x^(1/3)*e + d)^9*e^(-8)*l
og(x^(1/3)*e + d)^2 - 28576800*(x^(1/3)*e + d)^8*d*e^(-8)*log(x^(1/3)*e + d
)^2 + 114307200*(x^(1/3)*e + d)^7*d^2*e^(-8)*log(x^(1/3)*e + d)^2 - 2667168
00*(x^(1/3)*e + d)^6*d^3*e^(-8)*log(x^(1/3)*e + d)^2 + 400075200*(x^(1/3)*e
 + d)^5*d^4*e^(-8)*log(x^(1/3)*e + d)^2 - 400075200*(x^(1/3)*e + d)^4*d^5*e
^(-8)*log(x^(1/3)*e + d)^2 + 266716800*(x^(1/3)*e + d)^3*d^6*e^(-8)*log(x^(
1/3)*e + d)^2 - 114307200*(x^(1/3)*e + d)^2*d^7*e^(-8)*log(x^(1/3)*e + d)^2
 + 28576800*(x^(1/3)*e + d)*d^8*e^(-8)*log(x^(1/3)*e + d)^2 - 705600*(x^(1/
3)*e + d)^9*e^(-8)*log(x^(1/3)*e + d) + 7144200*(x^(1/3)*e + d)^8*d*e^(-8)*
log(x^(1/3)*e + d) - 32659200*(x^(1/3)*e + d)^7*d^2*e^(-8)*log(x^(1/3)*e +
d) + 88905600*(x^(1/3)*e + d)^6*d^3*e^(-8)*log(x^(1/3)*e + d) - 160030080*(
x^(1/3)*e + d)^5*d^4*e^(-8)*log(x^(1/3)*e + d) + 200037600*(x^(1/3)*e + d)^
4*d^5*e^(-8)*log(x^(1/3)*e + d) - 177811200*(x^(1/3)*e + d)^3*d^6*e^(-8)*lo
g(x^(1/3)*e + d) + 114307200*(x^(1/3)*e + d)^2*d^7*e^(-8)*log(x^(1/3)*e + d
) - 57153600*(x^(1/3)*e + d)*d^8*e^(-8)*log(x^(1/3)*e + d) + 78400*(x^(1/3)
*e + d)^9*e^(-8) - 893025*(x^(1/3)*e + d)^8*d*e^(-8) + 4665600*(x^(1/3)*e +
 d)^7*d^2*e^(-8) - 14817600*(x^(1/3)*e + d)^6*d^3*e^(-8) + 32006016*(x^(1/3
)*e + d)^5*d^4*e^(-8) - 50009400*(x^(1/3)*e + d)^4*d^5*e^(-8) + 59270400*(x
^(1/3)*e + d)^3*d^6*e^(-8) - 57153600*(x^(1/3)*e + d)^2*d^7*e^(-8) + 571536
00*(x^(1/3)*e + d)*d^8*e^(-8))*b^3*n^2*log(c) + 3175200*(2520*(x^(1/3)*e +
d)^9*e^(-8)*log(x^(1/3)*e + d) - 22680*(x^(1/3)*e + d)^8*d*e^(-8)*log(x^(1/
3)*e + d) + 90720*(x^(1/3)*e + d)^7*d^2*e^(-8)*log(x^(1/3)*e + d) - 211680*
(x^(1/3)*e + d)^6*d^3*e^(-8)*log(x^(1/3)*e + d) + 317520*(x^(1/3)*e + d)^5*
d^4*e^(-8)*log(x^(1/3)*e + d) - 317520*(x^(1/3)*e + d)^4*d^5*e^(-8)*log(x^(
1/3)*e + d) + 211680*(x^(1/3)*e + d)^3*d^6*e^(-8)*log(x^(1/3)*e + d) - 9072
0*(x^(1/3)*e + d)^2*d^7*e^(-8)*log(x^(1/3)*e + d) + 22680*(x^(1/3)*e + d)*d
^8*e^(-8)*log(x^(1/3)*e + d) - 280*(x^(1/3)*e + d)^9*e^(-8) + 2835*(x^(1/3)
*e + d)^8*d*e^(-8) - 12960*(x^(1/3)*e + d)^7*d^2*e^(-8) + 35280*(x^(1/3)*e
+ d)^6*d^3*e^(-8) - 63504*(x^(1/3)*e + d)^5*d^4*e^(-8) + 79380*(x^(1/3)*e +

```
d)^4*d^5*e^(-8) - 70560*(x^(1/3)*e + d)^3*d^6*e^(-8) + 45360*(x^(1/3)*e +
d)^2*d^7*e^(-8) - 22680*(x^(1/3)*e + d)*d^8*e^(-8))*b^3*n*log(c)^2 + 2520*(
3175200*(x^(1/3)*e + d)^9*e^(-8)*log(x^(1/3)*e + d)^2 - 28576800*(x^(1/3)*e
 + d)^8*d*e^(-8)*log(x^(1/3)*e + d)^2 + 114307200*(x^(1/3)*e + d)^7*d^2*e^(
-8)*log(x^(1/3)*e + d)^2 - 266716800*(x^(1/3)*e + d)^6*d^3*e^(-8)*log(x^(1/
3)*e + d)^2 + 400075200*(x^(1/3)*e + d)^5*d^4*e^(-8)*log(x^(1/3)*e + d)^2 -
 400075200*(x^(1/3)*e + d)^4*d^5*e^(-8)*log(x^(1/3)*e + d)^2 + 266716800*(x
^(1/3)*e + d)^3*d^6*e^(-8)*log(x^(1/3)*e + d)^2 - 114307200*(x^(1/3)*e + d)
^2*d^7*e^(-8)*log(x^(1/3)*e + d)^2 + 28576800*(...
```

Mupad [B]
time = 8.25, size = 1386, normalized size = 1.02

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e*x^(1/3))^n))^3,x)
```

```
[Out] (a^3*x^3)/3 + (b^3*x^3*log(c*(d + e*x^(1/3))^n)^3)/3 - (2*b^3*n^3*x^3)/729
+ a*b^2*x^3*log(c*(d + e*x^(1/3))^n)^2 - (b^3*n*x^3*log(c*(d + e*x^(1/3))^n
)^2)/9 + (2*b^3*n^2*x^3*log(c*(d + e*x^(1/3))^n))/81 + (2*a*b^2*n^2*x^3)/81
 + (b^3*d^9*log(c*(d + e*x^(1/3))^n)^3)/(3*e^9) + a^2*b*x^3*log(c*(d + e*x^
(1/3))^n) - (a^2*b*n*x^3)/9 - (2*a*b^2*n*x^3*log(c*(d + e*x^(1/3))^n))/9 +
(30300391*b^3*d^9*n^3*log(d + e*x^(1/3)))/(3175200*e^9) + (47485*b^3*d^3*n^
3*x^2)/(762048*e^3) - (24385*b^3*d^2*n^3*x^(7/3))/(889056*e^2) - (2134141*b
^3*d^4*n^3*x^(5/3))/(15876000*e^4) + (3714811*b^3*d^5*n^3*x^(4/3))/(1270080
0*e^5) + (12335311*b^3*d^7*n^3*x^(2/3))/(6350400*e^7) - (30300391*b^3*d^8*n
^3*x^(1/3))/(3175200*e^8) + (a*b^2*d^9*log(c*(d + e*x^(1/3))^n)^2)/e^9 - (7
129*b^3*d^9*n*log(c*(d + e*x^(1/3))^n)^2)/(2520*e^9) + (217*b^3*d*n^3*x^(8/
3))/(20736*e) - (6527971*b^3*d^6*n^3*x)/(9525600*e^6) + (a^2*b*d^9*n*log(d
+ e*x^(1/3)))/e^9 + (b^3*d*n*x^(8/3)*log(c*(d + e*x^(1/3))^n)^2)/(8*e) - (1
7*b^3*d*n^2*x^(8/3)*log(c*(d + e*x^(1/3))^n))/(288*e) - (b^3*d^6*n*x*log(c*
(d + e*x^(1/3))^n)^2)/(3*e^6) + (3349*b^3*d^6*n^2*x*log(c*(d + e*x^(1/3))^n
))/(3780*e^6) + (a^2*b*d^3*n*x^2)/(6*e^3) - (17*a*b^2*d*n^2*x^(8/3))/(288*e
) + (3349*a*b^2*d^6*n^2*x)/(3780*e^6) - (a^2*b*d^2*n*x^(7/3))/(7*e^2) - (a^
2*b*d^4*n*x^(5/3))/(5*e^4) + (a^2*b*d^5*n*x^(4/3))/(4*e^5) + (a^2*b*d^7*n*x
^(2/3))/(2*e^7) - (a^2*b*d^8*n*x^(1/3))/e^8 - (7129*a*b^2*d^9*n^2*log(d + e
*x^(1/3)))/(1260*e^9) + (b^3*d^3*n*x^2*log(c*(d + e*x^(1/3))^n)^2)/(6*e^3)
- (275*b^3*d^3*n^2*x^2*log(c*(d + e*x^(1/3))^n))/(1512*e^3) - (b^3*d^2*n*x^
(7/3)*log(c*(d + e*x^(1/3))^n)^2)/(7*e^2) + (191*b^3*d^2*n^2*x^(7/3)*log(c*
(d + e*x^(1/3))^n))/(1764*e^2) - (b^3*d^4*n*x^(5/3)*log(c*(d + e*x^(1/3))^n
)^2)/(5*e^4) + (1879*b^3*d^4*n^2*x^(5/3)*log(c*(d + e*x^(1/3))^n))/(6300*e^
4) + (b^3*d^5*n*x^(4/3)*log(c*(d + e*x^(1/3))^n)^2)/(4*e^5) - (2509*b^3*d^5
*n^2*x^(4/3)*log(c*(d + e*x^(1/3))^n))/(5040*e^5) + (b^3*d^7*n*x^(2/3)*log(
c*(d + e*x^(1/3))^n)^2)/(2*e^7) - (4609*b^3*d^7*n^2*x^(2/3)*log(c*(d + e*x^
```

(1/3))^n))/(2520*e^7) - (b^3*d^8*n*x^(1/3)*log(c*(d + e*x^(1/3))^n)^2)/e^8 + (7129*b^3*d^8*n^2*x^(1/3)*log(c*(d + e*x^(1/3))^n))/(1260*e^8) - (275*a*b^2*d^3*n^2*x^2)/(1512*e^3) + (191*a*b^2*d^2*n^2*x^(7/3))/(1764*e^2) + (1879*a*b^2*d^4*n^2*x^(5/3))/(6300*e^4) - (2509*a*b^2*d^5*n^2*x^(4/3))/(5040*e^5) - (4609*a*b^2*d^7*n^2*x^(2/3))/(2520*e^7) + (7129*a*b^2*d^8*n^2*x^(1/3))/(1260*e^8) + (a^2*b*d*n*x^(8/3))/(8*e) - (a^2*b*d^6*n*x)/(3*e^6) + (a*b^2*d*n*x^(8/3)*log(c*(d + e*x^(1/3))^n))/(4*e) - (2*a*b^2*d^6*n*x*log(c*(d + e*x^(1/3))^n))/(3*e^6) + (a*b^2*d^3*n*x^2*log(c*(d + e*x^(1/3))^n))/(3*e^3) - (2*a*b^2*d^2*n*x^(7/3)*log(c*(d + e*x^(1/3))^n))/(7*e^2) - (2*a*b^2*d^4*n*x^(5/3)*log(c*(d + e*x^(1/3))^n))/(5*e^4) + (a*b^2*d^5*n*x^(4/3)*log(c*(d + e*x^(1/3))^n))/(2*e^5) + (a*b^2*d^7*n*x^(2/3)*log(c*(d + e*x^(1/3))^n))/e^7 - (2*a*b^2*d^8*n*x^(1/3)*log(c*(d + e*x^(1/3))^n))/e^8

3.458 $\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=907

$$-\frac{45b^3d^4n^3\left(d+e\sqrt[3]{x}\right)^2}{8e^6}+\frac{20b^3d^3n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^6}-\frac{45b^3d^2n^3\left(d+e\sqrt[3]{x}\right)^4}{64e^6}+\frac{18b^3dn^3\left(d+e\sqrt[3]{x}\right)^5}{125e^6}-\frac{b^3n^3\left(d+e\sqrt[3]{x}\right)^6}{72e^6}$$

```
[Out] 1/2*(d+e*x^(1/3))^6*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^6-18*a*b^2*d^5*n^2*x^(1
/3)/e^5-18*b^3*d^5*n^2*(d+e*x^(1/3))*ln(c*(d+e*x^(1/3))^n)/e^6+45/4*b^2*d^4
*n^2*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6-20/3*b^2*d^3*n^2*(d+e*
x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6+45/16*b^2*d^2*n^2*(d+e*x^(1/3))^
4*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6-18/25*b^2*d*n^2*(d+e*x^(1/3))^5*(a+b*ln(c
*(d+e*x^(1/3))^n))/e^6+9*b*d^5*n*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))^
2/e^6-45/4*b*d^4*n*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^6+10*b*d
^3*n*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^6-45/8*b*d^2*n*(d+e*x^
(1/3))^4*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^6+9/5*b*d*n*(d+e*x^(1/3))^5*(a+b*l
n(c*(d+e*x^(1/3))^n))^2/e^6+1/12*b^2*n^2*(d+e*x^(1/3))^6*(a+b*ln(c*(d+e*x^(
1/3))^n))/e^6-1/4*b*n*(d+e*x^(1/3))^6*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^6-45/
8*b^3*d^4*n^3*(d+e*x^(1/3))^2/e^6+20/9*b^3*d^3*n^3*(d+e*x^(1/3))^3/e^6-45/6
4*b^3*d^2*n^3*(d+e*x^(1/3))^4/e^6+18/125*b^3*d*n^3*(d+e*x^(1/3))^5/e^6+18*b
^3*d^5*n^3*x^(1/3)/e^5-3*d^5*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^
6+15/2*d^4*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^6-10*d^3*(d+e*x^
(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^6+15/2*d^2*(d+e*x^(1/3))^4*(a+b*ln
(c*(d+e*x^(1/3))^n))^3/e^6-3*d*(d+e*x^(1/3))^5*(a+b*ln(c*(d+e*x^(1/3))^n))^
3/e^6-1/72*b^3*n^3*(d+e*x^(1/3))^6/e^6
```

Rubi [A]
time = 0.63, antiderivative size = 907, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.364$, Rules used = {2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

```
[Out] (-45*b^3*d^4*n^3*(d + e*x^(1/3))^2)/(8*e^6) + (20*b^3*d^3*n^3*(d + e*x^(1/3
))^3)/(9*e^6) - (45*b^3*d^2*n^3*(d + e*x^(1/3))^4)/(64*e^6) + (18*b^3*d*n^3
*(d + e*x^(1/3))^5)/(125*e^6) - (b^3*n^3*(d + e*x^(1/3))^6)/(72*e^6) - (18*
a*b^2*d^5*n^2*x^(1/3))/e^5 + (18*b^3*d^5*n^3*x^(1/3))/e^5 - (18*b^3*d^5*n^2
*(d + e*x^(1/3))*Log[c*(d + e*x^(1/3))^n])/e^6 + (45*b^2*d^4*n^2*(d + e*x^(
1/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/(4*e^6) - (20*b^2*d^3*n^2*(d + e*
x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n]))/(3*e^6) + (45*b^2*d^2*n^2*(d +
 e*x^(1/3))^4*(a + b*Log[c*(d + e*x^(1/3))^n]))/(16*e^6) - (18*b^2*d*n^2*(d
 + e*x^(1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n]))/(25*e^6) + (b^2*n^2*(d +
```

```
e*x^(1/3))^6*(a + b*Log[c*(d + e*x^(1/3))^n]))/(12*e^6) + (9*b*d^5*n*(d + e
*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^6 - (45*b*d^4*n*(d + e*x^(1
/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(4*e^6) + (10*b*d^3*n*(d + e*x^(
1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^6 - (45*b*d^2*n*(d + e*x^(1/3
))^4*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(8*e^6) + (9*b*d*n*(d + e*x^(1/3))
^5*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(5*e^6) - (b*n*(d + e*x^(1/3))^6*(a
+ b*Log[c*(d + e*x^(1/3))^n])^2)/(4*e^6) - (3*d^5*(d + e*x^(1/3))*(a + b*Lo
g[c*(d + e*x^(1/3))^n])^3)/e^6 + (15*d^4*(d + e*x^(1/3))^2*(a + b*Log[c*(d
+ e*x^(1/3))^n])^3)/(2*e^6) - (10*d^3*(d + e*x^(1/3))^3*(a + b*Log[c*(d + e
*x^(1/3))^n])^3)/e^6 + (15*d^2*(d + e*x^(1/3))^4*(a + b*Log[c*(d + e*x^(1/3
))^n])^3)/(2*e^6) - (3*d*(d + e*x^(1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n])
^3)/e^6 + ((d + e*x^(1/3))^6*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/(2*e^6)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
```

```
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx &= 3\text{Subst}\left(\int x^5(a+b\log(c(d+ex)^n))^3\,dx,x,\sqrt[3]{x}\right)\\
&= 3\text{Subst}\left(\int\left(-\frac{d^5(a+b\log(c(d+ex)^n))^3}{e^5}+\frac{5d^4(d+ex)\,(a+b\log(c\right.\right.}{e^5}\\
&= \frac{3\text{Subst}\left(\int(d+ex)^5\,(a+b\log(c(d+ex)^n))^3\,dx,x,\sqrt[3]{x}\right)}{e^5}-\frac{(15d)\text{Subs}}{}\\
&= \frac{3\text{Subst}\left(\int x^5(a+b\log(cx^n))^3\,dx,x,d+e\sqrt[3]{x}\right)}{e^6}-\frac{(15d)\text{Subst}\left(\int x^4(a\right.}{}\\
&= -\frac{3d^5\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^6}+\frac{15d^4\left(d+e\sqrt[3]{x}\right)^2(a}{}\\
&= \frac{9bd^5n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^6}-\frac{45bd^4n\left(d+e\sqrt[3]{x}\right)^2}{}\\
&= -\frac{45b^3d^4n^3\left(d+e\sqrt[3]{x}\right)^2}{8e^6}+\frac{20b^3d^3n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^6}-\frac{45b^3d^2n^3\left(d+e\sqrt[3]{x}\right.}{64e^6}\\
&= -\frac{45b^3d^4n^3\left(d+e\sqrt[3]{x}\right)^2}{8e^6}+\frac{20b^3d^3n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^6}-\frac{45b^3d^2n^3\left(d+e\sqrt[3]{x}\right.}{64e^6}
\end{aligned}$$

Mathematica [A]
time = 0.50, size = 673, normalized size = 0.74

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

```
[Out] (-36000*b^3*d^6*n^3*Log[d + e*x^(1/3)]^3 + 5400*b^2*d^6*n^2*Log[d + e*x^(1/
3)]^2*(20*a - 49*b*n + 20*b*Log[c*(d + e*x^(1/3))^n]) - 60*b*d^6*n*Log[d +
e*x^(1/3)]*(1800*a^2 - 8820*a*b*n + 13489*b^2*n^2 + 180*b*(20*a - 49*b*n)*L
og[c*(d + e*x^(1/3))^n] + 1800*b^2*Log[c*(d + e*x^(1/3))^n]^2) + e*x^(1/3)*
(36000*a^3*e^5*x^(5/3) + b^3*n^3*(809340*d^5 - 140070*d^4*e*x^(1/3) + 41180
*d^3*e^2*x^(2/3) - 13785*d^2*e^3*x + 4368*d*e^4*x^(4/3) - 1000*e^5*x^(5/3))
 - 60*a*b^2*n^2*(8820*d^5 - 2610*d^4*e*x^(1/3) + 1140*d^3*e^2*x^(2/3) - 555
*d^2*e^3*x + 264*d*e^4*x^(4/3) - 100*e^5*x^(5/3)) + 1800*a^2*b*n*(60*d^5 -
30*d^4*e*x^(1/3) + 20*d^3*e^2*x^(2/3) - 15*d^2*e^3*x + 12*d*e^4*x^(4/3) - 1
0*e^5*x^(5/3)) + 60*b*(1800*a^2*e^5*x^(5/3) + 60*a*b*n*(60*d^5 - 30*d^4*e*x
^(1/3) + 20*d^3*e^2*x^(2/3) - 15*d^2*e^3*x + 12*d*e^4*x^(4/3) - 10*e^5*x^(5
/3)) + b^2*n^2*(-8820*d^5 + 2610*d^4*e*x^(1/3) - 1140*d^3*e^2*x^(2/3) + 555
*d^2*e^3*x - 264*d*e^4*x^(4/3) + 100*e^5*x^(5/3)))*Log[c*(d + e*x^(1/3))^n]
 + 1800*b^2*(60*a*e^5*x^(5/3) + b*n*(60*d^5 - 30*d^4*e*x^(1/3) + 20*d^3*e^2
*x^(2/3) - 15*d^2*e^3*x + 12*d*e^4*x^(4/3) - 10*e^5*x^(5/3)))*Log[c*(d + e*
x^(1/3))^n]^2 + 36000*b^3*e^5*x^(5/3)*Log[c*(d + e*x^(1/3))^n]^3))/(72000*e
^6)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int x\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^{3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e*x^(1/3))^n))^3,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e*x^(1/3))^n))^3,x)
```

Maxima [A]
time = 0.30, size = 657, normalized size = 0.72

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*x^2*log((x^(1/3)*e + d)^n*c)^3 + 3/2*a*b^2*x^2*log((x^(1/3)*e + d)^
n*c)^2 - 1/40*(60*d^6*e^(-7)*log(x^(1/3)*e + d) + (30*d^4*x^(2/3)*e - 60*d^
5*x^(1/3) - 20*d^3*x*e^2 + 15*d^2*x^(4/3)*e^3 - 12*d*x^(5/3)*e^4 + 10*x^2*e
^5)*e^(-6))*a^2*b*n*e + 3/2*a^2*b*x^2*log((x^(1/3)*e + d)^n*c) + 1/2*a^3*x^
2 + 1/1200*((1800*d^6*log(x^(1/3)*e + d)^2 + 8820*d^6*log(x^(1/3)*e + d) -
8820*d^5*x^(1/3)*e + 2610*d^4*x^(2/3)*e^2 - 1140*d^3*x*e^3 + 555*d^2*x^(4/3
```

```
)*e^4 - 264*d*x^(5/3)*e^5 + 100*x^2*e^6)*n^2*e^(-6) - 60*(60*d^6*e^(-7)*log
(x^(1/3)*e + d) + (30*d^4*x^(2/3)*e - 60*d^5*x^(1/3) - 20*d^3*x*e^2 + 15*d^
2*x^(4/3)*e^3 - 12*d*x^(5/3)*e^4 + 10*x^2*e^5)*e^(-6))*n*e*log((x^(1/3)*e +
 d)^n*c))*a*b^2 - 1/72000*(1800*(60*d^6*e^(-7)*log(x^(1/3)*e + d) + (30*d^4
*x^(2/3)*e - 60*d^5*x^(1/3) - 20*d^3*x*e^2 + 15*d^2*x^(4/3)*e^3 - 12*d*x^(5
/3)*e^4 + 10*x^2*e^5)*e^(-6))*n*e*log((x^(1/3)*e + d)^n*c)^2 + ((36000*d^6*
log(x^(1/3)*e + d)^3 + 264600*d^6*log(x^(1/3)*e + d)^2 + 809340*d^6*log(x^(
1/3)*e + d) - 809340*d^5*x^(1/3)*e + 140070*d^4*x^(2/3)*e^2 - 41180*d^3*x*e
^3 + 13785*d^2*x^(4/3)*e^4 - 4368*d*x^(5/3)*e^5 + 1000*x^2*e^6)*n^2*e^(-7)
- 60*(1800*d^6*log(x^(1/3)*e + d)^2 + 8820*d^6*log(x^(1/3)*e + d) - 8820*d^
5*x^(1/3)*e + 2610*d^4*x^(2/3)*e^2 - 1140*d^3*x*e^3 + 555*d^2*x^(4/3)*e^4 -
 264*d*x^(5/3)*e^5 + 100*x^2*e^6)*n*e^(-7)*log((x^(1/3)*e + d)^n*c))*n*e)*b
^3
```

Fricas [A]
time = 0.41, size = 1100, normalized size = 1.21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="fricas")
```

```
[Out] 1/72000*(36000*b^3*x^2*e^6*log(c)^3 - 1000*(b^3*n^3 - 6*a*b^2*n^2 + 18*a^2*
b*n - 36*a^3)*x^2*e^6 - 36000*(b^3*d^6*n^3 - b^3*n^3*x^2*e^6)*log(x^(1/3)*e
 + d)^3 + 20*(2059*b^3*d^3*n^3 - 3420*a*b^2*d^3*n^2 + 1800*a^2*b*d^3*n)*x*e
^3 + 1800*(147*b^3*d^6*n^3 - 60*a*b^2*d^6*n^2 + 20*b^3*d^3*n^3*x*e^3 - 10*(
b^3*n^3 - 6*a*b^2*n^2)*x^2*e^6 - 60*(b^3*d^6*n^2 - b^3*n^2*x^2*e^6)*log(c)
- 6*(5*b^3*d^4*n^3*e^2 - 2*b^3*d*n^3*x*e^5)*x^(2/3) + 15*(4*b^3*d^5*n^3*e -
 b^3*d^2*n^3*x*e^4)*x^(1/3))*log(x^(1/3)*e + d)^2 + 18000*(2*b^3*d^3*n*x*e^
3 - (b^3*n - 6*a*b^2)*x^2*e^6)*log(c)^2 - 60*(13489*b^3*d^6*n^3 - 8820*a*b^
2*d^6*n^2 + 1800*a^2*b*d^6*n - 100*(b^3*n^3 - 6*a*b^2*n^2 + 18*a^2*b*n)*x^2
*e^6 + 60*(19*b^3*d^3*n^3 - 20*a*b^2*d^3*n^2)*x*e^3 + 1800*(b^3*d^6*n - b^3
*n*x^2*e^6)*log(c)^2 - 60*(147*b^3*d^6*n^2 - 60*a*b^2*d^6*n + 20*b^3*d^3*n^
2*x*e^3 - 10*(b^3*n^2 - 6*a*b^2*n)*x^2*e^6)*log(c) + 6*(4*(11*b^3*d*n^3 - 3
0*a*b^2*d*n^2)*x*e^5 - 15*(29*b^3*d^4*n^3 - 20*a*b^2*d^4*n^2)*e^2 + 60*(5*b
^3*d^4*n^2*e^2 - 2*b^3*d*n^2*x*e^5)*log(c))*x^(2/3) - 15*((37*b^3*d^2*n^3 -
 60*a*b^2*d^2*n^2)*x*e^4 - 12*(49*b^3*d^5*n^3 - 20*a*b^2*d^5*n^2)*e + 60*(4
*b^3*d^5*n^2*e - b^3*d^2*n^2*x*e^4)*log(c))*x^(1/3))*log(x^(1/3)*e + d) + 1
200*(5*(b^3*n^2 - 6*a*b^2*n + 18*a^2*b)*x^2*e^6 - 3*(19*b^3*d^3*n^2 - 20*a*
b^2*d^3*n)*x*e^3)*log(c) + 6*(8*(91*b^3*d*n^3 - 330*a*b^2*d*n^2 + 450*a^2*b
*d*n)*x*e^5 - 1800*(5*b^3*d^4*n*e^2 - 2*b^3*d*n*x*e^5)*log(c)^2 - 5*(4669*b
^3*d^4*n^3 - 5220*a*b^2*d^4*n^2 + 1800*a^2*b*d^4*n)*e^2 - 60*(4*(11*b^3*d*n
^2 - 30*a*b^2*d*n)*x*e^5 - 15*(29*b^3*d^4*n^2 - 20*a*b^2*d^4*n)*e^2)*log(c)
)*x^(2/3) - 15*((919*b^3*d^2*n^3 - 2220*a*b^2*d^2*n^2 + 1800*a^2*b*d^2*n)*x
*e^4 - 1800*(4*b^3*d^5*n*e - b^3*d^2*n*x*e^4)*log(c)^2 - 4*(13489*b^3*d^5*n
```

```
^3 - 8820*a*b^2*d^5*n^2 + 1800*a^2*b*d^5*n)*e - 60*((37*b^3*d^2*n^2 - 60*a*
b^2*d^2*n)*x*e^4 - 12*(49*b^3*d^5*n^2 - 20*a*b^2*d^5*n)*e)*log(c))*x^(1/3))
*e^(-6)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/3))**n))**3,x)
```

```
[Out] Integral(x*(a + b*log(c*(d + e*x**(1/3))**n))**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2223 vs. 2(803) = 1606.
time = 3.03, size = 2223, normalized size = 2.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="giac")
```

```
[Out] 1/72000*(36000*b^3*x^2*e*log(c)^3 + 108000*a*b^2*x^2*e*log(c)^2 + (36000*(x
^(1/3)*e + d)^6*e^(-5)*log(x^(1/3)*e + d)^3 - 216000*(x^(1/3)*e + d)^5*d*e^
(-5)*log(x^(1/3)*e + d)^3 + 540000*(x^(1/3)*e + d)^4*d^2*e^(-5)*log(x^(1/3)
*e + d)^3 - 720000*(x^(1/3)*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e + d)^3 + 5400
00*(x^(1/3)*e + d)^2*d^4*e^(-5)*log(x^(1/3)*e + d)^3 - 216000*(x^(1/3)*e +
d)*d^5*e^(-5)*log(x^(1/3)*e + d)^3 - 18000*(x^(1/3)*e + d)^6*e^(-5)*log(x^(
1/3)*e + d)^2 + 129600*(x^(1/3)*e + d)^5*d*e^(-5)*log(x^(1/3)*e + d)^2 - 40
5000*(x^(1/3)*e + d)^4*d^2*e^(-5)*log(x^(1/3)*e + d)^2 + 720000*(x^(1/3)*e
+ d)^3*d^3*e^(-5)*log(x^(1/3)*e + d)^2 - 810000*(x^(1/3)*e + d)^2*d^4*e^(-5
)*log(x^(1/3)*e + d)^2 + 648000*(x^(1/3)*e + d)*d^5*e^(-5)*log(x^(1/3)*e +
d)^2 + 6000*(x^(1/3)*e + d)^6*e^(-5)*log(x^(1/3)*e + d) - 51840*(x^(1/3)*e
+ d)^5*d*e^(-5)*log(x^(1/3)*e + d) + 202500*(x^(1/3)*e + d)^4*d^2*e^(-5)*lo
g(x^(1/3)*e + d) - 480000*(x^(1/3)*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e + d) +
 810000*(x^(1/3)*e + d)^2*d^4*e^(-5)*log(x^(1/3)*e + d) - 1296000*(x^(1/3)*
e + d)*d^5*e^(-5)*log(x^(1/3)*e + d) - 1000*(x^(1/3)*e + d)^6*e^(-5) + 1036
8*(x^(1/3)*e + d)^5*d*e^(-5) - 50625*(x^(1/3)*e + d)^4*d^2*e^(-5) + 160000*
(x^(1/3)*e + d)^3*d^3*e^(-5) - 405000*(x^(1/3)*e + d)^2*d^4*e^(-5) + 129600
0*(x^(1/3)*e + d)*d^5*e^(-5))*b^3*n^3 + 60*(1800*(x^(1/3)*e + d)^6*e^(-5)*l
og(x^(1/3)*e + d)^2 - 10800*(x^(1/3)*e + d)^5*d*e^(-5)*log(x^(1/3)*e + d)^2
 + 27000*(x^(1/3)*e + d)^4*d^2*e^(-5)*log(x^(1/3)*e + d)^2 - 36000*(x^(1/3)
*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e + d)^2 + 27000*(x^(1/3)*e + d)^2*d^4*e^(
-5)*log(x^(1/3)*e + d)^2 - 10800*(x^(1/3)*e + d)*d^5*e^(-5)*log(x^(1/3)*e +
```

```
d)^2 - 600*(x^(1/3)*e + d)^6*e^(-5)*log(x^(1/3)*e + d) + 4320*(x^(1/3)*e +
d)^5*d*e^(-5)*log(x^(1/3)*e + d) - 13500*(x^(1/3)*e + d)^4*d^2*e^(-5)*log(
x^(1/3)*e + d) + 24000*(x^(1/3)*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e + d) - 27
000*(x^(1/3)*e + d)^2*d^4*e^(-5)*log(x^(1/3)*e + d) + 21600*(x^(1/3)*e + d)
*d^5*e^(-5)*log(x^(1/3)*e + d) + 100*(x^(1/3)*e + d)^6*e^(-5) - 864*(x^(1/3
)*e + d)^5*d*e^(-5) + 3375*(x^(1/3)*e + d)^4*d^2*e^(-5) - 8000*(x^(1/3)*e +
d)^3*d^3*e^(-5) + 13500*(x^(1/3)*e + d)^2*d^4*e^(-5) - 21600*(x^(1/3)*e +
d)*d^5*e^(-5))*b^3*n^2*log(c) + 108000*a^2*b*x^2*e*log(c) + 1800*(60*(x^(1/
3)*e + d)^6*e^(-5)*log(x^(1/3)*e + d) - 360*(x^(1/3)*e + d)^5*d*e^(-5)*log(
x^(1/3)*e + d) + 900*(x^(1/3)*e + d)^4*d^2*e^(-5)*log(x^(1/3)*e + d) - 1200
*(x^(1/3)*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e + d) + 900*(x^(1/3)*e + d)^2*d^
4*e^(-5)*log(x^(1/3)*e + d) - 360*(x^(1/3)*e + d)*d^5*e^(-5)*log(x^(1/3)*e
+ d) - 10*(x^(1/3)*e + d)^6*e^(-5) + 72*(x^(1/3)*e + d)^5*d*e^(-5) - 225*(x
^(1/3)*e + d)^4*d^2*e^(-5) + 400*(x^(1/3)*e + d)^3*d^3*e^(-5) - 450*(x^(1/3
)*e + d)^2*d^4*e^(-5) + 360*(x^(1/3)*e + d)*d^5*e^(-5))*b^3*n*log(c)^2 + 60
*(1800*(x^(1/3)*e + d)^6*e^(-5)*log(x^(1/3)*e + d)^2 - 10800*(x^(1/3)*e + d
)^5*d*e^(-5)*log(x^(1/3)*e + d)^2 + 27000*(x^(1/3)*e + d)^4*d^2*e^(-5)*log(
x^(1/3)*e + d)^2 - 36000*(x^(1/3)*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e + d)^2
+ 27000*(x^(1/3)*e + d)^2*d^4*e^(-5)*log(x^(1/3)*e + d)^2 - 10800*(x^(1/3)*
e + d)*d^5*e^(-5)*log(x^(1/3)*e + d)^2 - 600*(x^(1/3)*e + d)^6*e^(-5)*log(x
^(1/3)*e + d) + 4320*(x^(1/3)*e + d)^5*d*e^(-5)*log(x^(1/3)*e + d) - 13500*
(x^(1/3)*e + d)^4*d^2*e^(-5)*log(x^(1/3)*e + d) + 24000*(x^(1/3)*e + d)^3*d
^3*e^(-5)*log(x^(1/3)*e + d) - 27000*(x^(1/3)*e + d)^2*d^4*e^(-5)*log(x^(1/
3)*e + d) + 21600*(x^(1/3)*e + d)*d^5*e^(-5)*log(x^(1/3)*e + d) + 100*(x^(1
/3)*e + d)^6*e^(-5) - 864*(x^(1/3)*e + d)^5*d*e^(-5) + 3375*(x^(1/3)*e + d)
^4*d^2*e^(-5) - 8000*(x^(1/3)*e + d)^3*d^3*e^(-5) + 13500*(x^(1/3)*e + d)^2
*d^4*e^(-5) - 21600*(x^(1/3)*e + d)*d^5*e^(-5))*a*b^2*n^2 + 36000*a^3*x^2*e
+ 3600*(60*(x^(1/3)*e + d)^6*e^(-5)*log(x^(1/3)*e + d) - 360*(x^(1/3)*e +
d)^5*d*e^(-5)*log(x^(1/3)*e + d) + 900*(x^(1/3)*e + d)^4*d^2*e^(-5)*log(x^(
1/3)*e + d) - 1200*(x^(1/3)*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e + d) + 900*(x
^(1/3)*e + d)^2*d^4*e^(-5)*log(x^(1/3)*e + d) - 360*(x^(1/3)*e + d)*d^5*e^(
-5)*log(x^(1/3)*e + d) - 10*(x^(1/3)*e + d)^6*e^(-5) + 72*(x^(1/3)*e + d)^5
*d*e^(-5) - 225*(x^(1/3)*e + d)^4*d^2*e^(-5) + 400*(x^(1/3)*e + d)^3*d^3*e^
(-5) - 450*(x^(1/3)*e + d)^2*d^4*e^(-5) + 360*(x^(1/3)*e + d)*d^5*e^(-5))*a
*b^2*n*log(c) + 1800*(60*(x^(1/3)*e + d)^6*e^(-5)*log(x^(1/3)*e + d) - 360*
(x^(1/3)*e + d)^5*d*e^(-5)*log(x^(1/3)*e + d) + 900*(x^(1/3)*e + d)^4*d^2*e
^(-5)*log(x^(1/3)*e + d) - 1200*(x^(1/3)*e + d)^3*d^3*e^(-5)*log(x^(1/3)*e
+ d) + 900*(x^(1/3)*e + d)^2*d^4*e^(-5)*log(x^(1/3)*e + d) - 360*(x^(1/3)*e
+ d)*d^5*e^(-5)*log(x^(1/3)*e + d) - 10*(x^(1/3)*e + d)^6*e^(-5) + 72*(x^(
1/3)*e + d)^5*d*e^(-5) - 225*(x^(1/3)*e + d)^4*d^2*e^(-5) + 400*(x^(1/3)*e
+ d)^3*d^3*e^(-5) - 450*(x^(1/3)*e + d)^2*d^4*e^(-5) + 360*(x^(1/3)*e + d)*
d^5*e^(-5))*a^2*b*n)*e^(-1)
```

Mupad [B]

time = 8.06, size = 979, normalized size = 1.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e*x^(1/3))^n))^3,x)
```

```
[Out] (a^3*x^2)/2 + (b^3*x^2*log(c*(d + e*x^(1/3))^n)^3)/2 - (b^3*n^3*x^2)/72 + (
3*a*b^2*x^2*log(c*(d + e*x^(1/3))^n)^2)/2 - (b^3*n*x^2*log(c*(d + e*x^(1/3)
)^n)^2)/4 + (b^3*n^2*x^2*log(c*(d + e*x^(1/3))^n))/12 + (a*b^2*n^2*x^2)/12
- (b^3*d^6*log(c*(d + e*x^(1/3))^n)^3)/(2*e^6) + (3*a^2*b*x^2*log(c*(d + e*
x^(1/3))^n))/2 - (a^2*b*n*x^2)/4 - (a*b^2*n*x^2*log(c*(d + e*x^(1/3))^n))/2
 - (13489*b^3*d^6*n^3*log(d + e*x^(1/3)))/(1200*e^6) - (919*b^3*d^2*n^3*x^(
4/3))/(4800*e^2) - (4669*b^3*d^4*n^3*x^(2/3))/(2400*e^4) + (13489*b^3*d^5*n
^3*x^(1/3))/(1200*e^5) - (3*a*b^2*d^6*log(c*(d + e*x^(1/3))^n)^2)/(2*e^6) +
 (147*b^3*d^6*n*log(c*(d + e*x^(1/3))^n)^2)/(40*e^6) + (2059*b^3*d^3*n^3*x)
/(3600*e^3) + (91*b^3*d*n^3*x^(5/3))/(1500*e) - (3*a^2*b*d^6*n*log(d + e*x^
(1/3)))/(2*e^6) + (b^3*d^3*n*x*log(c*(d + e*x^(1/3))^n)^2)/(2*e^3) - (19*b^
3*d^3*n^2*x*log(c*(d + e*x^(1/3))^n))/(20*e^3) + (3*b^3*d*n*x^(5/3)*log(c*(
d + e*x^(1/3))^n)^2)/(10*e) - (11*b^3*d*n^2*x^(5/3)*log(c*(d + e*x^(1/3))^n
))/(50*e) - (19*a*b^2*d^3*n^2*x)/(20*e^3) - (11*a*b^2*d*n^2*x^(5/3))/(50*e)
 - (3*a^2*b*d^2*n*x^(4/3))/(8*e^2) - (3*a^2*b*d^4*n*x^(2/3))/(4*e^4) + (3*a
^2*b*d^5*n*x^(1/3))/(2*e^5) + (147*a*b^2*d^6*n^2*log(d + e*x^(1/3)))/(20*e^
6) - (3*b^3*d^2*n*x^(4/3)*log(c*(d + e*x^(1/3))^n)^2)/(8*e^2) + (37*b^3*d^2
*n^2*x^(4/3)*log(c*(d + e*x^(1/3))^n))/(80*e^2) - (3*b^3*d^4*n*x^(2/3)*log(
c*(d + e*x^(1/3))^n)^2)/(4*e^4) + (87*b^3*d^4*n^2*x^(2/3)*log(c*(d + e*x^(1
/3))^n))/(40*e^4) + (3*b^3*d^5*n*x^(1/3)*log(c*(d + e*x^(1/3))^n)^2)/(2*e^5
) - (147*b^3*d^5*n^2*x^(1/3)*log(c*(d + e*x^(1/3))^n))/(20*e^5) + (37*a*b^2
*d^2*n^2*x^(4/3))/(80*e^2) + (87*a*b^2*d^4*n^2*x^(2/3))/(40*e^4) - (147*a*b
^2*d^5*n^2*x^(1/3))/(20*e^5) + (a^2*b*d^3*n*x)/(2*e^3) + (3*a^2*b*d*n*x^(5/
3))/(10*e) + (a*b^2*d^3*n*x*log(c*(d + e*x^(1/3))^n))/e^3 + (3*a*b^2*d*n*x^
(5/3)*log(c*(d + e*x^(1/3))^n))/(5*e) - (3*a*b^2*d^2*n*x^(4/3)*log(c*(d + e
*x^(1/3))^n))/(4*e^2) - (3*a*b^2*d^4*n*x^(2/3)*log(c*(d + e*x^(1/3))^n))/(2
*e^4) + (3*a*b^2*d^5*n*x^(1/3)*log(c*(d + e*x^(1/3))^n))/e^5
```

3.459 $\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=438

$$\frac{9b^3dn^3\left(d+e\sqrt[3]{x}\right)^2}{4e^3}-\frac{2b^3n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^3}+\frac{18ab^2d^2n^2\sqrt[3]{x}}{e^2}-\frac{18b^3d^2n^3\sqrt[3]{x}}{e^2}+\frac{18b^3d^2n^2\left(d+e\sqrt[3]{x}\right)\log\left(c\left(d+e\sqrt[3]{x}\right.\right.}{e^3}$$

```
[Out] 9/4*b^3*d*n^3*(d+e*x^(1/3))^2/e^3-2/9*b^3*n^3*(d+e*x^(1/3))^3/e^3+18*a*b^2*
d^2*n^2*x^(1/3)/e^2-18*b^3*d^2*n^3*x^(1/3)/e^2+18*b^3*d^2*n^2*(d+e*x^(1/3))
*ln(c*(d+e*x^(1/3))^n)/e^3-9/2*b^2*d*n^2*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(
1/3))^n))/e^3+2/3*b^2*n^2*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))/e^3-9
*b*d^2*n*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^3+9/2*b*d*n*(d+e*x^(
1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^3-b*n*(d+e*x^(1/3))^3*(a+b*ln(c*(d+
e*x^(1/3))^n))^2/e^3+3*d^2*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^3-
3*d*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^3+(d+e*x^(1/3))^3*(a+b*
ln(c*(d+e*x^(1/3))^n))^3/e^3
```

Rubi [A]
time = 0.29, antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2501, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

$$\frac{2b^2n^2\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^3}-\frac{9b^2dn^2\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{2e^3}+\frac{18ab^2d^2n^2\sqrt[3]{x}}{e^2}-\frac{9bd^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^3}+\frac{3d^2\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^3}-\frac{bn\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^3}+\frac{9bdn\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2e^3}+\frac{\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^3}-\frac{3d\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^3}+\frac{18b^3d^2n^2\left(d+e\sqrt[3]{x}\right)\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{e^3}-\frac{18b^3d^2n^3\sqrt[3]{x}}{e^2}-\frac{2b^3n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^3}+\frac{9b^3dn^3\left(d+e\sqrt[3]{x}\right)^2}{4e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

```
[Out] (9*b^3*d*n^3*(d + e*x^(1/3))^2)/(4*e^3) - (2*b^3*n^3*(d + e*x^(1/3))^3)/(9*
e^3) + (18*a*b^2*d^2*n^2*x^(1/3))/e^2 - (18*b^3*d^2*n^3*x^(1/3))/e^2 + (18*
b^3*d^2*n^2*(d + e*x^(1/3))*Log[c*(d + e*x^(1/3))^n])/e^3 - (9*b^2*d*n^2*(d
 + e*x^(1/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/(2*e^3) + (2*b^2*n^2*(d +
 e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n]))/(3*e^3) - (9*b*d^2*n*(d + e
*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^3 + (9*b*d*n*(d + e*x^(1/3)
)^2*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(2*e^3) - (b*n*(d + e*x^(1/3))^3*(a
 + b*Log[c*(d + e*x^(1/3))^n])^2)/e^3 + (3*d^2*(d + e*x^(1/3))*(a + b*Log[c
*(d + e*x^(1/3))^n])^3)/e^3 - (3*d*(d + e*x^(1/3))^2*(a + b*Log[c*(d + e*x^
(1/3))^n])^3)/e^3 + ((d + e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/
e^3
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2501

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rubi steps

$$\int \left(a+b\log \left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3 dx = 3\text{Subst}\left(\int x^2(a+b\log (c(d+ex)^n))^3\, dx, x, \sqrt[3]{x}\right)$$

$$= 3\text{Subst}\left(\int \left(\frac{d^2(a+b\log (c(d+ex)^n))^3}{e^2} - \frac{2d(d+ex)\,(a+b\log (c(d+\right.\right.$$

$$= \frac{3\text{Subst}\left(\int (d+ex)^2\,(a+b\log (c(d+ex)^n))^3\, dx, x, \sqrt[3]{x}\right)}{e^2} - \frac{(6d)\text{Subst}\left(\right.}{}$$

$$= \frac{3\text{Subst}\left(\int x^2(a+b\log (cx^n))^3\, dx, x, d+e\sqrt[3]{x}\right)}{e^3} - \frac{(6d)\text{Subst}\left(\int x(a+b\right.}{}$$

$$= \frac{3d^2\left(d+e\sqrt[3]{x}\right)\left(a+b\log \left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^3} - \frac{3d\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\right.}{e^3}$$

$$= -\frac{9bd^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log \left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^3} + \frac{9bdn\left(d+e\sqrt[3]{x}\right)^2\left(\right.}{}$$

$$= \frac{9b^3dn^3\left(d+e\sqrt[3]{x}\right)^2}{4e^3} - \frac{2b^3n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^3} + \frac{18ab^2d^2n^2\sqrt[3]{x}}{e^2} - \frac{9b^2dn^2\left(\right.}{}$$

$$= \frac{9b^3dn^3\left(d+e\sqrt[3]{x}\right)^2}{4e^3} - \frac{2b^3n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^3} + \frac{18ab^2d^2n^2\sqrt[3]{x}}{e^2} - \frac{18b^3d^2n^3}{e^2}$$

Mathematica [A]
time = 0.26, size = 463, normalized size = 1.06

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

```
[Out] (36*b^3*d^3*n^3*Log[d + e*x^(1/3)]^3 + 18*b^2*d^3*n^2*Log[d + e*x^(1/3)]^2*
(-6*a + 11*b*n - 6*b*Log[c*(d + e*x^(1/3))^n]) + 6*b*d^3*n*Log[d + e*x^(1/3
)]*(18*a^2 - 66*a*b*n + 85*b^2*n^2 + 6*b*(6*a - 11*b*n)*Log[c*(d + e*x^(1/3
))^n] + 18*b^2*Log[c*(d + e*x^(1/3))^n]^2) + e*x^(1/3)*(b^3*n^3*(-510*d^2 +
 57*d*e*x^(1/3) - 8*e^2*x^(2/3)) - 18*a^2*b*n*(6*d^2 - 3*d*e*x^(1/3) + 2*e^
2*x^(2/3)) + 6*a*b^2*n^2*(66*d^2 - 15*d*e*x^(1/3) + 4*e^2*x^(2/3)) + 36*a^3
*e^2*x^(2/3) + 6*b*(-6*a*b*n*(6*d^2 - 3*d*e*x^(1/3) + 2*e^2*x^(2/3)) + b^2*
n^2*(66*d^2 - 15*d*e*x^(1/3) + 4*e^2*x^(2/3)) + 18*a^2*e^2*x^(2/3))*Log[c*(
d + e*x^(1/3))^n] - 18*b^2*(b*n*(6*d^2 - 3*d*e*x^(1/3) + 2*e^2*x^(2/3)) - 6
*a*e^2*x^(2/3))*Log[c*(d + e*x^(1/3))^n]^2 + 36*b^3*e^2*x^(2/3)*Log[c*(d +
e*x^(1/3))^n]^3))/(36*e^3)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(a+b\ln \left(c\left(d+e\,x^{\frac{1}{3}}\right)^n\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))^3,x)
```

Maxima [A]
time = 0.31, size = 459, normalized size = 1.05

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((6*d^3*e^(-4)*log(x^(1/3)*e + d) + (3*d*x^(2/3)*e - 6*d^2*x^(1/3) - 2*
x*e^2)*e^(-3))*n*e + 6*x*log((x^(1/3)*e + d)^n*c))*a^2*b - 1/6*((18*d^3*log
(x^(1/3)*e + d)^2 + 66*d^3*log(x^(1/3)*e + d) - 66*d^2*x^(1/3)*e + 15*d*x^(
2/3)*e^2 - 4*x*e^3)*n^2*e^(-3) - 6*(6*d^3*e^(-4)*log(x^(1/3)*e + d) + (3*d*
x^(2/3)*e - 6*d^2*x^(1/3) - 2*x*e^2)*e^(-3))*n*e*log((x^(1/3)*e + d)^n*c) -
 18*x*log((x^(1/3)*e + d)^n*c)^2)*a*b^2 + 1/36*(18*(6*d^3*e^(-4)*log(x^(1/3
)*e + d) + (3*d*x^(2/3)*e - 6*d^2*x^(1/3) - 2*x*e^2)*e^(-3))*n*e*log((x^(1/
3)*e + d)^n*c)^2 + 36*x*log((x^(1/3)*e + d)^n*c)^3 + ((36*d^3*log(x^(1/3)*e
 + d)^3 + 198*d^3*log(x^(1/3)*e + d)^2 + 510*d^3*log(x^(1/3)*e + d) - 510*d
^2*x^(1/3)*e + 57*d*x^(2/3)*e^2 - 8*x*e^3)*n^2*e^(-4) - 6*(18*d^3*log(x^(1/
3)*e + d)^2 + 66*d^3*log(x^(1/3)*e + d) - 66*d^2*x^(1/3)*e + 15*d*x^(2/3)*e
^2 - 4*x*e^3)*n*e^(-4)*log((x^(1/3)*e + d)^n*c))*n*e)*b^3 + a^3*x
```

Fricas [A]
time = 0.40, size = 646, normalized size = 1.47

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="fricas")
```

```
[Out] 1/36*(36*b^3*x*e^3*log(c)^3 - 36*(b^3*n - 3*a*b^2)*x*e^3*log(c)^2 + 36*(b^3
*d^3*n^3 + b^3*n^3*x*e^3)*log(x^(1/3)*e + d)^3 + 12*(2*b^3*n^2 - 6*a*b^2*n
+ 9*a^2*b)*x*e^3*log(c) - 4*(2*b^3*n^3 - 6*a*b^2*n^2 + 9*a^2*b*n - 9*a^3)*x
*e^3 - 18*(6*b^3*d^2*n^3*x^(1/3)*e + 11*b^3*d^3*n^3 - 3*b^3*d*n^3*x^(2/3)*e
^2 - 6*a*b^2*d^3*n^2 + 2*(b^3*n^3 - 3*a*b^2*n^2)*x*e^3 - 6*(b^3*d^3*n^2 + b
^3*n^2*x*e^3)*log(c))*log(x^(1/3)*e + d)^2 + 6*(85*b^3*d^3*n^3 - 66*a*b^2*d
^3*n^2 + 18*a^2*b*d^3*n + 2*(2*b^3*n^3 - 6*a*b^2*n^2 + 9*a^2*b*n)*x*e^3 + 1
8*(b^3*d^3*n + b^3*n*x*e^3)*log(c)^2 - 6*(11*b^3*d^3*n^2 - 6*a*b^2*d^3*n +
2*(b^3*n^2 - 3*a*b^2*n)*x*e^3)*log(c) + 3*(6*b^3*d*n^2*e^2*log(c) - (5*b^3*
d*n^3 - 6*a*b^2*d*n^2)*e^2)*x^(2/3) - 6*(6*b^3*d^2*n^2*e*log(c) - (11*b^3*d
^2*n^3 - 6*a*b^2*d^2*n^2)*e)*x^(1/3))*log(x^(1/3)*e + d) + 3*(18*b^3*d*n*e^
```

```
2*log(c)^2 - 6*(5*b^3*d*n^2 - 6*a*b^2*d*n)*e^2*log(c) + (19*b^3*d*n^3 - 30*
a*b^2*d*n^2 + 18*a^2*b*d*n)*e^2)*x^(2/3) - 6*(18*b^3*d^2*n*e*log(c)^2 - 6*(
11*b^3*d^2*n^2 - 6*a*b^2*d^2*n)*e*log(c) + (85*b^3*d^2*n^3 - 66*a*b^2*d^2*n
^2 + 18*a^2*b*d^2*n)*e)*x^(1/3))*e^(-3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \log \left(c \left(d + e \sqrt[3]{x}\right)^{n}\right)\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))**3,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**(1/3))**n))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1105 vs. 2(391) = 782.
time = 3.35, size = 1105, normalized size = 2.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="giac")
```

```
[Out] 1/36*(36*b^3*x*e*log(c)^3 + (36*(x^(1/3)*e + d)^3*e^(-2)*log(x^(1/3)*e + d)
^3 - 108*(x^(1/3)*e + d)^2*d*e^(-2)*log(x^(1/3)*e + d)^3 + 108*(x^(1/3)*e +
 d)*d^2*e^(-2)*log(x^(1/3)*e + d)^3 - 36*(x^(1/3)*e + d)^3*e^(-2)*log(x^(1/
3)*e + d)^2 + 162*(x^(1/3)*e + d)^2*d*e^(-2)*log(x^(1/3)*e + d)^2 - 324*(x^
(1/3)*e + d)*d^2*e^(-2)*log(x^(1/3)*e + d)^2 + 24*(x^(1/3)*e + d)^3*e^(-2)*
log(x^(1/3)*e + d) - 162*(x^(1/3)*e + d)^2*d*e^(-2)*log(x^(1/3)*e + d) + 64
8*(x^(1/3)*e + d)*d^2*e^(-2)*log(x^(1/3)*e + d) - 8*(x^(1/3)*e + d)^3*e^(-2
) + 81*(x^(1/3)*e + d)^2*d*e^(-2) - 648*(x^(1/3)*e + d)*d^2*e^(-2))*b^3*n^3
 + 6*(18*(x^(1/3)*e + d)^3*e^(-2)*log(x^(1/3)*e + d)^2 - 54*(x^(1/3)*e + d)
^2*d*e^(-2)*log(x^(1/3)*e + d)^2 + 54*(x^(1/3)*e + d)*d^2*e^(-2)*log(x^(1/3
)*e + d)^2 - 12*(x^(1/3)*e + d)^3*e^(-2)*log(x^(1/3)*e + d) + 54*(x^(1/3)*e
 + d)^2*d*e^(-2)*log(x^(1/3)*e + d) - 108*(x^(1/3)*e + d)*d^2*e^(-2)*log(x^
(1/3)*e + d) + 4*(x^(1/3)*e + d)^3*e^(-2) - 27*(x^(1/3)*e + d)^2*d*e^(-2) +
 108*(x^(1/3)*e + d)*d^2*e^(-2))*b^3*n^2*log(c) + 18*(6*(x^(1/3)*e + d)^3*e
^(-2)*log(x^(1/3)*e + d) - 18*(x^(1/3)*e + d)^2*d*e^(-2)*log(x^(1/3)*e + d)
 + 18*(x^(1/3)*e + d)*d^2*e^(-2)*log(x^(1/3)*e + d) - 2*(x^(1/3)*e + d)^3*e
^(-2) + 9*(x^(1/3)*e + d)^2*d*e^(-2) - 18*(x^(1/3)*e + d)*d^2*e^(-2))*b^3*n
*log(c)^2 + 108*a*b^2*x*e*log(c)^2 + 6*(18*(x^(1/3)*e + d)^3*e^(-2)*log(x^(
1/3)*e + d)^2 - 54*(x^(1/3)*e + d)^2*d*e^(-2)*log(x^(1/3)*e + d)^2 + 54*(x^
(1/3)*e + d)*d^2*e^(-2)*log(x^(1/3)*e + d)^2 - 12*(x^(1/3)*e + d)^3*e^(-2)*
log(x^(1/3)*e + d) + 54*(x^(1/3)*e + d)^2*d*e^(-2)*log(x^(1/3)*e + d) - 108
```

```
*(x^(1/3)*e + d)*d^2*e^(-2)*log(x^(1/3)*e + d) + 4*(x^(1/3)*e + d)^3*e^(-2)
 - 27*(x^(1/3)*e + d)^2*d*e^(-2) + 108*(x^(1/3)*e + d)*d^2*e^(-2))*a*b^2*n^
2 + 36*(6*(x^(1/3)*e + d)^3*e^(-2)*log(x^(1/3)*e + d) - 18*(x^(1/3)*e + d)^
2*d*e^(-2)*log(x^(1/3)*e + d) + 18*(x^(1/3)*e + d)*d^2*e^(-2)*log(x^(1/3)*e
 + d) - 2*(x^(1/3)*e + d)^3*e^(-2) + 9*(x^(1/3)*e + d)^2*d*e^(-2) - 18*(x^(
1/3)*e + d)*d^2*e^(-2))*a*b^2*n*log(c) + 108*a^2*b*x*e*log(c) + 18*(6*(x^(1
/3)*e + d)^3*e^(-2)*log(x^(1/3)*e + d) - 18*(x^(1/3)*e + d)^2*d*e^(-2)*log(
x^(1/3)*e + d) + 18*(x^(1/3)*e + d)*d^2*e^(-2)*log(x^(1/3)*e + d) - 2*(x^(1
/3)*e + d)^3*e^(-2) + 9*(x^(1/3)*e + d)^2*d*e^(-2) - 18*(x^(1/3)*e + d)*d^2
*e^(-2))*a^2*b*n + 36*a^3*x*e)*e^(-1)
```

Mupad [B]
time = 0.69, size = 558, normalized size = 1.27

$$x\left(a^3-a^2bn+\frac{2ab^2n^2}{3}-\frac{2b^3n^3}{9}\right)-x^{2/3}\left(\frac{d\left(3a^3-3a^2bn+2ab^2n^2-\frac{2b^3n^3}{3}\right)}{2e}-\frac{d\left(6a^3-6ab^2n^2+5b^3n^3\right)}{4e}\right)+\ln\left(c\left(d+ex^{1/3}\right)^n\right)^3\left(b^3x+\frac{b^3d^3}{e^3}\right)+\ln\left(c\left(d+ex^{1/3}\right)^n\right)^2\left(\frac{d\left(6ab^2d^2-11b^3d^2n\right)}{2e^3}-x^{2/3}\left(\frac{3b^2d\left(3a-bn\right)}{2e}-\frac{9ab^2d}{2e}\right)+b^2x\left(3a-bn\right)+\frac{dx^{1/3}\left(\frac{3b^2d\left(3a-bn\right)}{e}-\frac{9ab^2d}{e}\right)}{e}\right)+x^{1/3}\left(\frac{d\left(\frac{d\left(3a^3-3a^2bn+2ab^2n^2-\frac{2b^3n^3}{3}\right)}{e}-\frac{d\left(6a^3-6ab^2n^2+5b^3n^3\right)}{2e}\right)}{e}+\frac{b^2d^2n^2\left(6a-11bn\right)}{e^2}\right)+\frac{\ln\left(d+ex^{1/3}\right)\left(18a^2bd^3n-66ab^2d^3n^2+85b^3d^3n^3\right)}{6e^3}+\frac{\ln\left(c\left(d+ex^{1/3}\right)^n\right)\left(\frac{x^{1/3}\left(\frac{d\left(bde\left(9a^2-6abn+2b^2n^2\right)-3bde\left(3a^2-b^2n^2\right)\right)}{e}+6b^3d^2n^2\right)}{e}-\frac{x^{2/3}\left(bde\left(9a^2-6abn+2b^2n^2\right)-3bde\left(3a^2-b^2n^2\right)\right)}{2e}+\frac{bex\left(9a^2-6abn+2b^2n^2\right)}{3}\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))^3,x)
```

```
[Out] x*(a^3 - (2*b^3*n^3)/9 + (2*a*b^2*n^2)/3 - a^2*b*n) - x^(2/3)*((d*(3*a^3 -
(2*b^3*n^3)/3 + 2*a*b^2*n^2 - 3*a^2*b*n))/(2*e) - (d*(6*a^3 + 5*b^3*n^3 - 6
*a*b^2*n^2))/(4*e)) + log(c*(d + e*x^(1/3))^n)^3*(b^3*x + (b^3*d^3)/e^3) +
log(c*(d + e*x^(1/3))^n)^2*((d*(6*a*b^2*d^2 - 11*b^3*d^2*n))/(2*e^3) - x^(2
/3)*((3*b^2*d*(3*a - b*n))/(2*e) - (9*a*b^2*d)/(2*e)) + b^2*x*(3*a - b*n) +
 (d*x^(1/3)*((3*b^2*d*(3*a - b*n))/e - (9*a*b^2*d)/e))/e) + x^(1/3)*((d*((d
*(3*a^3 - (2*b^3*n^3)/3 + 2*a*b^2*n^2 - 3*a^2*b*n))/e - (d*(6*a^3 + 5*b^3*n
^3 - 6*a*b^2*n^2))/(2*e)))/e + (b^2*d^2*n^2*(6*a - 11*b*n))/e^2) + (log(d +
 e*x^(1/3))*(85*b^3*d^3*n^3 - 66*a*b^2*d^3*n^2 + 18*a^2*b*d^3*n))/(6*e^3) +
 (log(c*(d + e*x^(1/3))^n)*((x^(1/3)*((d*(b*d*e*(9*a^2 + 2*b^2*n^2 - 6*a*b*
n) - 3*b*d*e*(3*a^2 - b^2*n^2)))/e + 6*b^3*d^2*n^2))/e - (x^(2/3)*(b*d*e*(9
*a^2 + 2*b^2*n^2 - 6*a*b*n) - 3*b*d*e*(3*a^2 - b^2*n^2)))/(2*e) + (b*e*x*(9
*a^2 + 2*b^2*n^2 - 6*a*b*n))/3))/e
```

3.460 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x}\,dx$

Optimal. Leaf size=135

$$3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt[3]{x}}{d}\right)+9bn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\operatorname{Li}_2\left(1+\frac{e\sqrt[3]{x}}{d}\right)-18b^2n^2(a+$$

```
[Out] 3*(a+b*ln(c*(d+e*x^(1/3))^n))^3*ln(-e*x^(1/3)/d)+9*b*n*(a+b*ln(c*(d+e*x^(1/
3))^n))^2*polylog(2,1+e*x^(1/3)/d)-18*b^2*n^2*(a+b*ln(c*(d+e*x^(1/3))^n))*p
olylog(3,1+e*x^(1/3)/d)+18*b^3*n^3*polylog(4,1+e*x^(1/3)/d)
```

Rubi [A]
time = 0.13, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2504, 2443, 2481, 2421, 2430, 6724}

$$-18b^2n^2\text{PolyLog}\left(3,\frac{e\sqrt[3]{x}}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)+9bn\text{PolyLog}\left(2,\frac{e\sqrt[3]{x}}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2+18b^3n^3\text{PolyLog}\left(4,\frac{e\sqrt[3]{x}}{d}+1\right)+3\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])^3/x,x]
```

```
[Out] 3*(a + b*Log[c*(d + e*x^(1/3))^n])^3*Log[-((e*x^(1/3))/d)] + 9*b*n*(a + b*L
og[c*(d + e*x^(1/3))^n])^2*PolyLog[2, 1 + (e*x^(1/3))/d] - 18*b^2*n^2*(a +
b*Log[c*(d + e*x^(1/3))^n])*PolyLog[3, 1 + (e*x^(1/3))/d] + 18*b^3*n^3*Poly
Log[4, 1 + (e*x^(1/3))/d]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
```

```
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x}\,dx = 3\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x}\,dx, x, \sqrt[3]{x}\right)$$
$$= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt[3]{x}}{d}\right) - (9ben)\text{Subst}\left(\int \frac{\log(-$$
$$= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt[3]{x}}{d}\right) - (9bn)\text{Subst}\left(\int \frac{(a+b\,l$$
$$= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt[3]{x}}{d}\right) + 9bn\left(a+b\log\left(c\left(d+e\sqrt[3]{}\right.\right.\right.$$
$$= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt[3]{x}}{d}\right) + 9bn\left(a+b\log\left(c\left(d+e\sqrt[3]{}\right.\right.\right.$$
$$= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt[3]{x}}{d}\right) + 9bn\left(a+b\log\left(c\left(d+e\sqrt[3]{}\right.\right.\right.$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 333 vs. $2(135) = 270$.
time = 0.10, size = 333, normalized size = 2.47

$$\left(a-bn\log\left(d+e\sqrt[3]{x}\right)+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\log(x)+3bn\left(a-bn\log\left(d+e\sqrt[3]{x}\right)+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\left(\left(\log\left(d+e\sqrt[3]{x}\right)-\log\left(1+\frac{e\sqrt[3]{x}}{d}\right)\right)\log(x)-3\text{Li}_2\left(-\frac{e\sqrt[3]{x}}{d}\right)\right)+9b^2n^2\left(a-bn\log\left(d+e\sqrt[3]{x}\right)+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\left(\log^2\left(d+e\sqrt[3]{x}\right)\log\left(-\frac{e\sqrt[3]{x}}{d}\right)+2\log\left(d+e\sqrt[3]{x}\right)\text{Li}_2\left(1+\frac{e\sqrt[3]{x}}{d}\right)-2\text{Li}_3\left(1+\frac{e\sqrt[3]{x}}{d}\right)\right)+3b^3n^3\left(\log^3\left(d+e\sqrt[3]{x}\right)\log\left(-\frac{e\sqrt[3]{x}}{d}\right)+3\log^2\left(d+e\sqrt[3]{x}\right)\text{Li}_2\left(1+\frac{e\sqrt[3]{x}}{d}\right)-6\log\left(d+e\sqrt[3]{x}\right)\text{Li}_3\left(1+\frac{e\sqrt[3]{x}}{d}\right)+6\text{Li}_4\left(1+\frac{e\sqrt[3]{x}}{d}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^3/x,x]
```

```
[Out] (a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^3*Log[x] + 3*b*n*
(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^2*((Log[d + e*x^(
1/3)] - Log[1 + (e*x^(1/3))/d])*Log[x] - 3*PolyLog[2, -((e*x^(1/3))/d)]) +
9*b^2*n^2*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])*(Log[d
+ e*x^(1/3)]^2*Log[-((e*x^(1/3))/d)] + 2*Log[d + e*x^(1/3)]*PolyLog[2, 1 +
(e*x^(1/3))/d] - 2*PolyLog[3, 1 + (e*x^(1/3))/d]) + 3*b^3*n^3*(Log[d + e*x^
(1/3)]^3*Log[-((e*x^(1/3))/d)] + 3*Log[d + e*x^(1/3)]^2*PolyLog[2, 1 + (e*x
^(1/3))/d] - 6*Log[d + e*x^(1/3)]*PolyLog[3, 1 + (e*x^(1/3))/d] + 6*PolyLog
[4, 1 + (e*x^(1/3))/d])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^n\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))^3/x,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))^3/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x,x, algorithm="maxima")
```

```
[Out] b^3*log((x^(1/3)*e + d)^n)^3*log(x) + integrate(((b^3*log(c)^3 + 3*a*b^2*lo
g(c)^2 + 3*a^2*b*log(c) + a^3)*x*e - (b^3*n*x*e*log(x) - 3*(b^3*log(c) + a*
b^2)*x*e - 3*(b^3*d*log(c) + a*b^2*d)*x^(2/3))*log((x^(1/3)*e + d)^n)^2 + 3
*((b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*b)*x*e + (b^3*d*log(c)^2 + 2*a*b^2*d
*log(c) + a^2*b*d)*x^(2/3))*log((x^(1/3)*e + d)^n) + (b^3*d*log(c)^3 + 3*a*
b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x^(2/3))/(x^2*e + d*x^(5/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((x^(1/3)*e + d)^n*c)^3 + 3*a*b^2*log((x^(1/3)*e + d)^n*c)
^2 + 3*a^2*b*log((x^(1/3)*e + d)^n*c) + a^3)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \log\left(c\left(d + e\sqrt[3]{x}\right)^{n}\right)\right)^{3}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))**3/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**(1/3))**n))**3/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x,x, algorithm="giac")
[Out] integrate((b*log((x^(1/3)*e + d)^n*c) + a)^3/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))^3/x,x)
[Out] int((a + b*log(c*(d + e*x^(1/3))^n))^3/x, x)
```

3.461 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^2}\,dx$

Optimal. Leaf size=439

$$-\frac{3b^2e^2n^2\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3\sqrt[3]{x}}-\frac{3b^2e^3n^2\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3}-$$

```
[Out] -3*b^2*e^2*n^2*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))/d^3/x^(1/3)-3*b^2*
e^3*n^2*ln(1-d/(d+e*x^(1/3)))*(a+b*ln(c*(d+e*x^(1/3))^n))/d^3-3/2*b*e*n*(a+
b*ln(c*(d+e*x^(1/3))^n))^2/d/x^(2/3)+3*b*e^2*n*(d+e*x^(1/3))*(a+b*ln(c*(d+e
*x^(1/3))^n))^2/d^3/x^(1/3)+3*b*e^3*n*ln(1-d/(d+e*x^(1/3)))*(a+b*ln(c*(d+e*
x^(1/3))^n))^2/d^3-(a+b*ln(c*(d+e*x^(1/3))^n))^3/x-6*b^2*e^3*n^2*(a+b*ln(c*
(d+e*x^(1/3))^n))*ln(-e*x^(1/3)/d)/d^3+b^3*e^3*n^3*ln(x)/d^3+3*b^3*e^3*n^3*
polylog(2,d/(d+e*x^(1/3)))/d^3-6*b^2*e^3*n^2*(a+b*ln(c*(d+e*x^(1/3))^n))*po
lylog(2,d/(d+e*x^(1/3)))/d^3-6*b^3*e^3*n^3*polylog(2,1+e*x^(1/3)/d)/d^3-6*b
^3*e^3*n^3*polylog(3,d/(d+e*x^(1/3)))/d^3
```

Rubi [A]
time = 0.55, antiderivative size = 439, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {2504, 2445, 2458, 2389, 2379, 2421, 6724, 2355, 2354, 2438, 2356, 2351, 31}

$$-\frac{6b^2e^3n^2\text{PolyLog}\left(2,\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3}+\frac{3b^3e^3n^3\text{PolyLog}\left(2,\frac{d}{d+e\sqrt[3]{x}}\right)}{d^3}-\frac{6b^3e^3n^3\text{PolyLog}\left(2,\frac{e\sqrt[3]{x}}{d}+1\right)}{d^3}-\frac{6b^3e^3n^3\text{PolyLog}\left(3,\frac{d}{d+e\sqrt[3]{x}}\right)}{d^3}-\frac{3b^2e^3n^2\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3}-\frac{6b^2e^3n^2\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3}-\frac{3b^2e^2n^2\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3\sqrt[3]{x}}+\frac{3be^3n\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{d^3}+\frac{3be^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{d^3\sqrt[3]{x}}-\frac{3ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2dx^{2/3}}-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x}+\frac{b^3e^3n^3\log(x)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])^3/x^2,x]
```

```
[Out] (-3*b^2*e^2*n^2*(d + e*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n]))/(d^3*x^(1
/3)) - (3*b^2*e^3*n^2*Log[1 - d/(d + e*x^(1/3))]*(a + b*Log[c*(d + e*x^(1/3
))^n]))/d^3 - (3*b*e*n*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(2*d*x^(2/3)) +
(3*b*e^2*n*(d + e*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(d^3*x^(1/3)
) + (3*b*e^3*n*Log[1 - d/(d + e*x^(1/3))]*(a + b*Log[c*(d + e*x^(1/3))^n])^
2)/d^3 - (a + b*Log[c*(d + e*x^(1/3))^n])^3/x - (6*b^2*e^3*n^2*(a + b*Log[c
*(d + e*x^(1/3))^n])*Log[-((e*x^(1/3))/d)])/d^3 + (b^3*e^3*n^3*Log[x])/d^3
+ (3*b^3*e^3*n^3*PolyLog[2, d/(d + e*x^(1/3))])/d^3 - (6*b^2*e^3*n^2*(a + b
*Log[c*(d + e*x^(1/3))^n])*PolyLog[2, d/(d + e*x^(1/3))])/d^3 - (6*b^3*e^3*
n^3*PolyLog[2, 1 + (e*x^(1/3))/d])/d^3 - (6*b^3*e^3*n^3*PolyLog[3, d/(d + e
*x^(1/3))])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^2}\,dx = 3\text{Subst}\left(\int \frac{(a+b\log\left(c(d+ex)^n\right))^3}{x^4}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x} + (3ben)\text{Subst}\left(\int \frac{(a+b\log\left(c(d+ex)\right.}{x^3(d+ex)}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x} + (3bn)\text{Subst}\left(\int \frac{(a+b\log\left(cx^n\right))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx,\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x} + \frac{(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+\right.}{d}$$

$$= -\frac{3ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2dx^{2/3}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x} - $$

$$= -\frac{3ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2dx^{2/3}} + \frac{3be^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c(d\right.\right.}{d^3\sqrt[3]{x}}$$

$$= -\frac{3b^2e^2n^2\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3\sqrt[3]{x}} - \frac{3ben\left(a+b\log\left(c(\right.\right.}{2dx^{2/}}$$

$$= -\frac{3b^2e^2n^2\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3\sqrt[3]{x}} + \frac{3be^3n\left(a+b\log\left(c(\right.\right.}{2d^3}$$

$$= -\frac{3b^2e^2n^2\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3\sqrt[3]{x}} + \frac{3be^3n\left(a+b\log\left(c(\right.\right.}{2d^3}$$

<u>Mathematica</u> [A]
time = 0.56, size = 733, normalized size = 1.67

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^3/x^2,x]
```

```
[Out] (-3*b*d^2*e*n*x^(1/3)*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))
^n])^2 + 6*b*d*e^2*n*x^(2/3)*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x
^(1/3))^n])^2 - 6*b*d^3*n*Log[d + e*x^(1/3)]*(a - b*n*Log[d + e*x^(1/3)] +
```

```
b*Log[c*(d + e*x^(1/3))^n])^2 - 6*b*e^3*n*x*Log[d + e*x^(1/3)]*(a - b*n*Log
[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^2 - 2*d^3*(a - b*n*Log[d + e*
x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^3 + 2*b*e^3*n*x*(a - b*n*Log[d + e*x
^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^2*Log[x] - 6*b^2*n^2*(a - b*n*Log[d +
 e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])*(d*e^2*x^(2/3) + (d^3 + e^3*x)*Lo
g[d + e*x^(1/3)]^2 + 3*e^3*x*Log[-((e*x^(1/3))/d)] + Log[d + e*x^(1/3)]*(d^
2*e*x^(1/3) - 2*d*e^2*x^(2/3) - 3*e^3*x - 2*e^3*x*Log[-((e*x^(1/3))/d)]) -
2*e^3*x*PolyLog[2, 1 + (e*x^(1/3))/d]) + b^3*n^3*(-6*d*e^2*x^(2/3)*Log[d +
e*x^(1/3)] - 6*e^3*x*Log[d + e*x^(1/3)] - 3*d^2*e*x^(1/3)*Log[d + e*x^(1/3)
]^2 + 6*d*e^2*x^(2/3)*Log[d + e*x^(1/3)]^2 + 9*e^3*x*Log[d + e*x^(1/3)]^2 -
 2*d^3*Log[d + e*x^(1/3)]^3 - 2*e^3*x*Log[d + e*x^(1/3)]^3 + 6*e^3*x*Log[-(
(e*x^(1/3))/d)] - 18*e^3*x*Log[d + e*x^(1/3)]*Log[-((e*x^(1/3))/d)] + 6*e^3
*x*Log[d + e*x^(1/3)]^2*Log[-((e*x^(1/3))/d)] + 6*e^3*x*(-3 + 2*Log[d + e*x
^(1/3)])*PolyLog[2, 1 + (e*x^(1/3))/d] - 12*e^3*x*PolyLog[3, 1 + (e*x^(1/3)
)/d]))/(2*d^3*x)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^{3}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))^3/x^2,x)
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))^3/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x^2,x, algorithm="maxima")
[Out] -1/2*(2*b^3*d^3*x^(2/3)*log((x^(1/3)*e + d)^n)^3 + (3*b^3*d^2*n*x*e + 6*b^3
*n*x^(5/3)*e^3*log(x^(1/3)*e + d) - 6*b^3*d*n*x^(4/3)*e^2 + 2*(3*b^3*d^3*lo
g(c) - b^3*n*x*e^3*log(x) + 3*a*b^2*d^3)*x^(2/3))*log((x^(1/3)*e + d)^n)^2)
/(d^3*x^(5/3)) + integrate(1/3*(3*(b^3*d^3*log(c)^3 + 3*a*b^2*d^3*log(c)^2
+ 3*a^2*b*d^3*log(c) + a^3*d^3)*x^(5/3)*e + 3*(b^3*d^4*log(c)^3 + 3*a*b^2*d
^4*log(c)^2 + 3*a^2*b*d^4*log(c) + a^3*d^4)*x^(4/3) + (3*b^3*d^2*n^2*x^2*e^
2 + 6*b^3*n^2*x^(8/3)*e^4*log(x^(1/3)*e + d) - 6*b^3*d*n^2*x^(7/3)*e^3 + 9*
(b^3*d^3*log(c)^2 + 2*a*b^2*d^3*log(c) + a^2*b*d^3)*x^(5/3)*e + 9*(b^3*d^4*
log(c)^2 + 2*a*b^2*d^4*log(c) + a^2*b*d^4)*x^(4/3) - 2*(b^3*n^2*x^2*e^4*log
(x) - 3*(b^3*d^3*n*log(c) + a*b^2*d^3*n)*x*e)*x^(2/3))*log((x^(1/3)*e + d)^
n))/(d^3*x^(11/3)*e + d^4*x^(10/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((x^(1/3)*e + d)^n*c)^3 + 3*a*b^2*log((x^(1/3)*e + d)^n*c)
^2 + 3*a^2*b*log((x^(1/3)*e + d)^n*c) + a^3)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\log\left(c\left(d + e\sqrt[3]{x}\right)^{n}\right)\right)^{3}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))**3/x**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**(1/3))**n))**3/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^(1/3)*e + d)^n*c) + a)^3/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)\right)^3}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))^3/x^2,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/3))^n))^3/x^2, x)
```

3.462 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^3}\,dx$

Optimal. Leaf size=765

$$-\frac{b^3e^3n^3}{20d^3x}+\frac{3b^3e^4n^3}{10d^4x^{2/3}}-\frac{71b^3e^5n^3}{40d^5\sqrt[3]{x}}+\frac{71b^3e^6n^3\log\left(d+e\sqrt[3]{x}\right)}{40d^6}-\frac{3b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{20d^2x^{4/3}}+\frac{9b^2e^3n^2(a}{}$$

```
[Out] -1/20*b^3*e^3*n^3/d^3/x+3/10*b^3*e^4*n^3/d^4/x^(2/3)-71/40*b^3*e^5*n^3/d^5/
x^(1/3)+71/40*b^3*e^6*n^3*ln(d+e*x^(1/3))/d^6-3/20*b^2*e^2*n^2*(a+b*ln(c*(d
+e*x^(1/3))^n))/d^2/x^(4/3)+9/20*b^2*e^3*n^2*(a+b*ln(c*(d+e*x^(1/3))^n))/d^
3/x-47/40*b^2*e^4*n^2*(a+b*ln(c*(d+e*x^(1/3))^n))/d^4/x^(2/3)+77/20*b^2*e^5
*n^2*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))/d^6/x^(1/3)+77/20*b^2*e^6*n^
2*ln(1-d/(d+e*x^(1/3)))*(a+b*ln(c*(d+e*x^(1/3))^n))/d^6-3/10*b*e*n*(a+b*ln(
c*(d+e*x^(1/3))^n))^2/d/x^(5/3)+3/8*b*e^2*n*(a+b*ln(c*(d+e*x^(1/3))^n))^2/d
^2/x^(4/3)-1/2*b*e^3*n*(a+b*ln(c*(d+e*x^(1/3))^n))^2/d^3/x+3/4*b*e^4*n*(a+b
*ln(c*(d+e*x^(1/3))^n))^2/d^4/x^(2/3)-3/2*b*e^5*n*(d+e*x^(1/3))*(a+b*ln(c*(
d+e*x^(1/3))^n))^2/d^6/x^(1/3)-3/2*b*e^6*n*ln(1-d/(d+e*x^(1/3)))*(a+b*ln(c*
(d+e*x^(1/3))^n))^2/d^6-1/2*(a+b*ln(c*(d+e*x^(1/3))^n))^3/x^2+3*b^2*e^6*n^2
*(a+b*ln(c*(d+e*x^(1/3))^n))*ln(-e*x^(1/3)/d)/d^6-15/8*b^3*e^6*n^3*ln(x)/d^
6-77/20*b^3*e^6*n^3*polylog(2,d/(d+e*x^(1/3)))/d^6+3*b^2*e^6*n^2*(a+b*ln(c*
(d+e*x^(1/3))^n))*polylog(2,d/(d+e*x^(1/3)))/d^6+3*b^3*e^6*n^3*polylog(2,1+
e*x^(1/3)/d)/d^6+3*b^3*e^6*n^3*polylog(3,d/(d+e*x^(1/3)))/d^6
```

Rubi [A]
time = 1.73, antiderivative size = 765, normalized size of antiderivative = 1.00, number of steps used = 62, number of rules used = 14, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {2504, 2445, 2458, 2389, 2379, 2421, 6724, 2355, 2354, 2438, 2356, 2351, 31, 46}

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])^3/x^3,x]
```

```
[Out] -1/20*(b^3*e^3*n^3)/(d^3*x) + (3*b^3*e^4*n^3)/(10*d^4*x^(2/3)) - (71*b^3*e^
5*n^3)/(40*d^5*x^(1/3)) + (71*b^3*e^6*n^3*Log[d + e*x^(1/3)])/(40*d^6) - (3
*b^2*e^2*n^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/(20*d^2*x^(4/3)) + (9*b^2*e^
3*n^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/(20*d^3*x) - (47*b^2*e^4*n^2*(a + b
*Log[c*(d + e*x^(1/3))^n]))/(40*d^4*x^(2/3)) + (77*b^2*e^5*n^2*(d + e*x^(1/
3))*(a + b*Log[c*(d + e*x^(1/3))^n]))/(20*d^6*x^(1/3)) + (77*b^2*e^6*n^2*Lo
g[1 - d/(d + e*x^(1/3))]*(a + b*Log[c*(d + e*x^(1/3))^n]))/(20*d^6) - (3*b*
e*n*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(10*d*x^(5/3)) + (3*b*e^2*n*(a + b*
Log[c*(d + e*x^(1/3))^n])^2)/(8*d^2*x^(4/3)) - (b*e^3*n*(a + b*Log[c*(d + e
```

```
*x^(1/3))^n])^2)/(2*d^3*x) + (3*b*e^4*n*(a + b*Log[c*(d + e*x^(1/3))^n])^2)
/(4*d^4*x^(2/3)) - (3*b*e^5*n*(d + e*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^
n])^2)/(2*d^6*x^(1/3)) - (3*b*e^6*n*Log[1 - d/(d + e*x^(1/3))]*(a + b*Log[c
*(d + e*x^(1/3))^n])^2)/(2*d^6) - (a + b*Log[c*(d + e*x^(1/3))^n])^3/(2*x^2
) + (3*b^2*e^6*n^2*(a + b*Log[c*(d + e*x^(1/3))^n])*Log[-((e*x^(1/3))/d)])/
d^6 - (15*b^3*e^6*n^3*Log[x])/(8*d^6) - (77*b^3*e^6*n^3*PolyLog[2, d/(d + e
*x^(1/3))])/(20*d^6) + (3*b^2*e^6*n^2*(a + b*Log[c*(d + e*x^(1/3))^n])*Poly
Log[2, d/(d + e*x^(1/3))])/d^6 + (3*b^3*e^6*n^3*PolyLog[2, 1 + (e*x^(1/3))/
d])/d^6 + (3*b^3*e^6*n^3*PolyLog[3, d/(d + e*x^(1/3))])/d^6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
```

```
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
```

```
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^3}\,dx = 3\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x^7}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2x^2} + \frac{1}{2}(3ben)\text{Subst}\left(\int \frac{(a+b\log(c(d+\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2x^2} + \frac{1}{2}(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2x^2} + \frac{(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d\right.}{2d}$$

$$= -\frac{3ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{10dx^{5/3}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2x^2} -$$

$$= -\frac{3ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{10dx^{5/3}} + \frac{3be^2n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right.\right.}{8d^2x^{4/3}}$$

$$= -\frac{3b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{20d^2x^{4/3}} - \frac{3ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right.\right.}{10dx^{5/3}}$$

$$= -\frac{3b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{20d^2x^{4/3}} + \frac{9b^2e^3n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right.\right.\right.}{20d^3x}$$

$$= -\frac{b^3e^3n^3}{20d^3x} + \frac{3b^3e^4n^3}{40d^4x^{2/3}} - \frac{3b^3e^5n^3}{20d^5\sqrt[3]{x}} + \frac{3b^3e^6n^3\log\left(d+e\sqrt[3]{x}\right)}{20d^6} - \frac{3b^2e^2n^2}{}$$

$$= -\frac{b^3e^3n^3}{20d^3x} + \frac{3b^3e^4n^3}{10d^4x^{2/3}} - \frac{3b^3e^5n^3}{5d^5\sqrt[3]{x}} + \frac{3b^3e^6n^3\log\left(d+e\sqrt[3]{x}\right)}{5d^6} - \frac{3b^2e^2n^2(}{}$$

$$= -\frac{b^3e^3n^3}{20d^3x} + \frac{3b^3e^4n^3}{10d^4x^{2/3}} - \frac{71b^3e^5n^3}{40d^5\sqrt[3]{x}} + \frac{71b^3e^6n^3\log\left(d+e\sqrt[3]{x}\right)}{40d^6} - \frac{3b^2e^2n}{}$$

$$= -\frac{b^3e^3n^3}{20d^3x} + \frac{3b^3e^4n^3}{10d^4x^{2/3}} - \frac{71b^3e^5n^3}{40d^5\sqrt[3]{x}} + \frac{71b^3e^6n^3\log\left(d+e\sqrt[3]{x}\right)}{40d^6} - \frac{3b^2e^2n}{}$$

Mathematica [A]

time = 1.15, size = 1074, normalized size = 1.40

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^3/x^3,x]
```

```
[Out] -1/40*(12*b*d^5*e*n*x^(1/3)*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^
(1/3))^n])^2 - 15*b*d^4*e^2*n*x^(2/3)*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c
*(d + e*x^(1/3))^n])^2 + 20*b*d^3*e^3*n*x*(a - b*n*Log[d + e*x^(1/3)] + b*L
og[c*(d + e*x^(1/3))^n])^2 - 30*b*d^2*e^4*n*x^(4/3)*(a - b*n*Log[d + e*x^(1
/3)] + b*Log[c*(d + e*x^(1/3))^n])^2 + 60*b*d*e^5*n*x^(5/3)*(a - b*n*Log[d
+ e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^2 + 60*b*d^6*n*Log[d + e*x^(1/3)
]*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^2 - 60*b*e^6*n*
x^2*Log[d + e*x^(1/3)]*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3)
)^n])^2 + 20*d^6*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^
3 + 20*b*e^6*n*x^2*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n]
)^2*Log[x] + b^2*n^2*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^
n])*(6*d^4*e^2*x^(2/3) - 18*d^3*e^3*x + 47*d^2*e^4*x^(4/3) - 154*d*e^5*x^(5
/3) + 60*(d^6 - e^6*x^2)*Log[d + e*x^(1/3)]^2 - 274*e^6*x^2*Log[-((e*x^(1/3
))/d)] + 2*Log[d + e*x^(1/3)]*(12*d^5*e*x^(1/3) - 15*d^4*e^2*x^(2/3) + 20*d
^3*e^3*x - 30*d^2*e^4*x^(4/3) + 60*d*e^5*x^(5/3) + 137*e^6*x^2 + 60*e^6*x^2
*Log[-((e*x^(1/3))/d)]) + 120*e^6*x^2*PolyLog[2, 1 + (e*x^(1/3))/d]) + b^3*
n^3*(3*d^4*e^2*x^(2/3)*(2 - 5*Log[d + e*x^(1/3)])*Log[d + e*x^(1/3)] + 12*d
^5*e*x^(1/3)*Log[d + e*x^(1/3)]^2 + 20*d^6*Log[d + e*x^(1/3)]^3 + 2*d^3*e^3
*x*(1 - 9*Log[d + e*x^(1/3)] + 10*Log[d + e*x^(1/3)]^2) - d^2*e^4*x^(4/3)*(
12 - 47*Log[d + e*x^(1/3)] + 30*Log[d + e*x^(1/3)]^2) + d*e^5*x^(5/3)*(71 -
 154*Log[d + e*x^(1/3)] + 60*Log[d + e*x^(1/3)]^2) + 225*e^6*x^2*(-Log[d +
e*x^(1/3)] + Log[-((e*x^(1/3))/d)]) + 137*e^6*x^2*(Log[d + e*x^(1/3)]*(Log[
d + e*x^(1/3)] - 2*Log[-((e*x^(1/3))/d)]) - 2*PolyLog[2, 1 + (e*x^(1/3))/d]
) - 20*e^6*x^2*(Log[d + e*x^(1/3)]^2*(Log[d + e*x^(1/3)] - 3*Log[-((e*x^(1/
3))/d)]) - 6*Log[d + e*x^(1/3)]*PolyLog[2, 1 + (e*x^(1/3))/d] + 6*PolyLog[3
, 1 + (e*x^(1/3))/d])))/(d^6*x^2)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^{3}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))^3/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))^3/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b^3*log((x^(1/3)*e + d)^n)^3/x^2 + integrate(1/2*(2*(b^3*log(c)^3 + 3*
a*b^2*log(c)^2 + 3*a^2*b*log(c) + a^3)*x*e + (b^3*n*x*e + 6*(b^3*log(c) + a
*b^2)*x*e + 6*(b^3*d*log(c) + a*b^2*d)*x^(2/3))*log((x^(1/3)*e + d)^n)^2 +
6*((b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*b)*x*e + (b^3*d*log(c)^2 + 2*a*b^2*
d*log(c) + a^2*b*d)*x^(2/3))*log((x^(1/3)*e + d)^n) + 2*(b^3*d*log(c)^3 + 3
*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x^(2/3))/(x^4*e + d*x^(11/3))
, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((x^(1/3)*e + d)^n*c)^3 + 3*a*b^2*log((x^(1/3)*e + d)^n*c)
^2 + 3*a^2*b*log((x^(1/3)*e + d)^n*c) + a^3)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\log\left(c\left(d + e\sqrt[3]{x}\right)^{n}\right)\right)^{3}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))**3/x**3,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**(1/3))**n))**3/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^(1/3)*e + d)^n*c) + a)^3/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))^3/x^3,x)
[Out] int((a + b*log(c*(d + e*x^(1/3))^n))^3/x^3, x)
```

3.463 $\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx$

Optimal. Leaf size=138

$$\frac{bd^5nx^{2/3}}{4e^5}-\frac{bd^4nx^{4/3}}{8e^4}+\frac{bd^3nx^2}{12e^3}-\frac{bd^2nx^{8/3}}{16e^2}+\frac{bdnx^{10/3}}{20e}-\frac{1}{24}bnx^4-\frac{bd^6n\log\left(d+ex^{2/3}\right)}{4e^6}+\frac{1}{4}x^4\left(a+b\log\left(c\left(d+e\right.\right.\right.$$

```
[Out] 1/4*b*d^5*n*x^(2/3)/e^5-1/8*b*d^4*n*x^(4/3)/e^4+1/12*b*d^3*n*x^2/e^3-1/16*b
*d^2*n*x^(8/3)/e^2+1/20*b*d*n*x^(10/3)/e-1/24*b*n*x^4-1/4*b*d^6*n*ln(d+e*x^
(2/3))/e^6+1/4*x^4*(a+b*ln(c*(d+e*x^(2/3))^n))
```

Rubi [A]

time = 0.07, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2504, 2442, 45}

$$\frac{1}{4}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{bd^6n\log\left(d+ex^{2/3}\right)}{4e^6}+\frac{bd^5nx^{2/3}}{4e^5}-\frac{bd^4nx^{4/3}}{8e^4}+\frac{bd^3nx^2}{12e^3}-\frac{bd^2nx^{8/3}}{16e^2}+\frac{bdnx^{10/3}}{20e}-\frac{1}{24}bnx^4$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(2/3))^n]),x]
```

```
[Out] (b*d^5*n*x^(2/3))/(4*e^5) - (b*d^4*n*x^(4/3))/(8*e^4) + (b*d^3*n*x^2)/(12*e
^3) - (b*d^2*n*x^(8/3))/(16*e^2) + (b*d*n*x^(10/3))/(20*e) - (b*n*x^4)/24 -
 (b*d^6*n*Log[d + e*x^(2/3)])/(4*e^6) + (x^4*(a + b*Log[c*(d + e*x^(2/3))^n
]))/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
```

```
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx &= \frac{3}{2}\text{Subst}\left(\int x^5(a+b\log\left(c(d+ex)^n\right))\,dx,x,x^{2/3}\right)\\
&= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{1}{4}(ben)\text{Subst}\left(\int\frac{x^6}{d+ex}\,dx,x,x^{2/}\right.\\
&= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{1}{4}(ben)\text{Subst}\left(\int\left(-\frac{d^5}{e^6}+\frac{d^4x}{e^5}-\right.\right.\\
&= \frac{bd^5nx^{2/3}}{4e^5}-\frac{bd^4nx^{4/3}}{8e^4}+\frac{bd^3nx^2}{12e^3}-\frac{bd^2nx^{8/3}}{16e^2}+\frac{bdnx^{10/3}}{20e}-\frac{1}{24}bnx^4-
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 135, normalized size = 0.98

$$\frac{ax^4}{4}-\frac{1}{4}ben\left(-\frac{d^5x^{2/3}}{e^6}+\frac{d^4x^{4/3}}{2e^5}-\frac{d^3x^2}{3e^4}+\frac{d^2x^{8/3}}{4e^3}-\frac{dx^{10/3}}{5e^2}+\frac{x^4}{6e}+\frac{d^6\log\left(d+ex^{2/3}\right)}{e^7}\right)+\frac{1}{4}bx^4\log\left(c\left(d+ex^{2/3}\right)^n\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(2/3))^n]),x]
```

```
[Out] (a*x^4)/4 - (b*e*n*(-((d^5*x^(2/3))/e^6) + (d^4*x^(4/3))/(2*e^5) - (d^3*x^2
)/(3*e^4) + (d^2*x^(8/3))/(4*e^3) - (d*x^(10/3))/(5*e^2) + x^4/(6*e) + (d^6
*Log[d + e*x^(2/3)])/e^7))/4 + (b*x^4*Log[c*(d + e*x^(2/3))^n])/4
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^3\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(2/3))^n)),x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e*x^(2/3))^n)),x)
```

Maxima [A]
time = 0.29, size = 106, normalized size = 0.77

$$\frac{1}{4}\,bx^4\log\left(\left(x^{\frac{2}{3}}e+d\right)^nc\right)+\frac{1}{4}\,ax^4-\frac{1}{240}\left(60\,d^6e^{(-7)}\log\left(x^{\frac{2}{3}}e+d\right)+\left(30\,d^4x^{\frac{4}{3}}e-20\,d^3x^2e^2-60\,d^5x^{\frac{2}{3}}+15\,d^2x^{\frac{8}{3}}e^3-12\,dx^{\frac{10}{3}}e^4+10\,x^4e^5\right)e^{(-6)}\right)bne$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4*log((x^(2/3)*e + d)^n*c) + 1/4*a*x^4 - 1/240*(60*d^6*e^(-7)*log(x
^(2/3)*e + d) + (30*d^4*x^(4/3)*e - 20*d^3*x^2*e^2 - 60*d^5*x^(2/3) + 15*d^
2*x^(8/3)*e^3 - 12*d*x^(10/3)*e^4 + 10*x^4*e^5)*e^(-6))*b*n*e
```

Fricas [A]
time = 0.38, size = 121, normalized size = 0.88

$$\frac{1}{240}\left(20\,bd^3nx^2e^3+60\,bx^4e^6\log\left(c\right)-10\left(bn-6\,a\right)x^4e^6-60\left(bd^6n-bnx^4e^6\right)\log\left(x^{\frac{2}{3}}e+d\right)+15\left(4\,bd^5ne-bd^2nx^2e^4\right)x^{\frac{2}{3}}-6\left(5\,bd^4nxe^2-2\,bdnx^3e^5\right)x^{\frac{1}{3}}\right)e^{(-6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="fricas")
```

```
[Out] 1/240*(20*b*d^3*n*x^2*e^3 + 60*b*x^4*e^6*log(c) - 10*(b*n - 6*a)*x^4*e^6 -
60*(b*d^6*n - b*n*x^4*e^6)*log(x^(2/3)*e + d) + 15*(4*b*d^5*n*e - b*d^2*n*x
^2*e^4)*x^(2/3) - 6*(5*b*d^4*n*x*e^2 - 2*b*d*n*x^3*e^5)*x^(1/3))*e^(-6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(2/3))**n)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(106) = 212.
time = 4.41, size = 261, normalized size = 1.89

$$\frac{1}{4}\,bx^4\log\left(c\right)+\frac{1}{4}\,ax^4+\frac{1}{240}\left(60\left(x^{\frac{2}{3}}e+d\right)^6e^{(-6)}\log\left(x^{\frac{2}{3}}e+d\right)-360\left(x^{\frac{2}{3}}e+d\right)^5de^{(-6)}\log\left(x^{\frac{2}{3}}e+d\right)+900\left(x^{\frac{2}{3}}e+d\right)^4d^2e^{(-6)}\log\left(x^{\frac{2}{3}}e+d\right)-1200\left(x^{\frac{2}{3}}e+d\right)^3d^3e^{(-6)}\log\left(x^{\frac{2}{3}}e+d\right)+900\left(x^{\frac{2}{3}}e+d\right)^2d^4e^{(-6)}\log\left(x^{\frac{2}{3}}e+d\right)-10\left(x^{\frac{2}{3}}e+d\right)^6e^{(-6)}+72\left(x^{\frac{2}{3}}e+d\right)^5de^{(-6)}-225\left(x^{\frac{2}{3}}e+d\right)^4d^2e^{(-6)}+400\left(x^{\frac{2}{3}}e+d\right)^3d^3e^{(-6)}-450\left(x^{\frac{2}{3}}e+d\right)^2d^4e^{(-6)}-360\left(\left(x^{\frac{2}{3}}e+d\right)\log\left(x^{\frac{2}{3}}e+d\right)-x^{\frac{2}{3}}e-d\right)d^5e^{(-6)}\right)bn$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="giac")
```

```
[Out] 1/4*b*x^4*log(c) + 1/4*a*x^4 + 1/240*(60*(x^(2/3)*e + d)^6*e^(-6)*log(x^(2/
3)*e + d) - 360*(x^(2/3)*e + d)^5*d*e^(-6)*log(x^(2/3)*e + d) + 900*(x^(2/3
)*e + d)^4*d^2*e^(-6)*log(x^(2/3)*e + d) - 1200*(x^(2/3)*e + d)^3*d^3*e^(-6
)*log(x^(2/3)*e + d) + 900*(x^(2/3)*e + d)^2*d^4*e^(-6)*log(x^(2/3)*e + d)
- 10*(x^(2/3)*e + d)^6*e^(-6) + 72*(x^(2/3)*e + d)^5*d*e^(-6) - 225*(x^(2/3
)*e + d)^4*d^2*e^(-6) + 400*(x^(2/3)*e + d)^3*d^3*e^(-6) - 450*(x^(2/3)*e +
 d)^2*d^4*e^(-6) - 360*((x^(2/3)*e + d)*log(x^(2/3)*e + d) - x^(2/3)*e - d)
*d^5*e^(-6))*b*n
```

Mupad [B]
time = 0.45, size = 113, normalized size = 0.82

$$\frac{a\,x^4}{4}-\frac{b\,n\,x^4}{24}+\frac{b\,x^4\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)}{4}+\frac{b\,d\,n\,x^{10/3}}{20\,e}-\frac{b\,d^6\,n\,\ln\left(d+e\,x^{2/3}\right)}{4\,e^6}+\frac{b\,d^3\,n\,x^2}{12\,e^3}-\frac{b\,d^2\,n\,x^{8/3}}{16\,e^2}-\frac{b\,d^4\,n\,x^{4/3}}{8\,e^4}+\frac{b\,d^5\,n\,x^{2/3}}{4\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e*x^(2/3))^n)),x)
```

```
[Out] (a*x^4)/4 - (b*n*x^4)/24 + (b*x^4*log(c*(d + e*x^(2/3))^n))/4 + (b*d*n*x^(1
0/3))/(20*e) - (b*d^6*n*log(d + e*x^(2/3)))/(4*e^6) + (b*d^3*n*x^2)/(12*e^3
) - (b*d^2*n*x^(8/3))/(16*e^2) - (b*d^4*n*x^(4/3))/(8*e^4) + (b*d^5*n*x^(2/
3))/(4*e^5)
```

3.464 $\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx$

Optimal. Leaf size=130

$$-\frac{2bd^4n\sqrt[3]{x}}{3e^4}+\frac{2bd^3nx}{9e^3}-\frac{2bd^2nx^{5/3}}{15e^2}+\frac{2bdnx^{7/3}}{21e}-\frac{2}{27}bnx^3+\frac{2bd^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{9/2}}+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\right.\right.\right.$$

```
[Out] -2/3*b*d^4*n*x^(1/3)/e^4+2/9*b*d^3*n*x/e^3-2/15*b*d^2*n*x^(5/3)/e^2+2/21*b*
d*n*x^(7/3)/e-2/27*b*n*x^3+2/3*b*d^(9/2)*n*arctan(x^(1/3)*e^(1/2)/d^(1/2))/
e^(9/2)+1/3*x^3*(a+b*ln(c*(d+e*x^(2/3))^n))
```

Rubi [A]
time = 0.06, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2505, 348, 308, 211}

$$\frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\frac{2bd^{9/2}n\text{ArcTan}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{9/2}}-\frac{2bd^4n\sqrt[3]{x}}{3e^4}+\frac{2bd^3nx}{9e^3}-\frac{2bd^2nx^{5/3}}{15e^2}+\frac{2bdnx^{7/3}}{21e}-\frac{2}{27}bnx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(2/3))^n]),x]
```

```
[Out] (-2*b*d^4*n*x^(1/3))/(3*e^4) + (2*b*d^3*n*x)/(9*e^3) - (2*b*d^2*n*x^(5/3))/
(15*e^2) + (2*b*d*n*x^(7/3))/(21*e) - (2*b*n*x^3)/27 + (2*b*d^(9/2)*n*ArcTa
n[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(3*e^(9/2)) + (x^3*(a + b*Log[c*(d + e*x^(2/3
))^n]))/3
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx &= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{1}{9}(2ben)\int\frac{x^{8/3}}{d+ex^{2/3}}\,dx\\ &= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{1}{3}(2ben)\text{Subst}\left(\int\frac{x^{10}}{d+ex^2}\,dx,x,\right.\\ &= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{1}{3}(2ben)\text{Subst}\left(\int\left(\frac{d^4}{e^5}-\frac{d^3x^2}{e^4}+\right.\right.\\ &= -\frac{2bd^4n\sqrt[3]{x}}{3e^4}+\frac{2bd^3nx}{9e^3}-\frac{2bd^2nx^{5/3}}{15e^2}+\frac{2bdnx^{7/3}}{21e}-\frac{2}{27}bnx^3+\frac{1}{3}x^3\left(a\right.\\ &= -\frac{2bd^4n\sqrt[3]{x}}{3e^4}+\frac{2bd^3nx}{9e^3}-\frac{2bd^2nx^{5/3}}{15e^2}+\frac{2bdnx^{7/3}}{21e}-\frac{2}{27}bnx^3+\frac{2bd^{9/2}n}{\quad}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 135, normalized size = 1.04

$$-\frac{2bd^4n\sqrt[3]{x}}{3e^4}+\frac{2bd^3nx}{9e^3}-\frac{2bd^2nx^{5/3}}{15e^2}+\frac{2bdnx^{7/3}}{21e}+\frac{ax^3}{3}-\frac{2}{27}bnx^3+\frac{2bd^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{9/2}}+\frac{1}{3}bx^3\log\left(c\left(d+ex^{2/3}\right)^n\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))^n]),x]
```

```
[Out] (-2*b*d^4*n*x^(1/3))/(3*e^4) + (2*b*d^3*n*x)/(9*e^3) - (2*b*d^2*n*x^(5/3))/
(15*e^2) + (2*b*d*n*x^(7/3))/(21*e) + (a*x^3)/3 - (2*b*n*x^3)/27 + (2*b*d^(
9/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(3*e^(9/2)) + (b*x^3*Log[c*(d + e
*x^(2/3))^n])/3
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(2/3))^n)),x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e*x^(2/3))^n)),x)
```

Maxima [A]
time = 0.51, size = 95, normalized size = 0.73

$$\frac{1}{3}bx^3\log\left(\left(x^{\frac{2}{3}}e+d\right)^n c\right)+\frac{1}{3}ax^3+\frac{2}{945}\left(315\,d^{\frac{9}{2}}\arctan\left(\frac{x^{\frac{1}{3}}e^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{11}{2}\right)}+\left(105\,d^3xe-63\,d^2x^{\frac{5}{3}}e^2-315\,d^4x^{\frac{1}{3}}+45\,dx^{\frac{7}{3}}e^3-35\,x^3e^4\right)e^{(-5)}\right)bne$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3*log((x^(2/3)*e + d)^n*c) + 1/3*a*x^3 + 2/945*(315*d^(9/2)*arctan(
x^(1/3)*e^(1/2)/sqrt(d))*e^(-11/2) + (105*d^3*x*e - 63*d^2*x^(5/3)*e^2 - 31
5*d^4*x^(1/3) + 45*d*x^(7/3)*e^3 - 35*x^3*e^4)*e^(-5))*b*n*e
```

Fricas [A]
time = 0.42, size = 307, normalized size = 2.36

$$\left[\frac{1}{945}\left(315\sqrt{-de^{(-1)}}\,bd^4n\log\left(-\frac{2\sqrt{-de^{(-1)}}\,dxe^2+d^3-x^2e^3-2\left(d^2e+\sqrt{-de^{(-1)}}\,xe^3\right)x^{\frac{2}{3}}-2\left(\sqrt{-de^{(-1)}}\,d^2e-dxe^2\right)x^{\frac{1}{3}}}{d^3+x^2e^3}\right)+210\,bd^3nxe+315\,bnx^3e^4\log\left(x^{\frac{2}{3}}e+d\right)-126\,bd^2nx^{\frac{5}{3}}e^2+315\,bx^3e^4\log\left(c\right)-35\,(2\,bn-9\,a)x^3e^4-90\left(7\,bd^4n-bdnx^2e^3\right)x^{\frac{1}{3}}\right)e^{(-4)},\ \frac{1}{945}\left(630\,bd^{\frac{9}{2}}n\arctan\left(\frac{x^{\frac{1}{3}}e^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}+210\,bd^3nxe+315\,bnx^3e^4\log\left(x^{\frac{2}{3}}e+d\right)-126\,bd^2nx^{\frac{5}{3}}e^2+315\,bx^3e^4\log\left(c\right)-35\,(2\,bn-9\,a)x^3e^4-90\left(7\,bd^4n-bdnx^2e^3\right)x^{\frac{1}{3}}\right)e^{(-4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="fricas")
```

```
[Out] [1/945*(315*sqrt(-d*e^(-1))*b*d^4*n*log(-(2*sqrt(-d*e^(-1))*d*x*e^2 + d^3 -
 x^2*e^3 - 2*(d^2*e + sqrt(-d*e^(-1))*x*e^3)*x^(2/3) - 2*(sqrt(-d*e^(-1))*d
^2*e - d*x*e^2)*x^(1/3))/(d^3 + x^2*e^3)) + 210*b*d^3*n*x*e + 315*b*n*x^3*e
^4*log(x^(2/3)*e + d) - 126*b*d^2*n*x^(5/3)*e^2 + 315*b*x^3*e^4*log(c) - 35
*(2*b*n - 9*a)*x^3*e^4 - 90*(7*b*d^4*n - b*d*n*x^2*e^3)*x^(1/3))*e^(-4), 1/
945*(630*b*d^(9/2)*n*arctan(x^(1/3)*e^(1/2)/sqrt(d))*e^(-1/2) + 210*b*d^3*n
*x*e + 315*b*n*x^3*e^4*log(x^(2/3)*e + d) - 126*b*d^2*n*x^(5/3)*e^2 + 315*b
*x^3*e^4*log(c) - 35*(2*b*n - 9*a)*x^3*e^4 - 90*(7*b*d^4*n - b*d*n*x^2*e^3)
*x^(1/3))*e^(-4)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(2/3))**n)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.09, size = 104, normalized size = 0.80

$$\frac{1}{3}bx^3\log\left(c\right)+\frac{1}{3}ax^3+\frac{1}{945}\left(315\,x^3\log\left(x^{\frac{2}{3}}e+d\right)+2\left(315\,d^{\frac{9}{2}}\arctan\left(\frac{x^{\frac{1}{3}}e^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{11}{2}\right)}-\left(315\,d^4x^{\frac{1}{3}}e^4-105\,d^3xe^5+63\,d^2x^{\frac{5}{3}}e^6-45\,dx^{\frac{7}{3}}e^7+35\,x^3e^8\right)e^{(-9)}\right)e\right)bn$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="giac")
```

```
[Out] 1/3*b*x^3*log(c) + 1/3*a*x^3 + 1/945*(315*x^3*log(x^(2/3)*e + d) + 2*(315*d
^(9/2)*arctan(x^(1/3)*e^(1/2)/sqrt(d))*e^(-11/2) - (315*d^4*x^(1/3)*e^4 - 1
05*d^3*x*e^5 + 63*d^2*x^(5/3)*e^6 - 45*d*x^(7/3)*e^7 + 35*x^3*e^8)*e^(-9))*
e)*b*n
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(a + b \ln \left(c \left(d + e\, x^{2/3}\right)^n\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e*x^(2/3))^n)),x)
```

```
[Out] int(x^2*(a + b*log(c*(d + e*x^(2/3))^n)), x)
```

3.465 $\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx$

Optimal. Leaf size=89

$$-\frac{bd^2nx^{2/3}}{2e^2}+\frac{bdnx^{4/3}}{4e}-\frac{1}{6}bnx^2+\frac{bd^3n\log\left(d+ex^{2/3}\right)}{2e^3}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)$$

```
[Out] -1/2*b*d^2*n*x^(2/3)/e^2+1/4*b*d*n*x^(4/3)/e-1/6*b*n*x^2+1/2*b*d^3*n*ln(d+e
*x^(2/3))/e^3+1/2*x^2*(a+b*ln(c*(d+e*x^(2/3))^n))
```

Rubi [A]
time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.150$, Rules used = {2504, 2442, 45}

$$\frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\frac{bd^3n\log\left(d+ex^{2/3}\right)}{2e^3}-\frac{bd^2nx^{2/3}}{2e^2}+\frac{bdnx^{4/3}}{4e}-\frac{1}{6}bnx^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*x^(2/3))^n]),x]
```

```
[Out] -1/2*(b*d^2*n*x^(2/3))/e^2 + (b*d*n*x^(4/3))/(4*e) - (b*n*x^2)/6 + (b*d^3*n
*Log[d + e*x^(2/3)])/(2*e^3) + (x^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx = \frac{3}{2}\text{Subst}\left(\int x^2(a+b\log\left(c(d+ex)^n\right))\,dx, x, x^{2/3}\right)$$
$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right) - \frac{1}{2}(ben)\text{Subst}\left(\int \frac{x^3}{d+ex}\,dx, x, x^{2/3}\right.$$
$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right) - \frac{1}{2}(ben)\text{Subst}\left(\int \left(\frac{d^2}{e^3} - \frac{dx}{e^2} + \frac{x^2}{e}\right.\right.$$
$$= -\frac{bd^2nx^{2/3}}{2e^2} + \frac{bdnx^{4/3}}{4e} - \frac{1}{6}bnx^2 + \frac{bd^3n\log\left(d+ex^{2/3}\right)}{2e^3} + \frac{1}{2}x^2\left(a+b\,\text{lo}\right.$$

Mathematica [A]
time = 0.02, size = 94, normalized size = 1.06

$$-\frac{bd^2nx^{2/3}}{2e^2} + \frac{bdnx^{4/3}}{4e} + \frac{ax^2}{2} - \frac{1}{6}bnx^2 + \frac{bd^3n\log\left(d+ex^{2/3}\right)}{2e^3} + \frac{1}{2}bx^2\log\left(c\left(d+ex^{2/3}\right)^n\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(2/3))^n]),x]
[Out] -1/2*(b*d^2*n*x^(2/3))/e^2 + (b*d*n*x^(4/3))/(4*e) + (a*x^2)/2 - (b*n*x^2)/
6 + (b*d^3*n*Log[d + e*x^(2/3)])/(2*e^3) + (b*x^2*Log[c*(d + e*x^(2/3))^n])
/2
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e*x^(2/3))^n)),x)
[Out] int(x*(a+b*ln(c*(d+e*x^(2/3))^n)),x)
```

Maxima [A]
time = 0.27, size = 76, normalized size = 0.85

$$\frac{1}{12}\left(6\,d^3e^{(-4)}\log\left(x^{\frac{2}{3}}e+d\right)+\left(3\,dx^{\frac{4}{3}}e-2\,x^2e^2-6\,d^2x^{\frac{2}{3}}\right)e^{(-3)}\right)bne+\frac{1}{2}\,bx^2\log\left(\left(x^{\frac{2}{3}}e+d\right)^nc\right)+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="maxima")
```

```
[Out] 1/12*(6*d^3*e^(-4)*log(x^(2/3)*e + d) + (3*d*x^(4/3)*e - 2*x^2*e^2 - 6*d^2*
x^(2/3))*e^(-3))*b*n*e + 1/2*b*x^2*log((x^(2/3)*e + d)^n*c) + 1/2*a*x^2
```

Fricas [A]
time = 0.40, size = 77, normalized size = 0.87

$$-\frac{1}{12}\left(6\,bd^2nx^{\frac{2}{3}}e-3\,bdnx^{\frac{4}{3}}e^2-6\,bx^2e^3\log\left(c\right)+2\left(bn-3\,a\right)x^2e^3-6\left(bd^3n+bnx^2e^3\right)\log\left(x^{\frac{2}{3}}e+d\right)\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="fricas")
```

```
[Out] -1/12*(6*b*d^2*n*x^(2/3)*e - 3*b*d*n*x^(4/3)*e^2 - 6*b*x^2*e^3*log(c) + 2*(
b*n - 3*a)*x^2*e^3 - 6*(b*d^3*n + b*n*x^2*e^3)*log(x^(2/3)*e + d))*e^(-3)
```

Sympy [A]
time = 54.92, size = 95, normalized size = 1.07

$$\frac{ax^2}{2}+b\left(-\frac{en\left(-\frac{3d^3\left(\begin{cases}\frac{x^{\frac{2}{3}}}{d} & \text{for}\ e=0\\ \frac{\log\left(d+ex^{\frac{2}{3}}\right)}{e} & \text{otherwise}\end{cases}\right)}{2e^3}+\frac{3d^2x^{\frac{2}{3}}}{2e^3}-\frac{3dx^{\frac{4}{3}}}{4e^2}+\frac{x^2}{2e}\right)}{3}+\frac{x^2\log\left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(2/3))**n)),x)
```

```
[Out] a*x**2/2 + b*(-e*n*(-3*d**3*Piecewise((x**(2/3)/d, Eq(e, 0)), (log(d + e*x*
*(2/3))/e, True))/(2*e**3) + 3*d**2*x**(2/3)/(2*e**3) - 3*d*x**(4/3)/(4*e**
2) + x**2/(2*e))/3 + x**2*log(c*(d + e*x**(2/3))**n)/2)
```

Giac [A]
time = 4.97, size = 82, normalized size = 0.92

$$\frac{1}{2}\,bx^2\log\left(c\right)+\frac{1}{12}\left(6\,x^2\log\left(x^{\frac{2}{3}}e+d\right)+\left(6\,d^3e^{(-4)}\log\left(\left|x^{\frac{2}{3}}e+d\right|\right)+\left(3\,dx^{\frac{4}{3}}e-2\,x^2e^2-6\,d^2x^{\frac{2}{3}}\right)e^{(-3)}\right)e\right)bn+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2*log(c) + 1/12*(6*x^2*log(x^(2/3)*e + d) + (6*d^3*e^(-4)*log(abs(x
^(2/3)*e + d)) + (3*d*x^(4/3)*e - 2*x^2*e^2 - 6*d^2*x^(2/3))*e^(-3))*e)*b*n
 + 1/2*a*x^2
```

Mupad [B]
time = 0.39, size = 74, normalized size = 0.83

$$\frac{a\,x^2}{2}-\frac{b\,n\,x^2}{6}+\frac{b\,x^2\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)}{2}+\frac{b\,d\,n\,x^{4/3}}{4\,e}+\frac{b\,d^3\,n\,\ln\left(d+e\,x^{2/3}\right)}{2\,e^3}-\frac{b\,d^2\,n\,x^{2/3}}{2\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e*x^(2/3))^n)),x)
```

```
[Out] (a*x^2)/2 - (b*n*x^2)/6 + (b*x^2*log(c*(d + e*x^(2/3))^n))/2 + (b*d*n*x^(4/
3))/(4*e) + (b*d^3*n*log(d + e*x^(2/3)))/(2*e^3) - (b*d^2*n*x^(2/3))/(2*e^2
)
```

3.466 $\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx$

Optimal. Leaf size=72

$$\frac{2bdn\sqrt[3]{x}}{e}+ax-\frac{2bnx}{3}-\frac{2bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{e^{3/2}}+bx\log\left(c\left(d+ex^{2/3}\right)^n\right)$$

```
[Out] 2*b*d*n*x^(1/3)/e+a*x-2/3*b*n*x-2*b*d^(3/2)*n*arctan(x^(1/3)*e^(1/2)/d^(1/2
))/e^(3/2)+b*x*ln(c*(d+e*x^(2/3))^n)
```

Rubi [A]
time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.222$, Rules used = {2498, 348, 308, 211}

$$ax-\frac{2bd^{3/2}n\text{ArcTan}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{e^{3/2}}+bx\log\left(c\left(d+ex^{2/3}\right)^n\right)+\frac{2bdn\sqrt[3]{x}}{e}-\frac{2bnx}{3}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Log[c*(d + e*x^(2/3))^n],x]
```

```
[Out] (2*b*d*n*x^(1/3))/e + a*x - (2*b*n*x)/3 - (2*b*d^(3/2)*n*ArcTan[(Sqrt[e]*x^
(1/3))/Sqrt[d]])/e^(3/2) + b*x*Log[c*(d + e*x^(2/3))^n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
```

```
e, n, p}, x]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx = ax+b\int \log\left(c\left(d+ex^{2/3}\right)^n\right)\,dx$$
$$= ax+bx\log\left(c\left(d+ex^{2/3}\right)^n\right)-\frac{1}{3}(2ben)\int \frac{x^{2/3}}{d+ex^{2/3}}\,dx$$
$$= ax+bx\log\left(c\left(d+ex^{2/3}\right)^n\right)-(2ben)\text{Subst}\left(\int \frac{x^4}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)$$
$$= ax+bx\log\left(c\left(d+ex^{2/3}\right)^n\right)-(2ben)\text{Subst}\left(\int \left(-\frac{d}{e^2}+\frac{x^2}{e}+\frac{d^2}{e^2\,(d+e$$
$$= \frac{2bdn\sqrt[3]{x}}{e}+ax-\frac{2bnx}{3}+bx\log\left(c\left(d+ex^{2/3}\right)^n\right)-\frac{(2bd^2n)\,\text{Subst}\left(\int \frac{}{d+}\right.}{e}$$
$$= \frac{2bdn\sqrt[3]{x}}{e}+ax-\frac{2bnx}{3}-\frac{2bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\,\sqrt[3]{x}}{\sqrt{d}}\right)}{e^{3/2}}+bx\log\left(c\left(d+e\right.\right.$$

Mathematica [A]
time = 0.02, size = 72, normalized size = 1.00

$$\frac{2bdn\sqrt[3]{x}}{e}+ax-\frac{2bnx}{3}-\frac{2bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\,\sqrt[3]{x}}{\sqrt{d}}\right)}{e^{3/2}}+bx\log\left(c\left(d+ex^{2/3}\right)^n\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Log[c*(d + e*x^(2/3))^n],x]
[Out] (2*b*d*n*x^(1/3))/e + a*x - (2*b*n*x)/3 - (2*b*d^(3/2)*n*ArcTan[(Sqrt[e]*x^
(1/3))/Sqrt[d]])/e^(3/2) + b*x*Log[c*(d + e*x^(2/3))^n]
```

Maple [A]
time = 0.03, size = 62, normalized size = 0.86

method	result	size
default	$ax+bx\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^n\right)-\frac{2bnx}{3}+\frac{2bdn\,x^{\frac{1}{3}}}{e}-\frac{2bn\,d^2\arctan\left(\frac{x^{\frac{1}{3}}e}{\sqrt{ed}}\right)}{e\sqrt{ed}}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*ln(c*(d+e*x^(2/3))^n),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*x*ln(c*(d+e*x^(2/3))^n)-2/3*b*n*x+2*b*d*n*x^(1/3)/e-2*b/e*n*d^2/(e*d)
^(1/2)*arctan(x^(1/3)*e/(e*d)^(1/2))
```

Maxima [A]
time = 0.51, size = 61, normalized size = 0.85

$$-\frac{1}{3}\left(2\left(3\,d^{\frac{3}{2}}\arctan\left(\frac{x^{\frac{1}{3}}e^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}+\left(xe-3\,dx^{\frac{1}{3}}\right)e^{\left(-2\right)}\right)ne-3\,x\log\left(\left(x^{\frac{2}{3}}e+d\right)^{n}c\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e*x^(2/3))^n),x, algorithm="maxima")
```

```
[Out] -1/3*(2*(3*d^(3/2)*arctan(x^(1/3)*e^(1/2)/sqrt(d))*e^(-5/2) + (x*e - 3*d*x^
(1/3))*e^(-2))*n*e - 3*x*log((x^(2/3)*e + d)^n*c))*b + a*x
```

Fricas [A]
time = 0.38, size = 218, normalized size = 3.03

$$\left[\frac{1}{3}\left(3\,bnxe\log\left(x^{\frac{2}{3}}e+d\right)+3\,\sqrt{-de^{(-1)}}\,bdn\log\left(\frac{2\,\sqrt{-de^{(-1)}}\,dxe^{2}-d^{3}+x^{2}e^{3}+2\left(d^{2}e-\sqrt{-de^{(-1)}}\,xe^{3}\right)x^{\frac{2}{3}}-2\left(\sqrt{-de^{(-1)}}\,d^{2}e+dxe^{2}\right)x^{\frac{1}{3}}}{d^{3}+x^{2}e^{3}}\right)+3\,bxe\log\left(c\right)+6\,bdnx^{\frac{1}{3}}-(2\,bn-3\,a)xe\right)e^{(-1)},-\frac{1}{3}\left(6\,bd^{\frac{3}{2}}n\arctan\left(\frac{x^{\frac{1}{3}}e^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}-3\,bnxe\log\left(x^{\frac{2}{3}}e+d\right)-3\,bxe\log\left(c\right)-6\,bdnx^{\frac{1}{3}}+(2\,bn-3\,a)xe\right)e^{(-1)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e*x^(2/3))^n),x, algorithm="fricas")
```

```
[Out] [1/3*(3*b*n*x*e*log(x^(2/3)*e + d) + 3*sqrt(-d*e^(-1))*b*d*n*log((2*sqrt(-d
*e^(-1))*d*x*e^2 - d^3 + x^2*e^3 + 2*(d^2*e - sqrt(-d*e^(-1))*x*e^3)*x^(2/3
) - 2*(sqrt(-d*e^(-1))*d^2*e + d*x*e^2)*x^(1/3))/(d^3 + x^2*e^3)) + 3*b*x*e
*log(c) + 6*b*d*n*x^(1/3) - (2*b*n - 3*a)*x*e)*e^(-1), -1/3*(6*b*d^(3/2)*n*
arctan(x^(1/3)*e^(1/2)/sqrt(d))*e^(-1/2) - 3*b*n*x*e*log(x^(2/3)*e + d) - 3
*b*x*e*log(c) - 6*b*d*n*x^(1/3) + (2*b*n - 3*a)*x*e)*e^(-1)]
```

Sympy [A]
time = 2.40, size = 124, normalized size = 1.72

$$ax+b\left(-\frac{2en\left(\begin{cases}\tilde{\infty}x & \text{for}\ d=0\wedge e=0\\ \frac{3x^{\frac{5}{3}}}{5d} & \text{for}\ e=0\\ \frac{x}{e} & \text{for}\ d=0\\ \frac{3d^{2}\log\left(\sqrt[3]{x}-\sqrt{-\frac{d}{e}}\right)}{2e^{3}\sqrt{-\frac{d}{e}}}-\frac{3d^{2}\log\left(\sqrt[3]{x}+\sqrt{-\frac{d}{e}}\right)}{2e^{3}\sqrt{-\frac{d}{e}}}-\frac{3d\sqrt[3]{x}}{e^{2}}+\frac{x}{e} & \text{otherwise}\end{cases}\right)}{3}+x\log\left(c\left(d+ex^{\frac{2}{3}}\right)^{n}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*ln(c*(d+e*x**(2/3))**n),x)
```

```
[Out] a*x + b*(-2*e*n*Piecewise((zoo*x, Eq(d, 0) & Eq(e, 0)), (3*x**(5/3)/(5*d),
Eq(e, 0)), (x/e, Eq(d, 0)), (3*d**2*log(x**(1/3) - sqrt(-d/e))/(2*e**3*sqrt
(-d/e)) - 3*d**2*log(x**(1/3) + sqrt(-d/e))/(2*e**3*sqrt(-d/e)) - 3*d*x**(1
/3)/e**2 + x/e, True))/3 + x*log(c*(d + e*x**(2/3))**n))
```

Giac [A]
time = 4.11, size = 68, normalized size = 0.94

$$-\frac{1}{3}\left(\left(2\left(3\,d^{\frac{3}{2}}\arctan\left(\frac{x^{\frac{1}{3}}e^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}-\left(3\,dx^{\frac{1}{3}}e-xe^{2}\right)e^{\left(-3\right)}\right)e-3\,x\log\left(x^{\frac{2}{3}}e+d\right)\right)n-3\,x\log\left(c\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e*x^(2/3))^n),x, algorithm="giac")
```

```
[Out] -1/3*((2*(3*d^(3/2)*arctan(x^(1/3)*e^(1/2)/sqrt(d))*e^(-5/2) - (3*d*x^(1/3)
*e - x*e^2)*e^(-3))*e - 3*x*log(x^(2/3)*e + d))*n - 3*x*log(c))*b + a*x
```

Mupad [B]
time = 0.39, size = 56, normalized size = 0.78

$$a\,x+b\,x\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)-\frac{2\,b\,n\,x}{3}+\frac{2\,b\,d\,n\,x^{1/3}}{e}-\frac{2\,b\,d^{3/2}\,n\,\mathrm{atan}\left(\frac{\sqrt{e}\,x^{1/3}}{\sqrt{d}}\right)}{e^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*log(c*(d + e*x^(2/3))^n),x)
```

```
[Out] a*x + b*x*log(c*(d + e*x^(2/3))^n) - (2*b*n*x)/3 + (2*b*d*n*x^(1/3))/e - (2
*b*d^(3/2)*n*atan((e^(1/2)*x^(1/3))/d^(1/2)))/e^(3/2)
```

3.467 $\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}\,dx$

Optimal. Leaf size=55

$$\frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\log\left(-\frac{ex^{2/3}}{d}\right)+\frac{3}{2}bn\text{Li}_2\left(1+\frac{ex^{2/3}}{d}\right)$$

```
[Out] 3/2*(a+b*ln(c*(d+e*x^(2/3))^n))*ln(-e*x^(2/3)/d)+3/2*b*n*polylog(2,1+e*x^(2
/3)/d)
```

Rubi [A]
time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2504, 2441, 2352}

$$\frac{3}{2}bn\text{PolyLog}\left(2,\frac{ex^{2/3}}{d}+1\right)+\frac{3}{2}\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])/x,x]
```

```
[Out] (3*(a + b*Log[c*(d + e*x^(2/3))^n])*Log[-((e*x^(2/3))/d)])/2 + (3*b*n*PolyL
og[2, 1 + (e*x^(2/3))/d])/2
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}\,dx = \frac{3}{2}\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x}\,dx, x, x^{2/3}\right)$$
$$= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\log\left(-\frac{ex^{2/3}}{d}\right) - \frac{1}{2}(3ben)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\right.$$
$$= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\log\left(-\frac{ex^{2/3}}{d}\right) + \frac{3}{2}bn\text{Li}_2\left(1+\frac{ex^{2/3}}{d}\right)$$

Mathematica [A]
time = 0.01, size = 55, normalized size = 1.00

$$a\log(x) + \frac{3}{2}b\left(\log\left(c\left(d+ex^{2/3}\right)^n\right)\log\left(-\frac{ex^{2/3}}{d}\right) + n\text{Li}_2\left(\frac{d+ex^{2/3}}{d}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])/x,x]
[Out] a*Log[x] + (3*b*(Log[c*(d + e*x^(2/3))^n]*Log[-((e*x^(2/3))/d)] + n*PolyLog
[2, (d + e*x^(2/3))/d]))/2
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))/x,x)
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))/x,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(47) = 94$.
time = 0.41, size = 128, normalized size = 2.33

$$-\frac{3}{2}\left(2\log\left(x^{\frac{1}{3}}\right)\log\left(\frac{e^{\left(\frac{2}{3}\log(x)+1\right)}}{d}+1\right) + \text{Li}_2\left(-\frac{e^{\left(\frac{2}{3}\log(x)+1\right)}}{d}\right)\right)bn + \frac{2\,bdn\log\left(x^{\frac{2}{3}}e+d\right)\log(x) + 2\left(bd\log(c)+ad\right)\log(x) - \frac{2\,bnxe\log(x)-3\,bnxe}{x^{\frac{1}{3}}}}{2\,d} + \frac{3\left(2\,bne^{\left(\frac{2}{3}\log(x)+1\right)}\log\left(x^{\frac{1}{3}}\right) - bne^{\left(\frac{2}{3}\log(x)+1\right)}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x,x, algorithm="maxima")
[Out] -3/2*(2*log(x^(1/3))*log(e^(2/3*log(x) + 1)/d + 1) + dilog(-e^(2/3*log(x) +
 1)/d))*b*n + 1/2*(2*b*d*n*log(x^(2/3)*e + d)*log(x) + 2*(b*d*log(c) + a*d)
```

```
*log(x) - (2*b*n*x*e*log(x) - 3*b*n*x*e)/x^(1/3))/d + 3/2*(2*b*n*e^(2/3*log
(x) + 1)*log(x^(1/3)) - b*n*e^(2/3*log(x) + 1))/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x,x, algorithm="fricas")
[Out] integral((b*log((x^(2/3)*e + d)^n*c) + a)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\log\left(c\left(d + ex^{\frac{2}{3}}\right)^{n}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))/x,x)
[Out] Integral((a + b*log(c*(d + e*x**(2/3))**n))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x,x, algorithm="giac")
[Out] integrate((b*log((x^(2/3)*e + d)^n*c) + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + b\,\ln\left(c\,{\left(d + e\,x^{2/3}\right)}^{n}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))/x,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))/x, x)
```

3.468 $\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^2}\,dx$

Optimal. Leaf size=68

$$-\frac{2ben}{d\sqrt[3]{x}}-\frac{2be^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}$$

```
[Out] -2*b*e*n/d/x^(1/3)-2*b*e^(3/2)*n*arctan(x^(1/3)*e^(1/2)/d^(1/2))/d^(3/2)+(-
a-b*ln(c*(d+e*x^(2/3))^n))/x
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2505, 348, 331, 211}

$$-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}-\frac{2be^{3/2}n\text{ArcTan}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2ben}{d\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])/x^2,x]
```

```
[Out] (-2*b*e*n)/(d*x^(1/3)) - (2*b*e^(3/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/
d^(3/2) - (a + b*Log[c*(d + e*x^(2/3))^n])/x
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^2}\,dx &= -\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}+\frac{1}{3}(2ben)\int \frac{1}{\left(d+ex^{2/3}\right)x^{4/3}}\,dx\\ &= -\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}+(2ben)\text{Subst}\left(\int \frac{1}{x^2\left(d+ex^2\right)}\,dx,x,\sqrt[3]{x}\right)\\ &= -\frac{2ben}{d\sqrt[3]{x}}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}-\frac{\left(2be^2n\right)\text{Subst}\left(\int \frac{1}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{d}\\ &= -\frac{2ben}{d\sqrt[3]{x}}-\frac{2be^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.01, size = 59, normalized size = 0.87

$$-\frac{a}{x}-\frac{2ben\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{ex^{2/3}}{d}\right)}{d\sqrt[3]{x}}-\frac{b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])/x^2,x]
[Out] -(a/x) - (2*b*e*n*Hypergeometric2F1[-1/2, 1, 1/2, -((e*x^(2/3))/d)])/(d*x^(
1/3)) - (b*Log[c*(d + e*x^(2/3))^n])/x
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))/x^2,x)
```

Maxima [A]
time = 0.50, size = 56, normalized size = 0.82

$$-2\,bn\left(\frac{\arctan\left(\frac{x^{\frac{1}{3}}e^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}}{d^{\frac{3}{2}}}+\frac{1}{dx^{\frac{1}{3}}}\right)e-\frac{b\log\left(\left(x^{\frac{2}{3}}e+d\right)^{n}c\right)}{x}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^2,x, algorithm="maxima")
```

```
[Out] -2*b*n*(arctan(x^(1/3)*e^(1/2)/sqrt(d))*e^(1/2)/d^(3/2) + 1/(d*x^(1/3)))*e
 - b*log((x^(2/3)*e + d)^n*c)/x - a/x
```

Fricas [A]
time = 0.38, size = 210, normalized size = 3.09

$$\left[\frac{bnx\sqrt{-\frac{e}{d}}\,e\log\left(\frac{2\,d^{2}x\sqrt{-\frac{e}{d}}\,e-d^{3}+x^{2}e^{3}-2\left(dx\sqrt{-\frac{e}{d}}\,e^{2}-d^{2}e\right)x^{\frac{2}{3}}-2\left(d^{3}\sqrt{-\frac{e}{d}}+dxe^{2}\right)x^{\frac{1}{3}}}{d^{3}+x^{2}e^{3}}\right)-bdn\log\left(x^{\frac{2}{3}}e+d\right)-2\,bnx^{\frac{2}{3}}e-bd\log\left(c\right)-ad}{dx},\ -\frac{\frac{2\,bnx\arctan\left(\frac{x^{\frac{1}{3}}e^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{3}{2}}}{\sqrt{d}}+bdn\log\left(x^{\frac{2}{3}}e+d\right)+2\,bnx^{\frac{2}{3}}e+bd\log\left(c\right)+ad}{dx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^2,x, algorithm="fricas")
```

```
[Out] [(b*n*x*sqrt(-e/d)*e*log((2*d^2*x*sqrt(-e/d)*e - d^3 + x^2*e^3 - 2*(d*x*sqr
t(-e/d)*e^2 - d^2*e)*x^(2/3) - 2*(d^3*sqrt(-e/d) + d*x*e^2)*x^(1/3))/(d^3 +
 x^2*e^3)) - b*d*n*log(x^(2/3)*e + d) - 2*b*n*x^(2/3)*e - b*d*log(c) - a*d)
/(d*x), -(2*b*n*x*arctan(x^(1/3)*e^(1/2)/sqrt(d))*e^(3/2)/sqrt(d) + b*d*n*l
og(x^(2/3)*e + d) + 2*b*n*x^(2/3)*e + b*d*log(c) + a*d)/(d*x)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))/x**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.90, size = 61, normalized size = 0.90

$$-\left(2\left(\frac{\arctan\left(\frac{x^{\frac{1}{3}}e^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}}{d^{\frac{3}{2}}}+\frac{1}{dx^{\frac{1}{3}}}\right)e+\frac{\log\left(x^{\frac{2}{3}}e+d\right)}{x}\right)bn-\frac{b\log\left(c\right)}{x}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^2,x, algorithm="giac")
```

```
[Out] -(2*(arctan(x^(1/3)*e^(1/2)/sqrt(d))*e^(1/2)/d^(3/2) + 1/(d*x^(1/3)))*e + l
og(x^(2/3)*e + d)/x)*b*n - b*log(c)/x - a/x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))/x^2,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))/x^2, x)
```

3.469 $\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^3}\,dx$

Optimal. Leaf size=94

$$-\frac{ben}{4dx^{4/3}}+\frac{be^2n}{2d^2x^{2/3}}-\frac{be^3n\log\left(d+ex^{2/3}\right)}{2d^3}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{2x^2}+\frac{be^3n\log(x)}{3d^3}$$

```
[Out] -1/4*b*e*n/d/x^(4/3)+1/2*b*e^2*n/d^2/x^(2/3)-1/2*b*e^3*n*ln(d+e*x^(2/3))/d^
3+1/2*(-a-b*ln(c*(d+e*x^(2/3))^n))/x^2+1/3*b*e^3*n*ln(x)/d^3
```

Rubi [A]
time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2504, 2442, 46}

$$-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{2x^2}-\frac{be^3n\log\left(d+ex^{2/3}\right)}{2d^3}+\frac{be^3n\log(x)}{3d^3}+\frac{be^2n}{2d^2x^{2/3}}-\frac{ben}{4dx^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])/x^3,x]
```

```
[Out] -1/4*(b*e*n)/(d*x^(4/3)) + (b*e^2*n)/(2*d^2*x^(2/3)) - (b*e^3*n*Log[d + e*x
^(2/3)])/(2*d^3) - (a + b*Log[c*(d + e*x^(2/3))^n])/(2*x^2) + (b*e^3*n*Log[
x])/(3*d^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
```

```
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{a+b\log \left(c\left(d+ex^{2/3}\right)^n\right)}{x^3}\,dx = \frac{3}{2}\text{Subst}\left(\int \frac{a+b\log \left(c(d+ex)^n\right)}{x^4}\,dx,x,x^{2/3}\right)$$
$$= -\frac{a+b\log \left(c\left(d+ex^{2/3}\right)^n\right)}{2x^2}+\frac{1}{2}(ben)\text{Subst}\left(\int \frac{1}{x^3(d+ex)}\,dx,x,x^{2/3}\right)$$
$$= -\frac{a+b\log \left(c\left(d+ex^{2/3}\right)^n\right)}{2x^2}+\frac{1}{2}(ben)\text{Subst}\left(\int \left(\frac{1}{dx^3}-\frac{e}{d^2x^2}+\frac{e^2}{d^3x}-\frac{}{d^3}\right.\right.$$
$$= -\frac{ben}{4dx^{4/3}}+\frac{be^2n}{2d^2x^{2/3}}-\frac{be^3n\log \left(d+ex^{2/3}\right)}{2d^3}-\frac{a+b\log \left(c\left(d+ex^{2/3}\right)^n\right)}{2x^2}+$$

Mathematica [A]
time = 0.03, size = 91, normalized size = 0.97

$$-\frac{a}{2x^2}-\frac{b\log \left(c\left(d+ex^{2/3}\right)^n\right)}{2x^2}+\frac{1}{2}ben\left(-\frac{1}{2dx^{4/3}}+\frac{e}{d^2x^{2/3}}-\frac{e^2\log \left(d+ex^{2/3}\right)}{d^3}+\frac{2e^2\log(x)}{3d^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])/x^3,x]
[Out] -1/2*a/x^2 - (b*Log[c*(d + e*x^(2/3))^n])/(2*x^2) + (b*e*n*(-1/2*1/(d*x^(4/
3)) + e/(d^2*x^(2/3)) - (e^2*Log[d + e*x^(2/3)])/d^3 + (2*e^2*Log[x])/(3*d^
3)))/2
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{a+b\ln \left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))/x^3,x)
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))/x^3,x)
```

Maxima [A]
time = 0.27, size = 79, normalized size = 0.84

$$-\frac{1}{4}\,bn\left(\frac{2\,e^2\log \left(x^{\frac{2}{3}}e+d\right)}{d^3}-\frac{2\,e^2\log \left(x^{\frac{2}{3}}\right)}{d^3}-\frac{2\,x^{\frac{2}{3}}e-d}{d^2x^{\frac{4}{3}}}\right)e-\frac{b\log \left(\left(x^{\frac{2}{3}}e+d\right)^{n}c\right)}{2\,x^2}-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*b*n*(2*e^2*log(x^(2/3)*e + d)/d^3 - 2*e^2*log(x^(2/3))/d^3 - (2*x^(2/3
)*e - d)/(d^2*x^(4/3)))*e - 1/2*b*log((x^(2/3)*e + d)^n*c)/x^2 - 1/2*a/x^2
```

Fricas [A]
time = 0.37, size = 84, normalized size = 0.89

$$\frac{4\,bnx^2e^3\log\left(x^{\frac{1}{3}}\right)-bd^2nx^{\frac{2}{3}}e+2\,bdnx^{\frac{4}{3}}e^2-2\,bd^3\log\left(c\right)-2\,ad^3-2\,\left(bd^3n+bnx^2e^3\right)\log\left(x^{\frac{2}{3}}e+d\right)}{4\,d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b*n*x^2*e^3*log(x^(1/3)) - b*d^2*n*x^(2/3)*e + 2*b*d*n*x^(4/3)*e^2 -
 2*b*d^3*log(c) - 2*a*d^3 - 2*(b*d^3*n + b*n*x^2*e^3)*log(x^(2/3)*e + d))/(
d^3*x^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))/x**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.21, size = 95, normalized size = 1.01

$$\frac{1}{4}\left(\left(\frac{2\,\log\left(x^{\frac{2}{3}}e\right)}{d^3}-\frac{2\,\log\left(\left|x^{\frac{2}{3}}e+d\right|\right)}{d^3}+\frac{\left(2\left(x^{\frac{2}{3}}e+d\right)d-3\,d^2\right)e^{(-2)}}{d^3x^{\frac{4}{3}}}\right)e^4-\frac{2\,e\log\left(x^{\frac{2}{3}}e+d\right)}{x^2}\right)bne^{(-1)}-\frac{b\log\left(c\right)}{2\,x^2}-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^3,x, algorithm="giac")
```

```
[Out] 1/4*((2*log(x^(2/3)*e)/d^3 - 2*log(abs(x^(2/3)*e + d))/d^3 + (2*(x^(2/3)*e
+ d)*d - 3*d^2)*e^(-2)/(d^3*x^(4/3)))*e^4 - 2*e*log(x^(2/3)*e + d)/x^2)*b*n
*e^(-1) - 1/2*b*log(c)/x^2 - 1/2*a/x^2
```

Mupad [B]
time = 0.61, size = 74, normalized size = 0.79

$$-\frac{\frac{b\,e\,n}{2\,d}-\frac{b\,e^2\,n\,x^{2/3}}{d^2}}{2\,x^{4/3}}-\frac{a}{2\,x^2}-\frac{b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)}{2\,x^2}-\frac{b\,e^3\,n\,\mathrm{atanh}\left(\frac{2\,e\,x^{2/3}}{d}+1\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))/x^3,x)
```

```
[Out] - ((b*e*n)/(2*d) - (b*e^2*n*x^(2/3))/d^2)/(2*x^(4/3)) - a/(2*x^2) - (b*log(
c*(d + e*x^(2/3))^n))/(2*x^2) - (b*e^3*n*atanh((2*e*x^(2/3))/d + 1))/d^3
```

3.470 $\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^4}\,dx$

Optimal. Leaf size=123

$$-\frac{2ben}{21dx^{7/3}}+\frac{2be^2n}{15d^2x^{5/3}}-\frac{2be^3n}{9d^3x}+\frac{2be^4n}{3d^4\sqrt[3]{x}}+\frac{2be^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3d^{9/2}}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}$$

```
[Out] -2/21*b*e*n/d/x^(7/3)+2/15*b*e^2*n/d^2/x^(5/3)-2/9*b*e^3*n/d^3/x+2/3*b*e^4*
n/d^4/x^(1/3)+2/3*b*e^(9/2)*n*arctan(x^(1/3)*e^(1/2)/d^(1/2))/d^(9/2)+1/3*(
-a-b*ln(c*(d+e*x^(2/3))^n))/x^3
```

Rubi [A]
time = 0.05, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2505, 348, 331, 211}

$$-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}+\frac{2be^{9/2}n\text{ArcTan}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3d^{9/2}}+\frac{2be^4n}{3d^4\sqrt[3]{x}}-\frac{2be^3n}{9d^3x}+\frac{2be^2n}{15d^2x^{5/3}}-\frac{2ben}{21dx^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])/x^4,x]
```

```
[Out] (-2*b*e*n)/(21*d*x^(7/3)) + (2*b*e^2*n)/(15*d^2*x^(5/3)) - (2*b*e^3*n)/(9*d
^3*x) + (2*b*e^4*n)/(3*d^4*x^(1/3)) + (2*b*e^(9/2)*n*ArcTan[(Sqrt[e]*x^(1/3
))/Sqrt[d]])/(3*d^(9/2)) - (a + b*Log[c*(d + e*x^(2/3))^n])/(3*x^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
```

```
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^4}\,dx = -\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}+\frac{1}{9}(2ben)\int \frac{1}{\left(d+ex^{2/3}\right)x^{10/3}}\,dx$$

$$= -\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}+\frac{1}{3}(2ben)\mathrm{Subst}\left(\int \frac{1}{x^8\left(d+ex^2\right)}\,dx,x,\sqrt[3]{x}\right)$$

$$= -\frac{2ben}{21dx^{7/3}}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}-\frac{\left(2be^2n\right)\mathrm{Subst}\left(\int \frac{1}{x^6(d+ex^2)}\,dx,x,\right.}{3d}$$

$$= -\frac{2ben}{21dx^{7/3}}+\frac{2be^2n}{15d^2x^{5/3}}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}+\frac{\left(2be^3n\right)\mathrm{Subst}\left(\int \frac{}{x^4(}\right.}{3d^2}$$

$$= -\frac{2ben}{21dx^{7/3}}+\frac{2be^2n}{15d^2x^{5/3}}-\frac{2be^3n}{9d^3x}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}-\frac{\left(2be^4n\right)\mathrm{Sul}}{}$$

$$= -\frac{2ben}{21dx^{7/3}}+\frac{2be^2n}{15d^2x^{5/3}}-\frac{2be^3n}{9d^3x}+\frac{2be^4n}{3d^4\sqrt[3]{x}}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}+$$

$$= -\frac{2ben}{21dx^{7/3}}+\frac{2be^2n}{15d^2x^{5/3}}-\frac{2be^3n}{9d^3x}+\frac{2be^4n}{3d^4\sqrt[3]{x}}+\frac{2be^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3d^{9/2}}-$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.01, size = 65, normalized size = 0.53

$$-\frac{a}{3x^3}-\frac{2ben\,{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};-\frac{ex^{2/3}}{d}\right)}{21dx^{7/3}}-\frac{b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])/x^4,x]
[Out] -1/3*a/x^3 - (2*b*e*n*Hypergeometric2F1[-7/2, 1, -5/2, -((e*x^(2/3))/d)])/(
21*d*x^(7/3)) - (b*Log[c*(d + e*x^(2/3))^n])/(3*x^3)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))/x^4,x)
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))/x^4,x)
```

Maxima [A]
time = 0.51, size = 88, normalized size = 0.72

$$\frac{2}{315}\,bn\left(\frac{105\arctan\left(\frac{x^{\frac{1}{3}}e^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{7}{2}}}{d^{\frac{9}{2}}}+\frac{21\,d^{2}x^{\frac{2}{3}}e-35\,dx^{\frac{4}{3}}e^{2}-15\,d^{3}+105\,x^{2}e^{3}}{d^{4}x^{\frac{7}{3}}}\right)e-\frac{b\log\left(\left(x^{\frac{2}{3}}e+d\right)^{n}c\right)}{3\,x^{3}}-\frac{a}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^4,x, algorithm="maxima")
[Out] 2/315*b*n*(105*arctan(x^(1/3)*e^(1/2)/sqrt(d))*e^(7/2)/d^(9/2) + (21*d^2*x^
(2/3)*e - 35*d*x^(4/3)*e^2 - 15*d^3 + 105*x^2*e^3)/(d^4*x^(7/3)))*e - 1/3*b
*log((x^(2/3)*e + d)^n*c)/x^3 - 1/3*a/x^3
```

Fricas [A]
time = 0.40, size = 303, normalized size = 2.46

$$\left[\frac{105\,bnx^{3}\sqrt{-\frac{e}{d}}\,e^{4}\log\left(-\frac{2\,d^{2}x\sqrt{-\frac{e}{d}}\,e+d^{3}-x^{2}e^{3}-2\left(dx\sqrt{-\frac{e}{d}}\,e^{2}+d^{2}e\right)x^{\frac{2}{3}}-2\left(d^{3}\sqrt{-\frac{e}{d}}-dxe^{2}\right)x^{\frac{1}{3}}}{d^{3}+x^{2}e^{3}}\right)-105\,bd^{4}n\log\left(x^{\frac{2}{3}}e+d\right)+42\,bd^{2}nx^{\frac{4}{3}}e^{2}-70\,bdnx^{2}e^{3}-105\,bd^{4}\log\left(c\right)-105\,ad^{4}-30\left(bd^{3}ne-7\,bnx^{2}e^{4}\right)x^{\frac{2}{3}}}{315\,d^{4}x^{3}},\ -\frac{105\,bd^{4}n\log\left(x^{\frac{2}{3}}e+d\right)-\frac{210\,bnx^{3}\arctan\left(\frac{x^{\frac{1}{3}}e^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{9}{2}}}{\sqrt{d}}-42\,bd^{2}nx^{\frac{4}{3}}e^{2}+70\,bdnx^{2}e^{3}+105\,bd^{4}\log\left(c\right)+105\,ad^{4}+30\left(bd^{3}ne-7\,bnx^{2}e^{4}\right)x^{\frac{2}{3}}}{315\,d^{4}x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^4,x, algorithm="fricas")
[Out] [1/315*(105*b*n*x^3*sqrt(-e/d)*e^4*log(-(2*d^2*x*sqrt(-e/d)*e + d^3 - x^2*e
^3 - 2*(d*x*sqrt(-e/d)*e^2 + d^2*e)*x^(2/3) - 2*(d^3*sqrt(-e/d) - d*x*e^2)*
x^(1/3))/(d^3 + x^2*e^3)) - 105*b*d^4*n*log(x^(2/3)*e + d) + 42*b*d^2*n*x^(
4/3)*e^2 - 70*b*d*n*x^2*e^3 - 105*b*d^4*log(c) - 105*a*d^4 - 30*(b*d^3*n*e
- 7*b*n*x^2*e^4)*x^(2/3))/(d^4*x^3), -1/315*(105*b*d^4*n*log(x^(2/3)*e + d)
 - 210*b*n*x^3*arctan(x^(1/3)*e^(1/2)/sqrt(d))*e^(9/2)/sqrt(d) - 42*b*d^2*n
*x^(4/3)*e^2 + 70*b*d*n*x^2*e^3 + 105*b*d^4*log(c) + 105*a*d^4 + 30*(b*d^3*
n*e - 7*b*n*x^2*e^4)*x^(2/3))/(d^4*x^3)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))/x**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [A]
time = 4.92, size = 94, normalized size = 0.76

$$\frac{1}{315}\left(2\left(\frac{105\arctan\left(\frac{x^{\frac{1}{3}}e^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{7}{2}}}{d^{\frac{9}{2}}}+\frac{21\,d^2x^{\frac{2}{3}}e-35\,dx^{\frac{4}{3}}e^2-15\,d^3+105\,x^2e^3}{d^4x^{\frac{7}{3}}}\right)e-\frac{105\,\log\left(x^{\frac{2}{3}}e+d\right)}{x^3}\right)bn-\frac{b\log\left(c\right)}{3\,x^3}-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^4,x, algorithm="giac")
```

```
[Out] 1/315*(2*(105*arctan(x^(1/3)*e^(1/2)/sqrt(d))*e^(7/2)/d^(9/2) + (21*d^2*x^(
2/3)*e - 35*d*x^(4/3)*e^2 - 15*d^3 + 105*x^2*e^3)/(d^4*x^(7/3)))*e - 105*lo
g(x^(2/3)*e + d)/x^3)*b*n - 1/3*b*log(c)/x^3 - 1/3*a/x^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))/x^4,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))/x^4, x)
```

3.471 $\int x^3 \left(a+b \log \left(c \left(d+e x^{2/3}\right)^n\right)\right)^2 \, dx$

Optimal. Leaf size=482

$$\frac{15 b^2 d^4 n^2 \left(d+e x^{2/3}\right)^2}{8 e^6}-\frac{10 b^2 d^3 n^2 \left(d+e x^{2/3}\right)^3}{9 e^6}+\frac{15 b^2 d^2 n^2 \left(d+e x^{2/3}\right)^4}{32 e^6}-\frac{3 b^2 d n^2 \left(d+e x^{2/3}\right)^5}{25 e^6}+\frac{b^2 n^2 \left(d+e x^{2/3}\right)^6}{72 e^6}$$

```
[Out] 15/8*b^2*d^4*n^2*(d+e*x^(2/3))^2/e^6-10/9*b^2*d^3*n^2*(d+e*x^(2/3))^3/e^6+1
5/32*b^2*d^2*n^2*(d+e*x^(2/3))^4/e^6-3/25*b^2*d*n^2*(d+e*x^(2/3))^5/e^6+1/7
2*b^2*n^2*(d+e*x^(2/3))^6/e^6-3*b^2*d^5*n^2*x^(2/3)/e^5+1/4*b^2*d^6*n^2*ln(
d+e*x^(2/3))^2/e^6+3*b*d^5*n*(d+e*x^(2/3))*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6-
15/4*b*d^4*n*(d+e*x^(2/3))^2*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6+10/3*b*d^3*n*(
d+e*x^(2/3))^3*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6-15/8*b*d^2*n*(d+e*x^(2/3))^4
*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6+3/5*b*d*n*(d+e*x^(2/3))^5*(a+b*ln(c*(d+e*x
^(2/3))^n))/e^6-1/12*b*n*(d+e*x^(2/3))^6*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6-1/
2*b*d^6*n*ln(d+e*x^(2/3))*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6+1/4*x^4*(a+b*ln(c
*(d+e*x^(2/3))^n))^2
```

Rubi [A]
time = 0.33, antiderivative size = 482, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2504, 2445, 2458, 45, 2372, 12, 14, 2338}

$$-\frac{b d^6 n \log \left(d+e x^{2/3}\right) \left(a+b \log \left(c \left(d+e x^{2/3}\right)^n\right)\right)}{2 e^6}+\frac{3 b d^5 n \left(d+e x^{2/3}\right) \left(a+b \log \left(c \left(d+e x^{2/3}\right)^n\right)\right)}{e^6}-\frac{15 b d^4 n \left(d+e x^{2/3}\right)^2 \left(a+b \log \left(c \left(d+e x^{2/3}\right)^n\right)\right)}{4 e^6}+\frac{10 b d^3 n \left(d+e x^{2/3}\right)^3 \left(a+b \log \left(c \left(d+e x^{2/3}\right)^n\right)\right)}{3 e^6}-\frac{15 b d^2 n \left(d+e x^{2/3}\right)^4 \left(a+b \log \left(c \left(d+e x^{2/3}\right)^n\right)\right)}{8 e^6}+\frac{3 b d n \left(d+e x^{2/3}\right)^5 \left(a+b \log \left(c \left(d+e x^{2/3}\right)^n\right)\right)}{5 e^6}-\frac{b n \left(d+e x^{2/3}\right)^6 \left(a+b \log \left(c \left(d+e x^{2/3}\right)^n\right)\right)}{12 e^6}+\frac{1}{4} x^4 \left(a+b \log \left(c \left(d+e x^{2/3}\right)^n\right)\right)^2+\frac{b^2 d^6 n^2 \log ^2\left(d+e x^{2/3}\right)}{4 e^6}-\frac{3 b^2 d^5 n^2 x^{2/3}}{e^5}+\frac{15 b^2 d^4 n^2 \left(d+e x^{2/3}\right)^2}{8 e^6}-\frac{10 b^2 d^3 n^2 \left(d+e x^{2/3}\right)^3}{9 e^6}+\frac{15 b^2 d^2 n^2 \left(d+e x^{2/3}\right)^4}{32 e^6}-\frac{3 b^2 d n^2 \left(d+e x^{2/3}\right)^5}{25 e^6}+\frac{b^2 n^2 \left(d+e x^{2/3}\right)^6}{72 e^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

```
[Out] (15*b^2*d^4*n^2*(d + e*x^(2/3))^2)/(8*e^6) - (10*b^2*d^3*n^2*(d + e*x^(2/3)
)^3)/(9*e^6) + (15*b^2*d^2*n^2*(d + e*x^(2/3))^4)/(32*e^6) - (3*b^2*d*n^2*(
d + e*x^(2/3))^5)/(25*e^6) + (b^2*n^2*(d + e*x^(2/3))^6)/(72*e^6) - (3*b^2*
d^5*n^2*x^(2/3))/e^5 + (b^2*d^6*n^2*Log[d + e*x^(2/3)]^2)/(4*e^6) + (3*b*d^
5*n*(d + e*x^(2/3))*(a + b*Log[c*(d + e*x^(2/3))^n]))/e^6 - (15*b*d^4*n*(d
+ e*x^(2/3))^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/(4*e^6) + (10*b*d^3*n*(d +
 e*x^(2/3))^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/(3*e^6) - (15*b*d^2*n*(d +
e*x^(2/3))^4*(a + b*Log[c*(d + e*x^(2/3))^n]))/(8*e^6) + (3*b*d*n*(d + e*x^
(2/3))^5*(a + b*Log[c*(d + e*x^(2/3))^n]))/(5*e^6) - (b*n*(d + e*x^(2/3))^6
*(a + b*Log[c*(d + e*x^(2/3))^n]))/(12*e^6) - (b*d^6*n*Log[d + e*x^(2/3)]*(
a + b*Log[c*(d + e*x^(2/3))^n]))/(2*e^6) + (x^4*(a + b*Log[c*(d + e*x^(2/3)
)^n])^2)/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx=\frac{3}{2}\text{Subst}\left(\int x^5(a+b\log\left(c(d+ex)^n\right))^2\,dx,x,x^{2/3}\right)$$

$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-\frac{1}{2}(ben)\text{Subst}\left(\int\frac{x^6(a+b\log(c}{d+e}\right.$$

$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-\frac{1}{2}(bn)\text{Subst}\left(\int\frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^6(a+}{x}\right.$$

$$=\frac{1}{120}bn\left(\frac{360d^5\left(d+ex^{2/3}\right)}{e^6}-\frac{450d^4\left(d+ex^{2/3}\right)^2}{e^6}+\frac{400d^3\left(d+ex^{2/3}\right)}{e^6}\right.$$

$$=\frac{1}{120}bn\left(\frac{360d^5\left(d+ex^{2/3}\right)}{e^6}-\frac{450d^4\left(d+ex^{2/3}\right)^2}{e^6}+\frac{400d^3\left(d+ex^{2/3}\right)}{e^6}\right.$$

$$=\frac{1}{120}bn\left(\frac{360d^5\left(d+ex^{2/3}\right)}{e^6}-\frac{450d^4\left(d+ex^{2/3}\right)^2}{e^6}+\frac{400d^3\left(d+ex^{2/3}\right)}{e^6}\right.$$

$$=\frac{15b^2d^4n^2\left(d+ex^{2/3}\right)^2}{8e^6}-\frac{10b^2d^3n^2\left(d+ex^{2/3}\right)^3}{9e^6}+\frac{15b^2d^2n^2\left(d+ex^{2/}\right.}{32e^6}$$

$$=\frac{15b^2d^4n^2\left(d+ex^{2/3}\right)^2}{8e^6}-\frac{10b^2d^3n^2\left(d+ex^{2/3}\right)^3}{9e^6}+\frac{15b^2d^2n^2\left(d+ex^{2/}\right.}{32e^6}$$

Mathematica [A]
time = 0.26, size = 328, normalized size = 0.68

$$\frac{ex^{2/3}\left(1800a^2e^5x^{10/3}+60abn\left(60d^5-30d^4ex^{2/3}+20d^3e^2x^{4/3}-15d^2e^3x^2+12de^4x^{8/3}-10e^5x^{10/3}\right)+b^2n^2\left(-8820d^5+2610d^4ex^{2/3}-1140d^3e^2x^{4/3}+555d^2e^3x^2-264de^4x^{8/3}+100e^5x^{10/3}\right)\right)+5220b^2d^6n^2\log\left(d+ex^{2/3}\right)+60b\left(bn\left(60d^6+60d^5ex^{2/3}-30d^4e^2x^{4/3}+20d^3e^3x^2-15d^2e^4x^{8/3}+12de^5x^{10/3}-10e^6x^4\right)-60a(d^6-e^6x^4)\right)\log\left(c\left(d+ex^{2/3}\right)^n\right)-1800b^2(d^6-e^6x^4)\log^2\left(c\left(d+ex^{2/3}\right)^n\right)}{7200e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

```
[Out] (e*x^(2/3)*(1800*a^2*e^5*x^(10/3) + 60*a*b*n*(60*d^5 - 30*d^4*e*x^(2/3) + 2
0*d^3*e^2*x^(4/3) - 15*d^2*e^3*x^2 + 12*d*e^4*x^(8/3) - 10*e^5*x^(10/3)) +
b^2*n^2*(-8820*d^5 + 2610*d^4*e*x^(2/3) - 1140*d^3*e^2*x^(4/3) + 555*d^2*e^
3*x^2 - 264*d*e^4*x^(8/3) + 100*e^5*x^(10/3))) + 5220*b^2*d^6*n^2*Log[d + e
*x^(2/3)] + 60*b*(b*n*(60*d^6 + 60*d^5*e*x^(2/3) - 30*d^4*e^2*x^(4/3) + 20*
```

```
d^3*e^3*x^2 - 15*d^2*e^4*x^(8/3) + 12*d*e^5*x^(10/3) - 10*e^6*x^4) - 60*a*(
d^6 - e^6*x^4))*Log[c*(d + e*x^(2/3))^n] - 1800*b^2*(d^6 - e^6*x^4)*Log[c*(
d + e*x^(2/3))^n]^2)/(7200*e^6)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^{3} \left(a +b \ln \left(c \left(d +e \,x^{\frac{2}{3}}\right)^{n}\right)\right)^{2}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(2/3))^n))^2,x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e*x^(2/3))^n))^2,x)
```

Maxima [A]
time = 0.32, size = 323, normalized size = 0.67

$$\frac{1}{4} b^{2} x^{4} \log \left(\left(x^{\frac{2}{3}} e + d\right)^{n} c\right)^{2} + \frac{1}{2} a b x^{4} \log \left(\left(x^{\frac{2}{3}} e + d\right)^{n} c\right) + \frac{1}{4} a^{2} x^{4} - \frac{1}{120} \left(60 d^{6} e^{(-7)} \log \left(x^{\frac{2}{3}} e + d\right) + \left(30 d^{4} x^{\frac{4}{3}} e - 20 d^{3} x^{2} e^{2} - 60 d^{5} x^{\frac{2}{3}} + 15 d^{2} x^{\frac{8}{3}} e^{3} - 12 d x^{\frac{10}{3}} e^{4} + 10 x^{4} e^{5}\right) e^{(-6)}\right) a b n e + \frac{1}{7200} \left(\left(1800 d^{6} \log \left(x^{\frac{2}{3}} e + d\right)^{2} + 8820 d^{6} \log \left(x^{\frac{2}{3}} e + d\right) - 8820 d^{5} x^{\frac{2}{3}} e + 2610 d^{4} x^{\frac{4}{3}} e^{2} - 1140 d^{3} x^{2} e^{3} + 555 d^{2} x^{\frac{8}{3}} e^{4} - 264 d x^{\frac{10}{3}} e^{5} + 100 x^{4} e^{6}\right) n^{2} e^{(-6)} - 60 \left(60 d^{6} e^{(-7)} \log \left(x^{\frac{2}{3}} e + d\right) + \left(30 d^{4} x^{\frac{4}{3}} e - 20 d^{3} x^{2} e^{2} - 60 d^{5} x^{\frac{2}{3}} + 15 d^{2} x^{\frac{8}{3}} e^{3} - 12 d x^{\frac{10}{3}} e^{4} + 10 x^{4} e^{5}\right) e^{(-6)}\right) n e \log \left(\left(x^{\frac{2}{3}} e + d\right)^{n} c\right)\right) b^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*log((x^(2/3)*e + d)^n*c)^2 + 1/2*a*b*x^4*log((x^(2/3)*e + d)^n*
c) + 1/4*a^2*x^4 - 1/120*(60*d^6*e^(-7)*log(x^(2/3)*e + d) + (30*d^4*x^(4/3
)*e - 20*d^3*x^2*e^2 - 60*d^5*x^(2/3) + 15*d^2*x^(8/3)*e^3 - 12*d*x^(10/3)*
e^4 + 10*x^4*e^5)*e^(-6))*a*b*n*e + 1/7200*((1800*d^6*log(x^(2/3)*e + d)^2
+ 8820*d^6*log(x^(2/3)*e + d) - 8820*d^5*x^(2/3)*e + 2610*d^4*x^(4/3)*e^2 -
 1140*d^3*x^2*e^3 + 555*d^2*x^(8/3)*e^4 - 264*d*x^(10/3)*e^5 + 100*x^4*e^6)
*n^2*e^(-6) - 60*(60*d^6*e^(-7)*log(x^(2/3)*e + d) + (30*d^4*x^(4/3)*e - 20
*d^3*x^2*e^2 - 60*d^5*x^(2/3) + 15*d^2*x^(8/3)*e^3 - 12*d*x^(10/3)*e^4 + 10
*x^4*e^5)*e^(-6))*n*e*log((x^(2/3)*e + d)^n*c))*b^2
```

Fricas [A]
time = 0.44, size = 473, normalized size = 0.98

$$\frac{1}{7200} \left(1800 b^{2} x^{4} e^{6} \log \left(c\right)^{2} + 100 \left(b^{2} n^{2} - 6 a b n + 18 a^{2}\right) x^{4} e^{6} - 60 \left(19 b^{2} d^{3} n^{2} - 20 a b d^{3} n\right) x^{2} e^{3} - 1800 \left(b^{2} d^{6} n^{2} - b^{2} n^{2} x^{4} e^{6}\right) \log \left(x^{\frac{2}{3}} e + d\right)^{2} + 60 \left(147 b^{2} d^{6} n^{2} + 20 b^{2} d^{3} n^{2} x^{2} e^{3} - 60 a b d^{6} n - 10 \left(b^{2} n^{2} - 6 a b n\right) x^{4} e^{6} - 60 \left(b^{2} d^{6} n - b^{2} n x^{4} e^{6}\right) \log \left(c\right) + 15 \left(4 b^{2} d^{5} n^{2} e - b^{2} d^{2} n^{2} x^{2} e^{4}\right) x^{\frac{2}{3}} - 6 \left(5 b^{2} d^{4} n^{2} x e^{2} - 2 b^{2} d n^{2} x^{3} e^{5}\right) x^{\frac{1}{3}}\right) \log \left(x^{\frac{2}{3}} e + d\right) + 600 \left(2 b^{2} d^{3} n x^{2} e^{3} - \left(b^{2} n - 6 a b\right) x^{4} e^{6}\right) \log \left(c\right) + 15 \left(\left(37 b^{2} d^{2} n^{2} - 60 a b d^{2} n\right) x^{2} e^{4} - 12 \left(49 b^{2} d^{5} n^{2} - 20 a b d^{5} n\right) e + 60 \left(4 b^{2} d^{5} n e - b^{2} d^{2} n x^{2} e^{4}\right) \log \left(c\right)\right) x^{\frac{2}{3}} - 6 \left(4 \left(11 b^{2} d n^{2} - 30 a b d n\right) x^{3} e^{5} - 15 \left(29 b^{2} d^{4} n^{2} - 20 a b d^{4} n\right) x e^{2} + 60 \left(5 b^{2} d^{4} n x e^{2} - 2 b^{2} d n x^{3} e^{5}\right) \log \left(c\right)\right) x^{\frac{1}{3}}\right) e^{(-6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="fricas")
```

```
[Out] 1/7200*(1800*b^2*x^4*e^6*log(c)^2 + 100*(b^2*n^2 - 6*a*b*n + 18*a^2)*x^4*e^
6 - 60*(19*b^2*d^3*n^2 - 20*a*b*d^3*n)*x^2*e^3 - 1800*(b^2*d^6*n^2 - b^2*n^
2*x^4*e^6)*log(x^(2/3)*e + d)^2 + 60*(147*b^2*d^6*n^2 + 20*b^2*d^3*n^2*x^2*
e^3 - 60*a*b*d^6*n - 10*(b^2*n^2 - 6*a*b*n)*x^4*e^6 - 60*(b^2*d^6*n - b^2*n
*x^4*e^6)*log(c) + 15*(4*b^2*d^5*n^2*e - b^2*d^2*n^2*x^2*e^4)*x^(2/3) - 6*(
```

```
5*b^2*d^4*n^2*x*e^2 - 2*b^2*d*n^2*x^3*e^5)*x^(1/3))*log(x^(2/3)*e + d) + 60
0*(2*b^2*d^3*n*x^2*e^3 - (b^2*n - 6*a*b)*x^4*e^6)*log(c) + 15*((37*b^2*d^2*
n^2 - 60*a*b*d^2*n)*x^2*e^4 - 12*(49*b^2*d^5*n^2 - 20*a*b*d^5*n)*e + 60*(4*
b^2*d^5*n*e - b^2*d^2*n*x^2*e^4)*log(c))*x^(2/3) - 6*(4*(11*b^2*d*n^2 - 30*
a*b*d*n)*x^3*e^5 - 15*(29*b^2*d^4*n^2 - 20*a*b*d^4*n)*x*e^2 + 60*(5*b^2*d^4
*n*x*e^2 - 2*b^2*d*n*x^3*e^5)*log(c))*x^(1/3))*e^(-6)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(2/3))**n))**2,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 933 vs. 2(419) = 838.
time = 4.44, size = 933, normalized size = 1.94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="giac")
[Out] 1/4*b^2*x^4*log(c)^2 + 1/2*a*b*x^4*log(c) + 1/4*a^2*x^4 + 1/7200*(1800*(x^(
2/3)*e + d)^6*e^(-6)*log(x^(2/3)*e + d)^2 - 10800*(x^(2/3)*e + d)^5*d*e^(-6
)*log(x^(2/3)*e + d)^2 + 27000*(x^(2/3)*e + d)^4*d^2*e^(-6)*log(x^(2/3)*e +
 d)^2 - 36000*(x^(2/3)*e + d)^3*d^3*e^(-6)*log(x^(2/3)*e + d)^2 + 27000*(x^
(2/3)*e + d)^2*d^4*e^(-6)*log(x^(2/3)*e + d)^2 - 600*(x^(2/3)*e + d)^6*e^(-
6)*log(x^(2/3)*e + d) + 4320*(x^(2/3)*e + d)^5*d*e^(-6)*log(x^(2/3)*e + d)
- 13500*(x^(2/3)*e + d)^4*d^2*e^(-6)*log(x^(2/3)*e + d) + 24000*(x^(2/3)*e
+ d)^3*d^3*e^(-6)*log(x^(2/3)*e + d) - 27000*(x^(2/3)*e + d)^2*d^4*e^(-6)*l
og(x^(2/3)*e + d) + 100*(x^(2/3)*e + d)^6*e^(-6) - 864*(x^(2/3)*e + d)^5*d*
e^(-6) + 3375*(x^(2/3)*e + d)^4*d^2*e^(-6) - 8000*(x^(2/3)*e + d)^3*d^3*e^(
-6) + 13500*(x^(2/3)*e + d)^2*d^4*e^(-6) - 10800*((x^(2/3)*e + d)*log(x^(2/
3)*e + d)^2 - 2*(x^(2/3)*e + d)*log(x^(2/3)*e + d) + 2*x^(2/3)*e + 2*d)*d^5
*e^(-6))*b^2*n^2 + 1/120*(60*(x^(2/3)*e + d)^6*e^(-6)*log(x^(2/3)*e + d) -
360*(x^(2/3)*e + d)^5*d*e^(-6)*log(x^(2/3)*e + d) + 900*(x^(2/3)*e + d)^4*d
^2*e^(-6)*log(x^(2/3)*e + d) - 1200*(x^(2/3)*e + d)^3*d^3*e^(-6)*log(x^(2/3
)*e + d) + 900*(x^(2/3)*e + d)^2*d^4*e^(-6)*log(x^(2/3)*e + d) - 10*(x^(2/3
)*e + d)^6*e^(-6) + 72*(x^(2/3)*e + d)^5*d*e^(-6) - 225*(x^(2/3)*e + d)^4*d
^2*e^(-6) + 400*(x^(2/3)*e + d)^3*d^3*e^(-6) - 450*(x^(2/3)*e + d)^2*d^4*e^
(-6) - 360*((x^(2/3)*e + d)*log(x^(2/3)*e + d) - x^(2/3)*e - d)*d^5*e^(-6))
*b^2*n*log(c) + 1/120*(60*(x^(2/3)*e + d)^6*e^(-6)*log(x^(2/3)*e + d) - 360
```

```
*(x^(2/3)*e + d)^5*d*e^(-6)*log(x^(2/3)*e + d) + 900*(x^(2/3)*e + d)^4*d^2*
e^(-6)*log(x^(2/3)*e + d) - 1200*(x^(2/3)*e + d)^3*d^3*e^(-6)*log(x^(2/3)*e
 + d) + 900*(x^(2/3)*e + d)^2*d^4*e^(-6)*log(x^(2/3)*e + d) - 10*(x^(2/3)*e
 + d)^6*e^(-6) + 72*(x^(2/3)*e + d)^5*d*e^(-6) - 225*(x^(2/3)*e + d)^4*d^2*
e^(-6) + 400*(x^(2/3)*e + d)^3*d^3*e^(-6) - 450*(x^(2/3)*e + d)^2*d^4*e^(-6
) - 360*((x^(2/3)*e + d)*log(x^(2/3)*e + d) - x^(2/3)*e - d)*d^5*e^(-6))*a*
b*n
```

Mupad [B]
time = 1.75, size = 440, normalized size = 0.91

$$\frac{a^2\,x^4}{4}+\frac{b^2\,x^4\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)^2}{4}+\frac{b^2\,n^2\,x^4}{72}+\frac{a\,b\,x^4\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)}{2}-\frac{b^2\,d^6\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)^2}{4\,e^6}-\frac{a\,b\,n\,x^4}{12}-\frac{b^2\,n\,x^4\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)}{12}+\frac{49\,b^2\,d^6\,n^2\,\ln\left(d+e\,x^{2/3}\right)}{40\,e^6}-\frac{19\,b^2\,d^3\,n^2\,x^2}{120\,e^3}+\frac{37\,b^2\,d^2\,n^2\,x^{8/3}}{480\,e^2}+\frac{29\,b^2\,d^4\,n^2\,x^{4/3}}{80\,e^4}-\frac{49\,b^2\,d^5\,n^2\,x^{2/3}}{40\,e^5}-\frac{11\,b^2\,d\,n^2\,x^{10/3}}{300\,e}+\frac{b^2\,d^3\,n\,x^2\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)}{6\,e^3}-\frac{b^2\,d^2\,n\,x^{8/3}\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)}{8\,e^2}-\frac{b^2\,d^4\,n\,x^{4/3}\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)}{4\,e^4}+\frac{b^2\,d^5\,n\,x^{2/3}\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)}{2\,e^5}+\frac{a\,b\,d\,n\,x^{10/3}}{10\,e}-\frac{a\,b\,d^6\,n\,\ln\left(d+e\,x^{2/3}\right)}{2\,e^6}+\frac{b^2\,d\,n\,x^{10/3}\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)}{10\,e}+\frac{a\,b\,d^3\,n\,x^2}{6\,e^3}-\frac{a\,b\,d^2\,n\,x^{8/3}}{8\,e^2}-\frac{a\,b\,d^4\,n\,x^{4/3}}{4\,e^4}+\frac{a\,b\,d^5\,n\,x^{2/3}}{2\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e*x^(2/3)))^n))^2,x)
```

```
[Out] (a^2*x^4)/4 + (b^2*x^4*log(c*(d + e*x^(2/3))^n)^2)/4 + (b^2*n^2*x^4)/72 + (
a*b*x^4*log(c*(d + e*x^(2/3))^n))/2 - (b^2*d^6*log(c*(d + e*x^(2/3))^n)^2)/
(4*e^6) - (a*b*n*x^4)/12 - (b^2*n*x^4*log(c*(d + e*x^(2/3))^n))/12 + (49*b^
2*d^6*n^2*log(d + e*x^(2/3)))/(40*e^6) - (19*b^2*d^3*n^2*x^2)/(120*e^3) + (
37*b^2*d^2*n^2*x^(8/3))/(480*e^2) + (29*b^2*d^4*n^2*x^(4/3))/(80*e^4) - (49
*b^2*d^5*n^2*x^(2/3))/(40*e^5) - (11*b^2*d*n^2*x^(10/3))/(300*e) + (b^2*d^3
*n*x^2*log(c*(d + e*x^(2/3))^n))/(6*e^3) - (b^2*d^2*n*x^(8/3)*log(c*(d + e*
x^(2/3))^n))/(8*e^2) - (b^2*d^4*n*x^(4/3)*log(c*(d + e*x^(2/3))^n))/(4*e^4)
 + (b^2*d^5*n*x^(2/3)*log(c*(d + e*x^(2/3))^n))/(2*e^5) + (a*b*d*n*x^(10/3)
)/(10*e) - (a*b*d^6*n*log(d + e*x^(2/3)))/(2*e^6) + (b^2*d*n*x^(10/3)*log(c
*(d + e*x^(2/3))^n))/(10*e) + (a*b*d^3*n*x^2)/(6*e^3) - (a*b*d^2*n*x^(8/3))
/(8*e^2) - (a*b*d^4*n*x^(4/3))/(4*e^4) + (a*b*d^5*n*x^(2/3))/(2*e^5)
```

3.472 $\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx$

Optimal. Leaf size=275

$$-\frac{3b^2dn^2\left(d+ex^{2/3}\right)^2}{4e^3}+\frac{b^2n^2\left(d+ex^{2/3}\right)^3}{9e^3}+\frac{3b^2d^2n^2x^{2/3}}{e^2}-\frac{b^2d^3n^2\log^2\left(d+ex^{2/3}\right)}{2e^3}-\frac{3bd^2n\left(d+ex^{2/3}\right)\left(a+b\log\right.}{e^3}$$

```
[Out] -3/4*b^2*d*n^2*(d+e*x^(2/3))^2/e^3+1/9*b^2*n^2*(d+e*x^(2/3))^3/e^3+3*b^2*d^
2*n^2*x^(2/3)/e^2-1/2*b^2*d^3*n^2*ln(d+e*x^(2/3))^2/e^3-3*b*d^2*n*(d+e*x^(2
/3))*(a+b*ln(c*(d+e*x^(2/3))^n))/e^3+3/2*b*d*n*(d+e*x^(2/3))^2*(a+b*ln(c*(d
+e*x^(2/3))^n))/e^3-1/3*b*n*(d+e*x^(2/3))^3*(a+b*ln(c*(d+e*x^(2/3))^n))/e^3
+b*d^3*n*ln(d+e*x^(2/3))*(a+b*ln(c*(d+e*x^(2/3))^n))/e^3+1/2*x^2*(a+b*ln(c*
(d+e*x^(2/3))^n))^2
```

Rubi [A]
time = 0.21, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.364$, Rules used = {2504, 2445, 2458, 45, 2372, 12, 14, 2338}

$$\frac{bd^3n\log\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^3}-\frac{3bd^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^3}+\frac{3bdn\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2e^3}-\frac{bn\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^3}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-\frac{b^2d^3n^2\log^2\left(d+ex^{2/3}\right)}{2e^3}+\frac{3b^2d^2n^2x^{2/3}}{e^2}-\frac{3b^2dn^2\left(d+ex^{2/3}\right)^2}{4e^3}+\frac{b^2n^2\left(d+ex^{2/3}\right)^3}{9e^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

```
[Out] (-3*b^2*d*n^2*(d + e*x^(2/3))^2)/(4*e^3) + (b^2*n^2*(d + e*x^(2/3))^3)/(9*e
^3) + (3*b^2*d^2*n^2*x^(2/3))/e^2 - (b^2*d^3*n^2*Log[d + e*x^(2/3)]^2)/(2*e
^3) - (3*b*d^2*n*(d + e*x^(2/3))*(a + b*Log[c*(d + e*x^(2/3))^n]))/e^3 + (3
*b*d*n*(d + e*x^(2/3))^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/(2*e^3) - (b*n*(
d + e*x^(2/3))^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/(3*e^3) + (b*d^3*n*Log[d
 + e*x^(2/3)]*(a + b*Log[c*(d + e*x^(2/3))^n]))/e^3 + (x^2*(a + b*Log[c*(d
+ e*x^(2/3))^n])^2)/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx &= \frac{3}{2}\text{Subst}\left(\int x^2(a+b\log\left(c(d+ex)^n\right))^2\,dx,x,x^{2/3}\right)\\
&= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-(ben)\text{Subst}\left(\int \frac{x^3(a+b\log(c(d}{d+ex}\right.\\
&= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-(bn)\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^3(a+b}{x}\right.\\
&= -\frac{1}{6}bn\left(\frac{18d^2\left(d+ex^{2/3}\right)}{e^3}-\frac{9d\left(d+ex^{2/3}\right)^2}{e^3}+\frac{2\left(d+ex^{2/3}\right)^3}{e^3}-\frac{6d^3\log}{}\right.\\
&= -\frac{1}{6}bn\left(\frac{18d^2\left(d+ex^{2/3}\right)}{e^3}-\frac{9d\left(d+ex^{2/3}\right)^2}{e^3}+\frac{2\left(d+ex^{2/3}\right)^3}{e^3}-\frac{6d^3\log}{}\right.\\
&= -\frac{1}{6}bn\left(\frac{18d^2\left(d+ex^{2/3}\right)}{e^3}-\frac{9d\left(d+ex^{2/3}\right)^2}{e^3}+\frac{2\left(d+ex^{2/3}\right)^3}{e^3}-\frac{6d^3\log}{}\right.\\
&= -\frac{3b^2dn^2\left(d+ex^{2/3}\right)^2}{4e^3}+\frac{b^2n^2\left(d+ex^{2/3}\right)^3}{9e^3}+\frac{3b^2d^2n^2x^{2/3}}{e^2}-\frac{1}{6}bn\left(\frac{18d}{}\right.\\
&= -\frac{3b^2dn^2\left(d+ex^{2/3}\right)^2}{4e^3}+\frac{b^2n^2\left(d+ex^{2/3}\right)^3}{9e^3}+\frac{3b^2d^2n^2x^{2/3}}{e^2}-\frac{b^2d^3n^2\log}{}
\end{aligned}
$$

<u>Mathematica</u> [A]
time = 0.11, size = 239, normalized size = 0.87

$$
\frac{18a^2d^3-36abd^2enx^{2/3}+66b^2d^2en^2x^{2/3}+18abde^2nx^{4/3}-15b^2de^2n^2x^{4/3}+18a^2e^3x^2-12abe^3nx^2+4b^2e^3n^2x^2-30b^2d^3n^2\log\left(d+ex^{2/3}\right)+6b(6a(d^3+e^3x^2)-bn(6d^3+6d^2ex^{2/3}-3de^2x^{4/3}+2e^3x^2))\log\left(c\left(d+ex^{2/3}\right)^n\right)+18b^2(d^3+e^3x^2)\log^2\left(c\left(d+ex^{2/3}\right)^n\right)}{36e^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

```
[Out] (18*a^2*d^3 - 36*a*b*d^2*e*n*x^(2/3) + 66*b^2*d^2*e*n^2*x^(2/3) + 18*a*b*d*
e^2*n*x^(4/3) - 15*b^2*d*e^2*n^2*x^(4/3) + 18*a^2*e^3*x^2 - 12*a*b*e^3*n*x^
2 + 4*b^2*e^3*n^2*x^2 - 30*b^2*d^3*n^2*Log[d + e*x^(2/3)] + 6*b*(6*a*(d^3 +
 e^3*x^2) - b*n*(6*d^3 + 6*d^2*e*x^(2/3) - 3*d*e^2*x^(4/3) + 2*e^3*x^2))*Lo
g[c*(d + e*x^(2/3))^n] + 18*b^2*(d^3 + e^3*x^2)*Log[c*(d + e*x^(2/3))^n]^2)
/(36*e^3)
```

<u>Maple</u> [F]
time = 0.03, size = 0, normalized size = 0.00

$$
\int x\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{2}dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e*x^(2/3))^n))^2,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e*x^(2/3))^n))^2,x)
```

Maxima [A]
time = 0.30, size = 232, normalized size = 0.84

$$\frac{1}{2}\,b^2x^2\log\left(\left(x^{\frac{2}{3}}e+d\right)^nc\right)^2+\frac{1}{6}\left(6\,d^3e^{(-4)}\log\left(x^{\frac{2}{3}}e+d\right)+\left(3\,dx^{\frac{4}{3}}e-2\,x^2e^2-6\,d^2x^{\frac{2}{3}}\right)e^{(-3)}\right)abne+abx^2\log\left(\left(x^{\frac{2}{3}}e+d\right)^nc\right)+\frac{1}{2}\,a^2x^2-\frac{1}{36}\left(\left(18\,d^3\log\left(x^{\frac{2}{3}}e+d\right)^2+66\,d^3\log\left(x^{\frac{2}{3}}e+d\right)-66\,d^2x^{\frac{2}{3}}e+15\,dx^{\frac{4}{3}}e^2-4\,x^2e^3\right)n^2e^{(-3)}-6\left(6\,d^3e^{(-4)}\log\left(x^{\frac{2}{3}}e+d\right)+\left(3\,dx^{\frac{4}{3}}e-2\,x^2e^2-6\,d^2x^{\frac{2}{3}}\right)e^{(-3)}\right)ne\log\left(\left(x^{\frac{2}{3}}e+d\right)^nc\right)\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*log((x^(2/3)*e + d)^n*c)^2 + 1/6*(6*d^3*e^(-4)*log(x^(2/3)*e +
d) + (3*d*x^(4/3)*e - 2*x^2*e^2 - 6*d^2*x^(2/3))*e^(-3))*a*b*n*e + a*b*x^2*
log((x^(2/3)*e + d)^n*c) + 1/2*a^2*x^2 - 1/36*((18*d^3*log(x^(2/3)*e + d)^2
 + 66*d^3*log(x^(2/3)*e + d) - 66*d^2*x^(2/3)*e + 15*d*x^(4/3)*e^2 - 4*x^2*
e^3)*n^2*e^(-3) - 6*(6*d^3*e^(-4)*log(x^(2/3)*e + d) + (3*d*x^(4/3)*e - 2*x
^2*e^2 - 6*d^2*x^(2/3))*e^(-3))*n*e*log((x^(2/3)*e + d)^n*c))*b^2
```

Fricas [A]
time = 0.41, size = 286, normalized size = 1.04

$$\frac{1}{36}\left(18\,b^2x^2e^3\log\left(c\right)^2-12\left(b^2n-3\,ab\right)x^2e^3\log\left(c\right)+2\left(2\,b^2n^2-6\,abn+9\,a^2\right)x^2e^3+18\left(b^2d^3n^2+b^2n^2x^2e^3\right)\log\left(x^{\frac{2}{3}}e+d\right)^2-6\left(6\,b^2d^2n^2x^{\frac{2}{3}}e-3\,b^2dn^2x^{\frac{4}{3}}e^2+11\,b^2d^3n^2-6\,abd^3n+2\left(b^2n^2-3\,abn\right)x^2e^3-6\left(b^2d^3n+b^2nx^2e^3\right)\log\left(c\right)\right)\log\left(x^{\frac{2}{3}}e+d\right)-6\left(6\,b^2d^2ne\log\left(c\right)-\left(11\,b^2d^2n^2-6\,abd^2n\right)e\right)x^{\frac{2}{3}}+3\left(6\,b^2dnxe^2\log\left(c\right)-\left(5\,b^2dn^2-6\,abdn\right)xe^2\right)x^{\frac{1}{3}}\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="fricas")
```

```
[Out] 1/36*(18*b^2*x^2*e^3*log(c)^2 - 12*(b^2*n - 3*a*b)*x^2*e^3*log(c) + 2*(2*b^
2*n^2 - 6*a*b*n + 9*a^2)*x^2*e^3 + 18*(b^2*d^3*n^2 + b^2*n^2*x^2*e^3)*log(x
^(2/3)*e + d)^2 - 6*(6*b^2*d^2*n^2*x^(2/3)*e - 3*b^2*d*n^2*x^(4/3)*e^2 + 11
*b^2*d^3*n^2 - 6*a*b*d^3*n + 2*(b^2*n^2 - 3*a*b*n)*x^2*e^3 - 6*(b^2*d^3*n +
 b^2*n*x^2*e^3)*log(c))*log(x^(2/3)*e + d) - 6*(6*b^2*d^2*n*e*log(c) - (11*
b^2*d^2*n^2 - 6*a*b*d^2*n)*e)*x^(2/3) + 3*(6*b^2*d*n*x*e^2*log(c) - (5*b^2*
d*n^2 - 6*a*b*d*n)*x*e^2)*x^(1/3))*e^(-3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\log\left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(2/3))**n))**2,x)
```

```
[Out] Integral(x*(a + b*log(c*(d + e*x**(2/3))**n))**2, x)
```

Giac [A]
time = 4.88, size = 316, normalized size = 1.15

$$\frac{1}{2}b^2x^2\log\left(c\right)^2+\frac{1}{36}\left(18\,x^2\log\left(x^{\frac{2}{3}}e+d\right)^2+\left(18\,d^3\log\left(x^{\frac{2}{3}}e+d\right)^2-12\left(x^{\frac{2}{3}}e+d\right)^3\log\left(x^{\frac{2}{3}}e+d\right)+54\left(x^{\frac{2}{3}}e+d\right)^2d\log\left(x^{\frac{2}{3}}e+d\right)-108\left(x^{\frac{2}{3}}e+d\right)d^2\log\left(x^{\frac{2}{3}}e+d\right)+4\left(x^{\frac{2}{3}}e+d\right)^3-27\left(x^{\frac{2}{3}}e+d\right)^2d+108\left(x^{\frac{2}{3}}e+d\right)d^2\right)e^{(-3)}\right)b^2n^2+\frac{1}{6}\left(6\,x^2\log\left(x^{\frac{2}{3}}e+d\right)+\left(6\,d^3e^{(-4)}\log\left(\left|x^{\frac{2}{3}}e+d\right|\right)+\left(3\,dx^{\frac{4}{3}}e-2\,x^2e^2-6\,d^2x^{\frac{2}{3}}\right)e^{(-3)}\right)e\right)b^2n\log\left(c\right)+abx^2\log\left(c\right)+\frac{1}{6}\left(6\,x^2\log\left(x^{\frac{2}{3}}e+d\right)+\left(6\,d^3e^{(-4)}\log\left(\left|x^{\frac{2}{3}}e+d\right|\right)+\left(3\,dx^{\frac{4}{3}}e-2\,x^2e^2-6\,d^2x^{\frac{2}{3}}\right)e^{(-3)}\right)e\right)abn+\frac{1}{2}a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="giac")
```

```
[Out] 1/2*b^2*x^2*log(c)^2 + 1/36*(18*x^2*log(x^(2/3)*e + d)^2 + (18*d^3*log(x^(2
/3)*e + d)^2 - 12*(x^(2/3)*e + d)^3*log(x^(2/3)*e + d) + 54*(x^(2/3)*e + d)
^2*d*log(x^(2/3)*e + d) - 108*(x^(2/3)*e + d)*d^2*log(x^(2/3)*e + d) + 4*(x
^(2/3)*e + d)^3 - 27*(x^(2/3)*e + d)^2*d + 108*(x^(2/3)*e + d)*d^2)*e^(-3))
*b^2*n^2 + 1/6*(6*x^2*log(x^(2/3)*e + d) + (6*d^3*e^(-4)*log(abs(x^(2/3)*e
+ d)) + (3*d*x^(4/3)*e - 2*x^2*e^2 - 6*d^2*x^(2/3))*e^(-3))*e)*b^2*n*log(c)
 + a*b*x^2*log(c) + 1/6*(6*x^2*log(x^(2/3)*e + d) + (6*d^3*e^(-4)*log(abs(x
^(2/3)*e + d)) + (3*d*x^(4/3)*e - 2*x^2*e^2 - 6*d^2*x^(2/3))*e^(-3))*e)*a*b
*n + 1/2*a^2*x^2
```

Mupad [B]
time = 0.53, size = 299, normalized size = 1.09

$$\ln\left(c\left(d+e\,x^{2/3}\right)^n\right)^2\left(\frac{b^2\,x^2}{2}+\frac{b^2\,d^3}{2\,e^3}\right)-x^{4/3}\left(\frac{d\left(\frac{3\,a^2}{2}-a\,b\,n+\frac{b^2\,n^2}{3}\right)}{2\,e}-\frac{d\left(3\,a^2-b^2\,n^2\right)}{4\,e}\right)+x^2\left(\frac{a^2}{2}-\frac{a\,b\,n}{3}+\frac{b^2\,n^2}{9}\right)+\ln\left(c\left(d+e\,x^{2/3}\right)^n\right)\left(\frac{b\,x^2\left(3\,a-b\,n\right)}{3}-x^{4/3}\left(\frac{b\,d\left(3\,a-b\,n\right)}{2\,e}-\frac{3\,a\,b\,d}{2\,e}\right)+\frac{d\,x^{2/3}\left(\frac{b\,d\left(3\,a-b\,n\right)}{e}-\frac{3\,a\,b\,d}{e}\right)}{e}\right)+x^{2/3}\left(\frac{d\left(\frac{d\left(\frac{3\,a^2}{2}-a\,b\,n+\frac{b^2\,n^2}{3}\right)}{e}-\frac{d\left(3\,a^2-b^2\,n^2\right)}{2\,e}\right)}{e}+\frac{b^2\,d^2\,n^2}{e^2}\right)-\frac{\ln\left(d+e\,x^{2/3}\right)\left(11\,b^2\,d^3\,n^2-6\,a\,b\,d^3\,n\right)}{6\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e*x^(2/3))^n))^2,x)
```

```
[Out] log(c*(d + e*x^(2/3))^n)^2*((b^2*x^2)/2 + (b^2*d^3)/(2*e^3)) - x^(4/3)*((d*
((3*a^2)/2 + (b^2*n^2)/3 - a*b*n))/(2*e) - (d*(3*a^2 - b^2*n^2))/(4*e)) + x
^2*(a^2/2 + (b^2*n^2)/9 - (a*b*n)/3) + log(c*(d + e*x^(2/3))^n)*((b*x^2*(3*
a - b*n))/3 - x^(4/3)*((b*d*(3*a - b*n))/(2*e) - (3*a*b*d)/(2*e)) + (d*x^(2
/3)*((b*d*(3*a - b*n))/e - (3*a*b*d)/e))/e) + x^(2/3)*((d*((d*((3*a^2)/2 +
(b^2*n^2)/3 - a*b*n))/e - (d*(3*a^2 - b^2*n^2))/(2*e)))/e + (b^2*d^2*n^2)/e
^2) - (log(d + e*x^(2/3))*(11*b^2*d^3*n^2 - 6*a*b*d^3*n))/(6*e^3)
```

3.473 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x}\,dx$

Optimal. Leaf size=95

$$\frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\log\left(-\frac{ex^{2/3}}{d}\right)+3bn\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\mathrm{Li}_2\left(1+\frac{ex^{2/3}}{d}\right)-3b^2n^2\mathrm{Li}_3\left(\right.$$

```
[Out] 3/2*(a+b*ln(c*(d+e*x^(2/3))^n))^2*ln(-e*x^(2/3)/d)+3*b*n*(a+b*ln(c*(d+e*x^(
2/3))^n))*polylog(2,1+e*x^(2/3)/d)-3*b^2*n^2*polylog(3,1+e*x^(2/3)/d)
```

Rubi [A]
time = 0.09, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2504, 2443, 2481, 2421, 6724}

$$3bn\text{PolyLog}\left(2,\frac{ex^{2/3}}{d}+1\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-3b^2n^2\text{PolyLog}\left(3,\frac{ex^{2/3}}{d}+1\right)+\frac{3}{2}\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x,x]
```

```
[Out] (3*(a + b*Log[c*(d + e*x^(2/3))^n])^2*Log[-((e*x^(2/3))/d)])/2 + 3*b*n*(a +
 b*Log[c*(d + e*x^(2/3))^n])*PolyLog[2, 1 + (e*x^(2/3))/d] - 3*b^2*n^2*Poly
Log[3, 1 + (e*x^(2/3))/d]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
```

```
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x}\,dx = \frac{3}{2}\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x}\,dx, x, x^{2/3}\right)$$

$$= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\log\left(-\frac{ex^{2/3}}{d}\right) - (3ben)\text{Subst}\left(\int \frac{\log(-$$

$$= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\log\left(-\frac{ex^{2/3}}{d}\right) - (3bn)\text{Subst}\left(\int \frac{(a+b$$

$$= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\log\left(-\frac{ex^{2/3}}{d}\right) + 3bn\left(a+b\log\left(c\left(d+e\right.\right.\right.$$

$$= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\log\left(-\frac{ex^{2/3}}{d}\right) + 3bn\left(a+b\log\left(c\left(d+e\right.\right.\right.$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 199 vs. 2(95) = 190.
time = 0.07, size = 199, normalized size = 2.09

$$\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\log(x)+2bn\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\left(\left(\log\left(d+ex^{2/3}\right)-\log\left(1+\frac{ex^{2/3}}{d}\right)\right)\log(x)-\frac{3}{2}\text{Li}_2\left(-\frac{ex^{2/3}}{d}\right)\right)+\frac{3}{2}b^2n^2\left(\log^2\left(d+ex^{2/3}\right)\log\left(-\frac{ex^{2/3}}{d}\right)+2\log\left(d+ex^{2/3}\right)\text{Li}_2\left(1+\frac{ex^{2/3}}{d}\right)-2\text{Li}_3\left(1+\frac{ex^{2/3}}{d}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x,x]
```

```
[Out] (a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2*Log[x] + 2*b*n*
(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])*((Log[d + e*x^(2/
3)] - Log[1 + (e*x^(2/3))/d])*Log[x] - (3*PolyLog[2, -((e*x^(2/3))/d)])/2)
+ (3*b^2*n^2*(Log[d + e*x^(2/3)]^2*Log[-((e*x^(2/3))/d)] + 2*Log[d + e*x^(2
/3)]*PolyLog[2, 1 + (e*x^(2/3))/d] - 2*PolyLog[3, 1 + (e*x^(2/3))/d]))/2
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x,x)
```

Maxima [A]
time = 0.34, size = 156, normalized size = 1.64

$$\frac{3}{2}\left(\log\left(x^{\frac{2}{3}}e+d\right)^{2}\log\left(-\frac{x^{\frac{2}{3}}e+d}{d}+1\right)+2\,\mathrm{Li}_2\left(\frac{x^{\frac{2}{3}}e+d}{d}\right)\log\left(x^{\frac{2}{3}}e+d\right)-2\,\mathrm{Li}_3\left(\frac{x^{\frac{2}{3}}e+d}{d}\right)\right)b^{2}n^{2}+a^{2}\log\left(x\right)+3\left(b^{2}n\log\left(c\right)+abn\right)\left(\log\left(x^{\frac{2}{3}}e+d\right)\log\left(-\frac{x^{\frac{2}{3}}e+d}{d}+1\right)+\mathrm{Li}_2\left(\frac{x^{\frac{2}{3}}e+d}{d}\right)\right)+\left(b^{2}\log\left(c\right)^{2}+2\,ab\log\left(c\right)\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x,x, algorithm="maxima")
```

```
[Out] 3/2*(log(x^(2/3)*e + d)^2*log(-(x^(2/3)*e + d)/d + 1) + 2*dilog((x^(2/3)*e
+ d)/d)*log(x^(2/3)*e + d) - 2*polylog(3, (x^(2/3)*e + d)/d))*b^2*n^2 + a^2
*log(x) + 3*(b^2*n*log(c) + a*b*n)*(log(x^(2/3)*e + d)*log(-(x^(2/3)*e + d)
/d + 1) + dilog((x^(2/3)*e + d)/d)) + (b^2*log(c)^2 + 2*a*b*log(c))*log(x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((x^(2/3)*e + d)^n*c)^2 + 2*a*b*log((x^(2/3)*e + d)^n*c) +
 a^2)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{\frac{2}{3}}\right)^{n}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**(2/3))**n))**2/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^(2/3)*e + d)^n*c) + a)^2/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x, x)
```

3.474 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=238

$$-\frac{b^2e^2n^2}{2d^2x^{2/3}}+\frac{b^2e^3n^2\log\left(d+ex^{2/3}\right)}{2d^3}-\frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2dx^{4/3}}+\frac{be^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right.\right.\right.}{d^3x^{2/3}}$$

```
[Out] -1/2*b^2*e^2*n^2/d^2/x^(2/3)+1/2*b^2*e^3*n^2*ln(d+e*x^(2/3))/d^3-1/2*b*e*n*
(a+b*ln(c*(d+e*x^(2/3))^n))/d/x^(4/3)+b*e^2*n*(d+e*x^(2/3))*(a+b*ln(c*(d+e*
x^(2/3))^n))/d^3/x^(2/3)+b*e^3*n*ln(1-d/(d+e*x^(2/3)))*(a+b*ln(c*(d+e*x^(2/
3))^n))/d^3-1/2*(a+b*ln(c*(d+e*x^(2/3))^n))^2/x^2-b^2*e^3*n^2*ln(x)/d^3-b^2
*e^3*n^2*polylog(2,d/(d+e*x^(2/3)))/d^3
```

Rubi [A]

time = 0.28, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2504, 2445, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$-\frac{b^2e^3n^2\text{PolyLog}\left(2,\frac{d}{d+ex^{2/3}}\right)}{d^3}+\frac{be^3n\log\left(1-\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^3}+\frac{be^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^3x^{2/3}}-\frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2dx^{4/3}}-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2x^2}+\frac{b^2e^3n^2\log\left(d+ex^{2/3}\right)}{2d^3}-\frac{b^2e^3n^2\log(x)}{d^3}-\frac{b^2e^2n^2}{2d^2x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^3,x]
```

```
[Out] -1/2*(b^2*e^2*n^2)/(d^2*x^(2/3)) + (b^2*e^3*n^2*Log[d + e*x^(2/3)])/(2*d^3)
 - (b*e*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(2*d*x^(4/3)) + (b*e^2*n*(d + e
*x^(2/3))*(a + b*Log[c*(d + e*x^(2/3))^n]))/(d^3*x^(2/3)) + (b*e^3*n*Log[1
- d/(d + e*x^(2/3))]*(a + b*Log[c*(d + e*x^(2/3))^n]))/d^3 - (a + b*Log[c*(
d + e*x^(2/3))^n])^2/(2*x^2) - (b^2*e^3*n^2*Log[x])/d^3 - (b^2*e^3*n^2*Poly
Log[2, d/(d + e*x^(2/3))])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
```

```
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
```

```
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^3}\,dx &= \frac{3}{2}\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^4}\,dx, x, x^{2/3}\right) \\
&= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2x^2} + (ben)\text{Subst}\left(\int \frac{a+b\log(c(d+ex)^n)}{x^3(d+ex)} \right. \\
&= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2x^2} + (bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, \right. \\
&= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2x^2} + \frac{(bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+ex \right.}{d} \\
&= -\frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2dx^{4/3}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2x^2} - \frac{(be}{} \\
&= -\frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2dx^{4/3}} + \frac{be^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+\right.\right.\right.}{d^3x^{2/3}} \\
&= -\frac{b^2e^2n^2}{2d^2x^{2/3}} + \frac{b^2e^3n^2\log\left(d+ex^{2/3}\right)}{2d^3} - \frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2dx^{4/3}} - \\
&= -\frac{b^2e^2n^2}{2d^2x^{2/3}} + \frac{b^2e^3n^2\log\left(d+ex^{2/3}\right)}{2d^3} - \frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2dx^{4/3}} -
\end{aligned}$$

Mathematica [A]
time = 0.21, size = 264, normalized size = 1.11

$$-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2+\frac{ex^{2/3}\left(3bd^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-6bdenx^{2/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+3e^2x^{4/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-2b^2e^2n^2x^{4/3}\left(3\log\left(d+ex^{2/3}\right)-2\log(x)\right)+b^2en^2x^{2/3}\left(3d-3ex^{2/3}\log\left(d+ex^{2/3}\right)+2ex^{2/3}\log(x)\right)-6be^2nx^{4/3}\left(\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\log\left(-\frac{ex^{2/3}}{d}\right)+bn\text{Li}_2\left(1+\frac{ex^{2/3}}{d}\right)\right)\right)}{d^3}}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^3,x]
```

```
[Out] -1/6*(3*(a + b*Log[c*(d + e*x^(2/3))^n])^2 + (e*x^(2/3)*(3*b*d^2*n*(a + b*L
og[c*(d + e*x^(2/3))^n]) - 6*b*d*e*n*x^(2/3)*(a + b*Log[c*(d + e*x^(2/3))^n
]) + 3*e^2*x^(4/3)*(a + b*Log[c*(d + e*x^(2/3))^n])^2 - 2*b^2*e^2*n^2*x^(4/
3)*(3*Log[d + e*x^(2/3)] - 2*Log[x]) + b^2*e*n^2*x^(2/3)*(3*d - 3*e*x^(2/3)
*Log[d + e*x^(2/3)] + 2*e*x^(2/3)*Log[x]) - 6*b*e^2*n*x^(4/3)*((a + b*Log[c
*(d + e*x^(2/3))^n])*Log[-((e*x^(2/3))/d)] + b*n*PolyLog[2, 1 + (e*x^(2/3))
/d])))/d^3)/x^2
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b^2*n^2*log(x^(2/3)*e + d)^2/x^2 + integrate(1/3*(3*(b^2*log(c)^2 + 2*
a*b*log(c) + a^2)*x*e + 2*(b^2*n*x*e + 3*(b^2*log(c) + a*b)*x*e + 3*(b^2*d*
log(c) + a*b*d)*x^(1/3))*n*log(x^(2/3)*e + d) + 3*(b^2*d*log(c)^2 + 2*a*b*d
*log(c) + a^2*d)*x^(1/3))/(x^4*e + d*x^(10/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((x^(2/3)*e + d)^n*c)^2 + 2*a*b*log((x^(2/3)*e + d)^n*c) +
 a^2)/x^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2/x**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^(2/3)*e + d)^n*c) + a)^2/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^3,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^3, x)
```

3.475 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^5}\,dx$

Optimal. Leaf size=412

$$-\frac{b^2e^2n^2}{40d^2x^{8/3}}+\frac{3b^2e^3n^2}{40d^3x^2}-\frac{47b^2e^4n^2}{240d^4x^{4/3}}+\frac{77b^2e^5n^2}{120d^5x^{2/3}}-\frac{77b^2e^6n^2\log\left(d+ex^{2/3}\right)}{120d^6}-\frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{10dx^{10/3}}+\frac{be^2}{}$$

```
[Out] -1/40*b^2*e^2*n^2/d^2/x^(8/3)+3/40*b^2*e^3*n^2/d^3/x^2-47/240*b^2*e^4*n^2/d
^4/x^(4/3)+77/120*b^2*e^5*n^2/d^5/x^(2/3)-77/120*b^2*e^6*n^2*ln(d+e*x^(2/3)
)/d^6-1/10*b*e*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d/x^(10/3)+1/8*b*e^2*n*(a+b*ln
(c*(d+e*x^(2/3))^n))/d^2/x^(8/3)-1/6*b*e^3*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d^
3/x^2+1/4*b*e^4*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d^4/x^(4/3)-1/2*b*e^5*n*(d+e*
x^(2/3))*(a+b*ln(c*(d+e*x^(2/3))^n))/d^6/x^(2/3)-1/2*b*e^6*n*ln(1-d/(d+e*x^
(2/3)))*(a+b*ln(c*(d+e*x^(2/3))^n))/d^6-1/4*(a+b*ln(c*(d+e*x^(2/3))^n))^2/x
^4+137/180*b^2*e^6*n^2*ln(x)/d^6+1/2*b^2*e^6*n^2*polylog(2,d/(d+e*x^(2/3)))
/d^6
```

Rubi [A]
time = 0.61, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2504, 2445, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$\frac{b^2e^6n^2\text{PolyLog}\left(2,\frac{d}{d+ex^{2/3}}\right)}{2d^6}-\frac{be^6n\log\left(1-\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^6}-\frac{be^5n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^6x^{2/3}}+\frac{be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{4d^4x^{4/3}}-\frac{be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{6d^3x^2}+\frac{be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{8d^2x^{8/3}}-\frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{10dx^{10/3}}-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4x^4}-\frac{77b^2e^6n^2\log\left(d+ex^{2/3}\right)}{120d^6}+\frac{137b^2e^6n^2\log(x)}{180d^6}+\frac{77b^2e^5n^2}{120d^5x^{2/3}}-\frac{47b^2e^4n^2}{240d^4x^{4/3}}+\frac{3b^2e^3n^2}{40d^3x^2}-\frac{b^2e^2n^2}{40d^2x^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^5,x]
```

```
[Out] -1/40*(b^2*e^2*n^2)/(d^2*x^(8/3)) + (3*b^2*e^3*n^2)/(40*d^3*x^2) - (47*b^2*
e^4*n^2)/(240*d^4*x^(4/3)) + (77*b^2*e^5*n^2)/(120*d^5*x^(2/3)) - (77*b^2*e
^6*n^2*Log[d + e*x^(2/3)])/(120*d^6) - (b*e*n*(a + b*Log[c*(d + e*x^(2/3))^
n]))/(10*d*x^(10/3)) + (b*e^2*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(8*d^2*x^
(8/3)) - (b*e^3*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(6*d^3*x^2) + (b*e^4*n*
(a + b*Log[c*(d + e*x^(2/3))^n]))/(4*d^4*x^(4/3)) - (b*e^5*n*(d + e*x^(2/3)
)*(a + b*Log[c*(d + e*x^(2/3))^n]))/(2*d^6*x^(2/3)) - (b*e^6*n*Log[1 - d/(d
 + e*x^(2/3))]*(a + b*Log[c*(d + e*x^(2/3))^n]))/(2*d^6) - (a + b*Log[c*(d
+ e*x^(2/3))^n])^2/(4*x^4) + (137*b^2*e^6*n^2*Log[x])/(180*d^6) + (b^2*e^6*
n^2*PolyLog[2, d/(d + e*x^(2/3))])/(2*d^6)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^5}\,dx = \frac{3}{2}\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^7}\,dx, x, x^{2/3}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4x^4} + \frac{1}{2}(ben)\text{Subst}\left(\int \frac{a+b\log(c(d+ex)}{x^6(d+ex)}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4x^4} + \frac{1}{2}(bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4x^4} + \frac{(bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+ex\right.}{2d}$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{10dx^{10/3}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4x^4} - \frac{(be}{}$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{10dx^{10/3}} + \frac{be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{8d^2x^{8/3}} -$$

$$= -\frac{b^2e^2n^2}{40d^2x^{8/3}} + \frac{b^2e^3n^2}{30d^3x^2} - \frac{b^2e^4n^2}{20d^4x^{4/3}} + \frac{b^2e^5n^2}{10d^5x^{2/3}} - \frac{b^2e^6n^2\log\left(d+ex^{2/3}\right)}{10d^6}$$

$$= -\frac{b^2e^2n^2}{40d^2x^{8/3}} + \frac{3b^2e^3n^2}{40d^3x^2} - \frac{9b^2e^4n^2}{80d^4x^{4/3}} + \frac{9b^2e^5n^2}{40d^5x^{2/3}} - \frac{9b^2e^6n^2\log\left(d+ex^{2/3}\right.}{40d^6}$$

$$= -\frac{b^2e^2n^2}{40d^2x^{8/3}} + \frac{3b^2e^3n^2}{40d^3x^2} - \frac{47b^2e^4n^2}{240d^4x^{4/3}} + \frac{47b^2e^5n^2}{120d^5x^{2/3}} - \frac{47b^2e^6n^2\log\left(d+e\right.}{120d^6}$$

$$= -\frac{b^2e^2n^2}{40d^2x^{8/3}} + \frac{3b^2e^3n^2}{40d^3x^2} - \frac{47b^2e^4n^2}{240d^4x^{4/3}} + \frac{77b^2e^5n^2}{120d^5x^{2/3}} - \frac{77b^2e^6n^2\log\left(d+e\right.}{120d^6}$$

$$= -\frac{b^2e^2n^2}{40d^2x^{8/3}} + \frac{3b^2e^3n^2}{40d^3x^2} - \frac{47b^2e^4n^2}{240d^4x^{4/3}} + \frac{77b^2e^5n^2}{120d^5x^{2/3}} - \frac{77b^2e^6n^2\log\left(d+e\right.}{120d^6}$$

Mathematica [A]
time = 0.20, size = 539, normalized size = 1.31

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^5,x]
```

```
[Out] -1/720*(180*a^2*d^6 + 72*a*b*d^5*e*n*x^(2/3) - 90*a*b*d^4*e^2*n*x^(4/3) + 1
8*b^2*d^4*e^2*n^2*x^(4/3) + 120*a*b*d^3*e^3*n*x^2 - 54*b^2*d^3*e^3*n^2*x^2
- 180*a*b*d^2*e^4*n*x^(8/3) + 141*b^2*d^2*e^4*n^2*x^(8/3) + 360*a*b*d*e^5*n
*x^(10/3) - 462*b^2*d*e^5*n^2*x^(10/3) + 822*b^2*e^6*n^2*x^4*Log[d + e*x^(2
/3)] + 360*a*b*d^6*Log[c*(d + e*x^(2/3))^n] + 72*b^2*d^5*e*n*x^(2/3)*Log[c*
(d + e*x^(2/3))^n] - 90*b^2*d^4*e^2*n*x^(4/3)*Log[c*(d + e*x^(2/3))^n] + 12
0*b^2*d^3*e^3*n*x^2*Log[c*(d + e*x^(2/3))^n] - 180*b^2*d^2*e^4*n*x^(8/3)*Lo
g[c*(d + e*x^(2/3))^n] + 360*b^2*d*e^5*n*x^(10/3)*Log[c*(d + e*x^(2/3))^n]
- 360*a*b*e^6*x^4*Log[c*(d + e*x^(2/3))^n] + 180*b^2*d^6*Log[c*(d + e*x^(2/
3))^n]^2 - 180*b^2*e^6*x^4*Log[c*(d + e*x^(2/3))^n]^2 + 360*a*b*e^6*n*x^4*L
og[-((e*x^(2/3))/d)] + 360*b^2*e^6*n*x^4*Log[c*(d + e*x^(2/3))^n]*Log[-((e*
x^(2/3))/d)] - 548*b^2*e^6*n^2*x^4*Log[x] + 360*b^2*e^6*n^2*x^4*PolyLog[2,
1 + (e*x^(2/3))/d])/(d^6*x^4)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\ln\left(c\left(d + e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{2}}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^5,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^5,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*n^2*log(x^(2/3)*e + d)^2/x^4 + integrate(1/3*(3*(b^2*log(c)^2 + 2*
a*b*log(c) + a^2)*x*e + (b^2*n*x*e + 6*(b^2*log(c) + a*b)*x*e + 6*(b^2*d*lo
g(c) + a*b*d)*x^(1/3))*n*log(x^(2/3)*e + d) + 3*(b^2*d*log(c)^2 + 2*a*b*d*l
og(c) + a^2*d)*x^(1/3))/(x^6*e + d*x^(16/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^5,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((x^(2/3)*e + d)^n*c)^2 + 2*a*b*log((x^(2/3)*e + d)^n*c) +
 a^2)/x^5, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2/x**5,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7318 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^(2/3)*e + d)^n*c) + a)^2/x^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^5,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^5, x)
```

3.476 $\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx$

Optimal. Leaf size=547

$$-\frac{4abd^4n\sqrt[3]{x}}{3e^4}+\frac{4504b^2d^4n^2\sqrt[3]{x}}{945e^4}-\frac{1984b^2d^3n^2x}{2835e^3}+\frac{1144b^2d^2n^2x^{5/3}}{4725e^2}-\frac{128b^2dn^2x^{7/3}}{1323e}+\frac{8}{243}b^2n^2x^3-\frac{4504b^2d^{9/2}n^2\,\mathrm{ta}}{945}$$

```
[Out] -4/3*a*b*d^4*n*x^(1/3)/e^4+4504/945*b^2*d^4*n^2*x^(1/3)/e^4-1984/2835*b^2*d
^3*n^2*x/e^3+1144/4725*b^2*d^2*n^2*x^(5/3)/e^2-128/1323*b^2*d*n^2*x^(7/3)/e
+8/243*b^2*n^2*x^3-4504/945*b^2*d^(9/2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))
/e^(9/2)+4/3*I*b^2*d^(9/2)*n^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x^(1/3)*e^(
1/2)))/e^(9/2)-4/3*b^2*d^4*n*x^(1/3)*ln(c*(d+e*x^(2/3))^n)/e^4+4/9*b*d^3*n*
x*(a+b*ln(c*(d+e*x^(2/3))^n))/e^3-4/15*b*d^2*n*x^(5/3)*(a+b*ln(c*(d+e*x^(2/
3))^n))/e^2+4/21*b*d*n*x^(7/3)*(a+b*ln(c*(d+e*x^(2/3))^n))/e-4/27*b*n*x^3*(
a+b*ln(c*(d+e*x^(2/3))^n))+4/3*b*d^(9/2)*n*arctan(x^(1/3)*e^(1/2)/d^(1/2))*
(a+b*ln(c*(d+e*x^(2/3))^n))/e^(9/2)+1/3*x^3*(a+b*ln(c*(d+e*x^(2/3))^n))^2+8
/3*b^2*d^(9/2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*
x^(1/3)*e^(1/2)))/e^(9/2)+4/3*I*b^2*d^(9/2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1
/2))^2/e^(9/2)
```

Rubi [A]
time = 0.50, antiderivative size = 547, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 14, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {2508, 2507, 2526, 2498, 327, 211, 2505, 308, 2520, 12, 5040, 4964, 2449, 2352}

$$\frac{4ib^2d^{9/2}n^2\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{3e^{9/2}}+\frac{4bd^{9/2}n\,\mathrm{ArcTan}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^{9/2}}+\frac{4bd^3nx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{9e^3}-\frac{4bd^2nx^{5/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15e^2}+\frac{4bdnx^{7/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{21e}+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-\frac{4}{27}bnx^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{4abd^4n\sqrt[3]{x}}{3e^4}+\frac{4ib^2d^{9/2}n^2\,\mathrm{ArcTan}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{3e^{9/2}}-\frac{4504b^2d^{9/2}n^2\,\mathrm{ArcTan}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{945e^{9/2}}+\frac{8b^2d^{9/2}n^2\,\mathrm{ArcTan}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{3e^{9/2}}-\frac{4b^2d^4n\sqrt[3]{x}\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3e^4}+\frac{4504b^2d^4n^2\sqrt[3]{x}}{945e^4}-\frac{1984b^2d^3n^2x}{2835e^3}+\frac{1144b^2d^2n^2x^{5/3}}{4725e^2}-\frac{128b^2dn^2x^{7/3}}{1323e}+\frac{8}{243}b^2n^2x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

```
[Out] (-4*a*b*d^4*n*x^(1/3))/(3*e^4) + (4504*b^2*d^4*n^2*x^(1/3))/(945*e^4) - (19
84*b^2*d^3*n^2*x)/(2835*e^3) + (1144*b^2*d^2*n^2*x^(5/3))/(4725*e^2) - (128
*b^2*d*n^2*x^(7/3))/(1323*e) + (8*b^2*n^2*x^3)/243 - (4504*b^2*d^(9/2)*n^2*
ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(945*e^(9/2)) + (((4*I)/3)*b^2*d^(9/2)*n
^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/e^(9/2) + (8*b^2*d^(9/2)*n^2*ArcTan
[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])
/(3*e^(9/2)) - (4*b^2*d^4*n*x^(1/3)*Log[c*(d + e*x^(2/3))^n])/(3*e^4) + (4*
b*d^3*n*x*(a + b*Log[c*(d + e*x^(2/3))^n]))/(9*e^3) - (4*b*d^2*n*x^(5/3)*(a
 + b*Log[c*(d + e*x^(2/3))^n]))/(15*e^2) + (4*b*d*n*x^(7/3)*(a + b*Log[c*(d
 + e*x^(2/3))^n]))/(21*e) - (4*b*n*x^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/27
 + (4*b*d^(9/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(
2/3))^n]))/(3*e^(9/2)) + (x^3*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/3 + (((4*
I)/3)*b^2*d^(9/2)*n^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/
3))])/e^(9/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
```

```
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2508

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1)
- 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
b, c, d, e, m, p, q}, x] && FractionQ[n]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 dx = 3\text{Subst}\left(\int x^8\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2 dx, x, \sqrt[3]{x}\right)$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 - \frac{1}{3}(4ben)\text{Subst}\left(\int \frac{x^{10}\left(a+b\log\right.}{d\,-}\right.$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 - \frac{1}{3}(4ben)\text{Subst}\left(\int\left(\frac{d^4\left(a+b\log\right.}{}\right.\right.$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 - \frac{1}{3}(4bn)\text{Subst}\left(\int x^8\left(a+b\log\left(c\right.\right.\right.$$

$$= -\frac{4abd^4n\sqrt[3]{x}}{3e^4} + \frac{4bd^3nx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{9e^3} - \frac{4bd^2nx^{5/3}\left(a+\right.}{}$$

$$= -\frac{4abd^4n\sqrt[3]{x}}{3e^4} - \frac{4b^2d^4n\sqrt[3]{x}\,\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3e^4} + \frac{4bd^3nx\left(a+b\log\right.}{9e}$$

$$= -\frac{4abd^4n\sqrt[3]{x}}{3e^4} + \frac{4504b^2d^4n^2\sqrt[3]{x}}{945e^4} - \frac{1984b^2d^3n^2x}{2835e^3} + \frac{1144b^2d^2n^2x^{5/3}}{4725e^2}$$

$$= -\frac{4abd^4n\sqrt[3]{x}}{3e^4} + \frac{4504b^2d^4n^2\sqrt[3]{x}}{945e^4} - \frac{1984b^2d^3n^2x}{2835e^3} + \frac{1144b^2d^2n^2x^{5/3}}{4725e^2}$$

$$= -\frac{4abd^4n\sqrt[3]{x}}{3e^4} + \frac{4504b^2d^4n^2\sqrt[3]{x}}{945e^4} - \frac{1984b^2d^3n^2x}{2835e^3} + \frac{1144b^2d^2n^2x^{5/3}}{4725e^2}$$

$$= -\frac{4abd^4n\sqrt[3]{x}}{3e^4} + \frac{4504b^2d^4n^2\sqrt[3]{x}}{945e^4} - \frac{1984b^2d^3n^2x}{2835e^3} + \frac{1144b^2d^2n^2x^{5/3}}{4725e^2}$$

Mathematica [A]
time = 0.33, size = 438, normalized size = 0.80

$$\frac{396900ib^2d^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2+1260bd^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(315a-1126bn+630bn\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)+315b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\sqrt{e}\sqrt[3]{x}\left(99225a^2e^4x^{8/3}-1260abn\left(315d^4-105d^3ex^{2/3}+63d^2e^2x^{4/3}-45de^3x^2+35e^4x^{8/3}\right)+8b^2n^2\left(177345d^4-26040d^3ex^{2/3}+9009d^2e^2x^{4/3}-3600de^3x^2+1225e^4x^{8/3}\right)-630b\left(-315ae^4x^{8/3}+2bn\left(315d^4-105d^3ex^{2/3}+63d^2e^2x^{4/3}-45de^3x^2+35e^4x^{8/3}\right)\right)\log\left(c\left(d+ex^{2/3}\right)^n\right)+99225b^2e^4x^{8/3}\log^2\left(c\left(d+ex^{2/3}\right)^n\right)\right)+396900ib^2d^{9/2}n^2\text{Li}_2\left(\frac{i\sqrt{d}+\sqrt{e}\sqrt[3]{x}}{-i\sqrt{d}+\sqrt{e}\sqrt[3]{x}}\right)}{297675e^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

```
[Out] ((396900*I)*b^2*d^(9/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2 + 1260*b*d^
(9/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(315*a - 1126*b*n + 630*b*n*Log[(
2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))] + 315*b*Log[c*(d + e*x^(2/3))^n])
 + Sqrt[e]*x^(1/3)*(99225*a^2*e^4*x^(8/3) - 1260*a*b*n*(315*d^4 - 105*d^3*e
*x^(2/3) + 63*d^2*e^2*x^(4/3) - 45*d*e^3*x^2 + 35*e^4*x^(8/3)) + 8*b^2*n^2*
(177345*d^4 - 26040*d^3*e*x^(2/3) + 9009*d^2*e^2*x^(4/3) - 3600*d*e^3*x^2 +
 1225*e^4*x^(8/3)) - 630*b*(-315*a*e^4*x^(8/3) + 2*b*n*(315*d^4 - 105*d^3*e
*x^(2/3) + 63*d^2*e^2*x^(4/3) - 45*d*e^3*x^2 + 35*e^4*x^(8/3)))*Log[c*(d +
e*x^(2/3))^n] + 99225*b^2*e^4*x^(8/3)*Log[c*(d + e*x^(2/3))^n]^2) + (396900
*I)*b^2*d^(9/2)*n^2*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x^(1/3))/((-I)*Sqrt[d]
+ Sqrt[e]*x^(1/3))])/(297675*e^(9/2))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(2/3))^n))^2,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e*x^(2/3))^n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*n^2*x^3*log(x^(2/3)*e + d)^2 + integrate(1/9*(9*(b^2*log(c)^2 + 2*a
*b*log(c) + a^2)*x^3*e + 9*(b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d)*x^(7/3
) - 2*(2*b^2*n*x^3*e - 9*(b^2*log(c) + a*b)*x^3*e - 9*(b^2*d*log(c) + a*b*d
)*x^(7/3))*n*log(x^(2/3)*e + d))/(x*e + d*x^(1/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*log((x^(2/3)*e + d)^n*c)^2 + 2*a*b*x^2*log((x^(2/3)*e + d)
^n*c) + a^2*x^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(2/3))**n))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^(2/3)*e + d)^n*c) + a)^2*x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e*x^(2/3))^n))^2,x)
```

```
[Out] int(x^2*(a + b*log(c*(d + e*x^(2/3))^n))^2, x)
```

3.477 $\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx$

Optimal. Leaf size=364

$$\frac{4abdn\sqrt[3]{x}}{e}-\frac{32b^2dn^2\sqrt[3]{x}}{3e}+\frac{8}{9}b^2n^2x+\frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\,\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{3/2}}-\frac{4ib^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\,\sqrt[3]{x}}{\sqrt{d}}\right)^2}{e^{3/2}}-\frac{8b^2d^{3/2}}{}$$

```
[Out] 4*a*b*d*n*x^(1/3)/e-32/3*b^2*d*n^2*x^(1/3)/e+8/9*b^2*n^2*x+32/3*b^2*d^(3/2)
*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))/e^(3/2)-4*I*b^2*d^(3/2)*n^2*arctan(x^(
1/3)*e^(1/2)/d^(1/2))^2/e^(3/2)+4*b^2*d*n*x^(1/3)*ln(c*(d+e*x^(2/3))^n)/e-4
/3*b*n*x*(a+b*ln(c*(d+e*x^(2/3))^n))-4*b*d^(3/2)*n*arctan(x^(1/3)*e^(1/2)/d
^(1/2))*(a+b*ln(c*(d+e*x^(2/3))^n))/e^(3/2)+x*(a+b*ln(c*(d+e*x^(2/3))^n))^2
-8*b^2*d^(3/2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*
x^(1/3)*e^(1/2)))/e^(3/2)-4*I*b^2*d^(3/2)*n^2*polylog(2,1-2*d^(1/2)/(d^(1/2
)+I*x^(1/3)*e^(1/2)))/e^(3/2)
```

Rubi [A]
time = 0.30, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {2501, 2507, 2526, 2498, 327, 211, 2505, 308, 2520, 12, 5040, 4964, 2449, 2352}

$$-\frac{4ib^2d^{3/2}n^2\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\,\sqrt[3]{x}}\right)}{e^{3/2}}-\frac{4bd^{3/2}n\text{ArcTan}\left(\frac{\sqrt{e}\,\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^{3/2}}-\frac{4}{3}bnx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2+\frac{4abdn\sqrt[3]{x}}{e}-\frac{4ib^2d^{3/2}n^2\text{ArcTan}\left(\frac{\sqrt{e}\,\sqrt[3]{x}}{\sqrt{d}}\right)^2}{e^{3/2}}+\frac{32b^2d^{3/2}n^2\text{ArcTan}\left(\frac{\sqrt{e}\,\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{3/2}}-\frac{8b^2d^{3/2}n^2\text{ArcTan}\left(\frac{\sqrt{e}\,\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\,\sqrt[3]{x}}\right)}{e^{3/2}}+\frac{4b^2dn\sqrt[3]{x}\log\left(c\left(d+ex^{2/3}\right)^n\right)}{e}-\frac{32b^2dn^2\sqrt[3]{x}}{3e}+\frac{8}{9}b^2n^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

```
[Out] (4*a*b*d*n*x^(1/3))/e - (32*b^2*d*n^2*x^(1/3))/(3*e) + (8*b^2*n^2*x)/9 + (3
2*b^2*d^(3/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(3*e^(3/2)) - ((4*I)*b
^2*d^(3/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/e^(3/2) - (8*b^2*d^(3/2
)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e
]*x^(1/3))])/e^(3/2) + (4*b^2*d*n*x^(1/3)*Log[c*(d + e*x^(2/3))^n])/e - (4*
b*n*x*(a + b*Log[c*(d + e*x^(2/3))^n]))/3 - (4*b*d^(3/2)*n*ArcTan[(Sqrt[e]*
x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2/3))^n]))/e^(3/2) + x*(a + b*Log[
c*(d + e*x^(2/3))^n])^2 - ((4*I)*b^2*d^(3/2)*n^2*PolyLog[2, 1 - (2*Sqrt[d])
/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/e^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2501

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx = 3\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2\,dx, x, \sqrt[3]{x}\right)$$

$$= x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 - (4ben)\text{Subst}\left(\int \frac{x^4\left(a+b\log\left(c\left(d+\right.\right.\right.}{d+ex^2}\right.$$

$$= x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 - (4ben)\text{Subst}\left(\int\left(-\frac{d\left(a+b\log\left(c\left(d\right.\right.\right.}{e^2}\right.\right.$$

$$= x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 - (4bn)\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+e\right.\right.\right.\right.$$

$$= \frac{4abdn\sqrt[3]{x}}{e} - \frac{4}{3}bnx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right) - \frac{4bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}}{\sqrt{}}\right.}{}$$

$$= \frac{4abdn\sqrt[3]{x}}{e} + \frac{4b^2dn\sqrt[3]{x}\,\log\left(c\left(d+ex^{2/3}\right)^n\right)}{e} - \frac{4}{3}bnx\left(a+b\log\left(c\left(d+\right.\right.\right.$$

$$= \frac{4abdn\sqrt[3]{x}}{e} - \frac{32b^2dn^2\sqrt[3]{x}}{3e} + \frac{8}{9}b^2n^2x - \frac{4ib^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\,\sqrt[3]{x}}{\sqrt{d}}\right)^2}{e^{3/2}}$$

$$= \frac{4abdn\sqrt[3]{x}}{e} - \frac{32b^2dn^2\sqrt[3]{x}}{3e} + \frac{8}{9}b^2n^2x + \frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\,\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{3/2}}$$

$$= \frac{4abdn\sqrt[3]{x}}{e} - \frac{32b^2dn^2\sqrt[3]{x}}{3e} + \frac{8}{9}b^2n^2x + \frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\,\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{3/2}}$$

$$= \frac{4abdn\sqrt[3]{x}}{e} - \frac{32b^2dn^2\sqrt[3]{x}}{3e} + \frac{8}{9}b^2n^2x + \frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\,\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{3/2}}$$

Mathematica [A]
time = 0.15, size = 319, normalized size = 0.88

$$\frac{-36ib^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2-12bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(3a-8bn+6bn\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)+3b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\sqrt{e}\sqrt[3]{x}\left(12abn\left(3d-ex^{2/3}\right)+8b^2n^2\left(-12d+ex^{2/3}\right)+9a^2ex^{2/3}+6b\left(6bdn+3aex^{2/3}-2benx^{2/3}\right)\log\left(c\left(d+ex^{2/3}\right)^n\right)+9b^2ex^{2/3}\log^2\left(c\left(d+ex^{2/3}\right)^n\right)\right)-36ib^2d^{3/2}n^2\text{Li}_2\left(\frac{i\sqrt{d}+\sqrt{e}\sqrt[3]{x}}{-i\sqrt{d}+\sqrt{e}\sqrt[3]{x}}\right)}{9e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

```
[Out] ((-36*I)*b^2*d^(3/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2 - 12*b*d^(3/2)
*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(3*a - 8*b*n + 6*b*n*Log[(2*Sqrt[d])/(
Sqrt[d] + I*Sqrt[e]*x^(1/3))] + 3*b*Log[c*(d + e*x^(2/3))^n]) + Sqrt[e]*x^(
1/3)*(12*a*b*n*(3*d - e*x^(2/3)) + 8*b^2*n^2*(-12*d + e*x^(2/3)) + 9*a^2*e*
x^(2/3) + 6*b*(6*b*d*n + 3*a*e*x^(2/3) - 2*b*e*n*x^(2/3))*Log[c*(d + e*x^(2
/3))^n] + 9*b^2*e*x^(2/3)*Log[c*(d + e*x^(2/3))^n]^2) - (36*I)*b^2*d^(3/2)*
n^2*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x^(1/3))/((-I)*Sqrt[d] + Sqrt[e]*x^(1/3
))])/(9*e^(3/2))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(a+b\ln \left(c{\left(d+e\,x^{\frac{2}{3}}\right)}^{n}\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^2,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(2*(3*d^(3/2)*arctan(x^(1/3)*e^(1/2)/sqrt(d))*e^(-5/2) + (x*e - 3*d*x^
(1/3))*e^(-2))*n*e - 3*x*log((x^(2/3)*e + d)^n*c))*a*b + (n^2*x*log(x^(2/3)
*e + d)^2 + integrate(1/3*(3*x*e*log(c)^2 + 3*d*x^(1/3)*log(c)^2 - 2*(2*n*x
*e - 3*x*e*log(c) - 3*d*x^(1/3)*log(c))*n*log(x^(2/3)*e + d))/(x*e + d*x^(1
/3)), x))*b^2 + a^2*x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*log((x^(2/3)*e + d)^n*c)^2 + 2*a*b*log((x^(2/3)*e + d)^n*c) +
a^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\log\left(c\left(d+ex^{\frac{2}{3}}\right)^{n}\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**(2/3))**n))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^(2/3)*e + d)^n*c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))^2,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^2, x)
```

3.478 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=298

$$\frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{4ib^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}}-\frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{d^{3/2}}$$

```
[Out] 8*b^2*e^(3/2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))/d^(3/2)-4*I*b^2*e^(3/2)*n
^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))^2/d^(3/2)-4*b*e*n*(a+b*ln(c*(d+e*x^(2/3)
)^n))/d/x^(1/3)-4*b*e^(3/2)*n*arctan(x^(1/3)*e^(1/2)/d^(1/2))*(a+b*ln(c*(d+
e*x^(2/3))^n))/d^(3/2)-(a+b*ln(c*(d+e*x^(2/3))^n))^2/x-8*b^2*e^(3/2)*n^2*ar
ctan(x^(1/3)*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x^(1/3)*e^(1/2)))/d^(
3/2)-4*I*b^2*e^(3/2)*n^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x^(1/3)*e^(1/2)))
/d^(3/2)
```

Rubi [A]

time = 0.27, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {2508, 2507, 2526, 2505, 211, 2520, 12, 5040, 4964, 2449, 2352}

$$-\frac{4ib^2e^{3/2}n^2\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{d^{3/2}}-\frac{4be^{3/2}n\text{ArcTan}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{3/2}}-\frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d\sqrt[3]{x}}-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x}-\frac{4ib^2e^{3/2}n^2\text{ArcTan}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}}+\frac{8b^2e^{3/2}n^2\text{ArcTan}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{8b^2e^{3/2}n^2\text{ArcTan}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^2,x]
```

```
[Out] (8*b^2*e^(3/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/d^(3/2) - ((4*I)*b^2*
e^(3/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/d^(3/2) - (8*b^2*e^(3/2)*n
^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x
^(1/3))])/d^(3/2) - (4*b*e*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(d*x^(1/3))
- (4*b*e^(3/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2
/3))^n]))/d^(3/2) - (a + b*Log[c*(d + e*x^(2/3))^n])^2/x - ((4*I)*b^2*e^(3/
2)*n^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/d^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2508

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1)
- 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
b, c, d, e, m, p, q}, x] && FractionQ[n]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^2}\,dx = 3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{x^4}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x} + (4ben)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^2\left(d+ex^2\right)}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x} + (4ben)\text{Subst}\left(\int \left(\frac{a+b\log\left(c\left(d+ex^2\right.\right.}{dx^2}\right.\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x} + \frac{(4ben)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^2}\,dx, x,\right.}{d}$$

$$= -\frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d\sqrt[3]{x}} - \frac{4be^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\right.}{d^{3/2}}$$

$$= \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d\sqrt[3]{x}} - \frac{4be^{3/}}{}$$

$$= \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{4ib^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} - \frac{4ben(a}{}$$

$$= \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{4ib^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} - \frac{8b^2e^{3/2}}{}$$

$$= \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{4ib^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} - \frac{8b^2e^{3/2}}{}$$

$$= \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{4ib^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} - \frac{8b^2e^{3/2}}{}$$

Mathematica [**A**]
time = 0.13, size = 247, normalized size = 0.83

$$\frac{-4ib^2e^{3/2}n^2x\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2-4be^{3/2}nx\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a-2bn+2bn\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\sqrt{d}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\left(ad+4benx^{2/3}+bd\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-4ib^2e^{3/2}n^2x\mathrm{Li}_2\left(\frac{i\sqrt{d}+\sqrt{e}\sqrt[3]{x}}{-i\sqrt{d}+\sqrt{e}\sqrt[3]{x}}\right)}{d^{3/2}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^2,x]
```

```
[Out] ((-4*I)*b^2*e^(3/2)*n^2*x*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2 - 4*b*e^(3/2)
*n*x*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a - 2*b*n + 2*b*n*Log[(2*Sqrt[d])/(
Sqrt[d] + I*Sqrt[e]*x^(1/3))] + b*Log[c*(d + e*x^(2/3))^n]) - Sqrt[d]*(a +
b*Log[c*(d + e*x^(2/3))^n])*(a*d + 4*b*e*n*x^(2/3) + b*d*Log[c*(d + e*x^(2/
3))^n]) - (4*I)*b^2*e^(3/2)*n^2*x*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x^(1/3))/
((-I)*Sqrt[d] + Sqrt[e]*x^(1/3))])/(d^(3/2)*x)
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^2,x, algorithm="maxima")
```

```
[Out] -b^2*n^2*log(x^(2/3)*e + d)^2/x + integrate(1/3*(3*(b^2*log(c)^2 + 2*a*b*lo
g(c) + a^2)*x*e + 2*(2*b^2*n*x*e + 3*(b^2*log(c) + a*b)*x*e + 3*(b^2*d*log(
c) + a*b*d)*x^(1/3))*n*log(x^(2/3)*e + d) + 3*(b^2*d*log(c)^2 + 2*a*b*d*log
(c) + a^2*d)*x^(1/3))/(x^3*e + d*x^(7/3)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((x^(2/3)*e + d)^n*c)^2 + 2*a*b*log((x^(2/3)*e + d)^n*c) +
 a^2)/x^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2/x**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^(2/3)*e + d)^n*c) + a)^2/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^2,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^2, x)
```

3.479 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^4}\,dx$

Optimal. Leaf size=476

$$-\frac{8b^2e^2n^2}{105d^2x^{5/3}}+\frac{32b^2e^3n^2}{105d^3x}-\frac{568b^2e^4n^2}{315d^4\sqrt[3]{x}}-\frac{1408b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{315d^{9/2}}+\frac{4ib^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{3d^{9/2}}+\frac{8b^2e}{}$$

```
[Out] -8/105*b^2*e^2*n^2/d^2/x^(5/3)+32/105*b^2*e^3*n^2/d^3/x-568/315*b^2*e^4*n^2
/d^4/x^(1/3)-1408/315*b^2*e^(9/2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))/d^(9/
2)+4/3*I*b^2*e^(9/2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))^2/d^(9/2)-4/21*b*e
*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d/x^(7/3)+4/15*b*e^2*n*(a+b*ln(c*(d+e*x^(2/3
))^n))/d^2/x^(5/3)-4/9*b*e^3*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d^3/x+4/3*b*e^4*
n*(a+b*ln(c*(d+e*x^(2/3))^n))/d^4/x^(1/3)+4/3*b*e^(9/2)*n*arctan(x^(1/3)*e^
(1/2)/d^(1/2))*(a+b*ln(c*(d+e*x^(2/3))^n))/d^(9/2)-1/3*(a+b*ln(c*(d+e*x^(2/
3))^n))^2/x^3+8/3*b^2*e^(9/2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))*ln(2*d^(1
/2)/(d^(1/2)+I*x^(1/3)*e^(1/2)))/d^(9/2)+4/3*I*b^2*e^(9/2)*n^2*polylog(2,1-
2*d^(1/2)/(d^(1/2)+I*x^(1/3)*e^(1/2)))/d^(9/2)
```

Rubi [A]
time = 0.41, antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2508, 2507, 2526, 2505, 331, 211, 2520, 12, 5040, 4964, 2449, 2352}

$$\frac{4ib^2e^{9/2}n^2\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{3d^{9/2}}+\frac{4be^{9/2}n\text{ArcTan}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3d^{9/2}}+\frac{4be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3d^4\sqrt[3]{x}}-\frac{4be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{9d^3x}+\frac{4be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15d^2x^{5/3}}-\frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{21dx^{7/3}}-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x^3}+\frac{4ib^2e^{9/2}n^2\text{ArcTan}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{3d^{9/2}}-\frac{1408b^2e^{9/2}n^2\text{ArcTan}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{315d^{9/2}}+\frac{8b^2e^{9/2}n^2\text{ArcTan}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{3d^{9/2}}-\frac{568b^2e^4n^2}{315d^4\sqrt[3]{x}}+\frac{32b^2e^3n^2}{105d^3x}-\frac{8b^2e^2n^2}{105d^2x^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^4,x]
```

```
[Out] (-8*b^2*e^2*n^2)/(105*d^2*x^(5/3)) + (32*b^2*e^3*n^2)/(105*d^3*x) - (568*b^
2*e^4*n^2)/(315*d^4*x^(1/3)) - (1408*b^2*e^(9/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3
))/Sqrt[d]])/(315*d^(9/2)) + (((4*I)/3)*b^2*e^(9/2)*n^2*ArcTan[(Sqrt[e]*x^(
1/3))/Sqrt[d]]^2)/d^(9/2) + (8*b^2*e^(9/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqr
t[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/(3*d^(9/2)) - (4*b*e*
n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(21*d*x^(7/3)) + (4*b*e^2*n*(a + b*Log[
c*(d + e*x^(2/3))^n]))/(15*d^2*x^(5/3)) - (4*b*e^3*n*(a + b*Log[c*(d + e*x^
(2/3))^n]))/(9*d^3*x) + (4*b*e^4*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(3*d^4
*x^(1/3)) + (4*b*e^(9/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c*(
d + e*x^(2/3))^n]))/(3*d^(9/2)) - (a + b*Log[c*(d + e*x^(2/3))^n])^2/(3*x^3
) + (((4*I)/3)*b^2*e^(9/2)*n^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt
[e]*x^(1/3))])/d^(9/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2508

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1)
```

```
- 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
b, c, d, e, m, p, q}, x] && FractionQ[n]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^4}\,dx = 3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{x^{10}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x^3} + \frac{1}{3}(4ben)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right.\right.}{x^8\left(d+ex^2\right)}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x^3} + \frac{1}{3}(4ben)\text{Subst}\left(\int \left(\frac{a+b\log\left(c\left(d+e\right.\right.}{dx^8}\right.\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x^3} + \frac{(4ben)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^8}\,dx, x,\right.}{3d}$$

$$= -\frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{21dx^{7/3}} + \frac{4be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15d^2x^{5/3}}$$

$$= -\frac{8b^2e^2n^2}{105d^2x^{5/3}} + \frac{8b^2e^3n^2}{45d^3x} - \frac{8b^2e^4n^2}{9d^4\sqrt[3]{x}} - \frac{8b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3d^{9/2}} - \frac{4be}{}$$

$$= -\frac{8b^2e^2n^2}{105d^2x^{5/3}} + \frac{32b^2e^3n^2}{105d^3x} - \frac{64b^2e^4n^2}{45d^4\sqrt[3]{x}} - \frac{32b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{9d^{9/2}} +$$

$$= -\frac{8b^2e^2n^2}{105d^2x^{5/3}} + \frac{32b^2e^3n^2}{105d^3x} - \frac{568b^2e^4n^2}{315d^4\sqrt[3]{x}} - \frac{184b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{45d^{9/2}}$$

$$= -\frac{8b^2e^2n^2}{105d^2x^{5/3}} + \frac{32b^2e^3n^2}{105d^3x} - \frac{568b^2e^4n^2}{315d^4\sqrt[3]{x}} - \frac{1408b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{315d^{9/2}}$$

$$= -\frac{8b^2e^2n^2}{105d^2x^{5/3}} + \frac{32b^2e^3n^2}{105d^3x} - \frac{568b^2e^4n^2}{315d^4\sqrt[3]{x}} - \frac{1408b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{315d^{9/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.37, size = 473, normalized size = 0.99

$$-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x^3}+\frac{4}{3}ben\left(-\frac{2be^{7/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{9/2}}-\frac{2ben\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\frac{ex^{2/3}}{d}\right)}{35d^2x^{5/3}}+\frac{2be^2n\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\frac{ex^{2/3}}{d}\right)}{15d^3x}-\frac{2be^3n\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{ex^{2/3}}{d}\right)}{3d^4\sqrt[3]{x}}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{7dx^{7/3}}+\frac{e\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5d^2x^{5/3}}-\frac{e^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3d^3x}+\frac{e^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^4\sqrt[3]{x}}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{9/2}}+\frac{ibe^{7/2}n\left(\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)-2i\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)\right)+\text{Li}_2\left(\frac{i\sqrt{d}+\sqrt{e}\sqrt[3]{x}}{-i\sqrt{d}+\sqrt{e}\sqrt[3]{x}}\right)\right)}{d^{9/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^4,x]
```

```
[Out] -1/3*(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^3 + (4*b*e*n*((-2*b*e^(7/2)*n*Arc
Tan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/d^(9/2) - (2*b*e*n*Hypergeometric2F1[-5/2,
1, -3/2, -((e*x^(2/3))/d)])/(35*d^2*x^(5/3)) + (2*b*e^2*n*Hypergeometric2F1
[-3/2, 1, -1/2, -((e*x^(2/3))/d)])/(15*d^3*x) - (2*b*e^3*n*Hypergeometric2F
1[-1/2, 1, 1/2, -((e*x^(2/3))/d)])/(3*d^4*x^(1/3)) - (a + b*Log[c*(d + e*x^
(2/3))^n])/(7*d*x^(7/3)) + (e*(a + b*Log[c*(d + e*x^(2/3))^n]))/(5*d^2*x^(5
/3)) - (e^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/(3*d^3*x) + (e^3*(a + b*Log[c
*(d + e*x^(2/3))^n]))/(d^4*x^(1/3)) + (e^(7/2)*ArcTan[(Sqrt[e]*x^(1/3))/Sqr
t[d]]*(a + b*Log[c*(d + e*x^(2/3))^n]))/d^(9/2) + (I*b*e^(7/2)*n*(ArcTan[(S
qrt[e]*x^(1/3))/Sqrt[d]]*(ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]] - (2*I)*Log[(2*
Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))]) + PolyLog[2, (I*Sqrt[d] + Sqrt[e]*
x^(1/3))/((-I)*Sqrt[d] + Sqrt[e]*x^(1/3))]))/d^(9/2)))/3
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{2}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^2*n^2*log(x^(2/3)*e + d)^2/x^3 + integrate(1/9*(9*(b^2*log(c)^2 + 2*
a*b*log(c) + a^2)*x*e + 2*(2*b^2*n*x*e + 9*(b^2*log(c) + a*b)*x*e + 9*(b^2*
d*log(c) + a*b*d)*x^(1/3))*n*log(x^(2/3)*e + d) + 9*(b^2*d*log(c)^2 + 2*a*b
*d*log(c) + a^2*d)*x^(1/3))/(x^5*e + d*x^(13/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^4,x, algorithm="fricas")
[Out] integral((b^2*log((x^(2/3)*e + d)^n*c)^2 + 2*a*b*log((x^(2/3)*e + d)^n*c) +
 a^2)/x^4, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2/x**4,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^4,x, algorithm="giac")
[Out] integrate((b*log((x^(2/3)*e + d)^n*c) + a)^2/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^4,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^4, x)
```

3.480 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^6}\,dx$

Optimal. Leaf size=640

$$-\frac{8b^2e^2n^2}{715d^2x^{11/3}}+\frac{64b^2e^3n^2}{2145d^3x^3}-\frac{2872b^2e^4n^2}{45045d^4x^{7/3}}+\frac{1216b^2e^5n^2}{9009d^5x^{5/3}}-\frac{224072b^2e^6n^2}{675675d^6x}+\frac{344192b^2e^7n^2}{225225d^7\sqrt[3]{x}}+\frac{704552b^2e^{15/2}n^2\tan^{-1}}{225225d^{15}}$$

```
[Out] -8/715*b^2*e^2*n^2/d^2/x^(11/3)+64/2145*b^2*e^3*n^2/d^3/x^3-2872/45045*b^2*
e^4*n^2/d^4/x^(7/3)+1216/9009*b^2*e^5*n^2/d^5/x^(5/3)-224072/675675*b^2*e^6
*n^2/d^6/x+344192/225225*b^2*e^7*n^2/d^7/x^(1/3)+704552/225225*b^2*e^(15/2)
*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))/d^(15/2)-4/5*I*b^2*e^(15/2)*n^2*polylo
g(2,1-2*d^(1/2)/(d^(1/2)+I*x^(1/3)*e^(1/2)))/d^(15/2)-4/65*b*e*n*(a+b*ln(c*
(d+e*x^(2/3))^n))/d/x^(13/3)+4/55*b*e^2*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d^2/x
^(11/3)-4/45*b*e^3*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d^3/x^3+4/35*b*e^4*n*(a+b*
ln(c*(d+e*x^(2/3))^n))/d^4/x^(7/3)-4/25*b*e^5*n*(a+b*ln(c*(d+e*x^(2/3))^n))
/d^5/x^(5/3)+4/15*b*e^6*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d^6/x-4/5*b*e^7*n*(a+
b*ln(c*(d+e*x^(2/3))^n))/d^7/x^(1/3)-4/5*b*e^(15/2)*n*arctan(x^(1/3)*e^(1/2
)/d^(1/2))*(a+b*ln(c*(d+e*x^(2/3))^n))/d^(15/2)-1/5*(a+b*ln(c*(d+e*x^(2/3))
^n))^2/x^5-8/5*b^2*e^(15/2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))*ln(2*d^(1/2
)/(d^(1/2)+I*x^(1/3)*e^(1/2)))/d^(15/2)-4/5*I*b^2*e^(15/2)*n^2*arctan(x^(1/
3)*e^(1/2)/d^(1/2))^2/d^(15/2)
```

Rubi [A]
time = 0.63, antiderivative size = 640, normalized size of antiderivative = 1.00, number of steps used = 45, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2508, 2507, 2526, 2505, 331, 211, 2520, 12, 5040, 4964, 2449, 2352}

$$-\frac{4ib^2e^{15/2}n^2\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{5d^{15/2}}-\frac{4be^{15/2}n\text{ArcTan}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5d^{15/2}}-\frac{4be^7n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5d^7\sqrt[3]{x}}+\frac{4be^6n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15d^6x}-\frac{4be^5n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{25d^5x^{5/3}}+\frac{4be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{35d^4x^{7/3}}-\frac{4be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{45d^3x^3}+\frac{4be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{55d^2x^{11/3}}-\frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{65dx^{13/3}}-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5x^5}-\frac{4ib^2e^{15/2}n^2\text{ArcTan}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{5d^{15/2}}+\frac{704552b^2e^{15/2}n^2\text{ArcTan}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{225225d^{15/2}}-\frac{8b^2e^{15/2}n^2\text{ArcTan}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{5d^{15/2}}+\frac{344192b^2e^7n^2}{225225d^7\sqrt[3]{x}}-\frac{224072b^2e^6n^2}{675675d^6x}+\frac{1216b^2e^5n^2}{9009d^5x^{5/3}}-\frac{2872b^2e^4n^2}{45045d^4x^{7/3}}+\frac{64b^2e^3n^2}{2145d^3x^3}-\frac{8b^2e^2n^2}{715d^2x^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^6,x]
```

```
[Out] (-8*b^2*e^2*n^2)/(715*d^2*x^(11/3)) + (64*b^2*e^3*n^2)/(2145*d^3*x^3) - (28
72*b^2*e^4*n^2)/(45045*d^4*x^(7/3)) + (1216*b^2*e^5*n^2)/(9009*d^5*x^(5/3))
 - (224072*b^2*e^6*n^2)/(675675*d^6*x) + (344192*b^2*e^7*n^2)/(225225*d^7*x
^(1/3)) + (704552*b^2*e^(15/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(2252
25*d^(15/2)) - (((4*I)/5)*b^2*e^(15/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]
]^2)/d^(15/2) - (8*b^2*e^(15/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(
2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/(5*d^(15/2)) - (4*b*e*n*(a + b*L
og[c*(d + e*x^(2/3))^n]))/(65*d*x^(13/3)) + (4*b*e^2*n*(a + b*Log[c*(d + e*
x^(2/3))^n]))/(55*d^2*x^(11/3)) - (4*b*e^3*n*(a + b*Log[c*(d + e*x^(2/3))^n
]))/(45*d^3*x^3) + (4*b*e^4*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(35*d^4*x^(
7/3)) - (4*b*e^5*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(25*d^5*x^(5/3)) + (4*
```

```
b*e^6*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(15*d^6*x) - (4*b*e^7*n*(a + b*Lo
g[c*(d + e*x^(2/3))^n]))/(5*d^7*x^(1/3)) - (4*b*e^(15/2)*n*ArcTan[(Sqrt[e]*
x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2/3))^n]))/(5*d^(15/2)) - (a + b*L
og[c*(d + e*x^(2/3))^n])^2/(5*x^5) - (((4*I)/5)*b^2*e^(15/2)*n^2*PolyLog[2,
 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/d^(15/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
```

```
b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2508

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1)
- 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
b, c, d, e, m, p, q}, x] && FractionQ[n]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^6}\,dx = 3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{x^{16}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5x^5} + \frac{1}{5}(4ben)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right.\right.}{x^{14}\left(d+ex^2\right)}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5x^5} + \frac{1}{5}(4ben)\text{Subst}\left(\int \left(\frac{a+b\log\left(c\left(d+e\right.\right.}{dx^{14}}\right.\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5x^5} + \frac{(4ben)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^{14}}\,dx, x,\right.}{5d}$$

$$= -\frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{65dx^{13/3}} + \frac{4be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{55d^2x^{11/3}}$$

$$= -\frac{8b^2e^2n^2}{715d^2x^{11/3}} + \frac{8b^2e^3n^2}{495d^3x^3} - \frac{8b^2e^4n^2}{315d^4x^{7/3}} + \frac{8b^2e^5n^2}{175d^5x^{5/3}} - \frac{8b^2e^6n^2}{75d^6x} + \frac{8b^2e^7n}{15d^7\sqrt[3]{}}$$

$$= -\frac{8b^2e^2n^2}{715d^2x^{11/3}} + \frac{64b^2e^3n^2}{2145d^3x^3} - \frac{32b^2e^4n^2}{693d^4x^{7/3}} + \frac{128b^2e^5n^2}{1575d^5x^{5/3}} - \frac{32b^2e^6n^2}{175d^6x} + \frac{64b}{75d}$$

$$= -\frac{8b^2e^2n^2}{715d^2x^{11/3}} + \frac{64b^2e^3n^2}{2145d^3x^3} - \frac{2872b^2e^4n^2}{45045d^4x^{7/3}} + \frac{1912b^2e^5n^2}{17325d^5x^{5/3}} - \frac{1144b^2e^6n^2}{4725d^6x}$$

$$= -\frac{8b^2e^2n^2}{715d^2x^{11/3}} + \frac{64b^2e^3n^2}{2145d^3x^3} - \frac{2872b^2e^4n^2}{45045d^4x^{7/3}} + \frac{1216b^2e^5n^2}{9009d^5x^{5/3}} - \frac{15104b^2e^6n^2}{51975d^6x}$$

$$= -\frac{8b^2e^2n^2}{715d^2x^{11/3}} + \frac{64b^2e^3n^2}{2145d^3x^3} - \frac{2872b^2e^4n^2}{45045d^4x^{7/3}} + \frac{1216b^2e^5n^2}{9009d^5x^{5/3}} - \frac{224072b^2e^6n^2}{675675d^6x}$$

$$\frac{8b^2e^2n^2}{} + \frac{64b^2e^3n^2}{} \quad \frac{2872b^2e^4n^2}{} + \frac{1216b^2e^5n^2}{} \quad \frac{224072b^2e^6n^2}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.75, size = 678, normalized size = 1.06

$$-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5x^5}+\frac{4}{5}ben\left(\frac{2be^{13/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{15/2}}-\frac{2ben\,{}_2F_1\left(-\frac{11}{2},1;-\frac{9}{2};-\frac{ex^{2/3}}{d}\right)}{143d^2x^{11/3}}+\frac{2be^2n\,{}_2F_1\left(-\frac{9}{2},1;-\frac{7}{2};-\frac{ex^{2/3}}{d}\right)}{99d^3x^3}-\frac{2be^3n\,{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};-\frac{ex^{2/3}}{d}\right)}{63d^4x^{7/3}}+\frac{2be^4n\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\frac{ex^{2/3}}{d}\right)}{35d^5x^{5/3}}-\frac{2be^5n\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\frac{ex^{2/3}}{d}\right)}{15d^6x}+\frac{2be^6n\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{ex^{2/3}}{d}\right)}{3d^7\sqrt[3]{x}}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{13dx^{13/3}}+\frac{e\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{11d^2x^{11/3}}-\frac{e^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{9d^3x^3}+\frac{e^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{7d^4x^{7/3}}-\frac{e^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5d^5x^{5/3}}+\frac{e^5\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3d^6x}-\frac{e^6\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^7\sqrt[3]{x}}-\frac{e^{13/2}\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{15/2}}-\frac{ibe^{13/2}n\left(\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)-2i\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)\right)+\text{Li}_2\left(\frac{i\sqrt{d}+\sqrt{e}\sqrt[3]{x}}{-i\sqrt{d}+\sqrt{e}\sqrt[3]{x}}\right)\right)}{d^{15/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^6,x]
```

```
[Out] -1/5*(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^5 + (4*b*e*n*((2*b*e^(13/2)*n*Arc
Tan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/d^(15/2) - (2*b*e*n*Hypergeometric2F1[-11/2
, 1, -9/2, -((e*x^(2/3))/d)])/(143*d^2*x^(11/3)) + (2*b*e^2*n*Hypergeometri
c2F1[-9/2, 1, -7/2, -((e*x^(2/3))/d)])/(99*d^3*x^3) - (2*b*e^3*n*Hypergeome
tric2F1[-7/2, 1, -5/2, -((e*x^(2/3))/d)])/(63*d^4*x^(7/3)) + (2*b*e^4*n*Hyp
ergeometric2F1[-5/2, 1, -3/2, -((e*x^(2/3))/d)])/(35*d^5*x^(5/3)) - (2*b*e^
5*n*Hypergeometric2F1[-3/2, 1, -1/2, -((e*x^(2/3))/d)])/(15*d^6*x) + (2*b*e
^6*n*Hypergeometric2F1[-1/2, 1, 1/2, -((e*x^(2/3))/d)])/(3*d^7*x^(1/3)) - (
a + b*Log[c*(d + e*x^(2/3))^n])/(13*d*x^(13/3)) + (e*(a + b*Log[c*(d + e*x^
(2/3))^n]))/(11*d^2*x^(11/3)) - (e^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/(9*d
^3*x^3) + (e^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/(7*d^4*x^(7/3)) - (e^4*(a
+ b*Log[c*(d + e*x^(2/3))^n]))/(5*d^5*x^(5/3)) + (e^5*(a + b*Log[c*(d + e*x
^(2/3))^n]))/(3*d^6*x) - (e^6*(a + b*Log[c*(d + e*x^(2/3))^n]))/(d^7*x^(1/3
)) - (e^(13/2)*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2/3
))^n]))/d^(15/2) - (I*b*e^(13/2)*n*(ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(ArcT
an[(Sqrt[e]*x^(1/3))/Sqrt[d]] - (2*I)*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*
x^(1/3))]) + PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x^(1/3))/((-I)*Sqrt[d] + Sqrt[
e]*x^(1/3))]))/d^(15/2)))/5
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{2}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^6,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^6,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^6,x, algorithm="maxima")
```

```
[Out] -1/5*b^2*n^2*log(x^(2/3)*e + d)^2/x^5 + integrate(1/15*(15*(b^2*log(c)^2 +
2*a*b*log(c) + a^2)*x*e + 2*(2*b^2*n*x*e + 15*(b^2*log(c) + a*b)*x*e + 15*(
b^2*d*log(c) + a*b*d)*x^(1/3))*n*log(x^(2/3)*e + d) + 15*(b^2*d*log(c)^2 +
2*a*b*d*log(c) + a^2*d)*x^(1/3))/(x^7*e + d*x^(19/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^6,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((x^(2/3)*e + d)^n*c)^2 + 2*a*b*log((x^(2/3)*e + d)^n*c) +
 a^2)/x^6, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2/x**6,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^(2/3)*e + d)^n*c) + a)^2/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^6,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^6, x)
```

3.481 $\int x^3 \left(a + b\log\left(c\left(d + ex^{2/3}\right)^n\right)\right)^3 \, dx$

Optimal. Leaf size=913

$$-\frac{45b^3d^4n^3\left(d+ex^{2/3}\right)^2}{16e^6}+\frac{10b^3d^3n^3\left(d+ex^{2/3}\right)^3}{9e^6}-\frac{45b^3d^2n^3\left(d+ex^{2/3}\right)^4}{128e^6}+\frac{9b^3dn^3\left(d+ex^{2/3}\right)^5}{125e^6}-\frac{b^3n^3\left(d+ex^{2/3}\right)^6}{144e^6}$$

```
[Out] 1/24*b^2*n^2*(d+e*x^(2/3))^6*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6-1/8*b*n*(d+e*x
^(2/3))^6*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e^6+1/4*(d+e*x^(2/3))^6*(a+b*ln(c*(
d+e*x^(2/3))^n))^3/e^6-10/3*b^2*d^3*n^2*(d+e*x^(2/3))^3*(a+b*ln(c*(d+e*x^(2
/3))^n))/e^6+45/32*b^2*d^2*n^2*(d+e*x^(2/3))^4*(a+b*ln(c*(d+e*x^(2/3))^n))/
e^6-9/25*b^2*d*n^2*(d+e*x^(2/3))^5*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6+9/2*b*d^
5*n*(d+e*x^(2/3))*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e^6-45/8*b*d^4*n*(d+e*x^(2/
3))^2*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e^6+5*b*d^3*n*(d+e*x^(2/3))^3*(a+b*ln(c
*(d+e*x^(2/3))^n))^2/e^6-45/16*b*d^2*n*(d+e*x^(2/3))^4*(a+b*ln(c*(d+e*x^(2/
3))^n))^2/e^6+9/10*b*d*n*(d+e*x^(2/3))^5*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e^6-
9*b^3*d^5*n^2*(d+e*x^(2/3))*ln(c*(d+e*x^(2/3))^n)/e^6+45/8*b^2*d^4*n^2*(d+e
*x^(2/3))^2*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6-3/2*d^5*(d+e*x^(2/3))*(a+b*ln(c
*(d+e*x^(2/3))^n))^3/e^6+15/4*d^4*(d+e*x^(2/3))^2*(a+b*ln(c*(d+e*x^(2/3))^n
))^3/e^6-5*d^3*(d+e*x^(2/3))^3*(a+b*ln(c*(d+e*x^(2/3))^n))^3/e^6+15/4*d^2*(
d+e*x^(2/3))^4*(a+b*ln(c*(d+e*x^(2/3))^n))^3/e^6-3/2*d*(d+e*x^(2/3))^5*(a+b
*ln(c*(d+e*x^(2/3))^n))^3/e^6-9*a*b^2*d^5*n^2*x^(2/3)/e^5-45/16*b^3*d^4*n^3
*(d+e*x^(2/3))^2/e^6+10/9*b^3*d^3*n^3*(d+e*x^(2/3))^3/e^6-45/128*b^3*d^2*n^
3*(d+e*x^(2/3))^4/e^6+9/125*b^3*d*n^3*(d+e*x^(2/3))^5/e^6+9*b^3*d^5*n^3*x^(
2/3)/e^5-1/144*b^3*n^3*(d+e*x^(2/3))^6/e^6
```

Rubi [A]
time = 0.67, antiderivative size = 913, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

```
[Out] (-45*b^3*d^4*n^3*(d + e*x^(2/3))^2)/(16*e^6) + (10*b^3*d^3*n^3*(d + e*x^(2/
3))^3)/(9*e^6) - (45*b^3*d^2*n^3*(d + e*x^(2/3))^4)/(128*e^6) + (9*b^3*d*n^
3*(d + e*x^(2/3))^5)/(125*e^6) - (b^3*n^3*(d + e*x^(2/3))^6)/(144*e^6) - (9
*a*b^2*d^5*n^2*x^(2/3))/e^5 + (9*b^3*d^5*n^3*x^(2/3))/e^5 - (9*b^3*d^5*n^2*
(d + e*x^(2/3))*Log[c*(d + e*x^(2/3))^n])/e^6 + (45*b^2*d^4*n^2*(d + e*x^(2
/3))^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/(8*e^6) - (10*b^2*d^3*n^2*(d + e*x
^(2/3))^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/(3*e^6) + (45*b^2*d^2*n^2*(d +
e*x^(2/3))^4*(a + b*Log[c*(d + e*x^(2/3))^n]))/(32*e^6) - (9*b^2*d*n^2*(d +
 e*x^(2/3))^5*(a + b*Log[c*(d + e*x^(2/3))^n]))/(25*e^6) + (b^2*n^2*(d + e*
```

```
x^(2/3))^6*(a + b*Log[c*(d + e*x^(2/3))^n]))/(24*e^6) + (9*b*d^5*n*(d + e*x
^(2/3))*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(2*e^6) - (45*b*d^4*n*(d + e*x^
(2/3))^2*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(8*e^6) + (5*b*d^3*n*(d + e*x^
(2/3))^3*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/e^6 - (45*b*d^2*n*(d + e*x^(2/
3))^4*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(16*e^6) + (9*b*d*n*(d + e*x^(2/3
))^5*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(10*e^6) - (b*n*(d + e*x^(2/3))^6*
(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(8*e^6) - (3*d^5*(d + e*x^(2/3))*(a + b
*Log[c*(d + e*x^(2/3))^n])^3)/(2*e^6) + (15*d^4*(d + e*x^(2/3))^2*(a + b*Lo
g[c*(d + e*x^(2/3))^n])^3)/(4*e^6) - (5*d^3*(d + e*x^(2/3))^3*(a + b*Log[c*
(d + e*x^(2/3))^n])^3)/e^6 + (15*d^2*(d + e*x^(2/3))^4*(a + b*Log[c*(d + e*
x^(2/3))^n])^3)/(4*e^6) - (3*d*(d + e*x^(2/3))^5*(a + b*Log[c*(d + e*x^(2/3
))^n])^3)/(2*e^6) + ((d + e*x^(2/3))^6*(a + b*Log[c*(d + e*x^(2/3))^n])^3)/
(4*e^6)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
```

```
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=\frac{3}{2}\text{Subst}\left(\int x^5(a+b\log\left(c(d+ex)^n\right))^3\,dx,x,x^{2/3}\right)$$

$$=\frac{3}{2}\text{Subst}\left(\int\left(-\frac{d^5(a+b\log\left(c(d+ex)^n\right))^3}{e^5}+\frac{5d^4(d+ex)\left(a+b\log\right.}{e^5}\right.\right.$$

$$=\frac{3\text{Subst}\left(\int(d+ex)^5\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx,x,x^{2/3}\right)}{2e^5}-\frac{(15d)\text{Su}}{}$$

$$=\frac{3\text{Subst}\left(\int x^5(a+b\log\left(cx^n\right))^3\,dx,x,d+ex^{2/3}\right)}{2e^6}-\frac{(15d)\text{Subst}\left(\int x^4\right.}{}$$

$$=-\frac{3d^5\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^6}+\frac{15d^4\left(d+ex^{2/3}\right)^2}{}$$

$$=\frac{9bd^5n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2e^6}-\frac{45bd^4n\left(d+ex^{2/3}\right.}{}$$

$$=-\frac{45b^3d^4n^3\left(d+ex^{2/3}\right)^2}{16e^6}+\frac{10b^3d^3n^3\left(d+ex^{2/3}\right)^3}{9e^6}-\frac{45b^3d^2n^3\left(d+e\right.}{128e^6}$$

$$=-\frac{45b^3d^4n^3\left(d+ex^{2/3}\right)^2}{16e^6}+\frac{10b^3d^3n^3\left(d+ex^{2/3}\right)^3}{9e^6}-\frac{45b^3d^2n^3\left(d+e\right.}{128e^6}$$

Mathematica [A]
time = 0.72, size = 598, normalized size = 0.65

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

```
[Out] (e*x^(2/3)*(36000*a^3*e^5*x^(10/3) + b^3*n^3*(809340*d^5 - 140070*d^4*e*x^(
2/3) + 41180*d^3*e^2*x^(4/3) - 13785*d^2*e^3*x^2 + 4368*d*e^4*x^(8/3) - 100
0*e^5*x^(10/3)) - 60*a*b^2*n^2*(8820*d^5 - 2610*d^4*e*x^(2/3) + 1140*d^3*e^
2*x^(4/3) - 555*d^2*e^3*x^2 + 264*d*e^4*x^(8/3) - 100*e^5*x^(10/3)) + 1800*
a^2*b*n*(60*d^5 - 30*d^4*e*x^(2/3) + 20*d^3*e^2*x^(4/3) - 15*d^2*e^3*x^2 +
12*d*e^4*x^(8/3) - 10*e^5*x^(10/3))) - 280140*b^3*d^6*n^3*Log[d + e*x^(2/3)
] - 60*b*(b^2*n^2*(8820*d^6 + 8820*d^5*e*x^(2/3) - 2610*d^4*e^2*x^(4/3) + 1
140*d^3*e^3*x^2 - 555*d^2*e^4*x^(8/3) + 264*d*e^5*x^(10/3) - 100*e^6*x^4) -
 60*a*b*n*(147*d^6 + 60*d^5*e*x^(2/3) - 30*d^4*e^2*x^(4/3) + 20*d^3*e^3*x^2
 - 15*d^2*e^4*x^(8/3) + 12*d*e^5*x^(10/3) - 10*e^6*x^4) + 1800*a^2*(d^6 - e
^6*x^4))*Log[c*(d + e*x^(2/3))^n] + 1800*b^2*(b*n*(147*d^6 + 60*d^5*e*x^(2/
3) - 30*d^4*e^2*x^(4/3) + 20*d^3*e^3*x^2 - 15*d^2*e^4*x^(8/3) + 12*d*e^5*x^
(10/3) - 10*e^6*x^4) - 60*a*(d^6 - e^6*x^4))*Log[c*(d + e*x^(2/3))^n]^2 - 3
6000*b^3*(d^6 - e^6*x^4)*Log[c*(d + e*x^(2/3))^n]^3)/(144000*e^6)
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^{3}\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(2/3))^n))^3,x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e*x^(2/3))^n))^3,x)
```

Maxima [**A**]
time = 0.31, size = 669, normalized size = 0.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*x^4*log((x^(2/3)*e + d)^n*c)^3 + 3/4*a*b^2*x^4*log((x^(2/3)*e + d)^
n*c)^2 + 3/4*a^2*b*x^4*log((x^(2/3)*e + d)^n*c) + 1/4*a^3*x^4 - 1/80*(60*d^
6*e^(-7)*log(x^(2/3)*e + d) + (30*d^4*x^(4/3)*e - 20*d^3*x^2*e^2 - 60*d^5*x
^(2/3) + 15*d^2*x^(8/3)*e^3 - 12*d*x^(10/3)*e^4 + 10*x^4*e^5)*e^(-6))*a^2*b
*n*e + 1/2400*((1800*d^6*log(x^(2/3)*e + d)^2 + 8820*d^6*log(x^(2/3)*e + d)
 - 8820*d^5*x^(2/3)*e + 2610*d^4*x^(4/3)*e^2 - 1140*d^3*x^2*e^3 + 555*d^2*x
^(8/3)*e^4 - 264*d*x^(10/3)*e^5 + 100*x^4*e^6)*n^2*e^(-6) - 60*(60*d^6*e^(-
7)*log(x^(2/3)*e + d) + (30*d^4*x^(4/3)*e - 20*d^3*x^2*e^2 - 60*d^5*x^(2/3)
 + 15*d^2*x^(8/3)*e^3 - 12*d*x^(10/3)*e^4 + 10*x^4*e^5)*e^(-6))*n*e*log((x^
```

```
(2/3)*e + d)^n*c))*a*b^2 - 1/144000*(1800*(60*d^6*e^(-7)*log(x^(2/3)*e + d)
 + (30*d^4*x^(4/3)*e - 20*d^3*x^2*e^2 - 60*d^5*x^(2/3) + 15*d^2*x^(8/3)*e^3
 - 12*d*x^(10/3)*e^4 + 10*x^4*e^5)*e^(-6))*n*e*log((x^(2/3)*e + d)^n*c)^2 +
 ((36000*d^6*log(x^(2/3)*e + d)^3 + 264600*d^6*log(x^(2/3)*e + d)^2 + 80934
0*d^6*log(x^(2/3)*e + d) - 809340*d^5*x^(2/3)*e + 140070*d^4*x^(4/3)*e^2 -
41180*d^3*x^2*e^3 + 13785*d^2*x^(8/3)*e^4 - 4368*d*x^(10/3)*e^5 + 1000*x^4*
e^6)*n^2*e^(-7) - 60*(1800*d^6*log(x^(2/3)*e + d)^2 + 8820*d^6*log(x^(2/3)*
e + d) - 8820*d^5*x^(2/3)*e + 2610*d^4*x^(4/3)*e^2 - 1140*d^3*x^2*e^3 + 555
*d^2*x^(8/3)*e^4 - 264*d*x^(10/3)*e^5 + 100*x^4*e^6)*n*e^(-7)*log((x^(2/3)*
e + d)^n*c))*n*e)*b^3
```

Fricas [A]
time = 0.45, size = 1142, normalized size = 1.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="fricas")
```

```
[Out] 1/144000*(36000*b^3*x^4*e^6*log(c)^3 - 1000*(b^3*n^3 - 6*a*b^2*n^2 + 18*a^2
*b*n - 36*a^3)*x^4*e^6 + 20*(2059*b^3*d^3*n^3 - 3420*a*b^2*d^3*n^2 + 1800*a
^2*b*d^3*n)*x^2*e^3 - 36000*(b^3*d^6*n^3 - b^3*n^3*x^4*e^6)*log(x^(2/3)*e +
 d)^3 + 1800*(147*b^3*d^6*n^3 + 20*b^3*d^3*n^3*x^2*e^3 - 60*a*b^2*d^6*n^2 -
 10*(b^3*n^3 - 6*a*b^2*n^2)*x^4*e^6 - 60*(b^3*d^6*n^2 - b^3*n^2*x^4*e^6)*lo
g(c) + 15*(4*b^3*d^5*n^3*e - b^3*d^2*n^3*x^2*e^4)*x^(2/3) - 6*(5*b^3*d^4*n^
3*x*e^2 - 2*b^3*d*n^3*x^3*e^5)*x^(1/3))*log(x^(2/3)*e + d)^2 + 18000*(2*b^3
*d^3*n*x^2*e^3 - (b^3*n - 6*a*b^2)*x^4*e^6)*log(c)^2 - 60*(13489*b^3*d^6*n^
3 - 8820*a*b^2*d^6*n^2 + 1800*a^2*b*d^6*n - 100*(b^3*n^3 - 6*a*b^2*n^2 + 18
*a^2*b*n)*x^4*e^6 + 60*(19*b^3*d^3*n^3 - 20*a*b^2*d^3*n^2)*x^2*e^3 + 1800*(
b^3*d^6*n - b^3*n*x^4*e^6)*log(c)^2 - 60*(147*b^3*d^6*n^2 + 20*b^3*d^3*n^2*
x^2*e^3 - 60*a*b^2*d^6*n - 10*(b^3*n^2 - 6*a*b^2*n)*x^4*e^6)*log(c) - 15*((
37*b^3*d^2*n^3 - 60*a*b^2*d^2*n^2)*x^2*e^4 - 12*(49*b^3*d^5*n^3 - 20*a*b^2*
d^5*n^2)*e + 60*(4*b^3*d^5*n^2*e - b^3*d^2*n^2*x^2*e^4)*log(c))*x^(2/3) + 6
*(4*(11*b^3*d*n^3 - 30*a*b^2*d*n^2)*x^3*e^5 - 15*(29*b^3*d^4*n^3 - 20*a*b^2
*d^4*n^2)*x*e^2 + 60*(5*b^3*d^4*n^2*x*e^2 - 2*b^3*d*n^2*x^3*e^5)*log(c))*x^
(1/3))*log(x^(2/3)*e + d) + 1200*(5*(b^3*n^2 - 6*a*b^2*n + 18*a^2*b)*x^4*e^
6 - 3*(19*b^3*d^3*n^2 - 20*a*b^2*d^3*n)*x^2*e^3)*log(c) - 15*((919*b^3*d^2*
n^3 - 2220*a*b^2*d^2*n^2 + 1800*a^2*b*d^2*n)*x^2*e^4 - 1800*(4*b^3*d^5*n*e
- b^3*d^2*n*x^2*e^4)*log(c)^2 - 4*(13489*b^3*d^5*n^3 - 8820*a*b^2*d^5*n^2 +
 1800*a^2*b*d^5*n)*e - 60*((37*b^3*d^2*n^2 - 60*a*b^2*d^2*n)*x^2*e^4 - 12*(
49*b^3*d^5*n^2 - 20*a*b^2*d^5*n)*e)*log(c))*x^(2/3) + 6*(8*(91*b^3*d*n^3 -
330*a*b^2*d*n^2 + 450*a^2*b*d*n)*x^3*e^5 - 5*(4669*b^3*d^4*n^3 - 5220*a*b^2
*d^4*n^2 + 1800*a^2*b*d^4*n)*x*e^2 - 1800*(5*b^3*d^4*n*x*e^2 - 2*b^3*d*n*x^
3*e^5)*log(c)^2 - 60*(4*(11*b^3*d*n^2 - 30*a*b^2*d*n)*x^3*e^5 - 15*(29*b^3*
d^4*n^2 - 20*a*b^2*d^4*n)*x*e^2)*log(c))*x^(1/3))*e^(-6)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(2/3))**n))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2174 vs. 2(803) = 1606.
time = 4.30, size = 2174, normalized size = 2.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="giac")
```

```
[Out] 1/4*b^3*x^4*log(c)^3 + 3/4*a*b^2*x^4*log(c)^2 + 3/4*a^2*b*x^4*log(c) + 1/14
4000*(36000*(x^(2/3)*e + d)^6*e^(-6)*log(x^(2/3)*e + d)^3 - 216000*(x^(2/3)
*e + d)^5*d*e^(-6)*log(x^(2/3)*e + d)^3 + 540000*(x^(2/3)*e + d)^4*d^2*e^(-
6)*log(x^(2/3)*e + d)^3 - 720000*(x^(2/3)*e + d)^3*d^3*e^(-6)*log(x^(2/3)*e
 + d)^3 + 540000*(x^(2/3)*e + d)^2*d^4*e^(-6)*log(x^(2/3)*e + d)^3 - 18000*
(x^(2/3)*e + d)^6*e^(-6)*log(x^(2/3)*e + d)^2 + 129600*(x^(2/3)*e + d)^5*d*
e^(-6)*log(x^(2/3)*e + d)^2 - 405000*(x^(2/3)*e + d)^4*d^2*e^(-6)*log(x^(2/
3)*e + d)^2 + 720000*(x^(2/3)*e + d)^3*d^3*e^(-6)*log(x^(2/3)*e + d)^2 - 81
0000*(x^(2/3)*e + d)^2*d^4*e^(-6)*log(x^(2/3)*e + d)^2 + 6000*(x^(2/3)*e +
d)^6*e^(-6)*log(x^(2/3)*e + d) - 51840*(x^(2/3)*e + d)^5*d*e^(-6)*log(x^(2/
3)*e + d) + 202500*(x^(2/3)*e + d)^4*d^2*e^(-6)*log(x^(2/3)*e + d) - 480000
*(x^(2/3)*e + d)^3*d^3*e^(-6)*log(x^(2/3)*e + d) + 810000*(x^(2/3)*e + d)^2
*d^4*e^(-6)*log(x^(2/3)*e + d) - 1000*(x^(2/3)*e + d)^6*e^(-6) + 10368*(x^(
2/3)*e + d)^5*d*e^(-6) - 50625*(x^(2/3)*e + d)^4*d^2*e^(-6) + 160000*(x^(2/
3)*e + d)^3*d^3*e^(-6) - 405000*(x^(2/3)*e + d)^2*d^4*e^(-6) - 216000*((x^(
2/3)*e + d)*log(x^(2/3)*e + d)^3 - 3*(x^(2/3)*e + d)*log(x^(2/3)*e + d)^2 +
 6*(x^(2/3)*e + d)*log(x^(2/3)*e + d) - 6*x^(2/3)*e - 6*d)*d^5*e^(-6))*b^3*
n^3 + 1/4*a^3*x^4 + 1/2400*(1800*(x^(2/3)*e + d)^6*e^(-6)*log(x^(2/3)*e + d
)^2 - 10800*(x^(2/3)*e + d)^5*d*e^(-6)*log(x^(2/3)*e + d)^2 + 27000*(x^(2/3
)*e + d)^4*d^2*e^(-6)*log(x^(2/3)*e + d)^2 - 36000*(x^(2/3)*e + d)^3*d^3*e^
(-6)*log(x^(2/3)*e + d)^2 + 27000*(x^(2/3)*e + d)^2*d^4*e^(-6)*log(x^(2/3)*
e + d)^2 - 600*(x^(2/3)*e + d)^6*e^(-6)*log(x^(2/3)*e + d) + 4320*(x^(2/3)*
e + d)^5*d*e^(-6)*log(x^(2/3)*e + d) - 13500*(x^(2/3)*e + d)^4*d^2*e^(-6)*l
og(x^(2/3)*e + d) + 24000*(x^(2/3)*e + d)^3*d^3*e^(-6)*log(x^(2/3)*e + d) -
 27000*(x^(2/3)*e + d)^2*d^4*e^(-6)*log(x^(2/3)*e + d) + 100*(x^(2/3)*e + d
)^6*e^(-6) - 864*(x^(2/3)*e + d)^5*d*e^(-6) + 3375*(x^(2/3)*e + d)^4*d^2*e^
```

```
(-6) - 8000*(x^(2/3)*e + d)^3*d^3*e^(-6) + 13500*(x^(2/3)*e + d)^2*d^4*e^(-
6) - 10800*((x^(2/3)*e + d)*log(x^(2/3)*e + d)^2 - 2*(x^(2/3)*e + d)*log(x^
(2/3)*e + d) + 2*x^(2/3)*e + 2*d)*d^5*e^(-6))*b^3*n^2*log(c) + 1/80*(60*(x^
(2/3)*e + d)^6*e^(-6)*log(x^(2/3)*e + d) - 360*(x^(2/3)*e + d)^5*d*e^(-6)*l
og(x^(2/3)*e + d) + 900*(x^(2/3)*e + d)^4*d^2*e^(-6)*log(x^(2/3)*e + d) - 1
200*(x^(2/3)*e + d)^3*d^3*e^(-6)*log(x^(2/3)*e + d) + 900*(x^(2/3)*e + d)^2
*d^4*e^(-6)*log(x^(2/3)*e + d) - 10*(x^(2/3)*e + d)^6*e^(-6) + 72*(x^(2/3)*
e + d)^5*d*e^(-6) - 225*(x^(2/3)*e + d)^4*d^2*e^(-6) + 400*(x^(2/3)*e + d)^
3*d^3*e^(-6) - 450*(x^(2/3)*e + d)^2*d^4*e^(-6) - 360*((x^(2/3)*e + d)*log(
x^(2/3)*e + d) - x^(2/3)*e - d)*d^5*e^(-6))*b^3*n*log(c)^2 + 1/2400*(1800*(
x^(2/3)*e + d)^6*e^(-6)*log(x^(2/3)*e + d)^2 - 10800*(x^(2/3)*e + d)^5*d*e^
(-6)*log(x^(2/3)*e + d)^2 + 27000*(x^(2/3)*e + d)^4*d^2*e^(-6)*log(x^(2/3)*
e + d)^2 - 36000*(x^(2/3)*e + d)^3*d^3*e^(-6)*log(x^(2/3)*e + d)^2 + 27000*
(x^(2/3)*e + d)^2*d^4*e^(-6)*log(x^(2/3)*e + d)^2 - 600*(x^(2/3)*e + d)^6*e
^(-6)*log(x^(2/3)*e + d) + 4320*(x^(2/3)*e + d)^5*d*e^(-6)*log(x^(2/3)*e +
d) - 13500*(x^(2/3)*e + d)^4*d^2*e^(-6)*log(x^(2/3)*e + d) + 24000*(x^(2/3)
*e + d)^3*d^3*e^(-6)*log(x^(2/3)*e + d) - 27000*(x^(2/3)*e + d)^2*d^4*e^(-6
)*log(x^(2/3)*e + d) + 100*(x^(2/3)*e + d)^6*e^(-6) - 864*(x^(2/3)*e + d)^5
*d*e^(-6) + 3375*(x^(2/3)*e + d)^4*d^2*e^(-6) - 8000*(x^(2/3)*e + d)^3*d^3*
e^(-6) + 13500*(x^(2/3)*e + d)^2*d^4*e^(-6) - 10800*((x^(2/3)*e + d)*log(x^
(2/3)*e + d)^2 - 2*(x^(2/3)*e + d)*log(x^(2/3)*e + d) + 2*x^(2/3)*e + 2*d)*
d^5*e^(-6))*a*b^2*n^2 + 1/40*(60*(x^(2/3)*e + d)^6*e^(-6)*log(x^(2/3)*e + d
) - 360*(x^(2/3)*e + d)^5*d*e^(-6)*log(x^(2/3)*e + d) + 900*(x^(2/3)*e + d)
^4*d^2*e^(-6)*log(x^(2/3)*e + d) - 1200*(x^(2/3)*e + d)^3*d^3*e^(-6)*log(x^
(2/3)*e + d) + 900*(x^(2/3)*e + d)^2*d^4*e^(-6)*log(x^(2/3)*e + d) - 10*(x^
(2/3)*e + d)^6*e^(-6) + 72*(x^(2/3)*e + d)^5*d*e^(-6) - 225*(x^(2/3)*e + d)
^4*d^2*e^(-6) + 400*(x^(2/3)*e + d)^3*d^3*e^(-6) - 450*(x^(2/3)*e + d)^2*d^
4*e^(-6) - 360*((x^(2/3)*e + d)*log(x^(2/3)*e + d) - x^(2/3)*e - d)*d^5*e^(
-6))*a*b^2*n*log(c) + 1/80*(60*(x^(2/3)*e + d)^6*e^(-6)*log(x^(2/3)*e + d)
- 360*(x^(2/3)*e + d)^5*d*e^(-6)*log(x^(2/3)*e + d) + 900*(x^(2/3)*e + d)^4
*d^2*e^(-6)*log(x^(2/3)*e + d) - 1200*(x^(2/3)*e + d)^3*d^3*e^(-6)*log(x^(2
/3)*e + d) + 900*(x^(2/3)*e + d)^2*d^4*e^(-6)*log(x^(2/3)*e + d) - 10*(x^(2
/3)*e + d)^6*e^(-6) + 72*(x^(2/3)*e + d)^5*d*e^(-6) - 225*(x^(2/3)*e + d)^4
*d^2*e^(-6) + 400*(x^(2/3)*e + d)^3*d^3*e^(-6) - 450*(x^(2/3)*e + d)^2*d^4*
e^(-6) - 360*((x^(2/3)*e + d)*log(x^(2/3)*e + d) - x^(2/3)*e - d)*d^5*e^(-6
))*a^2*b*n
```

Mupad [B]
time = 8.10, size = 992, normalized size = 1.09

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e*x^(2/3))^n))^3,x)
```

```
[Out] (a^3*x^4)/4 + (b^3*x^4*log(c*(d + e*x^(2/3))^n)^3)/4 - (b^3*n^3*x^4)/144 +
(3*a*b^2*x^4*log(c*(d + e*x^(2/3))^n)^2)/4 - (b^3*n*x^4*log(c*(d + e*x^(2/3
))^n)^2)/8 + (b^3*n^2*x^4*log(c*(d + e*x^(2/3))^n))/24 + (a*b^2*n^2*x^4)/24
 - (b^3*d^6*log(c*(d + e*x^(2/3))^n)^3)/(4*e^6) + (3*a^2*b*x^4*log(c*(d + e
*x^(2/3))^n))/4 - (a^2*b*n*x^4)/8 - (a*b^2*n*x^4*log(c*(d + e*x^(2/3))^n))/
4 - (13489*b^3*d^6*n^3*log(d + e*x^(2/3)))/(2400*e^6) + (2059*b^3*d^3*n^3*x
^2)/(7200*e^3) - (919*b^3*d^2*n^3*x^(8/3))/(9600*e^2) - (4669*b^3*d^4*n^3*x
^(4/3))/(4800*e^4) + (13489*b^3*d^5*n^3*x^(2/3))/(2400*e^5) - (3*a*b^2*d^6*
log(c*(d + e*x^(2/3))^n)^2)/(4*e^6) + (147*b^3*d^6*n*log(c*(d + e*x^(2/3))^
n)^2)/(80*e^6) + (91*b^3*d*n^3*x^(10/3))/(3000*e) - (3*a^2*b*d^6*n*log(d +
e*x^(2/3)))/(4*e^6) + (3*b^3*d*n*x^(10/3)*log(c*(d + e*x^(2/3))^n)^2)/(20*e
) - (11*b^3*d*n^2*x^(10/3)*log(c*(d + e*x^(2/3))^n))/(100*e) + (a^2*b*d^3*n
*x^2)/(4*e^3) - (3*a^2*b*d^2*n*x^(8/3))/(16*e^2) - (3*a^2*b*d^4*n*x^(4/3))/
(8*e^4) + (3*a^2*b*d^5*n*x^(2/3))/(4*e^5) - (11*a*b^2*d*n^2*x^(10/3))/(100*
e) + (147*a*b^2*d^6*n^2*log(d + e*x^(2/3)))/(40*e^6) + (b^3*d^3*n*x^2*log(c
*(d + e*x^(2/3))^n)^2)/(4*e^3) - (19*b^3*d^3*n^2*x^2*log(c*(d + e*x^(2/3))^
n))/(40*e^3) - (3*b^3*d^2*n*x^(8/3)*log(c*(d + e*x^(2/3))^n)^2)/(16*e^2) +
(37*b^3*d^2*n^2*x^(8/3)*log(c*(d + e*x^(2/3))^n))/(160*e^2) - (3*b^3*d^4*n*
x^(4/3)*log(c*(d + e*x^(2/3))^n)^2)/(8*e^4) + (87*b^3*d^4*n^2*x^(4/3)*log(c
*(d + e*x^(2/3))^n))/(80*e^4) + (3*b^3*d^5*n*x^(2/3)*log(c*(d + e*x^(2/3))^
n)^2)/(4*e^5) - (147*b^3*d^5*n^2*x^(2/3)*log(c*(d + e*x^(2/3))^n))/(40*e^5)
 - (19*a*b^2*d^3*n^2*x^2)/(40*e^3) + (37*a*b^2*d^2*n^2*x^(8/3))/(160*e^2) +
 (87*a*b^2*d^4*n^2*x^(4/3))/(80*e^4) - (147*a*b^2*d^5*n^2*x^(2/3))/(40*e^5)
 + (3*a^2*b*d*n*x^(10/3))/(20*e) + (3*a*b^2*d*n*x^(10/3)*log(c*(d + e*x^(2/
3))^n))/(10*e) + (a*b^2*d^3*n*x^2*log(c*(d + e*x^(2/3))^n))/(2*e^3) - (3*a*
b^2*d^2*n*x^(8/3)*log(c*(d + e*x^(2/3))^n))/(8*e^2) - (3*a*b^2*d^4*n*x^(4/3
)*log(c*(d + e*x^(2/3))^n))/(4*e^4) + (3*a*b^2*d^5*n*x^(2/3)*log(c*(d + e*x
^(2/3))^n))/(2*e^5)
```

3.482 $\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=449

$$\frac{9b^3dn^3\left(d+ex^{2/3}\right)^2}{8e^3}-\frac{b^3n^3\left(d+ex^{2/3}\right)^3}{9e^3}+\frac{9ab^2d^2n^2x^{2/3}}{e^2}-\frac{9b^3d^2n^3x^{2/3}}{e^2}+\frac{9b^3d^2n^2\left(d+ex^{2/3}\right)\log\left(c\left(d+ex^{2/3}\right)^n\right.}{e^3}$$

```
[Out] 9/8*b^3*d*n^3*(d+e*x^(2/3))^2/e^3-1/9*b^3*n^3*(d+e*x^(2/3))^3/e^3+9*a*b^2*d
^2*n^2*x^(2/3)/e^2-9*b^3*d^2*n^3*x^(2/3)/e^2+9*b^3*d^2*n^2*(d+e*x^(2/3))*ln
(c*(d+e*x^(2/3))^n)/e^3-9/4*b^2*d*n^2*(d+e*x^(2/3))^2*(a+b*ln(c*(d+e*x^(2/3
))^n))/e^3+1/3*b^2*n^2*(d+e*x^(2/3))^3*(a+b*ln(c*(d+e*x^(2/3))^n))/e^3-9/2*
b*d^2*n*(d+e*x^(2/3))*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e^3+9/4*b*d*n*(d+e*x^(2
/3))^2*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e^3-1/2*b*n*(d+e*x^(2/3))^3*(a+b*ln(c*
(d+e*x^(2/3))^n))^2/e^3+3/2*d^2*(d+e*x^(2/3))*(a+b*ln(c*(d+e*x^(2/3))^n))^3
/e^3-3/2*d*(d+e*x^(2/3))^2*(a+b*ln(c*(d+e*x^(2/3))^n))^3/e^3+1/2*(d+e*x^(2/
3))^3*(a+b*ln(c*(d+e*x^(2/3))^n))^3/e^3
```

Rubi [A]
time = 0.30, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

$$\frac{b^2n^2\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^3}-\frac{9b^2dn^2\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{4e^3}+\frac{9ab^2d^2n^2x^{2/3}}{e^2}-\frac{9bd^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2e^3}+\frac{3d^2\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^3}-\frac{bn\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2e^3}+\frac{9bdn\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4e^3}+\frac{\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^3}-\frac{3d\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^3}+\frac{9b^3d^2n^2\left(d+ex^{2/3}\right)\log\left(c\left(d+ex^{2/3}\right)^n\right)}{e^3}-\frac{9b^3d^2n^3x^{2/3}}{e^2}-\frac{b^3n^3\left(d+ex^{2/3}\right)^3}{9e^3}+\frac{9b^3dn^3\left(d+ex^{2/3}\right)^2}{8e^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

```
[Out] (9*b^3*d*n^3*(d + e*x^(2/3))^2)/(8*e^3) - (b^3*n^3*(d + e*x^(2/3))^3)/(9*e^
3) + (9*a*b^2*d^2*n^2*x^(2/3))/e^2 - (9*b^3*d^2*n^3*x^(2/3))/e^2 + (9*b^3*d
^2*n^2*(d + e*x^(2/3))*Log[c*(d + e*x^(2/3))^n])/e^3 - (9*b^2*d*n^2*(d + e*
x^(2/3))^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/(4*e^3) + (b^2*n^2*(d + e*x^(2
/3))^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/(3*e^3) - (9*b*d^2*n*(d + e*x^(2/3
))*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(2*e^3) + (9*b*d*n*(d + e*x^(2/3))^2
*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(4*e^3) - (b*n*(d + e*x^(2/3))^3*(a +
b*Log[c*(d + e*x^(2/3))^n])^2)/(2*e^3) + (3*d^2*(d + e*x^(2/3))*(a + b*Log[
c*(d + e*x^(2/3))^n])^3)/(2*e^3) - (3*d*(d + e*x^(2/3))^2*(a + b*Log[c*(d +
 e*x^(2/3))^n])^3)/(2*e^3) + ((d + e*x^(2/3))^3*(a + b*Log[c*(d + e*x^(2/3)
)^n])^3)/(2*e^3)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx = \frac{3}{2}\text{Subst}\left(\int x^2(a+b\log(c(d+ex)^n))^3\,dx,x,x^{2/3}\right)$$
$$= \frac{3}{2}\text{Subst}\left(\int\left(\frac{d^2(a+b\log(c(d+ex)^n))^3}{e^2}-\frac{2d(d+ex)\,(a+b\log(c\ldots}{e^2}\right.\right.$$
$$= \frac{3\text{Subst}\left(\int(d+ex)^2\,(a+b\log(c(d+ex)^n))^3\,dx,x,x^{2/3}\right)}{2e^2}-\frac{(3d)\text{Sub}\ldots}{}$$
$$= \frac{3\text{Subst}\left(\int x^2(a+b\log(cx^n))^3\,dx,x,d+ex^{2/3}\right)}{2e^3}-\frac{(3d)\text{Subst}\left(\int x(a\ldots\right.}{}$$
$$= \frac{3d^2\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^3}-\frac{3d\left(d+ex^{2/3}\right)^2(a+\ldots}{2\ldots}$$
$$= -\frac{9bd^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2e^3}+\frac{9bdn\left(d+ex^{2/3}\right)\ldots}{}$$
$$= \frac{9b^3dn^3\left(d+ex^{2/3}\right)^2}{8e^3}-\frac{b^3n^3\left(d+ex^{2/3}\right)^3}{9e^3}+\frac{9ab^2d^2n^2x^{2/3}}{e^2}-\frac{9b^2dn^2(\ldots}{}$$
$$= \frac{9b^3dn^3\left(d+ex^{2/3}\right)^2}{8e^3}-\frac{b^3n^3\left(d+ex^{2/3}\right)^3}{9e^3}+\frac{9ab^2d^2n^2x^{2/3}}{e^2}-\frac{9b^3d^2n^3\ldots}{e^2}$$

Mathematica [A]
time = 0.29, size = 428, normalized size = 0.95

$$\frac{36a^3d^3-198a^2bd^3n-108a^2bd^2enx^{2/3}+396ab^2d^2en^2x^{2/3}-510b^3d^2en^3x^{2/3}+54a^2bde^2nx^{4/3}-90ab^2de^2n^2x^{4/3}+57b^3de^2n^3x^{4/3}+36a^3e^3x^2-36a^2be^3nx^2+24ab^2e^3n^2x^2-8b^3e^3n^3x^2+114b^3d^3n^3\log\left(d+ex^{2/3}\right)+6b\left(18a^2\left(d^3+e^3x^2\right)-6abn\left(11d^3+6d^2ex^{2/3}-3de^2x^{4/3}+2e^3x^2\right)+b^2n^2\left(66d^3+66d^2ex^{2/3}-15de^2x^{4/3}+4e^3x^2\right)\right)\log\left(c\left(d+ex^{2/3}\right)^n\right)+18b^2\left(6a\left(d^3+e^3x^2\right)-bn\left(11d^3+6d^2ex^{2/3}-3de^2x^{4/3}+2e^3x^2\right)\right)\log^2\left(c\left(d+ex^{2/3}\right)^n\right)+36b^3\left(d^3+e^3x^2\right)\log^3\left(c\left(d+ex^{2/3}\right)^n\right)}{72e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

```
[Out] (36*a^3*d^3 - 198*a^2*b*d^3*n - 108*a^2*b*d^2*e*n*x^(2/3) + 396*a*b^2*d^2*e
*n^2*x^(2/3) - 510*b^3*d^2*e*n^3*x^(2/3) + 54*a^2*b*d*e^2*n*x^(4/3) - 90*a*
b^2*d*e^2*n^2*x^(4/3) + 57*b^3*d*e^2*n^3*x^(4/3) + 36*a^3*e^3*x^2 - 36*a^2*
b*e^3*n*x^2 + 24*a*b^2*e^3*n^2*x^2 - 8*b^3*e^3*n^3*x^2 + 114*b^3*d^3*n^3*Lo
g[d + e*x^(2/3)] + 6*b*(18*a^2*(d^3 + e^3*x^2) - 6*a*b*n*(11*d^3 + 6*d^2*e*
x^(2/3) - 3*d*e^2*x^(4/3) + 2*e^3*x^2) + b^2*n^2*(66*d^3 + 66*d^2*e*x^(2/3)
 - 15*d*e^2*x^(4/3) + 4*e^3*x^2))*Log[c*(d + e*x^(2/3))^n] + 18*b^2*(6*a*(d
^3 + e^3*x^2) - b*n*(11*d^3 + 6*d^2*e*x^(2/3) - 3*d*e^2*x^(4/3) + 2*e^3*x^2
))*Log[c*(d + e*x^(2/3))^n]^2 + 36*b^3*(d^3 + e^3*x^2)*Log[c*(d + e*x^(2/3)
)^n]^3)/(72*e^3)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int x\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e*x^(2/3))^n))^3,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e*x^(2/3))^n))^3,x)
```

Maxima [A]
time = 0.31, size = 489, normalized size = 1.09

$$\frac{1}{2}\,b^{3}x^{2}\log\left(\left(x^{\frac{2}{3}}e+d\right)^{n}c\right)^{3}+\frac{3}{2}\,ab^{2}x^{2}\log\left(\left(x^{\frac{2}{3}}e+d\right)^{n}c\right)^{2}+\frac{1}{4}\left(6\,d^{3}e^{(-4)}\log\left(x^{\frac{2}{3}}e+d\right)+\left(3\,dx^{\frac{4}{3}}e-2\,x^{2}e^{2}-6\,d^{2}x^{\frac{2}{3}}\right)e^{(-3)}\right)a^{2}bne+\frac{3}{2}\,a^{2}bx^{2}\log\left(\left(x^{\frac{2}{3}}e+d\right)^{n}c\right)+\frac{1}{2}\,a^{3}x^{2}-\frac{1}{12}\left(\left(18\,d^{3}\log\left(x^{\frac{2}{3}}e+d\right)^{2}+66\,d^{3}\log\left(x^{\frac{2}{3}}e+d\right)-66\,d^{2}x^{\frac{2}{3}}e+15\,dx^{\frac{4}{3}}e^{2}-4\,x^{2}e^{3}\right)n^{2}e^{(-3)}-6\left(6\,d^{3}e^{(-4)}\log\left(x^{\frac{2}{3}}e+d\right)+\left(3\,dx^{\frac{4}{3}}e-2\,x^{2}e^{2}-6\,d^{2}x^{\frac{2}{3}}\right)e^{(-3)}\right)ne\log\left(\left(x^{\frac{2}{3}}e+d\right)^{n}c\right)\right)ab^{2}+\frac{1}{72}\left(18\left(6\,d^{3}e^{(-4)}\log\left(x^{\frac{2}{3}}e+d\right)+\left(3\,dx^{\frac{4}{3}}e-2\,x^{2}e^{2}-6\,d^{2}x^{\frac{2}{3}}\right)e^{(-3)}\right)ne\log\left(\left(x^{\frac{2}{3}}e+d\right)^{n}c\right)^{2}+\left(\left(36\,d^{3}\log\left(x^{\frac{2}{3}}e+d\right)^{3}+198\,d^{3}\log\left(x^{\frac{2}{3}}e+d\right)^{2}+510\,d^{3}\log\left(x^{\frac{2}{3}}e+d\right)-510\,d^{2}x^{\frac{2}{3}}e+57\,dx^{\frac{4}{3}}e^{2}-8\,x^{2}e^{3}\right)n^{2}e^{(-4)}-6\left(18\,d^{3}\log\left(x^{\frac{2}{3}}e+d\right)^{2}+66\,d^{3}\log\left(x^{\frac{2}{3}}e+d\right)-66\,d^{2}x^{\frac{2}{3}}e+15\,dx^{\frac{4}{3}}e^{2}-4\,x^{2}e^{3}\right)ne^{(-4)}\log\left(\left(x^{\frac{2}{3}}e+d\right)^{n}c\right)\right)ne\right)b^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*x^2*log((x^(2/3)*e + d)^n*c)^3 + 3/2*a*b^2*x^2*log((x^(2/3)*e + d)^
n*c)^2 + 1/4*(6*d^3*e^(-4)*log(x^(2/3)*e + d) + (3*d*x^(4/3)*e - 2*x^2*e^2
- 6*d^2*x^(2/3))*e^(-3))*a^2*b*n*e + 3/2*a^2*b*x^2*log((x^(2/3)*e + d)^n*c)
 + 1/2*a^3*x^2 - 1/12*((18*d^3*log(x^(2/3)*e + d)^2 + 66*d^3*log(x^(2/3)*e
+ d) - 66*d^2*x^(2/3)*e + 15*d*x^(4/3)*e^2 - 4*x^2*e^3)*n^2*e^(-3) - 6*(6*d
^3*e^(-4)*log(x^(2/3)*e + d) + (3*d*x^(4/3)*e - 2*x^2*e^2 - 6*d^2*x^(2/3))*
e^(-3))*n*e*log((x^(2/3)*e + d)^n*c))*a*b^2 + 1/72*(18*(6*d^3*e^(-4)*log(x^
(2/3)*e + d) + (3*d*x^(4/3)*e - 2*x^2*e^2 - 6*d^2*x^(2/3))*e^(-3))*n*e*log(
(x^(2/3)*e + d)^n*c)^2 + ((36*d^3*log(x^(2/3)*e + d)^3 + 198*d^3*log(x^(2/3
)*e + d)^2 + 510*d^3*log(x^(2/3)*e + d) - 510*d^2*x^(2/3)*e + 57*d*x^(4/3)*
e^2 - 8*x^2*e^3)*n^2*e^(-4) - 6*(18*d^3*log(x^(2/3)*e + d)^2 + 66*d^3*log(x
^(2/3)*e + d) - 66*d^2*x^(2/3)*e + 15*d*x^(4/3)*e^2 - 4*x^2*e^3)*n*e^(-4)*l
og((x^(2/3)*e + d)^n*c))*n*e)*b^3
```

Fricas [A]
time = 0.46, size = 671, normalized size = 1.49

$$\frac{1}{72}\left(36\,b^{3}x^{2}e^{3}\log\left(c\right)^{3}-36\left(b^{3}n-3\,ab^{2}\right)x^{2}e^{3}\log\left(c\right)^{2}+12\left(2\,b^{3}n^{2}-6\,ab^{2}n+9\,a^{2}b\right)x^{2}e^{3}\log\left(c\right)-4\left(2\,b^{3}n^{3}-6\,ab^{2}n^{2}+9\,a^{2}bn-9\,a^{3}\right)x^{2}e^{3}+36\left(b^{3}d^{3}n^{3}+b^{3}n^{3}x^{2}e^{3}\right)\log\left(x^{\frac{2}{3}}e+d\right)^{3}-18\left(6\,b^{3}d^{2}n^{3}x^{\frac{2}{3}}e-3\,b^{3}dn^{3}x^{\frac{4}{3}}e^{2}+11\,b^{3}d^{3}n^{3}-6\,ab^{2}d^{3}n^{2}+2\left(b^{3}n^{3}-3\,ab^{2}n^{2}\right)x^{2}e^{3}-6\left(b^{3}d^{3}n^{2}+b^{3}n^{2}x^{2}e^{3}\right)\log\left(c\right)\right)\log\left(x^{\frac{2}{3}}e+d\right)^{2}+6\left(85\,b^{3}d^{3}n^{3}-66\,ab^{2}d^{3}n^{2}+18\,a^{2}bd^{3}n+2\left(2\,b^{3}n^{3}-6\,ab^{2}n^{2}+9\,a^{2}bn\right)x^{2}e^{3}+18\left(b^{3}d^{3}n+b^{3}nx^{2}e^{3}\right)\log\left(c\right)^{2}-6\left(11\,b^{3}d^{3}n^{2}-6\,ab^{2}d^{3}n+2\left(b^{3}n^{2}-3\,ab^{2}n\right)x^{2}e^{3}\right)\log\left(c\right)-6\left(6\,b^{3}d^{2}n^{2}e\log\left(c\right)-\left(11\,b^{3}d^{2}n^{3}-6\,ab^{2}d^{2}n^{2}\right)e\right)x^{\frac{2}{3}}+3\left(6\,b^{3}dn^{2}xe^{2}\log\left(c\right)-\left(5\,b^{3}dn^{3}-6\,ab^{2}dn^{2}\right)xe^{2}\right)x^{\frac{1}{3}}\right)\log\left(x^{\frac{2}{3}}e+d\right)-6\left(18\,b^{3}d^{2}ne\log\left(c\right)^{2}-6\left(11\,b^{3}d^{2}n^{2}-6\,ab^{2}d^{2}n\right)e\log\left(c\right)+\left(85\,b^{3}d^{2}n^{3}-66\,ab^{2}d^{2}n^{2}+18\,a^{2}bd^{2}n\right)e\right)x^{\frac{2}{3}}+3\left(18\,b^{3}dnxe^{2}\log\left(c\right)^{2}-6\left(5\,b^{3}dn^{2}-6\,ab^{2}dn\right)xe^{2}\log\left(c\right)+\left(19\,b^{3}dn^{3}-30\,ab^{2}dn^{2}+18\,a^{2}bdn\right)xe^{2}\right)x^{\frac{1}{3}}\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="fricas")
```

```
[Out] 1/72*(36*b^3*x^2*e^3*log(c)^3 - 36*(b^3*n - 3*a*b^2)*x^2*e^3*log(c)^2 + 12*
(2*b^3*n^2 - 6*a*b^2*n + 9*a^2*b)*x^2*e^3*log(c) - 4*(2*b^3*n^3 - 6*a*b^2*n
^2 + 9*a^2*b*n - 9*a^3)*x^2*e^3 + 36*(b^3*d^3*n^3 + b^3*n^3*x^2*e^3)*log(x^
(2/3)*e + d)^3 - 18*(6*b^3*d^2*n^3*x^(2/3)*e - 3*b^3*d*n^3*x^(4/3)*e^2 + 11
*b^3*d^3*n^3 - 6*a*b^2*d^3*n^2 + 2*(b^3*n^3 - 3*a*b^2*n^2)*x^2*e^3 - 6*(b^3
*d^3*n^2 + b^3*n^2*x^2*e^3)*log(c))*log(x^(2/3)*e + d)^2 + 6*(85*b^3*d^3*n^
3 - 66*a*b^2*d^3*n^2 + 18*a^2*b*d^3*n + 2*(2*b^3*n^3 - 6*a*b^2*n^2 + 9*a^2*
b*n)*x^2*e^3 + 18*(b^3*d^3*n + b^3*n*x^2*e^3)*log(c)^2 - 6*(11*b^3*d^3*n^2
- 6*a*b^2*d^3*n + 2*(b^3*n^2 - 3*a*b^2*n)*x^2*e^3)*log(c) - 6*(6*b^3*d^2*n^
2*e*log(c) - (11*b^3*d^2*n^3 - 6*a*b^2*d^2*n^2)*e)*x^(2/3) + 3*(6*b^3*d*n^2
```

```
*x*e^2*log(c) - (5*b^3*d*n^3 - 6*a*b^2*d*n^2)*x*e^2)*x^(1/3))*log(x^(2/3)*e
 + d) - 6*(18*b^3*d^2*n*e*log(c)^2 - 6*(11*b^3*d^2*n^2 - 6*a*b^2*d^2*n)*e*l
og(c) + (85*b^3*d^2*n^3 - 66*a*b^2*d^2*n^2 + 18*a^2*b*d^2*n)*e)*x^(2/3) + 3
*(18*b^3*d*n*x*e^2*log(c)^2 - 6*(5*b^3*d*n^2 - 6*a*b^2*d*n)*x*e^2*log(c) +
(19*b^3*d*n^3 - 30*a*b^2*d*n^2 + 18*a^2*b*d*n)*x*e^2)*x^(1/3))*e^(-3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(2/3))**n))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.91, size = 778, normalized size = 1.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^2*log(c)^3 + 1/72*(36*x^2*log(x^(2/3)*e + d)^3 + (36*d^3*log(x^(2
/3)*e + d)^3 - 36*(x^(2/3)*e + d)^3*log(x^(2/3)*e + d)^2 + 162*(x^(2/3)*e +
 d)^2*d*log(x^(2/3)*e + d)^2 - 324*(x^(2/3)*e + d)*d^2*log(x^(2/3)*e + d)^2
 + 24*(x^(2/3)*e + d)^3*log(x^(2/3)*e + d) - 162*(x^(2/3)*e + d)^2*d*log(x^
(2/3)*e + d) + 648*(x^(2/3)*e + d)*d^2*log(x^(2/3)*e + d) - 8*(x^(2/3)*e +
d)^3 + 81*(x^(2/3)*e + d)^2*d - 648*(x^(2/3)*e + d)*d^2)*e^(-3))*b^3*n^3 +
1/12*(18*x^2*log(x^(2/3)*e + d)^2 + (18*d^3*log(x^(2/3)*e + d)^2 - 12*(x^(2
/3)*e + d)^3*log(x^(2/3)*e + d) + 54*(x^(2/3)*e + d)^2*d*log(x^(2/3)*e + d)
 - 108*(x^(2/3)*e + d)*d^2*log(x^(2/3)*e + d) + 4*(x^(2/3)*e + d)^3 - 27*(x
^(2/3)*e + d)^2*d + 108*(x^(2/3)*e + d)*d^2)*e^(-3))*b^3*n^2*log(c) + 1/4*(
6*x^2*log(x^(2/3)*e + d) + (6*d^3*e^(-4)*log(abs(x^(2/3)*e + d)) + (3*d*x^(
4/3)*e - 2*x^2*e^2 - 6*d^2*x^(2/3))*e^(-3))*e)*b^3*n*log(c)^2 + 3/2*a*b^2*x
^2*log(c)^2 + 1/12*(18*x^2*log(x^(2/3)*e + d)^2 + (18*d^3*log(x^(2/3)*e + d
)^2 - 12*(x^(2/3)*e + d)^3*log(x^(2/3)*e + d) + 54*(x^(2/3)*e + d)^2*d*log(
x^(2/3)*e + d) - 108*(x^(2/3)*e + d)*d^2*log(x^(2/3)*e + d) + 4*(x^(2/3)*e
+ d)^3 - 27*(x^(2/3)*e + d)^2*d + 108*(x^(2/3)*e + d)*d^2)*e^(-3))*a*b^2*n^
2 + 1/2*(6*x^2*log(x^(2/3)*e + d) + (6*d^3*e^(-4)*log(abs(x^(2/3)*e + d)) +
 (3*d*x^(4/3)*e - 2*x^2*e^2 - 6*d^2*x^(2/3))*e^(-3))*e)*a*b^2*n*log(c) + 3/
2*a^2*b*x^2*log(c) + 1/4*(6*x^2*log(x^(2/3)*e + d) + (6*d^3*e^(-4)*log(abs(
x^(2/3)*e + d)) + (3*d*x^(4/3)*e - 2*x^2*e^2 - 6*d^2*x^(2/3))*e^(-3))*e)*a^
2*b*n + 1/2*a^3*x^2
```

Mupad [B]
time = 0.72, size = 575, normalized size = 1.28

$$\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)^3\,\left(\frac{b^3\,x^2}{2}+\frac{b^3\,d^3}{2\,e^3}\right)-x^{4/3}\,\left(\frac{d\,\left(\frac{3\,a^3}{2}-\frac{3\,a^2\,b\,n}{2}+a\,b^2\,n^2-\frac{b^3\,n^3}{3}\right)}{2\,e}-\frac{d\,\left(6\,a^3-6\,a\,b^2\,n^2+5\,b^3\,n^3\right)}{8\,e}\right)+\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)^2\,\left(\frac{b^2\,x^2\,\left(3\,a-b\,n\right)}{2}-\frac{x^{4/3}\,\left(\frac{3\,b^2\,d\,\left(3\,a-b\,n\right)}{2\,e}-\frac{9\,a\,b^2\,d}{2\,e}\right)}{2}+\frac{d\,\left(6\,a\,b^2\,d^2-11\,b^3\,d^2\,n\right)}{4\,e^3}+\frac{d\,x^{2/3}\,\left(\frac{6\,b^2\,d\,\left(3\,a-b\,n\right)}{e}-\frac{18\,a\,b^2\,d}{e}\right)}{4\,e}\right)+x^{2/3}\,\left(\frac{d\,\left(\frac{d\,\left(\frac{3\,a^3}{2}-\frac{3\,a^2\,b\,n}{2}+a\,b^2\,n^2-\frac{b^3\,n^3}{3}\right)}{e}-\frac{d\,\left(6\,a^3-6\,a\,b^2\,n^2+5\,b^3\,n^3\right)}{4\,e}\right)}{e}+\frac{b^2\,d^2\,n^2\,\left(6\,a-11\,b\,n\right)}{2\,e^2}\right)+x^2\,\left(\frac{a^3}{2}-\frac{a^2\,b\,n}{2}+\frac{a\,b^2\,n^2}{3}-\frac{b^3\,n^3}{9}\right)+\frac{\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)\,\left(\frac{x^{2/3}\,\left(\frac{d\,\left(2\,b\,d\,e\,\left(9\,a^2-6\,a\,b\,n+2\,b^2\,n^2\right)-6\,b\,d\,e\,\left(3\,a^2-b^2\,n^2\right)\right)}{e}+12\,b^3\,d^2\,n^2\right)}{2\,e}-\frac{x^{4/3}\,\left(2\,b\,d\,e\,\left(9\,a^2-6\,a\,b\,n+2\,b^2\,n^2\right)-6\,b\,d\,e\,\left(3\,a^2-b^2\,n^2\right)\right)}{4\,e}+\frac{b\,e\,x^2\,\left(9\,a^2-6\,a\,b\,n+2\,b^2\,n^2\right)}{3}\right)}{2\,e}+\frac{\ln\left(d+e\,x^{2/3}\right)\,\left(18\,a^2\,b\,d^3\,n-66\,a\,b^2\,d^3\,n^2+85\,b^3\,d^3\,n^3\right)}{12\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e*x^(2/3))^n))^3,x)
```

```
[Out] log(c*(d + e*x^(2/3))^n)^3*((b^3*x^2)/2 + (b^3*d^3)/(2*e^3)) - x^(4/3)*((d*
((3*a^3)/2 - (b^3*n^3)/3 + a*b^2*n^2 - (3*a^2*b*n)/2))/(2*e) - (d*(6*a^3 +
5*b^3*n^3 - 6*a*b^2*n^2))/(8*e)) + log(c*(d + e*x^(2/3))^n)^2*((b^2*x^2*(3*
a - b*n))/2 - (x^(4/3)*((3*b^2*d*(3*a - b*n))/(2*e) - (9*a*b^2*d)/(2*e)))/2
 + (d*(6*a*b^2*d^2 - 11*b^3*d^2*n))/(4*e^3) + (d*x^(2/3)*((6*b^2*d*(3*a - b
*n))/e - (18*a*b^2*d)/e))/(4*e)) + x^(2/3)*((d*((d*((3*a^3)/2 - (b^3*n^3)/3
 + a*b^2*n^2 - (3*a^2*b*n)/2))/e - (d*(6*a^3 + 5*b^3*n^3 - 6*a*b^2*n^2))/(4
*e)))/e + (b^2*d^2*n^2*(6*a - 11*b*n))/(2*e^2)) + x^2*(a^3/2 - (b^3*n^3)/9
+ (a*b^2*n^2)/3 - (a^2*b*n)/2) + (log(c*(d + e*x^(2/3))^n)*((x^(2/3)*((d*(2
*b*d*e*(9*a^2 + 2*b^2*n^2 - 6*a*b*n) - 6*b*d*e*(3*a^2 - b^2*n^2)))/e + 12*b
^3*d^2*n^2))/(2*e) - (x^(4/3)*(2*b*d*e*(9*a^2 + 2*b^2*n^2 - 6*a*b*n) - 6*b*
d*e*(3*a^2 - b^2*n^2)))/(4*e) + (b*e*x^2*(9*a^2 + 2*b^2*n^2 - 6*a*b*n))/3))
/(2*e) + (log(d + e*x^(2/3))*(85*b^3*d^3*n^3 - 66*a*b^2*d^3*n^2 + 18*a^2*b*
d^3*n))/(12*e^3)
```

3.483 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x}\,dx$

Optimal. Leaf size=139

$$\frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\log\left(-\frac{ex^{2/3}}{d}\right)+\frac{9}{2}bn\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\operatorname{Li}_2\left(1+\frac{ex^{2/3}}{d}\right)-9b^2n^2\left(a\right.$$

```
[Out] 3/2*(a+b*ln(c*(d+e*x^(2/3))^n))^3*ln(-e*x^(2/3)/d)+9/2*b*n*(a+b*ln(c*(d+e*x
^(2/3))^n))^2*polylog(2,1+e*x^(2/3)/d)-9*b^2*n^2*(a+b*ln(c*(d+e*x^(2/3))^n)
)*polylog(3,1+e*x^(2/3)/d)+9*b^3*n^3*polylog(4,1+e*x^(2/3)/d)
```

Rubi [A]
time = 0.13, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2504, 2443, 2481, 2421, 2430, 6724}

$$-9b^2n^2\text{PolyLog}\left(3,\frac{ex^{2/3}}{d}+1\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\frac{9}{2}bn\text{PolyLog}\left(2,\frac{ex^{2/3}}{d}+1\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2+9b^3n^3\text{PolyLog}\left(4,\frac{ex^{2/3}}{d}+1\right)+\frac{3}{2}\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x,x]
```

```
[Out] (3*(a + b*Log[c*(d + e*x^(2/3))^n])^3*Log[-((e*x^(2/3))/d)])/2 + (9*b*n*(a
+ b*Log[c*(d + e*x^(2/3))^n])^2*PolyLog[2, 1 + (e*x^(2/3))/d])/2 - 9*b^2*n^
2*(a + b*Log[c*(d + e*x^(2/3))^n])*PolyLog[3, 1 + (e*x^(2/3))/d] + 9*b^3*n^
3*PolyLog[4, 1 + (e*x^(2/3))/d]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
```

```
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x}\,dx &= \frac{3}{2}\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{x}\,dx, x, x^{2/3}\right)\\
&= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\log\left(-\frac{ex^{2/3}}{d}\right)-\frac{1}{2}(9ben)\text{Subst}\left(\int \frac{\log}{}\right.\\
&= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\log\left(-\frac{ex^{2/3}}{d}\right)-\frac{1}{2}(9bn)\text{Subst}\left(\int \frac{(a}{}\right.\\
&= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\log\left(-\frac{ex^{2/3}}{d}\right)+\frac{9}{2}bn\left(a+b\log\left(c(d+\right.\right.\\
&= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\log\left(-\frac{ex^{2/3}}{d}\right)+\frac{9}{2}bn\left(a+b\log\left(c(d+\right.\right.\\
&= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\log\left(-\frac{ex^{2/3}}{d}\right)+\frac{9}{2}bn\left(a+b\log\left(c(d+\right.\right.
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 339 vs. 2(139) = 278.
time = 0.12, size = 339, normalized size = 2.44

$$\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\log(x)+3bn\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\left(\left(\log\left(d+ex^{2/3}\right)-\log\left(1+\frac{ex^{2/3}}{d}\right)\right)\log(x)-\frac{3}{2}\text{Li}_2\left(-\frac{ex^{2/3}}{d}\right)\right)+\frac{9}{2}b^2n^2\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\left(\log^2\left(d+ex^{2/3}\right)\log\left(-\frac{ex^{2/3}}{d}\right)+2\log\left(d+ex^{2/3}\right)\text{Li}_2\left(1+\frac{ex^{2/3}}{d}\right)-2\text{Li}_3\left(1+\frac{ex^{2/3}}{d}\right)\right)+\frac{3}{2}b^3n^3\left(\log^3\left(d+ex^{2/3}\right)\log\left(-\frac{ex^{2/3}}{d}\right)+3\log^2\left(d+ex^{2/3}\right)\text{Li}_2\left(1+\frac{ex^{2/3}}{d}\right)-6\log\left(d+ex^{2/3}\right)\text{Li}_3\left(1+\frac{ex^{2/3}}{d}\right)+6\text{Li}_4\left(1+\frac{ex^{2/3}}{d}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x,x]
```

```
[Out] (a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^3*Log[x] + 3*b*n*
(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2*((Log[d + e*x^(
2/3)] - Log[1 + (e*x^(2/3))/d])*Log[x] - (3*PolyLog[2, -((e*x^(2/3))/d)])/2
) + (9*b^2*n^2*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])*(L
og[d + e*x^(2/3)]^2*Log[-((e*x^(2/3))/d)] + 2*Log[d + e*x^(2/3)]*PolyLog[2,
 1 + (e*x^(2/3))/d] - 2*PolyLog[3, 1 + (e*x^(2/3))/d]))/2 + (3*b^3*n^3*(Log
[d + e*x^(2/3)]^3*Log[-((e*x^(2/3))/d)] + 3*Log[d + e*x^(2/3)]^2*PolyLog[2,
 1 + (e*x^(2/3))/d] - 6*Log[d + e*x^(2/3)]*PolyLog[3, 1 + (e*x^(2/3))/d] +
6*PolyLog[4, 1 + (e*x^(2/3))/d]))/2
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^3/x,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^3/x,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(127) = 254$.
time = 0.38, size = 297, normalized size = 2.14

$$\frac{3}{2}\left(\log\left(x^{\frac{2}{3}}e+d\right)^{3}\log\left(-\frac{x^{\frac{2}{3}}e+d}{d}+1\right)+3\,\mathrm{Li}_2\left(\frac{x^{\frac{2}{3}}e+d}{d}\right)\log\left(x^{\frac{2}{3}}e+d\right)^{2}-6\,\log\left(x^{\frac{2}{3}}e+d\right)\mathrm{Li}_3\left(\frac{x^{\frac{2}{3}}e+d}{d}\right)+6\,\mathrm{Li}_4\left(\frac{x^{\frac{2}{3}}e+d}{d}\right)\right)b^3n^3+a^3\log\left(x\right)+\frac{9}{2}\left(b^3n^2\log\left(c\right)+ab^2n^2\right)\left(\log\left(x^{\frac{2}{3}}e+d\right)^{2}\log\left(-\frac{x^{\frac{2}{3}}e+d}{d}+1\right)+2\,\mathrm{Li}_2\left(\frac{x^{\frac{2}{3}}e+d}{d}\right)\log\left(x^{\frac{2}{3}}e+d\right)-2\,\mathrm{Li}_3\left(\frac{x^{\frac{2}{3}}e+d}{d}\right)\right)+\frac{9}{2}\left(b^3n\log\left(c\right)^2+2\,ab^2n\log\left(c\right)+a^2bn\right)\left(\log\left(x^{\frac{2}{3}}e+d\right)\log\left(-\frac{x^{\frac{2}{3}}e+d}{d}+1\right)+\mathrm{Li}_2\left(\frac{x^{\frac{2}{3}}e+d}{d}\right)\right)+\left(b^3\log\left(c\right)^3+3\,ab^2\log\left(c\right)^2+3\,a^2b\log\left(c\right)\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x,x, algorithm="maxima")
```

```
[Out] 3/2*(log(x^(2/3)*e + d)^3*log(-(x^(2/3)*e + d)/d + 1) + 3*dilog((x^(2/3)*e
+ d)/d)*log(x^(2/3)*e + d)^2 - 6*log(x^(2/3)*e + d)*polylog(3, (x^(2/3)*e +
 d)/d) + 6*polylog(4, (x^(2/3)*e + d)/d))*b^3*n^3 + a^3*log(x) + 9/2*(b^3*n
^2*log(c) + a*b^2*n^2)*(log(x^(2/3)*e + d)^2*log(-(x^(2/3)*e + d)/d + 1) +
2*dilog((x^(2/3)*e + d)/d)*log(x^(2/3)*e + d) - 2*polylog(3, (x^(2/3)*e + d
)/d)) + 9/2*(b^3*n*log(c)^2 + 2*a*b^2*n*log(c) + a^2*b*n)*(log(x^(2/3)*e +
d)*log(-(x^(2/3)*e + d)/d + 1) + dilog((x^(2/3)*e + d)/d)) + (b^3*log(c)^3
+ 3*a*b^2*log(c)^2 + 3*a^2*b*log(c))*log(x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((x^(2/3)*e + d)^n*c)^3 + 3*a*b^2*log((x^(2/3)*e + d)^n*c)
^2 + 3*a^2*b*log((x^(2/3)*e + d)^n*c) + a^3)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\log\left(c\left(d + ex^{\frac{2}{3}}\right)^{n}\right)\right)^{3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**3/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**(2/3))**n))**3/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x,x, algorithm="giac")
[Out] integrate((b*log((x^(2/3)*e + d)^n*c) + a)^3/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))^3/x,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^3/x, x)
```

3.484 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^3}\,dx$

Optimal. Leaf size=451

$$-\frac{3b^2e^2n^2\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^3x^{2/3}}-\frac{3b^2e^3n^2\log\left(1-\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^3}-\frac{3be}{}$$

```
[Out] -3/2*b^2*e^2*n^2*(d+e*x^(2/3))*(a+b*ln(c*(d+e*x^(2/3))^n))/d^3/x^(2/3)-3/2*
b^2*e^3*n^2*ln(1-d/(d+e*x^(2/3)))*(a+b*ln(c*(d+e*x^(2/3))^n))/d^3-3/4*b*e*n
*(a+b*ln(c*(d+e*x^(2/3))^n))^2/d/x^(4/3)+3/2*b*e^2*n*(d+e*x^(2/3))*(a+b*ln(
c*(d+e*x^(2/3))^n))^2/d^3/x^(2/3)+3/2*b*e^3*n*ln(1-d/(d+e*x^(2/3)))*(a+b*ln
(c*(d+e*x^(2/3))^n))^2/d^3-1/2*(a+b*ln(c*(d+e*x^(2/3))^n))^3/x^2-3*b^2*e^3*
n^2*(a+b*ln(c*(d+e*x^(2/3))^n))*ln(-e*x^(2/3)/d)/d^3+b^3*e^3*n^3*ln(x)/d^3+
3/2*b^3*e^3*n^3*polylog(2,d/(d+e*x^(2/3)))/d^3-3*b^2*e^3*n^2*(a+b*ln(c*(d+e
*x^(2/3))^n))*polylog(2,d/(d+e*x^(2/3)))/d^3-3*b^3*e^3*n^3*polylog(2,1+e*x^
(2/3)/d)/d^3-3*b^3*e^3*n^3*polylog(3,d/(d+e*x^(2/3)))/d^3
```

Rubi [A]
time = 0.57, antiderivative size = 451, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {2504, 2445, 2458, 2389, 2379, 2421, 6724, 2355, 2354, 2438, 2356, 2351, 31}

$$-\frac{3b^2e^3n^2\text{PolyLog}\left(2,\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^3}+\frac{3b^3e^3n^3\text{PolyLog}\left(2,\frac{d}{d+ex^{2/3}}\right)}{2d^3}-\frac{3b^3e^3n^3\text{PolyLog}\left(2,\frac{ex^{2/3}}{d}+1\right)}{d^3}-\frac{3b^3e^3n^3\text{PolyLog}\left(3,\frac{d}{d+ex^{2/3}}\right)}{d^3}-\frac{3b^2e^3n^2\log\left(1-\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^3}-\frac{3b^2e^3n^2\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^3}-\frac{3b^2e^2n^2\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^3x^{2/3}}+\frac{3be^3n\log\left(1-\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2d^3}+\frac{3be^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2d^3x^{2/3}}-\frac{3ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4dx^{4/3}}-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2x^2}+\frac{b^3e^3n^3\log(x)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x^3,x]
```

```
[Out] (-3*b^2*e^2*n^2*(d + e*x^(2/3))*(a + b*Log[c*(d + e*x^(2/3))^n]))/(2*d^3*x^
(2/3)) - (3*b^2*e^3*n^2*Log[1 - d/(d + e*x^(2/3))]*(a + b*Log[c*(d + e*x^(2
/3))^n]))/(2*d^3) - (3*b*e*n*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(4*d*x^(4/
3)) + (3*b*e^2*n*(d + e*x^(2/3))*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(2*d^3
*x^(2/3)) + (3*b*e^3*n*Log[1 - d/(d + e*x^(2/3))]*(a + b*Log[c*(d + e*x^(2/
3))^n])^2)/(2*d^3) - (a + b*Log[c*(d + e*x^(2/3))^n])^3/(2*x^2) - (3*b^2*e^
3*n^2*(a + b*Log[c*(d + e*x^(2/3))^n])*Log[-((e*x^(2/3))/d)])/d^3 + (b^3*e^
3*n^3*Log[x])/d^3 + (3*b^3*e^3*n^3*PolyLog[2, d/(d + e*x^(2/3))])/(2*d^3) -
 (3*b^2*e^3*n^2*(a + b*Log[c*(d + e*x^(2/3))^n])*PolyLog[2, d/(d + e*x^(2/3
))])/d^3 - (3*b^3*e^3*n^3*PolyLog[2, 1 + (e*x^(2/3))/d])/d^3 - (3*b^3*e^3*n
^3*PolyLog[3, d/(d + e*x^(2/3))])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^3}\,dx = \frac{3}{2}\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x^4}\,dx, x, x^{2/3}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2x^2} + \frac{1}{2}(3ben)\text{Subst}\left(\int \frac{(a+b\log(c(d+e}{x^3(d+ex)}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2x^2} + \frac{1}{2}(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,d\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2x^2} + \frac{(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+\right.}{2d}$$

$$= -\frac{3ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4dx^{4/3}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2x^2} -$$

$$= -\frac{3ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4dx^{4/3}} + \frac{3be^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c(d\right.\right.}{2d^3x^{2/3}}$$

$$= -\frac{3b^2e^2n^2\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^3x^{2/3}} - \frac{3ben\left(a+b\log\left(c(\right.\right.}{4dx^{4/}}$$

$$= -\frac{3b^2e^2n^2\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^3x^{2/3}} + \frac{3be^3n\left(a+b\log\left(c\right.\right.}{4d^3}$$

$$= -\frac{3b^2e^2n^2\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^3x^{2/3}} + \frac{3be^3n\left(a+b\log\left(c\right.\right.}{4d^3}$$

Mathematica [A]
time = 0.55, size = 764, normalized size = 1.69

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x^3,x]
```

```
[Out] (-3*b*d^2*e*n*x^(2/3)*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))
^n])^2 + 6*b*d*e^2*n*x^(4/3)*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x
^(2/3))^n])^2 - 6*b*d^3*n*Log[d + e*x^(2/3)]*(a - b*n*Log[d + e*x^(2/3)] +
b*Log[c*(d + e*x^(2/3))^n])^2 - 6*b*e^3*n*x^2*Log[d + e*x^(2/3)]*(a - b*n*L
og[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 - 2*d^3*(a - b*n*Log[d +
```

```
e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^3 + 4*b*e^3*n*x^2*(a - b*n*Log[d +
 e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2*Log[x] - 6*b^2*n^2*(a - b*n*Log
[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])*((d^3 + e^3*x^2)*Log[d + e*x^
(2/3)]^2 + e^2*x^(4/3)*(d + 3*e*x^(2/3)*Log[-((e*x^(2/3))/d)]) + Log[d + e*
x^(2/3)]*(d^2*e*x^(2/3) - 2*d*e^2*x^(4/3) - 3*e^3*x^2 - 2*e^3*x^2*Log[-((e*
x^(2/3))/d)]) - 2*e^3*x^2*PolyLog[2, 1 + (e*x^(2/3))/d]) + b^3*n^3*(-6*d*e^
2*x^(4/3)*Log[d + e*x^(2/3)] - 6*e^3*x^2*Log[d + e*x^(2/3)] - 3*d^2*e*x^(2/
3)*Log[d + e*x^(2/3)]^2 + 6*d*e^2*x^(4/3)*Log[d + e*x^(2/3)]^2 + 9*e^3*x^2*
Log[d + e*x^(2/3)]^2 - 2*d^3*Log[d + e*x^(2/3)]^3 - 2*e^3*x^2*Log[d + e*x^(
2/3)]^3 + 6*e^3*x^2*Log[-((e*x^(2/3))/d)] - 18*e^3*x^2*Log[d + e*x^(2/3)]*L
og[-((e*x^(2/3))/d)] + 6*e^3*x^2*Log[d + e*x^(2/3)]^2*Log[-((e*x^(2/3))/d)]
 + 6*e^3*x^2*(-3 + 2*Log[d + e*x^(2/3)])*PolyLog[2, 1 + (e*x^(2/3))/d] - 12
*e^3*x^2*PolyLog[3, 1 + (e*x^(2/3))/d]))/(4*d^3*x^2)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{3}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^3/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^3/x^3,x)
```

Maxima [A]
time = 0.41, size = 690, normalized size = 1.53

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^3,x, algorithm="maxima")
```

```
[Out] -3/4*a^2*b*n*(2*e^2*log(x^(2/3)*e + d)/d^3 - 2*e^2*log(x^(2/3))/d^3 - (2*x^
(2/3)*e - d)/(d^2*x^(4/3)))*e + 3/2*(log(x^(2/3)*e + d)^2*log(-(x^(2/3)*e +
 d)/d + 1) + 2*dilog((x^(2/3)*e + d)/d)*log(x^(2/3)*e + d) - 2*polylog(3, (
x^(2/3)*e + d)/d))*b^3*n^3*e^3/d^3 - 3/2*a^2*b*log((x^(2/3)*e + d)^n*c)/x^2
 - 1/2*a^3/x^2 + 3/2*(2*a*b^2*n^2 - (3*n^3 - 2*n^2*log(c))*b^3)*(log(x^(2/3
)*e + d)*log(-(x^(2/3)*e + d)/d + 1) + dilog((x^(2/3)*e + d)/d))*e^3/d^3 -
((3*n^2 - 2*n*log(c))*a*b^2 - (n^3 - 3*n^2*log(c) + n*log(c)^2)*b^3)*e^3*lo
g(x)/d^3 - 1/4*(2*b^3*d^3*log(c)^3 + 6*a*b^2*d^3*log(c)^2 + 2*(b^3*d^3*n^3
+ b^3*n^3*x^2*e^3)*log(x^(2/3)*e + d)^3 + 6*((d*n^2 - 2*d*n*log(c))*a*b^2 +
 (d*n^2*log(c) - d*n*log(c)^2)*b^3)*x^(4/3)*e^2 + 3*(b^3*d^2*n^3*x^(2/3)*e
- 2*b^3*d*n^3*x^(4/3)*e^2 + 2*b^3*d^3*n^2*log(c) + 2*a*b^2*d^3*n^2 + (2*a*b
^2*n^2 - (3*n^3 - 2*n^2*log(c))*b^3)*x^2*e^3)*log(x^(2/3)*e + d)^2 + 3*(b^3
```

```
*d^2*n*log(c)^2 + 2*a*b^2*d^2*n*log(c))*x^(2/3)*e + 6*(b^3*d^3*n*log(c)^2 +
 2*a*b^2*d^3*n*log(c) - ((3*n^2 - 2*n*log(c))*a*b^2 - (n^3 - 3*n^2*log(c) +
 n*log(c)^2)*b^3)*x^2*e^3 - (2*a*b^2*d*n^2 - (d*n^3 - 2*d*n^2*log(c))*b^3)*
x^(4/3)*e^2 + (b^3*d^2*n^2*log(c) + a*b^2*d^2*n^2)*x^(2/3)*e)*log(x^(2/3)*e
 + d))/(d^3*x^2)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^3,x, algorithm="fricas")
[Out] integral((b^3*log((x^(2/3)*e + d)^n*c)^3 + 3*a*b^2*log((x^(2/3)*e + d)^n*c)
^2 + 3*a^2*b*log((x^(2/3)*e + d)^n*c) + a^3)/x^3, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**3/x**3,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^3,x, algorithm="giac")
[Out] integrate((b*log((x^(2/3)*e + d)^n*c) + a)^3/x^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))^3/x^3,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^3/x^3, x)
```

3.485 $\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=794

$$\frac{4504ab^2d^4n^2\sqrt[3]{x}}{315e^4}-\frac{3475504b^3d^4n^3\sqrt[3]{x}}{99225e^4}+\frac{637984b^3d^3n^3x}{297675e^3}-\frac{221344b^3d^2n^3x^{5/3}}{496125e^2}+\frac{3088b^3dn^3x^{7/3}}{27783e}-\frac{16}{729}b^3n^3x^3+\frac{34}{}$$

```
[Out] 4504/315*a*b^2*d^4*n^2*x^(1/3)/e^4-3475504/99225*b^3*d^4*n^3*x^(1/3)/e^4+63
7984/297675*b^3*d^3*n^3*x/e^3-221344/496125*b^3*d^2*n^3*x^(5/3)/e^2+3088/27
783*b^3*d*n^3*x^(7/3)/e-16/729*b^3*n^3*x^3+3475504/99225*b^3*d^(9/2)*n^3*ar
ctan(x^(1/3)*e^(1/2)/d^(1/2))/e^(9/2)-4504/315*I*b^3*d^(9/2)*n^3*arctan(x^(
1/3)*e^(1/2)/d^(1/2))^2/e^(9/2)+4504/315*b^3*d^4*n^2*x^(1/3)*ln(c*(d+e*x^(2
/3))^n)/e^4-1984/945*b^2*d^3*n^2*x*(a+b*ln(c*(d+e*x^(2/3))^n))/e^3+1144/157
5*b^2*d^2*n^2*x^(5/3)*(a+b*ln(c*(d+e*x^(2/3))^n))/e^2-128/441*b^2*d*n^2*x^(
7/3)*(a+b*ln(c*(d+e*x^(2/3))^n))/e+8/81*b^2*n^2*x^3*(a+b*ln(c*(d+e*x^(2/3))
^n))-4504/315*b^2*d^(9/2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))*(a+b*ln(c*(d+
e*x^(2/3))^n))/e^(9/2)-2*b*d^4*n*x^(1/3)*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e^4+
2/3*b*d^3*n*x*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e^3-2/5*b*d^2*n*x^(5/3)*(a+b*ln
(c*(d+e*x^(2/3))^n))^2/e^2+2/7*b*d*n*x^(7/3)*(a+b*ln(c*(d+e*x^(2/3))^n))^2/
e-2/9*b*n*x^3*(a+b*ln(c*(d+e*x^(2/3))^n))^2+1/3*x^3*(a+b*ln(c*(d+e*x^(2/3))
^n))^3-9008/315*b^3*d^(9/2)*n^3*arctan(x^(1/3)*e^(1/2)/d^(1/2))*ln(2*d^(1/2
)/(d^(1/2)+I*x^(1/3)*e^(1/2)))/e^(9/2)-4504/315*I*b^3*d^(9/2)*n^3*polylog(2
,1-2*d^(1/2)/(d^(1/2)+I*x^(1/3)*e^(1/2)))/e^(9/2)+2/3*b*d^5*n*Unintegrable(
(a+b*ln(c*(d+e*x^(2/3))^n))^2/(d+e*x^(2/3))/x^(2/3),x)/e^4
```

Rubi [A]
time = 1.94, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

```
[Out] (4504*a*b^2*d^4*n^2*x^(1/3))/(315*e^4) - (3475504*b^3*d^4*n^3*x^(1/3))/(992
25*e^4) + (637984*b^3*d^3*n^3*x)/(297675*e^3) - (221344*b^3*d^2*n^3*x^(5/3)
)/(496125*e^2) + (3088*b^3*d*n^3*x^(7/3))/(27783*e) - (16*b^3*n^3*x^3)/729
+ (3475504*b^3*d^(9/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(99225*e^(9/2
)) - (((4504*I)/315)*b^3*d^(9/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/e
^(9/2) - (9008*b^3*d^(9/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqr
t[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/(315*e^(9/2)) + (4504*b^3*d^4*n^2*x^(
1/3)*Log[c*(d + e*x^(2/3))^n])/(315*e^4) - (1984*b^2*d^3*n^2*x*(a + b*Log[c
```

```
*(d + e*x^(2/3))^n]))/(945*e^3) + (1144*b^2*d^2*n^2*x^(5/3)*(a + b*Log[c*(d
 + e*x^(2/3))^n]))/(1575*e^2) - (128*b^2*d*n^2*x^(7/3)*(a + b*Log[c*(d + e*
x^(2/3))^n]))/(441*e) + (8*b^2*n^2*x^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/81
 - (4504*b^2*d^(9/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d
+ e*x^(2/3))^n]))/(315*e^(9/2)) - (2*b*d^4*n*x^(1/3)*(a + b*Log[c*(d + e*x^
(2/3))^n])^2)/e^4 + (2*b*d^3*n*x*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(3*e^3
) - (2*b*d^2*n*x^(5/3)*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(5*e^2) + (2*b*d
*n*x^(7/3)*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(7*e) - (2*b*n*x^3*(a + b*Lo
g[c*(d + e*x^(2/3))^n])^2)/9 + (x^3*(a + b*Log[c*(d + e*x^(2/3))^n])^3)/3 -
 (((4504*I)/315)*b^3*d^(9/2)*n^3*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sq
rt[e]*x^(1/3))])/e^(9/2) + (2*b*d^5*n*Defer[Subst][Defer[Int][(a + b*Log[c*
(d + e*x^2)^n])^2/(d + e*x^2), x], x, x^(1/3)])/e^4
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx = 3\text{Subst}\left(\int x^8\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^3\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 - (2ben)\text{Subst}\left(\int \frac{x^{10}\left(a+b\log\left(\right.\right.}{d+}\right.$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 - (2ben)\text{Subst}\left(\int\left(\frac{d^4\left(a+b\log\right.}{}\right.\right.$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 - (2bn)\text{Subst}\left(\int x^8\left(a+b\log\left(c\left(\right.\right.\right.\right.$$

$$= -\frac{2bd^4n\sqrt[3]{x}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e^4} + \frac{2bd^3nx\left(a+b\log\left(c\left(d+\right.\right.\right.}{3e^3}$$

$$= -\frac{2bd^4n\sqrt[3]{x}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e^4} + \frac{2bd^3nx\left(a+b\log\left(c\left(d+\right.\right.\right.}{3e^3}$$

$$= -\frac{2bd^4n\sqrt[3]{x}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e^4} + \frac{2bd^3nx\left(a+b\log\left(c\left(d+\right.\right.\right.}{3e^3}$$

$$= \frac{4504ab^2d^4n^2\sqrt[3]{x}}{315e^4} - \frac{1984b^2d^3n^2x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{945e^3} + \frac{1144}{}$$

$$= \frac{4504ab^2d^4n^2\sqrt[3]{x}}{315e^4} + \frac{4504b^3d^4n^2\sqrt[3]{x}\,\log\left(c\left(d+ex^{2/3}\right)^n\right)}{315e^4} - \frac{1984b^2d^3}{}$$

$$= \frac{4504ab^2d^4n^2\sqrt[3]{x}}{315e^4} - \frac{3475504b^3d^4n^3\sqrt[3]{x}}{99225e^4} + \frac{637984b^3d^3n^3x}{297675e^3} - \frac{221344}{49}$$

$$= \frac{4504ab^2d^4n^2\sqrt[3]{x}}{315e^4} - \frac{3475504b^3d^4n^3\sqrt[3]{x}}{99225e^4} + \frac{637984b^3d^3n^3x}{297675e^3} - \frac{221344}{49}$$

$$= \frac{4504ab^2d^4n^2\sqrt[3]{x}}{315e^4} - \frac{3475504b^3d^4n^3\sqrt[3]{x}}{99225e^4} + \frac{637984b^3d^3n^3x}{297675e^3} - \frac{221344}{49}$$

Mathematica [A] Leaf count is larger than twice the leaf count of optimal. 3146 vs. $2(794) = 1588$.
time = 8.19, size = 3146, normalized size = 3.96

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

```
[Out] (b^3*n^3*x^(1/3)*(32*d^4 - 32*d^4*Sqrt[1 - (d + e*x^(2/3))/d] + 128*d^3*Sqr
t[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3)) - 192*d^2*Sqrt[1 - (d + e*x^(2/3))
/d]*(d + e*x^(2/3))^2 + 128*d*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^3
 - 32*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^4 + 1584*d^3*(d + e*x^(2/
3))*HypergeometricPFQ[{-7/2, 1, 1, 1}, {2, 2, 2}, (d + e*x^(2/3))/d] - 4536
*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-5/2, 1, 1, 1}, {2, 2, 2}, (d + e*x
^(2/3))/d] + 3780*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-3/2, 1, 1, 1}, {2
, 2, 2}, (d + e*x^(2/3))/d] - 864*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-7
/2, 1, 1, 1, 1}, {2, 2, 2, 2}, (d + e*x^(2/3))/d] + 3024*d^3*(d + e*x^(2/3)
)*HypergeometricPFQ[{-5/2, 1, 1, 1, 1}, {2, 2, 2, 2}, (d + e*x^(2/3))/d] -
3780*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-3/2, 1, 1, 1, 1}, {2, 2, 2, 2}
, (d + e*x^(2/3))/d] + 1890*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-1/2, 1,
 1, 1, 1}, {2, 2, 2, 2}, (d + e*x^(2/3))/d] - 240*d^4*Log[d + e*x^(2/3)] +
240*d^4*Sqrt[1 - (d + e*x^(2/3))/d]*Log[d + e*x^(2/3)] - 672*d^3*Sqrt[1 - (
d + e*x^(2/3))/d]*(d + e*x^(2/3))*Log[d + e*x^(2/3)] + 576*d^2*Sqrt[1 - (d
+ e*x^(2/3))/d]*(d + e*x^(2/3))^2*Log[d + e*x^(2/3)] - 96*d*Sqrt[1 - (d + e
*x^(2/3))/d]*(d + e*x^(2/3))^3*Log[d + e*x^(2/3)] - 48*Sqrt[1 - (d + e*x^(2
/3))/d]*(d + e*x^(2/3))^4*Log[d + e*x^(2/3)] - 3780*d^3*(d + e*x^(2/3))*Hyp
ergeometricPFQ[{-3/2, 1, 1}, {2, 2}, (d + e*x^(2/3))/d]*Log[d + e*x^(2/3)]
+ 864*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-7/2, 1, 1, 1}, {2, 2, 2}, (d
+ e*x^(2/3))/d]*Log[d + e*x^(2/3)] - 3024*d^3*(d + e*x^(2/3))*Hypergeometri
cPFQ[{-5/2, 1, 1, 1}, {2, 2, 2}, (d + e*x^(2/3))/d]*Log[d + e*x^(2/3)] + 37
80*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-3/2, 1, 1, 1}, {2, 2, 2}, (d + e
*x^(2/3))/d]*Log[d + e*x^(2/3)] - 1890*d^3*(d + e*x^(2/3))*HypergeometricPF
Q[{-1/2, 1, 1, 1}, {2, 2, 2}, (d + e*x^(2/3))/d]*Log[d + e*x^(2/3)] + 284*d
^4*Log[d + e*x^(2/3)]^2 - 284*d^4*Sqrt[1 - (d + e*x^(2/3))/d]*Log[d + e*x^(
2/3)]^2 + 668*d^3*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))*Log[d + e*x^(
2/3)]^2 - 552*d^2*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^2*Log[d + e*x
^(2/3)]^2 + 236*d*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^3*Log[d + e*x
^(2/3)]^2 - 68*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^4*Log[d + e*x^(2
/3)]^2 - 1890*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-3/2, 1, 1}, {2, 2}, (
d + e*x^(2/3))/d]*Log[d + e*x^(2/3)]^2 + 945*d^3*(d + e*x^(2/3))*Hypergeome
tricPFQ[{-1/2, 1, 1}, {2, 2}, (d + e*x^(2/3))/d]*Log[d + e*x^(2/3)]^2 - 70*
d^4*Log[d + e*x^(2/3)]^3 + 70*d^4*Sqrt[1 - (d + e*x^(2/3))/d]*Log[d + e*x^(
2/3)]^3 - 280*d^3*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))*Log[d + e*x^(
2/3)]^3 + 420*d^2*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^2*Log[d + e*x
```

```
^(2/3)]^3 - 280*d*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^3*Log[d + e*x
^(2/3)]^3 + 70*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^4*Log[d + e*x^(2
/3)]^3 + 1512*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-5/2, 1, 1}, {2, 2}, (
d + e*x^(2/3))/d]*(1 + 3*Log[d + e*x^(2/3)] + Log[d + e*x^(2/3)]^2) - 144*d
^3*(d + e*x^(2/3))*HypergeometricPFQ[{-7/2, 1, 1}, {2, 2}, (d + e*x^(2/3))/
d]*(6 + 11*Log[d + e*x^(2/3)] + 3*Log[d + e*x^(2/3)]^2)))/(210*e^4*Sqrt[1 -
 (d + e*x^(2/3))/d]) + (b^2*n^2*x^(1/3)*(-120*d^4 + 120*d^4*Sqrt[1 - (d + e
*x^(2/3))/d] - 336*d^3*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3)) + 288*d^
2*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^2 - 48*d*Sqrt[1 - (d + e*x^(2
/3))/d]*(d + e*x^(2/3))^3 - 24*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^
4 - 1890*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-3/2, 1, 1}, {2, 2}, (d + e
*x^(2/3))/d] + 432*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-7/2, 1, 1, 1}, {
2, 2, 2}, (d + e*x^(2/3))/d] - 1512*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{
-5/2, 1, 1, 1}, {2, 2, 2}, (d + e*x^(2/3))/d] + 1890*d^3*(d + e*x^(2/3))*Hy
pergeometricPFQ[{-3/2, 1, 1, 1}, {2, 2, 2}, (d + e*x^(2/3))/d] - 945*d^3*(d
 + e*x^(2/3))*HypergeometricPFQ[{-1/2, 1, 1, 1}, {2, 2, 2}, (d + e*x^(2/3))
/d] + 284*d^4*Log[d + e*x^(2/3)] - 284*d^4*Sqrt[1 - (d + e*x^(2/3))/d]*Log[
d + e*x^(2/3)] + 668*d^3*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))*Log[d
+ e*x^(2/3)] - 552*d^2*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^2*Log[d
+ e*x^(2/3)] + 236*d*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^3*Log[d +
e*x^(2/3)] - 68*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^4*Log[d + e*x^(
2/3)] - 1890*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-3/2, 1, 1}, {2, 2}, (d
 + e*x^(2/3))/d]*Log[d + e*x^(2/3)] + 945*d^3*(d + e*x^(2/3))*Hypergeometri
cPFQ[{-1/2, 1, 1}, {2, 2}, (d + e*x^(2/3))/d]*Log[d + e*x^(2/3)] - 105*d^4*
Log[d + e*x^(2/3)]^2 + 105*d^4*Sqrt[1 - (d + e*x^(2/3))/d]*Log[d + e*x^(2/3
)]^2 - 420*d^3*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))*Log[d + e*x^(2/3
)]^2 + 630*d^2*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^2*Log[d + e*x^(2
/3)]^2 - 420*d*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^3*Log[d + e*x^(2
/3)]^2 + 105*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^4*Log[d + e*x^(2/3
)]^2 + 756*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-5/2, 1, 1}, {2, 2}, (d +
 e*x^(2/3))/d]*(3 + 2*Log[d + e*x^(2/3)]) - 72*...
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int x^{2} \left(a +b \ln \left(c \left(d +e \,x^{\frac{2}{3}}\right)^{n}\right)\right)^{3}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(2/3))^n))^3,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e*x^(2/3))^n))^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*n^3*x^3*log(x^(2/3)*e + d)^3 + integrate(1/3*(3*(b^3*log(c)^3 + 3*a
*b^2*log(c)^2 + 3*a^2*b*log(c) + a^3)*x^3*e - (2*b^3*n*x^3*e - 9*(b^3*log(c
) + a*b^2)*x^3*e - 9*(b^3*d*log(c) + a*b^2*d)*x^(7/3))*n^2*log(x^(2/3)*e +
d)^2 + 3*(b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x
^(7/3) + 9*((b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*b)*x^3*e + (b^3*d*log(c)^2
 + 2*a*b^2*d*log(c) + a^2*b*d)*x^(7/3))*n*log(x^(2/3)*e + d))/(x*e + d*x^(1
/3)), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*log((x^(2/3)*e + d)^n*c)^3 + 3*a*b^2*x^2*log((x^(2/3)*e +
d)^n*c)^2 + 3*a^2*b*x^2*log((x^(2/3)*e + d)^n*c) + a^3*x^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(2/3))**n))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^(2/3)*e + d)^n*c) + a)^3*x^2, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e*x^(2/3))^n))^3,x)
```

```
[Out] int(x^2*(a + b*log(c*(d + e*x^(2/3))^n))^3, x)
```

3.486 $\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=486

$$-\frac{32ab^2dn^2\sqrt[3]{x}}{e}+\frac{208b^3dn^3\sqrt[3]{x}}{3e}-\frac{16}{9}b^3n^3x-\frac{208b^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{32ib^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{e^{3/2}}-$$

```
[Out] -32*a*b^2*d*n^2*x^(1/3)/e+208/3*b^3*d*n^3*x^(1/3)/e-16/9*b^3*n^3*x-208/3*b^
3*d^(3/2)*n^3*arctan(x^(1/3)*e^(1/2)/d^(1/2))/e^(3/2)+32*I*b^3*d^(3/2)*n^3*
arctan(x^(1/3)*e^(1/2)/d^(1/2))^2/e^(3/2)-32*b^3*d*n^2*x^(1/3)*ln(c*(d+e*x^
(2/3))^n)/e+8/3*b^2*n^2*x*(a+b*ln(c*(d+e*x^(2/3))^n))+32*b^2*d^(3/2)*n^2*ar
ctan(x^(1/3)*e^(1/2)/d^(1/2))*(a+b*ln(c*(d+e*x^(2/3))^n))/e^(3/2)+6*b*d*n*x
^(1/3)*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e-2*b*n*x*(a+b*ln(c*(d+e*x^(2/3))^n))^
2+x*(a+b*ln(c*(d+e*x^(2/3))^n))^3+64*b^3*d^(3/2)*n^3*arctan(x^(1/3)*e^(1/2)
/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x^(1/3)*e^(1/2)))/e^(3/2)+32*I*b^3*d^(3/2
)*n^3*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x^(1/3)*e^(1/2)))/e^(3/2)-2*b*d^2*n*
Unintegrable((a+b*ln(c*(d+e*x^(2/3))^n))^2/(d+e*x^(2/3))/x^(2/3),x)/e
```

Rubi [A]
time = 0.71, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

```
[Out] (-32*a*b^2*d*n^2*x^(1/3))/e + (208*b^3*d*n^3*x^(1/3))/(3*e) - (16*b^3*n^3*x
)/9 - (208*b^3*d^(3/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(3*e^(3/2)) +
 ((32*I)*b^3*d^(3/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/e^(3/2) + (64
*b^3*d^(3/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d]
 + I*Sqrt[e]*x^(1/3))])/e^(3/2) - (32*b^3*d*n^2*x^(1/3)*Log[c*(d + e*x^(2/3
))^n])/e + (8*b^2*n^2*x*(a + b*Log[c*(d + e*x^(2/3))^n]))/3 + (32*b^2*d^(3/
2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2/3))^n]))/
e^(3/2) + (6*b*d*n*x^(1/3)*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/e - 2*b*n*x*
(a + b*Log[c*(d + e*x^(2/3))^n])^2 + x*(a + b*Log[c*(d + e*x^(2/3))^n])^3 +
 ((32*I)*b^3*d^(3/2)*n^3*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^
(1/3))])/e^(3/2) - (6*b*d^2*n*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x
^2)^n])^2/(d + e*x^2), x], x, x^(1/3)])/e
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx = 3\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^3\,dx, x, \sqrt[3]{x}\right)$$

$$= x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 - (6ben)\text{Subst}\left(\int \frac{x^4(a+b\log(c(d-}{d+ex^2}\right.$$

$$= x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 - (6ben)\text{Subst}\left(\int\left(-\frac{d(a+b\log(c(}{e^2}\right.\right.$$

$$= x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 - (6bn)\text{Subst}\left(\int x^2(a+b\log(c(d+\right.$$

$$= \frac{6bdn\sqrt[3]{x}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e} - 2bnx\left(a+b\log\left(c\left(d+ex^{2/3}\right.\right.\right.$$

$$= \frac{6bdn\sqrt[3]{x}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e} - 2bnx\left(a+b\log\left(c\left(d+ex^{2/3}\right.\right.\right.$$

$$= \frac{6bdn\sqrt[3]{x}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e} - 2bnx\left(a+b\log\left(c\left(d+ex^{2/3}\right.\right.\right.$$

$$= -\frac{32ab^2dn^2\sqrt[3]{x}}{e} + \frac{8}{3}b^2n^2x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right) + \frac{32b^2d^{3/2}n^2}{}$$

$$= -\frac{32ab^2dn^2\sqrt[3]{x}}{e} - \frac{32b^3dn^2\sqrt[3]{x}\log\left(c\left(d+ex^{2/3}\right)^n\right)}{e} + \frac{8}{3}b^2n^2x\left(a+b\right.$$

$$= -\frac{32ab^2dn^2\sqrt[3]{x}}{e} + \frac{208b^3dn^3\sqrt[3]{x}}{3e} - \frac{16}{9}b^3n^3x + \frac{32ib^3d^{3/2}n^3\tan^{-1}\left(\right.}{e^{3/2}}$$

$$= -\frac{32ab^2dn^2\sqrt[3]{x}}{e} + \frac{208b^3dn^3\sqrt[3]{x}}{3e} - \frac{16}{9}b^3n^3x - \frac{208b^3d^{3/2}n^3\tan^{-1}\left(\right.}{3e^{3/2}}$$

$$= -\frac{32ab^2dn^2\sqrt[3]{x}}{e} + \frac{208b^3dn^3\sqrt[3]{x}}{3e} - \frac{16}{9}b^3n^3x - \frac{208b^3d^{3/2}n^3\tan^{-1}\left(\right.}{3e^{3/2}}$$

Mathematica [A]
time = 0.90, size = 598, normalized size = 1.23

$$-\frac{b^3n^3x\left(-18\left(d+ex^{2/3}\right)\,{}_5F_4\left(-\frac{1}{2},1,1,1,1;2,2,2,2;1+\frac{ex^{2/3}}{d}\right)+\log\left(d+ex^{2/3}\right)\left(18\left(d+ex^{2/3}\right)\,{}_4F_3\left(-\frac{1}{2},1,1,1;2,2,2;1+\frac{ex^{2/3}}{d}\right)+\log\left(d+ex^{2/3}\right)\left(-9\left(d+ex^{2/3}\right)\,{}_3F_2\left(-\frac{1}{2},1,1;2,2;1+\frac{ex^{2/3}}{d}\right)+2\left(d-d\left(-\frac{ex^{2/3}}{d}\right)^{3/2}\right)\log\left(d+ex^{2/3}\right)\right)\right)\right)}{2d\left(-\frac{ex^{2/3}}{d}\right)^{3/2}}+\frac{3b^2n^2x\left(3\left(d+ex^{2/3}\right)\,{}_4F_3\left(-\frac{1}{2},1,1,1;2,2,2;1+\frac{ex^{2/3}}{d}\right)+\log\left(d+ex^{2/3}\right)\left(-3\left(d+ex^{2/3}\right)\,{}_3F_2\left(-\frac{1}{2},1,1;2,2;1+\frac{ex^{2/3}}{d}\right)+\left(d-d\left(-\frac{ex^{2/3}}{d}\right)^{3/2}\right)\log\left(d+ex^{2/3}\right)\right)\right)\left(-a+bn\log\left(d+ex^{2/3}\right)-b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d\left(-\frac{ex^{2/3}}{d}\right)^{3/2}}+\frac{6bdn\sqrt[3]{x}\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e}-\frac{6bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e^{3/2}}+3bnx\log\left(d+ex^{2/3}\right)\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2+x\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\left(a-2bn-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

```
[Out] -1/2*(b^3*n^3*x*(-18*(d + e*x^(2/3))*HypergeometricPFQ[{-1/2, 1, 1, 1, 1},
{2, 2, 2, 2}, 1 + (e*x^(2/3))/d] + Log[d + e*x^(2/3)]*(18*(d + e*x^(2/3))*H
ypergeometricPFQ[{-1/2, 1, 1, 1}, {2, 2, 2}, 1 + (e*x^(2/3))/d] + Log[d + e
*x^(2/3)]*(-9*(d + e*x^(2/3))*HypergeometricPFQ[{-1/2, 1, 1}, {2, 2}, 1 + (
e*x^(2/3))/d] + 2*(d - d*(-((e*x^(2/3))/d))^(3/2))*Log[d + e*x^(2/3)]))))/(
d*(-((e*x^(2/3))/d))^(3/2)) + (3*b^2*n^2*x*(3*(d + e*x^(2/3))*Hypergeometri
cPFQ[{-1/2, 1, 1, 1}, {2, 2, 2}, 1 + (e*x^(2/3))/d] + Log[d + e*x^(2/3)]*(-
3*(d + e*x^(2/3))*HypergeometricPFQ[{-1/2, 1, 1}, {2, 2}, 1 + (e*x^(2/3))/d
] + (d - d*(-((e*x^(2/3))/d))^(3/2))*Log[d + e*x^(2/3)]))*(-a + b*n*Log[d +
 e*x^(2/3)] - b*Log[c*(d + e*x^(2/3))^n]))/(d*(-((e*x^(2/3))/d))^(3/2)) + (
6*b*d*n*x^(1/3)*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2
)/e - (6*b*d^(3/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a - b*n*Log[d + e*x
^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2)/e^(3/2) + 3*b*n*x*Log[d + e*x^(2/3
)]*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 + x*(a - b*n
*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2*(a - 2*b*n - b*n*Log[d
+ e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="maxima")
```

```
[Out] b^3*n^3*x*log(x^(2/3)*e + d)^3 - (2*(3*d^(3/2)*arctan(x^(1/3)*e^(1/2)/sqrt(
d))*e^(-5/2) + (x*e - 3*d*x^(1/3))*e^(-2))*n*e - 3*x*log((x^(2/3)*e + d)^n*
```

```
c))*a^2*b + a^3*x + integrate(-((2*b^3*n*x*e - 3*(b^3*log(c) + a*b^2)*x*e -
 3*(b^3*d*log(c) + a*b^2*d)*x^(1/3))*n^2*log(x^(2/3)*e + d)^2 - (b^3*log(c)
^3 + 3*a*b^2*log(c)^2)*x*e - 3*((b^3*log(c)^2 + 2*a*b^2*log(c))*x*e + (b^3*
d*log(c)^2 + 2*a*b^2*d*log(c))*x^(1/3))*n*log(x^(2/3)*e + d) - (b^3*d*log(c
)^3 + 3*a*b^2*d*log(c)^2)*x^(1/3))/(x*e + d*x^(1/3)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*log((x^(2/3)*e + d)^n*c)^3 + 3*a*b^2*log((x^(2/3)*e + d)^n*c)^
2 + 3*a^2*b*log((x^(2/3)*e + d)^n*c) + a^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\log\left(c\left(d + ex^{\frac{2}{3}}\right)^{n}\right)\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**3,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**(2/3))**n))**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^(2/3)*e + d)^n*c) + a)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))^3,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^3, x)
```

3.487 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^2}\,dx$

Optimal. Leaf size=319

$$\frac{24ib^3e^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}}+\frac{48b^3e^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{d^{3/2}}+\frac{24b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}}{\sqrt{}}\right.}{}$$

```
[Out] 24*I*b^3*e^(3/2)*n^3*arctan(x^(1/3)*e^(1/2)/d^(1/2))^2/d^(3/2)+24*b^2*e^(3/
2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))*(a+b*ln(c*(d+e*x^(2/3))^n))/d^(3/2)-
6*b*e*n*(a+b*ln(c*(d+e*x^(2/3))^n))^2/d/x^(1/3)-(a+b*ln(c*(d+e*x^(2/3))^n))
^3/x+48*b^3*e^(3/2)*n^3*arctan(x^(1/3)*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/
2)+I*x^(1/3)*e^(1/2)))/d^(3/2)+24*I*b^3*e^(3/2)*n^3*polylog(2,1-2*d^(1/2)/(
d^(1/2)+I*x^(1/3)*e^(1/2)))/d^(3/2)-2*b*e^2*n*Unintegrable((a+b*ln(c*(d+e*x
^(2/3))^n))^2/(d+e*x^(2/3))/x^(2/3),x)/d
```

Rubi [A]
time = 0.33, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x^2,x]
```

```
[Out] ((24*I)*b^3*e^(3/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/d^(3/2) + (48*
b^3*e^(3/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d]
+ I*Sqrt[e]*x^(1/3))])/d^(3/2) + (24*b^2*e^(3/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3
))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2/3))^n]))/d^(3/2) - (6*b*e*n*(a + b*Log
[c*(d + e*x^(2/3))^n])^2)/(d*x^(1/3)) - (a + b*Log[c*(d + e*x^(2/3))^n])^3/
x + ((24*I)*b^3*e^(3/2)*n^3*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]
*x^(1/3))])/d^(3/2) - (6*b*e^2*n*Defer[Subst][Defer[Int][(a + b*Log[c*(d +
e*x^2)^n])^2/(d + e*x^2), x], x, x^(1/3)])/d
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^2}\,dx = 3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^3}{x^4}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x} + (6ben)\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex\right.\right.\right.}{x^2\left(d+ex^2\right)}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x} + (6ben)\text{Subst}\left(\int \left(\frac{\left(a+b\log\left(c\left(d+\right.\right.\right.}{dx^2}\right.\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x} + \frac{(6ben)\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{x^2}\,dx\right.}{d}$$

$$= -\frac{6ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{d\sqrt[3]{x}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x} -$$

$$= \frac{24b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\,\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{3/2}} - \frac{6ben\left(a+\right.}{}$$

$$= \frac{24b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\,\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{3/2}} - \frac{6ben\left(a+\right.}{}$$

$$= \frac{24ib^3e^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\,\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} + \frac{24b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\,\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\right.}{d^{3/2}}$$

$$= \frac{24ib^3e^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\,\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} + \frac{48b^3e^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\,\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\right.}{d^{3/2}}$$

$$= \frac{24ib^3e^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\,\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} + \frac{48b^3e^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\,\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\right.}{d^{3/2}}$$

$$= \frac{24ib^3e^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\,\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} + \frac{48b^3e^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\,\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\right.}{d^{3/2}}$$

Mathematica [A] Leaf count is larger than twice the leaf count of optimal. 1028 vs. $2(319) = 638$.
time = 5.02, size = 1028, normalized size = 3.22

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x^2,x]
```

```
[Out] (-3*b^2*n^2*(-3*d*(d + e*x^(2/3))*(-((e*x^(2/3))/d))^(3/2)*HypergeometricPF
Q[{1, 1, 1, 5/2}, {2, 2, 2}, 1 + (e*x^(2/3))/d] - d*Log[d + e*x^(2/3)]*(-4*
e*(-1 + Sqrt[-((e*x^(2/3))/d)])*x^(2/3) + 4*d*(-((e*x^(2/3))/d))^(3/2)*Log[
(1 + Sqrt[-((e*x^(2/3))/d)])/2] + (d - d*(-((e*x^(2/3))/d))^(3/2))*Log[d +
e*x^(2/3)]))*(-a + b*n*Log[d + e*x^(2/3)] - b*Log[c*(d + e*x^(2/3))^n]))/(d
^2*x) - (6*b*e*n*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^
2)/(d*x^(1/3)) - (6*b*e^(3/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a - b*n*
Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2)/d^(3/2) - (3*b*n*Log[d
+ e*x^(2/3)]*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2)/x
 - (a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^3/x + (b^3*n^3
*(48*Sqrt[-d^2]*e*Sqrt[(e*x^(2/3))/(d + e*x^(2/3))]*x^(2/3)*HypergeometricP
FQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}, d/(d + e*x^(2/3))] - 12*d*Sqrt[-d
^2]*(-((e*x^(2/3))/d))^(3/2)*Log[(1 + Sqrt[-((e*x^(2/3))/d)])/2]^2 - 24*Sqr
t[d]*(e*x^(2/3))^(3/2)*ArcTanh[Sqrt[e*x^(2/3)]/Sqrt[-d]]*Log[d + e*x^(2/3)]
 + 24*Sqrt[-d^2]*e*Sqrt[(e*x^(2/3))/(d + e*x^(2/3))]*x^(2/3)*Hypergeometric
PFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, d/(d + e*x^(2/3))]*Log[d + e*x^(2/3)] - 6*
Sqrt[-d^2]*e*x^(2/3)*Log[d + e*x^(2/3)]^2 + 6*Sqrt[-d]*(d + e*x^(2/3))^(3/2
)*((e*x^(2/3))/(d + e*x^(2/3)))^(3/2)*ArcSin[Sqrt[d]/Sqrt[d + e*x^(2/3)]]*L
og[d + e*x^(2/3)]^2 + (d^(5/2)*Log[d + e*x^(2/3)]^3)/Sqrt[-d] + 24*Sqrt[d]*
(e*x^(2/3))^(3/2)*ArcTanh[Sqrt[e*x^(2/3)]/Sqrt[-d]]*Log[1 + (e*x^(2/3))/d]
+ 24*d*Sqrt[-d^2]*(-((e*x^(2/3))/d))^(3/2)*Log[(1 + Sqrt[-((e*x^(2/3))/d)])
/2]*Log[1 + (e*x^(2/3))/d] - 6*d*Sqrt[-d^2]*(-((e*x^(2/3))/d))^(3/2)*Log[1
+ (e*x^(2/3))/d]^2 + 24*d*Sqrt[-d^2]*(-((e*x^(2/3))/d))^(3/2)*PolyLog[2, 1/
2 - Sqrt[-((e*x^(2/3))/d)]/2]))/(Sqrt[-d]*d^(3/2)*x)
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{3}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^3/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^3/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^2,x, algorithm="maxima")
```

```
[Out] -b^3*n^3*log(x^(2/3)*e + d)^3/x + integrate(((2*b^3*n*x*e + 3*(b^3*log(c) +
 a*b^2)*x*e + 3*(b^3*d*log(c) + a*b^2*d)*x^(1/3))*n^2*log(x^(2/3)*e + d)^2
+ (b^3*log(c)^3 + 3*a*b^2*log(c)^2 + 3*a^2*b*log(c) + a^3)*x*e + 3*((b^3*lo
g(c)^2 + 2*a*b^2*log(c) + a^2*b)*x*e + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c) +
 a^2*b*d)*x^(1/3))*n*log(x^(2/3)*e + d) + (b^3*d*log(c)^3 + 3*a*b^2*d*log(c
)^2 + 3*a^2*b*d*log(c) + a^3*d)*x^(1/3))/(x^3*e + d*x^(7/3)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((x^(2/3)*e + d)^n*c)^3 + 3*a*b^2*log((x^(2/3)*e + d)^n*c)
^2 + 3*a^2*b*log((x^(2/3)*e + d)^n*c) + a^3)/x^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**3/x**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^(2/3)*e + d)^n*c) + a)^3/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{2/3}\right)^n\right)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))^3/x^2,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^3/x^2, x)
```

3.488 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^4}\,dx$

Optimal. Leaf size=632

$$-\frac{16b^3e^3n^3}{105d^3x}+\frac{16b^3e^4n^3}{7d^4\sqrt[3]{x}}+\frac{1376b^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{105d^{9/2}}-\frac{1408ib^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{105d^{9/2}}-\frac{2816b^3e^{9/2}n^3\tan}{}$$

```
[Out] -16/105*b^3*e^3*n^3/d^3/x+16/7*b^3*e^4*n^3/d^4/x^(1/3)+1376/105*b^3*e^(9/2)
*n^3*arctan(x^(1/3)*e^(1/2)/d^(1/2))/d^(9/2)-1408/105*I*b^3*e^(9/2)*n^3*arc
tan(x^(1/3)*e^(1/2)/d^(1/2))^2/d^(9/2)-8/35*b^2*e^2*n^2*(a+b*ln(c*(d+e*x^(2
/3))^n))/d^2/x^(5/3)+32/35*b^2*e^3*n^2*(a+b*ln(c*(d+e*x^(2/3))^n))/d^3/x-56
8/105*b^2*e^4*n^2*(a+b*ln(c*(d+e*x^(2/3))^n))/d^4/x^(1/3)-1408/105*b^2*e^(9
/2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))*(a+b*ln(c*(d+e*x^(2/3))^n))/d^(9/2)
-2/7*b*e*n*(a+b*ln(c*(d+e*x^(2/3))^n))^2/d/x^(7/3)+2/5*b*e^2*n*(a+b*ln(c*(d
+e*x^(2/3))^n))^2/d^2/x^(5/3)-2/3*b*e^3*n*(a+b*ln(c*(d+e*x^(2/3))^n))^2/d^3
/x+2*b*e^4*n*(a+b*ln(c*(d+e*x^(2/3))^n))^2/d^4/x^(1/3)-1/3*(a+b*ln(c*(d+e*x
^(2/3))^n))^3/x^3-2816/105*b^3*e^(9/2)*n^3*arctan(x^(1/3)*e^(1/2)/d^(1/2))*
ln(2*d^(1/2)/(d^(1/2)+I*x^(1/3)*e^(1/2)))/d^(9/2)-1408/105*I*b^3*e^(9/2)*n^
3*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x^(1/3)*e^(1/2)))/d^(9/2)+2/3*b*e^5*n*Un
integrable((a+b*ln(c*(d+e*x^(2/3))^n))^2/(d+e*x^(2/3))/x^(2/3),x)/d^4
```

Rubi [A]
time = 1.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x^4,x]
```

```
[Out] (-16*b^3*e^3*n^3)/(105*d^3*x) + (16*b^3*e^4*n^3)/(7*d^4*x^(1/3)) + (1376*b^
3*e^(9/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(105*d^(9/2)) - (((1408*I)
/105)*b^3*e^(9/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/d^(9/2) - (2816*
b^3*e^(9/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d]
+ I*Sqrt[e]*x^(1/3))])/(105*d^(9/2)) - (8*b^2*e^2*n^2*(a + b*Log[c*(d + e*x
^(2/3))^n]))/(35*d^2*x^(5/3)) + (32*b^2*e^3*n^2*(a + b*Log[c*(d + e*x^(2/3)
)^n]))/(35*d^3*x) - (568*b^2*e^4*n^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/(105
*d^4*x^(1/3)) - (1408*b^2*e^(9/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a
+ b*Log[c*(d + e*x^(2/3))^n]))/(105*d^(9/2)) - (2*b*e*n*(a + b*Log[c*(d + e
*x^(2/3))^n])^2)/(7*d*x^(7/3)) + (2*b*e^2*n*(a + b*Log[c*(d + e*x^(2/3))^n]
```

```
)^2)/(5*d^2*x^(5/3)) - (2*b*e^3*n*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(3*d^
3*x) + (2*b*e^4*n*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(d^4*x^(1/3)) - (a +
b*Log[c*(d + e*x^(2/3))^n])^3/(3*x^3) - (((1408*I)/105)*b^3*e^(9/2)*n^3*Pol
yLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/d^(9/2) + (2*b*e^5*
n*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x^2)^n])^2/(d + e*x^2), x], x
, x^(1/3)])/d^4
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^4}\,dx = 3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^3}{x^{10}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{3x^3} + (2ben)\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex\right.\right.\right.}{x^8\left(d+ex^2\right)}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{3x^3} + (2ben)\text{Subst}\left(\int \left(\frac{\left(a+b\log\left(c\left(d+\right.\right.\right.}{dx^8}\right.\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{3x^3} + \frac{(2ben)\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{x^8}\,dx\right.}{d}$$

$$= -\frac{2ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{7dx^{7/3}} + \frac{2be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right.}{5d^2x^{5/3}}$$

$$= -\frac{8b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\,\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{9/2}} - \frac{2ben\left(a+\right.}{}$$

$$= -\frac{8b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\,\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{9/2}} - \frac{2ben\left(a+\right.}{}$$

$$= -\frac{8ib^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\,\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{9/2}} - \frac{8b^2e^2n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{35d^2x^{5/3}}$$

$$= -\frac{16b^3e^3n^3}{105d^3x} + \frac{64b^3e^4n^3}{35d^4\sqrt[3]{x}} + \frac{1136b^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\,\sqrt[3]{x}}{\sqrt{d}}\right)}{105d^{9/2}} - \frac{8ib^3e^{9/2}n}{}$$

$$= -\frac{16b^3e^3n^3}{105d^3x} + \frac{16b^3e^4n^3}{7d^4\sqrt[3]{x}} + \frac{1328b^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\,\sqrt[3]{x}}{\sqrt{d}}\right)}{105d^{9/2}} - \frac{1408ib^3e}{}$$

Mathematica [A]
time = 2.53, size = 803, normalized size = 1.27

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x^4,x]
```

```
[Out] (35*b^3*n^3*(54*e^4*(d + e*x^(2/3))*Sqrt[-((e*x^(2/3))/d)]*x^(8/3)*Hypergeo
metricPFQ[{1, 1, 1, 1, 11/2}, {2, 2, 2, 2}, 1 + (e*x^(2/3))/d] + Log[d + e*
x^(2/3)]*(54*d*e^3*(d + e*x^(2/3))*(-((e*x^(2/3))/d))^(3/2)*x^2*Hypergeomet
ricPFQ[{1, 1, 1, 11/2}, {2, 2, 2}, 1 + (e*x^(2/3))/d] + Log[d + e*x^(2/3)]*
(27*e^4*(d + e*x^(2/3))*Sqrt[-((e*x^(2/3))/d)]*x^(8/3)*HypergeometricPFQ[{1
, 1, 11/2}, {2, 2}, 1 + (e*x^(2/3))/d] - 2*d*(d^4 + d*e^3*(-((e*x^(2/3))/d)
)^(3/2)*x^2)*Log[d + e*x^(2/3)]))) + (210*b^2*n^2*(-9*e^5*(d + e*x^(2/3))*x
^(10/3)*HypergeometricPFQ[{1, 1, 1, 11/2}, {2, 2, 2}, 1 + (e*x^(2/3))/d] +
Log[d + e*x^(2/3)]*(9*e^5*(d + e*x^(2/3))*x^(10/3)*HypergeometricPFQ[{1, 1,
 11/2}, {2, 2}, 1 + (e*x^(2/3))/d] + d*(d^5*Sqrt[-((e*x^(2/3))/d)] + e^5*x^
(10/3))*Log[d + e*x^(2/3)]))*(-a + b*n*Log[d + e*x^(2/3)] - b*Log[c*(d + e*
x^(2/3))^n]))/(d*Sqrt[-((e*x^(2/3))/d)]) - 60*b*d^4*e*n*x^(2/3)*(a - b*n*Lo
g[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 + 84*b*d^3*e^2*n*x^(4/3)*(
a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 - 140*b*d^2*e^3*
n*x^2*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 + 420*b*d
*e^4*n*x^(8/3)*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2
+ 420*b*Sqrt[d]*e^(9/2)*n*x^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a - b*n*Lo
g[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 - 210*b*d^5*n*Log[d + e*x^
(2/3)]*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 - 70*d^5
*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^3)/(210*d^5*x^3)
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{3}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^3/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^3/x^4,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^3*n^3*log(x^(2/3)*e + d)^3/x^3 + integrate(1/3*((2*b^3*n*x*e + 9*(b^
3*log(c) + a*b^2)*x*e + 9*(b^3*d*log(c) + a*b^2*d)*x^(1/3))*n^2*log(x^(2/3)
*e + d)^2 + 3*(b^3*log(c)^3 + 3*a*b^2*log(c)^2 + 3*a^2*b*log(c) + a^3)*x*e
+ 9*((b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*b)*x*e + (b^3*d*log(c)^2 + 2*a*b^
2*d*log(c) + a^2*b*d)*x^(1/3))*n*log(x^(2/3)*e + d) + 3*(b^3*d*log(c)^3 + 3
*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x^(1/3))/(x^5*e + d*x^(13/3))
, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((x^(2/3)*e + d)^n*c)^3 + 3*a*b^2*log((x^(2/3)*e + d)^n*c)
^2 + 3*a^2*b*log((x^(2/3)*e + d)^n*c) + a^3)/x^4, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**3/x**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^(2/3)*e + d)^n*c) + a)^3/x^4, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))^3/x^4,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^3/x^4, x)
```

3.489 $\int x^3 \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx$

Optimal. Leaf size=239

$$\frac{be^{11}n\sqrt[3]{x}}{4d^{11}}-\frac{be^{10}nx^{2/3}}{8d^{10}}+\frac{be^9nx}{12d^9}-\frac{be^8nx^{4/3}}{16d^8}+\frac{be^7nx^{5/3}}{20d^7}-\frac{be^6nx^2}{24d^6}+\frac{be^5nx^{7/3}}{28d^5}-\frac{be^4nx^{8/3}}{32d^4}+\frac{be^3nx^3}{36d^3}-\frac{be^2nx^{10/3}}{40d^2}+\frac{ben\ldots}{4\ldots}$$

```
[Out] 1/4*b*e^11*n*x^(1/3)/d^11-1/8*b*e^10*n*x^(2/3)/d^10+1/12*b*e^9*n*x/d^9-1/16
*b*e^8*n*x^(4/3)/d^8+1/20*b*e^7*n*x^(5/3)/d^7-1/24*b*e^6*n*x^2/d^6+1/28*b*e
^5*n*x^(7/3)/d^5-1/32*b*e^4*n*x^(8/3)/d^4+1/36*b*e^3*n*x^3/d^3-1/40*b*e^2*n
*x^(10/3)/d^2+1/44*b*e*n*x^(11/3)/d-1/4*b*e^12*n*ln(d+e/x^(1/3))/d^12+1/4*x
^4*(a+b*ln(c*(d+e/x^(1/3))^n))-1/12*b*e^12*n*ln(x)/d^12
```

Rubi [A]
time = 0.12, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2504, 2442, 46}

$$\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{be^{12}n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{4d^{12}}-\frac{be^{12}n\log(x)}{12d^{12}}+\frac{be^{11}n\sqrt[3]{x}}{4d^{11}}-\frac{be^{10}nx^{2/3}}{8d^{10}}+\frac{be^9nx}{12d^9}-\frac{be^8nx^{4/3}}{16d^8}+\frac{be^7nx^{5/3}}{20d^7}-\frac{be^6nx^2}{24d^6}+\frac{be^5nx^{7/3}}{28d^5}-\frac{be^4nx^{8/3}}{32d^4}+\frac{be^3nx^3}{36d^3}-\frac{be^2nx^{10/3}}{40d^2}+\frac{benx^{11/3}}{44d}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e/x^(1/3))^n]),x]
```

```
[Out] (b*e^11*n*x^(1/3))/(4*d^11) - (b*e^10*n*x^(2/3))/(8*d^10) + (b*e^9*n*x)/(12
*d^9) - (b*e^8*n*x^(4/3))/(16*d^8) + (b*e^7*n*x^(5/3))/(20*d^7) - (b*e^6*n*
x^2)/(24*d^6) + (b*e^5*n*x^(7/3))/(28*d^5) - (b*e^4*n*x^(8/3))/(32*d^4) + (
b*e^3*n*x^3)/(36*d^3) - (b*e^2*n*x^(10/3))/(40*d^2) + (b*e*n*x^(11/3))/(44*
d) - (b*e^12*n*Log[d + e/x^(1/3)])/(4*d^12) + (x^4*(a + b*Log[c*(d + e/x^(1
/3))^n]))/4 - (b*e^12*n*Log[x])/(12*d^12)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
```

```
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx=-\left(3\text{Subst}\left(\int\frac{a+b\log\left(c(d+ex)^n\right)}{x^{13}}\,dx,x,\frac{1}{\sqrt[3]{x}}\right)\right)$$

$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{1}{4}(ben)\text{Subst}\left(\int\frac{1}{x^{12}(d+ex)}\right.$$

$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{1}{4}(ben)\text{Subst}\left(\int\left(\frac{1}{dx^{12}}-\right.\right.$$

$$=\frac{be^{11}n\sqrt[3]{x}}{4d^{11}}-\frac{be^{10}nx^{2/3}}{8d^{10}}+\frac{be^9nx}{12d^9}-\frac{be^8nx^{4/3}}{16d^8}+\frac{be^7nx^{5/3}}{20d^7}-\frac{be^6nx^2}{24d^6}$$

Mathematica [A]
time = 0.16, size = 218, normalized size = 0.91

$$\frac{ax^4}{4}+\frac{1}{4}bx^4\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)-\frac{1}{4}ben\left(-\frac{e^{10}\sqrt[3]{x}}{d^{11}}+\frac{e^9x^{2/3}}{2d^{10}}-\frac{e^8x}{3d^9}+\frac{e^7x^{4/3}}{4d^8}-\frac{e^6x^{5/3}}{5d^7}+\frac{e^5x^2}{6d^6}-\frac{e^4x^{7/3}}{7d^5}+\frac{e^3x^{8/3}}{8d^4}-\frac{e^2x^3}{9d^3}+\frac{ex^{10/3}}{10d^2}-\frac{x^{11/3}}{11d}+\frac{e^{11}\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^{12}}+\frac{e^{11}\log(x)}{3d^{12}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e/x^(1/3))^n]),x]
```

```
[Out] (a*x^4)/4 + (b*x^4*Log[c*(d + e/x^(1/3))^n])/4 - (b*e*n*(-((e^10*x^(1/3))/d
^11) + (e^9*x^(2/3))/(2*d^10) - (e^8*x)/(3*d^9) + (e^7*x^(4/3))/(4*d^8) - (
e^6*x^(5/3))/(5*d^7) + (e^5*x^2)/(6*d^6) - (e^4*x^(7/3))/(7*d^5) + (e^3*x^(
8/3))/(8*d^4) - (e^2*x^3)/(9*d^3) + (e*x^(10/3))/(10*d^2) - x^(11/3)/(11*d)
 + (e^11*Log[d + e/x^(1/3)])/d^12 + (e^11*Log[x])/(3*d^12)))/4
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^3\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e/x^(1/3))^n)),x)
[Out] int(x^3*(a+b*ln(c*(d+e/x^(1/3))^n)),x)
```

Maxima [A]
time = 0.28, size = 155, normalized size = 0.65

$$\frac{1}{4}\,bx^4\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+\frac{1}{4}\,ax^4+\frac{1}{110880}\,bn\left(\frac{2520\,d^{10}x^{\frac{11}{3}}-2772\,d^9x^{\frac{10}{3}}e+3080\,d^8x^3e^2-3465\,d^7x^{\frac{8}{3}}e^3+3960\,d^6x^{\frac{7}{3}}e^4-4620\,d^5x^2e^5+5544\,d^4x^{\frac{5}{3}}e^6-6930\,d^3x^{\frac{4}{3}}e^7+9240\,d^2xe^8-13860\,dx^{\frac{2}{3}}e^9+27720\,x^{\frac{1}{3}}e^{10}}{d^{11}}-\frac{27720\,e^{11}\log\left(dx^{\frac{1}{3}}+e\right)}{d^{12}}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="maxima")
[Out] 1/4*b*x^4*log(c*(d + e/x^(1/3))^n) + 1/4*a*x^4 + 1/110880*b*n*((2520*d^10*x
^(11/3) - 2772*d^9*x^(10/3)*e + 3080*d^8*x^3*e^2 - 3465*d^7*x^(8/3)*e^3 + 3
960*d^6*x^(7/3)*e^4 - 4620*d^5*x^2*e^5 + 5544*d^4*x^(5/3)*e^6 - 6930*d^3*x^
(4/3)*e^7 + 9240*d^2*x*e^8 - 13860*d*x^(2/3)*e^9 + 27720*x^(1/3)*e^10)/d^11
 - 27720*e^11*log(d*x^(1/3) + e)/d^12)*e
```

Fricas [A]
time = 0.38, size = 225, normalized size = 0.94

$$\frac{27720\,bd^{12}x^4\log\left(c\right)+27720\,ad^{12}x^4+3080\,bd^9nx^3e^3-27720\,bd^{12}n\log\left(x^{\frac{1}{3}}\right)-4620\,bd^6nx^2e^6+9240\,bd^3nxe^9+27720\,(bd^{12}n-bne^{12})\log\left(dx^{\frac{1}{3}}+e\right)+27720\,(bd^{12}nx^4-bd^{12}n)\log\left(\frac{dx+x^{\frac{2}{3}}e}{x}\right)+63\,(40\,bd^{11}nx^3e-55\,bd^8nx^2e^4+88\,bd^5nxe^7-220\,bd^2ne^{10})x^{\frac{2}{3}}-198\,(14\,bd^{10}nx^3e^2-20\,bd^7nx^2e^5+35\,bd^4nxe^8-140\,bdne^{11})x^{\frac{1}{3}}}{110880\,d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="fricas")
[Out] 1/110880*(27720*b*d^12*x^4*log(c) + 27720*a*d^12*x^4 + 3080*b*d^9*n*x^3*e^3
 - 27720*b*d^12*n*log(x^(1/3)) - 4620*b*d^6*n*x^2*e^6 + 9240*b*d^3*n*x*e^9
+ 27720*(b*d^12*n - b*n*e^12)*log(d*x^(1/3) + e) + 27720*(b*d^12*n*x^4 - b*
d^12*n)*log((d*x + x^(2/3)*e)/x) + 63*(40*b*d^11*n*x^3*e - 55*b*d^8*n*x^2*e
^4 + 88*b*d^5*n*x*e^7 - 220*b*d^2*n*e^10)*x^(2/3) - 198*(14*b*d^10*n*x^3*e^
2 - 20*b*d^7*n*x^2*e^5 + 35*b*d^4*n*x*e^8 - 140*b*d*n*e^11)*x^(1/3))/d^12
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e/x**(1/3))**n)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [A]
time = 4.40, size = 161, normalized size = 0.67

$$\frac{1}{4}\,bx^4\log\left(c\right)+\frac{1}{4}\,ax^4+\frac{1}{110880}\left(27720\,x^4\log\left(d+\frac{e}{x^{\frac{1}{3}}}\right)+\left(\frac{2520\,d^{10}x^{\frac{11}{3}}-2772\,d^9x^{\frac{10}{3}}e+3080\,d^8x^3e^2-3465\,d^7x^{\frac{8}{3}}e^3+3960\,d^6x^{\frac{7}{3}}e^4-4620\,d^5x^2e^5+5544\,d^4x^{\frac{5}{3}}e^6-6930\,d^3x^{\frac{4}{3}}e^7+9240\,d^2xe^8-13860\,dx^{\frac{2}{3}}e^9+27720\,x^{\frac{1}{3}}e^{10}}{d^{11}}-\frac{27720\,e^{11}\log\left(\left|dx^{\frac{1}{3}}+e\right|\right)}{d^{12}}\right)e\right)bn$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="giac")
```

```
[Out] 1/4*b*x^4*log(c) + 1/4*a*x^4 + 1/110880*(27720*x^4*log(d + e/x^(1/3)) + ((2
520*d^10*x^(11/3) - 2772*d^9*x^(10/3)*e + 3080*d^8*x^3*e^2 - 3465*d^7*x^(8/
3)*e^3 + 3960*d^6*x^(7/3)*e^4 - 4620*d^5*x^2*e^5 + 5544*d^4*x^(5/3)*e^6 - 6
930*d^3*x^(4/3)*e^7 + 9240*d^2*x*e^8 - 13860*d*x^(2/3)*e^9 + 27720*x^(1/3)*
e^10)/d^11 - 27720*e^11*log(abs(d*x^(1/3) + e))/d^12)*e)*b*n
```

Mupad [B]
time = 0.80, size = 191, normalized size = 0.80

$$\frac{\frac{a\,d^{12}\,x^4}{4}-\frac{b\,e^{12}\,n\,\mathrm{atanh}\left(\frac{2\,e}{d\,x^{1/3}}+1\right)}{2}+\frac{b\,d^{12}\,x^4\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^n\right)}{4}+\frac{b\,d^3\,e^9\,n\,x}{12}+\frac{b\,d\,e^{11}\,n\,x^{1/3}}{4}+\frac{b\,d^{11}\,e\,n\,x^{11/3}}{44}-\frac{b\,d^6\,e^6\,n\,x^2}{24}+\frac{b\,d^9\,e^3\,n\,x^3}{36}-\frac{b\,d^2\,e^{10}\,n\,x^{2/3}}{8}-\frac{b\,d^4\,e^8\,n\,x^{4/3}}{16}+\frac{b\,d^5\,e^7\,n\,x^{5/3}}{20}+\frac{b\,d^7\,e^5\,n\,x^{7/3}}{28}-\frac{b\,d^8\,e^4\,n\,x^{8/3}}{32}-\frac{b\,d^{10}\,e^2\,n\,x^{10/3}}{40}}{d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e/x^(1/3))^n)),x)
```

```
[Out] ((a*d^12*x^4)/4 - (b*e^12*n*atanh((2*e)/(d*x^(1/3)) + 1))/2 + (b*d^12*x^4*l
og(c*(d + e/x^(1/3))^n))/4 + (b*d^3*e^9*n*x)/12 + (b*d*e^11*n*x^(1/3))/4 +
(b*d^11*e*n*x^(11/3))/44 - (b*d^6*e^6*n*x^2)/24 + (b*d^9*e^3*n*x^3)/36 - (b
*d^2*e^10*n*x^(2/3))/8 - (b*d^4*e^8*n*x^(4/3))/16 + (b*d^5*e^7*n*x^(5/3))/2
0 + (b*d^7*e^5*n*x^(7/3))/28 - (b*d^8*e^4*n*x^(8/3))/32 - (b*d^10*e^2*n*x^(
10/3))/40)/d^12
```

3.490 $\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx$

Optimal. Leaf size=190

$$-\frac{be^8n\sqrt[3]{x}}{3d^8}+\frac{be^7nx^{2/3}}{6d^7}-\frac{be^6nx}{9d^6}+\frac{be^5nx^{4/3}}{12d^5}-\frac{be^4nx^{5/3}}{15d^4}+\frac{be^3nx^2}{18d^3}-\frac{be^2nx^{7/3}}{21d^2}+\frac{benx^{8/3}}{24d}+\frac{be^9n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{3d^9}+\frac{1}{3}x$$

```
[Out] -1/3*b*e^8*n*x^(1/3)/d^8+1/6*b*e^7*n*x^(2/3)/d^7-1/9*b*e^6*n*x/d^6+1/12*b*e
^5*n*x^(4/3)/d^5-1/15*b*e^4*n*x^(5/3)/d^4+1/18*b*e^3*n*x^2/d^3-1/21*b*e^2*n
*x^(7/3)/d^2+1/24*b*e*n*x^(8/3)/d+1/3*b*e^9*n*ln(d+e/x^(1/3))/d^9+1/3*x^3*(
a+b*ln(c*(d+e/x^(1/3))^n))+1/9*b*e^9*n*ln(x)/d^9
```

Rubi [A]
time = 0.09, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2504, 2442, 46}

$$\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)+\frac{be^9n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{3d^9}+\frac{be^9n\log(x)}{9d^9}-\frac{be^8n\sqrt[3]{x}}{3d^8}+\frac{be^7nx^{2/3}}{6d^7}-\frac{be^6nx}{9d^6}+\frac{be^5nx^{4/3}}{12d^5}-\frac{be^4nx^{5/3}}{15d^4}+\frac{be^3nx^2}{18d^3}-\frac{be^2nx^{7/3}}{21d^2}+\frac{benx^{8/3}}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e/x^(1/3))^n]),x]
```

```
[Out] -1/3*(b*e^8*n*x^(1/3))/d^8 + (b*e^7*n*x^(2/3))/(6*d^7) - (b*e^6*n*x)/(9*d^6
) + (b*e^5*n*x^(4/3))/(12*d^5) - (b*e^4*n*x^(5/3))/(15*d^4) + (b*e^3*n*x^2)
/(18*d^3) - (b*e^2*n*x^(7/3))/(21*d^2) + (b*e*n*x^(8/3))/(24*d) + (b*e^9*n*
Log[d + e/x^(1/3)])/(3*d^9) + (x^3*(a + b*Log[c*(d + e/x^(1/3))^n]))/3 + (b
*e^9*n*Log[x])/(9*d^9)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)dx=-\left(3\text{Subst}\left(\int\frac{a+b\log\left(c(d+ex)^n\right)}{x^{10}}dx,x,\frac{1}{\sqrt[3]{x}}\right)\right)$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{1}{3}(ben)\text{Subst}\left(\int\frac{1}{x^9(d+ex$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{1}{3}(ben)\text{Subst}\left(\int\left(\frac{1}{dx^9}-\frac{}{d$$

$$=-\frac{be^8n\sqrt[3]{x}}{3d^8}+\frac{be^7nx^{2/3}}{6d^7}-\frac{be^6nx}{9d^6}+\frac{be^5nx^{4/3}}{12d^5}-\frac{be^4nx^{5/3}}{15d^4}+\frac{be^3nx}{18d^3}$$

Mathematica [A]
time = 0.09, size = 175, normalized size = 0.92

$$\frac{ax^3}{3}+\frac{1}{3}bx^3\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)-\frac{1}{3}ben\left(\frac{e^7\sqrt[3]{x}}{d^8}-\frac{e^6x^{2/3}}{2d^7}+\frac{e^5x}{3d^6}-\frac{e^4x^{4/3}}{4d^5}+\frac{e^3x^{5/3}}{5d^4}-\frac{e^2x^2}{6d^3}+\frac{ex^{7/3}}{7d^2}-\frac{x^{8/3}}{8d}-\frac{e^8\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^9}-\frac{e^8\log(x)}{3d^9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/x^(1/3))^n]),x]
```

```
[Out] (a*x^3)/3 + (b*x^3*Log[c*(d + e/x^(1/3))^n])/3 - (b*e*n*((e^7*x^(1/3))/d^8
- (e^6*x^(2/3))/(2*d^7) + (e^5*x)/(3*d^6) - (e^4*x^(4/3))/(4*d^5) + (e^3*x^
(5/3))/(5*d^4) - (e^2*x^2)/(6*d^3) + (e*x^(7/3))/(7*d^2) - x^(8/3)/(8*d) -
(e^8*Log[d + e/x^(1/3)])/d^9 - (e^8*Log[x])/(3*d^9)))/3
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e/x^(1/3))^n)),x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e/x^(1/3))^n)),x)
```

Maxima [A]
time = 0.29, size = 125, normalized size = 0.66

$$\frac{1}{3}\,bx^3\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+\frac{1}{3}\,ax^3+\frac{1}{2520}\,bn\left(\frac{105\,d^7x^{\frac{8}{3}}-120\,d^6x^{\frac{7}{3}}e+140\,d^5x^2e^2-168\,d^4x^{\frac{5}{3}}e^3+210\,d^3x^{\frac{4}{3}}e^4-280\,d^2xe^5+420\,dx^{\frac{2}{3}}e^6-840\,x^{\frac{1}{3}}e^7}{d^8}+\frac{840\,e^8\log\left(dx^{\frac{1}{3}}+e\right)}{d^9}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3*log(c*(d + e/x^(1/3))^n) + 1/3*a*x^3 + 1/2520*b*n*((105*d^7*x^(8/
3) - 120*d^6*x^(7/3)*e + 140*d^5*x^2*e^2 - 168*d^4*x^(5/3)*e^3 + 210*d^3*x^
(4/3)*e^4 - 280*d^2*x*e^5 + 420*d*x^(2/3)*e^6 - 840*x^(1/3)*e^7)/d^8 + 840*
e^8*log(d*x^(1/3) + e)/d^9)*e
```

Fricas [A]
time = 0.38, size = 188, normalized size = 0.99

$$\frac{840\,bd^9x^3\log\left(c\right)+840\,ad^9x^3-840\,bd^9n\log\left(x^{\frac{1}{3}}\right)+140\,bd^6nx^2e^3-280\,bd^3nxe^6+840\,\left(bd^9n+bne^9\right)\log\left(dx^{\frac{1}{3}}+e\right)+840\,\left(bd^9nx^3-bd^9n\right)\log\left(\frac{dx+x^{\frac{2}{3}}e}{x}\right)+21\,\left(5\,bd^8nx^2e-8\,bd^5nxe^4+20\,bd^2ne^7\right)x^{\frac{2}{3}}-30\,\left(4\,bd^7nx^2e^2-7\,bd^4nxe^5+28\,bdne^8\right)x^{\frac{1}{3}}}{2520\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="fricas")
```

```
[Out] 1/2520*(840*b*d^9*x^3*log(c) + 840*a*d^9*x^3 - 840*b*d^9*n*log(x^(1/3)) + 1
40*b*d^6*n*x^2*e^3 - 280*b*d^3*n*x*e^6 + 840*(b*d^9*n + b*n*e^9)*log(d*x^(1
/3) + e) + 840*(b*d^9*n*x^3 - b*d^9*n)*log((d*x + x^(2/3)*e)/x) + 21*(5*b*d
^8*n*x^2*e - 8*b*d^5*n*x*e^4 + 20*b*d^2*n*e^7)*x^(2/3) - 30*(4*b*d^7*n*x^2*
e^2 - 7*b*d^4*n*x*e^5 + 28*b*d*n*e^8)*x^(1/3))/d^9
```

Sympy [A]
time = 62.18, size = 180, normalized size = 0.95

$$\frac{ax^3}{3}+b\left(\frac{en\left(\frac{3x^{\frac{8}{3}}}{8d}-\frac{3ex^{\frac{7}{3}}}{7d^2}+\frac{e^2x^2}{2d^3}-\frac{3e^3x^{\frac{5}{3}}}{5d^4}+\frac{3e^4x^{\frac{4}{3}}}{4d^5}-\frac{e^5x}{d^6}+\frac{3e^6x^{\frac{2}{3}}}{2d^7}-\frac{3e^7\sqrt[3]{x}}{d^8}+\frac{3e^9\left(\begin{cases}\frac{1}{d\sqrt[3]{x}} & \text{for}\: e=0\\\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e} & \text{otherwise}\end{cases}\right)}{d^9}-\frac{3e^8\log\left(\frac{1}{\sqrt[3]{x}}\right)}{d^9}\right)}{9}+\frac{x^3\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(1/3))**n)),x)
```

```
[Out] a*x**3/3 + b*(e*n*(3*x**(8/3)/(8*d) - 3*e*x**(7/3)/(7*d**2) + e**2*x**2/(2*
d**3) - 3*e**3*x**(5/3)/(5*d**4) + 3*e**4*x**(4/3)/(4*d**5) - e**5*x/d**6 +
 3*e**6*x**(2/3)/(2*d**7) - 3*e**7*x**(1/3)/d**8 + 3*e**9*Piecewise((1/(d*x
**(1/3)), Eq(e, 0)), (log(d + e/x**(1/3))/e, True))/d**9 - 3*e**8*log(x**(-
1/3))/d**9)/9 + x**3*log(c*(d + e/x**(1/3))**n)/3)
```

Giac [A]
time = 4.73, size = 131, normalized size = 0.69

$$\frac{1}{3}\,bx^{3}\log\left(c\right) + \frac{1}{3}\,ax^{3} + \frac{1}{2520}\,\left(840\,x^{3}\log\left(d + \frac{e}{x^{\frac{1}{3}}}\right) + \left(\frac{105\,d^{7}x^{\frac{8}{3}} - 120\,d^{6}x^{\frac{7}{3}}e + 140\,d^{5}x^{2}e^{2} - 168\,d^{4}x^{\frac{5}{3}}e^{3} + 210\,d^{3}x^{\frac{4}{3}}e^{4} - 280\,d^{2}xe^{5} + 420\,dx^{\frac{2}{3}}e^{6} - 840\,x^{\frac{1}{3}}e^{7}}{d^{8}} + \frac{840\,e^{8}\log\left(\left|dx^{\frac{1}{3}} + e\right|\right)}{d^{9}}\right)e\right)bn$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="giac")
```

```
[Out] 1/3*b*x^3*log(c) + 1/3*a*x^3 + 1/2520*(840*x^3*log(d + e/x^(1/3)) + ((105*d
^7*x^(8/3) - 120*d^6*x^(7/3)*e + 140*d^5*x^2*e^2 - 168*d^4*x^(5/3)*e^3 + 21
0*d^3*x^(4/3)*e^4 - 280*d^2*x*e^5 + 420*d*x^(2/3)*e^6 - 840*x^(1/3)*e^7)/d^
8 + 840*e^8*log(abs(d*x^(1/3) + e))/d^9)*e)*b*n
```

Mupad [B]
time = 0.68, size = 153, normalized size = 0.81

$$\frac{840\,a\,d^{9}\,x^{3} + 1680\,b\,e^{9}\,n\,\mathrm{atanh}\left(\frac{2\,e}{d\,x^{1/3}} + 1\right) + 840\,b\,d^{9}\,x^{3}\,\ln\left(c\,{\left(d + \frac{e}{x^{1/3}}\right)}^{n}\right) - 280\,b\,d^{3}\,e^{6}\,n\,x - 840\,b\,d\,e^{8}\,n\,x^{1/3} + 105\,b\,d^{8}\,e\,n\,x^{8/3} + 140\,b\,d^{6}\,e^{3}\,n\,x^{2} + 420\,b\,d^{2}\,e^{7}\,n\,x^{2/3} + 210\,b\,d^{4}\,e^{5}\,n\,x^{4/3} - 168\,b\,d^{5}\,e^{4}\,n\,x^{5/3} - 120\,b\,d^{7}\,e^{2}\,n\,x^{7/3}}{2520\,d^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e/x^(1/3))^n)),x)
```

```
[Out] (840*a*d^9*x^3 + 1680*b*e^9*n*atanh((2*e)/(d*x^(1/3)) + 1) + 840*b*d^9*x^3*
log(c*(d + e/x^(1/3))^n) - 280*b*d^3*e^6*n*x - 840*b*d*e^8*n*x^(1/3) + 105*
b*d^8*e*n*x^(8/3) + 140*b*d^6*e^3*n*x^2 + 420*b*d^2*e^7*n*x^(2/3) + 210*b*d
^4*e^5*n*x^(4/3) - 168*b*d^5*e^4*n*x^(5/3) - 120*b*d^7*e^2*n*x^(7/3))/(2520
*d^9)
```

3.491 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx$

Optimal. Leaf size=141

$$\frac{be^5n\sqrt[3]{x}}{2d^5}-\frac{be^4nx^{2/3}}{4d^4}+\frac{be^3nx}{6d^3}-\frac{be^2nx^{4/3}}{8d^2}+\frac{benx^{5/3}}{10d}-\frac{be^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{2d^6}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-$$

```
[Out] 1/2*b*e^5*n*x^(1/3)/d^5-1/4*b*e^4*n*x^(2/3)/d^4+1/6*b*e^3*n*x/d^3-1/8*b*e^2
*n*x^(4/3)/d^2+1/10*b*e*n*x^(5/3)/d-1/2*b*e^6*n*ln(d+e/x^(1/3))/d^6+1/2*x^2
*(a+b*ln(c*(d+e/x^(1/3))^n))-1/6*b*e^6*n*ln(x)/d^6
```

Rubi [A]
time = 0.06, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2504, 2442, 46}

$$\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{be^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{2d^6}-\frac{be^6n\log(x)}{6d^6}+\frac{be^5n\sqrt[3]{x}}{2d^5}-\frac{be^4nx^{2/3}}{4d^4}+\frac{be^3nx}{6d^3}-\frac{be^2nx^{4/3}}{8d^2}+\frac{benx^{5/3}}{10d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e/x^(1/3))^n]),x]
```

```
[Out] (b*e^5*n*x^(1/3))/(2*d^5) - (b*e^4*n*x^(2/3))/(4*d^4) + (b*e^3*n*x)/(6*d^3)
 - (b*e^2*n*x^(4/3))/(8*d^2) + (b*e*n*x^(5/3))/(10*d) - (b*e^6*n*Log[d + e/
x^(1/3)])/(2*d^6) + (x^2*(a + b*Log[c*(d + e/x^(1/3))^n]))/2 - (b*e^6*n*Log
[x])/(6*d^6)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx=-\left(3\text{Subst}\left(\int\frac{a+b\log\left(c(d+ex)^n\right)}{x^7}\,dx,x,\frac{1}{\sqrt[3]{x}}\right)\right)$$
$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{1}{2}(ben)\text{Subst}\left(\int\frac{1}{x^6(d+ex)}\right.$$
$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{1}{2}(ben)\text{Subst}\left(\int\left(\frac{1}{dx^6}-\frac{e}{d^2}\right.\right.$$
$$=\frac{be^5n\sqrt[3]{x}}{2d^5}-\frac{be^4nx^{2/3}}{4d^4}+\frac{be^3nx}{6d^3}-\frac{be^2nx^{4/3}}{8d^2}+\frac{benx^{5/3}}{10d}-\frac{be^6n\log\left(\right.}{2}$$

Mathematica [A]
time = 0.07, size = 132, normalized size = 0.94

$$\frac{ax^2}{2}+\frac{1}{2}bx^2\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)-\frac{1}{2}ben\left(-\frac{e^4\sqrt[3]{x}}{d^5}+\frac{e^3x^{2/3}}{2d^4}-\frac{e^2x}{3d^3}+\frac{ex^{4/3}}{4d^2}-\frac{x^{5/3}}{5d}+\frac{e^5\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^6}+\frac{e^5\log(x)}{3d^6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(1/3))^n]),x]
[Out] (a*x^2)/2 + (b*x^2*Log[c*(d + e/x^(1/3))^n])/2 - (b*e*n*(-((e^4*x^(1/3))/d^
5) + (e^3*x^(2/3))/(2*d^4) - (e^2*x)/(3*d^3) + (e*x^(4/3))/(4*d^2) - x^(5/3
)/(5*d) + (e^5*Log[d + e/x^(1/3)])/d^6 + (e^5*Log[x])/(3*d^6)))/2
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(1/3))^n)),x)
[Out] int(x*(a+b*ln(c*(d+e/x^(1/3))^n)),x)
```

Maxima [A]
time = 0.29, size = 95, normalized size = 0.67

$$\frac{1}{120}\,bn\left(\frac{12\,d^4x^{\frac{5}{3}}-15\,d^3x^{\frac{4}{3}}e+20\,d^2xe^2-30\,dx^{\frac{2}{3}}e^3+60\,x^{\frac{1}{3}}e^4}{d^5}-\frac{60\,e^5\log\left(dx^{\frac{1}{3}}+e\right)}{d^6}\right)e+\frac{1}{2}\,bx^2\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="maxima")
```

```
[Out] 1/120*b*n*((12*d^4*x^(5/3) - 15*d^3*x^(4/3)*e + 20*d^2*x*e^2 - 30*d*x^(2/3)
*e^3 + 60*x^(1/3)*e^4)/d^5 - 60*e^5*log(d*x^(1/3) + e)/d^6)*e + 1/2*b*x^2*l
og(c*(d + e/x^(1/3))^n) + 1/2*a*x^2
```

Fricas [A]
time = 0.38, size = 152, normalized size = 1.08

$$\frac{60\,bd^6x^2\log\left(c\right)+60\,ad^6x^2-60\,bd^6n\log\left(x^{\frac{1}{3}}\right)+20\,bd^3nxe^3+60\,(bd^6n-bne^6)\log\left(dx^{\frac{1}{3}}+e\right)+60\,(bd^6nx^2-bd^6n)\log\left(\frac{dx+x^{\frac{2}{3}}e}{x}\right)+6\,(2\,bd^5nxe-5\,bd^2ne^4)x^{\frac{2}{3}}-15\,(bd^4nxe^2-4\,bdne^5)x^{\frac{1}{3}}}{120\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="fricas")
```

```
[Out] 1/120*(60*b*d^6*x^2*log(c) + 60*a*d^6*x^2 - 60*b*d^6*n*log(x^(1/3)) + 20*b*
d^3*n*x*e^3 + 60*(b*d^6*n - b*n*e^6)*log(d*x^(1/3) + e) + 60*(b*d^6*n*x^2 -
 b*d^6*n)*log((d*x + x^(2/3)*e)/x) + 6*(2*b*d^5*n*x*e - 5*b*d^2*n*e^4)*x^(2
/3) - 15*(b*d^4*n*x*e^2 - 4*b*d*n*e^5)*x^(1/3))/d^6
```

Sympy [A]
time = 13.52, size = 138, normalized size = 0.98

$$\frac{ax^2}{2}+b\left(\frac{en\left(\frac{3x^{\frac{5}{3}}}{5d}-\frac{3ex^{\frac{4}{3}}}{4d^2}+\frac{e^2x}{d^3}-\frac{3e^3x^{\frac{2}{3}}}{2d^4}+\frac{3e^4\sqrt[3]{x}}{d^5}-\frac{3e^6\left(\begin{cases}\frac{1}{d\sqrt[3]{x}} & \text{for } e=0\\ \frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e} & \text{otherwise}\end{cases}\right)}{d^6}+\frac{3e^5\log\left(\frac{1}{\sqrt[3]{x}}\right)}{d^6}\right)}{6}+\frac{x^2\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/3))**n)),x)
```

```
[Out] a*x**2/2 + b*(e*n*(3*x**(5/3)/(5*d) - 3*e*x**(4/3)/(4*d**2) + e**2*x/d**3 -
 3*e**3*x**(2/3)/(2*d**4) + 3*e**4*x**(1/3)/d**5 - 3*e**6*Piecewise((1/(d*x
**(1/3)), Eq(e, 0)), (log(d + e/x**(1/3))/e, True))/d**6 + 3*e**5*log(x**(-
1/3))/d**6)/6 + x**2*log(c*(d + e/x**(1/3))**n)/2)
```

Giac [A]
time = 4.46, size = 101, normalized size = 0.72

$$\frac{1}{2}\,bx^2\log\left(c\right)+\frac{1}{120}\left(60\,x^2\log\left(d+\frac{e}{x^{\frac{1}{3}}}\right)+\left(\frac{12\,d^4x^{\frac{5}{3}}-15\,d^3x^{\frac{4}{3}}e+20\,d^2xe^2-30\,dx^{\frac{2}{3}}e^3+60\,x^{\frac{1}{3}}e^4}{d^5}-\frac{60\,e^5\log\left(\left|dx^{\frac{1}{3}}+e\right|\right)}{d^6}\right)e\right)bn+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2*log(c) + 1/120*(60*x^2*log(d + e/x^(1/3)) + ((12*d^4*x^(5/3) - 15
*d^3*x^(4/3)*e + 20*d^2*x*e^2 - 30*d*x^(2/3)*e^3 + 60*x^(1/3)*e^4)/d^5 - 60
*e^5*log(abs(d*x^(1/3) + e))/d^6)*e)*b*n + 1/2*a*x^2
```

Mupad [B]
time = 0.79, size = 112, normalized size = 0.79

$$\frac{x^{5/3}\left(\frac{b\,e\,n}{5\,d}-\frac{b\,e^2\,n}{4\,d^2\,x^{1/3}}-\frac{b\,e^4\,n}{2\,d^4\,x}+\frac{b\,e^3\,n}{3\,d^3\,x^{2/3}}+\frac{b\,e^5\,n}{d^5\,x^{4/3}}\right)}{2}+\frac{a\,x^2}{2}+\frac{b\,x^2\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)}{2}-\frac{b\,e^6\,n\,\mathrm{atanh}\left(\frac{2\,e}{d\,x^{1/3}}+1\right)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e/x^(1/3))^n)),x)
```

```
[Out] (x^(5/3)*((b*e*n)/(5*d) - (b*e^2*n)/(4*d^2*x^(1/3)) - (b*e^4*n)/(2*d^4*x) +
 (b*e^3*n)/(3*d^3*x^(2/3)) + (b*e^5*n)/(d^5*x^(4/3))))/2 + (a*x^2)/2 + (b*x
^2*log(c*(d + e/x^(1/3))^n))/2 - (b*e^6*n*atanh((2*e)/(d*x^(1/3)) + 1))/d^6
```

3.492 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx$

Optimal. Leaf size=70

$$-\frac{be^2n\sqrt[3]{x}}{d^2}+\frac{benx^{2/3}}{2d}+ax+bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)+\frac{be^3n\log\left(e+d\sqrt[3]{x}\right)}{d^3}$$

```
[Out] -b*e^2*n*x^(1/3)/d^2+1/2*b*e*n*x^(2/3)/d+a*x+b*x*ln(c*(d+e/x^(1/3))^n)+b*e^
3*n*ln(e+d*x^(1/3))/d^3
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2498, 269, 196, 45}

$$ax+bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)+\frac{be^3n\log\left(d\sqrt[3]{x}+e\right)}{d^3}-\frac{be^2n\sqrt[3]{x}}{d^2}+\frac{benx^{2/3}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Log[c*(d + e/x^(1/3))^n],x]
```

```
[Out] -((b*e^2*n*x^(1/3))/d^2) + (b*e*n*x^(2/3))/(2*d) + a*x + b*x*Log[c*(d + e/x
^(1/3))^n] + (b*e^3*n*Log[e + d*x^(1/3)])/d^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
```

```
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)dx &= ax+b\int \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)dx\\ &= ax+bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)+\frac{1}{3}(ben)\int \frac{1}{\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}}\,dx\\ &= ax+bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)+\frac{1}{3}(ben)\int \frac{1}{e+d\sqrt[3]{x}}\,dx\\ &= ax+bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)+(ben)\text{Subst}\left(\int \frac{x^2}{e+dx}\,dx,x,\sqrt[3]{x}\right)\\ &= ax+bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)+(ben)\text{Subst}\left(\int \left(-\frac{e}{d^2}+\frac{x}{d}+\frac{e}{d^2(e\ldots}\right.\right.\\ &= -\frac{be^2n\sqrt[3]{x}}{d^2}+\frac{benx^{2/3}}{2d}+ax+bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)+\frac{be^3n\log(\ldots}{\ldots}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 79, normalized size = 1.13

$$ax+bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)-ben\left(\frac{e\sqrt[3]{x}}{d^2}-\frac{x^{2/3}}{2d}-\frac{e^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^3}-\frac{e^2\log(x)}{3d^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Log[c*(d + e/x^(1/3))^n],x]
[Out] a*x + b*x*Log[c*(d + e/x^(1/3))^n] - b*e*n*((e*x^(1/3))/d^2 - x^(2/3)/(2*d)
 - (e^2*Log[d + e/x^(1/3)])/d^3 - (e^2*Log[x])/(3*d^3))
```

Maple [A]
time = 0.05, size = 115, normalized size = 1.64

method	result
default	$ax+xb\ln\left(c\left(\frac{e+d\,x^{\frac{1}{3}}}{x^{\frac{1}{3}}}\right)^n\right)+\frac{b\,e^3 n\ln(d^3x+e^3)}{3d^3}+\frac{ben\,x^{\frac{2}{3}}}{2d}-\frac{b\,e^3 n\ln\left(d^2x^{\frac{2}{3}}-ed\,x^{\frac{1}{3}}+e^2\right)}{3d^3}+\frac{2b\,e^3 n\ln\left(e+d\,x^{\frac{1}{3}}\right)}{3d^3}-\frac{b\,e^2\ldots}{d\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*ln(c*(d+e/x^(1/3))^n),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+x*b*ln(c*((e+d*x^(1/3))/x^(1/3))^n)+1/3*b*e^3*n*ln(d^3*x+e^3)/d^3+1/2*b
*e*n*x^(2/3)/d-1/3*b*e^3*n/d^3*ln(d^2*x^(2/3)-e*d*x^(1/3)+e^2)+2/3*b*e^3*n*
ln(e+d*x^(1/3))/d^3-b*e^2*n*x^(1/3)/d^2
```

Maxima [A]
time = 0.27, size = 62, normalized size = 0.89

$$\frac{1}{2}\left(n\left(\frac{dx^{\frac{2}{3}}-2\,x^{\frac{1}{3}}e}{d^2}+\frac{2\,e^2\log\left(dx^{\frac{1}{3}}+e\right)}{d^3}\right)e+2\,x\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/x^(1/3))^n),x, algorithm="maxima")
```

```
[Out] 1/2*(n*((d*x^(2/3) - 2*x^(1/3)*e)/d^2 + 2*e^2*log(d*x^(1/3) + e)/d^3)*e + 2
*x*log(c*(d + e/x^(1/3))^n))*b + a*x
```

Fricas [A]
time = 0.39, size = 109, normalized size = 1.56

$$\frac{2\,bd^3x\log\left(c\right)-2\,bd^3n\log\left(x^{\frac{1}{3}}\right)+bd^2nx^{\frac{2}{3}}e+2\,ad^3x-2\,bdnx^{\frac{1}{3}}e^2+2\left(bd^3n+bne^3\right)\log\left(dx^{\frac{1}{3}}+e\right)+2\left(bd^3nx-bd^3n\right)\log\left(\frac{dx+x^{\frac{2}{3}}e}{x}\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/x^(1/3))^n),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d^3*x*log(c) - 2*b*d^3*n*log(x^(1/3)) + b*d^2*n*x^(2/3)*e + 2*a*d^
3*x - 2*b*d*n*x^(1/3)*e^2 + 2*(b*d^3*n + b*n*e^3)*log(d*x^(1/3) + e) + 2*(b
*d^3*n*x - b*d^3*n)*log((d*x + x^(2/3)*e)/x))/d^3
```

Sympy [A]
time = 3.67, size = 92, normalized size = 1.31

$$ax+b\left(\frac{en\left(\frac{3x^{\frac{2}{3}}}{2d}-\frac{3e\sqrt[3]{x}}{d^2}+\frac{3e^3\left(\begin{cases}\frac{1}{d\sqrt[3]{x}} & \text{for}\ e=0\\ \frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e} & \text{otherwise}\end{cases}\right)}{d^3}-\frac{3e^2\log\left(\frac{1}{\sqrt[3]{x}}\right)}{d^3}\right)}{3}+x\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*ln(c*(d+e/x**(1/3))**n),x)
```

```
[Out] a*x + b*(e*n*(3*x**(2/3)/(2*d) - 3*e*x**(1/3)/d**2 + 3*e**3*Piecewise((1/(d
*x**(1/3)), Eq(e, 0)), (log(d + e/x**(1/3))/e, True))/d**3 - 3*e**2*log(x**
(-1/3))/d**3)/3 + x*log(c*(d + e/x**(1/3))**n))
```

Giac [A]
time = 4.64, size = 66, normalized size = 0.94

$$\frac{1}{2}\left(\left(\left(\left(\frac{dx^{\frac{2}{3}}-2\,x^{\frac{1}{3}}e}{d^2}+\frac{2\,e^2\log\left(\left|dx^{\frac{1}{3}}+e\right|\right)}{d^3}\right)e+2\,x\log\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)n+2\,x\log\left(c\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/x^(1/3))^n),x, algorithm="giac")
```

```
[Out] 1/2*((((d*x^(2/3) - 2*x^(1/3)*e)/d^2 + 2*e^2*log(abs(d*x^(1/3) + e))/d^3)*e
 + 2*x*log(d + e/x^(1/3)))*n + 2*x*log(c))*b + a*x
```

Mupad [B]
time = 0.47, size = 59, normalized size = 0.84

$$a\,x+b\,x\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^n\right)+\frac{b\,\left(2\,e^3\,n\,\ln\left(e+d\,x^{1/3}\right)-2\,d\,e^2\,n\,x^{1/3}+d^2\,e\,n\,x^{2/3}\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*log(c*(d + e/x^(1/3))^n),x)
```

```
[Out] a*x + b*x*log(c*(d + e/x^(1/3))^n) + (b*(2*e^3*n*log(e + d*x^(1/3)) - 2*d*e
^2*n*x^(1/3) + d^2*e*n*x^(2/3)))/(2*d^3)
```

3.493 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}\,dx$

Optimal. Leaf size=51

$$-3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt[3]{x}}\right)-3bn\text{Li}_2\left(1+\frac{e}{d\sqrt[3]{x}}\right)$$

```
[Out] -3*(a+b*ln(c*(d+e/x^(1/3))^n))*ln(-e/d/x^(1/3))-3*b*n*polylog(2,1+e/d/x^(1/
3))
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2504, 2441, 2352}

$$-3bn\text{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)-3\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])/x,x]
```

```
[Out] -3*(a + b*Log[c*(d + e/x^(1/3))^n])*Log[-(e/(d*x^(1/3)))] - 3*b*n*PolyLog[2
, 1 + e/(d*x^(1/3))]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}\,dx = -\left(3\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt[3]{x}}\right) + (3ben)\text{Subst}\left(\int \frac{\text{lo}}{}\right.$$

$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt[3]{x}}\right) - 3bn\text{Li}_2\left(1+\frac{e}{d\sqrt[3]{x}}\right.$$

Mathematica [A]
time = 0.00, size = 53, normalized size = 1.04

$$-3b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\log\left(-\frac{e}{d\sqrt[3]{x}}\right)+a\log(x)-3bn\text{Li}_2\left(\frac{d+\frac{e}{\sqrt[3]{x}}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])/x,x]
```

```
[Out] -3*b*Log[c*(d + e/x^(1/3))^n]*Log[-(e/(d*x^(1/3)))] + a*Log[x] - 3*b*n*Poly
Log[2, (d + e/x^(1/3))/d]
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^n))/x,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))/x,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(47) = 94.
time = 0.73, size = 183, normalized size = 3.59

$$-3\left(\log\left(de^{\left(\frac{1}{3}\log(x)-1\right)}+1\right)\log\left(x^{\frac{1}{3}}\right)+\text{Li}_2\left(-de^{\left(\frac{1}{3}\log(x)-1\right)}\right)\right)bn+\frac{1}{12}\left(2\,bne^2\log(x)^2+9\,bd^2nx^{\frac{2}{3}}-36\,bdnx^{\frac{1}{3}}e+12\,be^2\log\left(\left(dx^{\frac{1}{3}}+e\right)^n\right)\log(x)-12\,be^2\log(x)\log\left(x^{\frac{1}{3}n}\right)+12\,(b\log(c)+a)e^2\log(x)-6\left(bd^2nx^{\frac{2}{3}}-2\,bdnx^{\frac{1}{3}}e\right)\log(x)+\frac{3\,(2\,bd^2nx\log(x)-3\,bd^2nx)}{x^{\frac{1}{3}}}-\frac{12\,(bdnxe\log(x)-3\,bdnxe)}{x^{\frac{2}{3}}}\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x,x, algorithm="maxima")
```

```
[Out] -3*(log(d*e^(1/3*log(x) - 1) + 1)*log(x^(1/3)) + dilog(-d*e^(1/3*log(x) - 1
)))*b*n + 1/12*(2*b*n*e^2*log(x)^2 + 9*b*d^2*n*x^(2/3) - 36*b*d*n*x^(1/3)*e
 + 12*b*e^2*log((d*x^(1/3) + e)^n)*log(x) - 12*b*e^2*log(x)*log(x^(1/3*n))
+ 12*(b*log(c) + a)*e^2*log(x) - 6*(b*d^2*n*x^(2/3) - 2*b*d*n*x^(1/3)*e)*lo
g(x) + 3*(2*b*d^2*n*x*log(x) - 3*b*d^2*n*x)/x^(1/3) - 12*(b*d*n*x*e*log(x)
- 3*b*d*n*x*e)/x^(2/3))*e^(-2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*((d*x + x^(2/3)*e)/x)^n) + a)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\log\left(c\left(d + \frac{e}{\sqrt[3]{x}}\right)^{n}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e/x**(1/3))**n))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + b\,\ln\left(c\,{\left(d + \frac{e}{x^{1/3}}\right)}^{n}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))/x,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(1/3))^n))/x, x)
```

3.494 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^2}\,dx$

Optimal. Leaf size=82

$$\frac{bn}{3x}-\frac{bdn}{2ex^{2/3}}+\frac{bd^2n}{e^2\sqrt[3]{x}}-\frac{bd^3n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}$$

```
[Out] 1/3*b*n/x-1/2*b*d*n/e/x^(2/3)+b*d^2*n/e^2/x^(1/3)-b*d^3*n*ln(d+e/x^(1/3))/e
^3+(-a-b*ln(c*(d+e/x^(1/3))^n))/x
```

Rubi [A]
time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2504, 2442, 45}

$$-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}-\frac{bd^3n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}+\frac{bd^2n}{e^2\sqrt[3]{x}}-\frac{bdn}{2ex^{2/3}}+\frac{bn}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])/x^2,x]
```

```
[Out] (b*n)/(3*x) - (b*d*n)/(2*e*x^(2/3)) + (b*d^2*n)/(e^2*x^(1/3)) - (b*d^3*n*Lo
g[d + e/x^(1/3)])/e^3 - (a + b*Log[c*(d + e/x^(1/3))^n])/x
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^2}\,dx &= -\left(3\text{Subst}\left(\int x^2(a+b\log\left(c(d+ex)^n\right))\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)\\
&= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x} + (ben)\text{Subst}\left(\int \frac{x^3}{d+ex}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\\
&= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x} + (ben)\text{Subst}\left(\int \left(\frac{d^2}{e^3} - \frac{dx}{e^2} + \frac{x^2}{e} - \frac{d}{e^3(d+
\end{aligned}$$

$$= \frac{bn}{3x} - \frac{bdn}{2ex^{2/3}} + \frac{bd^2n}{e^2\sqrt[3]{x}} - \frac{bd^3n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3} - \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right.\right.}{x}$$

Mathematica [**A**]
time = 0.03, size = 85, normalized size = 1.04

$$-\frac{a}{x}+\frac{bn}{3x}-\frac{bdn}{2ex^{2/3}}+\frac{bd^2n}{e^2\sqrt[3]{x}}-\frac{bd^3n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}-\frac{b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])/x^2,x]
```

```
[Out] -(a/x) + (b*n)/(3*x) - (b*d*n)/(2*e*x^(2/3)) + (b*d^2*n)/(e^2*x^(1/3)) - (b
*d^3*n*Log[d + e/x^(1/3)])/e^3 - (b*Log[c*(d + e/x^(1/3))^n])/x
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^n))/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))/x^2,x)
```

Maxima [A]
time = 0.28, size = 86, normalized size = 1.05

$$-\frac{1}{6}\left(6\,d^3e^{(-4)}\log\left(dx^{\frac{1}{3}}+e\right)-2\,d^3e^{(-4)}\log\left(x\right)-\frac{\left(6\,d^2x^{\frac{2}{3}}-3\,dx^{\frac{1}{3}}e+2\,e^2\right)e^{(-3)}}{x}\right)bne-\frac{b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)}{x}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^2,x, algorithm="maxima")
```

```
[Out] -1/6*(6*d^3*e^(-4)*log(d*x^(1/3) + e) - 2*d^3*e^(-4)*log(x) - (6*d^2*x^(2/3
) - 3*d*x^(1/3)*e + 2*e^2)*e^(-3)/x)*b*n*e - b*log(c*(d + e/x^(1/3))^n)/x -
 a/x
```

Fricas [A]
time = 0.35, size = 94, normalized size = 1.15

$$\frac{\left(6\,bd^2nx^{\frac{2}{3}}e-3\,bdnx^{\frac{1}{3}}e^2+6\,(bx-b)e^3\log\left(c\right)+2\,(bn-(bn-3\,a)x-3\,a)e^3-6\,(bd^3nx+bne^3)\log\left(\frac{dx+x^{\frac{2}{3}}e}{x}\right)\right)e^{(-3)}}{6\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(6*b*d^2*n*x^(2/3)*e - 3*b*d*n*x^(1/3)*e^2 + 6*(b*x - b)*e^3*log(c) + 2
*(b*n - (b*n - 3*a)*x - 3*a)*e^3 - 6*(b*d^3*n*x + b*n*e^3)*log((d*x + x^(2/
3)*e)/x))*e^(-3)/x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(75) = 150$.
time = 237.55, size = 398, normalized size = 4.85

$$\begin{cases} -\frac{6ade^3x^{\frac{7}{3}}}{6de^3x^{\frac{10}{3}}+6e^4x^3}-\frac{6ae^4x^2}{6de^3x^{\frac{10}{3}}+6e^4x^3}-\frac{6bd^4x^{\frac{10}{3}}\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{6de^3x^{\frac{10}{3}}+6e^4x^3}+\frac{6bd^3enx^3}{6de^3x^{\frac{10}{3}}+6e^4x^3}-\frac{6bd^3ex^3\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{6de^3x^{\frac{10}{3}}+6e^4x^3}+\frac{3bd^2e^2nx^{\frac{8}{3}}}{6de^3x^{\frac{10}{3}}+6e^4x^3}-\frac{bde^3nx^{\frac{7}{3}}}{6de^3x^{\frac{10}{3}}+6e^4x^3}-\frac{6bde^3x^{\frac{7}{3}}\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{6de^3x^{\frac{10}{3}}+6e^4x^3}+\frac{2be^4nx^2}{6de^3x^{\frac{10}{3}}+6e^4x^3}-\frac{6be^4x^2\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{6de^3x^{\frac{10}{3}}+6e^4x^3} & \text{for}\: e\neq 0 \\ -\frac{a+b\log\left(cd^n\right)}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))/x**2,x)
```

```
[Out] Piecewise((-6*a*d*e**3*x**(7/3)/(6*d*e**3*x**(10/3) + 6*e**4*x**3) - 6*a*e*
*4*x**2/(6*d*e**3*x**(10/3) + 6*e**4*x**3) - 6*b*d**4*x**(10/3)*log(c*(d +
e/x**(1/3))**n)/(6*d*e**3*x**(10/3) + 6*e**4*x**3) + 6*b*d**3*e*n*x**3/(6*d
*e**3*x**(10/3) + 6*e**4*x**3) - 6*b*d**3*e*x**3*log(c*(d + e/x**(1/3))**n)
/(6*d*e**3*x**(10/3) + 6*e**4*x**3) + 3*b*d**2*e**2*n*x**(8/3)/(6*d*e**3*x*
*(10/3) + 6*e**4*x**3) - b*d*e**3*n*x**(7/3)/(6*d*e**3*x**(10/3) + 6*e**4*x
**3) - 6*b*d*e**3*x**(7/3)*log(c*(d + e/x**(1/3))**n)/(6*d*e**3*x**(10/3) +
 6*e**4*x**3) + 2*b*e**4*n*x**2/(6*d*e**3*x**(10/3) + 6*e**4*x**3) - 6*b*e*
```

```
*4*x**2*log(c*(d + e/x**(1/3))**n)/(6*d*e**3*x**(10/3) + 6*e**4*x**3), Ne(e
, 0)), (-(a + b*log(c*d**n))/x, True))
```

Giac [A]
time = 3.99, size = 95, normalized size = 1.16

$$-\frac{1}{6}\left(\left(\left(6\,d^3e^{(-4)}\log\left(\left|dx^{\frac{1}{3}}+e\right|\right)-2\,d^3e^{(-4)}\log\left(|x|\right)-\frac{\left(6\,d^2x^{\frac{2}{3}}e-3\,dx^{\frac{1}{3}}e^2+2\,e^3\right)e^{(-4)}}{x}\right)e+\frac{6\,\log\left(d+\frac{e}{x^{\frac{1}{3}}}\right)}{x}\right)bn-\frac{b\log\left(c\right)}{x}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^2,x, algorithm="giac")
```

```
[Out] -1/6*((6*d^3*e^(-4)*log(abs(d*x^(1/3) + e)) - 2*d^3*e^(-4)*log(abs(x)) - (6
*d^2*x^(2/3)*e - 3*d*x^(1/3)*e^2 + 2*e^3)*e^(-4)/x)*e + 6*log(d + e/x^(1/3)
)/x)*b*n - b*log(c)/x - a/x
```

Mupad [B]
time = 0.43, size = 73, normalized size = 0.89

$$\frac{b\,n}{3\,x}-\frac{a}{x}-\frac{b\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^n\right)}{x}-\frac{b\,d\,n}{2\,e\,x^{2/3}}-\frac{b\,d^3\,n\,\ln\left(d+\frac{e}{x^{1/3}}\right)}{e^3}+\frac{b\,d^2\,n}{e^2\,x^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))/x^2,x)
```

```
[Out] (b*n)/(3*x) - a/x - (b*log(c*(d + e/x^(1/3))^n))/x - (b*d*n)/(2*e*x^(2/3))
- (b*d^3*n*log(d + e/x^(1/3)))/e^3 + (b*d^2*n)/(e^2*x^(1/3))
```

3.495 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^3}\,dx$

Optimal. Leaf size=138

$$\frac{bn}{12x^2}-\frac{bdn}{10ex^{5/3}}+\frac{bd^2n}{8e^2x^{4/3}}-\frac{bd^3n}{6e^3x}+\frac{bd^4n}{4e^4x^{2/3}}-\frac{bd^5n}{2e^5\sqrt[3]{x}}+\frac{bd^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{2e^6}-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{2x^2}$$

```
[Out] 1/12*b*n/x^2-1/10*b*d*n/e/x^(5/3)+1/8*b*d^2*n/e^2/x^(4/3)-1/6*b*d^3*n/e^3/x
+1/4*b*d^4*n/e^4/x^(2/3)-1/2*b*d^5*n/e^5/x^(1/3)+1/2*b*d^6*n*ln(d+e/x^(1/3)
)/e^6+1/2*(-a-b*ln(c*(d+e/x^(1/3))^n))/x^2
```

Rubi [A]
time = 0.07, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2504, 2442, 45}

$$-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{2x^2}+\frac{bd^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{2e^6}-\frac{bd^5n}{2e^5\sqrt[3]{x}}+\frac{bd^4n}{4e^4x^{2/3}}-\frac{bd^3n}{6e^3x}+\frac{bd^2n}{8e^2x^{4/3}}-\frac{bdn}{10ex^{5/3}}+\frac{bn}{12x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])/x^3,x]
```

```
[Out] (b*n)/(12*x^2) - (b*d*n)/(10*e*x^(5/3)) + (b*d^2*n)/(8*e^2*x^(4/3)) - (b*d^
3*n)/(6*e^3*x) + (b*d^4*n)/(4*e^4*x^(2/3)) - (b*d^5*n)/(2*e^5*x^(1/3)) + (b
*d^6*n*Log[d + e/x^(1/3)])/(2*e^6) - (a + b*Log[c*(d + e/x^(1/3))^n])/(2*x^
2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^3}\,dx = -\left(3\text{Subst}\left(\int x^5(a+b\log\left(c(d+ex)^n\right))\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{2x^2}+\frac{1}{2}(ben)\text{Subst}\left(\int\frac{x^6}{d+ex}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)$$

$$= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{2x^2}+\frac{1}{2}(ben)\text{Subst}\left(\int\left(-\frac{d^5}{e^6}+\frac{d^4x}{e^5}-\frac{d^3x^2}{e^4}+\right.\right.$$

$$= \frac{bn}{12x^2}-\frac{bdn}{10ex^{5/3}}+\frac{bd^2n}{8e^2x^{4/3}}-\frac{bd^3n}{6e^3x}+\frac{bd^4n}{4e^4x^{2/3}}-\frac{bd^5n}{2e^5\sqrt[3]{x}}+\frac{bd^6n\log\left(d+\right.}{2e^6}$$

Mathematica [A]
time = 0.07, size = 135, normalized size = 0.98

$$-\frac{a}{2x^2}+\frac{1}{2}ben\left(\frac{1}{6ex^2}-\frac{d}{5e^2x^{5/3}}+\frac{d^2}{4e^3x^{4/3}}-\frac{d^3}{3e^4x}+\frac{d^4}{2e^5x^{2/3}}-\frac{d^5}{e^6\sqrt[3]{x}}+\frac{d^6\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^7}\right)-\frac{b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])/x^3,x]
[Out] -1/2*a/x^2 + (b*e*n*(1/(6*e*x^2) - d/(5*e^2*x^(5/3)) + d^2/(4*e^3*x^(4/3))
- d^3/(3*e^4*x) + d^4/(2*e^5*x^(2/3)) - d^5/(e^6*x^(1/3)) + (d^6*Log[d + e/
x^(1/3)])/e^7))/2 - (b*Log[c*(d + e/x^(1/3))^n])/(2*x^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^n))/x^3,x)
[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))/x^3,x)
```

Maxima [A]
time = 0.32, size = 114, normalized size = 0.83

$$\frac{1}{120}\left(60\,d^6e^{(-7)}\log\left(dx^{\frac{1}{3}}+e\right)-20\,d^6e^{(-7)}\log\left(x\right)-\frac{\left(60\,d^5x^{\frac{5}{3}}-30\,d^4x^{\frac{4}{3}}e+20\,d^3xe^2-15\,d^2x^{\frac{2}{3}}e^3+12\,dx^{\frac{1}{3}}e^4-10\,e^5\right)e^{(-6)}}{x^2}\right)bne-\frac{b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)}{2\,x^2}-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^3,x, algorithm="maxima")
[Out] 1/120*(60*d^6*e^(-7)*log(d*x^(1/3) + e) - 20*d^6*e^(-7)*log(x) - (60*d^5*x^
(5/3) - 30*d^4*x^(4/3)*e + 20*d^3*x*e^2 - 15*d^2*x^(2/3)*e^3 + 12*d*x^(1/3)
*e^4 - 10*e^5)*e^(-6)/x^2)*b*n*e - 1/2*b*log(c*(d + e/x^(1/3))^n)/x^2 - 1/2
*a/x^2
```

Fricas [A]
time = 0.39, size = 149, normalized size = 1.08

$$\frac{\left(60\,(bx^2-b)e^6\log\left(c\right)-10\,((bn-6\,a)x^2-bn+6\,a)e^6+20\,(bd^3nx^2-bd^3nx)e^3+60\,(bd^6nx^2-bne^6)\log\left(\frac{dx+x^{\frac{2}{3}}e}{x}\right)-15\,(4\,bd^5nxe-bd^2ne^4)x^{\frac{2}{3}}+6\,(5\,bd^4nxe^2-2\,bdne^5)x^{\frac{1}{3}}\right)e^{(-6)}}{120\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^3,x, algorithm="fricas")
[Out] 1/120*(60*(b*x^2 - b)*e^6*log(c) - 10*((b*n - 6*a)*x^2 - b*n + 6*a)*e^6 + 2
0*(b*d^3*n*x^2 - b*d^3*n*x)*e^3 + 60*(b*d^6*n*x^2 - b*n*e^6)*log((d*x + x^(
2/3)*e)/x) - 15*(4*b*d^5*n*x*e - b*d^2*n*e^4)*x^(2/3) + 6*(5*b*d^4*n*x*e^2
- 2*b*d*n*e^5)*x^(1/3))*e^(-6)/x^2
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))/x**3,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [A]
time = 4.58, size = 123, normalized size = 0.89

$$\frac{1}{120}\left(\left(60\,d^6e^{(-7)}\log\left(\left|dx^{\frac{1}{3}}+e\right|\right)-20\,d^6e^{(-7)}\log\left(|x|\right)-\frac{\left(60\,d^5x^{\frac{5}{3}}e-30\,d^4x^{\frac{4}{3}}e^2+20\,d^3xe^3-15\,d^2x^{\frac{2}{3}}e^4+12\,dx^{\frac{1}{3}}e^5-10\,e^6\right)e^{(-7)}}{x^2}\right)e-\frac{60\,\log\left(d+\frac{e}{x^{\frac{1}{3}}}\right)}{x^2}\right)bn-\frac{b\log\left(c\right)}{2\,x^2}-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^3,x, algorithm="giac")
```

```
[Out] 1/120*((60*d^6*e^(-7)*log(abs(d*x^(1/3) + e)) - 20*d^6*e^(-7)*log(abs(x)) -
 (60*d^5*x^(5/3)*e - 30*d^4*x^(4/3)*e^2 + 20*d^3*x*e^3 - 15*d^2*x^(2/3)*e^4
 + 12*d*x^(1/3)*e^5 - 10*e^6)*e^(-7)/x^2)*e - 60*log(d + e/x^(1/3))/x^2)*b*
n - 1/2*b*log(c)/x^2 - 1/2*a/x^2
```

Mupad [B]
time = 0.46, size = 113, normalized size = 0.82

$$\frac{b\,n}{12\,x^2}-\frac{a}{2\,x^2}-\frac{b\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^n\right)}{2\,x^2}-\frac{b\,d\,n}{10\,e\,x^{5/3}}+\frac{b\,d^6\,n\,\ln\left(d+\frac{e}{x^{1/3}}\right)}{2\,e^6}-\frac{b\,d^3\,n}{6\,e^3\,x}+\frac{b\,d^2\,n}{8\,e^2\,x^{4/3}}+\frac{b\,d^4\,n}{4\,e^4\,x^{2/3}}-\frac{b\,d^5\,n}{2\,e^5\,x^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))/x^3,x)
```

```
[Out] (b*n)/(12*x^2) - a/(2*x^2) - (b*log(c*(d + e/x^(1/3))^n))/(2*x^2) - (b*d*n)
/(10*e*x^(5/3)) + (b*d^6*n*log(d + e/x^(1/3)))/(2*e^6) - (b*d^3*n)/(6*e^3*x
) + (b*d^2*n)/(8*e^2*x^(4/3)) + (b*d^4*n)/(4*e^4*x^(2/3)) - (b*d^5*n)/(2*e^
5*x^(1/3))
```

3.496 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^4}\,dx$

Optimal. Leaf size=187

$$\frac{bn}{27x^3}-\frac{bdn}{24ex^{8/3}}+\frac{bd^2n}{21e^2x^{7/3}}-\frac{bd^3n}{18e^3x^2}+\frac{bd^4n}{15e^4x^{5/3}}-\frac{bd^5n}{12e^5x^{4/3}}+\frac{bd^6n}{9e^6x}-\frac{bd^7n}{6e^7x^{2/3}}+\frac{bd^8n}{3e^8\sqrt[3]{x}}-\frac{bd^9n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{3e^9}-$$

```
[Out] 1/27*b*n/x^3-1/24*b*d*n/e/x^(8/3)+1/21*b*d^2*n/e^2/x^(7/3)-1/18*b*d^3*n/e^3
/x^2+1/15*b*d^4*n/e^4/x^(5/3)-1/12*b*d^5*n/e^5/x^(4/3)+1/9*b*d^6*n/e^6/x-1/
6*b*d^7*n/e^7/x^(2/3)+1/3*b*d^8*n/e^8/x^(1/3)-1/3*b*d^9*n*ln(d+e/x^(1/3))/e
^9+1/3*(-a-b*ln(c*(d+e/x^(1/3))^n))/x^3
```

Rubi [A]

time = 0.09, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2504, 2442, 45}

$$-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{3x^3}-\frac{bd^9n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{3e^9}+\frac{bd^8n}{3e^8\sqrt[3]{x}}-\frac{bd^7n}{6e^7x^{2/3}}+\frac{bd^6n}{9e^6x}-\frac{bd^5n}{12e^5x^{4/3}}+\frac{bd^4n}{15e^4x^{5/3}}-\frac{bd^3n}{18e^3x^2}+\frac{bd^2n}{21e^2x^{7/3}}-\frac{bdn}{24ex^{8/3}}+\frac{bn}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])/x^4,x]
```

```
[Out] (b*n)/(27*x^3) - (b*d*n)/(24*e*x^(8/3)) + (b*d^2*n)/(21*e^2*x^(7/3)) - (b*d
^3*n)/(18*e^3*x^2) + (b*d^4*n)/(15*e^4*x^(5/3)) - (b*d^5*n)/(12*e^5*x^(4/3)
) + (b*d^6*n)/(9*e^6*x) - (b*d^7*n)/(6*e^7*x^(2/3)) + (b*d^8*n)/(3*e^8*x^(1
/3)) - (b*d^9*n*Log[d + e/x^(1/3)])/(3*e^9) - (a + b*Log[c*(d + e/x^(1/3))^
n])/(3*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^4}\,dx = -\left(3\text{Subst}\left(\int x^8(a+b\log(c(d+ex)^n))\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{3x^3} + \frac{1}{3}(ben)\text{Subst}\left(\int \frac{x^9}{d+ex}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)$$

$$= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{3x^3} + \frac{1}{3}(ben)\text{Subst}\left(\int \left(\frac{d^8}{e^9} - \frac{d^7x}{e^8} + \frac{d^6x^2}{e^7} - \right.\right.$$

$$= \frac{bn}{27x^3} - \frac{bdn}{24ex^{8/3}} + \frac{bd^2n}{21e^2x^{7/3}} - \frac{bd^3n}{18e^3x^2} + \frac{bd^4n}{15e^4x^{5/3}} - \frac{bd^5n}{12e^5x^{4/3}} + \frac{bd^6n}{9e^6x} -$$

Mathematica [A]
time = 0.11, size = 178, normalized size = 0.95

$$-\frac{a}{3x^3} + \frac{1}{3}ben\left(\frac{1}{9ex^3} - \frac{d}{8e^2x^{8/3}} + \frac{d^2}{7e^3x^{7/3}} - \frac{d^3}{6e^4x^2} + \frac{d^4}{5e^5x^{5/3}} - \frac{d^5}{4e^6x^{4/3}} + \frac{d^6}{3e^7x} - \frac{d^7}{2e^8x^{2/3}} + \frac{d^8}{e^9\sqrt[3]{x}} - \frac{d^9\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^{10}}\right) - \frac{b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])/x^4,x]
```

```
[Out] -1/3*a/x^3 + (b*e*n*(1/(9*e*x^3) - d/(8*e^2*x^(8/3)) + d^2/(7*e^3*x^(7/3))
- d^3/(6*e^4*x^2) + d^4/(5*e^5*x^(5/3)) - d^5/(4*e^6*x^(4/3)) + d^6/(3*e^7*
x) - d^7/(2*e^8*x^(2/3)) + d^8/(e^9*x^(1/3)) - (d^9*Log[d + e/x^(1/3)])/e^1
0))/3 - (b*Log[c*(d + e/x^(1/3))^n])/(3*x^3)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^n))/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))/x^4,x)
```

Maxima [A]
time = 0.29, size = 144, normalized size = 0.77

$$-\frac{1}{7560}\left(2520\,d^{9}e^{(-10)}\log\left(dx^{\frac{1}{3}}+e\right)-840\,d^{9}e^{(-10)}\log\left(x\right)-\frac{\left(2520\,d^{8}x^{\frac{8}{3}}-1260\,d^{7}x^{\frac{7}{3}}e+840\,d^{6}x^{2}e^{2}-630\,d^{5}x^{\frac{5}{3}}e^{3}+504\,d^{4}x^{\frac{4}{3}}e^{4}-420\,d^{3}xe^{5}+360\,d^{2}x^{\frac{2}{3}}e^{6}-315\,dx^{\frac{1}{3}}e^{7}+280\,e^{8}\right)e^{(-9)}}{x^{3}}\right)bne-\frac{b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)}{3\,x^{3}}-\frac{a}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^4,x, algorithm="maxima")
```

```
[Out] -1/7560*(2520*d^9*e^(-10)*log(d*x^(1/3) + e) - 840*d^9*e^(-10)*log(x) - (25
20*d^8*x^(8/3) - 1260*d^7*x^(7/3)*e + 840*d^6*x^2*e^2 - 630*d^5*x^(5/3)*e^3
 + 504*d^4*x^(4/3)*e^4 - 420*d^3*x*e^5 + 360*d^2*x^(2/3)*e^6 - 315*d*x^(1/3
)*e^7 + 280*e^8)*e^(-9)/x^3)*b*n*e - 1/3*b*log(c*(d + e/x^(1/3))^n)/x^3 - 1
/3*a/x^3
```

Fricas [A]
time = 0.39, size = 196, normalized size = 1.05

$$\frac{\left(2520\,(bx^{3}-b)e^{9}\log\left(c\right)-280\,((bn-9\,a)x^{3}-bn+9\,a)e^{9}+420\,(bd^{3}nx^{3}-bd^{3}nx)e^{6}-840\,(bd^{6}nx^{3}-bd^{6}nx^{2})e^{3}-2520\,(bd^{9}nx^{3}+bne^{9})\log\left(\frac{dx+x^{\frac{2}{3}}e}{x}\right)+90\,(28\,bd^{8}nx^{2}e-7\,bd^{5}nxe^{4}+4\,bd^{2}ne^{7})x^{\frac{2}{3}}-63\,(20\,bd^{7}nx^{2}e^{2}-8\,bd^{4}nxe^{5}+5\,bdne^{8})x^{\frac{1}{3}}\right)e^{(-9)}}{7560\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^4,x, algorithm="fricas")
```

```
[Out] 1/7560*(2520*(b*x^3 - b)*e^9*log(c) - 280*((b*n - 9*a)*x^3 - b*n + 9*a)*e^9
 + 420*(b*d^3*n*x^3 - b*d^3*n*x)*e^6 - 840*(b*d^6*n*x^3 - b*d^6*n*x^2)*e^3
- 2520*(b*d^9*n*x^3 + b*n*e^9)*log((d*x + x^(2/3)*e)/x) + 90*(28*b*d^8*n*x^
2*e - 7*b*d^5*n*x*e^4 + 4*b*d^2*n*e^7)*x^(2/3) - 63*(20*b*d^7*n*x^2*e^2 - 8
*b*d^4*n*x*e^5 + 5*b*d*n*e^8)*x^(1/3))*e^(-9)/x^3
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))/x**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5987 deep
```

Giac [A]
time = 3.43, size = 153, normalized size = 0.82

$$-\frac{1}{7560}\left(\left(\left(2520\,d^{9}e^{(-10)}\log\left(\left|dx^{\frac{1}{3}}+e\right|\right)-840\,d^{9}e^{(-10)}\log\left(\left|x\right|\right)-\frac{\left(2520\,d^{8}x^{\frac{8}{3}}e-1260\,d^{7}x^{\frac{7}{3}}e^{2}+840\,d^{6}x^{2}e^{3}-630\,d^{5}x^{\frac{5}{3}}e^{4}+504\,d^{4}x^{\frac{4}{3}}e^{5}-420\,d^{3}xe^{6}+360\,d^{2}x^{\frac{2}{3}}e^{7}-315\,dx^{\frac{1}{3}}e^{8}+280\,e^{9}\right)e^{(-10)}}{x^{3}}\right)e+\frac{2520\,\log\left(d+\frac{e}{x^{\frac{1}{3}}}\right)}{x^{3}}\right)bn-\frac{b\log\left(c\right)}{3\,x^{3}}-\frac{a}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^4,x, algorithm="giac")
```

```
[Out] -1/7560*((2520*d^9*e^(-10)*log(abs(d*x^(1/3) + e)) - 840*d^9*e^(-10)*log(ab
s(x)) - (2520*d^8*x^(8/3)*e - 1260*d^7*x^(7/3)*e^2 + 840*d^6*x^2*e^3 - 630*
d^5*x^(5/3)*e^4 + 504*d^4*x^(4/3)*e^5 - 420*d^3*x*e^6 + 360*d^2*x^(2/3)*e^7
 - 315*d*x^(1/3)*e^8 + 280*e^9)*e^(-10)/x^3)*e + 2520*log(d + e/x^(1/3))/x^
3)*b*n - 1/3*b*log(c)/x^3 - 1/3*a/x^3
```

Mupad [B]
time = 0.53, size = 152, normalized size = 0.81

$$\frac{b\,n}{27\,x^3}-\frac{a}{3\,x^3}-\frac{b\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^n\right)}{3\,x^3}-\frac{b\,d\,n}{24\,e\,x^{8/3}}-\frac{b\,d^9\,n\,\ln\left(d+\frac{e}{x^{1/3}}\right)}{3\,e^9}-\frac{b\,d^3\,n}{18\,e^3\,x^2}+\frac{b\,d^6\,n}{9\,e^6\,x}+\frac{b\,d^2\,n}{21\,e^2\,x^{7/3}}+\frac{b\,d^4\,n}{15\,e^4\,x^{5/3}}-\frac{b\,d^5\,n}{12\,e^5\,x^{4/3}}-\frac{b\,d^7\,n}{6\,e^7\,x^{2/3}}+\frac{b\,d^8\,n}{3\,e^8\,x^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))/x^4,x)
```

```
[Out] (b*n)/(27*x^3) - a/(3*x^3) - (b*log(c*(d + e/x^(1/3))^n))/(3*x^3) - (b*d*n)
/(24*e*x^(8/3)) - (b*d^9*n*log(d + e/x^(1/3)))/(3*e^9) - (b*d^3*n)/(18*e^3*
x^2) + (b*d^6*n)/(9*e^6*x) + (b*d^2*n)/(21*e^2*x^(7/3)) + (b*d^4*n)/(15*e^4
*x^(5/3)) - (b*d^5*n)/(12*e^5*x^(4/3)) - (b*d^7*n)/(6*e^7*x^(2/3)) + (b*d^8
*n)/(3*e^8*x^(1/3))
```

3.497 $\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx$

Optimal. Leaf size=572

$$\frac{481b^2e^8n^2\sqrt[3]{x}}{420d^8}-\frac{341b^2e^7n^2x^{2/3}}{840d^7}+\frac{743b^2e^6n^2x}{3780d^6}-\frac{533b^2e^5n^2x^{4/3}}{5040d^5}+\frac{73b^2e^4n^2x^{5/3}}{1260d^4}-\frac{5b^2e^3n^2x^2}{168d^3}+\frac{b^2e^2n^2x^{7/3}}{84d^2}-\frac{481b}{}$$

```
[Out] 481/420*b^2*e^8*n^2*x^(1/3)/d^8-341/840*b^2*e^7*n^2*x^(2/3)/d^7+743/3780*b^
2*e^6*n^2*x/d^6-533/5040*b^2*e^5*n^2*x^(4/3)/d^5+73/1260*b^2*e^4*n^2*x^(5/3
)/d^4-5/168*b^2*e^3*n^2*x^2/d^3+1/84*b^2*e^2*n^2*x^(7/3)/d^2-481/420*b^2*e^
9*n^2*ln(d+e/x^(1/3))/d^9-2/3*b*e^8*n*(d+e/x^(1/3))*x^(1/3)*(a+b*ln(c*(d+e/
x^(1/3))^n))/d^9+1/3*b*e^7*n*x^(2/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^7-2/9*b*
e^6*n*x*(a+b*ln(c*(d+e/x^(1/3))^n))/d^6+1/6*b*e^5*n*x^(4/3)*(a+b*ln(c*(d+e/
x^(1/3))^n))/d^5-2/15*b*e^4*n*x^(5/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^4+1/9*b
*e^3*n*x^2*(a+b*ln(c*(d+e/x^(1/3))^n))/d^3-2/21*b*e^2*n*x^(7/3)*(a+b*ln(c*(
d+e/x^(1/3))^n))/d^2+1/12*b*e*n*x^(8/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d-2/3*b
*e^9*n*ln(1-d/(d+e/x^(1/3)))*(a+b*ln(c*(d+e/x^(1/3))^n))/d^9+1/3*x^3*(a+b*l
n(c*(d+e/x^(1/3))^n))^2-761/1260*b^2*e^9*n^2*ln(x)/d^9+2/3*b^2*e^9*n^2*poly
log(2,d/(d+e/x^(1/3)))/d^9
```

Rubi [A]
time = 1.08, antiderivative size = 572, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2504, 2445, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46}

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e/x^(1/3))^n])^2,x]
```

```
[Out] (481*b^2*e^8*n^2*x^(1/3))/(420*d^8) - (341*b^2*e^7*n^2*x^(2/3))/(840*d^7) +
 (743*b^2*e^6*n^2*x)/(3780*d^6) - (533*b^2*e^5*n^2*x^(4/3))/(5040*d^5) + (7
3*b^2*e^4*n^2*x^(5/3))/(1260*d^4) - (5*b^2*e^3*n^2*x^2)/(168*d^3) + (b^2*e^
2*n^2*x^(7/3))/(84*d^2) - (481*b^2*e^9*n^2*Log[d + e/x^(1/3)])/(420*d^9) -
(2*b*e^8*n*(d + e/x^(1/3))*x^(1/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*d^9
) + (b*e^7*n*x^(2/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*d^7) - (2*b*e^6*n
*x*(a + b*Log[c*(d + e/x^(1/3))^n]))/(9*d^6) + (b*e^5*n*x^(4/3)*(a + b*Log[
c*(d + e/x^(1/3))^n]))/(6*d^5) - (2*b*e^4*n*x^(5/3)*(a + b*Log[c*(d + e/x^(
1/3))^n]))/(15*d^4) + (b*e^3*n*x^2*(a + b*Log[c*(d + e/x^(1/3))^n]))/(9*d^3
) - (2*b*e^2*n*x^(7/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(21*d^2) + (b*e*n*
x^(8/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(12*d) - (2*b*e^9*n*Log[1 - d/(d
```

```
+ e/x^(1/3))]*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*d^9) + (x^3*(a + b*Log[c
*(d + e/x^(1/3))^n])^2)/3 - (761*b^2*e^9*n^2*Log[x])/(1260*d^9) + (2*b^2*e^
9*n^2*PolyLog[2, d/(d + e/x^(1/3))])/(3*d^9)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2dx = -\left(3\text{Subst}\left(\int\frac{(a+b\log(c(d+ex)^n))^2}{x^{10}}dx,x,\frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - \frac{1}{3}(2ben)\text{Subst}\left(\int\frac{a+b\log}{x^9(}\right.$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - \frac{1}{3}(2bn)\text{Subst}\left(\int\frac{a+b\log}{x\left(-\frac{d}{e}+\right.}\right.$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - \frac{(2bn)\text{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^9}d\right.}{3d}$$

$$= \frac{benx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{12d} + \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\right.\right.\right.$$

$$= -\frac{2be^2nx^{7/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{21d^2} + \frac{benx^{8/3}\left(a+b\log\left(\right.\right.}{12}$$

$$= \frac{b^2e^8n^2\sqrt[3]{x}}{12d^8} - \frac{b^2e^7n^2x^{2/3}}{24d^7} + \frac{b^2e^6n^2x}{36d^6} - \frac{b^2e^5n^2x^{4/3}}{48d^5} + \frac{b^2e^4n^2x^{5/3}}{60d^4} -$$

$$= \frac{5b^2e^8n^2\sqrt[3]{x}}{28d^8} - \frac{5b^2e^7n^2x^{2/3}}{56d^7} + \frac{5b^2e^6n^2x}{84d^6} - \frac{5b^2e^5n^2x^{4/3}}{112d^5} + \frac{b^2e^4n^2x}{28d^4}$$

$$= \frac{73b^2e^8n^2\sqrt[3]{x}}{252d^8} - \frac{73b^2e^7n^2x^{2/3}}{504d^7} + \frac{73b^2e^6n^2x}{756d^6} - \frac{73b^2e^5n^2x^{4/3}}{1008d^5} + 73b$$

$$= \frac{533b^2e^8n^2\sqrt[3]{x}}{1260d^8} - \frac{533b^2e^7n^2x^{2/3}}{2520d^7} + \frac{533b^2e^6n^2x}{3780d^6} - \frac{533b^2e^5n^2x^{4/3}}{5040d^5} +$$

$$= \frac{743b^2e^8n^2\sqrt[3]{x}}{1260d^8} - \frac{743b^2e^7n^2x^{2/3}}{2520d^7} + \frac{743b^2e^6n^2x}{3780d^6} - \frac{533b^2e^5n^2x^{4/3}}{5040d^5} +$$

$$= \frac{341b^2e^8n^2\sqrt[3]{x}}{420d^8} - \frac{341b^2e^7n^2x^{2/3}}{840d^7} + \frac{743b^2e^6n^2x}{3780d^6} - \frac{533b^2e^5n^2x^{4/3}}{5040d^5} +$$

$$= \frac{481b^2e^8n^2\sqrt[3]{x}}{420d^8} - \frac{341b^2e^7n^2x^{2/3}}{840d^7} + \frac{743b^2e^6n^2x}{3780d^6} - \frac{533b^2e^5n^2x^{4/3}}{5040d^5} +$$

$$= \frac{481b^2e^8n^2\sqrt[3]{x}}{420d^8} - \frac{341b^2e^7n^2x^{2/3}}{840d^7} + \frac{743b^2e^6n^2x}{3780d^6} - \frac{533b^2e^5n^2x^{4/3}}{5040d^5} +$$

Mathematica [A]
time = 1.06, size = 495, normalized size = 0.87

$$-\frac{-5040 d^9 x^3 \left(a-b n \log \left(d+\frac{e}{\sqrt[3]{x}}\right)+b \log \left(c \left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2+12 b n \left(a-b n \log \left(d+\frac{e}{\sqrt[3]{x}}\right)+b \log \left(c \left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right) \left(d e \sqrt[3]{x} \left(840 e^7-420 d e^6 \sqrt[3]{x}+280 d^2 e^5 x^{2/3}-210 d^3 e^4 x+168 d^4 e^3 x^{4/3}-140 d^5 e^2 x^{5/3}+120 d^6 e x^2-105 d^7 x^{7/3}\right)-840 (e^9+d^9 x^3) \log \left(d+\frac{e}{\sqrt[3]{x}}\right)+840 e^9 \log \left(\frac{e}{\sqrt[3]{x}}\right)\right)+b^2 n^2 \left(-5040 (e^9+d^9 x^3) \log^2\left(d+\frac{e}{\sqrt[3]{x}}\right)-e^2 \left(17316 d e^6 \sqrt[3]{x}-6138 d^2 e^5 x^{2/3}+2972 d^3 e^4 x-1599 d^4 e^3 x^{4/3}+876 d^5 e^2 x^{5/3}-450 d^6 e x^2+180 d^7 x^{7/3}+27396 e^7 \log \left(-\frac{e}{d \sqrt[3]{x}}\right)\right)+12 e \log \left(d+\frac{e}{\sqrt[3]{x}}\right) \left(2283 e^8+840 d e^7 \sqrt[3]{x}-420 d^2 e^6 x^{2/3}+280 d^3 e^5 x-210 d^4 e^4 x^{4/3}+168 d^5 e^3 x^{5/3}-140 d^6 e^2 x^2+120 d^7 e x^{7/3}-105 d^8 x^{8/3}+840 e^8 \log \left(-\frac{e}{d \sqrt[3]{x}}\right)\right)+10080 e^9 \mathrm{Li}_2\left(1+\frac{e}{d \sqrt[3]{x}}\right)\right)}{15120 d^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/x^(1/3))^n])^2,x]
```

```
[Out] -1/15120*(-5040*d^9*x^3*(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3
))^n])^2 + 12*b*n*(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])
*(d*e*x^(1/3)*(840*e^7 - 420*d*e^6*x^(1/3) + 280*d^2*e^5*x^(2/3) - 210*d^3*
e^4*x + 168*d^4*e^3*x^(4/3) - 140*d^5*e^2*x^(5/3) + 120*d^6*e*x^2 - 105*d^7
*x^(7/3)) - 840*(e^9 + d^9*x^3)*Log[d + e/x^(1/3)] + 840*e^9*Log[e/x^(1/3)]
) + b^2*n^2*(-5040*(e^9 + d^9*x^3)*Log[d + e/x^(1/3)]^2 - e^2*(17316*d*e^6*
x^(1/3) - 6138*d^2*e^5*x^(2/3) + 2972*d^3*e^4*x - 1599*d^4*e^3*x^(4/3) + 87
6*d^5*e^2*x^(5/3) - 450*d^6*e*x^2 + 180*d^7*x^(7/3) + 27396*e^7*Log[-(e/(d*
x^(1/3)))]) + 12*e*Log[d + e/x^(1/3)]*(2283*e^8 + 840*d*e^7*x^(1/3) - 420*d
^2*e^6*x^(2/3) + 280*d^3*e^5*x - 210*d^4*e^4*x^(4/3) + 168*d^5*e^3*x^(5/3)
- 140*d^6*e^2*x^2 + 120*d^7*e*x^(7/3) - 105*d^8*x^(8/3) + 840*e^8*Log[-(e/(
d*x^(1/3)))]) + 10080*e^9*PolyLog[2, 1 + e/(d*x^(1/3))]))/d^9
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^2 \left(a+b \ln \left(c \left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e/x^(1/3))^n))^2,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e/x^(1/3))^n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3*log((d*x^(1/3) + e)^n)^2 - integrate(-1/9*(9*(b^2*log(c)^2 + 2*
a*b*log(c) + a^2)*x^(8/3)*e + 9*(b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d)*x
^3 + 9*(b^2*d*x^3 + b^2*x^(8/3)*e)*log(x^(1/3*n))^2 - 2*(b^2*d*n*x^3 - 9*(b
^2*log(c) + a*b)*x^(8/3)*e - 9*(b^2*d*log(c) + a*b*d)*x^3 + 9*(b^2*d*x^3 +
b^2*x^(8/3)*e)*log(x^(1/3*n)))*log((d*x^(1/3) + e)^n) - 18*((b^2*log(c) + a
```

```
*b)*x^(8/3)*e + (b^2*d*log(c) + a*b*d)*x^3)*log(x^(1/3*n)))/(d*x + x^(2/3)*
e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="fricas")
[Out] integral(b^2*x^2*log(c*((d*x + x^(2/3)*e)/x)^n)^2 + 2*a*b*x^2*log(c*((d*x +
 x^(2/3)*e)/x)^n) + a^2*x^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(1/3))**n))**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)^2*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(a + b \ln\left(c \left(d + \frac{e}{x^{1/3}}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e/x^(1/3))^n))^2,x)
[Out] int(x^2*(a + b*log(c*(d + e/x^(1/3))^n))^2, x)
```

3.498 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\,dx$

Optimal. Leaf size=400

$$-\frac{77b^2e^5n^2\sqrt[3]{x}}{60d^5}+\frac{47b^2e^4n^2x^{2/3}}{120d^4}-\frac{3b^2e^3n^2x}{20d^3}+\frac{b^2e^2n^2x^{4/3}}{20d^2}+\frac{77b^2e^6n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{60d^6}+\frac{be^5n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(\right.}{}$$

```
[Out] -77/60*b^2*e^5*n^2*x^(1/3)/d^5+47/120*b^2*e^4*n^2*x^(2/3)/d^4-3/20*b^2*e^3*
n^2*x/d^3+1/20*b^2*e^2*n^2*x^(4/3)/d^2+77/60*b^2*e^6*n^2*ln(d+e/x^(1/3))/d^
6+b*e^5*n*(d+e/x^(1/3))*x^(1/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^6-1/2*b*e^4*n
*x^(2/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^4+1/3*b*e^3*n*x*(a+b*ln(c*(d+e/x^(1/
3))^n))/d^3-1/4*b*e^2*n*x^(4/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^2+1/5*b*e*n*x
^(5/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d+b*e^6*n*ln(1-d/(d+e/x^(1/3)))*(a+b*ln(
c*(d+e/x^(1/3))^n))/d^6+1/2*x^2*(a+b*ln(c*(d+e/x^(1/3))^n))^2+137/180*b^2*e
^6*n^2*ln(x)/d^6-b^2*e^6*n^2*polylog(2,d/(d+e/x^(1/3)))/d^6
```

Rubi [A]
time = 0.62, antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2504, 2445, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$-\frac{b^2e^6n^2\text{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)}{d^6}+\frac{be^6n\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^6}+\frac{be^5n\sqrt[3]{x}\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^6}-\frac{be^4nx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{2d^4}+\frac{be^3nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3d^3}-\frac{be^2nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{4d^2}+\frac{benx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{5d}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2+\frac{77b^2e^6n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{60d^6}+\frac{137b^2e^6n^2\log(x)}{180d^6}-\frac{77b^2e^5n^2\sqrt[3]{x}}{60d^5}+\frac{47b^2e^4n^2x^{2/3}}{120d^4}-\frac{3b^2e^3n^2x}{20d^3}+\frac{b^2e^2n^2x^{4/3}}{20d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e/x^(1/3))^n])^2,x]
```

```
[Out] (-77*b^2*e^5*n^2*x^(1/3))/(60*d^5) + (47*b^2*e^4*n^2*x^(2/3))/(120*d^4) - (
3*b^2*e^3*n^2*x)/(20*d^3) + (b^2*e^2*n^2*x^(4/3))/(20*d^2) + (77*b^2*e^6*n^
2*Log[d + e/x^(1/3)])/(60*d^6) + (b*e^5*n*(d + e/x^(1/3))*x^(1/3)*(a + b*Lo
g[c*(d + e/x^(1/3))^n]))/d^6 - (b*e^4*n*x^(2/3)*(a + b*Log[c*(d + e/x^(1/3)
)^n]))/(2*d^4) + (b*e^3*n*x*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*d^3) - (b*
e^2*n*x^(4/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(4*d^2) + (b*e*n*x^(5/3)*(a
 + b*Log[c*(d + e/x^(1/3))^n]))/(5*d) + (b*e^6*n*Log[1 - d/(d + e/x^(1/3))]
*(a + b*Log[c*(d + e/x^(1/3))^n]))/d^6 + (x^2*(a + b*Log[c*(d + e/x^(1/3))^
n])^2)/2 + (137*b^2*e^6*n^2*Log[x])/(180*d^6) - (b^2*e^6*n^2*PolyLog[2, d/(
d + e/x^(1/3))])/d^6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
```

```
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2dx=-\left(3\text{Subst}\left(\int\frac{(a+b\log(c(d+ex)^n))^2}{x^7}dx,x,\frac{1}{\sqrt[3]{x}}\right)\right)$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2-(ben)\text{Subst}\Bigg(\int\frac{a+b\log(c(\cdots}{x^6(d+\cdots}$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2-(bn)\text{Subst}\Bigg(\int\frac{a+b\log(cx^n\cdots}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6\cdots}$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2-\frac{(bn)\text{Subst}\Bigg(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}dx,\cdots}{d}$$

$$=\frac{benx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{5d}+\frac{1}{2}x^2\Bigg(a+b\log\Bigg(c\Bigg(d+\cdots$$

$$=-\frac{be^2nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{4d^2}+\frac{benx^{5/3}\Bigg(a+b\log\Bigg(c\Bigg(\cdots}{5d}$$

$$=-\frac{b^2e^5n^2\sqrt[3]{x}}{5d^5}+\frac{b^2e^4n^2x^{2/3}}{10d^4}-\frac{b^2e^3n^2x}{15d^3}+\frac{b^2e^2n^2x^{4/3}}{20d^2}+\frac{b^2e^6n^2\log\Bigg(\cdots}{5d\cdots}$$

$$=-\frac{9b^2e^5n^2\sqrt[3]{x}}{20d^5}+\frac{9b^2e^4n^2x^{2/3}}{40d^4}-\frac{3b^2e^3n^2x}{20d^3}+\frac{b^2e^2n^2x^{4/3}}{20d^2}+\frac{9b^2e^6n^2\cdots}{\cdots}$$

$$=-\frac{47b^2e^5n^2\sqrt[3]{x}}{60d^5}+\frac{47b^2e^4n^2x^{2/3}}{120d^4}-\frac{3b^2e^3n^2x}{20d^3}+\frac{b^2e^2n^2x^{4/3}}{20d^2}+\frac{47b^2e^{\cdots}}{\cdots}$$

$$=-\frac{77b^2e^5n^2\sqrt[3]{x}}{60d^5}+\frac{47b^2e^4n^2x^{2/3}}{120d^4}-\frac{3b^2e^3n^2x}{20d^3}+\frac{b^2e^2n^2x^{4/3}}{20d^2}+\frac{77b^2e^{\cdots}}{\cdots}$$

$$=-\frac{77b^2e^5n^2\sqrt[3]{x}}{60d^5}+\frac{47b^2e^4n^2x^{2/3}}{120d^4}-\frac{3b^2e^3n^2x}{20d^3}+\frac{b^2e^2n^2x^{4/3}}{20d^2}+\frac{77b^2e^{\cdots}}{\cdots}$$

Mathematica [A]
time = 0.64, size = 383, normalized size = 0.96

$$\frac{60d^6x^2\left(a-bn\log\left(d+\frac{e}{\sqrt[3]{x}}\right)+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2+2bn\left(a-bn\log\left(d+\frac{e}{\sqrt[3]{x}}\right)+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\left(de\sqrt[3]{x}\left(60e^4-30de^3\sqrt[3]{x}+20d^2e^2x^{2/3}-15d^3ex+12d^4x^{4/3}\right)-60(e^6-d^6x^2)\log\left(d+\frac{e}{\sqrt[3]{x}}\right)+60e^6\log\left(\frac{e}{\sqrt[3]{x}}\right)\right)+b^2n^2\left(de^2\sqrt[3]{x}\left(-154e^3+47de^2\sqrt[3]{x}-18d^2ex^{2/3}+6d^3x\right)-60(e^6-d^6x^2)\log^2\left(d+\frac{e}{\sqrt[3]{x}}\right)-274e^6\log\left(-\frac{e}{d\sqrt[3]{x}}\right)+2e\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(137e^5+60de^4\sqrt[3]{x}-30d^2e^3x^{2/3}+20d^3e^2x-15d^4ex^{4/3}+12d^5x^{5/3}+60e^5\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\right)+120e^6\text{Li}_2\left(1+\frac{e}{d\sqrt[3]{x}}\right)\right)}{120d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(1/3))^n])^2,x]
```

```
[Out] (60*d^6*x^2*(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])^2 + 2
*b*n*(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])*(d*e*x^(1/3)
*(60*e^4 - 30*d*e^3*x^(1/3) + 20*d^2*e^2*x^(2/3) - 15*d^3*e*x + 12*d^4*x^(4
/3)) - 60*(e^6 - d^6*x^2)*Log[d + e/x^(1/3)] + 60*e^6*Log[e/x^(1/3)]) + b^2
*n^2*(d*e^2*x^(1/3)*(-154*e^3 + 47*d*e^2*x^(1/3) - 18*d^2*e*x^(2/3) + 6*d^3
*x) - 60*(e^6 - d^6*x^2)*Log[d + e/x^(1/3)]^2 - 274*e^6*Log[-(e/(d*x^(1/3))
)] + 2*e*Log[d + e/x^(1/3)]*(137*e^5 + 60*d*e^4*x^(1/3) - 30*d^2*e^3*x^(2/3
) + 20*d^3*e^2*x - 15*d^4*e*x^(4/3) + 12*d^5*x^(5/3) + 60*e^5*Log[-(e/(d*x^
(1/3)))]) + 120*e^6*PolyLog[2, 1 + e/(d*x^(1/3))]))/(120*d^6)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(1/3))^n))^2,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e/x^(1/3))^n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*log((d*x^(1/3) + e)^n)^2 - integrate(-1/3*(3*(b^2*log(c)^2 + 2*
a*b*log(c) + a^2)*x^(5/3)*e + 3*(b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d)*x
^2 + 3*(b^2*d*x^2 + b^2*x^(5/3)*e)*log(x^(1/3*n))^2 - (b^2*d*n*x^2 - 6*(b^2
*log(c) + a*b)*x^(5/3)*e - 6*(b^2*d*log(c) + a*b*d)*x^2 + 6*(b^2*d*x^2 + b^
2*x^(5/3)*e)*log(x^(1/3*n)))*log((d*x^(1/3) + e)^n) - 6*((b^2*log(c) + a*b)
*x^(5/3)*e + (b^2*d*log(c) + a*b*d)*x^2)*log(x^(1/3*n)))/(d*x + x^(2/3)*e),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x*log(c*((d*x + x^(2/3)*e)/x)^n)^2 + 2*a*b*x*log(c*((d*x + x^(
2/3)*e)/x)^n) + a^2*x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/3))**n))**2,x)
```

```
[Out] Integral(x*(a + b*log(c*(d + e/x**(1/3))**n))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)^2*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{1/3}}\right)^{n}\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e/x^(1/3))^n))^2,x)
```

```
[Out] int(x*(a + b*log(c*(d + e/x^(1/3))^n))^2, x)
```

3.499 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx$

Optimal. Leaf size=227

$$\frac{b^2e^2n^2\sqrt[3]{x}}{d^2}-\frac{b^2e^3n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^3}-\frac{2be^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3}+\frac{benx^{2/3}\left(a+\right.}{}$$

```
[Out] b^2*e^2*n^2*x^(1/3)/d^2-b^2*e^3*n^2*ln(d+e/x^(1/3))/d^3-2*b*e^2*n*(d+e/x^(1
/3))*x^(1/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^3+b*e*n*x^(2/3)*(a+b*ln(c*(d+e/x
^(1/3))^n))/d-2*b*e^3*n*ln(1-d/(d+e/x^(1/3)))*(a+b*ln(c*(d+e/x^(1/3))^n))/d
^3+x*(a+b*ln(c*(d+e/x^(1/3))^n))^2-b^2*e^3*n^2*ln(x)/d^3+2*b^2*e^3*n^2*poly
log(2,d/(d+e/x^(1/3)))/d^3
```

Rubi [A]
time = 0.30, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {2501, 2504, 2445, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$\frac{2b^2e^3n^2\text{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)}{d^3}-\frac{2be^3n\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3}-\frac{2be^2n\sqrt[3]{x}\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3}+\frac{benx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d}+x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2-\frac{b^2e^3n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^3}-\frac{b^2e^3n^2\log(x)}{d^3}+\frac{b^2e^2n^2\sqrt[3]{x}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])^2,x]
```

```
[Out] (b^2*e^2*n^2*x^(1/3))/d^2 - (b^2*e^3*n^2*Log[d + e/x^(1/3)])/d^3 - (2*b*e^2
*n*(d + e/x^(1/3))*x^(1/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/d^3 + (b*e*n*x
^(2/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/d - (2*b*e^3*n*Log[1 - d/(d + e/x^
(1/3))]*(a + b*Log[c*(d + e/x^(1/3))^n]))/d^3 + x*(a + b*Log[c*(d + e/x^(1/
3))^n])^2 - (b^2*e^3*n^2*Log[x])/d^3 + (2*b^2*e^3*n^2*PolyLog[2, d/(d + e/x
^(1/3))])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2501

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx = 3\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^n\right)\right)^2 dx, x, \sqrt[3]{x}\right)$$

$$= -\left(3\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^4}dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - (2ben)\text{Subst}\left(\int \frac{a+b\log(c(d+}{x^3(d+ex}\right.$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - (2bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\right.$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - \frac{(2bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}dx, x,\right.}{d}$$

$$= \frac{benx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right.\right.\right.$$

$$= -\frac{2be^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3} + \frac{benx^{2/3}\left(\right.}{}$$

$$= \frac{b^2e^2n^2\sqrt[3]{x}}{d^2} - \frac{b^2e^3n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^3} - \frac{2be^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+\right.}{d^3}$$

$$= \frac{b^2e^2n^2\sqrt[3]{x}}{d^2} - \frac{b^2e^3n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^3} - \frac{2be^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+\right.}{d^3}$$

Mathematica [A]
time = 0.27, size = 336, normalized size = 1.48

$$x\left(a+b\left(-n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)+\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\right)^2 - 6be^3n\left(a+b\left(-n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)+\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\right)\left(\frac{\sqrt[3]{x}}{3d^2e}-\frac{x^{2/3}}{6de^2}-\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{3d^3}-\frac{x\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{3e^3}+\frac{\log\left(\frac{e}{\sqrt[3]{x}}\right)}{3d^3}\right) - b^2e^3n^2\left(-\frac{\sqrt[3]{x}\left(1-2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{d^2e}-\frac{x^{2/3}\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{de^2}-\frac{x\log^2\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}+\frac{3\left(-\log\left(1-\frac{d+\frac{e}{\sqrt[3]{x}}}{d}\right)+\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{d^3}+\frac{-\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(-2\log\left(1-\frac{d+\frac{e}{\sqrt[3]{x}}}{d}\right)+\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)+2\text{Li}_2\left(\frac{d+\frac{e}{\sqrt[3]{x}}}{d}\right)}{d^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^2,x]
```

```
[Out] x*(a + b*(-(n*Log[d + e/x^(1/3)]) + Log[c*(d + e/x^(1/3))^n]))^2 - 6*b*e^3*
n*(a + b*(-(n*Log[d + e/x^(1/3)]) + Log[c*(d + e/x^(1/3))^n]))*(x^(1/3)/(3*
d^2*e) - x^(2/3)/(6*d*e^2) - Log[d + e/x^(1/3)]/(3*d^3) - (x*Log[d + e/x^(1
/3)])/(3*e^3) + Log[e/x^(1/3)]/(3*d^3)) - b^2*e^3*n^2*(-((x^(1/3)*(1 - 2*Lo
```

```
g[d + e/x^(1/3)]))/(d^2*e)) - (x^(2/3)*Log[d + e/x^(1/3)])/(d*e^2) - (x*Log
[d + e/x^(1/3)]^2)/e^3 + (3*(-Log[1 - (d + e/x^(1/3))/d] + Log[d + e/x^(1/3
)]))/d^3 + (-(Log[d + e/x^(1/3)]*(-2*Log[1 - (d + e/x^(1/3))/d] + Log[d + e
/x^(1/3)])) + 2*PolyLog[2, (d + e/x^(1/3))/d])/d^3)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^n))^2,x)
[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="maxima")
[Out] (n*((d*x^(2/3) - 2*x^(1/3)*e)/d^2 + 2*e^2*log(d*x^(1/3) + e)/d^3)*e + 2*x*l
og(c*(d + e/x^(1/3))^n))*a*b + (x*log((d*x^(1/3) + e)^n)^2 - integrate(-1/3
*(3*d*x*log(c)^2 + 3*x^(2/3)*e*log(c)^2 + 3*(d*x + x^(2/3)*e)*log(x^(1/3*n)
)^2 - 2*(d*n*x - 3*d*x*log(c) - 3*x^(2/3)*e*log(c) + 3*(d*x + x^(2/3)*e)*lo
g(x^(1/3*n)))*log((d*x^(1/3) + e)^n) - 6*(d*x*log(c) + x^(2/3)*e*log(c))*lo
g(x^(1/3*n)))/(d*x + x^(2/3)*e), x))*b^2 + a^2*x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="fricas")
[Out] integral(b^2*log(c*((d*x + x^(2/3)*e)/x)^n)^2 + 2*a*b*log(c*((d*x + x^(2/3)
*e)/x)^n) + a^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e/x**(1/3))**n))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^n\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))^2,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(1/3))^n))^2, x)
```

3.500 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x}\,dx$

Optimal. Leaf size=93

$$-3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt[3]{x}}\right)-6bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\text{Li}_2\left(1+\frac{e}{d\sqrt[3]{x}}\right)+6b$$

```
[Out] -3*(a+b*ln(c*(d+e/x^(1/3))^n))^2*ln(-e/d/x^(1/3))-6*b*n*(a+b*ln(c*(d+e/x^(1
/3))^n))*polylog(2,1+e/d/x^(1/3))+6*b^2*n^2*polylog(3,1+e/d/x^(1/3))
```

Rubi [A]
time = 0.09, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2504, 2443, 2481, 2421, 6724}

$$-6bn\text{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)+6b^2n^2\text{PolyLog}\left(3,\frac{e}{d\sqrt[3]{x}}+1\right)-3\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])^2/x,x]
```

```
[Out] -3*(a + b*Log[c*(d + e/x^(1/3))^n])^2*Log[-(e/(d*x^(1/3)))] - 6*b*n*(a + b*
Log[c*(d + e/x^(1/3))^n])*PolyLog[2, 1 + e/(d*x^(1/3))] + 6*b^2*n^2*PolyLog
[3, 1 + e/(d*x^(1/3))]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
```

```
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x}\,dx = -\left(3\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt[3]{x}}\right)+(6ben)\text{Subst}\Bigg(\int$$

$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt[3]{x}}\right)+(6bn)\text{Subst}\Bigg(\int$$

$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt[3]{x}}\right)-6bn\Bigg(a+b\log\Bigg($$

$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt[3]{x}}\right)-6bn\Bigg(a+b\log\Bigg($$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 389 vs. 2(93) = 186.
time = 0.14, size = 389, normalized size = 4.18

$$\left(a-bn\log\left(d+\frac{e}{\sqrt[3]{x}}\right)+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\log(x)+2bn\left(a-bn\log\left(d+\frac{e}{\sqrt[3]{x}}\right)+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\left(\left(\log\left(d+\frac{e}{\sqrt[3]{x}}\right)-\log\left(1+\frac{e}{d\sqrt[3]{x}}\right)\right)\log(x)+3\text{Li}_2\left(-\frac{e}{d\sqrt[3]{x}}\right)\right)+3b^2n^2\left(2\log\left(\frac{e}{d}+\sqrt[3]{x}\right)\text{Li}_2\left(1+\frac{d\sqrt[3]{x}}{e}\right)-2\left(\log\left(d+\frac{e}{\sqrt[3]{x}}\right)-\log\left(\frac{e}{d}+\sqrt[3]{x}\right)\right)\text{Li}_2\left(-\frac{d\sqrt[3]{x}}{e}\right)+\frac{1}{81}\left(81\log^2\left(\frac{e}{d}+\sqrt[3]{x}\right)\log\left(-\frac{d\sqrt[3]{x}}{e}\right)+27\log^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\log(x)-27\log^2\left(\frac{e}{d}+\sqrt[3]{x}\right)\log(x)-54\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\log\left(1+\frac{d\sqrt[3]{x}}{e}\right)\log(x)+54\log\left(\frac{e}{d}+\sqrt[3]{x}\right)\log\left(1+\frac{d\sqrt[3]{x}}{e}\right)\log(x)+9\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\log^2(x)-9\log\left(1+\frac{d\sqrt[3]{x}}{e}\right)\log^2(x)+\log^3(x)-162\text{Li}_3\left(1+\frac{d\sqrt[3]{x}}{e}\right)-162\text{Li}_3\left(-\frac{d\sqrt[3]{x}}{e}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^2/x,x]
```

```
[Out] (a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])^2*Log[x] + 2*b*n*
(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])*((Log[d + e/x^(1/
3)] - Log[1 + e/(d*x^(1/3))])*Log[x] + 3*PolyLog[2, -(e/(d*x^(1/3)))]) + 3*
b^2*n^2*(2*Log[e/d + x^(1/3)]*PolyLog[2, 1 + (d*x^(1/3))/e] - 2*(Log[d + e/
x^(1/3)] - Log[e/d + x^(1/3)])*PolyLog[2, -((d*x^(1/3))/e)] + (81*Log[e/d +
 x^(1/3)]^2*Log[-((d*x^(1/3))/e)] + 27*Log[d + e/x^(1/3)]^2*Log[x] - 27*Log
[e/d + x^(1/3)]^2*Log[x] - 54*Log[d + e/x^(1/3)]*Log[1 + (d*x^(1/3))/e]*Log
[x] + 54*Log[e/d + x^(1/3)]*Log[1 + (d*x^(1/3))/e]*Log[x] + 9*Log[d + e/x^(
1/3)]*Log[x]^2 - 9*Log[1 + (d*x^(1/3))/e]*Log[x]^2 + Log[x]^3 - 162*PolyLog
[3, 1 + (d*x^(1/3))/e] - 162*PolyLog[3, -((d*x^(1/3))/e)])/81)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^n))^2/x,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))^2/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x,x, algorithm="maxima")
```

```
[Out] b^2*log((d*x^(1/3) + e)^n)^2*log(x) - integrate(-1/3*(3*(b^2*d*x + b^2*x^(2
/3)*e)*log(x^(1/3*n))^2 + 3*(b^2*log(c)^2 + 2*a*b*log(c) + a^2)*x^(2/3)*e +
 3*(b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d)*x - 2*(b^2*d*n*x*log(x) - 3*(b
^2*log(c) + a*b)*x^(2/3)*e - 3*(b^2*d*log(c) + a*b*d)*x + 3*(b^2*d*x + b^2*
x^(2/3)*e)*log(x^(1/3*n)))*log((d*x^(1/3) + e)^n) - 6*((b^2*log(c) + a*b)*x
^(2/3)*e + (b^2*d*log(c) + a*b*d)*x)*log(x^(1/3*n)))/(d*x^2 + x^(5/3)*e), x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*((d*x + x^(2/3)*e)/x)^n)^2 + 2*a*b*log(c*((d*x + x^(2/3
)*e)/x)^n) + a^2)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\log\left(c\left(d + \frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{2}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))**2/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e/x**(1/3))**n))**2/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)^2/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^n\right)\right)}^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))^2/x,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(1/3))^n))^2/x, x)
```

3.501 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=269

$$\frac{3b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{2e^3}-\frac{2b^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^3}-\frac{6b^2d^2n^2}{e^2\sqrt[3]{x}}+\frac{b^2d^3n^2\log^2\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}+\frac{6bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\cdots\right.}{e^3}$$

```
[Out] 3/2*b^2*d*n^2*(d+e/x^(1/3))^2/e^3-2/9*b^2*n^2*(d+e/x^(1/3))^3/e^3-6*b^2*d^2
*n^2/e^2/x^(1/3)+b^2*d^3*n^2*ln(d+e/x^(1/3))^2/e^3+6*b*d^2*n*(d+e/x^(1/3))*
(a+b*ln(c*(d+e/x^(1/3))^n))/e^3-3*b*d*n*(d+e/x^(1/3))^2*(a+b*ln(c*(d+e/x^(1
/3))^n))/e^3+2/3*b*n*(d+e/x^(1/3))^3*(a+b*ln(c*(d+e/x^(1/3))^n))/e^3-2*b*d^
3*n*ln(d+e/x^(1/3))*(a+b*ln(c*(d+e/x^(1/3))^n))/e^3-(a+b*ln(c*(d+e/x^(1/3))
^n))^2/x
```

Rubi [A]

time = 0.21, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2504, 2445, 2458, 45, 2372, 12, 14, 2338}

$$-\frac{2bd^3n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^3}+\frac{6bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^3}-\frac{3bdn\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^3}+\frac{2bn\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3e^3}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x}+\frac{b^2d^3n^2\log^2\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}-\frac{6b^2d^2n^2}{e^2\sqrt[3]{x}}+\frac{3b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{2e^3}-\frac{2b^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])^2/x^2,x]
```

```
[Out] (3*b^2*d*n^2*(d + e/x^(1/3))^2)/(2*e^3) - (2*b^2*n^2*(d + e/x^(1/3))^3)/(9*
e^3) - (6*b^2*d^2*n^2)/(e^2*x^(1/3)) + (b^2*d^3*n^2*Log[d + e/x^(1/3)]^2)/e
^3 + (6*b*d^2*n*(d + e/x^(1/3))*(a + b*Log[c*(d + e/x^(1/3))^n]))/e^3 - (3*
b*d*n*(d + e/x^(1/3))^2*(a + b*Log[c*(d + e/x^(1/3))^n]))/e^3 + (2*b*n*(d +
 e/x^(1/3))^3*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*e^3) - (2*b*d^3*n*Log[d
+ e/x^(1/3)]*(a + b*Log[c*(d + e/x^(1/3))^n]))/e^3 - (a + b*Log[c*(d + e/x^
(1/3))^n])^2/x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^2}\,dx = -\left(3\text{Subst}\left(\int x^2(a+b\log\left(c(d+ex)^n\right))^2\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x} + (2ben)\text{Subst}\left(\int \frac{x^3(a+b\log(}{d+}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x} + (2bn)\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^3(a}{x}\right.$$

$$= \frac{1}{3}bn\left(\frac{18d^2\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3} - \frac{9d\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{e^3} + \frac{2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{e^3} - \frac{6d}{}\right.$$

$$= \frac{1}{3}bn\left(\frac{18d^2\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3} - \frac{9d\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{e^3} + \frac{2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{e^3} - \frac{6d}{}\right.$$

$$= \frac{1}{3}bn\left(\frac{18d^2\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3} - \frac{9d\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{e^3} + \frac{2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{e^3} - \frac{6d}{}\right.$$

$$= \frac{3b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{2e^3} - \frac{2b^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^3} - \frac{6b^2d^2n^2}{e^2\sqrt[3]{x}} + \frac{1}{3}bn\left(\frac{18d}{}\right.$$

$$= \frac{3b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{2e^3} - \frac{2b^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^3} - \frac{6b^2d^2n^2}{e^2\sqrt[3]{x}} + \frac{b^2d^3n^2\log}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.27, size = 374, normalized size = 1.39

$$\frac{-18\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2+\frac{bn\left(-2ben\left(2e^2-3de\sqrt[3]{x}+6d^2x^{2/3}\right)+9bden\left(e-2d\sqrt[3]{x}\right)\sqrt[3]{x}+36ad^2ex^{2/3}-36bd^2enx^{2/3}+30bd^3nx\log\left(d+\frac{e}{\sqrt[3]{x}}\right)+36bd^2\left(e+d\sqrt[3]{x}\right)x^{2/3}\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)+12e^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-18de^2\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-36d^3x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\log\left(e+d\sqrt[3]{x}\right)-36d^3x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt[3]{x}}\right)+18bd^3nx\log\left(e+d\sqrt[3]{x}\right)\left(\log\left(e+d\sqrt[3]{x}\right)-2\log\left(-\frac{d\sqrt[3]{x}}{e}\right)\right)-36bd^3nx\text{Li}_2\left(1+\frac{e}{d\sqrt[3]{x}}\right)-36bd^3nx\text{Li}_2\left(1+\frac{d\sqrt[3]{x}}{e}\right)\right)}{e^3}}{18x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^2/x^2,x]
```

```
[Out] (-18*(a + b*Log[c*(d + e/x^(1/3))^n])^2 + (b*n*(-2*b*e*n*(2*e^2 - 3*d*e*x^(
1/3) + 6*d^2*x^(2/3)) + 9*b*d*e*n*(e - 2*d*x^(1/3))*x^(1/3) + 36*a*d^2*e*x^
(2/3) - 36*b*d^2*e*n*x^(2/3) + 30*b*d^3*n*x*Log[d + e/x^(1/3)] + 36*b*d^2*(
e + d*x^(1/3))*x^(2/3)*Log[c*(d + e/x^(1/3))^n] + 12*e^3*(a + b*Log[c*(d +
e/x^(1/3))^n]) - 18*d*e^2*x^(1/3)*(a + b*Log[c*(d + e/x^(1/3))^n]) - 36*d^3
*x*(a + b*Log[c*(d + e/x^(1/3))^n])*Log[e + d*x^(1/3)] - 36*d^3*x*(a + b*Lo
g[c*(d + e/x^(1/3))^n])*Log[-(e/(d*x^(1/3)))] + 18*b*d^3*n*x*Log[e + d*x^(1
/3)]*(Log[e + d*x^(1/3)] - 2*Log[-((d*x^(1/3))/e)]) - 36*b*d^3*n*x*PolyLog[
2, 1 + e/(d*x^(1/3))] - 36*b*d^3*n*x*PolyLog[2, 1 + (d*x^(1/3))/e]))/e^3)/(
18*x)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)^{2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^n))^2/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))^2/x^2,x)
```

Maxima [A]
time = 0.29, size = 285, normalized size = 1.06

$$-\frac{1}{3}\left(6\,d^{3}e^{(-4)}\log\left(dx^{\frac{1}{3}}+e\right)-2\,d^{3}e^{(-4)}\log\left(x\right)-\frac{\left(6\,d^{2}x^{\frac{2}{3}}-3\,dx^{\frac{1}{3}}e+2\,e^{2}\right)e^{(-3)}}{x}\right)abne-\frac{1}{18}\left(6\left(6\,d^{3}e^{(-4)}\log\left(dx^{\frac{1}{3}}+e\right)-2\,d^{3}e^{(-4)}\log\left(x\right)-\frac{\left(6\,d^{2}x^{\frac{2}{3}}-3\,dx^{\frac{1}{3}}e+2\,e^{2}\right)e^{(-3)}}{x}\right)ne\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)-\frac{\left(18\,d^{3}x\log\left(dx^{\frac{1}{3}}+e\right)^{2}+2\,d^{3}x\log\left(x\right)^{2}-22\,d^{3}x\log\left(x\right)-66\,d^{2}x^{\frac{2}{3}}e+15\,dx^{\frac{1}{3}}e^{2}-6\left(2\,d^{3}x\log\left(x\right)-11\,d^{3}x\right)\log\left(dx^{\frac{1}{3}}+e\right)-4\,e^{3}\right)n^{2}e^{(-3)}}{x}\right)b^{2}-\frac{b^{2}\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)^{2}}{x}-\frac{2\,ab\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)}{x}-\frac{a^{2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x^2,x, algorithm="maxima")
```

```
[Out] -1/3*(6*d^3*e^(-4)*log(d*x^(1/3) + e) - 2*d^3*e^(-4)*log(x) - (6*d^2*x^(2/3
) - 3*d*x^(1/3)*e + 2*e^2)*e^(-3)/x)*a*b*n*e - 1/18*(6*(6*d^3*e^(-4)*log(d*
x^(1/3) + e) - 2*d^3*e^(-4)*log(x) - (6*d^2*x^(2/3) - 3*d*x^(1/3)*e + 2*e^2
)*e^(-3)/x)*n*e*log(c*(d + e/x^(1/3))^n) - (18*d^3*x*log(d*x^(1/3) + e)^2 +
 2*d^3*x*log(x)^2 - 22*d^3*x*log(x) - 66*d^2*x^(2/3)*e + 15*d*x^(1/3)*e^2 -
 6*(2*d^3*x*log(x) - 11*d^3*x)*log(d*x^(1/3) + e) - 4*e^3)*n^2*e^(-3)/x)*b^
2 - b^2*log(c*(d + e/x^(1/3))^n)^2/x - 2*a*b*log(c*(d + e/x^(1/3))^n)/x - a
^2/x
```

Fricas [A]
time = 0.41, size = 329, normalized size = 1.22

$$\frac{\left(18\,(b^{2}x-b^{2})e^{3}\log\left(c\right)^{2}+12\,(b^{2}n-3\,ab-(b^{2}n-3\,ab)x)e^{3}\log\left(c\right)-18\,(b^{2}d^{3}n^{2}x+b^{2}n^{2}e^{3})\log\left(\frac{dx+x^{\frac{2}{3}}e}{x}\right)^{2}-2\,(2\,b^{2}n^{2}-6\,abn+9\,a^{2}-(2\,b^{2}n^{2}-6\,abn+9\,a^{2})x)e^{3}+6\left(6\,b^{2}d^{2}n^{2}x^{\frac{2}{3}}e-3\,b^{2}dn^{2}x^{\frac{1}{3}}e^{2}+(11\,b^{2}d^{3}n^{2}-6\,abd^{3}n)x+2\,(b^{2}n^{2}-3\,abn)e^{3}-6\,(b^{2}d^{3}nx+b^{2}ne^{3})\log\left(c\right)\right)\log\left(\frac{dx+x^{\frac{2}{3}}e}{x}\right)+6\,(6\,b^{2}d^{2}ne\log\left(c\right)-(11\,b^{2}d^{2}n^{2}-6\,abd^{2}n)e)x^{\frac{2}{3}}-3\,(6\,b^{2}dne^{2}\log\left(c\right)-(5\,b^{2}dn^{2}-6\,abdn)e^{2})x^{\frac{1}{3}}\right)e^{(-3)}}{18\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/18*(18*(b^2*x - b^2)*e^3*log(c)^2 + 12*(b^2*n - 3*a*b - (b^2*n - 3*a*b)*x
)*e^3*log(c) - 18*(b^2*d^3*n^2*x + b^2*n^2*e^3)*log((d*x + x^(2/3)*e)/x)^2
- 2*(2*b^2*n^2 - 6*a*b*n + 9*a^2 - (2*b^2*n^2 - 6*a*b*n + 9*a^2)*x)*e^3 + 6
*(6*b^2*d^2*n^2*x^(2/3)*e - 3*b^2*d*n^2*x^(1/3)*e^2 + (11*b^2*d^3*n^2 - 6*a
*b*d^3*n)*x + 2*(b^2*n^2 - 3*a*b*n)*e^3 - 6*(b^2*d^3*n*x + b^2*n*e^3)*log(c
))*log((d*x + x^(2/3)*e)/x) + 6*(6*b^2*d^2*n*e*log(c) - (11*b^2*d^2*n^2 - 6
*a*b*d^2*n)*e)*x^(2/3) - 3*(6*b^2*d*n*e^2*log(c) - (5*b^2*d*n^2 - 6*a*b*d*n
)*e^2)*x^(1/3))*e^(-3)/x
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\log\left(c\left(d + \frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))**2/x**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e/x**(1/3))**n))**2/x**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 787 vs. 2(241) = 482.
time = 3.83, size = 787, normalized size = 2.93

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x^2,x, algorithm="giac")
```

```
[Out] -1/18*(54*(d*x^(1/3) + e)*b^2*d^2*n^2*log((d*x^(1/3) + e)/x^(1/3))^2/x^(1/3
) - 54*(d*x^(1/3) + e)^2*b^2*d*n^2*log((d*x^(1/3) + e)/x^(1/3))^2/x^(2/3) +
 18*(d*x^(1/3) + e)^3*b^2*n^2*log((d*x^(1/3) + e)/x^(1/3))^2/x - 108*(d*x^(
1/3) + e)*b^2*d^2*n^2*log((d*x^(1/3) + e)/x^(1/3))/x^(1/3) + 108*(d*x^(1/3)
 + e)*b^2*d^2*n*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x^(1/3) + 54*(d*x^(1/3)
 + e)^2*b^2*d*n^2*log((d*x^(1/3) + e)/x^(1/3))/x^(2/3) - 108*(d*x^(1/3) + e
)^2*b^2*d*n*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x^(2/3) - 12*(d*x^(1/3) + e
)^3*b^2*n^2*log((d*x^(1/3) + e)/x^(1/3))/x + 36*(d*x^(1/3) + e)^3*b^2*n*log
(c)*log((d*x^(1/3) + e)/x^(1/3))/x + 108*(d*x^(1/3) + e)*b^2*d^2*n^2/x^(1/3
) - 108*(d*x^(1/3) + e)*b^2*d^2*n*log(c)/x^(1/3) + 54*(d*x^(1/3) + e)*b^2*d
^2*log(c)^2/x^(1/3) + 108*(d*x^(1/3) + e)*a*b*d^2*n*log((d*x^(1/3) + e)/x^(
1/3))/x^(1/3) - 27*(d*x^(1/3) + e)^2*b^2*d*n^2/x^(2/3) + 54*(d*x^(1/3) + e)
^2*b^2*d*n*log(c)/x^(2/3) - 54*(d*x^(1/3) + e)^2*b^2*d*log(c)^2/x^(2/3) - 1
08*(d*x^(1/3) + e)^2*a*b*d*n*log((d*x^(1/3) + e)/x^(1/3))/x^(2/3) + 4*(d*x^
```

```
(1/3) + e)^3*b^2*n^2/x - 12*(d*x^(1/3) + e)^3*b^2*n*log(c)/x + 18*(d*x^(1/3
) + e)^3*b^2*log(c)^2/x + 36*(d*x^(1/3) + e)^3*a*b*n*log((d*x^(1/3) + e)/x^
(1/3))/x - 108*(d*x^(1/3) + e)*a*b*d^2*n/x^(1/3) + 108*(d*x^(1/3) + e)*a*b*
d^2*log(c)/x^(1/3) + 54*(d*x^(1/3) + e)^2*a*b*d*n/x^(2/3) - 108*(d*x^(1/3)
+ e)^2*a*b*d*log(c)/x^(2/3) - 12*(d*x^(1/3) + e)^3*a*b*n/x + 36*(d*x^(1/3)
+ e)^3*a*b*log(c)/x + 54*(d*x^(1/3) + e)*a^2*d^2/x^(1/3) - 54*(d*x^(1/3) +
e)^2*a^2*d/x^(2/3) + 18*(d*x^(1/3) + e)^3*a^2/x)*e^(-3)
```

Mupad [B]
time = 0.56, size = 299, normalized size = 1.11

$$\frac{\frac{d\left(3a^2-2abn+\frac{2b^2n^2}{3}\right)}{2e}-\frac{d\left(3a^2-b^2n^2\right)}{2e}}{x^{2/3}}-\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)^2\left(\frac{b^2}{x}+\frac{b^2d^3}{e^3}\right)-\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)\left(\frac{2b(3a-bn)}{3x}-\frac{\frac{bd(3a-bn)}{e}-\frac{3abd}{e}}{x^{2/3}}+\frac{d\left(\frac{2bd(3a-bn)}{e}-\frac{6abd}{e}\right)}{e\,x^{1/3}}\right)-\frac{\frac{d\left(\frac{d\left(3a^2-2abn+\frac{2b^2n^2}{3}\right)}{e}-\frac{d\left(3a^2-b^2n^2\right)}{e}\right)}{e}+\frac{2b^2d^2n^2}{e^2}}{x^{1/3}}-\frac{a^2-\frac{2abn}{3}+\frac{2b^2n^2}{9}}{x}+\frac{\ln\left(d+\frac{e}{x^{1/3}}\right)\left(11b^2d^3n^2-6abd^3n\right)}{3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))^2/x^2,x)
```

```
[Out] ((d*(3*a^2 + (2*b^2*n^2)/3 - 2*a*b*n))/(2*e) - (d*(3*a^2 - b^2*n^2))/(2*e))
/x^(2/3) - log(c*(d + e/x^(1/3))^n)^2*(b^2/x + (b^2*d^3)/e^3) - log(c*(d +
e/x^(1/3))^n)*((2*b*(3*a - b*n))/(3*x) - ((b*d*(3*a - b*n))/e - (3*a*b*d)/e
)/x^(2/3) + (d*((2*b*d*(3*a - b*n))/e - (6*a*b*d)/e))/(e*x^(1/3))) - ((d*((
d*(3*a^2 + (2*b^2*n^2)/3 - 2*a*b*n))/e - (d*(3*a^2 - b^2*n^2))/e))/e + (2*b
^2*d^2*n^2)/e^2)/x^(1/3) - (a^2 + (2*b^2*n^2)/9 - (2*a*b*n)/3)/x + (log(d +
 e/x^(1/3))*(11*b^2*d^3*n^2 - 6*a*b*d^3*n))/(3*e^3)
```

3.502 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=479

$$-\frac{15b^2d^4n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{4e^6}+\frac{20b^2d^3n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^6}-\frac{15b^2d^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^4}{16e^6}+\frac{6b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^5}{25e^6}-\frac{b^2n^2\left(d+\right.}{36e}$$

```
[Out] -15/4*b^2*d^4*n^2*(d+e/x^(1/3))^2/e^6+20/9*b^2*d^3*n^2*(d+e/x^(1/3))^3/e^6-
15/16*b^2*d^2*n^2*(d+e/x^(1/3))^4/e^6+6/25*b^2*d*n^2*(d+e/x^(1/3))^5/e^6-1/
36*b^2*n^2*(d+e/x^(1/3))^6/e^6+6*b^2*d^5*n^2/e^5/x^(1/3)-1/2*b^2*d^6*n^2*ln
(d+e/x^(1/3))^2/e^6-6*b*d^5*n*(d+e/x^(1/3))*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6
+15/2*b*d^4*n*(d+e/x^(1/3))^2*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6-20/3*b*d^3*n*
(d+e/x^(1/3))^3*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6+15/4*b*d^2*n*(d+e/x^(1/3))^
4*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6-6/5*b*d*n*(d+e/x^(1/3))^5*(a+b*ln(c*(d+e/
x^(1/3))^n))/e^6+1/6*b*n*(d+e/x^(1/3))^6*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6+b*
d^6*n*ln(d+e/x^(1/3))*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6-1/2*(a+b*ln(c*(d+e/x^
(1/3))^n))^2/x^2
```

Rubi [A]
time = 0.32, antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2504, 2445, 2458, 45, 2372, 12, 14, 2338}

$$\frac{bd^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^6}-\frac{6bd^5n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^6}+\frac{15bd^4n\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{2e^6}-\frac{20bd^3n\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3e^6}+\frac{15bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{4e^6}-\frac{6bdn\left(d+\frac{e}{\sqrt[3]{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{5e^6}+\frac{bn\left(d+\frac{e}{\sqrt[3]{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{6e^6}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2x^2}-\frac{b^2d^6n^2\log^2\left(d+\frac{e}{\sqrt[3]{x}}\right)}{2e^6}+\frac{6b^2d^5n^2}{e^5\sqrt[3]{x}}-\frac{15b^2d^4n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{4e^6}+\frac{20b^2d^3n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^6}-\frac{15b^2d^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^4}{16e^6}+\frac{6b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^5}{25e^6}-\frac{b^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^6}{36e^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])^2/x^3,x]
```

```
[Out] (-15*b^2*d^4*n^2*(d + e/x^(1/3))^2)/(4*e^6) + (20*b^2*d^3*n^2*(d + e/x^(1/3
))^3)/(9*e^6) - (15*b^2*d^2*n^2*(d + e/x^(1/3))^4)/(16*e^6) + (6*b^2*d*n^2*
(d + e/x^(1/3))^5)/(25*e^6) - (b^2*n^2*(d + e/x^(1/3))^6)/(36*e^6) + (6*b^2
*d^5*n^2)/(e^5*x^(1/3)) - (b^2*d^6*n^2*Log[d + e/x^(1/3)]^2)/(2*e^6) - (6*b
*d^5*n*(d + e/x^(1/3))*(a + b*Log[c*(d + e/x^(1/3))^n]))/e^6 + (15*b*d^4*n*
(d + e/x^(1/3))^2*(a + b*Log[c*(d + e/x^(1/3))^n]))/(2*e^6) - (20*b*d^3*n*(
d + e/x^(1/3))^3*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*e^6) + (15*b*d^2*n*(d
 + e/x^(1/3))^4*(a + b*Log[c*(d + e/x^(1/3))^n]))/(4*e^6) - (6*b*d*n*(d + e
/x^(1/3))^5*(a + b*Log[c*(d + e/x^(1/3))^n]))/(5*e^6) + (b*n*(d + e/x^(1/3)
)^6*(a + b*Log[c*(d + e/x^(1/3))^n]))/(6*e^6) + (b*d^6*n*Log[d + e/x^(1/3)]
*(a + b*Log[c*(d + e/x^(1/3))^n]))/e^6 - (a + b*Log[c*(d + e/x^(1/3))^n])^2
/(2*x^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^3}\,dx = -\left(3\text{Subst}\left(\int x^5(a+b\log(c(d+ex)^n))^2\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2x^2} + (ben)\text{Subst}\left(\int \frac{x^6(a+b\log(c(\cdots}{d+ex\cdots}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2x^2} + (bn)\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^6(a+\cdots}{x}\right.$$

$$= -\frac{1}{60}bn\left(\frac{360d^5\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^6} - \frac{450d^4\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{e^6} + \frac{400d^3\left(d+\frac{e}{\sqrt[3]{x}}\cdots\right.}{e^6}\right.$$

$$= -\frac{1}{60}bn\left(\frac{360d^5\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^6} - \frac{450d^4\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{e^6} + \frac{400d^3\left(d+\frac{e}{\sqrt[3]{x}}\cdots\right.}{e^6}\right.$$

$$= -\frac{1}{60}bn\left(\frac{360d^5\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^6} - \frac{450d^4\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{e^6} + \frac{400d^3\left(d+\frac{e}{\sqrt[3]{x}}\cdots\right.}{e^6}\right.$$

$$= -\frac{15b^2d^4n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{4e^6} + \frac{20b^2d^3n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^6} - \frac{15b^2d^2n^2\left(d+\cdots\right.}{16e^6}$$

$$= -\frac{15b^2d^4n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{4e^6} + \frac{20b^2d^3n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^6} - \frac{15b^2d^2n^2\left(d+\cdots\right.}{16e^6}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.24, size = 698, normalized size = 1.46

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^2/x^3,x]
```

```
[Out] (-1800*a^2*e^6 + 600*a*b*e^6*n - 100*b^2*e^6*n^2 - 720*a*b*d*e^5*n*x^(1/3)
+ 264*b^2*d*e^5*n^2*x^(1/3) + 900*a*b*d^2*e^4*n*x^(2/3) - 555*b^2*d^2*e^4*n
^2*x^(2/3) - 1200*a*b*d^3*e^3*n*x + 1140*b^2*d^3*e^3*n^2*x + 1800*a*b*d^4*e
^2*n*x^(4/3) - 2610*b^2*d^4*e^2*n^2*x^(4/3) - 3600*a*b*d^5*e*n*x^(5/3) + 88
20*b^2*d^5*e*n^2*x^(5/3) - 5220*b^2*d^6*n^2*x^2*Log[d + e/x^(1/3)] - 3600*a
*b*e^6*Log[c*(d + e/x^(1/3))^n] + 600*b^2*e^6*n*Log[c*(d + e/x^(1/3))^n] -
720*b^2*d*e^5*n*x^(1/3)*Log[c*(d + e/x^(1/3))^n] + 900*b^2*d^2*e^4*n*x^(2/3
)*Log[c*(d + e/x^(1/3))^n] - 1200*b^2*d^3*e^3*n*x*Log[c*(d + e/x^(1/3))^n]
+ 1800*b^2*d^4*e^2*n*x^(4/3)*Log[c*(d + e/x^(1/3))^n] - 3600*b^2*d^5*e*n*x^
(5/3)*Log[c*(d + e/x^(1/3))^n] - 3600*b^2*d^6*n*x^2*Log[c*(d + e/x^(1/3))^n
] - 1800*b^2*e^6*Log[c*(d + e/x^(1/3))^n]^2 + 3600*a*b*d^6*n*x^2*Log[e + d*
x^(1/3)] + 3600*b^2*d^6*n*x^2*Log[c*(d + e/x^(1/3))^n]*Log[e + d*x^(1/3)] -
 1800*b^2*d^6*n^2*x^2*Log[e + d*x^(1/3)]^2 + 3600*a*b*d^6*n*x^2*Log[-(e/(d*
x^(1/3)))] + 3600*b^2*d^6*n*x^2*Log[c*(d + e/x^(1/3))^n]*Log[-(e/(d*x^(1/3)
))] + 3600*b^2*d^6*n^2*x^2*Log[e + d*x^(1/3)]*Log[-((d*x^(1/3))/e)] + 3600*
b^2*d^6*n^2*x^2*PolyLog[2, 1 + e/(d*x^(1/3))] + 3600*b^2*d^6*n^2*x^2*PolyLo
g[2, 1 + (d*x^(1/3))/e])/(3600*e^6*x^2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)^{2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^n))^2/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))^2/x^3,x)
```

Maxima [A]
time = 0.30, size = 379, normalized size = 0.79

$$\frac{1}{60}\left(60\,d^{6}e^{(-7)}\log\left(dx^{\frac{1}{3}}+e\right)-20\,d^{6}e^{(-7)}\log\left(x\right)-\frac{\left(60\,d^{5}x^{\frac{5}{3}}-30\,d^{4}x^{\frac{4}{3}}e+20\,d^{3}xe^{2}-15\,d^{2}x^{\frac{2}{3}}e^{3}+12\,dx^{\frac{1}{3}}e^{4}-10\,e^{5}\right)e^{(-6)}}{x^{2}}\right)abne+\frac{1}{3600}\left(60\left(60\,d^{6}e^{(-7)}\log\left(dx^{\frac{1}{3}}+e\right)-20\,d^{6}e^{(-7)}\log\left(x\right)-\frac{\left(60\,d^{5}x^{\frac{5}{3}}-30\,d^{4}x^{\frac{4}{3}}e+20\,d^{3}xe^{2}-15\,d^{2}x^{\frac{2}{3}}e^{3}+12\,dx^{\frac{1}{3}}e^{4}-10\,e^{5}\right)e^{(-6)}}{x^{2}}\right)ne\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)-\frac{\left(1800\,d^{6}x^{2}\log\left(dx^{\frac{1}{3}}+e\right)^{2}+200\,d^{6}x^{2}\log\left(x\right)^{2}-2940\,d^{6}x^{2}\log\left(x\right)-8820\,d^{5}x^{\frac{5}{3}}e+2610\,d^{4}x^{\frac{4}{3}}e^{2}-1140\,d^{3}xe^{3}+555\,d^{2}x^{\frac{2}{3}}e^{4}-264\,dx^{\frac{1}{3}}e^{5}-60\left(20\,d^{6}x^{2}\log\left(x\right)-147\,d^{6}x^{2}\right)\log\left(dx^{\frac{1}{3}}+e\right)+100\,e^{6}\right)n^{2}e^{(-6)}}{x^{2}}\right)b^{2}-\frac{b^{2}\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)^{2}}{2\,x^{2}}-\frac{ab\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)}{x^{2}}-\frac{a^{2}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/60*(60*d^6*e^(-7)*log(d*x^(1/3) + e) - 20*d^6*e^(-7)*log(x) - (60*d^5*x^(
5/3) - 30*d^4*x^(4/3)*e + 20*d^3*x*e^2 - 15*d^2*x^(2/3)*e^3 + 12*d*x^(1/3)*
e^4 - 10*e^5)*e^(-6)/x^2)*a*b*n*e + 1/3600*(60*(60*d^6*e^(-7)*log(d*x^(1/3)
 + e) - 20*d^6*e^(-7)*log(x) - (60*d^5*x^(5/3) - 30*d^4*x^(4/3)*e + 20*d^3*
x*e^2 - 15*d^2*x^(2/3)*e^3 + 12*d*x^(1/3)*e^4 - 10*e^5)*e^(-6)/x^2)*n*e*log
(c*(d + e/x^(1/3))^n) - (1800*d^6*x^2*log(d*x^(1/3) + e)^2 + 200*d^6*x^2*lo
g(x)^2 - 2940*d^6*x^2*log(x) - 8820*d^5*x^(5/3)*e + 2610*d^4*x^(4/3)*e^2 -
```

```
1140*d^3*x*e^3 + 555*d^2*x^(2/3)*e^4 - 264*d*x^(1/3)*e^5 - 60*(20*d^6*x^2*l
og(x) - 147*d^6*x^2)*log(d*x^(1/3) + e) + 100*e^6)*n^2*e^(-6)/x^2)*b^2 - 1/
2*b^2*log(c*(d + e/x^(1/3))^n)^2/x^2 - a*b*log(c*(d + e/x^(1/3))^n)/x^2 - 1
/2*a^2/x^2
```

Fricas [A]
time = 0.50, size = 550, normalized size = 1.15

$$\frac{\left(1800\,(b^2x^2-b^2)e^6\log(c)^2+1800\,(b^2d^6n^2x^2-b^2n^2e^6)\log\left(\frac{dx+x^{\frac{2}{3}}e}{x}\right)^2-100\,(b^2n^2-6\,abn-(b^2n^2-6\,abn+18\,a^2)x^2+18\,a^2)e^6-60\,((19\,b^2d^3n^2-20\,abd^3n)x^2-(19\,b^2d^3n^2-20\,abd^3n)x)e^3+600\,((b^2n-(b^2n-6\,ab)x^2-6\,ab)e^6+2\,(b^2d^3nx^2-b^2d^3nx)e^3)\log(c)-60\left(20\,b^2d^3n^2xe^3+3\,(49\,b^2d^6n^2-20\,abd^6n)x^2-10\,(b^2n^2-6\,abn)e^6-60\,(b^2d^6nx^2-b^2ne^6)\log(c)+15\,(4\,b^2d^5n^2xe-b^2d^2n^2e^4)x^{\frac{2}{3}}-6\,(5\,b^2d^4n^2xe^2-2\,b^2dn^2e^5)x^{\frac{1}{3}}\right)\log\left(\frac{dx+x^{\frac{2}{3}}e}{x}\right)+15\,(12\,(49\,b^2d^5n^2-20\,abd^5n)xe-(37\,b^2d^2n^2-60\,abd^2n)e^4-60\,(4\,b^2d^5nxe-b^2d^2ne^4)\log(c))x^{\frac{2}{3}}-6\,(15\,(29\,b^2d^4n^2-20\,abd^4n)xe^2-4\,(11\,b^2dn^2-30\,abdn)e^5-60\,(5\,b^2d^4nxe^2-2\,b^2dne^5)\log(c))x^{\frac{1}{3}}\right)e^{(-6)}}{3600\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/3600*(1800*(b^2*x^2 - b^2)*e^6*log(c)^2 + 1800*(b^2*d^6*n^2*x^2 - b^2*n^2
*e^6)*log((d*x + x^(2/3)*e)/x)^2 - 100*(b^2*n^2 - 6*a*b*n - (b^2*n^2 - 6*a*
b*n + 18*a^2)*x^2 + 18*a^2)*e^6 - 60*((19*b^2*d^3*n^2 - 20*a*b*d^3*n)*x^2 -
 (19*b^2*d^3*n^2 - 20*a*b*d^3*n)*x)*e^3 + 600*((b^2*n - (b^2*n - 6*a*b)*x^2
 - 6*a*b)*e^6 + 2*(b^2*d^3*n*x^2 - b^2*d^3*n*x)*e^3)*log(c) - 60*(20*b^2*d^
3*n^2*x*e^3 + 3*(49*b^2*d^6*n^2 - 20*a*b*d^6*n)*x^2 - 10*(b^2*n^2 - 6*a*b*n
)*e^6 - 60*(b^2*d^6*n*x^2 - b^2*n*e^6)*log(c) + 15*(4*b^2*d^5*n^2*x*e - b^2
*d^2*n^2*e^4)*x^(2/3) - 6*(5*b^2*d^4*n^2*x*e^2 - 2*b^2*d*n^2*e^5)*x^(1/3))*
log((d*x + x^(2/3)*e)/x) + 15*(12*(49*b^2*d^5*n^2 - 20*a*b*d^5*n)*x*e - (37
*b^2*d^2*n^2 - 60*a*b*d^2*n)*e^4 - 60*(4*b^2*d^5*n*x*e - b^2*d^2*n*e^4)*log
(c))*x^(2/3) - 6*(15*(29*b^2*d^4*n^2 - 20*a*b*d^4*n)*x*e^2 - 4*(11*b^2*d*n^
2 - 30*a*b*d*n)*e^5 - 60*(5*b^2*d^4*n*x*e^2 - 2*b^2*d*n*e^5)*log(c))*x^(1/3
))*e^(-6)/x^2
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))**2/x**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1639 vs. 2(418) = 836.
time = 5.46, size = 1639, normalized size = 3.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x^3,x, algorithm="giac")
```

[Out] 1/3600*(10800*(d*x^(1/3) + e)*b^2*d^5*n^2*log((d*x^(1/3) + e)/x^(1/3))^2/x^(1/3) - 27000*(d*x^(1/3) + e)^2*b^2*d^4*n^2*log((d*x^(1/3) + e)/x^(1/3))^2/x^(2/3) + 36000*(d*x^(1/3) + e)^3*b^2*d^3*n^2*log((d*x^(1/3) + e)/x^(1/3))^2/x - 21600*(d*x^(1/3) + e)*b^2*d^5*n^2*log((d*x^(1/3) + e)/x^(1/3))/x^(1/3) + 21600*(d*x^(1/3) + e)*b^2*d^5*n*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x^(1/3) - 27000*(d*x^(1/3) + e)^4*b^2*d^2*n^2*log((d*x^(1/3) + e)/x^(1/3))^2/x^(4/3) + 27000*(d*x^(1/3) + e)^2*b^2*d^4*n^2*log((d*x^(1/3) + e)/x^(1/3))/x^(2/3) - 54000*(d*x^(1/3) + e)^2*b^2*d^4*n*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x^(2/3) + 10800*(d*x^(1/3) + e)^5*b^2*d*n^2*log((d*x^(1/3) + e)/x^(1/3))^2/x^(5/3) - 24000*(d*x^(1/3) + e)^3*b^2*d^3*n^2*log((d*x^(1/3) + e)/x^(1/3))/x + 72000*(d*x^(1/3) + e)^3*b^2*d^3*n*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x - 1800*(d*x^(1/3) + e)^6*b^2*n^2*log((d*x^(1/3) + e)/x^(1/3))^2/x^2 + 21600*(d*x^(1/3) + e)*b^2*d^5*n^2/x^(1/3) - 21600*(d*x^(1/3) + e)*b^2*d^5*n*log(c)/x^(1/3) + 10800*(d*x^(1/3) + e)*b^2*d^5*log(c)^2/x^(1/3) + 13500*(d*x^(1/3) + e)^4*b^2*d^2*n^2*log((d*x^(1/3) + e)/x^(1/3))/x^(4/3) + 21600*(d*x^(1/3) + e)*a*b*d^5*n*log((d*x^(1/3) + e)/x^(1/3))/x^(1/3) - 54000*(d*x^(1/3) + e)^4*b^2*d^2*n*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x^(4/3) - 13500*(d*x^(1/3) + e)^2*b^2*d^4*n^2/x^(2/3) + 27000*(d*x^(1/3) + e)^2*b^2*d^4*n*log(c)/x^(2/3) - 27000*(d*x^(1/3) + e)^2*b^2*d^4*log(c)^2/x^(2/3) - 4320*(d*x^(1/3) + e)^5*b^2*d*n^2*log((d*x^(1/3) + e)/x^(1/3))/x^(5/3) - 54000*(d*x^(1/3) + e)^2*a*b*d^4*n*log((d*x^(1/3) + e)/x^(1/3))/x^(2/3) + 21600*(d*x^(1/3) + e)^5*b^2*d*n*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x^(5/3) + 8000*(d*x^(1/3) + e)^3*b^2*d^3*n^2/x - 24000*(d*x^(1/3) + e)^3*b^2*d^3*n*log(c)/x + 36000*(d*x^(1/3) + e)^3*b^2*d^3*log(c)^2/x + 600*(d*x^(1/3) + e)^6*b^2*n^2*log((d*x^(1/3) + e)/x^(1/3))/x^2 + 72000*(d*x^(1/3) + e)^3*a*b*d^3*n*log((d*x^(1/3) + e)/x^(1/3))/x - 3600*(d*x^(1/3) + e)^6*b^2*n*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x^2 - 3375*(d*x^(1/3) + e)^4*b^2*d^2*n^2/x^(4/3) - 21600*(d*x^(1/3) + e)*a*b*d^5*n/x^(1/3) + 13500*(d*x^(1/3) + e)^4*b^2*d^2*n*log(c)/x^(4/3) + 21600*(d*x^(1/3) + e)*a*b*d^5*log(c)/x^(1/3) - 27000*(d*x^(1/3) + e)^4*b^2*d^2*log(c)^2/x^(4/3) - 54000*(d*x^(1/3) + e)^4*a*b*d^2*n*log((d*x^(1/3) + e)/x^(1/3))/x^(4/3) + 864*(d*x^(1/3) + e)^5*b^2*d*n^2/x^(5/3) + 27000*(d*x^(1/3) + e)^2*a*b*d^4*n/x^(2/3) - 4320*(d*x^(1/3) + e)^5*b^2*d*n*log(c)/x^(5/3) - 54000*(d*x^(1/3) + e)^2*a*b*d^4*log(c)/x^(2/3) + 10800*(d*x^(1/3) + e)^5*b^2*d*log(c)^2/x^(5/3) + 21600*(d*x^(1/3) + e)^5*a*b*d*n*log((d*x^(1/3) + e)/x^(1/3))/x^(5/3) - 100*(d*x^(1/3) + e)^6*b^2*n^2/x^2 - 24000*(d*x^(1/3) + e)^3*a*b*d^3*n/x + 600*(d*x^(1/3) + e)^6*b^2*n*log(c)/x^2 + 72000*(d*x^(1/3) + e)^3*a*b*d^3*log(c)/x - 1800*(d*x^(1/3) + e)^6*b^2*log(c)^2/x^2 - 3600*(d*x^(1/3) + e)^6*a*b*n*log((d*x^(1/3) + e)/x^(1/3))/x^2 + 13500*(d*x^(1/3) + e)^4*a*b*d^2*n/x^(4/3) + 10800*(d*x^(1/3) + e)*a^2*d^5/x^(1/3) - 54000*(d*x^(1/3) + e)^4*a*b*d^2*log(c)/x^(4/3) - 4320*(d*x^(1/3) + e)^5*a*b*d*n/x^(5/3) - 27000*(d*x^(1/3) + e)^2*a^2*d^4/x^(2/3) + 21600*(d*x^(1/3) + e)^5*a*b*d*log(c)/x^(5/3) + 600*(d*x^(1/3) + e)^6*a*b*n/x^2 + 36000*(d*x^(1/3) + e)^3*a^2*d^3/x - 3600*(d*x^(1/3) + e)^6*a*b*log(c)/x^2 - 27000*(d*x^(1/3) + e)^4*a^2*d^2/x^(4/3) + 10800*(d*x^(1/3) + e)^5*a^2*d/x^(5/3) - 1800*(d*x^(1/3) + e)^6*a^2/x^2)*e^(-6)

Mupad [B]
time = 1.76, size = 439, normalized size = 0.92

$$\frac{b^2\,d^6\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^n\right)^2}{2\,e^6}-\frac{b^2\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^n\right)^2}{2\,x^2}-\frac{b^2\,n^2}{36\,x^2}-\frac{a\,b\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^n\right)}{x^2}-\frac{a^2}{2\,x^2}+\frac{a\,b\,n}{6\,x^2}+\frac{b^2\,n\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^n\right)}{6\,x^2}-\frac{49\,b^2\,d^6\,n^2\,\ln\left(d+\frac{e}{x^{1/3}}\right)}{20\,e^6}+\frac{19\,b^2\,d^3\,n^2}{60\,e^3\,x}-\frac{37\,b^2\,d^2\,n^2}{240\,e^2\,x^{4/3}}-\frac{29\,b^2\,d^4\,n^2}{40\,e^4\,x^{2/3}}+\frac{49\,b^2\,d^5\,n^2}{20\,e^5\,x^{1/3}}+\frac{11\,b^2\,d\,n^2}{150\,e\,x^{5/3}}-\frac{b^2\,d^3\,n\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^n\right)}{3\,e^3\,x}+\frac{b^2\,d^2\,n\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^n\right)}{4\,e^2\,x^{4/3}}+\frac{b^2\,d^4\,n\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^n\right)}{2\,e^4\,x^{2/3}}-\frac{b^2\,d^5\,n\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^n\right)}{e^5\,x^{1/3}}-\frac{a\,b\,d\,n}{5\,e\,x^{5/3}}+\frac{a\,b\,d^6\,n\,\ln\left(d+\frac{e}{x^{1/3}}\right)}{e^6}-\frac{b^2\,d\,n\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^n\right)}{5\,e\,x^{5/3}}-\frac{a\,b\,d^3\,n}{3\,e^3\,x}+\frac{a\,b\,d^2\,n}{4\,e^2\,x^{4/3}}+\frac{a\,b\,d^4\,n}{2\,e^4\,x^{2/3}}-\frac{a\,b\,d^5\,n}{e^5\,x^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))^2/x^3,x)
```

```
[Out] (b^2*d^6*log(c*(d + e/x^(1/3))^n)^2)/(2*e^6) - (b^2*log(c*(d + e/x^(1/3))^n
)^2)/(2*x^2) - (b^2*n^2)/(36*x^2) - (a*b*log(c*(d + e/x^(1/3))^n))/x^2 - a^
2/(2*x^2) + (a*b*n)/(6*x^2) + (b^2*n*log(c*(d + e/x^(1/3))^n))/(6*x^2) - (4
9*b^2*d^6*n^2*log(d + e/x^(1/3)))/(20*e^6) + (19*b^2*d^3*n^2)/(60*e^3*x) -
(37*b^2*d^2*n^2)/(240*e^2*x^(4/3)) - (29*b^2*d^4*n^2)/(40*e^4*x^(2/3)) + (4
9*b^2*d^5*n^2)/(20*e^5*x^(1/3)) + (11*b^2*d*n^2)/(150*e*x^(5/3)) - (b^2*d^3
*n*log(c*(d + e/x^(1/3))^n))/(3*e^3*x) + (b^2*d^2*n*log(c*(d + e/x^(1/3))^n
))/(4*e^2*x^(4/3)) + (b^2*d^4*n*log(c*(d + e/x^(1/3))^n))/(2*e^4*x^(2/3)) -
 (b^2*d^5*n*log(c*(d + e/x^(1/3))^n))/(e^5*x^(1/3)) - (a*b*d*n)/(5*e*x^(5/3
)) + (a*b*d^6*n*log(d + e/x^(1/3)))/e^6 - (b^2*d*n*log(c*(d + e/x^(1/3))^n)
)/(5*e*x^(5/3)) - (a*b*d^3*n)/(3*e^3*x) + (a*b*d^2*n)/(4*e^2*x^(4/3)) + (a*
b*d^4*n)/(2*e^4*x^(2/3)) - (a*b*d^5*n)/(e^5*x^(1/3))
```

3.503 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=759

$$\frac{71b^3e^5n^3\sqrt[3]{x}}{40d^5}-\frac{3b^3e^4n^3x^{2/3}}{10d^4}+\frac{b^3e^3n^3x}{20d^3}-\frac{71b^3e^6n^3\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{40d^6}-\frac{77b^2e^5n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(\right.\right.\right.}{20d^6}$$

```
[Out] 71/40*b^3*e^5*n^3*x^(1/3)/d^5-3/10*b^3*e^4*n^3*x^(2/3)/d^4+1/20*b^3*e^3*n^3
*x/d^3-71/40*b^3*e^6*n^3*ln(d+e/x^(1/3))/d^6-77/20*b^2*e^5*n^2*(d+e/x^(1/3)
)*x^(1/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^6+47/40*b^2*e^4*n^2*x^(2/3)*(a+b*ln
(c*(d+e/x^(1/3))^n))/d^4-9/20*b^2*e^3*n^2*x*(a+b*ln(c*(d+e/x^(1/3))^n))/d^3
+3/20*b^2*e^2*n^2*x^(4/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^2-77/20*b^2*e^6*n^2
*ln(1-d/(d+e/x^(1/3)))*(a+b*ln(c*(d+e/x^(1/3))^n))/d^6+3/2*b*e^5*n*(d+e/x^(
1/3))*x^(1/3)*(a+b*ln(c*(d+e/x^(1/3))^n))^2/d^6-3/4*b*e^4*n*x^(2/3)*(a+b*ln
(c*(d+e/x^(1/3))^n))^2/d^4+1/2*b*e^3*n*x*(a+b*ln(c*(d+e/x^(1/3))^n))^2/d^3-
3/8*b*e^2*n*x^(4/3)*(a+b*ln(c*(d+e/x^(1/3))^n))^2/d^2+3/10*b*e*n*x^(5/3)*(a
+b*ln(c*(d+e/x^(1/3))^n))^2/d+3/2*b*e^6*n*ln(1-d/(d+e/x^(1/3)))*(a+b*ln(c*(
d+e/x^(1/3))^n))^2/d^6+1/2*x^2*(a+b*ln(c*(d+e/x^(1/3))^n))^3-3*b^2*e^6*n^2*
(a+b*ln(c*(d+e/x^(1/3))^n))*ln(-e/d/x^(1/3))/d^6-15/8*b^3*e^6*n^3*ln(x)/d^6
+77/20*b^3*e^6*n^3*polylog(2,d/(d+e/x^(1/3)))/d^6-3*b^2*e^6*n^2*(a+b*ln(c*(
d+e/x^(1/3))^n))*polylog(2,d/(d+e/x^(1/3)))/d^6-3*b^3*e^6*n^3*polylog(2,1+e
/d/x^(1/3))/d^6-3*b^3*e^6*n^3*polylog(3,d/(d+e/x^(1/3)))/d^6
```

Rubi [A]
time = 1.75, antiderivative size = 759, normalized size of antiderivative = 1.00, number of steps used = 62, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {2504, 2445, 2458, 2389, 2379, 2421, 6724, 2355, 2354, 2438, 2356, 2351, 31, 46}

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e/x^(1/3))^n])^3,x]
```

```
[Out] (71*b^3*e^5*n^3*x^(1/3))/(40*d^5) - (3*b^3*e^4*n^3*x^(2/3))/(10*d^4) + (b^3
*e^3*n^3*x)/(20*d^3) - (71*b^3*e^6*n^3*Log[d + e/x^(1/3)])/(40*d^6) - (77*b
^2*e^5*n^2*(d + e/x^(1/3))*x^(1/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(20*d^
6) + (47*b^2*e^4*n^2*x^(2/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(40*d^4) - (
9*b^2*e^3*n^2*x*(a + b*Log[c*(d + e/x^(1/3))^n]))/(20*d^3) + (3*b^2*e^2*n^2
*x^(4/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(20*d^2) - (77*b^2*e^6*n^2*Log[1
 - d/(d + e/x^(1/3))]*(a + b*Log[c*(d + e/x^(1/3))^n]))/(20*d^6) + (3*b*e^5
```

```
*n*(d + e/x^(1/3))*x^(1/3)*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(2*d^6) - (3
*b*e^4*n*x^(2/3)*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(4*d^4) + (b*e^3*n*x*(
a + b*Log[c*(d + e/x^(1/3))^n])^2)/(2*d^3) - (3*b*e^2*n*x^(4/3)*(a + b*Log[
c*(d + e/x^(1/3))^n])^2)/(8*d^2) + (3*b*e*n*x^(5/3)*(a + b*Log[c*(d + e/x^(
1/3))^n])^2)/(10*d) + (3*b*e^6*n*Log[1 - d/(d + e/x^(1/3))]*(a + b*Log[c*(d
 + e/x^(1/3))^n])^2)/(2*d^6) + (x^2*(a + b*Log[c*(d + e/x^(1/3))^n])^3)/2 -
 (3*b^2*e^6*n^2*(a + b*Log[c*(d + e/x^(1/3))^n])*Log[-(e/(d*x^(1/3)))])/d^6
 - (15*b^3*e^6*n^3*Log[x])/(8*d^6) + (77*b^3*e^6*n^3*PolyLog[2, d/(d + e/x^
(1/3))])/(20*d^6) - (3*b^2*e^6*n^2*(a + b*Log[c*(d + e/x^(1/3))^n])*PolyLog
[2, d/(d + e/x^(1/3))])/d^6 - (3*b^3*e^6*n^3*PolyLog[2, 1 + e/(d*x^(1/3))])
/d^6 - (3*b^3*e^6*n^3*PolyLog[3, d/(d + e/x^(1/3))])/d^6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
```

```
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3dx=-\left(3\text{Subst}\left(\int\frac{(a+b\log(c(d+ex)^n))^3}{x^7}dx,x,\frac{1}{\sqrt[3]{x}}\right)\right)$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3-\frac{1}{2}(3ben)\text{Subst}\left(\int\frac{(a+b\log}{x}\right.$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3-\frac{1}{2}(3bn)\text{Subst}\left(\int\frac{(a+b\log}{x\left(-\frac{d}{e}\right.}\right.$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3-\frac{(3bn)\text{Subst}\left(\int\frac{(a+b\log(cx^n))}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\right.}{2d}$$

$$=\frac{3benx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{10d}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\right.\right.\right.$$

$$=-\frac{3be^2nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{8d^2}+\frac{3benx^{5/3}\left(a+b\log\right.}{}$$

$$=\frac{3b^2e^2n^2x^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{20d^2}+\frac{be^3nx\left(a+b\log\left(c\right.\right.}{2d}$$

$$=-\frac{9b^2e^3n^2x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{20d^3}+\frac{3b^2e^2n^2x^{4/3}\left(a+b\log\right.}{2}$$

$$=\frac{3b^3e^5n^3\sqrt[3]{x}}{20d^5}-\frac{3b^3e^4n^3x^{2/3}}{40d^4}+\frac{b^3e^3n^3x}{20d^3}-\frac{3b^3e^6n^3\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{20d^6}$$

$$=\frac{3b^3e^5n^3\sqrt[3]{x}}{5d^5}-\frac{3b^3e^4n^3x^{2/3}}{10d^4}+\frac{b^3e^3n^3x}{20d^3}-\frac{3b^3e^6n^3\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{5d^6}$$

$$=\frac{71b^3e^5n^3\sqrt[3]{x}}{40d^5}-\frac{3b^3e^4n^3x^{2/3}}{10d^4}+\frac{b^3e^3n^3x}{20d^3}-\frac{71b^3e^6n^3\log\left(d+\frac{e}{\sqrt[3]{x}}\right.}{40d^6}$$

$$=\frac{71b^3e^5n^3\sqrt[3]{x}}{40d^5}-\frac{3b^3e^4n^3x^{2/3}}{10d^4}+\frac{b^3e^3n^3x}{20d^3}-\frac{71b^3e^6n^3\log\left(d+\frac{e}{\sqrt[3]{x}}\right.}{40d^6}$$

Mathematica [A]
time = 1.03, size = 1006, normalized size = 1.33

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(1/3))^n])^3,x]
```

```
[Out] (60*b*d*e^5*n*x^(1/3)*(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))
^n])^2 - 30*b*d^2*e^4*n*x^(2/3)*(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d +
e/x^(1/3))^n])^2 + 20*b*d^3*e^3*n*x*(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(
d + e/x^(1/3))^n])^2 - 15*b*d^4*e^2*n*x^(4/3)*(a - b*n*Log[d + e/x^(1/3)] +
 b*Log[c*(d + e/x^(1/3))^n])^2 + 12*b*d^5*e*n*x^(5/3)*(a - b*n*Log[d + e/x^
(1/3)] + b*Log[c*(d + e/x^(1/3))^n])^2 + 60*b*d^6*n*x^2*Log[d + e/x^(1/3)]*
(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])^2 + 20*d^6*x^2*(a
 - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])^3 - 60*b*e^6*n*(a -
 b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])^2*Log[e + d*x^(1/3)]
+ b^2*n^2*(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])*(d*e^2*
x^(1/3)*(-154*e^3 + 47*d*e^2*x^(1/3) - 18*d^2*e*x^(2/3) + 6*d^3*x) - 60*(e^
6 - d^6*x^2)*Log[d + e/x^(1/3)]^2 - 274*e^6*Log[-(e/(d*x^(1/3)))] + 2*e*Log
[d + e/x^(1/3)]*(137*e^5 + 60*d*e^4*x^(1/3) - 30*d^2*e^3*x^(2/3) + 20*d^3*e
^2*x - 15*d^4*e*x^(4/3) + 12*d^5*x^(5/3) + 60*e^5*Log[-(e/(d*x^(1/3)))]) +
120*e^6*PolyLog[2, 1 + e/(d*x^(1/3))]) + b^3*n^3*(3*d^4*e^2*x^(4/3)*(2 - 5*
Log[d + e/x^(1/3)])*Log[d + e/x^(1/3)] + 12*d^5*e*x^(5/3)*Log[d + e/x^(1/3)
]^2 + 20*d^6*x^2*Log[d + e/x^(1/3)]^3 + 2*d^3*e^3*x*(1 - 9*Log[d + e/x^(1/3
)] + 10*Log[d + e/x^(1/3)]^2) - d^2*e^4*x^(2/3)*(12 - 47*Log[d + e/x^(1/3)]
 + 30*Log[d + e/x^(1/3)]^2) + d*e^5*x^(1/3)*(71 - 154*Log[d + e/x^(1/3)] +
60*Log[d + e/x^(1/3)]^2) + 225*e^6*(-Log[d + e/x^(1/3)] + Log[-(e/(d*x^(1/3
)))]) + 137*e^6*(Log[d + e/x^(1/3)]*(Log[d + e/x^(1/3)] - 2*Log[-(e/(d*x^(1
/3)))]) - 2*PolyLog[2, 1 + e/(d*x^(1/3))]) - 20*e^6*(Log[d + e/x^(1/3)]^2*(
Log[d + e/x^(1/3)] - 3*Log[-(e/(d*x^(1/3)))]) - 6*Log[d + e/x^(1/3)]*PolyLo
g[2, 1 + e/(d*x^(1/3))] + 6*PolyLog[3, 1 + e/(d*x^(1/3))])))/(40*d^6)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int x \left(a + b \ln \left(c \left(d + \frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)^{3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(1/3))^n))^3,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e/x^(1/3))^n))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*x^2*log((d*x^(1/3) + e)^n)^3 - integrate(1/2*(2*(b^3*d*x^2 + b^3*x^
(5/3)*e)*log(x^(1/3*n))^3 - 2*(b^3*log(c)^3 + 3*a*b^2*log(c)^2 + 3*a^2*b*lo
g(c) + a^3)*x^(5/3)*e - 2*(b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*
log(c) + a^3*d)*x^2 + (b^3*d*n*x^2 - 6*(b^3*log(c) + a*b^2)*x^(5/3)*e - 6*(
b^3*d*log(c) + a*b^2*d)*x^2 + 6*(b^3*d*x^2 + b^3*x^(5/3)*e)*log(x^(1/3*n)))
*log((d*x^(1/3) + e)^n)^2 - 6*((b^3*log(c) + a*b^2)*x^(5/3)*e + (b^3*d*log(
c) + a*b^2*d)*x^2)*log(x^(1/3*n))^2 - 6*((b^3*log(c)^2 + 2*a*b^2*log(c) + a
^2*b)*x^(5/3)*e + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x^2 + (b^3*
d*x^2 + b^3*x^(5/3)*e)*log(x^(1/3*n))^2 - 2*((b^3*log(c) + a*b^2)*x^(5/3)*e
 + (b^3*d*log(c) + a*b^2*d)*x^2)*log(x^(1/3*n)))*log((d*x^(1/3) + e)^n) + 6
*((b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*b)*x^(5/3)*e + (b^3*d*log(c)^2 + 2*a
*b^2*d*log(c) + a^2*b*d)*x^2)*log(x^(1/3*n)))/(d*x + x^(2/3)*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*log(c*((d*x + x^(2/3)*e)/x)^n)^3 + 3*a*b^2*x*log(c*((d*x + x
^(2/3)*e)/x)^n)^2 + 3*a^2*b*x*log(c*((d*x + x^(2/3)*e)/x)^n) + a^3*x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(a + b \log \left(c \left(d + \frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/3))**n))**3,x)
```

```
[Out] Integral(x*(a + b*log(c*(d + e/x**(1/3))**n))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)^3*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e/x^(1/3))^n))^3,x)
[Out] int(x*(a + b*log(c*(d + e/x^(1/3))^n))^3, x)
```

3.504 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=436

$$\frac{3b^2e^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3}+\frac{3b^2e^3n^2\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right.\right.\right.}{d^3}$$

```
[Out] 3*b^2*e^2*n^2*(d+e/x^(1/3))*x^(1/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^3+3*b^2*e
^3*n^2*ln(1-d/(d+e/x^(1/3)))*(a+b*ln(c*(d+e/x^(1/3))^n))/d^3-3*b*e^2*n*(d+e
/x^(1/3))*x^(1/3)*(a+b*ln(c*(d+e/x^(1/3))^n))^2/d^3+3/2*b*e*n*x^(2/3)*(a+b*
ln(c*(d+e/x^(1/3))^n))^2/d-3*b*e^3*n*ln(1-d/(d+e/x^(1/3)))*(a+b*ln(c*(d+e/x
^(1/3))^n))^2/d^3+x*(a+b*ln(c*(d+e/x^(1/3))^n))^3+6*b^2*e^3*n^2*(a+b*ln(c*(
d+e/x^(1/3))^n))*ln(-e/d/x^(1/3))/d^3+b^3*e^3*n^3*ln(x)/d^3-3*b^3*e^3*n^3*p
olylog(2,d/(d+e/x^(1/3)))/d^3+6*b^2*e^3*n^2*(a+b*ln(c*(d+e/x^(1/3))^n))*pol
ylog(2,d/(d+e/x^(1/3)))/d^3+6*b^3*e^3*n^3*polylog(2,1+e/d/x^(1/3))/d^3+6*b^
3*e^3*n^3*polylog(3,d/(d+e/x^(1/3)))/d^3
```

Rubi [A]
time = 0.59, antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {2501, 2504, 2445, 2458, 2389, 2379, 2421, 6724, 2355, 2354, 2438, 2356, 2351, 31}

$$\frac{6b^2e^3n^2\text{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3}-\frac{3b^3e^3n^3\text{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)}{d^3}+\frac{6b^3e^3n^3\text{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)}{d^3}+\frac{6b^3e^3n^3\text{PolyLog}\left(3,\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)}{d^3}+\frac{3b^2e^3n^2\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3}+\frac{6b^2e^3n^2\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3}+\frac{3b^2e^2n^2\sqrt[3]{x}\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3}-\frac{3be^3n\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{d^3}-\frac{3be^2n\sqrt[3]{x}\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{d^3}+\frac{3benx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2d}+x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3+\frac{b^3e^3n^3\log(x)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])^3,x]
```

```
[Out] (3*b^2*e^2*n^2*(d + e/x^(1/3))*x^(1/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/d^
3 + (3*b^2*e^3*n^2*Log[1 - d/(d + e/x^(1/3))]*(a + b*Log[c*(d + e/x^(1/3))^
n]))/d^3 - (3*b*e^2*n*(d + e/x^(1/3))*x^(1/3)*(a + b*Log[c*(d + e/x^(1/3))^
n])^2)/d^3 + (3*b*e*n*x^(2/3)*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(2*d) - (
3*b*e^3*n*Log[1 - d/(d + e/x^(1/3))]*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/d^
3 + x*(a + b*Log[c*(d + e/x^(1/3))^n])^3 + (6*b^2*e^3*n^2*(a + b*Log[c*(d +
 e/x^(1/3))^n])*Log[-(e/(d*x^(1/3)))])/d^3 + (b^3*e^3*n^3*Log[x])/d^3 - (3*
b^3*e^3*n^3*PolyLog[2, d/(d + e/x^(1/3))])/d^3 + (6*b^2*e^3*n^2*(a + b*Log[
c*(d + e/x^(1/3))^n])*PolyLog[2, d/(d + e/x^(1/3))])/d^3 + (6*b^3*e^3*n^3*P
olyLog[2, 1 + e/(d*x^(1/3))])/d^3 + (6*b^3*e^3*n^3*PolyLog[3, d/(d + e/x^(1
/3))])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2501

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 dx = 3\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^n\right)\right)^3 dx, x, \sqrt[3]{x}\right)$$

$$= -\left(3\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x^4} dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 - (3ben)\text{Subst}\left(\int \frac{(a+b\log(c(d}{x^3(d+e}\right.$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 - (3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\right.$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 - \frac{(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3} dx,\right.}{d}$$

$$= \frac{3benx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2d} + x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right.\right.\right.$$

$$= -\frac{3be^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{d^3} + \frac{3benx^{2/3}}{}$$

$$= \frac{3b^2e^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3} - \frac{3be^2n\left(d\right.}{}$$

$$= \frac{3b^2e^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3} - \frac{3be^3n\left(a\right.}{}$$

$$= \frac{3b^2e^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3} - \frac{3be^3n\left(a\right.}{}$$

Mathematica [A]
time = 0.46, size = 675, normalized size = 1.55

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^3,x]
```

```
[Out] (-6*b*d*e^2*n*x^(1/3)*(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))
^n])^2 + 3*b*d^2*e*n*x^(2/3)*(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x
^(1/3))^n])^2 + 6*b*d^3*n*x*Log[d + e/x^(1/3)]*(a - b*n*Log[d + e/x^(1/3)]
+ b*Log[c*(d + e/x^(1/3))^n])^2 + 2*d^3*x*(a - b*n*Log[d + e/x^(1/3)] + b*L
og[c*(d + e/x^(1/3))^n])^3 + 6*b*e^3*n*(a - b*n*Log[d + e/x^(1/3)] + b*Log[
c*(d + e/x^(1/3))^n])^2*Log[e + d*x^(1/3)] + 6*b^2*n^2*(a - b*n*Log[d + e/x
^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])*((e^3 + d^3*x)*Log[d + e/x^(1/3)]^2 +
 e^2*(d*x^(1/3) + 3*e*Log[-(e/(d*x^(1/3)))]) + e*Log[d + e/x^(1/3)]*(-3*e^2
 - 2*d*e*x^(1/3) + d^2*x^(2/3) - 2*e^2*Log[-(e/(d*x^(1/3)))]) - 2*e^3*PolyL
og[2, 1 + e/(d*x^(1/3))]) - b^3*n^3*(-6*e^3*Log[d + e/x^(1/3)] - 6*d*e^2*x^
(1/3)*Log[d + e/x^(1/3)] + 9*e^3*Log[d + e/x^(1/3)]^2 + 6*d*e^2*x^(1/3)*Log
[d + e/x^(1/3)]^2 - 3*d^2*e*x^(2/3)*Log[d + e/x^(1/3)]^2 - 2*e^3*Log[d + e/
x^(1/3)]^3 - 2*d^3*x*Log[d + e/x^(1/3)]^3 + 6*e^3*Log[-(e/(d*x^(1/3)))] - 1
8*e^3*Log[d + e/x^(1/3)]*Log[-(e/(d*x^(1/3)))] + 6*e^3*Log[d + e/x^(1/3)]^2
*Log[-(e/(d*x^(1/3)))] + 6*e^3*(-3 + 2*Log[d + e/x^(1/3)])*PolyLog[2, 1 + e
/(d*x^(1/3))] - 12*e^3*PolyLog[3, 1 + e/(d*x^(1/3))]))/(2*d^3)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \left(a+b\ln \left(c{\left(d+\frac{e}{x^{\frac{1}{3}}}\right)}^{n}\right)\right)^{3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^n))^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3,x, algorithm="maxima")
```

```
[Out] b^3*x*log((d*x^(1/3) + e)^n)^3 + 3/2*(n*((d*x^(2/3) - 2*x^(1/3)*e)/d^2 + 2*
e^2*log(d*x^(1/3) + e)/d^3)*e + 2*x*log(c*(d + e/x^(1/3))^n))*a^2*b + a^3*x
 - integrate(((b^3*d*x + b^3*x^(2/3)*e)*log(x^(1/3*n))^3 + (b^3*d*n*x - 3*(
b^3*log(c) + a*b^2)*x^(2/3)*e - 3*(b^3*d*log(c) + a*b^2*d)*x + 3*(b^3*d*x +
 b^3*x^(2/3)*e)*log(x^(1/3*n)))*log((d*x^(1/3) + e)^n)^2 - 3*((b^3*log(c) +
 a*b^2)*x^(2/3)*e + (b^3*d*log(c) + a*b^2*d)*x)*log(x^(1/3*n))^2 - (b^3*log
```

```
(c)^3 + 3*a*b^2*log(c)^2)*x^(2/3)*e - (b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2)
*x - 3*((b^3*d*x + b^3*x^(2/3)*e)*log(x^(1/3*n))^2 + (b^3*log(c)^2 + 2*a*b^
2*log(c))*x^(2/3)*e + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c))*x - 2*((b^3*log(c
) + a*b^2)*x^(2/3)*e + (b^3*d*log(c) + a*b^2*d)*x)*log(x^(1/3*n)))*log((d*x
^(1/3) + e)^n) + 3*((b^3*log(c)^2 + 2*a*b^2*log(c))*x^(2/3)*e + (b^3*d*log(
c)^2 + 2*a*b^2*d*log(c))*x)*log(x^(1/3*n)))/(d*x + x^(2/3)*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3,x, algorithm="fricas")
[Out] integral(b^3*log(c*((d*x + x^(2/3)*e)/x)^n)^3 + 3*a*b^2*log(c*((d*x + x^(2/
3)*e)/x)^n)^2 + 3*a^2*b*log(c*((d*x + x^(2/3)*e)/x)^n) + a^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \log \left(c \left(d + \frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))**3,x)
[Out] Integral((a + b*log(c*(d + e/x**(1/3))**n))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^n\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))^3,x)
[Out] int((a + b*log(c*(d + e/x^(1/3))^n))^3, x)
```

3.505 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x}\,dx$

Optimal. Leaf size=135

$$-3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt[3]{x}}\right)-9bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\text{Li}_2\left(1+\frac{e}{d\sqrt[3]{x}}\right)+1$$

```
[Out] -3*(a+b*ln(c*(d+e/x^(1/3))^n))^3*ln(-e/d/x^(1/3))-9*b*n*(a+b*ln(c*(d+e/x^(1
/3))^n))^2*polylog(2,1+e/d/x^(1/3))+18*b^2*n^2*(a+b*ln(c*(d+e/x^(1/3))^n))*
polylog(3,1+e/d/x^(1/3))-18*b^3*n^3*polylog(4,1+e/d/x^(1/3))
```

Rubi [A]
time = 0.13, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2504, 2443, 2481, 2421, 2430, 6724}

$$18b^2n^2\text{PolyLog}\left(3,\frac{e}{d\sqrt[3]{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-9bn\text{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2-18b^3n^3\text{PolyLog}\left(4,\frac{e}{d\sqrt[3]{x}}+1\right)-3\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])^3/x,x]
```

```
[Out] -3*(a + b*Log[c*(d + e/x^(1/3))^n])^3*Log[-(e/(d*x^(1/3)))] - 9*b*n*(a + b*
Log[c*(d + e/x^(1/3))^n])^2*PolyLog[2, 1 + e/(d*x^(1/3))] + 18*b^2*n^2*(a +
 b*Log[c*(d + e/x^(1/3))^n])*PolyLog[3, 1 + e/(d*x^(1/3))] - 18*b^3*n^3*Pol
yLog[4, 1 + e/(d*x^(1/3))]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
```

```
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x}\,dx = -\left(3\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt[3]{x}}\right) + (9ben)\text{Subst}\left(\right.$$

$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt[3]{x}}\right) + (9bn)\text{Subst}\left(\right.$$

$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt[3]{x}}\right) - 9bn\left(a+b\log\right.$$

$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt[3]{x}}\right) - 9bn\left(a+b\log\right.$$

$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt[3]{x}}\right) - 9bn\left(a+b\log\right.$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 527 vs. 2(135) = 270.
time = 0.20, size = 527, normalized size = 3.90

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^3/x,x]
```

```
[Out] (a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])^3*Log[x] + 3*b*n*
(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])^2*((Log[d + e/x^(
1/3)] - Log[1 + e/(d*x^(1/3))])*Log[x] + 3*PolyLog[2, -(e/(d*x^(1/3)))]) +
9*b^2*n^2*(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])*(2*Log[
e/d + x^(1/3)]*PolyLog[2, 1 + (d*x^(1/3))/e] - 2*(Log[d + e/x^(1/3)] - Log[
e/d + x^(1/3)])*PolyLog[2, -((d*x^(1/3))/e)] + (81*Log[e/d + x^(1/3)]^2*Log
[-((d*x^(1/3))/e)] + 27*Log[d + e/x^(1/3)]^2*Log[x] - 27*Log[e/d + x^(1/3)]
^2*Log[x] - 54*Log[d + e/x^(1/3)]*Log[1 + (d*x^(1/3))/e]*Log[x] + 54*Log[e/
d + x^(1/3)]*Log[1 + (d*x^(1/3))/e]*Log[x] + 9*Log[d + e/x^(1/3)]*Log[x]^2
- 9*Log[1 + (d*x^(1/3))/e]*Log[x]^2 + Log[x]^3 - 162*PolyLog[3, 1 + (d*x^(1
/3))/e] - 162*PolyLog[3, -((d*x^(1/3))/e)])/81) - 3*b^3*n^3*(Log[d + e/x^(1
/3)]^3*Log[-(e/(d*x^(1/3)))] + 3*Log[d + e/x^(1/3)]^2*PolyLog[2, 1 + e/(d*x
^(1/3))] - 6*Log[d + e/x^(1/3)]*PolyLog[3, 1 + e/(d*x^(1/3))] + 6*PolyLog[4
, 1 + e/(d*x^(1/3))])
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)^{3}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^n))^3/x,x)
[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))^3/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x,x, algorithm="maxima")
[Out] b^3*log((d*x^(1/3) + e)^n)^3*log(x) - integrate(((b^3*d*x + b^3*x^(2/3)*e)*
log(x^(1/3*n))^3 + (b^3*d*n*x*log(x) - 3*(b^3*log(c) + a*b^2)*x^(2/3)*e - 3
*(b^3*d*log(c) + a*b^2*d)*x + 3*(b^3*d*x + b^3*x^(2/3)*e)*log(x^(1/3*n)))*l
og((d*x^(1/3) + e)^n)^2 - 3*((b^3*log(c) + a*b^2)*x^(2/3)*e + (b^3*d*log(c)
 + a*b^2*d)*x)*log(x^(1/3*n))^2 - (b^3*log(c)^3 + 3*a*b^2*log(c)^2 + 3*a^2*
b*log(c) + a^3)*x^(2/3)*e - (b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*
d*log(c) + a^3*d)*x - 3*((b^3*d*x + b^3*x^(2/3)*e)*log(x^(1/3*n))^2 + (b^3*
log(c)^2 + 2*a*b^2*log(c) + a^2*b)*x^(2/3)*e + (b^3*d*log(c)^2 + 2*a*b^2*d*
log(c) + a^2*b*d)*x - 2*((b^3*log(c) + a*b^2)*x^(2/3)*e + (b^3*d*log(c) + a
*b^2*d)*x)*log(x^(1/3*n)))*log((d*x^(1/3) + e)^n) + 3*((b^3*log(c)^2 + 2*a*
b^2*log(c) + a^2*b)*x^(2/3)*e + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*
d)*x)*log(x^(1/3*n)))/(d*x^2 + x^(5/3)*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x,x, algorithm="fricas")
[Out] integral((b^3*log(c*((d*x + x^(2/3)*e)/x)^n)^3 + 3*a*b^2*log(c*((d*x + x^(2
/3)*e)/x)^n)^2 + 3*a^2*b*log(c*((d*x + x^(2/3)*e)/x)^n) + a^3)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{3}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))**3/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e/x**(1/3))**n))**3/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)^3/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^n\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))^3/x,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(1/3))^n))^3/x, x)
```

3.506 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^2}\,dx$

Optimal. Leaf size=438

$$-\frac{9b^3dn^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{4e^3}+\frac{2b^3n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^3}-\frac{18ab^2d^2n^2}{e^2\sqrt[3]{x}}+\frac{18b^3d^2n^3}{e^2\sqrt[3]{x}}-\frac{18b^3d^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right.\right.}{e^3}$$

```
[Out] -9/4*b^3*d*n^3*(d+e/x^(1/3))^2/e^3+2/9*b^3*n^3*(d+e/x^(1/3))^3/e^3-18*a*b^2
*d^2*n^2/e^2/x^(1/3)+18*b^3*d^2*n^3/e^2/x^(1/3)-18*b^3*d^2*n^2*(d+e/x^(1/3)
)*ln(c*(d+e/x^(1/3))^n)/e^3+9/2*b^2*d*n^2*(d+e/x^(1/3))^2*(a+b*ln(c*(d+e/x^
(1/3))^n))/e^3-2/3*b^2*n^2*(d+e/x^(1/3))^3*(a+b*ln(c*(d+e/x^(1/3))^n))/e^3+
9*b*d^2*n*(d+e/x^(1/3))*(a+b*ln(c*(d+e/x^(1/3))^n))^2/e^3-9/2*b*d*n*(d+e/x^
(1/3))^2*(a+b*ln(c*(d+e/x^(1/3))^n))^2/e^3+b*n*(d+e/x^(1/3))^3*(a+b*ln(c*(d
+e/x^(1/3))^n))^2/e^3-3*d^2*(d+e/x^(1/3))*(a+b*ln(c*(d+e/x^(1/3))^n))^3/e^3
+3*d*(d+e/x^(1/3))^2*(a+b*ln(c*(d+e/x^(1/3))^n))^3/e^3-(d+e/x^(1/3))^3*(a+b
*ln(c*(d+e/x^(1/3))^n))^3/e^3
```

Rubi [A]
time = 0.30, antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

$$-\frac{2b^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3e^3}+\frac{9b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{2e^3}-\frac{18ab^2d^2n^2}{e^2\sqrt[3]{x}}+\frac{9bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{e^3}-\frac{3d^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^3}+\frac{bn\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{e^3}-\frac{9bdn\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2e^3}-\frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^3}+\frac{3d\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^3}-\frac{18b^3d^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{e^3}+\frac{18b^3d^2n^3}{e^2\sqrt[3]{x}}+\frac{2b^3n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^3}-\frac{9b^3dn^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{4e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])^3/x^2,x]
```

```
[Out] (-9*b^3*d*n^3*(d + e/x^(1/3))^2)/(4*e^3) + (2*b^3*n^3*(d + e/x^(1/3))^3)/(9
*e^3) - (18*a*b^2*d^2*n^2)/(e^2*x^(1/3)) + (18*b^3*d^2*n^3)/(e^2*x^(1/3)) -
 (18*b^3*d^2*n^2*(d + e/x^(1/3))*Log[c*(d + e/x^(1/3))^n])/e^3 + (9*b^2*d*n
^2*(d + e/x^(1/3))^2*(a + b*Log[c*(d + e/x^(1/3))^n]))/(2*e^3) - (2*b^2*n^2
*(d + e/x^(1/3))^3*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*e^3) + (9*b*d^2*n*(
d + e/x^(1/3))*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/e^3 - (9*b*d*n*(d + e/x^
(1/3))^2*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(2*e^3) + (b*n*(d + e/x^(1/3))
^3*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/e^3 - (3*d^2*(d + e/x^(1/3))*(a + b*
Log[c*(d + e/x^(1/3))^n])^3)/e^3 + (3*d*(d + e/x^(1/3))^2*(a + b*Log[c*(d +
 e/x^(1/3))^n])^3)/e^3 - ((d + e/x^(1/3))^3*(a + b*Log[c*(d + e/x^(1/3))^n]
)^3)/e^3
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^2}\,dx = -\left(3\text{Subst}\left(\int x^2(a+b\log(c(d+ex)^n))^3\,dx,x,\frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\left(3\text{Subst}\left(\int\left(\frac{d^2(a+b\log(c(d+ex)^n))^3}{e^2}-\frac{2d(d+ex)(a+b\log}{e^2}\right.\right.\right.$$

$$= -\frac{3\text{Subst}\left(\int(d+ex)^2(a+b\log(c(d+ex)^n))^3\,dx,x,\frac{1}{\sqrt[3]{x}}\right)}{e^2}+\frac{(6d)\text{S}}{}$$

$$= -\frac{3\text{Subst}\left(\int x^2(a+b\log(cx^n))^3\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}+\frac{(6d)\text{Subst}\left(\int x\right.}{}$$

$$= -\frac{3d^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^3}+\frac{3d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{}$$

$$= \frac{9bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{e^3}-\frac{9bdn\left(d+\frac{e}{\sqrt[3]{x}}\right.}{}$$

$$= -\frac{9b^3dn^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{4e^3}+\frac{2b^3n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^3}-\frac{18ab^2d^2n^2}{e^2\sqrt[3]{x}}+\frac{9b^2dn^2}{}$$

$$= -\frac{9b^3dn^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{4e^3}+\frac{2b^3n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^3}-\frac{18ab^2d^2n^2}{e^2\sqrt[3]{x}}+\frac{18b^3d^2}{e^2\sqrt[3]{x}}$$

Mathematica [A]
time = 0.51, size = 666, normalized size = 1.52

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^3/x^2,x]
```

```
[Out] (-36*a^3*e^3 + 36*a^2*b*e^3*n - 24*a*b^2*e^3*n^2 + 8*b^3*e^3*n^3 - 54*a^2*b
*d*e^2*n*x^(1/3) + 90*a*b^2*d*e^2*n^2*x^(1/3) - 57*b^3*d*e^2*n^3*x^(1/3) +
108*a^2*b*d^2*e*n*x^(2/3) - 396*a*b^2*d^2*e*n^2*x^(2/3) + 510*b^3*d^2*e*n^3
*x^(2/3) + 72*b^3*d^3*n^3*x*Log[d + e/x^(1/3)]^3 - 36*b^3*e^3*Log[c*(d + e/
x^(1/3))^n]^3 - 108*a^2*b*d^3*n*x*Log[e + d*x^(1/3)] + 396*a*b^2*d^3*n^2*x*
```

```
Log[e + d*x^(1/3)] - 510*b^3*d^3*n^3*x*Log[e + d*x^(1/3)] + 12*b^2*d^3*n^2*
x*Log[d + e/x^(1/3)]*(6*a - 11*b*n + 6*b*Log[c*(d + e/x^(1/3))^n])*(3*Log[e
 + d*x^(1/3)] - Log[x]) + 36*a^2*b*d^3*n*x*Log[x] - 132*a*b^2*d^3*n^2*x*Log
[x] + 170*b^3*d^3*n^3*x*Log[x] - 18*b^2*d^3*n^2*x*Log[d + e/x^(1/3)]^2*(6*a
 - 11*b*n + 6*b*Log[c*(d + e/x^(1/3))^n] + 6*b*n*Log[e + d*x^(1/3)] - 2*b*n
*Log[x]) + 18*b^2*Log[c*(d + e/x^(1/3))^n]^2*(e*(-6*a*e^2 + 2*b*e^2*n - 3*b
*d*e*n*x^(1/3) + 6*b*d^2*n*x^(2/3)) - 6*b*d^3*n*x*Log[e + d*x^(1/3)] + 2*b*
d^3*n*x*Log[x]) - 6*b*Log[c*(d + e/x^(1/3))^n]*(18*a^2*e^3 - 6*a*b*e*n*(2*e
^2 - 3*d*e*x^(1/3) + 6*d^2*x^(2/3)) + b^2*e*n^2*(4*e^2 - 15*d*e*x^(1/3) + 6
6*d^2*x^(2/3)) + 6*b*d^3*n*(6*a - 11*b*n)*x*Log[e + d*x^(1/3)] + 2*b*d^3*n*
(-6*a + 11*b*n)*x*Log[x]))/(36*e^3*x)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)^{3}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^n))^3/x^2,x)
[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))^3/x^2,x)
```

Maxima [A]
time = 0.32, size = 645, normalized size = 1.47

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x^2,x, algorithm="maxima")
[Out] -1/2*(6*d^3*e^(-4)*log(d*x^(1/3) + e) - 2*d^3*e^(-4)*log(x) - (6*d^2*x^(2/3
) - 3*d*x^(1/3)*e + 2*e^2)*e^(-3)/x)*a^2*b*n*e - b^3*log(c*(d + e/x^(1/3))^
n)^3/x - 1/6*(6*(6*d^3*e^(-4)*log(d*x^(1/3) + e) - 2*d^3*e^(-4)*log(x) - (6
*d^2*x^(2/3) - 3*d*x^(1/3)*e + 2*e^2)*e^(-3)/x)*n*e*log(c*(d + e/x^(1/3))^n
) - (18*d^3*x*log(d*x^(1/3) + e)^2 + 2*d^3*x*log(x)^2 - 22*d^3*x*log(x) - 6
6*d^2*x^(2/3)*e + 15*d*x^(1/3)*e^2 - 6*(2*d^3*x*log(x) - 11*d^3*x)*log(d*x^
(1/3) + e) - 4*e^3)*n^2*e^(-3)/x)*a*b^2 - 1/108*(54*(6*d^3*e^(-4)*log(d*x^(
1/3) + e) - 2*d^3*e^(-4)*log(x) - (6*d^2*x^(2/3) - 3*d*x^(1/3)*e + 2*e^2)*e
^(-3)/x)*n*e*log(c*(d + e/x^(1/3))^n)^2 + ((108*d^3*x*log(d*x^(1/3) + e)^3
- 4*d^3*x*log(x)^3 + 66*d^3*x*log(x)^2 - 510*d^3*x*log(x) - 1530*d^2*x^(2/3
)*e - 54*(2*d^3*x*log(x) - 11*d^3*x)*log(d*x^(1/3) + e)^2 + 171*d*x^(1/3)*e
^2 + 18*(2*d^3*x*log(x)^2 - 22*d^3*x*log(x) + 85*d^3*x)*log(d*x^(1/3) + e)
- 24*e^3)*n^2*e^(-4)/x - 18*(18*d^3*x*log(d*x^(1/3) + e)^2 + 2*d^3*x*log(x)
^2 - 22*d^3*x*log(x) - 66*d^2*x^(2/3)*e + 15*d*x^(1/3)*e^2 - 6*(2*d^3*x*log
```

```
(x) - 11*d^3*x)*log(d*x^(1/3) + e) - 4*e^3)*n*e^(-4)*log(c*(d + e/x^(1/3))^
n)/x)*n*e)*b^3 - 3*a*b^2*log(c*(d + e/x^(1/3))^n)^2/x - 3*a^2*b*log(c*(d +
e/x^(1/3))^n)/x - a^3/x
```

Fricas [A]
time = 0.42, size = 750, normalized size = 1.71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/36*(36*(b^3*x - b^3)*e^3*log(c)^3 + 36*(b^3*n - 3*a*b^2 - (b^3*n - 3*a*b^
2)*x)*e^3*log(c)^2 - 36*(b^3*d^3*n^3*x + b^3*n^3*e^3)*log((d*x + x^(2/3)*e)
/x)^3 - 12*(2*b^3*n^2 - 6*a*b^2*n + 9*a^2*b - (2*b^3*n^2 - 6*a*b^2*n + 9*a^
2*b)*x)*e^3*log(c) + 18*(6*b^3*d^2*n^3*x^(2/3)*e - 3*b^3*d*n^3*x^(1/3)*e^2
+ (11*b^3*d^3*n^3 - 6*a*b^2*d^3*n^2)*x + 2*(b^3*n^3 - 3*a*b^2*n^2)*e^3 - 6*
(b^3*d^3*n^2*x + b^3*n^2*e^3)*log(c))*log((d*x + x^(2/3)*e)/x)^2 + 4*(2*b^3
*n^3 - 6*a*b^2*n^2 + 9*a^2*b*n - 9*a^3 - (2*b^3*n^3 - 6*a*b^2*n^2 + 9*a^2*b
*n - 9*a^3)*x)*e^3 - 6*(18*(b^3*d^3*n*x + b^3*n*e^3)*log(c)^2 + (85*b^3*d^3
*n^3 - 66*a*b^2*d^3*n^2 + 18*a^2*b*d^3*n)*x + 2*(2*b^3*n^3 - 6*a*b^2*n^2 +
9*a^2*b*n)*e^3 - 6*((11*b^3*d^3*n^2 - 6*a*b^2*d^3*n)*x + 2*(b^3*n^2 - 3*a*b
^2*n)*e^3)*log(c) - 6*(6*b^3*d^2*n^2*e*log(c) - (11*b^3*d^2*n^3 - 6*a*b^2*d
^2*n^2)*e)*x^(2/3) + 3*(6*b^3*d*n^2*e^2*log(c) - (5*b^3*d*n^3 - 6*a*b^2*d*n
^2)*e^2)*x^(1/3))*log((d*x + x^(2/3)*e)/x) + 6*(18*b^3*d^2*n*e*log(c)^2 - 6
*(11*b^3*d^2*n^2 - 6*a*b^2*d^2*n)*e*log(c) + (85*b^3*d^2*n^3 - 66*a*b^2*d^2
*n^2 + 18*a^2*b*d^2*n)*e)*x^(2/3) - 3*(18*b^3*d*n*e^2*log(c)^2 - 6*(5*b^3*d
*n^2 - 6*a*b^2*d*n)*e^2*log(c) + (19*b^3*d*n^3 - 30*a*b^2*d*n^2 + 18*a^2*b*
d*n)*e^2)*x^(1/3))*e^(-3)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\log\left(c\left(d + \frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))**3/x**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e/x**(1/3))**n))**3/x**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1758 vs. 2(391) = 782.
time = 3.97, size = 1758, normalized size = 4.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x^2,x, algorithm="giac")

[Out] -1/36*(108*(d*x^(1/3) + e)*b^3*d^2*n^3*log((d*x^(1/3) + e)/x^(1/3))^3/x^(1/3) - 108*(d*x^(1/3) + e)^2*b^3*d*n^3*log((d*x^(1/3) + e)/x^(1/3))^3/x^(2/3) + 36*(d*x^(1/3) + e)^3*b^3*n^3*log((d*x^(1/3) + e)/x^(1/3))^3/x - 324*(d*x^(1/3) + e)*b^3*d^2*n^3*log((d*x^(1/3) + e)/x^(1/3))^2/x^(1/3) + 324*(d*x^(1/3) + e)*b^3*d^2*n^2*log(c)*log((d*x^(1/3) + e)/x^(1/3))^2/x^(1/3) + 162*(d*x^(1/3) + e)^2*b^3*d*n^3*log((d*x^(1/3) + e)/x^(1/3))^2/x^(2/3) - 324*(d*x^(1/3) + e)^2*b^3*d*n^2*log(c)*log((d*x^(1/3) + e)/x^(1/3))^2/x^(2/3) - 36*(d*x^(1/3) + e)^3*b^3*n^3*log((d*x^(1/3) + e)/x^(1/3))^2/x + 108*(d*x^(1/3) + e)^3*b^3*n^2*log(c)*log((d*x^(1/3) + e)/x^(1/3))^2/x + 648*(d*x^(1/3) + e)*b^3*d^2*n^3*log((d*x^(1/3) + e)/x^(1/3))/x^(1/3) - 648*(d*x^(1/3) + e)*b^3*d^2*n^2*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x^(1/3) + 324*(d*x^(1/3) + e)*b^3*d^2*n*log(c)^2*log((d*x^(1/3) + e)/x^(1/3))/x^(1/3) + 324*(d*x^(1/3) + e)*a*b^2*d^2*n^2*log((d*x^(1/3) + e)/x^(1/3))^2/x^(1/3) - 162*(d*x^(1/3) + e)^2*b^3*d*n^3*log((d*x^(1/3) + e)/x^(1/3))/x^(2/3) + 324*(d*x^(1/3) + e)^2*b^3*d*n^2*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x^(2/3) - 324*(d*x^(1/3) + e)^2*b^3*d*n*log(c)^2*log((d*x^(1/3) + e)/x^(1/3))/x^(2/3) - 324*(d*x^(1/3) + e)^2*a*b^2*d*n^2*log((d*x^(1/3) + e)/x^(1/3))^2/x^(2/3) + 24*(d*x^(1/3) + e)^3*b^3*n^3*log((d*x^(1/3) + e)/x^(1/3))/x - 72*(d*x^(1/3) + e)^3*b^3*n^2*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x + 108*(d*x^(1/3) + e)^3*b^3*n*log(c)^2*log((d*x^(1/3) + e)/x^(1/3))/x + 108*(d*x^(1/3) + e)^3*a*b^2*n^2*log((d*x^(1/3) + e)/x^(1/3))^2/x - 648*(d*x^(1/3) + e)*b^3*d^2*n^3/x^(1/3) + 648*(d*x^(1/3) + e)*b^3*d^2*n^2*log(c)/x^(1/3) - 324*(d*x^(1/3) + e)*b^3*d^2*n*log(c)^2/x^(1/3) + 108*(d*x^(1/3) + e)*b^3*d^2*log(c)^3/x^(1/3) - 648*(d*x^(1/3) + e)*a*b^2*d^2*n^2*log((d*x^(1/3) + e)/x^(1/3))/x^(1/3) + 648*(d*x^(1/3) + e)*a*b^2*d^2*n*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x^(1/3) + 81*(d*x^(1/3) + e)^2*b^3*d*n^3/x^(2/3) - 162*(d*x^(1/3) + e)^2*b^3*d*n^2*log(c)/x^(2/3) + 162*(d*x^(1/3) + e)^2*b^3*d*n*log(c)^2/x^(2/3) - 108*(d*x^(1/3) + e)^2*b^3*d*log(c)^3/x^(2/3) + 324*(d*x^(1/3) + e)^2*a*b^2*d*n^2*log((d*x^(1/3) + e)/x^(1/3))/x^(2/3) - 648*(d*x^(1/3) + e)^2*a*b^2*d*n*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x^(2/3) - 8*(d*x^(1/3) + e)^3*b^3*n^3/x + 24*(d*x^(1/3) + e)^3*b^3*n^2*log(c)/x - 36*(d*x^(1/3) + e)^3*b^3*n*log(c)^2/x + 36*(d*x^(1/3) + e)^3*b^3*log(c)^3/x - 72*(d*x^(1/3) + e)^3*a*b^2*n^2*log((d*x^(1/3) + e)/x^(1/3))/x + 216*(d*x^(1/3) + e)^3*a*b^2*n*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x + 648*(d*x^(1/3) + e)*a*b^2*d^2*n^2/x^(1/3) - 648*(d*x^(1/3) + e)*a*b^2*d^2*n*log(c)/x^(1/3) + 324*(d*x^(1/3) + e)*a*b^2*d^2*log(c)^2/x^(1/3) + 324*(d*x^(1/3) + e)*a^2*b*d^2*n*log((d*x^(1/3) + e)/x^(1/3))/x^(1/3) - 162*(d*x^(1/3) + e)^2*a*b^2*d*n^2/x^(2/3) + 324*(d*x^(1/3) + e)^2*a*b^2*d*n*log(c)/x^(2/3) - 324*(d*x^(1/3) + e)^2*a*b^2*d*log(c)^2/x^(2/3) - 324*(d*x^(1/3) + e)^2*a^2*b*d*n*log((d*x^(1/3) + e)/x^(1/3))/x^(2/3) + 24*(d*x^(1/3) + e)^3*a*b^2*n^2/x - 72*(d*x^(1/3) + e)^3*a*b^2*n*log(c)/x + 108*(d*x^(1/3) + e)^3*a*b^2*log(c)^2/x + 108*(d*x^(1/3) + e)^3*a^2*b*n*log((d*x^(1/3

```
) + e)/x^(1/3))/x - 324*(d*x^(1/3) + e)*a^2*b*d^2*n/x^(1/3) + 324*(d*x^(1/3
) + e)*a^2*b*d^2*log(c)/x^(1/3) + 162*(d*x^(1/3) + e)^2*a^2*b*d*n/x^(2/3) -
 324*(d*x^(1/3) + e)^2*a^2*b*d*log(c)/x^(2/3) - 36*(d*x^(1/3) + e)^3*a^2*b*
n/x + 108*(d*x^(1/3) + e)^3*a^2*b*log(c)/x + 108*(d*x^(1/3) + e)*a^3*d^2/x^
(1/3) - 108*(d*x^(1/3) + e)^2*a^3*d/x^(2/3) + 36*(d*x^(1/3) + e)^3*a^3/x)*e
^(-3)
```

Mupad [B]
time = 0.74, size = 570, normalized size = 1.30

$$\frac{\frac{d\left(3a^3-3a^2bn+2ab^2n^2-\frac{2b^3n^3}{3}\right)}{2e}-\frac{d\left(6a^3-6ab^2n^2+5b^3n^3\right)}{4e}}{x^{2/3}}-\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)^3\left(\frac{b^3}{x}+\frac{b^3d^3}{e^3}\right)-\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)^2\left(\frac{b^2(3a-bn)}{x}-\frac{\frac{3b^2d(3a-bn)}{2e}-\frac{9ab^2d}{2e}}{x^{2/3}}+\frac{d\left(6ab^2d^2-11b^3d^2n\right)}{2e^3}+\frac{d\left(\frac{3b^2d(3a-bn)}{e}-\frac{9ab^2d}{e}\right)}{e\,x^{1/3}}\right)-\frac{a^3-a^2bn+\frac{2ab^2n^2}{3}-\frac{2b^3n^3}{9}}{x}-\frac{\frac{d\left(\frac{d\left(3a^3-3a^2bn+2ab^2n^2-\frac{2b^3n^3}{3}\right)}{e}-\frac{d\left(6a^3-6ab^2n^2+5b^3n^3\right)}{2e}\right)}{e}+\frac{b^2d^2n^2(6a-11bn)}{e^2}}{x^{1/3}}-\frac{\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)\left(\frac{\frac{d\left(bde\left(9a^2-6abn+2b^2n^2\right)-3bde\left(3a^2-b^2n^2\right)\right)}{e}+6b^3d^2n^2}{e\,x^{1/3}}-\frac{bde\left(9a^2-6abn+2b^2n^2\right)-3bde\left(3a^2-b^2n^2\right)}{2e\,x^{2/3}}+\frac{be\left(9a^2-6abn+2b^2n^2\right)}{3x}\right)}{e}-\frac{\ln\left(d+\frac{e}{x^{1/3}}\right)\left(18a^2bd^3n-66ab^2d^3n^2+85b^3d^3n^3\right)}{6e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))^3/x^2,x)
```

```
[Out] ((d*(3*a^3 - (2*b^3*n^3)/3 + 2*a*b^2*n^2 - 3*a^2*b*n))/(2*e) - (d*(6*a^3 +
5*b^3*n^3 - 6*a*b^2*n^2))/(4*e))/x^(2/3) - log(c*(d + e/x^(1/3))^n)^3*(b^3/
x + (b^3*d^3)/e^3) - log(c*(d + e/x^(1/3))^n)^2*((b^2*(3*a - b*n))/x - ((3*
b^2*d*(3*a - b*n))/(2*e) - (9*a*b^2*d)/(2*e))/x^(2/3) + (d*(6*a*b^2*d^2 - 1
1*b^3*d^2*n))/(2*e^3) + (d*((3*b^2*d*(3*a - b*n))/e - (9*a*b^2*d)/e))/(e*x^
(1/3))) - (a^3 - (2*b^3*n^3)/9 + (2*a*b^2*n^2)/3 - a^2*b*n)/x - ((d*((d*(3*
a^3 - (2*b^3*n^3)/3 + 2*a*b^2*n^2 - 3*a^2*b*n))/e - (d*(6*a^3 + 5*b^3*n^3 -
 6*a*b^2*n^2))/(2*e)))/e + (b^2*d^2*n^2*(6*a - 11*b*n))/e^2)/x^(1/3) - (log
(c*(d + e/x^(1/3))^n)*(((d*(b*d*e*(9*a^2 + 2*b^2*n^2 - 6*a*b*n) - 3*b*d*e*(
3*a^2 - b^2*n^2)))/e + 6*b^3*d^2*n^2)/(e*x^(1/3)) - (b*d*e*(9*a^2 + 2*b^2*n
^2 - 6*a*b*n) - 3*b*d*e*(3*a^2 - b^2*n^2))/(2*e*x^(2/3)) + (b*e*(9*a^2 + 2*
b^2*n^2 - 6*a*b*n))/(3*x)))/e - (log(d + e/x^(1/3))*(85*b^3*d^3*n^3 - 66*a*
b^2*d^3*n^2 + 18*a^2*b*d^3*n))/(6*e^3)
```

3.507 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^3}\,dx$

Optimal. Leaf size=907

$$\frac{45b^3d^4n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{8e^6}-\frac{20b^3d^3n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^6}+\frac{45b^3d^2n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^4}{64e^6}-\frac{18b^3dn^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^5}{125e^6}+\frac{b^3n^3\left(d+\right.}{72e}$$

```
[Out] -1/12*b^2*n^2*(d+e/x^(1/3))^6*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6+1/4*b*n*(d+e/
x^(1/3))^6*(a+b*ln(c*(d+e/x^(1/3))^n))^2/e^6+45/8*b^3*d^4*n^3*(d+e/x^(1/3))
^2/e^6-20/9*b^3*d^3*n^3*(d+e/x^(1/3))^3/e^6+45/64*b^3*d^2*n^3*(d+e/x^(1/3))
^4/e^6-18/125*b^3*d*n^3*(d+e/x^(1/3))^5/e^6-18*b^3*d^5*n^3/e^5/x^(1/3)-1/2*
(d+e/x^(1/3))^6*(a+b*ln(c*(d+e/x^(1/3))^n))^3/e^6-9*b*d^5*n*(d+e/x^(1/3))*(
a+b*ln(c*(d+e/x^(1/3))^n))^2/e^6+45/4*b*d^4*n*(d+e/x^(1/3))^2*(a+b*ln(c*(d+
e/x^(1/3))^n))^2/e^6-10*b*d^3*n*(d+e/x^(1/3))^3*(a+b*ln(c*(d+e/x^(1/3))^n))
^2/e^6+45/8*b*d^2*n*(d+e/x^(1/3))^4*(a+b*ln(c*(d+e/x^(1/3))^n))^2/e^6-9/5*b
*d*n*(d+e/x^(1/3))^5*(a+b*ln(c*(d+e/x^(1/3))^n))^2/e^6+18*b^3*d^5*n^2*(d+e/
x^(1/3))*ln(c*(d+e/x^(1/3))^n)/e^6-45/4*b^2*d^4*n^2*(d+e/x^(1/3))^2*(a+b*ln
(c*(d+e/x^(1/3))^n))/e^6+20/3*b^2*d^3*n^2*(d+e/x^(1/3))^3*(a+b*ln(c*(d+e/x^
(1/3))^n))/e^6-45/16*b^2*d^2*n^2*(d+e/x^(1/3))^4*(a+b*ln(c*(d+e/x^(1/3))^n)
)/e^6+18/25*b^2*d*n^2*(d+e/x^(1/3))^5*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6+18*a*
b^2*d^5*n^2/e^5/x^(1/3)+3*d^5*(d+e/x^(1/3))*(a+b*ln(c*(d+e/x^(1/3))^n))^3/e
^6-15/2*d^4*(d+e/x^(1/3))^2*(a+b*ln(c*(d+e/x^(1/3))^n))^3/e^6+10*d^3*(d+e/x
^(1/3))^3*(a+b*ln(c*(d+e/x^(1/3))^n))^3/e^6-15/2*d^2*(d+e/x^(1/3))^4*(a+b*l
n(c*(d+e/x^(1/3))^n))^3/e^6+3*d*(d+e/x^(1/3))^5*(a+b*ln(c*(d+e/x^(1/3))^n))
^3/e^6+1/72*b^3*n^3*(d+e/x^(1/3))^6/e^6
```

Rubi [A]
time = 0.64, antiderivative size = 907, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])^3/x^3,x]
```

```
[Out] (45*b^3*d^4*n^3*(d + e/x^(1/3))^2)/(8*e^6) - (20*b^3*d^3*n^3*(d + e/x^(1/3)
)^3)/(9*e^6) + (45*b^3*d^2*n^3*(d + e/x^(1/3))^4)/(64*e^6) - (18*b^3*d*n^3*
(d + e/x^(1/3))^5)/(125*e^6) + (b^3*n^3*(d + e/x^(1/3))^6)/(72*e^6) + (18*a
*b^2*d^5*n^2)/(e^5*x^(1/3)) - (18*b^3*d^5*n^3)/(e^5*x^(1/3)) + (18*b^3*d^5*
n^2*(d + e/x^(1/3))*Log[c*(d + e/x^(1/3))^n])/e^6 - (45*b^2*d^4*n^2*(d + e/
```

```
x^(1/3))^2*(a + b*Log[c*(d + e/x^(1/3))^n]))/(4*e^6) + (20*b^2*d^3*n^2*(d +
 e/x^(1/3))^3*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*e^6) - (45*b^2*d^2*n^2*(
d + e/x^(1/3))^4*(a + b*Log[c*(d + e/x^(1/3))^n]))/(16*e^6) + (18*b^2*d*n^2
*(d + e/x^(1/3))^5*(a + b*Log[c*(d + e/x^(1/3))^n]))/(25*e^6) - (b^2*n^2*(d
 + e/x^(1/3))^6*(a + b*Log[c*(d + e/x^(1/3))^n]))/(12*e^6) - (9*b*d^5*n*(d
+ e/x^(1/3))*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/e^6 + (45*b*d^4*n*(d + e/x
^(1/3))^2*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(4*e^6) - (10*b*d^3*n*(d + e/
x^(1/3))^3*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/e^6 + (45*b*d^2*n*(d + e/x^(
1/3))^4*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(8*e^6) - (9*b*d*n*(d + e/x^(1/
3))^5*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(5*e^6) + (b*n*(d + e/x^(1/3))^6*
(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(4*e^6) + (3*d^5*(d + e/x^(1/3))*(a + b
*Log[c*(d + e/x^(1/3))^n])^3)/e^6 - (15*d^4*(d + e/x^(1/3))^2*(a + b*Log[c*
(d + e/x^(1/3))^n])^3)/(2*e^6) + (10*d^3*(d + e/x^(1/3))^3*(a + b*Log[c*(d
+ e/x^(1/3))^n])^3)/e^6 - (15*d^2*(d + e/x^(1/3))^4*(a + b*Log[c*(d + e/x^(
1/3))^n])^3)/(2*e^6) + (3*d*(d + e/x^(1/3))^5*(a + b*Log[c*(d + e/x^(1/3))^
n])^3)/e^6 - ((d + e/x^(1/3))^6*(a + b*Log[c*(d + e/x^(1/3))^n])^3)/(2*e^6)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^3}\,dx = -\left(3\text{Subst}\left(\int x^5(a+b\log\left(c(d+ex)^n\right))^3\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\left(3\text{Subst}\left(\int \left(-\frac{d^5(a+b\log(c(d+ex)^n))^3}{e^5} + \frac{5d^4(d+ex)\,(a+b}{e^5}\right.\right.\right.$$

$$= -\frac{3\text{Subst}\left(\int (d+ex)^5\,(a+b\log(c(d+ex)^n))^3\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^5} + \frac{(15d)}{}$$

$$= -\frac{3\text{Subst}\left(\int x^5(a+b\log(cx^n))^3\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^6} + \frac{(15d)\text{Subst}\left(\int\right.}{}$$

$$= \frac{3d^5\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^6} - \frac{15d^4\left(d+\frac{e}{\sqrt[3]{x}}\right)}{}$$

$$= -\frac{9bd^5n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{e^6} + \frac{45bd^4n\left(d+\right.}{}$$

$$= \frac{45b^3d^4n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{8e^6} - \frac{20b^3d^3n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^6} + \frac{45b^3d^2n^3\left(d+\frac{}{\sqrt[3]{}}\right.}{64e^6}$$

$$= \frac{45b^3d^4n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{8e^6} - \frac{20b^3d^3n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^6} + \frac{45b^3d^2n^3\left(d+\frac{}{\sqrt[3]{}}\right.}{64e^6}$$

Mathematica [A]
time = 1.08, size = 962, normalized size = 1.06

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^3/x^3,x]
```

```
[Out] (-36000*a^3*e^6 + 18000*a^2*b*e^6*n - 6000*a*b^2*e^6*n^2 + 1000*b^3*e^6*n^3
 - 21600*a^2*b*d*e^5*n*x^(1/3) + 15840*a*b^2*d*e^5*n^2*x^(1/3) - 4368*b^3*d
*e^5*n^3*x^(1/3) + 27000*a^2*b*d^2*e^4*n*x^(2/3) - 33300*a*b^2*d^2*e^4*n^2*
x^(2/3) + 13785*b^3*d^2*e^4*n^3*x^(2/3) - 36000*a^2*b*d^3*e^3*n*x + 68400*a
*b^2*d^3*e^3*n^2*x - 41180*b^3*d^3*e^3*n^3*x + 54000*a^2*b*d^4*e^2*n*x^(4/3
) - 156600*a*b^2*d^4*e^2*n^2*x^(4/3) + 140070*b^3*d^4*e^2*n^3*x^(4/3) - 108
000*a^2*b*d^5*e*n*x^(5/3) + 529200*a*b^2*d^5*e*n^2*x^(5/3) - 809340*b^3*d^5
```

```
*e*n^3*x^(5/3) - 72000*b^3*d^6*n^3*x^2*Log[d + e/x^(1/3)]^3 - 36000*b^3*e^6
*Log[c*(d + e/x^(1/3))^n]^3 + 108000*a^2*b*d^6*n*x^2*Log[e + d*x^(1/3)] - 5
29200*a*b^2*d^6*n^2*x^2*Log[e + d*x^(1/3)] + 809340*b^3*d^6*n^3*x^2*Log[e +
 d*x^(1/3)] + 3600*b^2*d^6*n^2*x^2*Log[d + e/x^(1/3)]*(-20*a + 49*b*n - 20*
b*Log[c*(d + e/x^(1/3))^n])*(3*Log[e + d*x^(1/3)] - Log[x]) - 36000*a^2*b*d
^6*n*x^2*Log[x] + 176400*a*b^2*d^6*n^2*x^2*Log[x] - 269780*b^3*d^6*n^3*x^2*
Log[x] + 1800*b^2*d^6*n^2*x^2*Log[d + e/x^(1/3)]^2*(60*a - 147*b*n + 60*b*L
og[c*(d + e/x^(1/3))^n] + 60*b*n*Log[e + d*x^(1/3)] - 20*b*n*Log[x]) + 1800
*b^2*Log[c*(d + e/x^(1/3))^n]^2*(e*(-60*a*e^5 + 10*b*e^5*n - 12*b*d*e^4*n*x
^(1/3) + 15*b*d^2*e^3*n*x^(2/3) - 20*b*d^3*e^2*n*x + 30*b*d^4*e*n*x^(4/3) -
 60*b*d^5*n*x^(5/3)) + 60*b*d^6*n*x^2*Log[e + d*x^(1/3)] - 20*b*d^6*n*x^2*L
og[x]) - 60*b*Log[c*(d + e/x^(1/3))^n]*(1800*a^2*e^6 + b^2*e*n^2*(100*e^5 -
 264*d*e^4*x^(1/3) + 555*d^2*e^3*x^(2/3) - 1140*d^3*e^2*x + 2610*d^4*e*x^(4
/3) - 8820*d^5*x^(5/3)) - 60*a*b*e*n*(10*e^5 - 12*d*e^4*x^(1/3) + 15*d^2*e^
3*x^(2/3) - 20*d^3*e^2*x + 30*d^4*e*x^(4/3) - 60*d^5*x^(5/3)) + 180*b*d^6*n
*(-20*a + 49*b*n)*x^2*Log[e + d*x^(1/3)] + 60*b*d^6*n*(20*a - 49*b*n)*x^2*L
og[x]))/(72000*e^6*x^2)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)^{3}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^n))^3/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))^3/x^3,x)
```

Maxima [A]
time = 0.34, size = 851, normalized size = 0.94

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/40*(60*d^6*e^(-7)*log(d*x^(1/3) + e) - 20*d^6*e^(-7)*log(x) - (60*d^5*x^(
5/3) - 30*d^4*x^(4/3)*e + 20*d^3*x*e^2 - 15*d^2*x^(2/3)*e^3 + 12*d*x^(1/3)*
e^4 - 10*e^5)*e^(-6)/x^2)*a^2*b*n*e + 1/1200*(60*(60*d^6*e^(-7)*log(d*x^(1/
3) + e) - 20*d^6*e^(-7)*log(x) - (60*d^5*x^(5/3) - 30*d^4*x^(4/3)*e + 20*d^
3*x*e^2 - 15*d^2*x^(2/3)*e^3 + 12*d*x^(1/3)*e^4 - 10*e^5)*e^(-6)/x^2)*n*e*l
og(c*(d + e/x^(1/3))^n) - (1800*d^6*x^2*log(d*x^(1/3) + e)^2 + 200*d^6*x^2*
log(x)^2 - 2940*d^6*x^2*log(x) - 8820*d^5*x^(5/3)*e + 2610*d^4*x^(4/3)*e^2
- 1140*d^3*x*e^3 + 555*d^2*x^(2/3)*e^4 - 264*d*x^(1/3)*e^5 - 60*(20*d^6*x^2
```

```
*log(x) - 147*d^6*x^2)*log(d*x^(1/3) + e) + 100*e^6)*n^2*e^(-6)/x^2)*a*b^2
+ 1/216000*(5400*(60*d^6*e^(-7)*log(d*x^(1/3) + e) - 20*d^6*e^(-7)*log(x) -
 (60*d^5*x^(5/3) - 30*d^4*x^(4/3)*e + 20*d^3*x*e^2 - 15*d^2*x^(2/3)*e^3 + 1
2*d*x^(1/3)*e^4 - 10*e^5)*e^(-6)/x^2)*n*e*log(c*(d + e/x^(1/3))^n)^2 + ((10
8000*d^6*x^2*log(d*x^(1/3) + e)^3 - 4000*d^6*x^2*log(x)^3 + 88200*d^6*x^2*l
og(x)^2 - 809340*d^6*x^2*log(x) - 2428020*d^5*x^(5/3)*e + 420210*d^4*x^(4/3
)*e^2 - 123540*d^3*x*e^3 + 41355*d^2*x^(2/3)*e^4 - 5400*(20*d^6*x^2*log(x)
- 147*d^6*x^2)*log(d*x^(1/3) + e)^2 - 13104*d*x^(1/3)*e^5 + 180*(200*d^6*x^
2*log(x)^2 - 2940*d^6*x^2*log(x) + 13489*d^6*x^2)*log(d*x^(1/3) + e) + 3000
*e^6)*n^2*e^(-7)/x^2 - 180*(1800*d^6*x^2*log(d*x^(1/3) + e)^2 + 200*d^6*x^2
*log(x)^2 - 2940*d^6*x^2*log(x) - 8820*d^5*x^(5/3)*e + 2610*d^4*x^(4/3)*e^2
 - 1140*d^3*x*e^3 + 555*d^2*x^(2/3)*e^4 - 264*d*x^(1/3)*e^5 - 60*(20*d^6*x^
2*log(x) - 147*d^6*x^2)*log(d*x^(1/3) + e) + 100*e^6)*n*e^(-7)*log(c*(d + e
/x^(1/3))^n)/x^2)*n*e)*b^3 - 1/2*b^3*log(c*(d + e/x^(1/3))^n)^3/x^2 - 3/2*a
*b^2*log(c*(d + e/x^(1/3))^n)^2/x^2 - 3/2*a^2*b*log(c*(d + e/x^(1/3))^n)/x^
2 - 1/2*a^3/x^2
```

Fricas [A]
time = 0.44, size = 1290, normalized size = 1.42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/72000*(36000*(b^3*x^2 - b^3)*e^6*log(c)^3 + 36000*(b^3*d^6*n^3*x^2 - b^3*
n^3*e^6)*log((d*x + x^(2/3)*e)/x)^3 + 18000*((b^3*n - 6*a*b^2 - (b^3*n - 6*
a*b^2)*x^2)*e^6 + 2*(b^3*d^3*n*x^2 - b^3*d^3*n*x)*e^3)*log(c)^2 - 1800*(20*
b^3*d^3*n^3*x*e^3 + 3*(49*b^3*d^6*n^3 - 20*a*b^2*d^6*n^2)*x^2 - 10*(b^3*n^3
 - 6*a*b^2*n^2)*e^6 - 60*(b^3*d^6*n^2*x^2 - b^3*n^2*e^6)*log(c) + 15*(4*b^3
*d^5*n^3*x*e - b^3*d^2*n^3*e^4)*x^(2/3) - 6*(5*b^3*d^4*n^3*x*e^2 - 2*b^3*d*
n^3*e^5)*x^(1/3))*log((d*x + x^(2/3)*e)/x)^2 + 1000*(b^3*n^3 - 6*a*b^2*n^2
+ 18*a^2*b*n - 36*a^3 - (b^3*n^3 - 6*a*b^2*n^2 + 18*a^2*b*n - 36*a^3)*x^2)*
e^6 + 20*((2059*b^3*d^3*n^3 - 3420*a*b^2*d^3*n^2 + 1800*a^2*b*d^3*n)*x^2 -
(2059*b^3*d^3*n^3 - 3420*a*b^2*d^3*n^2 + 1800*a^2*b*d^3*n)*x)*e^3 - 1200*(5
*(b^3*n^2 - 6*a*b^2*n + 18*a^2*b - (b^3*n^2 - 6*a*b^2*n + 18*a^2*b)*x^2)*e^
6 + 3*((19*b^3*d^3*n^2 - 20*a*b^2*d^3*n)*x^2 - (19*b^3*d^3*n^2 - 20*a*b^2*d
^3*n)*x)*e^3)*log(c) + 60*((13489*b^3*d^6*n^3 - 8820*a*b^2*d^6*n^2 + 1800*a
^2*b*d^6*n)*x^2 + 60*(19*b^3*d^3*n^3 - 20*a*b^2*d^3*n^2)*x*e^3 + 1800*(b^3*
d^6*n*x^2 - b^3*n*e^6)*log(c)^2 - 100*(b^3*n^3 - 6*a*b^2*n^2 + 18*a^2*b*n)*
e^6 - 60*(20*b^3*d^3*n^2*x*e^3 + 3*(49*b^3*d^6*n^2 - 20*a*b^2*d^6*n)*x^2 -
10*(b^3*n^2 - 6*a*b^2*n)*e^6)*log(c) + 15*(12*(49*b^3*d^5*n^3 - 20*a*b^2*d^
5*n^2)*x*e - (37*b^3*d^2*n^3 - 60*a*b^2*d^2*n^2)*e^4 - 60*(4*b^3*d^5*n^2*x*
e - b^3*d^2*n^2*e^4)*log(c))*x^(2/3) - 6*(15*(29*b^3*d^4*n^3 - 20*a*b^2*d^4
*n^2)*x*e^2 - 4*(11*b^3*d*n^3 - 30*a*b^2*d*n^2)*e^5 - 60*(5*b^3*d^4*n^2*x*e
```

```
^2 - 2*b^3*d*n^2*e^5)*log(c))*x^(1/3))*log((d*x + x^(2/3)*e)/x) - 15*(4*(13
489*b^3*d^5*n^3 - 8820*a*b^2*d^5*n^2 + 1800*a^2*b*d^5*n)*x*e + 1800*(4*b^3*
d^5*n*x*e - b^3*d^2*n*e^4)*log(c)^2 - (919*b^3*d^2*n^3 - 2220*a*b^2*d^2*n^2
 + 1800*a^2*b*d^2*n)*e^4 - 60*(12*(49*b^3*d^5*n^2 - 20*a*b^2*d^5*n)*x*e - (
37*b^3*d^2*n^2 - 60*a*b^2*d^2*n)*e^4)*log(c))*x^(2/3) + 6*(5*(4669*b^3*d^4*
n^3 - 5220*a*b^2*d^4*n^2 + 1800*a^2*b*d^4*n)*x*e^2 + 1800*(5*b^3*d^4*n*x*e^
2 - 2*b^3*d*n*e^5)*log(c)^2 - 8*(91*b^3*d*n^3 - 330*a*b^2*d*n^2 + 450*a^2*b
*d*n)*e^5 - 60*(15*(29*b^3*d^4*n^2 - 20*a*b^2*d^4*n)*x*e^2 - 4*(11*b^3*d*n^
2 - 30*a*b^2*d*n)*e^5)*log(c))*x^(1/3))*e^(-6)/x^2
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))**3/x**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3651 vs. 2(803) = 1606.
time = 3.53, size = 3651, normalized size = 4.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x^3,x, algorithm="giac")
```

```
[Out] 1/72000*(216000*(d*x^(1/3) + e)*b^3*d^5*n^3*log((d*x^(1/3) + e)/x^(1/3))^3/
x^(1/3) - 540000*(d*x^(1/3) + e)^2*b^3*d^4*n^3*log((d*x^(1/3) + e)/x^(1/3))
^3/x^(2/3) + 720000*(d*x^(1/3) + e)^3*b^3*d^3*n^3*log((d*x^(1/3) + e)/x^(1/
3))^3/x - 648000*(d*x^(1/3) + e)*b^3*d^5*n^3*log((d*x^(1/3) + e)/x^(1/3))^2
/x^(1/3) + 648000*(d*x^(1/3) + e)*b^3*d^5*n^2*log(c)*log((d*x^(1/3) + e)/x^
(1/3))^2/x^(1/3) - 540000*(d*x^(1/3) + e)^4*b^3*d^2*n^3*log((d*x^(1/3) + e)
/x^(1/3))^3/x^(4/3) + 810000*(d*x^(1/3) + e)^2*b^3*d^4*n^3*log((d*x^(1/3) +
 e)/x^(1/3))^2/x^(2/3) - 1620000*(d*x^(1/3) + e)^2*b^3*d^4*n^2*log(c)*log((
d*x^(1/3) + e)/x^(1/3))^2/x^(2/3) + 216000*(d*x^(1/3) + e)^5*b^3*d*n^3*log(
(d*x^(1/3) + e)/x^(1/3))^3/x^(5/3) - 720000*(d*x^(1/3) + e)^3*b^3*d^3*n^3*l
og((d*x^(1/3) + e)/x^(1/3))^2/x + 2160000*(d*x^(1/3) + e)^3*b^3*d^3*n^2*log
(c)*log((d*x^(1/3) + e)/x^(1/3))^2/x - 36000*(d*x^(1/3) + e)^6*b^3*n^3*log(
(d*x^(1/3) + e)/x^(1/3))^3/x^2 + 1296000*(d*x^(1/3) + e)*b^3*d^5*n^3*log((d
*x^(1/3) + e)/x^(1/3))/x^(1/3) - 1296000*(d*x^(1/3) + e)*b^3*d^5*n^2*log(c)
*log((d*x^(1/3) + e)/x^(1/3))/x^(1/3) + 648000*(d*x^(1/3) + e)*b^3*d^5*n*lo
g(c)^2*log((d*x^(1/3) + e)/x^(1/3))/x^(1/3) + 405000*(d*x^(1/3) + e)^4*b^3*
d^2*n^3*log((d*x^(1/3) + e)/x^(1/3))^2/x^(4/3) + 648000*(d*x^(1/3) + e)*a*b
```

^2*d^5*n^2*log((d*x^(1/3) + e)/x^(1/3))^2/x^(1/3) - 1620000*(d*x^(1/3) + e)
^4*b^3*d^2*n^2*log(c)*log((d*x^(1/3) + e)/x^(1/3))^2/x^(4/3) - 810000*(d*x^
(1/3) + e)^2*b^3*d^4*n^3*log((d*x^(1/3) + e)/x^(1/3))/x^(2/3) + 1620000*(d*
x^(1/3) + e)^2*b^3*d^4*n^2*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x^(2/3) - 16
20000*(d*x^(1/3) + e)^2*b^3*d^4*n*log(c)^2*log((d*x^(1/3) + e)/x^(1/3))/x^(
2/3) - 129600*(d*x^(1/3) + e)^5*b^3*d*n^3*log((d*x^(1/3) + e)/x^(1/3))^2/x^
(5/3) - 1620000*(d*x^(1/3) + e)^2*a*b^2*d^4*n^2*log((d*x^(1/3) + e)/x^(1/3)
)^2/x^(2/3) + 648000*(d*x^(1/3) + e)^5*b^3*d*n^2*log(c)*log((d*x^(1/3) + e)
/x^(1/3))^2/x^(5/3) + 480000*(d*x^(1/3) + e)^3*b^3*d^3*n^3*log((d*x^(1/3) +
 e)/x^(1/3))/x - 1440000*(d*x^(1/3) + e)^3*b^3*d^3*n^2*log(c)*log((d*x^(1/3
) + e)/x^(1/3))/x + 2160000*(d*x^(1/3) + e)^3*b^3*d^3*n*log(c)^2*log((d*x^(
1/3) + e)/x^(1/3))/x + 18000*(d*x^(1/3) + e)^6*b^3*n^3*log((d*x^(1/3) + e)/
x^(1/3))^2/x^2 + 2160000*(d*x^(1/3) + e)^3*a*b^2*d^3*n^2*log((d*x^(1/3) + e
)/x^(1/3))^2/x - 108000*(d*x^(1/3) + e)^6*b^3*n^2*log(c)*log((d*x^(1/3) + e
)/x^(1/3))^2/x^2 - 1296000*(d*x^(1/3) + e)*b^3*d^5*n^3/x^(1/3) + 1296000*(d
*x^(1/3) + e)*b^3*d^5*n^2*log(c)/x^(1/3) - 648000*(d*x^(1/3) + e)*b^3*d^5*n
log(c)^2/x^(1/3) + 216000(d*x^(1/3) + e)*b^3*d^5*log(c)^3/x^(1/3) - 20250
0*(d*x^(1/3) + e)^4*b^3*d^2*n^3*log((d*x^(1/3) + e)/x^(1/3))/x^(4/3) - 1296
000*(d*x^(1/3) + e)*a*b^2*d^5*n^2*log((d*x^(1/3) + e)/x^(1/3))/x^(1/3) + 81
0000*(d*x^(1/3) + e)^4*b^3*d^2*n^2*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x^(4
/3) + 1296000*(d*x^(1/3) + e)*a*b^2*d^5*n*log(c)*log((d*x^(1/3) + e)/x^(1/3
))/x^(1/3) - 1620000*(d*x^(1/3) + e)^4*b^3*d^2*n*log(c)^2*log((d*x^(1/3) +
e)/x^(1/3))/x^(4/3) - 1620000*(d*x^(1/3) + e)^4*a*b^2*d^2*n^2*log((d*x^(1/3
) + e)/x^(1/3))^2/x^(4/3) + 405000*(d*x^(1/3) + e)^2*b^3*d^4*n^3/x^(2/3) -
810000*(d*x^(1/3) + e)^2*b^3*d^4*n^2*log(c)/x^(2/3) + 810000*(d*x^(1/3) + e
)^2*b^3*d^4*n*log(c)^2/x^(2/3) - 540000*(d*x^(1/3) + e)^2*b^3*d^4*log(c)^3/
x^(2/3) + 51840*(d*x^(1/3) + e)^5*b^3*d*n^3*log((d*x^(1/3) + e)/x^(1/3))/x^
(5/3) + 1620000*(d*x^(1/3) + e)^2*a*b^2*d^4*n^2*log((d*x^(1/3) + e)/x^(1/3)
)/x^(2/3) - 259200*(d*x^(1/3) + e)^5*b^3*d*n^2*log(c)*log((d*x^(1/3) + e)/x
^(1/3))/x^(5/3) - 3240000*(d*x^(1/3) + e)^2*a*b^2*d^4*n*log(c)*log((d*x^(1/
3) + e)/x^(1/3))/x^(2/3) + 648000*(d*x^(1/3) + e)^5*b^3*d*n*log(c)^2*log((d
x^(1/3) + e)/x^(1/3))/x^(5/3) + 648000(d*x^(1/3) + e)^5*a*b^2*d*n^2*log((
d*x^(1/3) + e)/x^(1/3))^2/x^(5/3) - 160000*(d*x^(1/3) + e)^3*b^3*d^3*n^3/x
+ 480000*(d*x^(1/3) + e)^3*b^3*d^3*n^2*log(c)/x - 720000*(d*x^(1/3) + e)^3*
b^3*d^3*n*log(c)^2/x + 720000*(d*x^(1/3) + e)^3*b^3*d^3*log(c)^3/x - 6000*(
d*x^(1/3) + e)^6*b^3*n^3*log((d*x^(1/3) + e)/x^(1/3))/x^2 - 1440000*(d*x^(1
/3) + e)^3*a*b^2*d^3*n^2*log((d*x^(1/3) + e)/x^(1/3))/x + 36000*(d*x^(1/3)
+ e)^6*b^3*n^2*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x^2 + 4320000*(d*x^(1/3)
 + e)^3*a*b^2*d^3*n*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x - 108000*(d*x^(1/
3) + e)^6*b^3*n*log(c)^2*log((d*x^(1/3) + e)/x^(1/3))/x^2 - 108000*(d*x^(1/
3) + e)^6*a*b^2*n^2*log((d*x^(1/3) + e)/x^(1/3))^2/x^2 + 50625*(d*x^(1/3) +
 e)^4*b^3*d^2*n^3/x^(4/3) + 1296000*(d*x^(1/3) + e)*a*b^2*d^5*n^2/x^(1/3) -
 202500*(d*x^(1/3) + e)^4*b^3*d^2*n^2*log(c)/x^(4/3) - 1296000*(d*x^(1/3) +
 e)*a*b^2*d^5*n*log(c)/x^(1/3) + 405000*(d*x^(1/3) + e)^4*b^3*d^2*n*log(c)^
2/x^(4/3) + 648000*(d*x^(1/3) + e)*a*b^2*d^5*log(c)^2/x^(1/3) - 540000*(d*x

```
^(1/3) + e)^4*b^3*d^2*log(c)^3/x^(4/3) + 810000*(d*x^(1/3) + e)^4*a*b^2*d^2
*n^2*log((d*x^(1/3) + e)/x^(1/3))/x^(4/3) + 648000*(d*x^(1/3) + e)*a^2*b*d^
5*n*log((d*x^(1/3) + e)/x^(1/3))/x^(1/3) - 3240000*(d*x^(1/3) + e)^4*a*b^2*
d^2*n*log(c)*log((d*x^(1/3) + e)/x^(1/3))/x^(4/3) - 10368*(d*x^(1/3) + e)^5
*b^3*d*n^3/x^(5/3) - 810000*(d*x^(1/3) + e)^2*a...
```

Mupad [B]
time = 8.20, size = 992, normalized size = 1.09

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))^3/x^3,x)
```

```
[Out] (b^3*n^3)/(72*x^2) - (b^3*log(c*(d + e/x^(1/3))^n)^3)/(2*x^2) - a^3/(2*x^2)
 - (3*a*b^2*log(c*(d + e/x^(1/3))^n)^2)/(2*x^2) + (b^3*n*log(c*(d + e/x^(1/
3))^n)^2)/(4*x^2) - (b^3*n^2*log(c*(d + e/x^(1/3))^n))/(12*x^2) - (a*b^2*n^
2)/(12*x^2) + (b^3*d^6*log(c*(d + e/x^(1/3))^n)^3)/(2*e^6) - (3*a^2*b*log(c
*(d + e/x^(1/3))^n))/(2*x^2) + (a^2*b*n)/(4*x^2) + (a*b^2*n*log(c*(d + e/x^
(1/3))^n))/(2*x^2) + (13489*b^3*d^6*n^3*log(d + e/x^(1/3)))/(1200*e^6) - (2
059*b^3*d^3*n^3)/(3600*e^3*x) + (919*b^3*d^2*n^3)/(4800*e^2*x^(4/3)) + (466
9*b^3*d^4*n^3)/(2400*e^4*x^(2/3)) - (13489*b^3*d^5*n^3)/(1200*e^5*x^(1/3))
+ (3*a*b^2*d^6*log(c*(d + e/x^(1/3))^n)^2)/(2*e^6) - (147*b^3*d^6*n*log(c*(
d + e/x^(1/3))^n)^2)/(40*e^6) - (91*b^3*d*n^3)/(1500*e*x^(5/3)) + (3*a^2*b*
d^6*n*log(d + e/x^(1/3)))/(2*e^6) - (3*b^3*d*n*log(c*(d + e/x^(1/3))^n)^2)/
(10*e*x^(5/3)) + (11*b^3*d*n^2*log(c*(d + e/x^(1/3))^n))/(50*e*x^(5/3)) - (
a^2*b*d^3*n)/(2*e^3*x) + (11*a*b^2*d*n^2)/(50*e*x^(5/3)) + (3*a^2*b*d^2*n)/
(8*e^2*x^(4/3)) + (3*a^2*b*d^4*n)/(4*e^4*x^(2/3)) - (3*a^2*b*d^5*n)/(2*e^5*
x^(1/3)) - (147*a*b^2*d^6*n^2*log(d + e/x^(1/3)))/(20*e^6) - (b^3*d^3*n*log
(c*(d + e/x^(1/3))^n)^2)/(2*e^3*x) + (19*b^3*d^3*n^2*log(c*(d + e/x^(1/3))^
n))/(20*e^3*x) + (3*b^3*d^2*n*log(c*(d + e/x^(1/3))^n)^2)/(8*e^2*x^(4/3)) -
 (37*b^3*d^2*n^2*log(c*(d + e/x^(1/3))^n))/(80*e^2*x^(4/3)) + (3*b^3*d^4*n*
log(c*(d + e/x^(1/3))^n)^2)/(4*e^4*x^(2/3)) - (87*b^3*d^4*n^2*log(c*(d + e/
x^(1/3))^n))/(40*e^4*x^(2/3)) - (3*b^3*d^5*n*log(c*(d + e/x^(1/3))^n)^2)/(2
*e^5*x^(1/3)) + (147*b^3*d^5*n^2*log(c*(d + e/x^(1/3))^n))/(20*e^5*x^(1/3))
 + (19*a*b^2*d^3*n^2)/(20*e^3*x) - (37*a*b^2*d^2*n^2)/(80*e^2*x^(4/3)) - (8
7*a*b^2*d^4*n^2)/(40*e^4*x^(2/3)) + (147*a*b^2*d^5*n^2)/(20*e^5*x^(1/3)) -
(3*a^2*b*d*n)/(10*e*x^(5/3)) - (3*a*b^2*d*n*log(c*(d + e/x^(1/3))^n))/(5*e*
x^(5/3)) - (a*b^2*d^3*n*log(c*(d + e/x^(1/3))^n))/(e^3*x) + (3*a*b^2*d^2*n*
log(c*(d + e/x^(1/3))^n))/(4*e^2*x^(4/3)) + (3*a*b^2*d^4*n*log(c*(d + e/x^(
1/3))^n))/(2*e^4*x^(2/3)) - (3*a*b^2*d^5*n*log(c*(d + e/x^(1/3))^n))/(e^5*x
^(1/3))
```

3.508 $\int x^3 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right) \, dx$

Optimal. Leaf size=143

$$\frac{be^5nx^{2/3}}{4d^5}-\frac{be^4nx^{4/3}}{8d^4}+\frac{be^3nx^2}{12d^3}-\frac{be^2nx^{8/3}}{16d^2}+\frac{benx^{10/3}}{20d}-\frac{be^6n\log\left(d+\frac{e}{x^{2/3}}\right)}{4d^6}+\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{be}{}$$

```
[Out] 1/4*b*e^5*n*x^(2/3)/d^5-1/8*b*e^4*n*x^(4/3)/d^4+1/12*b*e^3*n*x^2/d^3-1/16*b
*e^2*n*x^(8/3)/d^2+1/20*b*e*n*x^(10/3)/d-1/4*b*e^6*n*ln(d+e/x^(2/3))/d^6+1/
4*x^4*(a+b*ln(c*(d+e/x^(2/3))^n))-1/6*b*e^6*n*ln(x)/d^6
```

Rubi [A]
time = 0.07, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2504, 2442, 46}

$$\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{be^6n\log\left(d+\frac{e}{x^{2/3}}\right)}{4d^6}-\frac{be^6n\log(x)}{6d^6}+\frac{be^5nx^{2/3}}{4d^5}-\frac{be^4nx^{4/3}}{8d^4}+\frac{be^3nx^2}{12d^3}-\frac{be^2nx^{8/3}}{16d^2}+\frac{benx^{10/3}}{20d}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e/x^(2/3))^n]),x]
```

```
[Out] (b*e^5*n*x^(2/3))/(4*d^5) - (b*e^4*n*x^(4/3))/(8*d^4) + (b*e^3*n*x^2)/(12*d
^3) - (b*e^2*n*x^(8/3))/(16*d^2) + (b*e*n*x^(10/3))/(20*d) - (b*e^6*n*Log[d
 + e/x^(2/3)])/(4*d^6) + (x^4*(a + b*Log[c*(d + e/x^(2/3))^n]))/4 - (b*e^6*
n*Log[x])/(6*d^6)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)dx = -\left(\frac{3}{2}\text{Subst}\left(\int\frac{a+b\log\left(c(d+ex)^n\right)}{x^7}\,dx,x,\frac{1}{x^{2/3}}\right)\right)$$
$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{1}{4}(ben)\text{Subst}\left(\int\frac{1}{x^6(d+ex)}\,dx,\right.$$
$$=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{1}{4}(ben)\text{Subst}\left(\int\left(\frac{1}{dx^6}-\frac{e}{d^2x^5}\right.\right.$$
$$=\frac{be^5nx^{2/3}}{4d^5}-\frac{be^4nx^{4/3}}{8d^4}+\frac{be^3nx^2}{12d^3}-\frac{be^2nx^{8/3}}{16d^2}+\frac{benx^{10/3}}{20d}-\frac{be^6n\log(}{4d}$$

Mathematica [A]
time = 0.08, size = 134, normalized size = 0.94

$$\frac{ax^4}{4}+\frac{1}{4}bx^4\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)-\frac{1}{4}ben\left(-\frac{e^4x^{2/3}}{d^5}+\frac{e^3x^{4/3}}{2d^4}-\frac{e^2x^2}{3d^3}+\frac{ex^{8/3}}{4d^2}-\frac{x^{10/3}}{5d}+\frac{e^5\log\left(d+\frac{e}{x^{2/3}}\right)}{d^6}+\frac{2e^5\log(x)}{3d^6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))^n]),x]
[Out] (a*x^4)/4 + (b*x^4*Log[c*(d + e/x^(2/3))^n])/4 - (b*e*n*(-((e^4*x^(2/3))/d^
5) + (e^3*x^(4/3))/(2*d^4) - (e^2*x^2)/(3*d^3) + (e*x^(8/3))/(4*d^2) - x^(1
0/3)/(5*d) + (e^5*Log[d + e/x^(2/3)])/d^6 + (2*e^5*Log[x])/(3*d^6)))/4
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^3\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e/x^(2/3))^n)),x)
[Out] int(x^3*(a+b*ln(c*(d+e/x^(2/3))^n)),x)
```

Maxima [A]
time = 0.28, size = 97, normalized size = 0.68

$$\frac{1}{4}bx^4\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+\frac{1}{4}ax^4+\frac{1}{240}bn\left(\frac{12\,d^4x^{\frac{10}{3}}-15\,d^3x^{\frac{8}{3}}e+20\,d^2x^2e^2-30\,dx^{\frac{4}{3}}e^3+60\,x^{\frac{2}{3}}e^4}{d^5}-\frac{60\,e^5\log\left(dx^{\frac{2}{3}}+e\right)}{d^6}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4*log(c*(d + e/x^(2/3))^n) + 1/4*a*x^4 + 1/240*b*n*((12*d^4*x^(10/3
) - 15*d^3*x^(8/3)*e + 20*d^2*x^2*e^2 - 30*d*x^(4/3)*e^3 + 60*x^(2/3)*e^4)/
d^5 - 60*e^5*log(d*x^(2/3) + e)/d^6)*e
```

Fricas [A]
time = 0.39, size = 159, normalized size = 1.11

$$\frac{60\,bd^6x^4\log\left(c\right)+60\,ad^6x^4-120\,bd^6n\log\left(x^{\frac{1}{3}}\right)+20\,bd^3nx^2e^3+60\,(bd^6n-bne^6)\log\left(dx^{\frac{2}{3}}+e\right)+60\,(bd^6nx^4-bd^6n)\log\left(\frac{dx+x^{\frac{1}{3}}e}{x}\right)-15\,(bd^4nx^2e^2-4\,bdne^5)x^{\frac{2}{3}}+6\,(2\,bd^5nx^3e-5\,bd^2nxe^4)x^{\frac{1}{3}}}{240\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="fricas")
```

```
[Out] 1/240*(60*b*d^6*x^4*log(c) + 60*a*d^6*x^4 - 120*b*d^6*n*log(x^(1/3)) + 20*b
*d^3*n*x^2*e^3 + 60*(b*d^6*n - b*n*e^6)*log(d*x^(2/3) + e) + 60*(b*d^6*n*x^
4 - b*d^6*n)*log((d*x + x^(1/3)*e)/x) - 15*(b*d^4*n*x^2*e^2 - 4*b*d*n*e^5)*
x^(2/3) + 6*(2*b*d^5*n*x^3*e - 5*b*d^2*n*x*e^4)*x^(1/3))/d^6
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e/x**(2/3))**n)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4848 deep
```

Giac [A]
time = 4.27, size = 103, normalized size = 0.72

$$\frac{1}{4}\,bx^4\log\left(c\right)+\frac{1}{4}\,ax^4+\frac{1}{240}\left(60\,x^4\log\left(d+\frac{e}{x^{\frac{2}{3}}}\right)+\left(\frac{12\,d^4x^{\frac{10}{3}}-15\,d^3x^{\frac{8}{3}}e+20\,d^2x^2e^2-30\,dx^{\frac{4}{3}}e^3+60\,x^{\frac{2}{3}}e^4}{d^5}-\frac{60\,e^5\log\left(\left|dx^{\frac{2}{3}}+e\right|\right)}{d^6}\right)e\right)bn$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="giac")
```

```
[Out] 1/4*b*x^4*log(c) + 1/4*a*x^4 + 1/240*(60*x^4*log(d + e/x^(2/3)) + ((12*d^4*
x^(10/3) - 15*d^3*x^(8/3)*e + 20*d^2*x^2*e^2 - 30*d*x^(4/3)*e^3 + 60*x^(2/3
)*e^4)/d^5 - 60*e^5*log(abs(d*x^(2/3) + e))/d^6)*e)*b*n
```

Mupad [B]
time = 0.70, size = 112, normalized size = 0.78

$$\frac{x^{10/3}\,\left(\frac{b\,e\,n}{5\,d}-\frac{b\,e^2\,n}{4\,d^2\,x^{2/3}}-\frac{b\,e^4\,n}{2\,d^4\,x^2}+\frac{b\,e^3\,n}{3\,d^3\,x^{4/3}}+\frac{b\,e^5\,n}{d^5\,x^{8/3}}\right)}{4}+\frac{a\,x^4}{4}+\frac{b\,x^4\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^n\right)}{4}-\frac{b\,e^6\,n\,\mathrm{atanh}\left(\frac{2\,e}{d\,x^{2/3}}+1\right)}{2\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e/x^(2/3))^n)),x)
```

```
[Out] (x^(10/3)*((b*e*n)/(5*d) - (b*e^2*n)/(4*d^2*x^(2/3)) - (b*e^4*n)/(2*d^4*x^2
) + (b*e^3*n)/(3*d^3*x^(4/3)) + (b*e^5*n)/(d^5*x^(8/3))))/4 + (a*x^4)/4 + (
b*x^4*log(c*(d + e/x^(2/3))^n))/4 - (b*e^6*n*atanh((2*e)/(d*x^(2/3)) + 1))/
(2*d^6)
```

3.509 $\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right) dx$

Optimal. Leaf size=121

$$-\frac{2be^4n\sqrt[3]{x}}{3d^4}+\frac{2be^3nx}{9d^3}-\frac{2be^2nx^{5/3}}{15d^2}+\frac{2benx^{7/3}}{21d}+\frac{2be^{9/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3d^{9/2}}+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)$$

```
[Out] -2/3*b*e^4*n*x^(1/3)/d^4+2/9*b*e^3*n*x/d^3-2/15*b*e^2*n*x^(5/3)/d^2+2/21*b*
e*n*x^(7/3)/d+2/3*b*e^(9/2)*n*arctan(x^(1/3)*d^(1/2)/e^(1/2))/d^(9/2)+1/3*x
^3*(a+b*ln(c*(d+e/x^(2/3))^n))
```

Rubi [A]
time = 0.05, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2505, 269, 348, 308, 211}

$$\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+\frac{2be^{9/2}n\text{ArcTan}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3d^{9/2}}-\frac{2be^4n\sqrt[3]{x}}{3d^4}+\frac{2be^3nx}{9d^3}-\frac{2be^2nx^{5/3}}{15d^2}+\frac{2benx^{7/3}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e/x^(2/3))^n]),x]
```

```
[Out] (-2*b*e^4*n*x^(1/3))/(3*d^4) + (2*b*e^3*n*x)/(9*d^3) - (2*b*e^2*n*x^(5/3))/
(15*d^2) + (2*b*e*n*x^(7/3))/(21*d) + (2*b*e^(9/2)*n*ArcTan[(Sqrt[d]*x^(1/3
))/Sqrt[e]])/(3*d^(9/2)) + (x^3*(a + b*Log[c*(d + e/x^(2/3))^n]))/3
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)dx &= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+\frac{1}{9}(2ben)\int\frac{x^{4/3}}{d+\frac{e}{x^{2/3}}}dx\\ &= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+\frac{1}{9}(2ben)\int\frac{x^2}{e+dx^{2/3}}dx\\ &= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+\frac{1}{3}(2ben)\text{Subst}\left(\int\frac{x^8}{e+dx^2}dx,x\right.\\ &= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+\frac{1}{3}(2ben)\text{Subst}\left(\int\left(-\frac{e^3}{d^4}+\frac{e^2x^2}{d^3}\right.\right.\\ &= -\frac{2be^4n\sqrt[3]{x}}{3d^4}+\frac{2be^3nx}{9d^3}-\frac{2be^2nx^{5/3}}{15d^2}+\frac{2benx^{7/3}}{21d}+\frac{1}{3}x^3\left(a+b\log\left(c\right.\right.\\ &= -\frac{2be^4n\sqrt[3]{x}}{3d^4}+\frac{2be^3nx}{9d^3}-\frac{2be^2nx^{5/3}}{15d^2}+\frac{2benx^{7/3}}{21d}+\frac{2be^{9/2}n\tan^{-1}\left(\right.}{3d^{9/2}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.01, size = 65, normalized size = 0.54

$$\frac{ax^3}{3}+\frac{2benx^{7/3}\,{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};-\frac{e}{dx^{2/3}}\right)}{21d}+\frac{1}{3}bx^3\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))^n]),x]
[Out] (a*x^3)/3 + (2*b*e*n*x^(7/3)*Hypergeometric2F1[-7/2, 1, -5/2, -(e/(d*x^(2/3
)))])/(21*d) + (b*x^3*Log[c*(d + e/x^(2/3))^n])/3
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a+b*ln(c*(d+e/x^(2/3))^n)),x)

[Out] int(x^2*(a+b*ln(c*(d+e/x^(2/3))^n)),x)

Maxima [A]
time = 0.52, size = 86, normalized size = 0.71

$$\frac{1}{3}\,bx^3\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+\frac{1}{3}\,ax^3+\frac{2}{315}\,bn\left(\frac{105\,\arctan\left(\sqrt{d}\,x^{\frac{1}{3}}e^{\left(-\frac{1}{2}\right)}\right)e^{\frac{7}{2}}}{d^{\frac{9}{2}}}+\frac{15\,d^3x^{\frac{7}{3}}-21\,d^2x^{\frac{5}{3}}e+35\,dxe^2-105\,x^{\frac{1}{3}}e^3}{d^4}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="maxima")

[Out] 1/3*b*x^3*log(c*(d + e/x^(2/3))^n) + 1/3*a*x^3 + 2/315*b*n*(105*arctan(sqrt(d)*x^(1/3)*e^(-1/2))*e^(7/2)/d^(9/2) + (15*d^3*x^(7/3) - 21*d^2*x^(5/3)*e + 35*d*x*e^2 - 105*x^(1/3)*e^3)/d^4)*e

Fricas [A]
time = 0.41, size = 389, normalized size = 3.21

$$\left[\frac{105\,bd^4x^3\log\left(c\right)+105\,ad^4x^3+105\,bd^4n\log\left(dx^{\frac{2}{3}}+e\right)-210\,bd^4n\log\left(x^{\frac{1}{3}}\right)-42\,bd^2nx^{\frac{5}{3}}e^2+70\,bdnxe^3+105\,bn\sqrt{-\frac{e}{d}}\,e^4\log\left(\frac{d^3x^2-2\,d^2x\sqrt{-\frac{e}{d}}\,e+2\left(d^3x\sqrt{-\frac{e}{d}}+de^2\right)x^{\frac{2}{3}}-2\left(d^2xe-d\sqrt{-\frac{e}{d}}\,e^2\right)x^{\frac{1}{3}}-e^3}{d^3x^2+e^3}\right)+105\left(bd^4nx^3-bd^4n\right)\log\left(\frac{dx+x^{\frac{1}{3}}e}{x}\right)+30\left(bd^3nx^2e-7\,bne^4\right)x^{\frac{1}{3}}}{315\,d^4},\ \frac{105\,bd^4x^3\log\left(c\right)+105\,ad^4x^3+105\,bd^4n\log\left(dx^{\frac{2}{3}}+e\right)-210\,bd^4n\log\left(x^{\frac{1}{3}}\right)-42\,bd^2nx^{\frac{5}{3}}e^2+70\,bdnxe^3+\frac{210\,bn\arctan\left(\sqrt{d}\,x^{\frac{1}{3}}e^{\left(-\frac{1}{2}\right)}\right)e^{\frac{9}{2}}}{\sqrt{d}}+105\left(bd^4nx^3-bd^4n\right)\log\left(\frac{dx+x^{\frac{1}{3}}e}{x}\right)+30\left(bd^3nx^2e-7\,bne^4\right)x^{\frac{1}{3}}}{315\,d^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="fricas")

[Out] [1/315*(105*b*d^4*x^3*log(c) + 105*a*d^4*x^3 + 105*b*d^4*n*log(d*x^(2/3) + e) - 210*b*d^4*n*log(x^(1/3)) - 42*b*d^2*n*x^(5/3)*e^2 + 70*b*d*n*x*e^3 + 105*b*n*sqrt(-e/d)*e^4*log((d^3*x^2 - 2*d^2*x*sqrt(-e/d)*e + 2*(d^3*x*sqrt(-e/d) + d*e^2)*x^(2/3) - 2*(d^2*x*e - d*sqrt(-e/d)*e^2)*x^(1/3) - e^3)/(d^3*x^2 + e^3)) + 105*(b*d^4*n*x^3 - b*d^4*n)*log((d*x + x^(1/3)*e)/x) + 30*(b*d^3*n*x^2*e - 7*b*n*e^4)*x^(1/3))/d^4, 1/315*(105*b*d^4*x^3*log(c) + 105*a*d^4*x^3 + 105*b*d^4*n*log(d*x^(2/3) + e) - 210*b*d^4*n*log(x^(1/3)) - 42*b*d^2*n*x^(5/3)*e^2 + 70*b*d*n*x*e^3 + 210*b*n*arctan(sqrt(d)*x^(1/3)*e^(-1/2))*e^(9/2)/sqrt(d) + 105*(b*d^4*n*x^3 - b*d^4*n)*log((d*x + x^(1/3)*e)/x) + 30*(b*d^3*n*x^2*e - 7*b*n*e^4)*x^(1/3))/d^4]

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(a+b*ln(c*(d+e/x**(2/3)))**n)),x)

[Out] Timed out

Giac [A]
time = 3.56, size = 97, normalized size = 0.80

$$\frac{1}{3}bx^3\log(c)+\frac{1}{3}ax^3+\frac{1}{315}\left(105x^3\log\left(d+\frac{e}{x^{\frac{2}{3}}}\right)+2\left(\frac{105\arctan\left(\sqrt{d}x^{\frac{1}{3}}e^{\left(-\frac{1}{2}\right)}\right)e^{\frac{7}{2}}}{d^{\frac{9}{2}}}+\frac{15d^6x^{\frac{7}{3}}-21d^5x^{\frac{5}{3}}e+35d^4xe^2-105d^3x^{\frac{1}{3}}e^3}{d^7}\right)e\right)bn$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="giac")
```

```
[Out] 1/3*b*x^3*log(c) + 1/3*a*x^3 + 1/315*(105*x^3*log(d + e/x^(2/3)) + 2*(105*a
rctan(sqrt(d)*x^(1/3)*e^(-1/2))*e^(7/2)/d^(9/2) + (15*d^6*x^(7/3) - 21*d^5*
x^(5/3)*e + 35*d^4*x*e^2 - 105*d^3*x^(1/3)*e^3)/d^7)*e)*b*n
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(a+b\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e/x^(2/3))^n)),x)
```

```
[Out] int(x^2*(a + b*log(c*(d + e/x^(2/3))^n)), x)
```

3.510 $\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx$

Optimal. Leaf size=94

$$-\frac{be^2nx^{2/3}}{2d^2}+\frac{benx^{4/3}}{4d}+\frac{be^3n\log\left(d+\frac{e}{x^{2/3}}\right)}{2d^3}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+\frac{be^3n\log(x)}{3d^3}$$

```
[Out] -1/2*b*e^2*n*x^(2/3)/d^2+1/4*b*e*n*x^(4/3)/d+1/2*b*e^3*n*ln(d+e/x^(2/3))/d^
3+1/2*x^2*(a+b*ln(c*(d+e/x^(2/3))^n))+1/3*b*e^3*n*ln(x)/d^3
```

Rubi [A]
time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.150$, Rules used = {2504, 2442, 46}

$$\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+\frac{be^3n\log\left(d+\frac{e}{x^{2/3}}\right)}{2d^3}+\frac{be^3n\log(x)}{3d^3}-\frac{be^2nx^{2/3}}{2d^2}+\frac{benx^{4/3}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e/x^(2/3))^n]),x]
```

```
[Out] -1/2*(b*e^2*n*x^(2/3))/d^2 + (b*e*n*x^(4/3))/(4*d) + (b*e^3*n*Log[d + e/x^(
2/3)])/(2*d^3) + (x^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/2 + (b*e^3*n*Log[x]
)/(3*d^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
```

```
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx = -\left(\frac{3}{2}\text{Subst}\left(\int\frac{a+b\log\left(c(d+ex)^n\right)}{x^4}\,dx,x,\frac{1}{x^{2/3}}\right)\right)$$
$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{1}{2}(ben)\text{Subst}\left(\int\frac{1}{x^3(d+ex)}\,dx,$$
$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{1}{2}(ben)\text{Subst}\left(\int\left(\frac{1}{dx^3}-\frac{e}{d^2x^2}+\right.\right.$$
$$=-\frac{be^2nx^{2/3}}{2d^2}+\frac{benx^{4/3}}{4d}+\frac{be^3n\log\left(d+\frac{e}{x^{2/3}}\right)}{2d^3}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\right.\right.\right.$$

Mathematica [A]
time = 0.02, size = 91, normalized size = 0.97

$$\frac{ax^2}{2}+\frac{1}{2}bx^2\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)-\frac{1}{2}ben\left(\frac{ex^{2/3}}{d^2}-\frac{x^{4/3}}{2d}-\frac{e^2\log\left(d+\frac{e}{x^{2/3}}\right)}{d^3}-\frac{2e^2\log(x)}{3d^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(2/3))^n]),x]
[Out] (a*x^2)/2 + (b*x^2*Log[c*(d + e/x^(2/3))^n])/2 - (b*e*n*((e*x^(2/3))/d^2 -
x^(4/3)/(2*d) - (e^2*Log[d + e/x^(2/3)])/d^3 - (2*e^2*Log[x])/(3*d^3)))/2
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(2/3))^n)),x)
[Out] int(x*(a+b*ln(c*(d+e/x^(2/3))^n)),x)
```

Maxima [A]
time = 0.29, size = 66, normalized size = 0.70

$$\frac{1}{4}\,bn\left(\frac{dx^{\frac{4}{3}}-2\,x^{\frac{2}{3}}e}{d^2}+\frac{2\,e^2\log\left(dx^{\frac{2}{3}}+e\right)}{d^3}\right)e+\frac{1}{2}\,bx^2\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="maxima")
```

```
[Out] 1/4*b*n*((d*x^(4/3) - 2*x^(2/3)*e)/d^2 + 2*e^2*log(d*x^(2/3) + e)/d^3)*e +
1/2*b*x^2*log(c*(d + e/x^(2/3))^n) + 1/2*a*x^2
```

Fricas [A]
time = 0.41, size = 115, normalized size = 1.22

$$\frac{2\,bd^3x^2\log\left(c\right)+bd^2nx^{\frac{4}{3}}e+2\,ad^3x^2-4\,bd^3n\log\left(x^{\frac{1}{3}}\right)-2\,bdnx^{\frac{2}{3}}e^2+2\,(bd^3n+bne^3)\log\left(dx^{\frac{2}{3}}+e\right)+2\,(bd^3nx^2-bd^3n)\log\left(\frac{dx+x^{\frac{1}{3}}e}{x}\right)}{4\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*d^3*x^2*log(c) + b*d^2*n*x^(4/3)*e + 2*a*d^3*x^2 - 4*b*d^3*n*log(x
^(1/3)) - 2*b*d*n*x^(2/3)*e^2 + 2*(b*d^3*n + b*n*e^3)*log(d*x^(2/3) + e) +
2*(b*d^3*n*x^2 - b*d^3*n)*log((d*x + x^(1/3)*e)/x))/d^3
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(2/3))**n)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.62, size = 72, normalized size = 0.77

$$\frac{1}{2}\,bx^2\log\left(c\right)+\frac{1}{4}\left(2\,x^2\log\left(d+\frac{e}{x^{\frac{2}{3}}}\right)+\left(\frac{dx^{\frac{4}{3}}-2\,x^{\frac{2}{3}}e}{d^2}+\frac{2\,e^2\log\left(\left|dx^{\frac{2}{3}}+e\right|\right)}{d^3}\right)e\right)bn+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2*log(c) + 1/4*(2*x^2*log(d + e/x^(2/3)) + ((d*x^(4/3) - 2*x^(2/3)*
e)/d^2 + 2*e^2*log(abs(d*x^(2/3) + e))/d^3)*e)*b*n + 1/2*a*x^2
```

Mupad [B]
time = 0.59, size = 73, normalized size = 0.78

$$\frac{x^{4/3}\,\left(\frac{b\,e\,n}{2\,d}-\frac{b\,e^2\,n}{d^2\,x^{2/3}}\right)}{2}+\frac{a\,x^2}{2}+\frac{b\,x^2\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^n\right)}{2}+\frac{b\,e^3\,n\,\mathrm{atanh}\left(\frac{2\,e}{d\,x^{2/3}}+1\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e/x^(2/3))^n)),x)
```

```
[Out] (x^(4/3)*((b*e*n)/(2*d) - (b*e^2*n)/(d^2*x^(2/3))))/2 + (a*x^2)/2 + (b*x^2*
log(c*(d + e/x^(2/3))^n))/2 + (b*e^3*n*atanh((2*e)/(d*x^(2/3)) + 1))/d^3
```

3.511 $\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)dx$

Optimal. Leaf size=65

$$\frac{2ben\sqrt[3]{x}}{d}+ax-\frac{2be^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{d^{3/2}}+bx\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)$$

```
[Out] 2*b*e*n*x^(1/3)/d+a*x-2*b*e^(3/2)*n*arctan(x^(1/3)*d^(1/2)/e^(1/2))/d^(3/2)
+b*x*ln(c*(d+e/x^(2/3))^n)
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.278$, Rules used = {2498, 269, 249, 327, 211}

$$ax-\frac{2be^{3/2}n\text{ArcTan}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{d^{3/2}}+bx\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)+\frac{2ben\sqrt[3]{x}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Log[c*(d + e/x^(2/3))^n],x]
```

```
[Out] (2*b*e*n*x^(1/3))/d + a*x - (2*b*e^(3/2)*n*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]
])/d^(3/2) + b*x*Log[c*(d + e/x^(2/3))^n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 249

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Free
Q[{a, b, p}, x] && FractionQ[n]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)dx &= ax+b\int \log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)dx\\
&= ax+bx\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)+\frac{1}{3}(2ben)\int \frac{1}{\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}}\,dx\\
&= ax+bx\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)+\frac{1}{3}(2ben)\int \frac{1}{e+dx^{2/3}}\,dx\\
&= ax+bx\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)+(2ben)\text{Subst}\left(\int \frac{x^2}{e+dx^2}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{2ben\sqrt[3]{x}}{d}+ax+bx\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)-\frac{(2be^2n)\,\text{Subst}\left(\int \frac{1}{e+dx^2}\,dx,\right.}{d}\\
&= \frac{2ben\sqrt[3]{x}}{d}+ax-\frac{2be^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt[3]{x}}{\sqrt{e}}\right)}{d^{3/2}}+bx\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.01, size = 53, normalized size = 0.82

$$ax+\frac{2ben\sqrt[3]{x}\;{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{e}{dx^{2/3}}\right)}{d}+bx\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Log[c*(d + e/x^(2/3))^n],x]
[Out] a*x + (2*b*e*n*x^(1/3)*Hypergeometric2F1[-1/2, 1, 1/2, -(e/(d*x^(2/3)))])/d
 + b*x*Log[c*(d + e/x^(2/3))^n]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(51) = 102.
time = 0.22, size = 168, normalized size = 2.58

method	result
default	$ax+xb\ln\left(c\left(\frac{e+d\,x^{\frac{2}{3}}}{x^{\frac{2}{3}}}\right)^{n}\right)+\frac{2b\,e^{2}n\arctan\left(\frac{x\,d^{2}}{e\sqrt{ed}}\right)}{3d\sqrt{ed}}+\frac{2ben\,x^{\frac{1}{3}}}{d}-\frac{4b\,e^{2}n\arctan\left(\frac{d\,x^{\frac{1}{3}}}{\sqrt{ed}}\right)}{3d\sqrt{ed}}+\frac{2b\,e^{2}n\arctan\left(\frac{\sqrt{3}\,\sqrt{}}{}\right.}{3d\sqrt{e}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*ln(c*(d+e/x^(2/3))^n),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+x*b*ln(c*((e+d*x^(2/3))/x^(2/3))^n)+2/3*b*e^2*n/d/(e*d)^(1/2)*arctan(x*
d^2/e/(e*d)^(1/2))+2*b*e*n*x^(1/3)/d-4/3*b*e^2*n/d/(e*d)^(1/2)*arctan(d*x^(
1/3)/(e*d)^(1/2))+2/3*b*e^2*n/d/(e*d)^(1/2)*arctan((3^(1/2)*d^(1/2)*e^(1/2)
-2*d*x^(1/3))/(e*d)^(1/2))-2/3*b*e^2*n/d/(e*d)^(1/2)*arctan((2*d*x^(1/3)+3^
(1/2)*d^(1/2)*e^(1/2))/(e*d)^(1/2))
```

Maxima [A]
time = 0.55, size = 54, normalized size = 0.83

$$-\left(2\,n\left(\frac{\arctan\left(\sqrt{d}\,x^{\frac{1}{3}}e^{\left(-\frac{1}{2}\right)}\right)e^{\frac{1}{2}}}{d^{\frac{3}{2}}}-\frac{x^{\frac{1}{3}}}{d}\right)e-x\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/x^(2/3))^n),x, algorithm="maxima")
```

```
[Out] -(2*n*(arctan(sqrt(d)*x^(1/3)*e^(-1/2))*e^(1/2)/d^(3/2) - x^(1/3)/d)*e - x*
log(c*(d + e/x^(2/3))^n))*b + a*x
```

Fricas [A]
time = 0.40, size = 274, normalized size = 4.22

$$\left[\frac{bn\sqrt{-\frac{e}{d}}\,e\log\left(\frac{d^{3}x^{2}+2\,d^{2}x\sqrt{-\frac{e}{d}}\,e-2\left(d^{3}x\sqrt{-\frac{e}{d}}-de^{2}\right)x^{\frac{2}{3}}-2\left(d^{2}xe+d\sqrt{-\frac{e}{d}}\,e^{2}\right)x^{\frac{1}{3}}-e^{3}}{d^{3}x^{2}+e^{3}}\right)+bdn\log\left(dx^{\frac{2}{3}}+e\right)+bdx\log\left(c\right)-2\,bdn\log\left(x^{\frac{1}{3}}\right)+2\,bnx^{\frac{1}{3}}e+adx+\left(bdnx-bdn\right)\log\left(\frac{dx+x^{\frac{1}{3}}e}{x}\right)}{d},\ \frac{bdn\log\left(dx^{\frac{2}{3}}+e\right)+bdx\log\left(c\right)-2\,bdn\log\left(x^{\frac{1}{3}}\right)-\frac{2\,bn\arctan\left(\sqrt{d}\,x^{\frac{1}{3}}e^{\left(-\frac{1}{2}\right)}\right)e^{\frac{3}{2}}}{\sqrt{d}}+2\,bnx^{\frac{1}{3}}e+adx+\left(bdnx-bdn\right)\log\left(\frac{dx+x^{\frac{1}{3}}e}{x}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/x^(2/3))^n),x, algorithm="fricas")
```

```
[Out] [(b*n*sqrt(-e/d)*e*log((d^3*x^2 + 2*d^2*x*sqrt(-e/d)*e - 2*(d^3*x*sqrt(-e/d
) - d*e^2)*x^(2/3) - 2*(d^2*x*e + d*sqrt(-e/d)*e^2)*x^(1/3) - e^3)/(d^3*x^2
 + e^3)) + b*d*n*log(d*x^(2/3) + e) + b*d*x*log(c) - 2*b*d*n*log(x^(1/3)) +
 2*b*n*x^(1/3)*e + a*d*x + (b*d*n*x - b*d*n)*log((d*x + x^(1/3)*e)/x))/d, (
b*d*n*log(d*x^(2/3) + e) + b*d*x*log(c) - 2*b*d*n*log(x^(1/3)) - 2*b*n*arct
an(sqrt(d)*x^(1/3)*e^(-1/2))*e^(3/2)/sqrt(d) + 2*b*n*x^(1/3)*e + a*d*x + (b
*d*n*x - b*d*n)*log((d*x + x^(1/3)*e)/x))/d]
```

Sympy [A]
time = 18.16, size = 112, normalized size = 1.72

$$ax+b\left(\frac{2en\left(\begin{cases}\tilde{\infty}x & \text{for } d=0\wedge e=0\\ \frac{x}{e} & \text{for } d=0\\ \frac{3\sqrt[3]{x}}{d} & \text{for } e=0\\ \frac{3\sqrt[3]{x}}{d}-\frac{3e\log\left(\sqrt[3]{x}-\sqrt{-\frac{e}{d}}\right)}{2d^2\sqrt{-\frac{e}{d}}}+\frac{3e\log\left(\sqrt[3]{x}+\sqrt{-\frac{e}{d}}\right)}{2d^2\sqrt{-\frac{e}{d}}} & \text{otherwise}\end{cases}\right)}{3}+x\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*ln(c*(d+e/x**(2/3))**n),x)
```

```
[Out] a*x + b*(2*e*n*Piecewise((zoo*x, Eq(d, 0) & Eq(e, 0)), (x/e, Eq(d, 0)), (3*
x**(1/3)/d, Eq(e, 0)), (3*x**(1/3)/d - 3*e*log(x**(1/3) - sqrt(-e/d))/(2*d*
*2*sqrt(-e/d)) + 3*e*log(x**(1/3) + sqrt(-e/d))/(2*d**2*sqrt(-e/d)), True))
/3 + x*log(c*(d + e/x**(2/3))**n))
```

Giac [A]
time = 3.11, size = 57, normalized size = 0.88

$$-\left(\left(2\left(\frac{\arctan\left(\sqrt{d}\,x^{\frac{1}{3}}e^{\left(-\frac{1}{2}\right)}\right)e^{\frac{1}{2}}}{d^{\frac{3}{2}}}-\frac{x^{\frac{1}{3}}}{d}\right)e-x\log\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)n-x\log\left(c\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/x^(2/3))^n),x, algorithm="giac")
```

```
[Out] -((2*(arctan(sqrt(d)*x^(1/3)*e^(-1/2))*e^(1/2)/d^(3/2) - x^(1/3)/d)*e - x*l
og(d + e/x^(2/3)))*n - x*log(c))*b + a*x
```

Mupad [B]
time = 0.43, size = 51, normalized size = 0.78

$$a\,x+b\,x\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^n\right)+\frac{2\,b\,e\,n\,x^{1/3}}{d}-\frac{2\,b\,e^{3/2}\,n\,\mathrm{atan}\left(\frac{\sqrt{d}\,x^{1/3}}{\sqrt{e}}\right)}{d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*log(c*(d + e/x^(2/3))^n),x)
```

```
[Out] a*x + b*x*log(c*(d + e/x^(2/3))^n) + (2*b*e*n*x^(1/3))/d - (2*b*e^(3/2)*n*a
tan((d^(1/2)*x^(1/3))/e^(1/2)))/d^(3/2)
```

3.512 $\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}\,dx$

Optimal. Leaf size=55

$$-\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(-\frac{e}{dx^{2/3}}\right)-\frac{3}{2}bn\text{Li}_2\left(1+\frac{e}{dx^{2/3}}\right)$$

```
[Out] -3/2*(a+b*ln(c*(d+e/x^(2/3))^n))*ln(-e/d/x^(2/3))-3/2*b*n*polylog(2,1+e/d/x
^(2/3))
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2504, 2441, 2352}

$$-\frac{3}{2}bn\text{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)-\frac{3}{2}\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])/x,x]
```

```
[Out] (-3*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[-(e/(d*x^(2/3)))])/2 - (3*b*n*Poly
Log[2, 1 + e/(d*x^(2/3))])/2
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}\,dx = -\left(\frac{3}{2}\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x}\,dx, x, \frac{1}{x^{2/3}}\right)\right)$$
$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(-\frac{e}{dx^{2/3}}\right)+\frac{1}{2}(3ben)\text{Subst}\left(\int \frac{\log\left(-\right.}{d+e}\right.$$
$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(-\frac{e}{dx^{2/3}}\right)-\frac{3}{2}bn\text{Li}_2\left(1+\frac{e}{dx^{2/3}}\right)$$

Mathematica [A]
time = 0.01, size = 55, normalized size = 1.00

$$a\log(x)-\frac{3}{2}b\left(\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\log\left(-\frac{e}{dx^{2/3}}\right)+n\text{Li}_2\left(\frac{d+\frac{e}{x^{2/3}}}{d}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])/x,x]
```

```
[Out] a*Log[x] - (3*b*(Log[c*(d + e/x^(2/3))^n]*Log[-(e/(d*x^(2/3)))] + n*PolyLog
[2, (d + e/x^(2/3))/d]))/2
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))/x,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))/x,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(47) = 94$.
time = 0.73, size = 130, normalized size = 2.36

$$-\frac{3}{2}\left(2\log\left(de^{\left(\frac{2}{3}\log(x)-1\right)}+1\right)\log\left(x^{\frac{1}{3}}\right)+\text{Li}_2\left(-de^{\left(\frac{2}{3}\log(x)-1\right)}\right)\right)bn+\frac{1}{6}\left(6\,bne\log\left(dx^{\frac{2}{3}}+e\right)\log(x)+2\,bne\log(x)^2+6\,bdnx^{\frac{2}{3}}\log(x)-12\,be\log(x)\log\left(x^{\frac{1}{3}n}\right)-9\,bdnx^{\frac{2}{3}}+6\,(b\log(c)+a)e\log(x)-\frac{3\,(2\,bdnx\log(x)-3\,bdnx)}{x^{\frac{1}{3}}}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x,x, algorithm="maxima")
```

```
[Out] -3/2*(2*log(d*e^(2/3*log(x) - 1) + 1)*log(x^(1/3)) + dilog(-d*e^(2/3*log(x)
 - 1)))*b*n + 1/6*(6*b*n*e*log(d*x^(2/3) + e)*log(x) + 2*b*n*e*log(x)^2 + 6
```

```
*b*d*n*x^(2/3)*log(x) - 12*b*e*log(x)*log(x^(1/3*n)) - 9*b*d*n*x^(2/3) + 6*
(b*log(c) + a)*e*log(x) - 3*(2*b*d*n*x*log(x) - 3*b*d*n*x)/x^(1/3))*e^(-1)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x,x, algorithm="fricas")
[Out] integral((b*log(c*((d*x + x^(1/3)*e)/x)^n) + a)/x, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))/x,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))/x,x)
[Out] int((a + b*log(c*(d + e/x^(2/3))^n))/x, x)
```

3.513 $\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^2}\,dx$

Optimal. Leaf size=77

$$\frac{2bn}{3x}-\frac{2bdn}{e\sqrt[3]{x}}-\frac{2bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{e^{3/2}}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}$$

```
[Out] 2/3*b*n/x-2*b*d*n/e/x^(1/3)-2*b*d^(3/2)*n*arctan(x^(1/3)*d^(1/2)/e^(1/2))/e
^(3/2)+(-a-b*ln(c*(d+e/x^(2/3))^n))/x
```

Rubi [A]
time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2505, 269, 348, 331, 211}

$$-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}-\frac{2bd^{3/2}n\text{ArcTan}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{e^{3/2}}-\frac{2bdn}{e\sqrt[3]{x}}+\frac{2bn}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])/x^2,x]
```

```
[Out] (2*b*n)/(3*x) - (2*b*d*n)/(e*x^(1/3)) - (2*b*d^(3/2)*n*ArcTan[(Sqrt[d]*x^(1
/3))/Sqrt[e]])/e^(3/2) - (a + b*Log[c*(d + e/x^(2/3))^n])/x
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^2}\,dx &= -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}-\frac{1}{3}(2ben)\int \frac{1}{\left(d+\frac{e}{x^{2/3}}\right)x^{8/3}}\,dx\\
&= -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}-\frac{1}{3}(2ben)\int \frac{1}{\left(e+dx^{2/3}\right)x^2}\,dx\\
&= -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}-(2ben)\text{Subst}\left(\int \frac{1}{x^4\left(e+dx^2\right)}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{2bn}{3x}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}+(2bdn)\text{Subst}\left(\int \frac{1}{x^2\left(e+dx^2\right)}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{2bn}{3x}-\frac{2bdn}{e\sqrt[3]{x}}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}-\frac{\left(2bd^2n\right)\text{Subst}\left(\int \frac{1}{e+dx^2}\,dx,x,\sqrt[3]{x}\right)}{e}\\
&= \frac{2bn}{3x}-\frac{2bdn}{e\sqrt[3]{x}}-\frac{2bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{e^{3/2}}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}
\end{aligned}$$

Mathematica [**A**]
time = 0.04, size = 80, normalized size = 1.04

$$-\frac{a}{x}+\frac{2bn}{3x}-\frac{2bdn}{e\sqrt[3]{x}}+\frac{2bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}}{\sqrt{d}\sqrt[3]{x}}\right)}{e^{3/2}}-\frac{b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])/x^2,x]
```

```
[Out] -(a/x) + (2*b*n)/(3*x) - (2*b*d*n)/(e*x^(1/3)) + (2*b*d^(3/2)*n*ArcTan[Sqrt
[e]/(Sqrt[d]*x^(1/3))])/e^(3/2) - (b*Log[c*(d + e/x^(2/3))^n])/x
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{a + b\ln\left(c\left(d + \frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))/x^2,x)
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))/x^2,x)
```

Maxima [A]
time = 0.51, size = 67, normalized size = 0.87

$$-\frac{2}{3}\left(3\,d^{\frac{3}{2}}\arctan\left(\sqrt{d}\,x^{\frac{1}{3}}e^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{5}{2}\right)} + \frac{\left(3\,dx^{\frac{2}{3}} - e\right)e^{(-2)}}{x}\right)bne - \frac{b\log\left(c\left(d + \frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)}{x} - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^2,x, algorithm="maxima")
[Out] -2/3*(3*d^(3/2)*arctan(sqrt(d)*x^(1/3)*e^(-1/2))*e^(-5/2) + (3*d*x^(2/3) -
e)*e^(-2)/x)*b*n*e - b*log(c*(d + e/x^(2/3))^n)/x - a/x
```

Fricas [A]
time = 0.48, size = 232, normalized size = 3.01

$$\left[\frac{\left(3\sqrt{-de^{(-1)}}\,bdnx\log\left(\frac{d^{3}x^{2} + 2\sqrt{-de^{(-1)}}\,dxe^{2} - 2\left(\sqrt{-de^{(-1)}}\,d^{2}xe - de^{2}\right)x^{\frac{2}{3}} - 2\left(d^{2}xe + \sqrt{-de^{(-1)}}\,e^{3}\right)x^{\frac{1}{3}} - e^{3}}{d^{3}x^{2} + e^{3}}\right) - 3\,bne\log\left(\frac{dx + x^{\frac{1}{3}}e}{x}\right) - 6\,bdnx^{\frac{2}{3}} - 3\,be\log\left(c\right) + \left(2\,bn - 3\,a\right)e\right)e^{(-1)}}{3\,x},\ -\frac{\left(6\,bd^{\frac{3}{2}}nx\arctan\left(\sqrt{d}\,x^{\frac{1}{3}}e^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{1}{2}\right)} + 3\,bne\log\left(\frac{dx + x^{\frac{1}{3}}e}{x}\right) + 6\,bdnx^{\frac{2}{3}} + 3\,be\log\left(c\right) - \left(2\,bn - 3\,a\right)e\right)e^{(-1)}}{3\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^2,x, algorithm="fricas")
[Out] [1/3*(3*sqrt(-d*e^(-1))*b*d*n*x*log((d^3*x^2 + 2*sqrt(-d*e^(-1))*d*x*e^2 -
2*(sqrt(-d*e^(-1))*d^2*x*e - d*e^2)*x^(2/3) - 2*(d^2*x*e + sqrt(-d*e^(-1))*
e^3)*x^(1/3) - e^3)/(d^3*x^2 + e^3)) - 3*b*n*e*log((d*x + x^(1/3)*e)/x) - 6
*b*d*n*x^(2/3) - 3*b*e*log(c) + (2*b*n - 3*a)*e)*e^(-1)/x, -1/3*(6*b*d^(3/2
)*n*x*arctan(sqrt(d)*x^(1/3)*e^(-1/2))*e^(-1/2) + 3*b*n*e*log((d*x + x^(1/3
)*e)/x) + 6*b*d*n*x^(2/3) + 3*b*e*log(c) - (2*b*n - 3*a)*e)*e^(-1)/x]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))/x**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.58, size = 73, normalized size = 0.95

$$-\frac{1}{3}\left(2\left(3\,d^{\frac{3}{2}}\arctan\left(\sqrt{d}\,x^{\frac{1}{3}}e^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{5}{2}\right)}+\frac{\left(3\,dx^{\frac{2}{3}}-e\right)e^{(-2)}}{x}\right)e+\frac{3\,\log\left(d+\frac{e}{x^{\frac{2}{3}}}\right)}{x}\right)bn-\frac{b\log\left(c\right)}{x}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^2,x, algorithm="giac")
```

```
[Out] -1/3*(2*(3*d^(3/2)*arctan(sqrt(d)*x^(1/3)*e^(-1/2))*e^(-5/2) + (3*d*x^(2/3)
 - e)*e^(-2)/x)*e + 3*log(d + e/x^(2/3))/x)*b*n - b*log(c)/x - a/x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^n\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))/x^2,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(2/3))^n))/x^2, x)
```

3.514 $\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^3}\,dx$

Optimal. Leaf size=89

$$\frac{bn}{6x^2}-\frac{bdn}{4ex^{4/3}}+\frac{bd^2n}{2e^2x^{2/3}}-\frac{bd^3n\log\left(d+\frac{e}{x^{2/3}}\right)}{2e^3}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2x^2}$$

```
[Out] 1/6*b*n/x^2-1/4*b*d*n/e/x^(4/3)+1/2*b*d^2*n/e^2/x^(2/3)-1/2*b*d^3*n*ln(d+e/
x^(2/3))/e^3+1/2*(-a-b*ln(c*(d+e/x^(2/3))^n))/x^2
```

Rubi [A]
time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2504, 2442, 45}

$$-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2x^2}-\frac{bd^3n\log\left(d+\frac{e}{x^{2/3}}\right)}{2e^3}+\frac{bd^2n}{2e^2x^{2/3}}-\frac{bdn}{4ex^{4/3}}+\frac{bn}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])/x^3,x]
```

```
[Out] (b*n)/(6*x^2) - (b*d*n)/(4*e*x^(4/3)) + (b*d^2*n)/(2*e^2*x^(2/3)) - (b*d^3*
n*Log[d + e/x^(2/3)])/(2*e^3) - (a + b*Log[c*(d + e/x^(2/3))^n])/(2*x^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
```

```
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^3}\,dx &= -\left(\frac{3}{2}\text{Subst}\left(\int x^2(a+b\log(c(d+ex)^n))\,dx, x, \frac{1}{x^{2/3}}\right)\right)\\ &= -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2x^2}+\frac{1}{2}(ben)\text{Subst}\left(\int \frac{x^3}{d+ex}\,dx, x, \frac{1}{x^{2/3}}\right)\\ &= -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2x^2}+\frac{1}{2}(ben)\text{Subst}\left(\int \left(\frac{d^2}{e^3}-\frac{dx}{e^2}+\frac{x^2}{e}-\frac{d^3}{e^3(d+ex)}\right.\right.\\ &= \frac{bn}{6x^2}-\frac{bdn}{4ex^{4/3}}+\frac{bd^2n}{2e^2x^{2/3}}-\frac{bd^3n\log\left(d+\frac{e}{x^{2/3}}\right)}{2e^3}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2x^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 94, normalized size = 1.06

$$-\frac{a}{2x^2}+\frac{bn}{6x^2}-\frac{bdn}{4ex^{4/3}}+\frac{bd^2n}{2e^2x^{2/3}}-\frac{bd^3n\log\left(d+\frac{e}{x^{2/3}}\right)}{2e^3}-\frac{b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])/x^3,x]
```

```
[Out] -1/2*a/x^2 + (b*n)/(6*x^2) - (b*d*n)/(4*e*x^(4/3)) + (b*d^2*n)/(2*e^2*x^(2/
3)) - (b*d^3*n*Log[d + e/x^(2/3)])/(2*e^3) - (b*Log[c*(d + e/x^(2/3))^n])/(
2*x^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))/x^3,x)
```

Maxima [A]
time = 0.30, size = 88, normalized size = 0.99

$$-\frac{1}{12}\left(6\,d^3e^{(-4)}\log\left(dx^{\frac{2}{3}}+e\right)-6\,d^3e^{(-4)}\log\left(x^{\frac{2}{3}}\right)-\frac{\left(6\,d^2x^{\frac{4}{3}}-3\,dx^{\frac{2}{3}}e+2\,e^2\right)e^{(-3)}}{x^2}\right)bne-\frac{b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)}{2\,x^2}-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^3,x, algorithm="maxima")
```

```
[Out] -1/12*(6*d^3*e^(-4)*log(d*x^(2/3) + e) - 6*d^3*e^(-4)*log(x^(2/3)) - (6*d^2
*x^(4/3) - 3*d*x^(2/3)*e + 2*e^2)*e^(-3)/x^2)*b*n*e - 1/2*b*log(c*(d + e/x^
(2/3))^n)/x^2 - 1/2*a/x^2
```

Fricas [A]
time = 0.41, size = 80, normalized size = 0.90

$$\frac{\left(6\,bd^2nx^{\frac{4}{3}}e - 3\,bdnx^{\frac{2}{3}}e^2 - 6\,be^3\log\left(c\right) + 2\,(bn - 3\,a)e^3 - 6\,(bd^3nx^2 + bne^3)\log\left(\frac{dx + x^{\frac{1}{3}}e}{x}\right)\right)e^{(-3)}}{12\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(6*b*d^2*n*x^(4/3)*e - 3*b*d*n*x^(2/3)*e^2 - 6*b*e^3*log(c) + 2*(b*n -
 3*a)*e^3 - 6*(b*d^3*n*x^2 + b*n*e^3)*log((d*x + x^(1/3)*e)/x))*e^(-3)/x^2
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))/x**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [A]
time = 4.33, size = 104, normalized size = 1.17

$$\frac{1}{12}\left(\left(12\,d^3e^{(-4)}\log\left(x^{\frac{1}{3}}\right) - 6\,d^3e^{(-4)}\log\left(\left|dx^{\frac{2}{3}} + e\right|\right) - \frac{\left(11\,d^3x^2 - 6\,d^2x^{\frac{4}{3}}e + 3\,dx^{\frac{2}{3}}e^2 - 2\,e^3\right)e^{(-4)}}{x^2}\right)e - \frac{6\,\log\left(d + \frac{e}{x^{\frac{2}{3}}}\right)}{x^2}\right)bn - \frac{b\log\left(c\right)}{2\,x^2} - \frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^3,x, algorithm="giac")
```

```
[Out] 1/12*((12*d^3*e^(-4)*log(x^(1/3)) - 6*d^3*e^(-4)*log(abs(d*x^(2/3) + e)) -
(11*d^3*x^2 - 6*d^2*x^(4/3)*e + 3*d*x^(2/3)*e^2 - 2*e^3)*e^(-4)/x^2)*e - 6*
log(d + e/x^(2/3))/x^2)*b*n - 1/2*b*log(c)/x^2 - 1/2*a/x^2
```

Mupad [B]
time = 0.44, size = 74, normalized size = 0.83

$$\frac{b\,n}{6\,x^2} - \frac{a}{2\,x^2} - \frac{b\,\ln\left(c\,{\left(d + \frac{e}{x^{2/3}}\right)}^n\right)}{2\,x^2} - \frac{b\,d\,n}{4\,e\,x^{4/3}} - \frac{b\,d^3\,n\,\ln\left(d + \frac{e}{x^{2/3}}\right)}{2\,e^3} + \frac{b\,d^2\,n}{2\,e^2\,x^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))/x^3,x)
```

```
[Out] (b*n)/(6*x^2) - a/(2*x^2) - (b*log(c*(d + e/x^(2/3))^n))/(2*x^2) - (b*d*n)/
(4*e*x^(4/3)) - (b*d^3*n*log(d + e/x^(2/3)))/(2*e^3) + (b*d^2*n)/(2*e^2*x^(
2/3))
```

3.515 $\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^4}\,dx$

Optimal. Leaf size=132

$$\frac{2bn}{27x^3}-\frac{2bdn}{21ex^{7/3}}+\frac{2bd^2n}{15e^2x^{5/3}}-\frac{2bd^3n}{9e^3x}+\frac{2bd^4n}{3e^4\sqrt[3]{x}}+\frac{2bd^{9/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3e^{9/2}}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}$$

```
[Out] 2/27*b*n/x^3-2/21*b*d*n/e/x^(7/3)+2/15*b*d^2*n/e^2/x^(5/3)-2/9*b*d^3*n/e^3/
x+2/3*b*d^4*n/e^4/x^(1/3)+2/3*b*d^(9/2)*n*arctan(x^(1/3)*d^(1/2)/e^(1/2))/e
^(9/2)+1/3*(-a-b*ln(c*(d+e/x^(2/3))^n))/x^3
```

Rubi [A]
time = 0.06, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2505, 269, 348, 331, 211}

$$-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}+\frac{2bd^{9/2}n\text{ArcTan}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3e^{9/2}}+\frac{2bd^4n}{3e^4\sqrt[3]{x}}-\frac{2bd^3n}{9e^3x}+\frac{2bd^2n}{15e^2x^{5/3}}-\frac{2bdn}{21ex^{7/3}}+\frac{2bn}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])/x^4,x]
```

```
[Out] (2*b*n)/(27*x^3) - (2*b*d*n)/(21*e*x^(7/3)) + (2*b*d^2*n)/(15*e^2*x^(5/3))
- (2*b*d^3*n)/(9*e^3*x) + (2*b*d^4*n)/(3*e^4*x^(1/3)) + (2*b*d^(9/2)*n*ArcT
an[(Sqrt[d]*x^(1/3))/Sqrt[e]])/(3*e^(9/2)) - (a + b*Log[c*(d + e/x^(2/3))^n
])/(3*x^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^4}\,dx = -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}-\frac{1}{9}(2ben)\int\frac{1}{\left(d+\frac{e}{x^{2/3}}\right)x^{14/3}}\,dx$$

$$= -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}-\frac{1}{9}(2ben)\int\frac{1}{\left(e+dx^{2/3}\right)x^4}\,dx$$

$$= -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}-\frac{1}{3}(2ben)\text{Subst}\left(\int\frac{1}{x^{10}\left(e+dx^2\right)}\,dx,x,\sqrt[3]{x}\right)$$

$$= \frac{2bn}{27x^3}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}+\frac{1}{3}(2bdn)\text{Subst}\left(\int\frac{1}{x^8\left(e+dx^2\right)}\,dx,x,\sqrt[3]{x}\right.$$

$$= \frac{2bn}{27x^3}-\frac{2bdn}{21ex^{7/3}}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}-\frac{\left(2bd^2n\right)\text{Subst}\left(\int\frac{1}{x^6\left(e+dx^2\right)}\,dx,\right.}{3e}$$

$$= \frac{2bn}{27x^3}-\frac{2bdn}{21ex^{7/3}}+\frac{2bd^2n}{15e^2x^{5/3}}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}+\frac{\left(2bd^3n\right)\text{Subst}\left(\int\right.}{3}$$

$$= \frac{2bn}{27x^3}-\frac{2bdn}{21ex^{7/3}}+\frac{2bd^2n}{15e^2x^{5/3}}-\frac{2bd^3n}{9e^3x}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}-\frac{\left(2bd^4n\right)}{}$$

$$= \frac{2bn}{27x^3}-\frac{2bdn}{21ex^{7/3}}+\frac{2bd^2n}{15e^2x^{5/3}}-\frac{2bd^3n}{9e^3x}+\frac{2bd^4n}{3e^4\sqrt[3]{x}}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}$$

$$= \frac{2bn}{27x^3}-\frac{2bdn}{21ex^{7/3}}+\frac{2bd^2n}{15e^2x^{5/3}}-\frac{2bd^3n}{9e^3x}+\frac{2bd^4n}{3e^4\sqrt[3]{x}}+\frac{2bd^{9/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right.}{3e^{9/2}}$$

Mathematica [A]

time = 0.05, size = 137, normalized size = 1.04

$$-\frac{a}{3x^3}+\frac{2bn}{27x^3}-\frac{2bdn}{21ex^{7/3}}+\frac{2bd^2n}{15e^2x^{5/3}}-\frac{2bd^3n}{9e^3x}+\frac{2bd^4n}{3e^4\sqrt[3]{x}}-\frac{2bd^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}}{\sqrt{d}\sqrt[3]{x}}\right)}{3e^{9/2}}-\frac{b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])/x^4,x]
```

```
[Out] -1/3*a/x^3 + (2*b*n)/(27*x^3) - (2*b*d*n)/(21*e*x^(7/3)) + (2*b*d^2*n)/(15*
e^2*x^(5/3)) - (2*b*d^3*n)/(9*e^3*x) + (2*b*d^4*n)/(3*e^4*x^(1/3)) - (2*b*d
^(9/2)*n*ArcTan[Sqrt[e]/(Sqrt[d]*x^(1/3))])/(3*e^(9/2)) - (b*Log[c*(d + e/x
^(2/3))^n])/(3*x^3)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))/x^4,x)
```

Maxima [A]
time = 0.54, size = 97, normalized size = 0.73

$$\frac{2}{945}\left(315\,d^{\frac{9}{2}}\arctan\left(\sqrt{d}\,x^{\frac{1}{3}}e^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{11}{2}\right)}+\frac{\left(315\,d^4x^{\frac{8}{3}}-105\,d^3x^2e+63\,d^2x^{\frac{4}{3}}e^2-45\,dx^{\frac{2}{3}}e^3+35\,e^4\right)e^{(-5)}}{x^3}\right)bne-\frac{b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)}{3\,x^3}-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^4,x, algorithm="maxima")
```

```
[Out] 2/945*(315*d^(9/2)*arctan(sqrt(d)*x^(1/3)*e^(-1/2))*e^(-11/2) + (315*d^4*x^
(8/3) - 105*d^3*x^2*e + 63*d^2*x^(4/3)*e^2 - 45*d*x^(2/3)*e^3 + 35*e^4)*e^(
-5)/x^3)*b*n*e - 1/3*b*log(c*(d + e/x^(2/3))^n)/x^3 - 1/3*a/x^3
```

Fricas [A]
time = 0.37, size = 317, normalized size = 2.40

$$\left[\frac{\left(315\,\sqrt{-de^{(-1)}}\,bd^4nx^3\log\left(\frac{d^3x^2-2\,\sqrt{-de^{(-1)}}\,dxe^2+2\left(\sqrt{-de^{(-1)}}\,d^2xe+de^2\right)x^{\frac{2}{3}}-2\left(d^2xe-\sqrt{-de^{(-1)}}\,e^3\right)x^{\frac{1}{3}}-e^3}{d^3x^2+e^3}\right)-210\,bd^3nx^2e+126\,bd^2nx^{\frac{4}{3}}e^2-315\,bne^4\log\left(\frac{dx+x^{\frac{1}{3}}e}{x}\right)-315\,be^4\log\left(c\right)+35\left(2\,bn-9\,a\right)e^4+90\left(7\,bd^4nx^2-bdne^3\right)x^{\frac{2}{3}}\right)e^{(-4)}}{945\,x^3},\ \frac{\left(630\,bd^{\frac{9}{2}}nx^3\arctan\left(\sqrt{d}\,x^{\frac{1}{3}}e^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{1}{2}\right)}-210\,bd^3nx^2e+126\,bd^2nx^{\frac{4}{3}}e^2-315\,bne^4\log\left(\frac{dx+x^{\frac{1}{3}}e}{x}\right)-315\,be^4\log\left(c\right)+35\left(2\,bn-9\,a\right)e^4+90\left(7\,bd^4nx^2-bdne^3\right)x^{\frac{2}{3}}\right)e^{(-4)}}{945\,x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^4,x, algorithm="fricas")
```

```
[Out] [1/945*(315*sqrt(-d*e^(-1))*b*d^4*n*x^3*log((d^3*x^2 - 2*sqrt(-d*e^(-1))*d*
x*e^2 + 2*(sqrt(-d*e^(-1))*d^2*x*e + d*e^2)*x^(2/3) - 2*(d^2*x*e - sqrt(-d*
e^(-1))*e^3)*x^(1/3) - e^3)/(d^3*x^2 + e^3)) - 210*b*d^3*n*x^2*e + 126*b*d^
2*n*x^(4/3)*e^2 - 315*b*n*e^4*log((d*x + x^(1/3)*e)/x) - 315*b*e^4*log(c) +
 35*(2*b*n - 9*a)*e^4 + 90*(7*b*d^4*n*x^2 - b*d*n*e^3)*x^(2/3))*e^(-4)/x^3,
 1/945*(630*b*d^(9/2)*n*x^3*arctan(sqrt(d)*x^(1/3)*e^(-1/2))*e^(-1/2) - 210
*b*d^3*n*x^2*e + 126*b*d^2*n*x^(4/3)*e^2 - 315*b*n*e^4*log((d*x + x^(1/3)*e
)/x) - 315*b*e^4*log(c) + 35*(2*b*n - 9*a)*e^4 + 90*(7*b*d^4*n*x^2 - b*d*n*
e^3)*x^(2/3))*e^(-4)/x^3]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))/x**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7318 deep
```

Giac [A]
time = 3.72, size = 103, normalized size = 0.78

$$\frac{1}{945}\left(2\left(315\,d^{\frac{9}{2}}\arctan\left(\sqrt{d}\,x^{\frac{1}{3}}e^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{11}{2}\right)}+\frac{\left(315\,d^{4}x^{\frac{8}{3}}-105\,d^{3}x^{2}e+63\,d^{2}x^{\frac{4}{3}}e^{2}-45\,dx^{\frac{2}{3}}e^{3}+35\,e^{4}\right)e^{(-5)}}{x^{3}}\right)e-\frac{315\,\log\left(d+\frac{e}{x^{\frac{2}{3}}}\right)}{x^{3}}\right)bn-\frac{b\log\left(c\right)}{3\,x^{3}}-\frac{a}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^4,x, algorithm="giac")
```

```
[Out] 1/945*(2*(315*d^(9/2)*arctan(sqrt(d)*x^(1/3)*e^(-1/2))*e^(-11/2) + (315*d^4
*x^(8/3) - 105*d^3*x^2*e + 63*d^2*x^(4/3)*e^2 - 45*d*x^(2/3)*e^3 + 35*e^4)*
e^(-5)/x^3)*e - 315*log(d + e/x^(2/3))/x^3)*b*n - 1/3*b*log(c)/x^3 - 1/3*a/
x^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^n\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))/x^4,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(2/3))^n))/x^4, x)
```

3.516 $\int x^3 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx$

Optimal. Leaf size=412

$$-\frac{77b^2e^5n^2x^{2/3}}{120d^5}+\frac{47b^2e^4n^2x^{4/3}}{240d^4}-\frac{3b^2e^3n^2x^2}{40d^3}+\frac{b^2e^2n^2x^{8/3}}{40d^2}+\frac{77b^2e^6n^2\log\left(d+\frac{e}{x^{2/3}}\right)}{120d^6}+\frac{be^5n\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}(a+b}{2d^6}$$

```
[Out] -77/120*b^2*e^5*n^2*x^(2/3)/d^5+47/240*b^2*e^4*n^2*x^(4/3)/d^4-3/40*b^2*e^3
*n^2*x^2/d^3+1/40*b^2*e^2*n^2*x^(8/3)/d^2+77/120*b^2*e^6*n^2*ln(d+e/x^(2/3)
)/d^6+1/2*b*e^5*n*(d+e/x^(2/3))*x^(2/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d^6-1/4
*b*e^4*n*x^(4/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d^4+1/6*b*e^3*n*x^2*(a+b*ln(c*
(d+e/x^(2/3))^n))/d^3-1/8*b*e^2*n*x^(8/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d^2+1
/10*b*e*n*x^(10/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d+1/2*b*e^6*n*ln(1-d/(d+e/x^
(2/3)))*(a+b*ln(c*(d+e/x^(2/3))^n))/d^6+1/4*x^4*(a+b*ln(c*(d+e/x^(2/3))^n))
^2+137/180*b^2*e^6*n^2*ln(x)/d^6-1/2*b^2*e^6*n^2*polylog(2,d/(d+e/x^(2/3)))
/d^6
```

Rubi [A]
time = 0.60, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2504, 2445, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$-\frac{b^2e^6n^2\text{PolyLog}\left(2,\frac{d}{d+\frac{e}{x^{2/3}}}\right)}{2d^6}+\frac{be^6n\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^6}+\frac{be^5nx^{2/3}\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^6}-\frac{be^4nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{4d^4}+\frac{be^3nx^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{6d^3}-\frac{be^2nx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{8d^2}+\frac{benx^{10/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{10d}+\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2+\frac{77b^2e^6n^2\log\left(d+\frac{e}{x^{2/3}}\right)}{120d^6}+\frac{137b^2e^6n^2\log(x)}{180d^6}-\frac{77b^2e^5n^2x^{2/3}}{120d^5}+\frac{47b^2e^4n^2x^{4/3}}{240d^4}-\frac{3b^2e^3n^2x^2}{40d^3}+\frac{b^2e^2n^2x^{8/3}}{40d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

```
[Out] (-77*b^2*e^5*n^2*x^(2/3))/(120*d^5) + (47*b^2*e^4*n^2*x^(4/3))/(240*d^4) -
(3*b^2*e^3*n^2*x^2)/(40*d^3) + (b^2*e^2*n^2*x^(8/3))/(40*d^2) + (77*b^2*e^6
*n^2*Log[d + e/x^(2/3)])/(120*d^6) + (b*e^5*n*(d + e/x^(2/3))*x^(2/3)*(a +
b*Log[c*(d + e/x^(2/3))^n]))/(2*d^6) - (b*e^4*n*x^(4/3)*(a + b*Log[c*(d + e
/x^(2/3))^n]))/(4*d^4) + (b*e^3*n*x^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(6*
d^3) - (b*e^2*n*x^(8/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(8*d^2) + (b*e*n*
x^(10/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(10*d) + (b*e^6*n*Log[1 - d/(d +
 e/x^(2/3))]*(a + b*Log[c*(d + e/x^(2/3))^n]))/(2*d^6) + (x^4*(a + b*Log[c*
(d + e/x^(2/3))^n])^2)/4 + (137*b^2*e^6*n^2*Log[x])/(180*d^6) - (b^2*e^6*n^
2*PolyLog[2, d/(d + e/x^(2/3))])/(2*d^6)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx = -\left(\frac{3}{2}\text{Subst}\left(\int\frac{(a+b\log(c(d+ex)^n))^2}{x^7}dx, x, \frac{1}{x^{2/3}}\right)\right)$$

$$= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 - \frac{1}{2}(ben)\text{Subst}\left(\int\frac{a+b\log(c(d\cdot}{x^6(d+e\cdot}\right.$$

$$= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 - \frac{1}{2}(bn)\text{Subst}\left(\int\frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\right.$$

$$= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 - \frac{(bn)\text{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}dx, x, d\right.}{2d}$$

$$= \frac{benx^{10/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{10d} + \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right.\right.$$

$$= -\frac{be^2nx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{8d^2} + \frac{benx^{10/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/\cdot}}\right.\right.\right.}{10d}$$

$$= -\frac{b^2e^5n^2x^{2/3}}{10d^5} + \frac{b^2e^4n^2x^{4/3}}{20d^4} - \frac{b^2e^3n^2x^2}{30d^3} + \frac{b^2e^2n^2x^{8/3}}{40d^2} + \frac{b^2e^6n^2\log(d}{10d^6}$$

$$= -\frac{9b^2e^5n^2x^{2/3}}{40d^5} + \frac{9b^2e^4n^2x^{4/3}}{80d^4} - \frac{3b^2e^3n^2x^2}{40d^3} + \frac{b^2e^2n^2x^{8/3}}{40d^2} + \frac{9b^2e^6n^2\,\mathrm{l}}{}$$

$$= -\frac{47b^2e^5n^2x^{2/3}}{120d^5} + \frac{47b^2e^4n^2x^{4/3}}{240d^4} - \frac{3b^2e^3n^2x^2}{40d^3} + \frac{b^2e^2n^2x^{8/3}}{40d^2} + \frac{47b^2e^6}{}$$

$$= -\frac{77b^2e^5n^2x^{2/3}}{120d^5} + \frac{47b^2e^4n^2x^{4/3}}{240d^4} - \frac{3b^2e^3n^2x^2}{40d^3} + \frac{b^2e^2n^2x^{8/3}}{40d^2} + \frac{77b^2e^6}{}$$

$$= -\frac{77b^2e^5n^2x^{2/3}}{120d^5} + \frac{47b^2e^4n^2x^{4/3}}{240d^4} - \frac{3b^2e^3n^2x^2}{40d^3} + \frac{b^2e^2n^2x^{8/3}}{40d^2} + \frac{77b^2e^6}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 968 vs. 2(412) = 824.
time = 0.32, size = 968, normalized size = 2.35

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

```
[Out] (360*a*b*d*e^5*n*x^(2/3) - 462*b^2*d*e^5*n^2*x^(2/3) - 180*a*b*d^2*e^4*n*x^
(4/3) + 141*b^2*d^2*e^4*n^2*x^(4/3) + 120*a*b*d^3*e^3*n*x^2 - 54*b^2*d^3*e^
3*n^2*x^2 - 90*a*b*d^4*e^2*n*x^(8/3) + 18*b^2*d^4*e^2*n^2*x^(8/3) + 72*a*b*
d^5*e*n*x^(10/3) + 180*a^2*d^6*x^4 + 822*b^2*e^6*n^2*Log[d + e/x^(2/3)] + 3
60*b^2*d*e^5*n*x^(2/3)*Log[c*(d + e/x^(2/3))^n] - 180*b^2*d^2*e^4*n*x^(4/3)
*Log[c*(d + e/x^(2/3))^n] + 120*b^2*d^3*e^3*n*x^2*Log[c*(d + e/x^(2/3))^n]
- 90*b^2*d^4*e^2*n*x^(8/3)*Log[c*(d + e/x^(2/3))^n] + 72*b^2*d^5*e*n*x^(10/
3)*Log[c*(d + e/x^(2/3))^n] + 360*a*b*d^6*x^4*Log[c*(d + e/x^(2/3))^n] + 18
0*b^2*d^6*x^4*Log[c*(d + e/x^(2/3))^n]^2 - 360*a*b*e^6*n*Log[Sqrt[e] - Sqrt
[-d]*x^(1/3)] - 360*b^2*e^6*n*Log[c*(d + e/x^(2/3))^n]*Log[Sqrt[e] - Sqrt[-
d]*x^(1/3)] + 180*b^2*e^6*n^2*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]^2 - 360*a*b*e
^6*n*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] - 360*b^2*e^6*n*Log[c*(d + e/x^(2/3))^
n]*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] + 180*b^2*e^6*n^2*Log[Sqrt[e] + Sqrt[-d]
*x^(1/3)]^2 + 360*b^2*e^6*n^2*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*Log[1/2 - (Sq
rt[-d]*x^(1/3))/(2*Sqrt[e])] + 360*b^2*e^6*n^2*Log[Sqrt[e] - Sqrt[-d]*x^(1/
3)]*Log[(1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 720*b^2*e^6*n^2*Log[Sqrt[e] +
 Sqrt[-d]*x^(1/3)]*Log[-((Sqrt[-d]*x^(1/3))/Sqrt[e])] - 720*b^2*e^6*n^2*Log
[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]] + 548*b^2*e^6*
n^2*Log[x] - 720*b^2*e^6*n^2*PolyLog[2, 1 - (Sqrt[-d]*x^(1/3))/Sqrt[e]] + 3
60*b^2*e^6*n^2*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] + 360*b^2*e
^6*n^2*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 720*b^2*e^6*n^2*Pol
yLog[2, 1 + (Sqrt[-d]*x^(1/3))/Sqrt[e]])/(720*d^6)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int x^{3} \left(a +b \ln \left(c \left(d +\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{2}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e/x^(2/3))^n))^2,x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e/x^(2/3))^n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*n^2*x^4*log(d*x^(2/3) + e)^2 - integrate(-1/3*(3*(b^2*log(c)^2 + 2*
a*b*log(c) + a^2)*x^(10/3)*e + 3*(b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d)*
```

```
x^4 - (b^2*d*n*x^4 - 6*(b^2*log(c) + a*b)*x^(10/3)*e - 6*(b^2*d*log(c) + a*
b*d)*x^4 + 12*(b^2*d*x^4 + b^2*x^(10/3)*e)*log(x^(1/3*n)))*n*log(d*x^(2/3)
+ e) + 12*(b^2*d*x^4 + b^2*x^(10/3)*e)*log(x^(1/3*n))^2 - 12*((b^2*log(c) +
 a*b)*x^(10/3)*e + (b^2*d*log(c) + a*b*d)*x^4)*log(x^(1/3*n)))/(d*x + x^(1/
3)*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="fricas")
[Out] integral(b^2*x^3*log(c*((d*x + x^(1/3)*e)/x)^n)^2 + 2*a*b*x^3*log(c*((d*x +
 x^(1/3)*e)/x)^n) + a^2*x^3, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e/x**(2/3))**n))**2,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5988 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \left(a+b\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e/x^(2/3))^n))^2,x)
[Out] int(x^3*(a + b*log(c*(d + e/x^(2/3))^n))^2, x)
```

3.517 $\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx$

Optimal. Leaf size=239

$$\frac{b^2e^2n^2x^{2/3}}{2d^2}-\frac{b^2e^3n^2\log\left(d+\frac{e}{x^{2/3}}\right)}{2d^3}-\frac{be^2n\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3}+\frac{benx^{4/3}\left(a+b\log\left(c\left(d+\right.\right.\right.}{2d}$$

```
[Out] 1/2*b^2*e^2*n^2*x^(2/3)/d^2-1/2*b^2*e^3*n^2*ln(d+e/x^(2/3))/d^3-b*e^2*n*(d+
e/x^(2/3))*x^(2/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d^3+1/2*b*e*n*x^(4/3)*(a+b*l
n(c*(d+e/x^(2/3))^n))/d-b*e^3*n*ln(1-d/(d+e/x^(2/3)))*(a+b*ln(c*(d+e/x^(2/3
))^n))/d^3+1/2*x^2*(a+b*ln(c*(d+e/x^(2/3))^n))^2-b^2*e^3*n^2*ln(x)/d^3+b^2*
e^3*n^2*polylog(2,d/(d+e/x^(2/3)))/d^3
```

Rubi [A]
time = 0.28, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2504, 2445, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$\frac{b^2e^3n^2\text{PolyLog}\left(2,\frac{d}{d+\frac{e}{x^{2/3}}}\right)}{d^3}-\frac{be^3n\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3}-\frac{be^2nx^{2/3}\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3}+\frac{benx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-\frac{b^2e^3n^2\log\left(d+\frac{e}{x^{2/3}}\right)}{2d^3}-\frac{b^2e^3n^2\log(x)}{d^3}+\frac{b^2e^2n^2x^{2/3}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

```
[Out] (b^2*e^2*n^2*x^(2/3))/(2*d^2) - (b^2*e^3*n^2*Log[d + e/x^(2/3)])/(2*d^3) -
(b*e^2*n*(d + e/x^(2/3))*x^(2/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/d^3 + (b
*e*n*x^(4/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(2*d) - (b*e^3*n*Log[1 - d/(
d + e/x^(2/3))]*(a + b*Log[c*(d + e/x^(2/3))^n]))/d^3 + (x^2*(a + b*Log[c*(
d + e/x^(2/3))^n])^2)/2 - (b^2*e^3*n^2*Log[x])/d^3 + (b^2*e^3*n^2*PolyLog[2
, d/(d + e/x^(2/3))])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
```

```
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx=-\left(\frac{3}{2}\text{Subst}\left(\int\frac{(a+b\log(c(d+ex)^n))^2}{x^4}\,dx,x,\frac{1}{x^{2/3}}\right)\right)$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-(ben)\text{Subst}\left(\int\frac{a+b\log(c(d+}{x^3(d+ex}\right.$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-(bn)\text{Subst}\left(\int\frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,d\right.$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-\frac{(bn)\text{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx,x,\right.}{d}$$

$$=\frac{benx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right.$$

$$=-\frac{be^2n\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3}+\frac{benx^{4/3}(a+b\log}{2d}$$

$$=\frac{b^2e^2n^2x^{2/3}}{2d^2}-\frac{b^2e^3n^2\log\left(d+\frac{e}{x^{2/3}}\right)}{2d^3}-\frac{be^2n\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}(a+b\log}{d^3}$$

$$=\frac{b^2e^2n^2x^{2/3}}{2d^2}-\frac{b^2e^3n^2\log\left(d+\frac{e}{x^{2/3}}\right)}{2d^3}-\frac{be^2n\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}(a+b\log}{d^3}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 542 vs. 2(239) = 478.
time = 0.31, size = 542, normalized size = 2.27

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

```
[Out] (x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/2 - (b*e*n*(6*d*e*x^(2/3)*(a + b*L
og[c*(d + e/x^(2/3))^n]) - 3*d^2*x^(4/3)*(a + b*Log[c*(d + e/x^(2/3))^n]) -
 6*e^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] - 6
*e^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] + 2*b
*e^2*n*(3*Log[d + e/x^(2/3)] + 2*Log[x]) + b*e*n*(-3*d*x^(2/3) + 3*e*Log[d
+ e/x^(2/3)] + 2*e*Log[x]) + 3*b*e^2*n*(Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*(Lo
g[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 2*Log[(1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] -
 4*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]]) - 4*PolyLog[2, 1 - (Sqrt[-d]*x^(1/3))/S
qrt[e]] + 2*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])]) + 3*b*e^2*n*(
Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*(Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] + 2*Log[1/
2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] - 4*Log[-((Sqrt[-d]*x^(1/3))/Sqrt[e])])
 + 2*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 4*PolyLog[2, 1 + (Sqr
t[-d]*x^(1/3))/Sqrt[e]])))/(6*d^3)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(2/3))^n))^2,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e/x^(2/3))^n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*n^2*x^2*log(d*x^(2/3) + e)^2 - integrate(-1/3*(3*(b^2*log(c)^2 + 2*
a*b*log(c) + a^2)*x^(4/3)*e + 3*(b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d)*x
^2 - 2*(b^2*d*n*x^2 - 3*(b^2*log(c) + a*b)*x^(4/3)*e - 3*(b^2*d*log(c) + a*
b*d)*x^2 + 6*(b^2*d*x^2 + b^2*x^(4/3)*e)*log(x^(1/3*n)))*n*log(d*x^(2/3) +
e) + 12*(b^2*d*x^2 + b^2*x^(4/3)*e)*log(x^(1/3*n))^2 - 12*((b^2*log(c) + a*
b)*x^(4/3)*e + (b^2*d*log(c) + a*b*d)*x^2)*log(x^(1/3*n)))/(d*x + x^(1/3)*e
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x*log(c*((d*x + x^(1/3)*e)/x)^n)^2 + 2*a*b*x*log(c*((d*x + x^(
1/3)*e)/x)^n) + a^2*x, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(2/3))**n))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2*x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e/x^(2/3))^n))^2,x)
```

```
[Out] int(x*(a + b*log(c*(d + e/x^(2/3))^n))^2, x)
```

3.518 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x}\,dx$

Optimal. Leaf size=95

$$-\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\log\left(-\frac{e}{dx^{2/3}}\right)-3bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\mathrm{Li}_2\left(1+\frac{e}{dx^{2/3}}\right)+3b^2n^2\mathrm{Li}_3\left(\right.$$

```
[Out] -3/2*(a+b*ln(c*(d+e/x^(2/3))^n))^2*ln(-e/d/x^(2/3))-3*b*n*(a+b*ln(c*(d+e/x^
(2/3))^n))*polylog(2,1+e/d/x^(2/3))+3*b^2*n^2*polylog(3,1+e/d/x^(2/3))
```

Rubi [A]
time = 0.09, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2504, 2443, 2481, 2421, 6724}

$$-3bn\text{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+3b^2n^2\text{PolyLog}\left(3,\frac{e}{dx^{2/3}}+1\right)-\frac{3}{2}\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x,x]
```

```
[Out] (-3*(a + b*Log[c*(d + e/x^(2/3))^n])^2*Log[-(e/(d*x^(2/3)))])/2 - 3*b*n*(a
+ b*Log[c*(d + e/x^(2/3))^n])*PolyLog[2, 1 + e/(d*x^(2/3))] + 3*b^2*n^2*Pol
yLog[3, 1 + e/(d*x^(2/3))]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
```

```
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x}\,dx = -\left(\frac{3}{2}\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x}\,dx, x, \frac{1}{x^{2/3}}\right)\right)$$

$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\log\left(-\frac{e}{dx^{2/3}}\right)+(3ben)\text{Subst}\left(\int \frac{\log(\right.$$

$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\log\left(-\frac{e}{dx^{2/3}}\right)+(3bn)\text{Subst}\left(\int \frac{(a+}{}\right.$$

$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\log\left(-\frac{e}{dx^{2/3}}\right)-3bn\left(a+b\log\left(c\left(d+\right.\right.\right.$$

$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\log\left(-\frac{e}{dx^{2/3}}\right)-3bn\left(a+b\log\left(c\left(d+\right.\right.\right.$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 199 vs. 2(95) = 190.
time = 0.13, size = 199, normalized size = 2.09

$$\left(a-bn\log\left(d+\frac{e}{x^{2/3}}\right)+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\log(x)+2bn\left(a-bn\log\left(d+\frac{e}{x^{2/3}}\right)+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\left(\left(\log\left(d+\frac{e}{x^{2/3}}\right)-\log\left(1+\frac{e}{dx^{2/3}}\right)\right)\log(x)+\frac{3}{2}\text{Li}_2\left(-\frac{e}{dx^{2/3}}\right)\right)-\frac{3}{2}b^2n^2\left(\log^2\left(d+\frac{e}{x^{2/3}}\right)\log\left(-\frac{e}{dx^{2/3}}\right)+2\log\left(d+\frac{e}{x^{2/3}}\right)\text{Li}_2\left(1+\frac{e}{dx^{2/3}}\right)-2\text{Li}_3\left(1+\frac{e}{dx^{2/3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x,x]
```

```
[Out] (a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2*Log[x] + 2*b*n*
(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])*((Log[d + e/x^(2/
3)] - Log[1 + e/(d*x^(2/3))])*Log[x] + (3*PolyLog[2, -(e/(d*x^(2/3)))])/2)
- (3*b^2*n^2*(Log[d + e/x^(2/3)]^2*Log[-(e/(d*x^(2/3)))] + 2*Log[d + e/x^(2
/3)]*PolyLog[2, 1 + e/(d*x^(2/3))] - 2*PolyLog[3, 1 + e/(d*x^(2/3))]))/2
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))^2/x,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))^2/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x,x, algorithm="maxima")
```

```
[Out] b^2*n^2*log(d*x^(2/3) + e)^2*log(x) - integrate(1/3*(2*(2*b^2*d*n*x*log(x)
- 3*(b^2*log(c) + a*b)*x^(1/3)*e - 3*(b^2*d*log(c) + a*b*d)*x + 6*(b^2*d*x
+ b^2*x^(1/3)*e)*log(x^(1/3*n)))*n*log(d*x^(2/3) + e) - 12*(b^2*d*x + b^2*x
^(1/3)*e)*log(x^(1/3*n))^2 - 3*(b^2*log(c)^2 + 2*a*b*log(c) + a^2)*x^(1/3)*
e - 3*(b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d)*x + 12*((b^2*log(c) + a*b)*
x^(1/3)*e + (b^2*d*log(c) + a*b*d)*x)*log(x^(1/3*n)))/(d*x^2 + x^(4/3)*e),
x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*((d*x + x^(1/3)*e)/x)^n)^2 + 2*a*b*log(c*((d*x + x^(1/3
)*e)/x)^n) + a^2)/x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**2/x,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b \ln\left(c \left(d + \frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))^2/x,x)
[Out] int((a + b*log(c*(d + e/x^(2/3))^n))^2/x, x)
```

3.519 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=276

$$\frac{3b^2dn^2\left(d+\frac{e}{x^{2/3}}\right)^2}{4e^3}-\frac{b^2n^2\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^3}-\frac{3b^2d^2n^2}{e^2x^{2/3}}+\frac{b^2d^3n^2\log^2\left(d+\frac{e}{x^{2/3}}\right)}{2e^3}+\frac{3bd^2n\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right.\right.\right.}{e^3}$$

```
[Out] 3/4*b^2*d*n^2*(d+e/x^(2/3))^2/e^3-1/9*b^2*n^2*(d+e/x^(2/3))^3/e^3-3*b^2*d^2
*n^2/e^2/x^(2/3)+1/2*b^2*d^3*n^2*ln(d+e/x^(2/3))^2/e^3+3*b*d^2*n*(d+e/x^(2/
3))*(a+b*ln(c*(d+e/x^(2/3))^n))/e^3-3/2*b*d*n*(d+e/x^(2/3))^2*(a+b*ln(c*(d+
e/x^(2/3))^n))/e^3+1/3*b*n*(d+e/x^(2/3))^3*(a+b*ln(c*(d+e/x^(2/3))^n))/e^3-
b*d^3*n*ln(d+e/x^(2/3))*(a+b*ln(c*(d+e/x^(2/3))^n))/e^3-1/2*(a+b*ln(c*(d+e/
x^(2/3))^n))^2/x^2
```

Rubi [A]
time = 0.21, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2504, 2445, 2458, 45, 2372, 12, 14, 2338}

$$-\frac{bd^3n\log\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^3}+\frac{3bd^2n\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^3}-\frac{3bdn\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2e^3}+\frac{bn\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3e^3}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2x^2}+\frac{b^2d^3n^2\log^2\left(d+\frac{e}{x^{2/3}}\right)}{2e^3}-\frac{3b^2d^2n^2}{e^2x^{2/3}}+\frac{3b^2dn^2\left(d+\frac{e}{x^{2/3}}\right)^2}{4e^3}-\frac{b^2n^2\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x^3,x]
```

```
[Out] (3*b^2*d*n^2*(d + e/x^(2/3))^2)/(4*e^3) - (b^2*n^2*(d + e/x^(2/3))^3)/(9*e^
3) - (3*b^2*d^2*n^2)/(e^2*x^(2/3)) + (b^2*d^3*n^2*Log[d + e/x^(2/3)]^2)/(2*
e^3) + (3*b*d^2*n*(d + e/x^(2/3))*(a + b*Log[c*(d + e/x^(2/3))^n]))/e^3 - (
3*b*d*n*(d + e/x^(2/3))^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(2*e^3) + (b*n*
(d + e/x^(2/3))^3*(a + b*Log[c*(d + e/x^(2/3))^n]))/(3*e^3) - (b*d^3*n*Log[
d + e/x^(2/3)]*(a + b*Log[c*(d + e/x^(2/3))^n]))/e^3 - (a + b*Log[c*(d + e/
x^(2/3))^n])^2/(2*x^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^3}\,dx = -\left(\frac{3}{2}\text{Subst}\left(\int x^2(a+b\log\left(c(d+ex)^n\right))^2\,dx, x, \frac{1}{x^{2/3}}\right)\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2x^2} + (ben)\text{Subst}\left(\int \frac{x^3(a+b\log\left(c(d+ex)^n\right))}{d+ex}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2x^2} + (bn)\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^3(a+b\log\left(cx^n\right.}{x}\right.$$

$$= \frac{1}{6}bn\left(\frac{18d^2\left(d+\frac{e}{x^{2/3}}\right)}{e^3} - \frac{9d\left(d+\frac{e}{x^{2/3}}\right)^2}{e^3} + \frac{2\left(d+\frac{e}{x^{2/3}}\right)^3}{e^3} - \frac{6d^3\log\left(d+\frac{e}{x^{2/3}}\right.}{e^3}\right.$$

$$= \frac{1}{6}bn\left(\frac{18d^2\left(d+\frac{e}{x^{2/3}}\right)}{e^3} - \frac{9d\left(d+\frac{e}{x^{2/3}}\right)^2}{e^3} + \frac{2\left(d+\frac{e}{x^{2/3}}\right)^3}{e^3} - \frac{6d^3\log\left(d+\frac{e}{x^{2/3}}\right.}{e^3}\right.$$

$$= \frac{1}{6}bn\left(\frac{18d^2\left(d+\frac{e}{x^{2/3}}\right)}{e^3} - \frac{9d\left(d+\frac{e}{x^{2/3}}\right)^2}{e^3} + \frac{2\left(d+\frac{e}{x^{2/3}}\right)^3}{e^3} - \frac{6d^3\log\left(d+\frac{e}{x^{2/3}}\right.}{e^3}\right.$$

$$= \frac{3b^2dn^2\left(d+\frac{e}{x^{2/3}}\right)^2}{4e^3} - \frac{b^2n^2\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^3} - \frac{3b^2d^2n^2}{e^2x^{2/3}} + \frac{1}{6}bn\left(\frac{18d^2\left(d+\frac{e}{x^{2/3}}\right)}{e^3}\right.$$

$$= \frac{3b^2dn^2\left(d+\frac{e}{x^{2/3}}\right)^2}{4e^3} - \frac{b^2n^2\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^3} - \frac{3b^2d^2n^2}{e^2x^{2/3}} + \frac{b^2d^3n^2\log^2\left(d+\frac{e}{x^{2/3}}\right)}{2e^3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.36, size = 691, normalized size = 2.50

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x^3,x]
```

```
[Out] (-18*e^3*(a + b*Log[c*(d + e/x^(2/3))^n])^2 + b*n*(9*b*d*n*x^(2/3)*(e*(e -
2*d*x^(2/3)) + 2*d^2*x^(4/3)*Log[d + e/x^(2/3)]) - 2*b*n*(e*(2*e^2 - 3*d*e*
x^(2/3) + 6*d^2*x^(4/3)) - 6*d^3*x^2*Log[d + e/x^(2/3)]) + 12*e^3*(a + b*Lo
g[c*(d + e/x^(2/3))^n]) - 18*d*e^2*x^(2/3)*(a + b*Log[c*(d + e/x^(2/3))^n])
 + 36*d^2*x^(4/3)*(e*(a - b*n) + b*(e + d*x^(2/3))*Log[c*(d + e/x^(2/3))^n]
) - 36*d^3*x^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] - Sqrt[-d]*x^(1
/3)] - 36*d^3*x^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] + Sqrt[-d]*x
^(1/3)] - 36*d^3*x^2*((a + b*Log[c*(d + e/x^(2/3))^n])*Log[-(e/(d*x^(2/3)))
] + b*n*PolyLog[2, 1 + e/(d*x^(2/3))]) + 18*b*d^3*n*x^2*(Log[Sqrt[e] - Sqrt
```

```
[-d]*x^(1/3)]*(Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 2*Log[(1 + (Sqrt[-d]*x^(1/
3))/Sqrt[e])/2] - 4*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]]) - 4*PolyLog[2, 1 - (Sq
rt[-d]*x^(1/3))/Sqrt[e]] + 2*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e]
)]) + 18*b*d^3*n*x^2*(Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*(Log[Sqrt[e] + Sqrt[-
d]*x^(1/3)] + 2*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] - 4*Log[-((Sqrt[-
d]*x^(1/3))/Sqrt[e])]) + 2*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] -
 4*PolyLog[2, 1 + (Sqrt[-d]*x^(1/3))/Sqrt[e]]))/(36*e^3*x^2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))^2/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))^2/x^3,x)
```

Maxima [A]
time = 0.30, size = 299, normalized size = 1.08

$$-\frac{1}{6}\left(6\,d^{3}e^{(-4)}\log\left(dx^{\frac{2}{3}}+e\right)-6\,d^{3}e^{(-4)}\log\left(x^{\frac{2}{3}}\right)-\frac{\left(6\,d^{2}x^{\frac{4}{3}}-3\,dx^{\frac{2}{3}}e+2\,e^{2}\right)e^{(-3)}}{x^{2}}\right)abne-\frac{1}{36}\left(6\left(6\,d^{3}e^{(-4)}\log\left(dx^{\frac{2}{3}}+e\right)-6\,d^{3}e^{(-4)}\log\left(x^{\frac{2}{3}}\right)-\frac{\left(6\,d^{2}x^{\frac{4}{3}}-3\,dx^{\frac{2}{3}}e+2\,e^{2}\right)e^{(-3)}}{x^{2}}\right)ne\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)-\frac{\left(18\,d^{3}x^{2}\log\left(dx^{\frac{2}{3}}+e\right)^{2}+8\,d^{3}x^{2}\log\left(x\right)^{2}-44\,d^{3}x^{2}\log\left(x\right)-66\,d^{2}x^{\frac{4}{3}}e+15\,dx^{\frac{2}{3}}e^{2}-6\left(4\,d^{3}x^{2}\log\left(x\right)-11\,d^{3}x^{2}\right)\log\left(dx^{\frac{2}{3}}+e\right)-4\,e^{3}\right)n^{2}e^{(-3)}}{x^{2}}\right)b^{2}-\frac{b^{2}\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)^{2}}{2\,x^{2}}-\frac{ab\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)}{x^{2}}-\frac{a^{2}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/6*(6*d^3*e^(-4)*log(d*x^(2/3) + e) - 6*d^3*e^(-4)*log(x^(2/3)) - (6*d^2*
x^(4/3) - 3*d*x^(2/3)*e + 2*e^2)*e^(-3)/x^2)*a*b*n*e - 1/36*(6*(6*d^3*e^(-4
)*log(d*x^(2/3) + e) - 6*d^3*e^(-4)*log(x^(2/3)) - (6*d^2*x^(4/3) - 3*d*x^(
2/3)*e + 2*e^2)*e^(-3)/x^2)*n*e*log(c*(d + e/x^(2/3))^n) - (18*d^3*x^2*log(
d*x^(2/3) + e)^2 + 8*d^3*x^2*log(x)^2 - 44*d^3*x^2*log(x) - 66*d^2*x^(4/3)*
e + 15*d*x^(2/3)*e^2 - 6*(4*d^3*x^2*log(x) - 11*d^3*x^2)*log(d*x^(2/3) + e)
 - 4*e^3)*n^2*e^(-3)/x^2)*b^2 - 1/2*b^2*log(c*(d + e/x^(2/3))^n)^2/x^2 - a*
b*log(c*(d + e/x^(2/3))^n)/x^2 - 1/2*a^2/x^2
```

Fricas [A]
time = 0.37, size = 294, normalized size = 1.07

$$-\frac{\left(18\,b^{2}e^{3}\log\left(c\right)^{2}-12\left(b^{2}n-3\,ab\right)e^{3}\log\left(c\right)+18\left(b^{2}d^{3}n^{2}x^{2}+b^{2}n^{2}e^{3}\right)\log\left(\frac{dx+x^{\frac{1}{3}}e}{x}\right)^{2}+2\left(2\,b^{2}n^{2}-6\,abn+9\,a^{2}\right)e^{3}-6\left(6\,b^{2}d^{2}n^{2}x^{\frac{4}{3}}e-3\,b^{2}dn^{2}x^{\frac{2}{3}}e^{2}+\left(11\,b^{2}d^{3}n^{2}-6\,abd^{3}n\right)x^{2}+2\left(b^{2}n^{2}-3\,abn\right)e^{3}-6\left(b^{2}d^{3}nx^{2}+b^{2}ne^{3}\right)\log\left(c\right)\right)\log\left(\frac{dx+x^{\frac{1}{3}}e}{x}\right)+3\left(6\,b^{2}dne^{2}\log\left(c\right)-\left(5\,b^{2}dn^{2}-6\,abdn\right)e^{2}\right)x^{\frac{2}{3}}-6\left(6\,b^{2}d^{2}nxe\log\left(c\right)-\left(11\,b^{2}d^{2}n^{2}-6\,abd^{2}n\right)xe\right)x^{\frac{1}{3}}\right)e^{(-3)}}{36\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/36*(18*b^2*e^3*log(c)^2 - 12*(b^2*n - 3*a*b)*e^3*log(c) + 18*(b^2*d^3*n^
2*x^2 + b^2*n^2*e^3)*log((d*x + x^(1/3)*e)/x)^2 + 2*(2*b^2*n^2 - 6*a*b*n +
```

```
9*a^2)*e^3 - 6*(6*b^2*d^2*n^2*x^(4/3)*e - 3*b^2*d*n^2*x^(2/3)*e^2 + (11*b^2
*d^3*n^2 - 6*a*b*d^3*n)*x^2 + 2*(b^2*n^2 - 3*a*b*n)*e^3 - 6*(b^2*d^3*n*x^2
+ b^2*n*e^3)*log(c))*log((d*x + x^(1/3)*e)/x) + 3*(6*b^2*d*n*e^2*log(c) - (
5*b^2*d*n^2 - 6*a*b*d*n)*e^2)*x^(2/3) - 6*(6*b^2*d^2*n*x*e*log(c) - (11*b^2
*d^2*n^2 - 6*a*b*d^2*n)*x*e)*x^(1/3))*e^(-3)/x^2
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**2/x**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2/x^3, x)
```

Mupad [B]
time = 0.57, size = 302, normalized size = 1.09

$$\frac{\frac{d\left(\frac{3a^2}{2}-a\,b\,n+\frac{b^2n^2}{3}\right)}{2e}-\frac{d\left(3a^2-b^2n^2\right)}{4e}}{x^{4/3}}-\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)^2\left(\frac{b^2}{2x^2}+\frac{b^2d^3}{2e^3}\right)-\frac{\frac{a^2}{2}-\frac{a\,b\,n}{3}+\frac{b^2n^2}{9}}{x^2}-\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\left(\frac{b\,(3a-b\,n)}{3x^2}-\frac{\frac{b\,d\,(3a-b\,n)}{2e}-\frac{3a\,b\,d}{2e}}{x^{4/3}}+\frac{d\left(\frac{b\,d\,(3a-b\,n)}{e}-\frac{3a\,b\,d}{e}\right)}{e\,x^{2/3}}\right)-\frac{\frac{d\left(\frac{d\left(\frac{3a^2}{2}-a\,b\,n+\frac{b^2n^2}{3}\right)}{e}-\frac{d\left(3a^2-b^2n^2\right)}{2e}\right)}{e}+\frac{b^2d^2n^2}{e^2}}{x^{2/3}}+\frac{\ln\left(d+\frac{e}{x^{2/3}}\right)\left(11b^2d^3n^2-6a\,b\,d^3n\right)}{6e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))^2/x^3,x)
```

```
[Out] ((d*((3*a^2)/2 + (b^2*n^2)/3 - a*b*n))/(2*e) - (d*(3*a^2 - b^2*n^2))/(4*e))
/x^(4/3) - log(c*(d + e/x^(2/3))^n)^2*(b^2/(2*x^2) + (b^2*d^3)/(2*e^3)) - (
a^2/2 + (b^2*n^2)/9 - (a*b*n)/3)/x^2 - log(c*(d + e/x^(2/3))^n)*((b*(3*a -
b*n))/(3*x^2) - ((b*d*(3*a - b*n))/(2*e) - (3*a*b*d)/(2*e))/x^(4/3) + (d*((
b*d*(3*a - b*n))/e - (3*a*b*d)/e))/(e*x^(2/3))) - ((d*((d*((3*a^2)/2 + (b^2
*n^2)/3 - a*b*n))/e - (d*(3*a^2 - b^2*n^2))/(2*e)))/e + (b^2*d^2*n^2)/e^2)/
x^(2/3) + (log(d + e/x^(2/3))*(11*b^2*d^3*n^2 - 6*a*b*d^3*n))/(6*e^3)
```

3.520 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^5}\,dx$

Optimal. Leaf size=482

$$-\frac{15b^2d^4n^2\left(d+\frac{e}{x^{2/3}}\right)^2}{8e^6}+\frac{10b^2d^3n^2\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^6}-\frac{15b^2d^2n^2\left(d+\frac{e}{x^{2/3}}\right)^4}{32e^6}+\frac{3b^2dn^2\left(d+\frac{e}{x^{2/3}}\right)^5}{25e^6}-\frac{b^2n^2\left(d+\frac{e}{x^{2/3}}\right)^6}{72e^6}+3$$

```
[Out] -15/8*b^2*d^4*n^2*(d+e/x^(2/3))^2/e^6+10/9*b^2*d^3*n^2*(d+e/x^(2/3))^3/e^6-
15/32*b^2*d^2*n^2*(d+e/x^(2/3))^4/e^6+3/25*b^2*d*n^2*(d+e/x^(2/3))^5/e^6-1/
72*b^2*n^2*(d+e/x^(2/3))^6/e^6+3*b^2*d^5*n^2/e^5/x^(2/3)-1/4*b^2*d^6*n^2*ln
(d+e/x^(2/3))^2/e^6-3*b*d^5*n*(d+e/x^(2/3))*(a+b*ln(c*(d+e/x^(2/3))^n))/e^6
+15/4*b*d^4*n*(d+e/x^(2/3))^2*(a+b*ln(c*(d+e/x^(2/3))^n))/e^6-10/3*b*d^3*n*
(d+e/x^(2/3))^3*(a+b*ln(c*(d+e/x^(2/3))^n))/e^6+15/8*b*d^2*n*(d+e/x^(2/3))^
4*(a+b*ln(c*(d+e/x^(2/3))^n))/e^6-3/5*b*d*n*(d+e/x^(2/3))^5*(a+b*ln(c*(d+e/
x^(2/3))^n))/e^6+1/12*b*n*(d+e/x^(2/3))^6*(a+b*ln(c*(d+e/x^(2/3))^n))/e^6+1
/2*b*d^6*n*ln(d+e/x^(2/3))*(a+b*ln(c*(d+e/x^(2/3))^n))/e^6-1/4*(a+b*ln(c*(d
+e/x^(2/3))^n))^2/x^4
```

Rubi [A]

time = 0.32, antiderivative size = 482, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2504, 2445, 2458, 45, 2372, 12, 14, 2338}

$$\frac{bd^6n\log\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2e^6}-\frac{3bd^5n\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^6}+\frac{15bd^4n\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{4e^6}-\frac{10bd^3n\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3e^6}+\frac{15bd^2n\left(d+\frac{e}{x^{2/3}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{8e^6}-\frac{3bdn\left(d+\frac{e}{x^{2/3}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{5e^6}+\frac{bn\left(d+\frac{e}{x^{2/3}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{12e^6}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4x^4}-\frac{b^2d^6n^2\log^2\left(d+\frac{e}{x^{2/3}}\right)}{4e^6}+\frac{3b^2d^5n^2}{e^5x^{2/3}}-\frac{15b^2d^4n^2\left(d+\frac{e}{x^{2/3}}\right)^2}{8e^6}+\frac{10b^2d^3n^2\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^6}-\frac{15b^2d^2n^2\left(d+\frac{e}{x^{2/3}}\right)^4}{32e^6}+\frac{3b^2dn^2\left(d+\frac{e}{x^{2/3}}\right)^5}{25e^6}-\frac{b^2n^2\left(d+\frac{e}{x^{2/3}}\right)^6}{72e^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x^5,x]
```

```
[Out] (-15*b^2*d^4*n^2*(d + e/x^(2/3))^2)/(8*e^6) + (10*b^2*d^3*n^2*(d + e/x^(2/3
))^3)/(9*e^6) - (15*b^2*d^2*n^2*(d + e/x^(2/3))^4)/(32*e^6) + (3*b^2*d*n^2*
(d + e/x^(2/3))^5)/(25*e^6) - (b^2*n^2*(d + e/x^(2/3))^6)/(72*e^6) + (3*b^2
*d^5*n^2)/(e^5*x^(2/3)) - (b^2*d^6*n^2*Log[d + e/x^(2/3)]^2)/(4*e^6) - (3*b
*d^5*n*(d + e/x^(2/3))*(a + b*Log[c*(d + e/x^(2/3))^n]))/e^6 + (15*b*d^4*n*
(d + e/x^(2/3))^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(4*e^6) - (10*b*d^3*n*(
d + e/x^(2/3))^3*(a + b*Log[c*(d + e/x^(2/3))^n]))/(3*e^6) + (15*b*d^2*n*(d
 + e/x^(2/3))^4*(a + b*Log[c*(d + e/x^(2/3))^n]))/(8*e^6) - (3*b*d*n*(d + e
/x^(2/3))^5*(a + b*Log[c*(d + e/x^(2/3))^n]))/(5*e^6) + (b*n*(d + e/x^(2/3)
)^6*(a + b*Log[c*(d + e/x^(2/3))^n]))/(12*e^6) + (b*d^6*n*Log[d + e/x^(2/3)
]*(a + b*Log[c*(d + e/x^(2/3))^n]))/(2*e^6) - (a + b*Log[c*(d + e/x^(2/3))^
n])^2/(4*x^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] && !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^5}\,dx = -\left(\frac{3}{2}\text{Subst}\left(\int x^5(a+b\log(c(d+ex)^n))^2\,dx, x, \frac{1}{x^{2/3}}\right)\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4x^4} + \frac{1}{2}(ben)\text{Subst}\left(\int \frac{x^6(a+b\log(c(d+ex)}{d+ex}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4x^4} + \frac{1}{2}(bn)\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^6(a+b\log(c}{x}\right.$$

$$= -\frac{1}{120}bn\left(\frac{360d^5\left(d+\frac{e}{x^{2/3}}\right)}{e^6} - \frac{450d^4\left(d+\frac{e}{x^{2/3}}\right)^2}{e^6} + \frac{400d^3\left(d+\frac{e}{x^{2/3}}\right)^3}{e^6} - \frac{2}{}\right.$$

$$= -\frac{1}{120}bn\left(\frac{360d^5\left(d+\frac{e}{x^{2/3}}\right)}{e^6} - \frac{450d^4\left(d+\frac{e}{x^{2/3}}\right)^2}{e^6} + \frac{400d^3\left(d+\frac{e}{x^{2/3}}\right)^3}{e^6} - \frac{2}{}\right.$$

$$= -\frac{1}{120}bn\left(\frac{360d^5\left(d+\frac{e}{x^{2/3}}\right)}{e^6} - \frac{450d^4\left(d+\frac{e}{x^{2/3}}\right)^2}{e^6} + \frac{400d^3\left(d+\frac{e}{x^{2/3}}\right)^3}{e^6} - \frac{2}{}\right.$$

$$= -\frac{15b^2d^4n^2\left(d+\frac{e}{x^{2/3}}\right)^2}{8e^6} + \frac{10b^2d^3n^2\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^6} - \frac{15b^2d^2n^2\left(d+\frac{e}{x^{2/3}}\right)^4}{32e^6} +$$

$$= -\frac{15b^2d^4n^2\left(d+\frac{e}{x^{2/3}}\right)^2}{8e^6} + \frac{10b^2d^3n^2\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^6} - \frac{15b^2d^2n^2\left(d+\frac{e}{x^{2/3}}\right)^4}{32e^6} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.58, size = 1021, normalized size = 2.12

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x^5,x]
```

```
[Out] (-1800*(a + b*Log[c*(d + e/x^(2/3))^n])^2 + (b*n*(600*a*e^6 - 100*b*e^6*n -
 720*a*d*e^5*x^(2/3) + 264*b*d*e^5*n*x^(2/3) + 900*a*d^2*e^4*x^(4/3) - 555*
b*d^2*e^4*n*x^(4/3) - 1200*a*d^3*e^3*x^2 + 1140*b*d^3*e^3*n*x^2 + 1800*a*d^
```

```
4*e^2*x^(8/3) - 2610*b*d^4*e^2*n*x^(8/3) - 3600*a*d^5*e*x^(10/3) + 8820*b*d
^5*e*n*x^(10/3) - 5220*b*d^6*n*x^4*Log[d + e/x^(2/3)] + 600*b*e^6*Log[c*(d
+ e/x^(2/3))^n] - 720*b*d*e^5*x^(2/3)*Log[c*(d + e/x^(2/3))^n] + 900*b*d^2*
e^4*x^(4/3)*Log[c*(d + e/x^(2/3))^n] - 1200*b*d^3*e^3*x^2*Log[c*(d + e/x^(2
/3))^n] + 1800*b*d^4*e^2*x^(8/3)*Log[c*(d + e/x^(2/3))^n] - 3600*b*d^5*e*x^
(10/3)*Log[c*(d + e/x^(2/3))^n] - 3600*b*d^6*x^4*Log[c*(d + e/x^(2/3))^n] +
 3600*a*d^6*x^4*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 3600*b*d^6*x^4*Log[c*(d +
 e/x^(2/3))^n]*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] - 1800*b*d^6*n*x^4*Log[Sqrt[
e] - Sqrt[-d]*x^(1/3)]^2 + 3600*a*d^6*x^4*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] +
 3600*b*d^6*x^4*Log[c*(d + e/x^(2/3))^n]*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] -
1800*b*d^6*n*x^4*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]^2 - 3600*b*d^6*n*x^4*Log[S
qrt[e] + Sqrt[-d]*x^(1/3)]*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] - 3600
*b*d^6*n*x^4*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log[(1 + (Sqrt[-d]*x^(1/3))/Sq
rt[e])/2] + 3600*a*d^6*x^4*Log[-(e/(d*x^(2/3)))] + 3600*b*d^6*x^4*Log[c*(d
+ e/x^(2/3))^n]*Log[-(e/(d*x^(2/3)))] + 7200*b*d^6*n*x^4*Log[Sqrt[e] + Sqrt
[-d]*x^(1/3)]*Log[-((Sqrt[-d]*x^(1/3))/Sqrt[e])] + 7200*b*d^6*n*x^4*Log[Sqr
t[e] - Sqrt[-d]*x^(1/3)]*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]] + 3600*b*d^6*n*x^4
*PolyLog[2, 1 + e/(d*x^(2/3))] + 7200*b*d^6*n*x^4*PolyLog[2, 1 - (Sqrt[-d]*
x^(1/3))/Sqrt[e]] - 3600*b*d^6*n*x^4*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2
*Sqrt[e])] - 3600*b*d^6*n*x^4*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2
] + 7200*b*d^6*n*x^4*PolyLog[2, 1 + (Sqrt[-d]*x^(1/3))/Sqrt[e]]))/e^6)/(720
0*x^4)
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \ln\left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{2}}{x^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))^2/x^5,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))^2/x^5,x)
```

Maxima [**A**]
time = 0.31, size = 389, normalized size = 0.81

$$\frac{1}{120} \left(60 \, d^{6} e^{(-7)} \log\left(d x^{\frac{2}{3}} + e\right) - 60 \, d^{6} e^{(-7)} \log\left(x^{\frac{2}{3}}\right) - \frac{\left(60 \, d^{5} x^{\frac{10}{3}} - 30 \, d^{4} x^{\frac{8}{3}} e + 20 \, d^{3} x^{2} e^{2} - 15 \, d^{2} x^{\frac{4}{3}} e^{3} + 12 \, d x^{\frac{2}{3}} e^{4} - 10 \, e^{5}\right) e^{(-6)}}{x^{4}}\right) a b n e + \frac{1}{7200} \left(60 \left(60 \, d^{6} e^{(-7)} \log\left(d x^{\frac{2}{3}} + e\right) - 60 \, d^{6} e^{(-7)} \log\left(x^{\frac{2}{3}}\right) - \frac{\left(60 \, d^{5} x^{\frac{10}{3}} - 30 \, d^{4} x^{\frac{8}{3}} e + 20 \, d^{3} x^{2} e^{2} - 15 \, d^{2} x^{\frac{4}{3}} e^{3} + 12 \, d x^{\frac{2}{3}} e^{4} - 10 \, e^{5}\right) e^{(-6)}}{x^{4}}\right) n e \log\left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)^{n}\right) - \frac{\left(1800 \, d^{6} x^{4} \log\left(d x^{\frac{2}{3}} + e\right)^{2} + 800 \, d^{6} x^{4} \log\left(x\right)^{2} - 5880 \, d^{6} x^{4} \log\left(x\right) - 8820 \, d^{5} x^{\frac{10}{3}} e + 2610 \, d^{4} x^{\frac{8}{3}} e^{2} - 1140 \, d^{3} x^{2} e^{3} + 555 \, d^{2} x^{\frac{4}{3}} e^{4} - 264 \, d x^{\frac{2}{3}} e^{5} - 60 \, \left(40 \, d^{6} x^{4} \log\left(x\right) - 147 \, d^{6} x^{4}\right) \log\left(d x^{\frac{2}{3}} + e\right) + 100 \, e^{6}\right) n^{2} e^{(-6)}}{x^{4}}\right) b^{2} - \frac{b^{2} \log\left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)^{2}}{4 \, x^{4}} - \frac{a b \log\left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)}{2 \, x^{4}} - \frac{a^{2}}{4 \, x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/120*(60*d^6*e^(-7)*log(d*x^(2/3) + e) - 60*d^6*e^(-7)*log(x^(2/3)) - (60*
d^5*x^(10/3) - 30*d^4*x^(8/3)*e + 20*d^3*x^2*e^2 - 15*d^2*x^(4/3)*e^3 + 12*
d*x^(2/3)*e^4 - 10*e^5)*e^(-6)/x^4)*a*b*n*e + 1/7200*(60*(60*d^6*e^(-7)*log
```

```
(d*x^(2/3) + e) - 60*d^6*e^(-7)*log(x^(2/3)) - (60*d^5*x^(10/3) - 30*d^4*x^
(8/3)*e + 20*d^3*x^2*e^2 - 15*d^2*x^(4/3)*e^3 + 12*d*x^(2/3)*e^4 - 10*e^5)*
e^(-6)/x^4)*n*e*log(c*(d + e/x^(2/3))^n) - (1800*d^6*x^4*log(d*x^(2/3) + e)
^2 + 800*d^6*x^4*log(x)^2 - 5880*d^6*x^4*log(x) - 8820*d^5*x^(10/3)*e + 261
0*d^4*x^(8/3)*e^2 - 1140*d^3*x^2*e^3 + 555*d^2*x^(4/3)*e^4 - 264*d*x^(2/3)*
e^5 - 60*(40*d^6*x^4*log(x) - 147*d^6*x^4)*log(d*x^(2/3) + e) + 100*e^6)*n^
2*e^(-6)/x^4)*b^2 - 1/4*b^2*log(c*(d + e/x^(2/3))^n)^2/x^4 - 1/2*a*b*log(c*
(d + e/x^(2/3))^n)/x^4 - 1/4*a^2/x^4
```

Fricas [**A**]
time = 0.37, size = 483, normalized size = 1.00

$$-\frac{\left(1800\,b^2e^6\log\left(c\right)^2-60\,(19\,b^2d^3n^2-20\,abd^3n)x^2e^3-1800\,(b^2d^6n^2x^4-b^2n^2e^6)\log\left(\frac{dx+x^{\frac{1}{3}}e}{x}\right)^2+100\,(b^2n^2-6\,abn+18\,a^2)e^6+600\,(2\,b^2d^3nx^2e^3-(b^2n-6\,ab)e^6)\log\left(c\right)+60\,\left(20\,b^2d^3n^2x^2e^3+3\,(49\,b^2d^6n^2-20\,abd^6n)x^4-10\,(b^2n^2-6\,abn)e^6-60\,(b^2d^6nx^4-b^2ne^6)\log\left(c\right)-6\,(5\,b^2d^4n^2x^2e^2-2\,b^2dn^2e^5)x^{\frac{2}{3}}+15\,(4\,b^2d^5n^2x^3e-b^2d^2n^2xe^4)x^{\frac{1}{3}}\right)\log\left(\frac{dx+x^{\frac{1}{3}}e}{x}\right)+6\,(15\,(29\,b^2d^4n^2-20\,abd^4n)x^2e^2-4\,(11\,b^2dn^2-30\,abdn)e^5-60\,(5\,b^2d^4nx^2e^2-2\,b^2dne^5)\log\left(c\right))x^{\frac{2}{3}}-15\,(12\,(49\,b^2d^5n^2-20\,abd^5n)x^3e-(37\,b^2d^2n^2-60\,abd^2n)xe^4-60\,(4\,b^2d^5nx^3e-b^2d^2nxe^4)\log\left(c\right))x^{\frac{1}{3}}\right)e^{(-6)}}{7200\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/7200*(1800*b^2*e^6*log(c)^2 - 60*(19*b^2*d^3*n^2 - 20*a*b*d^3*n)*x^2*e^3
 - 1800*(b^2*d^6*n^2*x^4 - b^2*n^2*e^6)*log((d*x + x^(1/3)*e)/x)^2 + 100*(b
^2*n^2 - 6*a*b*n + 18*a^2)*e^6 + 600*(2*b^2*d^3*n*x^2*e^3 - (b^2*n - 6*a*b)
*e^6)*log(c) + 60*(20*b^2*d^3*n^2*x^2*e^3 + 3*(49*b^2*d^6*n^2 - 20*a*b*d^6*
n)*x^4 - 10*(b^2*n^2 - 6*a*b*n)*e^6 - 60*(b^2*d^6*n*x^4 - b^2*n*e^6)*log(c)
 - 6*(5*b^2*d^4*n^2*x^2*e^2 - 2*b^2*d*n^2*e^5)*x^(2/3) + 15*(4*b^2*d^5*n^2*
x^3*e - b^2*d^2*n^2*x*e^4)*x^(1/3))*log((d*x + x^(1/3)*e)/x) + 6*(15*(29*b^
2*d^4*n^2 - 20*a*b*d^4*n)*x^2*e^2 - 4*(11*b^2*d*n^2 - 30*a*b*d*n)*e^5 - 60*
(5*b^2*d^4*n*x^2*e^2 - 2*b^2*d*n*e^5)*log(c))*x^(2/3) - 15*(12*(49*b^2*d^5*
n^2 - 20*a*b*d^5*n)*x^3*e - (37*b^2*d^2*n^2 - 60*a*b*d^2*n)*x*e^4 - 60*(4*b
^2*d^5*n*x^3*e - b^2*d^2*n*x*e^4)*log(c))*x^(1/3))*e^(-6)/x^4
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**2/x**5,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2/x^5, x)
```

Mupad [B]
time = 1.81, size = 440, normalized size = 0.91

$$\frac{b^2\,d^6\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^n\right)^2}{4\,e^6}-\frac{b^2\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^n\right)^2}{4\,x^4}-\frac{b^2\,n^2}{72\,x^4}-\frac{a\,b\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^n\right)}{2\,x^4}-\frac{a^2}{4\,x^4}+\frac{a\,b\,n}{12\,x^4}+\frac{b^2\,n\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^n\right)}{12\,x^4}-\frac{49\,b^2\,d^6\,n^2\,\ln\left(d+\frac{e}{x^{2/3}}\right)}{40\,e^6}+\frac{19\,b^2\,d^3\,n^2}{120\,e^3\,x^2}-\frac{37\,b^2\,d^2\,n^2}{480\,e^2\,x^{8/3}}-\frac{29\,b^2\,d^4\,n^2}{80\,e^4\,x^{4/3}}+\frac{49\,b^2\,d^5\,n^2}{40\,e^5\,x^{2/3}}+\frac{11\,b^2\,d\,n^2}{300\,e\,x^{10/3}}-\frac{b^2\,d^3\,n\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^n\right)}{6\,e^3\,x^2}+\frac{b^2\,d^2\,n\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^n\right)}{8\,e^2\,x^{8/3}}+\frac{b^2\,d^4\,n\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^n\right)}{4\,e^4\,x^{4/3}}-\frac{b^2\,d^5\,n\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^n\right)}{2\,e^5\,x^{2/3}}-\frac{a\,b\,d\,n}{10\,e\,x^{10/3}}+\frac{a\,b\,d^6\,n\,\ln\left(d+\frac{e}{x^{2/3}}\right)}{2\,e^6}-\frac{b^2\,d\,n\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^n\right)}{10\,e\,x^{10/3}}-\frac{a\,b\,d^3\,n}{6\,e^3\,x^2}+\frac{a\,b\,d^2\,n}{8\,e^2\,x^{8/3}}+\frac{a\,b\,d^4\,n}{4\,e^4\,x^{4/3}}-\frac{a\,b\,d^5\,n}{2\,e^5\,x^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))^2/x^5,x)
```

```
[Out] (b^2*d^6*log(c*(d + e/x^(2/3))^n)^2)/(4*e^6) - (b^2*log(c*(d + e/x^(2/3))^n
)^2)/(4*x^4) - (b^2*n^2)/(72*x^4) - (a*b*log(c*(d + e/x^(2/3))^n))/(2*x^4)
- a^2/(4*x^4) + (a*b*n)/(12*x^4) + (b^2*n*log(c*(d + e/x^(2/3))^n))/(12*x^4
) - (49*b^2*d^6*n^2*log(d + e/x^(2/3)))/(40*e^6) + (19*b^2*d^3*n^2)/(120*e^
3*x^2) - (37*b^2*d^2*n^2)/(480*e^2*x^(8/3)) - (29*b^2*d^4*n^2)/(80*e^4*x^(4
/3)) + (49*b^2*d^5*n^2)/(40*e^5*x^(2/3)) + (11*b^2*d*n^2)/(300*e*x^(10/3))
- (b^2*d^3*n*log(c*(d + e/x^(2/3))^n))/(6*e^3*x^2) + (b^2*d^2*n*log(c*(d +
e/x^(2/3))^n))/(8*e^2*x^(8/3)) + (b^2*d^4*n*log(c*(d + e/x^(2/3))^n))/(4*e^
4*x^(4/3)) - (b^2*d^5*n*log(c*(d + e/x^(2/3))^n))/(2*e^5*x^(2/3)) - (a*b*d*
n)/(10*e*x^(10/3)) + (a*b*d^6*n*log(d + e/x^(2/3)))/(2*e^6) - (b^2*d*n*log(
c*(d + e/x^(2/3))^n))/(10*e*x^(10/3)) - (a*b*d^3*n)/(6*e^3*x^2) + (a*b*d^2*
n)/(8*e^2*x^(8/3)) + (a*b*d^4*n)/(4*e^4*x^(4/3)) - (a*b*d^5*n)/(2*e^5*x^(2/
3))
```

3.521 $\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 \, dx$

Optimal. Leaf size=490

$$-\frac{4abe^4n\sqrt[3]{x}}{3d^4}+\frac{568b^2e^4n^2\sqrt[3]{x}}{315d^4}-\frac{32b^2e^3n^2x}{105d^3}+\frac{8b^2e^2n^2x^{5/3}}{105d^2}-\frac{1408b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt[3]{x}}{\sqrt{e}}\right)}{315d^{9/2}}-\frac{4ib^2e^{9/2}n^2\tan^{-}}{3d}$$

```
[Out] -4/3*a*b*e^4*n*x^(1/3)/d^4+568/315*b^2*e^4*n^2*x^(1/3)/d^4-32/105*b^2*e^3*n
^2*x/d^3+8/105*b^2*e^2*n^2*x^(5/3)/d^2-1408/315*b^2*e^(9/2)*n^2*arctan(x^(1
/3)*d^(1/2)/e^(1/2))/d^(9/2)-4/3*I*b^2*e^(9/2)*n^2*arctan(x^(1/3)*d^(1/2)/e
^(1/2))^2/d^(9/2)-4/3*b^2*e^4*n*x^(1/3)*ln(c*(d+e/x^(2/3))^n)/d^4+4/9*b*e^3
*n*x*(a+b*ln(c*(d+e/x^(2/3))^n))/d^3-4/15*b*e^2*n*x^(5/3)*(a+b*ln(c*(d+e/x^
(2/3))^n))/d^2+4/21*b*e*n*x^(7/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d+4/3*b*e^(9/
2)*n*arctan(x^(1/3)*d^(1/2)/e^(1/2))*(a+b*ln(c*(d+e/x^(2/3))^n))/d^(9/2)+1/
3*x^3*(a+b*ln(c*(d+e/x^(2/3))^n))^2+8/3*b^2*e^(9/2)*n^2*arctan(x^(1/3)*d^(1
/2)/e^(1/2))*ln(2-2*e^(1/2)/(-I*x^(1/3)*d^(1/2)+e^(1/2)))/d^(9/2)-4/3*I*b^2
*e^(9/2)*n^2*polylog(2,-1+2*e^(1/2)/(-I*x^(1/3)*d^(1/2)+e^(1/2)))/d^(9/2)
```

Rubi [A]
time = 0.53, antiderivative size = 490, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 17, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.708$, Rules used = {2508, 2507, 2526, 2498, 269, 211, 2505, 199, 327, 308, 2520, 12, 266, 6820, 5044, 4988, 2497}

$$-\frac{4ib^2e^{9/2}n^2\text{PolyLog}\left(2,-1+\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\,\sqrt[3]{x}}\right)}{3d^{9/2}}+\frac{4be^{9/2}n\text{ArcTan}\left(\frac{\sqrt{d}\,\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3d^{9/2}}+\frac{4be^3nx\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{9d^3}-\frac{4be^2nx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{15d^2}+\frac{4benx^{7/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{21d}+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-\frac{4abe^4n\sqrt[3]{x}}{3d^4}-\frac{4ib^2e^{9/2}n^2\text{ArcTan}\left(\frac{\sqrt{d}\,\sqrt[3]{x}}{\sqrt{e}}\right)^2}{3d^{9/2}}-\frac{1408b^2e^{9/2}n^2\text{ArcTan}\left(\frac{\sqrt{d}\,\sqrt[3]{x}}{\sqrt{e}}\right)}{315d^{9/2}}+\frac{8b^2e^{9/2}n^2\text{ArcTan}\left(\frac{\sqrt{d}\,\sqrt[3]{x}}{\sqrt{e}}\right)\log\left(2-\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\,\sqrt[3]{x}}\right)}{3d^{9/2}}-\frac{4b^2e^4n\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3d^4}+\frac{568b^2e^4n^2\sqrt[3]{x}}{315d^4}-\frac{32b^2e^3n^2x}{105d^3}+\frac{8b^2e^2n^2x^{5/3}}{105d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

```
[Out] (-4*a*b*e^4*n*x^(1/3))/(3*d^4) + (568*b^2*e^4*n^2*x^(1/3))/(315*d^4) - (32*
b^2*e^3*n^2*x)/(105*d^3) + (8*b^2*e^2*n^2*x^(5/3))/(105*d^2) - (1408*b^2*e^
(9/2)*n^2*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]])/(315*d^(9/2)) - (((4*I)/3)*b^2
*e^(9/2)*n^2*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]^2)/d^(9/2) + (8*b^2*e^(9/2)*
n^2*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]*Log[2 - (2*Sqrt[e])/(Sqrt[e] - I*Sqrt
[d]*x^(1/3))])/(3*d^(9/2)) - (4*b^2*e^4*n*x^(1/3)*Log[c*(d + e/x^(2/3))^n])
/(3*d^4) + (4*b*e^3*n*x*(a + b*Log[c*(d + e/x^(2/3))^n]))/(9*d^3) - (4*b*e^
2*n*x^(5/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(15*d^2) + (4*b*e*n*x^(7/3)*(
a + b*Log[c*(d + e/x^(2/3))^n]))/(21*d) + (4*b*e^(9/2)*n*ArcTan[(Sqrt[d]*x^
(1/3))/Sqrt[e]]*(a + b*Log[c*(d + e/x^(2/3))^n]))/(3*d^(9/2)) + (x^3*(a + b
*Log[c*(d + e/x^(2/3))^n])^2)/3 - (((4*I)/3)*b^2*e^(9/2)*n^2*PolyLog[2, -1
+ (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/d^(9/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2508

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1)
- 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
b, c, d, e, m, p, q}, x] && FractionQ[n]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
```

```
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx = 3\text{Subst}\left(\int x^8\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2 dx, x, \sqrt[3]{x}\right)$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 + \frac{1}{3}(4ben)\text{Subst}\left(\int \frac{x^6\left(a+b\log\right.}{d}\right.$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 + \frac{1}{3}(4ben)\text{Subst}\left(\int\left(-\frac{e^3\left(a+\right.}{}\right.\right.$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 + \frac{(4ben)\text{Subst}\left(\int x^6\left(a+b\log\left(c\right.\right.\right.}{3d}$$

$$= -\frac{4abe^4n\sqrt[3]{x}}{3d^4} + \frac{4be^3nx\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{9d^3} - \frac{4be^2nx^{5/3}\left(a+\right.}{}$$

$$= -\frac{4abe^4n\sqrt[3]{x}}{3d^4} - \frac{4b^2e^4n\sqrt[3]{x}\,\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3d^4} + \frac{4be^3nx\left(a+b\log\right.}{9d}$$

$$= -\frac{4abe^4n\sqrt[3]{x}}{3d^4} + \frac{8b^2e^4n^2\sqrt[3]{x}}{9d^4} - \frac{4b^2e^4n\sqrt[3]{x}\,\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3d^4} + \frac{4b}{}$$

$$= -\frac{4abe^4n\sqrt[3]{x}}{3d^4} + \frac{568b^2e^4n^2\sqrt[3]{x}}{315d^4} - \frac{32b^2e^3n^2x}{105d^3} + \frac{8b^2e^2n^2x^{5/3}}{105d^2} - \frac{32b^2}{}$$

$$= -\frac{4abe^4n\sqrt[3]{x}}{3d^4} + \frac{568b^2e^4n^2\sqrt[3]{x}}{315d^4} - \frac{32b^2e^3n^2x}{105d^3} + \frac{8b^2e^2n^2x^{5/3}}{105d^2} - \frac{140}{}$$

$$= -\frac{4abe^4n\sqrt[3]{x}}{3d^4} + \frac{568b^2e^4n^2\sqrt[3]{x}}{315d^4} - \frac{32b^2e^3n^2x}{105d^3} + \frac{8b^2e^2n^2x^{5/3}}{105d^2} - \frac{140}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.46, size = 735, normalized size = 1.50

$$\frac{1}{3}\left(x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-4ben\left(\frac{ae^3\sqrt[3]{x}}{d^4}-\frac{2be^{7/2}n\tan^{-1}\left(\frac{\sqrt{e}}{\sqrt{d}\sqrt[3]{x}}\right)}{d^{9/2}}-\frac{2benx^{5/3}\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\frac{e}{dx^{2/3}}\right)}{35d^2}+\frac{2be^2nx\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\frac{e}{dx^{2/3}}\right)}{15d^3}-\frac{2be^3n\sqrt[3]{x}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{e}{dx^{2/3}}\right)}{3d^4}+\frac{be^3\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{d^4}-\frac{e^2x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3d^3}+\frac{ex^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{5d^2}-\frac{x^{7/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{7d}+\frac{e^{7/2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)}{2(-d)^{9/2}}-\frac{e^{7/2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(\sqrt{e}+\sqrt{-d}\sqrt[3]{x}\right)}{2(-d)^{9/2}}-\frac{be^{7/2}n\left(\log\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)\left(\log\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)+2\log\left(\frac{1}{2}\left(1+\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)\right)-4\log\left(\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)\right)-4\text{Li}_2\left(1-\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)+2\text{Li}_2\left(\frac{1}{2}-\frac{\sqrt{-d}\sqrt[3]{x}}{2\sqrt{e}}\right)\right)}{4(-d)^{9/2}}+\frac{be^{7/2}n\left(\log\left(\sqrt{e}+\sqrt{-d}\sqrt[3]{x}\right)\left(\log\left(\sqrt{e}+\sqrt{-d}\sqrt[3]{x}\right)+2\log\left(\frac{1}{2}-\frac{\sqrt{-d}\sqrt[3]{x}}{2\sqrt{e}}\right)-4\log\left(-\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)\right)+2\text{Li}_2\left(\frac{1}{2}\left(1+\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)\right)-4\text{Li}_2\left(1+\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)\right)}{4(-d)^{9/2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

```
[Out] (x^3*(a + b*Log[c*(d + e/x^(2/3))^n])^2 - 4*b*e*n*((a*e^3*x^(1/3))/d^4 - (2
*b*e^(7/2)*n*ArcTan[Sqrt[e]/(Sqrt[d]*x^(1/3))])/d^(9/2) - (2*b*e*n*x^(5/3)*
Hypergeometric2F1[-5/2, 1, -3/2, -(e/(d*x^(2/3)))])/(35*d^2) + (2*b*e^2*n*x
*Hypergeometric2F1[-3/2, 1, -1/2, -(e/(d*x^(2/3)))])/(15*d^3) - (2*b*e^3*n*
x^(1/3)*Hypergeometric2F1[-1/2, 1, 1/2, -(e/(d*x^(2/3)))])/(3*d^4) + (b*e^3
*x^(1/3)*Log[c*(d + e/x^(2/3))^n])/d^4 - (e^2*x*(a + b*Log[c*(d + e/x^(2/3)
)^n]))/(3*d^3) + (e*x^(5/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(5*d^2) - (x^
(7/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(7*d) + (e^(7/2)*(a + b*Log[c*(d +
e/x^(2/3))^n])*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)])/(2*(-d)^(9/2)) - (e^(7/2)*(
a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)])/(2*(-d)^(9
/2)) - (b*e^(7/2)*n*(Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*(Log[Sqrt[e] - Sqrt[-d
]*x^(1/3)] + 2*Log[(1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 4*Log[(Sqrt[-d]*x^
(1/3))/Sqrt[e]]) - 4*PolyLog[2, 1 - (Sqrt[-d]*x^(1/3))/Sqrt[e]] + 2*PolyLog
[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])]))/(4*(-d)^(9/2)) + (b*e^(7/2)*n*(
Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*(Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] + 2*Log[1/
2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] - 4*Log[-((Sqrt[-d]*x^(1/3))/Sqrt[e])])
 + 2*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 4*PolyLog[2, 1 + (Sqr
t[-d]*x^(1/3))/Sqrt[e]]))/(4*(-d)^(9/2))))/3
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)\right)^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e/x^(2/3))^n))^2,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e/x^(2/3))^n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*n^2*x^3*log(d*x^(2/3) + e)^2 - integrate(-1/9*(9*(b^2*log(c)^2 + 2*
a*b*log(c) + a^2)*x^(7/3)*e + 9*(b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d)*x
^3 - 2*(2*b^2*d*n*x^3 - 9*(b^2*log(c) + a*b)*x^(7/3)*e - 9*(b^2*d*log(c) +
a*b*d)*x^3 + 18*(b^2*d*x^3 + b^2*x^(7/3)*e)*log(x^(1/3*n)))*n*log(d*x^(2/3)
 + e) + 36*(b^2*d*x^3 + b^2*x^(7/3)*e)*log(x^(1/3*n))^2 - 36*((b^2*log(c) +
 a*b)*x^(7/3)*e + (b^2*d*log(c) + a*b*d)*x^3)*log(x^(1/3*n)))/(d*x + x^(1/3
)*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*log(c*((d*x + x^(1/3)*e)/x)^n)^2 + 2*a*b*x^2*log(c*((d*x +
 x^(1/3)*e)/x)^n) + a^2*x^2, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(2/3))**n))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^n\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e/x^(2/3))^n))^2,x)
```

```
[Out] int(x^2*(a + b*log(c*(d + e/x^(2/3))^n))^2, x)
```

3.522 $\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx$

Optimal. Leaf size=309

$$\frac{4aben\sqrt[3]{x}}{d}+\frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{d^{3/2}}+\frac{4ib^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{d^{3/2}}-\frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\log\left(\right.}{d^{3/2}}$$

```
[Out] 4*a*b*e*n*x^(1/3)/d+8*b^2*e^(3/2)*n^2*arctan(x^(1/3)*d^(1/2)/e^(1/2))/d^(3/
2)+4*I*b^2*e^(3/2)*n^2*arctan(x^(1/3)*d^(1/2)/e^(1/2))^2/d^(3/2)+4*b^2*e*n*
x^(1/3)*ln(c*(d+e/x^(2/3))^n)/d-4*b*e^(3/2)*n*arctan(x^(1/3)*d^(1/2)/e^(1/2
))*(a+b*ln(c*(d+e/x^(2/3))^n))/d^(3/2)+x*(a+b*ln(c*(d+e/x^(2/3))^n))^2-8*b^
2*e^(3/2)*n^2*arctan(x^(1/3)*d^(1/2)/e^(1/2))*ln(2-2*e^(1/2)/(-I*x^(1/3)*d^
(1/2)+e^(1/2)))/d^(3/2)+4*I*b^2*e^(3/2)*n^2*polylog(2,-1+2*e^(1/2)/(-I*x^(1
/3)*d^(1/2)+e^(1/2)))/d^(3/2)
```

Rubi [A]
time = 0.29, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {2501, 2507, 2521, 2498, 269, 211, 2520, 12, 266, 6820, 5044, 4988, 2497}

$$\frac{4ib^2e^{3/2}n^2\text{PolyLog}\left(2,-1+\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{d^{3/2}}-\frac{4be^{3/2}n\text{ArcTan}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^{3/2}}+x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2+\frac{4aben\sqrt[3]{x}}{d}+\frac{4ib^2e^{3/2}n^2\text{ArcTan}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{d^{3/2}}+\frac{8b^2e^{3/2}n^2\text{ArcTan}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{d^{3/2}}-\frac{8b^2e^{3/2}n^2\text{ArcTan}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\log\left(2-\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{d^{3/2}}+\frac{4b^2en\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

```
[Out] (4*a*b*e*n*x^(1/3))/d + (8*b^2*e^(3/2)*n^2*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]
])/d^(3/2) + ((4*I)*b^2*e^(3/2)*n^2*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]^2)/d^
(3/2) - (8*b^2*e^(3/2)*n^2*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]*Log[2 - (2*Sqr
t[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/d^(3/2) + (4*b^2*e*n*x^(1/3)*Log[c*(d
 + e/x^(2/3))^n])/d - (4*b*e^(3/2)*n*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]*(a +
 b*Log[c*(d + e/x^(2/3))^n]))/d^(3/2) + x*(a + b*Log[c*(d + e/x^(2/3))^n])^
2 + ((4*I)*b^2*e^(3/2)*n^2*PolyLog[2, -1 + (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]
*x^(1/3))])/d^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2501

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx = 3\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2 dx, x, \sqrt[3]{x}\right)$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 + (4ben)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right.\right.}{d+\frac{e}{x^2}}\right.$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 + (4ben)\text{Subst}\left(\int\left(\frac{a+b\log\left(c\left(d+\right.\right.}{d}\right.\right.$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 + \frac{(4ben)\text{Subst}\left(\int\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right.\right.\right.\right.}{d}$$

$$= \frac{4aben\sqrt[3]{x}}{d} - \frac{4be^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^{3/2}} +$$

$$= \frac{4aben\sqrt[3]{x}}{d} + \frac{4b^2en\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{d} - \frac{4be^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right.}{}$$

$$= \frac{4aben\sqrt[3]{x}}{d} + \frac{4b^2en\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{d} - \frac{4be^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right.}{}$$

$$= \frac{4aben\sqrt[3]{x}}{d} + \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{d^{3/2}} + \frac{4ib^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}}{\sqrt{e}}\right.}{d^{3/2}}$$

$$= \frac{4aben\sqrt[3]{x}}{d} + \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{d^{3/2}} + \frac{4ib^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}}{\sqrt{e}}\right.}{d^{3/2}}$$

$$= \frac{4aben\sqrt[3]{x}}{d} + \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{d^{3/2}} + \frac{4ib^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}}{\sqrt{e}}\right.}{d^{3/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.70, size = 296, normalized size = 0.96

$$\frac{-3b^2en^2\left(e+dx^{2/3}\right)\sqrt{-\frac{e}{dx^{2/3}}}\,{}_4F_3\left(1,1,1,\frac{5}{2};2,2,2;1+\frac{e}{dx^{2/3}}\right)+\frac{4b\sqrt{d}\,e^2n\tan^{-1}\left(\frac{\sqrt{\frac{e}{x^{2/3}}}}{\sqrt{d}}\right)\left(a-bn\log\left(d+\frac{e}{x^{2/3}}\right)+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{\sqrt{\frac{e}{x^{2/3}}}}+dx^{2/3}\left(-4b^2en^2\sqrt{-\frac{e}{dx^{2/3}}}\left(1+\log\left(\frac{1}{2}\left(1+\sqrt{-\frac{e}{dx^{2/3}}}\right)\right)\right)\log\left(d+\frac{e}{x^{2/3}}\right)+b^2en^2\sqrt{-\frac{e}{dx^{2/3}}}\log^2\left(d+\frac{e}{x^{2/3}}\right)+\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\left(4ben+adx^{2/3}+bdx^{2/3}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\right)}{d^2\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

```
[Out] (-3*b^2*e*n^2*(e + d*x^(2/3))*Sqrt[-(e/(d*x^(2/3)))]*HypergeometricPFQ[{1,
1, 1, 5/2}, {2, 2, 2}, 1 + e/(d*x^(2/3))] + (4*b*Sqrt[d]*e^2*n*ArcTan[Sqrt[
e/x^(2/3)]/Sqrt[d]]*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n
]))/Sqrt[e/x^(2/3)] + d*x^(2/3)*(-4*b^2*e*n^2*Sqrt[-(e/(d*x^(2/3)))]*(1 + L
og[(1 + Sqrt[-(e/(d*x^(2/3)))])/2])*Log[d + e/x^(2/3)] + b^2*e*n^2*Sqrt[-(e
/(d*x^(2/3)))]*Log[d + e/x^(2/3)]^2 + (a + b*Log[c*(d + e/x^(2/3))^n])*(4*b
*e*n + a*d*x^(2/3) + b*d*x^(2/3)*Log[c*(d + e/x^(2/3))^n])))/(d^2*x^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="maxima")
```

```
[Out] -2*(2*n*(arctan(sqrt(d)*x^(1/3)*e^(-1/2))*e^(1/2)/d^(3/2) - x^(1/3)/d)*e -
x*log(c*(d + e/x^(2/3))^n))*a*b + (n^2*x*log(d*x^(2/3) + e)^2 - integrate(-
1/3*(3*d*x*log(c)^2 + 3*x^(1/3)*e*log(c)^2 - 2*(2*d*n*x - 3*d*x*log(c) - 3*
x^(1/3)*e*log(c) + 6*(d*x + x^(1/3)*e)*log(x^(1/3*n)))*n*log(d*x^(2/3) + e)
 + 12*(d*x + x^(1/3)*e)*log(x^(1/3*n))^2 - 12*(d*x*log(c) + x^(1/3)*e*log(c
))*log(x^(1/3*n)))/(d*x + x^(1/3)*e), x))*b^2 + a^2*x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*log(c*((d*x + x^(1/3)*e)/x)^n)^2 + 2*a*b*log(c*((d*x + x^(1/3)
*e)/x)^n) + a^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \log \left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e/x**(2/3))**n))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^n\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))^2,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(2/3))^n))^2, x)
```

3.523 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=361

$$-\frac{8b^2n^2}{9x}+\frac{32b^2dn^2}{3e\sqrt[3]{x}}+\frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3e^{3/2}}+\frac{4ib^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{e^{3/2}}-\frac{8b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{}$$

```
[Out] -8/9*b^2*n^2/x+32/3*b^2*d*n^2/e/x^(1/3)+32/3*b^2*d^(3/2)*n^2*arctan(x^(1/3)
*d^(1/2)/e^(1/2))/e^(3/2)+4*I*b^2*d^(3/2)*n^2*arctan(x^(1/3)*d^(1/2)/e^(1/2
))^2/e^(3/2)+4/3*b*n*(a+b*ln(c*(d+e/x^(2/3))^n))/x-4*b*d*n*(a+b*ln(c*(d+e/x
^(2/3))^n))/e/x^(1/3)-4*b*d^(3/2)*n*arctan(x^(1/3)*d^(1/2)/e^(1/2))*(a+b*ln
(c*(d+e/x^(2/3))^n))/e^(3/2)-(a+b*ln(c*(d+e/x^(2/3))^n))^2/x-8*b^2*d^(3/2)*
n^2*arctan(x^(1/3)*d^(1/2)/e^(1/2))*ln(2-2*e^(1/2)/(-I*x^(1/3)*d^(1/2)+e^(1
/2)))/e^(3/2)+4*I*b^2*d^(3/2)*n^2*polylog(2,-1+2*e^(1/2)/(-I*x^(1/3)*d^(1/2
)+e^(1/2)))/e^(3/2)
```

Rubi [A]
time = 0.38, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {2508, 2507, 2526, 2505, 269, 331, 211, 2520, 12, 266, 6820, 5044, 4988, 2497}

$$\frac{4ib^2d^{3/2}n^2\text{PolyLog}\left(2,-1+\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{e^{3/2}}-\frac{4bd^{3/2}n\text{ArcTan}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^{3/2}}-\frac{4bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e\sqrt[3]{x}}+\frac{4bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x}+\frac{4ib^2d^{3/2}n^2\text{ArcTan}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{e^{3/2}}+\frac{32b^2d^{3/2}n^2\text{ArcTan}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3e^{3/2}}-\frac{8b^2d^{3/2}n^2\text{ArcTan}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\log\left(2-\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{e^{3/2}}+\frac{32b^2dn^2}{3e\sqrt[3]{x}}-\frac{8b^2n^2}{9x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x^2,x]
```

```
[Out] (-8*b^2*n^2)/(9*x) + (32*b^2*d*n^2)/(3*e*x^(1/3)) + (32*b^2*d^(3/2)*n^2*Arc
Tan[(Sqrt[d]*x^(1/3))/Sqrt[e]])/(3*e^(3/2)) + ((4*I)*b^2*d^(3/2)*n^2*ArcTan
[(Sqrt[d]*x^(1/3))/Sqrt[e]]^2)/e^(3/2) - (8*b^2*d^(3/2)*n^2*ArcTan[(Sqrt[d]
*x^(1/3))/Sqrt[e]]*Log[2 - (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/e^(3
/2) + (4*b*n*(a + b*Log[c*(d + e/x^(2/3))^n]))/(3*x) - (4*b*d*n*(a + b*Log[
c*(d + e/x^(2/3))^n]))/(e*x^(1/3)) - (4*b*d^(3/2)*n*ArcTan[(Sqrt[d]*x^(1/3)
)/Sqrt[e]]*(a + b*Log[c*(d + e/x^(2/3))^n]))/e^(3/2) - (a + b*Log[c*(d + e/
x^(2/3))^n])^2/x + ((4*I)*b^2*d^(3/2)*n^2*PolyLog[2, -1 + (2*Sqrt[e])/(Sqrt
[e] - I*Sqrt[d]*x^(1/3))])/e^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2508

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1)
```

```
- 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
b, c, d, e, m, p, q}, x] && FractionQ[n]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^2}\,dx = 3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{x^4}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} - (4ben)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)}{\left(d+\frac{e}{x^2}\right)x^6}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} - (4ben)\text{Subst}\left(\int \left(\frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right.}{ex^4}\right.\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} - (4bn)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)}{x^4}\right.$$

$$= \frac{4bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x} - \frac{4bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e\sqrt[3]{x}} - \frac{4bd^{3}}{}$$

$$= \frac{4bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x} - \frac{4bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e\sqrt[3]{x}} - \frac{4bd^{3}}{}$$

$$= -\frac{8b^2n^2}{9x} + \frac{8b^2dn^2}{e\sqrt[3]{x}} + \frac{4bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x} - \frac{4bdn\left(a+b\log\left(c\right.\right.}{e\sqrt[3]{x}}$$

$$= -\frac{8b^2n^2}{9x} + \frac{32b^2dn^2}{3e\sqrt[3]{x}} + \frac{8b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{e^{3/2}} + \frac{4ib^2d^{3/2}n^2\tan^{-1}\left(\right.}{e^{3/2}}$$

$$= -\frac{8b^2n^2}{9x} + \frac{32b^2dn^2}{3e\sqrt[3]{x}} + \frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3e^{3/2}} + \frac{4ib^2d^{3/2}n^2\tan^{-1}}{e^{3/2}}$$

$$= -\frac{8b^2n^2}{9x} + \frac{32b^2dn^2}{3e\sqrt[3]{x}} + \frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3e^{3/2}} + \frac{4ib^2d^{3/2}n^2\tan^{-1}}{e^{3/2}}$$

Mathematica [A]
time = 0.89, size = 598, normalized size = 1.66

$$\frac{-9\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2+\frac{bn\left(72bd\sqrt{e}\,nx^{2/3}-72bd^{3/2}nx\tan^{-1}\left(\frac{\sqrt{e}}{\sqrt{d}\sqrt[3]{x}}\right)-8bn\left(\sqrt{e}\left(e-3dx^{2/3}\right)+3d^{3/2}x\tan^{-1}\left(\frac{\sqrt{e}}{\sqrt{d}\sqrt[3]{x}}\right)\right)+12e^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-36d\sqrt{e}\,x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+18(-d)^{3/2}x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)+18\sqrt{-d}\,dx\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(\sqrt{e}+\sqrt{-d}\sqrt[3]{x}\right)+9b\sqrt{-d}\,dnx\left(\log\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)\left(\log\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)+2\log\left(\frac{1}{2}\left(1+\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)\right)-4\log\left(\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)\right)-4\operatorname{Li}_2\left(1-\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)+2\operatorname{Li}_2\left(\frac{1}{2}-\frac{\sqrt{-d}\sqrt[3]{x}}{2\sqrt{e}}\right)\right)+9b(-d)^{3/2}nx\left(\log\left(\sqrt{e}+\sqrt{-d}\sqrt[3]{x}\right)\left(\log\left(\sqrt{e}+\sqrt{-d}\sqrt[3]{x}\right)+2\log\left(\frac{1}{2}-\frac{\sqrt{-d}\sqrt[3]{x}}{2\sqrt{e}}\right)-4\log\left(-\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)\right)+2\operatorname{Li}_2\left(\frac{1}{2}\left(1+\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)\right)-4\operatorname{Li}_2\left(1+\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)\right)\right)}{e^{3/2}}}{9x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x^2,x]
```

```
[Out] (-9*(a + b*Log[c*(d + e/x^(2/3))^n])^2 + (b*n*(72*b*d*Sqrt[e]*n*x^(2/3) - 7
2*b*d^(3/2)*n*x*ArcTan[Sqrt[e]/(Sqrt[d]*x^(1/3))] - 8*b*n*(Sqrt[e]*(e - 3*d
*x^(2/3)) + 3*d^(3/2)*x*ArcTan[Sqrt[e]/(Sqrt[d]*x^(1/3))]) + 12*e^(3/2)*(a
+ b*Log[c*(d + e/x^(2/3))^n]) - 36*d*Sqrt[e]*x^(2/3)*(a + b*Log[c*(d + e/x^
(2/3))^n]) + 18*(-d)^(3/2)*x*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] -
 Sqrt[-d]*x^(1/3)] + 18*Sqrt[-d]*d*x*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[S
qrt[e] + Sqrt[-d]*x^(1/3)] + 9*b*Sqrt[-d]*d*n*x*(Log[Sqrt[e] - Sqrt[-d]*x^(
1/3)]*(Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 2*Log[(1 + (Sqrt[-d]*x^(1/3))/Sqrt
[e])/2] - 4*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]]) - 4*PolyLog[2, 1 - (Sqrt[-d]*x
^(1/3))/Sqrt[e]] + 2*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])]) + 9*
b*(-d)^(3/2)*n*x*(Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*(Log[Sqrt[e] + Sqrt[-d]*x
^(1/3)] + 2*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] - 4*Log[-((Sqrt[-d]*x
^(1/3))/Sqrt[e])]) + 2*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 4*P
olyLog[2, 1 + (Sqrt[-d]*x^(1/3))/Sqrt[e]])))/e^(3/2))/(9*x)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))^2/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))^2/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^2,x, algorithm="maxima")
```

```
[Out] -b^2*n^2*log(d*x^(2/3) + e)^2/x - integrate(-1/3*(2*(2*b^2*d*n*x + 3*(b^2*l
og(c) + a*b)*x^(1/3)*e + 3*(b^2*d*log(c) + a*b*d)*x - 6*(b^2*d*x + b^2*x^(1
/3)*e)*log(x^(1/3*n)))*n*log(d*x^(2/3) + e) + 12*(b^2*d*x + b^2*x^(1/3)*e)*
```

```
log(x^(1/3*n))^2 + 3*(b^2*log(c)^2 + 2*a*b*log(c) + a^2)*x^(1/3)*e + 3*(b^2
*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d)*x - 12*((b^2*log(c) + a*b)*x^(1/3)*e
+ (b^2*d*log(c) + a*b*d)*x)*log(x^(1/3*n)))/(d*x^3 + x^(7/3)*e), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^2,x, algorithm="fricas")
[Out] integral((b^2*log(c*((d*x + x^(1/3)*e)/x)^n)^2 + 2*a*b*log(c*((d*x + x^(1/3
)*e)/x)^n) + a^2)/x^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**2/x**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^2,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^n\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))^2/x^2,x)
[Out] int((a + b*log(c*(d + e/x^(2/3))^n))^2/x^2, x)
```

3.524 $\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=773

$$\frac{71b^3e^5n^3x^{2/3}}{80d^5}-\frac{3b^3e^4n^3x^{4/3}}{20d^4}+\frac{b^3e^3n^3x^2}{40d^3}-\frac{71b^3e^6n^3\log\left(d+\frac{e}{x^{2/3}}\right)}{80d^6}-\frac{77b^2e^5n^2\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right.\right.\right.}{40d^6}$$

```
[Out] 71/80*b^3*e^5*n^3*x^(2/3)/d^5-3/20*b^3*e^4*n^3*x^(4/3)/d^4+1/40*b^3*e^3*n^3
*x^2/d^3-71/80*b^3*e^6*n^3*ln(d+e/x^(2/3))/d^6-77/40*b^2*e^5*n^2*(d+e/x^(2/
3))*x^(2/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d^6+47/80*b^2*e^4*n^2*x^(4/3)*(a+b*
ln(c*(d+e/x^(2/3))^n))/d^4-9/40*b^2*e^3*n^2*x^2*(a+b*ln(c*(d+e/x^(2/3))^n))
/d^3+3/40*b^2*e^2*n^2*x^(8/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d^2-77/40*b^2*e^6
*n^2*ln(1-d/(d+e/x^(2/3)))*(a+b*ln(c*(d+e/x^(2/3))^n))/d^6+3/4*b*e^5*n*(d+e
/x^(2/3))*x^(2/3)*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d^6-3/8*b*e^4*n*x^(4/3)*(a+
b*ln(c*(d+e/x^(2/3))^n))^2/d^4+1/4*b*e^3*n*x^2*(a+b*ln(c*(d+e/x^(2/3))^n))^
2/d^3-3/16*b*e^2*n*x^(8/3)*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d^2+3/20*b*e*n*x^(
10/3)*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d+3/4*b*e^6*n*ln(1-d/(d+e/x^(2/3)))*(a+
b*ln(c*(d+e/x^(2/3))^n))^2/d^6+1/4*x^4*(a+b*ln(c*(d+e/x^(2/3))^n))^3-3/2*b^
2*e^6*n^2*(a+b*ln(c*(d+e/x^(2/3))^n))*ln(-e/d/x^(2/3))/d^6-15/8*b^3*e^6*n^3
*ln(x)/d^6+77/40*b^3*e^6*n^3*polylog(2,d/(d+e/x^(2/3)))/d^6-3/2*b^2*e^6*n^2
*(a+b*ln(c*(d+e/x^(2/3))^n))*polylog(2,d/(d+e/x^(2/3)))/d^6-3/2*b^3*e^6*n^3
*polylog(2,1+e/d/x^(2/3))/d^6-3/2*b^3*e^6*n^3*polylog(3,d/(d+e/x^(2/3)))/d^
6
```

Rubi [A]
time = 1.78, antiderivative size = 773, normalized size of antiderivative = 1.00, number of steps used = 62, number of rules used = 14, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {2504, 2445, 2458, 2389, 2379, 2421, 6724, 2355, 2354, 2438, 2356, 2351, 31, 46}

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

```
[Out] (71*b^3*e^5*n^3*x^(2/3))/(80*d^5) - (3*b^3*e^4*n^3*x^(4/3))/(20*d^4) + (b^3
*e^3*n^3*x^2)/(40*d^3) - (71*b^3*e^6*n^3*Log[d + e/x^(2/3)])/(80*d^6) - (77
*b^2*e^5*n^2*(d + e/x^(2/3))*x^(2/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(40*
d^6) + (47*b^2*e^4*n^2*x^(4/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(80*d^4) -
 (9*b^2*e^3*n^2*x^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(40*d^3) + (3*b^2*e^2
*n^2*x^(8/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(40*d^2) - (77*b^2*e^6*n^2*L
og[1 - d/(d + e/x^(2/3))]*(a + b*Log[c*(d + e/x^(2/3))^n]))/(40*d^6) + (3*b
*e^5*n*(d + e/x^(2/3))*x^(2/3)*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(4*d^6)
```

```
- (3*b*e^4*n*x^(4/3)*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(8*d^4) + (b*e^3*n
*x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(4*d^3) - (3*b*e^2*n*x^(8/3)*(a +
b*Log[c*(d + e/x^(2/3))^n])^2)/(16*d^2) + (3*b*e*n*x^(10/3)*(a + b*Log[c*(d
 + e/x^(2/3))^n])^2)/(20*d) + (3*b*e^6*n*Log[1 - d/(d + e/x^(2/3))]*(a + b*
Log[c*(d + e/x^(2/3))^n])^2)/(4*d^6) + (x^4*(a + b*Log[c*(d + e/x^(2/3))^n]
)^3)/4 - (3*b^2*e^6*n^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[-(e/(d*x^(2/3)
))])/(2*d^6) - (15*b^3*e^6*n^3*Log[x])/(8*d^6) + (77*b^3*e^6*n^3*PolyLog[2,
 d/(d + e/x^(2/3))])/(40*d^6) - (3*b^2*e^6*n^2*(a + b*Log[c*(d + e/x^(2/3))
^n])*PolyLog[2, d/(d + e/x^(2/3))])/(2*d^6) - (3*b^3*e^6*n^3*PolyLog[2, 1 +
 e/(d*x^(2/3))])/(2*d^6) - (3*b^3*e^6*n^3*PolyLog[3, d/(d + e/x^(2/3))])/(2
*d^6)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
```

```
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx = -\left(\frac{3}{2}\text{Subst}\left(\int\frac{(a+b\log(c(d+ex)^n))^3}{x^7}\,dx,x,\frac{1}{x^{2/3}}\right)\right)$$

$$= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 - \frac{1}{4}(3ben)\text{Subst}\left(\int\frac{(a+b\log(c(}{x^6(d+}\right.$$

$$= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 - \frac{1}{4}(3bn)\text{Subst}\left(\int\frac{(a+b\log(cx^}{x\left(-\frac{d}{e}+\frac{x}{e}\right)}\right.$$

$$= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 - \frac{(3bn)\text{Subst}\left(\int\frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx,\right.}{4d}$$

$$= \frac{3benx^{10/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{20d} + \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right.\right.$$

$$= -\frac{3be^2nx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{16d^2} + \frac{3benx^{10/3}\left(a+b\log\left(c\left(d+\right.\right.\right.}{20d}$$

$$= \frac{3b^2e^2n^2x^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{40d^2} + \frac{be^3nx^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/}}\right.\right.\right.}{4d^3}$$

$$= -\frac{9b^2e^3n^2x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{40d^3} + \frac{3b^2e^2n^2x^{8/3}\left(a+b\log\left(c\left(d\right.\right.\right.}{40d^2}$$

$$= \frac{3b^3e^5n^3x^{2/3}}{40d^5} - \frac{3b^3e^4n^3x^{4/3}}{80d^4} + \frac{b^3e^3n^3x^2}{40d^3} - \frac{3b^3e^6n^3\log\left(d+\frac{e}{x^{2/3}}\right)}{40d^6} +$$

$$= \frac{3b^3e^5n^3x^{2/3}}{10d^5} - \frac{3b^3e^4n^3x^{4/3}}{20d^4} + \frac{b^3e^3n^3x^2}{40d^3} - \frac{3b^3e^6n^3\log\left(d+\frac{e}{x^{2/3}}\right)}{10d^6} -$$

$$= \frac{71b^3e^5n^3x^{2/3}}{80d^5} - \frac{3b^3e^4n^3x^{4/3}}{20d^4} + \frac{b^3e^3n^3x^2}{40d^3} - \frac{71b^3e^6n^3\log\left(d+\frac{e}{x^{2/3}}\right)}{80d^6} -$$

$$= \frac{71b^3e^5n^3x^{2/3}}{80d^5} - \frac{3b^3e^4n^3x^{4/3}}{20d^4} + \frac{b^3e^3n^3x^2}{40d^3} - \frac{71b^3e^6n^3\log\left(d+\frac{e}{x^{2/3}}\right)}{80d^6} -$$

Mathematica [A]
time = 1.44, size = 1014, normalized size = 1.31

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

```
[Out] (60*b*d*e^5*n*x^(2/3)*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))
^n])^2 - 30*b*d^2*e^4*n*x^(4/3)*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d +
e/x^(2/3))^n])^2 + 20*b*d^3*e^3*n*x^2*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c
*(d + e/x^(2/3))^n])^2 - 15*b*d^4*e^2*n*x^(8/3)*(a - b*n*Log[d + e/x^(2/3)]
 + b*Log[c*(d + e/x^(2/3))^n])^2 + 12*b*d^5*e*n*x^(10/3)*(a - b*n*Log[d + e
/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2 + 60*b*d^6*n*x^4*Log[d + e/x^(2/3
)]*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2 + 20*d^6*x^4
*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^3 - 60*b*e^6*n*(
a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2*Log[e + d*x^(2/3
)] + b^2*n^2*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])*(d*e
^2*x^(2/3)*(-154*e^3 + 47*d*e^2*x^(2/3) - 18*d^2*e*x^(4/3) + 6*d^3*x^2) - 6
0*(e^6 - d^6*x^4)*Log[d + e/x^(2/3)]^2 - 274*e^6*Log[-(e/(d*x^(2/3)))] + 2*
e*Log[d + e/x^(2/3)]*(137*e^5 + 60*d*e^4*x^(2/3) - 30*d^2*e^3*x^(4/3) + 20*
d^3*e^2*x^2 - 15*d^4*e*x^(8/3) + 12*d^5*x^(10/3) + 60*e^5*Log[-(e/(d*x^(2/3
)))]) + 120*e^6*PolyLog[2, 1 + e/(d*x^(2/3))]) + b^3*n^3*(3*d^4*e^2*x^(8/3)
*(2 - 5*Log[d + e/x^(2/3)])*Log[d + e/x^(2/3)] + 12*d^5*e*x^(10/3)*Log[d +
e/x^(2/3)]^2 + 20*d^6*x^4*Log[d + e/x^(2/3)]^3 + 2*d^3*e^3*x^2*(1 - 9*Log[d
 + e/x^(2/3)] + 10*Log[d + e/x^(2/3)]^2) - d^2*e^4*x^(4/3)*(12 - 47*Log[d +
 e/x^(2/3)] + 30*Log[d + e/x^(2/3)]^2) + d*e^5*x^(2/3)*(71 - 154*Log[d + e/
x^(2/3)] + 60*Log[d + e/x^(2/3)]^2) + 225*e^6*(-Log[d + e/x^(2/3)] + Log[-(
e/(d*x^(2/3)))]) + 137*e^6*(Log[d + e/x^(2/3)]*(Log[d + e/x^(2/3)] - 2*Log[
-(e/(d*x^(2/3)))]) - 2*PolyLog[2, 1 + e/(d*x^(2/3))]) - 20*e^6*(Log[d + e/x
^(2/3)]^2*(Log[d + e/x^(2/3)] - 3*Log[-(e/(d*x^(2/3)))]) - 6*Log[d + e/x^(2
/3)]*PolyLog[2, 1 + e/(d*x^(2/3))] + 6*PolyLog[3, 1 + e/(d*x^(2/3))])))/(80
*d^6)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int x^{3} \left(a + b\ln \left(c\left(d + \frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e/x^(2/3))^n))^3,x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e/x^(2/3))^n))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*n^3*x^4*log(d*x^(2/3) + e)^3 - integrate(-1/2*(2*(b^3*log(c)^3 + 3*
a*b^2*log(c)^2 + 3*a^2*b*log(c) + a^3)*x^(10/3)*e + 2*(b^3*d*log(c)^3 + 3*a
*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x^4 - (b^3*d*n*x^4 - 6*(b^3*log
(c) + a*b^2)*x^(10/3)*e - 6*(b^3*d*log(c) + a*b^2*d)*x^4 + 12*(b^3*d*x^4 +
b^3*x^(10/3)*e)*log(x^(1/3*n)))*n^2*log(d*x^(2/3) + e)^2 - 16*(b^3*d*x^4 +
b^3*x^(10/3)*e)*log(x^(1/3*n))^3 + 6*((b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*
b)*x^(10/3)*e + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x^4 + 4*(b^3*
d*x^4 + b^3*x^(10/3)*e)*log(x^(1/3*n))^2 - 4*((b^3*log(c) + a*b^2)*x^(10/3)
*e + (b^3*d*log(c) + a*b^2*d)*x^4)*log(x^(1/3*n)))*n*log(d*x^(2/3) + e) + 2
4*((b^3*log(c) + a*b^2)*x^(10/3)*e + (b^3*d*log(c) + a*b^2*d)*x^4)*log(x^(1
/3*n))^2 - 12*((b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*b)*x^(10/3)*e + (b^3*d*
log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x^4)*log(x^(1/3*n)))/(d*x + x^(1/3)*
e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*log(c*((d*x + x^(1/3)*e)/x)^n)^3 + 3*a*b^2*x^3*log(c*((d*x
 + x^(1/3)*e)/x)^n)^2 + 3*a^2*b*x^3*log(c*((d*x + x^(1/3)*e)/x)^n) + a^3*x^
3, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e/x**(2/3))**n))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7318 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \left(a + b \ln \left(c \left(d + \frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e/x^(2/3))^n))^3,x)
[Out] int(x^3*(a + b*log(c*(d + e/x^(2/3))^n))^3, x)
```

3.525 $\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=451

$$\frac{3b^2e^2n^2\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^3}+\frac{3b^2e^3n^2\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^3}-\frac{3be^2n}{}$$

```
[Out] 3/2*b^2*e^2*n^2*(d+e/x^(2/3))*x^(2/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d^3+3/2*b
^2*e^3*n^2*ln(1-d/(d+e/x^(2/3)))*(a+b*ln(c*(d+e/x^(2/3))^n))/d^3-3/2*b*e^2*
n*(d+e/x^(2/3))*x^(2/3)*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d^3+3/4*b*e*n*x^(4/3)
*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d-3/2*b*e^3*n*ln(1-d/(d+e/x^(2/3)))*(a+b*ln(
c*(d+e/x^(2/3))^n))^2/d^3+1/2*x^2*(a+b*ln(c*(d+e/x^(2/3))^n))^3+3*b^2*e^3*n
^2*(a+b*ln(c*(d+e/x^(2/3))^n))*ln(-e/d/x^(2/3))/d^3+b^3*e^3*n^3*ln(x)/d^3-3
/2*b^3*e^3*n^3*polylog(2,d/(d+e/x^(2/3)))/d^3+3*b^2*e^3*n^2*(a+b*ln(c*(d+e/
x^(2/3))^n))*polylog(2,d/(d+e/x^(2/3)))/d^3+3*b^3*e^3*n^3*polylog(2,1+e/d/x
^(2/3))/d^3+3*b^3*e^3*n^3*polylog(3,d/(d+e/x^(2/3)))/d^3
```

Rubi [A]
time = 0.57, antiderivative size = 451, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {2504, 2445, 2458, 2389, 2379, 2421, 6724, 2355, 2354, 2438, 2356, 2351, 31}

$$\frac{3b^2e^3n^2\text{PolyLog}\left(2,\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3}-\frac{3b^3e^3n^3\text{PolyLog}\left(2,\frac{d}{d+\frac{e}{x^{2/3}}}\right)}{2d^3}+\frac{3b^3e^3n^3\text{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)}{d^3}+\frac{3b^3e^3n^3\text{PolyLog}\left(3,\frac{d}{d+\frac{e}{x^{2/3}}}\right)}{d^3}+\frac{3b^2e^3n^2\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^3}+\frac{3b^2e^3n^2\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3}+\frac{3b^2e^2n^2x^{2/3}\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^3}-\frac{3be^3n\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2d^3}-\frac{3be^2nx^{2/3}\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2d^3}+\frac{3benx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4d}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3+\frac{b^3e^3n^3\log(x)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

```
[Out] (3*b^2*e^2*n^2*(d + e/x^(2/3))*x^(2/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(2
*d^3) + (3*b^2*e^3*n^2*Log[1 - d/(d + e/x^(2/3))]*(a + b*Log[c*(d + e/x^(2/
3))^n]))/(2*d^3) - (3*b*e^2*n*(d + e/x^(2/3))*x^(2/3)*(a + b*Log[c*(d + e/x
^(2/3))^n])^2)/(2*d^3) + (3*b*e*n*x^(4/3)*(a + b*Log[c*(d + e/x^(2/3))^n])^
2)/(4*d) - (3*b*e^3*n*Log[1 - d/(d + e/x^(2/3))]*(a + b*Log[c*(d + e/x^(2/3
))^n])^2)/(2*d^3) + (x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^3)/2 + (3*b^2*e^3
*n^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[-(e/(d*x^(2/3)))])/d^3 + (b^3*e^3
*n^3*Log[x])/d^3 - (3*b^3*e^3*n^3*PolyLog[2, d/(d + e/x^(2/3))])/(2*d^3) +
(3*b^2*e^3*n^2*(a + b*Log[c*(d + e/x^(2/3))^n])*PolyLog[2, d/(d + e/x^(2/3)
)])/d^3 + (3*b^3*e^3*n^3*PolyLog[2, 1 + e/(d*x^(2/3))])/d^3 + (3*b^3*e^3*n^
3*PolyLog[3, d/(d + e/x^(2/3))])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx = -\left(\frac{3}{2}\text{Subst}\left(\int\frac{(a+b\log(c(d+ex)^n))^3}{x^4}\,dx, x, \frac{1}{x^{2/3}}\right)\right)$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 - \frac{1}{2}(3ben)\text{Subst}\left(\int\frac{(a+b\log(c\ldots}{x^3(d\ldots}\right.$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 - \frac{1}{2}(3bn)\text{Subst}\left(\int\frac{(a+b\log(cx\ldots}{x\left(-\frac{d}{e}+\frac{x}{e}\ldots\right.}\right.$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 - \frac{(3bn)\text{Subst}\left(\int\frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx,\ldots\right.}{2d}$$

$$= \frac{3benx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4d} + \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\ldots\right.\right.$$

$$= -\frac{3be^2n\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2d^3} + \frac{3benx^{4/3}(a+b\ldots}{\ldots}$$

$$= \frac{3b^2e^2n^2\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^3} - \frac{3be^2n\left(d+\frac{e}{x^{2/3}}\right)\ldots}{\ldots}$$

$$= \frac{3b^2e^2n^2\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^3} - \frac{3be^3n(a+b\log\ldots}{4\ldots}$$

$$= \frac{3b^2e^2n^2\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^3} - \frac{3be^3n(a+b\log\ldots}{4\ldots}$$

Mathematica [A]
time = 0.70, size = 683, normalized size = 1.51

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

```
[Out] (-6*b*d*e^2*n*x^(2/3)*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))
^n])^2 + 3*b*d^2*e*n*x^(4/3)*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x
^(2/3))^n])^2 + 6*b*d^3*n*x^2*Log[d + e/x^(2/3)]*(a - b*n*Log[d + e/x^(2/3)
] + b*Log[c*(d + e/x^(2/3))^n])^2 + 2*d^3*x^2*(a - b*n*Log[d + e/x^(2/3)] +
 b*Log[c*(d + e/x^(2/3))^n])^3 + 6*b*e^3*n*(a - b*n*Log[d + e/x^(2/3)] + b*
Log[c*(d + e/x^(2/3))^n])^2*Log[e + d*x^(2/3)] + 6*b^2*n^2*(a - b*n*Log[d +
```

```
e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])*((e^3 + d^3*x^2)*Log[d + e/x^(2/3
)]^2 + e^2*(d*x^(2/3) + 3*e*Log[-(e/(d*x^(2/3)))]) + e*Log[d + e/x^(2/3)]*(
-3*e^2 - 2*d*e*x^(2/3) + d^2*x^(4/3) - 2*e^2*Log[-(e/(d*x^(2/3)))]) - 2*e^3
*PolyLog[2, 1 + e/(d*x^(2/3))]) - b^3*n^3*(-6*e^3*Log[d + e/x^(2/3)] - 6*d*
e^2*x^(2/3)*Log[d + e/x^(2/3)] + 9*e^3*Log[d + e/x^(2/3)]^2 + 6*d*e^2*x^(2/
3)*Log[d + e/x^(2/3)]^2 - 3*d^2*e*x^(4/3)*Log[d + e/x^(2/3)]^2 - 2*e^3*Log[
d + e/x^(2/3)]^3 - 2*d^3*x^2*Log[d + e/x^(2/3)]^3 + 6*e^3*Log[-(e/(d*x^(2/3
)))] - 18*e^3*Log[d + e/x^(2/3)]*Log[-(e/(d*x^(2/3)))] + 6*e^3*Log[d + e/x^
(2/3)]^2*Log[-(e/(d*x^(2/3)))] + 6*e^3*(-3 + 2*Log[d + e/x^(2/3)])*PolyLog[
2, 1 + e/(d*x^(2/3))] - 12*e^3*PolyLog[3, 1 + e/(d*x^(2/3))]))/(4*d^3)
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(2/3))^n))^3,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e/x^(2/3))^n))^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*n^3*x^2*log(d*x^(2/3) + e)^3 - integrate(((b^3*d*n*x^2 - 3*(b^3*log
(c) + a*b^2)*x^(4/3)*e - 3*(b^3*d*log(c) + a*b^2*d)*x^2 + 6*(b^3*d*x^2 + b^
3*x^(4/3)*e)*log(x^(1/3*n)))*n^2*log(d*x^(2/3) + e)^2 + 8*(b^3*d*x^2 + b^3*
x^(4/3)*e)*log(x^(1/3*n))^3 - (b^3*log(c)^3 + 3*a*b^2*log(c)^2 + 3*a^2*b*lo
g(c) + a^3)*x^(4/3)*e - (b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*lo
g(c) + a^3*d)*x^2 - 3*((b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*b)*x^(4/3)*e +
(b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x^2 + 4*(b^3*d*x^2 + b^3*x^(4
/3)*e)*log(x^(1/3*n))^2 - 4*((b^3*log(c) + a*b^2)*x^(4/3)*e + (b^3*d*log(c)
 + a*b^2*d)*x^2)*log(x^(1/3*n)))*n*log(d*x^(2/3) + e) - 12*((b^3*log(c) + a
*b^2)*x^(4/3)*e + (b^3*d*log(c) + a*b^2*d)*x^2)*log(x^(1/3*n))^2 + 6*((b^3*
log(c)^2 + 2*a*b^2*log(c) + a^2*b)*x^(4/3)*e + (b^3*d*log(c)^2 + 2*a*b^2*d*
log(c) + a^2*b*d)*x^2)*log(x^(1/3*n)))/(d*x + x^(1/3)*e), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*log(c*((d*x + x^(1/3)*e)/x)^n)^3 + 3*a*b^2*x*log(c*((d*x + x
^(1/3)*e)/x)^n)^2 + 3*a^2*b*x*log(c*((d*x + x^(1/3)*e)/x)^n) + a^3*x, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(2/3))**n))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3*x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e/x^(2/3))^n))^3,x)
```

```
[Out] int(x*(a + b*log(c*(d + e/x^(2/3))^n))^3, x)
```

3.526 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x}\,dx$

Optimal. Leaf size=139

$$-\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\log\left(-\frac{e}{dx^{2/3}}\right)-\frac{9}{2}bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\text{Li}_2\left(1+\frac{e}{dx^{2/3}}\right)+9b^2n^2\left(a\right.$$

```
[Out] -3/2*(a+b*ln(c*(d+e/x^(2/3))^n))^3*ln(-e/d/x^(2/3))-9/2*b*n*(a+b*ln(c*(d+e/
x^(2/3))^n))^2*polylog(2,1+e/d/x^(2/3))+9*b^2*n^2*(a+b*ln(c*(d+e/x^(2/3))^n
))*polylog(3,1+e/d/x^(2/3))-9*b^3*n^3*polylog(4,1+e/d/x^(2/3))
```

Rubi [A]
time = 0.13, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2504, 2443, 2481, 2421, 2430, 6724}

$$9b^2n^2\text{PolyLog}\left(3,\frac{e}{dx^{2/3}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{9}{2}bn\text{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-9b^3n^3\text{PolyLog}\left(4,\frac{e}{dx^{2/3}}+1\right)-\frac{3}{2}\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x,x]
```

```
[Out] (-3*(a + b*Log[c*(d + e/x^(2/3))^n])^3*Log[-(e/(d*x^(2/3)))])/2 - (9*b*n*(a
 + b*Log[c*(d + e/x^(2/3))^n])^2*PolyLog[2, 1 + e/(d*x^(2/3))])/2 + 9*b^2*n
^2*(a + b*Log[c*(d + e/x^(2/3))^n])*PolyLog[3, 1 + e/(d*x^(2/3))] - 9*b^3*n
^3*PolyLog[4, 1 + e/(d*x^(2/3))]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
```

```
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x}\,dx = -\left(\frac{3}{2}\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x}\,dx, x, \frac{1}{x^{2/3}}\right)\right)$$

$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\log\left(-\frac{e}{dx^{2/3}}\right)+\frac{1}{2}(9ben)\text{Subst}\left(\int \frac{\log}{}\right.$$

$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\log\left(-\frac{e}{dx^{2/3}}\right)+\frac{1}{2}(9bn)\text{Subst}\left(\int \frac{(a+}{}\right.$$

$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\log\left(-\frac{e}{dx^{2/3}}\right)-\frac{9}{2}bn\left(a+b\log\left(c\left(d+\right.\right.\right.$$

$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\log\left(-\frac{e}{dx^{2/3}}\right)-\frac{9}{2}bn\left(a+b\log\left(c\left(d+\right.\right.\right.$$

$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\log\left(-\frac{e}{dx^{2/3}}\right)-\frac{9}{2}bn\left(a+b\log\left(c\left(d+\right.\right.\right.$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 341 vs. $2(139) = 278$.
time = 0.24, size = 341, normalized size = 2.45

$$\left(a-bn\log\left(d+\frac{e}{x^{2/3}}\right)+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\log(x)+3bn\left(a-bn\log\left(d+\frac{e}{x^{2/3}}\right)+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\left(\left(\log\left(d+\frac{e}{x^{2/3}}\right)-\log\left(1+\frac{e}{dx^{2/3}}\right)\right)\log(x)+\frac{3}{2}\text{Li}_2\left(-\frac{e}{dx^{2/3}}\right)\right)+\frac{9}{2}b^2n^2\left(-a+bn\log\left(d+\frac{e}{x^{2/3}}\right)-b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\left(\log^2\left(d+\frac{e}{x^{2/3}}\right)\log\left(-\frac{e}{dx^{2/3}}\right)+2\log\left(d+\frac{e}{x^{2/3}}\right)\text{Li}_2\left(1+\frac{e}{dx^{2/3}}\right)-2\text{Li}_3\left(1+\frac{e}{dx^{2/3}}\right)\right)-\frac{3}{2}b^3n^3\left(\log^3\left(d+\frac{e}{x^{2/3}}\right)\log\left(-\frac{e}{dx^{2/3}}\right)+3\log^2\left(d+\frac{e}{x^{2/3}}\right)\text{Li}_2\left(1+\frac{e}{dx^{2/3}}\right)-6\log\left(d+\frac{e}{x^{2/3}}\right)\text{Li}_3\left(1+\frac{e}{dx^{2/3}}\right)+6\text{Li}_4\left(1+\frac{e}{dx^{2/3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x,x]
```

```
[Out] (a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^3*Log[x] + 3*b*n*
(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2*((Log[d + e/x^(
2/3)] - Log[1 + e/(d*x^(2/3))])*Log[x] + (3*PolyLog[2, -(e/(d*x^(2/3)))])/2
) + (9*b^2*n^2*(-a + b*n*Log[d + e/x^(2/3)] - b*Log[c*(d + e/x^(2/3))^n])*(
Log[d + e/x^(2/3)]^2*Log[-(e/(d*x^(2/3)))] + 2*Log[d + e/x^(2/3)]*PolyLog[2
, 1 + e/(d*x^(2/3))] - 2*PolyLog[3, 1 + e/(d*x^(2/3))]))/2 - (3*b^3*n^3*(Lo
g[d + e/x^(2/3)]^3*Log[-(e/(d*x^(2/3)))] + 3*Log[d + e/x^(2/3)]^2*PolyLog[2
, 1 + e/(d*x^(2/3))] - 6*Log[d + e/x^(2/3)]*PolyLog[3, 1 + e/(d*x^(2/3))] +
 6*PolyLog[4, 1 + e/(d*x^(2/3))]))/2
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))^3/x,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))^3/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x,x, algorithm="maxima")
```

```
[Out] b^3*n^3*log(d*x^(2/3) + e)^3*log(x) - integrate(((2*b^3*d*n*x*log(x) - 3*(b
^3*log(c) + a*b^2)*x^(1/3)*e - 3*(b^3*d*log(c) + a*b^2*d)*x + 6*(b^3*d*x +
b^3*x^(1/3)*e)*log(x^(1/3*n)))*n^2*log(d*x^(2/3) + e)^2 + 8*(b^3*d*x + b^3*
x^(1/3)*e)*log(x^(1/3*n))^3 - 3*(4*(b^3*d*x + b^3*x^(1/3)*e)*log(x^(1/3*n))
^2 + (b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*b)*x^(1/3)*e + (b^3*d*log(c)^2 +
2*a*b^2*d*log(c) + a^2*b*d)*x - 4*((b^3*log(c) + a*b^2)*x^(1/3)*e + (b^3*d*
log(c) + a*b^2*d)*x)*log(x^(1/3*n)))*n*log(d*x^(2/3) + e) - 12*((b^3*log(c)
 + a*b^2)*x^(1/3)*e + (b^3*d*log(c) + a*b^2*d)*x)*log(x^(1/3*n))^2 - (b^3*l
og(c)^3 + 3*a*b^2*log(c)^2 + 3*a^2*b*log(c) + a^3)*x^(1/3)*e - (b^3*d*log(c
)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x + 6*((b^3*log(c)^2 +
 2*a*b^2*log(c) + a^2*b)*x^(1/3)*e + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a
^2*b*d)*x)*log(x^(1/3*n)))/(d*x^2 + x^(4/3)*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*((d*x + x^(1/3)*e)/x)^n)^3 + 3*a*b^2*log(c*((d*x + x^(1
/3)*e)/x)^n)^2 + 3*a^2*b*log(c*((d*x + x^(1/3)*e)/x)^n) + a^3)/x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**3/x,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^n\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))^3/x,x)
[Out] int((a + b*log(c*(d + e/x^(2/3))^n))^3/x, x)
```

3.527 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^3}\,dx$

Optimal. Leaf size=449

$$-\frac{9b^3dn^3\left(d+\frac{e}{x^{2/3}}\right)^2}{8e^3}+\frac{b^3n^3\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^3}-\frac{9ab^2d^2n^2}{e^2x^{2/3}}+\frac{9b^3d^2n^3}{e^2x^{2/3}}-\frac{9b^3d^2n^2\left(d+\frac{e}{x^{2/3}}\right)\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{e^3}+\frac{9b^2dn^2(}{}$$

```
[Out] -9/8*b^3*d*n^3*(d+e/x^(2/3))^2/e^3+1/9*b^3*n^3*(d+e/x^(2/3))^3/e^3-9*a*b^2*
d^2*n^2/e^2/x^(2/3)+9*b^3*d^2*n^3/e^2/x^(2/3)-9*b^3*d^2*n^2*(d+e/x^(2/3))*l
n(c*(d+e/x^(2/3))^n)/e^3+9/4*b^2*d*n^2*(d+e/x^(2/3))^2*(a+b*ln(c*(d+e/x^(2/
3))^n))/e^3-1/3*b^2*n^2*(d+e/x^(2/3))^3*(a+b*ln(c*(d+e/x^(2/3))^n))/e^3+9/2
*b*d^2*n*(d+e/x^(2/3))*(a+b*ln(c*(d+e/x^(2/3))^n))^2/e^3-9/4*b*d*n*(d+e/x^(
2/3))^2*(a+b*ln(c*(d+e/x^(2/3))^n))^2/e^3+1/2*b*n*(d+e/x^(2/3))^3*(a+b*ln(c
*(d+e/x^(2/3))^n))^2/e^3-3/2*d^2*(d+e/x^(2/3))*(a+b*ln(c*(d+e/x^(2/3))^n))^
3/e^3+3/2*d*(d+e/x^(2/3))^2*(a+b*ln(c*(d+e/x^(2/3))^n))^3/e^3-1/2*(d+e/x^(2
/3))^3*(a+b*ln(c*(d+e/x^(2/3))^n))^3/e^3
```

Rubi [A]

time = 0.30, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x^3,x]
```

```
[Out] (-9*b^3*d*n^3*(d + e/x^(2/3))^2)/(8*e^3) + (b^3*n^3*(d + e/x^(2/3))^3)/(9*e
^3) - (9*a*b^2*d^2*n^2)/(e^2*x^(2/3)) + (9*b^3*d^2*n^3)/(e^2*x^(2/3)) - (9*
b^3*d^2*n^2*(d + e/x^(2/3))*Log[c*(d + e/x^(2/3))^n])/e^3 + (9*b^2*d*n^2*(d
 + e/x^(2/3))^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(4*e^3) - (b^2*n^2*(d + e
/x^(2/3))^3*(a + b*Log[c*(d + e/x^(2/3))^n]))/(3*e^3) + (9*b*d^2*n*(d + e/x
^(2/3))*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(2*e^3) - (9*b*d*n*(d + e/x^(2/
3))^2*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(4*e^3) + (b*n*(d + e/x^(2/3))^3*
(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(2*e^3) - (3*d^2*(d + e/x^(2/3))*(a + b
*Log[c*(d + e/x^(2/3))^n])^3)/(2*e^3) + (3*d*(d + e/x^(2/3))^2*(a + b*Log[c
*(d + e/x^(2/3))^n])^3)/(2*e^3) - ((d + e/x^(2/3))^3*(a + b*Log[c*(d + e/x^
(2/3))^n])^3)/(2*e^3)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^3}\,dx = -\left(\frac{3}{2}\text{Subst}\left(\int x^2(a+b\log(c(d+ex)^n))^3\,dx, x, \frac{1}{x^{2/3}}\right)\right)$$

$$= -\left(\frac{3}{2}\text{Subst}\left(\int\left(\frac{d^2(a+b\log(c(d+ex)^n))^3}{e^2} - \frac{2d(d+ex)\,(a+b\log(c\cdots}{e^2}\cdots\right.\right.\right.$$

$$= -\frac{3\text{Subst}\left(\int(d+ex)^2\,(a+b\log(c(d+ex)^n))^3\,dx, x, \frac{1}{x^{2/3}}\right)}{2e^2} + \frac{(3d)\text{Subst}\left(\cdots\right.}{\cdots}$$

$$= -\frac{3\text{Subst}\left(\int x^2(a+b\log(cx^n))^3\,dx, x, d+\frac{e}{x^{2/3}}\right)}{2e^3} + \frac{(3d)\text{Subst}\left(\int x(a+b\cdots\right.}{\cdots}$$

$$= -\frac{3d^2\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{2e^3} + \frac{3d\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(\cdots\right.\right.}{2e^3}$$

$$= \frac{9bd^2n\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2e^3} - \frac{9bdn\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\cdots\right.}{4e^3}$$

$$= -\frac{9b^3dn^3\left(d+\frac{e}{x^{2/3}}\right)^2}{8e^3} + \frac{b^3n^3\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^3} - \frac{9ab^2d^2n^2}{e^2x^{2/3}} + \frac{9b^2dn^2\left(d+\frac{e}{x^{2/3}}\right)\cdots}{\cdots}$$

$$= -\frac{9b^3dn^3\left(d+\frac{e}{x^{2/3}}\right)^2}{8e^3} + \frac{b^3n^3\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^3} - \frac{9ab^2d^2n^2}{e^2x^{2/3}} + \frac{9b^3d^2n^3}{e^2x^{2/3}} - \frac{9b^3d^2\cdots}{\cdots}$$

Mathematica [**A**]
time = 0.74, size = 692, normalized size = 1.54

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x^3,x]
```

```
[Out] (-36*a^3*e^3 + 36*a^2*b*e^3*n - 24*a*b^2*e^3*n^2 + 8*b^3*e^3*n^3 - 54*a^2*b
*d*e^2*n*x^(2/3) + 90*a*b^2*d*e^2*n^2*x^(2/3) - 57*b^3*d*e^2*n^3*x^(2/3) +
108*a^2*b*d^2*e*n*x^(4/3) - 396*a*b^2*d^2*e*n^2*x^(4/3) + 510*b^3*d^2*e*n^3
*x^(4/3) + 72*b^3*d^3*n^3*x^2*Log[d + e/x^(2/3)]^3 - 36*b^3*e^3*Log[c*(d +
e/x^(2/3))^n]^3 - 108*a^2*b*d^3*n*x^2*Log[e + d*x^(2/3)] + 396*a*b^2*d^3*n^
2*x^2*Log[e + d*x^(2/3)] - 510*b^3*d^3*n^3*x^2*Log[e + d*x^(2/3)] + 12*b^2*
d^3*n^2*x^2*Log[d + e/x^(2/3)]*(6*a - 11*b*n + 6*b*Log[c*(d + e/x^(2/3))^n]
)*(3*Log[e + d*x^(2/3)] - 2*Log[x]) + 72*a^2*b*d^3*n*x^2*Log[x] - 264*a*b^2
*d^3*n^2*x^2*Log[x] + 340*b^3*d^3*n^3*x^2*Log[x] - 18*b^2*d^3*n^2*x^2*Log[d
 + e/x^(2/3)]^2*(6*a - 11*b*n + 6*b*Log[c*(d + e/x^(2/3))^n] + 6*b*n*Log[e
+ d*x^(2/3)] - 4*b*n*Log[x]) + 18*b^2*Log[c*(d + e/x^(2/3))^n]^2*(e*(-6*a*e
^2 + 2*b*e^2*n - 3*b*d*e*n*x^(2/3) + 6*b*d^2*n*x^(4/3)) - 6*b*d^3*n*x^2*Log
[e + d*x^(2/3)] + 4*b*d^3*n*x^2*Log[x]) - 6*b*Log[c*(d + e/x^(2/3))^n]*(18*
a^2*e^3 - 6*a*b*e*n*(2*e^2 - 3*d*e*x^(2/3) + 6*d^2*x^(4/3)) + b^2*e*n^2*(4*
```

```
e^2 - 15*d*e*x^(2/3) + 66*d^2*x^(4/3)) + 6*b*d^3*n*(6*a - 11*b*n)*x^2*Log[e
 + d*x^(2/3)] + 4*b*d^3*n*(-6*a + 11*b*n)*x^2*Log[x]))/(72*e^3*x^2)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{3}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))^3/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))^3/x^3,x)
```

Maxima [A]
time = 0.32, size = 689, normalized size = 1.53

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*(6*d^3*e^(-4)*log(d*x^(2/3) + e) - 6*d^3*e^(-4)*log(x^(2/3)) - (6*d^2*
x^(4/3) - 3*d*x^(2/3)*e + 2*e^2)*e^(-3)/x^2)*a^2*b*n*e - 1/12*(6*(6*d^3*e^(
-4)*log(d*x^(2/3) + e) - 6*d^3*e^(-4)*log(x^(2/3)) - (6*d^2*x^(4/3) - 3*d*x
^(2/3)*e + 2*e^2)*e^(-3)/x^2)*n*e*log(c*(d + e/x^(2/3))^n) - (18*d^3*x^2*lo
g(d*x^(2/3) + e)^2 + 8*d^3*x^2*log(x)^2 - 44*d^3*x^2*log(x) - 66*d^2*x^(4/3
)*e + 15*d*x^(2/3)*e^2 - 6*(4*d^3*x^2*log(x) - 11*d^3*x^2)*log(d*x^(2/3) +
e) - 4*e^3)*n^2*e^(-3)/x^2)*a*b^2 - 1/216*(54*(6*d^3*e^(-4)*log(d*x^(2/3) +
 e) - 6*d^3*e^(-4)*log(x^(2/3)) - (6*d^2*x^(4/3) - 3*d*x^(2/3)*e + 2*e^2)*e
^(-3)/x^2)*n*e*log(c*(d + e/x^(2/3))^n)^2 + ((108*d^3*x^2*log(d*x^(2/3) + e
)^3 - 32*d^3*x^2*log(x)^3 + 264*d^3*x^2*log(x)^2 - 1020*d^3*x^2*log(x) - 15
30*d^2*x^(4/3)*e - 54*(4*d^3*x^2*log(x) - 11*d^3*x^2)*log(d*x^(2/3) + e)^2
+ 171*d*x^(2/3)*e^2 + 18*(8*d^3*x^2*log(x)^2 - 44*d^3*x^2*log(x) + 85*d^3*x
^2)*log(d*x^(2/3) + e) - 24*e^3)*n^2*e^(-4)/x^2 - 18*(18*d^3*x^2*log(d*x^(2
/3) + e)^2 + 8*d^3*x^2*log(x)^2 - 44*d^3*x^2*log(x) - 66*d^2*x^(4/3)*e + 15
*d*x^(2/3)*e^2 - 6*(4*d^3*x^2*log(x) - 11*d^3*x^2)*log(d*x^(2/3) + e) - 4*e
^3)*n*e^(-4)*log(c*(d + e/x^(2/3))^n)/x^2)*n*e)*b^3 - 1/2*b^3*log(c*(d + e/
x^(2/3))^n)^3/x^2 - 3/2*a*b^2*log(c*(d + e/x^(2/3))^n)^2/x^2 - 3/2*a^2*b*lo
g(c*(d + e/x^(2/3))^n)/x^2 - 1/2*a^3/x^2
```

Fricas [A]
time = 0.40, size = 686, normalized size = 1.53

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^3,x, algorithm="fricas")

[Out] -1/72*(36*b^3*e^3*log(c)^3 - 36*(b^3*n - 3*a*b^2)*e^3*log(c)^2 + 36*(b^3*d^3*n^3*x^2 + b^3*n^3*e^3)*log((d*x + x^(1/3)*e)/x)^3 + 12*(2*b^3*n^2 - 6*a*b^2*n + 9*a^2*b)*e^3*log(c) - 18*(6*b^3*d^2*n^3*x^(4/3)*e - 3*b^3*d*n^3*x^(2/3)*e^2 + (11*b^3*d^3*n^3 - 6*a*b^2*d^3*n^2)*x^2 + 2*(b^3*n^3 - 3*a*b^2*n^2)*e^3 - 6*(b^3*d^3*n^2*x^2 + b^3*n^2*e^3)*log(c))*log((d*x + x^(1/3)*e)/x)^2 - 4*(2*b^3*n^3 - 6*a*b^2*n^2 + 9*a^2*b*n - 9*a^3)*e^3 + 6*((85*b^3*d^3*n^3 - 66*a*b^2*d^3*n^2 + 18*a^2*b*d^3*n)*x^2 + 18*(b^3*d^3*n*x^2 + b^3*n*e^3)*log(c)^2 + 2*(2*b^3*n^3 - 6*a*b^2*n^2 + 9*a^2*b*n)*e^3 - 6*((11*b^3*d^3*n^2 - 6*a*b^2*d^3*n)*x^2 + 2*(b^3*n^2 - 3*a*b^2*n)*e^3)*log(c) + 3*(6*b^3*d*n^2*e^2*log(c) - (5*b^3*d*n^3 - 6*a*b^2*d*n^2)*e^2)*x^(2/3) - 6*(6*b^3*d^2*n^2*x*e*log(c) - (11*b^3*d^2*n^3 - 6*a*b^2*d^2*n^2)*x*e)*x^(1/3))*log((d*x + x^(1/3)*e)/x) + 3*(18*b^3*d*n*e^2*log(c)^2 - 6*(5*b^3*d*n^2 - 6*a*b^2*d*n)*e^2*log(c) + (19*b^3*d*n^3 - 30*a*b^2*d*n^2 + 18*a^2*b*d*n)*e^2)*x^(2/3) - 6*(18*b^3*d^2*n*x*e*log(c)^2 - 6*(11*b^3*d^2*n^2 - 6*a*b^2*d^2*n)*x*e*log(c) + (85*b^3*d^2*n^3 - 66*a*b^2*d^2*n^2 + 18*a^2*b*d^2*n)*x*e)*x^(1/3))*e^(-3)/x^2

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**3/x**3,x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^3,x, algorithm="giac")

[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3/x^3, x)

Mupad [B]
time = 0.75, size = 578, normalized size = 1.29

$$\frac{\frac{d\left(\frac{3a^3}{2}-\frac{3a^2bn}{2}+ab^2n^2-\frac{b^3n^3}{3}\right)}{2e}-\frac{d\left(6a^3-6ab^2n^2+5b^3n^3\right)}{8e}}{x^{4/3}}-\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)^3\left(\frac{b^3}{2x^2}+\frac{b^3d^3}{2e^3}\right)-\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)^2\left(\frac{b^2\left(3a-bn\right)}{2x^2}-\frac{\frac{3b^2d\left(3a-bn\right)}{2e}-\frac{9ab^2d}{2e}}{2x^{4/3}}+\frac{d\left(6ab^2d^2-11b^3d^2n\right)}{4e^3}+\frac{d\left(\frac{6b^2d\left(3a-bn\right)}{e}-\frac{18ab^2d}{e}\right)}{4ex^{2/3}}\right)-\frac{\frac{d\left(\frac{d\left(\frac{3a^3}{2}-\frac{3a^2bn}{2}+ab^2n^2-\frac{b^3n^3}{3}\right)}{e}-\frac{d\left(6a^3-6ab^2n^2+5b^3n^3\right)}{4e}\right)}{e}+\frac{b^2d^2n^2\left(6a-11bn\right)}{2e^2}}{x^{2/3}}-\frac{\frac{a^3}{2}-\frac{a^2bn}{2}+\frac{ab^2n^2}{3}-\frac{b^3n^3}{9}}{x^2}-\frac{\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\left(\frac{\frac{d\left(2bde\left(9a^2-6abn+2b^2n^2\right)-6bde\left(3a^2-b^2n^2\right)\right)}{e}+12b^3d^2n^2}{2ex^{2/3}}-\frac{2bde\left(9a^2-6abn+2b^2n^2\right)-6bde\left(3a^2-b^2n^2\right)}{4ex^{4/3}}+\frac{be\left(9a^2-6abn+2b^2n^2\right)}{3x^2}\right)}{2e}-\frac{\ln\left(d+\frac{e}{x^{2/3}}\right)\left(18a^2bd^3n-66ab^2d^3n^2+85b^3d^3n^3\right)}{12e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))^3/x^3,x)
```

```
[Out] ((d*((3*a^3)/2 - (b^3*n^3)/3 + a*b^2*n^2 - (3*a^2*b*n)/2))/(2*e) - (d*(6*a^
3 + 5*b^3*n^3 - 6*a*b^2*n^2))/(8*e))/x^(4/3) - log(c*(d + e/x^(2/3))^n)^3*(
b^3/(2*x^2) + (b^3*d^3)/(2*e^3)) - log(c*(d + e/x^(2/3))^n)^2*((b^2*(3*a -
b*n))/(2*x^2) - ((3*b^2*d*(3*a - b*n))/(2*e) - (9*a*b^2*d)/(2*e))/(2*x^(4/3
)) + (d*(6*a*b^2*d^2 - 11*b^3*d^2*n))/(4*e^3) + (d*((6*b^2*d*(3*a - b*n))/e
 - (18*a*b^2*d)/e))/(4*e*x^(2/3))) - ((d*((d*((3*a^3)/2 - (b^3*n^3)/3 + a*b
^2*n^2 - (3*a^2*b*n)/2))/e - (d*(6*a^3 + 5*b^3*n^3 - 6*a*b^2*n^2))/(4*e)))/
e + (b^2*d^2*n^2*(6*a - 11*b*n))/(2*e^2))/x^(2/3) - (a^3/2 - (b^3*n^3)/9 +
(a*b^2*n^2)/3 - (a^2*b*n)/2)/x^2 - (log(c*(d + e/x^(2/3))^n)*(((d*(2*b*d*e*
(9*a^2 + 2*b^2*n^2 - 6*a*b*n) - 6*b*d*e*(3*a^2 - b^2*n^2)))/e + 12*b^3*d^2*
n^2)/(2*e*x^(2/3)) - (2*b*d*e*(9*a^2 + 2*b^2*n^2 - 6*a*b*n) - 6*b*d*e*(3*a^
2 - b^2*n^2))/(4*e*x^(4/3)) + (b*e*(9*a^2 + 2*b^2*n^2 - 6*a*b*n))/(3*x^2)))
/(2*e) - (log(d + e/x^(2/3))*(85*b^3*d^3*n^3 - 66*a*b^2*d^3*n^2 + 18*a^2*b*
d^3*n))/(12*e^3)
```

3.528 $\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx$

Optimal. Leaf size=1278

$$\frac{568ab^2e^4n^2\sqrt[3]{x}}{105d^4}-\frac{16b^3e^4n^3\sqrt[3]{x}}{7d^4}+\frac{16b^3e^3n^3x}{105d^3}+\frac{1376b^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{105d^{9/2}}+\frac{568ib^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right.}{105d^{9/2}}$$

```
[Out] 16/105*b^3*e^3*n^3*x/d^3+1376/105*b^3*e^(9/2)*n^3*arctan(x^(1/3)*d^(1/2)/e^
(1/2))/d^(9/2)-2*b^3*e^(9/2)*n^3*ln(-x^(1/3)*(-d)^(1/2)+e^(1/2))^2/(-d)^(9/
2)+2*b^3*e^(9/2)*n^3*ln(x^(1/3)*(-d)^(1/2)+e^(1/2))^2/(-d)^(9/2)+8*b^3*e^(9
/2)*n^3*polylog(2,1-x^(1/3)*(-d)^(1/2)/e^(1/2))/(-d)^(9/2)-4*b^3*e^(9/2)*n^
3*polylog(2,1/2-1/2*x^(1/3)*(-d)^(1/2)/e^(1/2))/(-d)^(9/2)+4*b^3*e^(9/2)*n^
3*polylog(2,1/2+1/2*x^(1/3)*(-d)^(1/2)/e^(1/2))/(-d)^(9/2)-8*b^3*e^(9/2)*n^
3*polylog(2,1+x^(1/3)*(-d)^(1/2)/e^(1/2))/(-d)^(9/2)+2/3*b*e^5*n*Unintegrab
le((a+b*ln(c*(d+e/x^(2/3))^n))^2/(e+d*x^(2/3))/x^(2/3),x)/d^4+568/105*I*b^3
*e^(9/2)*n^3*arctan(x^(1/3)*d^(1/2)/e^(1/2))^2/d^(9/2)+568/105*I*b^3*e^(9/2
)*n^3*polylog(2,-1+2*e^(1/2)/(-I*x^(1/3)*d^(1/2)+e^(1/2)))/d^(9/2)+568/105*
b^3*e^4*n^2*x^(1/3)*ln(c*(d+e/x^(2/3))^n)/d^4-32/35*b^2*e^3*n^2*x*(a+b*ln(c
*(d+e/x^(2/3))^n))/d^3+8/35*b^2*e^2*n^2*x^(5/3)*(a+b*ln(c*(d+e/x^(2/3))^n))
/d^2-568/105*b^2*e^(9/2)*n^2*arctan(x^(1/3)*d^(1/2)/e^(1/2))*(a+b*ln(c*(d+e
/x^(2/3))^n))/d^(9/2)-2*b*e^4*n*x^(1/3)*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d^4+2
/3*b*e^3*n*x*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d^3-2/5*b*e^2*n*x^(5/3)*(a+b*ln(
c*(d+e/x^(2/3))^n))^2/d^2+2/7*b*e*n*x^(7/3)*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d
+4*b^2*e^(9/2)*n^2*(a+b*ln(c*(d+e/x^(2/3))^n))*ln(-x^(1/3)*(-d)^(1/2)+e^(1/
2))/(-d)^(9/2)-4*b^3*e^(9/2)*n^3*ln(1/2+1/2*x^(1/3)*(-d)^(1/2)/e^(1/2))*ln(
-x^(1/3)*(-d)^(1/2)+e^(1/2))/(-d)^(9/2)+8*b^3*e^(9/2)*n^3*ln(x^(1/3)*(-d)^(
1/2)/e^(1/2))*ln(-x^(1/3)*(-d)^(1/2)+e^(1/2))/(-d)^(9/2)-4*b^2*e^(9/2)*n^2*
(a+b*ln(c*(d+e/x^(2/3))^n))*ln(x^(1/3)*(-d)^(1/2)+e^(1/2))/(-d)^(9/2)+4*b^3
*e^(9/2)*n^3*ln(1/2-1/2*x^(1/3)*(-d)^(1/2)/e^(1/2))*ln(x^(1/3)*(-d)^(1/2)+e
^(1/2))/(-d)^(9/2)-8*b^3*e^(9/2)*n^3*ln(-x^(1/3)*(-d)^(1/2)/e^(1/2))*ln(x^(
1/3)*(-d)^(1/2)+e^(1/2))/(-d)^(9/2)-1136/105*b^3*e^(9/2)*n^3*arctan(x^(1/3)
*d^(1/2)/e^(1/2))*ln(2-2*e^(1/2)/(-I*x^(1/3)*d^(1/2)+e^(1/2)))/d^(9/2)-16/7
*b^3*e^4*n^3*x^(1/3)/d^4+568/105*a*b^2*e^4*n^2*x^(1/3)/d^4+1/3*x^3*(a+b*ln(
c*(d+e/x^(2/3))^n))^3
```

Rubi [A]
time = 1.99, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

```
[Out] (568*a*b^2*e^4*n^2*x^(1/3))/(105*d^4) - (16*b^3*e^4*n^3*x^(1/3))/(7*d^4) +
(16*b^3*e^3*n^3*x)/(105*d^3) + (1376*b^3*e^(9/2)*n^3*ArcTan[(Sqrt[d]*x^(1/3
))/Sqrt[e]])/(105*d^(9/2)) + (((568*I)/105)*b^3*e^(9/2)*n^3*ArcTan[(Sqrt[d]
*x^(1/3))/Sqrt[e]]^2)/d^(9/2) - (1136*b^3*e^(9/2)*n^3*ArcTan[(Sqrt[d]*x^(1/
3))/Sqrt[e]]*Log[2 - (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/(105*d^(9/
2)) + (568*b^3*e^4*n^2*x^(1/3)*Log[c*(d + e/x^(2/3))^n])/(105*d^4) - (32*b^
2*e^3*n^2*x*(a + b*Log[c*(d + e/x^(2/3))^n]))/(35*d^3) + (8*b^2*e^2*n^2*x^(
5/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(35*d^2) - (568*b^2*e^(9/2)*n^2*ArcT
an[(Sqrt[d]*x^(1/3))/Sqrt[e]]*(a + b*Log[c*(d + e/x^(2/3))^n]))/(105*d^(9/2
)) - (2*b*e^4*n*x^(1/3)*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/d^4 + (2*b*e^3*
n*x*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(3*d^3) - (2*b*e^2*n*x^(5/3)*(a + b
*Log[c*(d + e/x^(2/3))^n])^2)/(5*d^2) + (2*b*e*n*x^(7/3)*(a + b*Log[c*(d +
e/x^(2/3))^n])^2)/(7*d) + (x^3*(a + b*Log[c*(d + e/x^(2/3))^n])^3)/3 + (4*b
^2*e^(9/2)*n^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] - Sqrt[-d]*x^(1
/3)])/(-d)^(9/2) - (2*b^3*e^(9/2)*n^3*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]^2)/(-
d)^(9/2) - (4*b^2*e^(9/2)*n^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e]
 + Sqrt[-d]*x^(1/3)])/(-d)^(9/2) + (2*b^3*e^(9/2)*n^3*Log[Sqrt[e] + Sqrt[-d]
*x^(1/3)]^2)/(-d)^(9/2) + (4*b^3*e^(9/2)*n^3*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)
]*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])])/(-d)^(9/2) - (4*b^3*e^(9/2)*n^
3*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log[(1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2])/
(-d)^(9/2) - (8*b^3*e^(9/2)*n^3*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*Log[-((Sqrt
[-d]*x^(1/3))/Sqrt[e])])/(-d)^(9/2) + (8*b^3*e^(9/2)*n^3*Log[Sqrt[e] - Sqrt
[-d]*x^(1/3)]*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]])/(-d)^(9/2) + (((568*I)/105)*
b^3*e^(9/2)*n^3*PolyLog[2, -1 + (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])
/d^(9/2) + (8*b^3*e^(9/2)*n^3*PolyLog[2, 1 - (Sqrt[-d]*x^(1/3))/Sqrt[e]])/(
-d)^(9/2) - (4*b^3*e^(9/2)*n^3*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[
e])])/(-d)^(9/2) + (4*b^3*e^(9/2)*n^3*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sq
rt[e])/2])/(-d)^(9/2) - (8*b^3*e^(9/2)*n^3*PolyLog[2, 1 + (Sqrt[-d]*x^(1/3)
)/Sqrt[e]])/(-d)^(9/2) + (2*b*e^5*n*Defer[Subst][Defer[Int][(a + b*Log[c*(d
 + e/x^2)^n])^2/(e + d*x^2), x], x, x^(1/3)])/d^4
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx=3\text{Subst}\left(\int x^8\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^3\,dx,x,\sqrt[3]{x}\right)$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3+(2ben)\text{Subst}\left(\int\frac{x^6\left(a+b\log\left(\right.\right.}{d}\right.$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3+(2ben)\text{Subst}\left(\int\left(-\frac{e^3\left(a+b\right.}{}\right.\right.$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3+\frac{(2ben)\text{Subst}\left(\int x^6\left(a+b\log\left(c\right.\right.\right.}{d}$$

$$=-\frac{2be^4n\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d^4}+\frac{2be^3nx\left(a+b\log\left(c\left(d+\right.\right.\right.}{3d^3}$$

$$=-\frac{2be^4n\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d^4}+\frac{2be^3nx\left(a+b\log\left(c\left(d+\right.\right.\right.}{3d^3}$$

$$=-\frac{2be^4n\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d^4}+\frac{2be^3nx\left(a+b\log\left(c\left(d+\right.\right.\right.}{3d^3}$$

$$=\frac{568ab^2e^4n^2\sqrt[3]{x}}{105d^4}-\frac{32b^2e^3n^2x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{35d^3}+\frac{8b^2e^2n^2x}{}$$

$$=\frac{568ab^2e^4n^2\sqrt[3]{x}}{105d^4}+\frac{568b^3e^4n^2\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{105d^4}-\frac{32b^2e^3n^2x\left(\right.}{}$$

$$=\frac{568ab^2e^4n^2\sqrt[3]{x}}{105d^4}-\frac{64b^3e^4n^3\sqrt[3]{x}}{35d^4}+\frac{568b^3e^4n^2\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right.}{105d^4}$$

$$=\frac{568ab^2e^4n^2\sqrt[3]{x}}{105d^4}-\frac{16b^3e^4n^3\sqrt[3]{x}}{7d^4}+\frac{16b^3e^3n^3x}{105d^3}+\frac{1328b^3e^{9/2}n^3\tan^{-}}{105d^{9}}$$

Mathematica [A]
time = 2.94, size = 764, normalized size = 0.60

$$\frac{b^3n^3\left(54e^5(e+dx^{2/3})\,{}_5F_4\left(1,1,1,1,\frac{11}{2};2,2,2,2;1+\frac{e}{dx^{2/3}}\right)+\log\left(d+\frac{e}{x^{2/3}}\right)\left(-54e^5(e+dx^{2/3})\,{}_4F_3\left(1,1,1,\frac{11}{2};2,2,2;1+\frac{e}{dx^{2/3}}\right)+\log\left(d+\frac{e}{x^{2/3}}\right)\left(27e^5(e+dx^{2/3})\,{}_3F_2\left(1,1,\frac{11}{2};2,2;1+\frac{e}{dx^{2/3}}\right)+2dx^{2/3}\left(e^5+d^5\sqrt{-\frac{e}{dx^{2/3}}}\,x^{10/3}\right)\log\left(d+\frac{e}{x^{2/3}}\right)\right)\right)\right)}{6d^6\sqrt{-\frac{e}{dx^{2/3}}}\,x}-\frac{b^2n^2\left(-9e^5(e+dx^{2/3})\,{}_4F_3\left(1,1,1,\frac{11}{2};2,2,2;1+\frac{e}{dx^{2/3}}\right)+\log\left(d+\frac{e}{x^{2/3}}\right)\left(9e^5(e+dx^{2/3})\,{}_3F_2\left(1,1,\frac{11}{2};2,2;1+\frac{e}{dx^{2/3}}\right)+dx^{2/3}\left(e^5+d^5\sqrt{-\frac{e}{dx^{2/3}}}\,x^{10/3}\right)\log\left(d+\frac{e}{x^{2/3}}\right)\right)\right)\left(-a+bn\log\left(d+\frac{e}{x^{2/3}}\right)-b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^6\sqrt{-\frac{e}{dx^{2/3}}}\,x}-\frac{2be^4n\sqrt[3]{x}\left(a-bn\log\left(d+\frac{e}{x^{2/3}}\right)+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d^4}+\frac{2be^3nx\left(a-bn\log\left(d+\frac{e}{x^{2/3}}\right)+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{3d^3}-\frac{2be^2nx^{5/3}\left(a-bn\log\left(d+\frac{e}{x^{2/3}}\right)+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{5d^2}+\frac{2benx^{7/3}\left(a-bn\log\left(d+\frac{e}{x^{2/3}}\right)+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{7d}+\frac{2be^{9/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a-bn\log\left(d+\frac{e}{x^{2/3}}\right)+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d^{9/2}}+bnx^3\log\left(d+\frac{e}{x^{2/3}}\right)\left(a-bn\log\left(d+\frac{e}{x^{2/3}}\right)+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2+\frac{1}{3}x^3\left(a-bn\log\left(d+\frac{e}{x^{2/3}}\right)+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

```
[Out] (b^3*n^3*(54*e^5*(e + d*x^(2/3))*HypergeometricPFQ[{1, 1, 1, 1, 11/2}, {2,
2, 2, 2}, 1 + e/(d*x^(2/3))] + Log[d + e/x^(2/3)]*(-54*e^5*(e + d*x^(2/3))*
HypergeometricPFQ[{1, 1, 1, 11/2}, {2, 2, 2}, 1 + e/(d*x^(2/3))] + Log[d +
e/x^(2/3)]*(27*e^5*(e + d*x^(2/3))*HypergeometricPFQ[{1, 1, 11/2}, {2, 2},
1 + e/(d*x^(2/3))] + 2*d*x^(2/3)*(e^5 + d^5*Sqrt[-(e/(d*x^(2/3)))]*x^(10/3)
)*Log[d + e/x^(2/3)]))))/(6*d^6*Sqrt[-(e/(d*x^(2/3)))]*x) - (b^2*n^2*(-9*e^
5*(e + d*x^(2/3))*HypergeometricPFQ[{1, 1, 1, 11/2}, {2, 2, 2}, 1 + e/(d*x^
(2/3))] + Log[d + e/x^(2/3)]*(9*e^5*(e + d*x^(2/3))*HypergeometricPFQ[{1, 1
, 11/2}, {2, 2}, 1 + e/(d*x^(2/3))] + d*x^(2/3)*(e^5 + d^5*Sqrt[-(e/(d*x^(2
/3)))]*x^(10/3))*Log[d + e/x^(2/3)]))*(-a + b*n*Log[d + e/x^(2/3)] - b*Log[
c*(d + e/x^(2/3))^n]))/(d^6*Sqrt[-(e/(d*x^(2/3)))]*x) - (2*b*e^4*n*x^(1/3)*
(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/d^4 + (2*b*e^3
*n*x*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/(3*d^3) -
 (2*b*e^2*n*x^(5/3)*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n
])^2)/(5*d^2) + (2*b*e*n*x^(7/3)*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d +
 e/x^(2/3))^n])^2)/(7*d) + (2*b*e^(9/2)*n*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]
*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/d^(9/2) + b*n
*x^3*Log[d + e/x^(2/3)]*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3
))^n])^2 + (x^3*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^3
)/3
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e/x^(2/3))^n))^3,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e/x^(2/3))^n))^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*n^3*x^3*log(d*x^(2/3) + e)^3 - integrate(1/3*((2*b^3*d*n*x^3 - 9*(b
^3*log(c) + a*b^2)*x^(7/3)*e - 9*(b^3*d*log(c) + a*b^2*d)*x^3 + 18*(b^3*d*x
^3 + b^3*x^(7/3)*e)*log(x^(1/3*n)))*n^2*log(d*x^(2/3) + e)^2 - 3*(b^3*log(c
)^3 + 3*a*b^2*log(c)^2 + 3*a^2*b*log(c) + a^3)*x^(7/3)*e - 3*(b^3*d*log(c)^
3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x^3 + 24*(b^3*d*x^3 + b^
3*x^(7/3)*e)*log(x^(1/3*n))^3 - 9*((b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*b)*
x^(7/3)*e + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x^3 + 4*(b^3*d*x^
3 + b^3*x^(7/3)*e)*log(x^(1/3*n))^2 - 4*((b^3*log(c) + a*b^2)*x^(7/3)*e + (
b^3*d*log(c) + a*b^2*d)*x^3)*log(x^(1/3*n)))*n*log(d*x^(2/3) + e) - 36*((b^
3*log(c) + a*b^2)*x^(7/3)*e + (b^3*d*log(c) + a*b^2*d)*x^3)*log(x^(1/3*n))^
2 + 18*((b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*b)*x^(7/3)*e + (b^3*d*log(c)^2
 + 2*a*b^2*d*log(c) + a^2*b*d)*x^3)*log(x^(1/3*n)))/(d*x + x^(1/3)*e), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*log(c*((d*x + x^(1/3)*e)/x)^n)^3 + 3*a*b^2*x^2*log(c*((d*x
 + x^(1/3)*e)/x)^n)^2 + 3*a^2*b*x^2*log(c*((d*x + x^(1/3)*e)/x)^n) + a^3*x^
2, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(2/3))**n))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3*x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(a + b \ln \left(c \left(d + \frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e/x^(2/3))^n))^3,x)
[Out] int(x^2*(a + b*log(c*(d + e/x^(2/3))^n))^3, x)
```

3.529 $\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx$

Optimal. Leaf size=738

$$\frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d}+x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3+\frac{12b^2e^{3/2}n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{(-d)^{3/2}}$$

```
[Out] 6*b*e*n*x^(1/3)*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d+x*(a+b*ln(c*(d+e/x^(2/3))^n
))^3+12*b^2*e^(3/2)*n^2*(a+b*ln(c*(d+e/x^(2/3))^n))*ln(-x^(1/3)*(-d)^(1/2)+
e^(1/2))/(-d)^(3/2)-12*b^3*e^(3/2)*n^3*ln(1/2+1/2*x^(1/3)*(-d)^(1/2)/e^(1/2
))*ln(-x^(1/3)*(-d)^(1/2)+e^(1/2))/(-d)^(3/2)+24*b^3*e^(3/2)*n^3*ln(x^(1/3)
*(-d)^(1/2)/e^(1/2))*ln(-x^(1/3)*(-d)^(1/2)+e^(1/2))/(-d)^(3/2)-6*b^3*e^(3/
2)*n^3*ln(-x^(1/3)*(-d)^(1/2)+e^(1/2))^2/(-d)^(3/2)-12*b^2*e^(3/2)*n^2*(a+b
*ln(c*(d+e/x^(2/3))^n))*ln(x^(1/3)*(-d)^(1/2)+e^(1/2))/(-d)^(3/2)+12*b^3*e^
(3/2)*n^3*ln(1/2-1/2*x^(1/3)*(-d)^(1/2)/e^(1/2))*ln(x^(1/3)*(-d)^(1/2)+e^(1
/2))/(-d)^(3/2)-24*b^3*e^(3/2)*n^3*ln(-x^(1/3)*(-d)^(1/2)/e^(1/2))*ln(x^(1/
3)*(-d)^(1/2)+e^(1/2))/(-d)^(3/2)+6*b^3*e^(3/2)*n^3*ln(x^(1/3)*(-d)^(1/2)+e
^(1/2))^2/(-d)^(3/2)+24*b^3*e^(3/2)*n^3*polylog(2,1-x^(1/3)*(-d)^(1/2)/e^(1
/2))/(-d)^(3/2)-12*b^3*e^(3/2)*n^3*polylog(2,1/2-1/2*x^(1/3)*(-d)^(1/2)/e^(
1/2))/(-d)^(3/2)+12*b^3*e^(3/2)*n^3*polylog(2,1/2+1/2*x^(1/3)*(-d)^(1/2)/e^
(1/2))/(-d)^(3/2)-24*b^3*e^(3/2)*n^3*polylog(2,1+x^(1/3)*(-d)^(1/2)/e^(1/2)
)/(-d)^(3/2)-2*b*e^2*n*Unintegrable((a+b*ln(c*(d+e/x^(2/3))^n))^2/(e+d*x^(2
/3))/x^(2/3),x)/d
```

Rubi [A]
time = 0.81, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

```
[Out] (6*b*e*n*x^(1/3)*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/d + x*(a + b*Log[c*(d
+ e/x^(2/3))^n])^3 + (12*b^2*e^(3/2)*n^2*(a + b*Log[c*(d + e/x^(2/3))^n])*L
og[Sqrt[e] - Sqrt[-d]*x^(1/3)])/(-d)^(3/2) - (6*b^3*e^(3/2)*n^3*Log[Sqrt[e]
 - Sqrt[-d]*x^(1/3)]^2)/(-d)^(3/2) - (12*b^2*e^(3/2)*n^2*(a + b*Log[c*(d +
e/x^(2/3))^n])*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)])/(-d)^(3/2) + (6*b^3*e^(3/2)
*n^3*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]^2)/(-d)^(3/2) + (12*b^3*e^(3/2)*n^3*Lo
g[Sqrt[e] + Sqrt[-d]*x^(1/3)]*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])])/(-
d)^(3/2) - (12*b^3*e^(3/2)*n^3*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log[(1 + (Sq
```

```
rt[-d]*x^(1/3))/Sqrt[e])/2])/(-d)^(3/2) - (24*b^3*e^(3/2)*n^3*Log[Sqrt[e] +
 Sqrt[-d]*x^(1/3)]*Log[-((Sqrt[-d]*x^(1/3))/Sqrt[e])])/(-d)^(3/2) + (24*b^3
*e^(3/2)*n^3*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]
])/(-d)^(3/2) + (24*b^3*e^(3/2)*n^3*PolyLog[2, 1 - (Sqrt[-d]*x^(1/3))/Sqrt[
e]])/(-d)^(3/2) - (12*b^3*e^(3/2)*n^3*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(
2*Sqrt[e])])/(-d)^(3/2) + (12*b^3*e^(3/2)*n^3*PolyLog[2, (1 + (Sqrt[-d]*x^(
1/3))/Sqrt[e])/2])/(-d)^(3/2) - (24*b^3*e^(3/2)*n^3*PolyLog[2, 1 + (Sqrt[-d
]*x^(1/3))/Sqrt[e]])/(-d)^(3/2) - (6*b*e^2*n*Defer[Subst][Defer[Int][(a + b
*Log[c*(d + e/x^2)^n])^2/(e + d*x^2), x], x, x^(1/3)])/d
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx = 3\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^3 dx, x, \sqrt[3]{x}\right)$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + (6ben)\text{Subst}\left(\int \frac{(a+b\log(c(d+}{d+\frac{e}{x^2}}\right.$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + (6ben)\text{Subst}\left(\int \left(\frac{(a+b\log(c(d-}{d}\right.\right.$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + \frac{(6ben)\text{Subst}\left(\int (a+b\log(c(d+\frac{e}{x^2}\right.}{d}$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)$$

Mathematica [A]
time = 3.52, size = 824, normalized size = 1.12

$$\frac{3b^2n^2\left(-3e^2\left(e+dx^{2/3}\right){}_4F_3\left(1,1,1,\frac{5}{2};2,2,2;1+\frac{e}{dx^{2/3}}\right)-dx^{2/3}\log\left(d+\frac{e}{x^{2/3}}\right)\left(4e\left(e-\frac{e}{\sqrt{-\frac{e}{dx^{2/3}}}}\right)+4e^2\log\left(\frac{1}{2}\left(1+\sqrt{-\frac{e}{dx^{2/3}}}\right)\right)+\left(-e^2+d^2\sqrt{-\frac{e}{dx^{2/3}}}\,x^{4/3}\right)\log\left(d+\frac{e}{x^{2/3}}\right)\right)\right)\left(-a+bn\log\left(d+\frac{e}{x^{2/3}}\right)-b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3\sqrt{-\frac{e}{dx^{2/3}}}\,x}+\frac{6ben\sqrt[3]{x}\left(a-bn\log\left(d+\frac{e}{x^{2/3}}\right)+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d}-\frac{6be^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a-bn\log\left(d+\frac{e}{x^{2/3}}\right)+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d^{3/2}}+3bnx\log\left(d+\frac{e}{x^{2/3}}\right)\left(a-bn\log\left(d+\frac{e}{x^{2/3}}\right)+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2+x\left(a-bn\log\left(d+\frac{e}{x^{2/3}}\right)+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3+\frac{b^3n^3\sqrt[3]{x}\left(\sqrt{d}\log^2\left(d+\frac{e}{x^{2/3}}\right)\left(6e+dx^{2/3}\log\left(d+\frac{e}{x^{2/3}}\right)\right)-6e\sqrt{\frac{e}{e+dx^{2/3}}}\left(8\sqrt{d}\,{}_4F_3\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2},\frac{3}{2};\frac{d}{d+\frac{e}{x^{2/3}}}\right)+\log\left(d+\frac{e}{x^{2/3}}\right)\left(4\sqrt{d}\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{d}{d+\frac{e}{x^{2/3}}}\right)+\sqrt{d+\frac{e}{x^{2/3}}}\sin^{-1}\left(\frac{\sqrt{d}}{\sqrt{d+\frac{e}{x^{2/3}}}}\right)\log\left(d+\frac{e}{x^{2/3}}\right)\right)\right)+6\sqrt{d}\,e\left(\frac{4\sqrt{\frac{e}{x^{2/3}}}\tanh^{-1}\left(\frac{\sqrt{\frac{e}{x^{2/3}}}}{\sqrt{-d}}\right)\left(\log\left(d+\frac{e}{x^{2/3}}\right)-\log\left(1+\frac{e}{dx^{2/3}}\right)\right)}{\sqrt{-d}}-\sqrt{-\frac{e}{dx^{2/3}}}\left(2\log^2\left(\frac{1}{2}\left(1+\sqrt{-\frac{e}{dx^{2/3}}}\right)\right)-4\log\left(\frac{1}{2}\left(1+\sqrt{-\frac{e}{dx^{2/3}}}\right)\right)\log\left(1+\frac{e}{dx^{2/3}}\right)+\log^2\left(1+\frac{e}{dx^{2/3}}\right)-4\,\mathrm{Li}_2\left(\frac{1}{2}-\frac{1}{2}\sqrt{-\frac{e}{dx^{2/3}}}\right)\right)\right)\right)}{d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

```
[Out] (3*b^2*n^2*(-3*e^2*(e + d*x^(2/3))*HypergeometricPFQ[{1, 1, 1, 5/2}, {2, 2,
 2}, 1 + e/(d*x^(2/3))] - d*x^(2/3)*Log[d + e/x^(2/3)]*(4*e*(e - e/Sqrt[-(e
/(d*x^(2/3)))]) + 4*e^2*Log[(1 + Sqrt[-(e/(d*x^(2/3)))])/2] + (-e^2 + d^2*S
qrt[-(e/(d*x^(2/3)))]*x^(4/3))*Log[d + e/x^(2/3)]))*(-a + b*n*Log[d + e/x^(
2/3)] - b*Log[c*(d + e/x^(2/3))^n]))/(d^3*Sqrt[-(e/(d*x^(2/3)))]*x) + (6*b*
e*n*x^(1/3)*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/d
- (6*b*e^(3/2)*n*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]*(a - b*n*Log[d + e/x^(2/
3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/d^(3/2) + 3*b*n*x*Log[d + e/x^(2/3)]*(
a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2 + x*(a - b*n*Log
[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^3 + (b^3*n^3*x^(1/3)*(Sqrt[d]
*Log[d + e/x^(2/3)]^2*(6*e + d*x^(2/3)*Log[d + e/x^(2/3)]) - 6*e*Sqrt[e/(e
+ d*x^(2/3))]*(8*Sqrt[d]*HypergeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2,
 3/2}, d/(d + e/x^(2/3))] + Log[d + e/x^(2/3)]*(4*Sqrt[d]*HypergeometricPFQ
[{1/2, 1/2, 1/2}, {3/2, 3/2}, d/(d + e/x^(2/3))] + Sqrt[d + e/x^(2/3)]*ArcS
in[Sqrt[d]/Sqrt[d + e/x^(2/3)]]*Log[d + e/x^(2/3)])) + 6*Sqrt[d]*e*((4*Sqrt
[e/x^(2/3)]*ArcTanh[Sqrt[e/x^(2/3)]/Sqrt[-d]]*(Log[d + e/x^(2/3)] - Log[1 +
 e/(d*x^(2/3))]))/Sqrt[-d] - Sqrt[-(e/(d*x^(2/3)))]*(2*Log[(1 + Sqrt[-(e/(d
*x^(2/3)))])/2]^2 - 4*Log[(1 + Sqrt[-(e/(d*x^(2/3)))])/2]*Log[1 + e/(d*x^(2
/3))] + Log[1 + e/(d*x^(2/3))]^2 - 4*PolyLog[2, 1/2 - Sqrt[-(e/(d*x^(2/3)))
]/2]))))/d^(3/2)
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="maxima")
```

```
[Out] b^3*n^3*x*log(d*x^(2/3) + e)^3 - 3*(2*n*(arctan(sqrt(d)*x^(1/3)*e^(-1/2))*e
^(1/2)/d^(3/2) - x^(1/3)/d)*e - x*log(c*(d + e/x^(2/3))^n))*a^2*b + a^3*x -
 integrate(((2*b^3*d*n*x - 3*(b^3*log(c) + a*b^2)*x^(1/3)*e - 3*(b^3*d*log(
c) + a*b^2*d)*x + 6*(b^3*d*x + b^3*x^(1/3)*e)*log(x^(1/3*n)))*n^2*log(d*x^(
2/3) + e)^2 + 8*(b^3*d*x + b^3*x^(1/3)*e)*log(x^(1/3*n))^3 - 3*(4*(b^3*d*x
+ b^3*x^(1/3)*e)*log(x^(1/3*n))^2 + (b^3*log(c)^2 + 2*a*b^2*log(c))*x^(1/3)
*e + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c))*x - 4*((b^3*log(c) + a*b^2)*x^(1/3
)*e + (b^3*d*log(c) + a*b^2*d)*x)*log(x^(1/3*n)))*n*log(d*x^(2/3) + e) - 12
*((b^3*log(c) + a*b^2)*x^(1/3)*e + (b^3*d*log(c) + a*b^2*d)*x)*log(x^(1/3*n
))^2 - (b^3*log(c)^3 + 3*a*b^2*log(c)^2)*x^(1/3)*e - (b^3*d*log(c)^3 + 3*a*
b^2*d*log(c)^2)*x + 6*((b^3*log(c)^2 + 2*a*b^2*log(c))*x^(1/3)*e + (b^3*d*l
og(c)^2 + 2*a*b^2*d*log(c))*x)*log(x^(1/3*n)))/(d*x + x^(1/3)*e), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*log(c*((d*x + x^(1/3)*e)/x)^n)^3 + 3*a*b^2*log(c*((d*x + x^(1/
3)*e)/x)^n)^2 + 3*a^2*b*log(c*((d*x + x^(1/3)*e)/x)^n) + a^3, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))^3,x)
[Out] int((a + b*log(c*(d + e/x^(2/3))^n))^3, x)
```

3.530 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^2}\,dx$

Optimal. Leaf size=483

$$\frac{16b^3n^3}{9x}-\frac{208b^3dn^3}{3e\sqrt[3]{x}}-\frac{208b^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3e^{3/2}}-\frac{32ib^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{e^{3/2}}+\frac{64b^3d^{3/2}n^3\tan^{-1}\left(\cdots\right.}{\cdots}$$

```
[Out] 16/9*b^3*n^3/x-208/3*b^3*d*n^3/e/x^(1/3)-208/3*b^3*d^(3/2)*n^3*arctan(x^(1/
3)*d^(1/2)/e^(1/2))/e^(3/2)-32*I*b^3*d^(3/2)*n^3*arctan(x^(1/3)*d^(1/2)/e^(
1/2))^2/e^(3/2)-8/3*b^2*n^2*(a+b*ln(c*(d+e/x^(2/3))^n))/x+32*b^2*d*n^2*(a+b
*ln(c*(d+e/x^(2/3))^n))/e/x^(1/3)+32*b^2*d^(3/2)*n^2*arctan(x^(1/3)*d^(1/2)
/e^(1/2))*(a+b*ln(c*(d+e/x^(2/3))^n))/e^(3/2)+2*b*n*(a+b*ln(c*(d+e/x^(2/3))
^n))^2/x-6*b*d*n*(a+b*ln(c*(d+e/x^(2/3))^n))^2/e/x^(1/3)-(a+b*ln(c*(d+e/x^(
2/3))^n))^3/x+64*b^3*d^(3/2)*n^3*arctan(x^(1/3)*d^(1/2)/e^(1/2))*ln(2-2*e^(
1/2)/(-I*x^(1/3)*d^(1/2)+e^(1/2)))/e^(3/2)-32*I*b^3*d^(3/2)*n^3*polylog(2,-
1+2*e^(1/2)/(-I*x^(1/3)*d^(1/2)+e^(1/2)))/e^(3/2)-2*b*d^2*n*Unintegrable((a
+b*ln(c*(d+e/x^(2/3))^n))^2/(e+d*x^(2/3))/x^(2/3),x)/e
```

Rubi [A]
time = 0.87, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x^2,x]
```

```
[Out] (16*b^3*n^3)/(9*x) - (208*b^3*d*n^3)/(3*e*x^(1/3)) - (208*b^3*d^(3/2)*n^3*A
rcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]])/(3*e^(3/2)) - ((32*I)*b^3*d^(3/2)*n^3*Arc
Tan[(Sqrt[d]*x^(1/3))/Sqrt[e]]^2)/e^(3/2) + (64*b^3*d^(3/2)*n^3*ArcTan[(Sqr
t[d]*x^(1/3))/Sqrt[e]]*Log[2 - (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/
e^(3/2) - (8*b^2*n^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(3*x) + (32*b^2*d*n^
2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(e*x^(1/3)) + (32*b^2*d^(3/2)*n^2*ArcTa
n[(Sqrt[d]*x^(1/3))/Sqrt[e]]*(a + b*Log[c*(d + e/x^(2/3))^n]))/e^(3/2) + (2
*b*n*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/x - (6*b*d*n*(a + b*Log[c*(d + e/x
^(2/3))^n])^2)/(e*x^(1/3)) - (a + b*Log[c*(d + e/x^(2/3))^n])^3/x - ((32*I)
*b^3*d^(3/2)*n^3*PolyLog[2, -1 + (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))]
)/e^(3/2) - (6*b*d^2*n*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x^2)^n])
^2/(e + d*x^2), x], x, x^(1/3)])/e
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^2}\,dx = 3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^3}{x^4}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x} - (6ben)\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right.}{\left(d+\frac{e}{x^2}\right)x^6}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x} - (6ben)\text{Subst}\left(\int \left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right.\right.}{ex^4}\right.\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x} - (6bn)\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{x^4}\right.$$

$$= \frac{2bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} - \frac{6bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{e\sqrt[3]{x}} - \frac{(a+}{}$$

$$= \frac{2bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} - \frac{6bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{e\sqrt[3]{x}} - \frac{(a+}{}$$

$$= \frac{2bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} - \frac{6bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{e\sqrt[3]{x}} - \frac{(a+}{}$$

$$= -\frac{8b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x} + \frac{32b^2dn^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e\sqrt[3]{x}} +$$

$$= -\frac{8b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x} + \frac{32b^2dn^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e\sqrt[3]{x}} +$$

$$= \frac{16b^3n^3}{9x} - \frac{64b^3dn^3}{e\sqrt[3]{x}} - \frac{8b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x} + \frac{32b^2dn^2\left(a+b\log\right.}{e\sqrt[3]{}}$$

$$= \frac{16b^3n^3}{9x} - \frac{208b^3dn^3}{3e\sqrt[3]{x}} - \frac{64b^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt[3]{x}}{\sqrt{e}}\right)}{e^{3/2}} - \frac{32ib^3d^{3/2}n^3\tan^{-1}}{e^{3/2}}$$

Mathematica [**A**] Leaf count is larger than twice the leaf count of optimal. 1097 vs. $2(483) = 966$.
time = 1.30, size = 1097, normalized size = 2.27

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x^2,x]
```

```
[Out] (b^3*n^3*(18*(e + d*x^(2/3))*HypergeometricPFQ[{-1/2, 1, 1, 1, 1}, {2, 2, 2
, 2}, 1 + e/(d*x^(2/3))] - Log[d + e/x^(2/3)]*(18*(e + d*x^(2/3))*Hypergeom
etricPFQ[{-1/2, 1, 1, 1}, {2, 2, 2}, 1 + e/(d*x^(2/3))] + Log[d + e/x^(2/3)
]*(-9*(e + d*x^(2/3))*HypergeometricPFQ[{-1/2, 1, 1}, {2, 2}, 1 + e/(d*x^(2
/3))] + 2*(e*Sqrt[-(e/(d*x^(2/3)))] + d*x^(2/3))*Log[d + e/x^(2/3)]))))/(2*
e*Sqrt[-(e/(d*x^(2/3)))]*x) - (6*b*d*n*(a - b*n*Log[d + e/x^(2/3)] + b*Log[
c*(d + e/x^(2/3))^n])^2)/(e*x^(1/3)) - (6*b*d^(3/2)*n*ArcTan[(Sqrt[d]*x^(1/
3))/Sqrt[e]]*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/e
^(3/2) - (3*b*n*Log[d + e/x^(2/3)]*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d
 + e/x^(2/3))^n])^2)/x - ((a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2
/3))^n])^2*(a - 2*b*n - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n]
))/x + (b^2*n^2*(-a + b*n*Log[d + e/x^(2/3)] - b*Log[c*(d + e/x^(2/3))^n])*
(8*e^(3/2) - 96*d*Sqrt[e]*x^(2/3) + 96*d^(3/2)*x*ArcTan[Sqrt[e]/(Sqrt[d]*x^
(1/3))] - 12*e^(3/2)*Log[d + e/x^(2/3)] + 36*d*Sqrt[e]*x^(2/3)*Log[d + e/x^
(2/3)] + 9*e^(3/2)*Log[d + e/x^(2/3)]^2 + 18*Sqrt[-d]*d*x*Log[d + e/x^(2/3)
]*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 9*(-d)^(3/2)*x*Log[Sqrt[e] - Sqrt[-d]*x
^(1/3)]^2 + 18*(-d)^(3/2)*x*Log[d + e/x^(2/3)]*Log[Sqrt[e] + Sqrt[-d]*x^(1/
3)] + 9*Sqrt[-d]*d*x*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]^2 + 18*Sqrt[-d]*d*x*Lo
g[Sqrt[e] + Sqrt[-d]*x^(1/3)]*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] + 1
8*(-d)^(3/2)*x*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log[(1 + (Sqrt[-d]*x^(1/3))/
Sqrt[e])/2] + 36*(-d)^(3/2)*x*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*Log[-((Sqrt[-
d]*x^(1/3))/Sqrt[e])] + 36*Sqrt[-d]*d*x*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log
[(Sqrt[-d]*x^(1/3))/Sqrt[e]] + 36*Sqrt[-d]*d*x*PolyLog[2, 1 - (Sqrt[-d]*x^(
1/3))/Sqrt[e]] + 18*(-d)^(3/2)*x*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqr
t[e])] + 18*Sqrt[-d]*d*x*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] + 3
6*(-d)^(3/2)*x*PolyLog[2, 1 + (Sqrt[-d]*x^(1/3))/Sqrt[e]]))/(3*e^(3/2)*x)
```

Maple [**A**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\ln\left(c\left(d + \frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{3}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))^3/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))^3/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^2,x, algorithm="maxima")
```

```
[Out] -b^3*n^3*log(d*x^(2/3) + e)^3/x - integrate(-((2*b^3*d*n*x + 3*(b^3*log(c)
+ a*b^2)*x^(1/3)*e + 3*(b^3*d*log(c) + a*b^2*d)*x - 6*(b^3*d*x + b^3*x^(1/3
)*e)*log(x^(1/3*n)))*n^2*log(d*x^(2/3) + e)^2 - 8*(b^3*d*x + b^3*x^(1/3)*e)
*log(x^(1/3*n))^3 + 3*(4*(b^3*d*x + b^3*x^(1/3)*e)*log(x^(1/3*n))^2 + (b^3*
log(c)^2 + 2*a*b^2*log(c) + a^2*b)*x^(1/3)*e + (b^3*d*log(c)^2 + 2*a*b^2*d*
log(c) + a^2*b*d)*x - 4*((b^3*log(c) + a*b^2)*x^(1/3)*e + (b^3*d*log(c) + a
*b^2*d)*x)*log(x^(1/3*n)))*n*log(d*x^(2/3) + e) + 12*((b^3*log(c) + a*b^2)*
x^(1/3)*e + (b^3*d*log(c) + a*b^2*d)*x)*log(x^(1/3*n))^2 + (b^3*log(c)^3 +
3*a*b^2*log(c)^2 + 3*a^2*b*log(c) + a^3)*x^(1/3)*e + (b^3*d*log(c)^3 + 3*a*
b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x - 6*((b^3*log(c)^2 + 2*a*b^2*l
og(c) + a^2*b)*x^(1/3)*e + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x)
*log(x^(1/3*n)))/(d*x^3 + x^(7/3)*e), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*((d*x + x^(1/3)*e)/x)^n)^3 + 3*a*b^2*log(c*((d*x + x^(1
/3)*e)/x)^n)^2 + 3*a^2*b*log(c*((d*x + x^(1/3)*e)/x)^n) + a^3)/x^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**3/x**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^n\right)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))^3/x^2,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(2/3))^n))^3/x^2, x)
```

3.531 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}\,dx$

Optimal. Leaf size=784

$$\frac{16b^3n^3}{729x^3}-\frac{3088b^3dn^3}{27783ex^{7/3}}+\frac{221344b^3d^2n^3}{496125e^2x^{5/3}}-\frac{637984b^3d^3n^3}{297675e^3x}+\frac{3475504b^3d^4n^3}{99225e^4\sqrt[3]{x}}+\frac{3475504b^3d^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{99225e^{9/2}}+$$

```
[Out] 16/729*b^3*n^3/x^3-3088/27783*b^3*d*n^3/e/x^(7/3)+221344/496125*b^3*d^2*n^3
/e^2/x^(5/3)-637984/297675*b^3*d^3*n^3/e^3/x+3475504/99225*b^3*d^4*n^3/e^4/
x^(1/3)+3475504/99225*b^3*d^(9/2)*n^3*arctan(x^(1/3)*d^(1/2)/e^(1/2))/e^(9/
2)+4504/315*I*b^3*d^(9/2)*n^3*polylog(2,-1+2*e^(1/2)/(-I*x^(1/3)*d^(1/2)+e^
(1/2)))/e^(9/2)-8/81*b^2*n^2*(a+b*ln(c*(d+e/x^(2/3))^n))/x^3+128/441*b^2*d*
n^2*(a+b*ln(c*(d+e/x^(2/3))^n))/e/x^(7/3)-1144/1575*b^2*d^2*n^2*(a+b*ln(c*(
d+e/x^(2/3))^n))/e^2/x^(5/3)+1984/945*b^2*d^3*n^2*(a+b*ln(c*(d+e/x^(2/3))^n
))/e^3/x-4504/315*b^2*d^4*n^2*(a+b*ln(c*(d+e/x^(2/3))^n))/e^4/x^(1/3)-4504/
315*b^2*d^(9/2)*n^2*arctan(x^(1/3)*d^(1/2)/e^(1/2))*(a+b*ln(c*(d+e/x^(2/3))
^n))/e^(9/2)+2/9*b*n*(a+b*ln(c*(d+e/x^(2/3))^n))^2/x^3-2/7*b*d*n*(a+b*ln(c*
(d+e/x^(2/3))^n))^2/e/x^(7/3)+2/5*b*d^2*n*(a+b*ln(c*(d+e/x^(2/3))^n))^2/e^2
/x^(5/3)-2/3*b*d^3*n*(a+b*ln(c*(d+e/x^(2/3))^n))^2/e^3/x+2*b*d^4*n*(a+b*ln(
c*(d+e/x^(2/3))^n))^2/e^4/x^(1/3)-1/3*(a+b*ln(c*(d+e/x^(2/3))^n))^3/x^3-900
8/315*b^3*d^(9/2)*n^3*arctan(x^(1/3)*d^(1/2)/e^(1/2))*ln(2-2*e^(1/2)/(-I*x^
(1/3)*d^(1/2)+e^(1/2)))/e^(9/2)+4504/315*I*b^3*d^(9/2)*n^3*arctan(x^(1/3)*d
^(1/2)/e^(1/2))^2/e^(9/2)+2/3*b*d^5*n*Unintegrable((a+b*ln(c*(d+e/x^(2/3))^
n))^2/(e+d*x^(2/3))/x^(2/3),x)/e^4
```

Rubi [A]
time = 2.27, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x^4,x]
```

```
[Out] (16*b^3*n^3)/(729*x^3) - (3088*b^3*d*n^3)/(27783*e*x^(7/3)) + (221344*b^3*d
^2*n^3)/(496125*e^2*x^(5/3)) - (637984*b^3*d^3*n^3)/(297675*e^3*x) + (34755
04*b^3*d^4*n^3)/(99225*e^4*x^(1/3)) + (3475504*b^3*d^(9/2)*n^3*ArcTan[(Sqrt
[d]*x^(1/3))/Sqrt[e]])/(99225*e^(9/2)) + (((4504*I)/315)*b^3*d^(9/2)*n^3*Ar
cTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]^2)/e^(9/2) - (9008*b^3*d^(9/2)*n^3*ArcTan[(
Sqrt[d]*x^(1/3))/Sqrt[e]]*Log[2 - (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))
```

```
])/(315*e^(9/2)) - (8*b^2*n^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(81*x^3) +
(128*b^2*d*n^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(441*e*x^(7/3)) - (1144*b^
2*d^2*n^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(1575*e^2*x^(5/3)) + (1984*b^2*
d^3*n^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(945*e^3*x) - (4504*b^2*d^4*n^2*(
a + b*Log[c*(d + e/x^(2/3))^n]))/(315*e^4*x^(1/3)) - (4504*b^2*d^(9/2)*n^2*
ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]*(a + b*Log[c*(d + e/x^(2/3))^n]))/(315*e^
(9/2)) + (2*b*n*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(9*x^3) - (2*b*d*n*(a +
 b*Log[c*(d + e/x^(2/3))^n])^2)/(7*e*x^(7/3)) + (2*b*d^2*n*(a + b*Log[c*(d
+ e/x^(2/3))^n])^2)/(5*e^2*x^(5/3)) - (2*b*d^3*n*(a + b*Log[c*(d + e/x^(2/3
))^n])^2)/(3*e^3*x) + (2*b*d^4*n*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(e^4*x
^(1/3)) - (a + b*Log[c*(d + e/x^(2/3))^n])^3/(3*x^3) + (((4504*I)/315)*b^3*
d^(9/2)*n^3*PolyLog[2, -1 + (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/e^(
9/2) + (2*b*d^5*n*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x^2)^n])^2/(e
 + d*x^2), x], x, x^(1/3)])/e^4
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}\,dx = 3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^3}{x^{10}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{3x^3} - (2ben)\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)}{\left(d+\frac{e}{x^2}\right)x^{12}}\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{3x^3} - (2ben)\text{Subst}\left(\int \left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right.\right.}{ex^{10}}\right.\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{3x^3} - (2bn)\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{x^{10}}\right.$$

$$= \frac{2bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{9x^3} - \frac{2bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{7ex^{7/3}} + \frac{2bd^2}{}$$

$$= \frac{2bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{9x^3} - \frac{2bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{7ex^{7/3}} + \frac{2bd^2}{}$$

$$= \frac{2bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{9x^3} - \frac{2bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{7ex^{7/3}} + \frac{2bd^2}{}$$

$$= -\frac{8b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{81x^3} + \frac{128b^2dn^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{441ex^{7/3}}$$

$$= -\frac{8b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{81x^3} + \frac{128b^2dn^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{441ex^{7/3}}$$

$$= \frac{16b^3n^3}{729x^3} - \frac{256b^3dn^3}{3087ex^{7/3}} + \frac{2288b^3d^2n^3}{7875e^2x^{5/3}} - \frac{3968b^3d^3n^3}{2835e^3x} + \frac{9008b^3d^4n^3}{315e^4\sqrt[3]{x}} - \frac{8b^2n^2}{}$$

$$= \frac{16b^3n^3}{729x^3} - \frac{3088b^3dn^3}{27783ex^{7/3}} + \frac{7472b^3d^2n^3}{18375e^2x^{5/3}} - \frac{26704b^3d^3n^3}{14175e^3x} + \frac{30992b^3d^4n^3}{945e^4\sqrt[3]{x}} + \frac{9}{}$$

Mathematica [A] Leaf count is larger than twice the leaf count of optimal. 2726 vs. $2(784) = 1568$.
time = 5.98, size = 2726, normalized size = 3.48

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x^4,x]
```

```
[Out] (b^3*n^3*(32*e^4*Sqrt[-(e/(d*x^(2/3)))] - 32*d^4*x^(8/3) - 1584*d^3*e*x^2*H
ypergeometricPFQ[{-7/2, 1, 1, 1}, {2, 2, 2}, 1 + e/(d*x^(2/3))] - 1584*d^4*
x^(8/3)*HypergeometricPFQ[{-7/2, 1, 1, 1}, {2, 2, 2}, 1 + e/(d*x^(2/3))] +
4536*d^3*e*x^2*HypergeometricPFQ[{-5/2, 1, 1, 1}, {2, 2, 2}, 1 + e/(d*x^(2/
3))] + 4536*d^4*x^(8/3)*HypergeometricPFQ[{-5/2, 1, 1, 1}, {2, 2, 2}, 1 + e
/(d*x^(2/3))] - 3780*d^3*e*x^2*HypergeometricPFQ[{-3/2, 1, 1, 1}, {2, 2, 2}
, 1 + e/(d*x^(2/3))] - 3780*d^4*x^(8/3)*HypergeometricPFQ[{-3/2, 1, 1, 1},
{2, 2, 2}, 1 + e/(d*x^(2/3))] + 864*d^3*e*x^2*HypergeometricPFQ[{-7/2, 1, 1
, 1, 1}, {2, 2, 2, 2}, 1 + e/(d*x^(2/3))] + 864*d^4*x^(8/3)*HypergeometricP
FQ[{-7/2, 1, 1, 1, 1}, {2, 2, 2, 2}, 1 + e/(d*x^(2/3))] - 3024*d^3*e*x^2*Hy
pergeometricPFQ[{-5/2, 1, 1, 1, 1}, {2, 2, 2, 2}, 1 + e/(d*x^(2/3))] - 3024
*d^4*x^(8/3)*HypergeometricPFQ[{-5/2, 1, 1, 1, 1}, {2, 2, 2, 2}, 1 + e/(d*x
^(2/3))] + 3780*d^3*e*x^2*HypergeometricPFQ[{-3/2, 1, 1, 1, 1}, {2, 2, 2, 2
}, 1 + e/(d*x^(2/3))] + 3780*d^4*x^(8/3)*HypergeometricPFQ[{-3/2, 1, 1, 1,
1}, {2, 2, 2, 2}, 1 + e/(d*x^(2/3))] - 1890*d^3*e*x^2*HypergeometricPFQ[{-1
/2, 1, 1, 1, 1}, {2, 2, 2, 2}, 1 + e/(d*x^(2/3))] - 1890*d^4*x^(8/3)*Hyperg
eometricPFQ[{-1/2, 1, 1, 1, 1}, {2, 2, 2, 2}, 1 + e/(d*x^(2/3))] - (288*e^4
*Log[d + e/x^(2/3)])/Sqrt[-(e/(d*x^(2/3)))] + 48*e^4*Sqrt[-(e/(d*x^(2/3)))]
*Log[d + e/x^(2/3)] + 240*d^4*x^(8/3)*Log[d + e/x^(2/3)] + 3780*d^3*e*x^2*H
ypergeometricPFQ[{-3/2, 1, 1}, {2, 2}, 1 + e/(d*x^(2/3))]*Log[d + e/x^(2/3)
] + 3780*d^4*x^(8/3)*HypergeometricPFQ[{-3/2, 1, 1}, {2, 2}, 1 + e/(d*x^(2/
3))]*Log[d + e/x^(2/3)] - 864*d^3*e*x^2*HypergeometricPFQ[{-7/2, 1, 1, 1},
{2, 2, 2}, 1 + e/(d*x^(2/3))]*Log[d + e/x^(2/3)] - 864*d^4*x^(8/3)*Hypergeo
metricPFQ[{-7/2, 1, 1, 1}, {2, 2, 2}, 1 + e/(d*x^(2/3))]*Log[d + e/x^(2/3)]
 + 3024*d^3*e*x^2*HypergeometricPFQ[{-5/2, 1, 1, 1}, {2, 2, 2}, 1 + e/(d*x^
(2/3))]*Log[d + e/x^(2/3)] + 3024*d^4*x^(8/3)*HypergeometricPFQ[{-5/2, 1, 1
, 1}, {2, 2, 2}, 1 + e/(d*x^(2/3))]*Log[d + e/x^(2/3)] - 3780*d^3*e*x^2*Hyp
ergeometricPFQ[{-3/2, 1, 1, 1}, {2, 2, 2}, 1 + e/(d*x^(2/3))]*Log[d + e/x^(
2/3)] - 3780*d^4*x^(8/3)*HypergeometricPFQ[{-3/2, 1, 1, 1}, {2, 2, 2}, 1 +
e/(d*x^(2/3))]*Log[d + e/x^(2/3)] + 1890*d^3*e*x^2*HypergeometricPFQ[{-1/2,
 1, 1, 1}, {2, 2, 2}, 1 + e/(d*x^(2/3))]*Log[d + e/x^(2/3)] + 1890*d^4*x^(8
/3)*HypergeometricPFQ[{-1/2, 1, 1, 1}, {2, 2, 2}, 1 + e/(d*x^(2/3))]*Log[d
+ e/x^(2/3)] + (252*e^4*Log[d + e/x^(2/3)]^2)/(-(e/(d*x^(2/3))))^(3/2) - (3
6*e^4*Log[d + e/x^(2/3)]^2)/Sqrt[-(e/(d*x^(2/3)))] + 68*e^4*Sqrt[-(e/(d*x^(
2/3)))]*Log[d + e/x^(2/3)]^2 - 284*d^4*x^(8/3)*Log[d + e/x^(2/3)]^2 + 1890*
d^3*e*x^2*HypergeometricPFQ[{-3/2, 1, 1}, {2, 2}, 1 + e/(d*x^(2/3))]*Log[d
```

```
+ e/x^(2/3)]^2 + 1890*d^4*x^(8/3)*HypergeometricPFQ[{-3/2, 1, 1}, {2, 2}, 1
 + e/(d*x^(2/3))]*Log[d + e/x^(2/3)]^2 - 945*d^3*e*x^2*HypergeometricPFQ[{-
1/2, 1, 1}, {2, 2}, 1 + e/(d*x^(2/3))]*Log[d + e/x^(2/3)]^2 - 945*d^4*x^(8/
3)*HypergeometricPFQ[{-1/2, 1, 1}, {2, 2}, 1 + e/(d*x^(2/3))]*Log[d + e/x^(
2/3)]^2 - 70*e^4*Sqrt[-(e/(d*x^(2/3)))]*Log[d + e/x^(2/3)]^3 + 70*d^4*x^(8/
3)*Log[d + e/x^(2/3)]^3 - 1512*d^3*(e + d*x^(2/3))*x^2*HypergeometricPFQ[{-
5/2, 1, 1}, {2, 2}, 1 + e/(d*x^(2/3))]*(1 + 3*Log[d + e/x^(2/3)] + Log[d +
e/x^(2/3)]^2) + 144*d^3*(e + d*x^(2/3))*x^2*HypergeometricPFQ[{-7/2, 1, 1},
 {2, 2}, 1 + e/(d*x^(2/3))]*(6 + 11*Log[d + e/x^(2/3)] + 3*Log[d + e/x^(2/3
)]^2)))/(210*e^4*Sqrt[-(e/(d*x^(2/3)))]*x^3) - (2*b*d*n*(a - b*n*Log[d + e/
x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/(7*e*x^(7/3)) + (2*b*d^2*n*(a - b
*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/(5*e^2*x^(5/3)) - (2
*b*d^3*n*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/(3*e^
3*x) + (2*b*d^4*n*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])
^2)/(e^4*x^(1/3)) + (2*b*d^(9/2)*n*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]*(a - b
*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/e^(9/2) - (b*n*Log[d
 + e/x^(2/3)]*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/
x^3 - ((a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2*(3*a - 2
*b*n - 3*b*n*Log[d + e/x^(2/3)] + 3*b*Log[c*(d + e/x^(2/3))^n]))/(9*x^3) +
(b^2*n^2*(-a + b*n*Log[d + e/x^(2/3)] - b*Log[c*(d + e/x^(2/3))^n])*(9800*e
^(9/2) - 28800*d*e^(7/2)*x^(2/3) + 72072*d^2*e^(5/2)*x^(4/3) - 208320*d^3*e
^(3/2)*x^2 + 1418760*d^4*Sqrt[e]*x^(8/3) - 1418760*d^(9/2)*x^3*ArcTan[Sqrt[
e]/(Sqrt[d]*x^(1/3))] - 44100*e^(9/2)*Log[d + e/x^(2/3)] + 56700*d*e^(7/2)*
x^(2/3)*Log[d + e/x^(2/3)] - 79380*d^2*e^(5/2)*x^(4/3)*Log[d + e/x^(2/3)] +
 132300*d^3*e^(3/2)*x^2*Log[d + e/x^(2/3)] - 396900*d^4*Sqrt[e]*x^(8/3)*Log
[d + e/x^(2/3)] + 99225*e^(9/2)*Log[d + e/x^(2/3)]^2 - 198450*(-d)^(9/2)*x^
3*Log[d + e/x^(2/3)]*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 99225*(-d)^(9/2)*x^3
*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]^2 + 198450*(-d)^(9/2)*x^3*Log[d + e/x^(2/3
)]*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] - 99225*(-d)^(9/2)*x^3*Log[Sqrt[e] + Sqr
t[-d]*x^(1/3)]^2 - 198450*(-d)^(9/2)*x^3*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*Lo
g[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] + 19845...
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{3}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))^3/x^4,x)
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))^3/x^4,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^3*n^3*log(d*x^(2/3) + e)^3/x^3 - integrate(-1/3*((2*b^3*d*n*x + 9*(b
^3*log(c) + a*b^2)*x^(1/3)*e + 9*(b^3*d*log(c) + a*b^2*d)*x - 18*(b^3*d*x +
 b^3*x^(1/3)*e)*log(x^(1/3*n)))*n^2*log(d*x^(2/3) + e)^2 - 24*(b^3*d*x + b^
3*x^(1/3)*e)*log(x^(1/3*n))^3 + 9*(4*(b^3*d*x + b^3*x^(1/3)*e)*log(x^(1/3*n
))^2 + (b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*b)*x^(1/3)*e + (b^3*d*log(c)^2
+ 2*a*b^2*d*log(c) + a^2*b*d)*x - 4*((b^3*log(c) + a*b^2)*x^(1/3)*e + (b^3*
d*log(c) + a*b^2*d)*x)*log(x^(1/3*n)))*n*log(d*x^(2/3) + e) + 36*((b^3*log(
c) + a*b^2)*x^(1/3)*e + (b^3*d*log(c) + a*b^2*d)*x)*log(x^(1/3*n))^2 + 3*(b
^3*log(c)^3 + 3*a*b^2*log(c)^2 + 3*a^2*b*log(c) + a^3)*x^(1/3)*e + 3*(b^3*d
*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x - 18*((b^3*log
(c)^2 + 2*a*b^2*log(c) + a^2*b)*x^(1/3)*e + (b^3*d*log(c)^2 + 2*a*b^2*d*log
(c) + a^2*b*d)*x)*log(x^(1/3*n)))/(d*x^5 + x^(13/3)*e), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*((d*x + x^(1/3)*e)/x)^n)^3 + 3*a*b^2*log(c*((d*x + x^(1
/3)*e)/x)^n)^2 + 3*a^2*b*log(c*((d*x + x^(1/3)*e)/x)^n) + a^3)/x^4, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**3/x**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7318 deep
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3/x^4, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))^3/x^4,x)
[Out] int((a + b*log(c*(d + e/x^(2/3))^n))^3/x^4, x)
```

3.532 $\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$

Optimal. Leaf size=730

$$\frac{2^{-2-3p}e^{-\frac{8a}{b}}\Gamma\left(1+p,-\frac{8\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^8e^8}-\ldots$$

```
[Out] 2^(-2-3*p)*GAMMA(1+p,-8*(a+b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2)
)))^p/c^8/e^8/exp(8*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))))/b)^p)-2*d*GAMMA(1+p,-
7*(a+b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(7^p)/c^7/e^8/ex
p(7*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))))/b)^p)+7*d^2*GAMMA(1+p,-6*(a+b*ln(c*(d
+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(6^p)/c^6/e^8/exp(6*a/b)/(((-a
-b*ln(c*(d+e*x^(1/2))))/b)^p)-14*d^3*GAMMA(1+p,-5*(a+b*ln(c*(d+e*x^(1/2))))
/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(5^p)/c^5/e^8/exp(5*a/b)/(((-a-b*ln(c*(d+e*
x^(1/2))))/b)^p)+35*2^(-1-2*p)*d^4*GAMMA(1+p,-4*(a+b*ln(c*(d+e*x^(1/2))))/b
)*(a+b*ln(c*(d+e*x^(1/2))))^p/c^4/e^8/exp(4*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))
))/b)^p)-14*d^5*GAMMA(1+p,-3*(a+b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^
(1/2))))^p/(3^p)/c^3/e^8/exp(3*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))))/b)^p)+7*d^
6*GAMMA(1+p,-2*(a+b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(2^
p)/c^2/e^8/exp(2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))))/b)^p)-2*d^7*GAMMA(1+p,(-
a-b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/c/e^8/exp(a/b)/(((-
a-b*ln(c*(d+e*x^(1/2))))/b)^p)
```

Rubi [A]

time = 0.87, antiderivative size = 730, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2504, 2448, 2436, 2336, 2212, 2437, 2346}

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

```
[Out] (2^(-2 - 3*p)*Gamma[1 + p, (-8*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Lo
g[c*(d + e*Sqrt[x])])^p)/(c^8*e^8*E^((8*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x
])])/b))^p) - (2*d*Gamma[1 + p, (-7*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a +
 b*Log[c*(d + e*Sqrt[x])])^p)/(7^p*c^7*e^8*E^((7*a)/b)*(-((a + b*Log[c*(d +
 e*Sqrt[x])])/b))^p) + (7*d^2*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*Sqrt[x])
]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(6^p*c^6*e^8*E^((6*a)/b)*(-((a + b
*Log[c*(d + e*Sqrt[x])])/b))^p) - (14*d^3*Gamma[1 + p, (-5*(a + b*Log[c*(d
+ e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(5^p*c^5*e^8*E^((5*a)/
b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p) + (35*2^(-1 - 2*p)*d^4*Gamma[1
+ p, (-4*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^
```

```
p)/(c^4*e^8*E^((4*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p) - (14*d^5*
Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sqr
t[x])])^p)/(3^p*c^3*e^8*E^((3*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p
) + (7*d^6*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c
*(d + e*Sqrt[x])])^p)/(2^p*c^2*e^8*E^((2*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[
x])])/b))^p) - (2*d^7*Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])])/b)]*(a
+ b*Log[c*(d + e*Sqrt[x])])^p)/(c*e^8*E^(a/b)*(-((a + b*Log[c*(d + e*Sqrt[x
])])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
```

```
d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx &= 2\text{Subst}\left(\int x^7(a+b\log(c(d+ex)))^p\,dx,x,\sqrt{x}\right)\\
&= 2\text{Subst}\left(\int\left(-\frac{d^7(a+b\log(c(d+ex)))^p}{e^7}+\frac{7d^6(d+ex)(a+b\log(c(\right.\right.}{e^7}\\
&= \frac{2\text{Subst}\left(\int(d+ex)^7(a+b\log(c(d+ex)))^p\,dx,x,\sqrt{x}\right)}{e^7}-\frac{(14d)\text{Subst}}{}\\
&= \frac{2\text{Subst}\left(\int x^7(a+b\log(cx))^p\,dx,x,d+e\sqrt{x}\right)}{e^8}-\frac{(14d)\text{Subst}\left(\int x^6(a\right.}{}\\
&= \frac{2\text{Subst}\left(\int e^{8x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{c^8e^8}-\frac{(14d)\text{Subst}\left(\int e^{7}\right.}{}\\
&= \frac{2^{-2-3p}e^{-\frac{8a}{b}}\Gamma\left(1+p,-\frac{8\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{\right.\right.\right.}{c^8e^8}
\end{aligned}$$

Mathematica [A]
time = 1.23, size = 435, normalized size = 0.60

$$\frac{2^{-2-3p}105^{-p}e^{-\frac{8a}{b}}\left(105^p\Gamma\left(1+p,-\frac{8\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)-8^{1+p}15^pcde^{a/b}\Gamma\left(1+p,-\frac{7\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)+5^p28^{1+p}c^2d^2e^{\frac{2a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)-3^p56^{1+p}c^3d^3e^{\frac{3a}{b}}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)+3^p70^{1+p}c^4d^4e^{\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)-5^p56^{1+p}c^5d^5e^{\frac{5a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)+15^p28^{1+p}c^6d^6e^{\frac{6a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)-8^{1+p}105^pc^7d^7e^{\frac{7a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^8e^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

```
[Out] (2^(-2 - 3*p)*(105^p*Gamma[1 + p, (-8*(a + b*Log[c*(d + e*Sqrt[x])]))/b] -
8^(1 + p)*15^p*c*d*E^(a/b)*Gamma[1 + p, (-7*(a + b*Log[c*(d + e*Sqrt[x])]))
/b] + 5^p*28^(1 + p)*c^2*d^2*E^((2*a)/b)*Gamma[1 + p, (-6*(a + b*Log[c*(d +
 e*Sqrt[x])]))/b] - 3^p*56^(1 + p)*c^3*d^3*E^((3*a)/b)*Gamma[1 + p, (-5*(a
+ b*Log[c*(d + e*Sqrt[x])]))/b] + 3^p*70^(1 + p)*c^4*d^4*E^((4*a)/b)*Gamma[
1 + p, (-4*(a + b*Log[c*(d + e*Sqrt[x])]))/b] - 5^p*56^(1 + p)*c^5*d^5*E^((
```

```
5*a)/b)*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])]))/b] + 15^p*28^(1 +
p)*c^6*d^6*E^((6*a)/b)*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])]))/b]
- 8^(1 + p)*105^p*c^7*d^7*E^((7*a)/b)*Gamma[1 + p, -((a + b*Log[c*(d + e*Sq
rt[x])])/b)])*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(105^p*c^8*e^8*E^((8*a)/b)*
(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int x^{3}\left(a+b\ln\left(c\left(d+e\sqrt{x}\,\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(1/2))))^p,x)
[Out] int(x^3*(a+b*ln(c*(d+e*x^(1/2))))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="maxima")
[Out] integrate((b*log((sqrt(x)*e + d)*c) + a)^p*x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="fricas")
[Out] integral((b*log(c*sqrt(x)*e + c*d) + a)^p*x^3, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(1/2))))**p,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8010 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="giac")
[Out] integrate((b*log((sqrt(x)*e + d)*c) + a)^p*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\left(a+b\,\ln\left(c\,\left(d+e\,\sqrt{x}\,\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e*x^(1/2))))^p,x)
[Out] int(x^3*(a + b*log(c*(d + e*x^(1/2))))^p, x)
```

3.533 $\int x^2 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p \, dx$

Optimal. Leaf size=551

$$\frac{2^{-p}3^{-1-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^6e^6} - 2 \cdots$$

```
[Out] 3^(-1-p)*GAMMA(1+p,-6*(a+b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2)))
)^p/(2^p)/c^6/e^6/exp(6*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))))/b)^p)-2*d*GAMMA(1
+p,-5*(a+b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(5^p)/c^5/e^
6/exp(5*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))))/b)^p)+5*d^2*GAMMA(1+p,-4*(a+b*ln(
c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(4^p)/c^4/e^6/exp(4*a/b)/(
((-a-b*ln(c*(d+e*x^(1/2))))/b)^p)-20*3^(-1-p)*d^3*GAMMA(1+p,-3*(a+b*ln(c*(d
+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/c^3/e^6/exp(3*a/b)/(((-a-b*ln(
c*(d+e*x^(1/2))))/b)^p)+5*d^4*GAMMA(1+p,-2*(a+b*ln(c*(d+e*x^(1/2))))/b)*(a+
b*ln(c*(d+e*x^(1/2))))^p/(2^p)/c^2/e^6/exp(2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2)
)))/b)^p)-2*d^5*GAMMA(1+p,(-a-b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1
/2))))^p/c/e^6/exp(a/b)/(((-a-b*ln(c*(d+e*x^(1/2))))/b)^p)
```

Rubi [A]
time = 0.57, antiderivative size = 551, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2504, 2448, 2436, 2336, 2212, 2437, 2346}

$$\frac{2^{-p}3^{-p-1}e^{-\frac{6a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^6e^6}-\frac{2d5^{-p}e^{-\frac{5a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^5e^6}+\frac{5d^24^{-p}e^{-\frac{4a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^4e^6}-\frac{20d^33^{-p-1}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^3e^6}+\frac{5d^42^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^2e^6}-\frac{2d^5e^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)}{ce^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

```
[Out] (3^(-1 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[
c*(d + e*Sqrt[x])])^p)/(2^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e*Sqrt
[x])])/b))^p) - (2*d*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a
 + b*Log[c*(d + e*Sqrt[x])])^p)/(5^p*c^5*e^6*E^((5*a)/b)*(-((a + b*Log[c*(d
 + e*Sqrt[x])])/b))^p) + (5*d^2*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*Sqrt[x
])]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(4^p*c^4*e^6*E^((4*a)/b)*(-((a +
 b*Log[c*(d + e*Sqrt[x])])/b))^p) - (20*3^(-1 - p)*d^3*Gamma[1 + p, (-3*(a
+ b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(c^3*e^6*
E^((3*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p) + (5*d^4*Gamma[1 + p,
(-2*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(2
^p*c^2*e^6*E^((2*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p) - (2*d^5*Ga
mma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])])/b)]*(a + b*Log[c*(d + e*Sqrt[x]
)])^p)/(c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx = 2\text{Subst}\left(\int x^5(a+b\log(c(d+ex)))^p\,dx,x,\sqrt{x}\right)$$
$$= 2\text{Subst}\left(\int\left(-\frac{d^5(a+b\log(c(d+ex)))^p}{e^5}+\frac{5d^4(d+ex)(a+b\log(c(d\right.\right.$$
$$= \frac{2\text{Subst}\left(\int(d+ex)^5(a+b\log(c(d+ex)))^p\,dx,x,\sqrt{x}\right)}{e^5}-\frac{(10d)\text{Subst}\left(\right.}{}$$
$$= \frac{2\text{Subst}\left(\int x^5(a+b\log(cx))^p\,dx,x,d+e\sqrt{x}\right)}{e^6}-\frac{(10d)\text{Subst}\left(\int x^4(a+\right.}{}$$
$$= \frac{2\text{Subst}\left(\int e^{6x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{c^6e^6}-\frac{(10d)\text{Subst}\left(\int e^{5x}\right.}{}$$
$$= \frac{2^{-p}3^{-1-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{\right.\right.\right.}{c^6e^6}$$

Mathematica [A]
time = 1.13, size = 325, normalized size = 0.59

$$\frac{3^{-1-p}20^{-p}e^{-\frac{6a}{b}}\left(10^p\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)-cde^{a/b}\left(2^{1+2p}3^{1+p}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)+5^pcde^{a/b}\left(-5\,3^{1+p}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)+2^pcde^{a/b}\left(5\,2^{2+p}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)+3^{1+p}cde^{a/b}\left(-5\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)+2^{1+p}cde^{a/b}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)\right)\right)\right)\right)\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^6e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

```
[Out] (3^(-1 - p)*(10^p*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*Sqrt[x])]))/b] - c*d
*E^(a/b)*(2^(1 + 2*p)*3^(1 + p)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*Sqrt[x
])]))/b] + 5^p*c*d*E^(a/b)*(-5*3^(1 + p)*Gamma[1 + p, (-4*(a + b*Log[c*(d +
 e*Sqrt[x])]))/b] + 2^p*c*d*E^(a/b)*(5*2^(2 + p)*Gamma[1 + p, (-3*(a + b*Lo
g[c*(d + e*Sqrt[x])]))/b] + 3^(1 + p)*c*d*E^(a/b)*(-5*Gamma[1 + p, (-2*(a +
 b*Log[c*(d + e*Sqrt[x])]))/b] + 2^(1 + p)*c*d*E^(a/b)*Gamma[1 + p, -((a +
b*Log[c*(d + e*Sqrt[x])])/b)])))))*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(20^p*
c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(1/2))))^p,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e*x^(1/2))))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="maxima")
[Out] integrate((b*log((sqrt(x)*e + d)*c) + a)^p*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="fricas")
[Out] integral((b*log(c*sqrt(x)*e + c*d) + a)^p*x^2, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/2))))**p,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="giac")
[Out] integrate((b*log((sqrt(x)*e + d)*c) + a)^p*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(a+b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)\right)\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e*x^(1/2))))^p,x)
[Out] int(x^2*(a + b*log(c*(d + e*x^(1/2))))^p, x)
```

3.534 $\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$

Optimal. Leaf size=360

$$\frac{2^{-1-2p}e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^4e^4}-\frac{2\,3^{-}\cdots}{\cdots}$$

```
[Out] 2^(-1-2*p)*GAMMA(1+p,-4*(a+b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2)
)))^p/c^4/e^4/exp(4*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))))/b)^p)-2*d*GAMMA(1+p,-
3*(a+b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(3^p)/c^3/e^4/ex
p(3*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))))/b)^p)+3*d^2*GAMMA(1+p,-2*(a+b*ln(c*(d
+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(2^p)/c^2/e^4/exp(2*a/b)/(((-a
-b*ln(c*(d+e*x^(1/2))))/b)^p)-2*d^3*GAMMA(1+p,(-a-b*ln(c*(d+e*x^(1/2))))/b)
*(a+b*ln(c*(d+e*x^(1/2))))^p/c/e^4/exp(a/b)/(((-a-b*ln(c*(d+e*x^(1/2))))/b)
^p)
```

Rubi [A]

time = 0.36, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2504, 2448, 2436, 2336, 2212, 2437, 2346}

$$\frac{2^{-2p-1}e^{-\frac{4a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^4e^4}-\frac{2d3^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^3e^4}+\frac{3d^22^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^2e^4}-\frac{2d^3e^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)}{ce^4}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

```
[Out] (2^(-1 - 2*p)*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Lo
g[c*(d + e*Sqrt[x])])^p)/(c^4*e^4*E^((4*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x
])])/b))^p) - (2*d*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a +
 b*Log[c*(d + e*Sqrt[x])])^p)/(3^p*c^3*e^4*E^((3*a)/b)*(-((a + b*Log[c*(d +
 e*Sqrt[x])])/b))^p) + (3*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])
]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(2^p*c^2*e^4*E^((2*a)/b)*(-((a + b
*Log[c*(d + e*Sqrt[x])])/b))^p) - (2*d^3*Gamma[1 + p, -((a + b*Log[c*(d + e
*Sqrt[x])])/b)]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(c*e^4*E^(a/b)*(-((a + b*
Log[c*(d + e*Sqrt[x])])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\,\right)\right)\right)^p\,dx = 2\text{Subst}\left(\int x^3(a+b\log(c(d+ex)))^p\,dx,x,\sqrt{x}\,\right)$$
$$= 2\text{Subst}\left(\int\left(-\frac{d^3(a+b\log(c(d+ex)))^p}{e^3}+\frac{3d^2(d+ex)(a+b\log(c(d+\right.\right.$$
$$= \frac{2\text{Subst}\left(\int(d+ex)^3(a+b\log(c(d+ex)))^p\,dx,x,\sqrt{x}\,\right)}{e^3}-\frac{(6d)\text{Subst}\left(\int(\right.}{}$$
$$= \frac{2\text{Subst}\left(\int x^3(a+b\log(cx))^p\,dx,x,d+e\sqrt{x}\,\right)}{e^4}-\frac{(6d)\text{Subst}\left(\int x^2(a+b\right.}{}$$
$$= \frac{2\text{Subst}\left(\int e^{4x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt{x}\,\right)\right)\right)}{c^4e^4}-\frac{(6d)\text{Subst}\left(\int e^{3x}(a\right.}{}$$
$$= \frac{2^{-1-2p}e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\,\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\,\right)\right.\right.}{c^4e^4}$$

Mathematica [A]
time = 0.64, size = 229, normalized size = 0.64

$$\frac{2^{-1-2p}3^{-p}e^{-\frac{4a}{b}}\left(3^p\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)-2^{1+p}cde^{a/b}\left(2^{1+p}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)+3^pcde^{a/b}\left(-3\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)+2^{1+p}cde^{a/b}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)\right)\right)\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^4e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

```
[Out] (2^(-1 - 2*p)*(3^p*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*Sqrt[x])]))/b] - 2^
(1 + p)*c*d*E^(a/b)*(2^(1 + p)*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x]
)]))/b] + 3^p*c*d*E^(a/b)*(-3*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])
]))/b] + 2^(1 + p)*c*d*E^(a/b)*Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])]
)/b)])))*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(3^p*c^4*e^4*E^((4*a)/b)*(-((a +
 b*Log[c*(d + e*Sqrt[x])])/b))^p)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\ln\left(c\left(d+e\sqrt{x}\,\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e*x^(1/2))))^p,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e*x^(1/2))))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="maxima")
[Out] integrate((b*log((sqrt(x)*e + d)*c) + a)^p*x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="fricas")
[Out] integral((b*log(c*sqrt(x)*e + c*d) + a)^p*x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/2))))**p,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="giac")
[Out] integrate((b*log((sqrt(x)*e + d)*c) + a)^p*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e*x^(1/2))))^p,x)
[Out] int(x*(a + b*log(c*(d + e*x^(1/2))))^p, x)
```

3.535 $\int \left(a+b\log \left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$

Optimal. Leaf size=174

$$\frac{2^{-p}e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^2e^2}-\frac{2de^{-\frac{a}{b}}\Gamma \cdots}{}$$

```
[Out] GAMMA(1+p,-2*(a+b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(2^p)
/c^2/e^2/exp(2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))))/b)^p)-2*d*GAMMA(1+p,(-a-b*
ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/c/e^2/exp(a/b)/(((-a-b*
ln(c*(d+e*x^(1/2))))/b)^p)
```

Rubi [A]
time = 0.15, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {2501, 2448, 2436, 2336, 2212, 2437, 2346}

$$\frac{2^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^2e^2}-\frac{2de^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)}{ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

```
[Out] (Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sq
rt[x])])^p)/(2^p*c^2*e^2*E^((2*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^
p) - (2*d*Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])])/b)]*(a + b*Log[c*(d
 + e*Sqrt[x])])^p)/(c*e^2*E^(a/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
```

```
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2501

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx = 2\text{Subst}\left(\int x(a+b\log(c(d+ex)))^p\,dx, x, \sqrt{x}\right)$$
$$= 2\text{Subst}\left(\int \left(-\frac{d(a+b\log(c(d+ex)))^p}{e} + \frac{(d+ex)(a+b\log(c(d+ex)))\right.$$
$$= \frac{2\text{Subst}\left(\int (d+ex)(a+b\log(c(d+ex)))^p\,dx, x, \sqrt{x}\right)}{e} - \frac{(2d)\text{Subst}\left(\int (a-\right.}{}$$
$$= \frac{2\text{Subst}\left(\int x(a+b\log(cx))^p\,dx, x, d+e\sqrt{x}\right)}{e^2} - \frac{(2d)\text{Subst}\left(\int (a+b\log(c\right.}{e^2}$$
$$= \frac{2\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{c^2e^2} - \frac{(2d)\text{Subst}\left(\int e^x(a+\right.}{}$$
$$= \frac{2^{-p}e^{-\frac{2a}{b}}\Gamma\left(1+p, -\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{c^2e^2}$$

Mathematica [A]
time = 0.33, size = 130, normalized size = 0.75

$$\frac{2^{-p}e^{-\frac{2a}{b}}\left(\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)-2^{1+p}cde^{a/b}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

```
[Out] ((Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])]))/b] - 2^(1 + p)*c*d*E^(a/
b)*Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])])/b)])*(a + b*Log[c*(d + e*S
qrt[x])])^p)/(2^p*c^2*e^2*E^((2*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))
^p)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\ln\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))))^p,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((sqrt(x)*e + d)*c) + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*sqrt(x)*e + c*d) + a)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \log \left(c \left(d + e \sqrt{x}\right)\right)\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))))**p,x)
```

```
[Out] Integral((a + b*log(c*(d + e*sqrt(x))))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((sqrt(x)*e + d)*c) + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)\right)\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))))^p,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/2))))^p, x)
```

3.536 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(1/2))))^p/x,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])])^p/x,x]
[Out] 2*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x)])^p/x, x], x, Sqrt[x]]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x}\,dx = 2\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)))^p}{x}\,dx,x,\sqrt{x}\right)$$

Mathematica [A]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])])^p/x,x]
[Out] Integrate[(a + b*Log[c*(d + e*Sqrt[x])])^p/x, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))))^p/x,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))))^p/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p/x,x, algorithm="maxima")
```

```
[Out] integrate((b*log((sqrt(x)*e + d)*c) + a)^p/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p/x,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*sqrt(x)*e + c*d) + a)^p/x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))))**p/x,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((sqrt(x)*e + d)*c) + a)^p/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,\sqrt{x}\,\right)\right)\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))))^p/x,x)
[Out] int((a + b*log(c*(d + e*x^(1/2))))^p/x, x)
```

3.537 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(1/2))))^p/x^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])])^p/x^2,x]
[Out] 2*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x)])^p/x^3, x], x, Sqrt[x]]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}\,dx = 2\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)))^p}{x^3}\,dx,x,\sqrt{x}\right)$$

Mathematica [A]
time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])])^p/x^2,x]
[Out] Integrate[(a + b*Log[c*(d + e*Sqrt[x])])^p/x^2, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))))^p/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))))^p/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*log((sqrt(x)*e + d)*c) + a)^p/x^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*sqrt(x)*e + c*d) + a)^p/x^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))))**p/x**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((sqrt(x)*e + d)*c) + a)^p/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))))^p/x^2,x)
[Out] int((a + b*log(c*(d + e*x^(1/2))))^p/x^2, x)
```

3.538 $\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=907

$$\frac{2^{-2(1+p)}e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{c^4e^8}$$

```
[Out] GAMMA(1+p,-4*(a+b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p/(
2^(2+2*p))/c^4/e^8/exp(4*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)+7*d^2*GA
MMA(1+p,-3*(a+b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p/(3^
p)/c^3/e^8/exp(3*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)+35*2^(-1-p)*d^4*
GAMMA(1+p,-2*(a+b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p/c
^2/e^8/exp(2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)+7*d^6*GAMMA(1+p,(-a-
b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p/c/e^8/exp(a/b)/((
(-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)-2^(1+p)*d*GAMMA(1+p,-7/2*(a+b*ln(c*(d+e*
x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p*(d+e*x^(1/2))^7/(7^p)/e^8/exp
(7/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)/(c*(d+e*x^(1/2))^2)^(7/2)-7*
2^(1+p)*d^3*GAMMA(1+p,-5/2*(a+b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^
(1/2))^2))^p*(d+e*x^(1/2))^5/(5^p)/e^8/exp(5/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/
2))^2))/b)^p)/(c*(d+e*x^(1/2))^2)^(5/2)-7*2^(1+p)*d^5*GAMMA(1+p,-3/2*(a+b*l
n(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p*(d+e*x^(1/2))^3/(3^p
)/e^8/exp(3/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)/(c*(d+e*x^(1/2))^2)
^(3/2)-2^(1+p)*d^7*GAMMA(1+p,1/2*(-a-b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*
(d+e*x^(1/2))^2))^p*(d+e*x^(1/2))/e^8/exp(1/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2
))^2))/b)^p)/(c*(d+e*x^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.94, antiderivative size = 907, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {2504, 2448, 2436, 2337, 2212, 2437, 2347}

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
```

```
[Out] (Gamma[1 + p, (-4*(a + b*Log[c*(d + e*Sqrt[x])^2]))/b]*(a + b*Log[c*(d + e*
Sqrt[x])^2])^p)/(2^(2*(1 + p))*c^4*e^8*E^((4*a)/b)*(-((a + b*Log[c*(d + e*S
qrt[x])^2])/b))^p) - (2^(1 + p)*d*(d + e*Sqrt[x])^7*Gamma[1 + p, (-7*(a + b
*Log[c*(d + e*Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(7^p
*e^8*E^((7*a)/(2*b))*(c*(d + e*Sqrt[x])^2)^(7/2)*(-((a + b*Log[c*(d + e*Sqr
```

```
t[x])^2])/b))^p) + (7*d^2*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])^2])
)/b]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(3^p*c^3*e^8*E^((3*a)/b)*(-((a + b
*Log[c*(d + e*Sqrt[x])^2])/b))^p) - (7*2^(1 + p)*d^3*(d + e*Sqrt[x])^5*Gamm
a[1 + p, (-5*(a + b*Log[c*(d + e*Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e*S
qrt[x])^2])^p)/(5^p*e^8*E^((5*a)/(2*b))*(c*(d + e*Sqrt[x])^2)^(5/2)*(-((a +
 b*Log[c*(d + e*Sqrt[x])^2])/b))^p) + (35*2^(-1 - p)*d^4*Gamma[1 + p, (-2*(
a + b*Log[c*(d + e*Sqrt[x])^2]))/b]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(c^
2*e^8*E^((2*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) - (7*2^(1 + p)
*d^5*(d + e*Sqrt[x])^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])^2]))/(
2*b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(3^p*e^8*E^((3*a)/(2*b))*(c*(d +
e*Sqrt[x])^2)^(3/2)*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) + (7*d^6*Gam
ma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])^2])/b)]*(a + b*Log[c*(d + e*Sqrt[x
])^2])^p)/(c*e^8*E^(a/b)*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) - (2^(1
 + p)*d^7*(d + e*Sqrt[x])*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e*Sqrt[x])^2]
)/b]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(e^8*E^(a/(2*b))*Sqrt[c*(d + e*Sqr
t[x])^2]*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
```

```
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=2\text{Subst}\left(\int x^7\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt{x}\right)$$

$$=2\text{Subst}\left(\int\left(-\frac{d^7\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^7}+\frac{7d^6(d+ex)\left(a+b\log\right.}{e^7}\right.\right.$$

$$=\frac{2\text{Subst}\left(\int(d+ex)^7\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt{x}\right)}{e^7}-\frac{(14d)\text{Sul}}{}$$

$$=\frac{2\text{Subst}\left(\int x^7\left(a+b\log\left(cx^2\right)\right)^p\,dx,x,d+e\sqrt{x}\right)}{e^8}-\frac{(14d)\text{Subst}\left(\int x^6\right.}{}$$

$$=\frac{\text{Subst}\left(\int e^{4x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{c^4e^8}+\frac{\left(21d^2\right)\text{Subst}\left(\right.}{}$$

$$=\frac{4^{-1-p}e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\right.\right.\right.}{c^4e^8}$$

Mathematica [F]
time = 0.35, size = 0, normalized size = 0.00

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
```

```
[Out] Integrate[x^3*(a + b*Log[c*(d + e*Sqrt[x])^2])^p, x]
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^{3}\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^{2}\right)\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(1/2))^2))^p,x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e*x^(1/2))^2))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((sqrt(x)*e + d)^2*c) + a)^p*x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(2*c*d*sqrt(x)*e + c*d^2 + c*x*e^2) + a)^p*x^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(1/2))**2))**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="giac")
[Out] integrate((b*log((sqrt(x)*e + d)^2*c) + a)^p*x^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \left(a + b \ln\left(c \left(d + e \sqrt{x}\right)^2\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e*x^(1/2))^2))^p,x)
[Out] int(x^3*(a + b*log(c*(d + e*x^(1/2))^2))^p, x)
```

3.539 $\int x^2 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=677

$$\frac{3^{-1-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{c^3e^6}$$

```
[Out] 3^(-1-p)*GAMMA(1+p,-3*(a+b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2)
)^2))^p/c^3/e^6/exp(3*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)+5*d^2*GAMMA
(1+p,-2*(a+b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p/(2^p)/
c^2/e^6/exp(2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)+5*d^4*GAMMA(1+p,(-a
-b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p/c/e^6/exp(a/b)/(
((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)-2^(1+p)*d*GAMMA(1+p,-5/2*(a+b*ln(c*(d+e
*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p*(d+e*x^(1/2))^5/(5^p)/e^6/ex
p(5/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)/(c*(d+e*x^(1/2))^2)^(5/2)-5
*2^(2+p)*3^(-1-p)*d^3*GAMMA(1+p,-3/2*(a+b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln
(c*(d+e*x^(1/2))^2))^p*(d+e*x^(1/2))^3/e^6/exp(3/2*a/b)/(((-a-b*ln(c*(d+e*x
^(1/2))^2))/b)^p)/(c*(d+e*x^(1/2))^2)^(3/2)-2^(1+p)*d^5*GAMMA(1+p,1/2*(-a-b
*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p*(d+e*x^(1/2))/e^6/
exp(1/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)/(c*(d+e*x^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.64, antiderivative size = 677, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {2504, 2448, 2436, 2337, 2212, 2437, 2347}

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
```

```
[Out] (3^(-1 - p)*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])^2]))/b]*(a + b*Lo
g[c*(d + e*Sqrt[x])^2])^p)/(c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e*Sqrt
[x])^2])/b))^p) - (2^(1 + p)*d*(d + e*Sqrt[x])^5*Gamma[1 + p, (-5*(a + b*Lo
g[c*(d + e*Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(5^p*e^
6*E^((5*a)/(2*b))*(c*(d + e*Sqrt[x])^2)^(5/2)*(-((a + b*Log[c*(d + e*Sqrt[x
])^2])/b))^p) + (5*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])^2]))/b
]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(2^p*c^2*e^6*E^((2*a)/b)*(-((a + b*Lo
g[c*(d + e*Sqrt[x])^2])/b))^p) - (5*2^(2 + p)*3^(-1 - p)*d^3*(d + e*Sqrt[x]
)^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*
(d + e*Sqrt[x])^2])^p)/(e^6*E^((3*a)/(2*b))*(c*(d + e*Sqrt[x])^2)^(3/2)*(-(
```

```
(a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) + (5*d^4*Gamma[1 + p, -((a + b*Log[
c*(d + e*Sqrt[x])^2])/b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(c*e^6*E^(a/b
)*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) - (2^(1 + p)*d^5*(d + e*Sqrt[x
])*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e*Sqrt[x])^2])/b]*(a + b*Log[c*(d +
e*Sqrt[x])^2])^p)/(e^6*E^(a/(2*b))*Sqrt[c*(d + e*Sqrt[x])^2]*(-((a + b*Log[
c*(d + e*Sqrt[x])^2])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx = 2\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt{x}\right)$$

$$= 2\text{Subst}\left(\int\left(-\frac{d^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5}+\frac{5d^4(d+ex)\left(a+b\log\right.}{e^5}\right.\right.$$

$$= \frac{2\text{Subst}\left(\int(d+ex)^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt{x}\right)}{e^5}-\frac{(10d)\text{S}}{}$$

$$= \frac{2\text{Subst}\left(\int x^5\left(a+b\log\left(cx^2\right)\right)^p\,dx,x,d+e\sqrt{x}\right)}{e^6}-\frac{(10d)\text{Subst}\left(\int x\right.}{}$$

$$= \frac{\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{c^3e^6}+\frac{\left(10d^2\right)\text{Subst}\left(\right.}{}$$

$$= \frac{3^{-1-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\right.\right.\right.}{c^3e^6}$$

Mathematica [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
[Out] Integrate[x^2*(a + b*Log[c*(d + e*Sqrt[x])^2])^p, x]
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(1/2))^2))^p,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e*x^(1/2))^2))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((sqrt(x)*e + d)^2*c) + a)^p*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(2*c*d*sqrt(x)*e + c*d^2 + c*x*e^2) + a)^p*x^2, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/2))**2))**p,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((sqrt(x)*e + d)^2*c) + a)^p*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(a+b \ln \left(c \left(d+e \sqrt{x}\right)^2\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e*x^(1/2))^2))^p,x)
[Out] int(x^2*(a + b*log(c*(d + e*x^(1/2))^2))^p, x)
```

3.540 $\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=445

$$\frac{2^{-1-p}e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{c^2e^4}-$$

```
[Out] 2^(-1-p)*GAMMA(1+p,-2*(a+b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2)
)^2))^p/c^2/e^4/exp(2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)+3*d^2*GAMMA
(1+p,(-a-b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p/c/e^4/ex
p(a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)-2^(1+p)*d*GAMMA(1+p,-3/2*(a+b*l
n(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p*(d+e*x^(1/2))^3/(3^p
)/e^4/exp(3/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)/(c*(d+e*x^(1/2))^2)
^(3/2)-2^(1+p)*d^3*GAMMA(1+p,1/2*(-a-b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*
(d+e*x^(1/2))^2))^p*(d+e*x^(1/2))/e^4/exp(1/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2
))^2))/b)^p)/(c*(d+e*x^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 445, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2504, 2448, 2436, 2337, 2212, 2437, 2347}

$$\frac{2^{-p-1}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)}{c^2e^4}-\frac{d^32^{p+1}e^{-\frac{a}{2b}}\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{2b}\right)}{e^4\sqrt{c\left(d+e\sqrt{x}\right)^2}}+\frac{3d^2e^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)}{ce^4}-\frac{d2^{p+1}3^{-p}e^{-\frac{3a}{2b}}\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{2b}\right)}{e^4\left(c\left(d+e\sqrt{x}\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
```

```
[Out] (2^(-1 - p)*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])^2]))/b]*(a + b*Lo
g[c*(d + e*Sqrt[x])^2])^p)/(c^2*e^4*E^((2*a)/b)*(-((a + b*Log[c*(d + e*Sqrt
[x])^2])/b))^p) - (2^(1 + p)*d*(d + e*Sqrt[x])^3*Gamma[1 + p, (-3*(a + b*Lo
g[c*(d + e*Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(3^p*e^
4*E^((3*a)/(2*b))*(c*(d + e*Sqrt[x])^2)^(3/2)*(-((a + b*Log[c*(d + e*Sqrt[x
])^2])/b))^p) + (3*d^2*Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])^2])/b)]*
(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(c*e^4*E^(a/b)*(-((a + b*Log[c*(d + e*S
qrt[x])^2])/b))^p) - (2^(1 + p)*d^3*(d + e*Sqrt[x])*Gamma[1 + p, -1/2*(a +
b*Log[c*(d + e*Sqrt[x])^2])/b]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(e^4*E^(
a/(2*b))*Sqrt[c*(d + e*Sqrt[x])^2]*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^
p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
```

```
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx = 2\text{Subst}\left(\int x^3(a+b\log(c(d+ex)^2))^p\,dx, x, \sqrt{x}\right)$$
$$= 2\text{Subst}\left(\int\left(-\frac{d^3(a+b\log(c(d+ex)^2))^p}{e^3}+\frac{3d^2(d+ex)(a+b\log(c\right.\right.$$
$$= \frac{2\text{Subst}\left(\int(d+ex)^3(a+b\log(c(d+ex)^2))^p\,dx, x, \sqrt{x}\right)}{e^3} - \frac{(6d)\text{Subs}}{}$$
$$= \frac{2\text{Subst}\left(\int x^3(a+b\log(cx^2))^p\,dx, x, d+e\sqrt{x}\right)}{e^4} - \frac{(6d)\text{Subst}\left(\int x^2(a\right.}{}$$
$$= \frac{\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{c^2e^4} + \frac{(3d^2)\,\text{Subst}\left(\int e\right.}{}$$
$$= \frac{2^{-1-p}e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{\right.\right.\right.}{c^2e^4}$$

Mathematica [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
[Out] Integrate[x*(a + b*Log[c*(d + e*Sqrt[x])^2])^p, x]
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e*x^(1/2))^2))^p,x)
[Out] int(x*(a+b*ln(c*(d+e*x^(1/2))^2))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((sqrt(x)*e + d)^2*c) + a)^p*x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(2*c*d*sqrt(x)*e + c*d^2 + c*x*e^2) + a)^p*x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/2))**2))**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((sqrt(x)*e + d)^2*c) + a)^p*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(a+b\,\ln\left(c\left(d+e\,\sqrt{x}\,\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e*x^(1/2))^2))^p,x)
```

```
[Out] int(x*(a + b*log(c*(d + e*x^(1/2))^2))^p, x)
```

3.541 $\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=213

$$\frac{e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{ce^2}-\frac{2^{1+p}de^{-\frac{a}{2b}}}{}$$

```
[Out] GAMMA(1+p,(-a-b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p/c/e
^2/exp(a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)-2^(1+p)*d*GAMMA(1+p,1/2*(-
a-b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p*(d+e*x^(1/2))/e
^2/exp(1/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)/(c*(d+e*x^(1/2))^2)^(1
/2)
```

Rubi [A]

time = 0.17, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2501, 2448, 2436, 2337, 2212, 2437, 2347}

$$\frac{e^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)}{ce^2}-\frac{d2^{p+1}e^{-\frac{a}{2b}}\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{2b}\right)}{e^2\sqrt{c\left(d+e\sqrt{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
```

```
[Out] (Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])^2])/b)]*(a + b*Log[c*(d + e*Sq
rt[x])^2])^p)/(c*e^2*E^(a/b)*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) - (
2^(1 + p)*d*(d + e*Sqrt[x])*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e*Sqrt[x])^
2])/b]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(e^2*E^(a/(2*b))*Sqrt[c*(d + e*S
qrt[x])^2]*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
```

```
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2501

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p dx = 2\text{Subst}\left(\int x\left(a+b\log\left(c(d+ex)^2\right)\right)^p dx, x, \sqrt{x}\right)$$
$$= 2\text{Subst}\left(\int \left(-\frac{d\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e}+\frac{(d+ex)\left(a+b\log\left(c(d+\right.\right.}{e}\right.\right.$$
$$= \frac{2\text{Subst}\left(\int (d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p dx, x, \sqrt{x}\right)}{e} - \frac{(2d)\text{Subst}\left(\right.}{}$$
$$= \frac{2\text{Subst}\left(\int x\left(a+b\log\left(cx^2\right)\right)^p dx, x, d+e\sqrt{x}\right)}{e^2} - \frac{(2d)\text{Subst}\left(\int (a+b\,\text{l}\right.}{}$$
$$= \frac{\text{Subst}\left(\int e^x(a+bx)^p\, dx, x, \log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{ce^2} - \frac{\left(d\left(d+e\sqrt{x}\right)\right)\text{Su}}{}$$
$$= \frac{e^{-\frac{a}{b}}\Gamma\left(1+p, -\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(\right.}{ce^2}$$

Mathematica [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
[Out] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^2])^p, x]
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^2))^p,x)
[Out] int((a+b*ln(c*(d+e*x^(1/2))^2))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log((sqrt(x)*e + d)^2*c) + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="fricas")
[Out] integral((b*log(2*c*d*sqrt(x)*e + c*d^2 + c*x*e^2) + a)^p, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**2))**p,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="giac")
[Out] integrate((b*log((sqrt(x)*e + d)^2*c) + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))^2))^p,x)
[Out] int((a + b*log(c*(d + e*x^(1/2))^2))^p, x)
```

3.542 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(1/2))^2))^p/x,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^2])^p/x,x]
```

```
[Out] 2*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x)^2])^p/x, x], x, Sqrt[x]]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}\,dx = 2\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{x}\,dx,x,\sqrt{x}\right)$$

Mathematica [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^2])^p/x,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^2])^p/x, x]
```

Maple [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\sqrt{x}\,\right)^{2}\right)\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^2))^p/x,x)
[Out] int((a+b*ln(c*(d+e*x^(1/2))^2))^p/x,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p/x,x, algorithm="maxima")
[Out] integrate((b*log((sqrt(x)*e + d)^2*c) + a)^p/x, x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p/x,x, algorithm="fricas")
[Out] integral((b*log(2*c*d*sqrt(x)*e + c*d^2 + c*x*e^2) + a)^p/x, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**2))**p/x,x)
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((sqrt(x)*e + d)^2*c) + a)^p/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^2\right)\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))^2))^p/x,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/2))^2))^p/x, x)
```

3.543 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(1/2))^2))^p/x^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^2])^p/x^2,x]
[Out] 2*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x)^2])^p/x^3, x], x, Sqrt[x]]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}\,dx = 2\mathrm{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{x^3}\,dx,x,\sqrt{x}\right)$$

Mathematica [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^2])^p/x^2,x]
[Out] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^2])^p/x^2, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\sqrt{x}\,\right)^{2}\right)\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^2))^p/x^2,x)
[Out] int((a+b*ln(c*(d+e*x^(1/2))^2))^p/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p/x^2,x, algorithm="maxima")
[Out] integrate((b*log((sqrt(x)*e + d)^2*c) + a)^p/x^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p/x^2,x, algorithm="fricas")
[Out] integral((b*log(2*c*d*sqrt(x)*e + c*d^2 + c*x*e^2) + a)^p/x^2, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**2))**p/x**2,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((sqrt(x)*e + d)^2*c) + a)^p/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,\sqrt{x}\,\right)}^2\right)\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/2))^2))^p/x^2,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/2))^2))^p/x^2, x)
```

3.544 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable(x*(a+b*ln(c*(d+e/x^(1/2))))^p,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[x*(a + b*Log[c*(d + e/Sqrt[x])])^p,x]
[Out] 2*Defer[Subst][Defer[Int][x^3*(a + b*Log[c*(d + e/x)])^p, x], x, Sqrt[x]]
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx = 2\text{Subst}\left(\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x}\right)\right)\right)^p\,dx,x,\sqrt{x}\right)$$

Mathematica [A]
time = 0.89, size = 0, normalized size = 0.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])])^p,x]
[Out] Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])])^p, x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(1/2))))^p,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e/x^(1/2))))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p*x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*sqrt(x)*e)/x) + a)^p*x, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/2))))**p,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p*x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x \left(a + b \ln \left(c \left(d + \frac{e}{\sqrt{x}}\right)\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e/x^(1/2))))^p,x)
[Out] int(x*(a + b*log(c*(d + e/x^(1/2))))^p, x)
```

3.545 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(1/2))))^p,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])])^p,x]
[Out] 2*Defer[Subst][Defer[Int][x*(a + b*Log[c*(d + e/x)])^p, x], x, Sqrt[x]]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx = 2\text{Subst}\left(\int x\left(a+b\log\left(c\left(d+\frac{e}{x}\right)\right)\right)^p\,dx,x,\sqrt{x}\right)$$

Mathematica [A]
time = 0.22, size = 0, normalized size = 0.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p,x]
[Out] Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))))^p,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*sqrt(x)*e)/x) + a)^p, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))))**p,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \left(a + b \ln\left(c \left(d + \frac{e}{\sqrt{x}}\right)\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))))^p,x)
[Out] int((a + b*log(c*(d + e/x^(1/2))))^p, x)
```

3.546 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(1/2))))^p/x,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])])^p/x,x]
[Out] 2*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x)])^p/x, x], x, Sqrt[x]]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx = 2\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x}\right)\right)\right)^p}{x}\,dx, x, \sqrt{x}\right)$$

Mathematica [A]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p/x,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p/x, x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))))^p/x,x)
[Out] int((a+b*ln(c*(d+e/x^(1/2))))^p/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x,x, algorithm="fricas")
[Out] integral((b*log((c*d*x + c*sqrt(x)*e)/x) + a)^p/x, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))))**p/x,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))))^p/x,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(1/2))))^p/x, x)
```

3.547 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=175

$$-\frac{2^{-p}e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^2e^2}+$$

```
[Out] -GAMMA(1+p,-2*(a+b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/(2^p
)/c^2/e^2/exp(2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))))/b)^p)+2*d*GAMMA(1+p,(-a-b
*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/c/e^2/exp(a/b)/(((-a-b
*ln(c*(d+e/x^(1/2))))/b)^p)
```

Rubi [A]
time = 0.16, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2504, 2448, 2436, 2336, 2212, 2437, 2346}

$$\frac{2de^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)}{ce^2}-\frac{2^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)}{c^2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])])^p/x^2,x]
```

```
[Out] -((Gamma[1 + p, (-2*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/
Sqrt[x])])^p)/(2^p*c^2*e^2*E^((2*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b)
)^p)) + (2*d*Gamma[1 + p, -((a + b*Log[c*(d + e/Sqrt[x])])/b)]*(a + b*Log[c
*(d + e/Sqrt[x])])^p)/(c*e^2*E^(a/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^
p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
```

```
c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^2}\,dx = -\left(2\text{Subst}\left(\int x(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\text{Subst}\left(\int \left(-\frac{d(a+b\log(c(d+ex)))^p}{e} + \frac{(d+ex)(a+b\log(c(\right.\right.\right.$$

$$= -\frac{2\text{Subst}\left(\int (d+ex)(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)}{e} + \frac{(2d)\text{Sub}}{}$$

$$= -\frac{2\text{Subst}\left(\int x(a+b\log(cx))^p\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^2} + \frac{(2d)\text{Subst}\left(\int (a+\right.}{}$$

$$= -\frac{2\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{c^2e^2} + \frac{(2d)\text{Subst}}{}$$

$$= -\frac{2^{-p}e^{-\frac{2a}{b}}\Gamma\left(1+p, -\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{}{}\right.\right.\right.}{c^2e^2}$$

Mathematica [A]
time = 0.28, size = 131, normalized size = 0.75

$$\frac{2^{-p}e^{-\frac{2a}{b}}\left(-\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)+2^{1+p}cde^{a/b}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p/x^2,x]
```

```
[Out] ((-Gamma[1 + p, (-2*(a + b*Log[c*(d + e/Sqrt[x])]))/b] + 2^(1 + p)*c*d*E^(a
/b)*Gamma[1 + p, -((a + b*Log[c*(d + e/Sqrt[x])])/b)])*(a + b*Log[c*(d + e/
Sqrt[x])])^p)/(2^p*c^2*e^2*E^((2*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b)
)^p)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))))^p/x^2,x)
[Out] int((a+b*ln(c*(d+e/x^(1/2))))^p/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^2,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^2,x, algorithm="fricas")
[Out] integral((b*log((c*d*x + c*sqrt(x)*e)/x) + a)^p/x^2, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))))**p/x**2,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8011 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^2,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))))^p/x^2,x)
[Out] int((a + b*log(c*(d + e/x^(1/2))))^p/x^2, x)
```

3.548 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^4}\,dx$

Optimal. Leaf size=552

$$-\frac{2^{-p}3^{-1-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)}{c^6e^6}$$

```
[Out] -3^(-1-p)*GAMMA(1+p,-6*(a+b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))
))^p/(2^p)/c^6/e^6/exp(6*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))))/b)^p)+2*d*GAMMA(
1+p,-5*(a+b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/(5^p)/c^5/e
^6/exp(5*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))))/b)^p)-5*d^2*GAMMA(1+p,-4*(a+b*ln
(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/(4^p)/c^4/e^6/exp(4*a/b)/
(((-a-b*ln(c*(d+e/x^(1/2))))/b)^p)+20*3^(-1-p)*d^3*GAMMA(1+p,-3*(a+b*ln(c*(
d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/c^3/e^6/exp(3*a/b)/(((-a-b*ln
(c*(d+e/x^(1/2))))/b)^p)-5*d^4*GAMMA(1+p,-2*(a+b*ln(c*(d+e/x^(1/2))))/b)*(a
+b*ln(c*(d+e/x^(1/2))))^p/(2^p)/c^2/e^6/exp(2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2
))))/b)^p)+2*d^5*GAMMA(1+p,(-a-b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(
1/2))))^p/c/e^6/exp(a/b)/(((-a-b*ln(c*(d+e/x^(1/2))))/b)^p)
```

Rubi [A]
time = 0.56, antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2504, 2448, 2436, 2336, 2212, 2437, 2346}

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])])^p/x^4,x]
```

```
[Out] -((3^(-1 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Lo
g[c*(d + e/Sqrt[x])])^p)/(2^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e/Sq
rt[x])])/b))^p)) + (2*d*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/Sqrt[x])]))/b]
*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(5^p*c^5*e^6*E^((5*a)/b)*(-((a + b*Log[c
*(d + e/Sqrt[x])])/b))^p) - (5*d^2*Gamma[1 + p, (-4*(a + b*Log[c*(d + e/Sqr
t[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(4^p*c^4*e^6*E^((4*a)/b)*(-((
a + b*Log[c*(d + e/Sqrt[x])])/b))^p) + (20*3^(-1 - p)*d^3*Gamma[1 + p, (-3*
(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(c^3*e
^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) - (5*d^4*Gamma[1 +
p, (-2*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)
/(2^p*c^2*e^6*E^((2*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) + (2*d^5
```

```
*Gamma[1 + p, -((a + b*Log[c*(d + e/Sqrt[x])])/b)]*(a + b*Log[c*(d + e/Sqrt
[x])])^p)/(c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^4}\,dx = -\left(2\text{Subst}\left(\int x^5(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\text{Subst}\left(\int \left(-\frac{d^5(a+b\log(c(d+ex)))^p}{e^5} + \frac{5d^4(d+ex)(a+b\log(\right.\right.\right.$$

$$= -\frac{2\text{Subst}\left(\int (d+ex)^5(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)}{e^5} + \frac{(10d)\text{Su}}{}$$

$$= -\frac{2\text{Subst}\left(\int x^5(a+b\log(cx))^p\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^6} + \frac{(10d)\text{Subst}\left(\int x^4(\right.}{}$$

$$= -\frac{2\text{Subst}\left(\int e^{6x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{c^6e^6} + \frac{(10d)\text{Subst}\left(\right.}{}$$

$$= -\frac{2^{-p}3^{-1-p}e^{-\frac{6a}{b}}\Gamma\left(1+p, -\frac{6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d\right.\right.\right.}{c^6e^6}$$

Mathematica [A]
time = 0.93, size = 325, normalized size = 0.59

$$\frac{3^{-1-p}20^{-p}e^{-\frac{6a}{b}}\left(-10^p\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)+cde^{a/b}\left(2^{1+2p}3^{1+p}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)+5^pcde^{a/b}\left(-53^{1+p}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)+2^pcde^{a/b}\left(5\,2^{2+p}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)+3^{1+p}cde^{a/b}\left(-5\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)+2^{1+p}cde^{a/b}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)\right)\right)\right)\right)\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^6e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p/x^4,x]
```

```
[Out] (3^(-1 - p)*(-(10^p*Gamma[1 + p, (-6*(a + b*Log[c*(d + e/Sqrt[x])]))/b]) +
c*d*E^(a/b)*(2^(1 + 2*p)*3^(1 + p)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/Sqr
t[x])]))/b] + 5^p*c*d*E^(a/b)*(-5*3^(1 + p)*Gamma[1 + p, (-4*(a + b*Log[c*(
d + e/Sqrt[x])]))/b] + 2^p*c*d*E^(a/b)*(5*2^(2 + p)*Gamma[1 + p, (-3*(a + b
*Log[c*(d + e/Sqrt[x])]))/b] + 3^(1 + p)*c*d*E^(a/b)*(-5*Gamma[1 + p, (-2*(
a + b*Log[c*(d + e/Sqrt[x])]))/b] + 2^(1 + p)*c*d*E^(a/b)*Gamma[1 + p, -((a
 + b*Log[c*(d + e/Sqrt[x])])/b)])))))*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(20
^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^{p}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))))^p/x^4,x)
[Out] int((a+b*ln(c*(d+e/x^(1/2))))^p/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^4,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^4,x, algorithm="fricas")
[Out] integral((b*log((c*d*x + c*sqrt(x)*e)/x) + a)^p/x^4, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))))**p/x**4,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))))^p/x^4,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(1/2))))^p/x^4, x)
```

3.549 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^6}\,dx$

Optimal. Leaf size=926

$$-\frac{2^{-p}5^{-1-p}e^{-\frac{10a}{b}}\Gamma\left(1+p,-\frac{10\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right.}{c^{10}e^{10}}$$

```
[Out] -5^(-1-p)*GAMMA(1+p,-10*(a+b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2)
)))^p/(2^p)/c^10/e^10/exp(10*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))))/b)^p)+2*d*GA
MMA(1+p,-9*(a+b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/(9^p)/c
^9/e^10/exp(9*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))))/b)^p)-9*d^2*GAMMA(1+p,-8*(a
+b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/(8^p)/c^8/e^10/exp(8
*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))))/b)^p)+24*d^3*GAMMA(1+p,-7*(a+b*ln(c*(d+e
/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/(7^p)/c^7/e^10/exp(7*a/b)/(((-a-
b*ln(c*(d+e/x^(1/2))))/b)^p)-7*6^(1-p)*d^4*GAMMA(1+p,-6*(a+b*ln(c*(d+e/x^(1
/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/c^6/e^10/exp(6*a/b)/(((-a-b*ln(c*(d+e
/x^(1/2))))/b)^p)+252*5^(-1-p)*d^5*GAMMA(1+p,-5*(a+b*ln(c*(d+e/x^(1/2))))/b
)*(a+b*ln(c*(d+e/x^(1/2))))^p/c^5/e^10/exp(5*a/b)/(((-a-b*ln(c*(d+e/x^(1/2)
)))/b)^p)-21*2^(1-2*p)*d^6*GAMMA(1+p,-4*(a+b*ln(c*(d+e/x^(1/2))))/b)*(a+b*l
n(c*(d+e/x^(1/2))))^p/c^4/e^10/exp(4*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))))/b)^p
)+8*3^(1-p)*d^7*GAMMA(1+p,-3*(a+b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^
(1/2))))^p/c^3/e^10/exp(3*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))))/b)^p)-9*d^8*GAM
MA(1+p,-2*(a+b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/(2^p)/c^
2/e^10/exp(2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))))/b)^p)+2*d^9*GAMMA(1+p,(-a-b*
ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/c/e^10/exp(a/b)/(((-a-b
*ln(c*(d+e/x^(1/2))))/b)^p)
```

Rubi [A]
time = 1.04, antiderivative size = 926, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2504, 2448, 2436, 2336, 2212, 2437, 2346}

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])])^p/x^6,x]
```

```
[Out] -((5^(-1 - p)*Gamma[1 + p, (-10*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*L
og[c*(d + e/Sqrt[x])])^p)/(2^p*c^10*e^10*E^((10*a)/b)*(-((a + b*Log[c*(d +
e/Sqrt[x])])/b))^p)) + (2*d*Gamma[1 + p, (-9*(a + b*Log[c*(d + e/Sqrt[x])])
```

```
)/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(9^p*c^9*e^10*E^((9*a)/b)*(-((a + b*
Log[c*(d + e/Sqrt[x])])/b))^p) - (9*d^2*Gamma[1 + p, (-8*(a + b*Log[c*(d +
e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(8^p*c^8*e^10*E^((8*a)/b
)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) + (24*d^3*Gamma[1 + p, (-7*(a +
b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(7^p*c^7*e^
10*E^((7*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) - (7*6^(1 - p)*d^4*
Gamma[1 + p, (-6*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqr
t[x])])^p)/(c^6*e^10*E^((6*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) +
 (252*5^(-1 - p)*d^5*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a
 + b*Log[c*(d + e/Sqrt[x])])^p)/(c^5*e^10*E^((5*a)/b)*(-((a + b*Log[c*(d +
e/Sqrt[x])])/b))^p) - (21*2^(1 - 2*p)*d^6*Gamma[1 + p, (-4*(a + b*Log[c*(d
+ e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(c^4*e^10*E^((4*a)/b)*
(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) + (8*3^(1 - p)*d^7*Gamma[1 + p, (-
3*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(c^3
*e^10*E^((3*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) - (9*d^8*Gamma[1
 + p, (-2*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])
^p)/(2^p*c^2*e^10*E^((2*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) + (2
*d^9*Gamma[1 + p, -((a + b*Log[c*(d + e/Sqrt[x])])/b)]*(a + b*Log[c*(d + e/
Sqrt[x])])^p)/(c*e^10*E^(a/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^6}\,dx = -\left(2\text{Subst}\left(\int x^9(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\text{Subst}\left(\int \left(-\frac{d^9(a+b\log(c(d+ex)))^p}{e^9} + \frac{9d^8(d+ex)(a+b\log\right.\right.\right.$$

$$= -\frac{2\text{Subst}\left(\int (d+ex)^9(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)}{e^9} + \frac{(18d)\text{S}}{}$$

$$= -\frac{2\text{Subst}\left(\int x^9(a+b\log(cx))^p\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^{10}} + \frac{(18d)\text{Subst}\left(\int x\right.}{}$$

$$= -\frac{2\text{Subst}\left(\int e^{10x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{c^{10}e^{10}} + \frac{(18d)\text{Subs}}{}$$

$$= -\frac{2^{-p}5^{-1-p}e^{-\frac{10a}{b}}\Gamma\left(1+p, -\frac{10\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\right.\right.}{c^{10}e^{10}}$$

Mathematica [A]
time = 3.74, size = 525, normalized size = 0.57

$$\frac{5^{-1-p}504^{-p}e^{-\frac{10a}{b}}\left(-252^p\Gamma\left(1+p,-\frac{10\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)+cde^{a/b}\left(2^{1+3p}5^{1+p}7^p\Gamma\left(1+p,-\frac{9\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)+cde^{a/b}\left(-7^p45^{1+p}\Gamma\left(1+p,-\frac{8\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)+2^pcde^{a/b}\left(2^{3+2p}3^{1+2p}5^{1+p}\Gamma\left(1+p,-\frac{7\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)+7^pcde^{a/b}\left(-7\,30^{1+p}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)+cde^{a/b}\left(7\,36^{1+p}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)+3^p5^{1+p}cde^{a/b}\left(-14\,3^{1+p}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)+2^pcde^{a/b}\left(3\,2^{3+p}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)+3^pcde^{a/b}\left(-9\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)+2^{1+p}cde^{a/b}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^{10}e^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p/x^6,x]
```

```
[Out] (5^(-1 - p)*(-(252^p*Gamma[1 + p, (-10*(a + b*Log[c*(d + e/Sqrt[x])]))/b])
+ c*d*E^(a/b)*(2^(1 + 3*p)*5^(1 + p)*7^p*Gamma[1 + p, (-9*(a + b*Log[c*(d +
 e/Sqrt[x])]))/b] + c*d*E^(a/b)*(-(7^p*45^(1 + p)*Gamma[1 + p, (-8*(a + b*L
og[c*(d + e/Sqrt[x])]))/b]) + 2^p*c*d*E^(a/b)*(2^(3 + 2*p)*3^(1 + 2*p)*5^(1
 + p)*Gamma[1 + p, (-7*(a + b*Log[c*(d + e/Sqrt[x])]))/b] + 7^p*c*d*E^(a/b)
*(-7*30^(1 + p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e/Sqrt[x])]))/b] + c*d*E
^(a/b)*(7*36^(1 + p)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/Sqrt[x])]))/b] +
3^p*5^(1 + p)*c*d*E^(a/b)*(-14*3^(1 + p)*Gamma[1 + p, (-4*(a + b*Log[c*(d +
 e/Sqrt[x])]))/b] + 2^p*c*d*E^(a/b)*(3*2^(3 + p)*Gamma[1 + p, (-3*(a + b*Lo
g[c*(d + e/Sqrt[x])]))/b] + 3^p*c*d*E^(a/b)*(-9*Gamma[1 + p, (-2*(a + b*Log
[c*(d + e/Sqrt[x])]))/b] + 2^(1 + p)*c*d*E^(a/b)*Gamma[1 + p, -((a + b*Log[
c*(d + e/Sqrt[x])])/b)])))))))))*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(504^p*c
^10*e^10*E^((10*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))))^p/x^6,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))))^p/x^6,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^6,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x^6, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^6,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*sqrt(x)*e)/x) + a)^p/x^6, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))))**p/x**6,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))))^p/x^6,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(1/2))))^p/x^6, x)
```

3.550 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p dx$

Optimal. Leaf size=25

$$\text{Int}\left(x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable(x*(a+b*ln(c*(d+e/x^(1/2))^2))^p,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p dx$$

Verification is not applicable to the result.

```
[In] Int[x*(a + b*Log[c*(d + e/Sqrt[x])^2])^p,x]
[Out] 2*Defer[Subst][Defer[Int][x^3*(a + b*Log[c*(d + e/x)^2])^p, x], x, Sqrt[x]]
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p dx = 2\text{Subst}\left(\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^2\right)\right)^p dx, x, \sqrt{x}\right)$$

Mathematica [A]
time = 0.17, size = 0, normalized size = 0.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])^2])^p,x]
[Out] Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])^2])^p, x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int x\left(a+b\ln\left(c{\left(d+\frac{e}{\sqrt{x}}\right)}^{2}\right)\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(1/2))^2))^p,x)
[Out] int(x*(a+b*ln(c*(d+e/x^(1/2))^2))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p*x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^2))^p,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x + 2*c*d*sqrt(x)*e + c*e^2)/x) + a)^p*x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/2))**2))**p,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^2))^p,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p*x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x\left(a+b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e/x^(1/2))^2))^p,x)
[Out] int(x*(a + b*log(c*(d + e/x^(1/2))^2))^p, x)
```

3.551 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(1/2))^2))^p,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^2])^p,x]
[Out] 2*Defer[Subst][Defer[Int][x*(a + b*Log[c*(d + e/x)^2])^p, x], x, Sqrt[x]]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx = 2\text{Subst}\left(\int x\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^2\right)\right)^p\,dx,x,\sqrt{x}\right)$$

Mathematica [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p,x]
[Out] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \ln \left(c\left(d + \frac{e}{\sqrt{x}}\right)^2\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^2))^p,x)
[Out] int((a+b*ln(c*(d+e/x^(1/2))^2))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x + 2*c*d*sqrt(x)*e + c*e^2)/x) + a)^p, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**2))**p,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \left(a+b\,\ln\left(c\,{\left(d+\frac{e}{\sqrt{x}}\right)}^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^2))^p,x)
[Out] int((a + b*log(c*(d + e/x^(1/2))^2))^p, x)
```

3.552 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(1/2))^2))^p/x,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x,x]
[Out] 2*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x)^2])^p/x, x], x, Sqrt[x]]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx = 2\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^2\right)\right)^p}{x}\,dx, x, \sqrt{x}\right)$$

Mathematica [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x,x]
[Out] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x, x]
```

Maple [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{2}\right)\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^2))^p/x,x)
[Out] int((a+b*ln(c*(d+e/x^(1/2))^2))^p/x,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x, x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x + 2*c*d*sqrt(x)*e + c*e^2)/x) + a)^p/x, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**2))**p/x,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{\sqrt{x}}\right)}^2\right)\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^2))^p/x,x)
[Out] int((a + b*log(c*(d + e/x^(1/2))^2))^p/x, x)
```

3.553 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=216

$$\frac{2^{1+p}de^{-\frac{a}{2b}}\left(d+\frac{e}{\sqrt{x}}\right)\Gamma\left(1+p,\frac{-a-b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right.\right.}{b}\right.}{e^2\sqrt{c\left(d+\frac{e}{\sqrt{x}}\right)^2}}$$

```
[Out] -GAMMA(1+p,(-a-b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p/c/
e^2/exp(a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)+2^(1+p)*d*GAMMA(1+p,1/2*(
-a-b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p*(d+e/x^(1/2))/
e^2/exp(1/2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)/(c*(d+e/x^(1/2))^2)^(
1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 213, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {2504, 2448, 2436, 2337, 2212, 2437, 2347}

$$\frac{d2^{p+1}e^{-\frac{a}{2b}}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{2b}\right)}{e^2\sqrt{c\left(d+\frac{e}{\sqrt{x}}\right)^2}}-\frac{e^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)}{ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^2,x]
```

```
[Out] -((Gamma[1 + p, -((a + b*Log[c*(d + e/Sqrt[x])^2])/b)]*(a + b*Log[c*(d + e/
Sqrt[x])^2])^p)/(c*e^2*E^(a/b)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p))
+ (2^(1 + p)*d*(d + e/Sqrt[x])*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e/Sqrt[x
])^2])/b]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(e^2*E^(a/(2*b))*Sqrt[c*(d +
e/Sqrt[x])^2]*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx = -\left(2\text{Subst}\left(\int x\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\text{Subst}\left(\int\left(-\frac{d(a+b\log\left(c(d+ex)^2\right))^p}{e} + \frac{(d+ex)\left(a+b\log\right.}{e}\right.\right.\right.$$

$$= -\frac{2\text{Subst}\left(\int (d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt{x}}\right)}{e} + \frac{(2d)}{}$$

$$= -\frac{2\text{Subst}\left(\int x\left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^2} + \frac{(2d)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\text{Subst}\left(\int e^x(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{ce^2} + \frac{\left(d\left(d+\right.\right.}{}$$

$$= -\frac{e^{-\frac{a}{b}}\Gamma\left(1+p, -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right.\right.}{ce^2}$$

Mathematica [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^2,x]
[Out] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^2, x]
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^2))^p/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^2))^p/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x + 2*c*d*sqrt(x)*e + c*e^2)/x) + a)^p/x^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**2))**p/x**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x^2, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^2))^p/x^2,x)
[Out] int((a + b*log(c*(d + e/x^(1/2))^2))^p/x^2, x)
```

3.554 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^4}\,dx$

Optimal. Leaf size=676

$$-\frac{3^{-1-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)}{c^3e^6}$$

```
[Out] -3^(-1-p)*GAMMA(1+p,-3*(a+b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2
))^2))^p/c^3/e^6/exp(3*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)-5*d^2*GAMM
A(1+p,-2*(a+b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p/(2^p)
/c^2/e^6/exp(2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)-5*d^4*GAMMA(1+p,(-
a-b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p/c/e^6/exp(a/b)/
(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)+2^(1+p)*d*GAMMA(1+p,-5/2*(a+b*ln(c*(d+
e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p*(d+e/x^(1/2))^5/(5^p)/e^6/e
xp(5/2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)/(c*(d+e/x^(1/2))^2)^(5/2)+
5*2^(2+p)*3^(-1-p)*d^3*GAMMA(1+p,-3/2*(a+b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*l
n(c*(d+e/x^(1/2))^2))^p*(d+e/x^(1/2))^3/e^6/exp(3/2*a/b)/(((-a-b*ln(c*(d+e/
x^(1/2))^2))/b)^p)/(c*(d+e/x^(1/2))^2)^(3/2)+2^(1+p)*d^5*GAMMA(1+p,1/2*(-a-
b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p*(d+e/x^(1/2))/e^6
/exp(1/2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)/(c*(d+e/x^(1/2))^2)^(1/2
)
```

Rubi [A]
time = 0.66, antiderivative size = 676, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {2504, 2448, 2436, 2337, 2212, 2437, 2347}

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^4,x]
```

```
[Out] -((3^(-1 - p)*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/Sqrt[x])^2]))/b]*(a + b*
Log[c*(d + e/Sqrt[x])^2])^p)/(c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e/Sq
rt[x])^2])/b))^p)) + (2^(1 + p)*d*(d + e/Sqrt[x])^5*Gamma[1 + p, (-5*(a + b
*Log[c*(d + e/Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(5^p
*e^6*E^((5*a)/(2*b))*(c*(d + e/Sqrt[x])^2)^(5/2)*(-((a + b*Log[c*(d + e/Sqr
```

```
t[x])^2])/b))^p) - (5*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d + e/Sqrt[x])^2])
)/b]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(2^p*c^2*e^6*E^((2*a)/b)*(-((a + b
*Log[c*(d + e/Sqrt[x])^2])/b))^p) + (5*2^(2 + p)*3^(-1 - p)*d^3*(d + e/Sqrt
[x])^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/Sqrt[x])^2]))/(2*b)]*(a + b*Log
[c*(d + e/Sqrt[x])^2])^p)/(e^6*E^((3*a)/(2*b))*(c*(d + e/Sqrt[x])^2)^(3/2)*
(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) - (5*d^4*Gamma[1 + p, -((a + b*L
og[c*(d + e/Sqrt[x])^2])/b)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(c*e^6*E^(
a/b)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) + (2^(1 + p)*d^5*(d + e/Sqr
t[x])*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e/Sqrt[x])^2])/b]*(a + b*Log[c*(d
 + e/Sqrt[x])^2])^p)/(e^6*E^(a/(2*b))*Sqrt[c*(d + e/Sqrt[x])^2]*(-((a + b*L
og[c*(d + e/Sqrt[x])^2])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^4}\,dx = -\left(2\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\text{Subst}\left(\int\left(-\frac{d^5(a+b\log(c(d+ex)^2))^p}{e^5}+\frac{5d^4(d+ex)\,(a+b\right.\right.\right.$$

$$= -\frac{2\text{Subst}\left(\int (d+ex)^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt{x}}\right)}{e^5}+\frac{(10d}{}$$

$$= -\frac{2\text{Subst}\left(\int x^5\left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^6}+\frac{(10d)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{c^3e^6}-\frac{(10d^2)\,\text{Sub}}{}$$

$$= -\frac{3^{-1-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d\right.\right.\right.}{c^3e^6}$$

Mathematica [F]

time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\log\left(c\left(d + \frac{e}{\sqrt{x}}\right)^{2}\right)\right)^{p}}{x^{4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^4,x]
[Out] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^4, x]
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\ln\left(c\left(d + \frac{e}{\sqrt{x}}\right)^{2}\right)\right)^{p}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^2))^p/x^4,x)
[Out] int((a+b*ln(c*(d+e/x^(1/2))^2))^p/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^4,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^4,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x + 2*c*d*sqrt(x)*e + c*e^2)/x) + a)^p/x^4, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**2))**p/x**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{\sqrt{x}}\right)}^2\right)\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^2))^p/x^4,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(1/2))^2))^p/x^4, x)
```

3.555 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}\,dx$

Optimal. Leaf size=1141

$$-\frac{5^{-1-p}e^{-\frac{5a}{b}}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right.}{c^5e^{10}}$$

```
[Out] -5^(-1-p)*GAMMA(1+p,-5*(a+b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2
))^2))^p/c^5/e^10/exp(5*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)-9*d^2*GAM
MA(1+p,-4*(a+b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p/(4^p
)/c^4/e^10/exp(4*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)-14*3^(1-p)*d^4*G
AMMA(1+p,-3*(a+b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p/c^
3/e^10/exp(3*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)-21*2^(1-p)*d^6*GAMMA
(1+p,-2*(a+b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p/c^2/e^
10/exp(2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)-9*d^8*GAMMA(1+p,(-a-b*ln
(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p/c/e^10/exp(a/b)/(((-a
-b*ln(c*(d+e/x^(1/2))^2))/b)^p)+2^(1+p)*d*GAMMA(1+p,-9/2*(a+b*ln(c*(d+e/x^(
1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p*(d+e/x^(1/2))^9/(9^p)/e^10/exp(9
/2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)/(c*(d+e/x^(1/2))^2)^(9/2)+3*2^
(3+p)*d^3*GAMMA(1+p,-7/2*(a+b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1
/2))^2))^p*(d+e/x^(1/2))^7/(7^p)/e^10/exp(7/2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2
))^2))/b)^p)/(c*(d+e/x^(1/2))^2)^(7/2)+63*2^(2+p)*5^(-1-p)*d^5*GAMMA(1+p,-5
/2*(a+b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p*(d+e/x^(1/2
))^5/e^10/exp(5/2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)/(c*(d+e/x^(1/2)
)^2)^(5/2)+2^(3+p)*3^(1-p)*d^7*GAMMA(1+p,-3/2*(a+b*ln(c*(d+e/x^(1/2))^2))/b
)*(a+b*ln(c*(d+e/x^(1/2))^2))^p*(d+e/x^(1/2))^3/e^10/exp(3/2*a/b)/(((-a-b*l
n(c*(d+e/x^(1/2))^2))/b)^p)/(c*(d+e/x^(1/2))^2)^(3/2)+2^(1+p)*d^9*GAMMA(1+p
,1/2*(-a-b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p*(d+e/x^(
1/2))/e^10/exp(1/2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)/(c*(d+e/x^(1/2
))^2)^(1/2)
```

Rubi [A]

time = 1.19, antiderivative size = 1141, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {2504, 2448, 2436, 2337, 2212, 2437, 2347}

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^6,x]
```

```
[Out] -((5^(-1 - p)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/Sqrt[x])^2]))/b]*(a + b*
Log[c*(d + e/Sqrt[x])^2])^p)/(c^5*e^10*E^((5*a)/b)*(-((a + b*Log[c*(d + e/S
qrt[x])^2])/b))^p)) + (2^(1 + p)*d*(d + e/Sqrt[x])^9*Gamma[1 + p, (-9*(a +
b*Log[c*(d + e/Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(9^
p*e^10*E^((9*a)/(2*b))*(c*(d + e/Sqrt[x])^2)^(9/2)*(-((a + b*Log[c*(d + e/S
qrt[x])^2])/b))^p) - (9*d^2*Gamma[1 + p, (-4*(a + b*Log[c*(d + e/Sqrt[x])^2
]))/b]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(4^p*c^4*e^10*E^((4*a)/b)*(-((a
+ b*Log[c*(d + e/Sqrt[x])^2])/b))^p) + (3*2^(3 + p)*d^3*(d + e/Sqrt[x])^7*G
amma[1 + p, (-7*(a + b*Log[c*(d + e/Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d +
e/Sqrt[x])^2])^p)/(7^p*e^10*E^((7*a)/(2*b))*(c*(d + e/Sqrt[x])^2)^(7/2)*(-(
(a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) - (14*3^(1 - p)*d^4*Gamma[1 + p, (-
3*(a + b*Log[c*(d + e/Sqrt[x])^2]))/b]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/
(c^3*e^10*E^((3*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) + (63*2^(2
 + p)*5^(-1 - p)*d^5*(d + e/Sqrt[x])^5*Gamma[1 + p, (-5*(a + b*Log[c*(d + e
/Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(e^10*E^((5*a)/(2
*b))*(c*(d + e/Sqrt[x])^2)^(5/2)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p)
 - (21*2^(1 - p)*d^6*Gamma[1 + p, (-2*(a + b*Log[c*(d + e/Sqrt[x])^2]))/b]*
(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(c^2*e^10*E^((2*a)/b)*(-((a + b*Log[c*(
d + e/Sqrt[x])^2])/b))^p) + (2^(3 + p)*3^(1 - p)*d^7*(d + e/Sqrt[x])^3*Gamm
a[1 + p, (-3*(a + b*Log[c*(d + e/Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e/S
qrt[x])^2])^p)/(e^10*E^((3*a)/(2*b))*(c*(d + e/Sqrt[x])^2)^(3/2)*(-((a + b*
Log[c*(d + e/Sqrt[x])^2])/b))^p) - (9*d^8*Gamma[1 + p, -((a + b*Log[c*(d +
e/Sqrt[x])^2])/b)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(c*e^10*E^(a/b)*(-((
a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) + (2^(1 + p)*d^9*(d + e/Sqrt[x])*Gam
ma[1 + p, -1/2*(a + b*Log[c*(d + e/Sqrt[x])^2])/b]*(a + b*Log[c*(d + e/Sqrt
[x])^2])^p)/(e^10*E^(a/(2*b))*Sqrt[c*(d + e/Sqrt[x])^2]*(-((a + b*Log[c*(d
+ e/Sqrt[x])^2])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}\,dx = -\left(2\text{Subst}\left(\int x^9\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\text{Subst}\left(\int\left(-\frac{d^9\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^9} + \frac{9d^8(d+ex)\left(a+b\right.}{e^9}\right.\right.\right.$$

$$= -\frac{2\text{Subst}\left(\int(d+ex)^9\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt{x}}\right)}{e^9} + \frac{(18d}{}$$

$$= -\frac{2\text{Subst}\left(\int x^9\left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^{10}} + \frac{(18d)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\text{Subst}\left(\int e^{5x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{c^5e^{10}} - \frac{(36d^2)\text{Sub}}{}$$

$$= -\frac{5^{-1-p}e^{-\frac{5a}{b}}\Gamma\left(1+p, -\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d\right.\right.\right.}{c^5e^{10}}$$

Mathematica [**F**]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^6,x]
[Out] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^6, x]
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^2))^p/x^6,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^2))^p/x^6,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^6,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x^6, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^6,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x + 2*c*d*sqrt(x)*e + c*e^2)/x) + a)^p/x^6, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**2))**p/x**6,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/2))^2))^p/x^6,x)
[Out] int((a + b*log(c*(d + e/x^(1/2))^2))^p/x^6, x)
```

3.556 $\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$

Optimal. Leaf size=1121

$$\frac{3^{-p}4^{-1-p}e^{-\frac{12a}{b}}\Gamma\left(1+p,-\frac{12\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^{12}e^{12}}-$$

```
[Out] 4^(-1-p)*GAMMA(1+p,-12*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))
))^p/(3^p)/c^12/e^12/exp(12*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)-3*d*GAM
MA(1+p,-11*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(11^p)/
c^11/e^12/exp(11*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)+33*2^(-1-p)*d^2*GA
MMA(1+p,-10*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(5^p)/
c^10/e^12/exp(10*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)-55*d^3*GAMMA(1+p,-
9*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(9^p)/c^9/e^12/e
xp(9*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)+495*2^(-2-3*p)*d^4*GAMMA(1+p,-
8*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/c^8/e^12/exp(8*a
/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)-198*d^5*GAMMA(1+p,-7*(a+b*ln(c*(d+e*
x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(7^p)/c^7/e^12/exp(7*a/b)/(((-a-b
*ln(c*(d+e*x^(1/3))))/b)^p)+77*3^(1-p)*d^6*GAMMA(1+p,-6*(a+b*ln(c*(d+e*x^(1
/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(2^p)/c^6/e^12/exp(6*a/b)/(((-a-b*ln(
c*(d+e*x^(1/3))))/b)^p)-198*d^7*GAMMA(1+p,-5*(a+b*ln(c*(d+e*x^(1/3))))/b)*(
a+b*ln(c*(d+e*x^(1/3))))^p/(5^p)/c^5/e^12/exp(5*a/b)/(((-a-b*ln(c*(d+e*x^(1
/3))))/b)^p)+495*4^(-1-p)*d^8*GAMMA(1+p,-4*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+
b*ln(c*(d+e*x^(1/3))))^p/c^4/e^12/exp(4*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b
)^p)-55*d^9*GAMMA(1+p,-3*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3
))))^p/(3^p)/c^3/e^12/exp(3*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)+33*2^(-
1-p)*d^10*GAMMA(1+p,-2*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))
))^p/c^2/e^12/exp(2*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)-3*d^11*GAMMA(1+
p,(-a-b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/c/e^12/exp(a/b)
/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)
```

Rubi [A]

time = 1.26, antiderivative size = 1121, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2504, 2448, 2436, 2336, 2212, 2437, 2346}

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

```
[Out] (4^(-1 - p)*Gamma[1 + p, (-12*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log
[c*(d + e*x^(1/3))])^p)/(3^p*c^12*e^12*E^((12*a)/b)*(-((a + b*Log[c*(d + e*
```

```
x^(1/3))])/b))^p) - (3*d*Gamma[1 + p, (-11*(a + b*Log[c*(d + e*x^(1/3))]))/
b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(11^p*c^11*e^12*E^((11*a)/b)*(-((a + b
*Log[c*(d + e*x^(1/3))])/b))^p) + (33*2^(-1 - p)*d^2*Gamma[1 + p, (-10*(a +
 b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(5^p*c^10*
e^12*E^((10*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) - (55*d^3*Gamma[
1 + p, (-9*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))]
)^p)/(9^p*c^9*e^12*E^((9*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) + (
495*2^(-2 - 3*p)*d^4*Gamma[1 + p, (-8*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a
 + b*Log[c*(d + e*x^(1/3))])^p)/(c^8*e^12*E^((8*a)/b)*(-((a + b*Log[c*(d +
e*x^(1/3))])/b))^p) - (198*d^5*Gamma[1 + p, (-7*(a + b*Log[c*(d + e*x^(1/3)
)]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(7^p*c^7*e^12*E^((7*a)/b)*(-((a +
 b*Log[c*(d + e*x^(1/3))])/b))^p) + (77*3^(1 - p)*d^6*Gamma[1 + p, (-6*(a +
 b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(2^p*c^6*e
^12*E^((6*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) - (198*d^7*Gamma[1
 + p, (-5*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])
^p)/(5^p*c^5*e^12*E^((5*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) + (4
95*4^(-1 - p)*d^8*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a +
b*Log[c*(d + e*x^(1/3))])^p)/(c^4*e^12*E^((4*a)/b)*(-((a + b*Log[c*(d + e*x
^(1/3))])/b))^p) - (55*d^9*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))]))
/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(3^p*c^3*e^12*E^((3*a)/b)*(-((a + b*L
og[c*(d + e*x^(1/3))])/b))^p) + (33*2^(-1 - p)*d^10*Gamma[1 + p, (-2*(a + b
*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(c^2*e^12*E^
((2*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) - (3*d^11*Gamma[1 + p, -
((a + b*Log[c*(d + e*x^(1/3))])/b)]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(c*e^
12*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx = 3\text{Subst}\left(\int x^{11}(a+b\log(c(d+ex)))^p\,dx,x,\sqrt[3]{x}\right)$$
$$= 3\text{Subst}\left(\int\left(-\frac{d^{11}(a+b\log(c(d+ex)))^p}{e^{11}}+\frac{11d^{10}(d+ex)(a+b\log(\right.\right.}{e^{11}}$$
$$= \frac{3\text{Subst}\left(\int(d+ex)^{11}(a+b\log(c(d+ex)))^p\,dx,x,\sqrt[3]{x}\right)}{e^{11}}-\frac{(33d)\text{Subs}}{}$$
$$= \frac{3\text{Subst}\left(\int x^{11}(a+b\log(cx))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}}-\frac{(33d)\text{Subst}\left(\int x^{10}(a\right.}{}$$
$$= \frac{3\text{Subst}\left(\int e^{12x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^{12}e^{12}}-\frac{(33d)\text{Subst}\left(\int e^{1}\right.}{}$$
$$= \frac{3^{-p}4^{-1-p}e^{-\frac{12a}{b}}\Gamma\left(1+p,-\frac{12\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c(d+\right.\right.}{c^{12}e^{12}}$$

Mathematica [A]
time = 3.35, size = 670, normalized size = 0.60

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

```
[Out] -((2^(-2 - 3*p)*(-(2310^p*Gamma[1 + p, (-12*(a + b*Log[c*(d + e*x^(1/3))]))
/b]) + 2^(2 + 3*p)*3^(1 + 2*p)*35^p*c*d*E^(a/b)*Gamma[1 + p, (-11*(a + b*Lo
g[c*(d + e*x^(1/3))]))/b] + c^2*d^2*E^((2*a)/b)*(-(6^(1 + 2*p)*7^p*11^(1 +
p)*Gamma[1 + p, (-10*(a + b*Log[c*(d + e*x^(1/3))]))/b]) + 2^(2 + 3*p)*7^p*
55^(1 + p)*c*d*E^(a/b)*Gamma[1 + p, (-9*(a + b*Log[c*(d + e*x^(1/3))]))/b]
+ c^2*d^2*E^((2*a)/b)*(-(7^p*495^(1 + p)*Gamma[1 + p, (-8*(a + b*Log[c*(d +
 e*x^(1/3))]))/b]) + 5^p*792^(1 + p)*c*d*E^(a/b)*Gamma[1 + p, (-7*(a + b*Lo
g[c*(d + e*x^(1/3))]))/b] - 5^p*924^(1 + p)*c^2*d^2*E^((2*a)/b)*Gamma[1 + p
, (-6*(a + b*Log[c*(d + e*x^(1/3))]))/b] + 7^p*792^(1 + p)*c^3*d^3*E^((3*a)
/b)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(1/3))]))/b] - 14^p*495^(1 + p)*
c^4*d^4*E^((4*a)/b)*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*x^(1/3))]))/b] + 2
^(2 + 3*p)*21^p*55^(1 + p)*c^5*d^5*E^((5*a)/b)*Gamma[1 + p, (-3*(a + b*Log[
c*(d + e*x^(1/3))]))/b] - 6^(1 + 2*p)*11^(1 + p)*35^p*c^6*d^6*E^((6*a)/b)*G
amma[1 + p, (-2*(a + b*Log[c*(d + e*x^(1/3))]))/b] + 2^(2 + 3*p)*3^(1 + 2*p
)*385^p*c^7*d^7*E^((7*a)/b)*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(1/3))])/b
)])))*(a + b*Log[c*(d + e*x^(1/3))])^p)/(3465^p*c^12*e^12*E^((12*a)/b)*(-((
a + b*Log[c*(d + e*x^(1/3))])/b))^p))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(1/3))))^p,x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e*x^(1/3))))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((x^(1/3)*e + d)*c) + a)^p*x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="fricas")
[Out] integral((b*log(c*x^(1/3)*e + c*d) + a)^p*x^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(1/3))))**p,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="giac")
[Out] integrate((b*log((x^(1/3)*e + d)*c) + a)^p*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\left(a+b\,\ln\left(c\,\left(d+e\,x^{1/3}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e*x^(1/3))))^p,x)
[Out] int(x^3*(a + b*log(c*(d + e*x^(1/3))))^p, x)
```

3.557 $\int x^2 \left(a+b \log \left(c \left(d+e \sqrt[3]{x}\right)\right)\right)^p \, dx$

Optimal. Leaf size=831

$$\frac{3^{-1-2p} e^{-\frac{9a}{b}} \Gamma\left(1+p,-\frac{9\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^9e^9} - \frac{3\,8^{-}\cdots}{\cdots}$$

```
[Out] 3^(-1-2*p)*GAMMA(1+p,-9*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3)
)))^p/c^9/e^9/exp(9*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)-3*d*GAMMA(1+p,-
8*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(8^p)/c^8/e^9/ex
p(8*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)+12*d^2*GAMMA(1+p,-7*(a+b*ln(c*(
d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(7^p)/c^7/e^9/exp(7*a/b)/(((-
a-b*ln(c*(d+e*x^(1/3))))/b)^p)-7*2^(2-p)*d^3*GAMMA(1+p,-6*(a+b*ln(c*(d+e*x^
(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(3^p)/c^6/e^9/exp(6*a/b)/(((-a-b*ln
(c*(d+e*x^(1/3))))/b)^p)+42*d^4*GAMMA(1+p,-5*(a+b*ln(c*(d+e*x^(1/3))))/b)*(
a+b*ln(c*(d+e*x^(1/3))))^p/(5^p)/c^5/e^9/exp(5*a/b)/(((-a-b*ln(c*(d+e*x^(1/
3))))/b)^p)-21*2^(1-2*p)*d^5*GAMMA(1+p,-4*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b
*ln(c*(d+e*x^(1/3))))^p/c^4/e^9/exp(4*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^
p)+28*d^6*GAMMA(1+p,-3*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))
))^p/(3^p)/c^3/e^9/exp(3*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)-3*2^(2-p)*
d^7*GAMMA(1+p,-2*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/c
^2/e^9/exp(2*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)+3*d^8*GAMMA(1+p,(-a-b*
ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/c/e^9/exp(a/b)/(((-a-b*
ln(c*(d+e*x^(1/3))))/b)^p)
```

Rubi [A]

time = 0.91, antiderivative size = 831, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2504, 2448, 2436, 2336, 2212, 2437, 2346}

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

```
[Out] (3^(-1 - 2*p)*Gamma[1 + p, (-9*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Lo
g[c*(d + e*x^(1/3))])^p)/(c^9*e^9*E^((9*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3
))])/b))^p) - (3*d*Gamma[1 + p, (-8*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a +
 b*Log[c*(d + e*x^(1/3))])^p)/(8^p*c^8*e^9*E^((8*a)/b)*(-((a + b*Log[c*(d +
 e*x^(1/3))])/b))^p) + (12*d^2*Gamma[1 + p, (-7*(a + b*Log[c*(d + e*x^(1/3)
)]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(7^p*c^7*e^9*E^((7*a)/b)*(-((a +
b*Log[c*(d + e*x^(1/3))])/b))^p) - (7*2^(2 - p)*d^3*Gamma[1 + p, (-6*(a + b
*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(3^p*c^6*e^9
```

```
*E^((6*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) + (42*d^4*Gamma[1 + p
, (-5*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/
(5^p*c^5*e^9*E^((5*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) - (21*2^(
1 - 2*p)*d^5*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log
[c*(d + e*x^(1/3))])^p)/(c^4*e^9*E^((4*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3)
)])/b))^p) + (28*d^6*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a
 + b*Log[c*(d + e*x^(1/3))])^p)/(3^p*c^3*e^9*E^((3*a)/b)*(-((a + b*Log[c*(d
 + e*x^(1/3))])/b))^p) - (3*2^(2 - p)*d^7*Gamma[1 + p, (-2*(a + b*Log[c*(d
+ e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(c^2*e^9*E^((2*a)/b)*(
-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) + (3*d^8*Gamma[1 + p, -((a + b*Log[
c*(d + e*x^(1/3))])/b)]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(c*e^9*E^(a/b)*(-
((a + b*Log[c*(d + e*x^(1/3))])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx = 3\text{Subst}\left(\int x^8(a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)$$

$$= 3\text{Subst}\left(\int\left(\frac{d^8(a+b\log(c(d+ex)))^p}{e^8}-\frac{8d^7(d+ex)(a+b\log(c(d+\right.\right.}{e^8}$$

$$= \frac{3\text{Subst}\left(\int(d+ex)^8(a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)}{e^8}-\frac{(24d)\text{Subst}\left(\right.}{}$$

$$= \frac{3\text{Subst}\left(\int x^8(a+b\log(cx))^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^9}-\frac{(24d)\text{Subst}\left(\int x^7(a+\right.}{}$$

$$= \frac{3\text{Subst}\left(\int e^{9x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^9e^9}-\frac{(24d)\text{Subst}\left(\int e^{8x}(\right.}{}$$

$$= \frac{3^{-1-2p}e^{-\frac{9a}{b}}\Gamma\left(1+p,-\frac{9\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right.\right.\right.}{c^9e^9}$$

Mathematica [A]
time = 1.26, size = 501, normalized size = 0.60

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

```
[Out] (3^(-1 - 2*p)*(280^p*Gamma[1 + p, (-9*(a + b*Log[c*(d + e*x^(1/3))]))/b] -
9^(1 + p)*35^p*c*d*E^(a/b)*Gamma[1 + p, (-8*(a + b*Log[c*(d + e*x^(1/3))]))
/b] + 2^(2 + 3*p)*5^p*9^(1 + p)*c^2*d^2*E^((2*a)/b)*Gamma[1 + p, (-7*(a + b
```

```
*Log[c*(d + e*x^(1/3))]))/b] - 5^p*84^(1 + p)*c^3*d^3*E^((3*a)/b)*Gamma[1 +
 p, (-6*(a + b*Log[c*(d + e*x^(1/3))]))/b] + 2^(1 + 3*p)*63^(1 + p)*c^4*d^4
*E^((4*a)/b)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(1/3))]))/b] - 5^p*126^
(1 + p)*c^5*d^5*E^((5*a)/b)*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*x^(1/3))])
)/b] + 2^(2 + 3*p)*5^p*21^(1 + p)*c^6*d^6*E^((6*a)/b)*Gamma[1 + p, (-3*(a +
 b*Log[c*(d + e*x^(1/3))]))/b] - 35^p*36^(1 + p)*c^7*d^7*E^((7*a)/b)*Gamma[
1 + p, (-2*(a + b*Log[c*(d + e*x^(1/3))]))/b] + 9^(1 + p)*280^p*c^8*d^8*E^(
(8*a)/b)*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(1/3))])/b)])*(a + b*Log[c*(d
 + e*x^(1/3))])^p)/(280^p*c^9*e^9*E^((9*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3
))])/b))^p)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2} \left(a+b \ln \left(c \left(d+e\,x^{\frac{1}{3}}\right)\right)\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(1/3))))^p,x)
[Out] int(x^2*(a+b*ln(c*(d+e*x^(1/3))))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="maxima")
[Out] integrate((b*log((x^(1/3)*e + d)*c) + a)^p*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="fricas")
[Out] integral((b*log(c*x^(1/3)*e + c*d) + a)^p*x^2, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/3))))**p,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8010 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^(1/3)*e + d)*c) + a)^p*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(a+b\,\ln\left(c\,\left(d+e\,x^{1/3}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e*x^(1/3))))^p,x)
```

```
[Out] int(x^2*(a + b*log(c*(d + e*x^(1/3))))^p, x)
```

3.558 $\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$

Optimal. Leaf size=553

$$\frac{2^{-1-p}3^{-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^6e^6}-\cdots$$

```
[Out] 2^(-1-p)*GAMMA(1+p,-6*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3)))
)^p/(3^p)/c^6/e^6/exp(6*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)-3*d*GAMMA(1
+p,-5*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(5^p)/c^5/e^
6/exp(5*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)+15*2^(-1-2*p)*d^2*GAMMA(1+p
,-4*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/c^4/e^6/exp(4*
a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)-10*d^3*GAMMA(1+p,-3*(a+b*ln(c*(d+e*
x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(3^p)/c^3/e^6/exp(3*a/b)/(((-a-b*
ln(c*(d+e*x^(1/3))))/b)^p)+15*2^(-1-p)*d^4*GAMMA(1+p,-2*(a+b*ln(c*(d+e*x^(1
/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/c^2/e^6/exp(2*a/b)/(((-a-b*ln(c*(d+e*
x^(1/3))))/b)^p)-3*d^5*GAMMA(1+p,(-a-b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d
+e*x^(1/3))))^p/c/e^6/exp(a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)
```

Rubi [A]

time = 0.56, antiderivative size = 553, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2504, 2448, 2436, 2336, 2212, 2437, 2346}

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

```
[Out] (2^(-1 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[
c*(d + e*x^(1/3))])^p)/(3^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e*x^(1
/3))])/b))^p) - (3*d*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a
 + b*Log[c*(d + e*x^(1/3))])^p)/(5^p*c^5*e^6*E^((5*a)/b)*(-((a + b*Log[c*(d
 + e*x^(1/3))])/b))^p) + (15*2^(-1 - 2*p)*d^2*Gamma[1 + p, (-4*(a + b*Log[c
*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(c^4*e^6*E^((4*a)/
b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) - (10*d^3*Gamma[1 + p, (-3*(a +
 b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(3^p*c^3*e
^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) + (15*2^(-1 - p)*d^
4*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x
^(1/3))])^p)/(c^2*e^6*E^((2*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p)
- (3*d^5*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(1/3))])/b)]*(a + b*Log[c*(d
+ e*x^(1/3))])^p)/(c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx = 3\text{Subst}\left(\int x^5(a+b\log(c(d+ex)))^p\,dx,x,\sqrt[3]{x}\right)$$
$$= 3\text{Subst}\left(\int\left(-\frac{d^5(a+b\log(c(d+ex)))^p}{e^5}+\frac{5d^4(d+ex)(a+b\log(c(d$$
$$= \frac{3\text{Subst}\left(\int(d+ex)^5(a+b\log(c(d+ex)))^p\,dx,x,\sqrt[3]{x}\right)}{e^5} - \frac{(15d)\text{Subst}(}{}$$
$$= \frac{3\text{Subst}\left(\int x^5(a+b\log(cx))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^6} - \frac{(15d)\text{Subst}\left(\int x^4(a+\right.}{}$$
$$= \frac{3\text{Subst}\left(\int e^{6x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^6e^6} - \frac{(15d)\text{Subst}\left(\int e^{5x}\right.}{}$$
$$= \frac{2^{-1-p}3^{-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{\right.\right.\right.}{c^6e^6}$$

Mathematica [A]
time = 1.11, size = 325, normalized size = 0.59

$$\frac{2^{-1-2p}15^{-p}e^{-\frac{6a}{b}}\left(10^p\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)-cde^{a/b}\left(2^{1+2p}3^{1+p}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)+5^pcde^{a/b}\left(-53^{1+p}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)+2^pcde^{a/b}\left(5\,2^{2+p}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)+3^{1+p}cde^{a/b}\left(-5\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)+2^{1+p}cde^{a/b}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)\right)\right)\right)\right)\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^6e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

```
[Out] (2^(-1 - 2*p)*(10^p*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*x^(1/3))]))/b] - c
*d*E^(a/b)*(2^(1 + 2*p)*3^(1 + p)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(1
/3))]))/b] + 5^p*c*d*E^(a/b)*(-5*3^(1 + p)*Gamma[1 + p, (-4*(a + b*Log[c*(d
 + e*x^(1/3))]))/b] + 2^p*c*d*E^(a/b)*(5*2^(2 + p)*Gamma[1 + p, (-3*(a + b*
Log[c*(d + e*x^(1/3))]))/b] + 3^(1 + p)*c*d*E^(a/b)*(-5*Gamma[1 + p, (-2*(a
 + b*Log[c*(d + e*x^(1/3))]))/b] + 2^(1 + p)*c*d*E^(a/b)*Gamma[1 + p, -((a
+ b*Log[c*(d + e*x^(1/3))])/b)])))))*(a + b*Log[c*(d + e*x^(1/3))])^p)/(15^
p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e*x^(1/3))))^p,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e*x^(1/3))))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="maxima")
[Out] integrate((b*log((x^(1/3)*e + d)*c) + a)^p*x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="fricas")
[Out] integral((b*log(c*x^(1/3)*e + c*d) + a)^p*x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/3))))**p,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="giac")
[Out] integrate((b*log((x^(1/3)*e + d)*c) + a)^p*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(a+b\ln\left(c\left(d+e\,x^{1/3}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e*x^(1/3))))^p,x)
[Out] int(x*(a + b*log(c*(d + e*x^(1/3))))^p, x)
```

3.559 $\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$

Optimal. Leaf size=266

$$\frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^3e^3}-\frac{3\ 2^{-p}a}{}$$

```
[Out] GAMMA(1+p,-3*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(3^p)
/c^3/e^3/exp(3*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)-3*d*GAMMA(1+p,-2*(a+
b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(2^p)/c^2/e^3/exp(2*a
/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)+3*d^2*GAMMA(1+p,(-a-b*ln(c*(d+e*x^(1
/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/c/e^3/exp(a/b)/(((-a-b*ln(c*(d+e*x^(1
/3))))/b)^p)
```

Rubi [A]

time = 0.25, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {2501, 2448, 2436, 2336, 2212, 2437, 2346}

$$\frac{3^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)}{c^3e^3}-\frac{3d2^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)}{c^2e^3}+\frac{3d^2e^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)}{ce^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

```
[Out] (Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^
(1/3))])^p)/(3^p*c^3*e^3*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^
p) - (3*d*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*
(d + e*x^(1/3))])^p)/(2^p*c^2*e^3*E^((2*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3
))])/b))^p) + (3*d^2*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(1/3))])/b)]*(a +
 b*Log[c*(d + e*x^(1/3))])^p)/(c*e^3*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3)
)])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
```

```
c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2501

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx &= 3\text{Subst}\left(\int x^2(a+b\log(c(d+ex)))^p\,dx,x,\sqrt[3]{x}\right)\\
&= 3\text{Subst}\left(\int\left(\frac{d^2(a+b\log(c(d+ex)))^p}{e^2}-\frac{2d(d+ex)(a+b\log(c(d+ex\\
&= \frac{3\text{Subst}\left(\int(d+ex)^2(a+b\log(c(d+ex)))^p\,dx,x,\sqrt[3]{x}\right)}{e^2}-\frac{(6d)\text{Subst}\left(\int(\right.}{}\\
&= \frac{3\text{Subst}\left(\int x^2(a+b\log(cx))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^3}-\frac{(6d)\text{Subst}\left(\int x(a+b\,\text{lo}\right.}{}\\
&= \frac{3\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^3e^3}-\frac{(6d)\text{Subst}\left(\int e^{2x}(a\right.}{}\\
&= \frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^{p}}{c^3e^3}
\end{aligned}
$$

Mathematica [A]
time = 0.39, size = 174, normalized size = 0.65

$$
\frac{6^{-p}e^{-\frac{3a}{b}}\left(2^p\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)+3^{1+p}cde^{a/b}\left(-\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)+2^pcde^{a/b}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)\right)\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^3e^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))])^p,x]
[Out] ((2^p*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))]))/b] + 3^(1 + p)*c*d*E
^(a/b)*(-Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(1/3))]))/b] + 2^p*c*d*E^(a
/b)*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(1/3))])/b)]))*(a + b*Log[c*(d + e
*x^(1/3))])^p)/(6^p*c^3*e^3*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b
))^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)\right)\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))))^p,x)
[Out] int((a+b*ln(c*(d+e*x^(1/3))))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="maxima")
[Out] integrate((b*log((x^(1/3)*e + d)*c) + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="fricas")
[Out] integral((b*log(c*x^(1/3)*e + c*d) + a)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\log\left(c\left(d + e\sqrt[3]{x}\right)\right)\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))))**p,x)
[Out] Integral((a + b*log(c*(d + e*x**(1/3))))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="giac")
[Out] integrate((b*log((x^(1/3)*e + d)*c) + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\ln\left(c\,\left(d+e\,x^{1/3}\right)\right)\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))))^p,x)
[Out] int((a + b*log(c*(d + e*x^(1/3))))^p, x)
```

3.560 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(1/3))))^p/x,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))])^p/x,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x)])^p/x, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x}\,dx = 3\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)))^p}{x}\,dx, x, \sqrt[3]{x}\right)$$

Mathematica [A]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))])^p/x,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(1/3))])^p/x, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)\right)\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))))^p/x,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))))^p/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p/x,x, algorithm="maxima")
```

```
[Out] integrate((b*log((x^(1/3)*e + d)*c) + a)^p/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p/x,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^(1/3)*e + c*d) + a)^p/x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))))**p/x,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^(1/3)*e + d)*c) + a)^p/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{1/3}\right)\right)\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))))^p/x,x)
[Out] int((a + b*log(c*(d + e*x^(1/3))))^p/x, x)
```

3.561 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(1/3))))^p/x^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))])^p/x^2,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x)])^p/x^4, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2}\,dx = 3\mathrm{Subst}\left(\int \frac{(a+b\log(c(d+ex)))^p}{x^4}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A]
time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))])^p/x^2,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(1/3))])^p/x^2, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)\right)\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))))^p/x^2,x)
[Out] int((a+b*ln(c*(d+e*x^(1/3))))^p/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p/x^2,x, algorithm="maxima")
[Out] integrate((b*log((x^(1/3)*e + d)*c) + a)^p/x^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p/x^2,x, algorithm="fricas")
[Out] integral((b*log(c*x^(1/3)*e + c*d) + a)^p/x^2, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))))**p/x**2,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p/x^2,x, algorithm="giac")
[Out] integrate((b*log((x^(1/3)*e + d)*c) + a)^p/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{1/3}\right)\right)\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))))^p/x^2,x)
[Out] int((a + b*log(c*(d + e*x^(1/3))))^p/x^2, x)
```

3.562 $\int x^3 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=1363

result too large to display

[Out] 2^(-2-p)*GAMMA(1+p,-6*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/(3^p)/c^6/e^12/exp(6*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)-3*(2/11)^p*d*(d+e*x^(1/3))^11*GAMMA(1+p,-11/2*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/e^12/exp(11/2*a/b)/(c*(d+e*x^(1/3))^2)^(11/2)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)+33/2*d^2*GAMMA(1+p,-5*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/(5^p)/c^5/e^12/exp(5*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)-55*(2/9)^p*d^3*(d+e*x^(1/3))^9*GAMMA(1+p,-9/2*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/e^12/exp(9/2*a/b)/(c*(d+e*x^(1/3))^2)^(9/2)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)+495*d^4*GAMMA(1+p,-4*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/(2^(2+2*p))/c^4/e^12/exp(4*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)-99*2^(1+p)*d^5*(d+e*x^(1/3))^7*GAMMA(1+p,-7/2*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/(7^p)/e^12/exp(7/2*a/b)/(c*(d+e*x^(1/3))^2)^(7/2)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)+77*3^(1-p)*d^6*GAMMA(1+p,-3*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/c^3/e^12/exp(3*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)-99*2^(1+p)*d^7*(d+e*x^(1/3))^5*GAMMA(1+p,-5/2*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/(5^p)/e^12/exp(5/2*a/b)/(c*(d+e*x^(1/3))^2)^(5/2)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)+495*2^(-2-p)*d^8*GAMMA(1+p,-2*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/c^2/e^12/exp(2*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)-55*(2/3)^p*d^9*(d+e*x^(1/3))^3*GAMMA(1+p,-3/2*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/e^12/exp(3/2*a/b)/(c*(d+e*x^(1/3))^2)^(3/2)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)+33/2*d^10*GAMMA(1+p,(-a-b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/c/e^12/exp(a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)-3*2^p*d^11*(d+e*x^(1/3))*GAMMA(1+p,1/2*(-a-b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/e^12/exp(1/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)/(c*(d+e*x^(1/3))^2)^(1/2)

Rubi [A]

time = 1.45, antiderivative size = 1363, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {2504, 2448, 2436, 2337, 2212, 2437, 2347}

Antiderivative was successfully verified.

[In] Int[x^3*(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]

[Out] (2^(-2 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*x^(1/3))^2]))/b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(3^p*c^6*e^12*E^((6*a)/b)*(-((a + b*Log[c*(d + e

```
*x^(1/3))^2])/b))^p) - (3*(2/11)^p*d*(d + e*x^(1/3))^11*Gamma[1 + p, (-11*(
a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)
/(e^12*E^((11*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(11/2)*(-((a + b*Log[c*(d + e
*x^(1/3))^2])/b))^p) + (33*d^2*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(1/3)
)^2]))/b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(2*5^p*c^5*e^12*E^((5*a)/b)*(
-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (55*(2/9)^p*d^3*(d + e*x^(1/3))
^9*Gamma[1 + p, (-9*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(
d + e*x^(1/3))^2])^p)/(e^12*E^((9*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(9/2)*(-(
(a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) + (495*d^4*Gamma[1 + p, (-4*(a + b*
Log[c*(d + e*x^(1/3))^2]))/b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(2^(2*(1
+ p))*c^4*e^12*E^((4*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (99
*2^(1 + p)*d^5*(d + e*x^(1/3))^7*Gamma[1 + p, (-7*(a + b*Log[c*(d + e*x^(1/
3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(7^p*e^12*E^((7*a)/(2*b
))*(c*(d + e*x^(1/3))^2)^(7/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) +
 (77*3^(1 - p)*d^6*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]))/b]*(a
 + b*Log[c*(d + e*x^(1/3))^2])^p)/(c^3*e^12*E^((3*a)/b)*(-((a + b*Log[c*(d
+ e*x^(1/3))^2])/b))^p) - (99*2^(1 + p)*d^7*(d + e*x^(1/3))^5*Gamma[1 + p,
(-5*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2
])^p)/(5^p*e^12*E^((5*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(5/2)*(-((a + b*Log[c
*(d + e*x^(1/3))^2])/b))^p) + (495*2^(-2 - p)*d^8*Gamma[1 + p, (-2*(a + b*L
og[c*(d + e*x^(1/3))^2]))/b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(c^2*e^12*
E^((2*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (55*(2/3)^p*d^9*(d
 + e*x^(1/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(
a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^12*E^((3*a)/(2*b))*(c*(d + e*x^(1/3))
^2)^(3/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) + (33*d^10*Gamma[1 + p
, -((a + b*Log[c*(d + e*x^(1/3))^2])/b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p
)/(2*c*e^12*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (3*2^p*d^1
1*(d + e*x^(1/3))*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e*x^(1/3))^2])/b]*(a
+ b*Log[c*(d + e*x^(1/3))^2])^p)/(e^12*E^(a/(2*b))*Sqrt[c*(d + e*x^(1/3))^2
]*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx = 3\text{Subst}\left(\int x^{11}\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \sqrt[3]{x}\right)$$

$$= 3\text{Subst}\left(\int\left(-\frac{d^{11}(a+b\log(c(d+ex)^2))^p}{e^{11}}+\frac{11d^{10}(d+ex)\,(a+b\,\mathrm{lo}}{e^{11}}\right.\right.$$

$$= \frac{3\text{Subst}\left(\int(d+ex)^{11}\,(a+b\log(c(d+ex)^2))^p\,dx, x, \sqrt[3]{x}\right)}{e^{11}} - \frac{(33d)\text{Su}}{}$$

$$= \frac{3\text{Subst}\left(\int x^{11}(a+b\log(cx^2))^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^{12}} - \frac{(33d)\text{Subst}\left(\int x^{1}\right.}{}$$

$$= \frac{3\text{Subst}\left(\int e^{6x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2c^6e^{12}} + \frac{(165d^2)\,\text{Subst}}{}$$

$$= \frac{2^{-2-p}3^{-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d\right.\right.\right.}{c^6e^{12}}$$

Mathematica [F]
time = 0.47, size = 0, normalized size = 0.00

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
[Out] Integrate[x^3*(a + b*Log[c*(d + e*x^(1/3))^2])^p, x]
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^3\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(1/3))^2))^p,x)
[Out] int(x^3*(a+b*ln(c*(d+e*x^(1/3))^2))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((x^(1/3)*e + d)^2*c) + a)^p*x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(2*c*d*x^(1/3)*e + c*d^2 + c*x^(2/3)*e^2) + a)^p*x^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(1/3))**2))**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^(1/3)*e + d)^2*c) + a)^p*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\left(a+b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e*x^(1/3))^2))^p,x)
```

```
[Out] int(x^3*(a + b*log(c*(d + e*x^(1/3))^2))^p, x)
```

3.563 $\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$

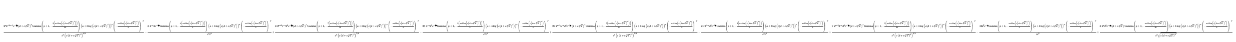
Optimal. Leaf size=1035

$$\frac{2^p 3^{-1-2p} e^{-\frac{9a}{2b}}\left(d+e\sqrt[3]{x}\right)^9\Gamma\left(1+p,-\frac{9\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\right.\right.}{b}\right.}{e^9\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{9/2}}$$

```
[Out] 2^p*3^(-1-2*p)*(d+e*x^(1/3))^9*GAMMA(1+p,-9/2*(a+b*ln(c*(d+e*x^(1/3))^2))/b
)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/e^9/exp(9/2*a/b)/(c*(d+e*x^(1/3))^2)^(9/2)/
(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)-3*d*GAMMA(1+p,-4*(a+b*ln(c*(d+e*x^(1/3
))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/(4^p)/c^4/e^9/exp(4*a/b)/(((-a-b*ln
(c*(d+e*x^(1/3))^2))/b)^p)+3*2^(2+p)*d^2*(d+e*x^(1/3))^7*GAMMA(1+p,-7/2*(a+
b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/(7^p)/e^9/exp(7/2
*a/b)/(c*(d+e*x^(1/3))^2)^(7/2)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)-28*d^3
*GAMMA(1+p,-3*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/
(3^p)/c^3/e^9/exp(3*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)+21*2^(1+p)*d^
4*(d+e*x^(1/3))^5*GAMMA(1+p,-5/2*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(
d+e*x^(1/3))^2))^p/(5^p)/e^9/exp(5/2*a/b)/(c*(d+e*x^(1/3))^2)^(5/2)/(((-a-b
*ln(c*(d+e*x^(1/3))^2))/b)^p)-21*2^(1-p)*d^5*GAMMA(1+p,-2*(a+b*ln(c*(d+e*x^
(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/c^2/e^9/exp(2*a/b)/(((-a-b*ln(c
*(d+e*x^(1/3))^2))/b)^p)+7*2^(2+p)*d^6*(d+e*x^(1/3))^3*GAMMA(1+p,-3/2*(a+b*
ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/(3^p)/e^9/exp(3/2*a
/b)/(c*(d+e*x^(1/3))^2)^(3/2)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)-12*d^7*G
AMMA(1+p,(-a-b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/c/e^
9/exp(a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)+3*2^p*d^8*(d+e*x^(1/3))*GAM
MA(1+p,1/2*(-a-b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/e^
9/exp(1/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)/(c*(d+e*x^(1/3))^2)^(1/
2)
```

Rubi [A]

time = 1.02, antiderivative size = 1035, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {2504, 2448, 2436, 2337, 2212, 2437, 2347}



Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
```

```
[Out] (2^p*3^(-1 - 2*p)*(d + e*x^(1/3))^9*Gamma[1 + p, (-9*(a + b*Log[c*(d + e*x^
(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^9*E^((9*a)/(2*b))
```

```
*(c*(d + e*x^(1/3))^2)^(9/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (
3*d*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*x^(1/3))^2]))/b]*(a + b*Log[c*(d +
 e*x^(1/3))^2])^p)/(4^p*c^4*e^9*E^((4*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))
^2])/b))^p) + (3*2^(2 + p)*d^2*(d + e*x^(1/3))^7*Gamma[1 + p, (-7*(a + b*Lo
g[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(7^p*e^
9*E^((7*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(7/2)*(-((a + b*Log[c*(d + e*x^(1/3
))^2])/b))^p) - (28*d^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]))/
b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(3^p*c^3*e^9*E^((3*a)/b)*(-((a + b*L
og[c*(d + e*x^(1/3))^2])/b))^p) + (21*2^(1 + p)*d^4*(d + e*x^(1/3))^5*Gamma
[1 + p, (-5*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^
(1/3))^2])^p)/(5^p*e^9*E^((5*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(5/2)*(-((a +
b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (21*2^(1 - p)*d^5*Gamma[1 + p, (-2*(a
+ b*Log[c*(d + e*x^(1/3))^2]))/b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(c^2*
e^9*E^((2*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) + (7*2^(2 + p)*d
^6*(d + e*x^(1/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*
b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(3^p*e^9*E^((3*a)/(2*b))*(c*(d + e*
x^(1/3))^2)^(3/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (12*d^7*Gamm
a[1 + p, -((a + b*Log[c*(d + e*x^(1/3))^2])/b)]*(a + b*Log[c*(d + e*x^(1/3)
)^2])^p)/(c*e^9*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) + (3*2^p
*d^8*(d + e*x^(1/3))*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e*x^(1/3))^2])/b]*
(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^9*E^(a/(2*b))*Sqrt[c*(d + e*x^(1/3))
^2]*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx = 3\text{Subst}\left(\int x^8\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \sqrt[3]{x}\right)$$

$$= 3\text{Subst}\left(\int\left(\frac{d^8\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^8}-\frac{8d^7(d+ex)\left(a+b\log\left(c(\right.\right.}{e^8}\right.\right.$$

$$= \frac{3\text{Subst}\left(\int(d+ex)^8\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \sqrt[3]{x}\right)}{e^8}-\frac{(24d)\text{Sul}}{}$$

$$= \frac{3\text{Subst}\left(\int x^8\left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^9}-\frac{(24d)\text{Subst}\left(\int x^7(\right.}{}$$

$$= -\frac{(12d)\text{Subst}\left(\int e^{4x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{c^4e^9}-\frac{(84d^3)\,\text{S}}{}$$

$$= \frac{2^p3^{-1-2p}e^{-\frac{9a}{2b}}\left(d+e\sqrt[3]{x}\right)^9\Gamma\left(1+p,-\frac{9\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right)\left(a\right.}{e^9\left(c\left(d+e\sqrt[3]{x}\right)^2\right.}$$

Mathematica [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
```

```
[Out] Integrate[x^2*(a + b*Log[c*(d + e*x^(1/3))^2])^p, x]
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(1/3))^2))^p,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e*x^(1/3))^2))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((x^(1/3)*e + d)^2*c) + a)^p*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(2*c*d*x^(1/3)*e + c*d^2 + c*x^(2/3)*e^2) + a)^p*x^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/3))**2))**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^(1/3)*e + d)^2*c) + a)^p*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\left(a+b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e*x^(1/3))^2))^p,x)
```

```
[Out] int(x^2*(a + b*log(c*(d + e*x^(1/3))^2))^p, x)
```

3.564 $\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=673

$$\frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)^{-p}}{2c^3e^6} - 3$$

```
[Out] 1/2*GAMMA(1+p,-3*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))
^p/(3^p)/c^3/e^6/exp(3*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)-3*(2/5)^p*
d*(d+e*x^(1/3))^5*GAMMA(1+p,-5/2*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(
d+e*x^(1/3))^2))^p/e^6/exp(5/2*a/b)/(c*(d+e*x^(1/3))^2)^(5/2)/(((-a-b*ln(c*
(d+e*x^(1/3))^2))/b)^p)+15*2^(-1-p)*d^2*GAMMA(1+p,-2*(a+b*ln(c*(d+e*x^(1/3)
)^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/c^2/e^6/exp(2*a/b)/(((-a-b*ln(c*(d+e
*x^(1/3))^2))/b)^p)-5*2^(1+p)*d^3*(d+e*x^(1/3))^3*GAMMA(1+p,-3/2*(a+b*ln(c*
(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/(3^p)/e^6/exp(3/2*a/b)/(
c*(d+e*x^(1/3))^2)^(3/2)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)+15/2*d^4*GAMM
A(1+p,(-a-b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/c/e^6/e
xp(a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)-3*2^p*d^5*(d+e*x^(1/3))*GAMMA(
1+p,1/2*(-a-b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/e^6/e
xp(1/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)/(c*(d+e*x^(1/3))^2)^(1/2)
```

Rubi [A]

time = 0.65, antiderivative size = 673, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2504, 2448, 2436, 2337, 2212, 2437, 2347}

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
```

```
[Out] (Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]))/b]*(a + b*Log[c*(d + e*
x^(1/3))^2])^p)/(2*3^p*c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))^
2])/b))^p) - (3*(2/5)^p*d*(d + e*x^(1/3))^5*Gamma[1 + p, (-5*(a + b*Log[c*(
d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^6*E^((5*a
)/(2*b))*(c*(d + e*x^(1/3))^2)^(5/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b)
)^p) + (15*2^(-1 - p)*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(1/3))^2])
)/b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(c^2*e^6*E^((2*a)/b)*(-((a + b*Log
[c*(d + e*x^(1/3))^2])/b))^p) - (5*2^(1 + p)*d^3*(d + e*x^(1/3))^3*Gamma[1
+ p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/
3))^2])^p)/(3^p*e^6*E^((3*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(3/2)*(-((a + b*L
```

```
og[c*(d + e*x^(1/3))^2])/b))^p) + (15*d^4*Gamma[1 + p, -((a + b*Log[c*(d +
e*x^(1/3))^2])/b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(2*c*e^6*E^(a/b)*(-(
(a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (3*2^p*d^5*(d + e*x^(1/3))*Gamma[
1 + p, -1/2*(a + b*Log[c*(d + e*x^(1/3))^2])/b]*(a + b*Log[c*(d + e*x^(1/3)
)^2])^p)/(e^6*E^(a/(2*b))*Sqrt[c*(d + e*x^(1/3))^2]*(-((a + b*Log[c*(d + e*
x^(1/3))^2])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx = 3\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

$$= 3\text{Subst}\left(\int\left(-\frac{d^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5}+\frac{5d^4(d+ex)\left(a+b\log\right.}{e^5}\right.\right.$$

$$= \frac{3\text{Subst}\left(\int(d+ex)^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)}{e^5}-\frac{(15d)\text{Su}}{}$$

$$= \frac{3\text{Subst}\left(\int x^5\left(a+b\log\left(cx^2\right)\right)^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^6}-\frac{(15d)\text{Subst}\left(\int x^4\right.}{}$$

$$= \frac{3\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2c^3e^6}+\frac{\left(15d^2\right)\text{Subst}}{}$$

$$= \frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{}\right.\right.\right.}{2c^3e^6}$$

Mathematica [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
[Out] Integrate[x*(a + b*Log[c*(d + e*x^(1/3))^2])^p, x]
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e*x^(1/3))^2))^p,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e*x^(1/3))^2))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((x^(1/3)*e + d)^2*c) + a)^p*x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(2*c*d*x^(1/3)*e + c*d^2 + c*x^(2/3)*e^2) + a)^p*x, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/3))**2))**p,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^(1/3)*e + d)^2*c) + a)^p*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x \left(a+b \ln \left(c \left(d+e x^{1/3}\right)^2\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e*x^(1/3))^2))^p,x)
[Out] int(x*(a + b*log(c*(d + e*x^(1/3))^2))^p, x)
```

3.565 $\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=338

$$\frac{\left(\frac{2}{3}\right)^p e^{-\frac{3a}{2b}}\left(d+e\sqrt[3]{x}\right)^3\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right.}{e^3\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{3/2}}$$

```
[Out] (2/3)^p*(d+e*x^(1/3))^3*GAMMA(1+p,-3/2*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*
ln(c*(d+e*x^(1/3))^2))^p/e^3/exp(3/2*a/b)/(c*(d+e*x^(1/3))^2)^(3/2)/(((-a-b
*ln(c*(d+e*x^(1/3))^2))/b)^p)-3*d*GAMMA(1+p,(-a-b*ln(c*(d+e*x^(1/3))^2))/b)
*(a+b*ln(c*(d+e*x^(1/3))^2))^p/c/e^3/exp(a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2)
)/b)^p)+3*2^p*d^2*(d+e*x^(1/3))*GAMMA(1+p,1/2*(-a-b*ln(c*(d+e*x^(1/3))^2))/
b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/e^3/exp(1/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/3)
)^2))/b)^p)/(c*(d+e*x^(1/3))^2)^(1/2)
```

Rubi [A]

time = 0.30, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2501, 2448, 2436, 2337, 2212, 2437, 2347}

$$\frac{3d^2 2^p e^{-\frac{a}{2b}}\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{2b}\right)}{e^3\sqrt{c\left(d+e\sqrt[3]{x}\right)^2}}+\frac{\left(\frac{2}{3}\right)^p e^{-\frac{3a}{2b}}\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right)}{e^3\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{3/2}}-\frac{3de^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)}{ce^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
```

```
[Out] ((2/3)^p*(d + e*x^(1/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]
))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^3*E^((3*a)/(2*b))*(c*(d +
e*x^(1/3))^2)^(3/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (3*d*Gamma
[1 + p, -((a + b*Log[c*(d + e*x^(1/3))^2])/b)]*(a + b*Log[c*(d + e*x^(1/3))
^2])^p)/(c*e^3*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) + (3*2^p*
d^2*(d + e*x^(1/3))*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e*x^(1/3))^2])/b]*(
a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^3*E^(a/(2*b))*Sqrt[c*(d + e*x^(1/3))^
2]*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2501

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p dx = 3\text{Subst}\left(\int x^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p dx, x, \sqrt[3]{x}\right)$$

$$= 3\text{Subst}\left(\int \left(\frac{d^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^2} - \frac{2d(d+ex)\left(a+b\log\left(c(d+\right.\right.}{e^2}\right.\right.$$

$$= \frac{3\text{Subst}\left(\int (d+ex)^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p dx, x, \sqrt[3]{x}\right)}{e^2} - \frac{(6d)\text{Subst}\left(\right.}{}$$

$$= \frac{3\text{Subst}\left(\int x^2\left(a+b\log\left(cx^2\right)\right)^p dx, x, d+e\sqrt[3]{x}\right)}{e^3} - \frac{(6d)\text{Subst}\left(\int x(a+\right.}{}$$

$$= -\frac{(3d)\text{Subst}\left(\int e^x(a+bx)^p dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{ce^3} + \frac{\left(3\left(d+e\sqrt[3]{x}\right.\right.}{}$$

$$= \frac{\left(\frac{2}{3}\right)^p e^{-\frac{3a}{2b}}\left(d+e\sqrt[3]{x}\right)^3\Gamma\left(1+p, -\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\right.}{e^3\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{3/2}}$$

Mathematica [**F**]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(1/3))^2])^p, x]
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^2\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^2))^p,x)
[Out] int((a+b*ln(c*(d+e*x^(1/3))^2))^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((x^(1/3)*e + d)^2*c) + a)^p, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(2*c*d*x^(1/3)*e + c*d^2 + c*x^(2/3)*e^2) + a)^p, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**2))**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^(1/3)*e + d)^2*c) + a)^p, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))^2))^p,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/3))^2))^p, x)
```

3.566 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x}\,dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(1/3))^2))^p/x,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^2])^p/x,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x)^2])^p/x, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x}\,dx = 3\mathrm{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{x}\,dx, x, \sqrt[3]{x}\right)$$

Mathematica [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^2])^p/x,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(1/3))^2])^p/x, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{2}\right)\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^2))^p/x,x)
[Out] int((a+b*ln(c*(d+e*x^(1/3))^2))^p/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p/x,x, algorithm="maxima")
[Out] integrate((b*log((x^(1/3)*e + d)^2*c) + a)^p/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p/x,x, algorithm="fricas")
[Out] integral((b*log(2*c*d*x^(1/3)*e + c*d^2 + c*x^(2/3)*e^2) + a)^p/x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**2))**p/x,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p/x,x, algorithm="giac")
[Out] integrate((b*log((x^(1/3)*e + d)^2*c) + a)^p/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^2\right)\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))^2))^p/x,x)
[Out] int((a + b*log(c*(d + e*x^(1/3))^2))^p/x, x)
```

3.567 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=27

$$\operatorname{Int}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(1/3))^2))^p/x^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^2])^p/x^2,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x)^2])^p/x^4, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2}\,dx = 3\operatorname{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{x^4}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^2])^p/x^2,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(1/3))^2])^p/x^2, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{2}\right)\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^2))^p/x^2,x)
[Out] int((a+b*ln(c*(d+e*x^(1/3))^2))^p/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p/x^2,x, algorithm="maxima")
[Out] integrate((b*log((x^(1/3)*e + d)^2*c) + a)^p/x^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p/x^2,x, algorithm="fricas")
[Out] integral((b*log(2*c*d*x^(1/3)*e + c*d^2 + c*x^(2/3)*e^2) + a)^p/x^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**2))**p/x**2,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p/x^2,x, algorithm="giac")
[Out] integrate((b*log((x^(1/3)*e + d)^2*c) + a)^p/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^2\right)\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(1/3))^2))^p/x^2,x)
[Out] int((a + b*log(c*(d + e*x^(1/3))^2))^p/x^2, x)
```

3.568 $\int x^3 \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$

Optimal. Leaf size=557

$$\frac{2^{-2-p}3^{-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}}{c^6e^6}-\frac{3\,5^{-p}de^{-\frac{5a}{b}}}{}$$

```
[Out] 2^(-2-p)*GAMMA(1+p,-6*(a+b*ln(c*(d+e*x^(2/3))))/b)*(a+b*ln(c*(d+e*x^(2/3)))
)^p/(3^p)/c^6/e^6/exp(6*a/b)/(((-a-b*ln(c*(d+e*x^(2/3))))/b)^p)-3/2*d*GAMMA
(1+p,-5*(a+b*ln(c*(d+e*x^(2/3))))/b)*(a+b*ln(c*(d+e*x^(2/3))))^p/(5^p)/c^5/
e^6/exp(5*a/b)/(((-a-b*ln(c*(d+e*x^(2/3))))/b)^p)+15*d^2*GAMMA(1+p,-4*(a+b*
ln(c*(d+e*x^(2/3))))/b)*(a+b*ln(c*(d+e*x^(2/3))))^p/(2^(2+2*p))/c^4/e^6/exp
(4*a/b)/(((-a-b*ln(c*(d+e*x^(2/3))))/b)^p)-5*d^3*GAMMA(1+p,-3*(a+b*ln(c*(d+
e*x^(2/3))))/b)*(a+b*ln(c*(d+e*x^(2/3))))^p/(3^p)/c^3/e^6/exp(3*a/b)/(((-a-
b*ln(c*(d+e*x^(2/3))))/b)^p)+15*2^(-2-p)*d^4*GAMMA(1+p,-2*(a+b*ln(c*(d+e*x^
(2/3))))/b)*(a+b*ln(c*(d+e*x^(2/3))))^p/c^2/e^6/exp(2*a/b)/(((-a-b*ln(c*(d+
e*x^(2/3))))/b)^p)-3/2*d^5*GAMMA(1+p,(-a-b*ln(c*(d+e*x^(2/3))))/b)*(a+b*ln(
c*(d+e*x^(2/3))))^p/c/e^6/exp(a/b)/(((-a-b*ln(c*(d+e*x^(2/3))))/b)^p)
```

Rubi [A]
time = 0.58, antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2504, 2448, 2436, 2336, 2212, 2437, 2346}

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(2/3))])^p,x]
```

```
[Out] (2^(-2 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*x^(2/3))]))/b]*(a + b*Log[
c*(d + e*x^(2/3))])^p)/(3^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e*x^(2
/3))])/b))^p) - (3*d*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(2/3))]))/b]*(a
 + b*Log[c*(d + e*x^(2/3))])^p)/(2*5^p*c^5*e^6*E^((5*a)/b)*(-((a + b*Log[c*
(d + e*x^(2/3))])/b))^p) + (15*d^2*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*x^(
2/3))]))/b]*(a + b*Log[c*(d + e*x^(2/3))])^p)/(2^(2*(1 + p))*c^4*e^6*E^((4*
a)/b)*(-((a + b*Log[c*(d + e*x^(2/3))])/b))^p) - (5*d^3*Gamma[1 + p, (-3*(a
 + b*Log[c*(d + e*x^(2/3))]))/b]*(a + b*Log[c*(d + e*x^(2/3))])^p)/(3^p*c^3
*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(2/3))])/b))^p) + (15*2^(-2 - p)*
d^4*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(2/3))]))/b]*(a + b*Log[c*(d + e
*x^(2/3))])^p)/(c^2*e^6*E^((2*a)/b)*(-((a + b*Log[c*(d + e*x^(2/3))])/b))^p
) - (3*d^5*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(2/3))])/b)]*(a + b*Log[c*(
d + e*x^(2/3))])^p)/(2*c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e*x^(2/3))])/b))^
p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx &= \frac{3}{2}\text{Subst}\left(\int x^5(a+b\log(c(d+ex)))^p\,dx,x,x^{2/3}\right)\\
&= \frac{3}{2}\text{Subst}\left(\int\left(-\frac{d^5(a+b\log(c(d+ex)))^p}{e^5}+\frac{5d^4(d+ex)(a+b\log(c(d
\\
&= \frac{3\text{Subst}\left(\int(d+ex)^5(a+b\log(c(d+ex)))^p\,dx,x,x^{2/3}\right)}{2e^5}-\frac{(15d)\text{Subst}\left(\int\right.}{}\\
&= \frac{3\text{Subst}\left(\int x^5(a+b\log(cx))^p\,dx,x,d+ex^{2/3}\right)}{2e^6}-\frac{(15d)\text{Subst}\left(\int x^4(a+\right.}{}\\
&= \frac{3\text{Subst}\left(\int e^{6x}(a+bx)^p\,dx,x,\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{2c^6e^6}-\frac{(15d)\text{Subst}\left(\int e^{5x}(\right.}{}\\
&= \frac{2^{-2-p}3^{-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right.}{c^6e^6}
\end{aligned}
$$

Mathematica [A]
time = 1.18, size = 325, normalized size = 0.58

$$
\frac{4^{-1-p}15^{-p}e^{-\frac{6a}{b}}\left(10^p\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)-cde^{a/b}\left(2^{1+2p}3^{1+p}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)+5^pcde^{a/b}\left(-53^{1+p}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)+2^pcde^{a/b}\left(5\,2^{2+p}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)+3^{1+p}cde^{a/b}\left(-5\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)+2^{1+p}cde^{a/b}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)\right)\right)\right)\right)\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}}{c^6e^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(2/3))])^p,x]
```

```
[Out] (4^(-1 - p)*(10^p*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*x^(2/3))]))/b] - c*d
*E^(a/b)*(2^(1 + 2*p)*3^(1 + p)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(2/3
))]))/b] + 5^p*c*d*E^(a/b)*(-5*3^(1 + p)*Gamma[1 + p, (-4*(a + b*Log[c*(d +
 e*x^(2/3))]))/b] + 2^p*c*d*E^(a/b)*(5*2^(2 + p)*Gamma[1 + p, (-3*(a + b*Lo
g[c*(d + e*x^(2/3))]))/b] + 3^(1 + p)*c*d*E^(a/b)*(-5*Gamma[1 + p, (-2*(a +
 b*Log[c*(d + e*x^(2/3))]))/b] + 2^(1 + p)*c*d*E^(a/b)*Gamma[1 + p, -((a +
b*Log[c*(d + e*x^(2/3))])/b)]))))))*(a + b*Log[c*(d + e*x^(2/3))])^p)/(15^p*
c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e*x^(2/3))])/b))^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^{3}\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="maxima")
[Out] integrate((b*log((x^(2/3)*e + d)*c) + a)^p*x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="fricas")
[Out] integral((b*log(c*x^(2/3)*e + c*d) + a)^p*x^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(2/3))))**p,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="giac")
[Out] integrate((b*log((x^(2/3)*e + d)*c) + a)^p*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e*x^(2/3))))^p,x)
[Out] int(x^3*(a + b*log(c*(d + e*x^(2/3))))^p, x)
```

3.569 $\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$

Optimal. Leaf size=273

$$\frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}}{2c^3e^3}-\frac{3\ 2^{-1-p}de^{-\frac{2a}{b}}\Gamma\left(\right.}{}$$

```
[Out] 1/2*GAMMA(1+p,-3*(a+b*ln(c*(d+e*x^(2/3))))/b)*(a+b*ln(c*(d+e*x^(2/3))))^p/(
3^p)/c^3/e^3/exp(3*a/b)/(((-a-b*ln(c*(d+e*x^(2/3))))/b)^p)-3*2^(-1-p)*d*GAM
MA(1+p,-2*(a+b*ln(c*(d+e*x^(2/3))))/b)*(a+b*ln(c*(d+e*x^(2/3))))^p/c^2/e^3/
exp(2*a/b)/(((-a-b*ln(c*(d+e*x^(2/3))))/b)^p)+3/2*d^2*GAMMA(1+p,(-a-b*ln(c*
(d+e*x^(2/3))))/b)*(a+b*ln(c*(d+e*x^(2/3))))^p/c/e^3/exp(a/b)/(((-a-b*ln(c*
(d+e*x^(2/3))))/b)^p)
```

Rubi [A]
time = 0.26, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2504, 2448, 2436, 2336, 2212, 2437, 2346}

$$\frac{3^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)}{2c^3e^3}-\frac{3d2^{-p-1}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)}{c^2e^3}+\frac{3d^2e^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)}{2ce^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*x^(2/3))])^p,x]
```

```
[Out] (Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(2/3))]))/b]*(a + b*Log[c*(d + e*x^
(2/3))])^p)/(2*3^p*c^3*e^3*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(2/3))])/b)
)^p) - (3*2^(-1 - p)*d*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(2/3))]))/b]*
(a + b*Log[c*(d + e*x^(2/3))])^p)/(c^2*e^3*E^((2*a)/b)*(-((a + b*Log[c*(d +
 e*x^(2/3))])/b))^p) + (3*d^2*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(2/3))])
/b)]*(a + b*Log[c*(d + e*x^(2/3))])^p)/(2*c*e^3*E^(a/b)*(-((a + b*Log[c*(d
+ e*x^(2/3))])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx &= \frac{3}{2}\text{Subst}\left(\int x^2(a+b\log(c(d+ex)))^p\,dx,x,x^{2/3}\right)\\ &= \frac{3}{2}\text{Subst}\left(\int\left(\frac{d^2(a+b\log(c(d+ex)))^p}{e^2}-\frac{2d(d+ex)(a+b\log(c(d+ex}{e^2}\right.\right.\\ &= \frac{3\text{Subst}\left(\int(d+ex)^2(a+b\log(c(d+ex)))^p\,dx,x,x^{2/3}\right)}{2e^2}-\frac{(3d)\text{Subst}\left(\int(a\right.}{}\\ &= \frac{3\text{Subst}\left(\int x^2(a+b\log(cx))^p\,dx,x,d+ex^{2/3}\right)}{2e^3}-\frac{(3d)\text{Subst}\left(\int x(a+b\log\right.}{e}\\ &= \frac{3\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx,x,\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{2c^3e^3}-\frac{(3d)\text{Subst}\left(\int e^{2x}(a-\right.}{}\\ &= \frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a}{}\right.}{2c^3e^3}\end{aligned}$$

Mathematica [A]
time = 0.41, size = 181, normalized size = 0.66

$$\frac{2^{-1-p}3^{-p}e^{-\frac{3a}{b}}\left(2^p\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)+3^{1+p}cde^{a/b}\left(-\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)+2^pcde^{a/b}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)\right)\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}}{c^3e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(2/3))])^p,x]
```

```
[Out] (2^(-1 - p)*(2^p*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(2/3))]))/b] + 3^(1
 + p)*c*d*E^(a/b)*(-Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(2/3))]))/b] + 2
^p*c*d*E^(a/b)*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(2/3))])/b)]))*(a + b*L
og[c*(d + e*x^(2/3))])^p)/(3^p*c^3*e^3*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x
^(2/3))])/b))^p)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((x^(2/3)*e + d)*c) + a)^p*x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^(2/3)*e + c*d) + a)^p*x, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(2/3))))**p,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^(2/3)*e + d)*c) + a)^p*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(a+b\ln\left(c\left(d+e\,x^{2/3}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e*x^(2/3))))^p,x)
```

```
[Out] int(x*(a + b*log(c*(d + e*x^(2/3))))^p, x)
```

3.570 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(2/3))))^p/x,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))])^p/x,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x^2)])^p/x, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x}\,dx = 3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)\right)\right)^p}{x}\,dx, x, \sqrt[3]{x}\right)$$

Mathematica [A]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p/x,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p/x, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)\right)\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))))^p/x,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))))^p/x,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x,x, algorithm="maxima")
```

```
[Out] integrate((b*log((x^(2/3)*e + d)*c) + a)^p/x, x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^(2/3)*e + c*d) + a)^p/x, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))))**p/x,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^(2/3)*e + d)*c) + a)^p/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)\right)\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))))^p/x,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))))^p/x, x)
```

3.571 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^3}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^3},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(2/3))))^p/x^3,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))])^p/x^3,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x^2)])^p/x^7, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^3}\,dx = 3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)\right)\right)^p}{x^7}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A]
time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p/x^3,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p/x^3, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)\right)\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))))^p/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))))^p/x^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((b*log((x^(2/3)*e + d)*c) + a)^p/x^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^(2/3)*e + c*d) + a)^p/x^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))))**p/x**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^(2/3)*e + d)*c) + a)^p/x^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{2/3}\right)\right)\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))))^p/x^3,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))))^p/x^3, x)
```

3.572 $\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable(x^2*(a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(2/3))])^p,x]
[Out] 3*Defer[Subst][Defer[Int][x^8*(a + b*Log[c*(d + e*x^2)])^p, x], x, x^(1/3)]
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx = 3\text{Subst}\left(\int x^8\left(a+b\log\left(c\left(d+ex^2\right)\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A]
time = 0.60, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))])^p,x]
[Out] Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))])^p, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(2/3))))^p,x)
[Out] int(x^2*(a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="maxima")
[Out] integrate((b*log((x^(2/3)*e + d)*c) + a)^p*x^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="fricas")
[Out] integral((b*log(c*x^(2/3)*e + c*d) + a)^p*x^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(2/3))))**p,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="giac")
[Out] integrate((b*log((x^(2/3)*e + d)*c) + a)^p*x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^2\,\left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e*x^(2/3))))^p,x)
[Out] int(x^2*(a + b*log(c*(d + e*x^(2/3))))^p, x)
```

3.573 $\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))])^p,x]
[Out] 3*Defer[Subst][Defer[Int][x^2*(a + b*Log[c*(d + e*x^2)])^p, x], x, x^(1/3)]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx = 3\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+ex^2\right)\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A]
time = 0.17, size = 0, normalized size = 0.00

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))))^p,x)
[Out] int((a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="maxima")
[Out] integrate((b*log((x^(2/3)*e + d)*c) + a)^p, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="fricas")
[Out] integral((b*log(c*x^(2/3)*e + c*d) + a)^p, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))))**p,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="giac")
[Out] integrate((b*log((x^(2/3)*e + d)*c) + a)^p, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))))^p,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))))^p, x)
```

3.574 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(2/3))))^p/x^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))])^p/x^2,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x^2)])^p/x^4, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2}\,dx = 3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)\right)\right)^p}{x^4}\,dx, x, \sqrt[3]{x}\right)$$

Mathematica [A]
time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p/x^2,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p/x^2, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)\right)\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))))^p/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))))^p/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*log((x^(2/3)*e + d)*c) + a)^p/x^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*x^(2/3)*e + c*d) + a)^p/x^2, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))))**p/x**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8011 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^(2/3)*e + d)*c) + a)^p/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{2/3}\right)\right)\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))))^p/x^2,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))))^p/x^2, x)
```

3.575 $\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=678

$$-\frac{3\,2^{-1+p}d^5e^{-\frac{a}{2b}}\left(d+ex^{2/3}\right)\Gamma\left(1+p,\frac{-a-b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right.}{e^6\sqrt{c\left(d+ex^{2/3}\right)^2}}$$

```
[Out] 1/4*GAMMA(1+p,-3*(a+b*ln(c*(d+e*x^(2/3))^2))/b)*(a+b*ln(c*(d+e*x^(2/3))^2))
^p/(3^p)/c^3/e^6/exp(3*a/b)/(((-a-b*ln(c*(d+e*x^(2/3))^2))/b)^p)-3*2^(-1+p)
*d*(d+e*x^(2/3))^5*GAMMA(1+p,-5/2*(a+b*ln(c*(d+e*x^(2/3))^2))/b)*(a+b*ln(c*
(d+e*x^(2/3))^2))^p/(5^p)/e^6/exp(5/2*a/b)/(c*(d+e*x^(2/3))^2)^(5/2)/(((-a-
b*ln(c*(d+e*x^(2/3))^2))/b)^p)+15*2^(-2-p)*d^2*GAMMA(1+p,-2*(a+b*ln(c*(d+e*
x^(2/3))^2))/b)*(a+b*ln(c*(d+e*x^(2/3))^2))^p/c^2/e^6/exp(2*a/b)/(((-a-b*ln
(c*(d+e*x^(2/3))^2))/b)^p)-5*(2/3)^p*d^3*(d+e*x^(2/3))^3*GAMMA(1+p,-3/2*(a+
b*ln(c*(d+e*x^(2/3))^2))/b)*(a+b*ln(c*(d+e*x^(2/3))^2))^p/e^6/exp(3/2*a/b)/
(c*(d+e*x^(2/3))^2)^(3/2)/(((-a-b*ln(c*(d+e*x^(2/3))^2))/b)^p)+15/4*d^4*GAM
MA(1+p,(-a-b*ln(c*(d+e*x^(2/3))^2))/b)*(a+b*ln(c*(d+e*x^(2/3))^2))^p/c/e^6/
exp(a/b)/(((-a-b*ln(c*(d+e*x^(2/3))^2))/b)^p)-3*2^(-1+p)*d^5*(d+e*x^(2/3))*
GAMMA(1+p,1/2*(-a-b*ln(c*(d+e*x^(2/3))^2))/b)*(a+b*ln(c*(d+e*x^(2/3))^2))^p
/e^6/exp(1/2*a/b)/(((-a-b*ln(c*(d+e*x^(2/3))^2))/b)^p)/(c*(d+e*x^(2/3))^2)^
(1/2)
```

Rubi [A]
time = 0.67, antiderivative size = 675, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {2504, 2448, 2436, 2337, 2212, 2437, 2347}

$$\frac{3^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{b}\right)}{4c^3e^6}+\frac{15d^22^{-p-2}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{b}\right)}{c^2e^6}-\frac{3d^52^{p-1}e^{-\frac{a}{2b}}\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{2b}\right)}{e^6\sqrt{c\left(d+ex^{2/3}\right)^2}}+\frac{15d^4e^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)}{4ce^6}-\frac{5d^3\left(\frac{2}{3}\right)^pe^{-\frac{3a}{2b}}\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{2b}\right)}{e^6\left(c\left(d+ex^{2/3}\right)^2\right)^{3/2}}-\frac{3d2^{p-1}5^{-p}e^{-\frac{5a}{2b}}\left(d+ex^{2/3}\right)^5\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{5\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{2b}\right)}{e^6\left(c\left(d+ex^{2/3}\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
```

```
[Out] (Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(2/3))^2]))/b]*(a + b*Log[c*(d + e*
x^(2/3))^2])^p)/(4*3^p*c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(2/3))^
2])/b))^p) - (3*2^(-1 + p)*d*(d + e*x^(2/3))^5*Gamma[1 + p, (-5*(a + b*Log[
c*(d + e*x^(2/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(2/3))^2])^p)/(5^p*e^6*
E^((5*a)/(2*b))*(c*(d + e*x^(2/3))^2)^(5/2)*(-((a + b*Log[c*(d + e*x^(2/3))
^2])/b))^p) + (15*2^(-2 - p)*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(2/
3))^2]))/b]*(a + b*Log[c*(d + e*x^(2/3))^2])^p)/(c^2*e^6*E^((2*a)/b)*(-((a
+ b*Log[c*(d + e*x^(2/3))^2])/b))^p) - (5*(2/3)^p*d^3*(d + e*x^(2/3))^3*Gam
ma[1 + p, (-3*(a + b*Log[c*(d + e*x^(2/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*
x^(2/3))^2])^p)/(e^6*E^((3*a)/(2*b))*(c*(d + e*x^(2/3))^2)^(3/2)*(-((a + b*
```

```
Log[c*(d + e*x^(2/3))^2])/b))^p) + (15*d^4*Gamma[1 + p, -((a + b*Log[c*(d +
 e*x^(2/3))^2])/b)]*(a + b*Log[c*(d + e*x^(2/3))^2])^p)/(4*c*e^6*E^(a/b)*(-
((a + b*Log[c*(d + e*x^(2/3))^2])/b))^p) - (3*2^(-1 + p)*d^5*(d + e*x^(2/3)
)*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e*x^(2/3))^2])/b]*(a + b*Log[c*(d + e
*x^(2/3))^2])^p)/(e^6*E^(a/(2*b))*Sqrt[c*(d + e*x^(2/3))^2]*(-((a + b*Log[c
*(d + e*x^(2/3))^2])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx = \frac{3}{2}\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,x^{2/3}\right)$$
$$= \frac{3}{2}\text{Subst}\left(\int\left(-\frac{d^5(a+b\log(c(d+ex)^2))^p}{e^5}+\frac{5d^4(d+ex)\,(a+b\log\right.\right.$$
$$= \frac{3\text{Subst}\left(\int(d+ex)^5\,(a+b\log(c(d+ex)^2))^p\,dx,x,x^{2/3}\right)}{2e^5}-\frac{(15d)\text{Su}}{}$$
$$= \frac{3\text{Subst}\left(\int x^5(a+b\log(cx^2))^p\,dx,x,d+ex^{2/3}\right)}{2e^6}-\frac{(15d)\text{Subst}\left(\int x^4\right.}{}$$
$$= \frac{3\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx,x,\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{4c^3e^6}+\frac{(15d^2)\,\text{Subst}}{}$$
$$= \frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right.\right.}{4c^3e^6}$$

Mathematica [F]
time = 0.36, size = 0, normalized size = 0.00

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
[Out] Integrate[x^3*(a + b*Log[c*(d + e*x^(2/3))^2])^p, x]
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^3\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
[Out] int(x^3*(a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log((x^(2/3)*e + d)^2*c) + a)^p*x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="fricas")
[Out] integral((b*log(2*c*d*x^(2/3)*e + c*x^(4/3)*e^2 + c*d^2) + a)^p*x^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(2/3))**2))**p,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="giac")
[Out] integrate((b*log((x^(2/3)*e + d)^2*c) + a)^p*x^3, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \left(a + b \ln \left(c \left(d + e\, x^{2/3}\right)^2\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e*x^(2/3))^2))^p,x)
[Out] int(x^3*(a + b*log(c*(d + e*x^(2/3))^2))^p, x)
```

3.576 $\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=350

$$\frac{3\,2^{-1+p}d^2e^{-\frac{a}{2b}}\left(d+ex^{2/3}\right)\Gamma\left(1+p,\frac{-a-b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)}{e^3\sqrt{c\left(d+ex^{2/3}\right)^2}}$$

```
[Out] 2^(-1+p)*(d+e*x^(2/3))^3*GAMMA(1+p,-3/2*(a+b*ln(c*(d+e*x^(2/3))^2))/b)*(a+b
*ln(c*(d+e*x^(2/3))^2))^p/(3^p)/e^3/exp(3/2*a/b)/(c*(d+e*x^(2/3))^2)^(3/2)/
(((-a-b*ln(c*(d+e*x^(2/3))^2))/b)^p)-3/2*d*GAMMA(1+p,(-a-b*ln(c*(d+e*x^(2/3
))^2))/b)*(a+b*ln(c*(d+e*x^(2/3))^2))^p/c/e^3/exp(a/b)/(((-a-b*ln(c*(d+e*x^
(2/3))^2))/b)^p)+3*2^(-1+p)*d^2*(d+e*x^(2/3))*GAMMA(1+p,1/2*(-a-b*ln(c*(d+e
*x^(2/3))^2))/b)*(a+b*ln(c*(d+e*x^(2/3))^2))^p/e^3/exp(1/2*a/b)/(((-a-b*ln(
c*(d+e*x^(2/3))^2))/b)^p)/(c*(d+e*x^(2/3))^2)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 347, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2504, 2448, 2436, 2337, 2212, 2437, 2347}

$$\frac{3d^2 2^{p-1}e^{-\frac{a}{2b}}\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{2b}\right)}{e^3\sqrt{c\left(d+ex^{2/3}\right)^2}}+\frac{2^{p-1}3^{-p}e^{-\frac{3a}{2b}}\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{2b}\right)}{e^3\left(c\left(d+ex^{2/3}\right)^2\right)^{3/2}}-\frac{3de^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)}{2ce^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
```

```
[Out] (2^(-1 + p)*(d + e*x^(2/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(2/3))
^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(2/3))^2])^p)/(3^p*e^3*E^((3*a)/(2*b))*(
c*(d + e*x^(2/3))^2)^(3/2)*(-((a + b*Log[c*(d + e*x^(2/3))^2])/b))^p) - (3*
d*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(2/3))^2])/b)]*(a + b*Log[c*(d + e*x
^(2/3))^2])^p)/(2*c*e^3*E^(a/b)*(-((a + b*Log[c*(d + e*x^(2/3))^2])/b))^p)
+ (3*2^(-1 + p)*d^2*(d + e*x^(2/3))*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e*x
^(2/3))^2])/b]*(a + b*Log[c*(d + e*x^(2/3))^2])^p)/(e^3*E^(a/(2*b))*Sqrt[c*
(d + e*x^(2/3))^2]*(-((a + b*Log[c*(d + e*x^(2/3))^2])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx = \frac{3}{2}\text{Subst}\left(\int x^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, x^{2/3}\right)$$

$$= \frac{3}{2}\text{Subst}\left(\int\left(\frac{d^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^2} - \frac{2d(d+ex)\left(a+b\log\left(c(d\right.\right.}{e^2}\right.\right.$$

$$= \frac{3\text{Subst}\left(\int(d+ex)^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, x^{2/3}\right)}{2e^2} - \frac{(3d)\text{Subst}}{}$$

$$= \frac{3\text{Subst}\left(\int x^2\left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+ex^{2/3}\right)}{2e^3} - \frac{(3d)\text{Subst}\left(\int x(a+\right.}{}$$

$$= -\frac{(3d)\text{Subst}\left(\int e^x(a+bx)^p\,dx, x, \log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{2ce^3} + \frac{\left(3\left(d+ex^2\right.\right.}{}$$

$$= \frac{2^{-1+p}3^{-p}e^{-\frac{3a}{2b}}\left(d+ex^{2/3}\right)^3\Gamma\left(1+p, -\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{2b}\right)\left(a+b\,\text{lo}\right.}{e^3\left(c\left(d+ex^{2/3}\right)^2\right)^{3/2}}$$

Mathematica [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
[Out] Integrate[x*(a + b*Log[c*(d + e*x^(2/3))^2])^p, x]
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
[Out] int(x*(a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((x^(2/3)*e + d)^2*c) + a)^p*x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(2*c*d*x^(2/3)*e + c*x^(4/3)*e^2 + c*d^2) + a)^p*x, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(2/3))**2))**p,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8010 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^(2/3)*e + d)^2*c) + a)^p*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e*x^(2/3))^2))^p,x)
```

```
[Out] int(x*(a + b*log(c*(d + e*x^(2/3))^2))^p, x)
```

3.577 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}\,dx$

Optimal. Leaf size=27

$$\operatorname{Int}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(2/3))^2))^p/x,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x^2)^2])^p/x, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}\,dx = 3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^2\right)\right)^p}{x}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{2}\right)\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^2))^p/x,x)
[Out] int((a+b*ln(c*(d+e*x^(2/3))^2))^p/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x,x, algorithm="maxima")
[Out] integrate((b*log((x^(2/3)*e + d)^2*c) + a)^p/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x,x, algorithm="fricas")
[Out] integral((b*log(2*c*d*x^(2/3)*e + c*x^(4/3)*e^2 + c*d^2) + a)^p/x, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**2))**p/x,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^(2/3)*e + d)^2*c) + a)^p/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^2\right)\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^2))^p/x,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(2/3))^2))^p/x, x)
```

3.578 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3}\,dx$

Optimal. Leaf size=27

$$\operatorname{Int}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(2/3))^2))^p/x^3,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x^3,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x^2)^2])^p/x^7, x], x, x^(1/3
)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3}\,dx = 3\operatorname{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^2\right)\right)^p}{x^7}\,dx, x, \sqrt[3]{x}\right)$$

Mathematica [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x^3,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x^3, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{2}\right)\right)^{p}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^2))^p/x^3,x)
[Out] int((a+b*ln(c*(d+e*x^(2/3))^2))^p/x^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x^3,x, algorithm="maxima")
[Out] integrate((b*log((x^(2/3)*e + d)^2*c) + a)^p/x^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x^3,x, algorithm="fricas")
[Out] integral((b*log(2*c*d*x^(2/3)*e + c*x^(4/3)*e^2 + c*d^2) + a)^p/x^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**2))**p/x**3,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^(2/3)*e + d)^2*c) + a)^p/x^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^2\right)\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^2))^p/x^3,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(2/3))^2))^p/x^3, x)
```

3.579 $\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable(x^2*(a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][x^8*(a + b*Log[c*(d + e*x^2)^2])^p, x], x, x^(1/3
)]
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx = 3\text{Subst}\left(\int x^8\left(a+b\log\left(c\left(d+ex^2\right)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
```

```
[Out] Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))^2])^p, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((x^(2/3)*e + d)^2*c) + a)^p*x^2, x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(2*c*d*x^(2/3)*e + c*x^(4/3)*e^2 + c*d^2) + a)^p*x^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(2/3))**2))**p,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^(2/3)*e + d)^2*c) + a)^p*x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^2 \left(a+b \ln \left(c \left(d+e x^{2/3}\right)^2\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e*x^(2/3))^2))^p,x)
[Out] int(x^2*(a + b*log(c*(d + e*x^(2/3))^2))^p, x)
```

3.580 $\int \left(a+b \log \left(c \left(d+e x^{2/3}\right)^2\right)\right)^p \, dx$

Optimal. Leaf size=23

$$\text{Int}\left(\left(a+b \log \left(c \left(d+e x^{2/3}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b \log \left(c \left(d+e x^{2/3}\right)^2\right)\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][x^2*(a + b*Log[c*(d + e*x^2)^2])^p, x], x, x^(1/3
)]
```

Rubi steps

$$\int \left(a+b \log \left(c \left(d+e x^{2/3}\right)^2\right)\right)^p \, dx = 3\text{Subst}\left(\int x^2 \left(a+b \log \left(c \left(d+e x^2\right)^2\right)\right)^p \, dx,x,\sqrt[3]{x}\right)$$

Mathematica [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \left(a+b \log \left(c \left(d+e x^{2/3}\right)^2\right)\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a +b \ln \left(c \left(d +e \,x^{\frac{2}{3}}\right)^{2}\right)\right)^{p}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log((x^(2/3)*e + d)^2*c) + a)^p, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(2*c*d*x^(2/3)*e + c*x^(4/3)*e^2 + c*d^2) + a)^p, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**2))**p,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^(2/3)*e + d)^2*c) + a)^p, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^2))^p,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))^2))^p, x)
```

3.581 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(2/3))^2))^p/x^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x^2,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x^2)^2])^p/x^4, x], x, x^(1/3
)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}\,dx = 3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^2\right)\right)^p}{x^4}\,dx, x, \sqrt[3]{x}\right)$$

Mathematica [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x^2,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x^2, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{2}\right)\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^2))^p/x^2,x)
[Out] int((a+b*ln(c*(d+e*x^(2/3))^2))^p/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x^2,x, algorithm="maxima")
[Out] integrate((b*log((x^(2/3)*e + d)^2*c) + a)^p/x^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x^2,x, algorithm="fricas")
[Out] integral((b*log(2*c*d*x^(2/3)*e + c*x^(4/3)*e^2 + c*d^2) + a)^p/x^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**2))**p/x**2,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^(2/3)*e + d)^2*c) + a)^p/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^2\right)\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^(2/3))^2))^p/x^2,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(2/3))^2))^p/x^2, x)
```

3.582 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable(x*(a+b*ln(c*(d+e/x^(1/3))))^p,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[x*(a + b*Log[c*(d + e/x^(1/3))])^p,x]
[Out] 3*Defer[Subst][Defer[Int][x^5*(a + b*Log[c*(d + e/x)])^p, x], x, x^(1/3)]
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx=3\text{Subst}\left(\int x^5\left(a+b\log\left(c\left(d+\frac{e}{x}\right)\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A]
time = 1.29, size = 0, normalized size = 0.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(1/3))])^p,x]
[Out] Integrate[x*(a + b*Log[c*(d + e/x^(1/3))])^p, x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(1/3))))^p,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e/x^(1/3))))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p*x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*x^(2/3)*e)/x) + a)^p*x, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/3))))**p,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8011 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p*x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x \left(a+b\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e/x^(1/3))))^p,x)
[Out] int(x*(a + b*log(c*(d + e/x^(1/3))))^p, x)
```

3.583 $\int \left(a+b \log \left(c \left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p \, dx$

Optimal. Leaf size=21

$$\text{Int}\left(\left(a+b \log \left(c \left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(1/3))))^p,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b \log \left(c \left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))])^p,x]
[Out] 3*Defer[Subst][Defer[Int][x^2*(a + b*Log[c*(d + e/x)])^p, x], x, x^(1/3)]
```

Rubi steps

$$\int \left(a+b \log \left(c \left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p \, dx = 3\text{Subst}\left(\int x^2 \left(a+b \log \left(c \left(d+\frac{e}{x}\right)\right)\right)^p \, dx,x,\sqrt[3]{x}\right)$$

Mathematica [A]
time = 0.24, size = 0, normalized size = 0.00

$$\int \left(a+b \log \left(c \left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p, x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(a+b \ln \left(c \left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))))^p,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))))^p,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p, x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*x^(2/3)*e)/x) + a)^p, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))))**p,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))))^p,x)
[Out] int((a + b*log(c*(d + e/x^(1/3))))^p, x)
```

3.584 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(1/3))))^p/x,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))])^p/x,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x)])^p/x, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x}\,dx = 3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x}\right)\right)\right)^p}{x}\,dx, x, \sqrt[3]{x}\right)$$

Mathematica [A]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p/x,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p/x, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))))^p/x,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))))^p/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*x^(2/3)*e)/x) + a)^p/x, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))))**p/x,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{1/3}}\right)\right)\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))))^p/x,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(1/3))))^p/x, x)
```

3.585 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=267

$$-\frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}}{c^3e^3}+\frac{3}{}$$

```
[Out] -GAMMA(1+p,-3*(a+b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/(3^p
)/c^3/e^3/exp(3*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))))/b)^p)+3*d*GAMMA(1+p,-2*(a
+b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/(2^p)/c^2/e^3/exp(2*
a/b)/(((-a-b*ln(c*(d+e/x^(1/3))))/b)^p)-3*d^2*GAMMA(1+p,(-a-b*ln(c*(d+e/x^(
1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/c/e^3/exp(a/b)/(((-a-b*ln(c*(d+e/x^(
1/3))))/b)^p)
```

Rubi [A]

time = 0.27, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2504, 2448, 2436, 2336, 2212, 2437, 2346}

$$-\frac{3^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)}{c^3e^3}+\frac{3d2^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)}{c^2e^3}-\frac{3d^2e^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)}{ce^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))])^p/x^2,x]
```

```
[Out] -((Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/
x^(1/3))])^p)/(3^p*c^3*e^3*E^((3*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b)
)^p)) + (3*d*Gamma[1 + p, (-2*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log
[c*(d + e/x^(1/3))])^p)/(2^p*c^2*e^3*E^((2*a)/b)*(-((a + b*Log[c*(d + e/x^(
1/3))])/b))^p) - (3*d^2*Gamma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))])/b)]*(
a + b*Log[c*(d + e/x^(1/3))])^p)/(c*e^3*E^(a/b)*(-((a + b*Log[c*(d + e/x^(1
/3))])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^2}\,dx = -\left(3\text{Subst}\left(\int x^2(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$
$$= -\left(3\text{Subst}\left(\int\left(\frac{d^2(a+b\log(c(d+ex)))^p}{e^2} - \frac{2d(d+ex)(a+b\log(c(\right.\right.\right.}{e^2}$$
$$= -\frac{3\text{Subst}\left(\int(d+ex)^2(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^2} + \frac{(6d)\text{Sub}}{}$$
$$= -\frac{3\text{Subst}\left(\int x^2(a+b\log(cx))^p\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^3} + \frac{(6d)\text{Subst}\left(\int x(a\right.}{}$$
$$= -\frac{3\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{c^3e^3} + \frac{(6d)\text{Subst}\left(\right.}{}$$
$$= -\frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p, -\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{}}\right.\right.\right.}{c^3e^3}$$

Mathematica [A]
time = 0.37, size = 175, normalized size = 0.66

$$-\frac{6^{-p}e^{-\frac{3a}{b}}\left(2^p\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)+3^{1+p}cde^{a/b}\left(-\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)+2^pcde^{a/b}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)\right)\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}}{c^3e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p/x^2,x]
```

```
[Out] -(((2^p*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))]))/b] + 3^(1 + p)*c*d
*E^(a/b)*(-Gamma[1 + p, (-2*(a + b*Log[c*(d + e/x^(1/3))]))/b] + 2^p*c*d*E^
(a/b)*Gamma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))])/b)]))*(a + b*Log[c*(d +
 e/x^(1/3))])^p)/(6^p*c^3*e^3*E^((3*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])
/b))^p))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))))^p/x^2,x)
[Out] int((a+b*ln(c*(d+e/x^(1/3))))^p/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^2,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^2,x, algorithm="fricas")
[Out] integral((b*log((c*d*x + c*x^(2/3)*e)/x) + a)^p/x^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))))**p/x**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^2,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{1/3}}\right)\right)\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))))^p/x^2,x)
[Out] int((a + b*log(c*(d + e/x^(1/3))))^p/x^2, x)
```

3.586 $\int \frac{\left(a+b \log \left(c \left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^3} \, dx$

Optimal. Leaf size=554

$$-\frac{2^{-1-p}3^{-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)}{c^6e^6}$$

```
[Out] -2^(-1-p)*GAMMA(1+p,-6*(a+b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))
))^p/(3^p)/c^6/e^6/exp(6*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))))/b)^p)+3*d*GAMMA(
1+p,-5*(a+b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/(5^p)/c^5/e
^6/exp(5*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))))/b)^p)-15*2^(-1-2*p)*d^2*GAMMA(1+
p,-4*(a+b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/c^4/e^6/exp(4
*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))))/b)^p)+10*d^3*GAMMA(1+p,-3*(a+b*ln(c*(d+e
/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/(3^p)/c^3/e^6/exp(3*a/b)/(((-a-b
*ln(c*(d+e/x^(1/3))))/b)^p)-15*2^(-1-p)*d^4*GAMMA(1+p,-2*(a+b*ln(c*(d+e/x^(
1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/c^2/e^6/exp(2*a/b)/(((-a-b*ln(c*(d+e
/x^(1/3))))/b)^p)+3*d^5*GAMMA(1+p,(-a-b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(
d+e/x^(1/3))))^p/c/e^6/exp(a/b)/(((-a-b*ln(c*(d+e/x^(1/3))))/b)^p)
```

Rubi [A]

time = 0.60, antiderivative size = 554, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2504, 2448, 2436, 2336, 2212, 2437, 2346}

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))])^p/x^3,x]
```

```
[Out] -((2^(-1 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Lo
g[c*(d + e/x^(1/3))])^p)/(3^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e/x^
(1/3))])/b))^p)) + (3*d*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/x^(1/3))]))/b]
*(a + b*Log[c*(d + e/x^(1/3))])^p)/(5^p*c^5*e^6*E^((5*a)/b)*(-((a + b*Log[c
*(d + e/x^(1/3))])/b))^p) - (15*2^(-1 - 2*p)*d^2*Gamma[1 + p, (-4*(a + b*Lo
g[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(c^4*e^6*E^((4*
a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p) + (10*d^3*Gamma[1 + p, (-3*(
a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(3^p*c^
3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p) - (15*2^(-1 - p)
*d^4*Gamma[1 + p, (-2*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d +
e/x^(1/3))])^p)/(c^2*e^6*E^((2*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^
```

```
p) + (3*d^5*Gamma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))])/b)]*(a + b*Log[c*
(d + e/x^(1/3))])^p)/(c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p
)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^3}\,dx = -\left(3\text{Subst}\left(\int x^5(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\left(3\text{Subst}\left(\int \left(-\frac{d^5(a+b\log(c(d+ex)))^p}{e^5} + \frac{5d^4(d+ex)(a+b\log(\right.\right.\right.$$

$$= -\frac{3\text{Subst}\left(\int (d+ex)^5(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^5} + \frac{(15d)\text{Sul}}{}$$

$$= -\frac{3\text{Subst}\left(\int x^5(a+b\log(cx))^p\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^6} + \frac{(15d)\text{Subst}\left(\int x^4(\right.}{}$$

$$= -\frac{3\text{Subst}\left(\int e^{6x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{c^6e^6} + \frac{(15d)\text{Subst}\left(\right.}{}$$

$$= -\frac{2^{-1-p}3^{-p}e^{-\frac{6a}{b}}\Gamma\left(1+p, -\frac{6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d\right.\right.\right.}{c^6e^6}$$

Mathematica [A]
time = 0.94, size = 325, normalized size = 0.59

$$\frac{2^{-1-2p}15^{-p}e^{-\frac{6a}{b}}\left(-10^p\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)+cde^{a/b}\left(2^{1+2p}3^{1+p}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)+5^pcde^{a/b}\left(-5\,3^{1+p}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)+2^pcde^{a/b}\left(5\,2^{2+p}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)+3^{1+p}cde^{a/b}\left(-5\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)+2^{1+p}cde^{a/b}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)\right)\right)\right)\right)\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}}{c^6e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p/x^3,x]
```

```
[Out] (2^(-1 - 2*p)*(-(10^p*Gamma[1 + p, (-6*(a + b*Log[c*(d + e/x^(1/3))]))/b])
+ c*d*E^(a/b)*(2^(1 + 2*p)*3^(1 + p)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/x
^(1/3))]))/b] + 5^p*c*d*E^(a/b)*(-5*3^(1 + p)*Gamma[1 + p, (-4*(a + b*Log[c
*(d + e/x^(1/3))]))/b] + 2^p*c*d*E^(a/b)*(5*2^(2 + p)*Gamma[1 + p, (-3*(a +
 b*Log[c*(d + e/x^(1/3))]))/b] + 3^(1 + p)*c*d*E^(a/b)*(-5*Gamma[1 + p, (-2
*(a + b*Log[c*(d + e/x^(1/3))]))/b] + 2^(1 + p)*c*d*E^(a/b)*Gamma[1 + p, -(
(a + b*Log[c*(d + e/x^(1/3))])/b)])))))*(a + b*Log[c*(d + e/x^(1/3))])^p)/(
15^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)\right)^{p}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))))^p/x^3,x)
[Out] int((a+b*ln(c*(d+e/x^(1/3))))^p/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^3,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^3,x, algorithm="fricas")
[Out] integral((b*log((c*d*x + c*x^(2/3)*e)/x) + a)^p/x^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))))**p/x**3,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{1/3}}\right)\right)\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))))^p/x^3,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(1/3))))^p/x^3, x)
```

3.587 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^4}\,dx$

Optimal. Leaf size=832

$$-\frac{3^{-1-2p}e^{-\frac{9a}{b}}\Gamma\left(1+p,-\frac{9\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}}{c^9e^9}$$

```
[Out] -3^(-1-2*p)*GAMMA(1+p,-9*(a+b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3
))))^p/c^9/e^9/exp(9*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))))/b)^p)+3*d*GAMMA(1+p,
-8*(a+b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/(8^p)/c^8/e^9/e
xp(8*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))))/b)^p)-12*d^2*GAMMA(1+p,-7*(a+b*ln(c*
(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/(7^p)/c^7/e^9/exp(7*a/b)/(((
-a-b*ln(c*(d+e/x^(1/3))))/b)^p)+7*2^(2-p)*d^3*GAMMA(1+p,-6*(a+b*ln(c*(d+e/x
^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/(3^p)/c^6/e^9/exp(6*a/b)/(((-a-b*l
n(c*(d+e/x^(1/3))))/b)^p)-42*d^4*GAMMA(1+p,-5*(a+b*ln(c*(d+e/x^(1/3))))/b)*
(a+b*ln(c*(d+e/x^(1/3))))^p/(5^p)/c^5/e^9/exp(5*a/b)/(((-a-b*ln(c*(d+e/x^(1
/3))))/b)^p)+21*2^(1-2*p)*d^5*GAMMA(1+p,-4*(a+b*ln(c*(d+e/x^(1/3))))/b)*(a+
b*ln(c*(d+e/x^(1/3))))^p/c^4/e^9/exp(4*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))))/b)
^p)-28*d^6*GAMMA(1+p,-3*(a+b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3)
)))^p/(3^p)/c^3/e^9/exp(3*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))))/b)^p)+3*2^(2-p)
*d^7*GAMMA(1+p,-2*(a+b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/
c^2/e^9/exp(2*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))))/b)^p)-3*d^8*GAMMA(1+p,(-a-b
*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/c/e^9/exp(a/b)/(((-a-b
*ln(c*(d+e/x^(1/3))))/b)^p)
```

Rubi [A]
time = 0.89, antiderivative size = 832, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2504, 2448, 2436, 2336, 2212, 2437, 2346}

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))])^p/x^4,x]
```

```
[Out] -((3^(-1 - 2*p)*Gamma[1 + p, (-9*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*
Log[c*(d + e/x^(1/3))])^p)/(c^9*e^9*E^((9*a)/b)*(-((a + b*Log[c*(d + e/x^(1
/3))])/b))^p)) + (3*d*Gamma[1 + p, (-8*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(
a + b*Log[c*(d + e/x^(1/3))])^p)/(8^p*c^8*e^9*E^((8*a)/b)*(-((a + b*Log[c*(
d + e/x^(1/3))])/b))^p) - (12*d^2*Gamma[1 + p, (-7*(a + b*Log[c*(d + e/x^(1
```

```
/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(7^p*c^7*e^9*E^((7*a)/b)*(-((a
 + b*Log[c*(d + e/x^(1/3))])/b))^p) + (7*2^(2 - p)*d^3*Gamma[1 + p, (-6*(a
+ b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(3^p*c^6*
e^9*E^((6*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p) - (42*d^4*Gamma[1
+ p, (-5*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^
p)/(5^p*c^5*e^9*E^((5*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p) + (21*
2^(1 - 2*p)*d^5*Gamma[1 + p, (-4*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*
Log[c*(d + e/x^(1/3))])^p)/(c^4*e^9*E^((4*a)/b)*(-((a + b*Log[c*(d + e/x^(1
/3))])/b))^p) - (28*d^6*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))]))/b]
*(a + b*Log[c*(d + e/x^(1/3))])^p)/(3^p*c^3*e^9*E^((3*a)/b)*(-((a + b*Log[c
*(d + e/x^(1/3))])/b))^p) + (3*2^(2 - p)*d^7*Gamma[1 + p, (-2*(a + b*Log[c*
(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(c^2*e^9*E^((2*a)/b
)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p) - (3*d^8*Gamma[1 + p, -((a + b*L
og[c*(d + e/x^(1/3))])/b)]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(c*e^9*E^(a/b)
*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
```

```
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^4}\,dx = -\left(3\text{Subst}\left(\int x^8(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\left(3\text{Subst}\left(\int \left(\frac{d^8(a+b\log(c(d+ex)))^p}{e^8} - \frac{8d^7(d+ex)(a+b\log(\right.\right.\right.}{e^8}$$

$$= -\frac{3\text{Subst}\left(\int (d+ex)^8(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^8} + \frac{(24d)\text{S}}{}$$

$$= -\frac{3\text{Subst}\left(\int x^8(a+b\log(cx))^p\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^9} + \frac{(24d)\text{Subst}\left(\int x\right.}{}$$

$$= -\frac{3\text{Subst}\left(\int e^{9x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{c^9e^9} + \frac{(24d)\text{Subs}}{}$$

$$= -\frac{3^{-1-2p}e^{-\frac{9a}{b}}\Gamma\left(1+p, -\frac{9\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\right.\right.\right.}{c^9e^9}$$

Mathematica [A]
time = 0.99, size = 502, normalized size = 0.60

$$-\frac{3^{-1-2p}280^{-p}e^{-\frac{9a}{b}}\left(280^p\Gamma\left(1+p,-\frac{9\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)-9^{1+p}35^pcde^{a/b}\Gamma\left(1+p,-\frac{8\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)+2^{2+3p}5^p9^{1+p}c^2d^2e^{\frac{2a}{b}}\Gamma\left(1+p,-\frac{7\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)-5^p84^{1+p}c^3d^3e^{\frac{3a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)+2^{1+3p}63^{1+p}c^4d^4e^{\frac{4a}{b}}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)-5^p126^{1+p}c^5d^5e^{\frac{5a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)+2^{2+3p}5^p21^{1+p}c^6d^6e^{\frac{6a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)-35^p36^{1+p}c^7d^7e^{\frac{7a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)+9^{1+p}280^pc^8d^8e^{\frac{8a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}}{c^9e^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p/x^4,x]
```

```
[Out] -((3^(-1 - 2*p)*(280^p*Gamma[1 + p, (-9*(a + b*Log[c*(d + e/x^(1/3))]))/b]
- 9^(1 + p)*35^p*c*d*E^(a/b)*Gamma[1 + p, (-8*(a + b*Log[c*(d + e/x^(1/3))]
))/b] + 2^(2 + 3*p)*5^p*9^(1 + p)*c^2*d^2*E^((2*a)/b)*Gamma[1 + p, (-7*(a +
 b*Log[c*(d + e/x^(1/3))]))/b] - 5^p*84^(1 + p)*c^3*d^3*E^((3*a)/b)*Gamma[1
 + p, (-6*(a + b*Log[c*(d + e/x^(1/3))]))/b] + 2^(1 + 3*p)*63^(1 + p)*c^4*d
^4*E^((4*a)/b)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/x^(1/3))]))/b] - 5^p*12
6^(1 + p)*c^5*d^5*E^((5*a)/b)*Gamma[1 + p, (-4*(a + b*Log[c*(d + e/x^(1/3))
]))/b] + 2^(2 + 3*p)*5^p*21^(1 + p)*c^6*d^6*E^((6*a)/b)*Gamma[1 + p, (-3*(a
 + b*Log[c*(d + e/x^(1/3))]))/b] - 35^p*36^(1 + p)*c^7*d^7*E^((7*a)/b)*Gamm
a[1 + p, (-2*(a + b*Log[c*(d + e/x^(1/3))]))/b] + 9^(1 + p)*280^p*c^8*d^8*E
^((8*a)/b)*Gamma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))])/b)])*(a + b*Log[c*
(d + e/x^(1/3))])^p)/(280^p*c^9*e^9*E^((9*a)/b)*(-((a + b*Log[c*(d + e/x^(1
/3))])/b))^p))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)\right)^{p}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))))^p/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))))^p/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*x^(2/3)*e)/x) + a)^p/x^4, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))))**p/x**4,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^4,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{1/3}}\right)\right)\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))))^p/x^4,x)
[Out] int((a + b*log(c*(d + e/x^(1/3))))^p/x^4, x)
```

3.588 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable(x*(a+b*ln(c*(d+e/x^(1/3))^2))^p,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[x*(a + b*Log[c*(d + e/x^(1/3))^2])^p,x]
[Out] 3*Defer[Subst][Defer[Int][x^5*(a + b*Log[c*(d + e/x)^2])^p, x], x, x^(1/3)]
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx = 3\text{Subst}\left(\int x^5\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A]
time = 0.17, size = 0, normalized size = 0.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(1/3))^2])^p,x]
[Out] Integrate[x*(a + b*Log[c*(d + e/x^(1/3))^2])^p, x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{2}\right)\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(1/3))^2))^p,x)
[Out] int(x*(a+b*ln(c*(d+e/x^(1/3))^2))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p*x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^2))^p,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x + 2*c*d*x^(2/3)*e + c*x^(1/3)*e^2)/x) + a)^p*x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/3))**2))**p,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p*x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e/x^(1/3))^2))^p,x)
```

```
[Out] int(x*(a + b*log(c*(d + e/x^(1/3))^2))^p, x)
```

3.589 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p dx$

Optimal. Leaf size=23

$$\text{Int}\left(\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(1/3))^2))^p,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^2])^p,x]
[Out] 3*Defer[Subst][Defer[Int][x^2*(a + b*Log[c*(d + e/x)^2])^p, x], x, x^(1/3)]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p dx = 3\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^2\right)\right)^p dx, x, \sqrt[3]{x}\right)$$

Mathematica [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{2}\right)\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^2))^p,x)
[Out] int((a+b*ln(c*(d+e/x^(1/3))^2))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x + 2*c*d*x^(2/3)*e + c*x^(1/3)*e^2)/x) + a)^p, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**2))**p,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^2))^p,x)
[Out] int((a + b*log(c*(d + e/x^(1/3))^2))^p, x)
```

3.590 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(1/3))^2))^p/x,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x)^2])^p/x, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x}\,dx = 3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^2\right)\right)^p}{x}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{2}\right)\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^2))^p/x,x)
[Out] int((a+b*ln(c*(d+e/x^(1/3))^2))^p/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x + 2*c*d*x^(2/3)*e + c*x^(1/3)*e^2)/x) + a)^p/x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**2))**p/x,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^2\right)\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^2))^p/x,x)
[Out] int((a + b*log(c*(d + e/x^(1/3))^2))^p/x, x)
```

3.591 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=342

$$-\frac{3\,2^p d^2 e^{-\frac{a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)\Gamma\left(1+p,\frac{-a-b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\cdots\right.\right.}{b}\right.}{e^3\sqrt{c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}}$$

```
[Out] -(2/3)^p*(d+e/x^(1/3))^3*GAMMA(1+p,-3/2*(a+b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b
*ln(c*(d+e/x^(1/3))^2))^p/e^3/exp(3/2*a/b)/(c*(d+e/x^(1/3))^2)^(3/2)/(((-a-
b*ln(c*(d+e/x^(1/3))^2))/b)^p)+3*d*GAMMA(1+p,(-a-b*ln(c*(d+e/x^(1/3))^2))/b
)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/c/e^3/exp(a/b)/(((-a-b*ln(c*(d+e/x^(1/3))^2
))/b)^p)-3*2^p*d^2*(d+e/x^(1/3))*GAMMA(1+p,1/2*(-a-b*ln(c*(d+e/x^(1/3))^2))
/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/e^3/exp(1/2*a/b)/(((-a-b*ln(c*(d+e/x^(1/3
))^2))/b)^p)/(c*(d+e/x^(1/3))^2)^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 339, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {2504, 2448, 2436, 2337, 2212, 2437, 2347}

$$-\frac{3d^2 2^p e^{-\frac{a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{2b}\right)}{e^3\sqrt{c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}}-\frac{\left(\frac{2}{3}\right)^p e^{-\frac{3a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2b}\right)}{e^3\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)^{3/2}}+\frac{3de^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)^{-p}\text{Gamma}\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)}{ce^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^2,x]
```

```
[Out] -(((2/3)^p*(d + e/x^(1/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))^
2]))/(2*b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(e^3*E^((3*a)/(2*b))*(c*(d
+ e/x^(1/3))^2)^(3/2)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p)) + (3*d*Ga
mma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))^2])/b)]*(a + b*Log[c*(d + e/x^(1/
3))^2])^p)/(c*e^3*E^(a/b)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p) - (3*2
^p*d^2*(d + e/x^(1/3))*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e/x^(1/3))^2])/b
]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(e^3*E^(a/(2*b))*Sqrt[c*(d + e/x^(1/3
))^2]*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
```

```
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^2}\,dx = -\left(3\text{Subst}\left(\int x^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\left(3\text{Subst}\left(\int \left(\frac{d^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^2} - \frac{2d(d+ex)\left(a+b\log\right.}{e^2}\right.\right.\right.$$

$$= -\frac{3\text{Subst}\left(\int (d+ex)^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^2} + \frac{(6d}{}$$

$$= -\frac{3\text{Subst}\left(\int x^2\left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^3} + \frac{(6d)\text{Subst}\left(\int\right.}{}$$

$$= \frac{(3d)\text{Subst}\left(\int e^x(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{ce^3} - \frac{\left(3\left(d\right.\right.}{}$$

$$= -\frac{\left(\frac{2}{3}\right)^p e^{-\frac{3a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\Gamma\left(1+p, -\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2b}\right)\left(a\right.}{e^3\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right.\right.}$$

Mathematica [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^2,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^2, x]
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^2))^p/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))^2))^p/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x + 2*c*d*x^(2/3)*e + c*x^(1/3)*e^2)/x) + a)^p/x^2,
x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**2))**p/x**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^2))^p/x^2,x)
[Out] int((a + b*log(c*(d + e/x^(1/3))^2))^p/x^2, x)
```

3.592 $\quad \int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^3}\,dx$

Optimal. Leaf size=673

$$-\frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)^{-p}}{2c^3e^6}$$

```
[Out] -1/2*GAMMA(1+p,-3*(a+b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2)
)^p/(3^p)/c^3/e^6/exp(3*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))^2))/b)^p)+3*(2/5)^p
*d*(d+e/x^(1/3))^5*GAMMA(1+p,-5/2*(a+b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*
(d+e/x^(1/3))^2))^p/e^6/exp(5/2*a/b)/(c*(d+e/x^(1/3))^2)^(5/2)/(((-a-b*ln(c
*(d+e/x^(1/3))^2))/b)^p)-15*2^(-1-p)*d^2*GAMMA(1+p,-2*(a+b*ln(c*(d+e/x^(1/3
))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/c^2/e^6/exp(2*a/b)/(((-a-b*ln(c*(d+
e/x^(1/3))^2))/b)^p)+5*2^(1+p)*d^3*(d+e/x^(1/3))^3*GAMMA(1+p,-3/2*(a+b*ln(c
*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/(3^p)/e^6/exp(3/2*a/b)/
(c*(d+e/x^(1/3))^2)^(3/2)/(((-a-b*ln(c*(d+e/x^(1/3))^2))/b)^p)-15/2*d^4*GAM
MA(1+p,(-a-b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/c/e^6/
exp(a/b)/(((-a-b*ln(c*(d+e/x^(1/3))^2))/b)^p)+3*2^p*d^5*(d+e/x^(1/3))*GAMMA
(1+p,1/2*(-a-b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/e^6/
exp(1/2*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))^2))/b)^p)/(c*(d+e/x^(1/3))^2)^(1/2)
```

Rubi [A]

time = 0.68, antiderivative size = 673, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {2504, 2448, 2436, 2337, 2212, 2437, 2347}

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^3,x]
```

```
[Out] -1/2*(Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))^2]))/b]*(a + b*Log[c*(d
 + e/x^(1/3))^2])^p)/(3^p*c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3
))^2])/b))^p) + (3*(2/5)^p*d*(d + e/x^(1/3))^5*Gamma[1 + p, (-5*(a + b*Log[
c*(d + e/x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(e^6*E^((
5*a)/(2*b))*(c*(d + e/x^(1/3))^2)^(5/2)*(-((a + b*Log[c*(d + e/x^(1/3))^2])
/b))^p) - (15*2^(-1 - p)*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d + e/x^(1/3))^
```

```
2]))/b]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(c^2*e^6*E^((2*a)/b)*(-((a + b*
Log[c*(d + e/x^(1/3))^2])/b))^p) + (5*2^(1 + p)*d^3*(d + e/x^(1/3))^3*Gamma
[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e/x^
(1/3))^2])^p)/(3^p*e^6*E^((3*a)/(2*b))*(c*(d + e/x^(1/3))^2)^(3/2)*(-((a +
b*Log[c*(d + e/x^(1/3))^2])/b))^p) - (15*d^4*Gamma[1 + p, -((a + b*Log[c*(d
 + e/x^(1/3))^2])/b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(2*c*e^6*E^(a/b)*
(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p) + (3*2^p*d^5*(d + e/x^(1/3))*Gam
ma[1 + p, -1/2*(a + b*Log[c*(d + e/x^(1/3))^2])/b]*(a + b*Log[c*(d + e/x^(1
/3))^2])^p)/(e^6*E^(a/(2*b))*Sqrt[c*(d + e/x^(1/3))^2]*(-((a + b*Log[c*(d +
 e/x^(1/3))^2])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
```

```
+ e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^3}\,dx = -\left(3\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\left(3\text{Subst}\left(\int\left(-\frac{d^5(a+b\log(c(d+ex)^2))^p}{e^5}+\frac{5d^4(d+ex)\,(a+b\right.\right.\right.$$

$$= -\frac{3\text{Subst}\left(\int (d+ex)^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^5}+\frac{(15d}{}$$

$$= -\frac{3\text{Subst}\left(\int x^5\left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^6}+\frac{(15d)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{3\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2c^3e^6}-\frac{(15d^2)\,\text{Su}}{}$$

$$= -\frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\right.\right.\right.}{2c^3e^6}$$

Mathematica [F]

time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^3,x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^3, x]
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^2))^p/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))^2))^p/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x + 2*c*d*x^(2/3)*e + c*x^(1/3)*e^2)/x) + a)^p/x^3,
x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**2))**p/x**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^2\right)\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^2))^p/x^3,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(1/3))^2))^p/x^3, x)
```

3.593 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^4}\,dx$

Optimal. Leaf size=1036

$$-\frac{2^p 3^{-1-2p} e^{-\frac{9a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)^9 \Gamma\left(1+p,-\frac{9\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b l}{\ldots}\right.}{e^9\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)^{9/2}}$$

```
[Out] -2^p*3^(-1-2*p)*(d+e/x^(1/3))^9*GAMMA(1+p,-9/2*(a+b*ln(c*(d+e/x^(1/3))^2))/
b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/e^9/exp(9/2*a/b)/(c*(d+e/x^(1/3))^2)^(9/2)
/(((-a-b*ln(c*(d+e/x^(1/3))^2))/b)^p)+3*d*GAMMA(1+p,-4*(a+b*ln(c*(d+e/x^(1/
3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/(4^p)/c^4/e^9/exp(4*a/b)/(((-a-b*l
n(c*(d+e/x^(1/3))^2))/b)^p)-3*2^(2+p)*d^2*(d+e/x^(1/3))^7*GAMMA(1+p,-7/2*(a
+b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/(7^p)/e^9/exp(7/
2*a/b)/(c*(d+e/x^(1/3))^2)^(7/2)/(((-a-b*ln(c*(d+e/x^(1/3))^2))/b)^p)+28*d^
3*GAMMA(1+p,-3*(a+b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p
/(3^p)/c^3/e^9/exp(3*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))^2))/b)^p)-21*2^(1+p)*d
^4*(d+e/x^(1/3))^5*GAMMA(1+p,-5/2*(a+b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*
(d+e/x^(1/3))^2))^p/(5^p)/e^9/exp(5/2*a/b)/(c*(d+e/x^(1/3))^2)^(5/2)/(((-a-
b*ln(c*(d+e/x^(1/3))^2))/b)^p)+21*2^(1-p)*d^5*GAMMA(1+p,-2*(a+b*ln(c*(d+e/x
^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/c^2/e^9/exp(2*a/b)/(((-a-b*ln(
c*(d+e/x^(1/3))^2))/b)^p)-7*2^(2+p)*d^6*(d+e/x^(1/3))^3*GAMMA(1+p,-3/2*(a+b
*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/(3^p)/e^9/exp(3/2*
a/b)/(c*(d+e/x^(1/3))^2)^(3/2)/(((-a-b*ln(c*(d+e/x^(1/3))^2))/b)^p)+12*d^7*
GAMMA(1+p,(-a-b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/c/e
^9/exp(a/b)/(((-a-b*ln(c*(d+e/x^(1/3))^2))/b)^p)-3*2^p*d^8*(d+e/x^(1/3))*GA
MMA(1+p,1/2*(-a-b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/e
^9/exp(1/2*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))^2))/b)^p)/(c*(d+e/x^(1/3))^2)^(1
/2)
```

Rubi [A]
time = 1.05, antiderivative size = 1036, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {2504, 2448, 2436, 2337, 2212, 2437, 2347}

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^4,x]
```

```
[Out] -((2^p*3^(-1 - 2*p)*(d + e/x^(1/3))^9*Gamma[1 + p, (-9*(a + b*Log[c*(d + e/
x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(e^9*E^((9*a)/(2*b
))*(c*(d + e/x^(1/3))^2)^(9/2)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p))
+ (3*d*Gamma[1 + p, (-4*(a + b*Log[c*(d + e/x^(1/3))^2]))/b]*(a + b*Log[c*(
d + e/x^(1/3))^2])^p)/(4^p*c^4*e^9*E^((4*a)/b)*(-((a + b*Log[c*(d + e/x^(1/
3))^2])/b))^p) - (3*2^(2 + p)*d^2*(d + e/x^(1/3))^7*Gamma[1 + p, (-7*(a + b
*Log[c*(d + e/x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(7^p
*e^9*E^((7*a)/(2*b))*(c*(d + e/x^(1/3))^2)^(7/2)*(-((a + b*Log[c*(d + e/x^(
1/3))^2])/b))^p) + (28*d^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))^2]
))/b]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(3^p*c^3*e^9*E^((3*a)/b)*(-((a +
b*Log[c*(d + e/x^(1/3))^2])/b))^p) - (21*2^(1 + p)*d^4*(d + e/x^(1/3))^5*Ga
mma[1 + p, (-5*(a + b*Log[c*(d + e/x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e
/x^(1/3))^2])^p)/(5^p*e^9*E^((5*a)/(2*b))*(c*(d + e/x^(1/3))^2)^(5/2)*(-((a
 + b*Log[c*(d + e/x^(1/3))^2])/b))^p) + (21*2^(1 - p)*d^5*Gamma[1 + p, (-2*
(a + b*Log[c*(d + e/x^(1/3))^2]))/b]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(c
^2*e^9*E^((2*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p) - (7*2^(2 + p
)*d^6*(d + e/x^(1/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))^2]))/
(2*b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(3^p*e^9*E^((3*a)/(2*b))*(c*(d +
 e/x^(1/3))^2)^(3/2)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p) + (12*d^7*G
amma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))^2])/b)]*(a + b*Log[c*(d + e/x^(1
/3))^2])^p)/(c*e^9*E^(a/b)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p) - (3*
2^p*d^8*(d + e/x^(1/3))*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e/x^(1/3))^2])/
b]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(e^9*E^(a/(2*b))*Sqrt[c*(d + e/x^(1/
3))^2]*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^4}\,dx = -\left(3\text{Subst}\left(\int x^8\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\left(3\text{Subst}\left(\int\left(\frac{d^8\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^8} - \frac{8d^7(d+ex)\left(a+b\log\right.}{e^8}\right.\right.\right.$$

$$= -\frac{3\text{Subst}\left(\int (d+ex)^8\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^8} + \frac{(24d}{}$$

$$= -\frac{3\text{Subst}\left(\int x^8\left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^9} + \frac{(24d)\text{Subst}\left(\int\right.}{}$$

$$= \frac{(12d)\text{Subst}\left(\int e^{4x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{c^4e^9} + \frac{(84d^3)}{}$$

$$= -\frac{2^p3^{-1-2p}e^{-\frac{9a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)^9\Gamma\left(1+p, -\frac{9\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2b}\right)}{e^9\left(c\left(d+\frac{e}{\sqrt[3]{}}\right.\right.}$$

Mathematica [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^4,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^4, x]
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^2))^p/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))^2))^p/x^4,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x^4, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x + 2*c*d*x^(2/3)*e + c*x^(1/3)*e^2)/x) + a)^p/x^4,
x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**2))**p/x**4,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^2\right)\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(1/3))^2))^p/x^4,x)
[Out] int((a + b*log(c*(d + e/x^(1/3))^2))^p/x^4, x)
```

3.594 $\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable(x^3*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^3*(a + b*Log[c*(d + e/x^(2/3))])^p,x]
[Out] 3*Defer[Subst][Defer[Int][x^11*(a + b*Log[c*(d + e/x^2)])^p, x], x, x^(1/3)
]
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=3\text{Subst}\left(\int x^{11}\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A]
time = 1.07, size = 0, normalized size = 0.00

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))])^p,x]
[Out] Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))])^p, x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^3\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p*x^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*x^(1/3)*e)/x) + a)^p*x^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e/x**(2/3))))**p,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p*x^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^3 \left(a + b \ln\left(c\left(d + \frac{e}{x^{2/3}}\right)\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e/x^(2/3))))^p,x)
[Out] int(x^3*(a + b*log(c*(d + e/x^(2/3))))^p, x)
```

3.595 $\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable(x^2*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*(a + b*Log[c*(d + e/x^(2/3))])^p,x]
[Out] 3*Defer[Subst][Defer[Int][x^8*(a + b*Log[c*(d + e/x^2)])^p, x], x, x^(1/3)]
```

Rubi steps

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx = 3\text{Subst}\left(\int x^8\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A]
time = 1.18, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))])^p,x]
[Out] Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))])^p, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a+b*ln(c*(d+e/x^(2/3))))^p,x)

[Out] int(x^2*(a+b*ln(c*(d+e/x^(2/3))))^p,x)

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="maxima")

[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p*x^2, x)

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="fricas")

[Out] integral((b*log((c*d*x + c*x^(1/3)*e)/x) + a)^p*x^2, x)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(a+b*ln(c*(d+e/x**(2/3))))**p,x)

[Out] Timed out

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="giac")

[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p*x^2, x)

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^2 \left(a + b \ln \left(c \left(d + \frac{e}{x^{2/3}}\right)\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e/x^(2/3))))^p,x)
[Out] int(x^2*(a + b*log(c*(d + e/x^(2/3))))^p, x)
```

3.596 $\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable(x*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[x*(a + b*Log[c*(d + e/x^(2/3))])^p,x]
[Out] 3*Defer[Subst][Defer[Int][x^5*(a + b*Log[c*(d + e/x^2)])^p, x], x, x^(1/3)]
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=3\text{Subst}\left(\int x^5\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A]
time = 0.60, size = 0, normalized size = 0.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(2/3))])^p,x]
[Out] Integrate[x*(a + b*Log[c*(d + e/x^(2/3))])^p, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p*x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*x^(1/3)*e)/x) + a)^p*x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(2/3))))**p,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p*x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e/x^(2/3))))^p,x)
[Out] int(x*(a + b*log(c*(d + e/x^(2/3))))^p, x)
```

3.597 $\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))])^p,x]
[Out] 3*Defer[Subst][Defer[Int][x^2*(a + b*Log[c*(d + e/x^2)])^p, x], x, x^(1/3)]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx = 3\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A]
time = 0.23, size = 0, normalized size = 0.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))])^p,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(2/3))])^p, x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*x^(1/3)*e)/x) + a)^p, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))))**p,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))))^p,x)
[Out] int((a + b*log(c*(d + e/x^(2/3))))^p, x)
```

3.598 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(2/3))))^p/x,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))])^p/x,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x^2)])^p/x, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx = 3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)\right)\right)^p}{x}\,dx, x, \sqrt[3]{x}\right)$$

Mathematica [A]
time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))])^p/x,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(2/3))])^p/x, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))))^p/x,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))))^p/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p/x,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p/x,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*x^(1/3)*e)/x) + a)^p/x, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))))**p/x,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a + b \ln\left(c\left(d + \frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))))^p/x,x)
[Out] int((a + b*log(c*(d + e/x^(2/3))))^p/x, x)
```

3.599 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(2/3))))^p/x^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))])^p/x^2,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x^2)])^p/x^4, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx = 3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)\right)\right)^p}{x^4}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A]
time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))])^p/x^2,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(2/3))])^p/x^2, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))))^p/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))))^p/x^2,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p/x^2, x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*x^(1/3)*e)/x) + a)^p/x^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))))**p/x**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a + b\,\ln\left(c\left(d + \frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))))^p/x^2,x)
[Out] int((a + b*log(c*(d + e/x^(2/3))))^p/x^2, x)
```

3.600 $\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable(x^3*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^3*(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
[Out] 3*Defer[Subst][Defer[Int][x^11*(a + b*Log[c*(d + e/x^2)^2])^p, x], x, x^(1/
3)]
```

Rubi steps

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx = 3\mathrm{Subst}\left(\int x^{11}\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^2\right)\right)^p\,dx, x, \sqrt[3]{x}\right)$$

Mathematica [A]
time = 0.21, size = 0, normalized size = 0.00

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
[Out] Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))^2])^p, x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^3\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p*x^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x^2 + 2*c*d*x^(4/3)*e + c*x^(2/3)*e^2)/x^2) + a)^p*x
^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*(d+e/x**(2/3))**2))**p,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p*x^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^3 \left(a + b \ln \left(c \left(d + \frac{e}{x^{2/3}}\right)^2\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*log(c*(d + e/x^(2/3))^2))^p,x)
[Out] int(x^3*(a + b*log(c*(d + e/x^(2/3))^2))^p, x)
```

3.601 $\int x^2 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p \, dx$

Optimal. Leaf size=27

$$\text{Int}\left(x^2 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable(x^2*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^2 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
[Out] 3*Defer[Subst][Defer[Int][x^8*(a + b*Log[c*(d + e/x^2)^2])^p, x], x, x^(1/3
)]
```

Rubi steps

$$\int x^2 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p \, dx = 3\text{Subst}\left(\int x^8 \left(a+b \log \left(c \left(d+\frac{e}{x^2}\right)^2\right)\right)^p \, dx,x,\sqrt[3]{x}\right)$$

Mathematica [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int x^2 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
[Out] Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))^2])^p, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a+b \ln \left(c \left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p*x^2, x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x^2 + 2*c*d*x^(4/3)*e + c*x^(2/3)*e^2)/x^2) + a)^p*x
^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(2/3))**2))**p,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p*x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^2 \left(a + b \ln \left(c \left(d + \frac{e}{x^{2/3}}\right)^2\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*(d + e/x^(2/3))^2))^p,x)
[Out] int(x^2*(a + b*log(c*(d + e/x^(2/3))^2))^p, x)
```

3.602 $\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable(x*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[x*(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
[Out] 3*Defer[Subst][Defer[Int][x^5*(a + b*Log[c*(d + e/x^2)^2])^p, x], x, x^(1/3
)]
```

Rubi steps

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=3\text{Subst}\left(\int x^5\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A]
time = 0.19, size = 0, normalized size = 0.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
[Out] Integrate[x*(a + b*Log[c*(d + e/x^(2/3))^2])^p, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p*x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x^2 + 2*c*d*x^(4/3)*e + c*x^(2/3)*e^2)/x^2) + a)^p*x
, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(2/3))**2))**p,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p*x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*(d + e/x^(2/3))^2))^p,x)
[Out] int(x*(a + b*log(c*(d + e/x^(2/3))^2))^p, x)
```

3.603 $\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
[Out] 3*Defer[Subst][Defer[Int][x^2*(a + b*Log[c*(d + e/x^2)^2])^p, x], x, x^(1/3
)]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx = 3\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^2\right)\right)^p\,dx, x, \sqrt[3]{x}\right)$$

Mathematica [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(2/3))^2])^p, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x^2 + 2*c*d*x^(4/3)*e + c*x^(2/3)*e^2)/x^2) + a)^p,
x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**2))**p,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^2))^p,x)
[Out] int((a + b*log(c*(d + e/x^(2/3))^2))^p, x)
```

3.604 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(2/3))^2))^p/x,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^2])^p/x,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x^2)^2])^p/x, x], x, x^(1/3)]
```

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x}\,dx = 3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^2\right)\right)^p}{x}\,dx, x, \sqrt[3]{x}\right)$$

Mathematica [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^2])^p/x,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(2/3))^2])^p/x, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln \left(c{\left(d+\frac{e}{x^{\frac{2}{3}}}\right)}^{2}\right)\right)^{p}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^2))^p/x,x)
[Out] int((a+b*ln(c*(d+e/x^(2/3))^2))^p/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p/x,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p/x,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x^2 + 2*c*d*x^(4/3)*e + c*x^(2/3)*e^2)/x^2) + a)^p/x
, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**2))**p/x,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^2\right)\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^2))^p/x,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(2/3))^2))^p/x, x)
```

3.605 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2},x\right)$$

[Out] Unintegrable((a+b*ln(c*(d+e/x^(2/3))^2))^p/x^2,x)

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification is not applicable to the result.

[In] Int[(a + b*Log[c*(d + e/x^(2/3))^2])^p/x^2,x]

[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x^2)^2])^p/x^4, x], x, x^(1/3
)]

Rubi steps

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx = 3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^2\right)\right)^p}{x^4}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^2])^p/x^2,x]

[Out] Integrate[(a + b*Log[c*(d + e/x^(2/3))^2])^p/x^2, x]

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{2}\right)\right)^{p}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^2))^p/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^2))^p/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p/x^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d^2*x^2 + 2*c*d*x^(4/3)*e + c*x^(2/3)*e^2)/x^2) + a)^p/x
^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**2))**p/x**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^2\right)\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/x^(2/3))^2))^p/x^2,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(2/3))^2))^p/x^2, x)
```

3.606 $\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{hx}}\,dx$

Optimal. Leaf size=631

$$\frac{2af\sqrt{hx}}{h}-\frac{8bfp\sqrt{hx}}{h}-\frac{8bgp(hx)^{3/2}}{9h^2}-\frac{2\sqrt{2}\,b\sqrt[4]{d}\,fp\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{\sqrt[4]{e}\,\sqrt{h}}-\frac{2\sqrt{2}\,bd^{3/4}gp\tan^{-1}\left(1-\right.}{3e^{3/4}\sqrt{h}}$$

[Out] -8/9*b*g*p*(h*x)^(3/2)/h^2+2/3*g*(h*x)^(3/2)*(a+b*ln(c*(e*x^2+d)^p))/h^2-2*b*d^(1/4)*f*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/e^(1/4)/h^(1/2)-2/3*b*d^(3/4)*g*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/e^(3/4)/h^(1/2)+2*b*d^(1/4)*f*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/e^(1/4)/h^(1/2)+2/3*b*d^(3/4)*g*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/e^(3/4)/h^(1/2)-b*d^(1/4)*f*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(1/4)/h^(1/2)+1/3*b*d^(3/4)*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(3/4)/h^(1/2)+b*d^(1/4)*f*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(1/4)/h^(1/2)-1/3*b*d^(3/4)*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(3/4)/h^(1/2)+2*a*f*(h*x)^(1/2)/h-8*b*f*p*(h*x)^(1/2)/h+2*b*f*ln(c*(e*x^2+d)^p)*(h*x)^(1/2)/h

Rubi [A]

time = 0.59, antiderivative size = 631, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {2517, 2521, 2498, 327, 217, 1179, 642, 1176, 631, 210, 2505, 303}

$$\frac{2g(hx)^{3/2}\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h^2}+\frac{2af\sqrt{hx}}{h}-\frac{2\sqrt{2}\,bd^{3/4}gp\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{3e^{3/4}\sqrt{h}}+\frac{2\sqrt{2}\,bd^{3/4}gp\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}+1\right)}{3e^{3/4}\sqrt{h}}-\frac{2\sqrt{2}\,b\sqrt[4]{d}\,fp\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{\sqrt[4]{e}\,\sqrt{h}}+\frac{2\sqrt{2}\,b\sqrt[4]{d}\,fp\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}+1\right)}{\sqrt[4]{e}\,\sqrt{h}}+\frac{2bf\sqrt{hx}\,\log\left(c\left(d+ex^2\right)^p\right)}{h}+\frac{\sqrt{2}\,bd^{3/4}gp\log\left(-\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{hx}+\sqrt{d}\,\sqrt{h}+\sqrt{e}\,\sqrt{h}\,x\right)}{3e^{3/4}\sqrt{h}}-\frac{\sqrt{2}\,bd^{3/4}gp\log\left(\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{hx}+\sqrt{d}\,\sqrt{h}+\sqrt{e}\,\sqrt{h}\,x\right)}{3e^{3/4}\sqrt{h}}-\frac{\sqrt{2}\,b\sqrt[4]{d}\,fp\log\left(-\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{hx}+\sqrt{d}\,\sqrt{h}+\sqrt{e}\,\sqrt{h}\,x\right)}{\sqrt[4]{e}\,\sqrt{h}}+\frac{\sqrt{2}\,b\sqrt[4]{d}\,fp\log\left(\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{hx}+\sqrt{d}\,\sqrt{h}+\sqrt{e}\,\sqrt{h}\,x\right)}{\sqrt[4]{e}\,\sqrt{h}}-\frac{8bfp\sqrt{hx}}{h}-\frac{8bgp(hx)^{3/2}}{9h^2}$$

Antiderivative was successfully verified.

[In] Int[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/Sqrt[h*x],x]

[Out] (2*a*f*Sqrt[h*x])/h - (8*b*f*p*Sqrt[h*x])/h - (8*b*g*p*(h*x)^(3/2))/(9*h^2) - (2*Sqrt[2]*b*d^(1/4)*f*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(e^(1/4)*Sqrt[h]) - (2*Sqrt[2]*b*d^(3/4)*g*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*e^(3/4)*Sqrt[h]) + (2*Sqrt[2]*b*d^(1/4)*f*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(e^(1/4)*Sqrt[h]) + (2*Sqrt[2]*b*d^(3/4)*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*e^(3/4)*Sqrt[h]) + (2*b*f*Sqrt[h*x]*Log[c*(d + e*x^2)^p])/h + (2*g*(h*x)^(3/2)*(a + b*Log[c*(d + e*x^2)^p]))/(3*h^2) - (Sqrt[2]*b*d^(1/4)*f*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*Sqrt[h]) + (Sqrt[2]*b*d^(3/4)*g*p*Log[Sqrt

```
[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*e^
(3/4)*Sqrt[h]) + (Sqrt[2]*b*d^(1/4)*f*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[
h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*Sqrt[h]) - (Sqrt[2]*b*d
^(3/4)*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4
)*Sqrt[h*x]])/(3*e^(3/4)*Sqrt[h])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2517

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*(
d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{hx}}\,dx = \frac{2\text{Subst}\left(\int \left(f+\frac{gx^2}{h}\right)\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)\,dx,x,\sqrt{hx}\right)}{h}$$

$$= \frac{2\text{Subst}\left(\int \left(f\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)+\frac{gx^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{h}\right.\right.}{h}$$

$$= \frac{(2g)\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)\,dx,x,\sqrt{hx}\right)}{h^2}+\frac{(2f)\text{S}}{}$$

$$= \frac{2af\sqrt{hx}}{h}+\frac{2g(hx)^{3/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2}+\frac{(2bf)\text{Subst}\left(\int\right.}{}$$

$$= \frac{2af\sqrt{hx}}{h}-\frac{8bgp(hx)^{3/2}}{9h^2}+\frac{2bf\sqrt{hx}\,\log\left(c(d+ex^2)^p\right)}{h}+\frac{2g(hx)}{}$$

$$= \frac{2af\sqrt{hx}}{h}-\frac{8bfp\sqrt{hx}}{h}-\frac{8bgp(hx)^{3/2}}{9h^2}+\frac{2bf\sqrt{hx}\,\log\left(c(d+ex\right.}{h}$$

$$= \frac{2af\sqrt{hx}}{h}-\frac{8bfp\sqrt{hx}}{h}-\frac{8bgp(hx)^{3/2}}{9h^2}+\frac{2bf\sqrt{hx}\,\log\left(c(d+ex\right.}{h}$$

$$= \frac{2af\sqrt{hx}}{h}-\frac{8bfp\sqrt{hx}}{h}-\frac{8bgp(hx)^{3/2}}{9h^2}+\frac{2bf\sqrt{hx}\,\log\left(c(d+ex\right.}{h}$$

$$= \frac{2af\sqrt{hx}}{h}-\frac{8bfp\sqrt{hx}}{h}-\frac{8bgp(hx)^{3/2}}{9h^2}-\frac{2\sqrt{2}\,bd^{3/4}gp\tan^{-1}\left(1\right.}{3e^{3/4}\sqrt{}}$$

$$= \frac{2af\sqrt{hx}}{h}-\frac{8bfp\sqrt{hx}}{h}-\frac{8bgp(hx)^{3/2}}{9h^2}-\frac{2\sqrt{2}\,b\sqrt[4]{d}\,fp\tan^{-1}\left(1\right.}{\sqrt[4]{e}\,\sqrt{}}$$

Mathematica [A]
time = 0.32, size = 344, normalized size = 0.55

$$\frac{2\sqrt{x}\left(af\sqrt{x}-\frac{2bgp\left(2\sqrt[4]{-d}\,e^{3/4}x^{3/2}-3d\tan^{-1}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{-d}}\right)+3d\tanh^{-1}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{-d}}\right)\right)}{9\sqrt[4]{-d}\,e^{3/4}}-\frac{bfp\left(8\sqrt[4]{e}\,\sqrt{x}+2\sqrt{2}\,\sqrt[4]{d}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)-2\sqrt{2}\,\sqrt[4]{d}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)+\sqrt{2}\,\sqrt[4]{d}\,\log\left(\sqrt{d}-\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{x}+\sqrt{e}\,x\right)-\sqrt{2}\,\sqrt[4]{d}\,\log\left(\sqrt{d}+\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{x}+\sqrt{e}\,x\right)\right)}{2\sqrt[4]{e}}+bf\sqrt{x}\,\log\left(c(d+ex^2)^p\right)+\frac{1}{3}gx^{3/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)\right)}{\sqrt{hx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/Sqrt[h*x],x]
```

```
[Out] (2*Sqrt[x]*(a*f*Sqrt[x] - (2*b*g*p*(2*(-d)^(1/4)*e^(3/4)*x^(3/2) - 3*d*ArcT
an[(e^(1/4)*Sqrt[x])/(-d)^(1/4)] + 3*d*ArcTanh[(e^(1/4)*Sqrt[x])/(-d)^(1/4)
]))/(9*(-d)^(1/4)*e^(3/4)) - (b*f*p*(8*e^(1/4)*Sqrt[x] + 2*Sqrt[2]*d^(1/4)*
ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - 2*Sqrt[2]*d^(1/4)*ArcTan[1
+ (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] + Sqrt[2]*d^(1/4)*Log[Sqrt[d] - Sqrt[2
]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x] - Sqrt[2]*d^(1/4)*Log[Sqrt[d] + Sqrt
[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x]))/(2*e^(1/4)) + b*f*Sqrt[x]*Log[c*
(d + e*x^2)^p] + (g*x^(3/2)*(a + b*Log[c*(d + e*x^2)^p]))/3))/Sqrt[h*x]
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)\left(a+b\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)\right)}{\sqrt{hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(1/2),x)
```

```
[Out] int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(1/2),x)
```

Maxima [A]
time = 0.51, size = 538, normalized size = 0.85

$$\frac{2\,bgx^{2}\log\left(\left(x^{2}e+d\right)^{p}c\right)}{3\,\sqrt{hx}}+\frac{2\,agx^{2}}{3\,\sqrt{hx}}+\frac{2\,\sqrt{hx}\,bf\log\left(\left(x^{2}e+d\right)^{p}c\right)}{h}-\frac{\left(8\,\sqrt{hx}\,h^{2}e^{(-1)}-\left(\frac{\sqrt{2}\,h^{4}e^{\left(-\frac{1}{4}\right)}\log\left(hxe^{\frac{1}{2}}+\sqrt{2}\,(dh^{2})^{\frac{1}{4}}\sqrt{hx}\,e^{\frac{1}{4}}+\sqrt{d}\,h\right)}{(dh^{2})^{\frac{3}{4}}}-\frac{\sqrt{2}\,h^{4}e^{\left(-\frac{1}{4}\right)}\log\left(hxe^{\frac{1}{2}}-\sqrt{2}\,(dh^{2})^{\frac{1}{4}}\sqrt{hx}\,e^{\frac{1}{4}}+\sqrt{d}\,h\right)}{(dh^{2})^{\frac{3}{4}}}+\frac{2\,\sqrt{2}\,h^{3}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(dh^{2})^{\frac{1}{4}}e^{\frac{1}{4}}+2\,\sqrt{hx}\,e^{\frac{1}{2}}\right)e^{\left(-\frac{1}{4}\right)}}{2\,\sqrt{\sqrt{d}\,h}}\right)e^{\left(-\frac{1}{4}\right)}}{\sqrt{\sqrt{d}\,h}\,\sqrt{d}}+\frac{2\,\sqrt{2}\,h^{3}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(dh^{2})^{\frac{1}{4}}e^{\frac{1}{4}}-2\,\sqrt{hx}\,e^{\frac{1}{2}}\right)e^{\left(-\frac{1}{4}\right)}}{2\,\sqrt{\sqrt{d}\,h}}\right)e^{\left(-\frac{1}{4}\right)}}{\sqrt{\sqrt{d}\,h}\,\sqrt{d}}\right)de^{(-1)}\right)bfpe}{h^{3}}+\frac{2\,\sqrt{hx}\,af}{h}-\frac{\left(3\left(\frac{\sqrt{2}\,e^{\left(-\frac{3}{4}\right)}\log\left(hxe^{\frac{1}{2}}+\sqrt{2}\,(dh^{2})^{\frac{1}{4}}\sqrt{hx}\,e^{\frac{1}{4}}+\sqrt{d}\,h\right)}{(dh^{2})^{\frac{1}{4}}}-\frac{\sqrt{2}\,e^{\left(-\frac{3}{4}\right)}\log\left(hxe^{\frac{1}{2}}-\sqrt{2}\,(dh^{2})^{\frac{1}{4}}\sqrt{hx}\,e^{\frac{1}{4}}+\sqrt{d}\,h\right)}{(dh^{2})^{\frac{1}{4}}}-\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(dh^{2})^{\frac{1}{4}}e^{\frac{1}{4}}+2\,\sqrt{hx}\,e^{\frac{1}{2}}\right)e^{\left(-\frac{1}{4}\right)}}{2\,\sqrt{\sqrt{d}\,h}}\right)e^{\left(-\frac{3}{4}\right)}}{\sqrt{\sqrt{d}\,h}}-\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(dh^{2})^{\frac{1}{4}}e^{\frac{1}{4}}-2\,\sqrt{hx}\,e^{\frac{1}{2}}\right)e^{\left(-\frac{1}{4}\right)}}{2\,\sqrt{\sqrt{d}\,h}}\right)e^{\left(-\frac{3}{4}\right)}}{\sqrt{\sqrt{d}\,h}}\right)dh^{4}e^{(-1)}+8\,(hx)^{\frac{3}{2}}h^{2}e^{(-1)}\right)bgpe}{9\,h^{4}}$$

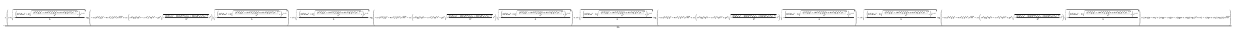
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2),x, algorithm="maxima
")
```

```
[Out] 2/3*b*g*x^2*log((x^2*e + d)^p*c)/sqrt(h*x) + 2/3*a*g*x^2/sqrt(h*x) + 2*sqrt
(h*x)*b*f*log((x^2*e + d)^p*c)/h - (8*sqrt(h*x)*h^2*e^(-1) - (sqrt(2)*h^4*e
^(-1/4)*log(h*x*e^(1/2) + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)
*h)/(d*h^2)^(3/4) - sqrt(2)*h^4*e^(-1/4)*log(h*x*e^(1/2) - sqrt(2)*(d*h^2)^
(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(3/4) + 2*sqrt(2)*h^3*arctan(1
/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*e^(1/2))*e^(-1/4)/s
qrt(sqrt(d)*h))*e^(-1/4)/(sqrt(sqrt(d)*h)*sqrt(d)) + 2*sqrt(2)*h^3*arctan(-
1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*e^(1/2))*e^(-1/4)/
sqrt(sqrt(d)*h))*e^(-1/4)/(sqrt(sqrt(d)*h)*sqrt(d)))*d*e^(-1))*b*f*p*e/h^3
+ 2*sqrt(h*x)*a*f/h - 1/9*(3*(sqrt(2)*e^(-3/4)*log(h*x*e^(1/2) + sqrt(2)*(d
*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(1/4) - sqrt(2)*e^(-3/4)
*log(h*x*e^(1/2) - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*
h^2)^(1/4) - 2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) +
```

```
2*sqrt(h*x)*e^(1/2))*e^(-1/4)/sqrt(sqrt(d)*h))*e^(-3/4)/sqrt(sqrt(d)*h) - 2
*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*e
^(1/2))*e^(-1/4)/sqrt(sqrt(d)*h))*e^(-3/4)/sqrt(sqrt(d)*h))*d*h^4*e^(-1) +
8*(h*x)^(3/2)*h^2*e^(-1))*b*g*p*e/h^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1263 vs. $2(423) = 846$.
time = 0.44, size = 1263, normalized size = 2.00



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2),x, algorithm="fricas
")
```

```
[Out] -2/9*(3*h*sqrt(-(6*b^2*d*f*g*p^2 + h*sqrt(-(b^4*d^3*g^4*p^4 - 18*b^4*d^2*f^
2*g^2*p^4*e + 81*b^4*d*f^4*p^4*e^2)*e^(-3)/h^2)*e)*e^(-1)/h)*log(-32*(b^3*d
^2*g^4*p^3 - 81*b^3*f^4*p^3*e^2)*sqrt(h*x) + 32*(3*b^2*d*f*g^2*h*p^2*e - 27
*b^2*f^3*h*p^2*e^2 - g*h^2*sqrt(-(b^4*d^3*g^4*p^4 - 18*b^4*d^2*f^2*g^2*p^4*
e + 81*b^4*d*f^4*p^4*e^2)*e^(-3)/h^2)*e^2)*sqrt(-(6*b^2*d*f*g*p^2 + h*sqrt(
-(b^4*d^3*g^4*p^4 - 18*b^4*d^2*f^2*g^2*p^4*e + 81*b^4*d*f^4*p^4*e^2)*e^(-3)
/h^2)*e)*e^(-1)/h)) - 3*h*sqrt(-(6*b^2*d*f*g*p^2 + h*sqrt(-(b^4*d^3*g^4*p^4
 - 18*b^4*d^2*f^2*g^2*p^4*e + 81*b^4*d*f^4*p^4*e^2)*e^(-3)/h^2)*e)*e^(-1)/h
)*log(-32*(b^3*d^2*g^4*p^3 - 81*b^3*f^4*p^3*e^2)*sqrt(h*x) - 32*(3*b^2*d*f*
g^2*h*p^2*e - 27*b^2*f^3*h*p^2*e^2 - g*h^2*sqrt(-(b^4*d^3*g^4*p^4 - 18*b^4*
d^2*f^2*g^2*p^4*e + 81*b^4*d*f^4*p^4*e^2)*e^(-3)/h^2)*e^2)*sqrt(-(6*b^2*d*f
*g*p^2 + h*sqrt(-(b^4*d^3*g^4*p^4 - 18*b^4*d^2*f^2*g^2*p^4*e + 81*b^4*d*f^4
*p^4*e^2)*e^(-3)/h^2)*e)*e^(-1)/h)) + 3*h*sqrt(-(6*b^2*d*f*g*p^2 - h*sqrt(-
(b^4*d^3*g^4*p^4 - 18*b^4*d^2*f^2*g^2*p^4*e + 81*b^4*d*f^4*p^4*e^2)*e^(-3)/
h^2)*e)*e^(-1)/h)*log(-32*(b^3*d^2*g^4*p^3 - 81*b^3*f^4*p^3*e^2)*sqrt(h*x)
+ 32*(3*b^2*d*f*g^2*h*p^2*e - 27*b^2*f^3*h*p^2*e^2 + g*h^2*sqrt(-(b^4*d^3*g
^4*p^4 - 18*b^4*d^2*f^2*g^2*p^4*e + 81*b^4*d*f^4*p^4*e^2)*e^(-3)/h^2)*e^2)*
sqrt(-(6*b^2*d*f*g*p^2 - h*sqrt(-(b^4*d^3*g^4*p^4 - 18*b^4*d^2*f^2*g^2*p^4*
e + 81*b^4*d*f^4*p^4*e^2)*e^(-3)/h^2)*e)*e^(-1)/h)) - 3*h*sqrt(-(6*b^2*d*f*
g*p^2 - h*sqrt(-(b^4*d^3*g^4*p^4 - 18*b^4*d^2*f^2*g^2*p^4*e + 81*b^4*d*f^4*
p^4*e^2)*e^(-3)/h^2)*e)*e^(-1)/h)*log(-32*(b^3*d^2*g^4*p^3 - 81*b^3*f^4*p^3
*e^2)*sqrt(h*x) - 32*(3*b^2*d*f*g^2*h*p^2*e - 27*b^2*f^3*h*p^2*e^2 + g*h^2*
sqrt(-(b^4*d^3*g^4*p^4 - 18*b^4*d^2*f^2*g^2*p^4*e + 81*b^4*d*f^4*p^4*e^2)*e
^(-3)/h^2)*e^2)*sqrt(-(6*b^2*d*f*g*p^2 - h*sqrt(-(b^4*d^3*g^4*p^4 - 18*b^4*
d^2*f^2*g^2*p^4*e + 81*b^4*d*f^4*p^4*e^2)*e^(-3)/h^2)*e)*e^(-1)/h)) + (36*b
*f*p - 9*a*f + (4*b*g*p - 3*a*g)*x - 3*(b*g*p*x + 3*b*f*p)*log(x^2*e + d) -
 3*(b*g*x + 3*b*f)*log(c))*sqrt(h*x))/h
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(1/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Giac [A]
time = 5.66, size = 514, normalized size = 0.81

$$\frac{6\sqrt{hx}\,bgx\log\left(c\right)+9\left(\left(2\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{4}\right)}+2\sqrt{hx}\right)e^{\frac{1}{4}}}{2\left(dh^2\right)^{\frac{1}{4}}}\right)e^{\left(-\frac{5}{4}\right)}+2\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{4}\right)}-2\sqrt{hx}\right)e^{\frac{1}{4}}}{2\left(dh^2\right)^{\frac{1}{4}}}\right)e^{\left(-\frac{5}{4}\right)}+\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}e^{\left(-\frac{5}{4}\right)}\log\left(\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}\sqrt{hx}\,e^{\left(-\frac{1}{4}\right)}+hx+\sqrt{dh^2}\,e^{\left(-\frac{1}{2}\right)}\right)-\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}e^{\left(-\frac{5}{4}\right)}\log\left(-\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}\sqrt{hx}\,e^{\left(-\frac{1}{4}\right)}+hx+\sqrt{dh^2}\,e^{\left(-\frac{1}{2}\right)}\right)-8\sqrt{hx}\,e^{\left(-1\right)}\right)e+2\sqrt{hx}\log\left(x^2e+d\right)\right)bfp+6\sqrt{hx}\,agx+18\sqrt{hx}\,bf\log\left(c\right)+\frac{\left(6\sqrt{hx}\,hx\log\left(x^2e+d\right)-\left(8\sqrt{hx}\,hxe^{\left(-1\right)}-6\sqrt{2}\left(dh^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{4}\right)}+2\sqrt{hx}\right)e^{\frac{1}{4}}}{2\left(dh^2\right)^{\frac{1}{4}}}\right)e^{\left(-\frac{7}{4}\right)}-6\sqrt{2}\left(dh^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{4}\right)}-2\sqrt{hx}\right)e^{\frac{1}{4}}}{2\left(dh^2\right)^{\frac{1}{4}}}\right)e^{\left(-\frac{7}{4}\right)}+3\sqrt{2}\left(dh^2\right)^{\frac{3}{4}}e^{\left(-\frac{7}{4}\right)}\log\left(\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}\sqrt{hx}\,e^{\left(-\frac{1}{4}\right)}+hx+\sqrt{dh^2}\,e^{\left(-\frac{1}{2}\right)}\right)-3\sqrt{2}\left(dh^2\right)^{\frac{3}{4}}e^{\left(-\frac{7}{4}\right)}\log\left(-\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}\sqrt{hx}\,e^{\left(-\frac{1}{4}\right)}+hx+\sqrt{dh^2}\,e^{\left(-\frac{1}{2}\right)}\right)\right)e\right)bgp}{h}+18\sqrt{hx}\,af}{9h}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/9*(6*sqrt(h*x)*b*g*x*log(c) + 9*((2*sqrt(2)*(d*h^2)^(1/4)*arctan(1/2*sqrt
(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/4))*e
^(-5/4) + 2*sqrt(2)*(d*h^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4
)*e^(-1/4) - 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-5/4) + sqrt(2)*(d*h^2)
^(1/4)*e^(-5/4)*log(sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d
*h^2)*e^(-1/2)) - sqrt(2)*(d*h^2)^(1/4)*e^(-5/4)*log(-sqrt(2)*(d*h^2)^(1/4)
*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2)) - 8*sqrt(h*x)*e^(-1))*e +
 2*sqrt(h*x)*log(x^2*e + d))*b*f*p + 6*sqrt(h*x)*a*g*x + 18*sqrt(h*x)*b*f*l
og(c) + (6*sqrt(h*x)*h*x*log(x^2*e + d) - (8*sqrt(h*x)*h*x*e^(-1) - 6*sqrt(
2)*(d*h^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqr
t(h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-7/4) - 6*sqrt(2)*(d*h^2)^(3/4)*arctan(-1
/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) - 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(
1/4))*e^(-7/4) + 3*sqrt(2)*(d*h^2)^(3/4)*e^(-7/4)*log(sqrt(2)*(d*h^2)^(1/4)
*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2)) - 3*sqrt(2)*(d*h^2)^(3/4)
*e^(-7/4)*log(-sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)
*e^(-1/2)))*e)*b*g*p/h + 18*sqrt(h*x)*a*f)/h
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{\sqrt{h\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(1/2),x)
```

```
[Out] int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(1/2), x)
```

3.607 $\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{3/2}}\,dx$

Optimal. Leaf size=603

$$\frac{2ag\sqrt{hx}}{h^2}-\frac{8bgp\sqrt{hx}}{h^2}-\frac{2\sqrt{2}\,b\sqrt[4]{e}\,fp\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{\sqrt[4]{d}\,h^{3/2}}-\frac{2\sqrt{2}\,b\sqrt[4]{d}\,gp\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{\sqrt[4]{e}\,h^{3/2}}$$

```
[Out] -2*b*e^(1/4)*f*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1
/2)/d^(1/4)/h^(3/2)-2*b*d^(1/4)*g*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^
(1/4)/h^(1/2))*2^(1/2)/e^(1/4)/h^(3/2)+2*b*e^(1/4)*f*p*arctan(1+e^(1/4)*2^(
1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(1/4)/h^(3/2)+2*b*d^(1/4)*g*p*a
rctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/e^(1/4)/h^(3/2
)+b*e^(1/4)*f*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2
)*(h*x)^(1/2))*2^(1/2)/d^(1/4)/h^(3/2)-b*d^(1/4)*g*p*ln(d^(1/2)*h^(1/2)+x*e
^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(1/4)/h^(3/2)
-b*e^(1/4)*f*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)
*(h*x)^(1/2))*2^(1/2)/d^(1/4)/h^(3/2)+b*d^(1/4)*g*p*ln(d^(1/2)*h^(1/2)+x*e^
(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(1/4)/h^(3/2)-
2*f*(a+b*ln(c*(e*x^2+d)^p))/h/(h*x)^(1/2)+2*a*g*(h*x)^(1/2)/h^2-8*b*g*p*(h*
x)^(1/2)/h^2+2*b*g*ln(c*(e*x^2+d)^p)*(h*x)^(1/2)/h^2
```

Rubi [A]
time = 0.53, antiderivative size = 603, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {2517, 2526, 2498, 327, 217, 1179, 642, 1176, 631, 210, 2505, 303}

$$-\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{h\sqrt{hx}}+\frac{2ag\sqrt{hx}}{h^2}-\frac{2\sqrt{2}\,b\sqrt[4]{e}\,fp\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{\sqrt[4]{d}\,h^{3/2}}+\frac{2\sqrt{2}\,b\sqrt[4]{e}\,fp\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}+1\right)}{\sqrt[4]{d}\,h^{3/2}}-\frac{2\sqrt{2}\,b\sqrt[4]{d}\,gp\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{\sqrt[4]{e}\,h^{3/2}}+\frac{2\sqrt{2}\,b\sqrt[4]{d}\,gp\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}+1\right)}{\sqrt[4]{e}\,h^{3/2}}+\frac{2bg\sqrt{hx}\,\log\left(c\left(d+ex^2\right)^p\right)}{h^2}+\frac{\sqrt{2}\,b\sqrt[4]{e}\,fp\log\left(-\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{hx}+\sqrt{d}\,\sqrt{h}+\sqrt{e}\,\sqrt{h}\,x\right)}{\sqrt[4]{d}\,h^{3/2}}-\frac{\sqrt{2}\,b\sqrt[4]{e}\,fp\log\left(\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{hx}+\sqrt{d}\,\sqrt{h}+\sqrt{e}\,\sqrt{h}\,x\right)}{\sqrt[4]{d}\,h^{3/2}}-\frac{\sqrt{2}\,b\sqrt[4]{d}\,gp\log\left(-\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{hx}+\sqrt{d}\,\sqrt{h}+\sqrt{e}\,\sqrt{h}\,x\right)}{\sqrt[4]{e}\,h^{3/2}}+\frac{\sqrt{2}\,b\sqrt[4]{d}\,gp\log\left(\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{hx}+\sqrt{d}\,\sqrt{h}+\sqrt{e}\,\sqrt{h}\,x\right)}{\sqrt[4]{e}\,h^{3/2}}-\frac{8bgp\sqrt{hx}}{h^2}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(3/2),x]
```

```
[Out] (2*a*g*Sqrt[h*x])/h^2 - (8*b*g*p*Sqrt[h*x])/h^2 - (2*Sqrt[2]*b*e^(1/4)*f*p*
ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])]])/(d^(1/4)*h^(3/2)
) - (2*Sqrt[2]*b*d^(1/4)*g*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4
)*Sqrt[h])]])/(e^(1/4)*h^(3/2)) + (2*Sqrt[2]*b*e^(1/4)*f*p*ArcTan[1 + (Sqrt[
2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])]])/(d^(1/4)*h^(3/2)) + (2*Sqrt[2]*b*
d^(1/4)*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])]])/(e^(
1/4)*h^(3/2)) + (2*b*g*Sqrt[h*x]*Log[c*(d + e*x^2)^p])/h^2 - (2*f*(a + b*Lo
g[c*(d + e*x^2)^p]))/(h*Sqrt[h*x]) + (Sqrt[2]*b*e^(1/4)*f*p*Log[Sqrt[d]*Sqr
t[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(d^(1/4)*h^(
3/2)) - (Sqrt[2]*b*d^(1/4)*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sq
rt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*h^(3/2)) - (Sqrt[2]*b*e^(1/4)*f*
```

```
p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*
x]])/(d^(1/4)*h^(3/2)) + (Sqrt[2]*b*d^(1/4)*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[
e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*h^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2517

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*(
d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{\left(f+\frac{gx^2}{h}\right)\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^2}\,dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{g\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{h} + \frac{f\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^2}\right)\,dx, x, \ldots}{h}$$

$$= \frac{(2g)\text{Subst}\left(\int \left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)\,dx, x, \sqrt{hx}\right)}{h^2} + \frac{(2f)\text{Su}\ldots}{\ldots}$$

$$= \frac{2ag\sqrt{hx}}{h^2} - \frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h\sqrt{hx}} + \frac{(2bg)\text{Subst}\left(\int \log\left(c\ldots\right.\right.}{\ldots}$$

$$= \frac{2ag\sqrt{hx}}{h^2} + \frac{2bg\sqrt{hx}\,\log\left(c(d+ex^2)^p\right)}{h^2} - \frac{2f\left(a+b\log\left(c(d+e\ldots\right.\right.}{h\sqrt{hx}}$$

$$= \frac{2ag\sqrt{hx}}{h^2} - \frac{8bgp\sqrt{hx}}{h^2} + \frac{2bg\sqrt{hx}\,\log\left(c(d+ex^2)^p\right)}{h^2} - \frac{2f\left(a+\ldots\right.}{\ldots}$$

$$= \frac{2ag\sqrt{hx}}{h^2} - \frac{8bgp\sqrt{hx}}{h^2} + \frac{2bg\sqrt{hx}\,\log\left(c(d+ex^2)^p\right)}{h^2} - \frac{2f\left(a+\ldots\right.}{\ldots}$$

$$= \frac{2ag\sqrt{hx}}{h^2} - \frac{8bgp\sqrt{hx}}{h^2} - \frac{2\sqrt{2}\,b\sqrt[4]{e}\,fp\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{h\ldots}}{\sqrt[4]{d}\,\sqrt{h}}\right.}{\sqrt[4]{d}\,h^{3/2}}$$

$$= \frac{2ag\sqrt{hx}}{h^2} - \frac{8bgp\sqrt{hx}}{h^2} - \frac{2\sqrt{2}\,b\sqrt[4]{e}\,fp\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{h\ldots}}{\sqrt[4]{d}\,\sqrt{h}}\right.}{\sqrt[4]{d}\,h^{3/2}}$$

$$= \frac{2ag\sqrt{hx}}{h^2} - \frac{8bgp\sqrt{hx}}{h^2} - \frac{2\sqrt{2}\,b\sqrt[4]{e}\,fp\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{h\ldots}}{\sqrt[4]{d}\,\sqrt{h}}\right.}{\sqrt[4]{d}\,h^{3/2}}$$

Mathematica [A]

time = 0.31, size = 316, normalized size = 0.52

$$\frac{2x^{3/2}\left(ag\sqrt{x} + \frac{2b\sqrt[4]{e}\,fp\left(\tan^{-1}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{-d}}\right)+\tanh^{-1}\left(\frac{d\sqrt[4]{e}\,\sqrt{x}}{(-d)^{5/4}}\right)\right)}{\sqrt[4]{-d}} - \frac{bgp\left(8\sqrt[4]{e}\,\sqrt{x}+2\sqrt{2}\,\sqrt[4]{d}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)-2\sqrt{2}\,\sqrt[4]{d}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)+\sqrt{2}\,\sqrt[4]{d}\,\log\left(\sqrt{d}-\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{x}+\sqrt{e}\,x\right)-\sqrt{2}\,\sqrt[4]{d}\,\log\left(\sqrt{d}+\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{x}+\sqrt{e}\,x\right)\right)}{2\sqrt[4]{e}} + bg\sqrt{x}\,\log\left(c(d+ex^2)^p\right) - \frac{f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{x}}\right)}{(hx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(3/2),x]
```

```
[Out] (2*x^(3/2)*(a*g*Sqrt[x] + (2*b*e^(1/4)*f*p*(ArcTan[(e^(1/4)*Sqrt[x])/(-d)^(
1/4)] + ArcTanh[(d*e^(1/4)*Sqrt[x])/(-d)^(5/4)]))/(-d)^(1/4) - (b*g*p*(8*e^
(1/4)*Sqrt[x] + 2*Sqrt[2]*d^(1/4)*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1
/4)] - 2*Sqrt[2]*d^(1/4)*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] + Sq
rt[2]*d^(1/4)*Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x] -
Sqrt[2]*d^(1/4)*Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x])
)/(2*e^(1/4)) + b*g*Sqrt[x]*Log[c*(d + e*x^2)^p] - (f*(a + b*Log[c*(d + e*x
^2)^p]))/Sqrt[x]))/(h*x)^(3/2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)\left(a+b\ln\left(c(e\,x^2+d)^p\right)\right)}{(hx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(3/2),x)
```

```
[Out] int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(3/2),x)
```

Maxima [A]
time = 0.52, size = 517, normalized size = 0.86

$$-\frac{\left(\frac{\sqrt{2}e^{\left(-\frac{3}{4}\right)}\log\left(hxe^{\frac{1}{2}}+\sqrt{2}(dh^2)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{(dh^2)^{\frac{1}{4}}}-\frac{\sqrt{2}e^{\left(-\frac{3}{4}\right)}\log\left(hxe^{\frac{1}{2}}-\sqrt{2}(dh^2)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{(dh^2)^{\frac{1}{4}}}-\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(dh^2)^{\frac{1}{4}}e^{\frac{1}{4}}+2\sqrt{hx}e^{\frac{1}{2}}\right)e^{\left(-\frac{1}{4}\right)}}{2\sqrt{\sqrt{d}h}}\right)e^{\left(-\frac{3}{4}\right)}}{\sqrt{\sqrt{d}h}}-\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(dh^2)^{\frac{1}{4}}e^{\frac{1}{4}}-2\sqrt{hx}e^{\frac{1}{2}}\right)e^{\left(-\frac{1}{4}\right)}}{2\sqrt{\sqrt{d}h}}\right)e^{\left(-\frac{3}{4}\right)}}{\sqrt{\sqrt{d}h}}\right)bfpe}{h}+\frac{2bgx^2\log\left((x^2e+d)^pc\right)}{(hx)^{\frac{3}{2}}}+\frac{2agx^2}{(hx)^{\frac{3}{2}}}-\frac{2bf\log\left((x^2e+d)^pc\right)}{\sqrt{hx}h}-\frac{\left(8\sqrt{hx}h^2e^{(-1)}-\left(\frac{\sqrt{2}h^4e^{\left(-\frac{1}{4}\right)}\log\left(hxe^{\frac{1}{2}}+\sqrt{2}(dh^2)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{(dh^2)^{\frac{3}{4}}}-\frac{\sqrt{2}h^4e^{\left(-\frac{1}{4}\right)}\log\left(hxe^{\frac{1}{2}}-\sqrt{2}(dh^2)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{(dh^2)^{\frac{3}{4}}}+\frac{2\sqrt{2}h^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(dh^2)^{\frac{1}{4}}e^{\frac{1}{4}}+2\sqrt{hx}e^{\frac{1}{2}}\right)e^{\left(-\frac{1}{4}\right)}}{2\sqrt{\sqrt{d}h}}\right)e^{\left(-\frac{1}{4}\right)}}{\sqrt{\sqrt{d}h}\sqrt{d}}+\frac{2\sqrt{2}h^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(dh^2)^{\frac{1}{4}}e^{\frac{1}{4}}-2\sqrt{hx}e^{\frac{1}{2}}\right)e^{\left(-\frac{1}{4}\right)}}{2\sqrt{\sqrt{d}h}}\right)e^{\left(-\frac{1}{4}\right)}}{\sqrt{\sqrt{d}h}\sqrt{d}}\right)de^{(-1)}\right)bgpe}{h^4}-\frac{2af}{\sqrt{hx}h}$$

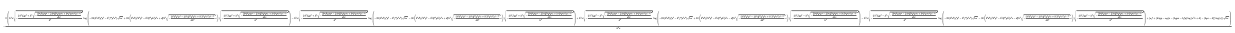
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2),x, algorithm="maxima
")
```

```
[Out] -(sqrt(2)*e^(-3/4)*log(h*x*e^(1/2) + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4
) + sqrt(d)*h)/(d*h^2)^(1/4) - sqrt(2)*e^(-3/4)*log(h*x*e^(1/2) - sqrt(2)*(
d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(1/4) - 2*sqrt(2)*arcta
n(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*e^(1/2))*e^(-1/4
)/sqrt(sqrt(d)*h))*e^(-3/4)/sqrt(sqrt(d)*h) - 2*sqrt(2)*arctan(-1/2*sqrt(2)
*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*e^(1/2))*e^(-1/4)/sqrt(sqrt(d
)*h))*e^(-3/4)/sqrt(sqrt(d)*h))*b*f*p*e/h + 2*b*g*x^2*log((x^2*e + d)^p*c)/
(h*x)^(3/2) + 2*a*g*x^2/(h*x)^(3/2) - 2*b*f*log((x^2*e + d)^p*c)/(sqrt(h*x)
*h) - (8*sqrt(h*x)*h^2*e^(-1) - (sqrt(2)*h^4*e^(-1/4)*log(h*x*e^(1/2) + sqr
t(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(3/4) - sqrt(2)*h
^4*e^(-1/4)*log(h*x*e^(1/2) - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqr
t(d)*h)/(d*h^2)^(3/4) + 2*sqrt(2)*h^3*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(
1/4)*e^(1/4) + 2*sqrt(h*x)*e^(1/2))*e^(-1/4)/sqrt(sqrt(d)*h))*e^(-1/4)/(sqr
```

```
t(sqrt(d)*h)*sqrt(d)) + 2*sqrt(2)*h^3*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^
(1/4)*e^(1/4) - 2*sqrt(h*x)*e^(1/2))*e^(-1/4)/sqrt(sqrt(d)*h))*e^(-1/4)/(sq
rt(sqrt(d)*h)*sqrt(d)))*d*e^(-1))*b*g*p*e/h^4 - 2*a*f/(sqrt(h*x)*h)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1254 vs. $2(409) = 818$.
time = 0.40, size = 1254, normalized size = 2.08



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2),x, algorithm="fricas
")
```

```
[Out] -2*(h^2*x*sqrt(-(2*b^2*f*g*p^2 + h^3*sqrt(-(b^4*d^2*g^4*p^4 - 2*b^4*d*f^2*g
^2*p^4*e + b^4*f^4*p^4*e^2)*e^(-1)/(d*h^6)))/h^3)*log(-32*(b^3*d^2*g^4*p^3
- b^3*f^4*p^3*e^2)*sqrt(h*x) + 32*(b^2*d^2*g^3*h^2*p^2 - b^2*d*f^2*g*h^2*p^
2*e + d*f*h^5*sqrt(-(b^4*d^2*g^4*p^4 - 2*b^4*d*f^2*g^2*p^4*e + b^4*f^4*p^4*
e^2)*e^(-1)/(d*h^6))*e)*sqrt(-(2*b^2*f*g*p^2 + h^3*sqrt(-(b^4*d^2*g^4*p^4 -
 2*b^4*d*f^2*g^2*p^4*e + b^4*f^4*p^4*e^2)*e^(-1)/(d*h^6)))/h^3)) - h^2*x*sq
rt(-(2*b^2*f*g*p^2 + h^3*sqrt(-(b^4*d^2*g^4*p^4 - 2*b^4*d*f^2*g^2*p^4*e + b
^4*f^4*p^4*e^2)*e^(-1)/(d*h^6)))/h^3)*log(-32*(b^3*d^2*g^4*p^3 - b^3*f^4*p^
3*e^2)*sqrt(h*x) - 32*(b^2*d^2*g^3*h^2*p^2 - b^2*d*f^2*g*h^2*p^2*e + d*f*h^
5*sqrt(-(b^4*d^2*g^4*p^4 - 2*b^4*d*f^2*g^2*p^4*e + b^4*f^4*p^4*e^2)*e^(-1)/
(d*h^6))*e)*sqrt(-(2*b^2*f*g*p^2 + h^3*sqrt(-(b^4*d^2*g^4*p^4 - 2*b^4*d*f^2
*g^2*p^4*e + b^4*f^4*p^4*e^2)*e^(-1)/(d*h^6)))/h^3)) + h^2*x*sqrt(-(2*b^2*f
*g*p^2 - h^3*sqrt(-(b^4*d^2*g^4*p^4 - 2*b^4*d*f^2*g^2*p^4*e + b^4*f^4*p^4*e
^2)*e^(-1)/(d*h^6)))/h^3)*log(-32*(b^3*d^2*g^4*p^3 - b^3*f^4*p^3*e^2)*sqrt(
h*x) + 32*(b^2*d^2*g^3*h^2*p^2 - b^2*d*f^2*g*h^2*p^2*e - d*f*h^5*sqrt(-(b^4
*d^2*g^4*p^4 - 2*b^4*d*f^2*g^2*p^4*e + b^4*f^4*p^4*e^2)*e^(-1)/(d*h^6))*e)*
sqrt(-(2*b^2*f*g*p^2 - h^3*sqrt(-(b^4*d^2*g^4*p^4 - 2*b^4*d*f^2*g^2*p^4*e +
 b^4*f^4*p^4*e^2)*e^(-1)/(d*h^6)))/h^3)) - h^2*x*sqrt(-(2*b^2*f*g*p^2 - h^3
*sqrt(-(b^4*d^2*g^4*p^4 - 2*b^4*d*f^2*g^2*p^4*e + b^4*f^4*p^4*e^2)*e^(-1)/(
d*h^6)))/h^3)*log(-32*(b^3*d^2*g^4*p^3 - b^3*f^4*p^3*e^2)*sqrt(h*x) - 32*(b
^2*d^2*g^3*h^2*p^2 - b^2*d*f^2*g*h^2*p^2*e - d*f*h^5*sqrt(-(b^4*d^2*g^4*p^4
 - 2*b^4*d*f^2*g^2*p^4*e + b^4*f^4*p^4*e^2)*e^(-1)/(d*h^6))*e)*sqrt(-(2*b^2
*f*g*p^2 - h^3*sqrt(-(b^4*d^2*g^4*p^4 - 2*b^4*d*f^2*g^2*p^4*e + b^4*f^4*p^4
*e^2)*e^(-1)/(d*h^6)))/h^3)) + (a*f + (4*b*g*p - a*g)*x - (b*g*p*x - b*f*p)
*log(x^2*e + d) - (b*g*x - b*f)*log(c))*sqrt(h*x))/(h^2*x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(3/2),x)

[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo

Giac [A]
time = 3.14, size = 431, normalized size = 0.71

$$\frac{\frac{2\left(\sqrt{2}\,(dh^2)^{\frac{1}{4}}bdghpe^{\frac{7}{4}}+\sqrt{2}\,(dh^2)^{\frac{3}{4}}bfpe^{\frac{9}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(dh^2)^{\frac{1}{4}}e^{\left(-\frac{1}{4}\right)}+2\sqrt{hx}\right)e^{\frac{1}{4}}}{2\,(dh^2)^{\frac{1}{4}}}\right)e^{(-2)}}{dh^2}+\frac{2\left(\sqrt{2}\,(dh^2)^{\frac{1}{4}}bdghpe^{\frac{7}{4}}+\sqrt{2}\,(dh^2)^{\frac{3}{4}}bfpe^{\frac{9}{4}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(dh^2)^{\frac{1}{4}}e^{\left(-\frac{1}{4}\right)}-2\sqrt{hx}\right)e^{\frac{1}{4}}}{2\,(dh^2)^{\frac{1}{4}}}\right)e^{(-2)}}{dh^2}+\frac{\left(\sqrt{2}\,(dh^2)^{\frac{1}{4}}bdghpe^{\frac{7}{4}}-\sqrt{2}\,(dh^2)^{\frac{3}{4}}bfpe^{\frac{9}{4}}\right)e^{(-2)}\log\left(\sqrt{2}\,(dh^2)^{\frac{1}{4}}\sqrt{hx}\,e^{\left(-\frac{1}{4}\right)}+hx+\sqrt{dh^2}\,e^{\left(-\frac{1}{2}\right)}\right)}{dh^2}-\frac{\left(\sqrt{2}\,(dh^2)^{\frac{1}{4}}bdghpe^{\frac{7}{4}}-\sqrt{2}\,(dh^2)^{\frac{3}{4}}bfpe^{\frac{9}{4}}\right)e^{(-2)}\log\left(-\sqrt{2}\,(dh^2)^{\frac{1}{4}}\sqrt{hx}\,e^{\left(-\frac{1}{4}\right)}+hx+\sqrt{dh^2}\,e^{\left(-\frac{1}{2}\right)}\right)}{dh^2}+\frac{2\,(bghpx\log\left(h^2x^2e+dh^2\right)-bghpx\log\left(h^2\right)-4\,bghpx-bfhp\log\left(h^2x^2e+dh^2\right)+bfhp\log\left(h^2\right)+bghx\log\left(c\right)+aghx-bfh\log\left(c\right)-afh)}{\sqrt{hx}\,h}}{h}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2),x, algorithm="giac")

[Out] (2*(sqrt(2)*(d*h^2)^(1/4)*b*d*g*h*p*e^(7/4) + sqrt(2)*(d*h^2)^(3/4)*b*f*p*e^(9/4))*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-2)/(d*h^2) + 2*(sqrt(2)*(d*h^2)^(1/4)*b*d*g*h*p*e^(7/4) + sqrt(2)*(d*h^2)^(3/4)*b*f*p*e^(9/4))*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) - 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-2)/(d*h^2) + (sqrt(2)*(d*h^2)^(1/4)*b*d*g*h*p*e^(7/4) - sqrt(2)*(d*h^2)^(3/4)*b*f*p*e^(9/4))*e^(-2)*log(sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2))/(d*h^2) - (sqrt(2)*(d*h^2)^(1/4)*b*d*g*h*p*e^(7/4) - sqrt(2)*(d*h^2)^(3/4)*b*f*p*e^(9/4))*e^(-2)*log(-sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2))/(d*h^2) + 2*(b*g*h*p*x*log(h^2*x^2*e + d*h^2) - b*g*h*p*x*log(h^2) - 4*b*g*h*p*x - b*f*h*p*log(h^2*x^2*e + d*h^2) + b*f*h*p*log(h^2) + b*g*h*x*log(c) + a*g*h*x - b*f*h*log(c) - a*f*h)/(sqrt(h*x)*h))/h

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{{(h\,x)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(3/2),x)

[Out] int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(3/2), x)

3.608 $\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{5/2}}\,dx$

Optimal. Leaf size=588

$$-\frac{2\sqrt{2}\,be^{3/4}fp\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{3d^{3/4}h^{5/2}}-\frac{2\sqrt{2}\,b\sqrt[4]{e}\,gp\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{\sqrt[4]{d}\,h^{5/2}}+\frac{2\sqrt{2}\,be^{3/4}fp\tan^{-1}}{3d}$$

```
[Out] -2/3*f*(a+b*ln(c*(e*x^2+d)^p))/h/(h*x)^(3/2)-2/3*b*e^(3/4)*f*p*arctan(1-e^(
1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(3/4)/h^(5/2)-2*b*e^(1/
4)*g*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(1/4
)/h^(5/2)+2/3*b*e^(3/4)*f*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^
(1/2))*2^(1/2)/d^(3/4)/h^(5/2)+2*b*e^(1/4)*g*p*arctan(1+e^(1/4)*2^(1/2)*(h*
x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(1/4)/h^(5/2)-1/3*b*e^(3/4)*f*p*ln(d^(1
/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/
d^(3/4)/h^(5/2)+b*e^(1/4)*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*
e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(1/4)/h^(5/2)+1/3*b*e^(3/4)*f*p*ln(d
^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/
2)/d^(3/4)/h^(5/2)-b*e^(1/4)*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/
4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(1/4)/h^(5/2)-2*g*(a+b*ln(c*(e*x^
2+d)^p))/h^2/(h*x)^(1/2)
```

Rubi [A]
time = 0.48, antiderivative size = 588, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2517, 2526, 2505, 217, 1179, 642, 1176, 631, 210, 303}

$$-\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h(hx)^{3/2}}-\frac{2g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{h^2\sqrt{hx}}-\frac{2\sqrt{2}\,be^{3/4}fp\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{3d^{3/4}h^{5/2}}+\frac{2\sqrt{2}\,be^{3/4}fp\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}+1\right)}{3d^{3/4}h^{5/2}}-\frac{2\sqrt{2}\,b\sqrt[4]{e}\,gp\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{\sqrt[4]{d}\,h^{5/2}}+\frac{2\sqrt{2}\,b\sqrt[4]{e}\,gp\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}+1\right)}{\sqrt[4]{d}\,h^{5/2}}-\frac{\sqrt{2}\,be^{3/4}fp\log\left(-\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{hx}+\sqrt{d}\,\sqrt{h}+\sqrt{e}\,\sqrt{h}\,x\right)}{3d^{3/4}h^{5/2}}+\frac{\sqrt{2}\,be^{3/4}fp\log\left(\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{hx}+\sqrt{d}\,\sqrt{h}+\sqrt{e}\,\sqrt{h}\,x\right)}{3d^{3/4}h^{5/2}}+\frac{\sqrt{2}\,b\sqrt[4]{e}\,gp\log\left(-\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{hx}+\sqrt{d}\,\sqrt{h}+\sqrt{e}\,\sqrt{h}\,x\right)}{\sqrt[4]{d}\,h^{5/2}}-\frac{\sqrt{2}\,b\sqrt[4]{e}\,gp\log\left(\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{hx}+\sqrt{d}\,\sqrt{h}+\sqrt{e}\,\sqrt{h}\,x\right)}{\sqrt[4]{d}\,h^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[2]*b*e^(3/4)*f*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*S
qrt[h])])/(3*d^(3/4)*h^(5/2)) - (2*Sqrt[2]*b*e^(1/4)*g*p*ArcTan[1 - (Sqrt[2
]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(5/2)) + (2*Sqrt[2]*b*e
^(3/4)*f*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*d^
(3/4)*h^(5/2)) + (2*Sqrt[2]*b*e^(1/4)*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[
h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(5/2)) - (2*f*(a + b*Log[c*(d + e*x^2)
^p]))/(3*h*(h*x)^(3/2)) - (2*g*(a + b*Log[c*(d + e*x^2)^p]))/(h^2*Sqrt[h*x]
) - (Sqrt[2]*b*e^(3/4)*f*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2
]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(5/2)) + (Sqrt[2]*b*e^(1/4)*g*p*
Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]
])/(d^(1/4)*h^(5/2)) + (Sqrt[2]*b*e^(3/4)*f*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]
```

```
*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(5/2)) - (Sqr
t[2]*b*e^(1/4)*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4
)*e^(1/4)*Sqrt[h*x]])/(d^(1/4)*h^(5/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2517

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*(
d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{\left(f+\frac{gx^2}{h}\right)\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^4}\,dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{f\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^4} + \frac{g\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{hx^2}\right)\,dx, x, \sqrt{\cdots}}{h}$$

$$= \frac{(2g)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^2}\,dx, x, \sqrt{hx}\right)}{h^2} + \frac{(2f)\text{Subst}\left(\int \frac{a+b\cdots}{\cdots}\right.}{\cdots}$$

$$= -\frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h(hx)^{3/2}} - \frac{2g\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h^2\sqrt{hx}} + \frac{(8\cdots}{\cdots}$$

$$= -\frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h(hx)^{3/2}} - \frac{2g\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h^2\sqrt{hx}} + \frac{(4\cdots}{\cdots}$$

$$= -\frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h(hx)^{3/2}} - \frac{2g\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h^2\sqrt{hx}} - \frac{\left(\cdots\right.}{\cdots}$$

$$= -\frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h(hx)^{3/2}} - \frac{2g\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h^2\sqrt{hx}} - \frac{\sqrt{\cdots}}{\cdots}$$

$$= -\frac{2\sqrt{2}\,be^{3/4}fp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{5/2}} - \frac{2\sqrt{2}\,b\sqrt[4]{e}\,gp\tan^{-1}\cdots}{\sqrt[4]{\cdots}}$$

Mathematica [A]
time = 0.28, size = 271, normalized size = 0.46

$$\frac{2x^{5/2}\left(\frac{2b\sqrt[4]{e}\,gp\left(\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{-d}}\right)+\tanh^{-1}\left(\frac{d\sqrt[4]{e}\sqrt{x}}{(-d)^{5/4}}\right)\right)}{\sqrt[4]{-d}} - \frac{be^{3/4}fp\left(2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)-2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)+\log\left(\sqrt{d}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\sqrt{e}\,x\right)-\log\left(\sqrt{d}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\sqrt{e}\,x\right)\right)}{3\sqrt{2}\,d^{3/4}} - \frac{f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3x^{3/2}} - \frac{g\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{x}}\right)}{(hx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(5/2),x]
```

```
[Out] (2*x^(5/2)*((2*b*e^(1/4)*g*p*(ArcTan[(e^(1/4)*Sqrt[x])/(-d)^(1/4)] + ArcTan
h[(d*e^(1/4)*Sqrt[x])/(-d)^(5/4)]))/(-d)^(1/4) - (b*e^(3/4)*f*p*(2*ArcTan[1
 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[
```

```
x])/d^(1/4)] + Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x] -
 Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x]))/(3*Sqrt[2]*d^
(3/4)) - (f*(a + b*Log[c*(d + e*x^2)^p]))/(3*x^(3/2)) - (g*(a + b*Log[c*(d
+ e*x^2)^p]))/Sqrt[x]))/(h*x)^(5/2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)\left(a+b\ln\left(c(e\,x^2+d)^p\right)\right)}{(hx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(5/2),x)
```

```
[Out] int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(5/2),x)
```

Maxima [A]
time = 0.52, size = 495, normalized size = 0.84

$$-\frac{\left(\frac{\sqrt{2}e^{\left(-\frac{3}{4}\right)}\log\left(hxe^{\frac{1}{2}}+\sqrt{2}(dh^2)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{(dh^2)^{\frac{1}{4}}}-\frac{\sqrt{2}e^{\left(-\frac{3}{4}\right)}\log\left(hxe^{\frac{1}{2}}-\sqrt{2}(dh^2)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{(dh^2)^{\frac{1}{4}}}-\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(dh^2)^{\frac{1}{4}}e^{\frac{1}{4}}+2\sqrt{hx}e^{\frac{1}{2}}\right)e^{\left(-\frac{1}{4}\right)}}{2\sqrt{\sqrt{d}h}}\right)e^{\left(-\frac{3}{4}\right)}}{\sqrt{\sqrt{d}h}}-\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(dh^2)^{\frac{1}{4}}e^{\frac{1}{4}}-2\sqrt{hx}e^{\frac{1}{2}}\right)e^{\left(-\frac{1}{4}\right)}}{2\sqrt{\sqrt{d}h}}\right)e^{\left(-\frac{3}{4}\right)}}{\sqrt{\sqrt{d}h}}\right)bgpe}{h^2}-\frac{2bgx^2\log\left((x^2e+d)^pc\right)}{(hx)^{\frac{5}{2}}}+\frac{\left(\frac{\sqrt{2}h^2e^{\left(-\frac{1}{4}\right)}\log\left(hxe^{\frac{1}{2}}+\sqrt{2}(dh^2)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{(dh^2)^{\frac{3}{4}}}-\frac{\sqrt{2}h^2e^{\left(-\frac{1}{4}\right)}\log\left(hxe^{\frac{1}{2}}-\sqrt{2}(dh^2)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{(dh^2)^{\frac{3}{4}}}+\frac{2\sqrt{2}h\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(dh^2)^{\frac{1}{4}}e^{\frac{1}{4}}+2\sqrt{hx}e^{\frac{1}{2}}\right)e^{\left(-\frac{1}{4}\right)}}{2\sqrt{\sqrt{d}h}}\right)e^{\left(-\frac{1}{4}\right)}}{\sqrt{\sqrt{d}h}\sqrt{d}}+\frac{2\sqrt{2}h\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(dh^2)^{\frac{1}{4}}e^{\frac{1}{4}}-2\sqrt{hx}e^{\frac{1}{2}}\right)e^{\left(-\frac{1}{4}\right)}}{2\sqrt{\sqrt{d}h}}\right)e^{\left(-\frac{1}{4}\right)}}{\sqrt{\sqrt{d}h}\sqrt{d}}\right)bfpe}{3h^3}-\frac{2agx^2}{(hx)^{\frac{5}{2}}}-\frac{2bf\log\left((x^2e+d)^pc\right)}{3(hx)^{\frac{3}{2}}h}-\frac{2af}{3(hx)^{\frac{3}{2}}h}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(5/2),x, algorithm="maxima
")
```

```
[Out] -(sqrt(2)*e^(-3/4)*log(h*x*e^(1/2) + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4
) + sqrt(d)*h)/(d*h^2)^(1/4) - sqrt(2)*e^(-3/4)*log(h*x*e^(1/2) - sqrt(2)*(
d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(1/4) - 2*sqrt(2)*arcta
n(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*e^(1/2))*e^(-1/4
)/sqrt(sqrt(d)*h))*e^(-3/4)/sqrt(sqrt(d)*h) - 2*sqrt(2)*arctan(-1/2*sqrt(2)
*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*e^(1/2))*e^(-1/4)/sqrt(sqrt(d
)*h))*e^(-3/4)/sqrt(sqrt(d)*h))*b*g*p*e/h^2 - 2*b*g*x^2*log((x^2*e + d)^p*c
)/(h*x)^(5/2) + 1/3*(sqrt(2)*h^2*e^(-1/4)*log(h*x*e^(1/2) + sqrt(2)*(d*h^2)
^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(3/4) - sqrt(2)*h^2*e^(-1/4)*
log(h*x*e^(1/2) - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h
^2)^(3/4) + 2*sqrt(2)*h*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) +
 2*sqrt(h*x)*e^(1/2))*e^(-1/4)/sqrt(sqrt(d)*h))*e^(-1/4)/(sqrt(sqrt(d)*h)*s
qrt(d)) + 2*sqrt(2)*h*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) -
2*sqrt(h*x)*e^(1/2))*e^(-1/4)/sqrt(sqrt(d)*h))*e^(-1/4)/(sqrt(sqrt(d)*h)*sq
rt(d)))*b*f*p*e/h^3 - 2*a*g*x^2/(h*x)^(5/2) - 2/3*b*f*log((x^2*e + d)^p*c)/
((h*x)^(3/2)*h) - 2/3*a*f/((h*x)^(3/2)*h)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1339 vs. $2(388) = 776$.
time = 0.39, size = 1339, normalized size = 2.28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(5/2),x, algorithm="fricas
")
```

```
[Out] -2/3*(h^3*x^2*sqrt(-(6*b^2*f*g*p^2*e + d*h^5*sqrt(-(81*b^4*d^2*g^4*p^4*e -
18*b^4*d*f^2*g^2*p^4*e^2 + b^4*f^4*p^4*e^3)/(d^3*h^10)))/(d*h^5))*log(-32*(
81*b^3*d^2*g^4*p^3*e - b^3*f^4*p^3*e^3)*sqrt(h*x) + 32*(9*b^2*d^2*f*g^2*h^3
*p^2*e - 3*d^3*g*h^8*sqrt(-(81*b^4*d^2*g^4*p^4*e - 18*b^4*d*f^2*g^2*p^4*e^2
 + b^4*f^4*p^4*e^3)/(d^3*h^10)) - b^2*d*f^3*h^3*p^2*e^2)*sqrt(-(6*b^2*f*g*p
^2*e + d*h^5*sqrt(-(81*b^4*d^2*g^4*p^4*e - 18*b^4*d*f^2*g^2*p^4*e^2 + b^4*f
^4*p^4*e^3)/(d^3*h^10)))/(d*h^5))) - h^3*x^2*sqrt(-(6*b^2*f*g*p^2*e + d*h^5
*sqrt(-(81*b^4*d^2*g^4*p^4*e - 18*b^4*d*f^2*g^2*p^4*e^2 + b^4*f^4*p^4*e^3)/
(d^3*h^10)))/(d*h^5))*log(-32*(81*b^3*d^2*g^4*p^3*e - b^3*f^4*p^3*e^3)*sqrt
(h*x) - 32*(9*b^2*d^2*f*g^2*h^3*p^2*e - 3*d^3*g*h^8*sqrt(-(81*b^4*d^2*g^4*p
^4*e - 18*b^4*d*f^2*g^2*p^4*e^2 + b^4*f^4*p^4*e^3)/(d^3*h^10)) - b^2*d*f^3*
h^3*p^2*e^2)*sqrt(-(6*b^2*f*g*p^2*e + d*h^5*sqrt(-(81*b^4*d^2*g^4*p^4*e - 1
8*b^4*d*f^2*g^2*p^4*e^2 + b^4*f^4*p^4*e^3)/(d^3*h^10)))/(d*h^5))) + h^3*x^2
*sqrt(-(6*b^2*f*g*p^2*e - d*h^5*sqrt(-(81*b^4*d^2*g^4*p^4*e - 18*b^4*d*f^2*
g^2*p^4*e^2 + b^4*f^4*p^4*e^3)/(d^3*h^10)))/(d*h^5))*log(-32*(81*b^3*d^2*g^
4*p^3*e - b^3*f^4*p^3*e^3)*sqrt(h*x) + 32*(9*b^2*d^2*f*g^2*h^3*p^2*e + 3*d^
3*g*h^8*sqrt(-(81*b^4*d^2*g^4*p^4*e - 18*b^4*d*f^2*g^2*p^4*e^2 + b^4*f^4*p^
4*e^3)/(d^3*h^10)) - b^2*d*f^3*h^3*p^2*e^2)*sqrt(-(6*b^2*f*g*p^2*e - d*h^5*
sqrt(-(81*b^4*d^2*g^4*p^4*e - 18*b^4*d*f^2*g^2*p^4*e^2 + b^4*f^4*p^4*e^3)/(
d^3*h^10)))/(d*h^5))) - h^3*x^2*sqrt(-(6*b^2*f*g*p^2*e - d*h^5*sqrt(-(81*b^
4*d^2*g^4*p^4*e - 18*b^4*d*f^2*g^2*p^4*e^2 + b^4*f^4*p^4*e^3)/(d^3*h^10)))/
(d*h^5))*log(-32*(81*b^3*d^2*g^4*p^3*e - b^3*f^4*p^3*e^3)*sqrt(h*x) - 32*(9
*b^2*d^2*f*g^2*h^3*p^2*e + 3*d^3*g*h^8*sqrt(-(81*b^4*d^2*g^4*p^4*e - 18*b^4
*d*f^2*g^2*p^4*e^2 + b^4*f^4*p^4*e^3)/(d^3*h^10)) - b^2*d*f^3*h^3*p^2*e^2)*
sqrt(-(6*b^2*f*g*p^2*e - d*h^5*sqrt(-(81*b^4*d^2*g^4*p^4*e - 18*b^4*d*f^2*g
^2*p^4*e^2 + b^4*f^4*p^4*e^3)/(d^3*h^10)))/(d*h^5))) + (3*a*g*x + a*f + (3*
b*g*p*x + b*f*p)*log(x^2*e + d) + (3*b*g*x + b*f)*log(c))*sqrt(h*x))/(h^3*x
^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(5/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Giac [A]
time = 4.40, size = 443, normalized size = 0.75

$$\frac{\frac{2\left(\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}bfhpe^{\frac{11}{4}}+3\sqrt{2}\left(dh^{2}\right)^{\frac{3}{4}}bgpe^{\frac{9}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}e^{\left(-\frac{1}{4}\right)}+2\sqrt{hx}\right)e^{\frac{1}{4}}}{2\left(dh^{2}\right)^{\frac{1}{4}}}\right)e^{\left(-2\right)}}{dh}+\frac{2\left(\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}bfhpe^{\frac{11}{4}}+3\sqrt{2}\left(dh^{2}\right)^{\frac{3}{4}}bgpe^{\frac{9}{4}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}e^{\left(-\frac{1}{4}\right)}-2\sqrt{hx}\right)e^{\frac{1}{4}}}{2\left(dh^{2}\right)^{\frac{1}{4}}}\right)e^{\left(-2\right)}}{dh}+\frac{\left(\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}bfhpe^{\frac{11}{4}}-3\sqrt{2}\left(dh^{2}\right)^{\frac{3}{4}}bgpe^{\frac{9}{4}}\right)e^{\left(-2\right)}\log\left(\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}\sqrt{hx}e^{\left(-\frac{1}{4}\right)}+hx+\sqrt{dh^{2}}e^{\left(-\frac{1}{2}\right)}\right)}{dh}-\frac{\left(\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}bfhpe^{\frac{11}{4}}-3\sqrt{2}\left(dh^{2}\right)^{\frac{3}{4}}bgpe^{\frac{9}{4}}\right)e^{\left(-2\right)}\log\left(-\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}\sqrt{hx}e^{\left(-\frac{1}{4}\right)}+hx+\sqrt{dh^{2}}e^{\left(-\frac{1}{2}\right)}\right)}{dh}-\frac{2\left(3bgh^{2}px\log\left(h^{2}x^{2}e+dh^{2}\right)-3bgh^{2}px\log\left(h^{2}\right)+bfh^{2}p\log\left(h^{2}x^{2}e+dh^{2}\right)-bfh^{2}p\log\left(h^{2}\right)+3bgh^{2}x\log\left(c\right)+3agh^{2}x+bfh^{2}\log\left(c\right)+afh^{2}\right)}{\sqrt{hx}hx}}{3h^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(5/2),x, algorithm="giac")

[Out] 1/3*(2*(sqrt(2)*(d*h^2)^(1/4)*b*f*h*p*e^(11/4) + 3*sqrt(2)*(d*h^2)^(3/4)*b*g*p*e^(9/4))*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-2)/(d*h) + 2*(sqrt(2)*(d*h^2)^(1/4)*b*f*h*p*e^(11/4) + 3*sqrt(2)*(d*h^2)^(3/4)*b*g*p*e^(9/4))*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) - 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-2)/(d*h) + (sqrt(2)*(d*h^2)^(1/4)*b*f*h*p*e^(11/4) - 3*sqrt(2)*(d*h^2)^(3/4)*b*g*p*e^(9/4))*e^(-2)*log(sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2))/(d*h) - (sqrt(2)*(d*h^2)^(1/4)*b*f*h*p*e^(11/4) - 3*sqrt(2)*(d*h^2)^(3/4)*b*g*p*e^(9/4))*e^(-2)*log(-sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2))/(d*h) - 2*(3*b*g*h^2*p*x*log(h^2*x^2*e + d*h^2) - 3*b*g*h^2*p*x*log(h^2) + b*f*h^2*p*log(h^2*x^2*e + d*h^2) - b*f*h^2*p*log(h^2) + 3*b*g*h^2*x*log(c) + 3*a*g*h^2*x + b*f*h^2*log(c) + a*f*h^2)/(sqrt(h*x)*h*x))/h^3

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{{\left(h\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(5/2),x)

[Out] int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(5/2), x)

3.609 $\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{7/2}}\,dx$

Optimal. Leaf size=620

$$-\frac{8befp}{5dh^3\sqrt{hx}}+\frac{2\sqrt{2}\,be^{5/4}fp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{7/2}}-\frac{2\sqrt{2}\,be^{3/4}gp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{7/2}}-\frac{2\sqrt{2}\,be}{}$$

```
[Out] -2/5*f*(a+b*ln(c*(e*x^2+d)^p))/h/(h*x)^(5/2)-2/3*g*(a+b*ln(c*(e*x^2+d)^p))/
h^2/(h*x)^(3/2)+2/5*b*e^(5/4)*f*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1
/4)/h^(1/2))*2^(1/2)/d^(5/4)/h^(7/2)-2/3*b*e^(3/4)*g*p*arctan(1-e^(1/4)*2^(
1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(3/4)/h^(7/2)-2/5*b*e^(5/4)*f*p
*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(5/4)/h^(7
/2)+2/3*b*e^(3/4)*g*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))
*2^(1/2)/d^(3/4)/h^(7/2)-1/5*b*e^(5/4)*f*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(
1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(5/4)/h^(7/2)-1/3*b*e^(
3/4)*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)
^(1/2))*2^(1/2)/d^(3/4)/h^(7/2)+1/5*b*e^(5/4)*f*p*ln(d^(1/2)*h^(1/2)+x*e^(1
/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(5/4)/h^(7/2)+1/
3*b*e^(3/4)*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2
)*(h*x)^(1/2))*2^(1/2)/d^(3/4)/h^(7/2)-8/5*b*e*f*p/d/h^3/(h*x)^(1/2)
```

Rubi [A]
time = 0.52, antiderivative size = 620, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {2517, 2526, 2505, 331, 303, 1176, 631, 210, 1179, 642, 217}

$$-\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5h(hx)^{5/2}}-\frac{2g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h^2(hx)^{3/2}}+\frac{2\sqrt{2}\,be^{5/4}fp\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{7/2}}-\frac{2\sqrt{2}\,be^{5/4}fp\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)}{5d^{5/4}h^{7/2}}-\frac{2\sqrt{2}\,be^{3/4}gp\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{7/2}}+\frac{2\sqrt{2}\,be^{3/4}gp\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)}{3d^{3/4}h^{7/2}}-\frac{\sqrt{2}\,be^{5/4}fp\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x\right)}{5d^{5/4}h^{7/2}}+\frac{\sqrt{2}\,be^{5/4}fp\log\left(\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x\right)}{5d^{5/4}h^{7/2}}-\frac{\sqrt{2}\,be^{3/4}gp\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x\right)}{3d^{3/4}h^{7/2}}+\frac{\sqrt{2}\,be^{3/4}gp\log\left(\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x\right)}{3d^{3/4}h^{7/2}}-\frac{8befp}{5dh^3\sqrt{hx}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(7/2),x]
```

```
[Out] (-8*b*e*f*p)/(5*d*h^3*Sqrt[h*x]) + (2*Sqrt[2]*b*e^(5/4)*f*p*ArcTan[1 - (Sqr
t[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(7/2)) - (2*Sqrt[2
]*b*e^(3/4)*g*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/
(3*d^(3/4)*h^(7/2)) - (2*Sqrt[2]*b*e^(5/4)*f*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*
Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(7/2)) + (2*Sqrt[2]*b*e^(3/4)*g
*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*d^(3/4)*h^
(7/2)) - (2*f*(a + b*Log[c*(d + e*x^2)^p]))/(5*h*(h*x)^(5/2)) - (2*g*(a + b
*Log[c*(d + e*x^2)^p]))/(3*h^2*(h*x)^(3/2)) - (Sqrt[2]*b*e^(5/4)*f*p*Log[Sq
rt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(5*
d^(5/4)*h^(7/2)) - (Sqrt[2]*b*e^(3/4)*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqr
t[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(7/2)) + (Sqrt[2]
```

```
*b*e^(5/4)*f*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^
(1/4)*Sqrt[h*x]])/(5*d^(5/4)*h^(7/2)) + (Sqrt[2]*b*e^(3/4)*g*p*Log[Sqrt[d]*
Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4
)*h^(7/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2517

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*(
d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{\left(f+\frac{gx^2}{h}\right)\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^6}\,dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{f\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^6} + \frac{g\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{hx^4}\right) dx, x,}{h}$$

$$= \frac{(2g)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^4}\,dx, x, \sqrt{hx}\right)}{h^2} + \frac{(2f)\text{Subst}\left(\int \frac{a+}{}\right.}{}$$

$$= -\frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h(hx)^{5/2}} - \frac{2g\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2(hx)^{3/2}} + \cdots$$

$$= -\frac{8befp}{5dh^3\sqrt{hx}} - \frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h(hx)^{5/2}} - \frac{2g\left(a+b\log\left(c(d\right.\right.}{3h^2(hx)^{3}}$$

$$= -\frac{8befp}{5dh^3\sqrt{hx}} - \frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h(hx)^{5/2}} - \frac{2g\left(a+b\log\left(c(d\right.\right.}{3h^2(hx)^{3}}$$

$$= -\frac{8befp}{5dh^3\sqrt{hx}} - \frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h(hx)^{5/2}} - \frac{2g\left(a+b\log\left(c(d\right.\right.}{3h^2(hx)^{3}}$$

$$= -\frac{8befp}{5dh^3\sqrt{hx}} - \frac{2\sqrt{2}\,be^{3/4}gp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{7/2}} + \frac{2\sqrt{2}}{}$$

$$= -\frac{8befp}{5dh^3\sqrt{hx}} + \frac{2\sqrt{2}\,be^{5/4}fp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{7/2}} - \frac{2\sqrt{2}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.12, size = 309, normalized size = 0.50

$$\frac{2x^{7/2}\left(-\frac{4befp\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\frac{ex^2}{d}\right)}{5d\sqrt{x}} - \frac{1}{6}bgp\left(\frac{2\left(\frac{\sqrt{2}\,e^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)}{\sqrt[4]{d}} - \frac{\sqrt{2}\,e^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)}{\sqrt[4]{d}}\right)}{\sqrt{d}} + \frac{\frac{\sqrt{2}\,e^{3/4}\log\left(\sqrt{d}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\sqrt{e}\,x\right)}{\sqrt[4]{d}} - \frac{\sqrt{2}\,e^{3/4}\log\left(\sqrt{d}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\sqrt{e}\,x\right)}{\sqrt[4]{d}}}{\sqrt{d}}\right) - \frac{f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5x^{5/2}} - \frac{g\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3x^{3/2}}\right)}{(hx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(7/2),x]
```

```
[Out] (2*x^(7/2)*((-4*b*e*f*p*Hypergeometric2F1[-1/4, 1, 3/4, -((e*x^2)/d)])/(5*d
*Sqrt[x]) - (b*g*p*((2*((Sqrt[2]*e^(3/4)*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x
])/d^(1/4)])/d^(1/4) - (Sqrt[2]*e^(3/4)*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x]
)/d^(1/4)])/d^(1/4)))/Sqrt[d] + ((Sqrt[2]*e^(3/4)*Log[Sqrt[d] - Sqrt[2]*d^(
1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x])/d^(1/4) - (Sqrt[2]*e^(3/4)*Log[Sqrt[d] +
 Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x])/d^(1/4))/Sqrt[d]))/6 - (f*(a
 + b*Log[c*(d + e*x^2)^p]))/(5*x^(5/2)) - (g*(a + b*Log[c*(d + e*x^2)^p]))/
(3*x^(3/2))))/(h*x)^(7/2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)\left(a+b\ln\left(c(e\,x^2+d)^p\right)\right)}{(hx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(7/2),x)
```

```
[Out] int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(7/2),x)
```

Maxima [A]
time = 0.53, size = 512, normalized size = 0.83

$$\frac{bfp\left(\frac{\left(\frac{\sqrt{2}e^{\left(-\frac{3}{4}\right)}\log\left(hxe^{\frac{1}{2}}+\sqrt{2}(dh^2)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{(dh^2)^{\frac{1}{4}}}-\frac{\sqrt{2}e^{\left(-\frac{3}{4}\right)}\log\left(hxe^{\frac{1}{2}}-\sqrt{2}(dh^2)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{(dh^2)^{\frac{1}{4}}}-\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(dh^2)^{\frac{1}{4}}e^{\frac{1}{4}}+2\sqrt{hx}e^{\frac{1}{2}}\right)e^{\left(-\frac{1}{4}\right)}}{2\sqrt{\sqrt{d}h}}\right)e^{\left(-\frac{3}{4}\right)}}{\sqrt{\sqrt{d}h}}-\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(dh^2)^{\frac{1}{4}}e^{\frac{1}{4}}-2\sqrt{hx}e^{\frac{1}{2}}\right)e^{\left(-\frac{1}{4}\right)}}{2\sqrt{\sqrt{d}h}}\right)e^{\left(-\frac{3}{4}\right)}}{\sqrt{\sqrt{d}h}}\right)e}{d}-\frac{8}{\sqrt{hx}d}\right)e}{5h^3}-\frac{2bgx^2\log\left((x^2e+d)^pc\right)}{3(hx)^{\frac{7}{2}}}+\frac{\left(\frac{\sqrt{2}h^2e^{\left(-\frac{1}{4}\right)}\log\left(hxe^{\frac{1}{2}}+\sqrt{2}(dh^2)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{(dh^2)^{\frac{3}{4}}}-\frac{\sqrt{2}h^2e^{\left(-\frac{1}{4}\right)}\log\left(hxe^{\frac{1}{2}}-\sqrt{2}(dh^2)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{(dh^2)^{\frac{3}{4}}}+\frac{2\sqrt{2}h\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(dh^2)^{\frac{1}{4}}e^{\frac{1}{4}}+2\sqrt{hx}e^{\frac{1}{2}}\right)e^{\left(-\frac{1}{4}\right)}}{2\sqrt{\sqrt{d}h}}\right)e^{\left(-\frac{1}{4}\right)}}{\sqrt{\sqrt{d}h}\sqrt{d}}+\frac{2\sqrt{2}h\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(dh^2)^{\frac{1}{4}}e^{\frac{1}{4}}-2\sqrt{hx}e^{\frac{1}{2}}\right)e^{\left(-\frac{1}{4}\right)}}{2\sqrt{\sqrt{d}h}}\right)e^{\left(-\frac{1}{4}\right)}}{\sqrt{\sqrt{d}h}\sqrt{d}}\right)bgpe}{3h^4}-\frac{2agx^2}{3(hx)^{\frac{7}{2}}}-\frac{2bf\log\left((x^2e+d)^pc\right)}{5(hx)^{\frac{5}{2}}h}-\frac{2af}{5(hx)^{\frac{5}{2}}h}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(7/2),x, algorithm="maxima
")
```

```
[Out] 1/5*b*f*p*((sqrt(2)*e^(-3/4)*log(h*x*e^(1/2) + sqrt(2)*(d*h^2)^(1/4)*sqrt(h
*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(1/4) - sqrt(2)*e^(-3/4)*log(h*x*e^(1/2) -
 sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(1/4) - 2*sqr
t(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*e^(1/2
))*e^(-1/4)/sqrt(sqrt(d)*h))*e^(-3/4)/sqrt(sqrt(d)*h) - 2*sqrt(2)*arctan(-1
/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*e^(1/2))*e^(-1/4)/s
qrt(sqrt(d)*h))*e^(-3/4)/sqrt(sqrt(d)*h))*e/d - 8/(sqrt(h*x)*d))*e/h^3 - 2/
3*b*g*x^2*log((x^2*e + d)^p*c)/(h*x)^(7/2) + 1/3*(sqrt(2)*h^2*e^(-1/4)*log(
h*x*e^(1/2) + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^
(3/4) - sqrt(2)*h^2*e^(-1/4)*log(h*x*e^(1/2) - sqrt(2)*(d*h^2)^(1/4)*sqrt(h
*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(3/4) + 2*sqrt(2)*h*arctan(1/2*sqrt(2)*(sq
```

```
rt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*e^(1/2))*e^(-1/4)/sqrt(sqrt(d)*h)
)*e^(-1/4)/(sqrt(sqrt(d)*h)*sqrt(d)) + 2*sqrt(2)*h*arctan(-1/2*sqrt(2)*(sqr
t(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*e^(1/2))*e^(-1/4)/sqrt(sqrt(d)*h))
*e^(-1/4)/(sqrt(sqrt(d)*h)*sqrt(d)))*b*g*p*e/h^4 - 2/3*a*g*x^2/(h*x)^(7/2)
- 2/5*b*f*log((x^2*e + d)^p*c)/((h*x)^(5/2)*h) - 2/5*a*f/((h*x)^(5/2)*h)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1393 vs. $2(407) = 814$.
time = 0.45, size = 1393, normalized size = 2.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(7/2),x, algorithm="fricas
")
```

```
[Out] -2/15*(d*h^4*x^3*sqrt((d^2*h^7*sqrt(-(625*b^4*d^2*g^4*p^4*e^3 - 450*b^4*d*f
^2*g^2*p^4*e^4 + 81*b^4*f^4*p^4*e^5)/(d^5*h^14)) + 30*b^2*f*g*p^2*e^2)/(d^2
*h^7))*log(-32*(625*b^3*d^2*g^4*p^3*e^2 - 81*b^3*f^4*p^3*e^4)*sqrt(h*x) + 3
2*(3*d^4*f*h^11*sqrt(-(625*b^4*d^2*g^4*p^4*e^3 - 450*b^4*d*f^2*g^2*p^4*e^4
+ 81*b^4*f^4*p^4*e^5)/(d^5*h^14)) + 125*b^2*d^3*g^3*h^4*p^2*e - 45*b^2*d^2*
f^2*g*h^4*p^2*e^2)*sqrt((d^2*h^7*sqrt(-(625*b^4*d^2*g^4*p^4*e^3 - 450*b^4*d
*f^2*g^2*p^4*e^4 + 81*b^4*f^4*p^4*e^5)/(d^5*h^14)) + 30*b^2*f*g*p^2*e^2)/(d
^2*h^7))) - d*h^4*x^3*sqrt((d^2*h^7*sqrt(-(625*b^4*d^2*g^4*p^4*e^3 - 450*b^
4*d*f^2*g^2*p^4*e^4 + 81*b^4*f^4*p^4*e^5)/(d^5*h^14)) + 30*b^2*f*g*p^2*e^2)
/(d^2*h^7))*log(-32*(625*b^3*d^2*g^4*p^3*e^2 - 81*b^3*f^4*p^3*e^4)*sqrt(h*x
) - 32*(3*d^4*f*h^11*sqrt(-(625*b^4*d^2*g^4*p^4*e^3 - 450*b^4*d*f^2*g^2*p^4
*e^4 + 81*b^4*f^4*p^4*e^5)/(d^5*h^14)) + 125*b^2*d^3*g^3*h^4*p^2*e - 45*b^2
*d^2*f^2*g*h^4*p^2*e^2)*sqrt((d^2*h^7*sqrt(-(625*b^4*d^2*g^4*p^4*e^3 - 450*
b^4*d*f^2*g^2*p^4*e^4 + 81*b^4*f^4*p^4*e^5)/(d^5*h^14)) + 30*b^2*f*g*p^2*e^
2)/(d^2*h^7))) - d*h^4*x^3*sqrt(-(d^2*h^7*sqrt(-(625*b^4*d^2*g^4*p^4*e^3 -
450*b^4*d*f^2*g^2*p^4*e^4 + 81*b^4*f^4*p^4*e^5)/(d^5*h^14)) - 30*b^2*f*g*p^
2*e^2)/(d^2*h^7))*log(-32*(625*b^3*d^2*g^4*p^3*e^2 - 81*b^3*f^4*p^3*e^4)*sq
rt(h*x) + 32*(3*d^4*f*h^11*sqrt(-(625*b^4*d^2*g^4*p^4*e^3 - 450*b^4*d*f^2*g
^2*p^4*e^4 + 81*b^4*f^4*p^4*e^5)/(d^5*h^14)) - 125*b^2*d^3*g^3*h^4*p^2*e +
45*b^2*d^2*f^2*g*h^4*p^2*e^2)*sqrt(-(d^2*h^7*sqrt(-(625*b^4*d^2*g^4*p^4*e^3
 - 450*b^4*d*f^2*g^2*p^4*e^4 + 81*b^4*f^4*p^4*e^5)/(d^5*h^14)) - 30*b^2*f*g
*p^2*e^2)/(d^2*h^7))) + d*h^4*x^3*sqrt(-(d^2*h^7*sqrt(-(625*b^4*d^2*g^4*p^4
*e^3 - 450*b^4*d*f^2*g^2*p^4*e^4 + 81*b^4*f^4*p^4*e^5)/(d^5*h^14)) - 30*b^2
*f*g*p^2*e^2)/(d^2*h^7))*log(-32*(625*b^3*d^2*g^4*p^3*e^2 - 81*b^3*f^4*p^3*
e^4)*sqrt(h*x) - 32*(3*d^4*f*h^11*sqrt(-(625*b^4*d^2*g^4*p^4*e^3 - 450*b^4*
d*f^2*g^2*p^4*e^4 + 81*b^4*f^4*p^4*e^5)/(d^5*h^14)) - 125*b^2*d^3*g^3*h^4*p
^2*e + 45*b^2*d^2*f^2*g*h^4*p^2*e^2)*sqrt(-(d^2*h^7*sqrt(-(625*b^4*d^2*g^4*
p^4*e^3 - 450*b^4*d*f^2*g^2*p^4*e^4 + 81*b^4*f^4*p^4*e^5)/(d^5*h^14)) - 30*
b^2*f*g*p^2*e^2)/(d^2*h^7))) + (12*b*f*p*x^2*e + 5*a*d*g*x + 3*a*d*f + (5*b
```

```
*d*g*p*x + 3*b*d*f*p)*log(x^2*e + d) + (5*b*d*g*x + 3*b*d*f)*log(c))*sqrt(h
*x))/(d*h^4*x^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep
```

Giac [A]
time = 3.09, size = 478, normalized size = 0.77

$$\frac{\frac{2\left(5\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}bdghpe^{\frac{7}{4}}-3\sqrt{2}\left(dh^2\right)^{\frac{3}{4}}bfpe^{\frac{9}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{4}\right)}+2\sqrt{hx}\right)e^{\frac{1}{4}}}{2\left(dh^2\right)^{\frac{1}{4}}}\right)e^{(-1)}}{d^2h}+\frac{2\left(5\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}bdghpe^{\frac{7}{4}}-3\sqrt{2}\left(dh^2\right)^{\frac{3}{4}}bfpe^{\frac{9}{4}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}e^{\left(-\frac{1}{4}\right)}-2\sqrt{hx}\right)e^{\frac{1}{4}}}{2\left(dh^2\right)^{\frac{1}{4}}}\right)e^{(-1)}}{d^2h}+\frac{\left(5\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}bdghpe^{\frac{7}{4}}+3\sqrt{2}\left(dh^2\right)^{\frac{3}{4}}bfpe^{\frac{9}{4}}\right)e^{(-1)}\log\left(\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}\sqrt{hx}e^{\left(-\frac{1}{4}\right)}+hx+\sqrt{dh^2}e^{\left(-\frac{1}{2}\right)}\right)}{d^2h}-\frac{\left(5\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}bdghpe^{\frac{7}{4}}+3\sqrt{2}\left(dh^2\right)^{\frac{3}{4}}bfpe^{\frac{9}{4}}\right)e^{(-1)}\log\left(-\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}\sqrt{hx}e^{\left(-\frac{1}{4}\right)}+hx+\sqrt{dh^2}e^{\left(-\frac{1}{2}\right)}\right)}{d^2h}-\frac{2\left(12bfh^3px^2e+5bdgh^3px\log\left(h^2x^2e+dh^2\right)-5bdgh^3px\log\left(h^2\right)+3bdfh^3p\log\left(h^2x^2e+dh^2\right)-3bdfh^3p\log\left(h^2\right)+5bdgh^3x\log(c)+5adgh^3x+3bdfh^3\log(c)+3adfh^3\right)}{\sqrt{hx}dh^2x^2}}{15h^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(2*(5*sqrt(2)*(d*h^2)^(1/4)*b*d*g*h*p*e^(7/4) - 3*sqrt(2)*(d*h^2)^(3/4
)*b*f*p*e^(9/4))*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqr
t(h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-1)/(d^2*h) + 2*(5*sqrt(2)*(d*h^2)^(1/4)*
b*d*g*h*p*e^(7/4) - 3*sqrt(2)*(d*h^2)^(3/4)*b*f*p*e^(9/4))*arctan(-1/2*sqrt
(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) - 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/4))*e
^(-1)/(d^2*h) + (5*sqrt(2)*(d*h^2)^(1/4)*b*d*g*h*p*e^(7/4) + 3*sqrt(2)*(d*h
^2)^(3/4)*b*f*p*e^(9/4))*e^(-1)*log(sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4
) + h*x + sqrt(d*h^2)*e^(-1/2))/(d^2*h) - (5*sqrt(2)*(d*h^2)^(1/4)*b*d*g*h*
p*e^(7/4) + 3*sqrt(2)*(d*h^2)^(3/4)*b*f*p*e^(9/4))*e^(-1)*log(-sqrt(2)*(d*h
^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2))/(d^2*h) - 2*(12*
b*f*h^3*p*x^2*e + 5*b*d*g*h^3*p*x*log(h^2*x^2*e + d*h^2) - 5*b*d*g*h^3*p*x*
log(h^2) + 3*b*d*f*h^3*p*log(h^2*x^2*e + d*h^2) - 3*b*d*f*h^3*p*log(h^2) +
5*b*d*g*h^3*x*log(c) + 5*a*d*g*h^3*x + 3*b*d*f*h^3*log(c) + 3*a*d*f*h^3)/(s
qrt(h*x)*d*h^2*x^2))/h^4
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{{\left(h\,x\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(7/2),x)
```

```
[Out] int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(7/2), x)
```

3.610 $\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{9/2}}\,dx$

Optimal. Leaf size=641

$$-\frac{8befp}{21dh^3(hx)^{3/2}}-\frac{8begp}{5dh^4\sqrt{hx}}+\frac{2\sqrt{2}\,be^{7/4}fp\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{7d^{7/4}h^{9/2}}+\frac{2\sqrt{2}\,be^{5/4}gp\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\ \cdots}{\sqrt[4]{d}\ \cdots}\right.}{5d^{5/4}h^{9/2}}$$

```
[Out] -8/21*b*e*f*p/d/h^3/(h*x)^(3/2)-2/7*f*(a+b*ln(c*(e*x^2+d)^p))/h/(h*x)^(7/2)
-2/5*g*(a+b*ln(c*(e*x^2+d)^p))/h^2/(h*x)^(5/2)+2/7*b*e^(7/4)*f*p*arctan(1-e
^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(7/4)/h^(9/2)+2/5*b*e
^(5/4)*g*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^
(5/4)/h^(9/2)-2/7*b*e^(7/4)*f*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4
)/h^(1/2))*2^(1/2)/d^(7/4)/h^(9/2)-2/5*b*e^(5/4)*g*p*arctan(1+e^(1/4)*2^(1/
2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(5/4)/h^(9/2)+1/7*b*e^(7/4)*f*p*l
n(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^
(1/2)/d^(7/4)/h^(9/2)-1/5*b*e^(5/4)*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2
)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(5/4)/h^(9/2)-1/7*b*e^(7/4
)*f*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1
/2))*2^(1/2)/d^(7/4)/h^(9/2)+1/5*b*e^(5/4)*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)
*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(5/4)/h^(9/2)-8/5*b
*e*g*p/d/h^4/(h*x)^(1/2)
```

Rubi [A]

time = 0.56, antiderivative size = 641, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {2517, 2526, 2505, 331, 217, 1179, 642, 1176, 631, 210, 303}

$$-\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{7h(hx)^{7/2}}-\frac{2g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5h^2(hx)^{5/2}}+\frac{2\sqrt{2}\,be^{7/4}fp\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{7d^{7/4}h^{9/2}}-\frac{2\sqrt{2}\,be^{7/4}fp\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}+1\right)}{7d^{7/4}h^{9/2}}+\frac{2\sqrt{2}\,be^{5/4}gp\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{5d^{5/4}h^{9/2}}-\frac{2\sqrt{2}\,be^{5/4}gp\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}+1\right)}{5d^{5/4}h^{9/2}}+\frac{\sqrt{2}\,be^{7/4}fp\log\left(-\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{hx}+\sqrt{d}\,\sqrt{h}+\sqrt{e}\,\sqrt{h}\,x\right)}{7d^{7/4}h^{9/2}}-\frac{\sqrt{2}\,be^{7/4}fp\log\left(\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{hx}+\sqrt{d}\,\sqrt{h}+\sqrt{e}\,\sqrt{h}\,x\right)}{7d^{7/4}h^{9/2}}-\frac{\sqrt{2}\,be^{5/4}gp\log\left(-\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{hx}+\sqrt{d}\,\sqrt{h}+\sqrt{e}\,\sqrt{h}\,x\right)}{5d^{5/4}h^{9/2}}+\frac{\sqrt{2}\,be^{5/4}gp\log\left(\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{hx}+\sqrt{d}\,\sqrt{h}+\sqrt{e}\,\sqrt{h}\,x\right)}{5d^{5/4}h^{9/2}}-\frac{8befp}{21dh^3(hx)^{3/2}}-\frac{8begp}{5dh^4\sqrt{hx}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(9/2),x]
```

```
[Out] (-8*b*e*f*p)/(21*d*h^3*(h*x)^(3/2)) - (8*b*e*g*p)/(5*d*h^4*Sqrt[h*x]) + (2*
Sqrt[2]*b*e^(7/4)*f*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[
h])])/(7*d^(7/4)*h^(9/2)) + (2*Sqrt[2]*b*e^(5/4)*g*p*ArcTan[1 - (Sqrt[2]*e^
(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(9/2)) - (2*Sqrt[2]*b*e^(
7/4)*f*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(7*d^(7
/4)*h^(9/2)) - (2*Sqrt[2]*b*e^(5/4)*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*
x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(9/2)) - (2*f*(a + b*Log[c*(d + e*x^2)
^p]))/(7*h*(h*x)^(7/2)) - (2*g*(a + b*Log[c*(d + e*x^2)^p]))/(5*h^2*(h*x)^(
5/2)) + (Sqrt[2]*b*e^(7/4)*f*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sq
rt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(7*d^(7/4)*h^(9/2)) - (Sqrt[2]*b*e^(5/4)*
```

```
g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[
h*x]])/(5*d^(5/4)*h^(9/2)) - (Sqrt[2]*b*e^(7/4)*f*p*Log[Sqrt[d]*Sqrt[h] + S
qrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(7*d^(7/4)*h^(9/2))
+ (Sqrt[2]*b*e^(5/4)*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*
d^(1/4)*e^(1/4)*Sqrt[h*x]])/(5*d^(5/4)*h^(9/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2517

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*(
d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{9/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{\left(f+\frac{gx^2}{h}\right)\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^8}\,dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{f\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^8} + \frac{g\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{hx^6}\right) dx, x, \sqrt{}\right.}{h}$$

$$= \frac{(2g)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^6}\,dx, x, \sqrt{hx}\right)}{h^2} + \frac{(2f)\text{Subst}\left(\int \frac{a+b}{}\right.}{}$$

$$= -\frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{7h(hx)^{7/2}} - \frac{2g\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h^2(hx)^{5/2}} + \frac{(8}{}$$

$$= -\frac{8befp}{21dh^3(hx)^{3/2}} - \frac{8begp}{5dh^4\sqrt{hx}} - \frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{7h(hx)^{7/2}} - \frac{2g}{}$$

$$= -\frac{8befp}{21dh^3(hx)^{3/2}} - \frac{8begp}{5dh^4\sqrt{hx}} - \frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{7h(hx)^{7/2}} - \frac{2g}{}$$

$$= -\frac{8befp}{21dh^3(hx)^{3/2}} - \frac{8begp}{5dh^4\sqrt{hx}} - \frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{7h(hx)^{7/2}} - \frac{2g}{}$$

$$= -\frac{8befp}{21dh^3(hx)^{3/2}} - \frac{8begp}{5dh^4\sqrt{hx}} - \frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{7h(hx)^{7/2}} - \frac{2g}{}$$

$$= -\frac{8befp}{21dh^3(hx)^{3/2}} - \frac{8begp}{5dh^4\sqrt{hx}} + \frac{2\sqrt{2}\,be^{7/4}fp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}}{\sqrt[4]{d}\,\sqrt{}}\right.}{7d^{7/4}h^{9/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.05, size = 100, normalized size = 0.16

$$-\frac{2\sqrt{hx}\left(20befpx^2\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\frac{ex^2}{d}\right)+84begpx^3\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\frac{ex^2}{d}\right)+3d(5f+7gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)\right)}{105dh^5x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(9/2),x]
```

[Out] (-2*Sqrt[h*x]*(20*b*e*f*p*x^2*Hypergeometric2F1[-3/4, 1, 1/4, -((e*x^2)/d)] + 84*b*e*g*p*x^3*Hypergeometric2F1[-1/4, 1, 3/4, -((e*x^2)/d)] + 3*d*(5*f + 7*g*x)*(a + b*Log[c*(d + e*x^2)^p])))/(105*d*h^5*x^4)

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)\left(a+b\ln\left(c(e\,x^2+d)^p\right)\right)}{(hx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(9/2),x)

[Out] int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(9/2),x)

Maxima [A]
time = 0.51, size = 526, normalized size = 0.82

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(9/2),x, algorithm="maxima")

[Out] -1/21*b*f*p*(3*(sqrt(2)*e^(3/4)*log(h*x*e^(1/2) + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(3/4) - sqrt(2)*e^(3/4)*log(h*x*e^(1/2) - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(3/4) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*e^(1/2))*e^(-1/4)/sqrt(sqrt(d)*h))*e^(3/4)/(sqrt(sqrt(d)*h)*sqrt(d)*h) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*e^(1/2))*e^(-1/4)/sqrt(sqrt(d)*h))*e^(3/4)/(sqrt(sqrt(d)*h)*sqrt(d)*h))/d + 8/((h*x)^(3/2)*d))*e/h^3 + 1/5*b*g*p*((sqrt(2)*e^(-3/4)*log(h*x*e^(1/2) + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(1/4) - sqrt(2)*e^(-3/4)*log(h*x*e^(1/2) - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(1/4) - 2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*e^(1/2))*e^(-1/4)/sqrt(sqrt(d)*h))*e^(-3/4)/sqrt(sqrt(d)*h) - 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*e^(1/2))*e^(-1/4)/sqrt(sqrt(d)*h))*e^(-3/4)/sqrt(sqrt(d)*h))*e/d - 8/(sqrt(h*x)*d))*e/h^4 - 2/5*b*g*x^2*log((x^2*e + d)^p*c)/(h*x)^(9/2) - 2/5*a*g*x^2/(h*x)^(9/2) - 2/7*b*f*log((x^2*e + d)^p*c)/((h*x)^(7/2)*h) - 2/7*a*f/((h*x)^(7/2)*h)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1408 vs. 2(425) = 850.
time = 0.42, size = 1408, normalized size = 2.20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(9/2),x, algorithm="fricas
")
```

```
[Out] -2/105*(3*d*h^5*x^4*sqrt(-(d^3*h^9*sqrt(-(2401*b^4*d^2*g^4*p^4*e^5 - 2450*b
^4*d*f^2*g^2*p^4*e^6 + 625*b^4*f^4*p^4*e^7)/(d^7*h^18)) + 70*b^2*f*g*p^2*e^
3)/(d^3*h^9))*log(-32*(2401*b^3*d^2*g^4*p^3*e^4 - 625*b^3*f^4*p^3*e^6)*sqrt
(h*x) + 32*(7*d^6*g*h^14*sqrt(-(2401*b^4*d^2*g^4*p^4*e^5 - 2450*b^4*d*f^2*g
^2*p^4*e^6 + 625*b^4*f^4*p^4*e^7)/(d^7*h^18)) - 245*b^2*d^3*f*g^2*h^5*p^2*e
^3 + 125*b^2*d^2*f^3*h^5*p^2*e^4)*sqrt(-(d^3*h^9*sqrt(-(2401*b^4*d^2*g^4*p^
4*e^5 - 2450*b^4*d*f^2*g^2*p^4*e^6 + 625*b^4*f^4*p^4*e^7)/(d^7*h^18)) + 70*
b^2*f*g*p^2*e^3)/(d^3*h^9))) - 3*d*h^5*x^4*sqrt(-(d^3*h^9*sqrt(-(2401*b^4*d
^2*g^4*p^4*e^5 - 2450*b^4*d*f^2*g^2*p^4*e^6 + 625*b^4*f^4*p^4*e^7)/(d^7*h^1
8)) + 70*b^2*f*g*p^2*e^3)/(d^3*h^9))*log(-32*(2401*b^3*d^2*g^4*p^3*e^4 - 62
5*b^3*f^4*p^3*e^6)*sqrt(h*x) - 32*(7*d^6*g*h^14*sqrt(-(2401*b^4*d^2*g^4*p^4
*e^5 - 2450*b^4*d*f^2*g^2*p^4*e^6 + 625*b^4*f^4*p^4*e^7)/(d^7*h^18)) - 245*
b^2*d^3*f*g^2*h^5*p^2*e^3 + 125*b^2*d^2*f^3*h^5*p^2*e^4)*sqrt(-(d^3*h^9*sqr
t(-(2401*b^4*d^2*g^4*p^4*e^5 - 2450*b^4*d*f^2*g^2*p^4*e^6 + 625*b^4*f^4*p^4
*e^7)/(d^7*h^18)) + 70*b^2*f*g*p^2*e^3)/(d^3*h^9))) - 3*d*h^5*x^4*sqrt((d^3
*h^9*sqrt(-(2401*b^4*d^2*g^4*p^4*e^5 - 2450*b^4*d*f^2*g^2*p^4*e^6 + 625*b^4
*f^4*p^4*e^7)/(d^7*h^18)) - 70*b^2*f*g*p^2*e^3)/(d^3*h^9))*log(-32*(2401*b^
3*d^2*g^4*p^3*e^4 - 625*b^3*f^4*p^3*e^6)*sqrt(h*x) + 32*(7*d^6*g*h^14*sqrt(
-(2401*b^4*d^2*g^4*p^4*e^5 - 2450*b^4*d*f^2*g^2*p^4*e^6 + 625*b^4*f^4*p^4*e
^7)/(d^7*h^18)) + 245*b^2*d^3*f*g^2*h^5*p^2*e^3 - 125*b^2*d^2*f^3*h^5*p^2*e
^4)*sqrt((d^3*h^9*sqrt(-(2401*b^4*d^2*g^4*p^4*e^5 - 2450*b^4*d*f^2*g^2*p^4*
e^6 + 625*b^4*f^4*p^4*e^7)/(d^7*h^18)) - 70*b^2*f*g*p^2*e^3)/(d^3*h^9))) +
3*d*h^5*x^4*sqrt((d^3*h^9*sqrt(-(2401*b^4*d^2*g^4*p^4*e^5 - 2450*b^4*d*f^2*
g^2*p^4*e^6 + 625*b^4*f^4*p^4*e^7)/(d^7*h^18)) - 70*b^2*f*g*p^2*e^3)/(d^3*h
^9))*log(-32*(2401*b^3*d^2*g^4*p^3*e^4 - 625*b^3*f^4*p^3*e^6)*sqrt(h*x) - 3
2*(7*d^6*g*h^14*sqrt(-(2401*b^4*d^2*g^4*p^4*e^5 - 2450*b^4*d*f^2*g^2*p^4*e^
6 + 625*b^4*f^4*p^4*e^7)/(d^7*h^18)) + 245*b^2*d^3*f*g^2*h^5*p^2*e^3 - 125*
b^2*d^2*f^3*h^5*p^2*e^4)*sqrt((d^3*h^9*sqrt(-(2401*b^4*d^2*g^4*p^4*e^5 - 24
50*b^4*d*f^2*g^2*p^4*e^6 + 625*b^4*f^4*p^4*e^7)/(d^7*h^18)) - 70*b^2*f*g*p^
2*e^3)/(d^3*h^9))) + (21*a*d*g*x + 15*a*d*f + 4*(21*b*g*p*x^3 + 5*b*f*p*x^2
)*e + 3*(7*b*d*g*p*x + 5*b*d*f*p)*log(x^2*e + d) + 3*(7*b*d*g*x + 5*b*d*f)*
log(c))*sqrt(h*x))/(d*h^5*x^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7319 deep
```

Giac [A]
time = 5.45, size = 488, normalized size = 0.76

$$-\frac{\frac{6\left(5\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}bfhpe^{\frac{11}{4}}+7\sqrt{2}\left(dh^{2}\right)^{\frac{3}{4}}bgpe^{\frac{9}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}e^{\left(-\frac{1}{4}\right)}+2\sqrt{hx}\right)e^{\frac{1}{4}}}{2\left(dh^{2}\right)^{\frac{1}{4}}}\right)e^{(-1)}}{d^{2}h}+\frac{6\left(5\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}bfhpe^{\frac{11}{4}}+7\sqrt{2}\left(dh^{2}\right)^{\frac{3}{4}}bgpe^{\frac{9}{4}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}e^{\left(-\frac{1}{4}\right)}-2\sqrt{hx}\right)e^{\frac{1}{4}}}{2\left(dh^{2}\right)^{\frac{1}{4}}}\right)e^{(-1)}}{d^{2}h}+\frac{3\left(5\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}bfhpe^{\frac{11}{4}}-7\sqrt{2}\left(dh^{2}\right)^{\frac{3}{4}}bgpe^{\frac{9}{4}}\right)e^{(-1)}\log\left(\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}\sqrt{hx}e^{\left(-\frac{1}{4}\right)}+hx+\sqrt{dh^{2}}e^{\left(-\frac{1}{2}\right)}\right)}{d^{2}h}-\frac{3\left(5\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}bfhpe^{\frac{11}{4}}-7\sqrt{2}\left(dh^{2}\right)^{\frac{3}{4}}bgpe^{\frac{9}{4}}\right)e^{(-1)}\log\left(-\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}\sqrt{hx}e^{\left(-\frac{1}{4}\right)}+hx+\sqrt{dh^{2}}e^{\left(-\frac{1}{2}\right)}\right)}{d^{2}h}+\frac{2\left(84bgh^{4}px^{3}e+20bfh^{4}px^{2}e+21bdgh^{4}px\log\left(h^{2}x^{2}e+dh^{2}\right)-21bdgh^{4}px\log\left(h^{2}\right)+15bdfh^{4}p\log\left(h^{2}x^{2}e+dh^{2}\right)-15bdfh^{4}p\log\left(h^{2}\right)+21bdgh^{4}x\log(c)+21adgh^{4}x+15bdfh^{4}\log(c)+15adfh^{4}\right)}{\sqrt{hx}dh^{3}x^{3}}}{105h^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*(6*(5*sqrt(2)*(d*h^2)^(1/4)*b*f*h*p*e^(11/4) + 7*sqrt(2)*(d*h^2)^(3/
4)*b*g*p*e^(9/4))*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sq
rt(h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-1)/(d^2*h) + 6*(5*sqrt(2)*(d*h^2)^(1/4)
*b*f*h*p*e^(11/4) + 7*sqrt(2)*(d*h^2)^(3/4)*b*g*p*e^(9/4))*arctan(-1/2*sqrt
(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) - 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/4))*e
^(-1)/(d^2*h) + 3*(5*sqrt(2)*(d*h^2)^(1/4)*b*f*h*p*e^(11/4) - 7*sqrt(2)*(d*
h^2)^(3/4)*b*g*p*e^(9/4))*e^(-1)*log(sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/
4) + h*x + sqrt(d*h^2)*e^(-1/2))/(d^2*h) - 3*(5*sqrt(2)*(d*h^2)^(1/4)*b*f*h
*p*e^(11/4) - 7*sqrt(2)*(d*h^2)^(3/4)*b*g*p*e^(9/4))*e^(-1)*log(-sqrt(2)*(d
*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2))/(d^2*h) + 2*(8
4*b*g*h^4*p*x^3*e + 20*b*f*h^4*p*x^2*e + 21*b*d*g*h^4*p*x*log(h^2*x^2*e + d
*h^2) - 21*b*d*g*h^4*p*x*log(h^2) + 15*b*d*f*h^4*p*log(h^2*x^2*e + d*h^2) -
 15*b*d*f*h^4*p*log(h^2) + 21*b*d*g*h^4*x*log(c) + 21*a*d*g*h^4*x + 15*b*d*
f*h^4*log(c) + 15*a*d*f*h^4)/(sqrt(h*x)*d*h^3*x^3))/h^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{{\left(h\,x\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(9/2),x)
```

```
[Out] int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(9/2), x)
```

3.611 $\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{hx}}\,dx$

Optimal. Leaf size=1002

$$\frac{2af^2\sqrt{hx}}{h}-\frac{8bf^2p\sqrt{hx}}{h}+\frac{8bdg^2p\sqrt{hx}}{5eh}-\frac{16bfgp(hx)^{3/2}}{9h^2}-\frac{8bg^2p(hx)^{5/2}}{25h^3}-\frac{2\sqrt{2}\,b\sqrt[4]{d}\,f^2p\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}}{\sqrt[4]{d}\,\sqrt{}}\right.}{\sqrt[4]{e}\,\sqrt{h}}$$

```
[Out] -16/9*b*f*g*p*(h*x)^(3/2)/h^2-8/25*b*g^2*p*(h*x)^(5/2)/h^3+4/3*f*g*(h*x)^(3
/2)*(a+b*ln(c*(e*x^2+d)^p))/h^2+2/5*g^2*(h*x)^(5/2)*(a+b*ln(c*(e*x^2+d)^p))
/h^3-2*b*d^(1/4)*f^2*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2)
)*2^(1/2)/e^(1/4)/h^(1/2)-4/3*b*d^(3/4)*f*g*p*arctan(1-e^(1/4)*2^(1/2)*(h*x
)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/e^(3/4)/h^(1/2)+2/5*b*d^(5/4)*g^2*p*arctan
(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/e^(5/4)/h^(1/2)+2*b
*d^(1/4)*f^2*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2
)/e^(1/4)/h^(1/2)+4/3*b*d^(3/4)*f*g*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/
d^(1/4)/h^(1/2))*2^(1/2)/e^(3/4)/h^(1/2)-2/5*b*d^(5/4)*g^2*p*arctan(1+e^(1/
4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/e^(5/4)/h^(1/2)-b*d^(1/4)*f
^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/
2))*2^(1/2)/e^(1/4)/h^(1/2)+2/3*b*d^(3/4)*f*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2
)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(3/4)/h^(1/2)+1/5*
b*d^(5/4)*g^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2
)*(h*x)^(1/2))*2^(1/2)/e^(5/4)/h^(1/2)+b*d^(1/4)*f^2*p*ln(d^(1/2)*h^(1/2)+x
*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(1/4)/h^(1/
2)-2/3*b*d^(3/4)*f*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)
*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(3/4)/h^(1/2)-1/5*b*d^(5/4)*g^2*p*ln(d^(1/2
)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^
(5/4)/h^(1/2)+2*a*f^2*(h*x)^(1/2)/h-8*b*f^2*p*(h*x)^(1/2)/h+8/5*b*d*g^2*p*(
h*x)^(1/2)/e/h+2*b*f^2*ln(c*(e*x^2+d)^p)*(h*x)^(1/2)/h
```

Rubi [A]

time = 0.88, antiderivative size = 1002, normalized size of antiderivative = 1.00, number of steps used = 38, number of rules used = 13, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {2517, 2521, 2498, 327, 217, 1179, 642, 1176, 631, 210, 2505, 303, 308}

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/Sqrt[h*x],x]
```

```
[Out] (2*a*f^2*Sqrt[h*x])/h - (8*b*f^2*p*Sqrt[h*x])/h + (8*b*d*g^2*p*Sqrt[h*x])/(
5*e*h) - (16*b*f*g*p*(h*x)^(3/2))/(9*h^2) - (8*b*g^2*p*(h*x)^(5/2))/(25*h^3
) - (2*Sqrt[2]*b*d^(1/4)*f^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1
```

```
/4)*Sqrt[h])])/(e^(1/4)*Sqrt[h]) - (4*Sqrt[2]*b*d^(3/4)*f*g*p*ArcTan[1 - (S
qrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*e^(3/4)*Sqrt[h]) + (2*Sqrt
[2]*b*d^(5/4)*g^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h]
)])/(5*e^(5/4)*Sqrt[h]) + (2*Sqrt[2]*b*d^(1/4)*f^2*p*ArcTan[1 + (Sqrt[2]*e^
(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(e^(1/4)*Sqrt[h]) + (4*Sqrt[2]*b*d^(3/
4)*f*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*e^(3
/4)*Sqrt[h]) - (2*Sqrt[2]*b*d^(5/4)*g^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[
h*x])/(d^(1/4)*Sqrt[h])])/(5*e^(5/4)*Sqrt[h]) + (2*b*f^2*Sqrt[h*x]*Log[c*(d
 + e*x^2)^p])/h + (4*f*g*(h*x)^(3/2)*(a + b*Log[c*(d + e*x^2)^p]))/(3*h^2)
+ (2*g^2*(h*x)^(5/2)*(a + b*Log[c*(d + e*x^2)^p]))/(5*h^3) - (Sqrt[2]*b*d^(
1/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4
)*Sqrt[h*x]])/(e^(1/4)*Sqrt[h]) + (2*Sqrt[2]*b*d^(3/4)*f*g*p*Log[Sqrt[d]*Sq
rt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*e^(3/4)*
Sqrt[h]) + (Sqrt[2]*b*d^(5/4)*g^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x
 - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(5*e^(5/4)*Sqrt[h]) + (Sqrt[2]*b*d^(
1/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4
)*Sqrt[h*x]])/(e^(1/4)*Sqrt[h]) - (2*Sqrt[2]*b*d^(3/4)*f*g*p*Log[Sqrt[d]*Sq
rt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*e^(3/4)*
Sqrt[h]) - (Sqrt[2]*b*d^(5/4)*g^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x
 + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(5*e^(5/4)*Sqrt[h])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
```

```
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2517

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*(
d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{hx}}\,dx = \frac{2\text{Subst}\left(\int\left(f+\frac{gx^2}{h}\right)^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)dx,x,\sqrt{hx}\right)}{h}$$

$$= \frac{2\text{Subst}\left(\int\left(f^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)+\frac{2fgx^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)\right.\right.}{h}\right.\right.}{h}$$

$$= \frac{(2g^2)\,\text{Subst}\left(\int x^4\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)dx,x,\sqrt{hx}\right)}{h^3} + \frac{(4f}{}$$

$$= \frac{2af^2\sqrt{hx}}{h} + \frac{4fg(hx)^{3/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2} + \frac{2g^2(hx)^{5/2}}{}$$

$$= \frac{2af^2\sqrt{hx}}{h} - \frac{16bfgp(hx)^{3/2}}{9h^2} + \frac{2bf^2\sqrt{hx}\,\log\left(c(d+ex^2)^p\right)}{h} + \frac{4}{}$$

$$= \frac{2af^2\sqrt{hx}}{h} - \frac{8bf^2p\sqrt{hx}}{h} + \frac{8bdg^2p\sqrt{hx}}{5eh} - \frac{16bfgp(hx)^{3/2}}{9h^2} - \frac{8b}{}$$

$$= \frac{2af^2\sqrt{hx}}{h} - \frac{8bf^2p\sqrt{hx}}{h} + \frac{8bdg^2p\sqrt{hx}}{5eh} - \frac{16bfgp(hx)^{3/2}}{9h^2} - \frac{8b}{}$$

$$= \frac{2af^2\sqrt{hx}}{h} - \frac{8bf^2p\sqrt{hx}}{h} + \frac{8bdg^2p\sqrt{hx}}{5eh} - \frac{16bfgp(hx)^{3/2}}{9h^2} - \frac{8b}{}$$

$$= \frac{2af^2\sqrt{hx}}{h} - \frac{8bf^2p\sqrt{hx}}{h} + \frac{8bdg^2p\sqrt{hx}}{5eh} - \frac{16bfgp(hx)^{3/2}}{9h^2} - \frac{8b}{}$$

$$= \frac{2af^2\sqrt{hx}}{h} - \frac{8bf^2p\sqrt{hx}}{h} + \frac{8bdg^2p\sqrt{hx}}{5eh} - \frac{16bfgp(hx)^{3/2}}{9h^2} - \frac{8b}{}$$

Mathematica [A]
time = 0.90, size = 588, normalized size = 0.59

$$\frac{2\sqrt{x}\left(af^2\sqrt{x}-\frac{4bfgp\left(2\sqrt[4]{-d}\,e^{3/4}x^{3/2}-3d\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{-d}}\right)+3d\tanh^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{-d}}\right)\right)}{9\sqrt[4]{-d}\,e^{3/4}}-\frac{bf^2p\left(8\sqrt[4]{e}\sqrt{x}+2\sqrt{2}\sqrt[4]{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)-2\sqrt{2}\sqrt[4]{d}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)+\sqrt{2}\sqrt[4]{d}\log\left(\sqrt{d}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\sqrt{e}x\right)-\sqrt{2}\sqrt[4]{d}\log\left(\sqrt{d}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\sqrt{e}x\right)\right)}{2\sqrt[4]{e}}-\frac{bg^2p\left(-40d\sqrt[4]{e}\sqrt{x}+8e^{5/4}x^{5/2}-10\sqrt{2}d^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)+10\sqrt{2}d^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)-5\sqrt{2}d^{5/4}\log\left(\sqrt{d}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\sqrt{e}x\right)+5\sqrt{2}d^{5/4}\log\left(\sqrt{d}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\sqrt{e}x\right)\right)}{50e^{5/4}}+bf^2\sqrt{x}\log\left(c(d+ex^2)^p\right)+\frac{2}{3}fgx^{3/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)+\frac{1}{5}g^2x^{5/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)\right)}{\sqrt{hx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/Sqrt[h*x],x]
```

```
[Out] (2*Sqrt[x]*(a*f^2*Sqrt[x] - (4*b*f*g*p*(2*(-d)^(1/4)*e^(3/4)*x^(3/2) - 3*d*
ArcTan[(e^(1/4)*Sqrt[x])/(-d)^(1/4)] + 3*d*ArcTanh[(e^(1/4)*Sqrt[x])/(-d)^(
1/4)]))/(9*(-d)^(1/4)*e^(3/4)) - (b*f^2*p*(8*e^(1/4)*Sqrt[x] + 2*Sqrt[2]*d^
(1/4)*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - 2*Sqrt[2]*d^(1/4)*Arc
Tan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] + Sqrt[2]*d^(1/4)*Log[Sqrt[d] -
Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x] - Sqrt[2]*d^(1/4)*Log[Sqrt[d]
+ Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x]))/(2*e^(1/4)) - (b*g^2*p*(-4
0*d*e^(1/4)*Sqrt[x] + 8*e^(5/4)*x^(5/2) - 10*Sqrt[2]*d^(5/4)*ArcTan[1 - (Sq
rt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] + 10*Sqrt[2]*d^(5/4)*ArcTan[1 + (Sqrt[2]*e^
(1/4)*Sqrt[x])/d^(1/4)] - 5*Sqrt[2]*d^(5/4)*Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e
^(1/4)*Sqrt[x] + Sqrt[e]*x] + 5*Sqrt[2]*d^(5/4)*Log[Sqrt[d] + Sqrt[2]*d^(1/
4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x]))/(50*e^(5/4)) + b*f^2*Sqrt[x]*Log[c*(d + e
*x^2)^p] + (2*f*g*x^(3/2)*(a + b*Log[c*(d + e*x^2)^p]))/3 + (g^2*x^(5/2)*(a
 + b*Log[c*(d + e*x^2)^p]))/5))/Sqrt[h*x]
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)^{2}\left(a+b\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)\right)}{\sqrt{hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(1/2),x)
```

```
[Out] int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(1/2),x)
```

Maxima [A]
time = 0.52, size = 847, normalized size = 0.85

$$\frac{2bg^{2}x^{3}\log\left(\left(x^{2}e+d\right)^{p}c\right)}{5\sqrt{hx}}+\frac{2ag^{2}x^{3}}{5\sqrt{hx}}+\frac{4bfgx^{2}\log\left(\left(x^{2}e+d\right)^{p}c\right)}{3\sqrt{hx}}+\frac{4afgx^{2}}{3\sqrt{hx}}+\frac{2\sqrt{hx}\,bf^{2}\log\left(\left(x^{2}e+d\right)^{p}c\right)}{h}-\ldots$$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 2/5*b*g^2*x^3*log((x^2*e + d)^p*c)/sqrt(h*x) + 2/5*a*g^2*x^3/sqrt(h*x) + 4/
3*b*f*g*x^2*log((x^2*e + d)^p*c)/sqrt(h*x) + 4/3*a*f*g*x^2/sqrt(h*x) + 2*sq
rt(h*x)*b*f^2*log((x^2*e + d)^p*c)/h - (8*sqrt(h*x)*h^2*e^(-1) - (sqrt(2)*h
^4*e^(-1/4)*log(h*x*e^(1/2) + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqr
t(d)*h)/(d*h^2)^(3/4) - sqrt(2)*h^4*e^(-1/4)*log(h*x*e^(1/2) - sqrt(2)*(d*h
^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(3/4) + 2*sqrt(2)*h^3*arct
an(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*e^(1/2))*e^(-1/
4)/sqrt(sqrt(d)*h))*e^(-1/4)/(sqrt(sqrt(d)*h)*sqrt(d)) + 2*sqrt(2)*h^3*arct
an(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*e^(1/2))*e^(-1
```

```
/4)/sqrt(sqrt(d)*h))*e^(-1/4)/(sqrt(sqrt(d)*h)*sqrt(d)))*d*e^(-1))*b*f^2*p*
e/h^3 + 2*sqrt(h*x)*a*f^2/h - 2/9*(3*(sqrt(2)*e^(-3/4)*log(h*x*e^(1/2) + sq
rt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(1/4) - sqrt(2)*
e^(-3/4)*log(h*x*e^(1/2) - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d
)*h)/(d*h^2)^(1/4) - 2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^
(1/4) + 2*sqrt(h*x)*e^(1/2))*e^(-1/4)/sqrt(sqrt(d)*h))*e^(-3/4)/sqrt(sqrt(d
)*h) - 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqr
t(h*x)*e^(1/2))*e^(-1/4)/sqrt(sqrt(d)*h))*e^(-3/4)/sqrt(sqrt(d)*h))*d*h^4*e
^(-1) + 8*(h*x)^(3/2)*h^2*e^(-1))*b*f*g*p*e/h^4 - 1/25*(5*(sqrt(2)*h^6*e^(-
1/4)*log(h*x*e^(1/2) + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)
/(d*h^2)^(3/4) - sqrt(2)*h^6*e^(-1/4)*log(h*x*e^(1/2) - sqrt(2)*(d*h^2)^(1/
4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(3/4) + 2*sqrt(2)*h^5*arctan(1/2*
sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*e^(1/2))*e^(-1/4)/sqrt
(sqrt(d)*h))*e^(-1/4)/(sqrt(sqrt(d)*h)*sqrt(d)) + 2*sqrt(2)*h^5*arctan(-1/2
*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*e^(1/2))*e^(-1/4)/sqr
t(sqrt(d)*h))*e^(-1/4)/(sqrt(sqrt(d)*h)*sqrt(d)))*d^2*e^(-2) - 8*(5*sqrt(h*
x)*d*h^4 - (h*x)^(5/2)*h^2*e)*e^(-2))*b*g^2*p*e/h^5
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2326 vs. $2(680) = 1360$.

time = 21.96, size = 2326, normalized size = 2.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2),x, algorithm="fric
as")
```

```
[Out] -2/225*(15*h*sqrt((60*b^2*d^2*f*g^3*p^2 - 300*b^2*d*f^3*g*p^2*e + h*sqrt(-(
81*b^4*d^5*g^8*p^4 - 3420*b^4*d^4*f^2*g^6*p^4*e + 40150*b^4*d^3*f^4*g^4*p^4
*e^2 - 85500*b^4*d^2*f^6*g^2*p^4*e^3 + 50625*b^4*d*f^8*p^4*e^4)*e^(-5)/h^2)
*e^2)/h)*log(16*(27*b^2*d^3*g^6*h*p^2*e - 705*b^2*d^2*f^2*g^4*h*p^2*e^2 + 3
525*b^2*d*f^4*g^2*h*p^2*e^3 - 3375*b^2*f^6*h*p^2*e^4 + 10*f*g*h^2*sqrt(-(81
*b^4*d^5*g^8*p^4 - 3420*b^4*d^4*f^2*g^6*p^4*e + 40150*b^4*d^3*f^4*g^4*p^4*e
^2 - 85500*b^4*d^2*f^6*g^2*p^4*e^3 + 50625*b^4*d*f^8*p^4*e^4)*e^(-5)/h^2)*e
^4)*sqrt((60*b^2*d^2*f*g^3*p^2 - 300*b^2*d*f^3*g*p^2*e + h*sqrt(-(81*b^4*d^
5*g^8*p^4 - 3420*b^4*d^4*f^2*g^6*p^4*e + 40150*b^4*d^3*f^4*g^4*p^4*e^2 - 85
500*b^4*d^2*f^6*g^2*p^4*e^3 + 50625*b^4*d*f^8*p^4*e^4)*e^(-5)/h^2)*e^2)/h)*
e^(-1) + 16*(81*b^3*d^4*g^8*p^3 - 1620*b^3*d^3*f^2*g^6*p^3*e + 2150*b^3*d^2
*f^4*g^4*p^3*e^2 - 40500*b^3*d*f^6*g^2*p^3*e^3 + 50625*b^3*f^8*p^3*e^4)*sqr
t(h*x)) - 15*h*sqrt((60*b^2*d^2*f*g^3*p^2 - 300*b^2*d*f^3*g*p^2*e + h*sqrt(
-(81*b^4*d^5*g^8*p^4 - 3420*b^4*d^4*f^2*g^6*p^4*e + 40150*b^4*d^3*f^4*g^4*p
^4*e^2 - 85500*b^4*d^2*f^6*g^2*p^4*e^3 + 50625*b^4*d*f^8*p^4*e^4)*e^(-5)/h^
2)*e^2)/h)*log(-16*(27*b^2*d^3*g^6*h*p^2*e - 705*b^2*d^2*f^2*g^4*h*p^2*e^2
+ 3525*b^2*d*f^4*g^2*h*p^2*e^3 - 3375*b^2*f^6*h*p^2*e^4 + 10*f*g*h^2*sqrt(-
```

```
(81*b^4*d^5*g^8*p^4 - 3420*b^4*d^4*f^2*g^6*p^4*e + 40150*b^4*d^3*f^4*g^4*p^
4*e^2 - 85500*b^4*d^2*f^6*g^2*p^4*e^3 + 50625*b^4*d*f^8*p^4*e^4)*e^(-5)/h^2
)*e^4)*sqrt((60*b^2*d^2*f*g^3*p^2 - 300*b^2*d*f^3*g*p^2*e + h*sqrt(-(81*b^4
*d^5*g^8*p^4 - 3420*b^4*d^4*f^2*g^6*p^4*e + 40150*b^4*d^3*f^4*g^4*p^4*e^2 -
 85500*b^4*d^2*f^6*g^2*p^4*e^3 + 50625*b^4*d*f^8*p^4*e^4)*e^(-5)/h^2)*e^2)/
h)*e^(-1) + 16*(81*b^3*d^4*g^8*p^3 - 1620*b^3*d^3*f^2*g^6*p^3*e + 2150*b^3*
d^2*f^4*g^4*p^3*e^2 - 40500*b^3*d*f^6*g^2*p^3*e^3 + 50625*b^3*f^8*p^3*e^4)*
sqrt(h*x)) + 15*h*sqrt((60*b^2*d^2*f*g^3*p^2 - 300*b^2*d*f^3*g*p^2*e - h*sq
rt(-(81*b^4*d^5*g^8*p^4 - 3420*b^4*d^4*f^2*g^6*p^4*e + 40150*b^4*d^3*f^4*g^
4*p^4*e^2 - 85500*b^4*d^2*f^6*g^2*p^4*e^3 + 50625*b^4*d*f^8*p^4*e^4)*e^(-5)
/h^2)*e^2)/h)*log(16*(27*b^2*d^3*g^6*h*p^2*e - 705*b^2*d^2*f^2*g^4*h*p^2*e^
2 + 3525*b^2*d*f^4*g^2*h*p^2*e^3 - 3375*b^2*f^6*h*p^2*e^4 - 10*f*g*h^2*sqrt
(-(81*b^4*d^5*g^8*p^4 - 3420*b^4*d^4*f^2*g^6*p^4*e + 40150*b^4*d^3*f^4*g^4*
p^4*e^2 - 85500*b^4*d^2*f^6*g^2*p^4*e^3 + 50625*b^4*d*f^8*p^4*e^4)*e^(-5)/h
^2)*e^4)*sqrt((60*b^2*d^2*f*g^3*p^2 - 300*b^2*d*f^3*g*p^2*e - h*sqrt(-(81*b
^4*d^5*g^8*p^4 - 3420*b^4*d^4*f^2*g^6*p^4*e + 40150*b^4*d^3*f^4*g^4*p^4*e^2
 - 85500*b^4*d^2*f^6*g^2*p^4*e^3 + 50625*b^4*d*f^8*p^4*e^4)*e^(-5)/h^2)*e^2
)/h)*e^(-1) + 16*(81*b^3*d^4*g^8*p^3 - 1620*b^3*d^3*f^2*g^6*p^3*e + 2150*b^
3*d^2*f^4*g^4*p^3*e^2 - 40500*b^3*d*f^6*g^2*p^3*e^3 + 50625*b^3*f^8*p^3*e^4
)*sqrt(h*x)) - 15*h*sqrt((60*b^2*d^2*f*g^3*p^2 - 300*b^2*d*f^3*g*p^2*e - h*
sqrt(-(81*b^4*d^5*g^8*p^4 - 3420*b^4*d^4*f^2*g^6*p^4*e + 40150*b^4*d^3*f^4*
g^4*p^4*e^2 - 85500*b^4*d^2*f^6*g^2*p^4*e^3 + 50625*b^4*d*f^8*p^4*e^4)*e^(-
5)/h^2)*e^2)/h)*log(-16*(27*b^2*d^3*g^6*h*p^2*e - 705*b^2*d^2*f^2*g^4*h*p^2
*e^2 + 3525*b^2*d*f^4*g^2*h*p^2*e^3 - 3375*b^2*f^6*h*p^2*e^4 - 10*f*g*h^2*s
qrt(-(81*b^4*d^5*g^8*p^4 - 3420*b^4*d^4*f^2*g^6*p^4*e + 40150*b^4*d^3*f^4*g
^4*p^4*e^2 - 85500*b^4*d^2*f^6*g^2*p^4*e^3 + 50625*b^4*d*f^8*p^4*e^4)*e^(-5
)/h^2)*e^4)*sqrt((60*b^2*d^2*f*g^3*p^2 - 300*b^2*d*f^3*g*p^2*e - h*sqrt(-(8
1*b^4*d^5*g^8*p^4 - 3420*b^4*d^4*f^2*g^6*p^4*e + 40150*b^4*d^3*f^4*g^4*p^4*
e^2 - 85500*b^4*d^2*f^6*g^2*p^4*e^3 + 50625*b^4*d*f^8*p^4*e^4)*e^(-5)/h^2)*
e^2)/h)*e^(-1) + 16*(81*b^3*d^4*g^8*p^3 - 1620*b^3*d^3*f^2*g^6*p^3*e + 2150
*b^3*d^2*f^4*g^4*p^3*e^2 - 40500*b^3*d*f^6*g^2*p^3*e^3 + 50625*b^3*f^8*p^3*
e^4)*sqrt(h*x)) - (180*b*d*g^2*p + 15*(3*b*g^2*p*x^2 + 10*b*f*g*p*x + 15*b*
f^2*p)*e*log(x^2*e + d) + 15*(3*b*g^2*x^2 + 10*b*f*g*x + 15*b*f^2)*e*log(c)
 - (900*b*f^2*p - 225*a*f^2 + 9*(4*b*g^2*p - 5*a*g^2)*x^2 + 50*(4*b*f*g*p -
 3*a*f*g)*x)*e)*sqrt(h*x))*e^(-1)/h
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(1/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Giac [A]
time = 4.02, size = 820, normalized size = 0.82

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2),x, algorithm="giac
")
```

```
[Out] 1/225*(90*sqrt(h*x)*b*g^2*x^2*log(c) + 90*sqrt(h*x)*a*g^2*x^2 + 300*sqrt(h*
x)*b*f*g*x*log(c) + 225*((2*sqrt(2)*(d*h^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(
2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-5/4) +
2*sqrt(2)*(d*h^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4)
 - 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-5/4) + sqrt(2)*(d*h^2)^(1/4)*e^(
-5/4)*log(sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-
1/2)) - sqrt(2)*(d*h^2)^(1/4)*e^(-5/4)*log(-sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)
*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2)) - 8*sqrt(h*x)*e^(-1))*e + 2*sqrt(h*
x)*log(x^2*e + d))*b*f^2*p + 9*(10*sqrt(h*x)*x^2*log(x^2*e + d) - (10*sqrt(
2)*(d*h^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*s
qrt(h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-9/4) + 10*sqrt(2)*(d*h^2)^(1/4)*d*arct
an(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) - 2*sqrt(h*x))*e^(1/4)/(d*h
^2)^(1/4))*e^(-9/4) + 5*sqrt(2)*(d*h^2)^(1/4)*d*e^(-9/4)*log(sqrt(2)*(d*h^2
)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2)) - 5*sqrt(2)*(d*h^2
)^(1/4)*d*e^(-9/4)*log(-sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sq
rt(d*h^2)*e^(-1/2)) + 8*(sqrt(h*x)*h^10*x^2*e^4 - 5*sqrt(h*x)*d*h^10*e^3)*e
^(-5)/h^10)*e)*b*g^2*p + 300*sqrt(h*x)*a*f*g*x + 450*sqrt(h*x)*b*f^2*log(c)
 + 50*(6*sqrt(h*x)*h*x*log(x^2*e + d) - (8*sqrt(h*x)*h*x*e^(-1) - 6*sqrt(2)
*(d*h^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqrt(
h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-7/4) - 6*sqrt(2)*(d*h^2)^(3/4)*arctan(-1/2
*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) - 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/
4))*e^(-7/4) + 3*sqrt(2)*(d*h^2)^(3/4)*e^(-7/4)*log(sqrt(2)*(d*h^2)^(1/4)*s
qrt(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2)) - 3*sqrt(2)*(d*h^2)^(3/4)*e
^(-7/4)*log(-sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e
^(-1/2)))*e)*b*f*g*p/h + 450*sqrt(h*x)*a*f^2)/h
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)^2\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{\sqrt{h\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(1/2),x)
```

```
[Out] int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(1/2), x)
```

3.612 $\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{3/2}}\,dx$

Optimal. Leaf size=949

$$\frac{4afg\sqrt{hx}}{h^2}-\frac{16bfgp\sqrt{hx}}{h^2}-\frac{8bg^2p(hx)^{3/2}}{9h^3}-\frac{2\sqrt{2}\,b\sqrt[4]{e}\,f^2p\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{\sqrt[4]{d}\,h^{3/2}}-\frac{4\sqrt{2}\,b\sqrt[4]{d}\,fgp\tan^{-}}{\sqrt[4]{}}$$

```
[Out] -8/9*b*g^2*p*(h*x)^(3/2)/h^3+2/3*g^2*(h*x)^(3/2)*(a+b*ln(c*(e*x^2+d)^p))/h^
3-2*b*e^(1/4)*f^2*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2
^(1/2)/d^(1/4)/h^(3/2)-4*b*d^(1/4)*f*g*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/
2)/d^(1/4)/h^(1/2))*2^(1/2)/e^(1/4)/h^(3/2)-2/3*b*d^(3/4)*g^2*p*arctan(1-e^
(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/e^(3/4)/h^(3/2)+2*b*e^(1
/4)*f^2*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(
1/4)/h^(3/2)+4*b*d^(1/4)*f*g*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)
/h^(1/2))*2^(1/2)/e^(1/4)/h^(3/2)+2/3*b*d^(3/4)*g^2*p*arctan(1+e^(1/4)*2^(1
/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/e^(3/4)/h^(3/2)+b*e^(1/4)*f^2*p*ln
(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(
1/2)/d^(1/4)/h^(3/2)-2*b*d^(1/4)*f*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)
-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(1/4)/h^(3/2)+1/3*b*d^(3/4)
*g^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(
1/2))*2^(1/2)/e^(3/4)/h^(3/2)-b*e^(1/4)*f^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*
h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(1/4)/h^(3/2)+2*b*d^
(1/4)*f*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h
*x)^(1/2))*2^(1/2)/e^(1/4)/h^(3/2)-1/3*b*d^(3/4)*g^2*p*ln(d^(1/2)*h^(1/2)+x
*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(3/4)/h^(3/
2)-2*f^2*(a+b*ln(c*(e*x^2+d)^p))/h/(h*x)^(1/2)+4*a*f*g*(h*x)^(1/2)/h^2-16*b
*f*g*p*(h*x)^(1/2)/h^2+4*b*f*g*ln(c*(e*x^2+d)^p)*(h*x)^(1/2)/h^2
```

Rubi [A]

time = 0.81, antiderivative size = 949, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {2517, 2526, 2498, 327, 217, 1179, 642, 1176, 631, 210, 2505, 303}

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(3/2),x]
```

```
[Out] (4*a*f*g*Sqrt[h*x])/h^2 - (16*b*f*g*p*Sqrt[h*x])/h^2 - (8*b*g^2*p*(h*x)^(3/
2))/(9*h^3) - (2*Sqrt[2]*b*e^(1/4)*f^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h
*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(3/2)) - (4*Sqrt[2]*b*d^(1/4)*f*g*p*Arc
Tan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(e^(1/4)*h^(3/2)) -
```

```
(2*Sqrt[2]*b*d^(3/4)*g^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)
*Sqrt[h])])/(3*e^(3/4)*h^(3/2)) + (2*Sqrt[2]*b*e^(1/4)*f^2*p*ArcTan[1 + (Sq
rt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(3/2)) + (4*Sqrt[2]
*b*d^(1/4)*f*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])
/(e^(1/4)*h^(3/2)) + (2*Sqrt[2]*b*d^(3/4)*g^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)
*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*e^(3/4)*h^(3/2)) + (4*b*f*g*Sqrt[h*x]*Lo
g[c*(d + e*x^2)^p])/h^2 - (2*f^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*Sqrt[h*x]
) + (2*g^2*(h*x)^(3/2)*(a + b*Log[c*(d + e*x^2)^p]))/(3*h^3) + (Sqrt[2]*b*e
^(1/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1
/4)*Sqrt[h*x]])/(d^(1/4)*h^(3/2)) - (2*Sqrt[2]*b*d^(1/4)*f*g*p*Log[Sqrt[d]*
Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*
h^(3/2)) + (Sqrt[2]*b*d^(3/4)*g^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x
 - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*e^(3/4)*h^(3/2)) - (Sqrt[2]*b*e^(
1/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4
)*Sqrt[h*x]])/(d^(1/4)*h^(3/2)) + (2*Sqrt[2]*b*d^(1/4)*f*g*p*Log[Sqrt[d]*Sq
rt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*h^
(3/2)) - (Sqrt[2]*b*d^(3/4)*g^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x +
 Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*e^(3/4)*h^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2517

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
```

```
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*(
d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int \frac{(f+gx)^2 \left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{\left(f+\frac{gx^2}{h}\right)^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^2}\,dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{2fg\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{h} + \frac{f^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^2} + \frac{g^2x \cdots}{\cdots}\right.\right.}{h}$$

$$= \frac{(2g^2)\,\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)\,dx, x, \sqrt{hx}\right)}{h^3} + \frac{(4 \cdots}{\cdots}$$

$$= \frac{4afg\sqrt{hx}}{h^2} - \frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h\sqrt{hx}} + \frac{2g^2(hx)^{3/2}\left(a+b \cdots\right.}{3 \cdots}$$

$$= \frac{4afg\sqrt{hx}}{h^2} - \frac{8bg^2p(hx)^{3/2}}{9h^3} + \frac{4bfg\sqrt{hx}\,\log\left(c(d+ex^2)^p\right)}{h^2} - \frac{2 \cdots}{\cdots}$$

$$= \frac{4afg\sqrt{hx}}{h^2} - \frac{16bfgp\sqrt{hx}}{h^2} - \frac{8bg^2p(hx)^{3/2}}{9h^3} + \frac{4bfg\sqrt{hx}\,\log\left(c \cdots\right.}{h^2}$$

$$= \frac{4afg\sqrt{hx}}{h^2} - \frac{16bfgp\sqrt{hx}}{h^2} - \frac{8bg^2p(hx)^{3/2}}{9h^3} + \frac{4bfg\sqrt{hx}\,\log\left(c \cdots\right.}{h^2}$$

$$= \frac{4afg\sqrt{hx}}{h^2} - \frac{16bfgp\sqrt{hx}}{h^2} - \frac{8bg^2p(hx)^{3/2}}{9h^3} - \frac{2\sqrt{2}\,b\sqrt[4]{e}\,f^2p\,\text{ta}\cdots}{\cdots}$$

$$= \frac{4afg\sqrt{hx}}{h^2} - \frac{16bfgp\sqrt{hx}}{h^2} - \frac{8bg^2p(hx)^{3/2}}{9h^3} - \frac{2\sqrt{2}\,b\sqrt[4]{e}\,f^2p\,\text{ta}\cdots}{\cdots}$$

$$= \frac{4afg\sqrt{hx}}{h^2} - \frac{16bfgp\sqrt{hx}}{h^2} - \frac{8bg^2p(hx)^{3/2}}{9h^3} - \frac{2\sqrt{2}\,b\sqrt[4]{e}\,f^2p\,\text{ta}\cdots}{\cdots}$$

Mathematica [A]

time = 0.58, size = 436, normalized size = 0.46

$$\frac{2x^{3/2}\left(2afg\sqrt{x}-\frac{2bg^2p\left(2\sqrt[4]{-d}\,e^{3/4}x^{3/2}-3d\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{-d}}\right)+3d\tanh^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{-d}}\right)\right)}{9\sqrt[4]{-d}\,e^{3/4}}+\frac{2b\sqrt[4]{e}\,f^2p\left(\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{-d}}\right)+\tanh^{-1}\left(\frac{d\sqrt[4]{e}\sqrt{x}}{(-d)^{5/4}}\right)\right)}{\sqrt[4]{-d}}-\frac{bfgp\left(8\sqrt[4]{e}\sqrt{x}+2\sqrt{2}\sqrt[4]{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)-2\sqrt{2}\sqrt[4]{d}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)+\sqrt{2}\sqrt[4]{d}\log\left(\sqrt{d}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\sqrt{e}x\right)-\sqrt{2}\sqrt[4]{d}\log\left(\sqrt{d}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\sqrt{e}x\right)\right)}{\sqrt[4]{e}}+2bfg\sqrt{x}\log\left(c(d+ex^2)^p\right)-\frac{f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{x}}+\frac{1}{3}g^2x^{3/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)\right)}{(hx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(3/2),x]
```

```
[Out] (2*x^(3/2)*(2*a*f*g*Sqrt[x] - (2*b*g^2*p*(2*(-d)^(1/4)*e^(3/4)*x^(3/2) - 3*
d*ArcTan[(e^(1/4)*Sqrt[x])/(-d)^(1/4)] + 3*d*ArcTanh[(e^(1/4)*Sqrt[x])/(-d)
^(1/4)]))/(9*(-d)^(1/4)*e^(3/4)) + (2*b*e^(1/4)*f^2*p*(ArcTan[(e^(1/4)*Sqrt
[x])/(-d)^(1/4)] + ArcTanh[(d*e^(1/4)*Sqrt[x])/(-d)^(5/4)]))/(-d)^(1/4) - (
b*f*g*p*(8*e^(1/4)*Sqrt[x] + 2*Sqrt[2]*d^(1/4)*ArcTan[1 - (Sqrt[2]*e^(1/4)*
Sqrt[x])/d^(1/4)] - 2*Sqrt[2]*d^(1/4)*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/
d^(1/4)] + Sqrt[2]*d^(1/4)*Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] +
Sqrt[e]*x] - Sqrt[2]*d^(1/4)*Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x]
+ Sqrt[e]*x]))/e^(1/4) + 2*b*f*g*Sqrt[x]*Log[c*(d + e*x^2)^p] - (f^2*(a + b
*Log[c*(d + e*x^2)^p]))/Sqrt[x] + (g^2*x^(3/2)*(a + b*Log[c*(d + e*x^2)^p])
)/3))/(h*x)^(3/2)
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)^2\left(a+b\ln\left(c(e\,x^2+d)^p\right)\right)}{(hx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(3/2),x)
```

```
[Out] int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(3/2),x)
```

Maxima [A]
time = 0.53, size = 793, normalized size = 0.84

$$\frac{2bg^2x^3\log\left((x^2e+d)^pc\right)}{3(hx)^{\frac{3}{2}}}-\frac{\left(\frac{\sqrt{2}e^{\left(-\frac{3}{4}\right)}\log\left(hxe^{\frac{1}{2}}+\sqrt{2}(dh^2)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{(dh^2)^{\frac{1}{4}}}-\frac{\sqrt{2}e^{\left(-\frac{3}{4}\right)}\log\left(hxe^{\frac{1}{2}}-\sqrt{2}(dh^2)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{(dh^2)^{\frac{1}{4}}}-\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(dh^2)^{\frac{1}{4}}e^{\frac{1}{4}}+2\sqrt{hx}e^{\frac{1}{2}}\right)e^{\left(-\frac{1}{4}\right)}}{2\sqrt{\sqrt{d}h}}\right)e^{\left(-\frac{3}{4}\right)}}{\sqrt{\sqrt{d}h}}-\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(dh^2)^{\frac{1}{4}}e^{\frac{1}{4}}-2\sqrt{hx}e^{\frac{1}{2}}\right)e^{\left(-\frac{1}{4}\right)}}{2\sqrt{\sqrt{d}h}}\right)e^{\left(-\frac{3}{4}\right)}}{\sqrt{\sqrt{d}h}}\right)bf^2pe}{h}+\frac{2ag^2x^3}{3(hx)^{\frac{3}{2}}}+\frac{4bfgx^2\log\left((x^2e+d)^pc\right)}{(hx)^{\frac{3}{2}}}+\frac{4afgx^2}{(hx)^{\frac{3}{2}}}-\frac{2bf^2\log\left((x^2e+d)^pc\right)}{\sqrt{hx}h}-\frac{2\left(8\sqrt{hx}h^2e^{(-1)}-\left(\frac{\sqrt{2}h^4e^{\left(-\frac{1}{4}\right)}\log\left(hxe^{\frac{1}{2}}+\sqrt{2}(dh^2)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{(dh^2)^{\frac{3}{4}}}-\frac{\sqrt{2}h^4e^{\left(-\frac{1}{4}\right)}\log\left(hxe^{\frac{1}{2}}-\sqrt{2}(dh^2)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{(dh^2)^{\frac{3}{4}}}+\frac{2\sqrt{2}h^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(dh^2)^{\frac{1}{4}}e^{\frac{1}{4}}+2\sqrt{hx}e^{\frac{1}{2}}\right)e^{\left(-\frac{1}{4}\right)}}{2\sqrt{\sqrt{d}h}}\right)e^{\left(-\frac{1}{4}\right)}}{\sqrt{\sqrt{d}h}\sqrt{d}}+\frac{2\sqrt{2}h^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(dh^2)^{\frac{1}{4}}e^{\frac{1}{4}}-2\sqrt{hx}e^{\frac{1}{2}}\right)e^{\left(-\frac{1}{4}\right)}}{2\sqrt{\sqrt{d}h}}\right)e^{\left(-\frac{1}{4}\right)}}{\sqrt{\sqrt{d}h}\sqrt{d}}\right)de^{(-1)}\right)bfgpe}{h^4}-\frac{2af^2}{\sqrt{hx}h}-\frac{\left(3\left(\frac{\sqrt{2}e^{\left(-\frac{3}{4}\right)}\log\left(hxe^{\frac{1}{2}}+\sqrt{2}(dh^2)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{(dh^2)^{\frac{1}{4}}}-\frac{\sqrt{2}e^{\left(-\frac{3}{4}\right)}\log\left(hxe^{\frac{1}{2}}-\sqrt{2}(dh^2)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{(dh^2)^{\frac{1}{4}}}-\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(dh^2)^{\frac{1}{4}}e^{\frac{1}{4}}+2\sqrt{hx}e^{\frac{1}{2}}\right)e^{\left(-\frac{1}{4}\right)}}{2\sqrt{\sqrt{d}h}}\right)e^{\left(-\frac{3}{4}\right)}}{\sqrt{\sqrt{d}h}}-\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(dh^2)^{\frac{1}{4}}e^{\frac{1}{4}}-2\sqrt{hx}e^{\frac{1}{2}}\right)e^{\left(-\frac{1}{4}\right)}}{2\sqrt{\sqrt{d}h}}\right)e^{\left(-\frac{3}{4}\right)}}{\sqrt{\sqrt{d}h}}\right)dh^4e^{(-1)}+8(hx)^{\frac{3}{2}}h^2e^{(-1)}\right)bg^2pe}{9h^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] 2/3*b*g^2*x^3*log((x^2*e + d)^p*c)/(h*x)^(3/2) - (sqrt(2)*e^(-3/4)*log(h*x*
e^(1/2) + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(1/4
) - sqrt(2)*e^(-3/4)*log(h*x*e^(1/2) - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1
/4) + sqrt(d)*h)/(d*h^2)^(1/4) - 2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h
^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*e^(1/2))*e^(-1/4)/sqrt(sqrt(d)*h))*e^(-3/4)
/sqrt(sqrt(d)*h) - 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(
```

```
1/4) - 2*sqrt(h*x)*e^(1/2))*e^(-1/4)/sqrt(sqrt(d)*h))*e^(-3/4)/sqrt(sqrt(d)
*h))*b*f^2*p*e/h + 2/3*a*g^2*x^3/(h*x)^(3/2) + 4*b*f*g*x^2*log((x^2*e + d)^
p*c)/(h*x)^(3/2) + 4*a*f*g*x^2/(h*x)^(3/2) - 2*b*f^2*log((x^2*e + d)^p*c)/(
sqrt(h*x)*h) - 2*(8*sqrt(h*x)*h^2*e^(-1) - (sqrt(2)*h^4*e^(-1/4)*log(h*x*e^
(1/2) + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(3/4)
- sqrt(2)*h^4*e^(-1/4)*log(h*x*e^(1/2) - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^
(1/4) + sqrt(d)*h)/(d*h^2)^(3/4) + 2*sqrt(2)*h^3*arctan(1/2*sqrt(2)*(sqrt(2
)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*e^(1/2))*e^(-1/4)/sqrt(sqrt(d)*h))*e^
(-1/4)/(sqrt(sqrt(d)*h)*sqrt(d)) + 2*sqrt(2)*h^3*arctan(-1/2*sqrt(2)*(sqrt(
2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*e^(1/2))*e^(-1/4)/sqrt(sqrt(d)*h))*e
^(-1/4)/(sqrt(sqrt(d)*h)*sqrt(d)))*d*e^(-1))*b*f*g*p*e/h^4 - 2*a*f^2/(sqrt(
h*x)*h) - 1/9*(3*(sqrt(2)*e^(-3/4)*log(h*x*e^(1/2) + sqrt(2)*(d*h^2)^(1/4)*
sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(1/4) - sqrt(2)*e^(-3/4)*log(h*x*e^(
1/2) - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(1/4) -
 2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*
e^(1/2))*e^(-1/4)/sqrt(sqrt(d)*h))*e^(-3/4)/sqrt(sqrt(d)*h) - 2*sqrt(2)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*e^(1/2))*e^(-
1/4)/sqrt(sqrt(d)*h))*e^(-3/4)/sqrt(sqrt(d)*h))*d*h^4*e^(-1) + 8*(h*x)^(3/2
)*h^2*e^(-1))*b*g^2*p*e/h^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2334 vs. $2(646) = 1292$.
time = 5.42, size = 2334, normalized size = 2.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2),x, algorithm="fric
as")
```

```
[Out] 2/9*(3*h^2*x*sqrt(-(12*b^2*d*f*g^3*p^2 + 36*b^2*f^3*g*p^2*e + h^3*sqrt(-(b^
4*d^4*g^8*p^4 - 60*b^4*d^3*f^2*g^6*p^4*e + 918*b^4*d^2*f^4*g^4*p^4*e^2 - 54
0*b^4*d*f^6*g^2*p^4*e^3 + 81*b^4*f^8*p^4*e^4)*e^(-3)/(d*h^6))*e)*e^(-1)/h^3
)*log(32*(b^3*d^4*g^8*p^3 + 12*b^3*d^3*f^2*g^6*p^3*e - 1242*b^3*d^2*f^4*g^4
*p^3*e^2 + 108*b^3*d*f^6*g^2*p^3*e^3 + 81*b^3*f^8*p^3*e^4)*sqrt(h*x) + 32*(
6*b^2*d^3*f*g^5*h^2*p^2*e - 180*b^2*d^2*f^3*g^3*h^2*p^2*e^2 + 54*b^2*d*f^5*
g*h^2*p^2*e^3 - (d^2*g^2*h^5*e^2 + 3*d*f^2*h^5*e^3)*sqrt(-(b^4*d^4*g^8*p^4
- 60*b^4*d^3*f^2*g^6*p^4*e + 918*b^4*d^2*f^4*g^4*p^4*e^2 - 540*b^4*d*f^6*g^
2*p^4*e^3 + 81*b^4*f^8*p^4*e^4)*e^(-3)/(d*h^6)))*sqrt(-(12*b^2*d*f*g^3*p^2
+ 36*b^2*f^3*g*p^2*e + h^3*sqrt(-(b^4*d^4*g^8*p^4 - 60*b^4*d^3*f^2*g^6*p^4*
e + 918*b^4*d^2*f^4*g^4*p^4*e^2 - 540*b^4*d*f^6*g^2*p^4*e^3 + 81*b^4*f^8*p^
4*e^4)*e^(-3)/(d*h^6))*e)*e^(-1)/h^3)) - 3*h^2*x*sqrt(-(12*b^2*d*f*g^3*p^2
+ 36*b^2*f^3*g*p^2*e + h^3*sqrt(-(b^4*d^4*g^8*p^4 - 60*b^4*d^3*f^2*g^6*p^4*
e + 918*b^4*d^2*f^4*g^4*p^4*e^2 - 540*b^4*d*f^6*g^2*p^4*e^3 + 81*b^4*f^8*p^
4*e^4)*e^(-3)/(d*h^6))*e)*e^(-1)/h^3)*log(32*(b^3*d^4*g^8*p^3 + 12*b^3*d^3*
```

```
f^2*g^6*p^3*e - 1242*b^3*d^2*f^4*g^4*p^3*e^2 + 108*b^3*d*f^6*g^2*p^3*e^3 +
81*b^3*f^8*p^3*e^4)*sqrt(h*x) - 32*(6*b^2*d^3*f*g^5*h^2*p^2*e - 180*b^2*d^2
*f^3*g^3*h^2*p^2*e^2 + 54*b^2*d*f^5*g*h^2*p^2*e^3 - (d^2*g^2*h^5*e^2 + 3*d*
f^2*h^5*e^3)*sqrt(-(b^4*d^4*g^8*p^4 - 60*b^4*d^3*f^2*g^6*p^4*e + 918*b^4*d^
2*f^4*g^4*p^4*e^2 - 540*b^4*d*f^6*g^2*p^4*e^3 + 81*b^4*f^8*p^4*e^4)*e^(-3)/
(d*h^6)))*sqrt(-(12*b^2*d*f*g^3*p^2 + 36*b^2*f^3*g*p^2*e + h^3*sqrt(-(b^4*d
^4*g^8*p^4 - 60*b^4*d^3*f^2*g^6*p^4*e + 918*b^4*d^2*f^4*g^4*p^4*e^2 - 540*b
^4*d*f^6*g^2*p^4*e^3 + 81*b^4*f^8*p^4*e^4)*e^(-3)/(d*h^6))*e)*e^(-1)/h^3))
+ 3*h^2*x*sqrt(-(12*b^2*d*f*g^3*p^2 + 36*b^2*f^3*g*p^2*e - h^3*sqrt(-(b^4*d
^4*g^8*p^4 - 60*b^4*d^3*f^2*g^6*p^4*e + 918*b^4*d^2*f^4*g^4*p^4*e^2 - 540*b
^4*d*f^6*g^2*p^4*e^3 + 81*b^4*f^8*p^4*e^4)*e^(-3)/(d*h^6))*e)*e^(-1)/h^3)*l
og(32*(b^3*d^4*g^8*p^3 + 12*b^3*d^3*f^2*g^6*p^3*e - 1242*b^3*d^2*f^4*g^4*p^
3*e^2 + 108*b^3*d*f^6*g^2*p^3*e^3 + 81*b^3*f^8*p^3*e^4)*sqrt(h*x) + 32*(6*b
^2*d^3*f*g^5*h^2*p^2*e - 180*b^2*d^2*f^3*g^3*h^2*p^2*e^2 + 54*b^2*d*f^5*g*h
^2*p^2*e^3 + (d^2*g^2*h^5*e^2 + 3*d*f^2*h^5*e^3)*sqrt(-(b^4*d^4*g^8*p^4 - 6
0*b^4*d^3*f^2*g^6*p^4*e + 918*b^4*d^2*f^4*g^4*p^4*e^2 - 540*b^4*d*f^6*g^2*p
^4*e^3 + 81*b^4*f^8*p^4*e^4)*e^(-3)/(d*h^6)))*sqrt(-(12*b^2*d*f*g^3*p^2 + 3
6*b^2*f^3*g*p^2*e - h^3*sqrt(-(b^4*d^4*g^8*p^4 - 60*b^4*d^3*f^2*g^6*p^4*e +
 918*b^4*d^2*f^4*g^4*p^4*e^2 - 540*b^4*d*f^6*g^2*p^4*e^3 + 81*b^4*f^8*p^4*e
^4)*e^(-3)/(d*h^6))*e)*e^(-1)/h^3)) - 3*h^2*x*sqrt(-(12*b^2*d*f*g^3*p^2 + 3
6*b^2*f^3*g*p^2*e - h^3*sqrt(-(b^4*d^4*g^8*p^4 - 60*b^4*d^3*f^2*g^6*p^4*e +
 918*b^4*d^2*f^4*g^4*p^4*e^2 - 540*b^4*d*f^6*g^2*p^4*e^3 + 81*b^4*f^8*p^4*e
^4)*e^(-3)/(d*h^6))*e)*e^(-1)/h^3)*log(32*(b^3*d^4*g^8*p^3 + 12*b^3*d^3*f^2
*g^6*p^3*e - 1242*b^3*d^2*f^4*g^4*p^3*e^2 + 108*b^3*d*f^6*g^2*p^3*e^3 + 81*
b^3*f^8*p^3*e^4)*sqrt(h*x) - 32*(6*b^2*d^3*f*g^5*h^2*p^2*e - 180*b^2*d^2*f^
3*g^3*h^2*p^2*e^2 + 54*b^2*d*f^5*g*h^2*p^2*e^3 + (d^2*g^2*h^5*e^2 + 3*d*f^2
*h^5*e^3)*sqrt(-(b^4*d^4*g^8*p^4 - 60*b^4*d^3*f^2*g^6*p^4*e + 918*b^4*d^2*f
^4*g^4*p^4*e^2 - 540*b^4*d*f^6*g^2*p^4*e^3 + 81*b^4*f^8*p^4*e^4)*e^(-3)/(d*
h^6)))*sqrt(-(12*b^2*d*f*g^3*p^2 + 36*b^2*f^3*g*p^2*e - h^3*sqrt(-(b^4*d^4*
g^8*p^4 - 60*b^4*d^3*f^2*g^6*p^4*e + 918*b^4*d^2*f^4*g^4*p^4*e^2 - 540*b^4*
d*f^6*g^2*p^4*e^3 + 81*b^4*f^8*p^4*e^4)*e^(-3)/(d*h^6))*e)*e^(-1)/h^3)) - (
9*a*f^2 + (4*b*g^2*p - 3*a*g^2)*x^2 + 18*(4*b*f*g*p - a*f*g)*x - 3*(b*g^2*p
*x^2 + 6*b*f*g*p*x - 3*b*f^2*p)*log(x^2*e + d) - 3*(b*g^2*x^2 + 6*b*f*g*x -
 3*b*f^2)*log(c))*sqrt(h*x))/(h^2*x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(3/2),x)
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Giac [A]

time = 6.28, size = 649, normalized size = 0.68

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2),x, algorithm="giac
")
```

```
[Out] 1/9*(6*(6*sqrt(2)*(d*h^2)^(1/4)*b*d*f*g*h*p*e^(11/4) + sqrt(2)*(d*h^2)^(3/4
)*b*d*g^2*p*e^(9/4) + 3*sqrt(2)*(d*h^2)^(3/4)*b*f^2*p*e^(13/4))*arctan(1/2*
sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/4
))*e^(-3)/(d*h^2) + 6*(6*sqrt(2)*(d*h^2)^(1/4)*b*d*f*g*h*p*e^(11/4) + sqrt(
2)*(d*h^2)^(3/4)*b*d*g^2*p*e^(9/4) + 3*sqrt(2)*(d*h^2)^(3/4)*b*f^2*p*e^(13/
4))*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) - 2*sqrt(h*x))*e^(1
/4)/(d*h^2)^(1/4))*e^(-3)/(d*h^2) + 3*(6*sqrt(2)*(d*h^2)^(1/4)*b*d*f*g*h*p*
e^(11/4) - sqrt(2)*(d*h^2)^(3/4)*b*d*g^2*p*e^(9/4) - 3*sqrt(2)*(d*h^2)^(3/4
)*b*f^2*p*e^(13/4))*e^(-3)*log(sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h
*x + sqrt(d*h^2)*e^(-1/2))/(d*h^2) - 3*(6*sqrt(2)*(d*h^2)^(1/4)*b*d*f*g*h*p
*e^(11/4) - sqrt(2)*(d*h^2)^(3/4)*b*d*g^2*p*e^(9/4) - 3*sqrt(2)*(d*h^2)^(3/
4)*b*f^2*p*e^(13/4))*e^(-3)*log(-sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) +
 h*x + sqrt(d*h^2)*e^(-1/2))/(d*h^2) + 2*(3*b*g^2*h^2*p*x^2*log(h^2*x^2*e +
 d*h^2) - 3*b*g^2*h^2*p*x^2*log(h^2) - 4*b*g^2*h^2*p*x^2 + 18*b*f*g*h^2*p*x
*log(h^2*x^2*e + d*h^2) - 18*b*f*g*h^2*p*x*log(h^2) + 3*b*g^2*h^2*x^2*log(c
) - 72*b*f*g*h^2*p*x + 3*a*g^2*h^2*x^2 - 9*b*f^2*h^2*p*log(h^2*x^2*e + d*h^
2) + 9*b*f^2*h^2*p*log(h^2) + 18*b*f*g*h^2*x*log(c) + 18*a*f*g*h^2*x - 9*b*
f^2*h^2*log(c) - 9*a*f^2*h^2)/(sqrt(h*x)*h^2))/h
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)^2\,\left(a+b\,\ln\left(c\,(e\,x^2+d)^p\right)\right)}{(h\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(3/2),x)
```

```
[Out] int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(3/2), x)
```

3.613 $\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{5/2}}\,dx$

Optimal. Leaf size=932

$$\frac{2ag^2\sqrt{hx}}{h^3}-\frac{8bg^2p\sqrt{hx}}{h^3}-\frac{2\sqrt{2}\,be^{3/4}f^2p\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{3d^{3/4}h^{5/2}}-\frac{4\sqrt{2}\,b\sqrt[4]{e}\,fgp\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{h}}{\sqrt[4]{d}\,\sqrt{h}}\right.}{\sqrt[4]{d}\,h^{5/2}}$$

```
[Out] -2/3*f^2*(a+b*ln(c*(e*x^2+d)^p))/h/(h*x)^(3/2)-2/3*b*e^(3/4)*f^2*p*arctan(1
-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(3/4)/h^(5/2)-4*b*e
^(1/4)*f*g*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/
d^(1/4)/h^(5/2)-2*b*d^(1/4)*g^2*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1
/4)/h^(1/2))*2^(1/2)/e^(1/4)/h^(5/2)+2/3*b*e^(3/4)*f^2*p*arctan(1+e^(1/4)*2
^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(3/4)/h^(5/2)+4*b*e^(1/4)*f*g
*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(1/4)/h^
(5/2)+2*b*d^(1/4)*g^2*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2
))*2^(1/2)/e^(1/4)/h^(5/2)-1/3*b*e^(3/4)*f^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)
*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(3/4)/h^(5/2)+2*b*e
^(1/4)*f*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(
h*x)^(1/2))*2^(1/2)/d^(1/4)/h^(5/2)-b*d^(1/4)*g^2*p*ln(d^(1/2)*h^(1/2)+x*e^
(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(1/4)/h^(5/2)+
1/3*b*e^(3/4)*f^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^
(1/2)*(h*x)^(1/2))*2^(1/2)/d^(3/4)/h^(5/2)-2*b*e^(1/4)*f*g*p*ln(d^(1/2)*h^(
1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(1/4)
/h^(5/2)+b*d^(1/4)*g^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/
4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(1/4)/h^(5/2)-4*f*g*(a+b*ln(c*(e*x^2+d)^p
))/h^2/(h*x)^(1/2)+2*a*g^2*(h*x)^(1/2)/h^3-8*b*g^2*p*(h*x)^(1/2)/h^3+2*b*g^
2*ln(c*(e*x^2+d)^p)*(h*x)^(1/2)/h^3
```

Rubi [A]
time = 0.78, antiderivative size = 932, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {2517, 2526, 2498, 327, 217, 1179, 642, 1176, 631, 210, 2505, 303}

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(5/2),x]
```

```
[Out] (2*a*g^2*Sqrt[h*x])/h^3 - (8*b*g^2*p*Sqrt[h*x])/h^3 - (2*Sqrt[2]*b*e^(3/4)*
f^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*d^(3/4)
*h^(5/2)) - (4*Sqrt[2]*b*e^(1/4)*f*g*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x
])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(5/2)) - (2*Sqrt[2]*b*d^(1/4)*g^2*p*ArcTa
```

```
n[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(e^(1/4)*h^(5/2)) + (
2*Sqrt[2]*b*e^(3/4)*f^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*S
qrt[h])])/(3*d^(3/4)*h^(5/2)) + (4*Sqrt[2]*b*e^(1/4)*f*g*p*ArcTan[1 + (Sqrt
[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(5/2)) + (2*Sqrt[2]*b
*d^(1/4)*g^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(
e^(1/4)*h^(5/2)) + (2*b*g^2*Sqrt[h*x]*Log[c*(d + e*x^2)^p])/h^3 - (2*f^2*(a
 + b*Log[c*(d + e*x^2)^p]))/(3*h*(h*x)^(3/2)) - (4*f*g*(a + b*Log[c*(d + e*
x^2)^p]))/(h^2*Sqrt[h*x]) - (Sqrt[2]*b*e^(3/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] +
Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(5/2))
 + (2*Sqrt[2]*b*e^(1/4)*f*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqr
t[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(d^(1/4)*h^(5/2)) - (Sqrt[2]*b*d^(1/4)*g^2
*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h
*x]])/(e^(1/4)*h^(5/2)) + (Sqrt[2]*b*e^(3/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sq
rt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(5/2)) -
 (2*Sqrt[2]*b*e^(1/4)*f*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[
2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(d^(1/4)*h^(5/2)) + (Sqrt[2]*b*d^(1/4)*g^2*p
*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x
]])/(e^(1/4)*h^(5/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2517

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
```

```
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*(
d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int \frac{(f+gx)^2 \left(a+b\log \left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{\left(f+\frac{gx^2}{h}\right)^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^4}\,dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{g^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{h^2} + \frac{f^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^4} + \frac{2fg\left(a\right.}{}\right.\right.}{h}$$

$$= \frac{(2g^2)\,\text{Subst}\left(\int \left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)\,dx, x, \sqrt{hx}\right)}{h^3} + \frac{(4fg)}{}$$

$$= \frac{2ag^2\sqrt{hx}}{h^3} - \frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h(hx)^{3/2}} - \frac{4fg\left(a+b\log\left(c(d\right.\right.}{h^2\sqrt{hx}}$$

$$= \frac{2ag^2\sqrt{hx}}{h^3} + \frac{2bg^2\sqrt{hx}\,\log\left(c(d+ex^2)^p\right)}{h^3} - \frac{2f^2\left(a+b\log\left(c(d+\right.\right.}{3h(hx)^{3/2}}$$

$$= \frac{2ag^2\sqrt{hx}}{h^3} - \frac{8bg^2p\sqrt{hx}}{h^3} + \frac{2bg^2\sqrt{hx}\,\log\left(c(d+ex^2)^p\right)}{h^3} - \frac{2f^2\left(a\right.}{}$$

$$= \frac{2ag^2\sqrt{hx}}{h^3} - \frac{8bg^2p\sqrt{hx}}{h^3} + \frac{2bg^2\sqrt{hx}\,\log\left(c(d+ex^2)^p\right)}{h^3} - \frac{2f^2\left(a\right.}{}$$

$$= \frac{2ag^2\sqrt{hx}}{h^3} - \frac{8bg^2p\sqrt{hx}}{h^3} - \frac{2\sqrt{2}\,be^{3/4}f^2p\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{}}{\sqrt[4]{d}\,\sqrt{h}}\right.}{3d^{3/4}h^{5/2}}$$

$$= \frac{2ag^2\sqrt{hx}}{h^3} - \frac{8bg^2p\sqrt{hx}}{h^3} - \frac{2\sqrt{2}\,be^{3/4}f^2p\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{}}{\sqrt[4]{d}\,\sqrt{h}}\right.}{3d^{3/4}h^{5/2}}$$

$$= \frac{2ag^2\sqrt{hx}}{h^3} - \frac{8bg^2p\sqrt{hx}}{h^3} - \frac{2\sqrt{2}\,be^{3/4}f^2p\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{}}{\sqrt[4]{d}\,\sqrt{h}}\right.}{3d^{3/4}h^{5/2}}$$

Mathematica [A]
time = 0.65, size = 503, normalized size = 0.54

$$\frac{2x^{5/2}\left(ag^2\sqrt{x} + \frac{4b\sqrt[4]{e}\,fgp\left(\tan^{-1}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{-d}}\right)+\tanh^{-1}\left(\frac{d\sqrt[4]{e}\,\sqrt{x}}{(-d)^{5/4}}\right)\right)}{\sqrt[4]{-d}} - \frac{be^{3/4}f^2p\left(2\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)-2\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)+\log\left(\sqrt{d}-\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{x}+\sqrt{e}\,x\right)-\log\left(\sqrt{d}+\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{x}+\sqrt{e}\,x\right)\right)}{3\sqrt{2}\,d^{3/4}} - \frac{bg^2p\left(8\sqrt[4]{e}\,\sqrt{x}+2\sqrt{2}\,\sqrt[4]{d}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)-2\sqrt{2}\,\sqrt[4]{d}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)+\sqrt{2}\,\sqrt[4]{d}\,\log\left(\sqrt{d}-\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{x}+\sqrt{e}\,x\right)-\sqrt{2}\,\sqrt[4]{d}\,\log\left(\sqrt{d}+\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{x}+\sqrt{e}\,x\right)\right)}{2\sqrt[4]{e}} + bg^2\sqrt{x}\,\log\left(c(d+ex^2)^p\right) - \frac{f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3x^{3/2}} - \frac{2fg\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{x}}\right)}{(hx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(5/2),x]
```

```
[Out] (2*x^(5/2)*(a*g^2*Sqrt[x] + (4*b*e^(1/4)*f*g*p*(ArcTan[(e^(1/4)*Sqrt[x])/(-
d)^(1/4)] + ArcTanh[(d*e^(1/4)*Sqrt[x])/(-d)^(5/4)]))/(-d)^(1/4) - (b*e^(3/
4)*f^2*p*(2*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - 2*ArcTan[1 + (S
qrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] + Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sq
rt[x] + Sqrt[e]*x] - Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e
]*x]))/(3*Sqrt[2]*d^(3/4)) - (b*g^2*p*(8*e^(1/4)*Sqrt[x] + 2*Sqrt[2]*d^(1/4
)*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - 2*Sqrt[2]*d^(1/4)*ArcTan[
1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] + Sqrt[2]*d^(1/4)*Log[Sqrt[d] - Sqrt
[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x] - Sqrt[2]*d^(1/4)*Log[Sqrt[d] + Sq
rt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x]))/(2*e^(1/4)) + b*g^2*Sqrt[x]*Lo
g[c*(d + e*x^2)^p] - (f^2*(a + b*Log[c*(d + e*x^2)^p]))/(3*x^(3/2)) - (2*f*
g*(a + b*Log[c*(d + e*x^2)^p]))/Sqrt[x]))/(h*x)^(5/2)
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)^2 \left(a+b\ln\left(c(e\,x^2+d)^p\right)\right)}{(hx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(5/2),x)
```

```
[Out] int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(5/2),x)
```

Maxima [A]
time = 0.53, size = 786, normalized size = 0.84

$$\frac{2bg^2x^3\log\left((x^2e+d)^pc\right)}{(hx)^{\frac{5}{2}}} - \frac{2\left(\ldots\right)bfgpe}{h^2} + \frac{2ag^2x^3}{(hx)^{\frac{5}{2}}} - \frac{4bfgx^2\log\left((x^2e+d)^pc\right)}{(hx)^{\frac{5}{2}}} + \ldots$$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] 2*b*g^2*x^3*log((x^2*e + d)^p*c)/(h*x)^(5/2) - 2*(sqrt(2)*e^(-3/4)*log(h*x*
e^(1/2) + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(1/4
) - sqrt(2)*e^(-3/4)*log(h*x*e^(1/2) - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1
/4) + sqrt(d)*h)/(d*h^2)^(1/4) - 2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h
^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*e^(1/2))*e^(-1/4)/sqrt(sqrt(d)*h))*e^(-3/4)
/sqrt(sqrt(d)*h) - 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(
1/4) - 2*sqrt(h*x)*e^(1/2))*e^(-1/4)/sqrt(sqrt(d)*h))*e^(-3/4)/sqrt(sqrt(d)
*h))*b*f*g*p*e/h^2 + 2*a*g^2*x^3/(h*x)^(5/2) - 4*b*f*g*x^2*log((x^2*e + d)^
p*c)/(h*x)^(5/2) + 1/3*(sqrt(2)*h^2*e^(-1/4)*log(h*x*e^(1/2) + sqrt(2)*(d*h
```

```
^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(3/4) - sqrt(2)*h^2*e^(-1/
4)*log(h*x*e^(1/2) - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(
d*h^2)^(3/4) + 2*sqrt(2)*h*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4
) + 2*sqrt(h*x)*e^(1/2))*e^(-1/4)/sqrt(sqrt(d)*h))*e^(-1/4)/(sqrt(sqrt(d)*h
)*sqrt(d)) + 2*sqrt(2)*h*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4)
 - 2*sqrt(h*x)*e^(1/2))*e^(-1/4)/sqrt(sqrt(d)*h))*e^(-1/4)/(sqrt(sqrt(d)*h)
*sqrt(d)))*b*f^2*p*e/h^3 - 4*a*f*g*x^2/(h*x)^(5/2) - 2/3*b*f^2*log((x^2*e +
 d)^p*c)/((h*x)^(3/2)*h) - (8*sqrt(h*x)*h^2*e^(-1) - (sqrt(2)*h^4*e^(-1/4)*
log(h*x*e^(1/2) + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h
^2)^(3/4) - sqrt(2)*h^4*e^(-1/4)*log(h*x*e^(1/2) - sqrt(2)*(d*h^2)^(1/4)*sq
rt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(3/4) + 2*sqrt(2)*h^3*arctan(1/2*sqrt(
2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*e^(1/2))*e^(-1/4)/sqrt(sqrt
(d)*h))*e^(-1/4)/(sqrt(sqrt(d)*h)*sqrt(d)) + 2*sqrt(2)*h^3*arctan(-1/2*sqrt
(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*e^(1/2))*e^(-1/4)/sqrt(sqr
t(d)*h))*e^(-1/4)/(sqrt(sqrt(d)*h)*sqrt(d)))*d*e^(-1))*b*g^2*p*e/h^5 - 2/3*
a*f^2/((h*x)^(3/2)*h)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2347 vs. $2(633) = 1266$.
time = 0.48, size = 2347, normalized size = 2.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(5/2),x, algorithm="fric
as")
```

```
[Out] 2/3*(h^3*x^2*sqrt(-(36*b^2*d*f*g^3*p^2 + 12*b^2*f^3*g*p^2*e + d*h^5*sqrt(-(
81*b^4*d^4*g^8*p^4 - 540*b^4*d^3*f^2*g^6*p^4*e + 918*b^4*d^2*f^4*g^4*p^4*e^
2 - 60*b^4*d*f^6*g^2*p^4*e^3 + b^4*f^8*p^4*e^4)*e^(-1)/(d^3*h^10)))/(d*h^5)
)*log(16*(81*b^3*d^4*g^8*p^3 + 108*b^3*d^3*f^2*g^6*p^3*e - 1242*b^3*d^2*f^4
*g^4*p^3*e^2 + 12*b^3*d*f^6*g^2*p^3*e^3 + b^3*f^8*p^3*e^4)*sqrt(h*x) + 16*(
27*b^2*d^4*g^6*h^3*p^2 - 81*b^2*d^3*f^2*g^4*h^3*p^2*e - 27*b^2*d^2*f^4*g^2*
h^3*p^2*e^2 + b^2*d*f^6*h^3*p^2*e^3 + 6*d^3*f*g*h^8*sqrt(-(81*b^4*d^4*g^8*p
^4 - 540*b^4*d^3*f^2*g^6*p^4*e + 918*b^4*d^2*f^4*g^4*p^4*e^2 - 60*b^4*d*f^6
*g^2*p^4*e^3 + b^4*f^8*p^4*e^4)*e^(-1)/(d^3*h^10))*e)*sqrt(-(36*b^2*d*f*g^3
*p^2 + 12*b^2*f^3*g*p^2*e + d*h^5*sqrt(-(81*b^4*d^4*g^8*p^4 - 540*b^4*d^3*f
^2*g^6*p^4*e + 918*b^4*d^2*f^4*g^4*p^4*e^2 - 60*b^4*d*f^6*g^2*p^4*e^3 + b^4
*f^8*p^4*e^4)*e^(-1)/(d^3*h^10)))/(d*h^5))) - h^3*x^2*sqrt(-(36*b^2*d*f*g^3
*p^2 + 12*b^2*f^3*g*p^2*e + d*h^5*sqrt(-(81*b^4*d^4*g^8*p^4 - 540*b^4*d^3*f
^2*g^6*p^4*e + 918*b^4*d^2*f^4*g^4*p^4*e^2 - 60*b^4*d*f^6*g^2*p^4*e^3 + b^4
*f^8*p^4*e^4)*e^(-1)/(d^3*h^10)))/(d*h^5))*log(16*(81*b^3*d^4*g^8*p^3 + 108
*b^3*d^3*f^2*g^6*p^3*e - 1242*b^3*d^2*f^4*g^4*p^3*e^2 + 12*b^3*d*f^6*g^2*p^
3*e^3 + b^3*f^8*p^3*e^4)*sqrt(h*x) - 16*(27*b^2*d^4*g^6*h^3*p^2 - 81*b^2*d^
3*f^2*g^4*h^3*p^2*e - 27*b^2*d^2*f^4*g^2*h^3*p^2*e^2 + b^2*d*f^6*h^3*p^2*e^
```

```
3 + 6*d^3*f*g*h^8*sqrt(-(81*b^4*d^4*g^8*p^4 - 540*b^4*d^3*f^2*g^6*p^4*e + 9
18*b^4*d^2*f^4*g^4*p^4*e^2 - 60*b^4*d*f^6*g^2*p^4*e^3 + b^4*f^8*p^4*e^4)*e^
(-1)/(d^3*h^10))*e)*sqrt(-(36*b^2*d*f*g^3*p^2 + 12*b^2*f^3*g*p^2*e + d*h^5*
sqrt(-(81*b^4*d^4*g^8*p^4 - 540*b^4*d^3*f^2*g^6*p^4*e + 918*b^4*d^2*f^4*g^4
*p^4*e^2 - 60*b^4*d*f^6*g^2*p^4*e^3 + b^4*f^8*p^4*e^4)*e^(-1)/(d^3*h^10)))/
(d*h^5))) + h^3*x^2*sqrt(-(36*b^2*d*f*g^3*p^2 + 12*b^2*f^3*g*p^2*e - d*h^5*
sqrt(-(81*b^4*d^4*g^8*p^4 - 540*b^4*d^3*f^2*g^6*p^4*e + 918*b^4*d^2*f^4*g^4
*p^4*e^2 - 60*b^4*d*f^6*g^2*p^4*e^3 + b^4*f^8*p^4*e^4)*e^(-1)/(d^3*h^10)))/
(d*h^5))*log(16*(81*b^3*d^4*g^8*p^3 + 108*b^3*d^3*f^2*g^6*p^3*e - 1242*b^3*
d^2*f^4*g^4*p^3*e^2 + 12*b^3*d*f^6*g^2*p^3*e^3 + b^3*f^8*p^3*e^4)*sqrt(h*x)
 + 16*(27*b^2*d^4*g^6*h^3*p^2 - 81*b^2*d^3*f^2*g^4*h^3*p^2*e - 27*b^2*d^2*f
^4*g^2*h^3*p^2*e^2 + b^2*d*f^6*h^3*p^2*e^3 - 6*d^3*f*g*h^8*sqrt(-(81*b^4*d^
4*g^8*p^4 - 540*b^4*d^3*f^2*g^6*p^4*e + 918*b^4*d^2*f^4*g^4*p^4*e^2 - 60*b^
4*d*f^6*g^2*p^4*e^3 + b^4*f^8*p^4*e^4)*e^(-1)/(d^3*h^10))*e)*sqrt(-(36*b^2*
d*f*g^3*p^2 + 12*b^2*f^3*g*p^2*e - d*h^5*sqrt(-(81*b^4*d^4*g^8*p^4 - 540*b^
4*d^3*f^2*g^6*p^4*e + 918*b^4*d^2*f^4*g^4*p^4*e^2 - 60*b^4*d*f^6*g^2*p^4*e^
3 + b^4*f^8*p^4*e^4)*e^(-1)/(d^3*h^10)))/(d*h^5))) - h^3*x^2*sqrt(-(36*b^2*
d*f*g^3*p^2 + 12*b^2*f^3*g*p^2*e - d*h^5*sqrt(-(81*b^4*d^4*g^8*p^4 - 540*b^
4*d^3*f^2*g^6*p^4*e + 918*b^4*d^2*f^4*g^4*p^4*e^2 - 60*b^4*d*f^6*g^2*p^4*e^
3 + b^4*f^8*p^4*e^4)*e^(-1)/(d^3*h^10)))/(d*h^5))*log(16*(81*b^3*d^4*g^8*p^
3 + 108*b^3*d^3*f^2*g^6*p^3*e - 1242*b^3*d^2*f^4*g^4*p^3*e^2 + 12*b^3*d*f^6
*g^2*p^3*e^3 + b^3*f^8*p^3*e^4)*sqrt(h*x) - 16*(27*b^2*d^4*g^6*h^3*p^2 - 81
*b^2*d^3*f^2*g^4*h^3*p^2*e - 27*b^2*d^2*f^4*g^2*h^3*p^2*e^2 + b^2*d*f^6*h^3
*p^2*e^3 - 6*d^3*f*g*h^8*sqrt(-(81*b^4*d^4*g^8*p^4 - 540*b^4*d^3*f^2*g^6*p^
4*e + 918*b^4*d^2*f^4*g^4*p^4*e^2 - 60*b^4*d*f^6*g^2*p^4*e^3 + b^4*f^8*p^4*
e^4)*e^(-1)/(d^3*h^10))*e)*sqrt(-(36*b^2*d*f*g^3*p^2 + 12*b^2*f^3*g*p^2*e -
 d*h^5*sqrt(-(81*b^4*d^4*g^8*p^4 - 540*b^4*d^3*f^2*g^6*p^4*e + 918*b^4*d^2*
f^4*g^4*p^4*e^2 - 60*b^4*d*f^6*g^2*p^4*e^3 + b^4*f^8*p^4*e^4)*e^(-1)/(d^3*h
^10)))/(d*h^5))) - (6*a*f*g*x + a*f^2 + 3*(4*b*g^2*p - a*g^2)*x^2 - (3*b*g^
2*p*x^2 - 6*b*f*g*p*x - b*f^2*p)*log(x^2*e + d) - (3*b*g^2*x^2 - 6*b*f*g*x
- b*f^2)*log(c))*sqrt(h*x))/(h^3*x^2)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(5/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Giac [**A**]
time = 3.32, size = 638, normalized size = 0.68

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(5/2),x, algorithm="giac
")
```

```
[Out] 1/3*(2*(3*sqrt(2)*(d*h^2)^(1/4)*b*d*g^2*h*p*e^(7/4) + sqrt(2)*(d*h^2)^(1/4)
*b*f^2*h*p*e^(11/4) + 6*sqrt(2)*(d*h^2)^(3/4)*b*f*g*p*e^(9/4))*arctan(1/2*s
qrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqrt(h*x))*e^(1/4)/(d*h^2)^(1/4)
)*e^(-2)/(d*h) + 2*(3*sqrt(2)*(d*h^2)^(1/4)*b*d*g^2*h*p*e^(7/4) + sqrt(2)*(
d*h^2)^(1/4)*b*f^2*h*p*e^(11/4) + 6*sqrt(2)*(d*h^2)^(3/4)*b*f*g*p*e^(9/4))*
arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) - 2*sqrt(h*x))*e^(1/4)/
(d*h^2)^(1/4))*e^(-2)/(d*h) + (3*sqrt(2)*(d*h^2)^(1/4)*b*d*g^2*h*p*e^(7/4)
+ sqrt(2)*(d*h^2)^(1/4)*b*f^2*h*p*e^(11/4) - 6*sqrt(2)*(d*h^2)^(3/4)*b*f*g*
p*e^(9/4))*e^(-2)*log(sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sqrt
(d*h^2)*e^(-1/2))/(d*h) - (3*sqrt(2)*(d*h^2)^(1/4)*b*d*g^2*h*p*e^(7/4) + sq
rt(2)*(d*h^2)^(1/4)*b*f^2*h*p*e^(11/4) - 6*sqrt(2)*(d*h^2)^(3/4)*b*f*g*p*e^
(9/4))*e^(-2)*log(-sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(-1/4) + h*x + sqrt(d*
h^2)*e^(-1/2))/(d*h) + 2*(3*b*g^2*h^2*p*x^2*log(h^2*x^2*e + d*h^2) - 3*b*g^
2*h^2*p*x^2*log(h^2) - 12*b*g^2*h^2*p*x^2 - 6*b*f*g*h^2*p*x*log(h^2*x^2*e +
 d*h^2) + 6*b*f*g*h^2*p*x*log(h^2) + 3*b*g^2*h^2*x^2*log(c) + 3*a*g^2*h^2*x
^2 - b*f^2*h^2*p*log(h^2*x^2*e + d*h^2) + b*f^2*h^2*p*log(h^2) - 6*b*f*g*h^
2*x*log(c) - 6*a*f*g*h^2*x - b*f^2*h^2*log(c) - a*f^2*h^2)/(sqrt(h*x)*h*x))
/h^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)^2\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{{\left(h\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(5/2),x)
```

```
[Out] int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(5/2), x)
```

3.614 $\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{7/2}}\,dx$

Optimal. Leaf size=935

$$-\frac{8bef^2p}{5dh^3\sqrt{hx}}+\frac{2\sqrt{2}\,be^{5/4}f^2p\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{5d^{5/4}h^{7/2}}-\frac{4\sqrt{2}\,be^{3/4}fgp\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{3d^{3/4}h^{7/2}}-\frac{2\sqrt{\cdots}}{\cdots}$$

```
[Out] -2/5*f^2*(a+b*ln(c*(e*x^2+d)^p))/h/(h*x)^(5/2)-4/3*f*g*(a+b*ln(c*(e*x^2+d)^
p))/h^2/(h*x)^(3/2)+2/5*b*e^(5/4)*f^2*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2
)/d^(1/4)/h^(1/2))*2^(1/2)/d^(5/4)/h^(7/2)-4/3*b*e^(3/4)*f*g*p*arctan(1-e^(
1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(3/4)/h^(7/2)-2*b*e^(1/
4)*g^2*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(1
/4)/h^(7/2)-2/5*b*e^(5/4)*f^2*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4
)/h^(1/2))*2^(1/2)/d^(5/4)/h^(7/2)+4/3*b*e^(3/4)*f*g*p*arctan(1+e^(1/4)*2^(
1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(3/4)/h^(7/2)+2*b*e^(1/4)*g^2*p
*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(1/4)/h^(7
/2)-1/5*b*e^(5/4)*f^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4
)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(5/4)/h^(7/2)-2/3*b*e^(3/4)*f*g*p*ln(d^(1/
2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d
^(3/4)/h^(7/2)+b*e^(1/4)*g^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)
*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(1/4)/h^(7/2)+1/5*b*e^(5/4)*f^2*p*l
n(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^
(1/2)/d^(5/4)/h^(7/2)+2/3*b*e^(3/4)*f*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1
/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(3/4)/h^(7/2)-b*e^(1/4)*
g^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1
/2))*2^(1/2)/d^(1/4)/h^(7/2)-8/5*b*e*f^2*p/d/h^3/(h*x)^(1/2)-2*g^2*(a+b*ln(
c*(e*x^2+d)^p))/h^3/(h*x)^(1/2)
```

Rubi [A]
time = 0.78, antiderivative size = 935, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {2517, 2526, 2505, 331, 303, 1176, 631, 210, 1179, 642, 217}

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(7/2),x]
```

```
[Out] (-8*b*e*f^2*p)/(5*d*h^3*Sqrt[h*x]) + (2*Sqrt[2]*b*e^(5/4)*f^2*p*ArcTan[1 -
(Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(7/2)) - (4*Sq
rt[2]*b*e^(3/4)*f*g*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[
h])])/(3*d^(3/4)*h^(7/2)) - (2*Sqrt[2]*b*e^(1/4)*g^2*p*ArcTan[1 - (Sqrt[2]*
```

```
e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(7/2)) - (2*Sqrt[2]*b*e^(
5/4)*f^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^
(5/4)*h^(7/2)) + (4*Sqrt[2]*b*e^(3/4)*f*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqr
t[h*x])/(d^(1/4)*Sqrt[h])])/(3*d^(3/4)*h^(7/2)) + (2*Sqrt[2]*b*e^(1/4)*g^2*
p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(7/
2)) - (2*f^2*(a + b*Log[c*(d + e*x^2)^p]))/(5*h*(h*x)^(5/2)) - (4*f*g*(a +
b*Log[c*(d + e*x^2)^p]))/(3*h^2*(h*x)^(3/2)) - (2*g^2*(a + b*Log[c*(d + e*x
^2)^p]))/(h^3*Sqrt[h*x]) - (Sqrt[2]*b*e^(5/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + S
qrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(5*d^(5/4)*h^(7/2))
- (2*Sqrt[2]*b*e^(3/4)*f*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt
[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(7/2)) + (Sqrt[2]*b*e^(1/4)*g^
2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[
h*x]])/(d^(1/4)*h^(7/2)) + (Sqrt[2]*b*e^(5/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + S
qrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(5*d^(5/4)*h^(7/2))
+ (2*Sqrt[2]*b*e^(3/4)*f*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt
[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(7/2)) - (Sqrt[2]*b*e^(1/4)*g^
2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[
h*x]])/(d^(1/4)*h^(7/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2517

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*(
d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{\left(f+\frac{gx^2}{h}\right)^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^6}\,dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{f^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^6} + \frac{2fg\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{hx^4} + \frac{g^2\left(a\right.}{}\right.\right.}{h}$$

$$= \frac{(2g^2)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^2}\,dx, x, \sqrt{hx}\right)}{h^3} + \frac{(4fg)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h(hx)^{5/2}} - \frac{4fg\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2(hx)^{3/2}} -$$

$$= -\frac{8bef^2p}{5dh^3\sqrt{hx}} - \frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h(hx)^{5/2}} - \frac{4fg\left(a+b\log\left(c(\right.\right.}{3h^2(hx)}$$

$$= -\frac{8bef^2p}{5dh^3\sqrt{hx}} - \frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h(hx)^{5/2}} - \frac{4fg\left(a+b\log\left(c(\right.\right.}{3h^2(hx)}$$

$$= -\frac{8bef^2p}{5dh^3\sqrt{hx}} - \frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h(hx)^{5/2}} - \frac{4fg\left(a+b\log\left(c(\right.\right.}{3h^2(hx)}$$

$$= -\frac{8bef^2p}{5dh^3\sqrt{hx}} - \frac{4\sqrt{2}\,be^{3/4}fgp\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{3d^{3/4}h^{7/2}} - \frac{2\sqrt{}}{}$$

$$= -\frac{8bef^2p}{5dh^3\sqrt{hx}} + \frac{2\sqrt{2}\,be^{5/4}f^2p\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{5d^{5/4}h^{7/2}} - \frac{4\sqrt{}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.63, size = 340, normalized size = 0.36

$$\frac{2x^{7/2}\left(\frac{2b\sqrt[4]{e}\,g^2p\left(\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{-d}}\right)+\tanh^{-1}\left(\frac{d\sqrt[4]{e}\sqrt{x}}{(-d)^{5/4}}\right)\right)}{\sqrt[4]{-d}}-\frac{4bef^2p\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\frac{ex^2}{d}\right)}{5d\sqrt{x}}-\frac{\sqrt{2}\,be^{3/4}fgp\left(2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)-2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)+\log\left(\sqrt{d}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\sqrt{e}\,x\right)-\log\left(\sqrt{d}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\sqrt{e}\,x\right)\right)}{3d^{3/4}}-\frac{f^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5x^{5/2}}-\frac{2fg\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3x^{3/2}}-\frac{g^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{x}}\right)}{(hx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(7/2),x]
```

```
[Out] (2*x^(7/2)*((2*b*e^(1/4)*g^2*p*(ArcTan[(e^(1/4)*Sqrt[x])/(-d)^(1/4)] + ArcT
anh[(d*e^(1/4)*Sqrt[x])/(-d)^(5/4)]))/(-d)^(1/4) - (4*b*e*f^2*p*Hypergeomet
ric2F1[-1/4, 1, 3/4, -((e*x^2)/d)])/(5*d*Sqrt[x]) - (Sqrt[2]*b*e^(3/4)*f*g*
p*(2*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*
e^(1/4)*Sqrt[x])/d^(1/4)] + Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] +
 Sqrt[e]*x] - Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x]))/
(3*d^(3/4)) - (f^2*(a + b*Log[c*(d + e*x^2)^p]))/(5*x^(5/2)) - (2*f*g*(a +
b*Log[c*(d + e*x^2)^p]))/(3*x^(3/2)) - (g^2*(a + b*Log[c*(d + e*x^2)^p]))/S
qrt[x]))/(h*x)^(7/2)
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)^2\left(a+b\ln\left(c(e\,x^2+d)^p\right)\right)}{(hx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(7/2),x)
```

```
[Out] int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(7/2),x)
```

Maxima [A]
time = 0.53, size = 767, normalized size = 0.82

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(7/2),x, algorithm="maxi
ma")
```

```
[Out] -2*b*g^2*x^3*log((x^2*e + d)^p*c)/(h*x)^(7/2) - (sqrt(2)*e^(-3/4)*log(h*x*e
^(1/2) + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(1/4)
 - sqrt(2)*e^(-3/4)*log(h*x*e^(1/2) - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/
4) + sqrt(d)*h)/(d*h^2)^(1/4) - 2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^
2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*e^(1/2))*e^(-1/4)/sqrt(sqrt(d)*h))*e^(-3/4)/
```

```
sqrt(sqrt(d)*h) - 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1
/4) - 2*sqrt(h*x)*e^(1/2))*e^(-1/4)/sqrt(sqrt(d)*h))*e^(-3/4)/sqrt(sqrt(d)*
h))*b*g^2*p*e/h^3 + 1/5*b*f^2*p*((sqrt(2)*e^(-3/4)*log(h*x*e^(1/2) + sqrt(2
)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(1/4) - sqrt(2)*e^(-
3/4)*log(h*x*e^(1/2) - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)
/(d*h^2)^(1/4) - 2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4
) + 2*sqrt(h*x)*e^(1/2))*e^(-1/4)/sqrt(sqrt(d)*h))*e^(-3/4)/sqrt(sqrt(d)*h)
 - 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*
x)*e^(1/2))*e^(-1/4)/sqrt(sqrt(d)*h))*e^(-3/4)/sqrt(sqrt(d)*h))*e/d - 8/(sq
rt(h*x)*d))*e/h^3 - 2*a*g^2*x^3/(h*x)^(7/2) - 4/3*b*f*g*x^2*log((x^2*e + d)
^p*c)/(h*x)^(7/2) + 2/3*(sqrt(2)*h^2*e^(-1/4)*log(h*x*e^(1/2) + sqrt(2)*(d*
h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(3/4) - sqrt(2)*h^2*e^(-1
/4)*log(h*x*e^(1/2) - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/
(d*h^2)^(3/4) + 2*sqrt(2)*h*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/
4) + 2*sqrt(h*x)*e^(1/2))*e^(-1/4)/sqrt(sqrt(d)*h))*e^(-1/4)/(sqrt(sqrt(d)*
h)*sqrt(d)) + 2*sqrt(2)*h*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4
) - 2*sqrt(h*x)*e^(1/2))*e^(-1/4)/sqrt(sqrt(d)*h))*e^(-1/4)/(sqrt(sqrt(d)*h
)*sqrt(d)))*b*f*g*p*e/h^4 - 4/3*a*f*g*x^2/(h*x)^(7/2) - 2/5*b*f^2*log((x^2*
e + d)^p*c)/((h*x)^(5/2)*h) - 2/5*a*f^2/((h*x)^(5/2)*h)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2420 vs. $2(627) = 1254$.
time = 6.23, size = 2420, normalized size = 2.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(7/2),x, algorithm="fric
as")
```

```
[Out] 2/15*(d*h^4*x^3*sqrt(-(300*b^2*d*f*g^3*p^2*e + d^2*h^7*sqrt(-(50625*b^4*d^4
*g^8*p^4*e - 85500*b^4*d^3*f^2*g^6*p^4*e^2 + 40150*b^4*d^2*f^4*g^4*p^4*e^3
- 3420*b^4*d*f^6*g^2*p^4*e^4 + 81*b^4*f^8*p^4*e^5)/(d^5*h^14)) - 60*b^2*f^3
*g*p^2*e^2)/(d^2*h^7))*log(32*(50625*b^3*d^4*g^8*p^3*e - 40500*b^3*d^3*f^2*
g^6*p^3*e^2 + 2150*b^3*d^2*f^4*g^4*p^3*e^3 - 1620*b^3*d*f^6*g^2*p^3*e^4 + 8
1*b^3*f^8*p^3*e^5)*sqrt(h*x) + 32*(2250*b^2*d^4*f*g^5*h^4*p^2*e - 1900*b^2*
d^3*f^3*g^3*h^4*p^2*e^2 + 90*b^2*d^2*f^5*g*h^4*p^2*e^3 - 3*(5*d^5*g^2*h^11
- d^4*f^2*h^11*e)*sqrt(-(50625*b^4*d^4*g^8*p^4*e - 85500*b^4*d^3*f^2*g^6*p^
4*e^2 + 40150*b^4*d^2*f^4*g^4*p^4*e^3 - 3420*b^4*d*f^6*g^2*p^4*e^4 + 81*b^4
*f^8*p^4*e^5)/(d^5*h^14)))*sqrt(-(300*b^2*d*f*g^3*p^2*e + d^2*h^7*sqrt(-(50
625*b^4*d^4*g^8*p^4*e - 85500*b^4*d^3*f^2*g^6*p^4*e^2 + 40150*b^4*d^2*f^4*g
^4*p^4*e^3 - 3420*b^4*d*f^6*g^2*p^4*e^4 + 81*b^4*f^8*p^4*e^5)/(d^5*h^14)) -
 60*b^2*f^3*g*p^2*e^2)/(d^2*h^7))) - d*h^4*x^3*sqrt(-(300*b^2*d*f*g^3*p^2*e
 + d^2*h^7*sqrt(-(50625*b^4*d^4*g^8*p^4*e - 85500*b^4*d^3*f^2*g^6*p^4*e^2 +
 40150*b^4*d^2*f^4*g^4*p^4*e^3 - 3420*b^4*d*f^6*g^2*p^4*e^4 + 81*b^4*f^8*p^
```

```
4*e^5)/(d^5*h^14)) - 60*b^2*f^3*g*p^2*e^2)/(d^2*h^7))*log(32*(50625*b^3*d^4
*g^8*p^3*e - 40500*b^3*d^3*f^2*g^6*p^3*e^2 + 2150*b^3*d^2*f^4*g^4*p^3*e^3 -
 1620*b^3*d*f^6*g^2*p^3*e^4 + 81*b^3*f^8*p^3*e^5)*sqrt(h*x) - 32*(2250*b^2*
d^4*f*g^5*h^4*p^2*e - 1900*b^2*d^3*f^3*g^3*h^4*p^2*e^2 + 90*b^2*d^2*f^5*g*h
^4*p^2*e^3 - 3*(5*d^5*g^2*h^11 - d^4*f^2*h^11*e)*sqrt(-(50625*b^4*d^4*g^8*p
^4*e - 85500*b^4*d^3*f^2*g^6*p^4*e^2 + 40150*b^4*d^2*f^4*g^4*p^4*e^3 - 3420
*b^4*d*f^6*g^2*p^4*e^4 + 81*b^4*f^8*p^4*e^5)/(d^5*h^14)))*sqrt(-(300*b^2*d*
f*g^3*p^2*e + d^2*h^7*sqrt(-(50625*b^4*d^4*g^8*p^4*e - 85500*b^4*d^3*f^2*g^
6*p^4*e^2 + 40150*b^4*d^2*f^4*g^4*p^4*e^3 - 3420*b^4*d*f^6*g^2*p^4*e^4 + 81
*b^4*f^8*p^4*e^5)/(d^5*h^14)) - 60*b^2*f^3*g*p^2*e^2)/(d^2*h^7))) + d*h^4*x
^3*sqrt(-(300*b^2*d*f*g^3*p^2*e - d^2*h^7*sqrt(-(50625*b^4*d^4*g^8*p^4*e -
85500*b^4*d^3*f^2*g^6*p^4*e^2 + 40150*b^4*d^2*f^4*g^4*p^4*e^3 - 3420*b^4*d*
f^6*g^2*p^4*e^4 + 81*b^4*f^8*p^4*e^5)/(d^5*h^14)) - 60*b^2*f^3*g*p^2*e^2)/(
d^2*h^7))*log(32*(50625*b^3*d^4*g^8*p^3*e - 40500*b^3*d^3*f^2*g^6*p^3*e^2 +
 2150*b^3*d^2*f^4*g^4*p^3*e^3 - 1620*b^3*d*f^6*g^2*p^3*e^4 + 81*b^3*f^8*p^3
*e^5)*sqrt(h*x) + 32*(2250*b^2*d^4*f*g^5*h^4*p^2*e - 1900*b^2*d^3*f^3*g^3*h
^4*p^2*e^2 + 90*b^2*d^2*f^5*g*h^4*p^2*e^3 + 3*(5*d^5*g^2*h^11 - d^4*f^2*h^1
1*e)*sqrt(-(50625*b^4*d^4*g^8*p^4*e - 85500*b^4*d^3*f^2*g^6*p^4*e^2 + 40150
*b^4*d^2*f^4*g^4*p^4*e^3 - 3420*b^4*d*f^6*g^2*p^4*e^4 + 81*b^4*f^8*p^4*e^5)
/(d^5*h^14)))*sqrt(-(300*b^2*d*f*g^3*p^2*e - d^2*h^7*sqrt(-(50625*b^4*d^4*g
^8*p^4*e - 85500*b^4*d^3*f^2*g^6*p^4*e^2 + 40150*b^4*d^2*f^4*g^4*p^4*e^3 -
3420*b^4*d*f^6*g^2*p^4*e^4 + 81*b^4*f^8*p^4*e^5)/(d^5*h^14)) - 60*b^2*f^3*g
*p^2*e^2)/(d^2*h^7))) - d*h^4*x^3*sqrt(-(300*b^2*d*f*g^3*p^2*e - d^2*h^7*sq
rt(-(50625*b^4*d^4*g^8*p^4*e - 85500*b^4*d^3*f^2*g^6*p^4*e^2 + 40150*b^4*d^
2*f^4*g^4*p^4*e^3 - 3420*b^4*d*f^6*g^2*p^4*e^4 + 81*b^4*f^8*p^4*e^5)/(d^5*h
^14)) - 60*b^2*f^3*g*p^2*e^2)/(d^2*h^7))*log(32*(50625*b^3*d^4*g^8*p^3*e -
40500*b^3*d^3*f^2*g^6*p^3*e^2 + 2150*b^3*d^2*f^4*g^4*p^3*e^3 - 1620*b^3*d*f
^6*g^2*p^3*e^4 + 81*b^3*f^8*p^3*e^5)*sqrt(h*x) - 32*(2250*b^2*d^4*f*g^5*h^4
*p^2*e - 1900*b^2*d^3*f^3*g^3*h^4*p^2*e^2 + 90*b^2*d^2*f^5*g*h^4*p^2*e^3 +
3*(5*d^5*g^2*h^11 - d^4*f^2*h^11*e)*sqrt(-(50625*b^4*d^4*g^8*p^4*e - 85500*
b^4*d^3*f^2*g^6*p^4*e^2 + 40150*b^4*d^2*f^4*g^4*p^4*e^3 - 3420*b^4*d*f^6*g^
2*p^4*e^4 + 81*b^4*f^8*p^4*e^5)/(d^5*h^14)))*sqrt(-(300*b^2*d*f*g^3*p^2*e -
 d^2*h^7*sqrt(-(50625*b^4*d^4*g^8*p^4*e - 85500*b^4*d^3*f^2*g^6*p^4*e^2 + 4
0150*b^4*d^2*f^4*g^4*p^4*e^3 - 3420*b^4*d*f^6*g^2*p^4*e^4 + 81*b^4*f^8*p^4*
e^5)/(d^5*h^14)) - 60*b^2*f^3*g*p^2*e^2)/(d^2*h^7))) - (12*b*f^2*p*x^2*e +
15*a*d*g^2*x^2 + 10*a*d*f*g*x + 3*a*d*f^2 + (15*b*d*g^2*p*x^2 + 10*b*d*f*g*
p*x + 3*b*d*f^2*p)*log(x^2*e + d) + (15*b*d*g^2*x^2 + 10*b*d*f*g*x + 3*b*d*
f^2)*log(c))*sqrt(h*x))/(d*h^4*x^3)
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep
```

Giac [A]
time = 4.90, size = 660, normalized size = 0.71

$$\frac{\frac{2\left(10\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}bdfghpe^{\frac{11}{4}}+15\sqrt{2}\left(dh^{2}\right)^{\frac{3}{4}}bdg^{2}pe^{\frac{9}{4}}-3\sqrt{2}\left(dh^{2}\right)^{\frac{3}{4}}bf^{2}pe^{\frac{13}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}e^{\left(-\frac{1}{4}\right)}+2\sqrt{hx}\right)e^{\frac{1}{4}}}{2\left(dh^{2}\right)^{\frac{1}{4}}}\right)e^{\left(-2\right)}}{d^{2}h}+\frac{2\left(10\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}bdfghpe^{\frac{11}{4}}+15\sqrt{2}\left(dh^{2}\right)^{\frac{3}{4}}bdg^{2}pe^{\frac{9}{4}}-3\sqrt{2}\left(dh^{2}\right)^{\frac{3}{4}}bf^{2}pe^{\frac{13}{4}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}e^{\left(-\frac{1}{4}\right)}-2\sqrt{hx}\right)e^{\frac{1}{4}}}{2\left(dh^{2}\right)^{\frac{1}{4}}}\right)e^{\left(-2\right)}}{d^{2}h}+\frac{\left(10\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}bdfghpe^{\frac{11}{4}}-15\sqrt{2}\left(dh^{2}\right)^{\frac{3}{4}}bdg^{2}pe^{\frac{9}{4}}+3\sqrt{2}\left(dh^{2}\right)^{\frac{3}{4}}bf^{2}pe^{\frac{13}{4}}\right)e^{\left(-2\right)}\log\left(\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}\sqrt{hx}e^{\left(-\frac{1}{4}\right)}+hx+\sqrt{dh^{2}}e^{\left(-\frac{1}{2}\right)}\right)}{d^{2}h}-\frac{\left(10\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}bdfghpe^{\frac{11}{4}}-15\sqrt{2}\left(dh^{2}\right)^{\frac{3}{4}}bdg^{2}pe^{\frac{9}{4}}+3\sqrt{2}\left(dh^{2}\right)^{\frac{3}{4}}bf^{2}pe^{\frac{13}{4}}\right)e^{\left(-2\right)}\log\left(-\sqrt{2}\left(dh^{2}\right)^{\frac{1}{4}}\sqrt{hx}e^{\left(-\frac{1}{4}\right)}+hx+\sqrt{dh^{2}}e^{\left(-\frac{1}{2}\right)}\right)}{d^{2}h}-\frac{2\left(15bdg^{2}h^{3}px^{2}\log\left(h^{2}x^{2}e+dh^{2}\right)-15bdg^{2}h^{3}px^{2}\log\left(h^{2}\right)+12bf^{2}h^{3}px^{2}e+10bdfgh^{3}px\log\left(h^{2}x^{2}e+dh^{2}\right)-10bdfgh^{3}px\log\left(h^{2}\right)+15bdg^{2}h^{3}x^{2}\log\left(c\right)+15adg^{2}h^{3}x^{2}+3bdf^{2}h^{3}p\log\left(h^{2}x^{2}e+dh^{2}\right)-3bdf^{2}h^{3}p\log\left(h^{2}\right)+10bdfgh^{3}x\log\left(c\right)+10adfgh^{3}x+3bdf^{2}h^{3}\log\left(c\right)+3adf^{2}h^{3}\right)}{\sqrt{hx}dh^{2}x^{2}}}{15h^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(7/2),x, algorithm="giac
")
```

```
[Out] 1/15*(2*(10*sqrt(2)*(d*h^2)^(1/4)*b*d*f*g*h*p*e^(11/4) + 15*sqrt(2)*(d*h^2)
^(3/4)*b*d*g^2*p*e^(9/4) - 3*sqrt(2)*(d*h^2)^(3/4)*b*f^2*p*e^(13/4))*arctan
(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqrt(h*x))*e^(1/4)/(d*h^2)
^(1/4))*e^(-2)/(d^2*h) + 2*(10*sqrt(2)*(d*h^2)^(1/4)*b*d*f*g*h*p*e^(11/4) +
 15*sqrt(2)*(d*h^2)^(3/4)*b*d*g^2*p*e^(9/4) - 3*sqrt(2)*(d*h^2)^(3/4)*b*f^2
*p*e^(13/4))*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) - 2*sqrt(h
*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-2)/(d^2*h) + (10*sqrt(2)*(d*h^2)^(1/4)*b*d*
f*g*h*p*e^(11/4) - 15*sqrt(2)*(d*h^2)^(3/4)*b*d*g^2*p*e^(9/4) + 3*sqrt(2)*(
d*h^2)^(3/4)*b*f^2*p*e^(13/4))*e^(-2)*log(sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e
^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2))/(d^2*h) - (10*sqrt(2)*(d*h^2)^(1/4)*b
*d*f*g*h*p*e^(11/4) - 15*sqrt(2)*(d*h^2)^(3/4)*b*d*g^2*p*e^(9/4) + 3*sqrt(2
)*(d*h^2)^(3/4)*b*f^2*p*e^(13/4))*e^(-2)*log(-sqrt(2)*(d*h^2)^(1/4)*sqrt(h*
x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2))/(d^2*h) - 2*(15*b*d*g^2*h^3*p*x^2
*log(h^2*x^2*e + d*h^2) - 15*b*d*g^2*h^3*p*x^2*log(h^2) + 12*b*f^2*h^3*p*x^
2*e + 10*b*d*f*g*h^3*p*x*log(h^2*x^2*e + d*h^2) - 10*b*d*f*g*h^3*p*x*log(h^
2) + 15*b*d*g^2*h^3*x^2*log(c) + 15*a*d*g^2*h^3*x^2 + 3*b*d*f^2*h^3*p*log(h
^2*x^2*e + d*h^2) - 3*b*d*f^2*h^3*p*log(h^2) + 10*b*d*f*g*h^3*x*log(c) + 10
*a*d*f*g*h^3*x + 3*b*d*f^2*h^3*log(c) + 3*a*d*f^2*h^3)/(sqrt(h*x)*d*h^2*x^2
))/h^4
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(f+g\,x\right)}^2\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{{\left(h\,x\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(7/2),x)
```

```
[Out] int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(7/2), x)
```

3.615 $\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{9/2}}\,dx$

Optimal. Leaf size=968

$$-\frac{8bef^2p}{21dh^3(hx)^{3/2}}-\frac{16befgp}{5dh^4\sqrt{hx}}+\frac{2\sqrt{2}\,be^{7/4}f^2p\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{7d^{7/4}h^{9/2}}+\frac{4\sqrt{2}\,be^{5/4}fgp\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{}}{\sqrt[4]{d}}\right.}{5d^{5/4}h^{9/2}}$$

```
[Out] -8/21*b*e*f^2*p/d/h^3/(h*x)^(3/2)-2/7*f^2*(a+b*ln(c*(e*x^2+d)^p))/h/(h*x)^(
7/2)-4/5*f*g*(a+b*ln(c*(e*x^2+d)^p))/h^2/(h*x)^(5/2)-2/3*g^2*(a+b*ln(c*(e*x
^2+d)^p))/h^3/(h*x)^(3/2)+2/7*b*e^(7/4)*f^2*p*arctan(1-e^(1/4)*2^(1/2)*(h*x
)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(7/4)/h^(9/2)+4/5*b*e^(5/4)*f*g*p*arctan
(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(5/4)/h^(9/2)-2/3
*b*e^(3/4)*g^2*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1
/2)/d^(3/4)/h^(9/2)-2/7*b*e^(7/4)*f^2*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2
)/d^(1/4)/h^(1/2))*2^(1/2)/d^(7/4)/h^(9/2)-4/5*b*e^(5/4)*f*g*p*arctan(1+e^(
1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(5/4)/h^(9/2)+2/3*b*e^(
3/4)*g^2*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^
(3/4)/h^(9/2)+1/7*b*e^(7/4)*f^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1
/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(7/4)/h^(9/2)-2/5*b*e^(5/4)*f*g*
p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))
*2^(1/2)/d^(5/4)/h^(9/2)-1/3*b*e^(3/4)*g^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h
^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(3/4)/h^(9/2)-1/7*b*e
^(7/4)*f^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(
h*x)^(1/2))*2^(1/2)/d^(7/4)/h^(9/2)+2/5*b*e^(5/4)*f*g*p*ln(d^(1/2)*h^(1/2)+
x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(5/4)/h^(9
/2)+1/3*b*e^(3/4)*g^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4
)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(3/4)/h^(9/2)-16/5*b*e*f*g*p/d/h^4/(h*x)^(
1/2)
```

Rubi [A]

time = 0.80, antiderivative size = 968, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {2517, 2526, 2505, 331, 217, 1179, 642, 1176, 631, 210, 303}

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(9/2),x]
```

```
[Out] (-8*b*e*f^2*p)/(21*d*h^3*(h*x)^(3/2)) - (16*b*e*f*g*p)/(5*d*h^4*Sqrt[h*x])
+ (2*Sqrt[2]*b*e^(7/4)*f^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4
)*Sqrt[h])])/(7*d^(7/4)*h^(9/2)) + (4*Sqrt[2]*b*e^(5/4)*f*g*p*ArcTan[1 - (S
```

```
qrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(9/2)) - (2*Sqrt
[2]*b*e^(3/4)*g^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h]
)])/(3*d^(3/4)*h^(9/2)) - (2*Sqrt[2]*b*e^(7/4)*f^2*p*ArcTan[1 + (Sqrt[2]*e^
(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(7*d^(7/4)*h^(9/2)) - (4*Sqrt[2]*b*e^(
5/4)*f*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^
(5/4)*h^(9/2)) + (2*Sqrt[2]*b*e^(3/4)*g^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqr
t[h*x])/(d^(1/4)*Sqrt[h])])/(3*d^(3/4)*h^(9/2)) - (2*f^2*(a + b*Log[c*(d +
e*x^2)^p]))/(7*h*(h*x)^(7/2)) - (4*f*g*(a + b*Log[c*(d + e*x^2)^p]))/(5*h^2
*(h*x)^(5/2)) - (2*g^2*(a + b*Log[c*(d + e*x^2)^p]))/(3*h^3*(h*x)^(3/2)) +
(Sqrt[2]*b*e^(7/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*
d^(1/4)*e^(1/4)*Sqrt[h*x]])/(7*d^(7/4)*h^(9/2)) - (2*Sqrt[2]*b*e^(5/4)*f*g*
p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*
x]])/(5*d^(5/4)*h^(9/2)) - (Sqrt[2]*b*e^(3/4)*g^2*p*Log[Sqrt[d]*Sqrt[h] + S
qrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(9/2))
- (Sqrt[2]*b*e^(7/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2
]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(7*d^(7/4)*h^(9/2)) + (2*Sqrt[2]*b*e^(5/4)*f*
g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[
h*x]])/(5*d^(5/4)*h^(9/2)) + (Sqrt[2]*b*e^(3/4)*g^2*p*Log[Sqrt[d]*Sqrt[h] +
 Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(9/2)
)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2517

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*(
d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\int \frac{(f+gx)^2 \left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{9/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{\left(f+\frac{gx^2}{h}\right)^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^8}\,dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{f^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^8} + \frac{2fg\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{hx^6} + \frac{g^2(\ldots}{\ldots}\right.\right.}{h}$$

$$= \frac{(2g^2)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^4}\,dx, x, \sqrt{hx}\right)}{h^3} + \frac{(4fg)\text{Subst}\left(\ldots\right.}{\ldots}$$

$$= -\frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{7h(hx)^{7/2}} - \frac{4fg\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h^2(hx)^{5/2}}$$

$$= -\frac{8bef^2p}{21dh^3(hx)^{3/2}} - \frac{16befgp}{5dh^4\sqrt{hx}} - \frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{7h(hx)^{7/2}} - $$

$$= -\frac{8bef^2p}{21dh^3(hx)^{3/2}} - \frac{16befgp}{5dh^4\sqrt{hx}} - \frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{7h(hx)^{7/2}} - $$

$$= -\frac{8bef^2p}{21dh^3(hx)^{3/2}} - \frac{16befgp}{5dh^4\sqrt{hx}} - \frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{7h(hx)^{7/2}} - $$

$$= -\frac{8bef^2p}{21dh^3(hx)^{3/2}} - \frac{16befgp}{5dh^4\sqrt{hx}} - \frac{2\sqrt{2}\,be^{3/4}g^2p\tan^{-1}\left(1-\frac{\sqrt{2}\ldots}{\sqrt[4]{\ldots}}\right.}{3d^{3/4}h^{9/2}}$$

$$= -\frac{8bef^2p}{21dh^3(hx)^{3/2}} - \frac{16befgp}{5dh^4\sqrt{hx}} + \frac{2\sqrt{2}\,be^{7/4}f^2p\tan^{-1}\left(1-\frac{\sqrt{2}\ldots}{\sqrt[4]{\ldots}}\right.}{7d^{7/4}h^{9/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.17, size = 294, normalized size = 0.30

$$\frac{x\left(-40bef^2px^2\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\frac{ex^2}{d}\right)-336befgpx^3\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\frac{ex^2}{d}\right)-35\sqrt{2}\,b\sqrt[4]{d}\,e^{3/4}g^2px^{7/2}\left(2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)-2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)+\log\left(\sqrt{d}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\sqrt{e}\,x\right)-\log\left(\sqrt{d}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\sqrt{e}\,x\right)\right)-30df^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)-84dfgx\left(a+b\log\left(c(d+ex^2)^p\right)\right)-70dg^2x^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)\right)}{105d(hx)^{9/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(9/2),x]

[Out] (x*(-40*b*e*f^2*p*x^2*Hypergeometric2F1[-3/4, 1, 1/4, -((e*x^2)/d)] - 336*b
*e*f*g*p*x^3*Hypergeometric2F1[-1/4, 1, 3/4, -((e*x^2)/d)] - 35*Sqrt[2]*b*d
^(1/4)*e^(3/4)*g^2*p*x^(7/2)*(2*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4
)] - 2*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] + Log[Sqrt[d] - Sqrt[2
]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x] - Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1
/4)*Sqrt[x] + Sqrt[e]*x]) - 30*d*f^2*(a + b*Log[c*(d + e*x^2)^p]) - 84*d*f*
g*x*(a + b*Log[c*(d + e*x^2)^p]) - 70*d*g^2*x^2*(a + b*Log[c*(d + e*x^2)^p]
)))/(105*d*(h*x)^(9/2))

Maple [**F**]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)^2 \left(a+b\ln\left(c(e\,x^2+d)^p\right)\right)}{(hx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(9/2),x)

[Out] int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(9/2),x)

Maxima [**A**]
time = 0.55, size = 795, normalized size = 0.82

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(9/2),x, algorithm="maxi
ma")

[Out] -1/21*b*f^2*p*(3*(sqrt(2)*e^(3/4)*log(h*x*e^(1/2) + sqrt(2)*(d*h^2)^(1/4)*s
qrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(3/4) - sqrt(2)*e^(3/4)*log(h*x*e^(1/
2) - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(3/4) + 2
*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*e^
(1/2))*e^(-1/4)/sqrt(sqrt(d)*h))*e^(3/4)/(sqrt(sqrt(d)*h)*sqrt(d)*h) + 2*sq
rt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*e^(1
/2))*e^(-1/4)/sqrt(sqrt(d)*h))*e^(3/4)/(sqrt(sqrt(d)*h)*sqrt(d)*h))/d + 8/(
(h*x)^(3/2)*d))*e/h^3 - 2/3*b*g^2*x^3*log((x^2*e + d)^p*c)/(h*x)^(9/2) + 2/
5*b*f*g*p*((sqrt(2)*e^(-3/4)*log(h*x*e^(1/2) + sqrt(2)*(d*h^2)^(1/4)*sqrt(h
*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(1/4) - sqrt(2)*e^(-3/4)*log(h*x*e^(1/2) -
 sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(1/4) - 2*sqr
t(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*e^(1/2
))*e^(-1/4)/sqrt(sqrt(d)*h))*e^(-3/4)/sqrt(sqrt(d)*h) - 2*sqrt(2)*arctan(-1
/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*e^(1/2))*e^(-1/4)/s

```
qrt(sqrt(d)*h))*e^(-3/4)/sqrt(sqrt(d)*h))*e/d - 8/(sqrt(h*x)*d))*e/h^4 - 2/
3*a*g^2*x^3/(h*x)^(9/2) - 4/5*b*f*g*x^2*log((x^2*e + d)^p*c)/(h*x)^(9/2) +
1/3*(sqrt(2)*h^2*e^(-1/4)*log(h*x*e^(1/2) + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)
*e^(1/4) + sqrt(d)*h)/(d*h^2)^(3/4) - sqrt(2)*h^2*e^(-1/4)*log(h*x*e^(1/2)
- sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(3/4) + 2*sq
rt(2)*h*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*e^(
1/2))*e^(-1/4)/sqrt(sqrt(d)*h))*e^(-1/4)/(sqrt(sqrt(d)*h)*sqrt(d)) + 2*sqrt
(2)*h*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*e^(1
/2))*e^(-1/4)/sqrt(sqrt(d)*h))*e^(-1/4)/(sqrt(sqrt(d)*h)*sqrt(d)))*b*g^2*p*
e/h^5 - 4/5*a*f*g*x^2/(h*x)^(9/2) - 2/7*b*f^2*log((x^2*e + d)^p*c)/((h*x)^(
7/2)*h) - 2/7*a*f^2/((h*x)^(7/2)*h)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2436 vs. $2(647) = 1294$.
time = 0.48, size = 2436, normalized size = 2.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(9/2),x, algorithm="fric
as")
```

```
[Out] 2/105*(d*h^5*x^4*sqrt((d^3*h^9*sqrt(-(1500625*b^4*d^4*g^8*p^4*e^3 - 6894300
*b^4*d^3*f^2*g^6*p^4*e^4 + 8469846*b^4*d^2*f^4*g^4*p^4*e^5 - 1266300*b^4*d*
f^6*g^2*p^4*e^6 + 50625*b^4*f^8*p^4*e^7)/(d^7*h^18)) + 2940*b^2*d*f*g^3*p^2
*e^2 - 1260*b^2*f^3*g*p^2*e^3)/(d^3*h^9))*log(16*(1500625*b^3*d^4*g^8*p^3*e
^2 - 2572500*b^3*d^3*f^2*g^6*p^3*e^3 - 1457946*b^3*d^2*f^4*g^4*p^3*e^4 - 47
2500*b^3*d*f^6*g^2*p^3*e^5 + 50625*b^3*f^8*p^3*e^6)*sqrt(h*x) + 16*(42*d^6*
f*g*h^14*sqrt(-(1500625*b^4*d^4*g^8*p^4*e^3 - 6894300*b^4*d^3*f^2*g^6*p^4*e
^4 + 8469846*b^4*d^2*f^4*g^4*p^4*e^5 - 1266300*b^4*d*f^6*g^2*p^4*e^6 + 5062
5*b^4*f^8*p^4*e^7)/(d^7*h^18)) + 42875*b^2*d^5*g^6*h^5*p^2*e - 116865*b^2*d
^4*f^2*g^4*h^5*p^2*e^2 + 50085*b^2*d^3*f^4*g^2*h^5*p^2*e^3 - 3375*b^2*d^2*f
^6*h^5*p^2*e^4)*sqrt((d^3*h^9*sqrt(-(1500625*b^4*d^4*g^8*p^4*e^3 - 6894300*
b^4*d^3*f^2*g^6*p^4*e^4 + 8469846*b^4*d^2*f^4*g^4*p^4*e^5 - 1266300*b^4*d*f
^6*g^2*p^4*e^6 + 50625*b^4*f^8*p^4*e^7)/(d^7*h^18)) + 2940*b^2*d*f*g^3*p^2*
e^2 - 1260*b^2*f^3*g*p^2*e^3)/(d^3*h^9))) - d*h^5*x^4*sqrt((d^3*h^9*sqrt(-(
1500625*b^4*d^4*g^8*p^4*e^3 - 6894300*b^4*d^3*f^2*g^6*p^4*e^4 + 8469846*b^4
*d^2*f^4*g^4*p^4*e^5 - 1266300*b^4*d*f^6*g^2*p^4*e^6 + 50625*b^4*f^8*p^4*e^
7)/(d^7*h^18)) + 2940*b^2*d*f*g^3*p^2*e^2 - 1260*b^2*f^3*g*p^2*e^3)/(d^3*h^
9))*log(16*(1500625*b^3*d^4*g^8*p^3*e^2 - 2572500*b^3*d^3*f^2*g^6*p^3*e^3 -
 1457946*b^3*d^2*f^4*g^4*p^3*e^4 - 472500*b^3*d*f^6*g^2*p^3*e^5 + 50625*b^3
*f^8*p^3*e^6)*sqrt(h*x) - 16*(42*d^6*f*g*h^14*sqrt(-(1500625*b^4*d^4*g^8*p^
4*e^3 - 6894300*b^4*d^3*f^2*g^6*p^4*e^4 + 8469846*b^4*d^2*f^4*g^4*p^4*e^5 -
 1266300*b^4*d*f^6*g^2*p^4*e^6 + 50625*b^4*f^8*p^4*e^7)/(d^7*h^18)) + 42875
*b^2*d^5*g^6*h^5*p^2*e - 116865*b^2*d^4*f^2*g^4*h^5*p^2*e^2 + 50085*b^2*d^3
```

```
*f^4*g^2*h^5*p^2*e^3 - 3375*b^2*d^2*f^6*h^5*p^2*e^4)*sqrt((d^3*h^9*sqrt(-(1
500625*b^4*d^4*g^8*p^4*e^3 - 6894300*b^4*d^3*f^2*g^6*p^4*e^4 + 8469846*b^4*
d^2*f^4*g^4*p^4*e^5 - 1266300*b^4*d*f^6*g^2*p^4*e^6 + 50625*b^4*f^8*p^4*e^7
)/(d^7*h^18)) + 2940*b^2*d*f*g^3*p^2*e^2 - 1260*b^2*f^3*g*p^2*e^3)/(d^3*h^9
))) - d*h^5*x^4*sqrt(-(d^3*h^9*sqrt(-(1500625*b^4*d^4*g^8*p^4*e^3 - 6894300
*b^4*d^3*f^2*g^6*p^4*e^4 + 8469846*b^4*d^2*f^4*g^4*p^4*e^5 - 1266300*b^4*d*
f^6*g^2*p^4*e^6 + 50625*b^4*f^8*p^4*e^7)/(d^7*h^18)) - 2940*b^2*d*f*g^3*p^2
*e^2 + 1260*b^2*f^3*g*p^2*e^3)/(d^3*h^9))*log(16*(1500625*b^3*d^4*g^8*p^3*e
^2 - 2572500*b^3*d^3*f^2*g^6*p^3*e^3 - 1457946*b^3*d^2*f^4*g^4*p^3*e^4 - 47
2500*b^3*d*f^6*g^2*p^3*e^5 + 50625*b^3*f^8*p^3*e^6)*sqrt(h*x) + 16*(42*d^6*
f*g*h^14*sqrt(-(1500625*b^4*d^4*g^8*p^4*e^3 - 6894300*b^4*d^3*f^2*g^6*p^4*e
^4 + 8469846*b^4*d^2*f^4*g^4*p^4*e^5 - 1266300*b^4*d*f^6*g^2*p^4*e^6 + 5062
5*b^4*f^8*p^4*e^7)/(d^7*h^18)) - 42875*b^2*d^5*g^6*h^5*p^2*e + 116865*b^2*d
^4*f^2*g^4*h^5*p^2*e^2 - 50085*b^2*d^3*f^4*g^2*h^5*p^2*e^3 + 3375*b^2*d^2*f
^6*h^5*p^2*e^4)*sqrt(-(d^3*h^9*sqrt(-(1500625*b^4*d^4*g^8*p^4*e^3 - 6894300
*b^4*d^3*f^2*g^6*p^4*e^4 + 8469846*b^4*d^2*f^4*g^4*p^4*e^5 - 1266300*b^4*d*
f^6*g^2*p^4*e^6 + 50625*b^4*f^8*p^4*e^7)/(d^7*h^18)) - 2940*b^2*d*f*g^3*p^2
*e^2 + 1260*b^2*f^3*g*p^2*e^3)/(d^3*h^9))) + d*h^5*x^4*sqrt(-(d^3*h^9*sqrt(
-(1500625*b^4*d^4*g^8*p^4*e^3 - 6894300*b^4*d^3*f^2*g^6*p^4*e^4 + 8469846*b
^4*d^2*f^4*g^4*p^4*e^5 - 1266300*b^4*d*f^6*g^2*p^4*e^6 + 50625*b^4*f^8*p^4*
e^7)/(d^7*h^18)) - 2940*b^2*d*f*g^3*p^2*e^2 + 1260*b^2*f^3*g*p^2*e^3)/(d^3*
h^9))*log(16*(1500625*b^3*d^4*g^8*p^3*e^2 - 2572500*b^3*d^3*f^2*g^6*p^3*e^3
 - 1457946*b^3*d^2*f^4*g^4*p^3*e^4 - 472500*b^3*d*f^6*g^2*p^3*e^5 + 50625*b
^3*f^8*p^3*e^6)*sqrt(h*x) - 16*(42*d^6*f*g*h^14*sqrt(-(1500625*b^4*d^4*g^8*
p^4*e^3 - 6894300*b^4*d^3*f^2*g^6*p^4*e^4 + 8469846*b^4*d^2*f^4*g^4*p^4*e^5
 - 1266300*b^4*d*f^6*g^2*p^4*e^6 + 50625*b^4*f^8*p^4*e^7)/(d^7*h^18)) - 428
75*b^2*d^5*g^6*h^5*p^2*e + 116865*b^2*d^4*f^2*g^4*h^5*p^2*e^2 - 50085*b^2*d
^3*f^4*g^2*h^5*p^2*e^3 + 3375*b^2*d^2*f^6*h^5*p^2*e^4)*sqrt(-(d^3*h^9*sqrt(
-(1500625*b^4*d^4*g^8*p^4*e^3 - 6894300*b^4*d^3*f^2*g^6*p^4*e^4 + 8469846*b
^4*d^2*f^4*g^4*p^4*e^5 - 1266300*b^4*d*f^6*g^2*p^4*e^6 + 50625*b^4*f^8*p^4*
e^7)/(d^7*h^18)) - 2940*b^2*d*f*g^3*p^2*e^2 + 1260*b^2*f^3*g*p^2*e^3)/(d^3*
h^9))) - (35*a*d*g^2*x^2 + 42*a*d*f*g*x + 15*a*d*f^2 + 4*(42*b*f*g*p*x^3 +
5*b*f^2*p*x^2)*e + (35*b*d*g^2*p*x^2 + 42*b*d*f*g*p*x + 15*b*d*f^2*p)*log(x
^2*e + d) + (35*b*d*g^2*x^2 + 42*b*d*f*g*x + 15*b*d*f^2)*log(c))*sqrt(h*x))
/(d*h^5*x^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7319 deep
```

Giac [A]

time = 5.68, size = 674, normalized size = 0.70

$$\frac{\frac{2\left(35\sqrt{2}\,(dh^2)^{\frac{1}{4}}bdg^2hpe^{\frac{7}{4}}-15\sqrt{2}\,(dh^2)^{\frac{1}{4}}bf^2hpe^{\frac{11}{4}}-42\sqrt{2}\,(dh^2)^{\frac{3}{4}}bfgpe^{\frac{9}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(dh^2)^{\frac{1}{4}}e^{\left(-\frac{1}{4}\right)}+2\sqrt{hx}\right)e^{\frac{1}{4}}}{2\,(dh^2)^{\frac{1}{4}}}\right)e^{(-1)}}{d^2h}+\frac{2\left(35\sqrt{2}\,(dh^2)^{\frac{1}{4}}bdg^2hpe^{\frac{7}{4}}-15\sqrt{2}\,(dh^2)^{\frac{1}{4}}bf^2hpe^{\frac{11}{4}}-42\sqrt{2}\,(dh^2)^{\frac{3}{4}}bfgpe^{\frac{9}{4}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(dh^2)^{\frac{1}{4}}e^{\left(-\frac{1}{4}\right)}-2\sqrt{hx}\right)e^{\frac{1}{4}}}{2\,(dh^2)^{\frac{1}{4}}}\right)e^{(-1)}}{d^2h}+\frac{\left(35\sqrt{2}\,(dh^2)^{\frac{1}{4}}bdg^2hpe^{\frac{7}{4}}-15\sqrt{2}\,(dh^2)^{\frac{1}{4}}bf^2hpe^{\frac{11}{4}}+42\sqrt{2}\,(dh^2)^{\frac{3}{4}}bfgpe^{\frac{9}{4}}\right)e^{(-1)}\log\left(\sqrt{2}\,(dh^2)^{\frac{1}{4}}\sqrt{hx}\,e^{\left(-\frac{1}{4}\right)}+hx+\sqrt{dh^2}\,e^{\left(-\frac{1}{2}\right)}\right)}{d^2h}-\frac{\left(35\sqrt{2}\,(dh^2)^{\frac{1}{4}}bdg^2hpe^{\frac{7}{4}}-15\sqrt{2}\,(dh^2)^{\frac{1}{4}}bf^2hpe^{\frac{11}{4}}+42\sqrt{2}\,(dh^2)^{\frac{3}{4}}bfgpe^{\frac{9}{4}}\right)e^{(-1)}\log\left(-\sqrt{2}\,(dh^2)^{\frac{1}{4}}\sqrt{hx}\,e^{\left(-\frac{1}{4}\right)}+hx+\sqrt{dh^2}\,e^{\left(-\frac{1}{2}\right)}\right)}{d^2h}-\frac{2\left(168bfgh^4px^3e+35bdg^2h^4px^2\log\left(h^2x^2e+dh^2\right)-35bdg^2h^4px^2\log\left(h^2\right)+20bf^2h^4px^2e+42bdfgh^4px\log\left(h^2x^2e+dh^2\right)-42bdfgh^4px\log\left(h^2\right)+35bdg^2h^4x^2\log\left(c\right)+35adg^2h^4x^2+15bdf^2h^4p\log\left(h^2x^2e+dh^2\right)-15bdf^2h^4p\log\left(h^2\right)+42bdfgh^4x\log\left(c\right)+42adfgh^4x+15bdf^2h^4\log\left(c\right)+15adf^2h^4\right)}{\sqrt{hx}\,dh^3x^3}}{105\,h^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(9/2),x, algorithm="giac
")
```

```
[Out] 1/105*(2*(35*sqrt(2)*(d*h^2)^(1/4)*b*d*g^2*h*p*e^(7/4) - 15*sqrt(2)*(d*h^2)
^(1/4)*b*f^2*h*p*e^(11/4) - 42*sqrt(2)*(d*h^2)^(3/4)*b*f*g*p*e^(9/4))*arcta
n(1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) + 2*sqrt(h*x))*e^(1/4)/(d*h^2
)^(1/4))*e^(-1)/(d^2*h) + 2*(35*sqrt(2)*(d*h^2)^(1/4)*b*d*g^2*h*p*e^(7/4) -
 15*sqrt(2)*(d*h^2)^(1/4)*b*f^2*h*p*e^(11/4) - 42*sqrt(2)*(d*h^2)^(3/4)*b*f
*g*p*e^(9/4))*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2)^(1/4)*e^(-1/4) - 2*sqrt(
h*x))*e^(1/4)/(d*h^2)^(1/4))*e^(-1)/(d^2*h) + (35*sqrt(2)*(d*h^2)^(1/4)*b*d
*g^2*h*p*e^(7/4) - 15*sqrt(2)*(d*h^2)^(1/4)*b*f^2*h*p*e^(11/4) + 42*sqrt(2)
*(d*h^2)^(3/4)*b*f*g*p*e^(9/4))*e^(-1)*log(sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*
e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2))/(d^2*h) - (35*sqrt(2)*(d*h^2)^(1/4)*
b*d*g^2*h*p*e^(7/4) - 15*sqrt(2)*(d*h^2)^(1/4)*b*f^2*h*p*e^(11/4) + 42*sqrt
(2)*(d*h^2)^(3/4)*b*f*g*p*e^(9/4))*e^(-1)*log(-sqrt(2)*(d*h^2)^(1/4)*sqrt(h
*x)*e^(-1/4) + h*x + sqrt(d*h^2)*e^(-1/2))/(d^2*h) - 2*(168*b*f*g*h^4*p*x^3
*e + 35*b*d*g^2*h^4*p*x^2*log(h^2*x^2*e + d*h^2) - 35*b*d*g^2*h^4*p*x^2*log
(h^2) + 20*b*f^2*h^4*p*x^2*e + 42*b*d*f*g*h^4*p*x*log(h^2*x^2*e + d*h^2) -
42*b*d*f*g*h^4*p*x*log(h^2) + 35*b*d*g^2*h^4*x^2*log(c) + 35*a*d*g^2*h^4*x^
2 + 15*b*d*f^2*h^4*p*log(h^2*x^2*e + d*h^2) - 15*b*d*f^2*h^4*p*log(h^2) + 4
2*b*d*f*g*h^4*x*log(c) + 42*a*d*f*g*h^4*x + 15*b*d*f^2*h^4*log(c) + 15*a*d*
f^2*h^4)/(sqrt(h*x)*d*h^3*x^3))/h^5
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(f+g\,x\right)}^2\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{{\left(h\,x\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(9/2),x)
```

```
[Out] int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(9/2), x)
```

3.616 $\int \frac{\sqrt{hx}\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{f+gx}\,dx$

Optimal. Leaf size=1680

$$\frac{2a\sqrt{hx}}{g}-\frac{8bp\sqrt{hx}}{g}-\frac{2\sqrt{2}\,b\sqrt[4]{d}\,\sqrt{h}\,p\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{\sqrt[4]{e}\,g}+\frac{2\sqrt{2}\,b\sqrt[4]{d}\,\sqrt{h}\,p\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{h}}{\sqrt[4]{d}\,\sqrt{h}}\right.}{\sqrt[4]{e}\,g}$$

```
[Out] -2*b*d^(1/4)*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2
)*h^(1/2)/e^(1/4)/g+2*b*d^(1/4)*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1
/4)/h^(1/2))*2^(1/2)*h^(1/2)/e^(1/4)/g-b*d^(1/4)*p*ln(d^(1/2)*h^(1/2)+x*e^(
1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)*h^(1/2)/e^(1/4)/g
+b*d^(1/4)*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(
h*x)^(1/2))*2^(1/2)*h^(1/2)/e^(1/4)/g-2*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/
h^(1/2))*(a+b*ln(c*(e*x^2+d)^p))*f^(1/2)*h^(1/2)/g^(3/2)-8*b*p*arctan(g^(1/
2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*h^(1/2)/(f^(1/2)*h^(1/2)-I*g^(
1/2)*(h*x)^(1/2)))*f^(1/2)*h^(1/2)/g^(3/2)+2*b*p*arctan(g^(1/2)*(h*x)^(1/2)
/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h)^(1/2)-e^(1/
4)*(h*x)^(1/2))/((-d)^(1/4)*g^(1/2)*(-h)^(1/2)-I*e^(1/4)*f^(1/2)*h^(1/2))/(
f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*f^(1/2)*h^(1/2)/g^(3/2)+2*b*p*arcta
n(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(-2*f^(1/2)*g^(1/2)*((-d)^(1/4)*h^
(1/2)-e^(1/4)*(h*x)^(1/2))/(I*e^(1/4)*f^(1/2)-(-d)^(1/4)*g^(1/2))/(f^(1/2)*
h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*f^(1/2)*h^(1/2)/g^(3/2)+2*b*p*arctan(g^(1/2
)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h
)^(1/2)+e^(1/4)*(h*x)^(1/2))/((-d)^(1/4)*g^(1/2)*(-h)^(1/2)+I*e^(1/4)*f^(1/
2)*h^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*f^(1/2)*h^(1/2)/g^(3/2
)+2*b*p*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*g^(1/2)*((
-d)^(1/4)*h^(1/2)+e^(1/4)*(h*x)^(1/2))/(I*e^(1/4)*f^(1/2)+(-d)^(1/4)*g^(1/2
))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*f^(1/2)*h^(1/2)/g^(3/2)-I*b*p*p
olylog(2,1-2*f^(1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h)^(1/2)-e^(1/4)*(h*x)^(
1/2))/((-d)^(1/4)*g^(1/2)*(-h)^(1/2)-I*e^(1/4)*f^(1/2)*h^(1/2))/(f^(1/2)*h^
(1/2)-I*g^(1/2)*(h*x)^(1/2)))*f^(1/2)*h^(1/2)/g^(3/2)-I*b*p*polylog(2,1-2*f
^(1/2)*g^(1/2)*((-d)^(1/4)*h^(1/2)+e^(1/4)*(h*x)^(1/2))/(I*e^(1/4)*f^(1/2)+
(-d)^(1/4)*g^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*f^(1/2)*h^(1/2
)/g^(3/2)+4*I*b*p*polylog(2,1-2*f^(1/2)*h^(1/2)/(f^(1/2)*h^(1/2)-I*g^(1/2)*
(h*x)^(1/2)))*f^(1/2)*h^(1/2)/g^(3/2)-I*b*p*polylog(2,1+2*f^(1/2)*g^(1/2)*(
(-d)^(1/4)*h^(1/2)-e^(1/4)*(h*x)^(1/2))/(I*e^(1/4)*f^(1/2)-(-d)^(1/4)*g^(1/
2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*f^(1/2)*h^(1/2)/g^(3/2)-I*b*p*
polylog(2,1-2*f^(1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h)^(1/2)+e^(1/4)*(h*x)^
(1/2))/((-d)^(1/4)*g^(1/2)*(-h)^(1/2)+I*e^(1/4)*f^(1/2)*h^(1/2))/(f^(1/2)*h
^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*f^(1/2)*h^(1/2)/g^(3/2)+2*a*(h*x)^(1/2)/g-8*
b*p*(h*x)^(1/2)/g+2*b*ln(c*(e*x^2+d)^p)*(h*x)^(1/2)/g
```

Rubi [A]

time = 2.03, antiderivative size = 1680, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 20, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.645$, Rules used = {2517, 2526, 2498, 327, 217, 1179, 642, 1176, 631, 210, 211, 2520, 12, 266, 6857, 5048, 4966, 2449, 2352, 2497}

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[h*x]*(a + b*Log[c*(d + e*x^2)^p]))/(f + g*x),x]
```

```
[Out] (2*a*Sqrt[h*x])/g - (8*b*p*Sqrt[h*x])/g - (2*Sqrt[2]*b*d^(1/4)*Sqrt[h]*p*Ar
cTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(e^(1/4)*g) + (2*S
qrt[2]*b*d^(1/4)*Sqrt[h]*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*
Sqrt[h])])/(e^(1/4)*g) + (2*b*Sqrt[h*x]*Log[c*(d + e*x^2)^p])/g - (2*Sqrt[f
]*Sqrt[h]*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*(a + b*Log[c*(d + e
*x^2)^p]))/g^(3/2) - (Sqrt[2]*b*d^(1/4)*Sqrt[h]*p*Log[Sqrt[d]*Sqrt[h] + Sqr
t[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*g) + (Sqrt[2]
*b*d^(1/4)*Sqrt[h]*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1
/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*g) - (8*b*Sqrt[f]*Sqrt[h]*p*ArcTan[(Sqrt[g
]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[h])/(Sqrt[f]*Sqrt[h] -
I*Sqrt[g]*Sqrt[h*x])])/g^(3/2) + (2*b*Sqrt[f]*Sqrt[h]*p*ArcTan[(Sqrt[g]*Sqr
t[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[
-h] - e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h] - I*e^(1/4)*Sqrt[f]
*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/g^(3/2) + (2*b*Sqrt[f]
*Sqrt[h]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(-2*Sqrt[f]*Sq
rt[g]*((-d)^(1/4)*Sqrt[h] - e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] - (-d)^
(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/g^(3/2) + (2*b*Sq
rt[f]*Sqrt[h]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[f
]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] + e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*S
qrt[g]*Sqrt[-h] + I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*S
qrt[h*x]))])/g^(3/2) + (2*b*Sqrt[f]*Sqrt[h]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(S
qrt[f]*Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] + e^(1/4)*Sqrt[
h*x]))/((I*e^(1/4)*Sqrt[f] + (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[
g]*Sqrt[h*x]))])/g^(3/2) + ((4*I)*b*Sqrt[f]*Sqrt[h]*p*PolyLog[2, 1 - (2*Sqr
t[f]*Sqrt[h])/(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x])])/g^(3/2) - (I*b*Sqrt
[f]*Sqrt[h]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h
] - e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h] - I*e^(1/4)*Sqrt[f]*S
qrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/g^(3/2) - (I*b*Sqrt[f]*S
qrt[h]*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] - e^(1/4)*Sq
rt[h*x]))/((I*e^(1/4)*Sqrt[f] - (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sq
rt[g]*Sqrt[h*x]))])/g^(3/2) - (I*b*Sqrt[f]*Sqrt[h]*p*PolyLog[2, 1 - (2*Sqrt
[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] + e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)
*Sqrt[g]*Sqrt[-h] + I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]
*Sqrt[h*x]))])/g^(3/2) - (I*b*Sqrt[f]*Sqrt[h]*p*PolyLog[2, 1 - (2*Sqrt[f]*S
```

```
qrt[g]*((-d)^(1/4)*Sqrt[h] + e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] + (-d)
^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/g^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2517

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
```

```
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*(
d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{hx}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{f+gx}\,dx = \frac{2\text{Subst}\left(\int \frac{x^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{f+\frac{gx^2}{h}}\,dx,x,\sqrt{hx}\right)}{h}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{h\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{g} - \frac{fh\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{g\left(f+\frac{gx^2}{h}\right)}\right)\,dx,x,\sqrt{h}\right.}{h}$$

$$= \frac{2\text{Subst}\left(\int \left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)\,dx,x,\sqrt{hx}\right)}{g} - \frac{(2f)\text{Subst}\left(\right.}{}$$

$$= \frac{2a\sqrt{hx}}{g} - \frac{2\sqrt{f}\sqrt{h}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{g^{3/2}}$$

$$= \frac{2a\sqrt{hx}}{g} + \frac{2b\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{g} - \frac{2\sqrt{f}\sqrt{h}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right.}{g}$$

$$= \frac{2a\sqrt{hx}}{g} - \frac{8bp\sqrt{hx}}{g} + \frac{2b\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{g} - \frac{2\sqrt{f}\sqrt{h}\tan}{}$$

$$= \frac{2a\sqrt{hx}}{g} - \frac{8bp\sqrt{hx}}{g} + \frac{2b\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{g} - \frac{2\sqrt{f}\sqrt{h}\tan}{}$$

$$= \frac{2a\sqrt{hx}}{g} - \frac{8bp\sqrt{hx}}{g} + \frac{2b\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{g} - \frac{2\sqrt{f}\sqrt{h}\tan}{}$$

$$= \frac{2a\sqrt{hx}}{g} - \frac{8bp\sqrt{hx}}{g} + \frac{2b\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{g} - \frac{2\sqrt{f}\sqrt{h}\tan}{}$$

Mathematica [A]
time = 0.91, size = 1471, normalized size = 0.88

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[h*x]*(a + b*Log[c*(d + e*x^2)^p]))/(f + g*x),x]
```

```
[Out] (Sqrt[h*x]*(2*a*Sqrt[g]*Sqrt[x] - (b*Sqrt[g]*p*(8*e^(1/4)*Sqrt[x] + 2*Sqrt[
2]*d^(1/4)*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - 2*Sqrt[2]*d^(1/4
)*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] + Sqrt[2]*d^(1/4)*Log[Sqrt[
d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x] - Sqrt[2]*d^(1/4)*Log[Sqr
t[d] + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x]))/e^(1/4) + 2*b*Sqrt[g]
*Sqrt[x]*Log[c*(d + e*x^2)^p] + Sqrt[-f]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]]*(a
 + b*Log[c*(d + e*x^2)^p]) - Sqrt[-f]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]]*(a +
b*Log[c*(d + e*x^2)^p]) - b*Sqrt[-f]*p*(Log[(Sqrt[g]*((-d)^(1/4) - e^(1/4)*
Sqrt[x]))/(-(e^(1/4)*Sqrt[-f]) + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g
]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) + I*e^(1/4)*Sqrt[x]))/(I*e^(1/4)*Sqrt
[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*
(I*(-d)^(1/4) + e^(1/4)*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(-d)^(1/4)*Sqrt[g])
]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) + e^(1/4)*Sqrt
[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[
x]] + PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] -
 (-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(
e^(1/4)*Sqrt[-f] - I*(-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f] -
 Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(-d)^(1/4)*Sqrt[g])] + PolyLog[2,
(e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[
g])]) + b*Sqrt[-f]*p*(Log[(Sqrt[g]*((-d)^(1/4) - e^(1/4)*Sqrt[x]))/(e^(1/4)
*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + Log[(Sqr
t[g]*((-d)^(1/4) - I*e^(1/4)*Sqrt[x]))/(I*e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqr
t[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) + I*e^(1/
4)*Sqrt[x]))/((-I)*e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + S
qrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) + e^(1/4)*Sqrt[x]))/(-(e^(1/4)*S
qrt[-f]) + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + PolyLog[2
, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] - (-d)^(1/4)*Sqr
t[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f
] - I*(-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x
]))/(e^(1/4)*Sqrt[-f] + I*(-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[
-f] + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])])))/(g^(3/2
)*Sqrt[x])
```

Maple [F]
time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{hx}\,\left(a+b\ln\left(c(e\,x^2+d)^p\right)\right)}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x)^(1/2)*(a+b*ln(c*(e*x^2+d)^p))/(g*x+f),x)
```

```
[Out] int((h*x)^(1/2)*(a+b*ln(c*(e*x^2+d)^p))/(g*x+f),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)^(1/2)*(a+b*log(c*(e*x^2+d)^p))/(g*x+f),x, algorithm="maxima
")
```

```
[Out] b*integrate((sqrt(h)*p*sqrt(x)*log(x^2*e + d) + sqrt(h)*sqrt(x)*log(c))/(g*
x + f), x) - 2*(f*h^2*arctan(sqrt(h*x)*g/sqrt(f*g*h))/(sqrt(f*g*h)*g) - sqr
t(h*x)*h/g)*a/h
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)^(1/2)*(a+b*log(c*(e*x^2+d)^p))/(g*x+f),x, algorithm="fricas
")
```

```
[Out] integral((sqrt(h*x)*b*log((x^2*e + d)^p*c) + sqrt(h*x)*a)/(g*x + f), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)**(1/2)*(a+b*ln(c*(e*x**2+d)**p))/(g*x+f),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x)^(1/2)*(a+b*log(c*(e*x^2+d)^p))/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate(sqrt(h*x)*(b*log((x^2*e + d)^p*c) + a)/(g*x + f), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{h\,x}\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{f+g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((h*x)^(1/2)*(a + b*log(c*(d + e*x^2)^p)))/(f + g*x),x)
[Out] int(((h*x)^(1/2)*(a + b*log(c*(d + e*x^2)^p)))/(f + g*x), x)
```

3.617 $\int \frac{a+b\log\left(c(d+ex^2)^p\right)}{\sqrt{hx}\,(f+gx)}\,dx$

Optimal. Leaf size=1361

$$\frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}}+\frac{8bp\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{f}\sqrt{h}}{\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}}-\frac{2bp}{}$$

```
[Out] 2*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*(a+b*ln(c*(e*x^2+d)^p))/f^(1/
2)/g^(1/2)/h^(1/2)+8*b*p*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2*f
^(1/2)*h^(1/2)/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))/f^(1/2)/g^(1/2)/h^(
1/2)-2*b*p*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*g^(1/2)
*h^(1/2)*((-d)^(1/4)*(-h)^(1/2)-e^(1/4)*(h*x)^(1/2))/((-d)^(1/4)*g^(1/2)*(-
h)^(1/2)-I*e^(1/4)*f^(1/2)*h^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2))
)/f^(1/2)/g^(1/2)/h^(1/2)-2*b*p*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))
*ln(-2*f^(1/2)*g^(1/2)*((-d)^(1/4)*h^(1/2)-e^(1/4)*(h*x)^(1/2))/(I*e^(1/4)*
f^(1/2)-(-d)^(1/4)*g^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))/f^(1/2
)/g^(1/2)/h^(1/2)-2*b*p*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2*f^
(1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h)^(1/2)+e^(1/4)*(h*x)^(1/2))/((-d)^(1/
4)*g^(1/2)*(-h)^(1/2)+I*e^(1/4)*f^(1/2)*h^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)
*(h*x)^(1/2)))/f^(1/2)/g^(1/2)/h^(1/2)-2*b*p*arctan(g^(1/2)*(h*x)^(1/2)/f^(
1/2)/h^(1/2))*ln(2*f^(1/2)*g^(1/2)*((-d)^(1/4)*h^(1/2)+e^(1/4)*(h*x)^(1/2))
/(I*e^(1/4)*f^(1/2)+(-d)^(1/4)*g^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1
/2)))/f^(1/2)/g^(1/2)/h^(1/2)-4*I*b*p*polylog(2,1-2*f^(1/2)*h^(1/2)/(f^(1/2
)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))/f^(1/2)/g^(1/2)/h^(1/2)+I*b*p*polylog(2,1
-2*f^(1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h)^(1/2)-e^(1/4)*(h*x)^(1/2))/((-d
)^(1/4)*g^(1/2)*(-h)^(1/2)-I*e^(1/4)*f^(1/2)*h^(1/2))/(f^(1/2)*h^(1/2)-I*g^
(1/2)*(h*x)^(1/2)))/f^(1/2)/g^(1/2)/h^(1/2)+I*b*p*polylog(2,1+2*f^(1/2)*g^(
1/2)*((-d)^(1/4)*h^(1/2)-e^(1/4)*(h*x)^(1/2))/(I*e^(1/4)*f^(1/2)-(-d)^(1/4)
*g^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))/f^(1/2)/g^(1/2)/h^(1/2)+
I*b*p*polylog(2,1-2*f^(1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h)^(1/2)+e^(1/4)*
(h*x)^(1/2))/((-d)^(1/4)*g^(1/2)*(-h)^(1/2)+I*e^(1/4)*f^(1/2)*h^(1/2))/(f^(
1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))/f^(1/2)/g^(1/2)/h^(1/2)+I*b*p*polylog(
2,1-2*f^(1/2)*g^(1/2)*((-d)^(1/4)*h^(1/2)+e^(1/4)*(h*x)^(1/2))/(I*e^(1/4)*f
^(1/2)+(-d)^(1/4)*g^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))/f^(1/2)
/g^(1/2)/h^(1/2)
```

Rubi [A]
time = 1.17, antiderivative size = 1361, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {2517, 211, 2520, 12, 266, 6857, 5048, 4966, 2449, 2352, 2497}

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^2)^p])/(Sqrt[h*x]*(f + g*x)),x]
```

```
[Out] (2*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*(a + b*Log[c*(d + e*x^2)^p
]))/(Sqrt[f]*Sqrt[g]*Sqrt[h]) + (8*b*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*
Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[h])/(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x])])
/(Sqrt[f]*Sqrt[g]*Sqrt[h]) - (2*b*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqr
t[h])]*Log[(2*Sqrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] - e^(1/4)*Sqrt[h
*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h] - I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*S
qrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) - (2*b*p*ArcTan[
(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(-2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*
Sqrt[h] - e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] - (-d)^(1/4)*Sqrt[g])*(Sq
rt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) - (2*b*p*
ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[g]*Sqrt[h
]*((-d)^(1/4)*Sqrt[-h] + e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h]
+ I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(Sq
rt[f]*Sqrt[g]*Sqrt[h]) - (2*b*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h]
)]*Log[(2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] + e^(1/4)*Sqrt[h*x]))/((I*e^(
1/4)*Sqrt[f] + (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))
])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) - ((4*I)*b*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[h]
)/(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x])])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) + (I*
b*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] - e^(1/4
)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h] - I*e^(1/4)*Sqrt[f]*Sqrt[h])*(S
qrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) + (I*b*p
*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] - e^(1/4)*Sqrt[h*x])
)/((I*e^(1/4)*Sqrt[f] - (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sq
rt[h*x]))])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) + (I*b*p*PolyLog[2, 1 - (2*Sqrt[f]*Sq
rt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] + e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[
g]*Sqrt[-h] + I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[
h*x]))])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) + (I*b*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[
g]*((-d)^(1/4)*Sqrt[h] + e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] + (-d)^(1/
4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(Sqrt[f]*Sqrt[g]*Sqr
t[h])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
```

```
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2517

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*(
d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{a+b\log\left(c(d+ex^2)^p\right)}{\sqrt{hx}\,(f+gx)}\,dx = \frac{2\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{f+\frac{gx^2}{h}}\,dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{hx}}{\sqrt{f}\,\sqrt{h}}\right)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{f}\,\sqrt{g}\,\sqrt{h}} - \frac{(8bep)\text{Subst}\left(\int \frac{\sqrt{h}\,x^3\tan\cdots}{\sqrt{f}\cdots}\right.}{\cdots}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{hx}}{\sqrt{f}\,\sqrt{h}}\right)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{f}\,\sqrt{g}\,\sqrt{h}} - \frac{(8bep)\text{Subst}\left(\int \frac{x^3\tan^{-1}\left(\cdots\right.}{d+\cdots}\right.}{\sqrt{f}\,\sqrt{\cdots}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{hx}}{\sqrt{f}\,\sqrt{h}}\right)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{f}\,\sqrt{g}\,\sqrt{h}} - \frac{(8bep)\text{Subst}\left(\int \left(\frac{h^2x\tan^{-}\cdots}{2\left(-\sqrt{-}\cdots\right.}\right.\right.}{\cdots}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{hx}}{\sqrt{f}\,\sqrt{h}}\right)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{f}\,\sqrt{g}\,\sqrt{h}} - \frac{(4bep)\text{Subst}\left(\int \frac{x\tan^{-1}\left(\cdots\right.}{-\sqrt{-d}\cdots}\right.}{\sqrt{f}\,\sqrt{\cdots}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{hx}}{\sqrt{f}\,\sqrt{h}}\right)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{f}\,\sqrt{g}\,\sqrt{h}} - \frac{(4bep)\text{Subst}\left(\int \left(-\frac{\text{t}\cdots}{2e^{3/4}\left(\cdots\right.}\right.\right.}{\cdots}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{hx}}{\sqrt{f}\,\sqrt{h}}\right)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{f}\,\sqrt{g}\,\sqrt{h}} + \frac{\left(2b\sqrt[4]{e}\,p\right)\text{Subst}\left(\int \frac{\tan^{-1}\cdots}{\sqrt[4]{-d}\cdots}\right.}{\sqrt{f}\cdots}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{hx}}{\sqrt{f}\,\sqrt{h}}\right)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{f}\,\sqrt{g}\,\sqrt{h}} + \frac{8bp\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{hx}}{\sqrt{f}\,\sqrt{h}}\right)\text{l}\cdots}{\sqrt{f}\cdots}$$

Mathematica [**A**]
time = 0.28, size = 1297, normalized size = 0.95

$$\frac{\sqrt{x}\left(a\log\left(\sqrt{-f}-\sqrt{g}\sqrt{x}\right)-bp\log\left(\frac{\sqrt{g}\left(\sqrt[4]{-d}-\sqrt[4]{e}\sqrt{x}\right)}{-\sqrt[4]{e}\sqrt{-f}+\sqrt[4]{-d}\sqrt{g}}\right)\log\left(\sqrt{-f}-\sqrt{g}\sqrt{x}\right)-bp\log\left(\frac{\sqrt{g}\left(\sqrt[4]{-d}+i\sqrt[4]{e}\sqrt{x}\right)}{i\sqrt[4]{e}\sqrt{-f}+\sqrt[4]{-d}\sqrt{g}}\right)\log\left(\sqrt{-f}-\sqrt{g}\sqrt{x}\right)-bp\log\left(\frac{\sqrt{g}\left(i\sqrt[4]{-d}+\sqrt[4]{e}\sqrt{x}\right)}{\sqrt[4]{e}\sqrt{-f}+i\sqrt[4]{-d}\sqrt{g}}\right)\log\left(\sqrt{-f}-\sqrt{g}\sqrt{x}\right)-bp\log\left(\frac{\sqrt{g}\left(\sqrt[4]{-d}+\sqrt[4]{e}\sqrt{x}\right)}{\sqrt[4]{e}\sqrt{-f}+\sqrt[4]{-d}\sqrt{g}}\right)\log\left(\sqrt{-f}-\sqrt{g}\sqrt{x}\right)-a\log\left(\sqrt{-f}+\sqrt{g}\sqrt{x}\right)+bp\log\left(\frac{\sqrt{g}\left(\sqrt[4]{-d}-\sqrt[4]{e}\sqrt{x}\right)}{\sqrt[4]{e}\sqrt{-f}+\sqrt[4]{-d}\sqrt{g}}\right)\log\left(\sqrt{-f}+\sqrt{g}\sqrt{x}\right)+bp\log\left(\frac{\sqrt{g}\left(\sqrt[4]{-d}-i\sqrt[4]{e}\sqrt{x}\right)}{i\sqrt[4]{e}\sqrt{-f}+\sqrt[4]{-d}\sqrt{g}}\right)\log\left(\sqrt{-f}+\sqrt{g}\sqrt{x}\right)+bp\log\left(\frac{\sqrt{g}\left(\sqrt[4]{-d}+i\sqrt[4]{e}\sqrt{x}\right)}{-i\sqrt[4]{e}\sqrt{-f}+\sqrt[4]{-d}\sqrt{g}}\right)\log\left(\sqrt{-f}+\sqrt{g}\sqrt{x}\right)+bp\log\left(\frac{\sqrt{g}\left(\sqrt[4]{-d}+\sqrt[4]{e}\sqrt{x}\right)}{-\sqrt[4]{e}\sqrt{-f}+\sqrt[4]{-d}\sqrt{g}}\right)\log\left(\sqrt{-f}+\sqrt{g}\sqrt{x}\right)+b\log\left(\sqrt{-f}-\sqrt{g}\sqrt{x}\right)\log\left(c\left(d+ex^2\right)^p\right)-b\log\left(\sqrt{-f}+\sqrt{g}\sqrt{x}\right)\log\left(c\left(d+ex^2\right)^p\right)-bp\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{e}\left(\sqrt{-f}-\sqrt{g}\sqrt{x}\right)}{\sqrt[4]{e}\sqrt{-f}-\sqrt[4]{-d}\sqrt{g}}\right)-bp\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{e}\left(\sqrt{-f}-\sqrt{g}\sqrt{x}\right)}{\sqrt[4]{e}\sqrt{-f}-i\sqrt[4]{-d}\sqrt{g}}\right)-bp\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{e}\left(\sqrt{-f}-\sqrt{g}\sqrt{x}\right)}{\sqrt[4]{e}\sqrt{-f}+i\sqrt[4]{-d}\sqrt{g}}\right)-bp\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{e}\left(\sqrt{-f}-\sqrt{g}\sqrt{x}\right)}{\sqrt[4]{e}\sqrt{-f}+\sqrt[4]{-d}\sqrt{g}}\right)+bp\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{e}\left(\sqrt{-f}+\sqrt{g}\sqrt{x}\right)}{\sqrt[4]{e}\sqrt{-f}-\sqrt[4]{-d}\sqrt{g}}\right)+bp\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{e}\left(\sqrt{-f}+\sqrt{g}\sqrt{x}\right)}{\sqrt[4]{e}\sqrt{-f}-i\sqrt[4]{-d}\sqrt{g}}\right)+bp\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{e}\left(\sqrt{-f}+\sqrt{g}\sqrt{x}\right)}{\sqrt[4]{e}\sqrt{-f}+i\sqrt[4]{-d}\sqrt{g}}\right)+bp\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{e}\left(\sqrt{-f}+\sqrt{g}\sqrt{x}\right)}{\sqrt[4]{e}\sqrt{-f}+\sqrt[4]{-d}\sqrt{g}}\right)\right)}{\sqrt{-f}\sqrt{g}\sqrt{hx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^2)^p])/(Sqrt[h*x]*(f + g*x)),x]
```

```
[Out] (Sqrt[x]*(a*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] - b*p*Log[(Sqrt[g]*((-d)^(1/4)
- e^(1/4)*Sqrt[x]))/(-(e^(1/4)*Sqrt[-f]) + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f
] - Sqrt[g]*Sqrt[x]] - b*p*Log[(Sqrt[g]*((-d)^(1/4) + I*e^(1/4)*Sqrt[x]))/(
I*e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] -
 b*p*Log[(Sqrt[g]*(I*(-d)^(1/4) + e^(1/4)*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(
-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] - b*p*Log[(Sqrt[g]*((-d
)^(1/4) + e^(1/4)*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sq
rt[-f] - Sqrt[g]*Sqrt[x]] - a*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + b*p*Log[(Sq
rt[g]*((-d)^(1/4) - e^(1/4)*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g
])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + b*p*Log[(Sqrt[g]*((-d)^(1/4) - I*e^(1
/4)*Sqrt[x]))/(I*e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqr
t[g]*Sqrt[x]] + b*p*Log[(Sqrt[g]*((-d)^(1/4) + I*e^(1/4)*Sqrt[x]))/((-I)*e^
(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + b*p
*Log[(Sqrt[g]*((-d)^(1/4) + e^(1/4)*Sqrt[x]))/(-(e^(1/4)*Sqrt[-f]) + (-d)^(
1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + b*Log[Sqrt[-f] - Sqrt[g]*S
qrt[x]]*Log[c*(d + e*x^2)^p] - b*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]]*Log[c*(d +
 e*x^2)^p] - b*p*PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)
*Sqrt[-f] - (-d)^(1/4)*Sqrt[g])] - b*p*PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt
[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] - I*(-d)^(1/4)*Sqrt[g])] - b*p*PolyLog[2, (
e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(-d)^(1/4)*Sqrt
[g])] - b*p*PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt
[-f] + (-d)^(1/4)*Sqrt[g])] + b*p*PolyLog[2, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*S
qrt[x]))/(e^(1/4)*Sqrt[-f] - (-d)^(1/4)*Sqrt[g])] + b*p*PolyLog[2, (e^(1/4)
*(Sqrt[-f] + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] - I*(-d)^(1/4)*Sqrt[g])] +
 b*p*PolyLog[2, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] +
I*(-d)^(1/4)*Sqrt[g])] + b*p*PolyLog[2, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x
]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]))/(Sqrt[-f]*Sqrt[g]*Sqrt[h*x])
```

Maple [**F**]
time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{a+b\ln\left(c\left(e\,x^2+d\right)^p\right)}{\sqrt{hx}\ \left(gx+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(e*x^2+d)^p))/(h*x)^(1/2)/(g*x+f),x)
```

[Out] int((a+b*ln(c*(e*x^2+d)^p))/(h*x)^(1/2)/(g*x+f),x)

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2)/(g*x+f),x, algorithm="maxima
")
```

```
[Out] b*integrate((sqrt(h)*p*log(x^2*e + d) + sqrt(h)*log(c))/(g*h*x^(3/2) + f*h*
sqrt(x)), x) + 2*a*arctan(sqrt(h*x)*g/sqrt(f*g*h))/sqrt(f*g*h)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2)/(g*x+f),x, algorithm="fricas
")
```

```
[Out] integral((sqrt(h*x)*b*log((x^2*e + d)^p*c) + sqrt(h*x)*a)/(g*h*x^2 + f*h*x)
, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(e*x**2+d)**p))/(h*x)**(1/2)/(g*x+f),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2)/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate((b*log((x^2*e + d)^p*c) + a)/((g*x + f)*sqrt(h*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{\left(f+g\,x\right)\,\sqrt{h\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^2)^p))/((f + g*x)*(h*x)^(1/2)),x)
[Out] int((a + b*log(c*(d + e*x^2)^p))/((f + g*x)*(h*x)^(1/2)), x)
```

3.618 $\int \frac{a+b\log\left(c(d+ex^2)^p\right)}{(hx)^{3/2}(f+gx)}\,dx$

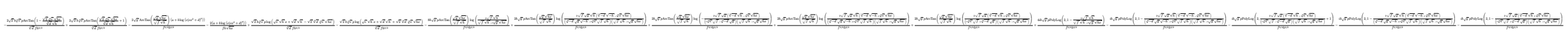
Optimal. Leaf size=1659

$$-\frac{2\sqrt{2}\,b\sqrt[4]{e}\,p\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{\sqrt[4]{d}\,fh^{3/2}}+\frac{2\sqrt{2}\,b\sqrt[4]{e}\,p\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{\sqrt[4]{d}\,fh^{3/2}}-\frac{2(a+b\log\left(c(d+ex\right.}{fh\sqrt{hx}}$$

```
[Out] -2*b*e^(1/4)*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2
)/d^(1/4)/f/h^(3/2)+2*b*e^(1/4)*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1
/4)/h^(1/2))*2^(1/2)/d^(1/4)/f/h^(3/2)+b*e^(1/4)*p*ln(d^(1/2)*h^(1/2)+x*e^(
1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(1/4)/f/h^(3/2)
-b*e^(1/4)*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(
h*x)^(1/2))*2^(1/2)/d^(1/4)/f/h^(3/2)-2*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/
h^(1/2))*(a+b*ln(c*(e*x^2+d)^p))*g^(1/2)/f^(3/2)/h^(3/2)-8*b*p*arctan(g^(1/
2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*h^(1/2)/(f^(1/2)*h^(1/2)-I*g^(
1/2)*(h*x)^(1/2)))*g^(1/2)/f^(3/2)/h^(3/2)+2*b*p*arctan(g^(1/2)*(h*x)^(1/2)
/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h)^(1/2)-e^(1/
4)*(h*x)^(1/2))/((-d)^(1/4)*g^(1/2)*(-h)^(1/2)-I*e^(1/4)*f^(1/2)*h^(1/2))/(
f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*g^(1/2)/f^(3/2)/h^(3/2)+2*b*p*arcta
n(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(-2*f^(1/2)*g^(1/2)*((-d)^(1/4)*h^
(1/2)-e^(1/4)*(h*x)^(1/2))/(I*e^(1/4)*f^(1/2)-(-d)^(1/4)*g^(1/2))/(f^(1/2)*
h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*g^(1/2)/f^(3/2)/h^(3/2)+2*b*p*arctan(g^(1/2
)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h
)^(1/2)+e^(1/4)*(h*x)^(1/2))/((-d)^(1/4)*g^(1/2)*(-h)^(1/2)+I*e^(1/4)*f^(1/
2)*h^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*g^(1/2)/f^(3/2)/h^(3/2
)+2*b*p*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*g^(1/2)*((
-d)^(1/4)*h^(1/2)+e^(1/4)*(h*x)^(1/2))/(I*e^(1/4)*f^(1/2)+(-d)^(1/4)*g^(1/2
))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*g^(1/2)/f^(3/2)/h^(3/2)-I*b*p*p
olylog(2,1-2*f^(1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h)^(1/2)+e^(1/4)*(h*x)^(
1/2))/((-d)^(1/4)*g^(1/2)*(-h)^(1/2)+I*e^(1/4)*f^(1/2)*h^(1/2))/(f^(1/2)*h^
(1/2)-I*g^(1/2)*(h*x)^(1/2)))*g^(1/2)/f^(3/2)/h^(3/2)+4*I*b*p*polylog(2,1-2
*f^(1/2)*h^(1/2)/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*g^(1/2)/f^(3/2)/h
^(3/2)-I*b*p*polylog(2,1-2*f^(1/2)*g^(1/2)*((-d)^(1/4)*h^(1/2)+e^(1/4)*(h*x
)^(1/2))/(I*e^(1/4)*f^(1/2)+(-d)^(1/4)*g^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*
(h*x)^(1/2)))*g^(1/2)/f^(3/2)/h^(3/2)-I*b*p*polylog(2,1-2*f^(1/2)*g^(1/2)*h
^(1/2)*((-d)^(1/4)*(-h)^(1/2)-e^(1/4)*(h*x)^(1/2))/((-d)^(1/4)*g^(1/2)*(-h)
^(1/2)-I*e^(1/4)*f^(1/2)*h^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*
g^(1/2)/f^(3/2)/h^(3/2)-I*b*p*polylog(2,1+2*f^(1/2)*g^(1/2)*((-d)^(1/4)*h^(
1/2)-e^(1/4)*(h*x)^(1/2))/(I*e^(1/4)*f^(1/2)-(-d)^(1/4)*g^(1/2))/(f^(1/2)*h
^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*g^(1/2)/f^(3/2)/h^(3/2)-2*(a+b*ln(c*(e*x^2+d
)^p))/f/h/(h*x)^(1/2)
```

<u>Rubi</u> [A]

time = 1.66, antiderivative size = 1659, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 19, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.613$, Rules used = {2517, 2526, 2505, 303, 1176, 631, 210, 1179, 642, 211, 2520, 12, 266, 6857, 5048, 4966, 2449, 2352, 2497}



Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^2)^p])/((h*x)^(3/2)*(f + g*x)),x]
```

```
[Out] (-2*Sqrt[2]*b*e^(1/4)*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqr
t[h])])/(d^(1/4)*f*h^(3/2)) + (2*Sqrt[2]*b*e^(1/4)*p*ArcTan[1 + (Sqrt[2]*e^
(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*f*h^(3/2)) - (2*(a + b*Log[c*
(d + e*x^2)^p]))/(f*h*Sqrt[h*x]) - (2*Sqrt[g]*ArcTan[(Sqrt[g]*Sqrt[h*x])/(S
qrt[f]*Sqrt[h])]*(a + b*Log[c*(d + e*x^2)^p]))/(f^(3/2)*h^(3/2)) + (Sqrt[2]
*b*e^(1/4)*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1
/4)*Sqrt[h*x]])/(d^(1/4)*f*h^(3/2)) - (Sqrt[2]*b*e^(1/4)*p*Log[Sqrt[d]*Sqrt
[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(d^(1/4)*f*h^
(3/2)) - (8*b*Sqrt[g]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(
2*Sqrt[f]*Sqrt[h])/(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x])])/(f^(3/2)*h^(3/
2)) + (2*b*Sqrt[g]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*S
qrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] - e^(1/4)*Sqrt[h*x]))/(((-d)^(1
/4)*Sqrt[g]*Sqrt[-h] - I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt
[g]*Sqrt[h*x]))])/(f^(3/2)*h^(3/2)) + (2*b*Sqrt[g]*p*ArcTan[(Sqrt[g]*Sqrt[h
*x])/(Sqrt[f]*Sqrt[h])]*Log[(-2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] - e^(1/
4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] - (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] -
 I*Sqrt[g]*Sqrt[h*x]))])/(f^(3/2)*h^(3/2)) + (2*b*Sqrt[g]*p*ArcTan[(Sqrt[g]
*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*S
qrt[-h] + e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h] + I*e^(1/4)*Sqr
t[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(f^(3/2)*h^(3/2))
+ (2*b*Sqrt[g]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[
f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] + e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] +
(-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(f^(3/2)*h^(
3/2)) + ((4*I)*b*Sqrt[g]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[h])/(Sqrt[f]*Sqrt
[h] - I*Sqrt[g]*Sqrt[h*x])])/(f^(3/2)*h^(3/2)) - (I*b*Sqrt[g]*p*PolyLog[2,
1 - (2*Sqrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] - e^(1/4)*Sqrt[h*x]))/(
((-d)^(1/4)*Sqrt[g]*Sqrt[-h] - I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h]
- I*Sqrt[g]*Sqrt[h*x]))])/(f^(3/2)*h^(3/2)) - (I*b*Sqrt[g]*p*PolyLog[2, 1 +
 (2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] - e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*S
qrt[f] - (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(f^
(3/2)*h^(3/2)) - (I*b*Sqrt[g]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*Sqrt[h]*(
(-d)^(1/4)*Sqrt[-h] + e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h] + I
*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(f^(3/
2)*h^(3/2)) - (I*b*Sqrt[g]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*
```

```
Sqrt[h] + e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] + (-d)^(1/4)*Sqrt[g])*(Sq
rt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(f^(3/2)*h^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2517

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*(
d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{a+b\log\left(c(d+ex^2)^p\right)}{(hx)^{3/2}(f+gx)}\,dx = \frac{2\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^2\left(f+\frac{gx^2}{h}\right)}\,dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{fx^2} - \frac{g\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{f(fh+gx^2)}\right)\,dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^2}\,dx, x, \sqrt{hx}\right)}{fh} - \frac{(2g)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{fh+gx^2}\,d\right.}{fh}$$

$$= -\frac{2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{fh\sqrt{hx}} - \frac{2\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{hx}}{\sqrt{f}\,\sqrt{h}}\right)\left(a+b\log\left(c(d+ex\right.\right.}{f^{3/2}h^{3/2}}$$

$$= -\frac{2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{fh\sqrt{hx}} - \frac{2\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{hx}}{\sqrt{f}\,\sqrt{h}}\right)\left(a+b\log\left(c(d+ex\right.\right.}{f^{3/2}h^{3/2}}$$

$$= -\frac{2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{fh\sqrt{hx}} - \frac{2\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{hx}}{\sqrt{f}\,\sqrt{h}}\right)\left(a+b\log\left(c(d+ex\right.\right.}{f^{3/2}h^{3/2}}$$

$$= -\frac{2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{fh\sqrt{hx}} - \frac{2\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{hx}}{\sqrt{f}\,\sqrt{h}}\right)\left(a+b\log\left(c(d+ex\right.\right.}{f^{3/2}h^{3/2}}$$

$$= -\frac{2\sqrt{2}\,b\sqrt[4]{e}\,p\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{\sqrt[4]{d}\,fh^{3/2}} + \frac{2\sqrt{2}\,b\sqrt[4]{e}\,p\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{e}}{\sqrt[4]{d}}\right.}{\sqrt[4]{d}\,fh^{3/2}}$$

$$= -\frac{2\sqrt{2}\,b\sqrt[4]{e}\,p\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{hx}}{\sqrt[4]{d}\,\sqrt{h}}\right)}{\sqrt[4]{d}\,fh^{3/2}} + \frac{2\sqrt{2}\,b\sqrt[4]{e}\,p\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{e}}{\sqrt[4]{d}}\right.}{\sqrt[4]{d}\,fh^{3/2}}$$

Mathematica [**A**]
time = 1.03, size = 1336, normalized size = 0.81

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^2)^p])/((h*x)^(3/2)*(f + g*x)),x]
```

```
[Out] (x^(3/2)*((4*b*e^(1/4)*p*(ArcTan[(e^(1/4)*Sqrt[x])/(-d)^(1/4)] + ArcTanh[(d
*e^(1/4)*Sqrt[x])/(-d)^(5/4)]))/(-d)^(1/4) - (2*(a + b*Log[c*(d + e*x^2)^p]
))/Sqrt[x] + (f*Sqrt[g]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]]*(a + b*Log[c*(d + e
*x^2)^p]))/(-f)^(3/2) + (Sqrt[g]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]]*(a + b*Log
[c*(d + e*x^2)^p]))/Sqrt[-f] + (b*Sqrt[g]*p*(Log[(Sqrt[g]*((-d)^(1/4) - e^(
1/4)*Sqrt[x]))/(-(e^(1/4)*Sqrt[-f]) + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - S
qrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) + I*e^(1/4)*Sqrt[x]))/(I*e^(1/4)
*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] + Log[(Sqr
t[g]*(I*(-d)^(1/4) + e^(1/4)*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(-d)^(1/4)*Sqr
t[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) + e^(1/4)
*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*
Sqrt[x]] + PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[
-f] - (-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x
]))/(e^(1/4)*Sqrt[-f] - I*(-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[
-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(-d)^(1/4)*Sqrt[g])] + PolyLo
g[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*
Sqrt[g])]))/Sqrt[-f] + (b*f*Sqrt[g]*p*(Log[(Sqrt[g]*((-d)^(1/4) - e^(1/4)*S
qrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sq
rt[x]] + Log[(Sqrt[g]*((-d)^(1/4) - I*e^(1/4)*Sqrt[x]))/(I*e^(1/4)*Sqrt[-f]
 + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d
)^(1/4) + I*e^(1/4)*Sqrt[x]))/((-I)*e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]
*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) + e^(1/4)*Sqrt[
x]))/(-(e^(1/4)*Sqrt[-f]) + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqr
t[x]] + PolyLog[2, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f]
 - (-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x]))
/(e^(1/4)*Sqrt[-f] - I*(-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f]
 + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(-d)^(1/4)*Sqrt[g])] + PolyLog[2
, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqr
t[g])]))/(-f)^(3/2)))/(f*(h*x)^(3/2))
```

Maple [**F**]
time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{a+b\ln \left(c(e\,x^{2}+d)^{p}\right)}{(hx)^{\frac{3}{2}}\,(gx+f)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(e*x^2+d)^p))/(h*x)^(3/2)/(g*x+f),x)
```

```
[Out] int((a+b*ln(c*(e*x^2+d)^p))/(h*x)^(3/2)/(g*x+f),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2)/(g*x+f),x, algorithm="maxima
")
```

```
[Out] b*integrate((sqrt(h)*p*log(x^2*e + d) + sqrt(h)*log(c))/(g*h^2*x^(5/2) + f*
h^2*x^(3/2)), x) - 2*a*(g*arctan(sqrt(h*x)*g/sqrt(f*g*h))/(sqrt(f*g*h)*f) +
 1/(sqrt(h*x)*f))/h
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2)/(g*x+f),x, algorithm="fricas
")
```

```
[Out] integral((sqrt(h*x)*b*log((x^2*e + d)^p*c) + sqrt(h*x)*a)/(g*h^2*x^3 + f*h^
2*x^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(e*x**2+d)**p))/(h*x)**(3/2)/(g*x+f),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2)/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate((b*log((x^2*e + d)^p*c) + a)/((g*x + f)*(h*x)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\ln\left(c\left(e\,x^2 + d\right)^p\right)}{\left(f + g\,x\right)\,\left(h\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^2)^p))/((f + g*x)*(h*x)^(3/2)),x)
[Out] int((a + b*log(c*(d + e*x^2)^p))/((f + g*x)*(h*x)^(3/2)), x)
```

3.619 $\int \frac{\log(fx^p)\log(1+ex^m)}{x} dx$

Optimal. Leaf size=33

$$-\frac{\log(fx^p)\,\text{Li}_2(-ex^m)}{m}+\frac{p\text{Li}_3(-ex^m)}{m^2}$$

```
[Out] -ln(f*x^p)*polylog(2,-e*x^m)/m+p*polylog(3,-e*x^m)/m^2
```

Rubi [A]

time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2421, 6724}

$$\frac{p\text{PolyLog}(3,-ex^m)}{m^2}-\frac{\log(fx^p)\,\text{PolyLog}(2,-ex^m)}{m}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[f*x^p]*Log[1 + e*x^m])/x,x]
```

```
[Out] -((Log[f*x^p]*PolyLog[2, -(e*x^m)])/m) + (p*PolyLog[3, -(e*x^m)])/m^2
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(fx^p)\log(1+ex^m)}{x}\,dx &= -\frac{\log(fx^p)\,\text{Li}_2(-ex^m)}{m}+\frac{p\int \frac{\text{Li}_2(-ex^m)}{x}\,dx}{m}\\ &= -\frac{\log(fx^p)\,\text{Li}_2(-ex^m)}{m}+\frac{p\text{Li}_3(-ex^m)}{m^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.00

$$-\frac{\log(fx^p)\operatorname{Li}_2(-ex^m)}{m}+\frac{p\operatorname{Li}_3(-ex^m)}{m^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[f*x^p]*Log[1 + e*x^m])/x,x]
```

```
[Out] -((Log[f*x^p]*PolyLog[2, -(e*x^m)])/m) + (p*PolyLog[3, -(e*x^m)])/m^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.45, size = 191, normalized size = 5.79

method	result
risch	$-\frac{p\ln(x)\,\mathrm{polylog}(2,-e\,x^m)}{m}+\frac{p\,\mathrm{polylog}(3,-e\,x^m)}{m^2}-\frac{(\ln(x^p)-\ln(x)p)\,\mathrm{dilog}(1+e\,x^m)}{m}+\frac{i\,\mathrm{dilog}(1+e\,x^m)\pi\,\mathrm{csgn}(if)\mathrm{csgn}(ix^p)\mathrm{csgn}}{2m}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(f*x^p)*ln(1+e*x^m)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/m*p*ln(x)*polylog(2,-e*x^m)+p*polylog(3,-e*x^m)/m^2-(ln(x^p)-ln(x)*p)/m*
dilog(1+e*x^m)+1/2*I/m*dilog(1+e*x^m)*Pi*csgn(I*f)*csgn(I*x^p)*csgn(I*f*x^p
)-1/2*I/m*dilog(1+e*x^m)*Pi*csgn(I*f)*csgn(I*f*x^p)^2-1/2*I/m*dilog(1+e*x^m
)*Pi*csgn(I*x^p)*csgn(I*f*x^p)^2+1/2*I/m*dilog(1+e*x^m)*Pi*csgn(I*f*x^p)^3-
1/m*dilog(1+e*x^m)*ln(f)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(f*x^p)*log(1+e*x^m)/x,x, algorithm="maxima")
```

```
[Out] -1/2*(p*log(x)^2 - 2*log(f)*log(x) - 2*log(x)*log(x^p))*log(e^(m*log(x) + 1
) + 1) - integrate(1/2*(2*m*e^(m*log(x) + 1)*log(x)*log(x^p) - (m*p*e*log(x
)^2 - 2*m*e*log(f)*log(x))*x^m)/(x*e^(m*log(x) + 1) + x), x)
```

Fricas [A]
time = 0.37, size = 37, normalized size = 1.12

$$-\frac{(mp\log(x)+m\log(f))\operatorname{Li}_2(-x^me)-p\mathrm{polylog}(3,-x^me)}{m^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(f*x^p)*log(1+e*x^m)/x,x, algorithm="fricas")
```

```
[Out] -((m*p*log(x) + m*log(f))*dilog(-x^m*e) - p*polylog(3, -x^m*e))/m^2
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(f*x**p)*ln(1+e*x**m)/x,x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(f*x^p)*log(1+e*x^m)/x,x, algorithm="giac")
```

```
[Out] integrate(log(f*x^p)*log(x^m*e + 1)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln(f\,x^p)\,\ln(e\,x^m+1)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(f*x^p)*log(e*x^m + 1))/x,x)
```

```
[Out] int((log(f*x^p)*log(e*x^m + 1))/x, x)
```

3.620 $\int \frac{x^{-1+m}\log^2(fx^p)}{d+ex^m}\,dx$

Optimal. Leaf size=75

$$\frac{\log^2(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{em}+\frac{2p\log(fx^p)\operatorname{Li}_2\left(-\frac{ex^m}{d}\right)}{em^2}-\frac{2p^2\operatorname{Li}_3\left(-\frac{ex^m}{d}\right)}{em^3}$$

```
[Out] ln(f*x^p)^2*ln(1+e*x^m/d)/e/m+2*p*ln(f*x^p)*polylog(2,-e*x^m/d)/e/m^2-2*p^2
*polylog(3,-e*x^m/d)/e/m^3
```

Rubi [A]
time = 0.07, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2375, 2421, 6724}

$$\frac{2p\log(fx^p)\operatorname{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{em^2}-\frac{2p^2\operatorname{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{em^3}+\frac{\log^2(fx^p)\log\left(\frac{ex^m}{d}+1\right)}{em}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + m)*Log[f*x^p]^2)/(d + e*x^m),x]
```

```
[Out] (Log[f*x^p]^2*Log[1 + (e*x^m)/d])/(e*m) + (2*p*Log[f*x^p]*PolyLog[2, -((e*x
^m)/d)])/(e*m^2) - (2*p^2*PolyLog[3, -((e*x^m)/d)])/(e*m^3)
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+m}\log^2(fx^p)}{d+ex^m}\,dx &= \frac{\log^2(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{em} - \frac{(2p)\int \frac{\log(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{x}\,dx}{em}\\ &= \frac{\log^2(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{em} + \frac{2p\log(fx^p)\,\mathrm{Li}_2\left(-\frac{ex^m}{d}\right)}{em^2} - \frac{(2p^2)\int \frac{\mathrm{Li}_2\left(-\frac{ex^m}{d}\right)}{x}\,dx}{em^2}\\ &= \frac{\log^2(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{em} + \frac{2p\log(fx^p)\,\mathrm{Li}_2\left(-\frac{ex^m}{d}\right)}{em^2} - \frac{2p^2\mathrm{Li}_3\left(-\frac{ex^m}{d}\right)}{em^3}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 220 vs. 2(75) = 150.
time = 0.17, size = 220, normalized size = 2.93

$$\frac{p^2\log^3(x)+3p\log^2(x)\left(-p\log(x)+\log(fx^p)\right)+3\log(x)\left(-p\log(x)+\log(fx^p)\right)^2-\frac{3(-p\log(x)+\log(fx^p))^2(\log(x^m)-\log(dm(d+ex^m)))}{m}-\frac{6p(-p\log(x)+\log(fx^p))\left(\frac{1}{2}m^2\log^2(x)+\left(-m\log(x)+\log\left(-\frac{ex^m}{d}\right)\right)\log(d+ex^m)+\mathrm{Li}_2\left(1+\frac{ex^m}{d}\right)\right)}{m^2}+\frac{3p^2\left(m^2\log^2(x)\log\left(1+\frac{dx^{-m}}{e}\right)-2m\log(x)\mathrm{Li}_2\left(-\frac{dx^{-m}}{e}\right)-2\mathrm{Li}_3\left(-\frac{dx^{-m}}{e}\right)\right)}{m^3}}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + m)*Log[f*x^p]^2)/(d + e*x^m),x]
```

```
[Out] (p^2*Log[x]^3 + 3*p*Log[x]^2*(-(p*Log[x]) + Log[f*x^p]) + 3*Log[x]*(-(p*Log
[x]) + Log[f*x^p])^2 - (3*(-(p*Log[x]) + Log[f*x^p])^2*(Log[x^m] - Log[d*m*
(d + e*x^m)]))/m - (6*p*(-(p*Log[x]) + Log[f*x^p])*((m^2*Log[x]^2)/2 + (-(m
*Log[x]) + Log[-((e*x^m)/d)])*Log[d + e*x^m] + PolyLog[2, 1 + (e*x^m)/d]))/
m^2 + (3*p^2*(m^2*Log[x]^2*Log[1 + d/(e*x^m)] - 2*m*Log[x]*PolyLog[2, -(d/(
e*x^m))] - 2*PolyLog[3, -(d/(e*x^m))]))/m^3)/(3*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.92, size = 1373, normalized size = 18.31

method	result	size
risch	Expression too large to display	1373

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+m)*ln(f*x^p)^2/(d+e*x^m),x,method=_RETURNVERBOSE)
```

```
[Out] -2*p^2*polylog(3,-e*x^m/d)/e/m^3-I/m*p*ln(x)*ln((d+e*x^m)/d)/e*Pi*csgn(I*f)
*csgn(I*x^p)*csgn(I*f*x^p)+I/m*ln(d+e*x^m)/e*p*ln(x)*Pi*csgn(I*f)*csgn(I*x^
p)*csgn(I*f*x^p)+I/m^2*p*dilog((d+e*x^m)/d)/e*Pi*csgn(I*f)*csgn(I*f*x^p)^2+
I/m^2*p*dilog((d+e*x^m)/d)/e*Pi*csgn(I*x^p)*csgn(I*f*x^p)^2-I/m*p*ln(x)*ln(
(d+e*x^m)/d)/e*Pi*csgn(I*f*x^p)^3+I/m*ln(d+e*x^m)/e*Pi*ln(f)*csgn(I*f)*csgn
(I*f*x^p)^2+I/m*ln(d+e*x^m)/e*Pi*ln(f)*csgn(I*x^p)*csgn(I*f*x^p)^2+I/m*ln(d
+e*x^m)/e*p*ln(x)*Pi*csgn(I*f*x^p)^3+I/m*ln(d+e*x^m)/e*ln(x^p)*Pi*csgn(I*f)
```

```
*csgn(I*f*x^p)^2+I/m*ln(d+e*x^m)/e*ln(x^p)*Pi*csgn(I*x^p)*csgn(I*f*x^p)^2+I
/m*p*ln(x)*ln((d+e*x^m)/d)/e*Pi*csgn(I*x^p)*csgn(I*f*x^p)^2-I/m*ln(d+e*x^m)
/e*ln(x^p)*Pi*csgn(I*f)*csgn(I*x^p)*csgn(I*f*x^p)-I/m^2*p*dilog((d+e*x^m)/d
)/e*Pi*csgn(I*f)*csgn(I*x^p)*csgn(I*f*x^p)-I/m*ln(d+e*x^m)/e*Pi*ln(f)*csgn(
I*f)*csgn(I*x^p)*csgn(I*f*x^p)+I/m*p*ln(x)*ln((d+e*x^m)/d)/e*Pi*csgn(I*f)*c
sgn(I*f*x^p)^2-I/m*ln(d+e*x^m)/e*p*ln(x)*Pi*csgn(I*f)*csgn(I*f*x^p)^2-I/m*l
n(d+e*x^m)/e*p*ln(x)*Pi*csgn(I*x^p)*csgn(I*f*x^p)^2+2/m*ln(d+e*x^m)/e*ln(x^
p)*ln(f)+2/m^2*p*dilog((d+e*x^m)/d)/e*ln(f)+1/m*p^2/e*ln(x)^2*ln(1+e*x^m/d)
+2/m^2*p^2/e*ln(x)*polylog(2,-e*x^m/d)+2/m^2*p*(ln(x^p)-ln(x)*p)*dilog((d+e
*x^m)/d)/e-1/4/m*ln(d+e*x^m)/e*Pi^2*csgn(I*f*x^p)^6+1/m*ln(d+e*x^m)/e*ln(f)
^2+1/m*(ln(x^p)-ln(x)*p)^2*ln(d+e*x^m)/e-I/m*ln(d+e*x^m)/e*ln(x^p)*Pi*csgn(
I*f*x^p)^3-I/m^2*p*dilog((d+e*x^m)/d)/e*Pi*csgn(I*f*x^p)^3+1/2/m*ln(d+e*x^m
)/e*Pi^2*csgn(I*f)*csgn(I*x^p)^2*csgn(I*f*x^p)^3-1/m*ln(d+e*x^m)/e*Pi^2*csg
n(I*f)*csgn(I*x^p)*csgn(I*f*x^p)^4-1/4/m*ln(d+e*x^m)/e*Pi^2*csgn(I*f)^2*csg
n(I*x^p)^2*csgn(I*f*x^p)^2-I/m*ln(d+e*x^m)/e*Pi*ln(f)*csgn(I*f*x^p)^3+1/2/m
*ln(d+e*x^m)/e*Pi^2*csgn(I*f)^2*csgn(I*x^p)*csgn(I*f*x^p)^3-1/4/m*ln(d+e*x^
m)/e*Pi^2*csgn(I*x^p)^2*csgn(I*f*x^p)^4+1/2/m*ln(d+e*x^m)/e*Pi^2*csgn(I*x^p
)*csgn(I*f*x^p)^5+2/m*p*(ln(x^p)-ln(x)*p)*ln(x)*ln((d+e*x^m)/d)/e-2/m*ln(d+
e*x^m)/e*p*ln(x)*ln(f)+2/m*p*ln(x)*ln((d+e*x^m)/d)/e*ln(f)-1/4/m*ln(d+e*x^m
)/e*Pi^2*csgn(I*f)^2*csgn(I*f*x^p)^4+1/2/m*ln(d+e*x^m)/e*Pi^2*csgn(I*f)*csg
n(I*f*x^p)^5
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*log(f*x^p)^2/(d+e*x^m),x, algorithm="maxima")
[Out] integrate(x^(m - 1)*log(f*x^p)^2/(x^m*e + d), x)
```

Fricas [**A**]
time = 0.36, size = 108, normalized size = 1.44

$$\frac{\left(m^2\log\left(x^m e+d\right)\log\left(f\right)^2-2\,p^2\mathrm{polylog}\left(3,-\frac{x^m e}{d}\right)+2\left(mp^2\log\left(x\right)+mp\log\left(f\right)\right)\mathrm{Li}_2\left(-\frac{x^m e+d}{d}+1\right)+\left(m^2p^2\log\left(x\right)^2+2\,m^2p\log\left(f\right)\log\left(x\right)\right)\log\left(\frac{x^m e+d}{d}\right)\right)e^{(-1)}}{m^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*log(f*x^p)^2/(d+e*x^m),x, algorithm="fricas")
[Out] (m^2*log(x^m*e + d)*log(f)^2 - 2*p^2*polylog(3, -x^m*e/d) + 2*(m*p^2*log(x)
 + m*p*log(f))*dilog(-(x^m*e + d)/d + 1) + (m^2*p^2*log(x)^2 + 2*m^2*p*log(
f)*log(x))*log((x^m*e + d)/d))*e^(-1)/m^3
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m-1}\log\left(fx^p\right)^2}{d+ex^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+m)*ln(f*x**p)**2/(d+e*x**m),x)
```

```
[Out] Integral(x**(m - 1)*log(f*x**p)**2/(d + e*x**m), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*log(f*x^p)^2/(d+e*x^m),x, algorithm="giac")
```

```
[Out] integrate(x^(m - 1)*log(f*x^p)^2/(x^m*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{m-1}\,\ln\left(f\,x^p\right)^2}{d+e\,x^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(m - 1)*log(f*x^p)^2)/(d + e*x^m),x)
```

```
[Out] int((x^(m - 1)*log(f*x^p)^2)/(d + e*x^m), x)
```

3.621 $\int \frac{\log^3(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx$

Optimal. Leaf size=161

$$\frac{\log^4(fx^p)(a+b\log(c(d+ex^m)^n))}{4p}-\frac{bn\log^4(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{4p}-\frac{bn\log^3(fx^p)\,\mathrm{Li}_2\left(-\frac{ex^m}{d}\right)}{m}+\frac{3bnp\log^2(fx^p)}{m^2}$$

```
[Out] 1/4*ln(f*x^p)^4*(a+b*ln(c*(d+e*x^m)^n))/p-1/4*b*n*ln(f*x^p)^4*ln(1+e*x^m/d)
/p-b*n*ln(f*x^p)^3*polylog(2,-e*x^m/d)/m+3*b*n*p*ln(f*x^p)^2*polylog(3,-e*x
^m/d)/m^2-6*b*n*p^2*ln(f*x^p)*polylog(4,-e*x^m/d)/m^3+6*b*n*p^3*polylog(5,-
e*x^m/d)/m^4
```

Rubi [A]
time = 0.15, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2531, 2375, 2421, 2430, 6724}

$$-\frac{6bnp^2\log(fx^p)\,\mathrm{PolyLog}\left(4,-\frac{ex^m}{d}\right)}{m^3}+\frac{3bnp\log^2(fx^p)\,\mathrm{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m^2}-\frac{bn\log^3(fx^p)\,\mathrm{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m}+\frac{6bnp^3\mathrm{PolyLog}\left(5,-\frac{ex^m}{d}\right)}{m^4}+\frac{\log^4(fx^p)(a+b\log(c(d+ex^m)^n))}{4p}-\frac{bn\log^4(fx^p)\log\left(\frac{ex^m}{d}+1\right)}{4p}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[f*x^p]^3*(a + b*Log[c*(d + e*x^m)^n]))/x,x]
```

```
[Out] (Log[f*x^p]^4*(a + b*Log[c*(d + e*x^m)^n]))/(4*p) - (b*n*Log[f*x^p]^4*Log[1
 + (e*x^m)/d])/(4*p) - (b*n*Log[f*x^p]^3*PolyLog[2, -((e*x^m)/d)])/m + (3*b
*n*p*Log[f*x^p]^2*PolyLog[3, -((e*x^m)/d)])/m^2 - (6*b*n*p^2*Log[f*x^p]*Pol
yLog[4, -((e*x^m)/d)])/m^3 + (6*b*n*p^3*PolyLog[5, -((e*x^m)/d)])/m^4
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
```

```
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2531

```
Int[(Log[(f_.)*(x_)^(q_.)]^(m_.)*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_)
)^(p_.)]*(b_.)))/(x_), x_Symbol] :> Simp[Log[f*x^q]^(m + 1)*((a + b*Log[c*(
d + e*x^n)^p])/(q*(m + 1))), x] - Dist[b*e*n*(p/(q*(m + 1))), Int[x^(n - 1)
*(Log[f*x^q]^(m + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n,
 p, q}, x] && NeQ[m, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log^3(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{x}\,dx = \frac{\log^4(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{4p} - \frac{(bemn)\int \frac{x^{-1+m}\log^4(fx^p)}{d+ex^m}\,dx}{4p}$$

$$= \frac{\log^4(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{4p} - \frac{bn\log^4(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{4p}$$

$$= \frac{\log^4(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{4p} - \frac{bn\log^4(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{4p}$$

$$= \frac{\log^4(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{4p} - \frac{bn\log^4(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{4p}$$

$$= \frac{\log^4(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{4p} - \frac{bn\log^4(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{4p}$$

$$= \frac{\log^4(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{4p} - \frac{bn\log^4(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{4p}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 659 vs. 2(161) = 322.
time = 0.18, size = 659, normalized size = 4.09

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(Log[f*x^p]^3*(a + b*Log[c*(d + e*x^m)^n]))/x,x]
```

```
[Out] (-3*b*m*n*p^3*Log[x]^5)/10 + (3*b*m*n*p^2*Log[x]^4*Log[f*x^p])/4 - (b*m*n*p
*Log[x]^3*Log[f*x^p]^2)/2 + (a*Log[f*x^p]^4)/(4*p) - (3*b*n*p^3*Log[x]^4*Lo
g[1 + d/(e*x^m)])/4 + 2*b*n*p^2*Log[x]^3*Log[f*x^p]*Log[1 + d/(e*x^m)] - (3
*b*n*p*Log[x]^2*Log[f*x^p]^2*Log[1 + d/(e*x^m)])/2 + b*n*p^3*Log[x]^4*Log[d
 + e*x^m] - (b*n*p^3*Log[x]^3*Log[-((e*x^m)/d)]*Log[d + e*x^m])/m - 3*b*n*p
^2*Log[x]^3*Log[f*x^p]*Log[d + e*x^m] + (3*b*n*p^2*Log[x]^2*Log[-((e*x^m)/d
)]*Log[f*x^p]*Log[d + e*x^m])/m + 3*b*n*p*Log[x]^2*Log[f*x^p]^2*Log[d + e*x
^m] - (3*b*n*p*Log[x]*Log[-((e*x^m)/d)]*Log[f*x^p]^2*Log[d + e*x^m])/m - b*
n*Log[x]*Log[f*x^p]^3*Log[d + e*x^m] + (b*n*Log[-((e*x^m)/d)]*Log[f*x^p]^3*
Log[d + e*x^m])/m - (b*p^3*Log[x]^4*Log[c*(d + e*x^m)^n])/4 + b*p^2*Log[x]^
3*Log[f*x^p]*Log[c*(d + e*x^m)^n] - (3*b*p*Log[x]^2*Log[f*x^p]^2*Log[c*(d +
 e*x^m)^n])/2 + b*Log[x]*Log[f*x^p]^3*Log[c*(d + e*x^m)^n] + (b*n*p*Log[x]*
(p^2*Log[x]^2 - 3*p*Log[x]*Log[f*x^p] + 3*Log[f*x^p]^2)*PolyLog[2, -(d/(e*x
^m))])/m - (b*n*(p*Log[x] - Log[f*x^p])^3*PolyLog[2, 1 + (e*x^m)/d])/m + (3
*b*n*p*Log[f*x^p]^2*PolyLog[3, -(d/(e*x^m))])/m^2 + (6*b*n*p^2*Log[f*x^p]*P
olyLog[4, -(d/(e*x^m))])/m^3 + (6*b*n*p^3*PolyLog[5, -(d/(e*x^m))])/m^4
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(f\,x^{p}\right)^{3} \left(a +b \ln \left(c \left(d +e\,x^{m}\right)^{n}\right)\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(f*x^p)^3*(a+b*ln(c*(d+e*x^m)^n))/x,x)
```

```
[Out] int(ln(f*x^p)^3*(a+b*ln(c*(d+e*x^m)^n))/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(f*x^p)^3*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="maxima")
```

```
[Out] -1/4*(b*p^3*log(x)^4 - 4*b*p^2*log(f)*log(x)^3 + 6*b*p*log(f)^2*log(x)^2 -
4*b*log(f)^3*log(x) - 4*b*log(x)*log(x^p)^3 + 6*(b*p*log(x)^2 - 2*b*log(f)*
log(x))*log(x^p)^2 - 4*(b*p^2*log(x)^3 - 3*b*p*log(f)*log(x)^2 + 3*b*log(f)
^2*log(x))*log(x^p))*log((d + e^(m*log(x) + 1))^n) - integrate(-1/4*(4*b*d*
log(c)*log(f)^3 + 4*a*d*log(f)^3 + 4*(b*d*log(c) + a*d - (b*m*n*e*log(x) -
(b*log(c) + a)*e)*x^m)*log(x^p)^3 + 6*(2*b*d*log(c)*log(f) + 2*a*d*log(f) +
 (b*m*n*p*e*log(x)^2 - 2*b*m*n*e*log(f)*log(x) + 2*(b*log(c)*log(f) + a*log
(f))*e)*x^m)*log(x^p)^2 + (b*m*n*p^3*e*log(x)^4 - 4*b*m*n*p^2*e*log(f)*log(
```

```
x)^3 + 6*b*m*n*p*e*log(f)^2*log(x)^2 - 4*b*m*n*e*log(f)^3*log(x) + 4*(b*log
(c)*log(f)^3 + a*log(f)^3)*e)*x^m + 4*(3*b*d*log(c)*log(f)^2 + 3*a*d*log(f)
^2 - (b*m*n*p^2*e*log(x)^3 - 3*b*m*n*p*e*log(f)*log(x)^2 + 3*b*m*n*e*log(f)
^2*log(x) - 3*(b*log(c)*log(f)^2 + a*log(f)^2)*e)*x^m)*log(x^p))/(d*x + x*e
^(m*log(x) + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(162) = 324$.
time = 0.36, size = 423, normalized size = 2.63

$$\frac{24\,bnp^3\operatorname{polylog}\left(5,-\frac{x^m e}{d}\right)+4\,(bm^4\log(c)+am^4)\log(f)^3\log(x)+6\,(bm^4p\log(c)+am^4p)\log(f)^2\log(x)^2+4\,(bm^4p^2\log(c)+am^4p^2)\log(f)\log(x)^3+(bm^4p^3\log(c)+am^4p^3)\log(x)^4-4\,(bm^3np^3\log(x)^3+3\,bm^3np^2\log(f)\log(x)^2+3\,bm^3np\log(f)^2\log(x)+bm^3n\log(f)^3)\operatorname{Li}_2\left(-\frac{x^m e+d}{d}+1\right)+(bm^4np^3\log(x)^4+4\,bm^4np^2\log(f)\log(x)^3+6\,bm^4np\log(f)^2\log(x)^2+4\,bm^4n\log(f)^3\log(x))\log(x^m e+d)-(bm^4np^3\log(x)^4+4\,bm^4np^2\log(f)\log(x)^3+6\,bm^4np\log(f)^2\log(x)^2+4\,bm^4n\log(f)^3\log(x))\log\left(\frac{x^m e+d}{d}\right)-24\,(bmnp^3\log(x)+bmnp^2\log(f))\operatorname{polylog}\left(4,-\frac{x^m e}{d}\right)+12\,(bm^2np^3\log(x)^2+2\,bm^2np^2\log(f)\log(x)+bm^2np\log(f)^2)\operatorname{polylog}\left(3,-\frac{x^m e}{d}\right)}{4\,m^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(f*x^p)^3*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="fricas")
```

```
[Out] 1/4*(24*b*n*p^3*polylog(5, -x^m*e/d) + 4*(b*m^4*log(c) + a*m^4)*log(f)^3*lo
g(x) + 6*(b*m^4*p*log(c) + a*m^4*p)*log(f)^2*log(x)^2 + 4*(b*m^4*p^2*log(c)
 + a*m^4*p^2)*log(f)*log(x)^3 + (b*m^4*p^3*log(c) + a*m^4*p^3)*log(x)^4 - 4
*(b*m^3*n*p^3*log(x)^3 + 3*b*m^3*n*p^2*log(f)*log(x)^2 + 3*b*m^3*n*p*log(f)
^2*log(x) + b*m^3*n*log(f)^3)*dilog(-(x^m*e + d)/d + 1) + (b*m^4*n*p^3*log(
x)^4 + 4*b*m^4*n*p^2*log(f)*log(x)^3 + 6*b*m^4*n*p*log(f)^2*log(x)^2 + 4*b*
m^4*n*log(f)^3*log(x))*log(x^m*e + d) - (b*m^4*n*p^3*log(x)^4 + 4*b*m^4*n*p
^2*log(f)*log(x)^3 + 6*b*m^4*n*p*log(f)^2*log(x)^2 + 4*b*m^4*n*log(f)^3*log
(x))*log((x^m*e + d)/d) - 24*(b*m*n*p^3*log(x) + b*m*n*p^2*log(f))*polylog(
4, -x^m*e/d) + 12*(b*m^2*n*p^3*log(x)^2 + 2*b*m^2*n*p^2*log(f)*log(x) + b*m
^2*n*p*log(f)^2)*polylog(3, -x^m*e/d))/m^4
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(f*x**p)**3*(a+b*ln(c*(d+e*x**m)**n))/x,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(f*x^p)^3*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^m*e + d)^n*c) + a)*log(f*x^p)^3/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln(f\,x^p)^3\,(a+b\,\ln(c\,(d+e\,x^m)^n))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(f*x^p)^3*(a + b*log(c*(d + e*x^m)^n)))/x,x)
[Out] int((log(f*x^p)^3*(a + b*log(c*(d + e*x^m)^n)))/x, x)
```

3.622 $\int \frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx$

Optimal. Leaf size=132

$$\frac{\log^3(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{3p}-\frac{bn\log^3(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{3p}-\frac{bn\log^2(fx^p)\,\text{Li}_2\left(-\frac{ex^m}{d}\right)}{m}+\frac{2bnp\log(fx^p)\,\text{L}}{m^2}$$

```
[Out] 1/3*ln(f*x^p)^3*(a+b*ln(c*(d+e*x^m)^n))/p-1/3*b*n*ln(f*x^p)^3*ln(1+e*x^m/d)
/p-b*n*ln(f*x^p)^2*polylog(2,-e*x^m/d)/m+2*b*n*p*ln(f*x^p)*polylog(3,-e*x^m
/d)/m^2-2*b*n*p^2*polylog(4,-e*x^m/d)/m^3
```

Rubi [A]
time = 0.12, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2531, 2375, 2421, 2430, 6724}

$$\frac{2bnp\log(fx^p)\,\text{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m^2}-\frac{bn\log^2(fx^p)\,\text{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m}-\frac{2bnp^2\,\text{PolyLog}\left(4,-\frac{ex^m}{d}\right)}{m^3}+\frac{\log^3(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{3p}-\frac{bn\log^3(fx^p)\log\left(\frac{ex^m}{d}+1\right)}{3p}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[f*x^p]^2*(a + b*Log[c*(d + e*x^m)^n]))/x,x]
```

```
[Out] (Log[f*x^p]^3*(a + b*Log[c*(d + e*x^m)^n]))/(3*p) - (b*n*Log[f*x^p]^3*Log[1
 + (e*x^m)/d])/(3*p) - (b*n*Log[f*x^p]^2*PolyLog[2, -((e*x^m)/d)])/m + (2*b
*n*p*Log[f*x^p]*PolyLog[3, -((e*x^m)/d)])/m^2 - (2*b*n*p^2*PolyLog[4, -((e*
x^m)/d)])/m^3
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
```

```
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2531

```
Int[(Log[(f_.)*(x_)^(q_.)]^(m_.)*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_)
)^(p_.)]*(b_.)))/(x_), x_Symbol] :> Simp[Log[f*x^q]^(m + 1)*((a + b*Log[c*(
d + e*x^n)^p])/(q*(m + 1))), x] - Dist[b*e*n*(p/(q*(m + 1))), Int[x^(n - 1)
*(Log[f*x^q]^(m + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n,
 p, q}, x] && NeQ[m, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx = \frac{\log^3(fx^p)(a+b\log(c(d+ex^m)^n))}{3p} - \frac{(bemn)\int \frac{x^{-1+m}\log^3(fx^p)}{d+ex^m}}{3p}$$
$$= \frac{\log^3(fx^p)(a+b\log(c(d+ex^m)^n))}{3p} - \frac{bn\log^3(fx^p)\log\left(1+\frac{ex}{d}\right.}{3p}$$
$$= \frac{\log^3(fx^p)(a+b\log(c(d+ex^m)^n))}{3p} - \frac{bn\log^3(fx^p)\log\left(1+\frac{ex}{d}\right.}{3p}$$
$$= \frac{\log^3(fx^p)(a+b\log(c(d+ex^m)^n))}{3p} - \frac{bn\log^3(fx^p)\log\left(1+\frac{ex}{d}\right.}{3p}$$
$$= \frac{\log^3(fx^p)(a+b\log(c(d+ex^m)^n))}{3p} - \frac{bn\log^3(fx^p)\log\left(1+\frac{ex}{d}\right.}{3p}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 456 vs. 2(132) = 264.
time = 0.16, size = 456, normalized size = 3.45

Antiderivative was successfully verified.

```
[In] Integrate[(Log[f*x^p]^2*(a + b*Log[c*(d + e*x^m)^n]))/x,x]
```

```
[Out] (b*m*n*p^2*Log[x]^4)/4 - (b*m*n*p*Log[x]^3*Log[f*x^p])/3 + (a*Log[f*x^p]^3)
/(3*p) + (2*b*n*p^2*Log[x]^3*Log[1 + d/(e*x^m)])/3 - b*n*p*Log[x]^2*Log[f*x
^p]*Log[1 + d/(e*x^m)] - b*n*p^2*Log[x]^3*Log[d + e*x^m] + (b*n*p^2*Log[x]^
2*Log[-((e*x^m)/d)]*Log[d + e*x^m])/m + 2*b*n*p*Log[x]^2*Log[f*x^p]*Log[d +
 e*x^m] - (2*b*n*p*Log[x]*Log[-((e*x^m)/d)]*Log[f*x^p]*Log[d + e*x^m])/m -
b*n*Log[x]*Log[f*x^p]^2*Log[d + e*x^m] + (b*n*Log[-((e*x^m)/d)]*Log[f*x^p]^
2*Log[d + e*x^m])/m + (b*p^2*Log[x]^3*Log[c*(d + e*x^m)^n])/3 - b*p*Log[x]^
2*Log[f*x^p]*Log[c*(d + e*x^m)^n] + b*Log[x]*Log[f*x^p]^2*Log[c*(d + e*x^m)
^n] - (b*n*p*Log[x]*(p*Log[x] - 2*Log[f*x^p])*PolyLog[2, -(d/(e*x^m))])/m +
 (b*n*(-(p*Log[x]) + Log[f*x^p])^2*PolyLog[2, 1 + (e*x^m)/d])/m + (2*b*n*p*
Log[f*x^p]*PolyLog[3, -(d/(e*x^m))])/m^2 + (2*b*n*p^2*PolyLog[4, -(d/(e*x^m
))])/m^3
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(f\,x^{p}\right)^{2} \left(a+b\ln \left(c(d+e\,x^{m})^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(f*x^p)^2*(a+b*ln(c*(d+e*x^m)^n))/x,x)
```

```
[Out] int(ln(f*x^p)^2*(a+b*ln(c*(d+e*x^m)^n))/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(f*x^p)^2*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="maxima")
```

```
[Out] 1/3*(b*p^2*log(x)^3 - 3*b*p*log(f)*log(x)^2 + 3*b*log(f)^2*log(x) + 3*b*log
(x)*log(x^p)^2 - 3*(b*p*log(x)^2 - 2*b*log(f)*log(x))*log(x^p))*log((d + e^
(m*log(x) + 1))^n) - integrate(-1/3*(3*b*d*log(c)*log(f)^2 + 3*a*d*log(f)^2
 + 3*(b*d*log(c) + a*d - (b*m*n*e*log(x) - (b*log(c) + a)*e)*x^m)*log(x^p)^
2 - (b*m*n*p^2*e*log(x)^3 - 3*b*m*n*p*e*log(f)*log(x)^2 + 3*b*m*n*e*log(f)^
2*log(x) - 3*(b*log(c)*log(f)^2 + a*log(f)^2)*e)*x^m + 3*(2*b*d*log(c)*log(
f) + 2*a*d*log(f) + (b*m*n*p*e*log(x)^2 - 2*b*m*n*e*log(f)*log(x) + 2*(b*lo
g(c)*log(f) + a*log(f))*e)*x^m)*log(x^p))/(d*x + x*e^(m*log(x) + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(132) = 264$.
time = 0.39, size = 286, normalized size = 2.17

$$-\frac{6\,bnp^{2}\mathrm{polylog}\left(4,-\frac{x^{m}e}{d}\right)-3\left(bm^{3}\log\left(c\right)+am^{3}\right)\log\left(f\right)^{2}\log\left(x\right)-3\left(bm^{3}p\log\left(c\right)+am^{3}p\right)\log\left(f\right)\log\left(x\right)^{2}-\left(bm^{3}p^{2}\log\left(c\right)+am^{3}p^{2}\right)\log\left(x\right)^{3}+3\left(bm^{2}np^{2}\log\left(x\right)^{2}+2\,bm^{2}np\log\left(f\right)\log\left(x\right)+bm^{2}n\log\left(f\right)^{2}\right){\rm Li}_2\left(-\frac{x^{m}e+d}{d}+1\right)-\left(bm^{3}np^{2}\log\left(x\right)^{3}+3\,bm^{3}np\log\left(f\right)\log\left(x\right)^{2}+3\,bm^{3}n\log\left(f\right)^{2}\log\left(x\right)\right)\log\left(x^{m}e+d\right)+\left(bm^{3}np^{2}\log\left(x\right)^{3}+3\,bm^{3}np\log\left(f\right)\log\left(x\right)^{2}+3\,bm^{3}n\log\left(f\right)^{2}\log\left(x\right)\right)\log\left(\frac{x^{m}e+d}{d}\right)-6\left(bmnp^{2}\log\left(x\right)+bmnp\log\left(f\right)\right)\mathrm{polylog}\left(3,-\frac{x^{m}e}{d}\right)}{3\,m^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(f*x^p)^2*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="fricas")
```

```
[Out] -1/3*(6*b*n*p^2*polylog(4, -x^m*e/d) - 3*(b*m^3*log(c) + a*m^3)*log(f)^2*lo
g(x) - 3*(b*m^3*p*log(c) + a*m^3*p)*log(f)*log(x)^2 - (b*m^3*p^2*log(c) + a
*m^3*p^2)*log(x)^3 + 3*(b*m^2*n*p^2*log(x)^2 + 2*b*m^2*n*p*log(f)*log(x) +
b*m^2*n*log(f)^2)*dilog(-(x^m*e + d)/d + 1) - (b*m^3*n*p^2*log(x)^3 + 3*b*m
^3*n*p*log(f)*log(x)^2 + 3*b*m^3*n*log(f)^2*log(x))*log(x^m*e + d) + (b*m^3
*n*p^2*log(x)^3 + 3*b*m^3*n*p*log(f)*log(x)^2 + 3*b*m^3*n*log(f)^2*log(x))*
log((x^m*e + d)/d) - 6*(b*m*n*p^2*log(x) + b*m*n*p*log(f))*polylog(3, -x^m*
e/d))/m^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\log(c(d + ex^m)^n))\log(fx^p)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(f*x**p)**2*(a+b*ln(c*(d+e*x**m)**n))/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**m)**n))*log(f*x**p)**2/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(f*x^p)^2*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^m*e + d)^n*c) + a)*log(f*x^p)^2/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln(f\,x^p)^2\,(a + b\,\ln(c\,(d + e\,x^m)^n))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(f*x^p)^2*(a + b*log(c*(d + e*x^m)^n)))/x,x)
```

```
[Out] int((log(f*x^p)^2*(a + b*log(c*(d + e*x^m)^n)))/x, x)
```

3.623 $\int \frac{\log(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx$

Optimal. Leaf size=102

$$\frac{\log^2(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{2p}-\frac{bn\log^2(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{2p}-\frac{bn\log(fx^p)\,\mathrm{Li}_2\left(-\frac{ex^m}{d}\right)}{m}+\frac{bnp\mathrm{Li}_3\left(-\frac{ex^m}{d}\right)}{m^2}$$

```
[Out] 1/2*ln(f*x^p)^2*(a+b*ln(c*(d+e*x^m)^n))/p-1/2*b*n*ln(f*x^p)^2*ln(1+e*x^m/d)
/p-b*n*ln(f*x^p)*polylog(2,-e*x^m/d)/m+b*n*p*polylog(3,-e*x^m/d)/m^2
```

Rubi [A]
time = 0.09, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2531, 2375, 2421, 6724}

$$-\frac{bn\log(fx^p)\,\mathrm{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m}+\frac{bnp\mathrm{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m^2}+\frac{\log^2(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{2p}-\frac{bn\log^2(fx^p)\log\left(\frac{ex^m}{d}+1\right)}{2p}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[f*x^p]*(a + b*Log[c*(d + e*x^m)^n]))/x,x]
```

```
[Out] (Log[f*x^p]^2*(a + b*Log[c*(d + e*x^m)^n]))/(2*p) - (b*n*Log[f*x^p]^2*Log[1
 + (e*x^m)/d])/(2*p) - (b*n*Log[f*x^p]*PolyLog[2, -((e*x^m)/d)])/m + (b*n*p
*PolyLog[3, -((e*x^m)/d)])/m^2
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2531

```
Int[(Log[(f_.)*(x_)^(q_.)]^(m_.)*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_)
)^(p_.)]*(b_.)))/(x_), x_Symbol] :> Simp[Log[f*x^q]^(m + 1)*((a + b*Log[c*(
d + e*x^n)^p])/(q*(m + 1))), x] - Dist[b*e*n*(p/(q*(m + 1))), Int[x^(n - 1)
*(Log[f*x^q]^(m + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n,
```

```
 p, q}, x] && NeQ[m, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx = \frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{2p} - \frac{(bemn)\int \frac{x^{-1+m}\log^2(fx^p)}{d+ex^m}\,d}{2p}$$
$$= \frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{2p} - \frac{bn\log^2(fx^p)\log\left(1+\frac{ex^m}{d}\right.}{2p}$$
$$= \frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{2p} - \frac{bn\log^2(fx^p)\log\left(1+\frac{ex^m}{d}\right.}{2p}$$
$$= \frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{2p} - \frac{bn\log^2(fx^p)\log\left(1+\frac{ex^m}{d}\right.}{2p}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 265 vs. 2(102) = 204.
time = 0.13, size = 265, normalized size = 2.60

$$-\frac{1}{6}bmnp\log^3(x)+\frac{a\log^2(fx^p)}{2p}-\frac{1}{2}bnp\log^2(x)\log\left(1+\frac{dx^{-m}}{e}\right)+bnp\log^2(x)\log(d+ex^m)-\frac{bnp\log(x)\log\left(-\frac{ex^m}{d}\right)\log(d+ex^m)}{m}-bn\log(x)\log(fx^p)\log(d+ex^m)+\frac{bn\log\left(-\frac{ex^m}{d}\right)\log(fx^p)\log(d+ex^m)}{m}-\frac{1}{2}bp\log^2(x)\log(c(d+ex^m)^n)+b\log(x)\log(fx^p)\log(c(d+ex^m)^n)+\frac{bnp\log(x)\mathrm{Li}_2\left(-\frac{dx^{-m}}{e}\right)}{m}-\frac{bn(p\log(x)-\log(fx^p))\mathrm{Li}_2\left(1+\frac{ex^m}{d}\right)}{m}+\frac{bnp\mathrm{Li}_3\left(-\frac{dx^{-m}}{e}\right)}{m^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[f*x^p]*(a + b*Log[c*(d + e*x^m)^n]))/x,x]
```

```
[Out] -1/6*(b*m*n*p*Log[x]^3) + (a*Log[f*x^p]^2)/(2*p) - (b*n*p*Log[x]^2*Log[1 +
d/(e*x^m)])/2 + b*n*p*Log[x]^2*Log[d + e*x^m] - (b*n*p*Log[x]*Log[-((e*x^m)
/d)]*Log[d + e*x^m])/m - b*n*Log[x]*Log[f*x^p]*Log[d + e*x^m] + (b*n*Log[-(
(e*x^m)/d)]*Log[f*x^p]*Log[d + e*x^m])/m - (b*p*Log[x]^2*Log[c*(d + e*x^m)^
n])/2 + b*Log[x]*Log[f*x^p]*Log[c*(d + e*x^m)^n] + (b*n*p*Log[x]*PolyLog[2,
 -(d/(e*x^m))])/m - (b*n*(p*Log[x] - Log[f*x^p])*PolyLog[2, 1 + (e*x^m)/d])
/m + (b*n*p*PolyLog[3, -(d/(e*x^m))])/m^2
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\ln(f\,x^p)(a+b\ln(c(d+e\,x^m)^n))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(f*x^p)*(a+b*ln(c*(d+e*x^m)^n))/x,x)
```

```
[Out] int(ln(f*x^p)*(a+b*ln(c*(d+e*x^m)^n))/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(f*x^p)*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="maxima")
```

```
[Out] -1/2*(b*p*log(x)^2 - 2*b*log(f)*log(x) - 2*b*log(x)*log(x^p))*log((d + e^(m
*log(x) + 1))^n) - integrate(-1/2*(2*b*d*log(c)*log(f) + 2*a*d*log(f) + (b*
m*n*p*e*log(x)^2 - 2*b*m*n*e*log(f)*log(x) + 2*(b*log(c)*log(f) + a*log(f))
*e)*x^m + 2*(b*d*log(c) + a*d - (b*m*n*e*log(x) - (b*log(c) + a)*e)*x^m)*lo
g(x^p))/(d*x + x*e^(m*log(x) + 1)), x)
```

Fricas [A]
time = 0.43, size = 165, normalized size = 1.62

$$\frac{2\,bnp\operatorname{polylog}\left(3,-\frac{x^m e}{d}\right)+2\,(bm^2\log(c)+am^2)\log(f)\log(x)+(bm^2p\log(c)+am^2p)\log(x)^2-2\,(bmnp\log(x)+bmn\log(f))\operatorname{Li}_2\left(-\frac{x^m e+d}{d}+1\right)+\left(bm^2np\log(x)^2+2\,bm^2n\log(f)\log(x)\right)\log(x^m e+d)-\left(bm^2np\log(x)^2+2\,bm^2n\log(f)\log(x)\right)\log\left(\frac{x^m e+d}{d}\right)}{2\,m^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(f*x^p)*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*n*p*polylog(3, -x^m*e/d) + 2*(b*m^2*log(c) + a*m^2)*log(f)*log(x)
+ (b*m^2*p*log(c) + a*m^2*p)*log(x)^2 - 2*(b*m*n*p*log(x) + b*m*n*log(f))*d
ilog(-(x^m*e + d)/d + 1) + (b*m^2*n*p*log(x)^2 + 2*b*m^2*n*log(f)*log(x))*l
og(x^m*e + d) - (b*m^2*n*p*log(x)^2 + 2*b*m^2*n*log(f)*log(x))*log((x^m*e +
 d)/d))/m^2
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(f*x**p)*(a+b*ln(c*(d+e*x**m)**n))/x,x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(f*x^p)*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^m*e + d)^n*c) + a)*log(f*x^p)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(f\,x^p\right)\,\left(a+b\,\ln\left(c\,{\left(d+e\,x^m\right)}^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(f*x^p)*(a + b*log(c*(d + e*x^m)^n)))/x,x)
```

```
[Out] int((log(f*x^p)*(a + b*log(c*(d + e*x^m)^n)))/x, x)
```

3.624 $\int \frac{a+b\log(c(d+ex^m)^n)}{x}\,dx$

Optimal. Leaf size=49

$$\frac{\log\left(-\frac{ex^m}{d}\right)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{m}+\frac{bn\mathrm{Li}_2\left(1+\frac{ex^m}{d}\right)}{m}$$

```
[Out] ln(-e*x^m/d)*(a+b*ln(c*(d+e*x^m)^n))/m+b*n*polylog(2,1+e*x^m/d)/m
```

Rubi [A]
time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2504, 2441, 2352}

$$\frac{bn\text{PolyLog}\left(2,\frac{ex^m}{d}+1\right)}{m}+\frac{\log\left(-\frac{ex^m}{d}\right)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{m}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e*x^m)^n])/x,x]
```

```
[Out] (Log[-((e*x^m)/d)]*(a + b*Log[c*(d + e*x^m)^n]))/m + (b*n*PolyLog[2, 1 + (e
*x^m)/d])/m
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\log(c(d+ex)^n)}{x}\,dx, x, x^m\right)}{m}$$
$$= \frac{\log\left(-\frac{ex^m}{d}\right)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{m} - \frac{(ben)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx, x, x^m\right)}{m}$$
$$= \frac{\log\left(-\frac{ex^m}{d}\right)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{m} + \frac{bn\text{Li}_2\left(1+\frac{ex^m}{d}\right)}{m}$$

Mathematica [A]
time = 0.01, size = 49, normalized size = 1.00

$$a\log(x) + \frac{b\left(\log\left(-\frac{ex^m}{d}\right)\log\left(c(d+ex^m)^n\right) + n\text{Li}_2\left(\frac{d+ex^m}{d}\right)\right)}{m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e*x^m)^n])/x,x]
```

```
[Out] a*Log[x] + (b*(Log[-((e*x^m)/d)]*Log[c*(d + e*x^m)^n] + n*PolyLog[2, (d + e
*x^m)/d]))/m
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.46, size = 189, normalized size = 3.86

method	result
risch	$b\ln(x)\ln\left((d+e\,x^m)^n\right) + \frac{i\ln(x)b\pi\,\text{csgn}(i(d+e\,x^m)^n)\text{csgn}(ic(d+e\,x^m)^n)^2}{2} - \frac{i\ln(x)b\pi\,\text{csgn}(i(d+e\,x^m)^n)\text{csgn}(ic(d+e\,x^m$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^m)^n))/x,x,method=_RETURNVERBOSE)
```

```
[Out] b*ln(x)*ln((d+e*x^m)^n)+1/2*I*ln(x)*b*Pi*csgn(I*(d+e*x^m)^n)*csgn(I*c*(d+e*
x^m)^n)^2-1/2*I*ln(x)*b*Pi*csgn(I*(d+e*x^m)^n)*csgn(I*c*(d+e*x^m)^n)*csgn(I
*c)-1/2*I*ln(x)*b*Pi*csgn(I*c*(d+e*x^m)^n)^3+1/2*I*ln(x)*b*Pi*csgn(I*c*(d+e
*x^m)^n)^2*csgn(I*c)+ln(c)*ln(x)*b+ln(x)*a-b*n/m*dilog((d+e*x^m)/d)-b*n*ln(
x)*ln((d+e*x^m)/d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="maxima")
```

```
[Out] 1/2*(2*d*m*n*integrate(log(x)/(d*x + x*e^(m*log(x) + 1)), x) - m*n*log(x)^2
 + 2*log(c)*log(x) + 2*log((d + e^(m*log(x) + 1))^n)*log(x))*b + a*log(x)
```

Fricas [A]
time = 0.35, size = 72, normalized size = 1.47

$$\frac{bmn\log\left(x^{m}e+d\right)\log\left(x\right)-bmn\log\left(x\right)\log\left(\frac{x^{m}e+d}{d}\right)-bn\text{Li}_2\left(-\frac{x^{m}e+d}{d}+1\right)+\left(bm\log\left(c\right)+am\right)\log\left(x\right)}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="fricas")
```

```
[Out] (b*m*n*log(x^m*e + d)*log(x) - b*m*n*log(x)*log((x^m*e + d)/d) - b*n*dilog(
-(x^m*e + d)/d + 1) + (b*m*log(c) + a*m)*log(x))/m
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\log\left(c(d+ex^{m})^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**m)**n))/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**m)**n))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^m*e + d)^n*c) + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{a+b\,\ln\left(c\,{\left(d+e\,x^{m}\right)}^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^m)^n))/x,x)
```

```
[Out] int((a + b*log(c*(d + e*x^m)^n))/x, x)
```

3.625 $\int \frac{a+b\log(c(d+ex^m)^n)}{x\log(fx^p)}\,dx$

Optimal. Leaf size=42

$$\frac{a\log(\log(fx^p))}{p}+b\text{Int}\left(\frac{\log(c(d+ex^m)^n)}{x\log(fx^p)},x\right)$$

```
[Out] a*ln(ln(f*x^p))/p+b*Unintegrable(ln(c*(d+e*x^m)^n)/x/ln(f*x^p),x)
```

Rubi [A]
time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\log(c(d+ex^m)^n)}{x\log(fx^p)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]),x]
```

```
[Out] (a*Log[Log[f*x^p]])/p + b*Defer[Int][Log[c*(d + e*x^m)^n]/(x*Log[f*x^p]), x
]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\log(c(d+ex^m)^n)}{x\log(fx^p)}\,dx &= \int \left(\frac{a}{x\log(fx^p)}+\frac{b\log(c(d+ex^m)^n)}{x\log(fx^p)}\right)dx\\ &= a\int \frac{1}{x\log(fx^p)}\,dx+b\int \frac{\log(c(d+ex^m)^n)}{x\log(fx^p)}\,dx\\ &= b\int \frac{\log(c(d+ex^m)^n)}{x\log(fx^p)}\,dx+\frac{a\text{Subst}\left(\int \frac{1}{x}\,dx,x,\log(fx^p)\right)}{p}\\ &= \frac{a\log(\log(fx^p))}{p}+b\int \frac{\log(c(d+ex^m)^n)}{x\log(fx^p)}\,dx\end{aligned}$$

Mathematica [A]
time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{a+b\log(c(d+ex^m)^n)}{x\log(fx^p)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]),x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]), x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{a+b\ln\left(c(d+e\,x^{m})^{n}\right)}{x\ln\left(f\,x^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^m)^n))/x/ln(f*x^p),x)
[Out] int((a+b*ln(c*(d+e*x^m)^n))/x/ln(f*x^p),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p),x, algorithm="maxima")
[Out] b*integrate((log(c) + log((d + e^(m*log(x) + 1))^n))/(x*log(f) + x*log(x^p)
), x) + a*log(log(f*x^p))/p
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p),x, algorithm="fricas")
[Out] integral((b*log((x^m*e + d)^n*c) + a)/(x*log(f*x^p)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\log\left(c(d+ex^{m})^{n}\right)}{x\log\left(fx^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**m)**n))/x/ln(f*x**p),x)
[Out] Integral((a + b*log(c*(d + e*x**m)**n))/(x*log(f*x**p)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p),x, algorithm="giac")
```

```
[Out] integrate((b*log((x^m*e + d)^n*c) + a)/(x*log(f*x^p)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x^m\right)}^n\right)}{x\,\ln\left(f\,x^p\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^m)^n))/(x*log(f*x^p)),x)
```

```
[Out] int((a + b*log(c*(d + e*x^m)^n))/(x*log(f*x^p)), x)
```

3.626 $\int \frac{a+b\log(c(d+ex^m)^n)}{x\log^2(fx^p)}\,dx$

Optimal. Leaf size=64

$$-\frac{a+b\log\left(c(d+ex^m)^n\right)}{p\log\left(fx^p\right)}+\frac{bemn\text{Int}\left(\frac{x^{-1+m}}{(d+ex^m)\log(fx^p)},x\right)}{p}$$

[Out] (-a-b*ln(c*(d+e*x^m)^n))/p/ln(f*x^p)+b*e*m*n*Unintegrable(x^(-1+m)/(d+e*x^m
)/ln(f*x^p),x)/p

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^2\left(fx^p\right)}\,dx$$

Verification is not applicable to the result.

[In] Int[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]^2),x]

[Out] -((a + b*Log[c*(d + e*x^m)^n])/(p*Log[f*x^p])) + (b*e*m*n*Defer[Int][x^(-1
+ m)/((d + e*x^m)*Log[f*x^p]), x])/p

Rubi steps

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^2\left(fx^p\right)}\,dx=-\frac{a+b\log\left(c(d+ex^m)^n\right)}{p\log\left(fx^p\right)}+\frac{(bemn)\int \frac{x^{-1+m}}{(d+ex^m)\log(fx^p)}\,dx}{p}$$

Mathematica [A]
time = 1.77, size = 0, normalized size = 0.00

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^2\left(fx^p\right)}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]^2),x]

[Out] Integrate[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]^2), x]

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{a+b\ln\left(c(d+e\,x^m)^n\right)}{x\ln\left(f\,x^p\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^m)^n))/x/ln(f*x^p)^2,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^m)^n))/x/ln(f*x^p)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p)^2,x, algorithm="maxima")
```

```
[Out] (m*n*integrate(e^(m*log(x) + 1)/(d*p*x*log(f) + p*x*e^(m*log(x) + 1)*log(f)
 + (d*p*x + p*x*e^(m*log(x) + 1))*log(x^p)), x) - (log(c) + log((d + e^(m*l
og(x) + 1))^n))/(p*log(f) + p*log(x^p)))*b - a/(p*log(f*x^p))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p)^2,x, algorithm="fricas")
```

```
[Out] integral((b*log((x^m*e + d)^n*c) + a)/(x*log(f*x^p)^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**m)**n))/x/ln(f*x**p)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p)^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((x^m*e + d)^n*c) + a)/(x*log(f*x^p)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x^m\right)}^n\right)}{x\,{\ln\left(f\,x^p\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^m)^n))/(x*log(f*x^p)^2),x)
[Out] int((a + b*log(c*(d + e*x^m)^n))/(x*log(f*x^p)^2), x)
```

3.627 $\int \frac{a+b\log(c(d+ex^m)^n)}{x\log^3(fx^p)}\,dx$

Optimal. Leaf size=69

$$-\frac{a+b\log\left(c(d+ex^m)^n\right)}{2p\log^2\left(fx^p\right)}+\frac{bemn\text{Int}\left(\frac{x^{-1+m}}{(d+ex^m)\log^2(fx^p)},x\right)}{2p}$$

```
[Out] 1/2*(-a-b*ln(c*(d+e*x^m)^n))/p/ln(f*x^p)^2+1/2*b*e*m*n*Unintegrable(x^(-1+m
)/(d+e*x^m)/ln(f*x^p)^2,x)/p
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^3\left(fx^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]^3),x]
```

```
[Out] -1/2*(a + b*Log[c*(d + e*x^m)^n])/(p*Log[f*x^p]^2) + (b*e*m*n*Defer[Int][x^
(-1 + m)/((d + e*x^m)*Log[f*x^p]^2), x])/(2*p)
```

Rubi steps

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^3\left(fx^p\right)}\,dx=-\frac{a+b\log\left(c(d+ex^m)^n\right)}{2p\log^2\left(fx^p\right)}+\frac{(bemn)\int \frac{x^{-1+m}}{(d+ex^m)\log^2(fx^p)}\,dx}{2p}$$

Mathematica [A]
time = 7.95, size = 0, normalized size = 0.00

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^3\left(fx^p\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]^3),x]
```

```
[Out] Integrate[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]^3), x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{a+b\ln\left(c(d+e\,x^m)^n\right)}{x\ln\left(f\,x^p\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e*x^m)^n))/x/ln(f*x^p)^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^m)^n))/x/ln(f*x^p)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*d*m^2*n*integrate(1/2*e^(m*log(x) + 1)/(d^2*p^2*x*log(f) + 2*d*p^2*x
*e^(m*log(x) + 1)*log(f) + p^2*x*e^(2*m*log(x) + 2)*log(f) + (d^2*p^2*x + 2
*d*p^2*x*e^(m*log(x) + 1) + p^2*x*e^(2*m*log(x) + 2))*log(x^p)), x) - (m*n*
e^(m*log(x) + 1)*log(x^p) + d*p*log(c) + (m*n*log(f) + p*log(c))*e^(m*log(x
) + 1) + (d*p + p*e^(m*log(x) + 1))*log((d + e^(m*log(x) + 1))^n))/(d*p^2*l
og(f)^2 + p^2*e^(m*log(x) + 1)*log(f)^2 + (d*p^2 + p^2*e^(m*log(x) + 1))*lo
g(x^p)^2 + 2*(d*p^2*log(f) + p^2*e^(m*log(x) + 1)*log(f))*log(x^p)))*b - 1/
2*a/(p*log(f*x^p)^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p)^3,x, algorithm="fricas")
```

```
[Out] integral((b*log((x^m*e + d)^n*c) + a)/(x*log(f*x^p)^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e*x**m)**n))/x/ln(f*x**p)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p)^3,x, algorithm="giac")
[Out] integrate((b*log((x^m*e + d)^n*c) + a)/(x*log(f*x^p)^3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x^m\right)}^n\right)}{x\,{\ln\left(f\,x^p\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e*x^m)^n))/(x*log(f*x^p)^3),x)
[Out] int((a + b*log(c*(d + e*x^m)^n))/(x*log(f*x^p)^3), x)
```

3.628 $\int \log \left(c(d+e(f+gx)^p)^q\right) \, dx$

Optimal. Leaf size=76

$$-\frac{epq(f+gx)^{1+p} {}_2F_1\left(1,1+\frac{1}{p};2+\frac{1}{p};-\frac{e(f+gx)^p}{d}\right)}{dg(1+p)}+\frac{(f+gx)\log\left(c(d+e(f+gx)^p)^q\right)}{g}$$

```
[Out] -e*p*q*(g*x+f)^(1+p)*hypergeom([1, 1+1/p],[2+1/p],-e*(g*x+f)^p/d)/d/g/(1+p)
+(g*x+f)*ln(c*(d+e*(g*x+f)^p)^q)/g
```

Rubi [A]
time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2533, 2498, 371}

$$\frac{(f+gx)\log\left(c(d+e(f+gx)^p)^q\right)}{g}-\frac{epq(f+gx)^{p+1} {}_2F_1\left(1,1+\frac{1}{p};2+\frac{1}{p};-\frac{e(f+gx)^p}{d}\right)}{dg(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*(f + g*x)^p)^q],x]
```

```
[Out] -((e*p*q*(f + g*x)^(1 + p)*Hypergeometric2F1[1, 1 + p^(-1), 2 + p^(-1), -((
e*(f + g*x)^p)/d)])/(d*g*(1 + p))) + ((f + g*x)*Log[c*(d + e*(f + g*x)^p)^q
])/g
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2533

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rubi steps

$$\int \log\left(c(d+e(f+gx)^p)^q\right)\,dx = \frac{\text{Subst}\left(\int \log\left(c(d+ex^p)^q\right)\,dx, x, f+gx\right)}{g}$$
$$= \frac{(f+gx)\log\left(c(d+e(f+gx)^p)^q\right)}{g} - \frac{(epq)\text{Subst}\left(\int \frac{x^p}{d+ex^p}\,dx, x, f+gx\right)}{g}$$
$$= -\frac{epq(f+gx)^{1+p}\,{}_2F_1\left(1,1+\frac{1}{p};2+\frac{1}{p};-\frac{e(f+gx)^p}{d}\right)}{dg(1+p)} + \frac{(f+gx)\log\left(c(d+e(\right.}{g}$$

Mathematica [A]
time = 0.02, size = 65, normalized size = 0.86

$$-pqx + \frac{pq(f+gx)\,{}_2F_1\left(1,\frac{1}{p};1+\frac{1}{p};-\frac{e(f+gx)^p}{d}\right)}{g} + \frac{(f+gx)\log\left(c(d+e(f+gx)^p)^q\right)}{g}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*(f + g*x)^p)^q],x]
[Out] -(p*q*x) + (p*q*(f + g*x)*Hypergeometric2F1[1, p^(-1), 1 + p^(-1), -((e*(f
+ g*x)^p)/d)])/g + ((f + g*x)*Log[c*(d + e*(f + g*x)^p)^q])/g
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \ln\left(c(d+e(gx+f)^p)^q\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*(g*x+f)^p)^q),x)
[Out] int(ln(c*(d+e*(g*x+f)^p)^q),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f)^p)^q),x, algorithm="maxima")
[Out] d*g*p*q*integrate(x/(d*g*x + (g*x*e + f*e)*(g*x + f)^p + d*f), x) + (f*p*q*
log(g*x + f) + g*x*log((d + e^(p*log(g*x + f) + 1))^q) - (g*p*q - g*log(c))
*x)/g
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f)^p)^q),x, algorithm="fricas")
[Out] integral(log(((g*x + f)^p*e + d)^q*c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log \left(c(d+e(f+gx)^p)^q\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*(g*x+f)**p)**q),x)
[Out] Integral(log(c*(d + e*(f + g*x)**p)**q), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f)^p)^q),x, algorithm="giac")
[Out] integrate(log(((g*x + f)^p*e + d)^q*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(c\,{\left(d+e\,{\left(f+g\,x\right)}^p\right)}^q\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*(f + g*x)^p)^q),x)
[Out] int(log(c*(d + e*(f + g*x)^p)^q), x)
```

3.629 $\int \log \left(c(d+e(f+gx)^3)^q\right) \, dx$

Optimal. Leaf size=169

$$-3qx-\frac{\sqrt{3}\,\sqrt[3]{d}\,q\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,(f+gx)}{\sqrt{3}\,\sqrt[3]{d}}\right)}{\sqrt[3]{e}\,g}+\frac{\sqrt[3]{d}\,q\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,(f+gx)\right)}{\sqrt[3]{e}\,g}-\frac{\sqrt[3]{d}\,q\log\left(d^{2/3}-\sqrt[3]{d}\,\sqrt[3]{e}\,(f$$

```
[Out] -3*q*x+d^(1/3)*q*ln(d^(1/3)+e^(1/3)*(g*x+f))/e^(1/3)/g-1/2*d^(1/3)*q*ln(d^(
2/3)-d^(1/3)*e^(1/3)*(g*x+f)+e^(2/3)*(g*x+f)^2)/e^(1/3)/g+(g*x+f)*ln(c*(d+e
*(g*x+f)^3)^q)/g-d^(1/3)*q*arctan(1/3*(d^(1/3)-2*e^(1/3)*(g*x+f))/d^(1/3)*3
^(1/2))*3^(1/2)/e^(1/3)/g
```

Rubi [A]
time = 0.14, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {2533, 2498, 327, 206, 31, 648, 631, 210, 642}

$$-\frac{\sqrt{3}\,\sqrt[3]{d}\,q\text{ArcTan}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,(f+gx)}{\sqrt{3}\,\sqrt[3]{d}}\right)}{\sqrt[3]{e}\,g}+\frac{(f+gx)\log\left(c(d+e(f+gx)^3)^q\right)}{g}-\frac{\sqrt[3]{d}\,q\log\left(d^{2/3}-\sqrt[3]{d}\,\sqrt[3]{e}\,(f+gx)+e^{2/3}(f+gx)^2\right)}{2\sqrt[3]{e}\,g}+\frac{\sqrt[3]{d}\,q\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,(f+gx)\right)}{\sqrt[3]{e}\,g}-3qx$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*(f + g*x)^3)^q],x]
```

```
[Out] -3*q*x - (Sqrt[3]*d^(1/3)*q*ArcTan[(d^(1/3) - 2*e^(1/3)*(f + g*x))/(Sqrt[3]
*d^(1/3))])/(e^(1/3)*g) + (d^(1/3)*q*Log[d^(1/3) + e^(1/3)*(f + g*x)])/(e^(
1/3)*g) - (d^(1/3)*q*Log[d^(2/3) - d^(1/3)*e^(1/3)*(f + g*x) + e^(2/3)*(f +
 g*x)^2])/(2*e^(1/3)*g) + ((f + g*x)*Log[c*(d + e*(f + g*x)^3)^q])/g
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2533

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rubi steps

$$\int \log\left(c\left(d+e(f+gx)^3\right)^q\right)\,dx = \frac{\text{Subst}\left(\int \log\left(c(d+ex^3)^q\right)\,dx,x,f+gx\right)}{g}$$
$$= \frac{(f+gx)\log\left(c(d+e(f+gx)^3)^q\right)}{g} - \frac{(3eq)\text{Subst}\left(\int \frac{x^3}{d+ex^3}\,dx,x,f+gx\right)}{g}$$
$$= -3qx + \frac{(f+gx)\log\left(c(d+e(f+gx)^3)^q\right)}{g} + \frac{(3dq)\text{Subst}\left(\int \frac{1}{d+ex^3}\,dx,x,f$$
$$= -3qx + \frac{(f+gx)\log\left(c(d+e(f+gx)^3)^q\right)}{g} + \frac{\left(\sqrt[3]{d}\,q\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{e}}\right.}{g}$$
$$= -3qx + \frac{\sqrt[3]{d}\,q\log\left(\sqrt[3]{d}+\sqrt[3]{e}(f+gx)\right)}{\sqrt[3]{e}\,g} + \frac{(f+gx)\log\left(c(d+e(f+gx)\right.}{g}$$
$$= -3qx + \frac{\sqrt[3]{d}\,q\log\left(\sqrt[3]{d}+\sqrt[3]{e}(f+gx)\right)}{\sqrt[3]{e}\,g} - \frac{\sqrt[3]{d}\,q\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}(f-\right.}{2\sqrt[3]{e}\,g}$$
$$= -3qx - \frac{\sqrt{3}\sqrt[3]{d}\,q\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{e}(f+gx)}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{\sqrt[3]{e}\,g} + \frac{\sqrt[3]{d}\,q\log\left(\sqrt[3]{d}+\sqrt[3]{e}(f+g\right.}{\sqrt[3]{e}\,g}$$

Mathematica [A]
time = 0.07, size = 147, normalized size = 0.87

$$-3qx+\frac{\sqrt[3]{d}\,q\left(2\sqrt{3}\tan^{-1}\left(\frac{-\sqrt[3]{d}+2\sqrt[3]{e}(f+gx)}{\sqrt{3}\sqrt[3]{d}}\right)+2\log\left(\sqrt[3]{d}+\sqrt[3]{e}(f+gx)\right)-\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+e^{2/3}(f+gx)^2\right)\right)}{2\sqrt[3]{e}\,g}+\frac{(f+gx)\log\left(c(d+e(f+gx)^3)^q\right)}{g}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*(f + g*x)^3)^q],x]
```

```
[Out] -3*q*x + (d^(1/3)*q*(2*Sqrt[3]*ArcTan[(-d^(1/3) + 2*e^(1/3)*(f + g*x))/(Sqr
t[3]*d^(1/3))] + 2*Log[d^(1/3) + e^(1/3)*(f + g*x)] - Log[d^(2/3) - d^(1/3)
*e^(1/3)*(f + g*x) + e^(2/3)*(f + g*x)^2]))/(2*e^(1/3)*g) + ((f + g*x)*Log[
c*(d + e*(f + g*x)^3)^q])/g
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.72, size = 154, normalized size = 0.91

method	result

default	$\ln\left(c(e\,g^3x^3+3ef\,g^2x^2+3e\,f^2gx+f^3e+d)^q\right)x-3egq\left(\frac{x}{eg}+\frac{\sum_{\_R=\mathrm{RootOf}\left(e\,g^3\_Z^3+3ef\,g^2\_Z^2+3f^2eg\_Z+f^3\right.}}{}\right.$
risch	$x\ln\left(\left(d+e(gx+f)^3\right)^q\right)-\frac{i\pi x\,\mathrm{csgn}\left(i\left(d+e(gx+f)^3\right)^q\right)\mathrm{csgn}\left(ic\left(d+e(gx+f)^3\right)^q\right)\mathrm{csgn}(ic)}{2}+\frac{i\mathrm{csgn}(ic)\mathrm{csgn}\left(ic\left(d+e(gx+\right.\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*(g*x+f)^3)^q),x,method=_RETURNVERBOSE)
```

```
[Out] ln(c*(e*g^3*x^3+3*e*f*g^2*x^2+3*e*f^2*g*x+e*f^3+d)^q)*x-3*e*g*q*(x/e/g+1/3/
e^2/g^2*sum((-_R^2*e*f*g^2-2*_R*e*f^2*g-e*f^3-d)/(_R^2*g^2+2*_R*f*g+f^2)*ln
(x-_R),_R=RootOf(_Z^3*e*g^3+3*_Z^2*e*f*g^2+3*_Z*e*f^2*g+e*f^3+d)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f)^3)^q),x, algorithm="maxima")
```

```
[Out] -(3*q - log(c))*x + 3*q*integrate((f*g^2*x^2*e + 2*f^2*g*x*e + f^3*e + d)/(
g^3*x^3*e + 3*f*g^2*x^2*e + 3*f^2*g*x*e + f^3*e + d), x) + x*log((g^3*x^3*e
 + 3*f*g^2*x^2*e + 3*f^2*g*x*e + f^3*e + d)^q)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.13, size = 1346, normalized size = 7.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f)^3)^q),x, algorithm="fricas")
```

```
[Out] 1/4*(4*g*q*x*log((g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3)*e + d) - 12*g*q*
x - 4*sqrt(3)*g*sqrt(((((-1/2*f^3*q^3/g^3 + 1/2*d*q^3*e^(-1)/g^3 + 1/2*(f^3*
q^3*e + d*q^3)*e^(-1)/g^3)^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)^2*g^2 + 4*((-1/
2*f^3*q^3/g^3 + 1/2*d*q^3*e^(-1)/g^3 + 1/2*(f^3*q^3*e + d*q^3)*e^(-1)/g^3)^
(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*f*g*q + 4*f^2*q^2)/g^2)*arctan(-1/24*(2*sq
rt(3)*sqrt(4*g^2*q^2*x^2 + 12*f*g*q^2*x + ((-1/2*f^3*q^3/g^3 + 1/2*d*q^3*e^
(-1)/g^3 + 1/2*(f^3*q^3*e + d*q^3)*e^(-1)/g^3)^(1/3)*(I*sqrt(3) + 1) - 2*f*
```

```
q/g)^2*g^2 + 12*f^2*q^2 + 2*(g^2*q*x + 3*f*g*q)*((-1/2*f^3*q^3/g^3 + 1/2*d*
q^3*e^(-1)/g^3 + 1/2*(f^3*q^3*e + d*q^3)*e^(-1)/g^3)^(1/3)*(I*sqrt(3) + 1)
- 2*f*q/g))*(((-1/2*f^3*q^3/g^3 + 1/2*d*q^3*e^(-1)/g^3 + 1/2*(f^3*q^3*e + d
*q^3)*e^(-1)/g^3)^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*g^2*e + 2*f*g*q*e)*sqrt(
(((-1/2*f^3*q^3/g^3 + 1/2*d*q^3*e^(-1)/g^3 + 1/2*(f^3*q^3*e + d*q^3)*e^(-1)
/g^3)^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)^2*g^2 + 4*((-1/2*f^3*q^3/g^3 + 1/2*d
*q^3*e^(-1)/g^3 + 1/2*(f^3*q^3*e + d*q^3)*e^(-1)/g^3)^(1/3)*(I*sqrt(3) + 1)
 - 2*f*q/g)*f*g*q + 4*f^2*q^2)/g^2) - sqrt(3)*(((-1/2*f^3*q^3/g^3 + 1/2*d*q
^3*e^(-1)/g^3 + 1/2*(f^3*q^3*e + d*q^3)*e^(-1)/g^3)^(1/3)*(I*sqrt(3) + 1) -
 2*f*q/g)^2*g^3*e + 4*(g^3*q*x + 2*f*g^2*q)*((-1/2*f^3*q^3/g^3 + 1/2*d*q^3*
e^(-1)/g^3 + 1/2*(f^3*q^3*e + d*q^3)*e^(-1)/g^3)^(1/3)*(I*sqrt(3) + 1) - 2*
f*q/g)*e + 4*(2*f*g^2*q^2*x + 3*f^2*g*q^2)*e)*sqrt((((-1/2*f^3*q^3/g^3 + 1/
2*d*q^3*e^(-1)/g^3 + 1/2*(f^3*q^3*e + d*q^3)*e^(-1)/g^3)^(1/3)*(I*sqrt(3) +
 1) - 2*f*q/g)^2*g^2 + 4*((-1/2*f^3*q^3/g^3 + 1/2*d*q^3*e^(-1)/g^3 + 1/2*(f
^3*q^3*e + d*q^3)*e^(-1)/g^3)^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*f*g*q + 4*f^
2*q^2)/g^2))/(d*q^3)) - 2*((-1/2*f^3*q^3/g^3 + 1/2*d*q^3*e^(-1)/g^3 + 1/2*(
f^3*q^3*e + d*q^3)*e^(-1)/g^3)^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*g*log(q*x -
 1/2*(-1/2*f^3*q^3/g^3 + 1/2*d*q^3*e^(-1)/g^3 + 1/2*(f^3*q^3*e + d*q^3)*e^(
-1)/g^3)^(1/3)*(I*sqrt(3) + 1) + f*q/g) + 4*g*x*log(c) + (((-1/2*f^3*q^3/g^
3 + 1/2*d*q^3*e^(-1)/g^3 + 1/2*(f^3*q^3*e + d*q^3)*e^(-1)/g^3)^(1/3)*(I*sqr
t(3) + 1) - 2*f*q/g)*g + 6*f*q)*log(4*g^2*q^2*x^2 + 12*f*g*q^2*x + ((-1/2*f
^3*q^3/g^3 + 1/2*d*q^3*e^(-1)/g^3 + 1/2*(f^3*q^3*e + d*q^3)*e^(-1)/g^3)^(1/
3)*(I*sqrt(3) + 1) - 2*f*q/g)^2*g^2 + 12*f^2*q^2 + 2*(g^2*q*x + 3*f*g*q)*((
-1/2*f^3*q^3/g^3 + 1/2*d*q^3*e^(-1)/g^3 + 1/2*(f^3*q^3*e + d*q^3)*e^(-1)/g^
3)^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)))/g
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*(g*x+f)**3)**q),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(130) = 260.
time = 6.35, size = 265, normalized size = 1.57

$$qx\log\left(g^3x^3e+3\,fg^2x^2e+3\,f^2gxe+f^3e+d\right)-3\,qx+x\log\left(c\right)+\frac{fq\log\left(\left|g^3x^3e+3\,fg^2x^2e+3\,f^2gxe+f^3e+d\right|\right)}{g}+\frac{\left(2\,\sqrt{3}\left(dg^6q^3\right)^{\frac{1}{3}}\arctan\left(-\frac{gxe+fe+d^{\frac{1}{3}}e^{\frac{2}{3}}}{\sqrt{3}\,gxe+\sqrt{3}\,fe-\sqrt{3}\,d^{\frac{1}{3}}e^{\frac{2}{3}}}\right)e^{\frac{2}{3}}-\left(dg^6q^3\right)^{\frac{1}{3}}e^{\frac{2}{3}}\log\left(4\left(\sqrt{3}\,gxe+\sqrt{3}\,fe-\sqrt{3}\,d^{\frac{1}{3}}e^{\frac{2}{3}}\right)^2+4\left(gxe+fe+d^{\frac{1}{3}}e^{\frac{2}{3}}\right)^2\right)+2\left(dg^6q^3\right)^{\frac{1}{3}}e^{\frac{2}{3}}\log\left(\left|gxe+fe+d^{\frac{1}{3}}e^{\frac{2}{3}}\right|\right)\right)e^{(-1)}}{2\,g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f)^3)^q),x, algorithm="giac")
```

```
[Out] q*x*log(g^3*x^3*e + 3*f*g^2*x^2*e + 3*f^2*g*x*e + f^3*e + d) - 3*q*x + x*lo
g(c) + f*q*log(abs(g^3*x^3*e + 3*f*g^2*x^2*e + 3*f^2*g*x*e + f^3*e + d))/g
+ 1/2*(2*sqrt(3)*(d*g^6*q^3)^(1/3)*arctan(-(g*x*e + f*e + d^(1/3)*e^(2/3))/
(sqrt(3)*g*x*e + sqrt(3)*f*e - sqrt(3)*d^(1/3)*e^(2/3)))*e^(2/3) - (d*g^6*q
^3)^(1/3)*e^(2/3)*log(4*(sqrt(3)*g*x*e + sqrt(3)*f*e - sqrt(3)*d^(1/3)*e^(2
/3))^2 + 4*(g*x*e + f*e + d^(1/3)*e^(2/3))^2) + 2*(d*g^6*q^3)^(1/3)*e^(2/3)
*log(abs(g*x*e + f*e + d^(1/3)*e^(2/3))))*e^(-1)/g^3
```

Mupad [B]
time = 0.28, size = 192, normalized size = 1.14

$$x\,\ln\left(c\,{\left(d+e\,{\left(f+g\,x\right)}^3\right)}^q\right)-\left(\sum_{k=1}^{3}\ln\left(d\,e^2\,g^5\,\left(\mathrm{root}\left(b^3\,e\,g^3+3\,b^2\,e\,f\,g^2\,q+3\,b\,e\,f^2\,g\,q^2+e\,f^3\,q^3+d\,q^3,b,k\right)\,g+f\,q\right)\,\left(\mathrm{root}\left(b^3\,e\,g^3+3\,b^2\,e\,f\,g^2\,q+3\,b\,e\,f^2\,g\,q^2+e\,f^3\,q^3+d\,q^3,b,k\right)-q\,x\right)\,9\right)\,\mathrm{root}\left(b^3\,e\,g^3+3\,b^2\,e\,f\,g^2\,q+3\,b\,e\,f^2\,g\,q^2+e\,f^3\,q^3+d\,q^3,b,k\right)\right)-3\,q\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*(f + g*x)^3)^q),x)
```

```
[Out] x*log(c*(d + e*(f + g*x)^3)^q) - symsum(log(9*d*e^2*g^5*(root(b^3*e*g^3 + 3
*b^2*e*f*g^2*q + 3*b*e*f^2*g*q^2 + e*f^3*q^3 + d*q^3, b, k)*g + f*q)*(root(
b^3*e*g^3 + 3*b^2*e*f*g^2*q + 3*b*e*f^2*g*q^2 + e*f^3*q^3 + d*q^3, b, k) -
q*x))*root(b^3*e*g^3 + 3*b^2*e*f*g^2*q + 3*b*e*f^2*g*q^2 + e*f^3*q^3 + d*q^
3, b, k), k, 1, 3) - 3*q*x
```

3.630 $\int \log \left(c(d+e(f+gx)^2)^q\right) \, dx$

Optimal. Leaf size=63

$$-2qx+\frac{2\sqrt{d}\, q \tan^{-1}\left(\frac{\sqrt{e}\,(f+gx)}{\sqrt{d}}\right)}{\sqrt{e}\, g}+\frac{(f+gx)\log\left(c(d+e(f+gx)^2)^q\right)}{g}$$

```
[Out] -2*q*x+(g*x+f)*ln(c*(d+e*(g*x+f)^2)^q)/g+2*q*arctan((g*x+f)*e^(1/2)/d^(1/2)
)*d^(1/2)/g/e^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2533, 2498, 327, 211}

$$\frac{2\sqrt{d}\, q\text{ArcTan}\left(\frac{\sqrt{e}\,(f+gx)}{\sqrt{d}}\right)}{\sqrt{e}\, g}+\frac{(f+gx)\log\left(c(d+e(f+gx)^2)^q\right)}{g}-2qx$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*(f + g*x)^2)^q],x]
```

```
[Out] -2*q*x + (2*Sqrt[d]*q*ArcTan[(Sqrt[e]*(f + g*x))/Sqrt[d]])/(Sqrt[e]*g) + ((
f + g*x)*Log[c*(d + e*(f + g*x)^2)^q])/g
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2533

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rubi steps

$$\int \log\left(c\left(d+e(f+gx)^2\right)^q\right)\,dx = \frac{\text{Subst}\left(\int \log\left(c\left(d+ex^2\right)^q\right)\,dx, x, f+gx\right)}{g}$$
$$= \frac{(f+gx)\log\left(c\left(d+e(f+gx)^2\right)^q\right)}{g} - \frac{(2eq)\text{Subst}\left(\int \frac{x^2}{d+ex^2}\,dx, x, f+gx\right)}{g}$$
$$= -2qx + \frac{(f+gx)\log\left(c\left(d+e(f+gx)^2\right)^q\right)}{g} + \frac{(2dq)\text{Subst}\left(\int \frac{1}{d+ex^2}\,dx, x, f+\right.}{g}$$
$$= -2qx + \frac{2\sqrt{d}\,q\tan^{-1}\left(\frac{\sqrt{e}\,(f+gx)}{\sqrt{d}}\right)}{\sqrt{e}\,g} + \frac{(f+gx)\log\left(c\left(d+e(f+gx)^2\right)^q\right)}{g}$$

Mathematica [A]
time = 0.03, size = 63, normalized size = 1.00

$$-2qx + \frac{2\sqrt{d}\,q\tan^{-1}\left(\frac{\sqrt{e}\,(f+gx)}{\sqrt{d}}\right)}{\sqrt{e}\,g} + \frac{(f+gx)\log\left(c\left(d+e(f+gx)^2\right)^q\right)}{g}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*(f + g*x)^2)^q],x]
```

```
[Out] -2*q*x + (2*Sqrt[d]*q*ArcTan[(Sqrt[e]*(f + g*x))/Sqrt[d]])/(Sqrt[e]*g) + ((
f + g*x)*Log[c*(d + e*(f + g*x)^2)^q])/g
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(55) = 110.
time = 0.31, size = 115, normalized size = 1.83

method	result
default	$\ln\left(c\left(e\,g^2x^2+2efgx+f^2e+d\right)^q\right)x-2egq\left(\frac{x}{eg}+\frac{-\frac{f\ln\left(e\,g^2x^2+2efgx+f^2e+d\right)}{2g}-\frac{d\arctan\left(\frac{2e\,g^2x+2efg}{2g\sqrt{ed}}\right)}{g\sqrt{ed}}}{eg}\right)$

risch	$x\ln\left(\left(d+e(gx+f)^2\right)^q\right)+\frac{i\,\mathrm{csgn}(ic)\,\mathrm{csgn}\left(ic\left(d+e(gx+f)^2\right)^q\right)^2 x\pi}{2}-\frac{i\pi x\,\mathrm{csgn}\left(i\left(d+e(gx+f)^2\right)^q\right)\mathrm{csgn}\left(ic\left(d+e(gx+f)\right.\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e*(g*x+f)^2)^q),x,method=_RETURNVERBOSE)
```

```
[Out] ln(c*(e*g^2*x^2+2*e*f*g*x+e*f^2+d)^q)*x-2*e*g*q*(x/e/g+1/e/g*(-1/2*f/g*ln(e
*g^2*x^2+2*e*f*g*x+e*f^2+d)-d/g/(e*d)^(1/2)*arctan(1/2*(2*e*g^2*x+2*e*f*g)/
g/(e*d)^(1/2))))
```

Maxima [A]
time = 0.52, size = 101, normalized size = 1.60

$$-gq\left(\frac{2\,xe^{(-1)}}{g}-\frac{fe^{(-1)}\log\left(g^2x^2e+2\,fgxe+f^2e+d\right)}{g^2}-\frac{2\,\sqrt{d}\,\arctan\left(\frac{(g^2xe+fge)e^{\left(-\frac{1}{2}\right)}}{\sqrt{d}\,g}\right)e^{\left(-\frac{3}{2}\right)}}{g^2}\right)e+x\log\left(\left((gx+f)^2e+d\right)^qc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f)^2)^q),x, algorithm="maxima")
```

```
[Out] -g*q*(2*x*e^(-1)/g - f*e^(-1)*log(g^2*x^2*e + 2*f*g*x*e + f^2*e + d)/g^2 -
2*sqrt(d)*arctan((g^2*x*e + f*g*e)*e^(-1/2)/(sqrt(d)*g))*e^(-3/2)/g^2)*e +
x*log(((g*x + f)^2*e + d)^q*c)
```

Fricas [A]
time = 0.34, size = 190, normalized size = 3.02

$$\left[-\frac{2\,gqx-gx\log(c)-\sqrt{-de^{(-1)}}\,q\log\left(\frac{2\,(gx+f)\sqrt{-de^{(-1)}}\,e+(g^2x^2+2\,fgx+f^2)e-d}{(g^2x^2+2\,fgx+f^2)e+d}\right)-(gqx+fq)\log\left((g^2x^2+2\,fgx+f^2)e+d\right)}{g},\ \frac{2\,\sqrt{d}\,q\arctan\left(\frac{(gx+f)e^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}-2\,gqx+gx\log(c)+(gqx+fq)\log\left((g^2x^2+2\,fgx+f^2)e+d\right)}{g}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f)^2)^q),x, algorithm="fricas")
```

```
[Out] [-(2*g*q*x - g*x*log(c) - sqrt(-d*e^(-1))*q*log((2*(g*x + f)*sqrt(-d*e^(-1)
)*e + (g^2*x^2 + 2*f*g*x + f^2)*e - d)/((g^2*x^2 + 2*f*g*x + f^2)*e + d)) -
 (g*q*x + f*q)*log((g^2*x^2 + 2*f*g*x + f^2)*e + d))/g, (2*sqrt(d)*q*arctan
((g*x + f)*e^(1/2)/sqrt(d))*e^(-1/2) - 2*g*q*x + g*x*log(c) + (g*q*x + f*q)
*log((g^2*x^2 + 2*f*g*x + f^2)*e + d))/g]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(58) = 116$.
time = 104.35, size = 235, normalized size = 3.73

$$\begin{cases} x\log\left(0^qc\right) & \text{for}\ d=0\wedge e=0\wedge g=0 \\ x\log\left(cd^q\right) & \text{for}\ e=0 \\ x\log\left(c(d+ef^2)^q\right) & \text{for}\ g=0 \\ \frac{f\log\left(c(ef^2+2efgx+eg^2x^2)^q\right)}{g}-2qx+x\log\left(c(ef^2+2efgx+eg^2x^2)^q\right) & \text{for}\ d=0 \\ \frac{2dq\log\left(\frac{f}{g}+x-\frac{\sqrt{-de}}{eg}\right)}{g\sqrt{-de}}-\frac{d\log\left(c(d+ef^2+2efgx+eg^2x^2)^q\right)}{g\sqrt{-de}}+\frac{f\log\left(c(d+ef^2+2efgx+eg^2x^2)^q\right)}{g}-2qx+x\log\left(c(d+ef^2+2efgx+eg^2x^2)^q\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*(g*x+f)**2)**q),x)
```

```
[Out] Piecewise((x*log(0**q*c), Eq(d, 0) & Eq(e, 0) & Eq(g, 0)), (x*log(c*d**q),
Eq(e, 0)), (x*log(c*(d + e*f**2)**q), Eq(g, 0)), (f*log(c*(e*f**2 + 2*e*f*g
*x + e*g**2*x**2)**q)/g - 2*q*x + x*log(c*(e*f**2 + 2*e*f*g*x + e*g**2*x**2
)**q), Eq(d, 0)), (2*d*q*log(f/g + x - sqrt(-d*e)/(e*g))/(g*sqrt(-d*e)) - d
*log(c*(d + e*f**2 + 2*e*f*g*x + e*g**2*x**2)**q)/(g*sqrt(-d*e)) + f*log(c*
(d + e*f**2 + 2*e*f*g*x + e*g**2*x**2)**q)/g - 2*q*x + x*log(c*(d + e*f**2
+ 2*e*f*g*x + e*g**2*x**2)**q), True))
```

Giac [A]
time = 7.28, size = 96, normalized size = 1.52

$$qx\log\left(g^2x^2e + 2\,fgxe + f^2e + d\right) + \frac{2\,\sqrt{d}\,q\arctan\left(\frac{(gxe+fe)e^{\left(-\frac{1}{2}\right)}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{g} - 2\,qx + \frac{fq\log\left(g^2x^2e + 2\,fgxe + f^2e + d\right)}{g} + x\log\left(c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f)^2)^q),x, algorithm="giac")
```

```
[Out] q*x*log(g^2*x^2*e + 2*f*g*x*e + f^2*e + d) + 2*sqrt(d)*q*arctan((g*x*e + f*
e)*e^(-1/2)/sqrt(d))*e^(-1/2)/g - 2*q*x + f*q*log(g^2*x^2*e + 2*f*g*x*e + f
^2*e + d)/g + x*log(c)
```

Mupad [B]
time = 0.13, size = 82, normalized size = 1.30

$$x\,\ln\left(c\,{\left(d+e\,{\left(f+g\,x\right)}^2\right)}^q\right) - 2\,q\,x + \frac{f\,q\,\ln\left(e\,f^2 + 2\,e\,f\,g\,x + e\,g^2\,x^2 + d\right)}{g} + \frac{2\,\sqrt{d}\,q\,\mathrm{atan}\left(\frac{\sqrt{e}\,f}{\sqrt{d}} + \frac{\sqrt{e}\,g\,x}{\sqrt{d}}\right)}{\sqrt{e}\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*(f + g*x)^2)^q),x)
```

```
[Out] x*log(c*(d + e*(f + g*x)^2)^q) - 2*q*x + (f*q*log(d + e*f^2 + e*g^2*x^2 + 2
*e*f*g*x))/g + (2*d^(1/2)*q*atan((e^(1/2)*f)/d^(1/2) + (e^(1/2)*g*x)/d^(1/2
)))/(e^(1/2)*g)
```

3.631 $\int \log \left(c(d+e(f+gx))^q\right) dx$

Optimal. Leaf size=35

$$-qx+\frac{(d+ef+egx)\log \left(c(d+e(f+gx))^q\right)}{eg}$$

```
[Out] -q*x+(e*g*x+e*f+d)*ln(c*(d+e*(g*x+f))^q)/e/g
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2494, 2436, 2332}

$$\frac{(d+ef+egx)\log \left(c(d+e(f+gx))^q\right)}{eg}-qx$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e*(f + g*x))^q],x]
```

```
[Out] -(q*x) + ((d + e*f + e*g*x)*Log[c*(d + e*(f + g*x))^q])/(e*g)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2494

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] &&  !LinearMatchQ[v, x] &&  !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])
```

Rubi steps

$$\int \log \left(c(d+e(f+gx))^q\right) dx = \int \log \left(c(d+ef+egx)^q\right) dx$$
$$= \frac{\text{Subst}\left(\int \log \left(cx^q\right) dx, x, d+ef+egx\right)}{eg}$$
$$= -qx + \frac{(d+ef+egx)\log \left(c(d+e(f+gx))^q\right)}{eg}$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 1.26

$$\frac{(d+ef)q\log(d+ef+egx)}{eg} + x\left(-q+\log \left(c(d+ef+egx)^q\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e*(f + g*x))^q],x]
[Out] ((d + e*f)*q*Log[d + e*f + e*g*x])/(e*g) + x*(-q + Log[c*(d + e*f + e*g*x)^
q])
```

Maple [A]
time = 0.22, size = 57, normalized size = 1.63

method	result
norman	$x\ln\left(c\,\mathrm{e}^{q\ln(d+(gx+f)e)}\right)+\frac{q(ef+d)\ln(d+(gx+f)e)}{eg}-qx$
default	$\ln\left(c(egx+ef+d)^q\right)x-egq\left(\frac{x}{eg}+\frac{(-ef-d)\ln(egx+ef+d)}{e^2g^2}\right)$
risch	$x\ln\left((egx+ef+d)^q\right)-\frac{i\pi x\,\mathrm{csgn}(i(egx+ef+d)^q)\mathrm{csgn}(ic(egx+ef+d)^q)\mathrm{csgn}(ic)}{2}+\frac{i\pi x\mathrm{csgn}(ic(egx+ef+d)^q)^2\mathrm{csgn}(ic)}{2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+(g*x+f)*e)^q),x,method=_RETURNVERBOSE)
[Out] ln(c*(e*g*x+e*f+d)^q)*x-e*g*q*(x/e/g+(-e*f-d)/e^2/g^2*ln(e*g*x+e*f+d))
```

Maxima [A]
time = 0.30, size = 57, normalized size = 1.63

$$-gq\left(\frac{xe^{(-1)}}{g} - \frac{(fe+d)e^{(-2)}\log(gxe+fe+d)}{g^2}\right)e + x\log\left(((gx+f)e+d)^q c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f))^q),x, algorithm="maxima")
```

```
[Out] -g*q*(x*e^(-1)/g - (f*e + d)*e^(-2)*log(g*x*e + f*e + d)/g^2)*e + x*log(((g
*x + f)*e + d)^q*c)
```

Fricas [A]
time = 0.35, size = 50, normalized size = 1.43

$$-\frac{(gqxe - gxe\log(c) - (dq + (gqx + fq)e)\log((gx + f)e + d))e^{(-1)}}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f))^q),x, algorithm="fricas")
```

```
[Out] -(g*q*x*e - g*x*e*log(c) - (d*q + (g*q*x + f*q)*e)*log((g*x + f)*e + d))*e^
(-1)/g
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(29) = 58$.
time = 0.37, size = 80, normalized size = 2.29

$$\begin{cases} x\log(cd^q) & \text{for } e = 0 \wedge (e = 0 \vee g = 0) \\ x\log(c(d + ef)^q) & \text{for } g = 0 \\ \frac{d\log(c(d+ef+egx)^q)}{eg} + \frac{f\log(c(d+ef+egx)^q)}{g} - qx + x\log(c(d + ef + egx)^q) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e*(g*x+f))**q),x)
```

```
[Out] Piecewise((x*log(c*d**q), Eq(e, 0) & (Eq(e, 0) | Eq(g, 0))), (x*log(c*(d +
e*f)**q), Eq(g, 0)), (d*log(c*(d + e*f + e*g*x)**q)/(e*g) + f*log(c*(d + e*
f + e*g*x)**q)/g - q*x + x*log(c*(d + e*f + e*g*x)**q), True))
```

Giac [A]
time = 5.05, size = 69, normalized size = 1.97

$$\frac{(gxe + fe + d)qe^{(-1)}\log(gxe + fe + d)}{g} - \frac{(gxe + fe + d)qe^{(-1)}}{g} + \frac{(gxe + fe + d)e^{(-1)}\log(c)}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e*(g*x+f))^q),x, algorithm="giac")
```

```
[Out] (g*x*e + f*e + d)*q*e^(-1)*log(g*x*e + f*e + d)/g - (g*x*e + f*e + d)*q*e^(
-1)/g + (g*x*e + f*e + d)*e^(-1)*log(c)/g
```

Mupad [B]
time = 0.31, size = 46, normalized size = 1.31

$$x\,\ln\left(c\,{\left(d+e\,\left(f+g\,x\right)\right)}^q\right)-q\,x+\frac{\ln\left(d+e\,f+e\,g\,x\right)\,\left(d\,q+e\,f\,q\right)}{e\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e*(f + g*x))^q),x)
```

```
[Out] x*log(c*(d + e*(f + g*x))^q) - q*x + (log(d + e*f + e*g*x)*(d*q + e*f*q))/(
e*g)
```

3.632 $\int \log \left(c\left(d+\frac{e}{f+gx}\right)^q\right) dx$

Optimal. Leaf size=45

$$\frac{(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^q\right)}{g}+\frac{eq\log(e+d(f+gx))}{dg}$$

```
[Out] (g*x+f)*ln(c*(d+e/(g*x+f))^q)/g+e*q*ln(e+d*(g*x+f))/d/g
```

Rubi [A]
time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2533, 2498, 269, 31}

$$\frac{(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^q\right)}{g}+\frac{eq\log(d(f+gx)+e)}{dg}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e/(f + g*x))^q],x]
```

```
[Out] ((f + g*x)*Log[c*(d + e/(f + g*x))^q])/g + (e*q*Log[e + d*(f + g*x)])/(d*g)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2533

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rubi steps

$$
\begin{aligned}
\int \log\left(c\left(d+\frac{e}{f+gx}\right)^q\right)dx &= \frac{\text{Subst}\left(\int \log\left(c\left(d+\frac{e}{x}\right)^q\right)\,dx,x,f+gx\right)}{g}\\
&= \frac{(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^q\right)}{g}+\frac{(eq)\text{Subst}\left(\int \frac{1}{\left(d+\frac{e}{x}\right)x}\,dx,x,f+gx\right)}{g}\\
&= \frac{(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^q\right)}{g}+\frac{(eq)\text{Subst}\left(\int \frac{1}{e+dx}\,dx,x,f+gx\right)}{g}\\
&= \frac{(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^q\right)}{g}+\frac{eq\log(e+d(f+gx))}{dg}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 56, normalized size = 1.24

$$\frac{-dfq\log(f+gx)+(e+df)q\log(e+df+dgx)+dgx\log\left(c\left(d+\frac{e}{f+gx}\right)^q\right)}{dg}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e/(f + g*x))^q],x]
```

```
[Out] (-(d*f*q*Log[f + g*x]) + (e + d*f)*q*Log[e + d*f + d*g*x] + d*g*x*Log[c*(d
+ e/(f + g*x))^q])/(d*g)
```

Maple [A]
time = 0.05, size = 71, normalized size = 1.58

method	result	size
default	$\ln\left(c\left(\frac{dgx+df+e}{gx+f}\right)^q\right)x+egq\left(\frac{(df+e)\ln(dgx+df+e)}{e\,g^2d}-\frac{f\ln(gx+f)}{g^2e}\right)$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e/(g*x+f))^q),x,method=_RETURNVERBOSE)
```

```
[Out] ln(c*((d*g*x+d*f+e)/(g*x+f))^q)*x+e*g*q*((d*f+e)/e/g^2/d*ln(d*g*x+d*f+e)-1/
g^2/e*f*ln(g*x+f))
```

Maxima [A]
time = 0.29, size = 67, normalized size = 1.49

$$-gq\left(\frac{fe^{(-1)}\log(gx+f)}{g^2}-\frac{(df+e)e^{(-1)}\log(dgx+df+e)}{dg^2}\right)e+x\log\left(c\left(d+\frac{e}{gx+f}\right)^q\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(g*x+f))^q),x, algorithm="maxima")
```

```
[Out] -g*q*(f*e^(-1)*log(g*x + f)/g^2 - (d*f + e)*e^(-1)*log(d*g*x + d*f + e)/(d*
g^2))*e + x*log(c*(d + e/(g*x + f))^q)
```

Fricas [A]
time = 0.34, size = 70, normalized size = 1.56

$$\frac{dgqx\log\left(\frac{dgx+df+e}{gx+f}\right)-dfq\log\left(gx+f\right)+dgx\log\left(c\right)+\left(dfq+qe\right)\log\left(dgx+df+e\right)}{dg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(g*x+f))^q),x, algorithm="fricas")
```

```
[Out] (d*g*q*x*log((d*g*x + d*f + e)/(g*x + f)) - d*f*q*log(g*x + f) + d*g*x*log(
c) + (d*f*q + q*e)*log(d*g*x + d*f + e))/(d*g)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(36) = 72$.
time = 0.65, size = 102, normalized size = 2.27

$$\begin{cases} x\log\left(c\left(\frac{e}{f}\right)^{q}\right) & \text{for}\: d=0 \wedge g=0 \\ \frac{f\log\left(c\left(\frac{e}{f+gx}\right)^{q}\right)}{g}+qx+x\log\left(c\left(\frac{e}{f+gx}\right)^{q}\right) & \text{for}\: d=0 \\ x\log\left(c\left(d+\frac{e}{f}\right)^{q}\right) & \text{for}\: g=0 \\ \frac{f\log\left(c\left(d+\frac{e}{f+gx}\right)^{q}\right)}{g}+x\log\left(c\left(d+\frac{e}{f+gx}\right)^{q}\right)+\frac{eq\log\left(df+dgx+e\right)}{dg} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e/(g*x+f))**q),x)
```

```
[Out] Piecewise((x*log(c*(e/f)**q), Eq(d, 0) & Eq(g, 0)), (f*log(c*(e/(f + g*x))*
*q)/g + q*x + x*log(c*(e/(f + g*x))**q), Eq(d, 0)), (x*log(c*(d + e/f)**q),
 Eq(g, 0)), (f*log(c*(d + e/(f + g*x))**q)/g + x*log(c*(d + e/(f + g*x))**q
) + e*q*log(d*f + d*g*x + e)/(d*g), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(48) = 96$.
time = 8.94, size = 172, normalized size = 3.82

$$\frac{\left(dfge^{(-2)}-(df+e)ge^{(-2)}\right)\left(dqe^{2}\log\left(-d+\frac{dgx+df+e}{gx+f}\right)+de^{2}\log\left(c\right)-\frac{(dgx+df+e)qe^{2}\log\left(-d+\frac{dgx+df+e}{gx+f}\right)}{gx+f}+\frac{(dgx+df+e)qe^{2}\log\left(\frac{dgx+df+e}{gx+f}\right)}{gx+f}\right)}{d^{2}g^{2}-\frac{(dgx+df+e)dg^{2}}{gx+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(g*x+f))^q),x, algorithm="giac")
```

```
[Out] (d*f*g*e^(-2) - (d*f + e)*g*e^(-2))*(d*q*e^2*log(-d + (d*g*x + d*f + e)/(g*
x + f)) + d*e^2*log(c) - (d*g*x + d*f + e)*q*e^2*log(-d + (d*g*x + d*f + e)
/(g*x + f))/(g*x + f) + (d*g*x + d*f + e)*q*e^2*log((d*g*x + d*f + e)/(g*x
+ f))/(g*x + f))/(d^2*g^2 - (d*g*x + d*f + e)*d*g^2/(g*x + f))
```

Mupad [B]
time = 0.18, size = 67, normalized size = 1.49

$$x\,\ln\left(c\,{\left(d+\frac{e}{f+g\,x}\right)}^q\right)-\frac{f\,q\,\ln\left(f+g\,x\right)}{g}+\frac{f\,q\,\ln\left(e+d\,f+d\,g\,x\right)}{g}+\frac{e\,q\,\ln\left(e+d\,f+d\,g\,x\right)}{d\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e/(f + g*x))^q),x)
```

```
[Out] x*log(c*(d + e/(f + g*x))^q) - (f*q*log(f + g*x))/g + (f*q*log(e + d*f + d*
g*x))/g + (e*q*log(e + d*f + d*g*x))/(d*g)
```

3.633 $\int \log \left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)\,dx$

Optimal. Leaf size=59

$$\frac{2\sqrt{e}\,q\tan^{-1}\left(\frac{\sqrt{d}\,(f+gx)}{\sqrt{e}}\right)}{\sqrt{d}\,g}+\frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)}{g}$$

[Out] (g*x+f)*ln(c*(d+e/(g*x+f)^2)^q)/g+2*q*arctan((g*x+f)*d^(1/2)/e^(1/2))*e^(1/2)/g/d^(1/2)

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2533, 2498, 269, 211}

$$\frac{2\sqrt{e}\,q\text{ArcTan}\left(\frac{\sqrt{d}\,(f+gx)}{\sqrt{e}}\right)}{\sqrt{d}\,g}+\frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)}{g}$$

Antiderivative was successfully verified.

[In] Int[Log[c*(d + e/(f + g*x)^2)^q],x]

[Out] (2*Sqrt[e]*q*ArcTan[(Sqrt[d]*(f + g*x))/Sqrt[e]])/(Sqrt[d]*g) + ((f + g*x)*Log[c*(d + e/(f + g*x)^2)^q])/g

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d, e, n, p}, x]
```

Rule 2533

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
```

```
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rubi steps

$$
\begin{aligned}
\int \log\left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)dx &= \frac{\text{Subst}\left(\int \log\left(c\left(d+\frac{e}{x^2}\right)^q\right)dx, x, f+gx\right)}{g}\\
&= \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)}{g}+\frac{(2eq)\text{Subst}\left(\int \frac{1}{\left(d+\frac{e}{x^2}\right)x^2}dx, x, f+gx\right)}{g}\\
&= \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)}{g}+\frac{(2eq)\text{Subst}\left(\int \frac{1}{e+dx^2}dx, x, f+gx\right)}{g}\\
&= \frac{2\sqrt{e}\,q\tan^{-1}\left(\frac{\sqrt{d}\,(f+gx)}{\sqrt{e}}\right)}{\sqrt{d}\,g}+\frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)}{g}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 61, normalized size = 1.03

$$
-\frac{2\sqrt{e}\,q\tan^{-1}\left(\frac{\sqrt{e}}{\sqrt{d}\,(f+gx)}\right)}{\sqrt{d}\,g}+\frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)}{g}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e/(f + g*x)^2)^q],x]
[Out] (-2*Sqrt[e]*q*ArcTan[Sqrt[e]/(Sqrt[d]*(f + g*x))])/(Sqrt[d]*g) + ((f + g*x)
*Log[c*(d + e/(f + g*x)^2)^q])/g
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(51) = 102$.
time = 0.13, size = 129, normalized size = 2.19

method	result	size
default	$\ln\left(c\left(\frac{d\,g^2x^2+2dfgx+d\,f^2+e}{(gx+f)^2}\right)^q\right)x+2egq\left(\frac{\frac{f\ln\left(d\,g^2x^2+2dfgx+d\,f^2+e\right)}{2g}+\frac{e\arctan\left(\frac{2d\,g^2x+2dfg}{2g\sqrt{ed}}\right)}{g\sqrt{ed}}}{eg}-\frac{f\ln(gx+f)}{g^2e}\right)$	129

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e/(g*x+f)^2)^q),x,method=_RETURNVERBOSE)
```

```
[Out] ln(c*((d*g^2*x^2+2*d*f*g*x+d*f^2+e)/(g*x+f)^2)^q)*x+2*e*g*q*(1/e/g*(1/2*f/g
*ln(d*g^2*x^2+2*d*f*g*x+d*f^2+e)+e/g/(e*d)^(1/2)*arctan(1/2*(2*d*g^2*x+2*d*
f*g)/g/(e*d)^(1/2)))-1/g^2/e*f*ln(g*x+f))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(50) = 100$.
time = 0.55, size = 101, normalized size = 1.71

$$gq\left(\frac{fe^{(-1)}\log\left(dg^2x^2+2\,dfgx+df^2+e\right)}{g^2}-\frac{2\,fe^{(-1)}\log\left(gx+f\right)}{g^2}+\frac{2\,\arctan\left(\frac{(dg^2x+dfg)e^{\left(-\frac{1}{2}\right)}}{\sqrt{d}\,g}\right)e^{\left(-\frac{1}{2}\right)}}{\sqrt{d}\,g^2}\right)e+x\log\left(c\left(d+\frac{e}{(gx+f)^2}\right)^q\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(g*x+f)^2)^q),x, algorithm="maxima")
```

```
[Out] g*q*(f*e^(-1)*log(d*g^2*x^2 + 2*d*f*g*x + d*f^2 + e)/g^2 - 2*f*e^(-1)*log(g
*x + f)/g^2 + 2*arctan((d*g^2*x + d*f*g)*e^(-1/2)/(sqrt(d)*g))*e^(-1/2)/(sq
rt(d)*g^2))*e + x*log(c*(d + e/(g*x + f)^2)^q)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(50) = 100$.
time = 0.40, size = 288, normalized size = 4.88

$$\left[\frac{gqx\log\left(\frac{dg^2x^2+2\,dfgx+df^2+e}{g^2x^2+2\,fgx+f^2}\right)+fq\log\left(dg^2x^2+2\,dfgx+df^2+e\right)-2\,fq\log\left(gx+f\right)+gx\log\left(c\right)+q\sqrt{-\frac{e}{d}}\log\left(\frac{dg^2x^2+2\,dfgx+df^2+2\,(dgx+df)\sqrt{-\frac{e}{d}}-e}{dg^2x^2+2\,dfgx+df^2+e}\right)}{g},\ \frac{gqx\log\left(\frac{dg^2x^2+2\,dfgx+df^2+e}{g^2x^2+2\,fgx+f^2}\right)+fq\log\left(dg^2x^2+2\,dfgx+df^2+e\right)-2\,fq\log\left(gx+f\right)+gx\log\left(c\right)+\frac{2\,q\arctan\left(\frac{(dgx+df)e^{\left(-\frac{1}{2}\right)}}{\sqrt{d}}\right)e^{\frac{1}{2}}}{\sqrt{d}}}{g}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(g*x+f)^2)^q),x, algorithm="fricas")
```

```
[Out] [(g*q*x*log((d*g^2*x^2 + 2*d*f*g*x + d*f^2 + e)/(g^2*x^2 + 2*f*g*x + f^2))
+ f*q*log(d*g^2*x^2 + 2*d*f*g*x + d*f^2 + e) - 2*f*q*log(g*x + f) + g*x*log
(c) + q*sqrt(-e/d)*log((d*g^2*x^2 + 2*d*f*g*x + d*f^2 + 2*(d*g*x + d*f)*sqr
t(-e/d) - e)/(d*g^2*x^2 + 2*d*f*g*x + d*f^2 + e)))/g, (g*q*x*log((d*g^2*x^2
 + 2*d*f*g*x + d*f^2 + e)/(g^2*x^2 + 2*f*g*x + f^2)) + f*q*log(d*g^2*x^2 +
2*d*f*g*x + d*f^2 + e) - 2*f*q*log(g*x + f) + g*x*log(c) + 2*q*arctan((d*g*
x + d*f)*e^(-1/2)/sqrt(d))*e^(1/2)/sqrt(d))/g]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e/(g*x+f)**2)**q),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(50) = 100.
time = 4.18, size = 137, normalized size = 2.32

$$dg^4q\left(\frac{fe^{(-1)}\log\left(dg^2x^2+2\,dfgx+df^2+e\right)}{dg^5}-\frac{2\,fe^{(-1)}\log\left(|gx+f|\right)}{dg^5}+\frac{2\,\arctan\left(\frac{(dgx+df)e^{\left(-\frac{1}{2}\right)}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{d^{\frac{3}{2}}g^5}\right)e+qx\log\left(dg^2x^2+2\,dfgx+df^2+e\right)-qx\log\left(g^2x^2+2\,fgx+f^2\right)+x\log\left(c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(g*x+f)^2)^q),x, algorithm="giac")
```

```
[Out] d*g^4*q*(f*e^(-1)*log(d*g^2*x^2 + 2*d*f*g*x + d*f^2 + e)/(d*g^5) - 2*f*e^(-
1)*log(abs(g*x + f))/(d*g^5) + 2*arctan((d*g*x + d*f)*e^(-1/2)/sqrt(d))*e^(
-1/2)/(d^(3/2)*g^5))*e + q*x*log(d*g^2*x^2 + 2*d*f*g*x + d*f^2 + e) - q*x*l
og(g^2*x^2 + 2*f*g*x + f^2) + x*log(c)
```

Mupad [B]
time = 0.52, size = 163, normalized size = 2.76

$$x\ln\left(c\left(d+\frac{e}{(f+g\,x)^2}\right)^q\right)-\frac{2\,f\,q\,\ln\left(f+g\,x\right)}{g}+\frac{\ln\left(e\,\sqrt{-d\,e}-3\,d\,f^2\,\sqrt{-d\,e}+4\,d\,e\,f+d\,e\,g\,x-3\,d\,f\,g\,x\,\sqrt{-d\,e}\right)\,\left(q\,\sqrt{-d\,e}+d\,f\,q\right)}{d\,g}-\frac{\ln\left(3\,d\,f^2\,\sqrt{-d\,e}-e\,\sqrt{-d\,e}+4\,d\,e\,f+d\,e\,g\,x+3\,d\,f\,g\,x\,\sqrt{-d\,e}\right)\,\left(q\,\sqrt{-d\,e}-d\,f\,q\right)}{d\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e/(f + g*x)^2)^q),x)
```

```
[Out] x*log(c*(d + e/(f + g*x)^2)^q) - (2*f*q*log(f + g*x))/g + (log(e*(-d*e)^(1/
2) - 3*d*f^2*(-d*e)^(1/2) + 4*d*e*f + d*e*g*x - 3*d*f*g*x*(-d*e)^(1/2))*(q*
(-d*e)^(1/2) + d*f*q))/(d*g) - (log(3*d*f^2*(-d*e)^(1/2) - e*(-d*e)^(1/2) +
 4*d*e*f + d*e*g*x + 3*d*f*g*x*(-d*e)^(1/2))*(q*(-d*e)^(1/2) - d*f*q))/(d*g
)
```

3.634 $\int \log \left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)\,dx$

Optimal. Leaf size=165

$$-\frac{\sqrt{3}\,\sqrt[3]{e}\,q\tan^{-1}\left(\frac{\sqrt[3]{e}-2\sqrt[3]{d}\,(f+gx)}{\sqrt{3}\,\sqrt[3]{e}}\right)}{\sqrt[3]{d}\,g}+\frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g}+\frac{\sqrt[3]{e}\,q\log\left(\sqrt[3]{e}+\sqrt[3]{d}\,(f+gx)\right)}{\sqrt[3]{d}\,g}-$$

```
[Out] (g*x+f)*ln(c*(d+e/(g*x+f)^3)^q)/g+e^(1/3)*q*ln(e^(1/3)+d^(1/3)*(g*x+f))/d^(
1/3)/g-1/2*e^(1/3)*q*ln(e^(2/3)-d^(1/3)*e^(1/3)*(g*x+f)+d^(2/3)*(g*x+f)^2)/
d^(1/3)/g-e^(1/3)*q*arctan(1/3*(e^(1/3)-2*d^(1/3)*(g*x+f))/e^(1/3)*3^(1/2))
*3^(1/2)/d^(1/3)/g
```

Rubi [A]
time = 0.11, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {2533, 2498, 269, 206, 31, 648, 631, 210, 642}

$$-\frac{\sqrt{3}\,\sqrt[3]{e}\,q\text{ArcTan}\left(\frac{\sqrt[3]{e}-2\sqrt[3]{d}\,(f+gx)}{\sqrt{3}\,\sqrt[3]{e}}\right)}{\sqrt[3]{d}\,g}+\frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g}-\frac{\sqrt[3]{e}\,q\log\left(d^{2/3}(f+gx)^2-\sqrt[3]{d}\,\sqrt[3]{e}\,(f+gx)+e^{2/3}\right)}{2\sqrt[3]{d}\,g}+\frac{\sqrt[3]{e}\,q\log\left(\sqrt[3]{d}\,(f+gx)+\sqrt[3]{e}\right)}{\sqrt[3]{d}\,g}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(d + e/(f + g*x)^3)^q],x]
```

```
[Out] -((Sqrt[3]*e^(1/3)*q*ArcTan[(e^(1/3) - 2*d^(1/3)*(f + g*x))/(Sqrt[3]*e^(1/3
))])/(d^(1/3)*g)) + ((f + g*x)*Log[c*(d + e/(f + g*x)^3)^q])/g + (e^(1/3)*q
*Log[e^(1/3) + d^(1/3)*(f + g*x)])/(d^(1/3)*g) - (e^(1/3)*q*Log[e^(2/3) - d
^(1/3)*e^(1/3)*(f + g*x) + d^(2/3)*(f + g*x)^2])/(2*d^(1/3)*g)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2533

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rubi steps

$$\int \log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)\,dx = \frac{\text{Subst}\left(\int \log\left(c\left(d+\frac{e}{x^3}\right)^q\right)\,dx,x,f+gx\right)}{g}$$

$$= \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g} + \frac{(3eq)\text{Subst}\left(\int \frac{1}{\left(d+\frac{e}{x^3}\right)x^3}\,dx,x,f+gx\right.}{g}$$

$$= \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g} + \frac{(3eq)\text{Subst}\left(\int \frac{1}{e+dx^3}\,dx,x,f+gx\right)}{g}$$

$$= \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g} + \frac{\left(\sqrt[3]{e}\,q\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{e}+\sqrt[3]{d}\,x}\,dx,x,f\right.}{g}$$

$$= \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g} + \frac{\sqrt[3]{e}\,q\log\left(\sqrt[3]{e}+\sqrt[3]{d}\,(f+gx)\right)}{\sqrt[3]{d}\,g} - \cdots$$

$$= \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g} + \frac{\sqrt[3]{e}\,q\log\left(\sqrt[3]{e}+\sqrt[3]{d}\,(f+gx)\right)}{\sqrt[3]{d}\,g} - \cdots$$

$$= -\frac{\sqrt{3}\,\sqrt[3]{e}\,q\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,(f+gx)}{\sqrt[3]{e}}}{\sqrt{3}}\right)}{\sqrt[3]{d}\,g} + \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g} + \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.08, size = 66, normalized size = 0.40

$$-\frac{3eq\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{e}{d(f+gx)^3}\right)}{2dg(f+gx)^2} + \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(d + e/(f + g*x)^3)^q],x]
```

```
[Out] (-3*e*q*Hypergeometric2F1[2/3, 1, 5/3, -(e/(d*(f + g*x)^3))])/(2*d*g*(f + g
*x)^2) + ((f + g*x)*Log[c*(d + e/(f + g*x)^3)^q])/g
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.06, size = 168, normalized size = 1.02

method	result
default	$\ln\left(c\left(\frac{d\,g^{3}x^{3}+3df\,g^{2}x^{2}+3d\,f^{2}gx+d\,f^{3}+e}{(gx+f)^{3}}\right)^{q}\right)x+3egq\left(-\frac{f\ln(gx+f)}{g^{2}e}+\frac{\sum_{\_R=\mathrm{RootOf}\left(d\,g^{3}\_Z^{3}+3df\,g^{2}\_Z^{2}+3dg\,f^{2}\_Z+d\,f^{3}+\right.}}{3d\,g}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(d+e/(g*x+f)^3)^q),x,method=_RETURNVERBOSE)
```

```
[Out] ln(c*((d*g^3*x^3+3*d*f*g^2*x^2+3*d*f^2*g*x+d*f^3+e)/(g*x+f)^3)^q)*x+3*e*g*q
*(-1/g^2/e*f*ln(g*x+f)+1/3/d/g^2*sum((_R^2*d*f*g^2+2*_R*d*f^2*g+d*f^3+e)/(_
R^2*g^2+2*_R*f*g+f^2)*ln(x-_R),_R=RootOf(_Z^3*d*g^3+3*_Z^2*d*f*g^2+3*_Z*d*f
^2*g+d*f^3+e))/e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(g*x+f)^3)^q),x, algorithm="maxima")
```

```
[Out] 3*q*integrate((d*f*g^2*x^2 + 2*d*f^2*g*x + d*f^3 + e)/(d*g^3*x^3 + 3*d*f*g^
2*x^2 + 3*d*f^2*g*x + d*f^3 + e), x) - (3*f*q*log(g*x + f) - g*x*log((d*g^3
*x^3 + 3*d*f*g^2*x^2 + 3*d*f^2*g*x + d*f^3 + e)^q) + 3*g*x*log((g*x + f)^q)
 - g*x*log(c))/g
```

Fricas [C] Result contains complex when optimal does not.
time = 1.11, size = 1424, normalized size = 8.63

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(g*x+f)^3)^q),x, algorithm="fricas")
```

```
[Out] 1/4*(4*g*q*x*log((d*g^3*x^3 + 3*d*f*g^2*x^2 + 3*d*f^2*g*x + d*f^3 + e)/(g^3
*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3)) - 4*sqrt(3)*g*sqrt((((-1/2*f^3*q^3/g
^3 + 1/2*q^3*e/(d*g^3) + 1/2*(d*f^3*q^3 + q^3*e)/(d*g^3))^(1/3)*(I*sqrt(3)
+ 1) - 2*f*q/g)^2*g^2 + 4*((-1/2*f^3*q^3/g^3 + 1/2*q^3*e/(d*g^3) + 1/2*(d*f
^3*q^3 + q^3*e)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*f*g*q + 4*f^2*q^2
)/g^2)*arctan(-1/24*(2*sqrt(3)*sqrt(4*g^2*q^2*x^2 + 12*f*g*q^2*x + ((-1/2*f
^3*q^3/g^3 + 1/2*q^3*e/(d*g^3) + 1/2*(d*f^3*q^3 + q^3*e)/(d*g^3))^(1/3)*(I*
sqrt(3) + 1) - 2*f*q/g)^2*g^2 + 12*f^2*q^2 + 2*(g^2*q*x + 3*f*g*q)*((-1/2*f
```

```
^3*q^3/g^3 + 1/2*q^3*e/(d*g^3) + 1/2*(d*f^3*q^3 + q^3*e)/(d*g^3))^(1/3)*(I*
sqrt(3) + 1) - 2*f*q/g))*(((-1/2*f^3*q^3/g^3 + 1/2*q^3*e/(d*g^3) + 1/2*(d*f
^3*q^3 + q^3*e)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*d*g^2 + 2*d*f*g*q
)*sqrt((((-1/2*f^3*q^3/g^3 + 1/2*q^3*e/(d*g^3) + 1/2*(d*f^3*q^3 + q^3*e)/(d
*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)^2*g^2 + 4*((-1/2*f^3*q^3/g^3 + 1/2*
q^3*e/(d*g^3) + 1/2*(d*f^3*q^3 + q^3*e)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*
f*q/g)*f*g*q + 4*f^2*q^2)/g^2) - sqrt(3)*(8*d*f*g^2*q^2*x + ((-1/2*f^3*q^3/
g^3 + 1/2*q^3*e/(d*g^3) + 1/2*(d*f^3*q^3 + q^3*e)/(d*g^3))^(1/3)*(I*sqrt(3)
 + 1) - 2*f*q/g)^2*d*g^3 + 12*d*f^2*g*q^2 + 4*(d*g^3*q*x + 2*d*f*g^2*q)*((-
1/2*f^3*q^3/g^3 + 1/2*q^3*e/(d*g^3) + 1/2*(d*f^3*q^3 + q^3*e)/(d*g^3))^(1/3
)*(I*sqrt(3) + 1) - 2*f*q/g))*sqrt((((-1/2*f^3*q^3/g^3 + 1/2*q^3*e/(d*g^3)
+ 1/2*(d*f^3*q^3 + q^3*e)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)^2*g^2 +
 4*((-1/2*f^3*q^3/g^3 + 1/2*q^3*e/(d*g^3) + 1/2*(d*f^3*q^3 + q^3*e)/(d*g^3)
)^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*f*g*q + 4*f^2*q^2)/g^2))*e^(-1)/q^3) - 1
2*f*q*log(g*x + f) - 2*((-1/2*f^3*q^3/g^3 + 1/2*q^3*e/(d*g^3) + 1/2*(d*f^3*
q^3 + q^3*e)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*g*log(q*x - 1/2*(-1/
2*f^3*q^3/g^3 + 1/2*q^3*e/(d*g^3) + 1/2*(d*f^3*q^3 + q^3*e)/(d*g^3))^(1/3)*
(I*sqrt(3) + 1) + f*q/g) + 4*g*x*log(c) + (((-1/2*f^3*q^3/g^3 + 1/2*q^3*e/(
d*g^3) + 1/2*(d*f^3*q^3 + q^3*e)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*
g + 6*f*q)*log(4*g^2*q^2*x^2 + 12*f*g*q^2*x + ((-1/2*f^3*q^3/g^3 + 1/2*q^3*
e/(d*g^3) + 1/2*(d*f^3*q^3 + q^3*e)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/
g)^2*g^2 + 12*f^2*q^2 + 2*(g^2*q*x + 3*f*g*q)*((-1/2*f^3*q^3/g^3 + 1/2*q^3*
e/(d*g^3) + 1/2*(d*f^3*q^3 + q^3*e)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/
g)))/g
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(d+e/(g*x+f)**3)**q),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 319 vs. 2(125) = 250.
time = 52.54, size = 319, normalized size = 1.93

$$\frac{1}{2}\,dg^5q\left(\frac{2\,fe^{(-1)}\log\left(|dg^3x^3+3\,dfg^2x^2+3\,df^2gx+df^3+e|\right)}{dg^6}-\frac{6\,fe^{(-1)}\log\left(|gx+f|\right)}{dg^6}+\frac{\left(2\,\sqrt{3}\,(d^5g^{21})^{\frac{1}{3}}\arctan\left(-\frac{dgx+df+(d^2)^{\frac{1}{3}}e^{\frac{1}{3}}}{\sqrt{3}\,dgx+\sqrt{3}\,df-\sqrt{3}\,(d^2)^{\frac{1}{3}}e^{\frac{1}{3}}}\right)e^{\frac{4}{3}}-(d^5g^{21})^{\frac{1}{3}}e^{\frac{4}{3}}\log\left(4\left(\sqrt{3}\,dgx+\sqrt{3}\,df-\sqrt{3}\,(d^2)^{\frac{1}{3}}e^{\frac{1}{3}}\right)^2+4\left(dgx+df+(d^2)^{\frac{1}{3}}e^{\frac{1}{3}}\right)^2\right)+2\,(d^5g^{21})^{\frac{1}{3}}e^{\frac{4}{3}}\log\left(\left|dgx+df+(d^2)^{\frac{1}{3}}e^{\frac{1}{3}}\right|\right)\right)e^{(-2)}}{d^3g^{13}}\right)e+qx\log\left(dg^3x^3+3\,dfg^2x^2+3\,df^2gx+df^3+e\right)-qx\log\left(g^3x^3+3\,fg^2x^2+3\,f^2gx+f^3\right)+x\log\left(c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(d+e/(g*x+f)^3)^q),x, algorithm="giac")
```

```
[Out] 1/2*d*g^5*q*(2*f*e^(-1)*log(abs(d*g^3*x^3 + 3*d*f*g^2*x^2 + 3*d*f^2*g*x + d
*f^3 + e))/(d*g^6) - 6*f*e^(-1)*log(abs(g*x + f))/(d*g^6) + (2*sqrt(3)*(d^5
```

```
*g^21)^(1/3)*arctan(-(d*g*x + d*f + (d^2)^(1/3)*e^(1/3))/(sqrt(3)*d*g*x + s
qrt(3)*d*f - sqrt(3)*(d^2)^(1/3)*e^(1/3)))*e^(4/3) - (d^5*g^21)^(1/3)*e^(4/
3)*log(4*(sqrt(3)*d*g*x + sqrt(3)*d*f - sqrt(3)*(d^2)^(1/3)*e^(1/3))^2 + 4*
(d*g*x + d*f + (d^2)^(1/3)*e^(1/3))^2) + 2*(d^5*g^21)^(1/3)*e^(4/3)*log(abs
(d*g*x + d*f + (d^2)^(1/3)*e^(1/3))))*e^(-2)/(d^3*g^13))*e + q*x*log(d*g^3*
x^3 + 3*d*f*g^2*x^2 + 3*d*f^2*g*x + d*f^3 + e) - q*x*log(g^3*x^3 + 3*f*g^2*
x^2 + 3*f^2*g*x + f^3) + x*log(c)
```

Mupad [B]

time = 0.63, size = 499, normalized size = 3.02

$$x\ln\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)-\left(\sum_{k=1}^{3}\ln\left(-d^2e^2g^{11}\left(3eq^3x+\mathrm{root}\left(dg^3z^3+3dfg^2qz^2+3df^2gq^2z+df^3q^3+eq^3,z,k\right)eq^2+\mathrm{root}\left(dg^3z^3+3dfg^2qz^2+3df^2gq^2z+df^3q^3+eq^3,z,k\right)^3dfg^2\,4+\mathrm{root}\left(dg^3z^3+3dfg^2qz^2+3df^2gq^2z+df^3q^3+eq^3,z,k\right)df^3q^2\,4+\mathrm{root}\left(dg^3z^3+3dfg^2qz^2+3df^2gq^2z+df^3q^3+eq^3,z,k\right)^3dg^3x\,4+\mathrm{root}\left(dg^3z^3+3dfg^2qz^2+3df^2gq^2z+df^3q^3+eq^3,z,k\right)^2df^2gq\,8+\mathrm{root}\left(dg^3z^3+3dfg^2qz^2+3df^2gq^2z+df^3q^3+eq^3,z,k\right)df^2gq^2x\,4+\mathrm{root}\left(dg^3z^3+3dfg^2qz^2+3df^2gq^2z+df^3q^3+eq^3,z,k\right)^2dfg^2qx\,8\right)9\right)\mathrm{root}\left(dg^3z^3+3dfg^2qz^2+3df^2gq^2z+df^3q^3+eq^3,z,k\right)\right)-\frac{3fq\ln(f+gx)}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(d + e/(f + g*x)^3)^q),x)
```

```
[Out] x*log(c*(d + e/(f + g*x)^3)^q) - symsum(log(-9*d^2*e^2*g^11*(3*e*q^3*x + ro
ot(d*g^3*z^3 + 3*d*f*g^2*q*z^2 + 3*d*f^2*g*q^2*z + d*f^3*q^3 + e*q^3, z, k)
*e*q^2 + 4*root(d*g^3*z^3 + 3*d*f*g^2*q*z^2 + 3*d*f^2*g*q^2*z + d*f^3*q^3 +
 e*q^3, z, k)^3*d*f*g^2 + 4*root(d*g^3*z^3 + 3*d*f*g^2*q*z^2 + 3*d*f^2*g*q^
2*z + d*f^3*q^3 + e*q^3, z, k)*d*f^3*q^2 + 4*root(d*g^3*z^3 + 3*d*f*g^2*q*z
^2 + 3*d*f^2*g*q^2*z + d*f^3*q^3 + e*q^3, z, k)^3*d*g^3*x + 8*root(d*g^3*z^
3 + 3*d*f*g^2*q*z^2 + 3*d*f^2*g*q^2*z + d*f^3*q^3 + e*q^3, z, k)^2*d*f^2*g*
q + 4*root(d*g^3*z^3 + 3*d*f*g^2*q*z^2 + 3*d*f^2*g*q^2*z + d*f^3*q^3 + e*q^
3, z, k)*d*f^2*g*q^2*x + 8*root(d*g^3*z^3 + 3*d*f*g^2*q*z^2 + 3*d*f^2*g*q^2
*z + d*f^3*q^3 + e*q^3, z, k)^2*d*f*g^2*q*x))*root(d*g^3*z^3 + 3*d*f*g^2*q*
z^2 + 3*d*f^2*g*q^2*z + d*f^3*q^3 + e*q^3, z, k), k, 1, 3) - (3*f*q*log(f +
 g*x))/g
```

3.635 $\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n dx$

Optimal. Leaf size=25

$$\text{Int}\left(\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n,x\right)$$

[Out] Unintegrable((a+b*ln(c*(d+e/(g*x+f))^p))^n,x)

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n dx$$

Verification is not applicable to the result.

[In] Int[(a + b*Log[c*(d + e/(f + g*x))^p])^n,x]

[Out] Defer[Int][(a + b*Log[c*(d + e/(f + g*x))^p])^n, x]

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n dx=\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n dx$$

Mathematica [A]
time = 0.26, size = 0, normalized size = 0.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n dx$$

Verification is not applicable to the result.

[In] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^n,x]

[Out] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^n, x]

Maple [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{gx+f}\right)^p\right)\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/(g*x+f))^p))^n,x)
[Out] int((a+b*ln(c*(d+e/(g*x+f))^p))^n,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^n,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/(g*x + f))^p) + a)^n, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^n,x, algorithm="fricas")
[Out] integral((b*log(c*((d*g*x + d*f + e)/(g*x + f))^p) + a)^n, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\log\left(c\left(d + \frac{e}{f + gx}\right)^{p}\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/(g*x+f))**p))**n,x)
[Out] Integral((a + b*log(c*(d + e/(f + g*x))**p))**n, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^n,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/(g*x + f))^p) + a)^n, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \left(a+b\,\ln\left(c\,{\left(d+\frac{e}{f+g\,x}\right)}^p\right)\right)^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/(f + g*x))^p))^n,x)
[Out] int((a + b*log(c*(d + e/(f + g*x))^p))^n, x)
```

3.636 $\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4\,dx$

Optimal. Leaf size=221

$$-\frac{4bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg}+\frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4}{dg}-\frac{12b^2ep^2}{}$$

```
[Out] -4*b*e*p*ln(-e/d/(g*x+f))*(a+b*ln(c*(d+e/(g*x+f))^p))^3/d/g+(e+d*(g*x+f))*(
a+b*ln(c*(d+e/(g*x+f))^p))^4/d/g-12*b^2*e*p^2*(a+b*ln(c*(d+e/(g*x+f))^p))^2
*polylog(2,1+e/d/(g*x+f))/d/g+24*b^3*e*p^3*(a+b*ln(c*(d+e/(g*x+f))^p))*poly
log(3,1+e/d/(g*x+f))/d/g-24*b^4*e*p^4*polylog(4,1+e/d/(g*x+f))/d/g
```

Rubi [A]
time = 0.18, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2533, 2499, 2504, 2443, 2481, 2421, 2430, 6724}

$$\frac{24b^3ep^3\text{PolyLog}\left(3,\frac{e}{d(f+gx)}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)}{dg}-\frac{12b^2ep^2\text{PolyLog}\left(2,\frac{e}{d(f+gx)}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg}-\frac{24b^4ep^4\text{PolyLog}\left(4,\frac{e}{d(f+gx)}+1\right)}{dg}-\frac{4bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg}+\frac{(d(f+gx)+e)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4}{dg}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/(f + g*x))^p])^4,x]
```

```
[Out] (-4*b*e*p*Log[-(e/(d*(f + g*x)))]*(a + b*Log[c*(d + e/(f + g*x))^p])^3)/(d*
g) + ((e + d*(f + g*x))*(a + b*Log[c*(d + e/(f + g*x))^p])^4)/(d*g) - (12*b
^2*e*p^2*(a + b*Log[c*(d + e/(f + g*x))^p])^2*PolyLog[2, 1 + e/(d*(f + g*x)
)])/(d*g) + (24*b^3*e*p^3*(a + b*Log[c*(d + e/(f + g*x))^p])*PolyLog[3, 1 +
 e/(d*(f + g*x))])/(d*g) - (24*b^4*e*p^4*PolyLog[4, 1 + e/(d*(f + g*x))])/(
d*g)
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2499

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)/(x_))^(p_.)]*(b_.))^(q_), x_Symbol] :>
 Simp[(e + d*x)*((a + b*Log[c*(d + e/x)^p])^q/d), x] + Dist[b*e*p*(q/d), In
t[(a + b*Log[c*(d + e/x)^p])^(q - 1)/x, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2533

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4 dx = \frac{\text{Subst}\left(\int \left(a+b\log\left(c\left(d+\frac{e}{x}\right)^p\right)\right)^4 dx, x, f+gx\right)}{g}$$

$$= \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4}{dg} + \frac{(4bep)\text{Subst}\left(\int \right.}{}$$

$$= \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4}{dg} - \frac{(4bep)\text{Subst}\left(\int \right.}{}$$

$$= -\frac{4bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} + \frac{(e+d(f+}{}$$

$$= -\frac{4bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} + \frac{(e+d(f+}{}$$

$$= -\frac{4bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} + \frac{(e+d(f+}{}$$

$$= -\frac{4bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} + \frac{(e+d(f+}{}$$

$$= -\frac{4bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} + \frac{(e+d(f+}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 739 vs. $2(221) = 442$.
time = 0.71, size = 739, normalized size = 3.34

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^4,x]
```

```
[Out] (-4*b*p*(d*f*Log[f + g*x] - (e + d*f)*Log[e + d*f + d*g*x] - d*g*x*Log[(e +
 d*f + d*g*x)/(f + g*x)])*(a - b*p*Log[d + e/(f + g*x)] + b*Log[c*(d + e/(f
 + g*x))^p])^3 + d*g*x*(a - b*p*Log[d + e/(f + g*x)] + b*Log[c*(d + e/(f +
g*x))^p])^4 + 6*b^2*p^2*(a - b*p*Log[d + e/(f + g*x)] + b*Log[c*(d + e/(f +
 g*x))^p])^2*(2*d*f*Log[-(e/(d*f + d*g*x))]*Log[(e + d*f + d*g*x)/(f + g*x)
```

```
] + 2*(e + d*f)*Log[e + d*f + d*g*x]*Log[(e + d*f + d*g*x)/(f + g*x)] + d*g
*x*Log[(e + d*f + d*g*x)/(f + g*x)]^2 - d*f*(Log[-(e/(d*f + d*g*x))]*(Log[-
(e/(d*f + d*g*x))] + 2*Log[(e + d*f + d*g*x)/e]) - 2*PolyLog[2, -((d*(f + g
*x))/e)]) + (e + d*f)*((2*Log[-((d*(f + g*x))/e)] - Log[e + d*f + d*g*x])*L
og[e + d*f + d*g*x] + 2*PolyLog[2, (e + d*f + d*g*x)/e])) + 4*b^3*p^3*(a -
b*p*Log[d + e/(f + g*x)] + b*Log[c*(d + e/(f + g*x))^p])*(Log[d + e/(f + g*
x)]^2*(-3*e*Log[-(e/(d*f + d*g*x))] + (e + d*f + d*g*x)*Log[d + e/(f + g*x)
]) - 6*e*Log[d + e/(f + g*x)]*PolyLog[2, 1 + e/(d*f + d*g*x)] + 6*e*PolyLog
[3, 1 + e/(d*f + d*g*x)]) - b^4*p^4*(4*e*Log[-(e/(d*f + d*g*x))]*Log[d + e/
(f + g*x)]^3 - e*Log[d + e/(f + g*x)]^4 - d*f*Log[d + e/(f + g*x)]^4 - d*g*
x*Log[d + e/(f + g*x)]^4 + 12*e*Log[d + e/(f + g*x)]^2*PolyLog[2, 1 + e/(d*
f + d*g*x)] - 24*e*Log[d + e/(f + g*x)]*PolyLog[3, 1 + e/(d*f + d*g*x)] + 2
4*e*PolyLog[4, 1 + e/(d*f + d*g*x)]))/(d*g)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)\right)^{4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/(g*x+f))^p))^4,x)
```

```
[Out] int((a+b*ln(c*(d+e/(g*x+f))^p))^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^4,x, algorithm="maxima")
```

```
[Out] -4*a^3*b*g*p*(f*e^(-1)*log(g*x + f)/g^2 - (d*f + e)*e^(-1)*log(d*g*x + d*f
+ e)/(d*g^2))*e + 4*a^3*b*x*log(c*(d + e/(g*x + f))^p) + a^4*x + (b^4*d*g*x
*log((d*g*x + d*f + e)^p)^4 - 4*(b^4*d*f*p*log(g*x + f) + b^4*d*g*x*log((g*
x + f)^p) - (b^4*d*g*log(c) + a*b^3*d*g)*x - (b^4*d*f*p + b^4*p*e)*log(d*g*
x + d*f + e))*log((d*g*x + d*f + e)^p)^3)/(d*g) + integrate((b^4*d*f*log(c)
^4 + 4*a*b^3*d*f*log(c)^3 + 6*a^2*b^2*d*f*log(c)^2 + (b^4*d*g*x + b^4*d*f +
 b^4*e)*log((g*x + f)^p)^4 - 4*(b^4*d*f*log(c) + a*b^3*d*f + (b^4*d*g*log(c
) + a*b^3*d*g)*x + (b^4*log(c) + a*b^3)*e)*log((g*x + f)^p)^3 + 6*(2*b^4*d*
f*p^2*log(g*x + f) + b^4*d*f*log(c)^2 + 2*a*b^3*d*f*log(c) + a^2*b^2*d*f +
(b^4*d*g*x + b^4*d*f + b^4*e)*log((g*x + f)^p)^2 + (a^2*b^2*d*g - 2*(d*g*p
- d*g*log(c))*a*b^3 - (2*d*g*p*log(c) - d*g*log(c)^2)*b^4)*x + (b^4*log(c)^
2 + 2*a*b^3*log(c) + a^2*b^2)*e - 2*(b^4*d*f*p^2 + b^4*p^2*e)*log(d*g*x + d
*f + e) - 2*(b^4*d*f*log(c) + a*b^3*d*f + (a*b^3*d*g - (d*g*p - d*g*log(c))
```

```
*b^4)*x + (b^4*log(c) + a*b^3)*e)*log((g*x + f)^p))*log((d*g*x + d*f + e)^p
)^2 + 6*(b^4*d*f*log(c)^2 + 2*a*b^3*d*f*log(c) + a^2*b^2*d*f + (b^4*d*g*log
(c)^2 + 2*a*b^3*d*g*log(c) + a^2*b^2*d*g)*x + (b^4*log(c)^2 + 2*a*b^3*log(c
) + a^2*b^2)*e)*log((g*x + f)^p)^2 + (b^4*d*g*log(c)^4 + 4*a*b^3*d*g*log(c)
^3 + 6*a^2*b^2*d*g*log(c)^2)*x + (b^4*log(c)^4 + 4*a*b^3*log(c)^3 + 6*a^2*b
^2*log(c)^2)*e + 4*(b^4*d*f*log(c)^3 + 3*a*b^3*d*f*log(c)^2 + 3*a^2*b^2*d*f
*log(c) - (b^4*d*g*x + b^4*d*f + b^4*e)*log((g*x + f)^p)^3 + 3*(b^4*d*f*log
(c) + a*b^3*d*f + (b^4*d*g*log(c) + a*b^3*d*g)*x + (b^4*log(c) + a*b^3)*e)*
log((g*x + f)^p)^2 + (b^4*d*g*log(c)^3 + 3*a*b^3*d*g*log(c)^2 + 3*a^2*b^2*d
*g*log(c))*x + (b^4*log(c)^3 + 3*a*b^3*log(c)^2 + 3*a^2*b^2*log(c))*e - 3*(
b^4*d*f*log(c)^2 + 2*a*b^3*d*f*log(c) + a^2*b^2*d*f + (b^4*d*g*log(c)^2 + 2
*a*b^3*d*g*log(c) + a^2*b^2*d*g)*x + (b^4*log(c)^2 + 2*a*b^3*log(c) + a^2*b
^2)*e)*log((g*x + f)^p))*log((d*g*x + d*f + e)^p) - 4*(b^4*d*f*log(c)^3 + 3
*a*b^3*d*f*log(c)^2 + 3*a^2*b^2*d*f*log(c) + (b^4*d*g*log(c)^3 + 3*a*b^3*d*
g*log(c)^2 + 3*a^2*b^2*d*g*log(c))*x + (b^4*log(c)^3 + 3*a*b^3*log(c)^2 + 3
*a^2*b^2*log(c))*e)*log((g*x + f)^p))/(d*g*x + d*f + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^4,x, algorithm="fricas")
```

```
[Out] integral(b^4*log(c*((d*g*x + d*f + e)/(g*x + f))^p)^4 + 4*a*b^3*log(c*((d*g
*x + d*f + e)/(g*x + f))^p)^3 + 6*a^2*b^2*log(c*((d*g*x + d*f + e)/(g*x + f
))^p)^2 + 4*a^3*b*log(c*((d*g*x + d*f + e)/(g*x + f))^p) + a^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \log \left(c \left(d + \frac{e}{f + gx}\right)^{p}\right)\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/(g*x+f))**p))**4,x)
```

```
[Out] Integral((a + b*log(c*(d + e/(f + g*x))**p))**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/(g*x + f))^p) + a)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\ln\left(c\,{\left(d+\frac{e}{f+g\,x}\right)}^p\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/(f + g*x))^p))^4,x)
```

```
[Out] int((a + b*log(c*(d + e/(f + g*x))^p))^4, x)
```

3.637 $\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3 dx$

Optimal. Leaf size=168

$$-\frac{3bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg}+\frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg}-\frac{6b^2ep^2(}{}$$

```
[Out] -3*b*e*p*ln(-e/d/(g*x+f))*(a+b*ln(c*(d+e/(g*x+f))^p))^2/d/g+(e+d*(g*x+f))*(
a+b*ln(c*(d+e/(g*x+f))^p))^3/d/g-6*b^2*e*p^2*(a+b*ln(c*(d+e/(g*x+f))^p))*po
lylog(2,1+e/d/(g*x+f))/d/g+6*b^3*e*p^3*polylog(3,1+e/d/(g*x+f))/d/g
```

Rubi [A]
time = 0.12, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2533, 2499, 2504, 2443, 2481, 2421, 6724}

$$-\frac{6b^2ep^2\text{PolyLog}\left(2,\frac{e}{d(f+gx)}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)}{dg}+\frac{6b^3ep^3\text{PolyLog}\left(3,\frac{e}{d(f+gx)}+1\right)}{dg}-\frac{3bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg}+\frac{(d(f+gx)+e)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/(f + g*x))^p])^3,x]
```

```
[Out] (-3*b*e*p*Log[-(e/(d*(f + g*x)))]*(a + b*Log[c*(d + e/(f + g*x))^p])^2)/(d*
g) + ((e + d*(f + g*x))*(a + b*Log[c*(d + e/(f + g*x))^p])^3)/(d*g) - (6*b^
2*e*p^2*(a + b*Log[c*(d + e/(f + g*x))^p])*PolyLog[2, 1 + e/(d*(f + g*x))])
/(d*g) + (6*b^3*e*p^3*PolyLog[3, 1 + e/(d*(f + g*x))])/(d*g)
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2499

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)/(x_))^(p_.)]*(b_.))^(q_), x_Symbol] :>
 Simp[(e + d*x)*((a + b*Log[c*(d + e/x)^p])^q/d), x] + Dist[b*e*p*(q/d), In
t[(a + b*Log[c*(d + e/x)^p])^(q - 1)/x, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2533

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3 dx = \frac{\text{Subst}\left(\int \left(a+b\log\left(c\left(d+\frac{e}{x}\right)^p\right)\right)^3 dx, x, f+gx\right)}{g}$$

$$= \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} + \frac{(3bep)\text{Subst}\left(\int \right.}{}$$

$$= \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} - \frac{(3bep)\text{Subst}\left(\int \right.}{}$$

$$= -\frac{3bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg} + \frac{(e+d(f+}{}$$

$$= -\frac{3bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg} + \frac{(e+d(f+}{}$$

$$= -\frac{3bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg} + \frac{(e+d(f+}{}$$

$$= -\frac{3bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg} + \frac{(e+d(f+}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 415 vs. 2(168) = 336.
time = 0.32, size = 415, normalized size = 2.47

$$\frac{3bdp(f+gx)\log\left(d+\frac{e}{f+gx}\right)\left(a-bp\log\left(d+\frac{e}{f+gx}\right)+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2+d(f+gx)\left(a-bp\log\left(d+\frac{e}{f+gx}\right)+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3+3bep\left(a-bp\log\left(d+\frac{e}{f+gx}\right)+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2\log(e+d(f+gx))+3b^2p^2\left(a-bp\log\left(d+\frac{e}{f+gx}\right)+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)\left(d(f+gx)\log^2\left(d+\frac{e}{f+gx}\right)+e\left(2\log\left(-\frac{d(f+gx)}{e}\right)-\log(e+df+dgx)+2\log\left(d+\frac{e}{f+gx}\right)\right)\log(e+d(f+gx))+2e\text{Li}_2\left(1+\frac{d(f+gx)}{e}\right)\right)+b^3p^3\left(\log^2\left(d+\frac{e}{f+gx}\right)\left(-3e\log\left(-\frac{e}{df+dgx}\right)+(e+df+dgx)\log\left(d+\frac{e}{f+gx}\right)\right)-6e\log\left(d+\frac{e}{f+gx}\right)\text{Li}_2\left(1+\frac{e}{df+dgx}\right)+6e\text{Li}_3\left(1+\frac{e}{df+dgx}\right)\right)}{dg}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^3,x]
```

```
[Out] (3*b*d*p*(f + g*x)*Log[d + e/(f + g*x)]*(a - b*p*Log[d + e/(f + g*x)] + b*L
og[c*(d + e/(f + g*x))^p])^2 + d*(f + g*x)*(a - b*p*Log[d + e/(f + g*x)] +
b*Log[c*(d + e/(f + g*x))^p])^3 + 3*b*e*p*(a - b*p*Log[d + e/(f + g*x)] + b
*Log[c*(d + e/(f + g*x))^p])^2*Log[e + d*(f + g*x)] + 3*b^2*p^2*(a - b*p*Lo
g[d + e/(f + g*x)] + b*Log[c*(d + e/(f + g*x))^p])*(d*(f + g*x)*Log[d + e/(
f + g*x)]^2 + e*(2*Log[-((d*(f + g*x))/e)] - Log[e + d*f + d*g*x] + 2*Log[d
 + e/(f + g*x)])*Log[e + d*(f + g*x)] + 2*e*PolyLog[2, 1 + (d*(f + g*x))/e]
) + b^3*p^3*(Log[d + e/(f + g*x)]^2*(-3*e*Log[-(e/(d*f + d*g*x))] + (e + d*
```

```
f + d*g*x)*Log[d + e/(f + g*x)]) - 6*e*Log[d + e/(f + g*x)]*PolyLog[2, 1 +
e/(d*f + d*g*x)] + 6*e*PolyLog[3, 1 + e/(d*f + d*g*x)]))/(d*g)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)\right)^{3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/(g*x+f))^p))^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/(g*x+f))^p))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^3,x, algorithm="maxima")
```

```
[Out] -3*a^2*b*g*p*(f*e^(-1)*log(g*x + f)/g^2 - (d*f + e)*e^(-1)*log(d*g*x + d*f
+ e)/(d*g^2))*e + 3*a^2*b*x*log(c*(d + e/(g*x + f))^p) + a^3*x + (b^3*d*g*x
*log((d*g*x + d*f + e)^p)^3 - 3*(b^3*d*f*p*log(g*x + f) + b^3*d*g*x*log((g*
x + f)^p) - (b^3*d*g*log(c) + a*b^2*d*g)*x - (b^3*d*f*p + b^3*p*e)*log(d*g*
x + d*f + e))*log((d*g*x + d*f + e)^p)^2)/(d*g) + integrate((b^3*d*f*log(c)
^3 + 3*a*b^2*d*f*log(c)^2 - (b^3*d*g*x + b^3*d*f + b^3*e)*log((g*x + f)^p)^
3 + 3*(b^3*d*f*log(c) + a*b^2*d*f + (b^3*d*g*log(c) + a*b^2*d*g)*x + (b^3*l
og(c) + a*b^2)*e)*log((g*x + f)^p)^2 + (b^3*d*g*log(c)^3 + 3*a*b^2*d*g*log(
c)^2)*x + (b^3*log(c)^3 + 3*a*b^2*log(c)^2)*e + 3*(2*b^3*d*f*p^2*log(g*x +
f) + b^3*d*f*log(c)^2 + 2*a*b^2*d*f*log(c) + (b^3*d*g*x + b^3*d*f + b^3*e)*
log((g*x + f)^p)^2 - (2*(d*g*p - d*g*log(c))*a*b^2 + (2*d*g*p*log(c) - d*g*
log(c)^2)*b^3)*x + (b^3*log(c)^2 + 2*a*b^2*log(c))*e - 2*(b^3*d*f*p^2 + b^3
*p^2*e)*log(d*g*x + d*f + e) - 2*(b^3*d*f*log(c) + a*b^2*d*f + (a*b^2*d*g -
 (d*g*p - d*g*log(c))*b^3)*x + (b^3*log(c) + a*b^2)*e)*log((g*x + f)^p))*lo
g((d*g*x + d*f + e)^p) - 3*(b^3*d*f*log(c)^2 + 2*a*b^2*d*f*log(c) + (b^3*d*
g*log(c)^2 + 2*a*b^2*d*g*log(c))*x + (b^3*log(c)^2 + 2*a*b^2*log(c))*e)*log
((g*x + f)^p))/(d*g*x + d*f + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*log(c*((d*g*x + d*f + e)/(g*x + f))^p)^3 + 3*a*b^2*log(c*((d*g
*x + d*f + e)/(g*x + f))^p)^2 + 3*a^2*b*log(c*((d*g*x + d*f + e)/(g*x + f))
^p) + a^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\log\left(c\left(d + \frac{e}{f + gx}\right)^{p}\right)\right)^{3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/(g*x+f))**p))**3,x)
```

```
[Out] Integral((a + b*log(c*(d + e/(f + g*x))**p))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/(g*x + f))^p) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{f+g\,x}\right)}^p\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/(f + g*x))^p))^3,x)
```

```
[Out] int((a + b*log(c*(d + e/(f + g*x))^p))^3, x)
```

3.638 $\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2\,dx$

Optimal. Leaf size=115

$$-\frac{2bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)}{dg}+\frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg}-\frac{2b^2ep^2}{}$$

```
[Out] -2*b*e*p*ln(-e/d/(g*x+f))*(a+b*ln(c*(d+e/(g*x+f))^p))/d/g+(e+d*(g*x+f))*(a+
b*ln(c*(d+e/(g*x+f))^p))^2/d/g-2*b^2*e*p^2*polylog(2,1+e/d/(g*x+f))/d/g
```

Rubi [A]

time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2533, 2499, 2504, 2441, 2352}

$$-\frac{2b^2ep^2\text{PolyLog}\left(2,\frac{e}{d(f+gx)}+1\right)}{dg}-\frac{2bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)}{dg}+\frac{(d(f+gx)+e)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*(d + e/(f + g*x))^p])^2,x]
```

```
[Out] (-2*b*e*p*Log[-(e/(d*(f + g*x)))]*(a + b*Log[c*(d + e/(f + g*x))^p]))/(d*g)
 + ((e + d*(f + g*x))*(a + b*Log[c*(d + e/(f + g*x))^p])^2)/(d*g) - (2*b^2*
e*p^2*PolyLog[2, 1 + e/(d*(f + g*x))])/(d*g)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2499

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)/(x_))^(p_.)]*(b_.))^(q_), x_Symbol] :>
 Simp[(e + d*x)*((a + b*Log[c*(d + e/x)^p])^q/d), x] + Dist[b*e*p*(q/d), In
t[(a + b*Log[c*(d + e/x)^p])^(q - 1)/x, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2533

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rubi steps

$$\int \left(a+b\log \left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2 dx = \frac{\text{Subst}\left(\int \left(a+b\log \left(c\left(d+\frac{e}{x}\right)^p\right)\right)^2 dx, x, f+gx\right)}{g}$$

$$= \frac{(e+d(f+gx))\left(a+b\log \left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg} + \frac{(2bep)\text{Subst}\left(\int \ldots\right.}{}$$

$$= \frac{(e+d(f+gx))\left(a+b\log \left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg} - \frac{(2bep)\text{Subst}\left(\int \ldots\right.}{}$$

$$= -\frac{2bep\log \left(-\frac{e}{d(f+gx)}\right)\left(a+b\log \left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)}{dg} + \frac{(e+d(f+\ldots}{}$$

$$= -\frac{2bep\log \left(-\frac{e}{d(f+gx)}\right)\left(a+b\log \left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)}{dg} + \frac{(e+d(f+\ldots}{}$$

Mathematica [A]
time = 0.18, size = 219, normalized size = 1.90

$$x\left(a+b\log \left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2-\frac{bp\left(2df\log(f+gx)\left(a+b\log \left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)-2(e+df)\left(a+b\log \left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)\log(e+d(f+gx))+bdfp\left(\log(f+gx)\left(\log(f+gx)-2\log \left(\frac{e+df+dgx}{e}\right)\right)-2\text{Li}_2\left(-\frac{d(f+gx)}{e}\right)\right)-b(e+df)p\left(\left(2\log \left(-\frac{d(f+gx)}{e}\right)-\log(e+df+dgx)\right)\log(e+d(f+gx))+2\text{Li}_2\left(\frac{e+df+dgx}{e}\right)\right)\right)}{dg}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^2,x]
```

```
[Out] x*(a + b*Log[c*(d + e/(f + g*x))^p])^2 - (b*p*(2*d*f*Log[f + g*x]*(a + b*Lo
g[c*(d + e/(f + g*x))^p]) - 2*(e + d*f)*(a + b*Log[c*(d + e/(f + g*x))^p])*
Log[e + d*(f + g*x)] + b*d*f*p*(Log[f + g*x]*(Log[f + g*x] - 2*Log[(e + d*f
 + d*g*x)/e]) - 2*PolyLog[2, -((d*(f + g*x))/e)]) - b*(e + d*f)*p*((2*Log[-
```

```
((d*(f + g*x))/e)] - Log[e + d*f + d*g*x])*Log[e + d*(f + g*x)] + 2*PolyLog
[2, (e + d*f + d*g*x)/e])))/(d*g)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*(d+e/(g*x+f))^p))^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/(g*x+f))^p))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*g*p*(f*e^(-1)*log(g*x + f)/g^2 - (d*f + e)*e^(-1)*log(d*g*x + d*f +
e)/(d*g^2))*e + 2*a*b*x*log(c*(d + e/(g*x + f))^p) + a^2*x + b^2*((d*g*x*lo
g((d*g*x + d*f + e)^p)^2 + d*g*x*log((g*x + f)^p)^2 - (d*f*p^2 + p^2*e)*log
(d*g*x + d*f + e)^2 + 2*(d*f*p^2 + p^2*e)*log(d*g*x + d*f + e)*log(g*x + f)
 - 2*(d*f*p*log(g*x + f) + d*g*x*log((g*x + f)^p) - d*g*x*log(c) - (d*f*p +
 p*e)*log(d*g*x + d*f + e))*log((d*g*x + d*f + e)^p) + 2*(d*f*p*log(g*x + f
) - d*g*x*log(c) - (d*f*p + p*e)*log(d*g*x + d*f + e))*log((g*x + f)^p))/(d
*g) - integrate(-(d*g^2*x^2*log(c)^2 + d*f^2*log(c)^2 + f*e*log(c)^2 + (2*d
*f*g*log(c)^2 + (2*g*p*log(c) + g*log(c)^2)*e)*x - 2*(d*f^2*p^2 + 2*f*p^2*e
 + (d*f*g*p^2 + g*p^2*e)*x)*log(g*x + f))/(d*g^2*x^2 + d*f^2 + (2*d*f*g + g
*e)*x + f*e), x))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*log(c*((d*g*x + d*f + e)/(g*x + f))^p)^2 + 2*a*b*log(c*((d*g*x
 + d*f + e)/(g*x + f))^p) + a^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \log\left(c\left(d + \frac{e}{f + gx}\right)^p\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*(d+e/(g*x+f))**p))**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e/(f + g*x))**p))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/(g*x + f))^p) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + b \ln\left(c\left(d + \frac{e}{f + g\,x}\right)^p\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*(d + e/(f + g*x))^p))^2,x)
```

```
[Out] int((a + b*log(c*(d + e/(f + g*x))^p))^2, x)
```

3.639 $\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)\,dx$

Optimal. Leaf size=50

$$ax+\frac{b(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}{g}+\frac{bep\log(e+d(f+gx))}{dg}$$

```
[Out] a*x+b*(g*x+f)*ln(c*(d+e/(g*x+f))^p)/g+b*e*p*ln(e+d*(g*x+f))/d/g
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2533, 2498, 269, 31}

$$ax+\frac{b(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}{g}+\frac{bep\log(d(f+gx)+e)}{dg}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Log[c*(d + e/(f + g*x))^p],x]
```

```
[Out] a*x + (b*(f + g*x)*Log[c*(d + e/(f + g*x))^p])/g + (b*e*p*Log[e + d*(f + g*
x)])/(d*g)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2533

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
```

```
&& (EqQ[q, 1] || IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\int \left(a+b\log \left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right) dx &= ax+b\int \log \left(c\left(d+\frac{e}{f+gx}\right)^p\right) dx\\ &= ax+\frac{b\text{Subst}\left(\int \log \left(c\left(d+\frac{e}{x}\right)^p\right) dx,x,f+gx\right)}{g}\\ &= ax+\frac{b(f+gx)\log \left(c\left(d+\frac{e}{f+gx}\right)^p\right)}{g}+\frac{(bep)\text{Subst}\left(\int \frac{1}{\left(d+\frac{e}{x}\right)x} dx,x,\right.}{g}\\ &= ax+\frac{b(f+gx)\log \left(c\left(d+\frac{e}{f+gx}\right)^p\right)}{g}+\frac{(bep)\text{Subst}\left(\int \frac{1}{e+dx} dx,x,f\right.}{g}\\ &= ax+\frac{b(f+gx)\log \left(c\left(d+\frac{e}{f+gx}\right)^p\right)}{g}+\frac{bep\log(e+d(f+gx))}{dg}\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 70, normalized size = 1.40

$$ax-begp\left(\frac{f\log(f+gx)}{eg^2}-\frac{(e+df)\log(e+df+dgx)}{deg^2}\right)+bx\log \left(c\left(d+\frac{e}{f+gx}\right)^p\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Log[c*(d + e/(f + g*x))^p],x]
```

```
[Out] a*x - b*e*g*p*((f*Log[f + g*x])/(e*g^2) - ((e + d*f)*Log[e + d*f + d*g*x])/
(d*e*g^2)) + b*x*Log[c*(d + e/(f + g*x))^p]
```

Maple [**A**]
time = 0.03, size = 81, normalized size = 1.62

method	result	size
default	$ax+b\ln\left(c\left(\frac{dgx+df+e}{gx+f}\right)^p\right)x-\frac{bpf\ln(gx+f)}{g}+\frac{bp\ln(dgx+df+e)f}{g}+\frac{bpe\ln(dgx+df+e)}{gd}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*ln(c*(d+e/(g*x+f))^p),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*ln(c*((d*g*x+d*f+e)/(g*x+f))^p)*x-b*p/g*f*ln(g*x+f)+b*p/g*ln(d*g*x+d*
f+e)*f+b*p*e/g/d*ln(d*g*x+d*f+e)
```

Maxima [A]
time = 0.39, size = 72, normalized size = 1.44

$$-bgp\left(\frac{fe^{(-1)}\log\left(gx+f\right)}{g^2}-\frac{(df+e)e^{(-1)}\log\left(dgx+df+e\right)}{dg^2}\right)e+bx\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/(g*x+f))^p),x, algorithm="maxima")
```

```
[Out] -b*g*p*(f*e^(-1)*log(g*x + f)/g^2 - (d*f + e)*e^(-1)*log(d*g*x + d*f + e)/(
d*g^2))*e + b*x*log(c*(d + e/(g*x + f))^p) + a*x
```

Fricas [A]
time = 0.40, size = 80, normalized size = 1.60

$$\frac{bdgpx\log\left(\frac{dgx+df+e}{gx+f}\right)-bdfp\log\left(gx+f\right)+bdgx\log\left(c\right)+adgx+\left(bdfp+bpe\right)\log\left(dgx+df+e\right)}{dg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/(g*x+f))^p),x, algorithm="fricas")
```

```
[Out] (b*d*g*p*x*log((d*g*x + d*f + e)/(g*x + f)) - b*d*f*p*log(g*x + f) + b*d*g*
x*log(c) + a*d*g*x + (b*d*f*p + b*p*e)*log(d*g*x + d*f + e))/(d*g)
```

Sympy [A]
time = 0.57, size = 107, normalized size = 2.14

$$ax+b\left(\begin{cases}x\log\left(c\left(\frac{e}{f}\right)^p\right) & \text{for}\: d=0\wedge g=0\\ \frac{f\log\left(c\left(\frac{e}{f+gx}\right)^p\right)}{g}+px+x\log\left(c\left(\frac{e}{f+gx}\right)^p\right) & \text{for}\: d=0\\ x\log\left(c\left(d+\frac{e}{f}\right)^p\right) & \text{for}\: g=0\\ \frac{f\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}{g}+x\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)+\frac{ep\log\left(df+dgx+e\right)}{dg} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*ln(c*(d+e/(g*x+f))**p),x)
```

```
[Out] a*x + b*Piecewise((x*log(c*(e/f)**p), Eq(d, 0) & Eq(g, 0)), (f*log(c*(e/(f
+ g*x))**p)/g + p*x + x*log(c*(e/(f + g*x))**p), Eq(d, 0)), (x*log(c*(d + e
/f)**p), Eq(g, 0)), (f*log(c*(d + e/(f + g*x))**p)/g + x*log(c*(d + e/(f +
g*x))**p) + e*p*log(d*f + d*g*x + e)/(d*g), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(53) = 106.
time = 5.63, size = 177, normalized size = 3.54

$$\frac{\left(dfge^{(-2)}-(df+e)ge^{(-2)}\right)\left(dpe^{2}\log\left(-d+\frac{dgx+df+e}{gx+f}\right)+de^{2}\log\left(c\right)-\frac{(dgx+df+e)pe^{2}\log\left(-d+\frac{dgx+df+e}{gx+f}\right)}{gx+f}+\frac{(dgx+df+e)pe^{2}\log\left(\frac{dgx+df+e}{gx+f}\right)}{gx+f}\right)b}{d^{2}g^{2}-\frac{(dgx+df+e)dg^{2}}{gx+f}}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*(d+e/(g*x+f))^p),x, algorithm="giac")
```

```
[Out] (d*f*g*e^(-2) - (d*f + e)*g*e^(-2))*(d*p*e^2*log(-d + (d*g*x + d*f + e)/(g*
x + f)) + d*e^2*log(c) - (d*g*x + d*f + e)*p*e^2*log(-d + (d*g*x + d*f + e)
/(g*x + f))/(g*x + f) + (d*g*x + d*f + e)*p*e^2*log((d*g*x + d*f + e)/(g*x
+ f))/(g*x + f))*b/(d^2*g^2 - (d*g*x + d*f + e)*d*g^2/(g*x + f)) + a*x
```

Mupad [B]
time = 0.42, size = 61, normalized size = 1.22

$$a\,x+b\,x\,\ln\left(c\,{\left(d+\frac{e}{f+g\,x}\right)}^p\right)-\frac{b\,f\,p\,\ln\left(f+g\,x\right)}{g}+\frac{b\,p\,\ln\left(e+d\,f+d\,g\,x\right)\,\left(e+d\,f\right)}{d\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*log(c*(d + e/(f + g*x))^p),x)
```

```
[Out] a*x + b*x*log(c*(d + e/(f + g*x))^p) - (b*f*p*log(f + g*x))/g + (b*p*log(e
+ d*f + d*g*x)*(e + d*f))/(d*g)
```

3.640 $\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)},x\right)$$

[Out] Unintegrable(1/(a+b*ln(c*(d+e/(g*x+f))^p)),x)

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx$$

Verification is not applicable to the result.

[In] Int[(a + b*Log[c*(d + e/(f + g*x))^p])^(-1),x]

[Out] Defer[Int][(a + b*Log[c*(d + e/(f + g*x))^p])^(-1), x]

Rubi steps

$$\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx = \int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx$$

Mathematica [A]
time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^(-1),x]

[Out] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^(-1), x]

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\ln\left(c\left(d+\frac{e}{gx+f}\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*ln(c*(d+e/(g*x+f))^p)),x)
```

```
[Out] int(1/(a+b*ln(c*(d+e/(g*x+f))^p)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*log(c*(d+e/(g*x+f))^p)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*log(c*(d + e/(g*x + f))^p) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*log(c*(d+e/(g*x+f))^p)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*log(c*((d*g*x + d*f + e)/(g*x + f))^p) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\log\left(c\left(d + \frac{e}{f+gx}\right)^{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*ln(c*(d+e/(g*x+f))**p)),x)
```

```
[Out] Integral(1/(a + b*log(c*(d + e/(f + g*x))**p)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*log(c*(d+e/(g*x+f))^p)),x, algorithm="giac")
```

```
[Out] integrate(1/(b*log(c*(d + e/(g*x + f))^p) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{a+b\ln\left(c\left(d+\frac{e}{f+g\,x}\right)^p\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*log(c*(d + e/(f + g*x))^p)),x)
[Out] int(1/(a + b*log(c*(d + e/(f + g*x))^p)), x)
```

3.641 $\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(a+b*ln(c*(d+e/(g*x+f))^p))^2,x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Log[c*(d + e/(f + g*x))^p])^(-2),x]
[Out] Defer[Int][(a + b*Log[c*(d + e/(f + g*x))^p])^(-2), x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx=\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx$$

Mathematica [A]
time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^(-2),x]
[Out] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^(-2), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\ln\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*ln(c*(d+e/(g*x+f))^p))^2,x)
```

```
[Out] int(1/(a+b*ln(c*(d+e/(g*x+f))^p))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*log(c*(d+e/(g*x+f))^p))^2,x, algorithm="maxima")
```

```
[Out] (d*g^2*x^2 + d*f^2 + (2*d*f*g + g*e)*x + f*e)/(b^2*g*p*e*log((d*g*x + d*f +
 e)^p) - b^2*g*p*e*log((g*x + f)^p) + (b^2*g*p*log(c) + a*b*g*p)*e) - integ
rate((2*d*g*x + 2*d*f + e)/(b^2*p*e*log((d*g*x + d*f + e)^p) - b^2*p*e*log(
(g*x + f)^p) + (b^2*p*log(c) + a*b*p)*e), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*log(c*(d+e/(g*x+f))^p))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*log(c*((d*g*x + d*f + e)/(g*x + f))^p)^2 + 2*a*b*log(c*((d*
g*x + d*f + e)/(g*x + f))^p) + a^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^{p}\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*ln(c*(d+e/(g*x+f))**p))**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e/(f + g*x))**p))**(-2), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*log(c*(d+e/(g*x+f))^p))^2,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/(g*x + f))^p) + a)^(-2), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{f+g\,x}\right)}^p\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*log(c*(d + e/(f + g*x))^p))^2,x)
[Out] int(1/(a + b*log(c*(d + e/(f + g*x))^p))^2, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(4,m1)    #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)    #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)    #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```